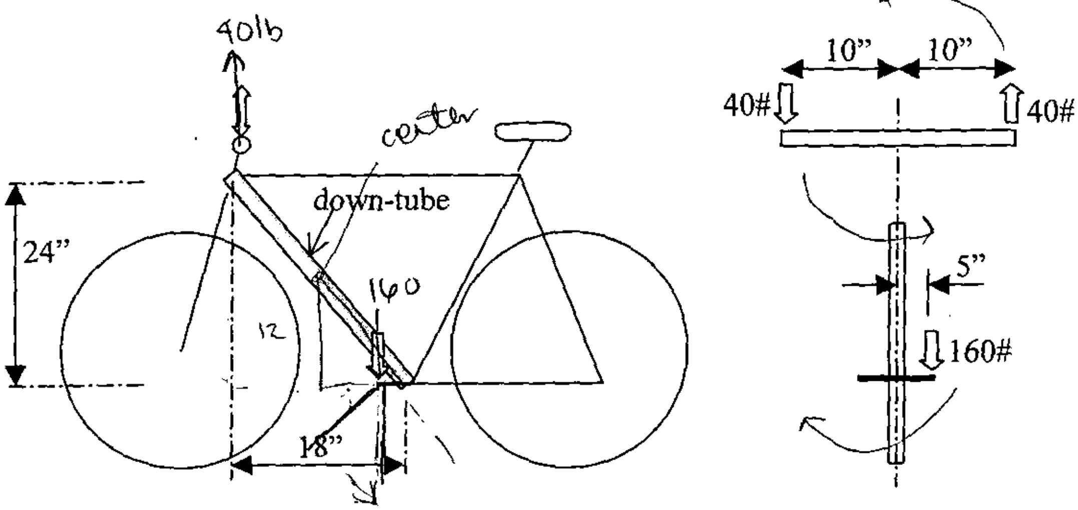
CEE 220 **Mid-term exam 2** **Autumn 2003**

Allowed: 1 page of notes, written on both sides.
Attempt all questions. Points per question are as shown. Use extra sheets as needed.

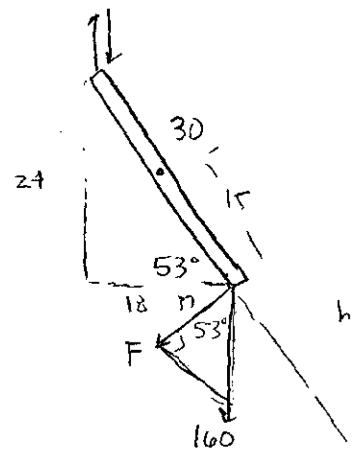
Write your name on every sheet.

1. (10 points). A cyclist applies the forces shown to the bike: (40 lbs up and down on the handlebars, and 160 lbs down on one pedal). Find the torque in the down-tube.



side view front view

Lateral $\Sigma T = 40lb(10in) + (40lb)(10in) - (160lb)(5in) = 0$
The torque side to side on the down tube is zero. ∴ It won't have a tendency to lean to either side, the forces balance.



*Find ⊥ force
$F = 160\cos 53°$
$= 96.3 lb$ (⊥ to tube) ✓

horizontal:
The 40 will cancel eachother out but the 160 lb force will cause it to rotate clockwise about the center of the tube (9,12) X

