

# Artificial Neural Networks

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[Read Ch. 4]

[Recommended exercises 4.1, 4.2, 4.5, 4.9, 4.11]

- Threshold units
- Gradient descent
- Multilayer networks
- Backpropagation
- Hidden layer representations
- Example: Face Recognition
- Advanced topics

# Connectionist Models

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Consider humans:

- Neuron switching time  $\sim .001$  second
- Number of neurons  $\sim 10^{10}$
- Connections per neuron  $\sim 10^{4-5}$
- Scene recognition time  $\sim .1$  second
- 100 inference steps doesn't seem like enough

→ much parallel computation

Properties of artificial neural nets (ANN's):

- Many neuron-like threshold switching units
- Many weighted interconnections among units
- Highly parallel, distributed process
- Emphasis on tuning weights automatically

# When to Consider Neural Networks

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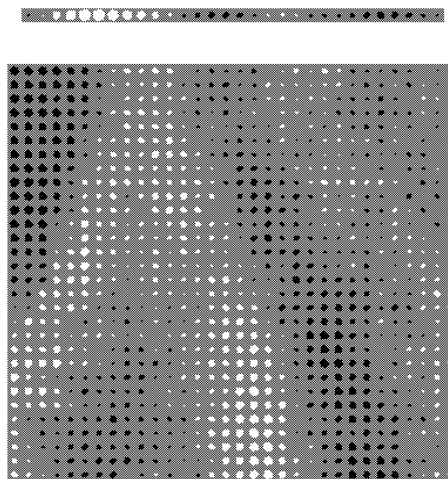
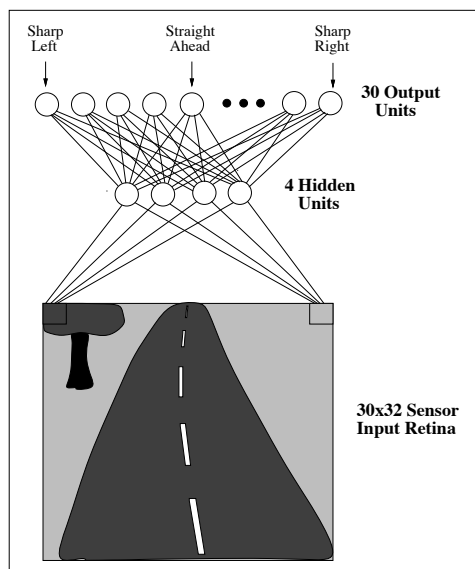
- Input is high-dimensional discrete or real-valued (e.g. raw sensor input)
- Output is discrete or real valued
- Output is a vector of values
- Possibly noisy data
- Form of target function is unknown
- Human readability of result is unimportant

Examples:

- Speech phoneme recognition [Waibel]
- Image classification [Kanade, Baluja, Rowley]
- Financial prediction

