Solutions WORKSHEET No: 3 (class - 5)

SEQUENCE SERIES

Ans: 1 → Let the three edges/sides of rectangular block are

$\frac{a}{r}, a, ar$

given volume = $216\ cm^3$

$\Rightarrow lbh = 216$

$\Rightarrow \frac{a}{\cancel{r}} \times a \times a\cancel{r} = 216$

$\Rightarrow a^3 = 216$

$\Rightarrow \boxed{a = 6}$

given surface area = $252\ cm^2$

$\Rightarrow 2(lb + bh + hl) = 252$

$\Rightarrow 2\left(\frac{a}{r} \times a + a \times ar + \frac{a}{r} \times ar\right) = 252$

$\Rightarrow \frac{a^2}{r} + a^2r + a^2 = 126$

$\Rightarrow a^2\left(\frac{1}{r} + r + 1\right) = 126$

put $a = 6$ $\Rightarrow 36\left(\frac{1 + r^2 + r}{r}\right) = 126$

$\Rightarrow \frac{1 + r^2 + r}{r} = \frac{\cancel{126}}{\cancel{36}} \frac{7}{2}$

$\Rightarrow 2 + 2r^2 + 2r = 7r$

$\Rightarrow 2r^2 - 5r + 2 = 0$

$\Rightarrow 2r^2 - 4r - r + 2 = 0$

$\Rightarrow 2r(r - 2) - 1(r - 2) = 0$

$\Rightarrow (2r - 1)(r - 2) = 0$ $\Rightarrow \boxed{r = 1/2}$ $\boxed{r = 2}$

for $a=6$ & $r=1/2$

Sides are $12\,cm, 6\,cm, 3\,cm$

for $a=6$ & $r=2$

Sides are $3\,cm, 6\,cm, 12\,cm$

$\therefore$ longest side is $12\,cm$ (A) Ans.

Ans: 2 →

let $a=4^x$ & $b=4^{1-x}$

we know that $A.M \geq G.M$

$$\Rightarrow \frac{a+b}{2} \geq \sqrt{ab}$$

$$\Rightarrow \frac{4^x+4^{1-x}}{2} \geq \sqrt{4^x \times 4^{1-x}}$$

$$\Rightarrow \frac{4^x+4^{1-x}}{2} \geq \sqrt{4}$$

$$\Rightarrow \frac{4^x+4^{1-x}}{2} \geq 2$$

$$\Rightarrow 4^x+4^{1-x} \geq 4$$

$\therefore$ Minimum value $=4$ Ans

Ans 3 → given $S_n = qn^2$

$$\frac{n}{2}[2a+(n-1)d] = qn^2$$

$$2a+(n-1)d = 2qn \quad \ldots (i)$$

$S_m = qm^2$

$$\frac{m}{2}(2a+(m-1)d) = qm^2$$

$$2a+(m-1)d = 2qm \quad \ldots (2)$$

(1) − (2)

$$d(\cancel{n-m}) = 2q(\cancel{n-m})$$

$\boxed{d=2q}$ put in (i)

$$2a+(n-1)(2q) = 2qn$$

$\Rightarrow \quad 2a + \cancel{2qn} - 2q = \cancel{2qn}$

$\Rightarrow \quad 2a = 2q$

$\Rightarrow \quad \boxed{a = q}$

Now $\quad S_q = \frac{q}{2}\left[2a + (q-1)d\right]$

$= \frac{q}{2}\left[2q + (q-1)(2q)\right]$

$= \frac{q}{2}\left[\cancel{2q} + 2q^2 - \cancel{2q}\right]$

$= \frac{q}{2}(2q^2)$

$\boxed{S_q = q^3}$ Ans

Ans 4 → given . $x, 2y, 3z$ are in AP

$\Rightarrow \boxed{4y = x + 3z}$ ---(1)

given $\quad x, y, z$ are in G-P

$\Rightarrow \boxed{y^2 = xz}$ ---(2)

$\Rightarrow \quad y^2 = (4y - 3z)z$

$\Rightarrow \quad y^2 = 4yz - 3z^2$

$\Rightarrow \quad y^2 - 4yz + 3z^2 = 0$

$\Rightarrow \quad y^2 - 3yz - yz + 3z^2 = 0$

$\Rightarrow \quad y(y-3z) - z(y-3z) = 0$

$\Rightarrow \quad (y-z)(y-3z) = 0$

$y = z \quad \Big| \quad y = 3z$ put in (1)

