XI $\boxed{\text{SOLUTIONS}}$ $\boxed{\text{Complex Numbers}}$ →

← class No: 3 (self wale Questions)

Ans 4 → given $\left|\frac{z-5i}{z+5i}\right| = 1$

$\Rightarrow \frac{|z-5i|}{|z+5i|} = 1$

$\Rightarrow |z-5i| = |z+5i|$

let $z = x+iy$

$\Rightarrow |x+iy-5i| = |x+iy+5i|$

$\Rightarrow |x+i(y-5)| = |x+i(y+5)|$

$\Rightarrow \sqrt{x^2+(y-5)^2} = \sqrt{x^2+(y+5)^2}$

Squaring $\cancel{x^2}+(y-5)^2 = \cancel{x^2}+(y+5)^2$

$\Rightarrow \cancel{y^2}+\cancel{25}-10y = \cancel{y^2}+\cancel{25}+10y$

$\Rightarrow 20y = 0$

$\Rightarrow y = 0$

$\therefore$ z lies on x-axis **Ans**

Ans 7 → given $\left|\frac{z+1-i}{z-1+i}\right| = 1$

$\Rightarrow \frac{|z+1-i|}{|z-1+i|} = 1$

$\Rightarrow |z+1-i| = |z-1+i|$

let $z = x+iy$

$\Rightarrow |x+iy+1-i| = |x+iy-1+i|$

$\Rightarrow |(x+1)+i(y-1)| = |(x-1)+i(y+1)|$

$\Rightarrow \sqrt{(x+1)^2+(y-1)^2} = \sqrt{(x-1)^2+(y+1)^2}$

Squaring

$\cancel{x^2}+\cancel{1}+2x+\cancel{y^2}+\cancel{1}-2y = \cancel{x^2}+\cancel{1}-2x+\cancel{y^2}+1+2y$

$\Rightarrow 4x-4y=0$

$\Rightarrow x-y=0$

this equation represents the equation of the st. line Ans

Ans 10 →

given $x = -1+\sqrt{-2}$

$\Rightarrow x = -1+\sqrt{2}\,i$

$\Rightarrow x+1 = \sqrt{2}\,i$

Squaring $x^2+1+2x = 2i^2$

$\Rightarrow x^2+1+2x = -2$

$\Rightarrow x^2+2x+3=0$

Now

$$
\begin{array}{r|l}
 & x^2+2x-1 \\
\hline
x^2+2x+3 & x^4+4x^3+6x^2+4x+9 \\
 & -(x^4+2x^3+3x^2) \\
\hline
 & 2x^3+3x^2+4x+9 \\
 & -(2x^3+4x^2+6x) \\
\hline
 & -x^2-2x+9 \\
 & -(-x^2-2x-3) \\
\hline
 & 12
\end{array}
$$

Now $x^4+4x^3+6x^2+4x+9 = (x^2+2x+3)(x^2+2x-1)+12$

$= 0(x^2+2x-1)+12 = 12$ Ans

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Qns 11) → given: $x = 1 + \sqrt{-4}$

$\Rightarrow x = 1 + 2i$

$\Rightarrow x - 1 = 2i$

Squaring

$x^2 + 1 - 2x = 4i^2$

$\Rightarrow x^2 - 2x + 5 = 0$

$$
\begin{array}{r|l}
 & x + 9 \\
\hline
x^2 - 2x + 5 & x^3 + 7x^2 - x + 16 \\
 & -(x^3 - 2x^2 + 5x) \\
\hline
 & 9x^2 - 6x + 16 \\
 & -(9x^2 - 18x + 45) \\
\hline
 & 12x - 29
\end{array}
$$

$\therefore x^3 + 7x^2 - x + 16 = (x^2 - 2x + 5)(x + 9) + (12x - 29)$

$= 0(x + 9) + (12x - 29)$

$= 12x - 29$

$= 12(1 + 2i) - (29)$

$= -17 + 24i$ Ans

Qns 12 → $Z = x + iy$ & $Z_2 = \frac{1 - iz}{z - i}$

given $|z_2| = 1$

$\Rightarrow \left| \frac{1 - iz}{z - i} \right| = 1$

$\Rightarrow \frac{|1 - iz|}{|z - i|} = 1$

$\Rightarrow |1-iz| = |z-i|$

put $z = x+iy$

$\Rightarrow |1-i(x+iy)| = |x+iy-i|$

$\Rightarrow |1-ix+y| = |x+iy-i|$

$\Rightarrow |(1+y)-ix| = |x+i(y-1)|$

$\Rightarrow \sqrt{(1+y)^2+x^2} = \sqrt{x^2+(y-1)^2}$

Squaring

$1+\cancel{y^2}+2y+\cancel{x^2} = \cancel{x^2}+\cancel{y^2}+1-2y$

$\Rightarrow 4y = 0$

$\Rightarrow y = 0$

$\therefore$ z is purely Real Ans

Ans 13 given $a+ib = \frac{c+i}{c-i}$

$\Rightarrow a+ib = \frac{c+i}{c-i} \times \frac{c+i}{c+i}$

$\Rightarrow a+ib = \frac{c^2+i^2+2ci}{c^2-i^2}$

$\Rightarrow a+ib = \frac{c^2-1}{c^2+1} + \frac{2ic}{c^2+1}$

equating Real & Imaginary parts

$a = \frac{c^2-1}{c^2+1}$ & $b = \frac{2c}{c^2+1}$

(i) Ans $\frac{b}{a} = \frac{\frac{2c}{\cancel{c^2+1}}}{\frac{c^2-1}{\cancel{c^2+1}}} = \frac{2c}{c^2-1}$ Ans

(ii) $a^2+b^2 = \left(\frac{c^2-1}{c^2+1}\right)^2 + \left(\frac{2c}{c^2+1}\right)^2 = \frac{c^4+1-2c^2}{(c^2+1)^2} + \frac{4c^2}{(c^2+1)^2}$

$= \frac{c^4+2c^2+1}{(c^2+1)^2}$

$= \frac{\cancel{(c^2+1)^2}}{\cancel{(c^2+1)^2}} = 1$ = Rhs Ans

Qn_14 $25x^2-30x+11=0$

Quadratic formula

$x = \frac{30 \pm \sqrt{900-1100}}{50}$

$x = \frac{30 \pm \sqrt{-200}}{50} = \frac{30 \pm 10\sqrt{2}\,i}{50}$

$x = \frac{3}{5} \pm \frac{\sqrt{2}i}{5}$ Ans

Qn-15 → $\sqrt{5}x^2 + x + \sqrt{5} = 0$

Quadratic formula

$x = \frac{-1 \pm \sqrt{1-20}}{2\sqrt{5}}$

$x = \frac{-1 \pm \sqrt{-19}}{2\sqrt{5}}$

$x = \frac{-1 \pm \sqrt{19}\,i}{2\sqrt{5}}$ Ans

—x—