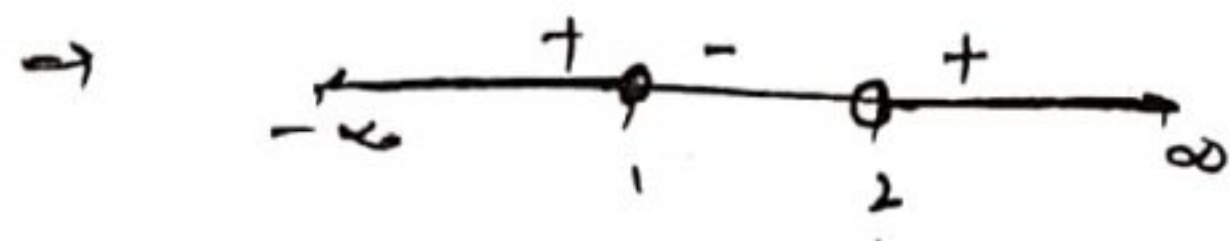SOLUTIONS: (WORKSHEET . No: 3)

← LINEAR INEQUALITIES → (solutions)

Ques 1

$$\frac{|x|-1}{|x|-2} \geq 0$$
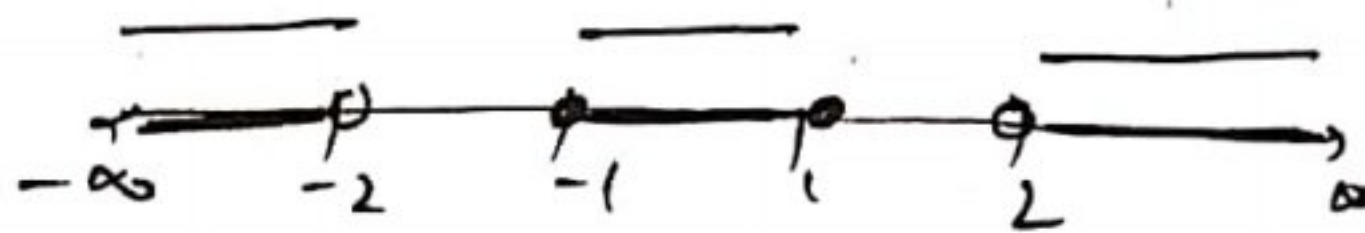
→

but $|x|-2 \neq 0$

$|x| \neq 2$

⟹ $|x| \leq 1$ (or) $|x| > 2$

⟹ $-1 \leq x \leq 1$ (or) $x < -2$ or $x > 2$

taking union

∴ $x \in (-\infty, -2) \cup [-1, 1] \cup (2, \infty)$ **Ans**

Ques 2 →

$$\frac{|x-2|-1}{|x-2|-2} \leq 0$$

and $|x-2|-2 \neq 0$

$|x-2| \neq 2$

⟹ $1 \leq |x-2| < 2$

Consider

$|x-2| \geq 1$ and $|x-2| < 2$

$(x-2) \leq -1$ (or) $(x-2) \geq 1$ and $-2 < (x-2) < 2$

$x \leq 1$ (or) $x \geq 3$ and $0 < x < 4$

∴ $x \in (0, 1] \cup [3, 4)$ **Ans**

Qus 3 → $\dfrac{1}{|x|-3} < \dfrac{1}{2}$

⟹ $\dfrac{1}{|x|-3} - \dfrac{1}{2} < 0$

⟹ $\dfrac{2 - |x| + 3}{2(|x|-3)} < 0$

⟹ $\dfrac{-|x| + 5}{2(|x|-3)} < 0$

⟹ $\dfrac{|x| - 5}{|x|-3} > 0$ → Sign change

$|x| < 3$ (or) $|x| > 5$

$-3 < x < 3$ ⓐ $x < -5$ (or) $x > 5$
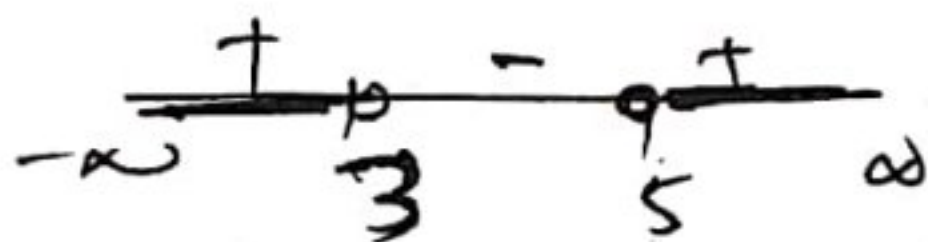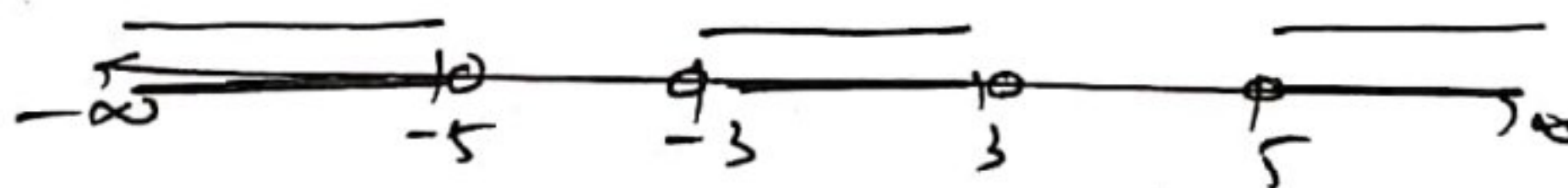
∴ $x \in (-\infty, -5) \cup (-3, 3) \cup (5, \infty)$ **Ans**

