← ULTIMATE MATHEMATICS →

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TRIGO CLASS NO: 7 (T-7)

19 ➤ Show that

$\cos^3 A + \cos^3(120° + A) + \cos^3(240° + A) = \frac{3}{4}\cos(3A)$

Take LHS

$\cos^3 A + \cos^3(120° + A) + \cos^3(240° + A)$

we know that $\cos(3\theta) = 4\cos^3\theta - 3\cos\theta$

$\Rightarrow \cos^3\theta = \frac{3\cos\theta + \cos(3\theta)}{4}$.

$\frac{3\cos A + \cos(3A)}{4} + \frac{3\cos(120° + A) + \cos(360° + 3A)}{4} + \frac{3\cos(240° + A) + \cos(720° + 3A)}{4}$

$\frac{1}{4}\left[3\cos A + \cos(3A) + 3\cos(120° + A) + \cos(3A) + 3\cos(240° + A) + \cos(3A)\right]$

$\frac{1}{4}\left[3\cos(3A) + 3\cos A + 3\cos(120° + A) + 3\cos(240° + A)\right]$

$\frac{3}{4}\left[\cos(3A) + \cos A + \cos(120° + A) + \cos(240° + A)\right]$

B Set-3 A

$\frac{3}{4}\left[\cos(3A) + 2\cos(120° + A)\cos(120°) + \cos(120° + A)\right]$

$\frac{3}{4}\left[\cos(3A) + 2\cos(120° + A)\cos(180° - 60°) + \cos(120° + A)\right]$

II

$\frac{3}{4}\left[\cos(3A) + \cancel{2}\cos(120° + A) \times \left(\frac{-1}{\cancel{2}}\right) + \cos(120° + A)\right]$

$\frac{3}{4}\left[\cos(3A) - \cancel{\cos(120° + A)} + \cancel{\cos(120° + A)}\right]$

$= \frac{3}{4}\cos(3A)$ Ans

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Q.N. 20 Show that
$\cos(5A) = 16\cos^5 A - 20\cos^3 A + 5\cos A$

Soln Taking LHS $\cos(5A)$

$= \cos[(3A) + (2A)]$

$= \cos(3A)\cos(2A) - \sin(3A)\sin(2A)$ ... {$\cos(A+B)$ formula}

$= (4\cos^3 A - 3\cos A)(2\cos^2 A - 1) - (3\sin A - 4\sin^3 A)(2\sin A\cos A)$

$= [8\cos^5 A - 4\cos^3 A - 6\cos^3 A + 3\cos A] - (3 - 4\sin^2 A)(2\sin^2 A\cos A)$

$= (8\cos^5 A - 10\cos^3 A + 3\cos A) - [3 - 4(1-\cos^2 A)][2(1-\cos^2 A)\cos A]$

$= (8\cos^5 A - 10\cos^3 A + 3\cos A) - [-1 + 4\cos^2 A][2\cos A - 2\cos^3 A]$

$= (8\cos^5 A - 10\cos^3 A + 3\cos A) - (-2\cos A + 2\cos^3 A + 8\cos^3 A - 8\cos^5 A)$

$= 8\cos^5 A - 10\cos^3 A + 3\cos A + 2\cos A - 10\cos^3 A + 8\cos^5 A$

$= 16\cos^5 A - 20\cos^3 A + 5\cos A$ Ans

Shortcut Method

LHS $\cos(5A)$ set-3

$= \cos(5A) + \cos A - \cos A$

$= 2\cos(3A)\cos(2A) - \cos A$

$= 2[4\cos^3 A - 3\cos A](2\cos^2 A - 1) - \cos A$

open all brackets
we will get Ans

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Qn 21 → Find the value of $\sin(18^\circ)$

Solution

Let $\theta = 18^\circ$

$\Rightarrow 5\theta = 90^\circ$

$\Rightarrow 3\theta + 2\theta = 90^\circ$

$\Rightarrow 2\theta = 90^\circ - 3\theta$

$\Rightarrow \sin(2\theta) = \sin(90^\circ - 3\theta)$

$\Rightarrow \sin(2\theta) = \cos(3\theta)$

$\Rightarrow 2\sin\theta\cos\theta = 4\cos^3\theta - 3\cos\theta$

$\Rightarrow 2\sin\theta\cancel{\cos\theta} = \cancel{\cos\theta}(4\cos^2\theta - 3)$

