!! જય શ્રી રાધે કૃષ્ણ !!

→ ULTIMATE MATHEMATICS →

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XI JEE MAINS: CLASS NO:1

CHAPTER LOGARITHMS (LOGARITHMS)

(·) $\log_a x$

(·) $a>0 \; ; a\neq 1$

(·) $\log(+x)$ ✓

(·) $\log(-x)$ X

(·) $\log(c)$ X

(·) log table $\log_{10} x$

(·) $a^x = N$ (exponential form)

Conversion

Ex! $a^x = N \Leftrightarrow \log_a N = x \Leftrightarrow a^x = N$

Ex $3^4 = 81 \Rightarrow \log_3 81 = 4 \Rightarrow 3^4 = 81$

PROPERTIES of Logarithms

① $\log_a(AB) = \log_a A + \log_a B$

(·) $\log(ABC) = \log_a A + \log_a B + \log_a C$

(·) $\log(x_1 \cdot x_2 \cdot x_3 \cdots x_n) = \log_a x_1 + \log_a x_2 + \log_a x_3 \cdots \log_a x_n$

② $\log_a(A/B) = \log_a A - \log_a B$

(·) $\log\left(\frac{AB}{C}\right) = \log A + \log B - \log C$

(3) $\log_a m^n = n \log_a m$

(.) $\log\left(\frac{\sqrt{A}\, B^{1/3}}{C}\right) = \frac{1}{2}\log A + \frac{1}{3}\log B - \log C$

(4) $\log_a 1 = 0$ (always)

