TRIGONOMETRY REVISION CLASS NO. 2 →

QNS: 1 If $x\cos\theta = y\cos\left(\theta+\frac{2\pi}{3}\right) = z\cos\left(\theta+\frac{4\pi}{3}\right)$

then find the value of $xy+yz+zx$

Soln let $x\cos\theta = y\cos\left(\theta+\frac{2\pi}{3}\right) = z\cos\left(\theta+\frac{4\pi}{3}\right) = K$

$\Rightarrow x = \frac{k}{\cos\theta}$; $y = \frac{k}{\cos\left(\theta+\frac{2\pi}{3}\right)}$; $z = \frac{k}{\cos\left(\theta+\frac{4\pi}{3}\right)}$

$\Rightarrow \frac{1}{x} = \frac{\cos\theta}{k}$; $\frac{1}{y} = \frac{\cos\left(\theta+\frac{2\pi}{3}\right)}{K}$: $\frac{1}{z} = \frac{\cos\left(\theta+\frac{4\pi}{3}\right)}{K}$

Tricky

Now

$xy+yz+zx = xyz\left(\frac{1}{x}+\frac{1}{y}+\frac{1}{z}\right)$

Now $\frac{1}{x}+\frac{1}{y}+\frac{1}{z} = \frac{1}{k}\left[\cos\theta+\cos(\theta+120^\circ)+\cos(\theta+240^\circ)\right]$

$= \frac{1}{k}(0)$

$\frac{1}{x}+\frac{1}{y}+\frac{1}{z} = 0$

$\therefore xy+yz+zx$

$= xyz(0)$

$= 0$ Ans

TRIGONOMETRY REVISION CLASS NO.2 →

Qns: 2 Find the minimum & Maximum value of $3\cos x + 4\sin x + 8$

Concept $a\cos\theta + b\sin\theta$

Max. value = $\sqrt{a^2+b^2}$

Min Value = $-\sqrt{a^2+b^2}$

Reason → M & D by $\sqrt{a^2+b^2}$

$= \sqrt{a^2+b^2}\left(\frac{a}{\sqrt{a^2+b^2}}\cos\theta + \frac{b}{\sqrt{a^2+b^2}}\sin\theta\right)$

$= \sqrt{a^2+b^2} \cdot \boxed{\sin(A+\theta)}$

-1 1

Soln

$3\cos x + 4\sin x + 8$

here $a = 3$, $b = 4$

Max value : $\sqrt{9+16} + 8$

$= 13$

Min value $= -\sqrt{9+16} +$

$= -5 + 8$

$= 3$

TRIGONOMETRY REVISION CLASS NO. 2 →

Find the minimum value of $2^{\sin\theta}+2^{\cos\theta}$

we have $\boxed{A.M \geq G.M}$

$$\frac{a+b}{2} \geq \sqrt{ab}$$

$$\Rightarrow \frac{2^{\sin\theta}+2^{\cos\theta}}{2} \geq \sqrt{2^{\sin\theta}\times 2^{\cos\theta}}$$

$$\Rightarrow \frac{2^{\sin\theta}+2^{\cos\theta}}{2} \geq \sqrt{2^{\sin\theta+\cos\theta}}$$

$$\Rightarrow \frac{2^{\sin\theta}+2^{\cos\theta}}{2} \geq 2^{\frac{1}{2}(\sin\theta+\cos\theta)}$$

$$\Rightarrow \frac{2^{\sin\theta}+2^{\cos\theta}}{2} \geq 2^{\frac{\sqrt{2}}{2}\left(\frac{1}{\sqrt{2}}\sin\theta+\frac{1}{\sqrt{2}}\cos\theta\right)}$$

$$\Rightarrow \;,, \geq 2^{\frac{1}{\sqrt{2}}\left(\cos\frac{\pi}{4}\sin\theta+\sin\frac{\pi}{4}\cos\theta\right)}$$

$$\Rightarrow \;,, \geq 2^{\frac{1}{\sqrt{2}}\cdot\sin\left(\theta+\frac{\pi}{4}\right)}$$

$$\rightarrow 2^{\sin\theta}+2^{\cos\theta} \geq 2\cdot 2^{\frac{1}{\sqrt{2}}\sin\left(\theta+\frac{\pi}{4}\right)}$$

