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!! जय श्री राधे कृष्णा !!

← ULTIMATE MATHEMATICS →

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Chapter: Complex - Numbers

CLASS - 2

Qu. 1 → If $x+iy = (u+iv)^{1/3}$

Show that $\frac{u}{x} + \frac{v}{y} = 4(x^2-y^2)$

Sol: Cubing both sides

$$(x+iy)^3 = u+iv$$

$$\Rightarrow x^3 + i^3y^3 + 3x^2iy + 3xi^2y^2 = u+iv$$

$$\Rightarrow x^3 - iy^3 + 3x^2iy - 3xy^2 = u+iv$$

$$\Rightarrow (x^3-3xy^2) + i(-y^3+3x^2y) = u+iv$$

equating real and imaginary parts

$$u = x^3-3xy^2 \quad \text{and} \quad v = 3x^2y - y^3$$

LHS $\frac{u}{x} + \frac{v}{y}$

$$= \frac{x^3-3xy^2}{x} + \frac{3x^2y-y^3}{y}$$

$$= x^2-3y^2 + 3x^2-y^2$$

$$= 4x^2-4y^2$$

$$= 4(x^2-y^2) = \text{RHS} \quad \text{Ans}$$

Qu. 2 → If $\left(\frac{1+i}{1-i}\right)^m = 1$ then find the least positive integral value of m

Sol: We have $\left(\frac{1+i}{1-i}\right)^m = 1$

$$\Rightarrow \left(\frac{1+i}{1-i} \times \frac{1+i}{1+i}\right)^m = 1$$

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$$\Rightarrow \left(\frac{(1+i)^2}{1-i^2}\right)^m = 1$$

$$\Rightarrow \left(\frac{\cancel{1}+\cancel{i^2}+2i}{1+1}\right)^m = 1$$

$$\Rightarrow (i)^m = 1$$

$$i^4 = 1$$

$\therefore$ least +ve Integral value of $m = 4$ Ans

Que 3 → If $x+iy = \frac{a+ib}{a-ib}$ Show that $x^2+y^2=1$

Soln

Concept $\boxed{(a+ib)(a-ib) = a^2-i^2b^2 = a^2+b^2}$

we have $x+iy = \frac{a+ib}{a-ib}$ --- (1)

taking Conjugate on both sides

$x-iy = \frac{a-ib}{a+ib}$ --- (2)

(1) × (2)

$$(x+iy)(x-iy) = \frac{a+ib}{a-ib} \times \frac{a-ib}{a+ib}$$

$$\Rightarrow x^2-i^2y^2 = \frac{a^2-i^2b^2}{a^2-i^2b^2}$$

$$\Rightarrow x^2+y^2 = \frac{\cancel{a^2+b^2}}{\cancel{a^2+b^2}} = 1$$ Ans

Que 4 If $a+ib = \frac{(x+i)^2}{2x^2+1}$ Show that $a^2+b^2 = \frac{(x^2+1)^2}{(2x^2+1)^2}$

Soln given $a+ib = \frac{(x+i)^2}{(2x^2+1)}$ ---- (1)

taking conjugate

Complex class no: 2

$$a - ib = \frac{(x-i)^2}{2x^2+1} \quad \text{---} \quad (2)$$

(1) × (2)

$$(a+ib)(a-ib) = \frac{(x+i)^2}{2x^2+1} \times \frac{(x-i)^2}{(2x^2+1)}$$

$$\Rightarrow a^2 - i^2b^2 = \frac{((x+i)(x-i))^2}{(2x^2+1)^2}$$

$$\Rightarrow a^2 + b^2 = \frac{(x^2 - i^2)^2}{(2x^2+1)^2}$$

$$\Rightarrow a^2 + b^2 = \frac{(x^2+1)^2}{(2x^2+1)^2} \quad \text{Ans}$$

Ques 5 → Find real values of x & y for which the complex numbers $-3 + ix^2y$ and $x^2 + y + 4i$ are conjugate of each other.

Sol: given

$$-3 + ix^2y = \overline{x^2 + y + 4i}$$

$$\Rightarrow -3 + ix^2y = (x^2+y) - 4i$$

$x^2 + y = -3$ and $x^2y = -4$

$x^2 = -3 - y$ put here

$$\therefore (-3-y)y = -4$$

$$-3y - y^2 = -4$$

$$y^2 + 3y - 4 = 0$$

$$(y+4)(y-1) = 0$$

$$y = -4 \; ; \; y = 1$$

put $y = -4$

$$x^2(-4) = -4$$

$$x^2 = 1$$

$$x = \pm 1$$

put $y = 1$

$$x^2 = -4$$

$$x = \pm\sqrt{-4} = \pm 2i$$

Reject.

