

ULTIMATE MATHEMATICS (BY: AJAY MITTAL)

Chapter: PERMUTATION & COMBINATION

→ CLASS NO: 2 →

WORDS

INVOLUTE

data. total = 8 ; vowels = I, O, U, E = 4

Consonants = N, V, L, T = 4

(1) total no of words ~~not~~ using all letters

$$n = 8, r = 8$$

$$\text{no of words} = {}^8P_8 = 8! = \boxed{} \text{ Ans}$$

(2) Using 5 letters

$$n = 8 ; r = 5$$

$$\text{no of words} = {}^8P_5 = \frac{8!}{3!} = \frac{8 \times 7 \times 6 \times 5 \times 4 \times \cancel{3!}}{\cancel{3!}} = \boxed{} \text{ Ans}$$

(3) Start with N and end with T

$$\frac{N}{(1)} \boxed{6!} \frac{T}{(1)}$$

$$\text{No of words} = 1 \times 6! \times 1 = 6! = 720 \text{ Ans}$$

(4) Using 5 letters. Start with N and end with T

$$\frac{N}{(1)} \boxed{{}^6P_3} \frac{T}{(1)}$$

$$\text{No of words} = 1 \times {}^6P_3 \times 1$$

$$= \frac{6!}{3!} = \frac{6 \times 5 \times 4 \times \cancel{3!}}{\cancel{3!}} = 120 \text{ Ans}$$

(5) starting & ending with a vowel

$$\underline{4} \boxed{6!} \underline{3} \quad \text{No. of words} \\ = 4 \times 6! \times 3 \\ = 12 \times 720 = \boxed{}$$

(6) Using 5 letters start and end with a Consonant

$$\underline{4} \boxed{6P_3} \underline{3} = 4 \times 6P_3 \times 3 = \boxed{}$$

(7) "All vowels occur together"

(i) Consider 4 vowels as 1 letter. $(I, O, U, E) = 1$

(ii) Now we have to arrange $(4+1) = 5$ letters

(iii) then 5 letters can be arranged in $= 5!$ ways

(iv) 4 vowels can mutually arranged in $= 4!$ ways

(v) Required no. of ways in which all the vowels occur together $= 5! \times 4!$

(8) all vowels occur together & all consonants occur together.

(i) $(I, O, U, E) = 1$ and $(N, V, L, T) = 1$

(ii) Now we have to arrange 2 letters =

(iii) then two letters can be arranged in $= 2!$ ways

(iv) 4 vowels can mutually arranged in $= 4!$ ways

(v) 4 consonants " " " " $= 4!$ ways

(vi) Required No. of words $= 2! \times 4! \times 4! = 2 \times 24 \times 24 = \boxed{}$

(9) N is always next to E

$$(EN) = 1$$

✓ Consider E & N as ¹ letter

✓ we have to arrange = $(6+1) = 7$ letters

✓ then 7 letters can be arranged in $7!$ ways

✓ Req number of words = $7! \times 1 = 7!$

(10) all vowels never together

Shortcut

$$\text{not together} = \text{total} - \text{together}$$

total no. of words = $8!$

all vowels together = $5! \times 4!$ --- (from (7))

$$\text{Req. no. of words} = 8! - 5! \times 4! = \boxed{}$$

(11) I & V never/not together

$$I \& V \text{ together} = (I, V) = 1 = 7! \times 2!$$

$$I \& V \text{ not together} = 8! - 7! \times 2!$$

(12) vowels occupy odd places

$$\begin{array}{cccccccc} V_1 & C_1 & V_2 & C_2 & V_3 & C_3 & V_4 & C_4 \\ \hline 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \end{array}$$

$$= 4! \times 4! = 24 \times 24 = \boxed{}$$

(13) Vowels and consonants are alternating:

Case (i) V C V C V C V C

Case (ii) C V C V C V C V

Case I = $4! \times 4! =$

Case II $4! \times 4! =$

Return No of ways = Case I + Case II = $(4! \times 4!) + (4! \times 4!)$
 $= \boxed{}$

(14) "No two" vowels are together

$- C_1 - C_2 - C_3 - C_4 -$

(i) there are 5 places available for 4 vowels
 which they can be arranged in $= {}^5P_4$ ways

(ii) 4 consonants can mutually arranged in $= 4!$ ways

(iii) Req. no of ways in which "No two" vowels
 are together = ${}^5P_4 \times 4!$

(15) there are always three letters b/w I & N

I	I	I	I	N	N	N	N
---	---	---	---	---	---	---	---

$= (6! \times 4) \times 2$

WORD

INEFFECTIVE

total letters: 11

Vowels = I, I, E, E, E = 5

Consonants = N, F, F, C, T, V = 6

(1) total no of words = $\frac{11!}{2! \cdot 3! \cdot 2!} = \boxed{}$

(2) all vowels occur together

$\boxed{I, I, E, E, E} = 1$
 $= \frac{7!}{2!} \times \frac{5!}{2! \cdot 3!}$

(3) "No two" vowels are together.

$- C_1 - C_2 - C_3 - C_4 - C_5 - C_6 -$
 $= \frac{7P_5}{2! \cdot 3!} \times \frac{6!}{2!}$

(4) Starting with a vowel

(X) ~~2~~ $\boxed{\text{Total}}$

Case I \underline{I} $\boxed{} = 1 \times \frac{10!}{3! \cdot 2!}$

Case II \underline{E} $\boxed{} = 1 \times \frac{10!}{2! \cdot 2! \cdot 2!}$

(.1) By no of ways = Case I + Case II Ans $-X-$

← Permutation & combination →

Ques 1 → How many words can be formed from the letters of the word DAUGHTER so that

- (i) the vowels always come together
- (2) the vowels never come together

Ans (1) 4320 (2) = 36000

Ques 2 → Word EQUATION. How many words

- (i) begin with ~~E~~ E?
- (2) begin with E and end with N?
- (3) the word begin and end with a constant?

Ans (1) 7! (2) 6! (3) 4320

Ques 3 → In how many ways can the letters of the word PERMUTATIONS be arranged if

(i) the words start with P and end with S Ans 1814400

(2) vowels are all together (2) Ans 2419200

(3) there are always 4 letters b/w P & S (3) Ans 25401600

Ques 4 → In how many ways can the letters of the word INTERMEDIATE be arranged so that:

- (i) the vowels always occupy even places?
- (2) the relative order of vowels and consonants do not alter?

Ans (1) 21600 (2) 21600

Q1-5 \rightarrow If $P(2n-1, n) : P(2n+1, n-1) = 22 : 7$
 Find n Ans $n=10$

Q1-6 \rightarrow If $n_{(r-1)} = 36$; $n_r = 84$ and $n_{(r+1)} = 126$
 Find r Ans $r=3$

Q1-7 In how many ways can the letters of the word ASSASSINATION be arranged so that all the S's are together?
Ans 151200

Q1-8 \rightarrow In how many of the distinct permutations or arrangements of the letter in MISSISSIPPI do the four I's not come together?
Ans 33810

-X-