(5) Change of Base

$$\log_a b = \frac{\log_c b}{\log_c a} \quad (\text{Imp})$$

(6) $\log_a b = \frac{1}{\log_b a}$

(7) $\log_a a = 1$ eg $\log_3 9 = \log_3 (3)^2 = 2\log_3(3) = 2$

(8) $\log_{a^p}(b)^q = \frac{q}{p}\log_a b$ ✓

eg $\log_{16} 9 = \log_{4^2}(3)^2 = \frac{2}{2}\log_4 3 = \log_4 3$

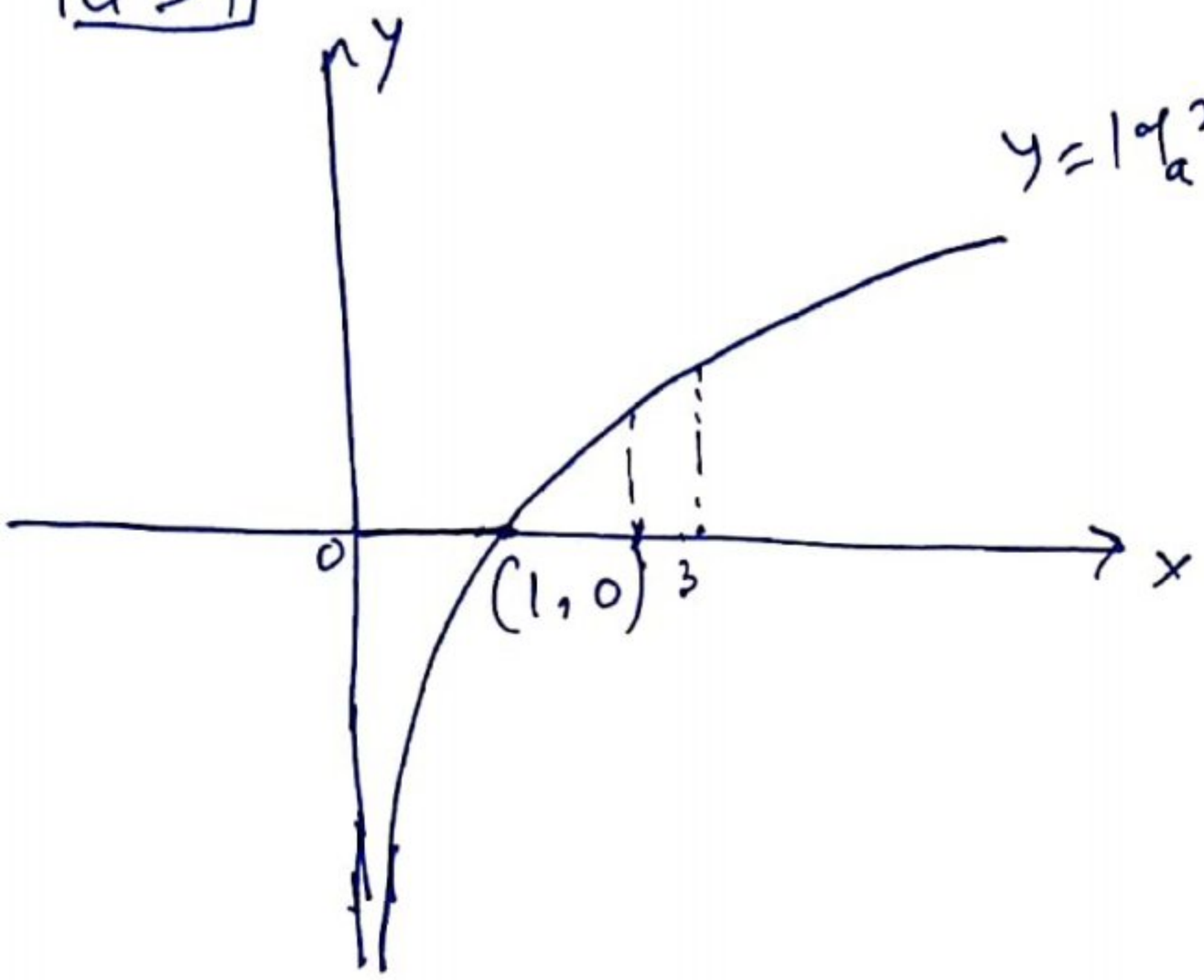
(9) $a^{\log_a x} = x$

eg (.) $2^{\log_2 4} = 4$

(10) $x^{\log_a y} = y^{\log_a x}$

GRAPH: $y = \log_a x$

Result (1) $\boxed{a > 1}$



(·) $\log_a x > 0 \quad ; x > 1$

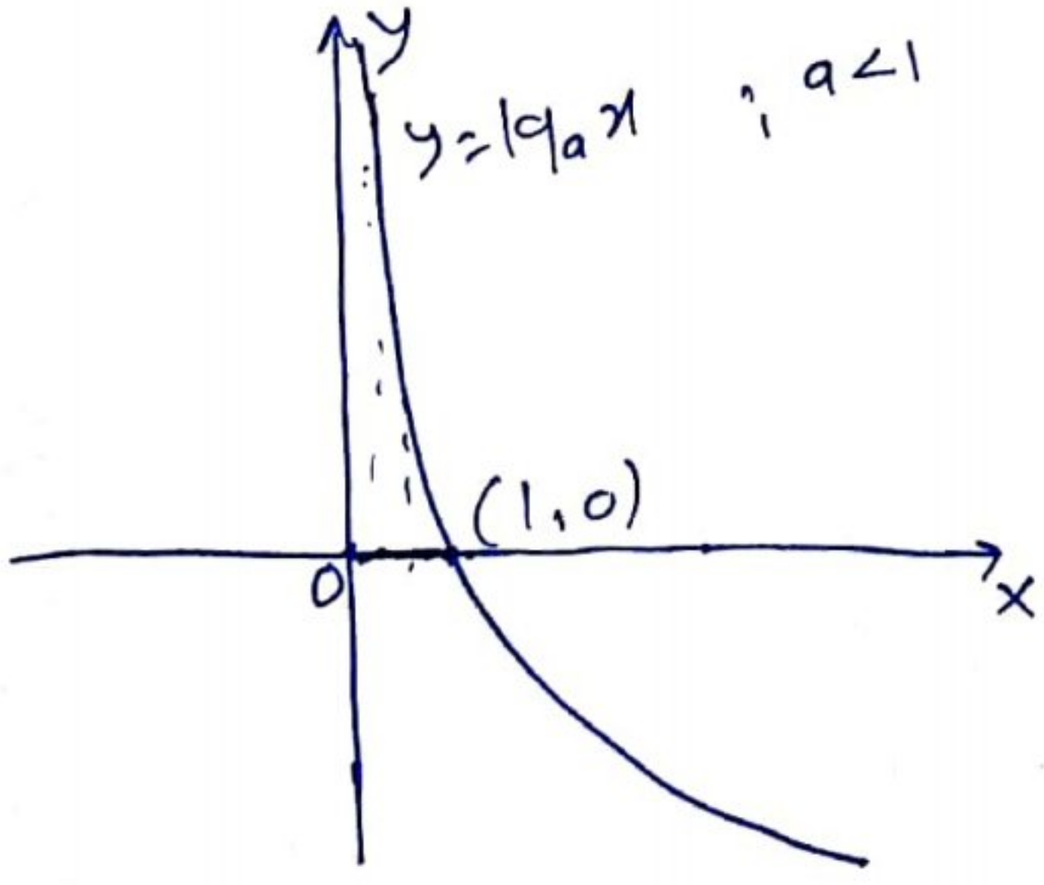
(·) $\log_a x < 0 \quad ; 0 < x < 1$

(·) $\log_a x = 0 \quad ; x = 1$

(·) If $x > y \Rightarrow \log_a x > \log_a y$ (Increasing funct.

$3 > 2 \Rightarrow \log 3 > \log 2$

Result 2 $\boxed{0 < a < 1}$



(·) $\log_a x > 0 \quad ; 0 < x < 1$

(·) $\log_a x \not< 0 \quad ; x > 1$

(·) $\log_a x = 0 \quad ; x = 1$

(·) $x > y \quad \log_a x < \log_a y$

(decreasing funct.)

