॥ जय श्री राधे कृष्णा। जय श्री गिरिराज जी महाराज ॥ 1

ULTIMATE MATHEMATICS: BY AJAY MITTAL

CHAPTER: LIMITS & DERIVATIVES | CLASS NO: 5

Type. facturization

Q: 1 $\lim_{x\to 1}\left(\frac{1}{x^2+x-2}-\frac{x}{x^3-1}\right)$

$= \lim_{x\to 1}\left(\frac{1}{(x+2)(x-1)}-\frac{x}{(x-1)(x^2+x+1)}\right)$

$= \lim_{x\to 1}\left(\frac{x^2+x+1-x(x+2)}{(x+2)(x-1)(x^2+x+1)}\right)$

$= \lim_{x\to 1}\left(\frac{\cancel{x^2}+x+1-\cancel{x^2}-2x}{(x+2)(x-1)(x^2+x+1)}\right)$

$= \lim_{x\to 1}\left(\frac{1-x}{(x+2)(x-1)(x^2+x+1)}\right)$

$= \lim_{x\to 1}\left(\frac{-1}{(x+2)(x^2+x+1)}\right)$

$= \frac{-1}{(3)(3)} = -\frac{1}{9}$ Ans

Qn: 2 $\lim_{x\to 2}\left(\frac{x^3-6x^2+11x-6}{x^2-6x+8}\right)$

$= \lim_{x\to 2}\left(\frac{\cancel{(x-2)}(x^2-4x+3)}{\cancel{(x-2)}(x-4)}\right) = \frac{1}{2}$ Ans

$$\begin{array}{r|l} & x^2-4x+3 \\ x-2 & x^3-6x^2+11x-6 \\ & -(x^3-2x^2) \\ \hline & -4x^2+11x-6 \\ & -(-4x^2+8x) \\ \hline & 3x-6 \\ & 3x-6 \end{array}$$

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Type Rationalization

$\sqrt{\ } - \sqrt{\ }$
$\sqrt{\ } - \square$
$\square - \sqrt{\ }$
$\sqrt{\ }$ ✗

Qn. 3 $\lim_{x\to 4}\left(\dfrac{x^2-16}{\sqrt{x^2+9}-5}\right)$

$= \lim_{x\to 4}\left(\dfrac{(x^2-16)(\sqrt{x^2+9}+5)}{x^2+9-25}\right)$

$= \lim_{x\to 4}\left(\dfrac{\cancel{(x^2-16)}(\sqrt{x^2+9}+5)}{\cancel{x^2-16}}\right)$

$= 5+5 = 10$ Ans

Qn. 4 evaluate $\lim_{x\to a}\left(\dfrac{\sqrt{a+2x}-\sqrt{3x}}{\sqrt{3a+x}-2\sqrt{x}}\right)$

Sol. $\lim_{x\to a}\left[\dfrac{(\sqrt{a+2x}-\sqrt{3x})(\sqrt{3a+x}+2\sqrt{x})(\sqrt{a+2x}+\sqrt{3x})}{(\sqrt{3a+x}-2\sqrt{x})(\sqrt{3a+x}+2\sqrt{x})(\sqrt{a+2x}+\sqrt{3x})}\right]$

$= \lim_{x\to a}\left[\dfrac{(a+2x-3x)(\sqrt{3a+x}+2\sqrt{x})}{(3a+x-4x)(\sqrt{a+2x}+\sqrt{3x})}\right]$

$= \lim_{x\to a}\left(\dfrac{(a-x)(\sqrt{3a+x}+2\sqrt{x})}{(3a-3x)(\sqrt{a+2x}+\sqrt{3x})}\right)$

$= \lim_{x\to a}\left(\dfrac{\sqrt{3a+x}+2\sqrt{x}}{3(\sqrt{a+2x}+\sqrt{3x})}\right)$

$= \dfrac{2\sqrt{a}+2\sqrt{a}}{3(\sqrt{3}\sqrt{a}+\sqrt{3}\sqrt{a})} = \dfrac{4\cancel{\sqrt{a}}}{6\sqrt{3}\cancel{\sqrt{a}}} = \dfrac{2}{3\sqrt{3}}$ Ans

Type

$$\lim_{x \to a} \left(\frac{x^n - a^n}{x - a} \right) = n a^{n-1}$$

Q=5 $\lim_{x \to 2} \left(\frac{x^{10} - 1024}{x - 2} \right)$

$= \lim_{x \to 2} \left(\frac{x^{10} - 2^{10}}{x - 2} \right)$

$= 10(2)^9 = 10 \times 512 = 5120$

Qn 6 $\lim_{z \to 1} \left(\frac{z^{1/3} - 1}{z^{1/6} - 1} \right)$

$= \lim_{z \to 1} \left(\frac{z^{1/3} - 1^{1/3}}{z^{1/6} - 1^{1/6}} \right)$

$= \lim_{z \to 1} \left(\frac{\frac{z^{1/3} - 1^{1/3}}{z - 1}}{\frac{z^{1/6} - 1^{1/6}}{z - 1}} \right)$

