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AA-210 STATICS QUIZ #1 (Closed-Book)

Thursday Oct 15, 2009 (Version A)

(1-doubled-sided page of notes and calculator are allowed)

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<u>**Problem 1**</u> (20 points)

Given the vectors $\vec{u} = 2\vec{i} - 3\vec{j}$ and $\vec{v} = 3\vec{i} + 4\vec{j} - 5\vec{k}$,

(1) (5 points) Find $\vec{u} \bullet \vec{v}$ (the dot product of $\vec{u}$ and $\vec{v}$).

$\vec{u}\cdot\vec{v} = (2i\cdot 3) + (4\cdot -3) + -5$

$6 \quad -12 \quad \cancel{-5}$

$\boxed{\vec{u}\cdot\vec{v} = -11}$

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(2) (5 points) Find $\vec{u} \otimes \vec{v}$ (the cross product of $\vec{u}$ and $\vec{v}$).

$\begin{vmatrix} i & j & k \\ 2 & -3 & 0 \\ 3 & 4 & -5 \end{vmatrix}$ $\left((-3\cdot -5) - 0\right)i - (-10 - 0)j + (8 + 9)k$

$\boxed{15i + 10j + 17k}$

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(3) (5 points) Find the angle between the vector $\vec{u}$ and $\vec{v}$.

$\theta = \cos^{-1} \dfrac{u\cdot v}{|u||v|} = \cos^{-1} \dfrac{-11}{\sqrt{2^2+3^2}\,\sqrt{3^2+4^2+5^2}}$

$(\sqrt{13})\ (\sqrt{50})$

$\cos^{-1}(-.43145)$

$\boxed{\theta = 115.56°}$

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(4) (5 points) Find a *unit* vector $\vec{e}$ that is perpendicular to <u>both</u> vectors $\vec{u}$ and $\vec{v}$.

$\dfrac{\vec{u}\times\vec{v}}{|\vec{u}\times\vec{v}|} = \dfrac{15i + 10j + 17k}{\sqrt{15^2 + 10^2 + 17^2}}$

24.78

$\vec{e} = \dfrac{15}{\sqrt{614}}i + \dfrac{10}{\sqrt{614}}j + \dfrac{17}{\sqrt{614}}k$

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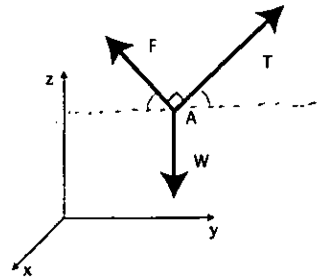
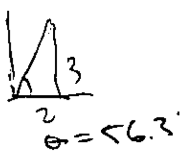
$\boxed{\vec{e} = .605i + .404j + .686k}$

Problem 2 (20 points) Three *concurrent* forces $\vec{T}$, $\vec{F}$ and $\vec{W}$ applied at point A are in equilibrium.





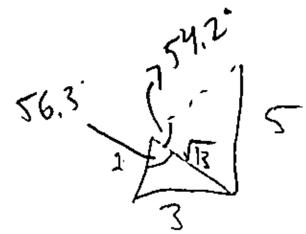


Figure 1: 3−D Forces $\vec{T}$, $\vec{F}$ and $\vec{W}$ in equilibrium.

The forces $\vec{T}$ and $\vec{F}$ are *orthogonal* (i.e., perpendicular) to each other where $\vec{T} = T\left(2\vec{i} + 3\vec{j} + 5\vec{k}\right)$ (N). The weight W is 100 (N).

(1) (10 points) Find the force $\vec{T}$.

$$\sum F_z = -\vec{W} + \frac{\vec{T}}{5} + \vec{F}(\cos 35.8°) = 0$$

$$\sum F_y = \frac{\vec{T}}{3} + \vec{F}\cos(33.7) = 0$$

$$\sum F_x - \frac{\vec{T}}{2} - \vec{F}\sin(33.7) = 0$$

$$F = \frac{\vec{T}}{2(\sin(90-56.3)}$$

$$100N = \frac{\vec{T}}{5} + \frac{\vec{T}}{2\sin(90-56.3)}\cos 35.8$$

$.73' T$

$.93\ T = 100N$

$\boxed{\vec{T} = 107.4\ N.}$

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(2) (10 points) Find the force $\vec{F}$.

$$\sum F_x : \quad \frac{107.42\ N}{2} = \vec{F}\sin(90-56.3)$$

$\boxed{\vec{F} = 96.8\ N}$

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<u>Problem 3</u> (10 points)

Let $\vec{u} = -2\vec{i} + 3\vec{j} - 7\vec{k}$, find a *unit* vector $\vec{v}$ that is *perpendicular* to the vector $\vec{u}$ and has NO component in the k-direction (i.e., $\vec{v} = v_x\vec{i} + v_y\vec{j}$).

$v_z = 0$

$\vec{u} \cdot \vec{v} = 0$

$= (-2 \cdot v_x)i + (3 \cdot v_y)j + \cancel{(-7 \cdot 0)}k$

$2v_x = 3v_y$

$v_x = 3$

$v_y = 2$

$\frac{\vec{v}}{|v|} = \frac{3i + 2j}{\sqrt{3^2 + 2^2}}$

$\sqrt{13}$

$\boxed{e_{\vec{v}} = .832i + .555j}$

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