Topic : ..... Date. : ..... Page No. : .....

← ULTIMATE MATHEMATICS →

BY: AJAY MITTAL: (9891067390)

← TRIGONOMETRY (CLASS NO: 8) →

Trigonometric equations → An equation which contains Trigonometric quantities.

Solution (value of x)

	I	II	III	IV
(1) Principal solution	θ	$\pi-\theta$	$\pi+\theta$	$2\pi-\theta$
(2) General solution	A	S	T	C

Q.1 Find the principal solution of the equation

$$\sin x = -1/2$$

III $\sin x = \sin(\pi + \frac{\pi}{6})$

$\sin x = \sin(\frac{7\pi}{6})$

$x = \frac{7\pi}{6}$

IV $\sin x = \sin(2\pi - \frac{\pi}{6})$

$\sin x = \sin(\frac{11\pi}{6})$

and $x = \frac{11\pi}{6}$ Ans

Q.2 Find the principal solution of the equation

$$\cos(3x) = -\frac{1}{\sqrt{2}}$$

Sol

II $\cos(3x) = \cos(\pi - \frac{\pi}{4})$

$\cos(3x) = \cos(\frac{3\pi}{4})$

$3x = \frac{3\pi}{4}$

$x = \frac{\pi}{4}$

IV $\cos(3x) = \cos(\pi + \frac{\pi}{4})$

$\cos(3x) = \cos(\frac{5\pi}{4})$

$3x = \frac{5\pi}{4}$

Ans $x = \frac{5\pi}{12}$ Ans

Q.3 Find the principal solution $2\cot(2x) = -2$

Sol $\cot(2x) = -1$

$\tan(2x) = -1$

II $\tan(2x) = \tan(\pi - \frac{\pi}{4})$

$2x = \frac{3\pi}{4}$

$x = \frac{3\pi}{8}$

IV $\tan(2x) = \tan(2\pi - \frac{\pi}{4})$

$2x = \frac{7\pi}{4}$

$x = \frac{7\pi}{8}$ Ans

Topic : ..... Date. : ..... Page No. : .....

Trigo (class - 8)

General solution:

Type 1

(1) $\sin\theta = 0$ $\theta = 0^\circ$, $\sin(\pi) = 0$ (180°) $\sin(2\pi) = 0$ $\sin(3\pi) = 0$

$\sin(-\pi) = 0$ $\sin(-2\pi) = 0$

then $\boxed{\theta = n\pi}$; $n \in Z$

(2) $\cos\theta = 0$ $\theta = \frac{\pi}{2}$ $\cos\left(\frac{3\pi}{2}\right) = 0$ $\cos\left(\frac{5\pi}{2}\right) = 0$ $\cos\left(\frac{7\pi}{2}\right) = 0$

$\cos\left(-\frac{3\pi}{2}\right) = 0$ $\cos\left(-\frac{5\pi}{2}\right) = 0$ - - -

$\boxed{\theta = (2n+1)\frac{\pi}{2}}$ $n \in Z$

(3) $\tan\theta = 0$

$\boxed{\theta = n\pi}$ $n \in Z$

Type 2

(1) $\sin\theta = \sin\alpha \Rightarrow \theta = n\pi + (-1)^n \alpha$

(2) $\cos\theta = \cos\alpha \Rightarrow \theta = 2n\pi \pm \alpha$

(3) $\tan\theta = \tan\alpha \Rightarrow \theta = n\pi + \alpha$

} $n \in Z$

Proof

$\cos\theta = \cos\alpha$

$\Rightarrow \cos\theta - \cos\alpha = 0$

$\Rightarrow -2\sin\left(\frac{\theta+\alpha}{2}\right) \cdot \sin\left(\frac{\theta-\alpha}{2}\right) = 0$

$\Rightarrow \sin\left(\frac{\theta+\alpha}{2}\right) \cdot \sin\left(\frac{\theta-\alpha}{2}\right) = 0$

$\Rightarrow \sin\left(\frac{\theta+\alpha}{2}\right) = 0 \quad | \quad \sin\left(\frac{\theta-\alpha}{2}\right) = 0$

$\Rightarrow \frac{\theta+\alpha}{2} = n\pi \quad | \quad \frac{\theta-\alpha}{2} = n\pi$

$\Rightarrow \theta = 2n\pi - \alpha \quad | \quad \theta = 2n\pi + \alpha$

$\theta = 2n\pi \pm \alpha \quad n \in Z$

CLASSTIME

Topic : ..... Date. : ..... Page No. : .....

