→ WORKSHEET No: 4 (S-4) → & solutions

Ques. 1→ Proof of Distributive law

Solution (i) T.P $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

let $x \in A \cup (B \cap C)$

$\Rightarrow x \in A$ or $x \in (B \cap C)$

$\Rightarrow x \in A$ or $(x \in B$ and $x \in C)$

$\Rightarrow (x \in A$ or $x \in B)$ and $(x \in A$ or $x \in C)$

$\Rightarrow x \in (A \cup B)$ and $x \in (A \cup C)$

$\Rightarrow x \in (A \cup B) \cap (A \cup C)$

$\Rightarrow A \cup (B \cap C) \subset (A \cup B) \cap (A \cup C)$ ....(i)

let $x \in (A \cup B) \cap (A \cup C)$

$\Rightarrow x \in (A \cup B)$ and $x \in (A \cup C)$

$\Rightarrow (x \in A$ or $x \in B)$ and $(x \in A$ or $x \in C)$

$\Rightarrow x \in A$ and $(x \in B$ or $x \in C)$

$\Rightarrow x \in A$ and $x \in (B \cup C)$

$\Rightarrow x \in A \cap (B \cup C)$.

$\Rightarrow (A \cup B) \cap (A \cup C) \subset A \cap (B \cup C)$ --(2)

from (1) & (2)

$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ proved

(ii) T.P $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

Do yourself (same as part (i))

Worksheet & Solution [5-4]

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Qn2 → let A and B be sets if $A \cup X = B \cup X$ and $A \cap X = B \cap X = \phi$. Show that $A = B$

Solution: we have $A \cup X = B \cup X$

$A \cap X = B \cap X = \phi$

Consider $A \cup X = B \cup X$

$\Rightarrow A \cap (A \cup X) = A \cap (B \cup X)$

$\Rightarrow (A \cap A) \cup (A \cap X) = (A \cap B) \cup (A \cap X)$

$\Rightarrow A \cup \phi = (A \cap B) \cup (A \cap X)$ ---- {given $A \cap X = \phi$}

$\Rightarrow A = (A \cap B) \cup (\phi)$

$\Rightarrow A = A \cap B$ ---- (1)

again consider, $A \cup X = B \cup X$

$\Rightarrow B \cap (A \cup X) = B \cap (B \cup X)$

$\Rightarrow (B \cap A) \cup (B \cap X) = (B \cap B) \cup (B \cap X)$

$\Rightarrow (A \cap B) \cup (\phi) = B \cup \phi$ --- {given $B \cap X = \phi$}

$\Rightarrow A \cap B = B$ --- (2)

from (1) (2)

$A = B$ proved Ans

Qns: 3 → Show that $A \cap B = A \cap C$ need not imply $B = C$

Solution: given $A \cap B = A \cap C$

To prove $B \neq C$

let $A = \{1, 2, 3\}$

$B = \{2, 3, 4, 5\}$

and $C = \{2, 3, 7, 8\}$

here $A \cap B = \{2, 3\}$

and $A \cap C = \{2, 3\}$

$\Rightarrow A \cap B = A \cap C$

But clearly $B \neq C$ Ans

Qns 4 (i) Show that $A \cup (A \cap B) = A$

Solution we have $A \cup (A \cap B)$

$= (A \cup A) \cap (A \cup B)$ --- {distributive prop}

$= A \cap (A \cup B)$

of course all element of A are always present in $(A \cup B)$ ∴ A is a subset of $A \cap B$

$\Rightarrow A \subset (A \cup B)$

$\therefore A \cap (A \cup B)$

$= A$ Proved

(ii) Show that $A \cap (A \cup B) = A$

Solution we have

$A \cap (A \cup B)$

$= (A \cap A) \cup (A \cap B)$ ---- {distributive law}

$= A \cup (A \cap B)$

of course all element of $(A \cap B)$ are always present in set A

$\therefore (A \cap B)$ is a subset of A

i.e $(A \cap B) \subset A$

$\therefore A \cup (A \cap B)$

$= A$ Proved

Qns 5 Find sets A, B and C such that $A \cap B$, $B \cap C$ and $A \cap C$ are non-empty sets and $A \cap B \cap C = \phi$

