ULTIMATE MATHEMATICS

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Chapter: COMPLEX NUMBERS

CLASS NO: 1

(·) $\sqrt{-4}$ Not real roots / Rejected / not possible

(·) $\boxed{\sqrt{-1} = i \text{ (iota)}}$ (imaginary)

eg $\sqrt{-4} = 2i$

$\sqrt{-100} = 10i$

(·)

$x-1=0$	$x^2-1=0$	$x^2+1=0$
$x=1$	$x^2=1 \Rightarrow x=\pm 1$	$x^2=-1$
		$x=\pm\sqrt{-1}$ (not real solution)
		$x=\pm i$

✓ (·) $i^2=-1$

✓ (·) $i^3=-i$ $\quad (i^3=i^2 \cdot i=(-1)i=-i)$

✓ (·) $i^4=1$ $\quad (i^4=i^2 \cdot i^2=(-1)(-1)=1)$

(·) $\dfrac{1}{i}=\dfrac{1}{i}\times\dfrac{i}{i}=\dfrac{i}{i^2}=\dfrac{i}{-1}=-i$

✓ $\boxed{\dfrac{1}{i}=-i}$

eg $i^{18}=i^2=-1$ $\quad \left\{ i^{18}=i^{16} \cdot i^2=(i^4)^4 \cdot i^2 = (1)^4 \cdot i^2 = i^2 \right\}$

$i^{23}=i^3=-i$

$i^{201}=i$

$i^{60}=1$

Complex (class No: 1)

(·) $i^{-18} = \frac{1}{i^{18}} = \frac{1}{i^2} = \frac{1}{-1} = -1$

(·) $i^{-43} = \frac{1}{i^{43}} = \frac{1}{i^3} = \frac{-1}{i} \times \frac{i}{i} = \frac{-i}{i^2} = \frac{-i}{-1} = i$

Qn 1 find $(1-i)^4$

$$= \left((1-i)^2\right)^2$$
$$= (1+i^2-2i)^2$$
$$= (\cancel{1}-\cancel{1}-2i)^2$$
$$= 4i^2$$
$$= 4(-1) = -4$$

Qn. 2 find $i^n + i^{n+1} + i^{n+2} + i^{n+3}$

Sol $i^n + i^n \cdot i + i^n \cdot i^2 + i^n \cdot i^3$

$$= i^n(1+i+i^2+i^3)$$
$$= i^n(\cancel{1}+\cancel{i}-\cancel{1}-\cancel{i})$$
$$= i^n(0) = 0 \text{ Ans}$$

Qn 3 find $1+i^2+i^4+i^6+\ldots\ldots i^{200}$

Soln $\underbrace{1-1+1-1+\ldots\ldots+1}_{100 \text{ term}}$

$= 1$ Ans

Qn 4 $\left\{i^{18} + \left(\frac{1}{i}\right)^{25}\right\}^3$

Sol $\left\{i^2 + \frac{1}{i}\right\}^3$

$$= (-1-i)^3$$

$$= -(1+i)^3$$

$$= -(1+i^3+3i+3i^2)$$

$$= -(1-i+3i-3)$$

$$= -(-2+2i)$$

$$= 2-2i \quad \underline{\underline{\text{Ans}}}$$

Complex number:

(·) $Z = a + ib$ (STANDARD form)

$a \rightarrow$ Real number, $ib \rightarrow$ Imaginary number

(·) Real part $Re(z) = a$

(·) Imaginary part $Im(z) = b$

(·) eg $z = 3-4i$

$Re(z) = 3$ & $Im(z) = -4$

(·) eg $z = 3i$ (purely Imaginary complex no.)

$z = 0+3i$

$\underline{\underline{Re(z) = 0}}$ & $Im(z) = 3$

(·) eg $z = 4$ (purely Real complex No.)

$z = 4+0i$

$Re(z) = 4$ $\underline{\underline{Im(z) = 0}}$

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Complex (class No: 1) (4)

(1) Operation on Complex numbers

(1) Addition of two complex no's

let $z_1 = 2+3i$ & $z_2 = 3-5i$

$z_1+z_2 = (2+3i)+(3-5i) = 5-2i$

(2) Subtraction

$z_1-z_2 = (2+3i)-(3-5i) = -1+8i$

(3) multiply

$z_1 z_2 = (2+3i)(3-5i) = 6-10i+9i-15i^2$

$= 6-10i+9i+15$

$= 21-i$

(4) division $\frac{z_1}{z_2} = \frac{2+3i}{3-5i}$

$= \frac{2+3i}{3-5i} \times \frac{3+5i}{3+5i}$

$= \frac{6+10i+9i+15i^2}{9-25i^2}$

$= \frac{-9+19i}{34}$ (separate)

$\frac{z_1}{z_2} = \frac{-9}{34} + \frac{19i}{34}$

(5) multiplicative Inverse (Reciprocal)

eg $z = 3-4i$

$\frac{1}{z} = \frac{1}{3-4i} \times \frac{3+4i}{3+4i} = \frac{3+4i}{9-16i^2} = \frac{3}{25} + \frac{4i}{25}$