$T = Fd$
$T = (96.3lb)(15in)$ 5"
$T \approx 1400$ lb·in. X

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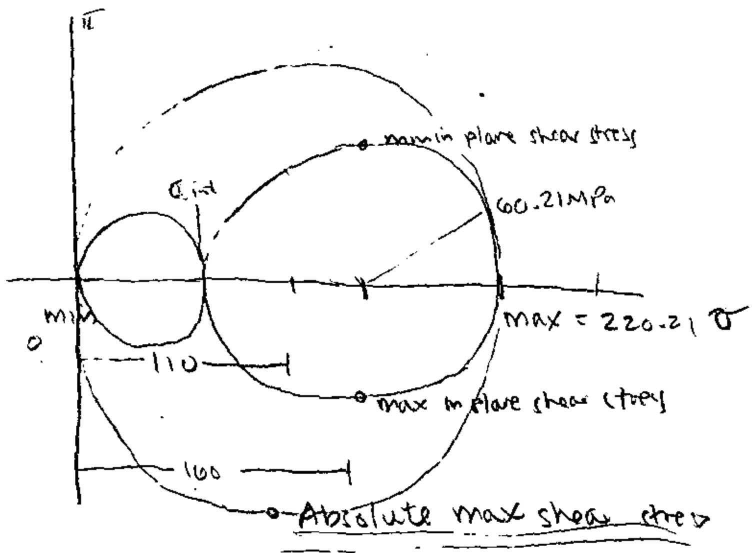
2. (15 points). A piece of sheet metal is subjected to in-plane stresses $\sigma_{xx} = 200$ MPa, $\sigma_{yy} = 120$ MPa, $\tau_{xy} = 45$ MPa. Find the absolute maximum shear stress.

$\sigma_{xx} = 200$ MPa $\quad \sigma_{yy} = 120$ MPa $\quad \tau_{xy} = 45$ MPa

$$\tau_{max\text{-}in\ plane} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + (\tau_{xy})^2}$$

$$= \left[\left(\frac{200\text{MPa} - 120\text{MPa}}{2}\right)^2 + (45\text{MPa})^2\right]^{1/2}$$

$$= 60.21 \text{ MPa} \rightarrow \text{max in-plane shear stress}$$



$\sigma_{ave} = \dfrac{\sigma_x + \sigma_y}{2} = \dfrac{120 + 200}{2} = 160$

$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + (\tau_{xy})^2}$

$= 60.21$

$$\tau_{abs\ max} = \frac{\sigma_{max} - \sigma_{min}}{2}$$

$$= \frac{220.21 - 0}{2}$$

$\tau_{abs\ max} = 110.105$ MPa

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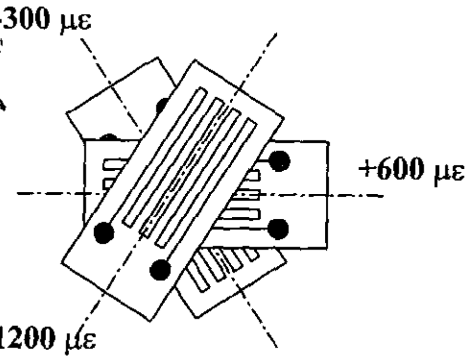
3. (20 points). A strain rosette has gages set at 0^0, 120^0, and 240^0. The measured strains are +600, -300 and +1200 $\mu\varepsilon$ respectively. Find the in-plane strains ε_{xx}, ε_{yy} and γ_{xy} and draw the Mohr's circle of strain.

* I didn't put rosette strain eqs on my note sheet so here is my "best remembered" version



$\sin\theta_a\cos\theta_a$

$\varepsilon_a = \varepsilon_{xx}\cos^2\theta_a + \varepsilon_{yy}\sin^2\theta_a + \gamma_{xy}\cos 2\theta_a$

$\varepsilon_b = \varepsilon_{xx}\cos^2\theta_b + \varepsilon_{yy}\sin^2\theta_b + \gamma_{xy}\cos 2\theta_b$

$\varepsilon_c = \varepsilon_{xx}\cos^2\theta_c + \varepsilon_{yy}\sin^2\theta_c + \gamma_{xy}\cos 2\theta_c$

