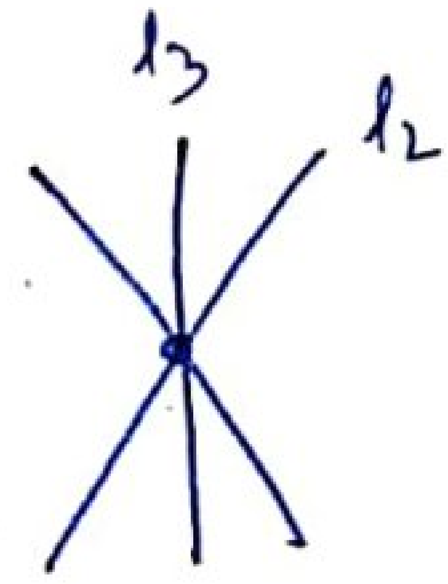
ULTIMATE MATHEMATICS: BY AJAY MITTAL

CHAPTER: STRAIGHT LINES : CLASS NO: 5

(·) Concurrent lines



Qn 1 If the lines $2x+y-3=0$;
$5x+ky-3=0$ & $3x-y-2=0$ are concurrent find the value of k

Soln Solve equation (1) & equation (3)
get $x=1,\ y=1$
(1, 1) also lies on $5x+ky-3=0$
$5+k-3=0 \Rightarrow \boxed{k=-2}$ Ans

Qn 2 → If the three lines $y=m_1x+c_1$; $y=m_2x+c_2$; $y=m_3x+c_3$ intersect at one point. Show that
$m_1(c_2-c_3)+m_2(c_3-c_1)+m_3(c_1-c_2)=0$

Soln
$y=m_1x+c_1$ --- (1)
$y=m_2x+c_2$ --- (2)
$y=m_3x+c_3$ --- (3)
Solve (1) & (2)
$m_1x+c_1=m_2x+c_2$
$x(m_1-m_2)=c_2-c_1$

$\boxed{x=\dfrac{c_2-c_1}{m_1-m_2}}$ put in eq (1)

$y=\dfrac{m_1c_2-m_1c_1}{m_1-m_2}+c_1$

$y=\dfrac{m_1c_2-\cancel{m_1c_1}+\cancel{m_1c_1}-m_2c_1}{m_1-m_2}$

$\boxed{y=\dfrac{m_1c_2-m_2c_1}{m_1-m_2}}$

put value of x & y in eq (3)

$$y = m_3 x + c_3$$

$$\frac{m_1 c_2 - m_2 c_1}{m_1 - m_2} = m_3\left(\frac{c_2 - c_1}{m_1 - m_2}\right) + c_3$$

Proceed

Ons:- 3 → Find the value of θ & p, if the equation $x\cos\theta + y\sin\theta = p$ is the normal form of the line $\sqrt{3}x + y + 2 = 0$

Sol

given general equation $\sqrt{3}x + y + 2 = 0$

$$\Rightarrow \sqrt{3}x + y = -2$$

$$\Rightarrow -\sqrt{3}x - y = 2$$

Divide both sides by $\sqrt{a^2+b^2}$

$$= \sqrt{3+1} = 2$$

$$\Rightarrow -\frac{\sqrt{3}}{2}x - \frac{1}{2}y = 1$$

$$\Rightarrow x\cos\left(\pi + \frac{\pi}{6}\right) + y\sin\left(\pi + \frac{\pi}{6}\right) = 1$$

