!! जय श्री गिरिराज जी महाराज जय श्री राधे कृष्णा !!

ULTIMATE MATHEMATICS: BY AJAY MITTAL

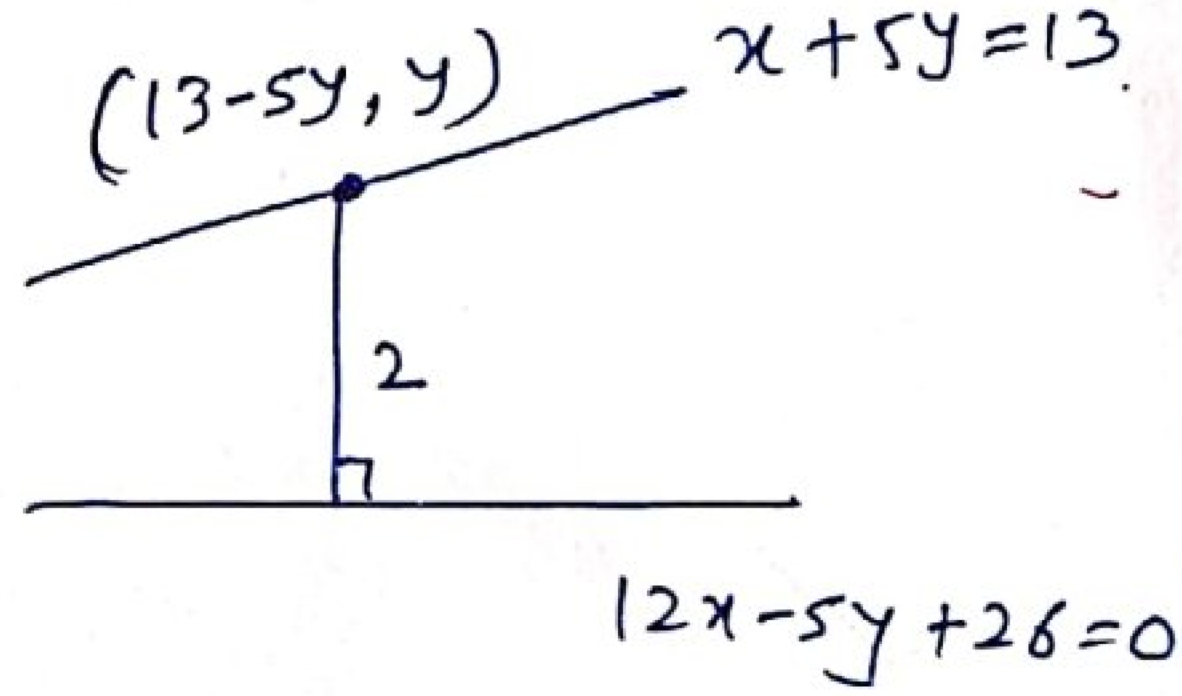
REVISION: STRAIGHT LINES

CLASS NO: 3

Que. 1 (i) Find the coordinates of the points P and Q on the line $x+5y=13$ which are at a distance of 2 units from the line $12x-5y+26=0$.

(ii) Also find the coordinates of the point (P & Q) on the line joining A(−2, 5) and B(3, 1) such that AP = PQ = QB

Solution



Distance b/w point and line

$$= \frac{|ax_1+by_1+c|}{\sqrt{a^2+b^2}}$$

$$\Rightarrow 2 = \frac{|12(13-5y)-5y+26|}{\sqrt{144+25}}$$

$$\Rightarrow 26 = |156-60y-5y+26|$$

$$\Rightarrow 26 = |182-65y|$$

$$\Rightarrow 26 = 182-65y$$
$$\Rightarrow 65y = 182-26$$
$$\Rightarrow 65y = 156$$
$$\Rightarrow y = \frac{156}{65}$$
$$\Rightarrow y = \frac{12}{5}$$