Qn. 4 → $|x-1| + |x-2| \geq 4$.

Soln   Let $f(x) = |x-1| + |x-2|$ ∴ $f(x) \geq 4$

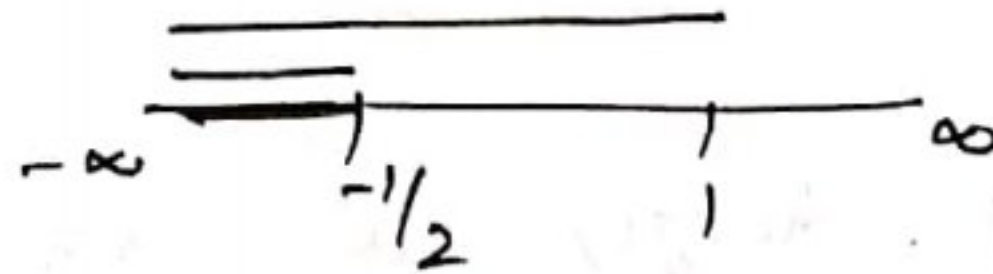Redefine the function   ← ① — ② →

$f(x) = \begin{cases} -(x-1)-(x-2) & ; \quad x < 1 \\ (x-1)-(x-2) & ; \quad 1 \leq x < 2 \\ (x-1)+(x-2) & ; \quad x \geq 2 \end{cases}$

$f(x) = \begin{cases} -2x+3 & : \quad x < 1 \\ 1 & : \quad 1 \leq x < 2 \\ 2x-3 & : \quad x \geq 2 \end{cases}$

Case I $-2x+3 \geq 4$ and $x < 1$

$-2x \geq 1$

$x \leq \dfrac{-1}{2}$ and $x < 1$

$$\underset{-\infty \qquad\quad -1/2 \qquad 1 \qquad\qquad \infty}{\rule{8cm}{0.4pt}}$$

$\therefore x \in \left(-\infty, \dfrac{-1}{2}\right]$ ----(i)

Case II

$1 \geq 4$ and $1 \leq x < 2$

$\underline{\text{False}}$ $x \in \phi$ ----(2)

Case III $2x-3 \geq 4$ and $x \geq 2$

$2x \geq 7$ and $x \geq 2$

$x \geq \dfrac{7}{2}$ and $x \geq 2$

$$\underset{-\infty \qquad\quad 2 \qquad 7/2 \qquad\qquad \infty}{\rule{8cm}{0.4pt}}$$
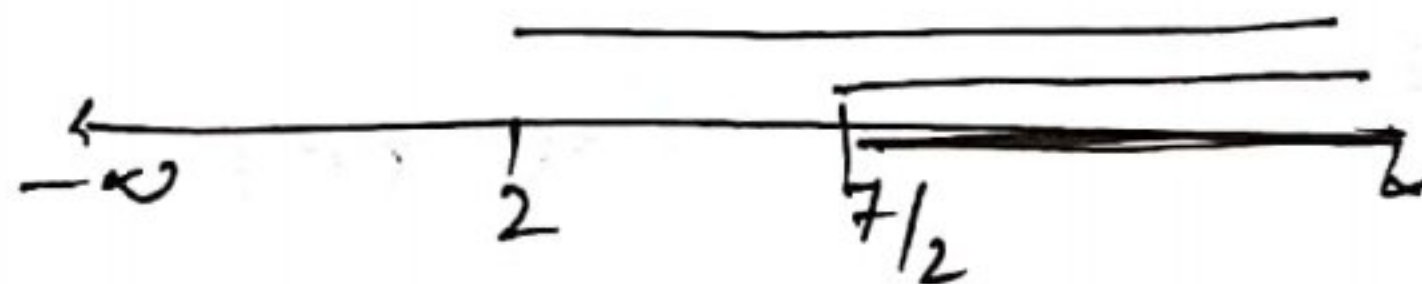
$\therefore x \in \left[\dfrac{7}{2}, \infty\right)$ ----③