Qn 1 Soln $\sin x = -\frac{1}{2}$

(I) $\sin x = \sin\left(\pi + \frac{\pi}{6}\right)$

$\sin x = \sin\left(\frac{7\pi}{6}\right)$

Compare with $\sin\theta = \sin\alpha$

here $\theta = x$; $\alpha = \frac{7\pi}{6}$

Soln $\theta = n\pi + (-1)^n \alpha$

$\boxed{x = n\pi + (-1)^n \frac{7\pi}{6}}$ $n \in Z$

OR

$\sin x = \sin\left(-\frac{\pi}{6}\right)$

here $\phi = x$

$\alpha = -\frac{\pi}{6}$

$\theta = n\pi + (-1)^n \alpha$

$x = n\pi + (-1)^n \left(-\frac{\pi}{6}\right)$

$x = n\pi + (-1)^{n+1} \cdot \frac{\pi}{6}$

$n \in Z$

Q 12 Soln $\cos(3x) = -\frac{1}{2}$

(II) $\cos(3x) = \cos\left(\pi - \frac{\pi}{3}\right)$

$\cos(3x) = \cos(2\pi/3)$

Comp with $\cos\theta = \cos\alpha$

$\theta = 3x$; $\alpha = 2\pi/3$

Soln $\theta = 2n\pi \pm \alpha$

$3x = 2n\pi \pm \frac{2\pi}{3}$

$x = \frac{2n\pi}{3} \pm \frac{2\pi}{9}$ $n \in Z$

Q 13 Soln $\sin(6x) - \sin(4x) + \sin(2x) = 0$

Soln $(\sin(6x) + \sin(2x)) - \sin(4x) = 0$

$\Rightarrow 2\sin(4x)\cos(3x) - \sin(4x) = 0$

$\Rightarrow \sin(4x)(2\cos(3x) - 1) = 0$

$\Rightarrow \sin(4x) = 0$

$4x = n\pi$

$x = \frac{n\pi}{4}$ $n \in Z$

$2\cos(3x) - 1 = 0$

$\cos(3x) = 1/2$

$\cos(3x) = \cos\left(\frac{\pi}{3}\right)$

CLASSTIME

Trigo (class-8)

Sol^{n} Here $\theta = 3x$; $\alpha = \pi/3$

$\theta = 2n\pi \pm \alpha$

$3x = 2n\pi \pm \pi/3$

$x = \frac{2n\pi}{3} \pm \frac{\pi}{9}$ $n \in Z$

Q.No 4 Solve $2\cos^2 x + 3\sin x = 0$

Sol^{n} $2(1-\sin^2 x) + 3\sin x = 0$

$\Rightarrow 2 - 2\sin^2 x + 3\sin x = 0$

$\Rightarrow 2\sin^2 x - 3\sin x - 2 = 0$

$\Rightarrow 2\sin^2 x - 4\sin x + \sin x - 2 = 0$

$\Rightarrow 2\sin x(\sin x - 2) + 1(\sin x - 2) = 0$

$\Rightarrow (\sin x - 2)(2\sin x + 1) = 0$

$\sin x = 2$	$2\sin x + 1 = 0$
(X) not possible	$\sin x = -\frac{1}{2}$
$-1 \le \sin\theta \le 1$	$\sin x = \sin(\pi + \frac{\pi}{6})$
	here $\theta = x$; $\alpha = \frac{7\pi}{6}$
	$\theta = n\pi + (-1)^n \alpha$
	$x = n\pi + (-1)^n (\frac{7\pi}{6})$ $n \in Z$ Ans

Q.No. 5 → Solve $\sin x - 3\sin(2x) + \sin(3x) = \cos x - 3\cos(2x) + \cos(3x)$

Sol^{n} $(\sin(3x) + \sin x) - 3\sin(2x) = (\cos(3x) + \cos x) - 3\cos(2x)$

$\Rightarrow 2\sin(2x).\cos(x) - 3\sin(2x) = 2\cos(2x).\cos(x) - 3\cos(2x)$

$\Rightarrow \sin(2x)(2\cos x - 3) = \cos(2x)(2\cos x - 3)$

$\Rightarrow \sin(2x)(2\cos x - 3) - \cos(2x)(2\cos x - 3) = 0$

$\Rightarrow (2\cos x - 3)(\sin(2x) - \cos(2x)) = 0$

$\Rightarrow \cos x = \frac{3}{2}$

Not possible

(X) $-1 \le \cos\theta \le 1$

$\sin(2x) = \cos(2x)$

$\tan(2x) = 1$

$\tan(2x) = \tan(\pi/4)$

$\theta = 2x \quad ; \alpha = \pi/4$

$\boxed{\theta = n\pi + \alpha}$

$2x = n\pi + \pi/4$

$x = \frac{n\pi}{2} + \frac{\pi}{8} \quad n \in Z$ Ans

Q1.6 Solve $\tan\theta + \tan(2\theta) + \sqrt{3}\tan\theta\tan(2\theta) = \sqrt{3}$

