← ULTIMATE MATHEMATICS: BY AJAY MITTAL →

Chapter: Permutation & Combination

CLASS NO: 3

Combination → Selection (ordy does not matter)

(·) ${}^nC_r = \frac{n!}{r!\,(n-r)!}$; $r \le n$

(·) Shortcuts

${}^nC_0 = 1$; ${}^6C_0 = 1$

${}^nC_1 = n$; ${}^7C_1 = 7$

${}^nC_2 = \frac{n(n-1)}{2}$; ${}^8C_2 = \frac{8\times 7}{2} = 28$

${}^nC_3 = \frac{n(n-1)(n-2)}{6}$; ${}^8C_3 = \frac{8\times 7\times 6}{6} = 56$

${}^nC_n = 1$; ${}^8C_8 = 1$

${}^{10}C_8 = {}^{10}C_2 = \frac{10\times 9}{2} = 45$

${}^{20}C_{19} = {}^{20}C_1 = 20$

Property $\boxed{{}^nC_r = {}^nC_{n-r}}$

Property If ${}^nC_x = {}^nC_y$
then either $x = y$ or $x + y = n$

Property ${}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$

eg ${}^7C_3 + {}^7C_4 = {}^8C_4$

Ques 1 A group consists of 4 boys and 5 girls. A team of 6 is to be formed. In how many ways if team consists of

(1) 2 boys and 4 girls

(·) 2 boys out of 4 boys can be selected in $= {}^4C_2$ ways

(·) 4 girls out of 5 girls can be selected in $= {}^5C_4$ ways

$\therefore$ Required no of ways $= {}^4C_2 \times {}^5C_4$

$= \frac{4\times3}{2} \times 5 = 30$

(2) there is no restriction

$= {}^9C_6 = {}^9C_3 = \frac{9\times8\times7}{6} =$ 84

(3) atleast 3 boys

Req = 6	(4) B	(5) G
	3	3
	4	2

Case I $= {}^4C_3 \times {}^5C_3$

Case II : ${}^4C_4 \times {}^5C_2$

Req no of ways $= ({}^4C_3 \times {}^5C_3) + ({}^4C_4 \times {}^5C_2) =$ ☐

(4) atmost 2 boys

Req = 6	(4) B	(5) G
	2	4
	1	5

Req ways $= ({}^4C_2 \times {}^5C_4) +$

$({}^4C_1 \times {}^5C_5) =$ ☐

(5) equal no. of boys and girls.

3B & 3G

Req. = $^4C_3 \times {}^5C_3$

(6) atleast 2 boy and atleast 2 girl

(4) B	(5) G
2	4
3	3
4	2

Req

Req no ways: $({}^4C_2 \times {}^5C_4) + ({}^4C_3 \times {}^5C_3) + ({}^4C_4 \times {}^5C_2)$

$= (6 \times 5) + (4 \times 10) + (1 \times 10)$

$= 30 + 40 + 10 = 80$ Ans

(7) Girls all in majority

(4) B	5 G
2	4
1	5

Req

Req no of ways $= ({}^4C_2 \times {}^5C_4) + ({}^4C_1 \times {}^5C_5)$

(8) a particular boy and a particular girl always chosen

$\therefore$ ways $= {}^1C_1 \times {}^1C_1 \times {}^7C_4$

(9) a particular boy and a particular girl never chosen

$= {}^7C_6$

Q No 2 → In an examination, a question paper consists of 12 questions divided in to two parts Part I and part II, containing 5 and 7 questions respectively. A student is required to attempt 8 questions in all, selecting atleast 3 from each part. In how many ways can a student select the questions?

Sol
= There are three cases

	(5)	(7)
Req. 8	I	II
	3	5
	4	4
	5	3

(1) Select 3 from part I & 5 from part II

no of ways of selection = $^5C_3 \times {}^7C_5$

(2) $^5C_4 \times {}^7C_4$

(3) $^5C_5 \times {}^7C_3$

Req no of ways = (Can I + (Can II + (Can III = [ ] Ans

Q No 3 → From a class of 25 students, 10 are to be chosen for an excursion party. There are 3 students who decide that either all of them will join or none of them will join. In how many ways can they be chosen?

Solution: Case I let the three students are not coming

$= {}^{20}C_{10}$

Case II let the three students are coming

$= {}^{3}C_{3} \times {}^{22}C_{7}$

Req no of ways $= {}^{22}C_{10} + {}^{22}C_{7}$ Ans

Qn 4 → we wish to select 6 persons out of 8 persons, but if the person A is chosen, then B must be chosen. In how many ways can the selection be made?