$= \frac{\frac{1}{3}(1)^{1/3 - 1}}{\frac{1}{6}(1)^{1/6 - 1}} = \frac{6}{3} = 2$ Ans

Qn. 7 → If $\lim_{x \to -a} \left(\frac{x^9 + a^9}{x + a} \right) = 9$ Find the value of a

Soln $\lim_{x \to -a} \left[\frac{x^9 - (-a)^9}{x - (-a)} \right] = 9$

$\Rightarrow 9(-a)^8 = 9$

$\Rightarrow 9a^8 = 9$

$\Rightarrow a^8 = 1$

$\Rightarrow a = \pm 1$

Q.No. 8 $\lim_{x \to a} \left(\frac{(x+2)^{5/3} - (a+2)^{5/3}}{x-a} \right)$

Soln put $x+2 = y$

when $x \to a$ then $y \to a+2$

$$\therefore \lim_{y \to (a+2)} \left[\frac{y^{5/3} - (a+2)^{5/3}}{y-2-a} \right]$$

$$= \lim_{y \to (a+2)} \left[\frac{y^{5/3} - (a+2)^{5/3}}{y-(a+2)} \right]$$

$$= \frac{5}{3}(a+2)^{5/3-1} = \frac{5}{3}(a+2)^{2/3} \text{ Ans}$$

Q.No. 9 → evaluate $\lim_{x \to 1} \left[\frac{(x+x^2+x^3+\cdots x^n) - n}{x-1} \right]$

Soln

$$= \lim_{x \to 1} \left[\frac{(x+x^2+x^3 \cdots x^n) - (1+1+1+1 \cdots n \text{ times})}{x-1} \right]$$

$$= \lim_{x \to 1} \left[\frac{(x-1) + (x^2-1) + (x^3-1) + \cdots (x^n-1)}{(x-1)} \right]$$

$$= \lim_{x \to 1} \left(\frac{\cancel{x-1}}{\cancel{x-1}} \right) + \lim_{x \to 1} \left(\frac{x^2-1^2}{x-1} \right) + \lim_{x \to 1} \left(\frac{x^3-1^3}{x-1} \right) + \cdots \lim_{x \to 1} \left(\frac{x^n-1^n}{x-1} \right)$$

$$= 1 + 2(1)^1 + 3(1)^2 + \cdots n(1)^{n-1}$$

$$= 1 + 2 + 3 + \cdots n$$

$$= \frac{n(n+1)}{2} \text{ Ans}$$

Types

(i) $\lim_{x \to 0} \left(\frac{e^x - 1}{x} \right) = 1$

(i) $\lim_{x \to 0} \left(\frac{a^x - 1}{x} \right) = \log a$ ... $\{\log e = 1\}$

(i) $\lim_{x \to 0} \left(\frac{\log(1+x)}{x} \right) = 1$

$\boxed{\text{If } \lim_{x \to a} (f(x)) \text{ put } x = a+h \ \& \ h \to 0}$

Qn. 10 Evaluate $\lim_{x \to 0} \left(\frac{2^{3x} - 1}{x} \right)$

$= \lim_{x \to 0} \left(\frac{2^{3x} - 1}{3x} \right) \times 3$

$= \log 2 \times 3 = 3\log 2$ ... $\left\{ \text{when } x \to 0 \atop 3x \to 0 \right\}$

Qn. 11 $\lim_{x \to 0} \left(\frac{a^x - b^x}{x} \right)$

$= \lim_{x \to 0} \left(\frac{a^x - b^x - 1 + 1}{x} \right)$

$= \lim_{x \to 0} \left(\frac{(a^x - 1)}{x} - \frac{(b^x - 1)}{x} \right)$

$= \log a - \log b$ Ans

$= \log\left(\frac{a}{b}\right)$ Ans

Q.N: 12 →

$$\lim_{x\to 0}\left(\frac{10^x-2^x-5^x+1}{\tan x}\right)$$

$$=\lim_{x\to 0}\left(\frac{10^x-2^x-5^x+1-1+1}{\tan x}\right)$$

$$=\lim_{x\to 0}\left(\frac{(10^x-1)-(2^x-1)-(5^x-1)}{\tan x}\right)$$

DIVIDED by x

$$=\lim_{x\to 0}\left(\frac{\frac{10^x-1}{x}-\frac{(2^x-1)}{x}-\frac{(5^x-1)}{x}}{\frac{\tan x}{x}}\right)$$