Soln

$\tan\theta + \tan(2\theta) = \sqrt{3} - \sqrt{3}\tan\theta\tan(2\theta)$

$\Rightarrow \tan\theta + \tan(2\theta) = \sqrt{3}(1 - \tan\theta\tan(2\theta))$

$\Rightarrow \frac{\tan\theta + \tan(2\theta)}{1 - \tan\theta\tan(2\theta)} = \sqrt{3}$

$\Rightarrow \tan(\theta + 2\theta) = \sqrt{3}$

$\Rightarrow \tan(3\theta) = \sqrt{3}$

$\Rightarrow \tan(3\theta) = \tan(\pi/3)$

Sol: $\boxed{\theta = n\pi + \alpha}$

$\Rightarrow 3\theta = n\pi + \pi/3$

$\Rightarrow \theta = \frac{n\pi}{3} + \frac{\pi}{9} \quad ; n \in Z$ Ans

Form

$a\cos\theta + b\sin\theta = c$

Step 1 → Divide both sides by $\sqrt{a^2+b^2}$

$\cos\theta = \cos\alpha$

Topic : ..... Date. : ..... Page No. : .....

Trigo (class 8) (6)

Q1.7 → Solve $\sqrt{3}\cos\theta - \sin\theta = \sqrt{2}$

form $a\cos\theta + b\sin\theta = c$

$a = \sqrt{3}$; $b = -1$ $c = \sqrt{2}$

divide both sides by $\sqrt{a^2+b^2} = \sqrt{3+1} = 2$

$\Rightarrow \frac{\sqrt{3}}{2}\cos\theta - \frac{1}{2}\sin\theta = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}$

$\Rightarrow \cos(\frac{\pi}{6})\cos\theta - \sin(\frac{\pi}{6})\sin\theta = \cos(\pi/4)$

$\Rightarrow \cos(\theta + \frac{\pi}{6}) = \cos\frac{\pi}{4}$

then $\alpha = \pi/4$

Soln

$\theta + \frac{\pi}{6} = 2n\pi \pm \pi/4$

$\theta = 2n\pi \pm \frac{\pi}{4} - \frac{\pi}{6}$

$\theta = 2n\pi + \frac{\pi}{4} - \frac{\pi}{6}$ | $\theta = 2n\pi - \frac{\pi}{4} - \frac{\pi}{6}$

$\theta = 2n\pi + \frac{A}{12}$ | $\theta = 2n\pi - \frac{5\pi}{12}$ $n \in Z$

$n \in Z$

Q1.8 → $\sqrt{2}\sec\theta - \tan\theta = 1$

Solve

Sol:

$\frac{\sqrt{2}}{\cos\theta} - \frac{\sin\theta}{\cos\theta} = 1$

$\Rightarrow \sqrt{2} - \sin\theta = \cos\theta$

$\Rightarrow \cos\theta + \sin\theta = \sqrt{2}$

form $a\cos\theta + b\sin\theta = c$

$a = 1$, $b = 1$

→ divide by $\sqrt{1+1} = \sqrt{2}$

$\frac{1}{\sqrt{2}}\cos\theta + \frac{1}{\sqrt{2}}\sin\theta = 1$

$\cos(\frac{\pi}{4})\cos\theta + \sin\frac{\pi}{4}\sin\theta = \cos(0)$

$\Rightarrow \cos(\theta - \frac{\pi}{4}) = \cos(0)$ ($\alpha = 0$)

Soln

$\theta - \frac{\pi}{4} = 2n\pi \pm 0$

$\theta = 2n\pi + \frac{\pi}{4}$ $n \in Z$

CLASSTIME

Topic : ..... Date. : ..... Page No. : .....