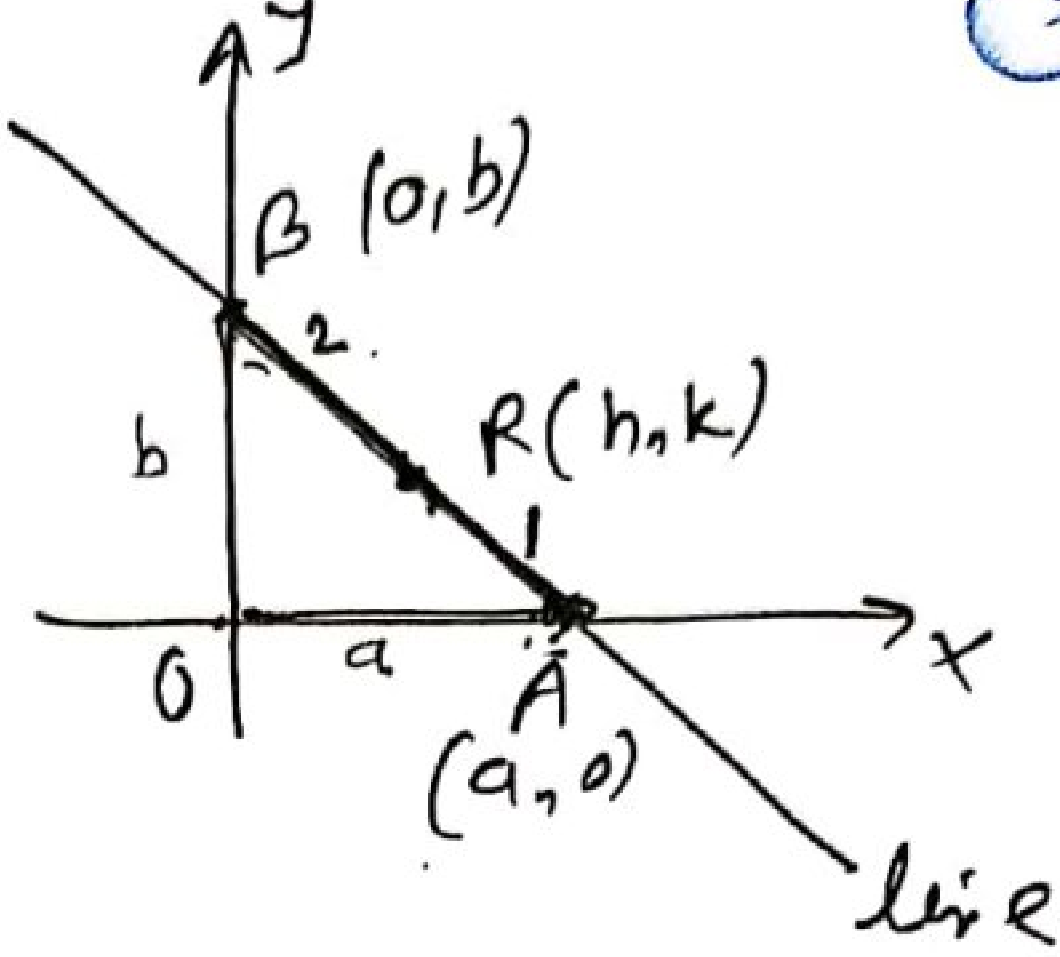
$$\Rightarrow x\cos(7\pi/6) + y\sin(7\pi/6) = 1$$

Comp with $x\cos\theta + y\sin\theta = p$

we get $\boxed{\theta = 7\pi/6}$ & $\boxed{p = 1}$ Ans

Ons:- 4 → Point $R(h,k)$ divides a line segment between the axes in the ratio $1:2$. Find the equation of the line.

