

Math III
Winter '12

8.5/20

Section BC

MATH 111

Activity: Tax as Cost

You produce and sell *Things*. The price per Thing is given by the demand function

$$p = h(q) = 15 - 0.375q,$$

where q is in thousands of Things and p is in dollars per Thing. You estimate that the variable cost to produce q Thousand Things is

$$VC(q) = 0.875q^2 + 5q \text{ thousand dollars.}$$

1. If you produce 1000 Things ($q = 1$), then your total cost is 11.875 thousand dollars. Compute the value of your fixed costs in thousands of dollars.

$$TC = FC + VC$$

$$11.875 = 0.875(1)^2 + 5(1) + FC$$

$$11.875 = 5.875 + FC$$

$$FC = 6$$

\$6000

2. Write out formulas for total revenue $TR(q)$, total cost $TC(q)$, and profit $P(q)$, each in thousands of dollars.

$$TR(q) = 15q - 0.375q^2$$

$$TC(q) = 0.875q^2 + 5q + 6$$

$$P(q) = [15q - 0.375q^2] - [0.875q^2 + 5q + 6]$$

3. Find the quantity that maximizes profit.

$$\text{Vertex} = \frac{-10}{2(-1.25)} = \frac{10}{2.5} = 4 \text{ things}$$

4. If you sell the quantity that maximizes profit, what is the price per Thing?

$$15 - 0.375(4) = p = h(4)$$

$$13.5 = \$$$

The government decides to tax your output, charging you t dollars for every Thing that you produce. This becomes an additional cost to you.

5. Write out new formulas for $TC(q)$ and $P(q)$, taking this new tax into account. (Assume that the price function and, therefore, the total revenue function do not change.)

$$TC(q) = 0.875q^2 + 5q + 6 + tq$$

$$P(q) = [15q - 0.375q^2] - [TC]$$

$$15q - 0.375q^2 - 0.875q^2 - (5+t)q - 6$$

$$15q - 1.25q^2 - (5+t)q - 6$$

$$(15q - (5+t)q) - 1.25q^2 - 6 = (10-t)q - 1.25q^2 - 6$$

6. Find the quantity that maximizes profit under the new tax system. (It will depend on t .)

$$\frac{15 - 10 + t}{2(-1.25)} = 4 - 0.4t = \text{quantity}$$

7. Let X be the amount of money that the government will collect from you in taxes (their tax revenue). If you produce q Things and the government collects $\$t$ per Thing, then $X = qt$. Assuming you produce the quantity that you found in #6, what value of t will maximize the government's tax revenue?

$$t = 5$$

$$X = qt = (4 - 0.4t)t$$

$$\text{vertex} = \frac{-4}{-2(0.4)} = 5 \text{ units}$$

8. If the government sets the value of t so that their tax revenues are maximized and you, in turn, set your production level q so that your profits are maximized, then what is the price per Thing? (Use your answers to #6 and #7.)

$$t = 5 \rightarrow q = 4 - 0.4(5)$$

$$P(2) = 15 - 1.375(2) = 14.25$$

Price

9. Assuming that the government sets the tax rate in order to maximize their tax revenues and that you always set your production level to maximize profits, your answer to #4 is the price per Thing with no output taxes and your answer to #8 is price per Thing after the tax has been imposed. What percentage of the tax are you passing on to your customers?

$$\% \text{ passed} = \frac{\Delta P}{t} = \frac{13.75 - 15}{5} \times 100 = 15\%$$