Sol Case I let A is chosen

then B must be chosen

$= {}^{1}C_{1} \times {}^{1}C_{1} \times {}^{6}C_{4}$

Case II let A is not chosen

$= {}^{7}C_{6}$

Req no of ways = Case I + Case II = Ans

Qn 5 A boy has 3 library tickets and 8 book of his interest in the library. of these 8 books, he does not want to borrow Chemistry part II, unless chemistry part I is also borrowed. In how many ways can he choose the three books?

Solⁿ Case I : If Chemistry part I is not balanced

$= {}^{6}C_3$ ways

Case II If Chemistry part I is balanced

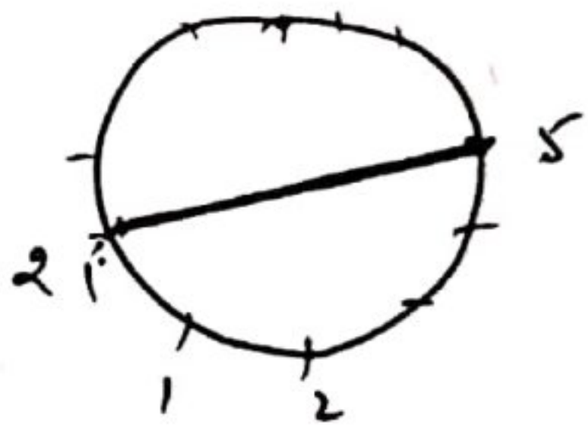
$= {}^{1}C_1 \times {}^{7}C_2$

No ways $= {}^{6}C_3 + {}^{7}C_2 =$ 56

Qn. 6 → How many Chords can be drawn through 21 points on a circle?

Soln

No of ways / no of Chord $= {}^{21}C_2$



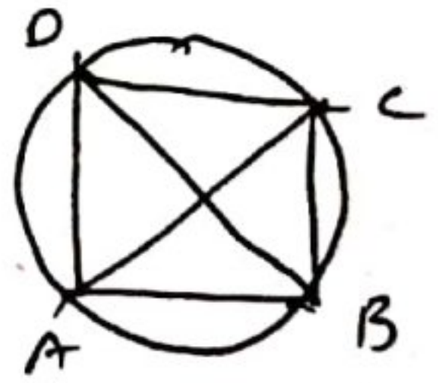
Explan

= 4 pairs available

to join 1 chord ; 2 points are required

No of way of select 2 point from 4 points $= {}^{4}C_2$

$= \frac{4 \times 3}{2} = 6$

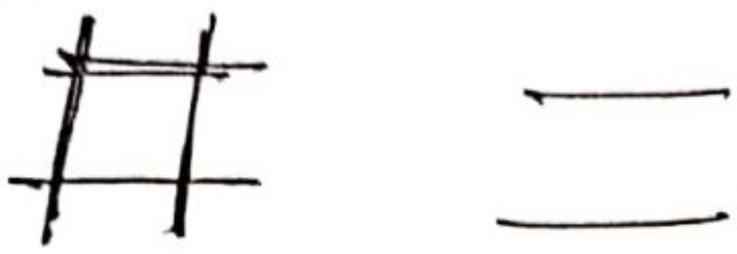


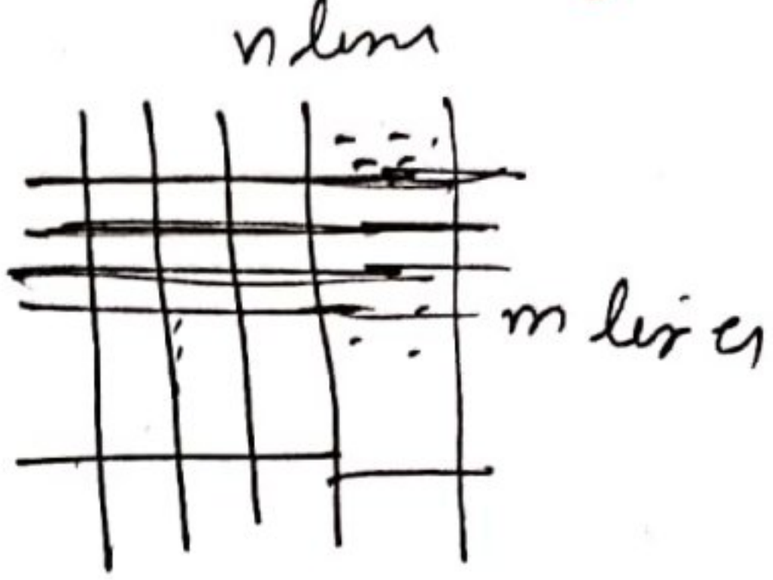
(i) No of triangle $= {}^{21}C_3$

(ii) No of quadrilateral $= {}^{21}C_4$

Q:7 → If 'm' parallel lines in plane are intersected by with 'n' parallel lines. Find the number of parallelograms formed?