Trigo (Class 8) (7)

Q1.9 → If $\sec x \cdot \cos(5x) + 1 = 0$ when $0 < x < \frac{\pi}{2}$
Find value of x

Sol =

$\sec x \cdot \cos(5x) + 1 = 0$

$\frac{1}{\cos x} \cdot \cos(5x) + 1 = 0$

$\Rightarrow \frac{\cos(5x) + \cos x}{\cos x} = 0$

$\Rightarrow \cos(5x) + \cos x = 0$

$\Rightarrow 2\cos(3x) \cdot \cos(2x) = 0$

$\Rightarrow \cos(3x) \cdot \cos(2x) = 0$

$\cos(3x) = 0$			$\cos(2x) = 0$		
$3x = \frac{\pi}{2}$	$3x = \frac{3\pi}{2}$	$3x = \frac{5\pi}{2}$	$2x = \frac{\pi}{2}$	$2x = \frac{3\pi}{2}$	$2x = \frac{5\pi}{2}$
$x = \frac{\pi}{6}$	$x = \frac{\pi}{2}$	$x = \frac{5\pi}{6}$ (X) $\notin (0, \frac{\pi}{2}]$	$x = \frac{\pi}{4}$	$x = \frac{3\pi}{4}$ (X) $\notin (0, \frac{\pi}{2}]$	(X)

∴ Any $\pi/6, \pi/2, \pi/4$ Any

Topic : ..... Date. : ..... Page No. : .....

TRIGONOMETRY (T-8)

(WORKSHEET NO:- 8) Class No=8

Q No 1 → Find the principal solution

(i) $\cot x = -\sqrt{3}$ (2) $2\sec x - 4 = 0$

Q.no 2 → Solve $\cos(3x) + \cos x - \cos(2x) = 0$

Q.no 3 → Solve $\sec^2(2x) = 1 - \tan(2x)$

Q.no 4 → Solve $\sin x + \sin(3x) + \sin(5x) = 0$

Q.no. 5 → Solve $5\cos^2\theta + 7\sin^2\theta - 6 = 0$

Q.no.6 → Solve $\cot\theta + \tan\theta = 2\operatorname{cosec}\theta$

Q.no.7 → If $2\sin^2\theta = 3\cos\theta$ where $0 \le \theta \le 2\pi$
Find the values θ

Q.no.8 Solve $\cos\theta + \sin\theta = 1$

Q.no.9 → Solve $\cot\theta + \operatorname{cosec}\theta = \sqrt{3}$

Q.no 10 → Solve $\tan\theta + \tan(2\theta) + \tan\theta\tan(2\theta) = 1$

Q.no 11 → Solve $\tan\theta + \tan(\theta + \frac{\pi}{3}) + \tan(\theta + \frac{2\pi}{3}) = 3$
Hint use formula $\tan(A+B)$

Q.no 12 → Solve $\tan^2\theta + (1-\sqrt{3})\tan\theta - \sqrt{3} = 0$

Q.no 13 → Solve $\cos(4x) = \cos(2x)$

Q.no 14 Solve $\sin(2x) + \cos x = 0$
Hint use formula $\sin(2\theta)$

Topic : .....

Date. : .....

Page No. : .....

Answers

(1) (i) $\frac{5\pi}{6}, \frac{11\pi}{6}$ (ii) $\frac{\pi}{3}, \frac{5\pi}{3}$

(2) $x = (2n+1)\frac{\pi}{4}$; $x = 2n\pi \pm \frac{\pi}{3}$ $n \in Z$

(3) $x = \frac{n\pi}{2}$; $x = \frac{n\pi}{2} + \frac{3\pi}{8}$ $n \in Z$

(4) $x = \frac{n\pi}{3}$; $x = n\pi \pm \frac{\pi}{3}$; $n \in Z$

(5) $x = n\pi \pm \pi/4$ $n \in Z$

(6) $\theta = 2n\pi \pm \pi/3$; $n \in Z$

(7) $\theta = \pi/3, 5\pi/3$

(8) $\theta = n\pi + (-1)^n \frac{\pi}{4} - \frac{\pi}{4}$

(OR) $\theta = 2n\pi$ (OR) $\theta = 2n\pi + \frac{\pi}{2}$ $n \in Z$

(9) $\theta = 2n\pi + \pi/3$; $\theta = 2n\pi - \pi$; $n \in Z$

(10) $\theta = \frac{n\pi}{3} + \frac{\pi}{12}$; $n \in Z$

(11) $\theta = \frac{n\pi}{3} + \frac{\pi}{12}$; $n \in Z$

(12) $\theta = n\pi - \frac{\pi}{4}$; $\theta = n\pi + \frac{\pi}{3}$; $n \in Z$

(13) $x = n\pi/3$; $x = n\pi$ $n \in Z$

(14) $x = n\pi + (-1)^n \frac{\pi}{6}$; $x = (2n+1)\frac{\pi}{2}$; $n \in Z$