$$=\frac{\log(10)-\log(2)-\log(5)}{1} \quad \cdots \left\{\lim_{x\to 0}\left(\frac{a^x-1}{x}\right)=\log a,\ \lim_{x\to 0}\left(\frac{\tan x}{x}\right)=1\right\}$$

$$=\log(10)-(\log 2+\log 5)$$

$$=\log(10)-\log(10)$$

$$=0 \quad \text{Ans}$$

$$\boxed{\begin{array}{l}\log(AB)=\log A+\log B\\ \log(A/B)=\log A-\log B\\ \log(m)^n=n\log m\end{array}}$$

Q. 13

$$\lim_{x\to 0}\left(\frac{10^x-2^x-5^x+1}{x^2}\right)$$

$$=\lim_{x\to 0}\left(\frac{2^x(5^x-1)-1(5^x-1)}{x^2}\right)$$

$$=\lim_{x\to 0}\left(\frac{(5^x-1)}{x}\cdot\frac{(2^x-1)}{x}\right)=\log 5\times\log 2 \quad \text{Ans}$$

Q.-14. Evaluate $\lim_{x \to 0}\left(\frac{2^x - 1}{\sqrt{1+x} - 1}\right)$

Sol $\lim_{x \to 0}\left(\frac{(2^x - 1)(\sqrt{1+x} + 1)}{\cancel{1}+x - \cancel{1}}\right)$

$= \lim_{x \to 0}\left(\frac{(2^x - 1)(\sqrt{x+1} + 1)}{x}\right)$

$= (\log 2)(1+1)$

$= 2\log 2$ Ans

Q. 15 $\lim_{x \to 1}\left(\frac{a^{x-1} - 1}{\sin(\pi x)}\right)$

put $x = 1+h$ & $h \to 0$

$= \lim_{h \to 0}\left(\frac{a^{1+h-1} - 1}{\sin(\pi(1+h))}\right)$

$= \lim_{h \to 0}\left(\frac{a^h - 1}{\sin(\pi + \pi h)}\right)$

$= \lim_{h \to 0}\left(\frac{a^h - 1}{-\sin(\pi h)}\right)$

$= \lim_{h \to 0}\left(\frac{\frac{a^h - 1}{h} \times \cancel{h}}{-\frac{\sin(\pi h)}{\pi h} \times \pi \cancel{h}}\right) = \frac{\log a}{-1 \times \pi}$

$= -\frac{\log a}{\pi}$ Ans

Qn. 16 $\lim_{x\to 0}\left(\frac{x(e^x-1)}{1-\cos x}\right)$

Soln: $\lim_{x\to 0}\left(\frac{x(e^x-1)}{2\sin^2(\frac{x}{2})}\right)$

$= \lim_{x\to 0}\left(\frac{\cancel{x}\frac{(e^x-1)}{x}\times\cancel{x}}{\frac{2\sin^2(x/2)}{\frac{x^2}{4}}\times\cancel{\frac{x^2}{4}}}\right)$

$= \frac{1}{2\times 1\times\frac{1}{4}}$ ... $\left\{\lim_{x\to 0}\left(\frac{e^x-1}{x}\right)=1,\ \lim_{x\to 0}\left(\frac{\sin x}{x}\right)=1\right\}$

$= 2$ Ans

Qn. 17 → $\lim_{x\to 0}\left(\frac{e^{3+x}-\sin x-e^3}{x}\right)$

$= \lim_{x\to 0}\left[\frac{(e^{3+x}-e^3)}{x}-\frac{\sin x}{x}\right]$

$= \lim_{x\to 0}\left(\frac{e^{3+x}-e^3}{x}\right)-\lim_{x\to 0}\left(\frac{\sin x}{x}\right)$

$= e^3\lim_{x\to 0}\left(\frac{e^x-1}{x}\right)-1$

$= e^3-1$ ... $\left\{\lim_{x\to 0}\left(\frac{e^x-1}{x}\right)=1\right\}$

$= e^3-1$ Ans

First principle method

Q. $f(x) = e^{\tan x}$

Soln $f'(x) = \lim_{h\to 0}\left(\dfrac{e^{\tan(x+h)} - e^{\tan x}}{h}\right)$

$= e^{\tan x}\lim_{h\to 0}\left(\dfrac{e^{\tan(x+h)-\tan x} - 1}{h}\right)$

$= e^{\tan x}\lim_{h\to 0}\left[\dfrac{\left(e^{\tan(x+h)-\tan x} - 1\right)\left(\tan(x+h) - \tan x\right)}{\left(\tan(x+h)-\tan x\right)h}\right]$