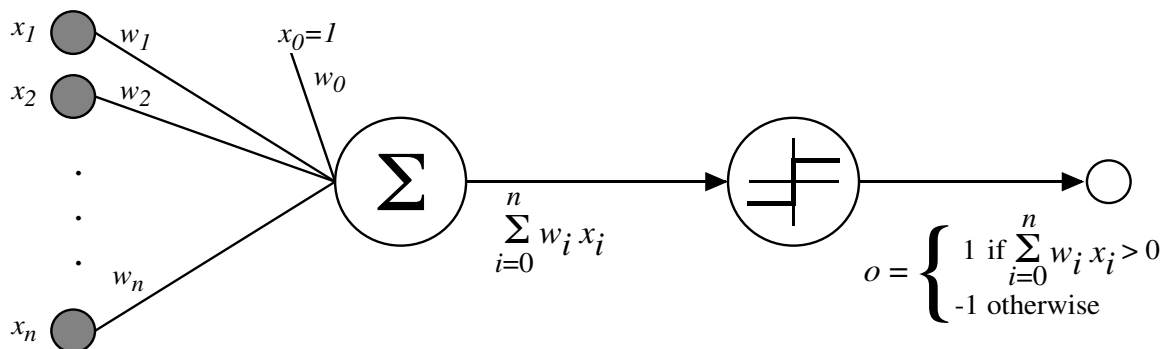
# ALVINN drives 70 mph on highways

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# Perceptron

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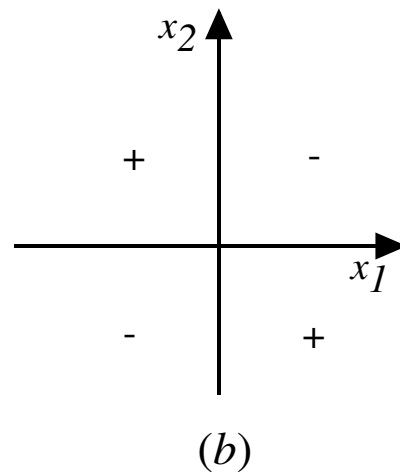
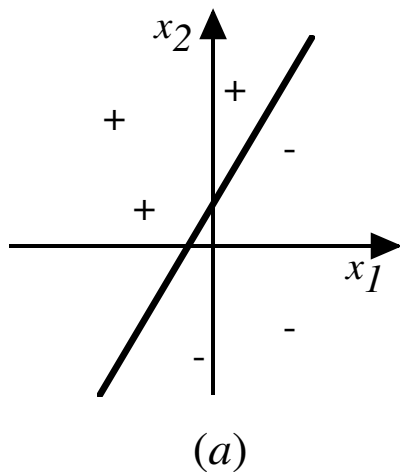
$$o(x_1, \dots, x_n) = \begin{cases} 1 & \text{if } w_0 + w_1 x_1 + \dots + w_n x_n > 0 \\ -1 & \text{otherwise.} \end{cases}$$

Sometimes we'll use simpler vector notation:

$$o(\vec{x}) = \begin{cases} 1 & \text{if } \vec{w} \cdot \vec{x} > 0 \\ -1 & \text{otherwise.} \end{cases}$$

# Decision Surface of a Perceptron

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Represents some useful functions

- What weights represent  
 $g(x_1, x_2) = AND(x_1, x_2)$ ?

But some functions not representable

- e.g., not linearly separable
- Therefore, we'll want networks of these...

# Perceptron training rule

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$$w_i \leftarrow w_i + \Delta w_i$$

where

$$\Delta w_i = \eta(t - o)x_i$$

Where:

- $t = c(\vec{x})$  is target value
- $o$  is perceptron output
- $\eta$  is small constant (e.g., .1) called *learning rate*

# Perceptron training rule

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Can prove it will converge

- If training data is linearly separable
- and  $\eta$  sufficiently small



# Gradient Descent

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To understand, consider simpler *linear unit*, where

$$o = w_0 + w_1x_1 + \cdots + w_nx_n$$

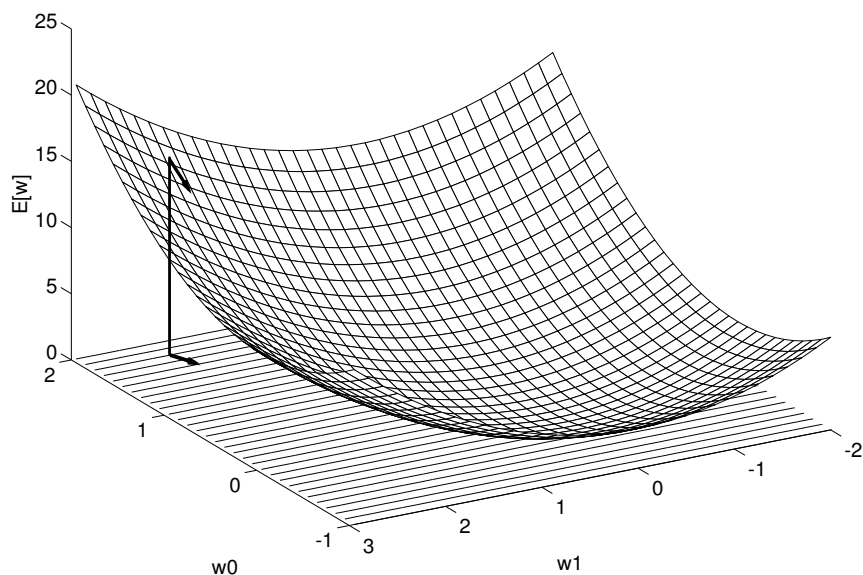
Let's learn  $w_i$ 's that minimize the squared error

$$E[\vec{w}] \equiv \frac{1}{2} \sum_{d \in D} (t_d - o_d)^2$$

Where  $D$  is set of training examples

# Gradient Descent

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Gradient

$$\nabla E[\vec{w}] \equiv \left[ \frac{\partial E}{\partial w_0}, \frac{\partial E}{\partial w_1}, \dots, \frac{\partial E}{\partial w_n} \right]$$

Training rule:

$$\Delta \vec{w} = -\eta \nabla E[\vec{w}]$$

i.e.,

$$\Delta w_i = -\eta \frac{\partial E}{\partial w_i}$$

# Gradient Descent

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$$\begin{aligned}\frac{\partial E}{\partial w_i} &= \frac{\partial}{\partial w_i} \frac{1}{2} \sum_d (t_d - o_d)^2 \\ &= \frac{1}{2} \sum_d \frac{\partial}{\partial w_i} (t_d - o_d)^2 \\ &= \frac{1}{2} \sum_d 2(t_d - o_d) \frac{\partial}{\partial w_i} (t_d - o_d) \\ &= \sum_d (t_d - o_d) \frac{\partial}{\partial w_i} (t_d - \vec{w} \cdot \vec{x}_d) \\ \frac{\partial E}{\partial w_i} &= \sum_d (t_d - o_d) (-x_{i,d})\end{aligned}$$

