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AA-210 STATICS MIDTERM (Closed-Book)

Monday Nov 2, 2009 (Version B)

(1-doubled-sided page of notes and calculator are allowed)

Problem 1 (20 points)

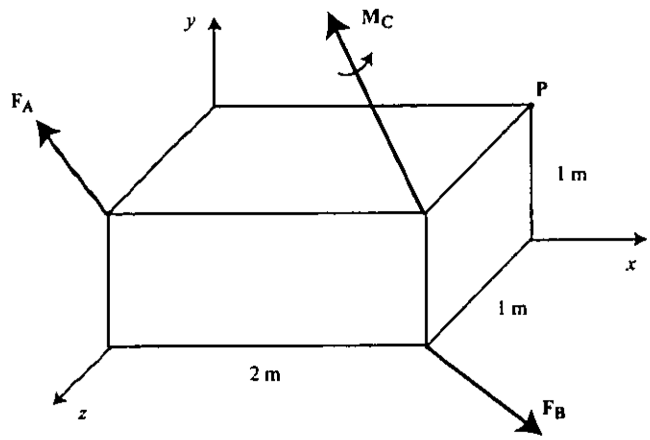

Figure 1: Forces and Couple acting on a Rectangular Block.

Two forces and a couple act on the rectangular block (Figure 1) where

$$\vec{F}_A = 30\,\vec{i} + 40\,\vec{j} - 10\,\vec{k}\ \text{(N)}$$

$$\vec{F}_B = 50\,\vec{i} - 10\,\vec{j} + 40\,\vec{k}\ \text{(N)}$$

and

$$\vec{M}_C = 40\,\vec{i} + 20\,\vec{j} + 70\,\vec{k}\ \text{(N-m)}$$

If you represent them by a force $\vec{F}$ and a couple $\vec{M}$ acting on the point P, what are $\vec{F}$ and $\vec{M}$?

$\vec{F} = \vec{F}_A + \vec{F}_B = 80i + 30j + 30k$ (N) ✓

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sign errors -1

$M_{F_A \text{ on } P} = (2i + 1z) \times (30i + 40j - 10k)$

$$\begin{vmatrix} 2 & 0 & 1 \\ 30 & 40 & -10 \end{vmatrix} = -40i - (\overset{-50}{-20 - 30})j + 80k$$

$-40i + 50j + 80k$ N-m

$M_{F_B \text{ on } P} = (0i + -j + k) \times (50i - 10j + 40k)$ ✓

$$\begin{vmatrix} 0 & -1 & 1 \\ 50 & -10 & 40 \end{vmatrix} = (-40+10)i - (-50)j + 50k$$

$-30i + 50j + 50k$ N-m ✓

$$\sum M_P = M_c + M_{F_A \text{ on } P} + M_{F_B \text{ on } P}$$

$$= (40i + 20j + 70k) + (-40i + 50j + 80k) + (-30i + 50j + 50k)$$

$$M_P = -30i + 120j + 200k \quad N\text{-}m$$

Problem 2 (35 points)

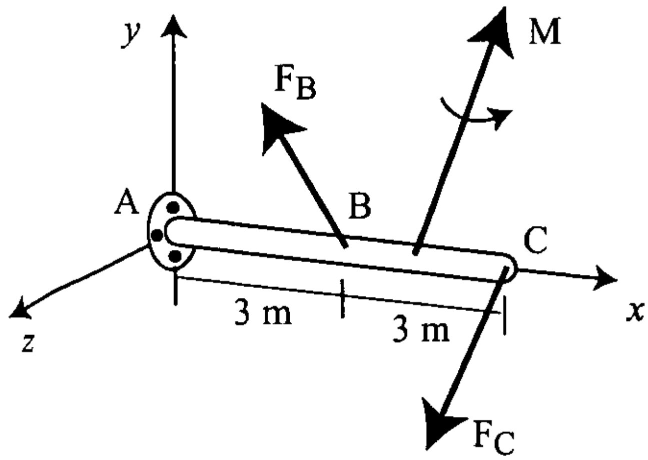


Figure 2: A Bar with Built-In Fix Support under Loading.

The bar $\boldsymbol{AB}$ has a built-in fix support at A and is loaded by the forces

$$\vec{F}_B = 3\,\vec{i} + 5\,\vec{j} - 3\,\vec{k}\ \text{(kN)}$$

$$\vec{F}_C = 2\,\vec{i} - 3\,\vec{j} + 5\,\vec{k}\ \text{(kN)}$$

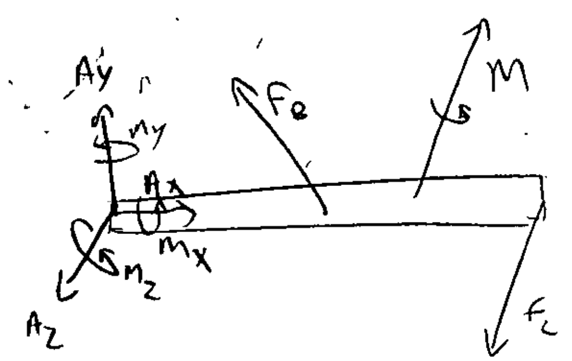
and a couple

$$\vec{M} = 14\,\vec{i} + 6\,\vec{j} + 5\,\vec{k}\ \text{(kN-m)}$$

(a) (5 points) Draw the free-body-diagram of the bar.