CLASSTIME

Complex (class No. 1)

(6) Conjugate of a complex number

$z = a+ib$

$\bar{z} = a-ib$

eg $z = -3-4i$

$\bar{z} = -3+4i$

Property

(·) $\overline{(\bar{z})} = z$

(·) $\overline{z_1+z_2} = \bar{z}_1+\bar{z}_2$

(·) $\overline{z_1-z_2} = \bar{z}_1-\bar{z}_2$

(·) $\overline{z_1 z_2} = \bar{z}_1 \cdot \bar{z}_2$

(·) $\overline{\left(\frac{z_1}{z_2}\right)} = \frac{\bar{z}_1}{\bar{z}_2}$

(7) Modulus of a complex number

$z = a+ib$

$|z| = \sqrt{a^2+b^2}$

eg $z = 3-4i$

$|z| = \sqrt{9+16} = 5$

Prop $|z_1 z_2| = |z_1|\,|z_2|$

$\left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}$

$|z_1 \pm z_2| \neq |z_1| \pm |z_2|$

$\boxed{z\bar{z} = |z|^2}$

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Complex No: (Class No: 1) (6)

(8) Equality of two complex numbers

$z_1 = z_2$

$a+ib = c+id$

$a=c$ & $b=d$

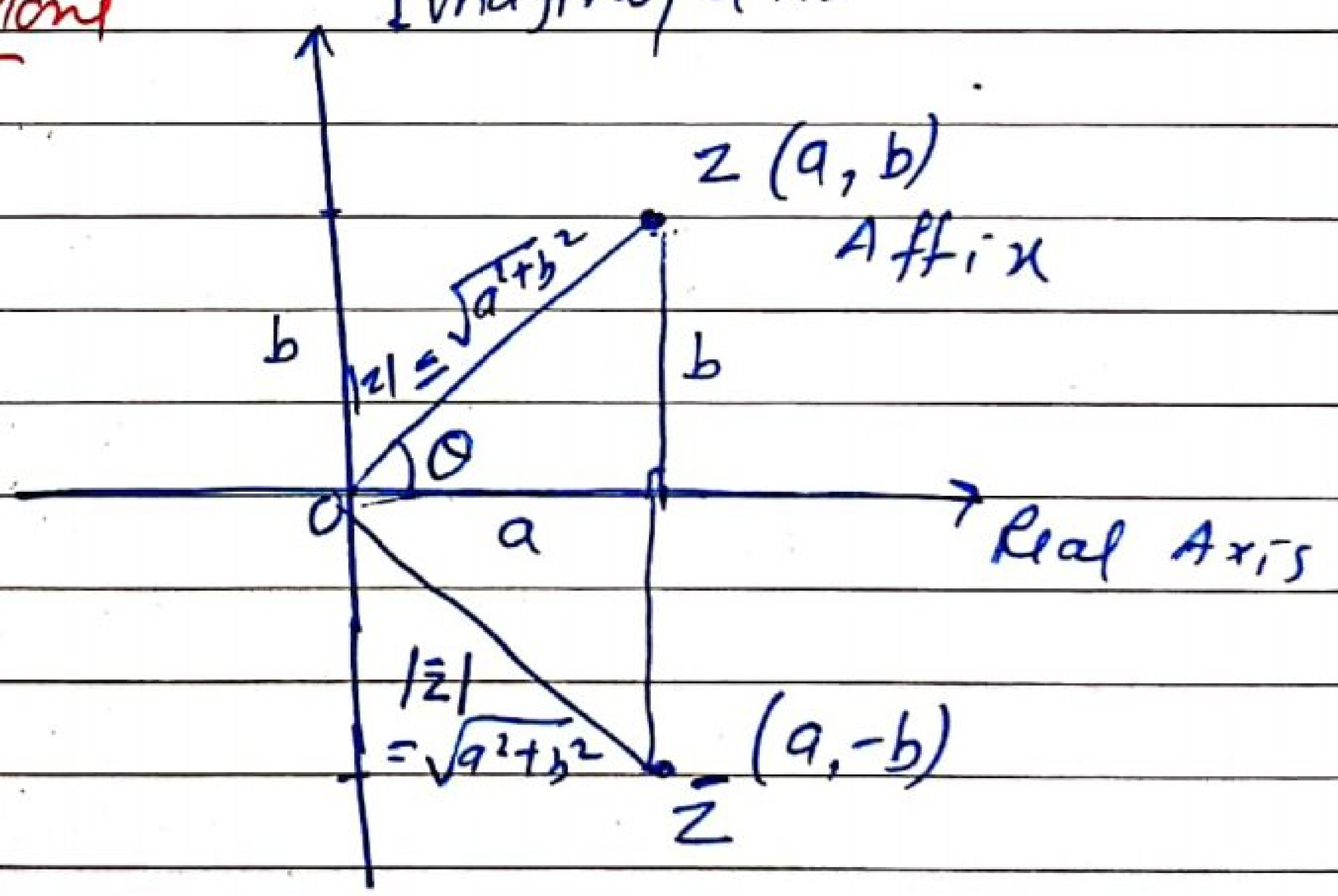
eg If $x-iy = (2+3i)^2$
find value of x & y