Ans $x = \pm 1$ & $y = -4$

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Q.6 **Imp** If α and β are different complex numbers with $|\beta| = 1$, then find $\left|\frac{\beta - \alpha}{1 - \bar{\alpha}\beta}\right|$

Soln: property $|z|^2 = z\bar{z}$

verify $z = a + ib$ | $z\bar{z} = (a+ib)(a-ib)$

$|z| = \sqrt{a^2 + b^2}$ | $z\bar{z} = a^2 + b^2$

$\bar{z} = a - ib$ | $|z|^2 = a^2 + b^2$

$\Rightarrow |z|^2 = z\bar{z}$

we have

$$\left|\underbrace{\frac{\beta - \alpha}{1 - \bar{\alpha}\beta}}_{z}\right|^2 = \left(\frac{\beta - \alpha}{1 - \bar{\alpha}\beta}\right)\overline{\left(\frac{\beta - \alpha}{1 - \bar{\alpha}\beta}\right)}$$

$$= \left(\frac{\beta - \alpha}{1 - \bar{\alpha}\beta}\right)\left(\frac{\bar{\beta} - \bar{\alpha}}{1 - \alpha\bar{\beta}}\right)$$

$$= \frac{\beta\bar{\beta} - \beta\bar{\alpha} - \alpha\bar{\beta} + \alpha\bar{\alpha}}{1 - \alpha\bar{\beta} - \bar{\alpha}\beta + \alpha\bar{\alpha}\beta\bar{\beta}}$$

$$= \frac{|\beta|^2 - \beta\bar{\alpha} - \alpha\bar{\beta} + |\alpha|^2}{1 - \alpha\bar{\beta} - \bar{\alpha}\beta + |\alpha|^2|\beta|^2}$$

$$= \frac{1 \cancel{- \beta\bar{\alpha} - \alpha\bar{\beta}} + |\alpha|^2}{1 \cancel{- \alpha\bar{\beta} - \bar{\alpha}\beta} + |\alpha|^2} \quad \cdots \{|\beta| = 1\}$$

$$\left|\frac{\beta - \alpha}{1 - \bar{\alpha}\beta}\right|^2 = 1$$

$$\Rightarrow \left|\frac{\beta - \alpha}{1 - \bar{\alpha}\beta}\right| = 1 \quad \text{Ans}$$

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(3)

Qn. 7 → Find all non-zero integral solutions of the equation $|1-i|^x = 2^x$

Soln

given $|1-i|^x = 2^x$

$\Rightarrow (\sqrt{1+1})^x = 2^x$

$\Rightarrow (\sqrt{2})^x = 2^x$

$\Rightarrow (2)^{x/2} = 2^x$

$\Rightarrow \frac{x}{2} = x$

$\Rightarrow x = 2x$

$\Rightarrow 2x - x = 0$

$\Rightarrow \boxed{x = 0}$

there is no non-zero solution Ans

Qn. 8 → Solve the equation $\bar{z} = z^2$
Imp
(or) Find z

Soln

let $z = x + iy$

given $\bar{z} = z^2$

$\Rightarrow$

$x - iy = (x+iy)^2$

$\Rightarrow x - iy = x^2 + i^2y^2 + 2ixy$

$\Rightarrow x - iy = (x^2 - y^2) + 2ixy$

equating Real & Imaginary parts

$x^2 - y^2 = x$ and $2xy = -y$

$\Rightarrow 2xy + y = 0$

$\Rightarrow y(2x+1) = 0$

$\Rightarrow y = 0$ (or) $x = -\frac{1}{2}$

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Put $y=0$ in $x^2-y^2=x$

$\Rightarrow x^2=x$

$\Rightarrow x^2-x=0$

$\Rightarrow x(x-1)=0$

$\Rightarrow x=0,\ x=1$

$\therefore\ x=0,\ y=0$ and $x=1,\ y=0$

Put $x=-1/2$ in $x^2-y^2=x$

$\Rightarrow \frac{1}{4}-y^2=\frac{-1}{2}$

$\Rightarrow \frac{1}{4}+\frac{1}{2}=y^2$

$\Rightarrow \frac{3}{4}=y^2$

$\Rightarrow y=\pm\frac{\sqrt{3}}{2}$

$x=-1/2,\ y=\frac{\sqrt{3}}{2}$ & $x=-1/2;\ y=-\frac{\sqrt{3}}{2}$

$\therefore\ 0+0i;\ 1+0i;\ \frac{-1}{2}+\frac{\sqrt{3}}{2}i;\ -\frac{1}{2}-\frac{\sqrt{3}}{2}i$ Ans