# Gradient Descent

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GRADIENT-DESCENT(*training\_examples*,  $\eta$ )

*Each training example is a pair of the form  $\langle \vec{x}, t \rangle$ , where  $\vec{x}$  is the vector of input values, and  $t$  is the target output value.  $\eta$  is the learning rate (e.g., .05).*

- Initialize each  $w_i$  to some small random value
- Until the termination condition is met, Do
  - Initialize each  $\Delta w_i$  to zero.
  - For each  $\langle \vec{x}, t \rangle$  in *training\_examples*, Do
    - \* Input the instance  $\vec{x}$  to the unit and compute the output  $o$
    - \* For each linear unit weight  $w_i$ , Do
  - For each linear unit weight  $w_i$ , Do

$$\Delta w_i \leftarrow \Delta w_i + \eta(t - o)x_i$$

$$w_i \leftarrow w_i + \Delta w_i$$

# Summary

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Perceptron training rule guaranteed to succeed if

- Training examples are linearly separable
- Sufficiently small learning rate  $\eta$

Linear unit training rule uses gradient descent

- Guaranteed to converge to hypothesis with minimum squared error
- Given sufficiently small learning rate  $\eta$
- Even when training data contains noise
- Even when training data not separable by  $H$

# Incremental (Stochastic) Gradient Descent

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## **Batch mode** Gradient Descent:

Do until satisfied

1. Compute the gradient  $\nabla E_D[\vec{w}]$
  2.  $\vec{w} \leftarrow \vec{w} - \eta \nabla E_D[\vec{w}]$
- 

## **Incremental mode** Gradient Descent:

Do until satisfied

- For each training example  $d$  in  $D$ 
    1. Compute the gradient  $\nabla E_d[\vec{w}]$
    2.  $\vec{w} \leftarrow \vec{w} - \eta \nabla E_d[\vec{w}]$
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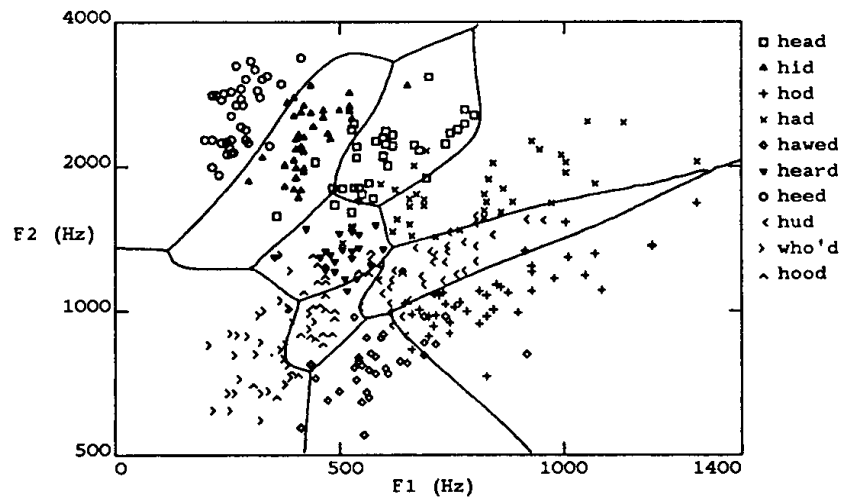
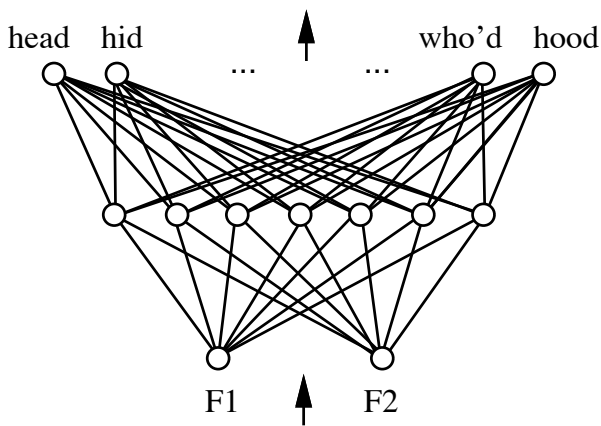
$$E_D[\vec{w}] \equiv \frac{1}{2} \sum_{d \in D} (t_d - o_d)^2$$

$$E_d[\vec{w}] \equiv \frac{1}{2} (t_d - o_d)^2$$

*Incremental Gradient Descent* can approximate  
*Batch Gradient Descent* arbitrarily closely if  $\eta$   
made small enough