$\theta_a = 0$ $\quad \varepsilon_a = 600\ \mu\varepsilon$

$\theta_b = 120$ $\quad \varepsilon_b = -300\ \mu\varepsilon$

$\theta_c = 240$ $\quad \varepsilon_c = 1200\ \mu\varepsilon$

$\cos^2\theta_a = 1$

$\cos^2\theta_b = .25$

$\cos^2\theta_c = .25$

$\sin^2\theta_a = 0$

$\sin^2\theta_b = .75$

$\sin^2\theta_c = .75$

$\cos 2\theta_a = 1$

$\cos 2\theta_b = -.5$

$\cos 2\theta_c = -.5$

$600 = \varepsilon_{xx}(1) + \varepsilon_{yy}(0) - \gamma_{xy}(1)$

$-300 = \varepsilon_{xx}(.25) + \varepsilon_{yy}(.75) - \gamma_{xy}(-.5)$

$1200 = \varepsilon_{xx}(.25) + \varepsilon_{yy}(.75) - \gamma_{xy}(-.5)$

(16)

$600 + \gamma_{xy} = \varepsilon_{yy}$

$-300 = .25\varepsilon_{xx} + .75(600 + \gamma_{xy}) + .5\gamma_{xy}$

$-300 = .25\varepsilon_{xx} + 450.5\gamma_{xy}$

$-300 - 450.5\gamma_{xy} = .25\varepsilon_{xx}$

$\varepsilon_{xx} = -1200 - 1802\gamma_{xy}$

$1200 = .25(-1200 - 1802\gamma_{xy}) + .75(600 + \gamma_{xy}) + .5\gamma_{xy}$

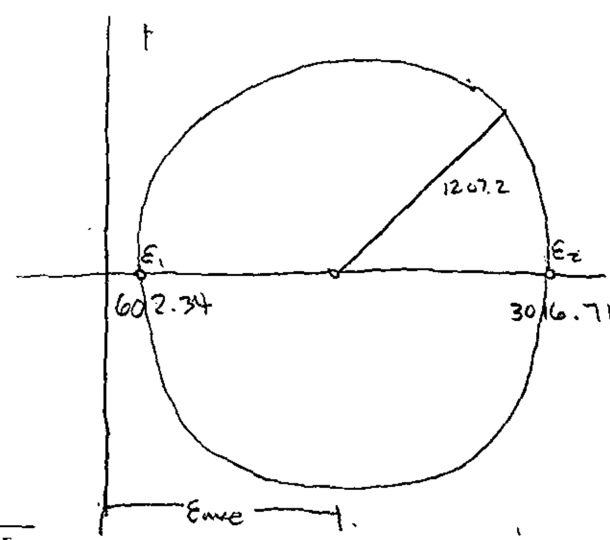
$1200 = -300 - 450.5\gamma_{xy} + 450 + .75\gamma_{xy} + .5\gamma_{xy}$

$1050 = -499.25\gamma_{xy}$

$\boxed{\gamma_{xy} = -2.34}$

$\boxed{\varepsilon_{yy} = 602.34}$

$\boxed{\varepsilon_{xx} = 3016.68}$



$R = \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\varepsilon_{xy}}{2}\right)^2}$

$= \left[\left(\frac{3016.68 - 602.34}{2}\right)^2 + \left(\frac{2.34}{2}\right)^2\right]^{1/2}$

$R = 1207.2$

center $= \varepsilon_{ave} = \frac{\varepsilon_{xx} + \varepsilon_{yy}}{2} = 1809.51$

