Name:____________________________________

Part I – Short Answer (3 pts each)

1. List the three fundamental relations used in formulating and solving problems in solid mechanics:

i. Equilibrium

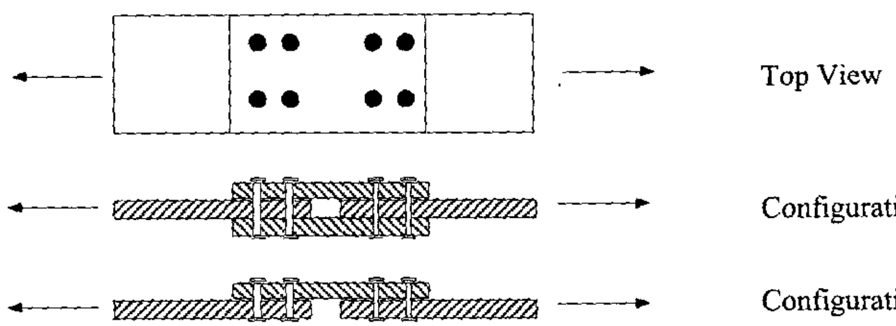
ii. Kinematics

iii. Constitutive relations

2. If any normal stress component (σ_x, σ_y, σ_z) in a given stress state is negative, then at least one of the principal stresses must be compressive: true or false?

True: easily shown by Mohr's circle (minimum principal stress always less than all other normal stresses)

3. What would you expect the ratio of the strengths of the two lap joint configurations shown below to be assuming that the same materials and fasteners are used throughout?



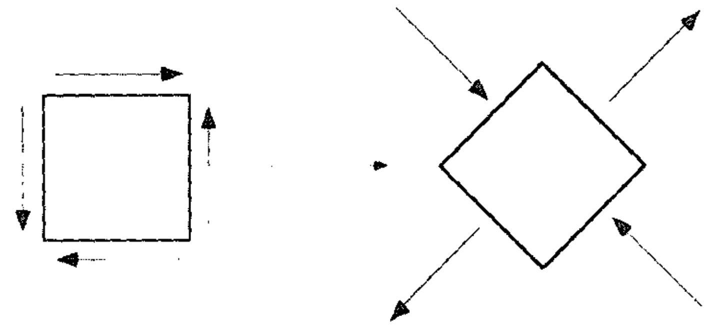
Configuration 1 should be at least 2x stronger than config 1, since the bolts are all in double shear for config 1.

4. A 1D bar with length L is deformed by displacing its left end by an amount u_1, and its right end by an amount u_2. What is the strain in the bar?

The length change comes from the *difference* of the displacements: $\varepsilon = \dfrac{u_2 - u_1}{L}$

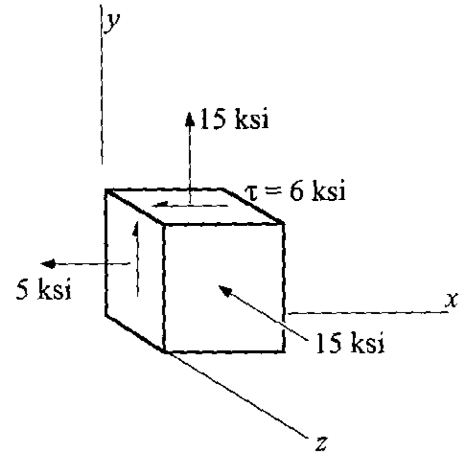
5. Explain why cracks caused by earthquakes in masonry or concrete walls tend to form according to a 45° X-shape pattern.

The horizontal shaking in an earthquake induces shear stress in the walls. The shear stresses are equivalent to tension (and compression) at 45°, which causes 45° cracks if there is insufficient steel reinforcement to carry the tension. Since an earthquake causes cyclic loading, the cracks form on both diagonals, thus forming an "X".



Part II – Problems

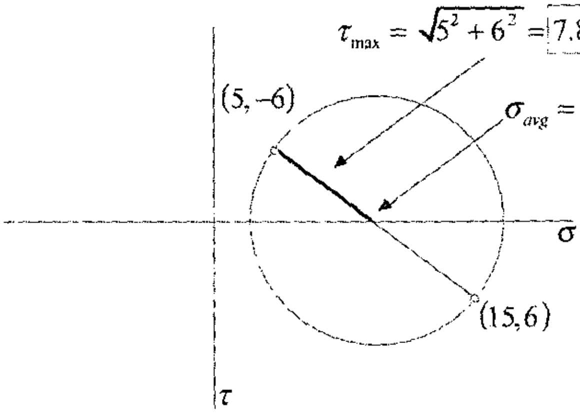
1. A stress state is given as shown below.



