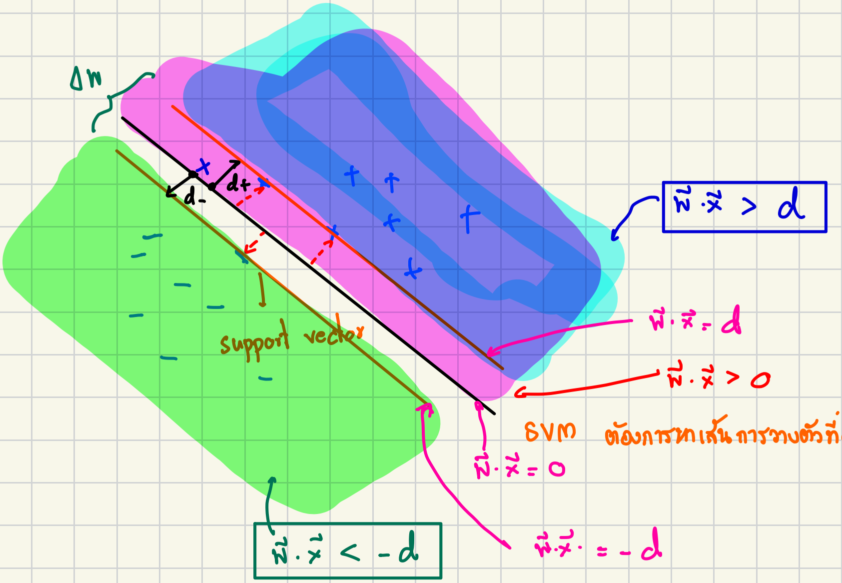
SVM



SVM ต้องการหาเส้นการวางตัวที่ดีที่สุด

อยากหา ระยะห่าง $\|\vec{x}_+ - \vec{x}\|$

$$\vec{w}\cdot\vec{x} > d = \frac{\vec{w}}{d}\cdot\vec{x} > 1 = \vec{w}\cdot\vec{x} > 1$$

$$\vec{w}\cdot\vec{x} < -d = \frac{\vec{w}}{d}\cdot\vec{x} < -1 = \vec{w}\cdot\vec{x} < -1$$

มอง 1 กับ -1 เป็น d

$d(\vec{w}\cdot\vec{x}) > 1$

$$d_+ = \|\vec{x}_+ - \vec{x}\|$$

$$w\cdot x_+ = 1 \quad \text{— ①}$$

$$w\cdot x = 0 \quad \text{— ②}$$

① - ② ; $(w\cdot x_+) - (w\cdot x) = 1 - 0$

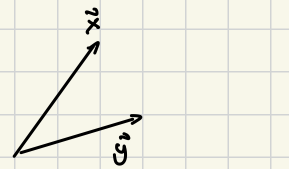
$$w\cdot(x_+ - x) = 1$$

$$\|w\|\,\|x_+ - x\|\,\cancel{\cos\theta}^{\,1} = 1$$

เพราะ $\vec{x}, \vec{y}$ ทิศเดียวกัน

$\|x_+ - x\| = \dfrac{1}{\|w\|}$

ระยะทาง



$\vec{x}\cdot\vec{y} = \|x\|\|y\|\cos\theta$

maximize $\longrightarrow \dfrac{2}{\|w\|} \longrightarrow$ minimize $\|w\|$

กลายเป็นปัญหา optimization

Optimization

minimize $\frac{1}{2}\|\vec{w}\|^2$ constant $d_i(\vec{w}_i \cdot \vec{x}_i) \geq 1$