$$-26 = 182-65y$$
$$65y = 208$$
$$y = \frac{208}{65}$$
$$y = \frac{16}{5}$$

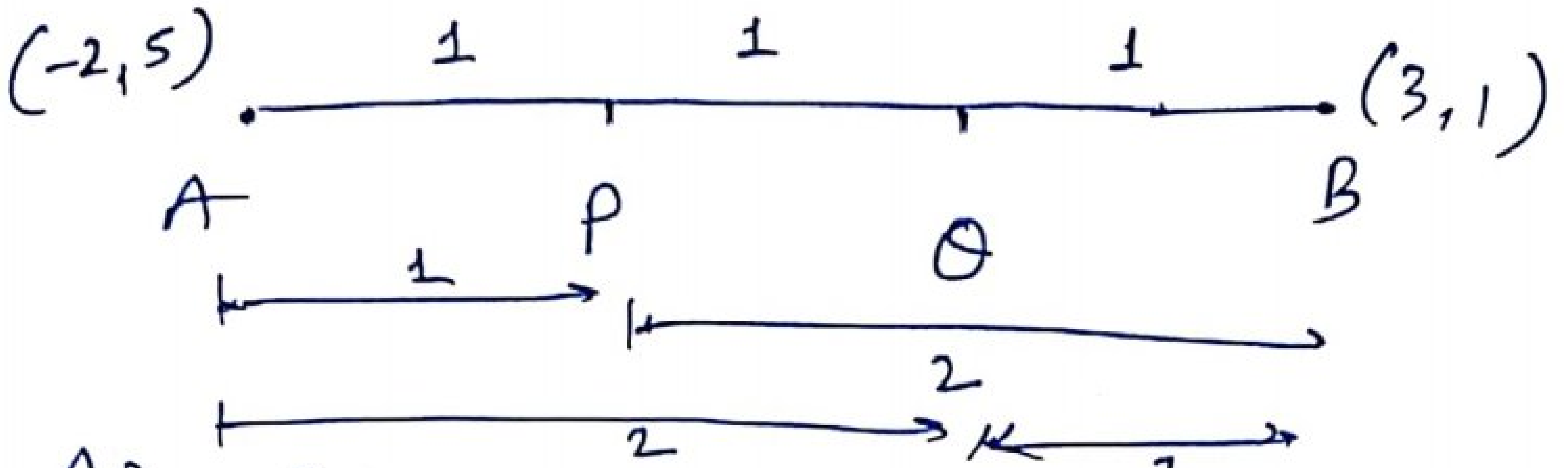
put in $x+5y=13$

when $y=\frac{12}{5}$ then $x=1$

when $y=\frac{16}{5}$, then $x=-3$

$\therefore P\left(1,\frac{12}{5}\right)$ & $Q\left(-3,\frac{16}{5}\right)$

(ii)



AP = PQ = QB

point P divides AB in the ratio 1:2

and point Q divides AB in the ratio 2:1

By section formula

Coordinate of point P is $\left(\frac{3-4}{1+2}, \frac{1+10}{1+2}\right) = P\left(\frac{-1}{3},\frac{11}{3}\right)$

& Coordinates of point Q is $\left(\frac{6-2}{1+2}, \frac{2+5}{1+2}\right) = Q\left(\frac{4}{3},\frac{7}{3}\right)$ Ans

Qn. 2 → Find the value of λ, so that the line

$(2x+3y+4)+\lambda(6x-y+12)=0$ is

(i) parallel to y-axis

(ii) $\perp^r$ to the line $7x+y-4=0$

(iii) passes through $(1,2)$

(iv) parallel to x-axis

Solution: given equation of line

$$(2x+3y+4) + \lambda(6x - y + 12) = 0$$

$$\Rightarrow \quad x(2+6\lambda) + y(3-\lambda) + (4+12\lambda) = 0$$

slope of this line $= -\dfrac{(2+6\lambda)}{3-\lambda}$

(i) slope of $y - ax = 0$ $\perp$

Since parallel

$\therefore$

$$-\frac{(2+6\lambda)}{3-\lambda} = \frac{1}{0}$$

$$\Rightarrow \quad 3-\lambda = 0 \quad \Rightarrow \boxed{\lambda = 3} \text{ Ans}$$