taking union of (1), (2),(3)

$\therefore x \in \left(-\infty, \dfrac{-1}{2}\right] \cup \left[\dfrac{7}{2}, \infty\right)$ $\underline{\underline{Ans}}$

Qn 5 → $\dfrac{|x-1|}{x+2} < 1$

$\Rightarrow \dfrac{|x-1|}{x+2} - 1 < 0$

$\Rightarrow \dfrac{|x-1| - x - 2}{x+2} < 0$

Case I $|x-1| = (x-1)$ when $x-1 \geqslant 0$; $\boxed{x \geqslant 1}$

$\therefore \quad \dfrac{x-1 - x - 2}{x+2} < 0$ and $x \geqslant 1$

$\Rightarrow \quad \dfrac{-3}{x+2} < 0$ and $x \geqslant 1$

$\Rightarrow \quad \dfrac{3}{x+2} > 0$ ... (Sign change)   and $x \geqslant 1$

$$\underset{-\infty \qquad\qquad -2 \qquad\qquad \infty}{\xleftarrow{\hspace{3cm}}}$$

$\therefore \quad x \in (-2, \infty)$ and $x \geqslant 1$

$$\underset{-\infty \qquad\qquad -2 \qquad\qquad 1 \qquad\qquad \infty}{\xleftarrow{\hspace{4cm}}}$$
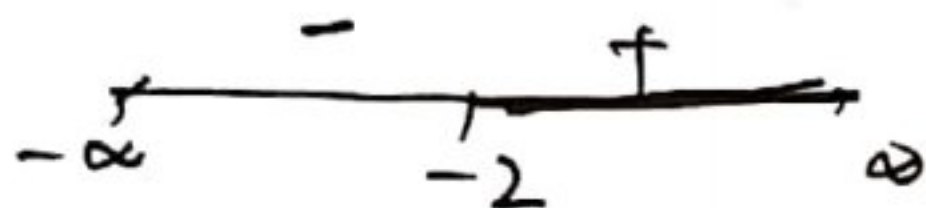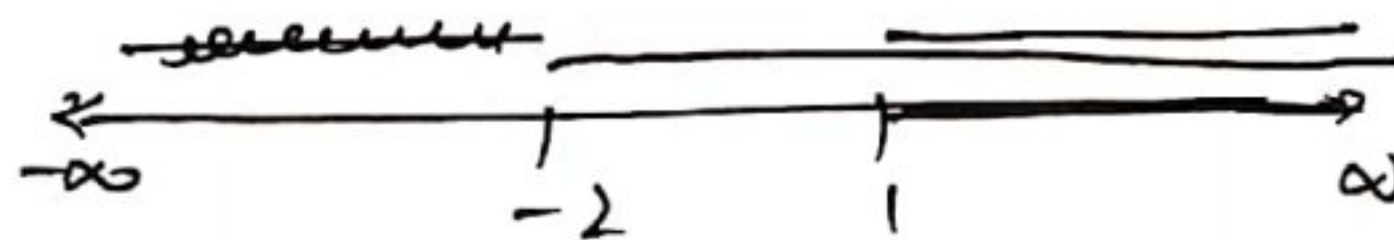
$\therefore \quad x \in [1, \infty)$ - - - ①

Case II $|x-1| = -(x-1)$; $x-1 < 0$; $\boxed{x < 1}$

$\therefore \quad \dfrac{-(x-1) - x - 2}{x+2} < 0$ and $x < 1$

$\Rightarrow \quad \dfrac{-x + 1 - x - 2}{x+2} < 0$ "

$\Rightarrow \quad \dfrac{-2x - 1}{x + 2} < 0$ "