Soln R(h, k) divides A & B in the ratio 1 : 2



By section formula

$$h = \frac{0 + 2a}{3} \quad ; \quad k = \frac{b + 0}{3}$$

$$\Rightarrow \quad a = \frac{3h}{2} \quad ; \quad b = 3k$$

By Intercept form $\frac{x}{a} + \frac{y}{b} = 1$

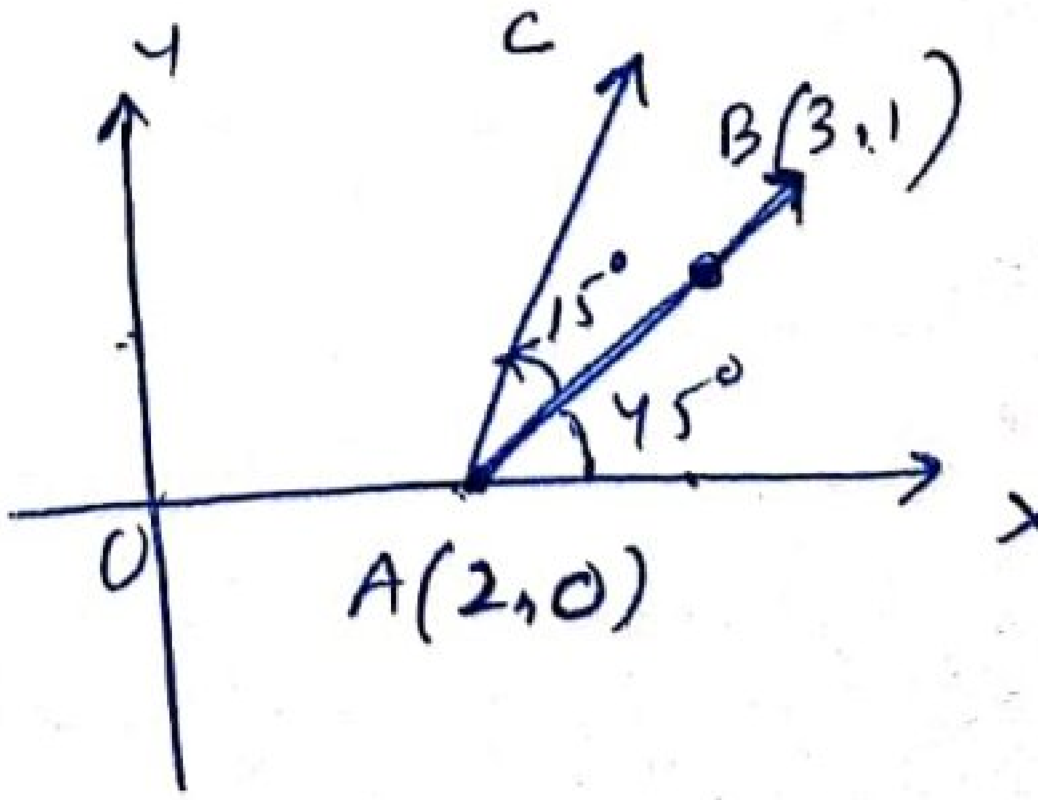
$$\Rightarrow \frac{x}{\frac{3h}{2}} + \frac{y}{3k} = 1$$

$$\Rightarrow \frac{2x}{3h} + \frac{y}{3k} = 1$$

$$\Rightarrow \boxed{\frac{2x}{h} + \frac{y}{k} = 3} \quad \text{Ans}$$

Qn: 5 → If the line joining two points A (2, 0) & B (3, 1) is "rotated" about 'A' in anticlock wise direction through an angle 15°. Find the equation of the line in new position.