(b) (30 points) Determine the reactions at the support $\boldsymbol{A}$.

FBD:



5

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$f_B = 3i + 5j - 3k$

$f_c = 2i - 3j + 5k$

$\vec{F}_B + \vec{f}_c + \vec{A} = 0$

$f_B + f_c = 5i + 2j + 2k$

$\vec{A}_x = -5 \text{ kN}$ $\qquad\qquad$ $\vec{A} = -5i - 2j - 2k \text{ (kN)}$

$A_y = -2 \text{ kN}$

$A_z = -2 \text{ kN}$

$M_A + M_i + M_{f_B \to A} + M_{f_c \to A} = 0$

$(14i + 6j + 5k) + r_1 \times f_B + r_2 \times f_2$

$+ \begin{vmatrix} 3 & 0 & 0 \\ 3 & 5 & -3 \end{vmatrix} + \begin{vmatrix} 6 & 0 & 0 \\ 2 & -3 & 5 \end{vmatrix}$

$\cancel{14}i + \cancel{6j} + 5k \quad (\cancel{0}i - (\cancel{-9})j + 15k) + (\cancel{0}i - \cancel{30}j + -18k)$

$-M_A = 14i - 15j + 2k \text{ N·m}$

$M_A = -14i + 15j - 2k \text{ (kN-m)}$

$M_{A_x} = -14 \text{ (kN-m)}$

$M_{A_y} = 15 \text{ (kN-m)}$

$M_{A_z} = -2 \text{ (kN-m)}$

Problem 3 (20 points)

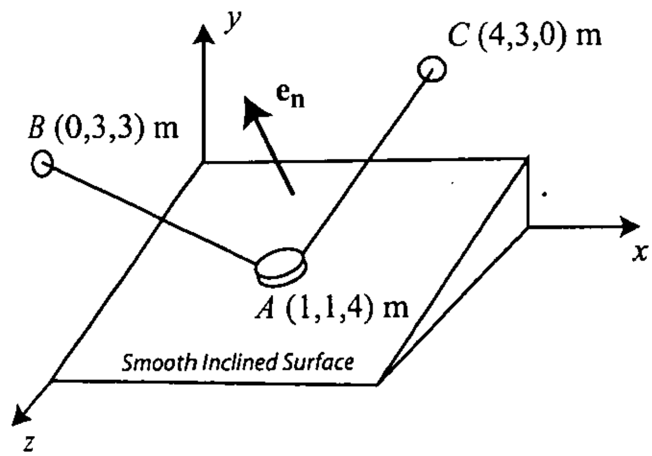
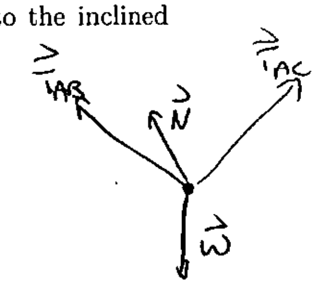


Figure 3: Metal Disk on a Smooth Inclined Surface.

The **10**-N metal disk $\boldsymbol{A}$ is supported by the smooth inclined surface and the strings $\boldsymbol{AB}$ and $\boldsymbol{AC}$ (Figure 3) where $e_n = \mathbf{0.31623}\ \vec{j} + \mathbf{0.94868}\ \vec{k}$ is the *unit vector* normal to the inclined surface. Find the tensions in the string.



$$\vec{W} + \vec{N} + \vec{T}_{AB} + \vec{T}_{AC} = 0$$

$$e_{AB} = \frac{r_{AB}}{|r_{AB}|} = \frac{-1i + 2j - 1k}{\sqrt{1^2+2^2+1^2}} = -.408i + .816j - .408k$$

$$e_{AC} = \frac{r_{AC}}{|r_{AC}|} = \frac{3i + 2j - 4k}{\sqrt{3^2+2^2+4^2}} = .557i + .371j - .743k$$

$\sqrt{29}$

$\vec{W} = -10j$ (N) ; $\vec{N} = |W| e_N = 3.1623j + 9.4868k$

eq are correct

$\Sigma F_x = -.408(T_{AB}) + .557(T_{AC}) = 0$ $(T_{AC} = .732 T_{AB})$

$\Sigma F_y = .816(T_{AB}) + .371(T_{AC}) + 3.1623 - 10 = 0$

$\Sigma F_z = -.408(T_{AB}) - .743(T_{AC}) + 9.4868 = 0$

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$T_{AC} = 8.86$ N

$T_{AB} = 12.1$ N

How? show