$= e^{\tan x}\lim_{h\to 0}\left(\dfrac{e^{\tan(x+h)-\tan x} - 1}{\tan(x+h)-\tan x}\right) \times \lim_{h\to 0}\left(\dfrac{\tan(x+h) - \tan x}{h}\right)$

$= e^{\tan x} \times 1 \times \lim_{h\to 0}\left[\dfrac{\tan(h)\left\{1+\tan(x+h)\tan x\right\}}{h}\right]$

$= e^{\tan x} \times (1 + \tan^2 x)$

$f'(x) = e^{\tan x} . \sec^2 x$ Ans

WORKSHEET No: 4 (class = 5)

LIMITS & DERIVATIVES

Ques: 1 Differentiate $f(x) = e^{\cos x}$ using first principle

Ans: $-\sin x \cdot e^{\cos x}$

Ques. 2 Differentiate $f(x) = e^{\sqrt{\tan x}}$ using first principle

Ans $\frac{1}{2\sqrt{\tan x}} \cdot \sec^2 x \cdot e^{\sqrt{\tan x}}$

Ques 3 → evaluate $\lim\limits_{x \to \frac{\pi}{2}} \left[\frac{2^{-\cos x} - 1}{x\left(x - \frac{\pi}{2}\right)} \right]$

Ans $\frac{2}{\pi} \log 2$

Ques 4 → evaluate $\lim\limits_{x \to 5} \left(\frac{e^x - e^5}{x - 5} \right)$

Ans $= e^5$

Qn. 5 → evaluate $\lim\limits_{x \to 0} \left(\frac{a^x + b^x - c^x - d^x}{x} \right)$

Ans $\log\left(\frac{ab}{cd}\right)$

Qn. 6 → $\lim\limits_{x \to 1} \left(\frac{x - 1}{\log x} \right)$

Ans 1

Qn. 7 → $\lim\limits_{x \to 0} \left(\frac{2^{3x} - 3^x}{\sin(3x)} \right)$

Ans $\log 2 - \frac{1}{3} \log 3$

Qn. 8 → evaluate $\lim\limits_{x \to 0} \left(\frac{12^x - 3^x - 4^x + 1}{\sin x} \right)$

Ans $= 0$

Qn. 9 evaluate $\lim\limits_{x \to 0} \left(\frac{12^x - 3^x - 4^x + 1}{x \sin x} \right)$

Ans $\log 3 \times \log 4$

Qn. 10 → $\lim\limits_{x \to \sqrt{2}} \left(\frac{x^4 - 4}{x^2 + 3\sqrt{2}x - 8} \right)$

Ans $\frac{8}{5}$

Qn. 11) $\lim_{x\to 2}\left(\frac{x^3+3x^2-9x-2}{x^3-x-6}\right)$ Ans $\frac{15}{11}$

Qn 12 → $\lim_{x\to 1}\left(\frac{x-2}{x^2-x}-\frac{1}{x^3-3x^2+2x}\right)$ Ans 2

Qn. 13 → $\lim_{x\to 4}\left(\frac{3-\sqrt{5+x}}{1-\sqrt{5-x}}\right)$ Ans $-\frac{1}{3}$

Qn. 14 → $\lim_{x\to 1}\left(\frac{(2x-3)(\sqrt{x}-1)}{2x^2+x-3}\right)$ Ans = $-\frac{1}{10}$

Qn 15 → $\lim_{x\to 0}\left(\frac{\sqrt{1+x^2}-\sqrt{1+x}}{\sqrt{1+x^3}-\sqrt{1+x}}\right)$ Ans = 1

Qn 16 → If $\lim_{x\to 2}\left(\frac{x^n-2^n}{x-2}\right)=80$ find value of n

Ans $n=5$

Qn 17 → Find value of 'k' if $\lim_{x\to 1}\left(\frac{x^4-1}{x-1}\right)=\lim_{x\to k}\left(\frac{x^3-k^3}{x^2-k^2}\right)$

Ans $k=8/3$

Qn. 18 → $\lim_{x\to 0}\left(\frac{(1-x)^n-1}{x}\right)$ Ans $=-n$

Qn. 19 → $\lim_{x\to 0}\left(\frac{(1+x)^6-1}{(1+x)^2-1}\right)$ Ans = 3

Qn. 20 → Differentiate $f(x)=x^{1/3}$ using first principle Method

Ans $\frac{1}{3}x^{-2/3}$