Solution: let $A = \{1, 2\}$

$B = \{2, 3\}$

$C = \{1, 3\}$

$A \cap B = \{2\} \neq \phi$

$B \cap C = \{3\} \neq \phi$

$A \cap C = \{1\} \neq \phi$

but $A \cap B \cap C = \phi$ Ans

Ques 6 → (i) Show that $(A \cap B) \cup (A - B) = A$

(ii) Show that $A \cup (B - A) = A \cup B$

Solution (i) Taking L.H.S

$(A \cap B) \cup (A - B)$

$= (A \cap B) \cup (A \cap B')$ --- $\{\because A - B = A \cap B'\}$

$= A \cap (B \cup B')$ --- {distributive prop}

$= A \cap U$ --- $\{\because A \cup A' = U$ universal$\}$

$= A =$ RHS

(ii) LHS $A \cup (B - A)$

$= A \cup (B \cap A')$

$= (A \cup B) \cap (A \cup A')$ --- {distributive Property}

$= (A \cup B) \cap U$ {∵ all element of $(A \cup B)$ are always present in universal set U}

$= A \cup B$ = RHS

Q.Nr 7 → Show that

$$A-(B-C) = (A-B) \cup (A \cap C)$$

Solution: Taking L.H.S

$= A-(B-C)$

$= A-(B \cap C')$ --- $\{\because A-B = A \cap B'\}$

$= A \cap (B \cap C')'$ --- $\{$again $A-B = A \cap B'\}$

$= A \cap (B' \cup C)$ --- $\{$demorgan's law $(A \cap B)' = A' \cup B'$ and $(A')' = A\}$

$= (A \cap B') \cup (A \cap C)$ --- $\{$distributive law$\}$

$= (A-B) \cup (A \cap C)$

$=$ RHS proved

Q.Nr 8 → Show that $A \cap (B-C) = (A \cap B) - (A \cap C)$
self

Q.Nr 9 → Show that $A-(B \cup C) = (A-B) \cap (A-C)$
self

Q.Nr 10 → Show that $A-(B \cap C) = (A-B) \cup (A-C)$
self

Q.Nr 11 → If $A \cup B = C$ and $A \cap B = \phi$ then show that $C-B = A$

Solution: given $A \cup B = C$ and $A \cap B = \phi$

T.P $C-B = A$

Taking L.H.S $C-B$

$= (A \cup B) - B$ --- {given $A \cup B = C$}

$= (A \cup B) \cap B'$ --- {$\because A-B = A \cap B'$}

$= (A \cap B') \cup (B \cap B')$ --- {distributive prop}

$= (A \cap B') \cup \phi$

$= A \cap B'$ --- {$\because A \cup \phi = A$}

$= A-B$

but we are given $A \cap B = \phi$

i.e No common element b/w A & B

$\Rightarrow A-B = A$ proved

Qn 12 → Show $(A \cup B) - (A \cap B) = (A-B) \cup (B-A)$

Solution LHS $(A \cup B) - (A \cap B)$

$= (A \cup B) \cap (A \cap B)'$ --- {$\because A-B = A \cap B'$}

$= (A \cup B) \cap (A' \cup B')$ --- {demorgan's law}

$= ((A \cup B) \cap A') \cup ((A \cup B) \cap B')$ ---- {distributive law}

$= ((A \cap A') \cup (B \cap A')) \cup ((A \cap B') \cup (B \cap B'))$ --- {distributive law}

$= (\phi \cup (B \cap A')) \cup ((A \cap B') \cup \phi)$

$= (B \cap A') \cup (A \cap B')$

$= (B-A) \cup (A-B) = (A-B) \cup (B-A)$ proved