$\Rightarrow x-iy = 4+9i^2+12i$

$x-iy = -5+12i$

$\Rightarrow x=-5$ & $y=-12$

(9) ARGAND plane



$z = a+ib$

$\bar{z} = a-ib$

(i) $\boxed{|z| = |\bar{z}|}$

Property

Property $\boxed{|z|^2 = z\bar{z}}$

let $z = a+ib$

$|z| = \sqrt{a^2+b^2}$

$|z|^2 = a^2+b^2$

$\bar{z} = a-ib$

$z\bar{z} = (a+ib)(a-ib)$

$z\bar{z} = a^2-i^2b^2$

$z\bar{z} = a^2+b^2 = |z|^2$

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Class No: 1 (Complex No.) (7)

Q.No 1 Convert in to standard form and then find Conjugate

$$z = \left(\frac{1}{1-4i} - \frac{2}{1+i}\right)\left(\frac{3-4i}{5+i}\right)$$

Sol:

$$z = \left(\frac{1}{1-4i} - \frac{2}{1+i}\right)\left(\frac{3-4i}{5+i}\right)$$

$$z = \frac{(1+i-2+8i)}{(1-4i)(1+i)}\left(\frac{3-4i}{5+i}\right)$$

$$z = \frac{(-1+9i)\,(3-4i)}{(1+i-4i-4i^2)\,(5+i)}$$

$$z = \frac{-3+4i+27i-36i^2}{(5-3i)(5+i)}$$

$$z = \frac{33+31i}{25+5i-15i-3i^2}$$

$$z = \frac{33+31i}{28-10i} \times \frac{28+10i}{28+10i}$$

$$z = \frac{924+330i+868i+310i^2}{(28)^2-100i^2}$$

$$z = \frac{614+1198i}{884}$$

$$z = \frac{307+599i}{442}$$

$$z = \frac{307}{442} + \frac{599i}{442} \text{ Ans}$$

$$\bar{z} = \frac{307}{442} - \frac{599i}{442} \text{ Ans}$$

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Ques 2 Find θ such that $\frac{3+2i\sin\theta}{1-2i\sin\theta}$ is purely Real.

Soln

Let $z = \frac{3+2i\sin\theta}{1-2i\sin\theta}$

$$z = \frac{3+2i\sin\theta}{1-2i\sin\theta} \times \frac{1+2i\sin\theta}{1+2i\sin\theta}$$

$$z = \frac{3+6i\sin\theta+2i\sin\theta+4i^2\sin^2\theta}{1-4i^2\sin^2\theta}$$

$$z = \frac{(3-4\sin^2\theta)+8i\sin\theta}{1+4\sin^2\theta}$$

$$z = \frac{3-4\sin^2\theta}{1+4\sin^2\theta} + \frac{8i\sin\theta}{1+4\sin^2\theta}$$

Since z is purely Real (given)

$\therefore Im(z) = 0$

$$\frac{8\sin\theta}{1+4\sin^2\theta} = 0$$

$8\sin\theta = 0$

$\sin\theta = 0$

$\boxed{\theta = n\pi}$; $n \in Z$

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WORKSHEET NO:-1

COMPLEX NUMBERS

Qn. 1 → Convert in to standard form

$$z = \frac{(3-2i)(2+3i)}{(1+2i)(2-i)}$$

Ans $\frac{63}{25} - \frac{16}{25} i$

Qn. 2 → Convert in to standard form and find its Modulus

$$\left(\frac{1}{1-2i} + \frac{3}{1+i}\right)\left(\frac{3+4i}{2-4i}\right)$$

Ans $z = \frac{1}{4} + \frac{9}{4} i$; $|z| = \frac{\sqrt{82}}{4}$

Qn. 3 → Find real values of x and y if

$$(1-i)x + (1+i)y = 1-3i$$

Ans $x=2, y=-1$

Qn. 4 → Find values of x & y if $\frac{x-1}{3+i} + \frac{y-1}{3-i} = i$

Ans $x=-4$, $y=6$

Qn. 5 → Find values of θ for which complex number $\frac{1+i\cos\theta}{1-2i\cos\theta}$ is purely real

Ans $\theta = (2n+1)\frac{\pi}{2}$; $n \in Z$

Qn. 6 → Find multiplicative Inverse of $z = (1+i\sqrt{3})^2$

Ans $\frac{1}{z} = -\frac{1}{8} - i\frac{\sqrt{3}}{8}$

Qn. 7 → If $z_1 = 2-i$; $z_2 = 1+i$ find $\left|\frac{z_1+z_2+1}{z_1-z_2+i}\right|$

Ans $= 2\sqrt{2}$

Qn. 8 → If $z_1 = 2-i$; $z_2 = -2+i$ find

(i) $Re\left(\frac{z_1 z_2}{\bar{z}_1}\right)$ (ii) $Im\left(\frac{1}{z_1\bar{z}_1}\right)$

Ans (i) $-\frac{2}{5}$ (ii) 0

— x —

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