

AA 210 Statics

Final Exam – Winter 2009

¹⁰⁰
(~~60~~ min, Open Book & Open Notes; show all work and FBD's)

Version A

1. Every horizontal member in a truss, as shown below, is each 1 m in length. F is 150 kN and h is 1 m.

- (a) Determine the axial forces in members BC, CF, and FG and indicate whether they are in tension (T) or compression (C). (12 pts)
- (b) Is there any zero-force member in the truss? If there is, indicate what member is. (3 pts)

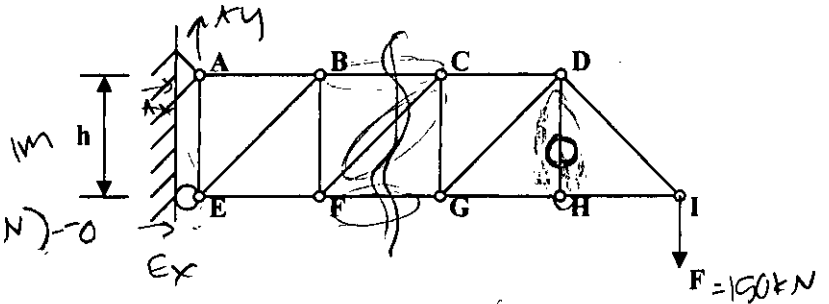
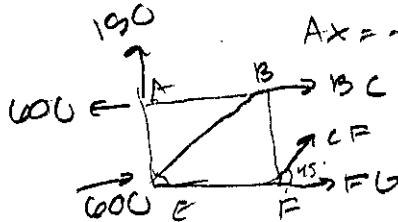
$$\sum F_x = A_x + E_y = 0$$

$$\sum F_y = A_y - 150 = 0 \quad \boxed{A_y = 150}$$

$$\sum M_A = (1\text{m})(E_y) - (4\text{m})(150\text{kN}) = 0$$

$$E_y = 600$$

$$A_x = -600$$



$$\sum F_x = -600 + 600 + BC + F_B + CF \cos 45^\circ = 0$$

$$\sum F_y = 150 + CF \sin 45^\circ = 0$$

$$CF = \frac{-150}{\sin 45^\circ} = -212.13 \text{ (C)}$$

$$\sum M_F = -(1\text{m})(BC) - (1\text{m})(150) + (1\text{m})(600) = 0$$

$$BC = 450 \text{ (T)}$$

$$-600 + 600 + 450 + F_B + (-212.13)(\cos 45^\circ) = 0$$

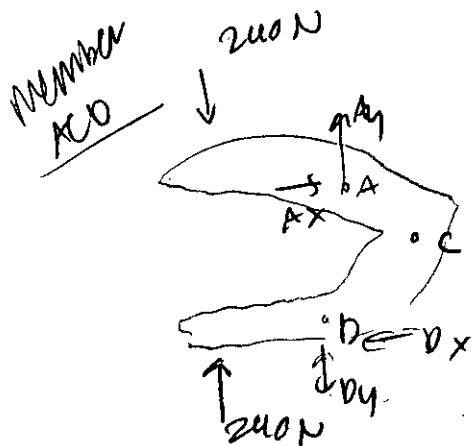
$$F_B = -300 \text{ (C)}$$

a.) $\boxed{\begin{aligned} CF &= -212.13 \text{ kN (C)} \\ BC &= 450 \text{ kN (T)} \\ F_B &= -300 \text{ kN (C)} \end{aligned}}$

b.) zero force members = \boxed{DH}

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2. Assume that pins A and D slide freely (no friction) in slots cut in the jaws. Determine the magnitude of the gripping force exerted along line a-a on the nut when two 240-N forces are applied to the handles as shown. (15 pts)