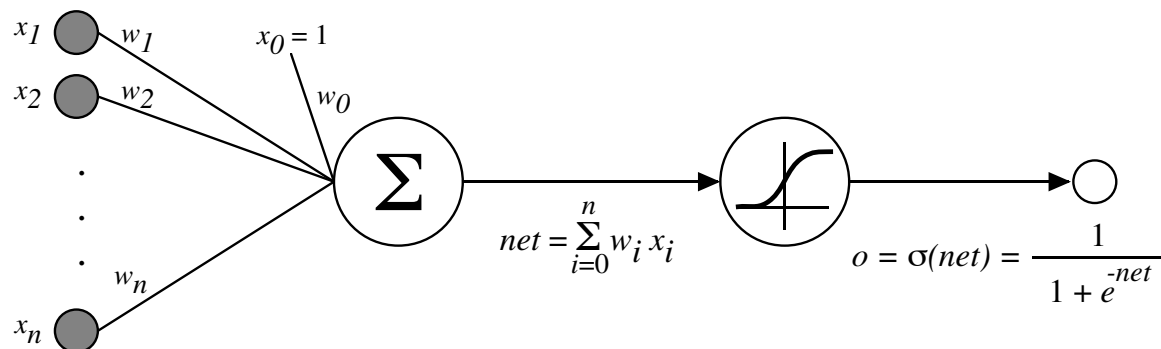
# Multilayer Networks of Sigmoid Units

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# Sigmoid Unit

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$\sigma(x)$  is the sigmoid function

$$\frac{1}{1 + e^{-x}}$$

Nice property:  $\frac{d\sigma(x)}{dx} = \sigma(x)(1 - \sigma(x))$

We can derive gradient decent rules to train

- One sigmoid unit
- *Multilayer networks* of sigmoid units  $\rightarrow$  Backpropagation



## Error Gradient for a Sigmoid Unit

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$$\begin{aligned}\frac{\partial E}{\partial w_i} &= \frac{\partial}{\partial w_i} \frac{1}{2} \sum_{d \in D} (t_d - o_d)^2 \\ &= \frac{1}{2} \sum_d \frac{\partial}{\partial w_i} (t_d - o_d)^2 \\ &= \frac{1}{2} \sum_d 2(t_d - o_d) \frac{\partial}{\partial w_i} (t_d - o_d) \\ &= \sum_d (t_d - o_d) \left( -\frac{\partial o_d}{\partial w_i} \right) \\ &= - \sum_d (t_d - o_d) \frac{\partial o_d}{\partial net_d} \frac{\partial net_d}{\partial w_i}\end{aligned}$$

But we know:

$$\begin{aligned}\frac{\partial o_d}{\partial net_d} &= \frac{\partial \sigma(net_d)}{\partial net_d} = o_d(1 - o_d) \\ \frac{\partial net_d}{\partial w_i} &= \frac{\partial (\vec{w} \cdot \vec{x}_d)}{\partial w_i} = x_{i,d}\end{aligned}$$

So:

$$\frac{\partial E}{\partial w_i} = - \sum_{d \in D} (t_d - o_d) o_d (1 - o_d) x_{i,d}$$

# Backpropagation Algorithm

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Initialize all weights to small random numbers.  
Until satisfied, Do

- For each training example, Do
  1. Input the training example to the network and compute the network outputs
  2. For each output unit  $k$

$$\delta_k \leftarrow o_k(1 - o_k)(t_k - o_k)$$

3. For each hidden unit  $h$

$$\delta_h \leftarrow o_h(1 - o_h) \sum_{k \in \text{outputs}} w_{h,k} \delta_k$$

4. Update each network weight  $w_{i,j}$

$$w_{i,j} \leftarrow w_{i,j} + \Delta w_{i,j}$$

where

$$\Delta w_{i,j} = \eta \delta_j x_{i,j}$$

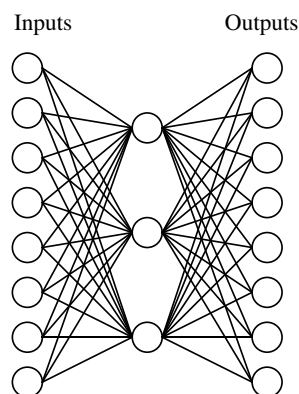
# More on Backpropagation

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- Gradient descent over entire *network* weight vector
- Easily generalized to arbitrary directed graphs
- Will find a local, not necessarily global error minimum
  - In practice, often works well (can run multiple times)
- Often include weight *momentum*  $\alpha$ 
$$\Delta w_{i,j}(n) = \eta \delta_j x_{i,j} + \alpha \Delta w_{i,j}(n-1)$$
- Minimizes error over *training* examples
  - Will it generalize well to subsequent examples?
- Training can take thousands of iterations  $\rightarrow$  slow!
- Using network after training is very fast