$$d_i(\vec{w}_i \cdot \vec{x}_i) - 1 \geq 0$$

Lagrange

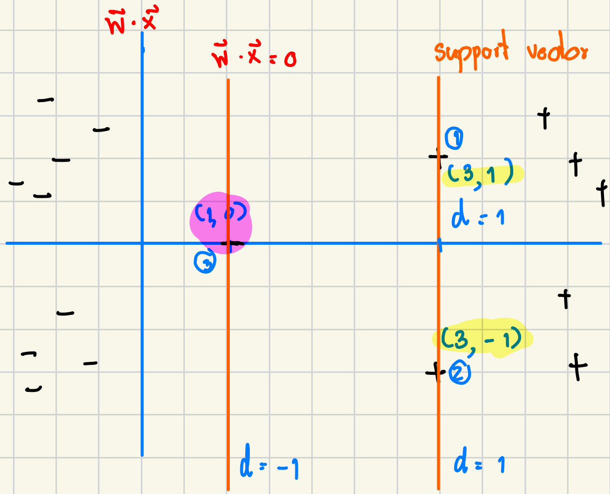
$$L_p(w, \Lambda) = \frac{1}{2}\|w\|^2 - \sum_{i=1}^{q} \lambda_i \left[d_i(\vec{w}_i \cdot \vec{x}_i) - 1 \right]$$

$$\frac{\partial L_p(w, \Lambda)}{\partial w} = \vec{w} - \sum_{i=1}^{q} \lambda_i (d_i x_i) = 0$$

$$\boxed{\vec{w} = \sum_{i=1}^{q} \lambda_i d_i x_i}$$

$$\frac{\partial L_p(w, w_0, \Lambda)}{\partial w_0} = \frac{\partial}{\partial w_0} \frac{1}{2}\|\vec{w}\|^2 - \sum_{i=1}^{q} \lambda_i \left[d_i(w_0) + d_i(\vec{w} \cdot \vec{x}) - 1 \right]$$

$$= \boxed{\sum_{i=1}^{q} \lambda_i d_i = 0}$$



ตัวอย่างบวก

$$\left\{ \begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 3 \\ -1 \end{bmatrix} \right\}$$

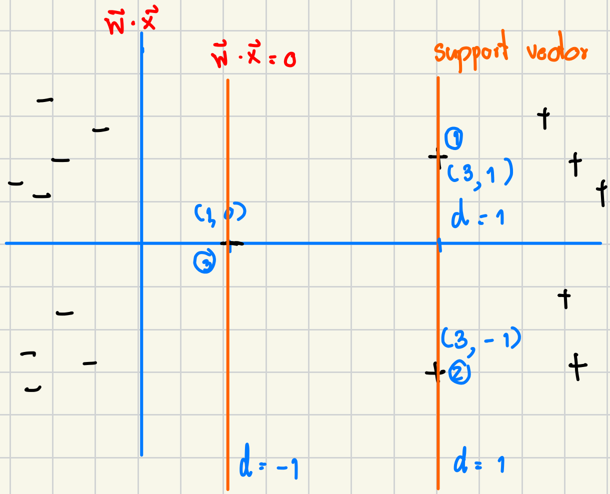
ตัวอย่างลบ

$$\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$$

$$\vec{w} = \sum_{i=1}^{q} \lambda_i d_i x_i$$

$$\vec{w} = \lambda_1(1)\begin{bmatrix}1\\3\\1\end{bmatrix} + \lambda_2(1)\begin{bmatrix}1\\3\\-1\end{bmatrix} + \lambda_3(-1)\begin{bmatrix}1\\1\\0\end{bmatrix}$$

$$= \begin{bmatrix}\lambda_1\\3\lambda_1\\\lambda_1\end{bmatrix} + \begin{bmatrix}\lambda_2\\3\lambda_2\\-\lambda_2\end{bmatrix} + \begin{bmatrix}-\lambda_3\\-\lambda_3\\0\end{bmatrix} = \begin{bmatrix}\lambda_1+\lambda_2-\lambda_3\\3\lambda_1+3\lambda_2-\lambda_3\\\lambda_1-\lambda_2\end{bmatrix}$$



1 เสมอ

① $$\begin{bmatrix}\lambda_1+\lambda_2-\lambda_3\\3\lambda_1+3\lambda_2-\lambda_3\\\lambda_1-\lambda_2\end{bmatrix}\begin{bmatrix}1\\3\\1\end{bmatrix} = 1$$

② $$\begin{bmatrix}\lambda_1+\lambda_2-\lambda_3\\3\lambda_1+3\lambda_2-\lambda_3\\\lambda_1-\lambda_2\end{bmatrix}\begin{bmatrix}1\\3\\-1\end{bmatrix} = 1$$