ii) given line: $7x + y - 4 = 0$

slope of this line $= -\frac{7}{1} = -7$

Since $\perp^r$

$$\therefore \left(-\frac{(2+6\lambda)}{3-\lambda}\right)(-7) = -1$$

$$\Rightarrow \quad 14 + 42\lambda = -3 + \lambda$$

$$\Rightarrow \quad 41\lambda = -17$$

$$\Rightarrow \boxed{\lambda = \frac{-17}{41}} \text{ Ans}$$

(iii) passes through $(1,2)$

$\therefore$ it satisfy the given equation

$$1(2+6\lambda) + 2(3-\lambda) + 4 + 12\lambda = 0$$

$$\Rightarrow \quad 16\lambda + 12 = 0 \quad \Rightarrow \boxed{\lambda = \frac{-3}{4}} \text{ Ans}$$

(iv) Slope of $x - a + 11 = 0$

Since parallel

$\therefore$

$$-\frac{(2+6\lambda)}{3-\lambda} = 0$$

$$\Rightarrow \boxed{\lambda = -1/3} \text{ Ans}$$

Ques 3 → Find the equation of the line passing through the intersection of the lines $2x-3y=0$ and $4x-5y=2$

(i) perpendicular to the line $x+2y+1=0$

(ii) passing through the ~~lo~~ point $(2,1)$

(iii) Parallel to the line $3x-4y+5=0$

(iv) equally inclined to the Axes

Solution given lines $2x-3y=0$ and $4x-5y=2$

Solving these equation

we get $x=3$ & $y=2$

$\therefore$ Int. point is $(3,2)$

(i) given line: $x+2y+1=0$

Slope of this line $= -1/2$

Since required line is $\perp^r$ to this line

$\therefore$ Slope of Required line $= 2$ {-ve reciprocal}

By point slope form

$$y-2 = 2(x-3)$$

$\Rightarrow y - 2 = 2x - 6$

$\Rightarrow \boxed{2x - y - 4 = 0}$ Ans

(ii) passing through $(2,1)$
already one point is $(3,2)$
By two point form

$$y - 1 = \frac{2-1}{3-2}(x-2)$$

$\Rightarrow$

$$y - 1 = x - 2$$

$\Rightarrow \boxed{x - y - 1 = 0}$ Ans

(iii) given line: $3x - 4y + 5 = 0$

Slope of this line $= \frac{-3}{-4} = \frac{3}{4}$

Since parallel

$\therefore$ slope of required line $= 3/4$

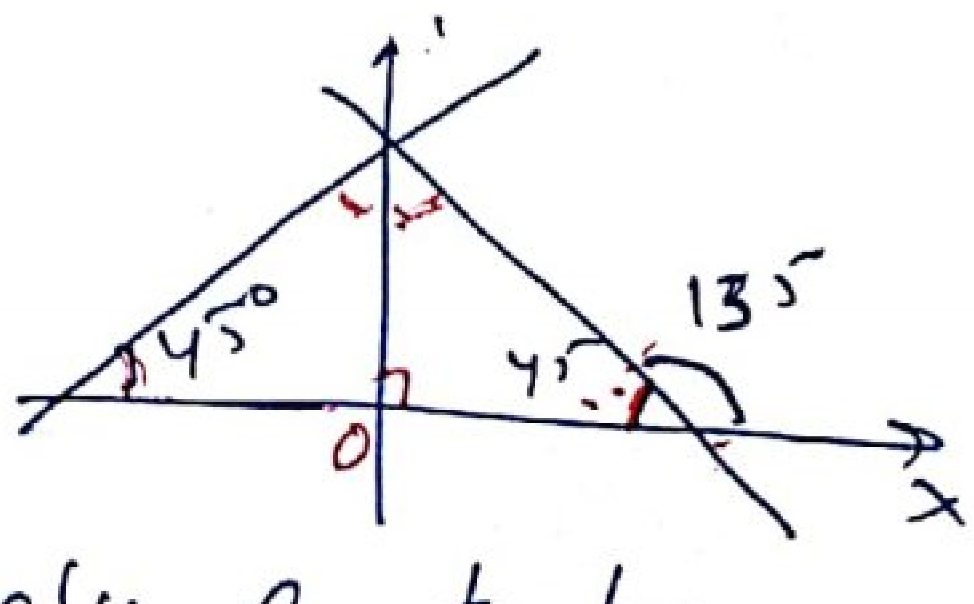
By point slope form

$$y - 2 = \frac{3}{4}(x-3)$$

$\Rightarrow 4y - 8 = 3x - 9$

$\Rightarrow \boxed{3x - 4y - 1 = 0}$ Ans

(iv) Equally inclined to the axes



Note Any line which is equally inclined to the axes, then $\theta = 45°$ or $135°$