# Learning Hidden Layer Representations

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A target function:

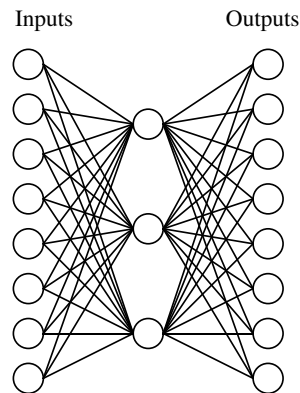
Input	Output
10000000	→ 10000000
01000000	→ 01000000
00100000	→ 00100000
00010000	→ 00010000
00001000	→ 00001000
00000100	→ 00000100
00000010	→ 00000010
00000001	→ 00000001

Can this be learned??

# Learning Hidden Layer Representations

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A network:

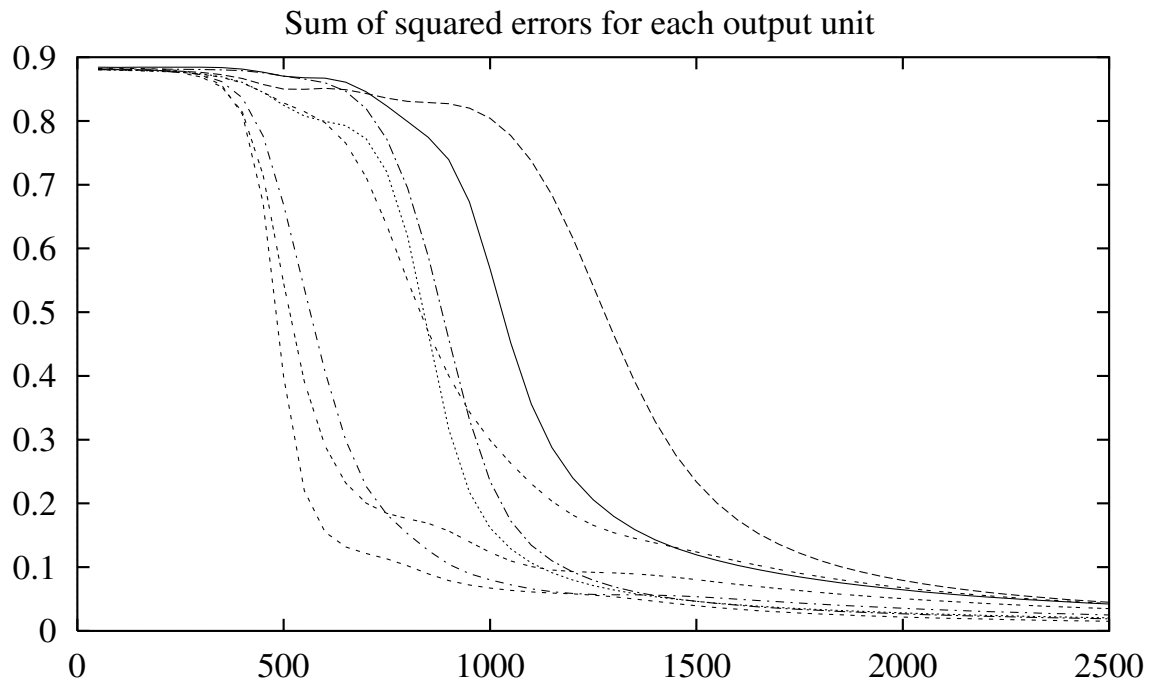


Learned hidden layer representation:

Input	Hidden Values			Output
10000000	→ .89	.04	.08	→ 10000000
01000000	→ .01	.11	.88	→ 01000000
00100000	→ .01	.97	.27	→ 00100000
00010000	→ .99	.97	.71	→ 00010000
00001000	→ .03	.05	.02	→ 00001000
00000100	→ .22	.99	.99	→ 00000100
00000010	→ .80	.01	.98	→ 00000010
00000001	→ .60	.94	.01	→ 00000001