$\Rightarrow \quad \dfrac{2x + 1}{x+2} > 0$ and $x < 1$

$$\underset{-\infty \qquad -2 \qquad -1/2 \qquad \infty}{\xleftarrow{\hspace{4cm}}}$$

$x \in (-\infty, -2) \cup (-1/2, \infty)$ and $x < 1$

$$\underset{-\infty \quad -2 \quad -1/2 \quad 1 \quad \infty}{\xleftarrow{\hspace{4cm}}}$$
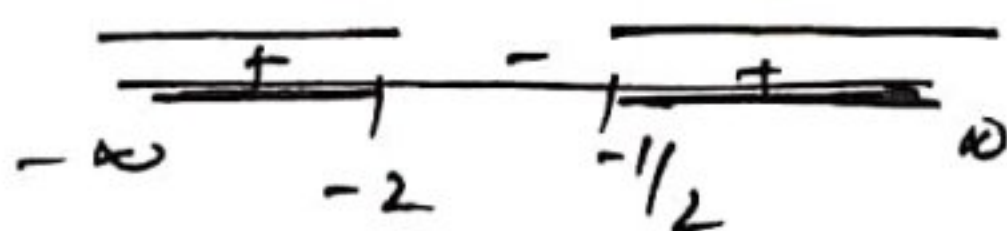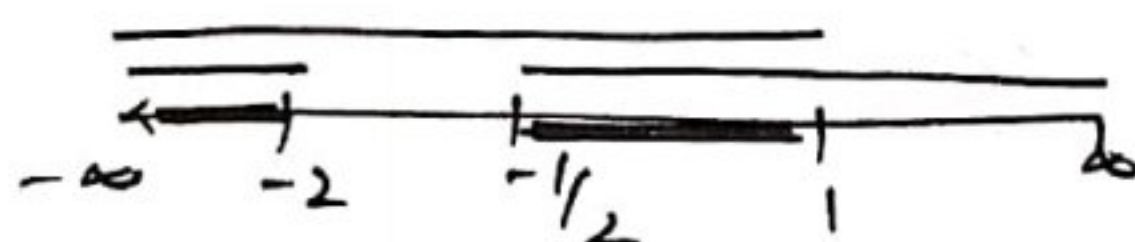
$\therefore \quad x \in (-\infty, -2) \cup (-1/2, 1)$ - - - - ②

Taking union of (1),(2)
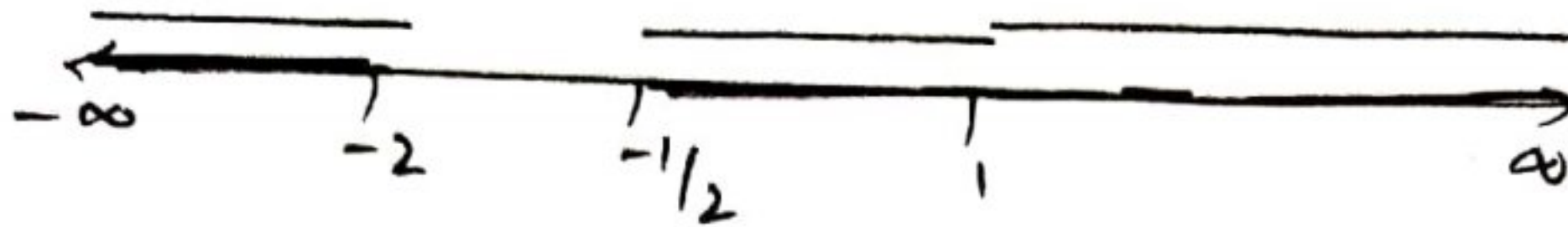
$\therefore x \in (-\infty, -2) \cup (-1/2, \infty)$ __Ans__

Qus 6

$$\left|\frac{3x-4}{2}\right| \le \frac{5}{12}$$

$$\Rightarrow \quad \frac{|3x-4|}{2} \le \frac{5}{12} \qquad \qquad ---\left\{\begin{array}{l}\left|\frac{a}{b}\right| = \frac{|a|}{|b|}\\ \text{and } |2| = 2\end{array}\right\}$$

$$\Rightarrow \quad |3x-4| \le \frac{5}{6}$$

$$\Rightarrow \quad -\frac{5}{6} \le 3x-4 \le \frac{5}{6}$$

$$\Rightarrow \quad -\frac{5}{6}+4 \le 3x \le \frac{5}{6}+4$$

$$\Rightarrow \quad \frac{19}{6} \le 3x \le \frac{29}{6}$$

$$\Rightarrow \quad \frac{19}{18} < x \le \frac{29}{18}$$

$$\therefore x \in \left[\frac{19}{18}, \frac{29}{18}\right) \quad \text{__Ans__}$$