$\Rightarrow 2\sin\theta = 4\cos^2\theta - 3$ --- $\{\because \cos\theta \neq 0$, $\cos(18^\circ) \neq 0\}$

$\Rightarrow 2\sin\theta = 4(1-\sin^2\theta) - 3$

$\Rightarrow 2\sin\theta = 4 - 4\sin^2\theta - 3$

$\Rightarrow 4\sin^2\theta + 2\sin\theta - 1 = 0$

Quadratic formula

$$\sin\theta = \frac{-2 \pm \sqrt{4+16}}{8}$$

$$\sin\theta = \frac{-2 \pm 2\sqrt{5}}{8}$$

$$\sin\theta = \frac{-1 \pm \sqrt{5}}{4}$$

$\sin(18^\circ) = \frac{-1+\sqrt{5}}{4}$ | $\sin(18^\circ) = \frac{-1-\sqrt{5}}{4}$

(Rejected since θ in Ist quad and $\sin\theta$ can not -ve)

$$\therefore \boxed{\sin(18^\circ) = \frac{\sqrt{5}-1}{4}}$$

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Qn-22 → find $\cos(36^\circ)$

Soln we know that

$$\cos(2\theta) = 1 - 2\sin^2\theta$$

put $\theta = 18^\circ$

$$\Rightarrow \cos(36^\circ) = 1 - 2\sin^2(18^\circ)$$

$$\Rightarrow \cos(36^\circ) = 1 - 2\sin^2(18^\circ) \quad \text{(Repeat)}$$

$$\Rightarrow \cos(36^\circ) = 1 - 2\left(\frac{\sqrt{5}-1}{4}\right)^2$$

$$\Rightarrow \cos(36^\circ) = 1 - 2\left(\frac{5+1-2\sqrt{5}}{16}\right)$$

$$\Rightarrow \cos(36^\circ) = \frac{16-12+4\sqrt{5}}{16}$$

$$\Rightarrow \cos(36^\circ) = \frac{4+4\sqrt{5}}{16}$$

$$\Rightarrow \boxed{\cos(36^\circ) = \frac{\sqrt{5}+1}{4}} \quad \text{Ans}$$

Qn 23 → Given $\tan\left(\frac{\theta}{2}\right) = \sqrt{\frac{1-e}{1+e}}\tan\left(\frac{\phi}{2}\right)$ $\quad \phi = \text{phi}$

Show that $\cos\phi = \dfrac{\cos\theta - e}{1 - e\cos\theta}$

Solution Squaring given equation

$$\tan^2\left(\frac{\theta}{2}\right) = \left(\frac{1-e}{1+e}\right)\tan^2\left(\frac{\phi}{2}\right)$$

$$\Rightarrow \tan^2\left(\frac{\phi}{2}\right) = \left(\frac{1+e}{1-e}\right)\tan^2\frac{\theta}{2}$$

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We know that

$$\cos(2\theta) = \frac{1-\tan^2\theta}{1+\tan^2\theta} \quad \text{-- \{ formula \}}$$

or

$$\cos\theta = \frac{1-\tan^2(\theta/2)}{1+\tan^2(\theta/2)}$$

Taking <u>LHS</u>

$\cos\phi$

$$\Rightarrow \cos\phi = \frac{1-\tan^2(\phi/2)}{1+\tan^2(\phi/2)}$$

$$\Rightarrow \cos\phi = \frac{1-\left(\frac{1+e}{1-e}\right)\tan^2\frac{\theta}{2}}{1+\left(\frac{1+e}{1-e}\right)\tan^2\frac{\theta}{2}}$$

$$\Rightarrow \cos\phi = \frac{(1-e)-(1+e)\tan^2(\theta/2)}{(1-e)+(1+e)\tan^2(\theta/2)}$$

$$\Rightarrow \cos\phi = \frac{1-e-\tan^2\frac{\theta}{2}-e\tan^2\frac{\theta}{2}}{1-e+\tan^2\frac{\theta}{2}+e\tan^2\frac{\theta}{2}}$$

$$\Rightarrow \cos\phi = \frac{\left(1-\tan^2\frac{\theta}{2}\right)-e\left(1+\tan^2\frac{\theta}{2}\right)}{\left(1+\tan^2\frac{\theta}{2}\right)-e\left(1-\tan^2\frac{\theta}{2}\right)}$$

