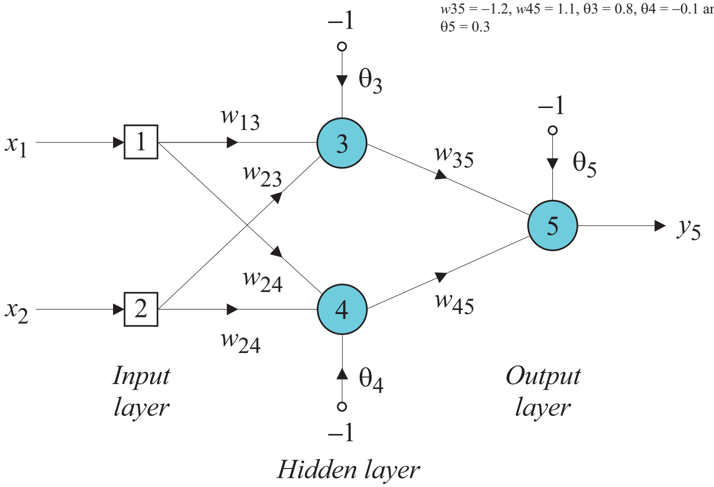
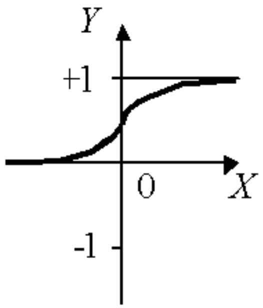
Three-layer network

$w13 = 0.5$, $w14 = 0.9$, $w23 = 0.4$, $w24 = 1.0$, $w35 = -1.2$, $w45 = 1.1$, $\theta 3 = 0.8$, $\theta 4 = -0.1$ and $\theta 5 = 0.3$



- The effect of the threshold applied to a neuron in the hidden or output layer is represented by its weight, θ, connected to a fixed input equal to -1.
- The initial weights and threshold levels are set randomly e.g., as follows:
  $w_{13} = 0.5$, $w_{14} = 0.9$, $w_{23} = 0.4$, $w_{24} = 1.0$, $w_{35} = -1.2$, $w_{45} = 1.1$, $\theta_3 = 0.8$, $\theta_4 = -0.1$ and $\theta_5 = 0.3$.

Assuming the sigmoid activation Function



$$Y^{sigmoid} = \frac{1}{1+e^{-X}}$$

<u>Step 2</u>: Activation

Activate the back-propagation neural network by applying inputs $x_1(p), x_2(p),\ldots, x_n(p)$ and desired outputs $y_{d,1}(p), y_{d,2}(p),\ldots, y_{d,n}(p)$.

(a) Calculate the actual outputs of the neurons in the hidden layer:

$$\boxed{y_j(p) = sigmoid\left[\sum_{i=1}^{n} x_i(p)\cdot w_{ij}(p) - \theta_j\right]}$$

where n is the number of inputs of neuron j in the hidden layer, and *sigmoid* is the *sigmoid* activation function.

<u>Step 2</u>: Activation (continued)

(*b*) Calculate the actual outputs of the neurons in the output layer:

$$y_k(p) = sigmoid\left[\sum_{j=1}^{m} x_{jk}(p) \cdot w_{jk}(p) - \theta_k\right]$$

where *m* is the number of inputs of neuron *k* in the output layer.

If the sigmoid activation function is used the output of the hidden layer is

$$\boxed{\begin{aligned} y_3 &= sigmoid\,(x_1 w_{13} + x_2 w_{23} - \theta_3) = 1/\left[1+e^{-(1\cdot 0.5+1\cdot 0.4-1\cdot 0.8)}\right] = 0.5250 \\ y_4 &= sigmoid\,(x_1 w_{14} + x_2 w_{24} - \theta_4) = 1/\left[1+e^{-(1\cdot 0.9+1\cdot 1.0+1\cdot 0.1)}\right] = 0.8808 \end{aligned}}$$

And the actual output of neuron 5 in the output layer is

$$\boxed{y_5 = sigmoid(y_3 w_{35} + y_4 w_{45} - \theta_5) = 1/\left[1+e^{-(-0.52501.2+0.88081.1-1\cdot 0.3)}\right] = 0.5097}$$

And the error is

$$e = y_{d,5} - y_5 = 0 - 0.5097 = -0.5097$$

What learning law applies in a multilayer neural network?

$$\Delta w_{jk}(p) = \alpha \cdot y_j(p) \cdot \delta_k(p)$$

<u>Step 3</u>: Weight training output layer

Update the weights in the back-propagation network propagating backward the errors associated with output neurons.