Qur 7

$$\frac{|x+2|-x}{x} < 2$$

$$\Rightarrow \quad \frac{|x+2|-x}{x} - 2 < 0$$

$$\Rightarrow \quad \frac{|x+2|-x-2x}{x} < 0$$

$$\Rightarrow \quad \frac{|x+2|-3x}{x} < 0$$

Case I $|x+2| = (x+2)$; $x+2 \geqslant 0$; $\boxed{x \geqslant -2}$

$\therefore$ $\dfrac{x+2 - 3x}{x} < 0$

$\Rightarrow$ $\dfrac{-2x +2}{x} < 0$

$\Rightarrow$ $\dfrac{x-1}{x} > 0$ $- -$ $\left(\begin{array}{l}\text{divide by } (-2) \\ \text{sign change}\end{array}\right)$
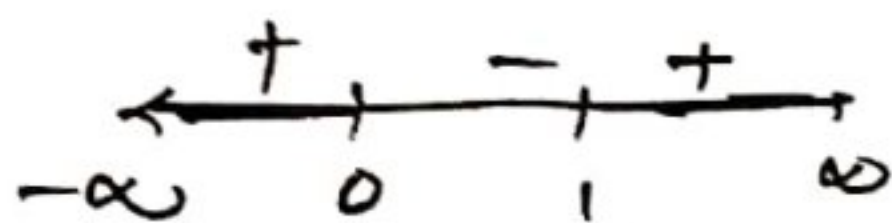
$x \in (-\infty, 0) \cup (1, \infty)$ and $\boxed{x \geqslant -2}$
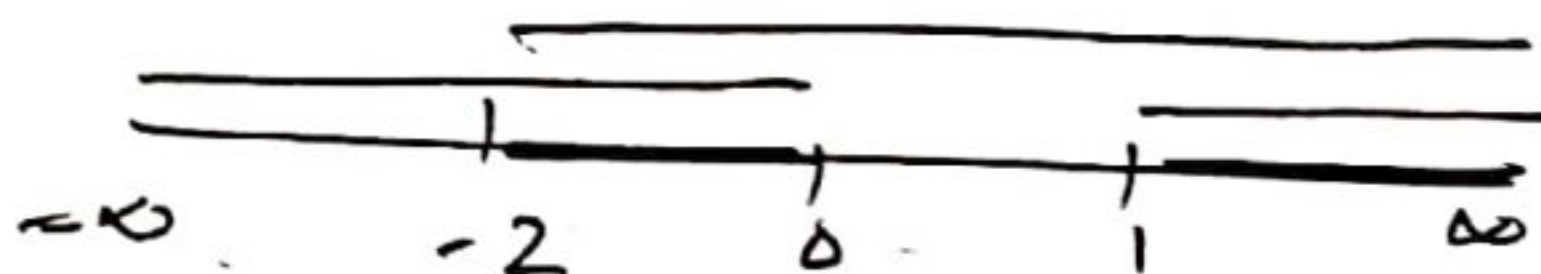
$\therefore$ $x \in [-2, 0) \cup (1, \infty)$ $- - \boxed{1}$