$\Rightarrow \frac{z}{y} = 1/3 \quad \longrightarrow \quad 4y = x + y$

$3y = x$

$\boxed{r = 1/3}$ Ans $\quad \frac{y}{x} = 1/3$ (ratio is same)

Qn. 5 →

LHS $\sec\theta_1 \cdot \sec\theta_2 + \sec\theta_2 \cdot \sec\theta_3 + \cdots \sec\theta_{n-1} \cdot \sec\theta_n$

$$= \frac{1}{\cos\theta_1 \cdot \cos\theta_2} + \frac{1}{\cos\theta_2 \cdot \cos\theta_3} + \cdots \frac{1}{\cos\theta_{n-1} \cdot \cos\theta_n}$$

Multiply & Divide by $\sin d$

$$= \frac{1}{\sin d}\left[\frac{\sin d}{\cos\theta_1 \cos\theta_2} + \frac{\sin d}{\cos\theta_2 \cos\theta_3} + \cdots \frac{\sin d}{\cos\theta_{n-1} \cdot \cos\theta_n}\right]$$

$$= \frac{1}{\sin d}\left[\frac{\sin(\theta_2 - \theta_1)}{\cos\theta_1 \cdot \cos\theta_2} + \frac{\sin(\theta_3 - \theta_2)}{\cos\theta_2 \cos\theta_3} + \cdots \frac{\sin(\theta_n - \theta_{n-1})}{\cos\theta_{n-1} \cdot \cos\theta_n}\right]$$

$$= \frac{1}{\sin d}\left[\frac{\sin\theta_2 \cos\theta_1 - \cos\theta_2 \cdot \sin\theta_1}{\cos\theta_1 \cdot \cos\theta_2} + \frac{\sin\theta_3 \cos\theta_2 - \cos\theta_3 \sin\theta_2}{\cos\theta_2 \cos\theta_3} + \right.$$

$$\left. + \cdots \frac{\sin(\theta_n) \cos\theta_{n-1} - \cos\theta_n \sin\theta_{n-1}}{\cos\theta_{n-1} \cdot \cos\theta_n}\right]$$

$$= \frac{1}{\sin d}\left[\tan\theta_2 - \tan\theta_1 + \tan\theta_3 - \tan\theta_2 + \cdots + \tan\theta_n - \tan\theta_{n-1}\right]$$

$$= \frac{1}{\sin d}\left(\tan\theta_n - \tan\theta_1\right) = \text{RHS} \quad \text{proved}$$

Qn 6 →

$S = a + ar + ar^2$

$S = a(1 + r + r^2)$

$$\boxed{S = a\left[\frac{r^3 - 1}{r - 1}\right]} \quad \cdots \left\{S_n = a\left(\frac{r^n - 1}{r - 1}\right)\right\}$$

$P = a \cdot ar \cdot ar^2$

$$\boxed{P = a^3 r^3}$$

$R = \frac{1}{a} + \frac{1}{ar} + \frac{1}{ar^2}$

$R = \frac{1}{a}\left(\frac{1 - \frac{1}{r^3}}{1 - \frac{1}{r}}\right) = \frac{1}{a}\left(\frac{r^3 - 1}{r - 1}\right) \cdot \frac{r}{r^3}$

LHS $\quad P^2 R^3$

$= (a^3 r^3)^2 \cdot \left[\frac{1}{a}\left(\frac{r^3-1}{r-1}\right) \cdot \frac{r}{r^3}\right]^3$

$= a^6 \cdot \cancel{r^6} \cdot \frac{1}{a^3} \cdot \left(\frac{r^3-1}{r-1}\right)^3 \cdot \frac{1}{\cancel{r^6}}$