(*a*) Calculate the error $\boxed{e_k(p) = y_{d,k}(p) - y_k(p)}$

and then the error gradient for the neurons in the output layer: $\boxed{\delta_k(p) = y_k(p) \cdot [1 - y_k(p)] \cdot e_k(p)}$

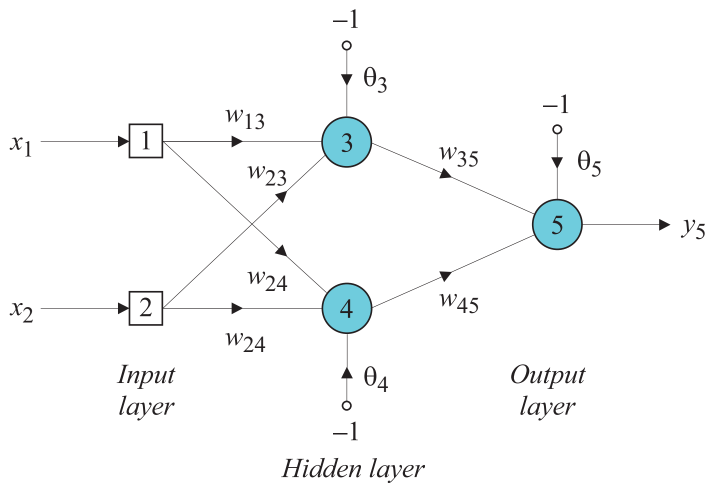
Then the weight corrections:

$$\Delta w_{jk}(p) = \alpha \cdot y_j(p) \cdot \delta_k(p)$$

Then the new weights at the output neurons:

$$\boxed{w_{jk}(p+1) = w_{jk}(p) + \Delta w_{jk}(p)}$$

Three-layer network for solving the Exclusive-OR operation



- The error gradient for neuron 5 in the output layer:

$$\delta_5 = y_5\,(1 - y_5)\,e = 0.5097 \cdot (1 - 0.5097) \cdot (-0.5097) = -0.1274$$

- Determine the weight corrections assuming that the learning rate parameter, α, is equal to 0.1:

$$\Delta w_{35} = \alpha \cdot y_3 \cdot \delta_5 = 0.1 \cdot 0.5250 \cdot (-0.1274) = -0.0067$$
$$\Delta w_{45} = \alpha \cdot y_4 \cdot \delta_5 = 0.1 \cdot 0.8808 \cdot (-0.1274) = -0.0112$$
$$\Delta \theta_5 = \alpha \cdot (-1) \cdot \delta_5 = 0.1 \cdot (-1) \cdot (-0.1274) = -0.0127$$

Apportioning error in the hidden layer

- Error is apportioned in proportion to the weights of the connecting arcs.
- Higher weight indicates higher error responsibility

<u>Step 3</u>: Weight training hidden layer

(b) Calculate the error gradient for the neurons in the hidden layer:

$$\delta_j(p) = y_j(p) \cdot [1 - y_j(p)] \cdot \sum_{k=1}^{l} \delta_k(p)\, w_{jk}(p)$$

Calculate the weight corrections:

$$\Delta w_{ij}(p) = \alpha \cdot x_i(p) \cdot \delta_j(p)$$

Update the weights at the hidden neurons:

$$w_{ij}(p+1) = w_{ij}(p) + \Delta w_{ij}(p)$$

- The error gradients for neurons 3 and 4 in the hidden layer:

$$\delta_3 = y_3(1-y_3)\cdot\delta_5\cdot w_{35} = 0.5250\cdot(1-0.5250)\cdot(-0.1274)\cdot(-1.2) = 0.0381$$
$$\delta_4 = y_4(1-y_4)\cdot\delta_5\cdot w_{45} = 0.8808\cdot(1-0.8808)\cdot(-0.1274)\cdot 1.1 = -0.0147$$