(·) $(a>1)$

$\log_a 3 > \log_a 1 \quad : x > a \quad ; \log_a x > 1$

$\log_3 2 < \log_3 1 \quad : x < a \quad ; \log_a x < 1$

$\log_2 2 = 1 \quad ; x < a \quad \log_a x = 1$

(·) $\boxed{0 < a < 1}$

$\log_{0.2} 3 < 1 \quad ; x > a \quad ; \log_a x < 1$

$\log_{0.2} 0.1 \geq 1 \quad ; x < a \quad ; \log_a x < 1$

$\log_{0.2} 0.2 = 1 \quad ; x = a \quad ; \log_a x = 1$

(·) $x = 2^5$

"taking log on both sides"

$\log_a x = \log_a 2^5$

$\log_a x = 5 \log_a 2$

— x —

QUESTIONS

Ques 1 If $\log_3 a \times \log_a x = 4$ then find x

Sol:

$$\frac{\log a}{\log 3} \times \frac{\log x}{\log a} = 4$$

$$\Rightarrow \frac{\log x}{\log 3} = 4$$

$$\Rightarrow \log_3 x = 4$$

$$\Rightarrow 3^4 = x \Rightarrow x = 81 \quad \underline{\text{Ans}}$$

Qn. 2 If a, b, c are +ve then find

$$\frac{1}{\log_{ab} abc} + \frac{1}{\log_{bc} abc} + \frac{1}{\log_{ca} abc} = ?$$

Sol:

$$\log_{abc}(ab) + \log_{abc}(bc) + \log_{abc}(ca)$$

$$= \log_{abc}(ab \cdot bc \cdot ca)$$

$$= \log_{abc}(abc)^2$$

$$= 2\log_{abc}(abc) = 2 \times 1 = 2$$

Qn. 3 → If $x = \log_{0.1}(0.001)$ & $y = \log_9 81$ find $\sqrt{x - 2\sqrt{y}} = ?$

Sol:

$$x = \log_{0.1}(0.001) = \log_{(0.1)}(0.1)^3 = 3\log_{(0.1)}(0.1) = 3$$

$$y = \log_9 81 = \log_9 (9)^2 = 2\log_9 9 = 2$$

$$\therefore \sqrt{x - 2\sqrt{y}} = \sqrt{3 - 2\sqrt{2}} = \sqrt{(\sqrt{2} - 1)^2} = \sqrt{2} - 1$$

Qn. 4 → If a, b, c are +ve real numbers, then find

$$a^{\log b - \log c} \times b^{\log c - \log a} \times c^{\log a - \log b}$$

Sol: Let $y = a^{\log b - \log c} \times b^{\log c - \log a} \times c^{\log a - \log b}$

taking log on both sides

$$\log y = \log \left[a^{\log b - \log c} \times b^{\log c - \log a} * c^{\log a - \log b} \right]$$

$$\log y = (\log b - \log c)\cdot \log a + (\log c - \log a)\log b + (\log a - \log b)\log c$$

$$\log y = 0$$

$$\Rightarrow \boxed{y = 1} \quad \text{Ans}$$

Q No 5 → If $x = \log_{2a}(a)$; $y = \log_{3a}(2a)$; $z = \log_{4a}(3a)$

then $xyz + 1 =$

(a) $2yz$ (b) $2xy$ (c) $2zx$ (d) none of these

Sol $xyz + 1 = \log_{2a}(a)\cdot \log_{3a}(2a)\cdot \log_{4a}(3a) + 1$

$$= \frac{\log a}{\log 2a} \times \frac{\log 2a}{\log 3a} \times \frac{\log 3a}{\log 4a} + 1$$

$$\Rightarrow \frac{\log a}{\log 4a} + 1$$

$xyz + 1 = \log_a a + 1$ (Main)

$xyz + 1 = \log_{4a} a + \log_{4a}(4a)$

$= \log_{4a}(a \cdot 4a)$

$= \log_{4a}(4a^2)$

$= \log_{4a}(2a)^2$

$xyz + 1 = 2\log_{4a}(2a)$

$= 2yz$

Rough

$yz = \log_{3a}(2a) \times \log_{4a}(3a)$

$= \frac{\log 2a}{\log 3a} \times \frac{\log 3a}{\log 4a}$

$= \log_{4a} 2a$

XI JEE MAINS (Class No 1)

Q1.6 If $5^x = (0.5)^y = 1000$ then $\frac{1}{x} - \frac{1}{y} = ?$

Sol:

$5^x = 1000$ | $(0.5)^y = 1000$

$\log_5 1000 = x$ | $\log_{(0.5)} 1000 = y$

$$\frac{1}{x} - \frac{1}{y} = \frac{1}{\log_5 1000} - \frac{1}{\log_{0.5} 1000}$$

$$= \log_{1000}(5) - \log_{1000}(0.5)$$

$$= \log_{1000}\left(\frac{5}{0.5}\right)$$

$$= \log_{1000}(10)$$

$$= \log_{(10)^3}(10)^1$$

$$= \frac{1}{3}\log_{10} 10$$

$$= 1/3 \text{ Ans}$$