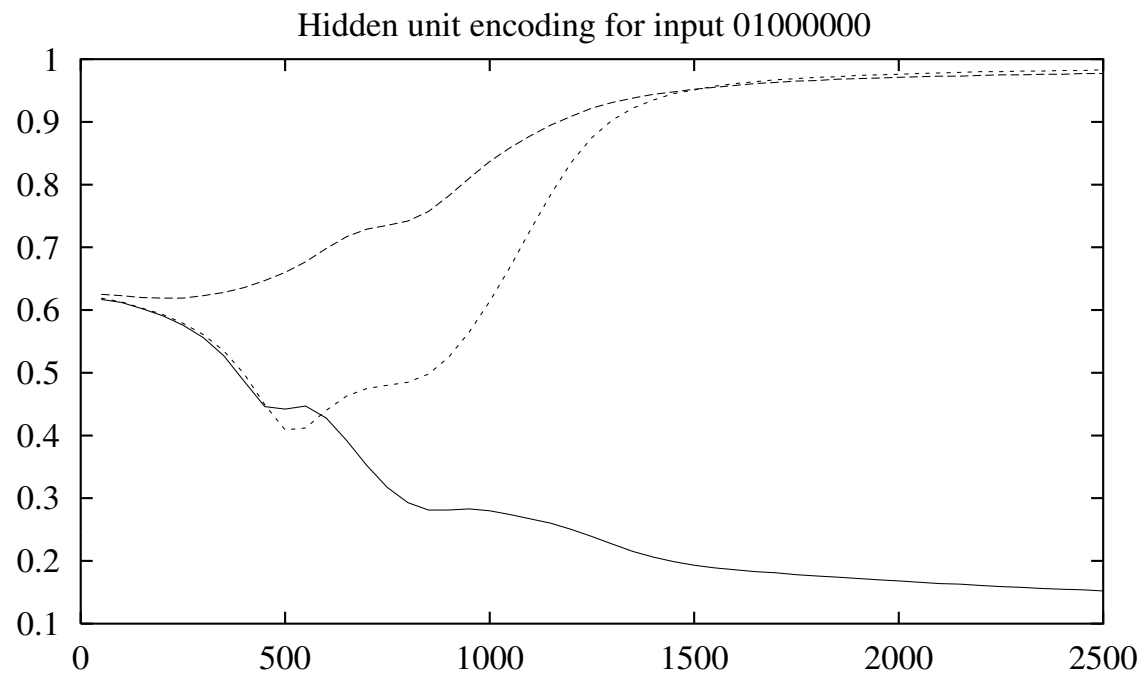
# Training

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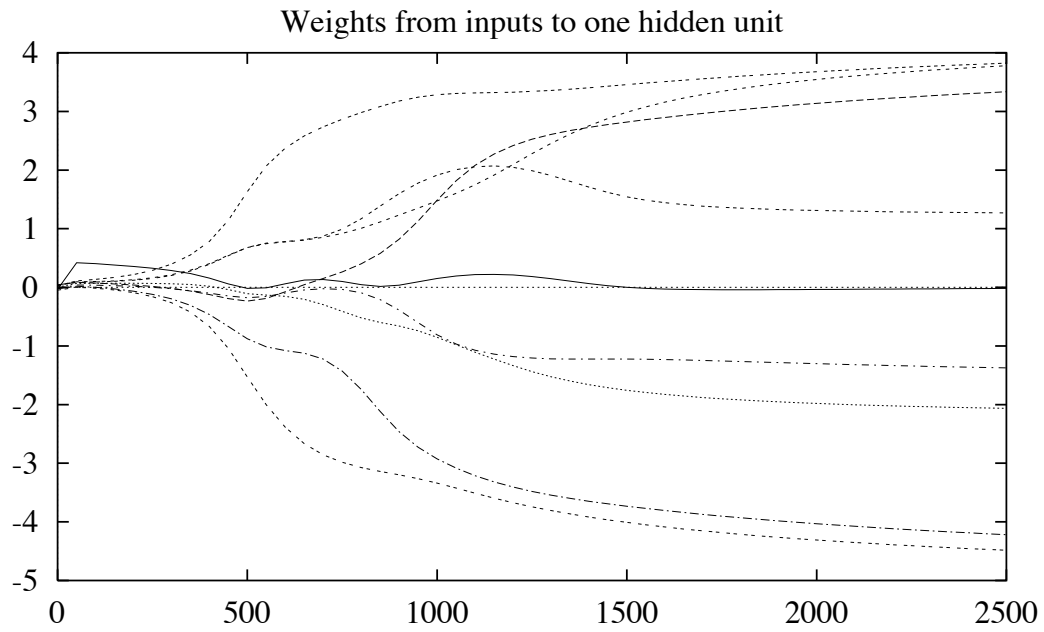
# Training

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# Training

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# Convergence of Backpropagation

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Gradient descent to some local minimum

- Perhaps not global minimum...
- Add momentum
- Stochastic gradient descent
- Train multiple nets with different initial weights

Nature of convergence

- Initialize weights near zero
- Therefore, initial networks near-linear
- Increasingly non-linear functions possible as training progresses

# Expressive Capabilities of ANNs

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## Boolean functions:

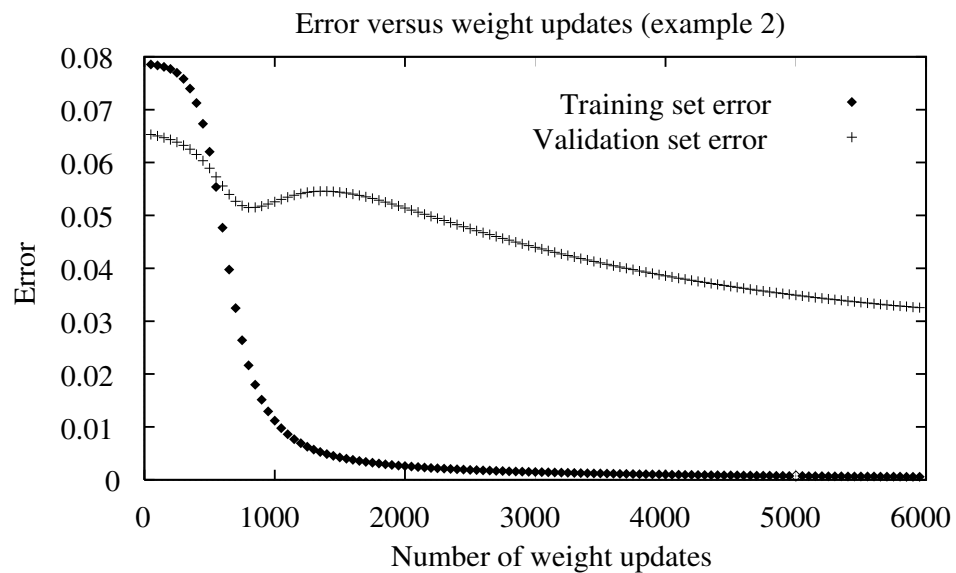
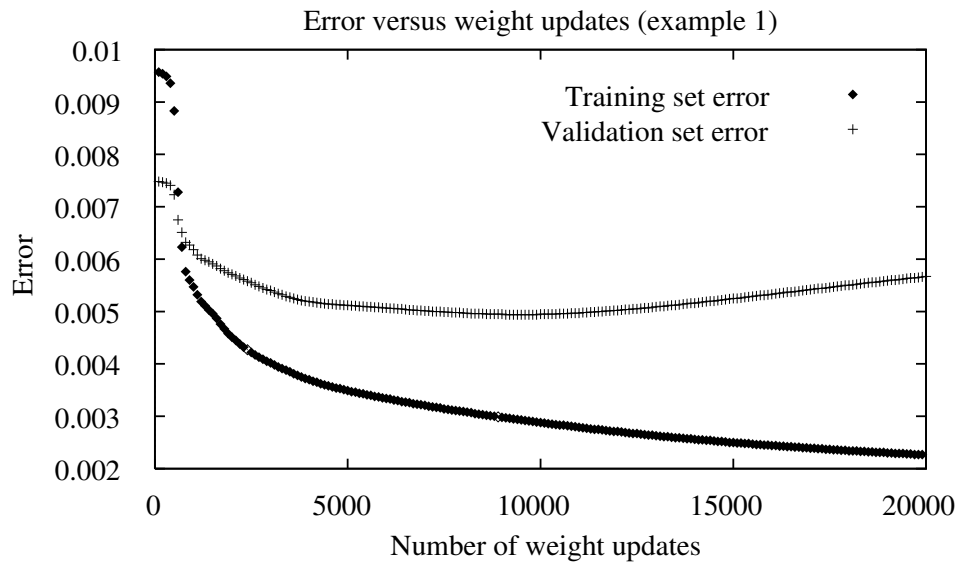
- Every boolean function can be represented by network with single hidden layer
- but might require exponential (in number of inputs) hidden units

## Continuous functions:

- Every bounded continuous function can be approximated with arbitrarily small error, by network with one hidden layer [Cybenko 1989; Hornik et al. 1989]
- Any function can be approximated to arbitrary accuracy by a network with two hidden layers [Cybenko 1988].

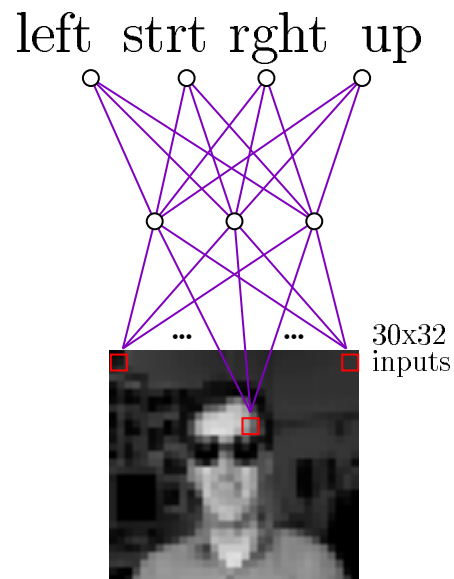
# Overfitting in ANNs

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# Neural Nets for Face Recognition