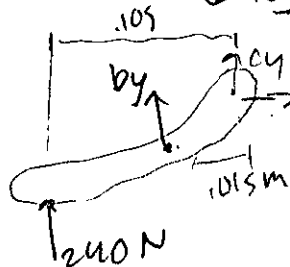


$$\sum F_x = Ax - Dx = 0 \quad Ax = Dx$$

$$\sum F_y = -240 + 240 + Ay - Dy = 0$$

$$Ay = Dy$$

$$\sum M_D = -(240)(90 \text{ mm}) + (240)(10 \text{ mm}) - d(Ax) = 0$$



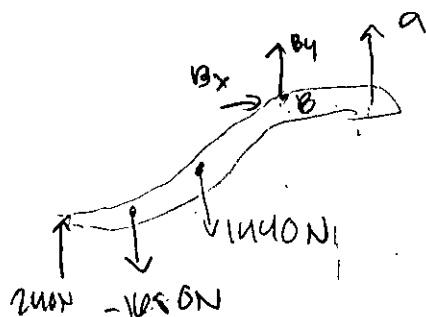
$$\sum F_x = Cx = 0 \text{ N}$$

$$\sum F_y = Cy + By + 240 = 0$$

$$\sum M_C = -(1015 \text{ mm})(By) - (240 \text{ N})(105) = 0$$

$$By = -1680 \text{ N}$$

$$Cy = 1440 \text{ N}$$



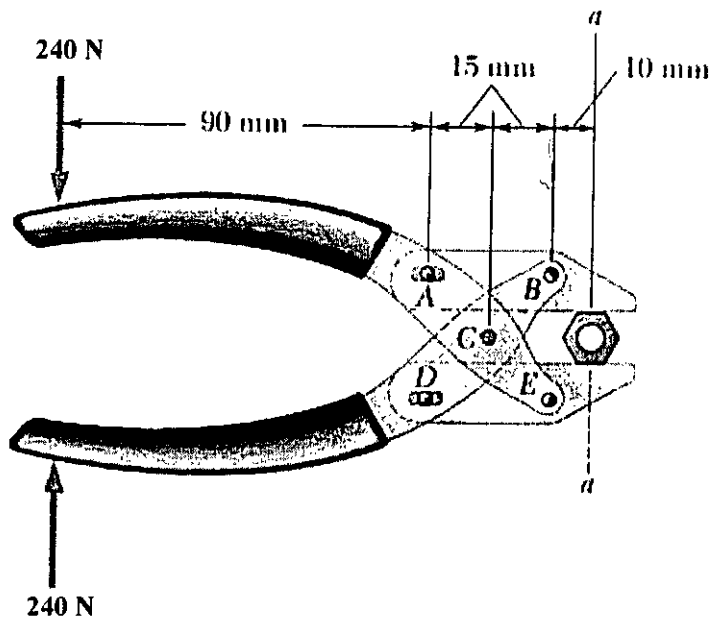
$$\sum F_x = Bx = 0$$

$$\sum F_y = 240 - (-1680) - (1440) + By + a = 480 + By + a$$

$$\sum M_B = (0.11 \text{ m})a + (0.015 \text{ m})(1440) + (0.03)(-1680) - (0.12)(240) = 0$$

