Physical principle used to define stress transf Equations?
Equilibrium.

CEE 220 **Exam 1** Spring, 2000

Name:______________________________

Part I – Short Answer (3 pts each)

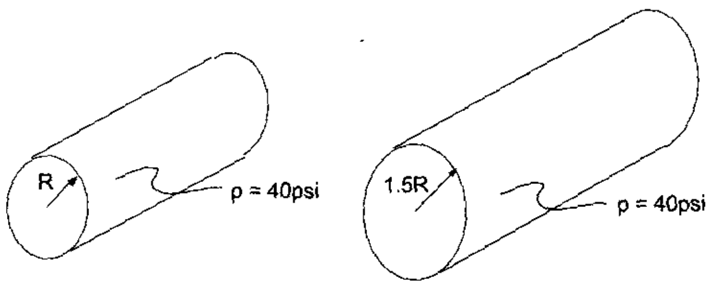
1. List the three fundamental relations used in formulating and solving problems in solid mechanics:

   i. Equilibrium

   ii. Kinematics/compatibility

   iii. Constitutive relations

2. The internal pressure and wall thickness are the same for the pressure vessels shown. Which vessel will experience higher hoop stresses?

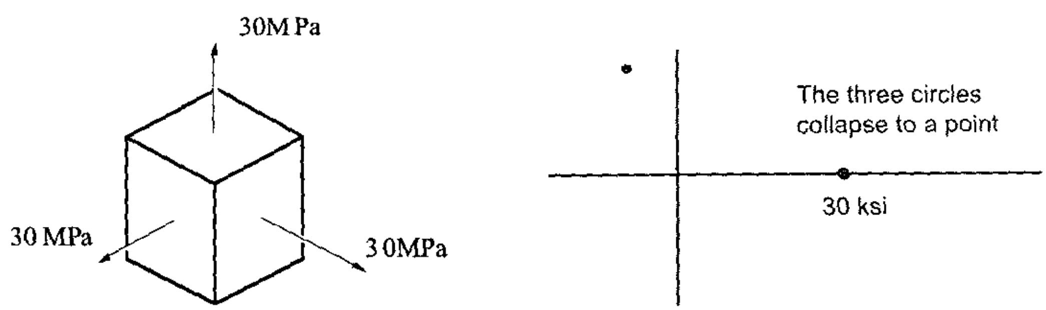


Bigger r bigger pr/t

3. We discussed in class how any stress state can be decomposed into a pressure part and a shearing part. What happens to a Mohr's circle plot if the pressure component of the stress state is increased?

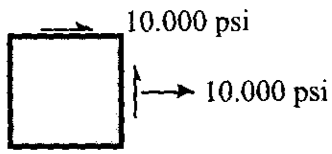
   a. The circles in the plot translate. this is the answer
   b. The circles in the plot grow.
   c. The circles in the plot both grow and translate.
   d. The circles in the plot remain unchanged.

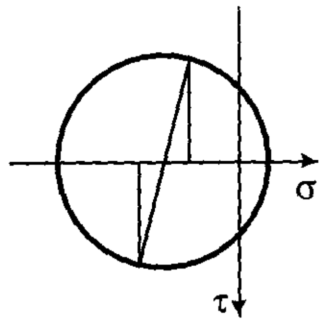
4. Sketch Mohr's circles for the stress state shown.



5. Match each stress state to the corresponding Mohr's circle:

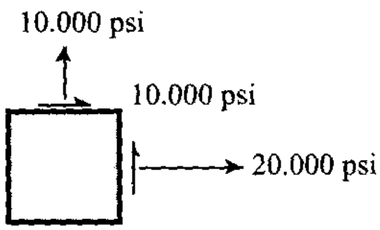
A

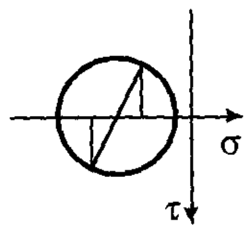




D

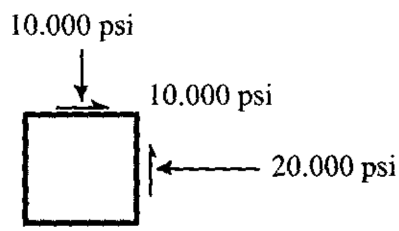
B

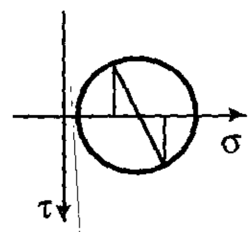




C

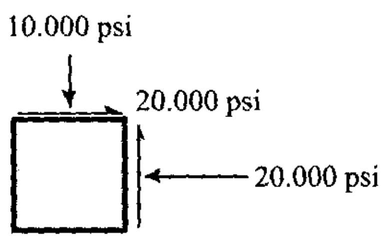
C

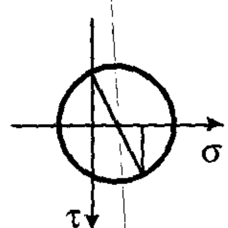




B

D

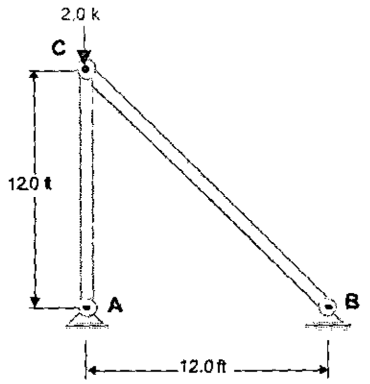




A

Part II – Problems (15 pts each)

1. A wooden 4 x 4 post is part of a simple structure as shown below (Assume the post's cross-sectional area is a full 4 inches by 4inches).



