AA 210. M1

AA 210 Statics

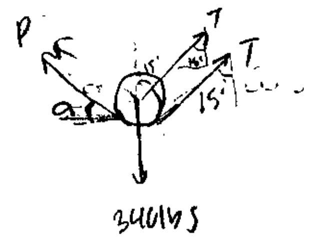
Midterm #1 – Winter 2009

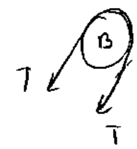
(60 min, Open Book & Open Notes; show all work and FBD's)

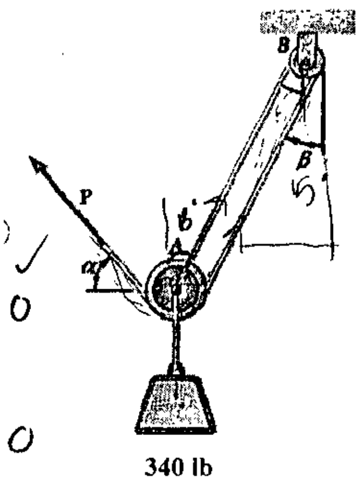
Version E

1. A 340-lb load is supported by the rope-and-pulley arrangement shown. Knowing that $\beta=15^\circ$, determine the magnitude and direction (only consider $\alpha>0$) of the force P which should be exerted on the free end of the rope to maintain equilibrium. Show the FBD. (25 pts)

$P=T$







$\Sigma F_x = -P\cos\alpha + 2(T\sin 15^\circ) = 0$ ✓

$\Sigma F_y = P\sin\alpha + 2(T\cos 15^\circ) - 340 = 0$

$\Sigma F_x' = -T\cos\alpha + 2(T\sin 15^\circ) = 0$

$\Sigma F_y = T\sin\alpha + 2(T\cos 15^\circ) = 340$

$= T(\sin\alpha + 2\cos 15) = 340$

$$T = \frac{340}{\sin\alpha + 2\cos 15} \quad (1)$$

$$\Sigma F_x = \left(\frac{-340}{\sin\alpha + 2\cos 15}\right)\cos\alpha + 2\left(\frac{340}{\sin\alpha + 2\cos 15}\right)\sin 15 = 0$$

$$= \frac{-340\cos\alpha}{\sin\alpha + 2\cos 15} + \frac{680\sin 15}{\sin\alpha + 2\cos 15} = 0$$

$$\frac{-340\cos\alpha + 680\sin 15}{\sin\alpha + 2\cos 15} = \frac{0}{1}$$

$-340\cos\alpha + 680\sin 15 = 0$

$-340\cos\alpha = -175.997$

$\cos\alpha = .51764$

$\boxed{\alpha = 58.8^\circ}$ ✓

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(1) $T = \dfrac{340}{\sin 58.8^\circ + 2\cos 15^\circ} = \boxed{P = 121.99\text{ lbs}}$ ✓

$$\Sigma F_x = -T\cos\alpha + 2(T\sin 15) = 0$$

$$\Sigma F_y = T\sin\alpha + 2T\cos 15 - 340 = 0$$

~~the a~~

$$T\sin\alpha + 2T\cos 15 = 340$$

$$T(\sin\alpha + 2\cos 15 = 340$$

$$T(\sin\alpha + 1.932) = 340$$

$$T = \frac{340}{\sin\alpha + 1.932}$$

$$-\frac{340}{\sin\alpha + 1.932}\cos\alpha + 2\left(\frac{340}{\sin\alpha + 1.932}\right)\sin 15) = 0$$

$$\frac{-340\cos\alpha + 680\sin 15}{\sin\alpha + 1.932} = \frac{0}{1}$$

$$\frac{-340\cos\alpha + 98.35}{\sin\alpha + 1.932} = 0$$

$$\frac{-340\cos\alpha}{\sin\alpha + 1.932} = -\frac{98.35}{\sin\alpha + 1.932}$$