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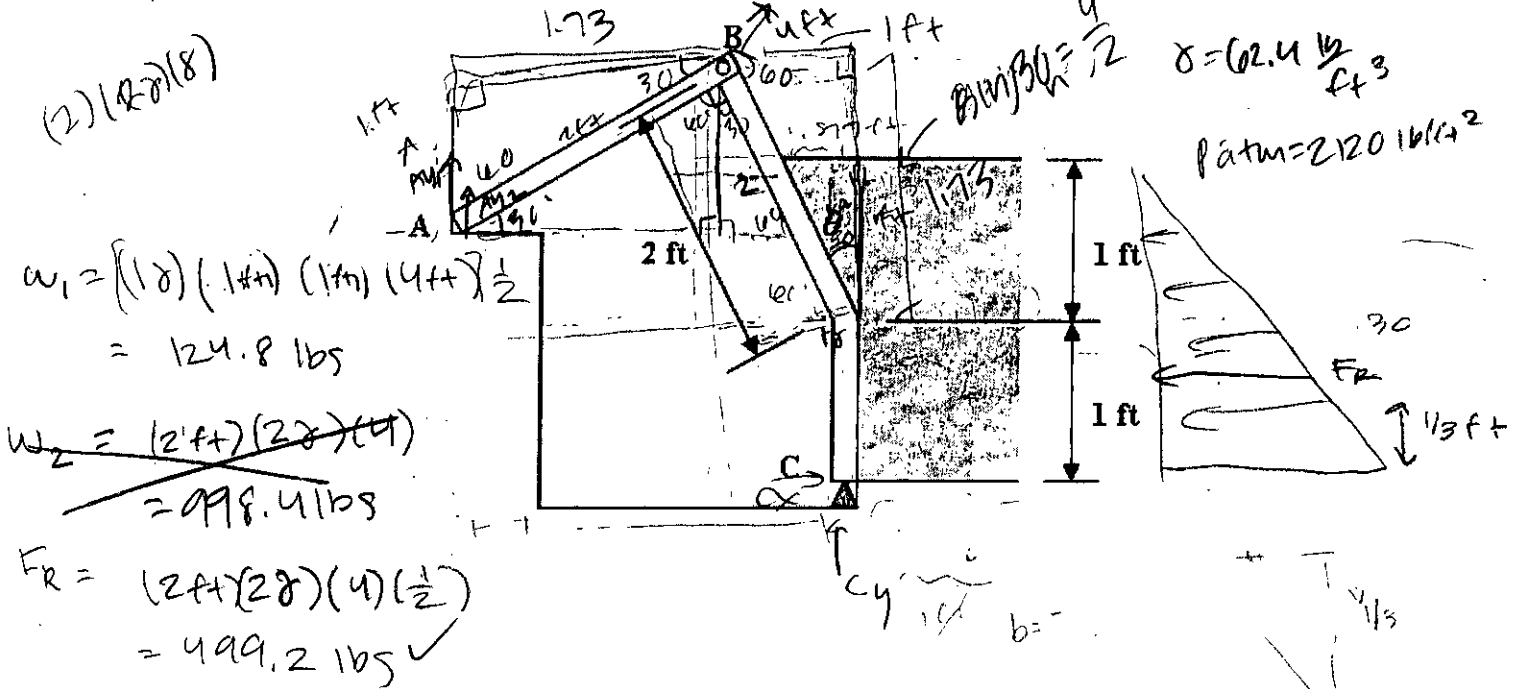
$$a = 5760 \text{ N} \quad \times \quad 3$$



$$\frac{x}{2} = \sin 30^\circ$$

3. The water gate BC is jointed with a member AB which has one end, A, supported by the ground and is perpendicular to water gate at B. The width of the gate and member AB (the dimension into the paper) is 4 ft. The weight density of the water is $\gamma = 62.4 \text{ lb/ft}^3$, and the atmospheric pressure $p_{atm} = 2120 \text{ lb/ft}^2$. Neglect the weights of the gate and member, and the friction between the member and ground. Assume $\theta = 30^\circ$, what are the reactions at A? (15 pts)

$$\bar{x} w_1 = \frac{1}{3} (57) = 19.23$$



$$(2)(28)(8)$$

$$w_1 = (18)(1.73)(1.73)(4) \frac{1}{2} = 124.8 \text{ lbs}$$

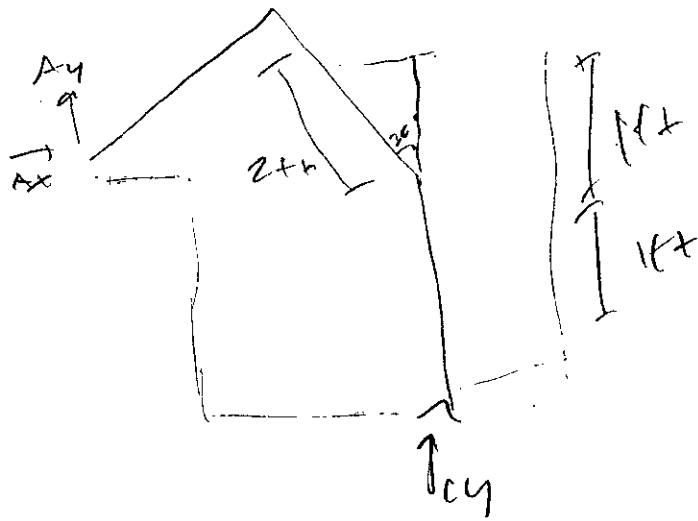
$$w_2 = (2.73)(28)(4) = 998.4 \text{ lbs}$$