Soln (∵) Slope of AB $= \frac{1-0}{3-2} = 1$



(∵) ⇒ $\tan\theta = 1$

$(\theta = 45°)$

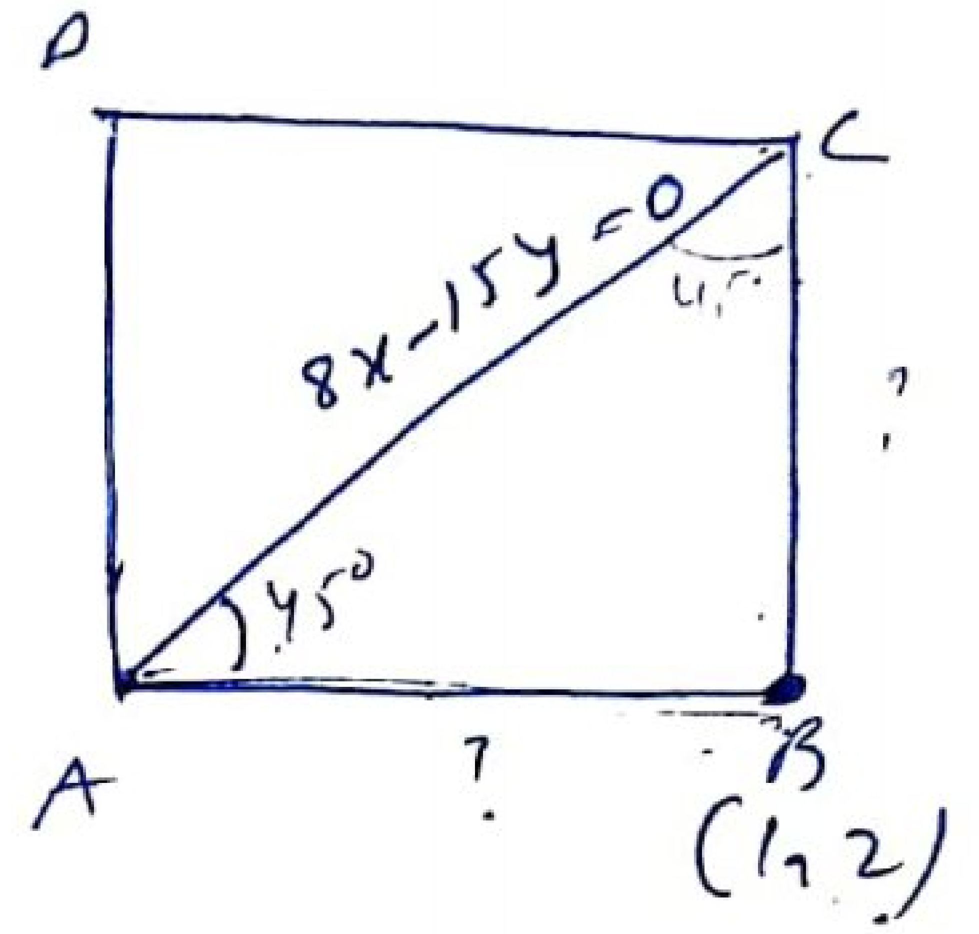
(∵) angle for AC $= 15° + 45° = 60°$

(∵) slope of AC $= \tan(60) = \sqrt{3}$

(∵) equation of AC $\quad y - 0 = \sqrt{3}(x-2)$ Ans

(4)

Qn: 6 → If one diagonal of a square is along the line $8x-15y=0$ and one of its vertex is at $(1,2)$. Then find the equation of sides of the square passing through this vertex