$\therefore \boxed{\text{Slope} = \pm 1}$

$\Rightarrow m = \tan 45$
$m = \tan(135°)$

By point slope form

$y-2 = \pm 1(x-3)$

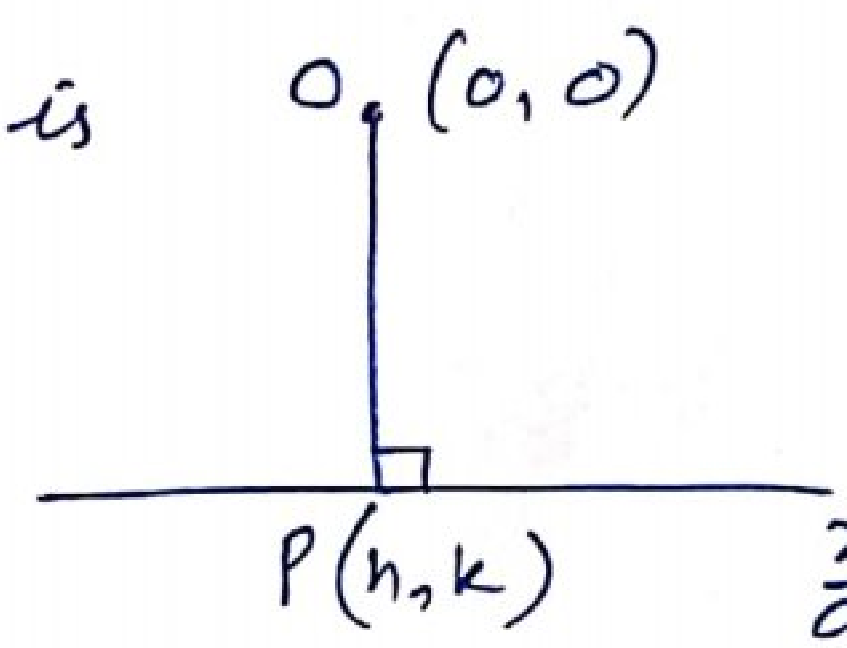
$y-2 = x-3$ | $y-2 = -x+3$

$\boxed{x-y-1=0}$ | $\boxed{x+y-5=0}$ Ans

Qn. 4 → The line $\frac{x}{a}+\frac{y}{b}=1$ moves in such a way that $\frac{1}{a^2}+\frac{1}{b^2}=\frac{1}{c^2}$, where c is a constant. Show that the locus of the foot of the perpendicular from the origin on the given line is $x^2+y^2=c^2$

Soln

let foot of $\perp^r$ is / moving point is $P(h,k)$



$\frac{x}{a}+\frac{y}{b}=1$

(or) $bx+ay-ab=0$

Now distance $OP = \sqrt{k^2+h^2}$

also distance $OP = \frac{|0+0-ab|}{\sqrt{b^2+a^2}}$

$\Rightarrow \sqrt{k^2+h^2} = \frac{|ab|}{\sqrt{b^2+a^2}}$

Squaring

$k^2+h^2 = \frac{a^2b^2}{b^2+a^2}$ ---(1)

given condition $\frac{1}{a^2}+\frac{1}{b^2}=\frac{1}{c^2}$

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$$\Rightarrow \quad \frac{b^2+a^2}{a^2b^2} = \frac{1}{c^2}$$

$$\Rightarrow \quad \frac{a^2b^2}{b^2+a^2} = c^2 \quad \text{--- (2)}$$

from (1) & (2)

$$k^2+h^2 = c^2$$

$$\Rightarrow \quad y^2+x^2 = c^2$$

$$\Rightarrow \quad \boxed{x^2+y^2=c^2}$$ is the required equation of locus

(circle centre $(0,0)$ Rad $= c$)

Q.No 5 → (i) If a, b, c are in A.P, then the straight lines $ax+by+c=0$ will always pass through ———

(ii) The points $(3,4)$ & $(2,-6)$ are situated on the ~~line~~ ——— of the line $3x-4y-8=0$

(iii) Equations of diagonals of the square formed by the lines $x=0$, $y=0$, $x=1$, $y=1$

Soln (i) a, b, c are in AP

i.e. $2b = a+c$

Given line $ax+by+c=0$

$$\Rightarrow \quad ax+by+(2b-a)=0 \quad \text{---} \{\because 2b=a+c\}$$

$$\Rightarrow \quad a(x-1)+b(y+2)=0$$

clearly $x=1$ & $y=-2$ satisfy this equation

∴ line must passes through the point $(1,-2)$ Ans

(ii) given line

$$3x-4y-8=0$$

given points $(3,4)$ & $(2,-6)$

put point $(3,4)$ in LHS

$$6-16-8 = -18 = -ve$$

put point $(2,-6)$ in LHS

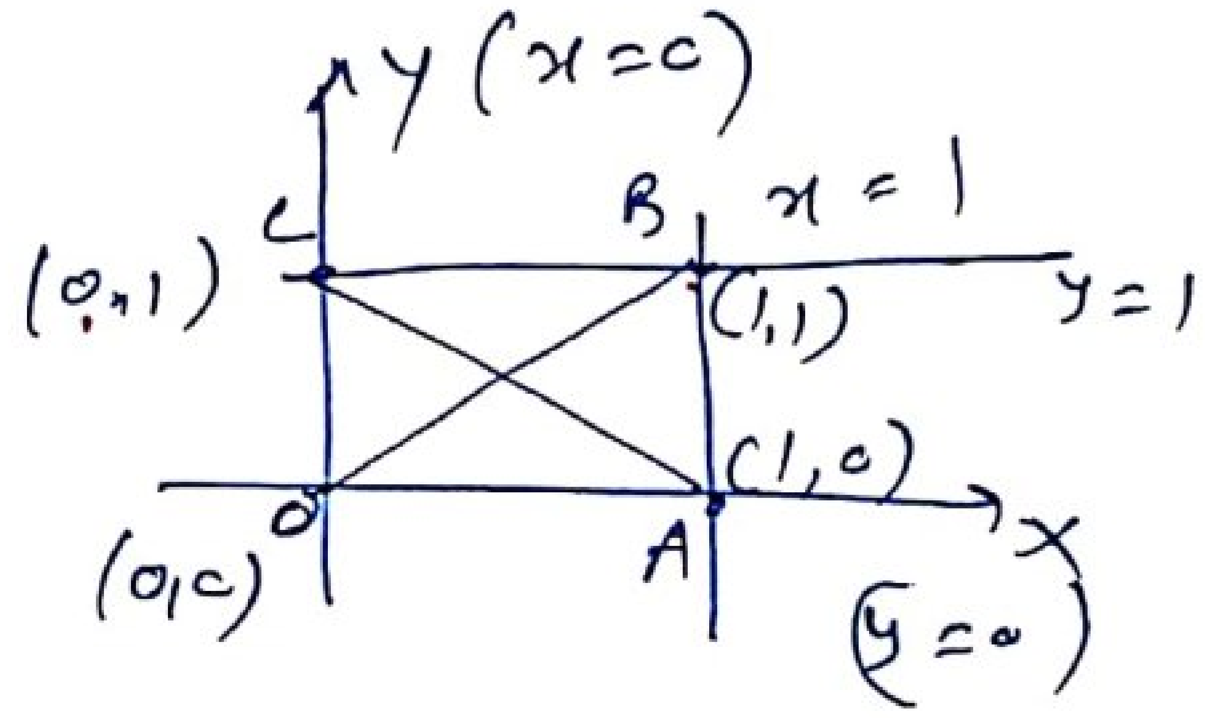
$$6+24-8 = 22 = +ve$$

opposite sign

Since they gives opposite signs

∴ the given points lies on the opposite sides of the given line Ans

(iii) from figure



$O(0,0)$, $B(1,1)$ & $C(0,1)$ & $A(1,0)$

By Two point form

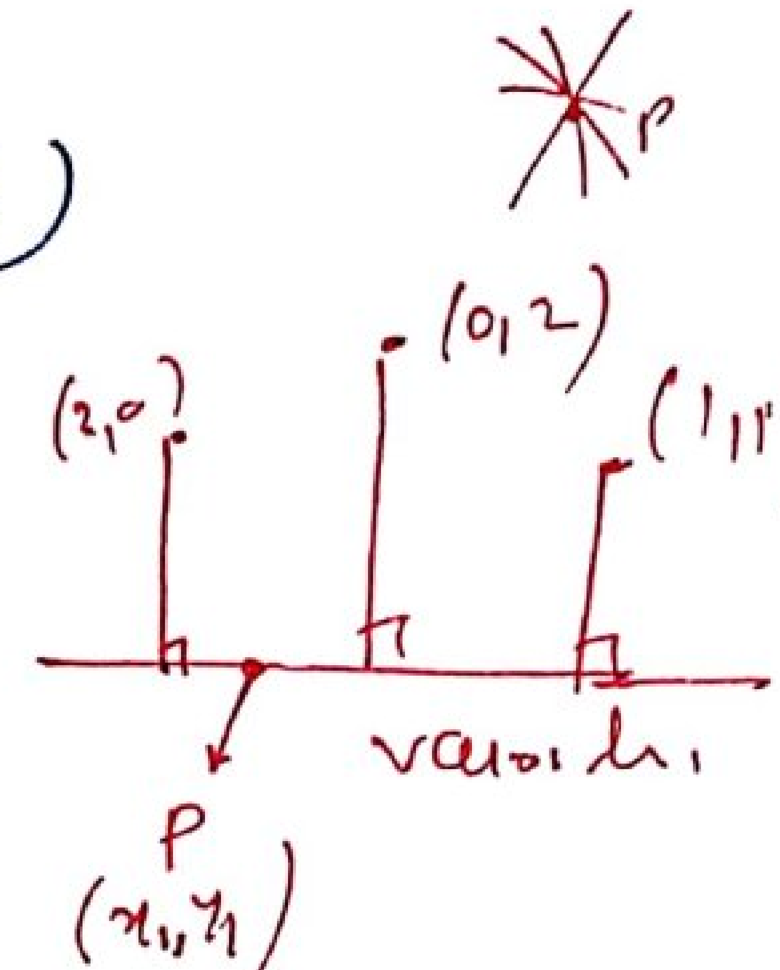
eq. of OB $\Rightarrow$ $y-0=\frac{1}{1}(x-0)$ $\Rightarrow$ $\boxed{y=x}$ Ans

eq of AC $\Rightarrow$ $y-0=\frac{1}{-1}(x-1)$

$\Rightarrow$ $y=-x+1$ $\Rightarrow$ $\boxed{x+y=1}$ Ans

Q no 6 → A variable line passes through a fixed point P. The algebraic sum of the perpendiculars drawn from the points (2, 0), (0, 2) & (1, 1) on the line is zero. Find the coordinates of point P

Solution Let the fixed point is $P(x_1, y_1)$



Let slope of variable line $= m$

By point slope form

Equation of variable line is

$$y - y_1 = m(x - x_1)$$

$$\Rightarrow \quad mx - y - mx_1 + y_1 = 0 \quad \text{--- (general form)}$$

Given that algebraic sum of the ⊥r drawn from the points (2, 0), (0, 2) & (1, 1) on this line = 0

$$\Rightarrow \left(\frac{2m - 0 - mx_1 + y_1}{\sqrt{m^2+1}}\right) + \left(\frac{0 - 2 - mx_1 + y_1}{\sqrt{m^2+1}}\right) + \left(\frac{m - 1 - mx_1 + y_1}{\sqrt{m^2+1}}\right) = 0$$

(don't use Modulus here: Reason: algebraic sum)

$$\Rightarrow \quad 3m - 3mx_1 + 3y_1 - 3 = 0$$

$$\Rightarrow \quad m - mx_1 + y_1 - 1 = 0$$

$$\Rightarrow \quad y_1 - 1 = mx_1 - m$$

$$\Rightarrow \quad \boxed{y_1 - 1 = m(x_1 - 1)}$$

Since line is variable

$\therefore$ m is also variable

to satisfy this equation for all values of m

x_1 & y_1 should be equal to 1

i.e. $x_1 = 1$ & $y_1 = 1$

$\therefore$ $\boxed{\text{fixed point } P(1,1)}$ Ans

Qu 7 → A point equidistant from the lines $4x+3y+10=0$, $5x-12y+26=0$ & $7x+24y-50=0$ is

(A) $(1,-1)$ (B) $(1,1)$ (C) $(0,0)$ (D) $(0,1)$

Soln

let required point is $P(a,b)$

given that,

$$\frac{|4a+3b+10|}{\sqrt{16+9}} = \frac{|5a-12b+26|}{\sqrt{25+144}} = \frac{|7a+24b-50|}{\sqrt{49+576}}$$

$$\Rightarrow \frac{|4a+3b+10|}{5} = \frac{|5a-12b+26|}{13} = \frac{|7a+24b-50|}{25}$$

we have to check every given option to satisfy these equations

(0,0) satisfy this

Since $\frac{|0+0+10|}{5} = \frac{|0-0+26|}{13} = \frac{|0+0-50|}{25}$

$\Rightarrow 2 = 2 = 2$

$\therefore$ (0,0) (C) Ans

REVISION : STRAIGHT LINE (WORKSHEET NO:- 2)

Qn 1 → Show that the line joining the point (3,5) to the point of Intersection of the lines $4x+y-1=0$ and $7x-3y-35=0$ is equidistant from the point (0,0) and (8,34)

Qn. 2 → The vertex of an equilateral triangle is (2,3) and the equation of the opposite side is $x+y=2$. Find the other two sides

ANS $y-3=(2\pm\sqrt{3})(x-2)$

Qn 3 → Find the locus of the mid point of the portion of the line $x\sin\theta + y\cos\theta = p$ ~~intercepted~~ Intercepted b/w the axes

Ans $4x^2y^2 = p^2(x^2+y^2)$

Qn 4 → A point moves so that square of its distance from the point (3,-2) is numerically equal to its distance from the line $5x-12y=3$. Find the equation of the locus

Ans $13x^2+13y^2-83x+64y+182=0$

Qn 5 → A line passes through (2,2) and is perpendicular to the line $3x+y=3$. Find its y-intercept

Ans $4/3$

Qn 6 → The point (4,1) undergoes the following two Successive transformations:

(i) Reflection about the line $y = x$

(ii) Translation through a distance 2 units along the +ve X-axis. Find the final coordinates of the point

Ans $(3,4)$

HINT First find image of point $(4,1)$ in the line $x = y$ (or) $x - y = 0$

then shift this image 2 units ~~towards~~ (in X-axis direction) (y-coordinate remaining same)

Q.N. 7 ★ For what values of a & b the intercepts cut off on the coordinate axes by the line $ax + by + 8 = 0$ are equal in length but opposite in signs to those cut off by the line $2x - 3y + 6 = 0$ on the axes

Ans $a = \frac{-8}{3}$, $b = 4$

Q.N. 8 ★ Find the angle b/w the lines
$y = (2-\sqrt{3})(x+5)$ and $y = (2+\sqrt{3})(x-7)$

Ans $60°$ or $120°$

Q.N. 9 ★ Show that the tangent of an angle b/w the lines $\frac{x}{a} + \frac{y}{b} = 1$ and $\frac{x}{a} - \frac{y}{b} = 1$ is $\frac{2ab}{a^2 - b^2}$

Q.N. 10 ★ Find the equation of line which passes through the point $(1,-2)$ & cut off equal equal intercepts from the axes Ans $x + y + 1 = 0$