$$F_R = (2.73)(28)(4)(\frac{1}{2}) = 499.2 \text{ lbs}$$

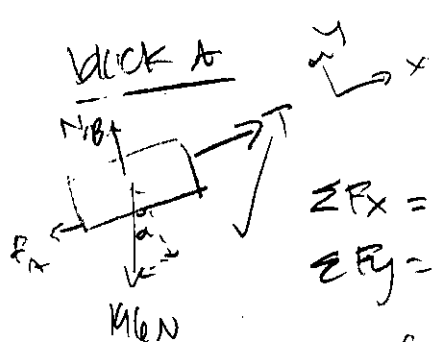
$$\sum F_x = -w_1(1.73) + C_x + AB \cos 30^\circ = 0$$

$$\sum F_y = -124.8 + AB \sin 30^\circ + C_y = 0$$

$$\begin{aligned} \sum M_C &= -(3.55)(124.8) - (2.73)(AB \sin 30^\circ) - (2.73)(AB \cos 30^\circ) = 0 \\ &= (48.048) - AB(2.73 \sin 30^\circ + 2.73 \cos 30^\circ) = 0 \\ AB &= -48.1 \text{ lbs} \end{aligned}$$



4. The masses of crates A and B are 20 kg and 25 kg, respectively. The coefficient of static friction (μ_s) between all contacting surfaces is 0.34. What is the largest value of α for which the crates will remain in equilibrium? (20 pts)

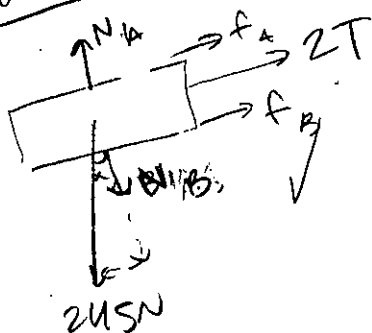


$$\sum F_x = f_A + T - 196 \sin \alpha = 0 \quad (1)$$

$$\sum F_y = N_B - 196 \cos \alpha = 0$$

$$f_A = (N_B)(0.34) = (196 \cos \alpha)(0.34) = 66.64 \cos \alpha$$

Block B



$$\sum F_x = f_A + f_B + 2T - 245 \sin \alpha = 0 \quad (2)$$

$$\sum F_y = N_A - N_B - 245 \cos \alpha = 0$$

$$= (196 \cos \alpha)(0.34) - N_B - 245 \cos \alpha = 0$$

$$f_B = (N_B)(0.34)$$

$$= (66.64)(\cos \alpha) - 245 \cos \alpha = f_B$$

$$(2) \sum F_y = \underbrace{66.64 \cos \alpha}_{f_A} + \underbrace{66.64 \cos \alpha}_{f_B} - 245 \cos \alpha + 2T - 245 \sin \alpha = 0$$

$$\frac{2T}{2} = \frac{-66.64 \cos \alpha - 66.64 \cos \alpha + 245 \cos \alpha + 245 \sin \alpha}{2}$$

$$T = -33.32 \cos \alpha - 33.32 \cos \alpha + 122.5 \cos \alpha + 122.5 \sin \alpha$$

(1)

$$-66.64 \cos \alpha + \underbrace{33.32 \cos \alpha - 33.32 \cos \alpha + 122.5 \cos \alpha + 122.5 \sin \alpha}_T$$

$$-196 \sin \alpha = 0$$

$$-10.78 \cos \alpha + 122.5 \sin \alpha - 196 \sin \alpha = 0$$

$$-10.78 \cos \alpha = -122.5 \sin \alpha + 196 \sin \alpha = 0$$

$$-10.78 \cos \alpha = 73.5 \sin \alpha$$

$$\frac{-10.78}{73.5} = \tan \alpha$$

$$\alpha = -8.34^\circ$$

(next pg)