③ $$\begin{bmatrix}\lambda_1+\lambda_2-\lambda_3\\3\lambda_1+3\lambda_2-\lambda_3\\\lambda_1-\lambda_2\end{bmatrix}\begin{bmatrix}1\\1\\0\end{bmatrix} = -1$$

① $(x_1+x_2-x_3)+(9x_1+9x_2-3x_3)+(x_1-x_2) = 1$

$$11x_1+9x_2-4x_3 = 1$$

② $(x_1+x_2-x_3)+(9x_1+9x_2-3x_3)+(-x_1+x_2) = 1$

$$9x_1+11x_2-4x_3 = 1$$

③ $(x_1+x_2-x_3)+(3x_1+3x_2-x_3) = -1$

$$4x_1+4x_2-2x_3 = -1$$

$$11x_1+9x_2-4x_3 = 1 \text{ —— ①}$$
$$9x_1+11x_2-4x_3 = 1 \text{ —— ②}$$
$$4x_1+4x_2-2x_3 = -1 \text{ —— ③}$$

① - ② ; $2x_1-2x_2 = 0$

$$x_1 = x_2$$

แทน $x_2 = x_1$ ใน ① ; $20x_1 - 4x_3 = 1$ —— ④

แทน $x_2 = x_1$ ใน ③ ; $8x_1 - 2x_3 = -1$ —— ⑤

⑤ × 2 ; $16x_1 - 4x_3 = -2$ —— ⑥

④ - ⑥ ; $4x_1 = 3$

$$x_1 = 0.75$$
$$x_2 = 0\ 75$$

แทน x_1, x_2 ใน ③ ; $-2x_3 = -7$

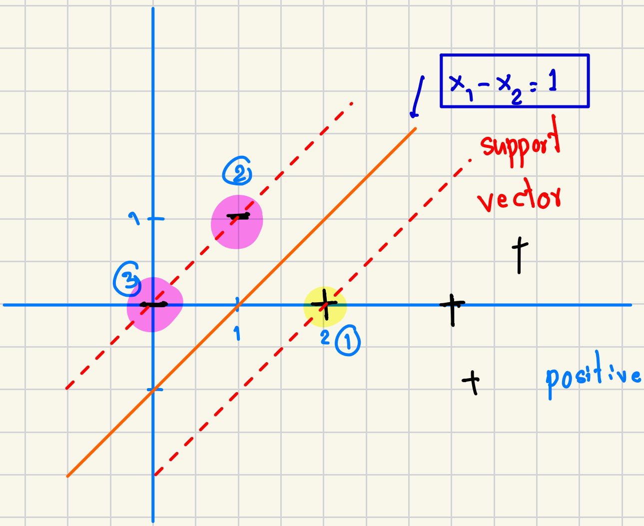
$$x_3 = 3.5$$

$$w = \begin{bmatrix} \lambda_1+\lambda_2-\lambda_3 \\ 3\lambda_1+3\lambda_2-\lambda_3 \\ \lambda_1-\lambda_2 \end{bmatrix} = \begin{bmatrix} 0.75+0.75-3.5 \\ 2.25+2.25-3.5 \\ 0.75-0.75 \end{bmatrix} = \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} w_0 \\ w_1 \\ w_0 \end{bmatrix} = \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix} \qquad -2 + w_1 + 0w_2 = 0$$

$$w_1 = 2$$

λ คือการหาค่า lagrenge multiplier



$$\vec{w} = \sum_{i=1}^{6} \lambda_i d_i \vec{x}_i$$

positive . $\left\{ \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} \right\}$ Negative $\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \right\}$

① $$\vec{w} = \lambda_1(1)\begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} + \lambda_2(-1)\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} + \lambda_3(-1)\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} \lambda_1 \\ 2\lambda_1 \\ 0 \end{bmatrix} + \begin{bmatrix} -\lambda_2 \\ -\lambda_2 \\ -\lambda_2 \end{bmatrix} + \begin{bmatrix} -\lambda_3 \\ 0 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} \lambda_1 - \lambda_2 - \lambda_3 \\ 2\lambda_1 - \lambda_2 \\ -\lambda_2 \end{bmatrix}$$