Soln (1,2) does not satisfy the given equation

∴ to find equations AB & BC

∴ slope of AC : $m_1 = \frac{-8}{-15} = \frac{8}{15}$

∴ let slope of AB : $m_2 = m$

∴ angle b/w them 45°

∴ $\tan(45°) = \left| \frac{\frac{8}{15} - m}{1 + \frac{8m}{15}} \right|$

$\Rightarrow \pm 1 = \frac{8-15m}{15+8m}$

$\Rightarrow 1 = \frac{8-15m}{15+8m}$ & $-1 = \frac{8-15m}{15+8m}$

$\Rightarrow 15+8m = 8-15m$ & $-15-8m = 8-15m$

$\Rightarrow 23m = -7$ & $7m = 23$

$m = -7/23$ & $m = 23/7$

∴ slope of AB $= -7/23$ & ∴ slope of BC $= 23/7$

∴ ~~equation~~

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Equation of AB

$$y - 2 = \frac{-7}{23}(x-1)$$

$$23y - 46 = -7x + 7$$

$$\boxed{7x + 23y - 53 = 0}$$

Equation of BC

$$y - 2 = \frac{23}{7}(x-1)$$

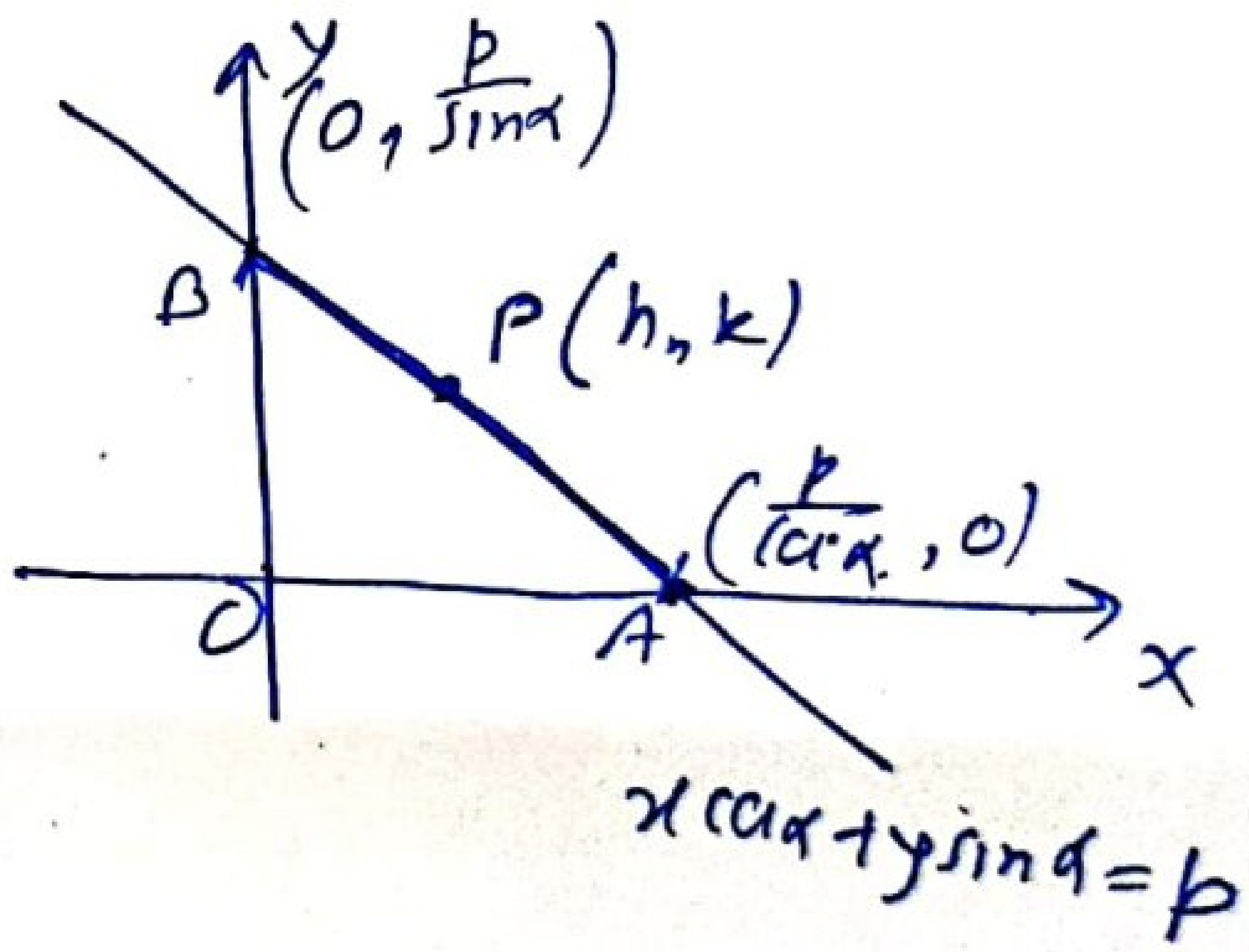
$$7y - 14 = 23x - 23$$

$$\boxed{23x - 7y - 9 = 0}$$

Ans

Qn: 7 Show that the locus of the mid point of the distance between the axes of the variable line $x\cos\alpha + y\sin\alpha = p$ is $\frac{1}{x^2} + \frac{1}{y^2} = \frac{4}{p^2}$ where p is any constant

Soln: ~~A(x,0)~~



point A lies on line (given)

put $y = 0$

$\Rightarrow x = \frac{p}{\cos\alpha}$

$\therefore A\left(\frac{p}{\cos\alpha}, 0\right)$

Similarly $B\left(0, \frac{p}{\sin\alpha}\right)$

P(h, k) is the mid point of AB

$$h = \frac{\frac{p}{\cos\alpha} + 0}{2} \quad \& \quad k = \frac{0 + \frac{p}{\sin\alpha}}{2}$$

$$h = \frac{p}{2\cos\alpha} \quad \& \quad k = \frac{p}{2\sin\alpha}$$

$$\Rightarrow \cos\alpha = \frac{p}{2h} \quad \& \quad \sin\alpha = \frac{p}{2k}$$