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Block A

$$\sum F_x = -0.34 N_B + T - 196 \sin \alpha = 0 \quad \checkmark$$

$$\sum F_y = N_B - 196 \cos \alpha = 0 \quad \checkmark$$

$$N_B = 196 \cos \alpha \quad f_A = 66.64 \cos \alpha$$

$$196 \cos \alpha - 66.64 \cos \alpha + T - 196 \sin \alpha = 0$$

Block B

$$\sum F_x = 66.64 \cos \alpha + f_B + 27 - 245 \sin \alpha = 0 \quad \checkmark$$

$$\sum F_y: N_A - 196 \cos \alpha - 245 \cos \alpha = 0 \quad /$$

$$(N_A)(9.81) \quad N_A = 196 \cos \alpha + 245 \cos \alpha$$

$$f_B = 66.64 \cos \alpha + 83.3 \cos \alpha = 0$$

$$66.64 \cos \alpha + 66.64 \cos \alpha + 83.3 \cos \alpha + 27 - 245 \sin \alpha = 0$$

$$T = -33.32 \cos \alpha - 33.32 \cos \alpha - 41.65 \cos \alpha + 122.5 \sin \alpha = 0$$

$$- 210 \cos \alpha + 122.5 \sin \alpha$$

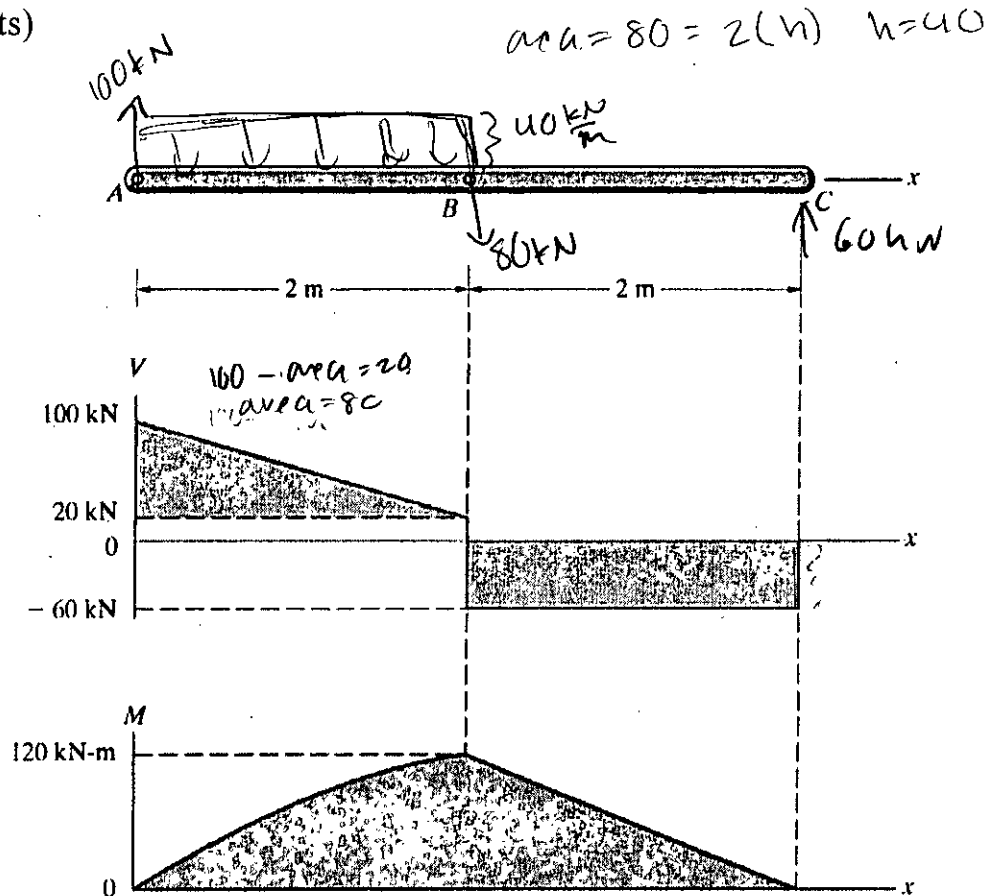
$$-66.64 \cos \alpha - 33.32 \cos \alpha - 33.32 \cos \alpha - 41.65 \cos \alpha + 122.5 \sin \alpha$$