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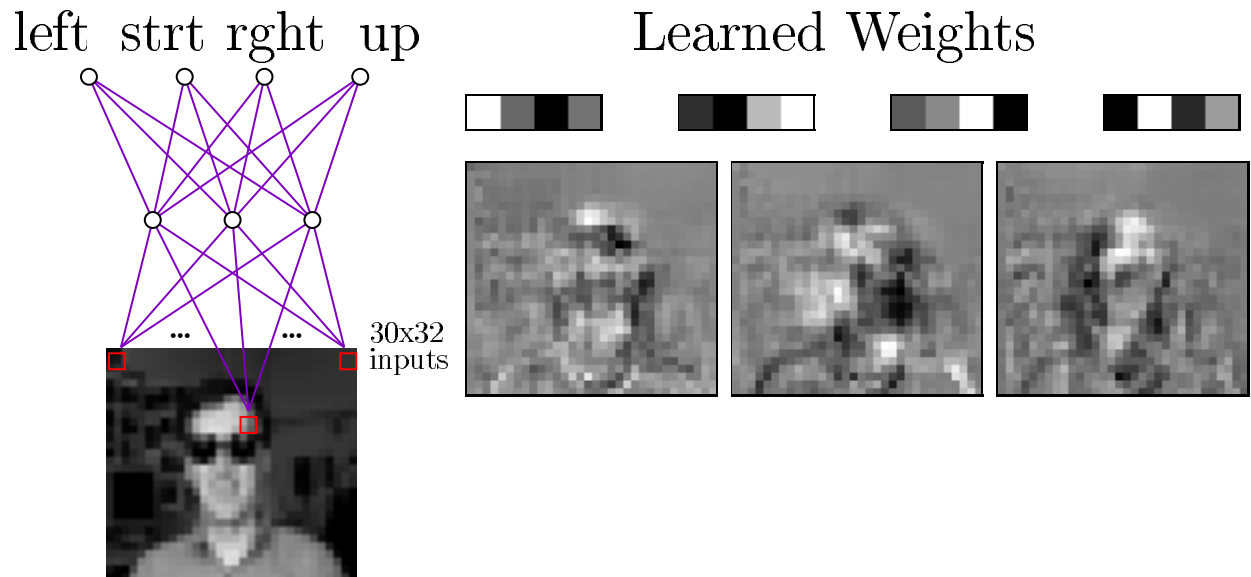


Typical input images

90% accurate learning head pose, and recognizing 1-of-20 faces

# Learned Hidden Unit Weights

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Typical input images

<http://www.cs.cmu.edu/~tom/faces.html>

# Alternative Error Functions

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Penalize large weights:

$$E(\vec{w}) \equiv \frac{1}{2} \sum_{d \in D} \sum_{k \in \text{outputs}} (t_{kd} - o_{kd})^2 + \gamma \sum_{i,j} w_{ji}^2$$

Train on target slopes as well as values:

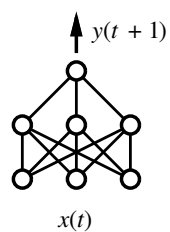
$$E(\vec{w}) \equiv \frac{1}{2} \sum_{d \in D} \sum_{k \in \text{outputs}} \left[ (t_{kd} - o_{kd})^2 + \mu \sum_{j \in \text{inputs}} \left( \frac{\partial t_{kd}}{\partial x_d^j} - \frac{\partial o_{kd}}{\partial x_d^j} \right)^2 \right]$$

Tie together weights:

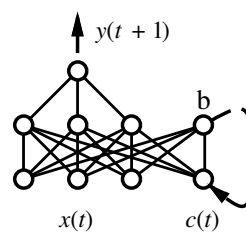
- e.g., in phoneme recognition network

# Recurrent Networks

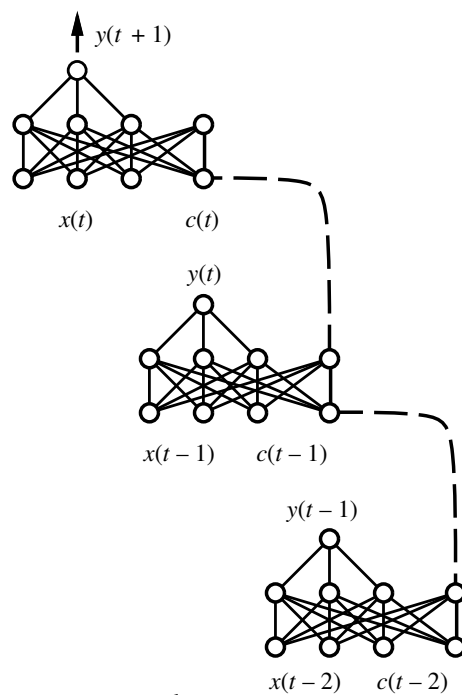
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(a) Feedforward network



(b) Recurrent network



(c) Recurrent network  
unfolded in time