Squaring & ~~the~~ adding these equation

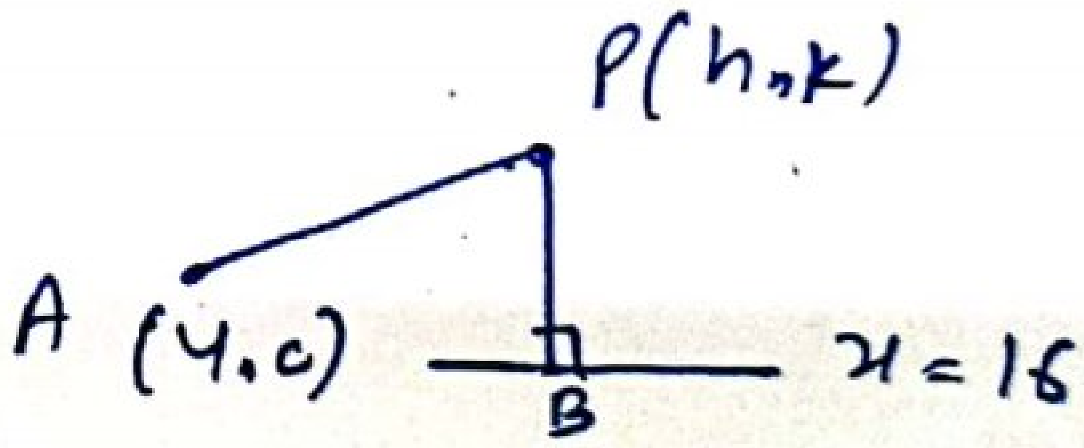
$$\cos^2\alpha + \sin^2\alpha = \frac{p^2}{4h^2} + \frac{p^2}{4k^2}$$

$$1 = \frac{p^2}{4h^2} + \frac{p^2}{4k^2}$$

$$\Rightarrow 1 = \frac{p^2}{4x^2} + \frac{p^2}{4y^2}$$

$$\Rightarrow \boxed{\frac{4}{p^2} = \frac{1}{x^2} + \frac{1}{y^2}}$$ Ans

Qn. 8 → A point moves such that its distance from the point (4, 0) is ~~half~~ half that of its distance from the line $x=16$. Find the locus of the point



OR $x-16=0$

Soln $PA = \frac{1}{2} PB$

$$\sqrt{(h-4)^2 + k^2} = \frac{1}{2}\frac{|h-16|}{\sqrt{1+0}}$$

Squaring

$$h^2 + 16 - 8h + k^2 = \frac{1}{4}(h^2 + 256 - 32h)$$

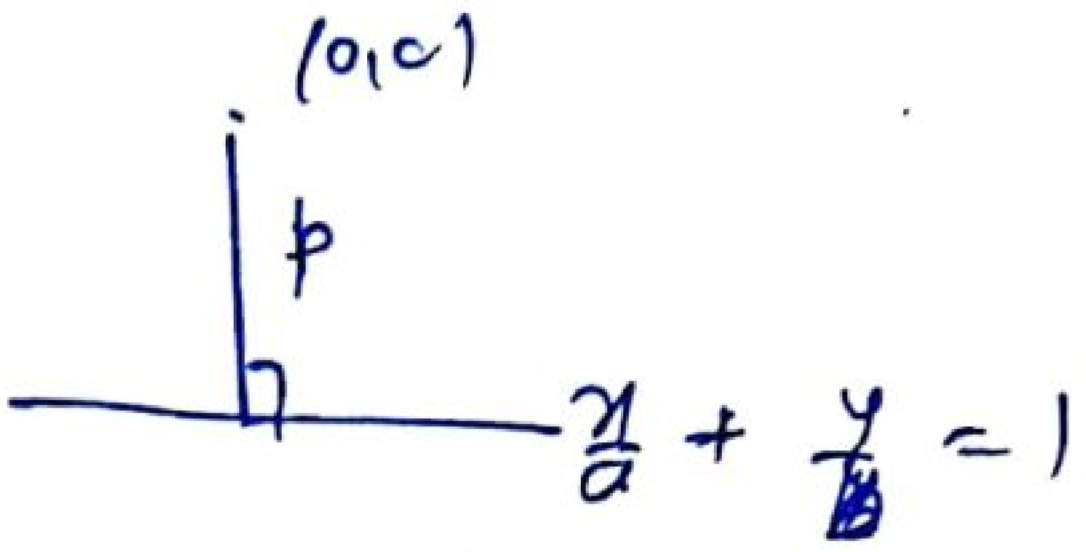
$$\Rightarrow 4h^2 + 64 - \cancel{32h} + 4k^2 = h^2 + 256 - \cancel{32h}$$

$$\Rightarrow 3h^2 + 4k^2 = \cancel{240}\ 192$$

$$\Rightarrow \boxed{3x^2 + 4y^2 = \cancel{240}\ 192}$$ Ans

Qn 1:9 → If p is the length of perpendicular from the origin on the line $\frac{x}{a}+\frac{y}{b}=1$ and a^2, p^2, b^2 are in AP, then show that $a^4+b^4=0$

