ULTIMATE MATHEMATICS

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CHAPTER: SEQUENCE & SERIES

CLASS NO: 1

Sequence
1, 2, 3, 4, - - -

Series
1+2+3+4, - -

A.P

(·) $a, a+d, a+2d, a+3d \ldots\ldots n$th term

(·) Common difference $d = a_2 - a_1 = a_3 - a_2$

(·) nth term / general term of AP

$a_n = a+(n-1)d$

$a_6 = a+5d \quad ; \quad a_8 = a+7d$

(·) always subtract

(·) Sum of n terms of A.P

$S_n = \frac{n}{2}\left(2a+(n-1)d\right)$

$S_n = \frac{n}{2}(a+l)$

Imp (·) If a, b, c are in A.P then $\boxed{2b = a+c}$

Reason:

$b - a = c - b$

$\Rightarrow 2b = a+c$

(·) Selection of three terms in AP

$a-d, a, a+d$ (use only when sum of three terms in AP is given)

Five terms

$a-2d, a-d, a, a+d, a+2d$

Four terms:

$a-3d, a-d, a+d, a+3d$

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(·) Arithmetic Mean (A.M)

$a, A, b \to A.P$

$\Rightarrow 2A = a+b$

$\Rightarrow \boxed{A = \frac{a+b}{2}}$

(·) Arithmetic Means: (AM's)

$a, A_1, A_2, A_3, \ldots A_n, b \rightsquigarrow A.P$

$\boxed{d = \frac{b-a}{n+1}}$; where $n \to$ No. of A.M's to be Inserted

$A_1 = a+d$; $A_2 = a+2d$, $A_3 = a+3d, \ldots$

Reason $b = a_{n+2}$

$\Rightarrow b = a+(n+1)d$

$\Rightarrow b-a = (n+1)d$

$\Rightarrow d = \frac{b-a}{n+1}$

Property: let $a_1, a_2, a_3, \ldots \ldots$ Ap.

$$\begin{cases} a_1+k,\ a_2+k,\ a_3+k \ldots & Ap \\ a_1-k,\ a_2-k,\ a_3-k, \ldots & Ap \\ a_1k,\ a_2k,\ a_3k, \ldots & Ap \\ \frac{a_1}{k},\ \frac{a_2}{k},\ \frac{a_3}{k}, \ldots & Ap \end{cases}$$

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GEOMETRIC PROGRESSION (G.P)

(•) G.P $a, ar, ar^2, ar^3, ar^4, ---$ n term

(•) r = common ratio

$r = \frac{a_2}{a_1} = \frac{a_3}{a_2} = \frac{a_4}{a_3}$

(•) eg. $3, 6, 12, 24, ----$

$r = \frac{a_2}{a_1} = \frac{6}{3} = 2$; $r = \frac{a_3}{a_2} = \frac{12}{6} = 2$ $\Rightarrow r = 2$

eg $1, \frac{-1}{2}, \frac{1}{4}, \frac{-1}{8}, -----$

$\frac{a_2}{a_1} = \frac{\frac{-1}{2}}{1} = \frac{-1}{2}$; $\frac{a_3}{a_2} = \frac{\frac{1}{4}}{\frac{-1}{2}} = \frac{-1}{2}$ $r = -1/2$

(•) n^{th} term of G.P

$$\boxed{a_n = ar^{n-1}}$$

(•) $a_4 = ar^3$; $a_7 = ar^6$

(•) always divide

(•) Sum of n terms in GP

$$S_n = a\left(\frac{r^n - 1}{r - 1}\right) \quad : r > 1$$

$$S_n = a\left(\frac{1 - r^n}{1 - r}\right) \quad : r < 1$$

(•) If a, b, c are in GP

then $\boxed{b^2 = ac}$

reason $\frac{b}{a} = \frac{c}{b} \Rightarrow b^2 = ac$

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(·) Selection of terms in GP

Three terms $\frac{a}{r}, a, ar$

Five terms $\frac{a}{r^2}, \frac{a}{r}, a, ar, ar^2$

{Use only when product of three terms in GP is given)