$$\delta = \frac{PL}{AE} = \frac{(2000)(12' \cdot 12''/')}{16\,in^2 \cdot 1500\,k/in^2}$$

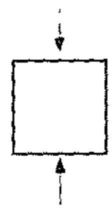
$$= \frac{288}{16 \cdot 1500} = \boxed{.012''}$$

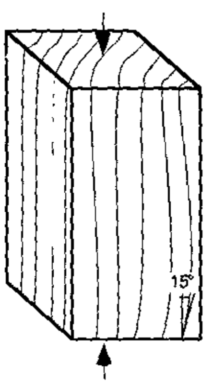
a. Determine the vertical deflection at point **C** ($E = 1{,}500$ ksi).

$$\delta = \frac{PL}{AE} = \frac{2k(12')(12''/')}{16in^2(1{,}500ksi)} = 0.012''$$

b. $AE \quad 16in \quad 1{,}500ksi$ The slope of the grain in the post has a maximum deviation from the axis of the member of 15°, and there is some concern that the relatively low shear strength of wood along its grain could cause problems. Calculate the shear stress along the grain for a 15° deviation as shown.

$$\tau = \text{shear Stress} = \frac{F_T}{A}$$

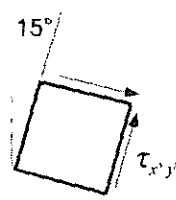




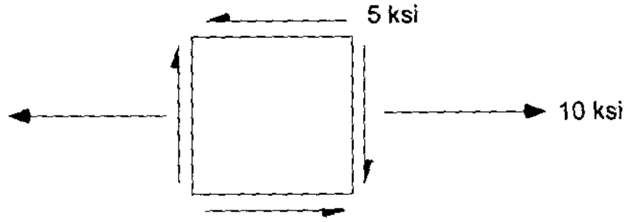
$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$= \frac{\sigma_y}{2}\sin 2\theta$$

$$= \frac{-2k/16\,in^2}{2}\sin(-30°) = \underline{0.0313\text{ksi}}$$

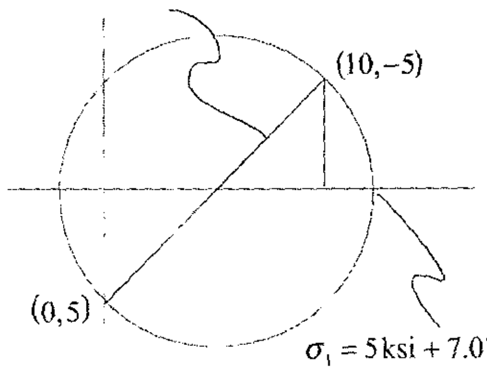


2. The stress state shown has been calculated at a particular point of a part that will be fabricated using a relatively brittle material.



a. Determine the maximum tensile stress experienced by the material at this point.

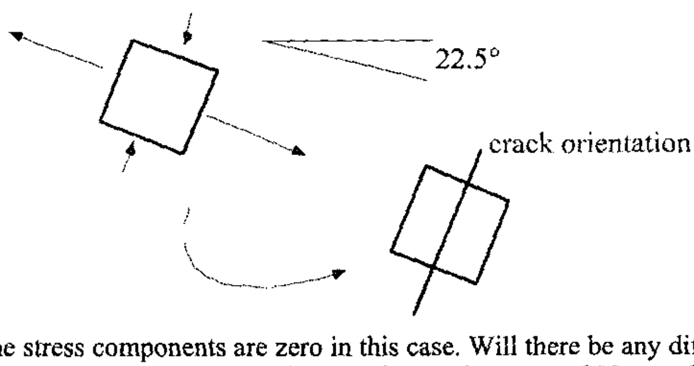
$$\tau_{max} = \sqrt{5^2 + 5^2} = 5\sqrt{2} = 7.07\,\text{ksi}$$



$$\sigma_1 = 5\,\text{ksi} + 7.07\,\text{ksi} = \underline{12.07\,\text{ksi}}$$

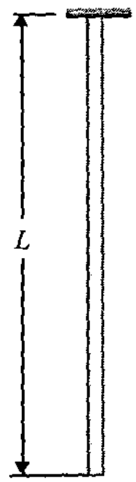
b. Determine the plane along which a crack would be most likely to form assuming the material is weak in tension, and show this plane on a sketch.

$$\theta_p = \frac{1}{2}\tan^{-1}\left(\frac{5\,\text{ksi}}{5\,\text{ksi}}\right) = 22.5° \text{ clockwise}$$



c. The out-of-plane stress components are zero in this case. Will there be any difference between the in-plane maximum shear stress and the absolute maximum shear stress? No—z-direction normal stress is intermediate.

3. Find the total elongation of a uniform rod with cross-sectional area, A, weight density, γ, length, L, and elastic modulus, E, hanging under its own weight.

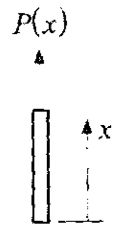


$$\delta = \int_0^L \frac{p(x)\,dx}{A(x)E} \qquad p(x) = \gamma L A \qquad A(x) = A$$

$$= \int_0^L \frac{\gamma L \cancel{A}\,dx}{\cancel{A}E} \qquad \gamma = \frac{lb}{\cancel{ft^3}} \cdot L\cancel{ft} \cdot A\cancel{ft^2} \qquad \gamma = lb = F = P$$

$$= \frac{\gamma}{E}\left[\frac{L^2}{2}\Big|_0^L\right]$$

$$= \boxed{\delta = \frac{\gamma L^2}{2E}}$$



$P(x) = \gamma x A$

$$\delta = \int_0^L \frac{P(x)dx}{AE} = \frac{1}{AE}\int_0^L \gamma x A\,dx = \underline{\frac{\gamma L^2}{2E}}$$