$$(-340\cos\alpha)(\sin\alpha) - 656.88\cos\alpha = 98.35\sin\alpha + 190.012$$

$r_{BC} = \dfrac{(3, 1.5, -4.33)}{5.477}$ $= e_{BC}$

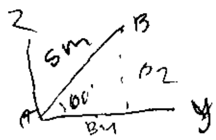
$(.5477, .2739, -.7906)$

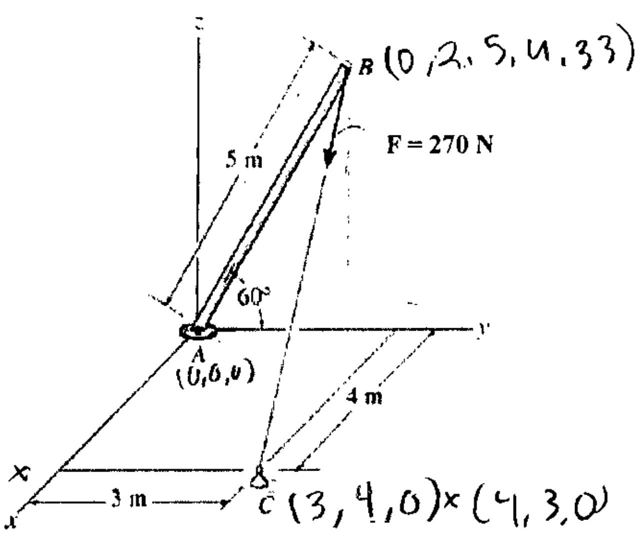
$$\begin{vmatrix} i & j & k \\ 0 & 7.5 & 4.33 \\ 147.08 & 73.95 & -213.46 \end{vmatrix}$$

$\frac{24}{25}$

2. The 5m long boom AB lies in the y-z plane and the cable exerts a force of F = 270 N at B.
   a) Determine the moment vector (M_A) of the force F about point A. (12 pts)
   b) Determine the shortest distance between the cable and point A. (7 pts)
   c) Determine the moment vector of the force about the y-axis. (6 pts)

a.) $\frac{11}{12}$





$B_y = 5\cos 60 = 2.5$

$B_z = 5\sin 60 = 4.33$

$r_{BC} = (3, 1.5, -4.33)$

$e_{BC} = \frac{(3, 1.5, -4.33)}{5.477}$

$= (.5477, .2739, -.7906)$ ×

$F_{BC} = 270(e_{BC}) = (147.88, 73.95, -213.46)$

$$r_{AB} \times F_{BC} = \begin{vmatrix} i & j & k \\ 0 & 2.5 & 4.33 \\ 147.88 & 73.95 & -213.46 \end{vmatrix} \checkmark$$