Soln



If a, b, c are in AP then $2b=a+c$

$b-a=c-b$
$2b=a+c$

or $bx+ay-ab=0$

$$p=\frac{|-ab|}{\sqrt{b^2+a^2}}$$

$$\Rightarrow p^2=\frac{a^2b^2}{b^2+a^2}$$

Now given that a^2, p^2, b^2 are in AP

$$\Rightarrow 2p^2=a^2+b^2$$

$$\Rightarrow \frac{2a^2b^2}{b^2+a^2}=a^2+b^2$$

$$\Rightarrow 2a^2b^2=(a^2+b^2)^2$$

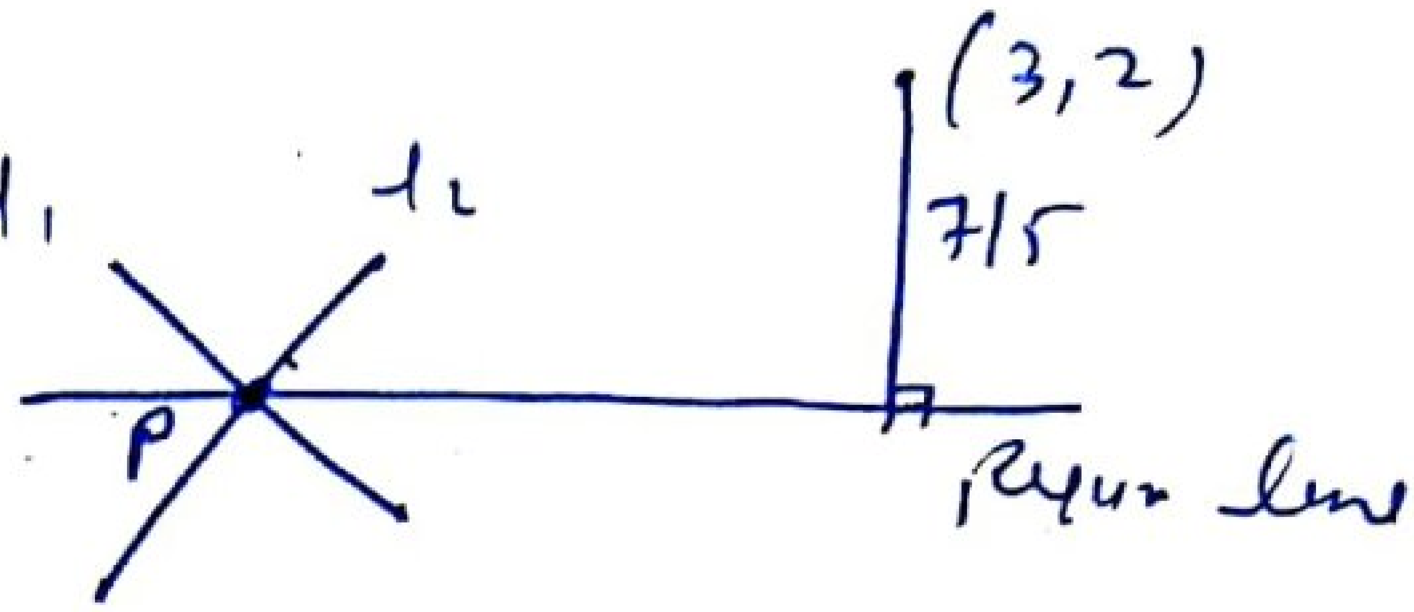
$$\Rightarrow 2a^2b^2=a^4+b^4+2a^2b^2$$

$$\Rightarrow \boxed{a^4+b^4=0}$$ Ans

Q No. 10 → Find the equations of the lines through the point of Intersection of the lines $x - y + 1 = 0$ and $2x - 3y + 5 = 0$ and whose distance from the point $(3, 2)$ is $\frac{7}{5}$.