$$-196 \sin \alpha = 0$$

$$-108.29 \cos \alpha = (-122.5 + 196) \sin \alpha$$

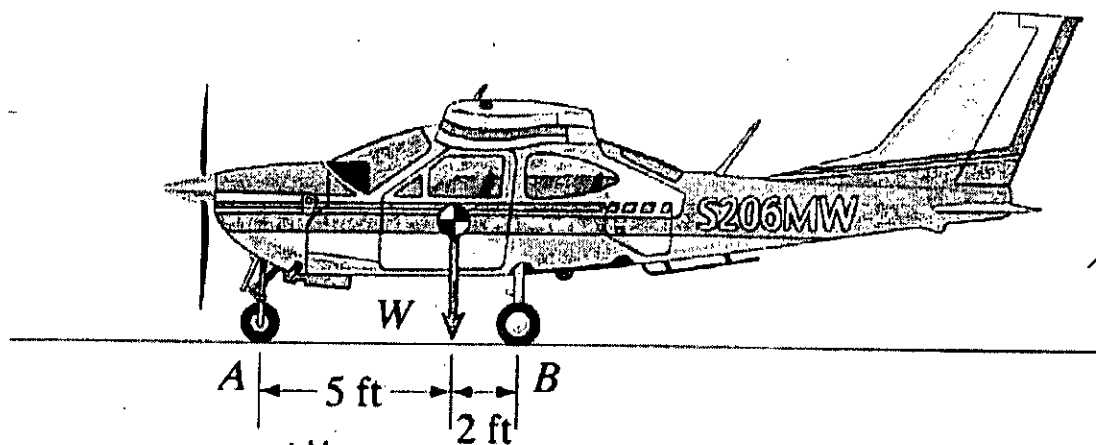
$$\alpha = -55.83^\circ \quad \times_2$$

5. A beam's shear force and bending moment diagrams are shown as below. $V_A = 100 \text{ kN}$, $V_{B-} = 20 \text{ kN}$, $V_{B+} = -60 \text{ kN}$, and the maximum bending moment M_{\max} is 120 kN-m . Determine values of the applied loads (point forces, point couples, or distributed loads) acting on the beam which result in these diagrams and draw them on beam ABC. (20 pts)



~~15~~ $\frac{15}{20}$

6. The empty airplane shown below has weight 2200 lb and center of gravity location 5 ft right to the point A.
- (a) For maintenance purpose, the engine has to be removed from the airplane. If the engine weighs 500 lb and has a center of gravity 0.25 ft right of point A, where will the airplane's center of gravity be in relation to A once the engine has been moved. (8 pts)
- (b) Now the airplane (complete with engine) is ready for flight. 240 lb of fuel are loaded. The fuel alone has a center of gravity 6 ft to the right of A. Also, two passengers board. They have a combined weight of 340 lb and a center of gravity 4.5 ft right of point A. Where is the center of gravity of fully loaded airplane located in relation A? (7 pts)



a.)

airplane w/ engine	W	\bar{x}
engine	2200	5 ft
	-500	0.25 ft

$$\bar{x} = \frac{(2200)(5) - (500)(0.25)}{2200 - 500}$$

$$\bar{x} = 6.39 \text{ ft (from A)}$$

b.)

airplane w/ engine	W	\bar{x}
fuel	2200 lbs	5 ft
passengers	240 lbs	6 ft
	340 lbs	4.5 ft

$$\bar{x} = \frac{(2200)(5) + (240)(6) + (340)(4.5)}{2200 + 240 + 340}$$

$$= 5.03 \text{ ft (from A)}$$

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