① ; $\begin{bmatrix} \lambda_1 - \lambda_2 - \lambda_3 \\ 2\lambda_1 - \lambda_2 \\ -\lambda_2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} = 1 \longrightarrow \lambda_1 - \lambda_2 - \lambda_3 + 4\lambda_1 - 2\lambda_2 = 1$

$$5\lambda_1 - 3\lambda_2 - \lambda_3 = 1$$

$\begin{bmatrix} \lambda_1 - \lambda_2 - \lambda_3 \\ -2\lambda_1 - \lambda_2 \\ -\lambda_2 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = -1 \longrightarrow \lambda_1 - \lambda_2 - \lambda_3 + 2\lambda_1 - \lambda_2 - \lambda_2 = -1$

$$3\lambda_1 - 3\lambda_2 - \lambda_3 = -1$$

$\begin{bmatrix} \lambda_1 - \lambda_2 - \lambda_3 \\ -2\lambda_1 - \lambda_2 \\ -\lambda_2 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = -1 \rightarrow \lambda_1 - \lambda_2 - \lambda_3 = 1$

$$5\lambda_1 - 3\lambda_2 - \lambda_3 = 1 \quad — ①$$
$$3\lambda_1 - 3\lambda_2 - \lambda_3 = -1 \quad — ②$$
$$\lambda_1 - \lambda_2 - \lambda_3 = -1 \quad — ③$$

① - ② ;

$$2\lambda_1 = 2$$
$$\lambda_1 = 1$$

แทน λ_1 ใน ① ; $\quad 5 - 3\lambda_2 - \lambda_3 = 1$

— " — ③ ; $\quad 1 - \lambda_2 - \lambda_3 = -1$

ย้ายข้าง

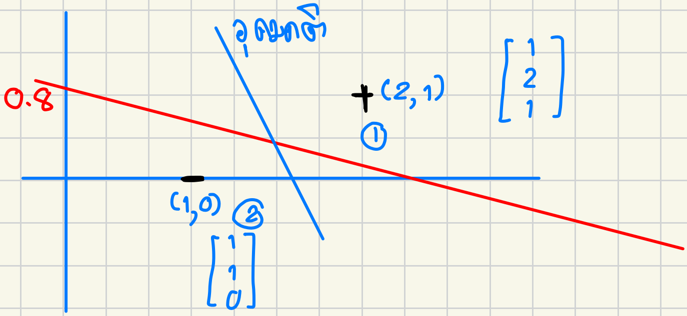
$$-3\lambda_2 - \lambda_3 = -4 \quad — ④$$
$$-\lambda_2 - \lambda_3 = -2 \quad — ⑤$$

⑤ - ④ ;

$$2\lambda_2 = 2$$
$$\lambda_2 = 1$$
$$\lambda_3 = 1$$

$$\vec{w} = \begin{bmatrix} \lambda_1 - \lambda_2 - \lambda_3 \\ 2\lambda_1 - \lambda_2 \\ -\lambda_2 \end{bmatrix} = \begin{bmatrix} 1 - 1 - 1 \\ 2 - 1 \\ -1 \end{bmatrix} = \begin{array}{c} W \\ \begin{bmatrix} -1 \\ 1 \\ -1 \end{bmatrix} \end{array} \begin{array}{l} \\ w_c \\ w_1 \\ w_2 \end{array} = -1 + x_1 - x_2 = 0$$

$$\boxed{x_1 - x_2 = 1}$$



$$\vec{w} = \alpha_1(1)\begin{bmatrix}1\\2\\1\end{bmatrix} + \alpha_2(-1)\begin{bmatrix}1\\1\\0\end{bmatrix}$$

$$= \begin{bmatrix}\alpha_1\\2\alpha_1\\\alpha_1\end{bmatrix} + \begin{bmatrix}-\alpha_2\\-\alpha_2\\0\end{bmatrix} = \begin{bmatrix}\alpha_1-\alpha_2\\2\alpha_1-\alpha_2\\\alpha_1\end{bmatrix}$$

