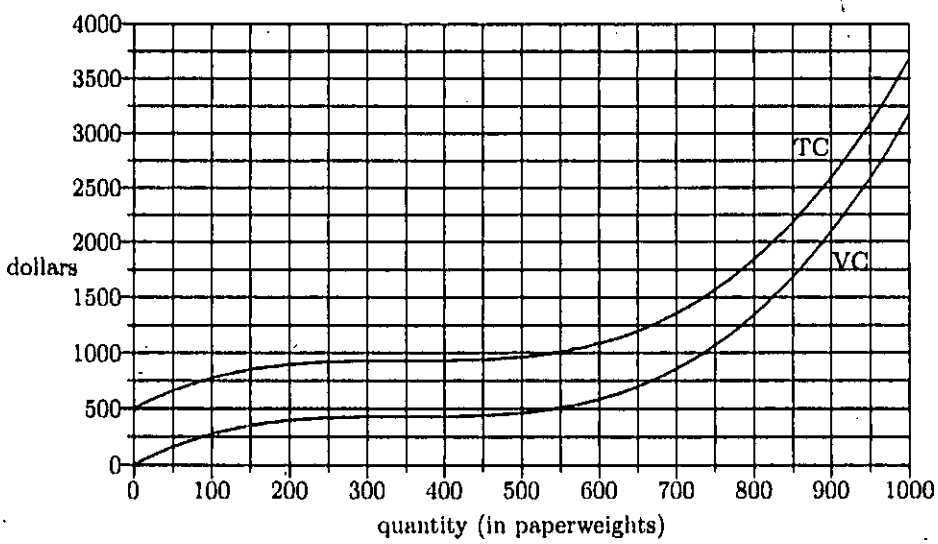
Math 111 Winter '12 Mcnifer

Section: BC 19/20

Math 111

Activity: The Cost of Max's Paperweights

Max sells paperweights at Pike Place market. The graph below shows Max's total cost (TC) for producing different quantities of paperweights from $q = 0$ to $q = 1000$.



1. Let $TC(q)$ represent the total cost of producing q paperweights.

   (a) Translate the following into English: $TC(550) = 1000$.

   The total cost of producing 550 paperweights is 1000 dollars

   (b) Estimate Max's total cost to produce 200 paperweights. Write a complete sentence in symbolic notation that expresses your answer.

   $TC(200) = 875$ $/unit, the total cost of producing 200 paperweights and is 875 dollars

2. Total cost is made up of two components: fixed cost and variable cost.

   (a) Fixed cost (FC) consists of all costs that must be paid regardless of how many paperweights Max makes. In particular, even if Max makes *no* paperweights, he will still have to pay his fixed costs. Fixed cost includes things like rent for the studio where Max makes the paperweights. You can find the value of FC on the graph since $FC = TC(0)$. What is the value of FC?

   $FC = TC - VC$

   $FC =$ $500 dollars

   (b) Variable cost (VC) makes up the rest of total cost. Variable cost consists of all costs that depend on how many paperweights Max produces. Variable cost includes things like the cost of the materials used to make the paperweights and the cost of labor. The graph of VC is given along with the graph of TC. Explain why the variable cost graph must always go through the origin.

   because if there is no q, then no materials or labor are being used to make zero paper weights

Let $VC(q)$ represent the value of the variable cost of producing q paperweights. Then, for any quantity q,

$$\boxed{TC(q) = VC(q) + FC.}$$

3. It may appear that the graphs of TC and VC get closer to each other for larger values of q. But that is an optical illusion. What can you say about the vertical distance between the two graphs at any quantity?

The distance is equal to FC for all values of q $FC = TC(8) - VC(8)$

4. The average cost (AC) of producing q paperweights is given by $AC(q) = \frac{TC(q)}{q}$ $\frac{\$}{q}$

   (a) Use the formula to determine the units on AC.

   AC is measured in dollars / paperweights

   (b) Use your answer to #1(b) to compute average cost at $q = 200$ paperweights. (Give your answer with the correct units.)

   $\frac{TC(200)}{200} = \frac{860}{200}$

   $AC(200) =$ 4.40 dollars / weight

   (c) Use a formula for slope to demonstrate that the value of $AC(x)$ is equal to the slope of the diagonal line through the TC graph at $q = x$. (HINT: First, find the coordinates of two points on that diagonal line.)

   $AC(x) = \frac{TC(x)}{x}$ (0;0), (200, 860) slope = 4.3 $\frac{860}{200} =$

   $TC(200) = VC(200) + FC$
   $= 400 + 500$
   $= 900/8 = 4.5$

   (d) Use the "rolling ruler" method to find the smallest value of AC.

   625 = q
   1100 = $ slope $\frac{1100}{625}$

   ANSWER: 1.77 $/units

5. The average variable cost (AVC) is given by $AVC(q) = \frac{VC(q)}{q}$

   (a) Compute average variable cost at $q = 900$ paperweights. (Give your answer with the correct units.)

   $AVC(900) = \frac{VC(900)}{900} = \frac{2100}{900}$

   $AVC(900) =$ 2.33 $/units

   (b) Give a formula relating $AVC(q)$ to the total cost function $TC(q)$.

   $TC(8) = VC(8) + FC$
   $VC(8) = AVC(8)\,8$

   $AVC(q) = \frac{(TC(8) - FC)}{8}$

   (c) Average variable cost can be seen on the graph as the slope of a diagonal line through the VC graph. Use the "rolling ruler" method to find the smallest value of AVC.

   q = 550
   $ = 500

   ANSWER: .91 dollars/units

   $\frac{500}{550}$ = slope of AVC line @ q = 550