$= a^3 \cdot \left(\frac{r^3-1}{r-1}\right)^3$

$= \left[a\left(\frac{r^3-1}{r-1}\right)\right]^3$

$= S^3 \quad \ldots \left\{\because S = a\left(\frac{r^3-1}{r-1}\right)\right\}$

$=$ RHS $\quad$ Ans

Ans 7 → given a, b, c, d are in GP

let $a = a$, $b = ar$, $c = ar^2$, $d = ar^3$

Toprove: $a^2 - b^2$, $b^2 - c^2$, $c^2 - d^2$ are in GP

i.e to prove $(b^2 - c^2)^2 = (a^2 - b^2)(c^2 - d^2)$

Taking LHS $\quad (b^2 - c^2)^2$

put value of b & c

$= (a^2r^2 - a^2r^4)^2$

$= a^4r^4(1-r^2)^2$

RHS $(a^2-b^2)(c^2-d^2)$

$= (a^2 - a^2r^2)(a^2r^4 - a^2r^6)$

$= a^2(1-r^2)\, a^2r^4(1-r^2)$

$= a^4r^4(1-r^2)^2$

Clearly LHS = RHS

$\therefore a^2-b^2,\ b^2-c^2,\ c^2-d^2$ are in GP Ans

Ans 8 → let the three numbers are $a-d, a, a+d$

given largest = 7 (smallest)

$a+d = 7(a-d)$

$a+d = 7a-7d$

$6a = 8d$

$\boxed{a = \frac{4}{3}d}$ or $d = \frac{3}{4}a$

given product = 224

$\Rightarrow (a-d)(a)(a+d) = 224$

$\Rightarrow (a^2-d^2)a = 224$

$\Rightarrow \left[a^2 - \frac{9}{16}a^2\right]a = 224$

$\Rightarrow \frac{(7a^2)a}{16} = 224$

Scanned with CamScanner

$\Rightarrow a^3 = \frac{224 \times 16}{7}$

$\Rightarrow a^3 = 32 \times 16$

$\Rightarrow a^3 = 8 \times 8 \times 8$

$\Rightarrow a = 8$

$\therefore d = \frac{3}{4}(a)$

$d = \frac{3}{4}(8) \Rightarrow d = 6$

$\therefore$ Numbers are 2, 8, 14 Ans

Ques 9 →

$a_1, a_2, a_3 \; \ldots \ldots \; a_{2n+1}$ ⇒ out of which $(n+1)$ odd term & (n) even terms

$S_n = a_1 + a_3 + a_5 + \ldots (n+1)$ term

$S_n = \frac{(n+1)}{2}\left[2a_1 + (n+\cancel{1}-\cancel{1})(2d)\right]$

$= \left(\frac{n+1}{2}\right)\left[2a + 2nd\right]$

$= (n+1)(a+nd)$

S'_n = sum of even term

$S'_n = a_2 + a_4 + a_6 + \ldots n$ term

$S'_n = \frac{n}{2}\left[2a_2 + (n-1)(2d)\right]$

$= \frac{n}{2}\left[2(a+d) + (n-1)(2d)\right]$

$= \frac{n}{2}\left[2a + \cancel{2d} + 2nd - \cancel{2d}\right]$

$= n(a+nd)$

Now

$\frac{S_n}{S'_n} = \frac{(n+1)\cancel{(a+nd)}}{n\cancel{(a+nd)}}$

$= (n+1) : n$

Proved

Scanned with CamScanner

Q.No 10 (8)

given $a_p = a$ & $a_q = b$

let $A \to$ Ist term ; $d \to$ common difference

$A + (p-1)d = a$ ---(i) and $A + (q-1)d = b$ - (2)

(1) - (2)

$d(p-q) = a-b$

$\boxed{d = \frac{a-b}{p-q}}$ put in eq (1)

$A + (p-1)\left(\frac{a-b}{p-q}\right) = a$

$A = a - \frac{(p-1)(a-b)}{p-q}$

Now $S_{p+q} = \frac{p+q}{2}\left[2A + (p+q-1)d\right]$

$= \frac{p+q}{2}\left[2a - \frac{2(p-1)(a-b)}{p-q} + (p+q-1)\left(\frac{a-b}{p-q}\right)\right]$

$-a+b$

$= \frac{p+q}{2}\left(\frac{\cancel{2ap} - 2aq - \cancel{2ap} + 2bp + 2a - 2b + ap - bp + aq - bq}{p-q}\right)$

$= \frac{p+q}{2}\left(\frac{-aq + bp + ap - bq + a - b}{p-q}\right)$

$= \frac{p+q}{2}\left(\frac{(a+b)(p-q) + a - b}{p-q}\right)$

separate

$S_{p+q} = \frac{p+q}{2}\left(a+b + \frac{a-b}{p-q}\right)$ Ans