-1

$$\Rightarrow 2^{\sin\theta}+2^{\cos\theta} \geq 2\cdot 2^{-\frac{1}{\sqrt{2}}}$$

$$,, \geq 2^{1-\frac{1}{\sqrt{2}}}$$

TRIGONOMETRY REVISION CLASS NO.2 →

QNS.: 4 → If $\tan(\lambda\cos\theta) = \cot(\lambda\sin\theta)$, then show that $\cos\left(\theta - \frac{\lambda}{4}\right) = \pm\frac{1}{2\sqrt{2}}$

Soln we have

$$\frac{\sin(\lambda\cos\theta)}{\cos(\lambda\cos\theta)} = \frac{\cos(\lambda\sin\theta)}{\sin(\lambda\sin\theta)}$$

$\Rightarrow \sin(\lambda\cos\theta)\cdot\sin(\lambda\sin\theta) = \cos(\lambda\sin\theta)\cdot\cos(\lambda\cos\theta)$

$\Rightarrow \cos(\overset{A}{\lambda\sin\theta})\cdot\cos(\overset{B}{\lambda\cos\theta}) - \sin(\overset{A}{\lambda\sin\theta})\sin(\overset{B}{\lambda\cos\theta}) = 0$

$\Rightarrow \cos(\lambda\sin\theta + \lambda\cos\theta) = 0$

$\Rightarrow \lambda\sin\theta + \lambda\cos\theta = \pm\frac{\lambda}{2}$

$\Rightarrow \cos\theta + \sin\theta = \pm\frac{1}{2}$

M & D by $\sqrt{1+1} = \sqrt{2}$

$\Rightarrow \sqrt{2}\left(\frac{1}{\sqrt{2}}\cos\theta + \frac{1}{\sqrt{2}}\sin\theta\right) = \pm\frac{1}{2}$

$\Rightarrow \cos\left(\theta - \frac{\lambda}{4}\right) = \pm\frac{1}{2\sqrt{2}}$ Ans

$\boxed{\text{TRIGONOMETRY}}$ REVISION CLASS NO. 2 →

5 → If $3\tan(\theta-15^\circ) = \tan(\theta+15^\circ)$; $0 < \theta < 90^\circ$. Find θ

Soln ⇒

$$\frac{\tan(\theta+15^\circ)}{\tan(\theta-15^\circ)} = \frac{3}{1}$$

Apply C & D

$$\Rightarrow \frac{\tan(\theta+15^\circ)+\tan(\theta-15^\circ)}{\tan(\theta+15^\circ)-\tan(\theta-15^\circ)} = \frac{3+1}{3-1}$$

$$\Rightarrow \frac{\frac{\sin(\theta+15)}{\cos(\theta+15)}+\frac{\sin(\theta-15)}{\cos(\theta-15)}}{\frac{\sin(\theta+15)}{\cos(\theta+15)}-\frac{\sin(\theta-15)}{\cos(\theta-15)}} = 2$$

$$\Rightarrow \frac{\sin(\theta+\cancel{15}+\theta-\cancel{15})}{\sin(\cancel{\theta}+15-\cancel{\theta}+15)} = 2$$

$$\Rightarrow \frac{\sin(2\theta)}{\frac{1}{2}} = 2$$

$$\Rightarrow \sin(2\theta) = 1$$

$$\boxed{2\theta = \frac{\pi}{2} \quad \theta = \frac{\pi}{4}} \text{ Ans.}$$

TRIGONOMETRY REVISION CLASS NO. 2 →

6 → If $\cos(\alpha+\beta) = \frac{4}{5}$ and $\sin(\alpha-\beta) = \frac{5}{13}$; $0 < \alpha, \beta < \frac{\pi}{4}$

find the value of $\tan(2\alpha)$

Soln we have $\tan(2\alpha) = \tan(\alpha+\alpha+\beta-\beta)$

$\Rightarrow \tan(2\alpha) = \tan((\overset{(A)}{\alpha+\beta}) + (\overset{(B)}{\alpha-\beta}))$

$\Rightarrow \tan(2\alpha) = \dfrac{\tan(\alpha+\beta) + \tan(\alpha-\beta)}{1 - \tan(\alpha+\beta)\tan(\alpha-\beta)}$