Sol: equ of l_1: $x - y = -1$

equ of l_2: $2x - 3y = -5$



$$\cancel{2}x - 2y = -2$$

$$-y = -3$$

$y = 3$ $\quad x = 2$ $\quad \therefore P(2, 3)$

let $m \to$ Slope of Required line

By point slope form: equation of Required line

$$y - 3 = m(x - 2)$$

$$\Rightarrow \quad mx - y - 2m + 3 = 0$$

distance of this line from $(3,2)$ is $7/5$

$$\frac{7}{5} = \frac{|3m - 2 - 2m + 3|}{\sqrt{m^2 + 1}}$$

$$\frac{7}{5} = \frac{|m + 1|}{\sqrt{m^2 + 1}}$$

Squaring

$$\frac{49}{25} = \frac{m^2 + 1 + 2m}{m^2 + 1}$$

$$\Rightarrow 49m^2 + 49 = 25m^2 + 25 + 50m$$

$$24m^2 - 50m + 24 = 0$$

$$12m^2 - 25m + 12 = 0$$

$$12m^2 - 16m - 9m + 12 = 0$$

$$4m(3m - 4) - 3(3m - 4) = 0$$

$m = 4/3$ $\quad m = 3/4$

Use point $P(2,3)$ point slope form find equations

WORKSHEET NO: 4 (class No: 5)

Straight lines

Ques: 1 → Find the value of p so that the three lines $3x+y-2=0$; $px+2y-3=0$ and $2x-y-3=0$ may Intersect at one point. ANS. $p=5$

Ques: 2 → Find the equation of the line drawn perpendicular to the line $\frac{x}{4}+\frac{y}{6}=1$, through the point, where it meets the Y-axis ANS $2x-3y+18=0$

Ques: 3 → Convert in to normal form and find α and p

$x-\sqrt{3}y+8=0$ ANS $x\cos\left(\frac{2\pi}{3}\right)+y\sin(2\pi/3)=4$

$\alpha=2\pi/3$, $p=4$

Ques: 4 → $P(a,b)$ is the Mid point of a line segment between axes. Show that equation of the line is $\frac{x}{a}+\frac{y}{b}=2$

Ques: 5 → Find the reflection (Image) of the point $(4,-13)$ about the line $5x+y+6=0$ ANS $(-1,-14)$

Ques: 6 → A line passes through point $P(1,2)$ such that its intercept between the axes is bisected at P. Find equation of the line ANS $2x+y-4=0$

Ques: 7 → The Intercept cut off by a line from Y-axis is twice than that from X-axis and the line passes through the point $(1,2)$. Find the equation of the line

ANS $2x+y=4$

Qn. 8 . Find the coordinates of the foot of perpendicular from the point $(2,3)$ ANS $(5,6)$

Qn. 9 → If two lines $ax+by=c$ and $a'x+b'y=c'$ are perpendicular show that $aa'+bb'=0$

Qn: 10 → Find the equation of the line passing through the point of Intersection of the lines $5x-6y-1=0$ and $3x+2y+5=0$ and perpendicular to the line $3x-5y+11=0$ ANS $5x+3y+8=0$

Qn: 11 → Find the equation of one of the sides of an isosceles Right angled triangle whose hypotenuse is given by $3x+4y=4$ and the opposite vertex of the hypotenuse is $(2,2)$ ANS $x-7y-12=0$

Qn. 12 → If the intercept of a line between the coordinate axes is divided by the point $(-5,4)$ in the Ratio $1:2$, then find the equation of the line

ANS $8x-5y+60=0$

Qn. 13 → In what direction should a line be drawn through the point $(1,2)$ so that its point of Intersection with the line $x+y=4$ is at a distance $\frac{\sqrt{6}}{3}$ from the given point ANS $15°$, or $75°$

Qn. 14 → If the equation of the base of an equilateral ~~B~~ triangle is $x+y=2$ and the vertex is $(2,-1)$. Then find the length of the side of the triangle ANS $\sqrt{\frac{2}{3}}$ unit

—x—