a. Determine the principal stresses and the maximum in-plane shear stress for the x-y plane. (9 pts)

$$\tau_{max} = \sqrt{5^2 + 6^2} = \boxed{7.81\ \text{ksi}}$$

$$\sigma_{avg} = 10\ \text{ksi}$$

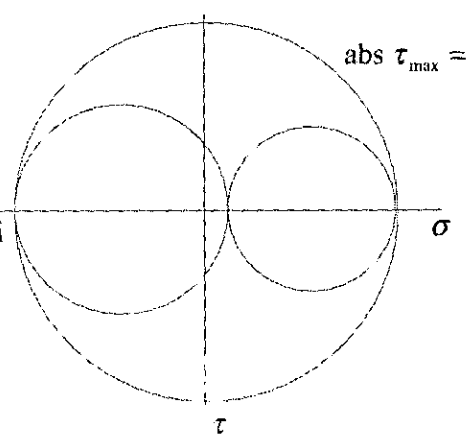


$$\sigma_1 = \sigma_{avg} + \tau_{max} = 17.81\ \text{ksi}$$
$$\sigma_2 = \sigma_{avg} - \tau_{max} = 2.2\ \text{ksi}$$

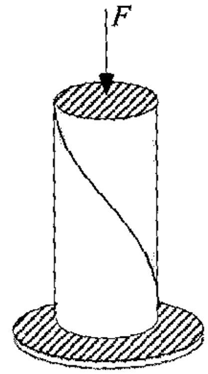
b. Determine the absolute maximum shear stress, and show a sketch of the 3D Mohr's circle plot. (6 pts)

$$\text{abs}\ \tau_{max} = \frac{17.81 + 15}{2} = \boxed{16.4\ \text{ksi}}$$

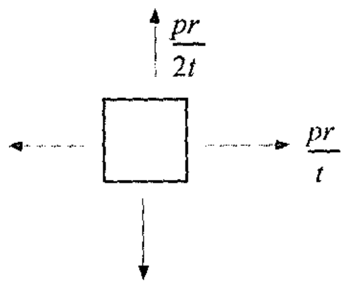
$$\sigma_z = \sigma_3 = 15\ \text{ksi}$$



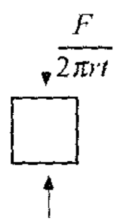
2. A pressure vessel with a 45° spiral seam has an internal pressure, p, and is also loaded axially by a force, F, as shown. The vessel has a wall thickness t and a radius r.



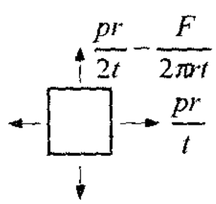
a. Determine the stress due to the internal pressure *alone* and show an appropriately labeled stress block.



b. Determine the stress due to the axial force *alone*, and again show an appropriately labeled stress block. (3 pts)



c. Show a stress block with the combined effect of both internal pressure and the axial force. (3 pts)

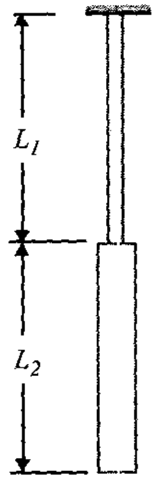


d. Determine the magnitude of the force, F, necessary such that the normal stress across the seam is zero. (6 pts)

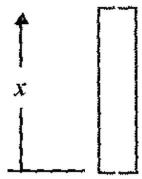
$$\sigma_{x'} = 0 = \left(\frac{pr}{t}\right)\cos^2\theta + \left(\frac{pr}{2t} - \frac{F}{2\pi rt}\right)\sin^2\theta \Rightarrow F = 2\pi rt\left[\left(\frac{pr}{t}\right)\cot^2\theta + \frac{pr}{2t}\right]$$

$$\theta = 45° \Rightarrow F = 3\pi pr^2$$

3. Find the total elongation of the rod system shown hanging under its own weight with cross-sectional areas, A_1 and A_2, weight density, γ, and elastic modulus, E. (10 pts)

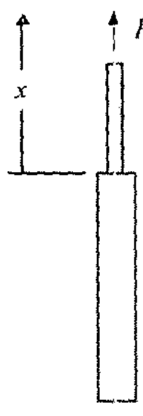


$P(x) = \gamma A x \ ; \ x < L_2$



$$\delta_2 = \int_0^{L_2} \frac{P(x)dx}{AE} = \int_0^{L_2} \frac{\gamma x}{E} dx = \underline{\frac{1}{2}\frac{\gamma}{E} L_2^2}$$

$P(x) = \gamma A_1 x + \gamma A_2 L_2 \ ; \ x < L_1$



$$\delta_1 = \int_0^{L_1} \frac{P(x)dx}{A_1 E} = \int_0^{L_1} \left[\frac{\gamma x}{E} + \frac{\gamma A_2 L_2}{A_1 E} \right] dx = \underline{\frac{1}{2}\frac{\gamma}{E} L_1^2 + \frac{\gamma A_2 L_2 L_1}{A_1 E}}$$

$$\boxed{\delta = \delta_1 + \delta_2 = \frac{\gamma}{E}\left[\frac{L_1^2 + L_2^2}{2} + \frac{A_2}{A_1} L_1 L_2 \right]}$$