(·) $\sin^2(\alpha+\beta) + \cos^2(\alpha+\beta) = 1$

$\Rightarrow \sin^2(\alpha+\beta) = 1 - \frac{16}{25}$

$\Rightarrow \sin^2(\alpha+\beta) = \frac{9}{25}$

$\Rightarrow \sin(\alpha+\beta) = \frac{3}{5}$

$\therefore \boxed{\tan(\alpha+\beta) = \frac{3}{4}}$

(·) $\sin^2(\alpha-\beta) + \cos^2(\alpha-\beta) = 1$

$\Rightarrow \cos^2(\alpha-\beta) = 1 - \frac{25}{169}$

$\Rightarrow \cos^2(\alpha-\beta) = \frac{144}{169}$

$\Rightarrow \cos(\alpha-\beta) = \frac{12}{13}$

$\Rightarrow \boxed{\tan(\alpha-\beta) = \frac{5}{12}}$

$\tan(2\alpha)$

$= \dfrac{\frac{3}{4} + \frac{5}{12}}{1 - \frac{3}{4} \times \frac{5}{12}}$

$= \frac{56}{33}$

Ans

TRIGONOMETRY REVISION CLASS NO. 2 →

Q.7 → In a ΔABC with $\angle C = 90°$. Find the Quadratic equation whose roots are $\tan A$ and $\tan B$

Soln $\angle C = 90°$

$\Rightarrow A + B = 90°$

Product of roots $= \tan A \cdot \tan B$

$= \tan A \cdot \tan(90° - A)$

$= \tan A \cdot \cot A$

$= \tan A \times \frac{1}{\tan A}$

$\boxed{\text{Product of roots} = 1}$

Sum of roots $= \tan A + \tan B$

$= \tan A + \tan(90 - A)$

$= \tan A + \cot A$

$= \frac{\sin A}{\cos A} + \frac{\cos A}{\sin A}$

$= \frac{1}{\sin A \cos A}$

$= \frac{1}{\frac{2 \sin A \cos A}{2}}$

$= \frac{2}{\sin(2A)}$

Q.E

$\boxed{x^2 - \frac{2}{\sin(2A)} x + 1 = 0}$

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TRIGONOMETRY REVISION CLASS NO.2 →

Q:: 8 → If $\frac{2\sin\alpha}{1+\cos\alpha+\sin\alpha} = y$, then show that $\frac{1-\cos\alpha+\sin\alpha}{1+\sin\alpha}$ is also equal to y.

Soln we have $\frac{1-\cos\alpha+\sin\alpha}{1+\sin\alpha}$

M & D by $1+\cos\alpha+\sin\alpha$

$$= \frac{1-\cos\alpha+\sin\alpha}{1+\sin\alpha} \times \frac{1+\cos\alpha+\sin\alpha}{1+\cos\alpha+\sin\alpha}$$

$$= \frac{(1+\sin\alpha)-\cos\alpha}{1+\sin\alpha} \times \frac{(1+\sin\alpha)+\cos\alpha}{1+\cos\alpha+\sin\alpha}$$

$$= \frac{1+\sin^2\alpha+2\sin\alpha-\cos^2\alpha}{(1+\sin\alpha)(1+\cos\alpha+\sin\alpha)}$$

$$= \frac{2\sin^2\alpha+2\sin\alpha}{(1+\sin\alpha)(1+\cos\alpha+\sin\alpha)}$$

$$= \frac{2\sin\alpha(\sin\alpha+1)}{(1+\sin\alpha)(1+\cos\alpha+\sin\alpha)}$$

$$= y$$

TRIGONOMETRY REVISION CLASS NO.2 →

ANS:- 9 → Show that $\cos(2\theta)\cdot\cos(2\phi) + \sin^2(\theta-\phi) - \sin^2(\theta+\phi) = \cos(2(\theta+\phi))$

Formula $\boxed{\sin^2 A - \sin^2 B = \sin(A+B)\cdot\sin(A-B)}$ (x)

LHS $\cos(2\theta)\cdot\cos(2\phi) + \frac{1-\cos(2\theta-2\phi)}{2} - \left(\frac{1-\cos(2\theta+2\phi)}{2}\right) \ldots \left\{\sin^2\theta = \frac{1-\cos(2\theta)}{2}\right.$

$= \cos(2\theta)\cos(2\phi) + \frac{\cos(\overset{A}{2\theta+2\phi}) - \cos(\overset{B}{2\theta-2\phi})}{2}$

$= \cos(2\theta)\cos(2\phi) - \frac{\cancel{2}\sin(2\theta)\cdot\sin(2\phi)}{\cancel{2}}$

$= \cos(2\theta)\cos(2\phi) - \sin(2\theta)\sin(2\phi)$

$= \cos(2\theta+2\phi)$