Divide N & D by $\left(1+\tan^2\frac{\theta}{2}\right)$

$$\Rightarrow \cos\phi = \frac{\left(\frac{1-\tan^2\frac{\theta}{2}}{1+\tan^2\frac{\theta}{2}}\right)-e}{1-e\left(\frac{1-\tan^2\theta/2}{1+\tan^2\theta/2}\right)}$$

$$\Rightarrow \cos\phi = \frac{\cos\theta-e}{1-e\cos\theta}$$

<u>PROVED</u>

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Qn 24 → given

(SELF)

$$\tan\left(\frac{\phi}{2}\right)\sqrt{\frac{a-b}{a+b}} = \tan\frac{\theta}{2}$$

Show that $\cos\theta = \dfrac{a\cos\phi + b}{a + b\cos\phi}$

Qns 25 → Show that

Tricky

$$\frac{\sin x}{\cos(3x)} + \frac{\sin(3x)}{\cos(9x)} + \frac{\sin(9x)}{\cos(27x)} = \frac{1}{2}\left(\tan(27x) - \tan x\right)$$

Soln LHS

$$\frac{\sin x}{\cos(3x)} + \frac{\sin(3x)}{\cos(9x)} + \frac{\sin(9x)}{\cos(27x)}$$

$$= \frac{2\sin x\cos x}{2\cos(3x)\cos x} + \frac{2\sin(3x)\cos(3x)}{2\cos(9x)\cos(3x)} + \frac{2\sin(9x)\cos(9x)}{2\cos(27x)\cos(9x)}$$

$$= \frac{\sin(2x)}{2\cos(3x)\cos x} + \frac{\sin(6x)}{2\cos(9x)\cos(3x)} + \frac{\sin(18x)}{2\cos(27x)\cos(9x)}$$

$$= \frac{1}{2}\left[\frac{\sin(\overset{A-B}{3x - x})}{\cos(3x)\cdot\cos x} + \frac{\sin(9x-3x)}{\cos(9x)\cos(3x)} + \frac{\sin(27x-9x)}{\cos(27x)\cos(9x)}\right]$$

$$= \frac{1}{2}\left[\frac{\sin(3x)\cos x - \cos(3x)\sin x}{\cos(3x)\cos x} + \frac{\sin(9x)\cos(3x) - \cos(9x)\sin(3x)}{\cos(9x)\cos(3x)} + \frac{\sin(27x)\cos(9x) - \cos(27x)\sin(9x)}{\cos(27x)\cos(9x)}\right]$$

Separate

$$= \frac{1}{2}\left[\cancel{\tan(3x)} - \tan x + \cancel{\tan(9x)} - \cancel{\tan(3x)} + \tan(27x) - \cancel{\tan(9x)}\right]$$

$$= \frac{1}{2}\left[\tan(27x) - \tan x\right]$$

Ans

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Qn 26 → Show that

$$\frac{\sec(8\theta)-1}{\sec(4\theta)-1} = \frac{\tan(8\theta)}{\tan(2\theta)}$$

Solution

LHS $\frac{\sec(8\theta)-1}{\sec(4\theta)-1}$

$$= \frac{\frac{1}{\cos(8\theta)}-1}{\frac{1}{\cos(4\theta)}-1}$$

$$= \frac{\frac{1-\cos(8\theta)}{\cos(8\theta)}}{\frac{1-\cos(4\theta)}{\cos(4\theta)}}$$

$$= \frac{(1-\cos(8\theta)).\cos(4\theta)}{(1-\cos(4\theta)).\cos(8\theta)} \quad ---- \{1-\cos(2\theta) = 2\sin^2\theta\}$$

$$= \frac{2\sin^2(4\theta).\cos(4\theta)}{2\sin^2(2\theta).\cos(8\theta)}$$

$$= \frac{[2\sin(4\theta).\cos(4\theta)].\sin(4\theta)}{2\sin^2(2\theta)\cos(8\theta)}$$

$$= \frac{\sin(8\theta) \times \cancel{2\sin(2\theta)}\cos(2\theta)}{\cancel{2\sin^2(2\theta)}\cos(8\theta)} \quad --- \{2\sin\theta\cos\theta = \sin(2\theta)\}$$

$$= \frac{\sin(8\theta).\cos(2\theta)}{\sin(2\theta)\cos(8\theta)}$$

$$= \frac{\tan(8\theta)}{\tan(2\theta)}$$ Ans