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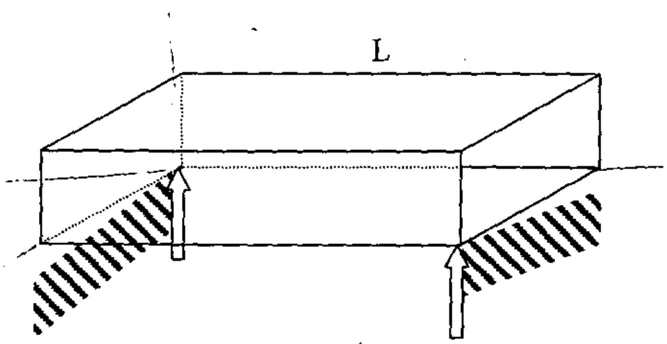
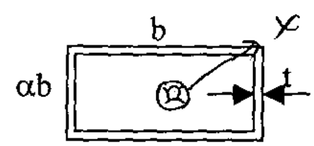
4. (25 points). A prefabricated box girder spans between two supports to form a bridge. Unfortunately, the supports were made with poor quality control, and one slopes one way and the other slopes in the opposite direction, with the result that, when the box girder is placed on them in a horizontal plane, it is supported on only two diagonally opposite corners, as shown. This arrangement induces torque in the member. The material has shear modulus G and weight density ρ. Dimensions are as shown. Dimensions b and αb are the average width and depth.

a) Find the torque along the member as function of x.
b) Find the angle of twist of one end of the girder relative to the other.

Assume that the angle of twist is small enough that the other two corners do not come into contact with the girder. The box girder may be treated as a thin-walled tube.





G : shear modulus
ρ = weight density
αb = ave depth
b = ave width

(a) $\boxed{\tau_{ave} = \dfrac{\text{Torque}}{2t\, A_m}}$

t = thickness
A_m = mean area enclosed

$A_m = (\alpha b)(b) = \alpha b^2$

$T(x) = T$

$x = \tau_{ave} = \dfrac{T(x)}{2t(\alpha b^2)}$ X

$T(x) = x[2t(\alpha b^2)]$

Torque will change along side $\tau_{ave} \Rightarrow x$ NO

(b) ϕ of twist $= \dfrac{TL}{4A_m^2 G} \cdot \oint \dfrac{ds}{t}$; the integral $\int \frac{ds}{t} \Rightarrow$ Length around center of the tube

X

$= \dfrac{T(x)L}{4\alpha b^2 G t} \cdot \oint ds \Rightarrow \dfrac{T(x)L}{4\alpha b^2 G t}(2b + 2\alpha b)$ thickness is constant

The angle of twist on either end of the girder will be the same in magnitude [assuming uniform density] but opposite in direction. Each end will rotate down toward the supports and thus the twist will be like a helix, with either end twisting the same amount only in opposite direction.

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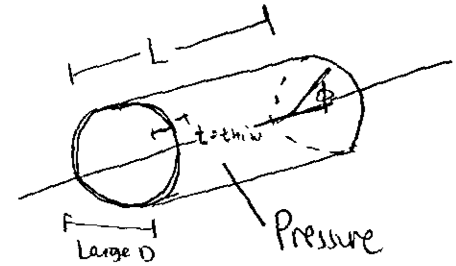


5. (30 points). NASA has asked your advice. They are trying to develop a super-light drive shaft for a motor to be used in space. The idea is to use a hollow circular tube with a relatively large diameter and a very thin wall. To prevent the tube wall from buckling, the tube is to be filled with gas under pressure. This will induce tension stresses. When torque is applied, it induces shear stresses that can be transformed into tension and compression stresses at an angle to the shaft's axis. These stresses are superimposed on the stresses caused by pressurization. The tube wall buckles when the normal stress in any direction drops to zero, and the material ruptures when the tension stress reaches a stress σ_0. NASA has asked you to find the optimum gas pressure that will allow the largest torque to be resisted, and the value of that torque in terms of σ_0, R and t (the tube average radius and wall thickness).

a) Draw the Mohr's circle for stress for the critical state, which is given by the material's reaching incipient buckling in one direction while it is on the point of rupturing in another. $\tau_{max} \longleftrightarrow \sigma_{max}$