1 ; $\begin{bmatrix}\alpha_1-\alpha_2\\2\alpha_1-\alpha_2\\\alpha_1\end{bmatrix}\begin{bmatrix}1\\2\\1\end{bmatrix} = 1 \rightarrow \alpha_1 - \alpha_2 + 4\alpha_1 - 2\alpha_2 + \alpha_1 = 1$

$$6\alpha_1 - 3\alpha_2 = 1 \quad — \ 1$$

2 ; $\begin{bmatrix}\alpha_1-\alpha_2\\2\alpha_1-\alpha_2\\\alpha_1\end{bmatrix}\begin{bmatrix}1\\1\\0\end{bmatrix} = -1 \longrightarrow \alpha_1 - \alpha_2 + 2\alpha_1 - \alpha_2 = -1$

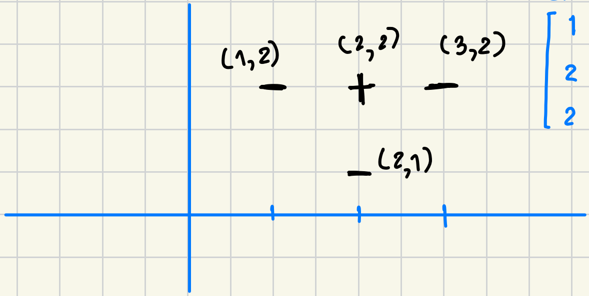
$$3\alpha_1 - 2\alpha_2 = -1 \quad — \ 2$$

2 × 2 ; $6\alpha_1 - 4\alpha_2 = -2 \quad —— \ 3$

1 - 3 ; $\alpha_2 = 3$

$$\alpha_1 = \frac{10}{6}$$

$$\vec{w} = \begin{bmatrix}\frac{10}{6}-3\\\frac{10}{3}-3\\\frac{10}{6}\end{bmatrix} = \begin{bmatrix}-4/3\\1/3\\5/3\end{bmatrix} = \begin{bmatrix}-4\\1\\5\end{bmatrix}$$



Positive

$$\begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}$$

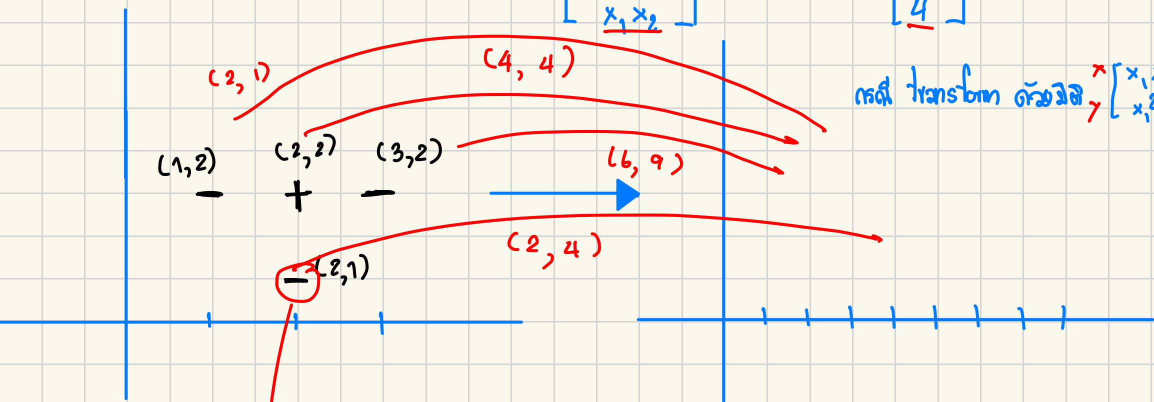
$$\underbrace{\begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}}_{\text{Negative}}$$