- Determine the weight corrections:

$$\Delta w_{13} = \alpha\cdot x_1\cdot\delta_3 = 0.1\cdot 1\cdot 0.0381 = 0.0038$$
$$\Delta w_{23} = \alpha\cdot x_2\cdot\delta_3 = 0.1\cdot 1\cdot 0.0381 = 0.0038$$
$$\Delta\theta_3 = \alpha\cdot(-1)\cdot\delta_3 = 0.1\cdot(-1)\cdot 0.0381 = -0.0038$$
$$\Delta w_{14} = \alpha\cdot x_1\cdot\delta_4 = 0.1\cdot 1\cdot(-0.0147) = -0.0015$$
$$\Delta w_{24} = \alpha\cdot x_2\cdot\delta_4 = 0.1\cdot 1\cdot(-0.0147) = -0.0015$$
$$\Delta\theta_4 = \alpha\cdot(-1)\cdot\delta_4 = 0.1\cdot(-1)\cdot(-0.0147) = 0.0015$$

- At last, we update all weights and threshold:

$$w_{13} = w_{13} + \Delta w_{13} = 0.5 + 0.0038 = 0.5038$$
$$w_{14} = w_{14} + \Delta w_{14} = 0.9 - 0.0015 = 0.8985$$
$$w_{23} = w_{23} + \Delta w_{23} = 0.4 + 0.0038 = 0.4038$$
$$w_{24} = w_{24} + \Delta w_{24} = 1.0 - 0.0015 = 0.9985$$
$$w_{35} = w_{35} + \Delta w_{35} = -1.2 - 0.0067 = -1.2067$$
$$w_{45} = w_{45} + \Delta w_{45} = 1.1 - 0.0112 = 1.0888$$
$$\theta_3 = \theta_3 + \Delta\theta_3 = 0.8 - 0.0038 = 0.7962$$
$$\theta_4 = \theta_4 + \Delta\theta_4 = -0.1 + 0.0015 = -0.0985$$
$$\theta_5 = \theta_5 + \Delta\theta_5 = 0.3 + 0.0127 = 0.3127$$

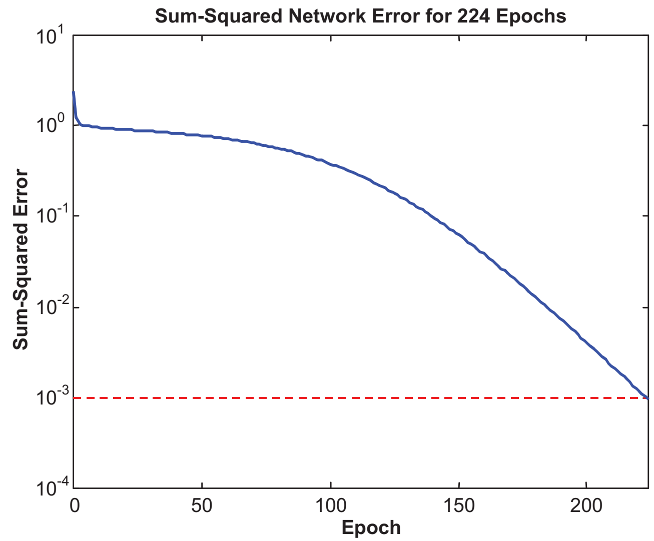
- The training process is repeated until the sum of squared errors is less than 0.001.

Step 4: Iteration

Increase iteration p by one, go back to *Step 2* and repeat the process until the selected error criterion is satisfied.

As an example, we may consider the three-layer back-propagation network. Suppose that the network is required to perform logical operation *Exclusive-OR*. Recall that a single-layer perceptron could not do this operation. Now we will apply the three-layer net.

Typical Learning Curve



Final results of three-layer network learning

Inputs x_1	Inputs x_2	Desired output y_d	Actual output y_5	Error e	Sum of squared errors
1	1	0	0.0155	–0.0155	0.0010
0	1	1	0.9849	0.0151	
1	0	1	0.9849	0.0151	
0	0	0	0.0175	–0.0175	

Network represented by McCulloch-Pitts model for solving the *Exclusive-OR* operation