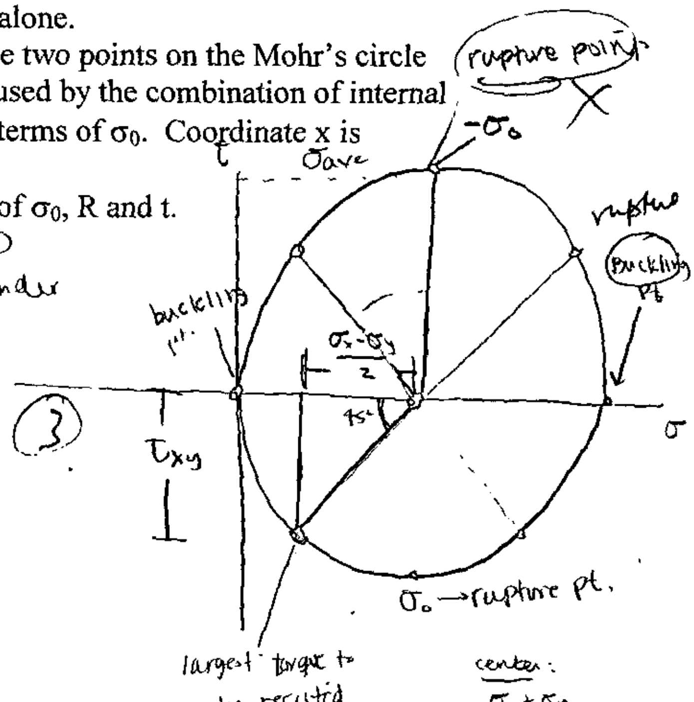
b) Find the stresses in the tube due to gas pressure, p, alone.

c) Using the results from part b or otherwise, locate the two points on the Mohr's circle that correspond to the total stresses in the xy plane (caused by the combination of internal pressure and torque). Compute the σ_{xx}, σ_{yy} and τ_{xy} in terms of σ_0. Coordinate x is longitudinal, y is circumferential.

d) Find the corresponding values of p and T, in terms of σ_0, R and t.

(6)

Max is at 45° on circle = 22.5° on cylinder
Why?



(3)

center: $\frac{\sigma_x+\sigma_y}{2}$

radius: $\sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2+(\tau_{xy})^2}$

(b) stress from gas pressure alone:

$\sigma_y = \sigma_{hoop} = \frac{pR}{t}$ $\quad \sigma_{diff} \to \tau_{x'y'} = \frac{\sigma_{hoop}-\sigma_{long}}{2}$

$\sigma_x = \sigma_{long} = \frac{pR}{2t}$ $\quad \sigma_{ave} = \frac{\sigma_{hoop}+\sigma_{long}}{2} \to \sigma_{x'}$

where p = pressure
R = radius
t = thickness

(6)

hoop → circumferential
long → longitudinal

(c) $\sigma_{xx} = \sigma_{long}$ ~~hoop~~ $= \frac{pR}{2t}$ When rotated

$\sigma_{yy} = \sigma_{hoop/long} + \left(\frac{F}{A}\right)$ → comes from axial tensile stresses

$\tau_{xy} = R\sin 45°$ - No. This is only if the torque is applied alone.

From torque T

(c) $\sigma_{xx} = \sigma_0 - R\cos 45° = \frac{pR}{2t}$

$\sigma_{yy} = \sigma_0 + R\cos 45° = \frac{pR}{t} + T$

(d) $\sigma_0 - R\cos 45 = \frac{pR}{2t}$

$2t(\sigma_0 - R\cos 45) = pR$ ← stress length. Must be wrong.

$\boxed{p = \frac{2t(\sigma_0 - R\cos 45°)}{R}}$

X.

$\sigma_0 + R\cos 45 = \frac{pR}{t} + T$

$\sigma_0 + R\cos 45 - \frac{\left(\frac{2t(\sigma_0 - R\cos 45°)}{R}\right)R}{t} = T$

$\boxed{\sigma_0 + R\cos 45 - \frac{2t(\sigma_0 - R\cos 45)}{t} = T}$