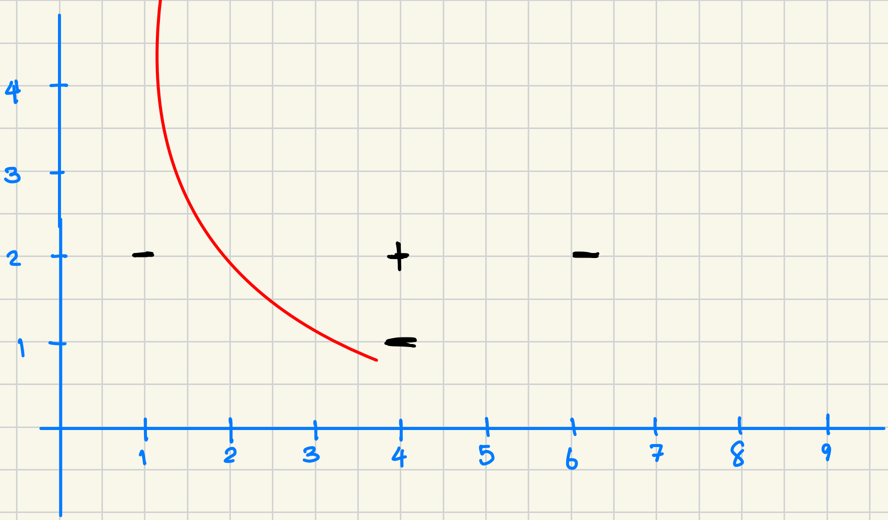
$$\begin{bmatrix} x_0 \\ x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ x_1 \\ x_2 \end{bmatrix}$$

$\Phi(\vec{x})$ transformation function

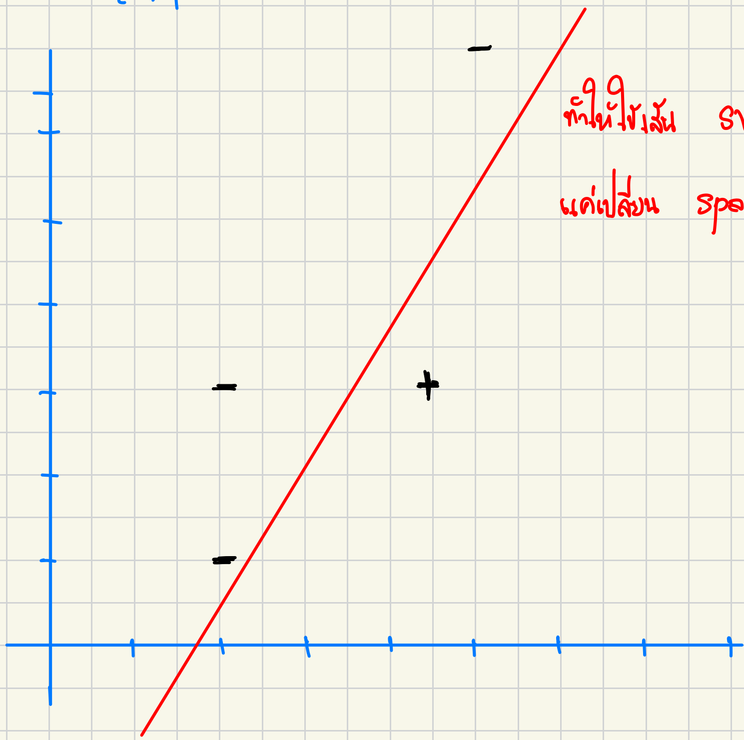
$$\begin{bmatrix} 1 \\ x_1 \\ x_2 \\ \underline{x_1^2} \\ x_2^2 \\ \underline{x_1 x_2} \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix} \xRightarrow{\Phi} \begin{bmatrix} 1 \\ 2 \\ 2 \\ \underline{4} \\ 4 \\ \underline{4} \end{bmatrix} x_1^2$$

กรณี transform ด้วยมิติ $\begin{bmatrix} x_1 x_2 \\ x_1^2 \end{bmatrix}$ $\begin{matrix} x \\ y \end{matrix}$



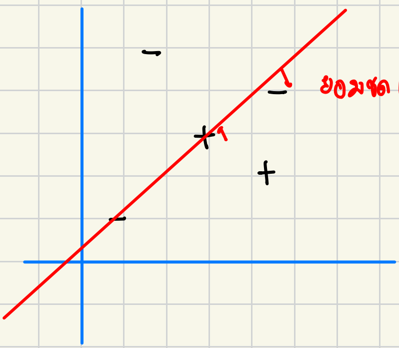


transform ด้วย $\begin{bmatrix} x_1 x_2 \\ x_1^2 \end{bmatrix}$



ทำให้ใช้เส้น SVM ได้เหมือนเดิม

แค่เปลี่ยน space (kernel)



ยอมชดเชยค่า ถ้าใช้เส้นตรงไม่ได้จริงๆ