Q.No 9 → If $|z^2-1|=|z|^2+1$, then show that z lies on Imaginary axis

Sol: let $z=x+iy$

we have $|z^2-1|=|z|^2+1$

$\Rightarrow |(x+iy)^2-1|=\left(\sqrt{x^2+y^2}\right)^2+1$

$\Rightarrow |x^2+i^2y^2+2ixy-1|=x^2+y^2+1$

$\Rightarrow |(x^2-y^2-1)+2ixy|=x^2+y^2+1$

$\Rightarrow \sqrt{(x^2-y^2-1)^2+4x^2y^2}=x^2+y^2+1$

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Squaring

$$(x^2-y^2-1)^2 + 4x^2y^2 = (x^2+y^2+1)^2$$

$$\Rightarrow \cancel{x^4}+\cancel{y^4}+\cancel{1} - \cancel{2x^2y^2} + \cancel{2y^2} - 2x^2 + \cancel{4x^2y^2} = \cancel{x^4}+\cancel{y^4}+\cancel{1}+\cancel{2x^2y^2}+\cancel{2y^2}+2x^2$$

$$\Rightarrow -2x^2 = 2x^2$$

$$\Rightarrow 4x^2 = 0$$

$$\Rightarrow x^2 = 0$$

$$\Rightarrow x = 0$$

$z = x+iy$

$z = iy$ Clearly z is purely Imaginary

(or) z lies on Imaginary Axis Ans

Qn. 10 → If $|z_1| = |z_2| = |z_3| = \cdots |z_n| = 1$

(Imp) then show that $|z_1+z_2+z_3 \cdots z_n| = \left|\frac{1}{z_1}+\frac{1}{z_2}+\frac{1}{z_3}+\cdots\frac{1}{z_n}\right|$

Soln RHS $\left|\frac{1}{z_1}+\frac{1}{z_2}+\frac{1}{z_3}+\cdots\frac{1}{z_n}\right|$

$$= \left|\frac{\bar z_1}{z_1\bar z_1}+\frac{\bar z_2}{z_2\bar z_2}+\frac{\bar z_3}{z_3\bar z_3}+\cdots\frac{\bar z_n}{z_n\bar z_n}\right|$$

$$= \left|\frac{\bar z_1}{|z_1|^2}+\frac{\bar z_2}{|z_2|^2}+\frac{\bar z_3}{|z_3|^2}+\cdots\frac{\bar z_n}{|z_n|^2}\right|$$

$$= |\bar z_1+\bar z_2+\bar z_3+\cdots\bar z_n| \quad \cdots \{|z_1|=|z_2|\cdots 1\}$$

$$= |\overline{z_1+z_2+z_3\cdots z_n}| \quad \cdots \{\overline{z_1+z_2} = \bar z_1+\bar z_2\}$$

$$= |z_1+z_2+z_3\cdots z_n| \quad \cdots \{|\bar z| = |z|\}$$

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Complex class no: 2

Qn 11 → If $(1+i)(1+2i)(1+3i) \cdots (1+ni) = x+iy$

Show that

$$2 \cdot 5 \cdot 10 \cdots (1+n^2) = x^2+y^2$$

Sol Given $(1+i)(1+2i)(1+3i) \cdots (1+ni) = x+iy \quad \cdots (1)$

taking Conjugate

$(1-i)(1-2i)(1-3i) \cdots (1-ni) = x-iy \quad \cdots (2)$

(1) × (2)

$\Rightarrow (1+i)(1-i)(1+2i)(1-2i)(1+3i)(1-3i) \cdots (1+ni)(1-ni)$

$= (x+iy)(x-iy)$

$\Rightarrow (1-i^2)(1-4i^2)(1-9i^2) \cdots (1-n^2i^2) = (x^2-i^2y^2)$

$\Rightarrow 2 \cdot 5 \cdot 10 \cdots (1+n^2) = (x^2+y^2)$ Ans

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Complex Numbers

WORKSHEET NO: 2

Qn 1 → Find the real numbers x and y if $(x-iy)(3+5i)$ is the conjugate of $-6-24i$

Ans $x=3$, $y=-3$

Qn 2 → If $(a+ib)(c+id)(e+if)(g+ih) = A+iB$ show that $(a^2+b^2)(c^2+d^2)(e^2+f^2)(g^2+h^2) = A^2+B^2$

Qn 3 → If $x-iy = \sqrt{\dfrac{a-ib}{c-id}}$ show that

$$x^2+y^2 = \frac{a^2+b^2}{c^2+d^2}$$

Qn. 4 → If $\left(\dfrac{1+i}{1-i}\right)^3 - \left(\dfrac{1-i}{1+i}\right)^3 = x+iy$ find (x,y)

Ans $(0,-2)$

Qn 5 → If $\left(\dfrac{1-i}{1+i}\right)^{100} = a+ib$ find (a,b) Ans $(1,0)$

Qn. 6 → Solve the equation

$$|z| = z+1+2i$$

Ans $z = \frac{3}{2} - 2i$

Qn 7 → Find value of $\dfrac{i^{4n+1} - i^{4n-1}}{2}$ Ans: i

Qn. 8 → If $|z_1| = |z_2| = |z_3| = 1$

and $\left|\dfrac{1}{z_1} + \dfrac{1}{z_2} + \dfrac{1}{z_3}\right| = 1$ then find value of

$$|z_1+z_2+z_3| = ?$$

Ans $= 1$

Qn. 9 → If $(x+iy)^{1/3} = a+ib$

show that $\dfrac{x}{a} - \dfrac{y}{b} = -2(a^2+b^2)$

CLASSTIME

— x —