Case II $|x+2| = -(x+2)$; $x+2 < 0$; $\boxed{x < -2}$

$\Rightarrow$ $\dfrac{-x-2-3x}{x} < 0$

$\Rightarrow$ $\dfrac{-4x-2}{x} < 0$

$\Rightarrow$ $\dfrac{2x+1}{x} > 0$ $- - -$ $\left(\begin{array}{l}\text{divide by } (-2) \\ \text{sign change}\end{array}\right)$
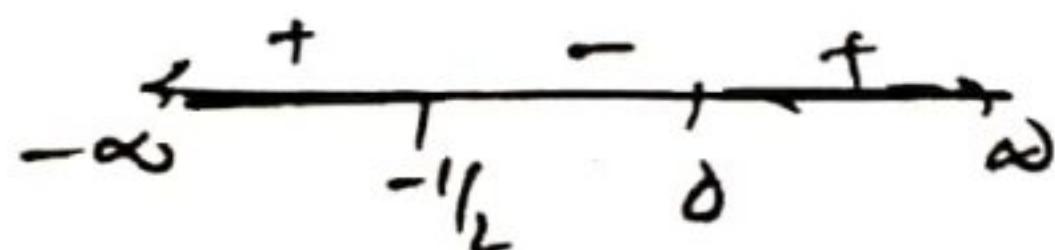
$x \in (-\infty, -\tfrac{1}{2}) \cup (0, \infty)$ and $\boxed{x < -2}$
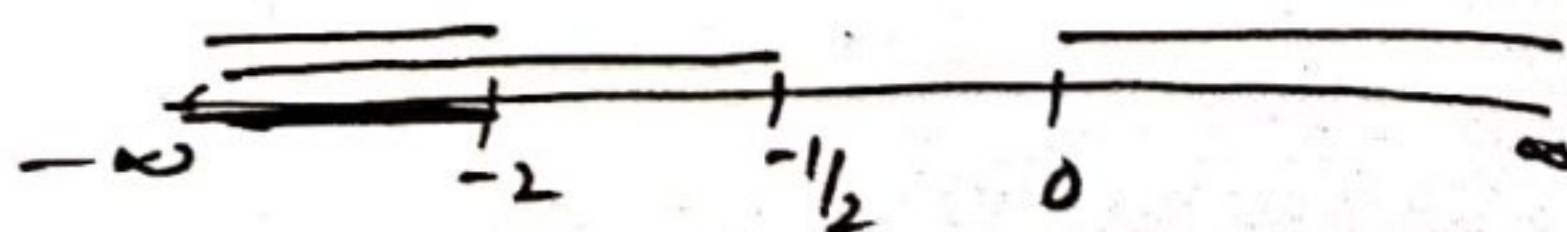
$\therefore$ $x \in (-\infty, -2)$ $- - \boxed{2}$

taking union of $(1), (2)$

$$\therefore \quad x \in (-\infty, 0) \cup (1, \infty) \quad \textcolor{red}{Ans}$$
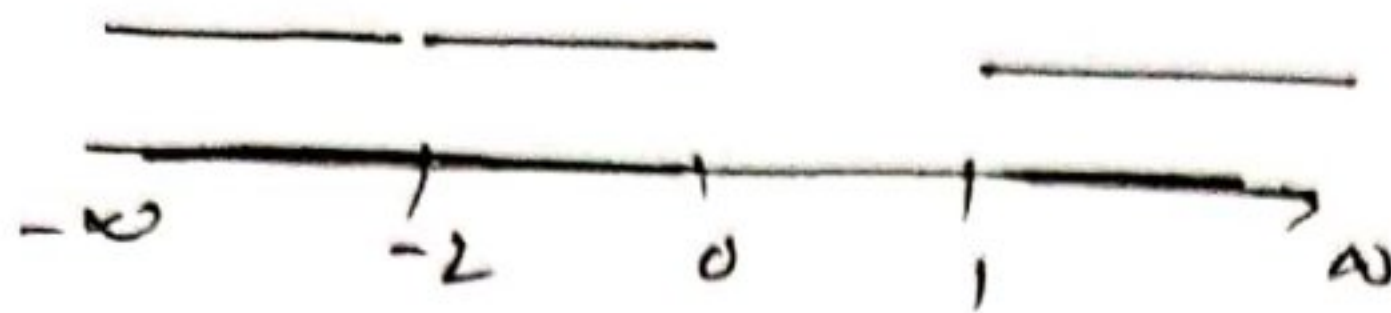
<u>Q.No 8</u>

$$\left| \frac{2}{x-4} \right| > 1$$

$$\Rightarrow \quad \frac{2}{|x-4|} > 1 \qquad - - - \left\{ \left| \frac{a}{b} \right| = \frac{|a|}{|b|} \right\}$$

$$\Rightarrow \quad 2 > |x-4| \qquad - - - \left\{ \begin{array}{l} \text{we can (Cross)multiply here} \\ \text{since } |x-4| \text{ is a} \\ \text{positive quanthy} \end{array} \right.$$

$$\Rightarrow \quad |x-4| < 2$$

$$-2 < x-4 < 2 \qquad \left\{ \begin{array}{l} \text{but } x-4 \neq 0 \\ \therefore x \neq 4 \end{array} \right.$$

$$\Rightarrow \quad 2 < x < 6 \qquad \text{as Denominator becomes 0}$$

$$\therefore \quad x \in (2,6) - \{4\} \qquad \textcolor{red}{Ans}$$

$$\textcolor{red}{-\times-}$$