Four terms $\frac{a}{r^3}, \frac{a}{r}, ar, ar^3$

$\boxed{\text{If product not given then let } a, ar, ar^2}$

(·) Geometric Mean (GM)

$a, \underline{G}, b \rightsquigarrow GP$

$\Rightarrow G^2 = ab$

$\Rightarrow \boxed{G = \sqrt{ab}}$

(·) Geometric Means (G-M's)

$a, G_1, G_2, G_3 \text{ - - - - - - - - - } G_n, b \longrightarrow G\text{-}P$

$\boxed{r = \left(\frac{b}{a}\right)^{\frac{1}{n+1}}}$ where $n \rightarrow$ No of Geometric Means to be inserted

$G_1 = ar \; ; \; G_2 = ar^2, \; G_3 = ar^3, \text{ - - - -}$

AP $a_4 = a+3d \; ; \; A_4 = a+4d$

GP $a_4 = ar^3 \; ; \; G_4 = ar^4$

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S&S (Class No. 1)

(·) Property

If $a_1, a_2, a_3, ---$ GP

then $a_1k, a_2k, a_3k, ---$ GP

then $\frac{a_1}{k}, \frac{a_2}{k}, \frac{a_3}{k}, ---$ GP

(·) Infinite G.P

$a_1, a_2, a_3 ------ \infty$

$a, ar, ar^2, ar^3, --- \infty$

Sum of infinite GP

$\boxed{S_\infty = \frac{a}{1-r}}$; $|r|<1$ or $-1<r<1$

Reason: $S_n = \frac{a(1-r^n)}{1-r}$; $r<1$

$(2)^4 = 16$; $(.2)^4 = .0016$ $(0.2)^\infty \approx 0$

Extra

Harmonic progression (H.P)

(·) If $a_1, a_2, a_3 ----$ are in AP

then $\frac{1}{a_1}, \frac{1}{a_2}, \frac{1}{a_3}, \frac{1}{a_4} ----$ are in H.P

(·) n^{th} term of HP $= \frac{1}{n^{th} \text{ term of AP}}$

(·) If $a, b, c \rightarrow$ H.P

$\Rightarrow \frac{1}{a}, \frac{1}{b}, \frac{1}{c} \rightarrow$ AP

$\Rightarrow \frac{2}{b} = \frac{1}{a} + \frac{1}{c}$

$\Rightarrow \frac{2}{b} = \frac{c+a}{ac}$

$\boxed{\frac{b}{2} = \frac{ac}{a+c}}$

(·) Harmonic Mean (HM)

$a, \underline{H}, b \rightsquigarrow H.P$

$\Rightarrow \frac{1}{a}, \frac{1}{H}, \frac{1}{b} \to AP$

$\Rightarrow \frac{2}{H} = \frac{1}{a} + \frac{1}{b}$

$\Rightarrow \frac{2}{H} = \frac{b+a}{ab}$

$\Rightarrow \frac{H}{2} = \frac{ab}{a+b}$

$\Rightarrow \boxed{H = \frac{2ab}{a+b}}$

SPECIAL SERIES

(·) $1+2+3+\dots n$ = sum of 1st n-natural numbers

$1+2+3\dots n = \frac{n(n+1)}{2} = \Sigma n$

(·) $1^2+2^2+3^2\dots n^2 = \frac{n(n+1)(2n+1)}{6} = \Sigma n^2$

(·) $1^3+2^3+3^3+\dots n^3 = \frac{n^2(n+1)^2}{4} = \Sigma n^3$

(·) Σ Summation

(·) $\Sigma 2 = 2+2+2+\dots$ n term $= 2n$

$\Sigma 3 = 3n$

$\boxed{\Sigma k = kn}$

(·) Property

$\Sigma(x \pm y) = \Sigma x \pm \Sigma y$

$\Sigma(xy) \neq \Sigma x \, \Sigma y$

$\Sigma\left(\frac{x}{y}\right) \neq \frac{\Sigma x}{\Sigma y}$

(·) $\Sigma 3n^2$

$= 3\Sigma n^2$

(·) $\Sigma 3 = 3n$