Лаба 3

Метод Рунге - Кутты

```
In [287]:
          def f(t, y):
              return 2 * t * y
In [288]: def Runge_Kutt4(t, y, h, count):
              k = [0] * 4
              for i in range(count):
                   k[0] = f(t[i], y[i])
                   k[1] = f(t[i] + h / 2, y[i] + h * k[0] / 2)
                   k[2] = f(t[i] + h / 2, y[i] + h * k[1] / 2)
                   k[3] = f(t[i] + h, y[i] + h * k[2])
                   kn = (k[0] + 2 * k[1] + 2 * k[2] + k[3]) / 6
                   y[i + 1] = y[i] + h * kn
              return y
In [328]:
          t max = 1
          t_min = 0
          h1 = 0.1
          h2 = 0.05
          h3 = 0.025
          print(h1, h2, h3)
          0.1 0.05 0.025
          count1 = t_max / h1
In [329]:
          count1 = int(count1)
          count2 = t_max / h2
          count2 = int(count2)
          count3 = t max / h3
          count3 = int(count3)
          y1 = [1] + [0] * count1
          y2 = [1] + [0] * count2
          y3 = [1] + [0] * count3
          t1 = [i / count1 for i in range(count1 + 1)]
          t2 = [i / count2 for i in range(count2 + 1)]
          t3 = [i / count3 for i in range(count3 + 1)]
In [330]: print(t1, t2, t3, sep="\n\n")
          [0.0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0]
          [0.0, 0.05, 0.1, 0.15, 0.2, 0.25, 0.3, 0.35, 0.4, 0.45, 0.5, 0.55, 0.6, 0.65, 0.7, 0.7]
          5, 0.8, 0.85, 0.9, 0.95, 1.0]
          [0.0, 0.025, 0.05, 0.075, 0.1, 0.125, 0.15, 0.175, 0.2, 0.225, 0.25, 0.275, 0.3, 0.32]
          5, 0.35, 0.375, 0.4, 0.425, 0.45, 0.475, 0.5, 0.525, 0.55, 0.575, 0.6, 0.625, 0.65, 0.
          675, 0.7, 0.725, 0.75, 0.775, 0.8, 0.825, 0.85, 0.875, 0.9, 0.925, 0.95, 0.975, 1.0]
In [331]: # решения для 3 сеток
          y1 = Runge_Kutt4(t1, y1, h1, count1)
          y2 = Runge_Kutt4(t2, y2, h2, count2)
          y3 = Runge_Kutt4(t3, y3, h3, count3)
```

```
In [332]:
          import numpy as np
           # истинные значения для каждой сетки (e ^ t ^ 2)
           a1 = [np.e ** ((i / count1) ** 2) for i in range(count1 + 1)]
           a2 = [np.e ** ((i / count2) ** 2) for i in range(count2 + 1)]
           a3 = [np.e ** ((i / count3) ** 2) for i in range(count3 + 1)]
In [333]:
          # абсолютная погрешность между практическим и теоретическим для каждой сетки
           b1 = list(abs(np.array(y1) - np.array(a1)))
           b2 = list(abs(np.array(y2) - np.array(a2)))
           b3 = list(abs(np.array(y3) - np.array(a3)))
In [334]:
           from matplotlib import pyplot as plt
           %matplotlib inline
In [375]:
          plt.subplots(figsize=(12, 8))
           plt.plot(t1, b1, label='h = 0.1', color='blue')
           plt.plot(t2, b2, label='h = 0.05', color='green')
           plt.plot(t3, b3, label='h = 0.025', color='black')
           plt.legend()
Out[375]: <matplotlib.legend.Legend at 0x160c3230>
           0.000012
                      h = 0.1
                      h = 0.05
                     h = 0.025
            0.000010
            0.000008
            0.000006
            0.000004
            0.000002
            0.000000
                                    0.2
                                                   0.4
                                                                  0.6
                                                                                 0.8
                                                                                                1.0
In [336]:
          print(b1[count1], b2[count2], b3[count3], sep='\n')
           1.1653075508988309e-05
           7.447471728205812e-07
           4.6923418040023535e-08
In [337]:
          print(b1[count1] / b2[count2], b2[count2] / b3[count3], sep='\n')
           15.647022149617046
```

Дополнительно - сетка с шагом 0.2

15.87154567864911

```
In [352]:
           h4 = 0.2
           count4 = t_max / h4
           count4 = int(count4)
           y4 = [1] + [0] * count4
           t4 = [i / count4 for i in range(count4 + 1)]
In [353]: y4 = Runge_Kutt4(t4, y4, h4, count4)
In [354]: b4 = list(abs(np.array(y4) - np.array([np.e ** (t4[i] ** 2) for i in range(count4 + 1)]
In [355]: plt.plot(t4, b4, color='red')
Out[355]: [<matplotlib.lines.Line2D at 0x13377cf0>]
            0.000175
            0.000150
            0.000125
            0.000100
            0.000075
            0.000050
            0.000025
            0.000000
                             0.2
                                      0.4
                     0.0
                                               0.6
                                                       0.8
                                                                1.0
In [356]: plt.plot(t4, y4, 'bo')
           plt.plot(t3, a3, color='k')
Out[356]: [<matplotlib.lines.Line2D at 0x133b4e50>]
            2.75
            2.50
            2.25
            2.00
            1.75
            1.50
            1.25
            1.00
                          0.2
                 0.0
                                   0.4
                                           0.6
                                                    0.8
                                                            1.0
```

In []: