SOLUTIONS

WORKSHEET No: 1

(class No: 2)

LIMITS & DERIVATIVES

Qn. 1

$$\lim_{x\to 0}\left(\frac{\tan x - \sin x}{\sin^3 x}\right)$$

$$= \lim_{x\to 0}\left(\frac{\frac{\sin x}{\cos x} - \sin x}{\sin^3 x}\right)$$

$$= \lim_{x\to 0}\left(\frac{\sin x - \sin x\cdot\cos x}{\sin^3 x\cdot\cos x}\right)$$

$$= \lim_{x\to 0}\left(\frac{\sin x\,(1-\cos x)}{\sin^3 x\cdot\cos x}\right)$$

$$= \lim_{x\to 0}\left(\frac{1-\cos x}{\sin^2 x\cdot\cos x}\right)$$

$$= \lim_{x\to 0}\left(\frac{2\sin^2(x/2)}{\sin^2 x\cdot\cos x}\right)$$

$$= \lim_{x\to 0}\left(\frac{\frac{2\sin^2(x/2)}{\frac{x^2}{4}}\times\cancel{\frac{x^2}{4}}}{\frac{\sin^2 x}{x^2}\times\cancel{x^2}}\cdot\frac{1}{\cos x}\right)$$

$$= \frac{2\times 1\times\frac{1}{4}}{1}\times\frac{1}{1} \quad \cdots \left\{\lim_{x\to 0}\left(\frac{\sin x}{x}\right)=1,\ \lim_{x\to 0}(\cos x)=1\right\}$$

$$= 1/2 \quad \underline{\text{Ans}}$$

Qns. 2 → $\lim_{x\to 0}\left(\dfrac{\sin(2x)+3x}{2x+\sin(3x)}\right)$

Divide N & D by x

$$=\lim_{x\to 0}\left(\dfrac{\dfrac{\sin(2x)}{x}+3}{2+\dfrac{\sin(3x)}{x}}\right)$$

$$=\lim_{x\to 0}\left(\dfrac{\dfrac{\sin(2x)}{2x}\times 2+3}{2+\dfrac{\sin(3x)}{3x}\times 3}\right)$$

$$=\dfrac{1\times 2+3}{2+1\times 3}=\dfrac{2+3}{2+3}=1 \text{ Ans}$$

Qn. 3 → $\lim_{x\to 0}\left(\dfrac{\sin(2x)+\sin(6x)}{\sin(5x)-\sin(3x)}\right)$

Divide N & D by x

$$=\lim_{x\to 0}\left(\dfrac{\dfrac{\sin(2x)}{x}+\dfrac{\sin(6x)}{x}}{\dfrac{\sin(5x)}{x}-\dfrac{\sin(3x)}{x}}\right)$$

$$=\lim_{x\to 0}\left(\dfrac{\dfrac{\sin(2x)}{2x}\times 2+\dfrac{\sin(6x)}{6x}\times 6}{\dfrac{\sin(5x)}{5x}\times 5-\dfrac{\sin(3x)}{3x}\times 3}\right)$$

$$=\dfrac{1\times 2+1\times 6}{1\times 5-1\times 3}=\dfrac{2+6}{5-3}=4 \text{ Ans}$$

$\because \left\{\because \lim_{x\to 0}\left(\dfrac{\sin x}{x}\right)=1\right\}$

Qn.4 → $\lim_{x\to 0}\left(\frac{\cot(2x)-\operatorname{cosec}(2x)}{x}\right)$

$$= \lim_{x\to 0}\left[\frac{\frac{\cos(2x)}{\sin(2x)} - \frac{1}{\sin(2x)}}{x}\right]$$

$$= \lim_{x\to 0}\left(\frac{\cos(2x)-1}{x\sin(2x)}\right)$$

$$= \lim_{x\to 0}\left(\frac{-\cancel{2}\sin^{\cancel{2}}x}{x\cdot\cancel{2}\cancel{\sin x}\cos x}\right) \quad \text{---} \{1-\cos(2\theta) = 2\sin^2\theta\}$$

$$= -\lim_{x\to 0}\left(\frac{\tan x}{x}\right)$$

$$= -1 \quad \text{---} \left\{\lim_{x\to 0}\left(\frac{\tan x}{x}\right) = 1\right\}$$

$= -1$ Ans

Qn. 5 → $\lim_{x\to 0}\left[\frac{\sin x - 2\sin(3x) + \sin(5x)}{x}\right]$

$$= \lim_{x\to 0}\left(\frac{\sin x}{x} - \frac{2\sin(3x)}{x} + \frac{\sin(5x)}{x}\right)$$

$$= \lim_{x\to 0}\left(\frac{\sin x}{x} - 2\frac{\sin(3x)}{3x}\times 3 + \frac{\sin(5x)}{5x}\times 5\right)$$

$$= 1 - 2\times 3 + 5 \quad \text{---} \left\{\lim_{x\to 0}\left(\frac{\sin x}{x}\right) = 1\right\}$$

$= 6 - 6 = 0$ Ans

Qn. 6 → $\lim_{x\to 0}\left(\frac{\cos(2x)-1}{\cos x - 1}\right)$

$$= \lim_{x \to 0} \left(\frac{1 - \cos(2x)}{1 - \cos x} \right)$$

$$= \lim_{x \to 0} \left(\frac{\cancel{2}\sin^2(x)}{\cancel{2}\sin^2\left(\frac{x}{2}\right)} \right) \quad \text{----} \{ 1 - \cos(2\theta) = 2\sin^2\theta \}$$

$$= \lim_{x \to 0} \left(\frac{\frac{\sin^2 x}{x^2} \times \cancel{x^2}}{\frac{\sin^2(x/2)}{\frac{x^2}{4}} \times \frac{\cancel{x^2}}{4}} \right)$$

$$= \frac{1}{1 \times \frac{1}{4}} \quad \text{---} \left\{ \because \lim_{x \to 0} \left(\frac{\sin x}{x} \right) = 1 \right\}$$

$= 4$ <u>ANS</u>

<u>QNS 7</u> → $\lim_{x \to 0} \left(\frac{\sqrt{1+\sin x} - \sqrt{1-\sin x}}{x} \right)$

Rationalize

$$= \lim_{x \to 0} \left(\frac{(1+\sin x) - (1-\sin x)}{x\left(\sqrt{1+\sin x} + \sqrt{1-\sin x}\right)} \right)$$

$$= \lim_{x \to 0} \left(\frac{2\sin x}{x\left(\sqrt{1+\sin x} + \sqrt{1-\sin x}\right)} \right)$$

$$= \frac{2}{\sqrt{1+0} + \sqrt{1-0}} \quad \text{--} \left\{ \lim_{x \to 0} \left(\frac{\sin x}{x} \right) = 1 \right\}$$

$= \frac{2}{2} = 1$ <u>ANS</u>

<u>QNS = 8</u> → Given $\lim_{x \to 0} \left(kx \operatorname{cosec} x \right) = \lim_{x \to 0} \left(x \operatorname{cosec}(kx) \right)$

$$\Rightarrow \lim_{x\to 0}\left(\frac{kx}{\sin x}\right) = \lim_{x\to 0}\left(\frac{x}{\sin(kx)}\right)$$

$$\Rightarrow k\times 1 = \lim_{x\to 0}\left(\frac{\cancel{x}}{\frac{\sin(kx)}{kx}\times k\cancel{x}}\right)$$

$$\Rightarrow k\times 1 = \frac{1}{1\times k} \quad \text{--} \quad \left\{\because \lim_{x\to 0}\left(\frac{\sin x}{x}\right) = 1 \text{ also } \lim_{x\to 0}\left(\frac{x}{\sin x}\right) = 1\right\}$$

$$\Rightarrow k^2 = 1$$

$$\Rightarrow \boxed{k = \pm 1} \text{ Ans}$$

Ans 9 → $$\lim_{x\to 0}\left(\frac{\sec(5x) - \sec(3x)}{\sec(3x) - \sec x}\right)$$

$$= \lim_{x\to 0}\left(\frac{\cos(3x) - \cos(5x)}{\cos x - \cos(3x)}\right)$$

$$= \lim_{x\to 0}\left(\frac{\cancel{-2}\sin(4x)\cdot \cancel{\sin(-x)}}{\cancel{-2}\sin(2x)\cdot \cancel{\sin(-x)}}\right)$$

$$= \lim_{x\to 0}\left(\frac{\frac{\sin(4x)}{4x}\times 4\cancel{x}}{\frac{\sin(2x)}{2x}\times 2\cancel{x}}\right)$$

$$= \frac{1\times 4}{1\times 2} \quad \text{---} \quad \left\{\lim_{x\to 0}\left(\frac{\sin x}{x}\right) = 1\right\}$$

$$= 2 \text{ Ans}$$

Ans 10 → $$\lim_{x\to 0}\left(\frac{\sin(2x)\{\cos(3x) - \cos x\}}{x^3}\right)$$

$$= \lim_{x\to 0}\left[\frac{\sin(2x).\left\{-2\sin(2x).\sin(x)\right\}}{x^3}\right]$$

$$= -2\lim_{x\to 0}\left(\frac{\sin(2x)}{x}.\frac{\sin(2x)}{x}.\frac{\sin x}{x}\right]$$

$$= -2\lim_{x\to 0}\left(\frac{\sin(2x)}{2x}\times 2.\frac{\sin(2x)}{2x}\times 2.\frac{\sin x}{x}\right]$$

$$= -2\left[(1)(2).(1)(2).(1)\right] \quad --- \left\{\lim_{x\to 0}\left(\frac{\sin x}{x}\right)=1\right\}$$

$$= -2(4)$$

$$= -8 \quad \underline{\text{Ans}}$$

Ans 11 → $\lim_{x\to 0}\left(\frac{\overset{(A)}{\sin(a+x)}+\overset{(B)}{\sin(a-x)}-2\sin a}{x\sin x}\right)$

$$= \lim_{x\to 0}\left(\frac{2\sin(a).\cos(x)-2\sin a}{x\sin x}\right)$$

$$= \lim_{x\to 0}\left(\frac{2\sin a.(\cos x-1)}{x\sin x}\right)$$

$$= -2\sin a.\lim_{x\to 0}\left(\frac{1-\cos x}{x\sin x}\right)$$

$$= -2\sin a.\lim_{x\to 0}\left(\frac{2\sin^2(x/2)}{x\sin x}\right)$$

$$= -2\sin a \lim_{x\to 0}\left[\frac{\frac{2\sin^2(x/2)}{\frac{x^2}{4}} \times \frac{\cancel{x^2}}{4}}{\cancel{x}\,\frac{\sin x}{x} \times \cancel{x}}\right]$$

$$= -2\sin a\left(\frac{2\times 1\times \frac{1}{4}}{1}\right) \quad \text{---} \left\{\because \lim_{x\to 0}\left(\frac{\sin x}{x}\right)=1\right\}$$

$$= -\cancel{2}\sin a\left(\frac{1}{\cancel{2}}\right)$$

$$= -\sin a \quad \underline{\text{Ans}}$$

Q No 12 → $\lim_{x\to\frac{\pi}{6}}\left[\frac{\sqrt{3}\sin x - \cos x}{x-\frac{\pi}{6}}\right]$

Put $x = \frac{\pi}{6}+h$ & $h\to 0$

$$= \lim_{h\to 0}\left(\frac{\sqrt{3}\sin\left(\frac{\pi}{6}+h\right) - \cos\left(\frac{\pi}{6}+h\right)}{\cancel{\frac{\pi}{6}}+h-\cancel{\frac{\pi}{6}}}\right)$$

$$= \lim_{h\to 0}\left(\frac{\sqrt{3}\left(\frac{1}{2}\cos h + \frac{\sqrt{3}}{2}\sin h\right) - \left(\frac{\sqrt{3}}{2}\cos h - \frac{1}{2}\sin h\right)}{h}\right)$$

$$= \lim_{h\to 0}\left(\frac{\cancel{\frac{\sqrt{3}}{2}\cos h} + \frac{3}{2}\sin h - \cancel{\frac{\sqrt{3}}{2}\cos h} + \frac{1}{2}\sin h}{h}\right)$$

$$= \lim_{h\to 0}\left(\frac{2\sin h}{h}\right)$$

$$= 2\times 1$$

$$= 2 \quad \underline{\text{Ans}}$$

Q.No. 13 → $\lim_{x \to \frac{\pi}{2}} \left(\frac{\tan(2x)}{x - \frac{\pi}{2}} \right)$

put $x = \frac{\pi}{2} + h$ & $h \to 0$

$= \lim_{h \to 0} \left(\frac{\tan\left(2\left(\frac{\pi}{2} + h\right)\right)}{\cancel{\frac{\pi}{2}} + h - \cancel{\frac{\pi}{2}}} \right)$

$= \lim_{h \to 0} \left(\frac{\tan(\pi + 2h)}{h} \right)$

$= \lim_{h \to 0} \left(\frac{\tan(2h)}{h} \right)$ --- $\{\tan(\pi + \theta) = \tan\theta\}$

$= \lim_{h \to 0} \left(\frac{\tan(2h)}{2h} \times 2 \right)$

$= 1 \times 2$

$= 2$ Ans

Que 14 → $\lim_{x \to \frac{\pi}{2}} \left(\frac{1 + \cos(2x)}{(\pi - 2x)^2} \right)$

$= \lim_{x \to \frac{\pi}{2}} \left(\frac{2\cos^2 x}{(\pi - 2x)^2} \right)$

put $x = \frac{\pi}{2} + h$ & $h \to 0$

$= \lim_{h \to 0} \left[\frac{2\cos^2\left(\frac{\pi}{2} + h\right)}{\left(\pi - 2\left(\frac{\pi}{2} + h\right)\right)^2} \right]$

$= \lim_{h \to 0} \left(\frac{2\sin^2 h}{(\cancel{\pi} - \cancel{\pi} - 2h)^2} \right)$

$$= \lim_{h \to 0} \left(\frac{2\sin^2 h}{4h^2} \right)$$

$$= \frac{1}{2} \lim_{h \to 0} \left(\frac{\sin^2 h}{h^2} \right)$$

$$= \frac{1}{2} \times 1$$

$$= 1/2 \quad \text{Ans}$$

Ans 15 → $\lim_{x \to \frac{\pi}{2}} \left[\frac{\cot x - \cos x}{(\pi - 2x)^3} \right]$

put $x = \frac{\pi}{2} + h$ & $h \to 0$

$$= \lim_{h \to 0} \left[\frac{\cot(\frac{\pi}{2}+h) - \cos(\frac{\pi}{2}+h)}{(\pi - 2(\frac{\pi}{2}+h))^3} \right]$$

$$= \lim_{h \to 0} \left(\frac{-\tan h + \sin h}{(\cancel{\pi} - \cancel{\pi} - 2h)^3} \right)$$

$$= \lim_{h \to 0} \left(\frac{-\tan h + \sin h}{-8h^3} \right)$$

$$= \frac{1}{8} \lim_{h \to 0} \left(\frac{\tan h - \sin h}{h^3} \right)$$

$$= \frac{1}{8} \lim_{h \to 0} \left(\frac{\frac{\sin h}{\cos h} - \sin h}{h^3} \right)$$

$$= \frac{1}{8} \lim_{h \to 0} \left(\frac{\sin h - \sin h \cdot \cos h}{h^3 \cdot \cos h} \right)$$

$$= \frac{1}{8} \lim_{h \to 0} \left(\frac{\sin h (1 - \cos h)}{h^3 \cdot \cos h} \right)$$

$$= \frac{1}{8} \lim_{h\to 0}\left[\frac{\sin h \cdot 2\sin^2\left(\frac{h}{2}\right)}{h^3 \cdot \cos h}\right]$$

$$= \frac{1}{8} \lim_{h\to 0}\left(\frac{\sin h}{h} \cdot \frac{2\sin^2\left(\frac{h}{2}\right)}{h^2} \cdot \frac{1}{\cos h}\right)$$

$$= \frac{1}{8} \lim_{h\to 0}\left(\frac{\sin h}{h} \cdot \frac{2\sin^2\left(\frac{h}{2}\right)}{\frac{h^2}{4}\times 4} \cdot \frac{1}{\cos h}\right)$$

$$= \frac{1}{8}\left(1\times 2\times 1\times \frac{1}{4}\times \frac{1}{1}\right)$$

$$= \frac{1}{8}\left(\frac{1}{2}\right)$$

$$= \frac{1}{16} \quad \text{Ans}$$

Ques 16 → $\lim_{x\to \frac{\pi}{2}}\left(\frac{\sqrt{2}-\sqrt{1+\sin x}}{\cos^2 x}\right)$

Rationalizing

$$= \lim_{x\to \frac{\pi}{2}}\left(\frac{2-1-\sin x}{\cos^2 x\left(\sqrt{2}+\sqrt{1+\sin x}\right)}\right)$$

$$= \lim_{x\to \frac{\pi}{2}}\left(\frac{1-\sin x}{\cos^2 x\left(\sqrt{2}+\sqrt{1+\sin x}\right)}\right)$$

put $x = \frac{\pi}{2}+h$ & $h\to 0$

$$= \lim_{h\to 0}\left[\frac{1-\sin\left(\frac{\pi}{2}+h\right)}{\cos^2\left(\frac{\pi}{2}+h\right)\left(\sqrt{2}+\sqrt{1+\sin\left(\frac{\pi}{2}+h\right)}\right)}\right]$$

$$= \lim_{h\to 0}\left(\frac{1-\cos h}{\sin^2 h\left(\sqrt{2}+\sqrt{1+\cos h}\right)}\right)$$

$$= \lim_{h\to 0}\left(\frac{2\sin^2(h/2)}{\sin^2 h\,(\sqrt{2}+\sqrt{1+\cos h})}\right)$$

$$= \lim_{h\to 0}\left(\frac{\dfrac{2\sin^2\left(\frac{h}{2}\right)}{\frac{h^2}{4}}\times \cancel{\frac{h^2}{4}}}{\dfrac{\sin^2 h}{h^2}\cdot \cancel{h^2}(\sqrt{2}+\sqrt{1+\cos h})}\right)$$

$$= \frac{2\times 1\times \frac{1}{4}}{1\times(\sqrt{2}+\sqrt{1+1})} \quad - \left\{\lim_{h\to 0}\left(\frac{\sin h}{h}\right)=1\right\}$$

$$= \frac{\frac{1}{2}}{2\sqrt{2}}$$

$$= \frac{1}{4} \quad \underline{\text{Ans}}$$

{ Note Misprint in worksheet Ans }

Qn 17 → $\lim_{x\to \frac{\pi}{4}}\left(\frac{\sqrt{2}-\cos x-\sin x}{(4x-\pi)^2}\right)$

put $x = \frac{\pi}{4}+h$ & $h\to 0$

$$= \lim_{h\to 0}\left(\frac{\sqrt{2}-\cos\left(\frac{\pi}{4}+h\right)-\sin\left(\frac{\pi}{4}+h\right)}{\left(4\left(\frac{\pi}{4}+h\right)-\pi\right)^2}\right)$$

$$= \lim_{h\to 0}\left(\frac{\sqrt{2}-\left(\frac{1}{\sqrt{2}}\cos h-\frac{1}{\sqrt{2}}\sin h\right)-\left(\frac{1}{\sqrt{2}}\cos h+\frac{1}{\sqrt{2}}\sin h\right)}{(\cancel{\pi}+4h-\cancel{\pi})^2}\right)$$

$$= \lim_{h\to 0}\left(\frac{\sqrt{2}-\frac{1}{\sqrt{2}}\cos h+\cancel{\frac{1}{\sqrt{2}}\sin h}-\frac{1}{\sqrt{2}}\cos h-\cancel{\frac{1}{\sqrt{2}}\sin h}}{16h^2}\right)$$

$$= \frac{1}{16} \lim_{h \to 0} \left(\frac{\sqrt{2} - \sqrt{2}\cos h}{h^2} \right)$$

$$= \frac{1}{16} \lim_{h \to 0} \left(\frac{\sqrt{2}(1 - \cos h)}{h^2} \right)$$

$$= \frac{1}{16} \lim_{h \to 0} \left(\frac{\sqrt{2} \cdot 2\sin^2(h/2)}{h^2} \right)$$

$$= \frac{1}{16} \lim_{h \to 0} \left(\frac{\sqrt{2} \times \frac{2\sin^2\left(\frac{h}{2}\right)}{\frac{h^2}{4}} \times \frac{\cancel{h^2}}{4}}{\cancel{h^2}} \right)$$

$$= \frac{1}{16}\left(\sqrt{2} \times 2 \times 1 \times \frac{1}{4}\right) \quad --- \left\{ \lim_{x \to 0} \left(\frac{\sin x}{x}\right) = 1 \right\}$$

$$= \frac{1}{16} \times \frac{1}{\sqrt{2}}$$

$$= \frac{1}{16\sqrt{2}} \quad \text{Ans}$$

Q.No 18 → $\lim_{x \to \frac{\pi}{6}} \left[\frac{2\sin^2 x + \sin x - 1}{2\sin^2 x - 3\sin x + 1} \right]$

$$= \lim_{x \to \frac{\pi}{6}} \left(\frac{2\sin^2 x + 2\sin x - \sin x - 1}{2\sin^2 x - 2\sin x - \sin x + 1} \right)$$

$$= \lim_{x \to \frac{\pi}{6}} \left(\frac{2\sin x(\sin x + 1) - 1(1 + \sin x)}{2\sin x(\sin x - 1) - 1(\sin x - 1)} \right)$$

$$= \lim_{x \to \frac{\pi}{6}} \left(\frac{\cancel{(2\sin x - 1)}(\sin x + 1)}{\cancel{(2\sin x - 1)}(\sin x - 1)} \right) = \frac{\frac{1}{2} + 1}{\frac{1}{2} - 1} = \frac{\frac{3}{2}}{-\frac{1}{2}}$$

$$= -3 \quad \text{Ans}$$

— x —