$= [(-213.46)(2.5) - (4.33)(73.45)]\, i$

$- [0 - (147.88)(4.33)]\, j + [0 - (2.5)(147.88)]\, k$

$$\boxed{M_A = -851.7i + 640.32j - 369.7k \text{ (N·m)}}$$ ×

$\frac{7}{7}$ b.) $|M_A| = |r||F|\sin\theta$ 90°

$|M_A| = \sqrt{(-851.72)^2 + (640.32)^2 - (369.7)^2} = 1127.87$

$|F| = \sqrt{147.88^2 + 73.95^2 + (-213.46^2)} = 270.00$

$|r| = \frac{|M_A|}{|F|} = \frac{1127.87}{270.00} = \boxed{4.18\text{m}}$ ✗

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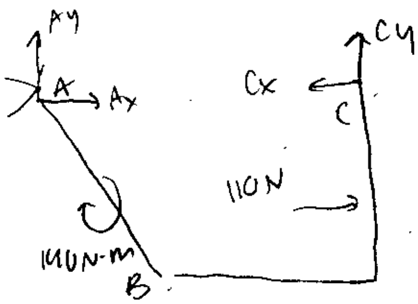
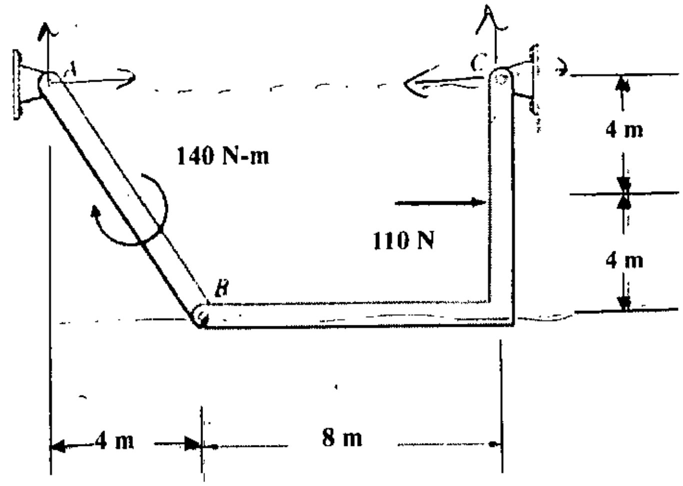
3. Determine the reactions at A and C. Show all FRD's used for solving this problem. (25 pts)





$\Sigma F_x = A_x + C_x + 110N = 0$ (1)

$\Sigma F_y = A_y + C_y = 0$ (2)

$A_y = -C_y$ (3)

$\Sigma M_A = (-140\ N\cdot m) + (12m)(C_y) - (4m)(110)$

$= -140 + 12C_y - 440$ (from (3))

$580 = 12C_y$ $\boxed{C_y = 48.3N}$ $\boxed{A_y = -48.3N}$

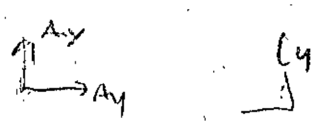
A and C = 2-force members $\alpha = 0$

A,

$\boxed{A_x = 0\,N}$

$\boxed{C_x = 0\,N}$

or $A_x = C_x = -55N$



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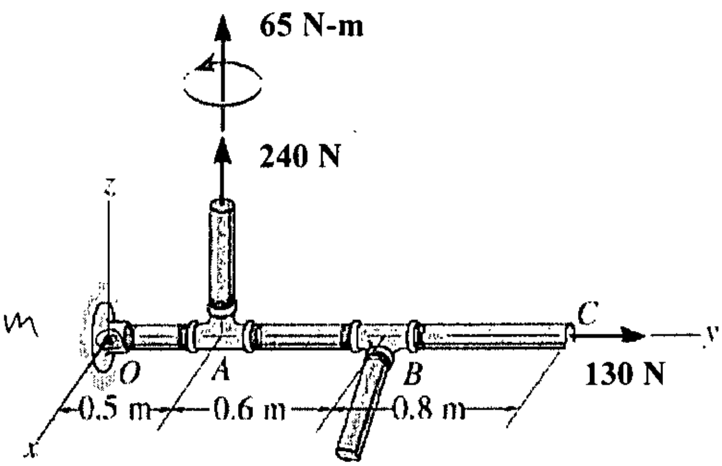
4. Replace the wrench and force acting on the pipe assembly by an equivalent force and couple moment at point O. (25 pts)

$\Sigma F_y = 130N$ ✓

$\Sigma F_z = 240\ N$



~~$\Sigma M_O = (.5m)(240) + 65 N\cdot m + (130N)(0m)$~~

~~$\Sigma M_O = 185\ N\cdot m$~~

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$\sqrt{240^2 + 130^2} = F_{eq.} = 272.95\ N$

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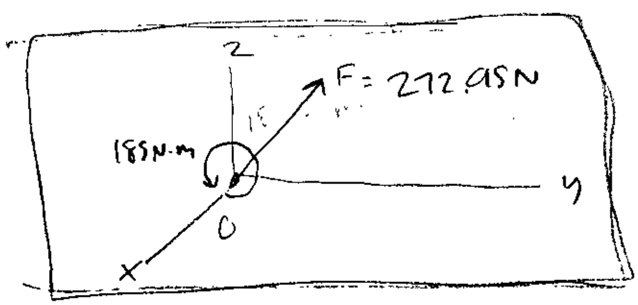

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