Soln





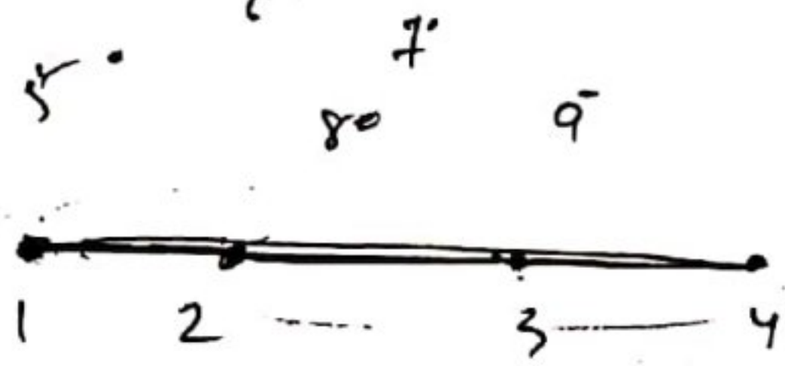
$= {}^{m}C_2 \times {}^{n}C_2$ = no of parallelogram

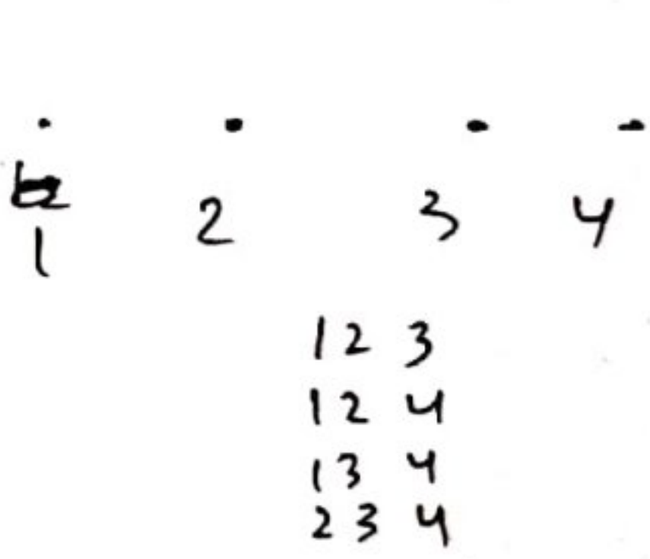
Q:8 There are 10 point in a plane out of which ~~4~~ 4 are collinear

(i) no of triangles?

(ii) no of St. lines?

Soln





(1) No of triangle = ${}^{10}C_3 - {}^{4}C_3$

(2) No of St line: ${}^{10}C_2 - {}^{4}C_2 + 1$

← WORKSHEET No: 3 →

"Permutation & Combination

Qn1. A committee of 7 has to be formed from 9 boys and 4 girls. In how many ways can this be done when the committe consists of:

(1) exactly 3 girls (2) atleast 3 girls (3) atmost 3 girls

Ans (1) 504 (2) 588 (3) 1632

Qn 2 → what is the number of ways of choosing 4 cards from a pack of 52 playing cards? In how many of these

(i) four cards are of the same suit

(2) four cards belongs to four different suits.

(3) all are face cards.

(4) two are red and two are black

(5) cards are of the same colour

Ans (1) 2860 (2) 13^4 (3) 495 (4) 105625 (5) 29900

Qn 3 → In how many ways can a Cricket eleven be chosen out of a batch of 15 players if

(1) there is no restriction on the selection

(2) a particular player is always chosen

(3) a particular player is never chosen

Ans (1) 1365 (2) 1001 (3) 364

Qu 4 → A box contains 5 different red and 6 different white balls. In how many ways can 6 balls be selected so that there are atleast two balls of each colour?

Ans 425

Qu 5 → out of 18 points in a plane, no three points are in the same straight line except five points which are collinear. How many

(i) Straight lines

(ii) triangles can be formed by joining them.

Ans (1) 144 (2) 806

Qu 6 → A student has to answer 10 questions, choosing atleast 4 from each ~~part~~ of part A and part B. If there are 6 questions in part A and 7 in part B. In how many ways can the student choose the 10 questions?

Ans 266

Qu 7 → How many triangles and how many diagonals can be formed by joining the vertices of a Hexagon?

Ans 20; 9

Qu 8 → In how many ways can one select a cricket team of eleven from 17 players in which only 5 players can bowl if each cricket team of 11 must include exactly 4 bowlers?

Ans 3960

—x—