

Ques: 1 (1) $36x^2 + 4y^2 = 144$

$$\Rightarrow \frac{x^2}{4} + \frac{y^2}{36} = 1$$

here $a=2, b=6$ ($b > a$)

(1) $e = \sqrt{1 - \frac{a^2}{b^2}} = \sqrt{1 - \frac{4}{36}} = \sqrt{1 - \frac{1}{9}} = \sqrt{\frac{8}{9}} = \frac{\sqrt{8}}{3} = \frac{2\sqrt{2}}{3}$

(2) foci = $(0, \pm be) = (0, \pm 4\sqrt{2})$

(3) vertices = $(0, \pm b) = (0, \pm 6)$

(4) Major Axis length = $2b = 12$

(5) Minor Axis length = $2a = 4$

(6) $LR = \frac{2a^2}{b} = \frac{2(4)}{6} = \frac{4}{3}$ Ans

(2) $4x^2 + 9y^2 = 36$

$$\Rightarrow \frac{x^2}{9} + \frac{y^2}{4} = 1$$

here $a=3$ & $b=2$ ($a > b$)

(1) $e = \sqrt{1 - \frac{b^2}{a^2}} = \sqrt{1 - \frac{4}{9}} = \sqrt{\frac{5}{9}} = \frac{\sqrt{5}}{3}$

(2) foci = $(\pm ae, 0) = (\pm \sqrt{5}, 0)$

(3) vertices = $(\pm a, 0) = (\pm 3, 0)$

(4) Major Axis length = $2a = 6$

(5) Minor Axis length = $2b = 4$

(6) $LR = \frac{2b^2}{a} = \frac{2(4)}{3} = \frac{8}{3}$ Ans

Ques 2 (i) $16x^2 - 9y^2 = 576$

$$\Rightarrow \frac{x^2}{36} - \frac{y^2}{64} = 1$$

It is a transverse hyperbola

with $a=6$ & $b=8$

$$(1) e = \sqrt{1 + \frac{b^2}{a^2}} = \sqrt{1 + \frac{64}{36}} = \sqrt{\frac{100}{36}} = \frac{10}{6} = \frac{5}{3}$$

$$(2) foci = (\pm ae, 0) = (\pm 10, 0)$$

$$(3) vertices = (\pm a, 0) = (\pm 6, 0)$$

$$(4) LR = \frac{2b^2}{a} = \frac{2(64)}{6} = \frac{64}{3} \quad \underline{\underline{Ans}}$$

(2) $49y^2 - 16x^2 = 784$

$$\Rightarrow \frac{-16x^2}{784} + \frac{49y^2}{784} = 1$$

$$\Rightarrow \frac{-x^2}{49} + \frac{y^2}{16} = 1$$

It is a conjugate hyperbola with $a=7$ & $b=4$

$$(1) e = \sqrt{1 + \frac{a^2}{b^2}} = \sqrt{1 + \frac{49}{16}} = \sqrt{\frac{65}{16}} = \frac{\sqrt{65}}{4}$$

$$(2) foci = (\pm ae, 0) = (0, \pm be) = (0, \pm \sqrt{65})$$

$$(3) vertices = (0, \pm b) = (0, \pm 4)$$

$$(4) LR = \frac{2a^2}{b} = \frac{2(49)}{4} = \frac{49}{2} \quad \underline{\underline{Ans}}$$

Ques 3 → ELLIPSE

(3)

Given: vertices $(0, \pm 13)$ & foci $(0, \pm 5)$

Comparing vertices with $(0, \pm b)$ & foci with $(0, \pm be)$

We have $\boxed{b = 13}$ & $\boxed{be = 5}$

Now, $e = \sqrt{1 - \frac{a^2}{b^2}}$

$$\Rightarrow be = \sqrt{b^2 - a^2}$$

$$\Rightarrow 5 = \sqrt{169 - a^2}$$

$$\Rightarrow 25 = 169 - a^2$$

$$\Rightarrow \boxed{a^2 = 144}$$

Equation of ellipse is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$$\Rightarrow \frac{x^2}{144} + \frac{y^2}{169} = 1 \quad \underline{\underline{\text{Ans}}}$$

Ques 4 → ELLIPSE

Given length of Major Axis = 26 & foci $(\pm 5, 0)$

Comparing Major axis with $2a$ & foci with $(\pm ae, 0)$

We have $2a = 26$ and $ae = 5$

$$\Rightarrow \boxed{a = 13}$$
 & $\boxed{ae = 5}$

Now, $e = \sqrt{1 - \frac{b^2}{a^2}}$

$$\Rightarrow ae = \sqrt{a^2 - b^2}$$

$$\Rightarrow 5 = \sqrt{169 - b^2}$$

$$25 = 169 - b^2$$

$$\boxed{b^2 = 144}$$

Equation of ellipse is

$$\frac{x^2}{169} + \frac{y^2}{144} = 1 \quad \underline{\underline{\text{Ans}}}$$

Q. No. 5 → HYPERBOLA

Given vertices $(\pm 2, 0)$ & foci $(\pm 3, 0)$

Comp. vertices with $(\pm a, 0)$ & foci with $(\pm ae, 0)$

we have $\boxed{a=2}$ & $\boxed{ae=3}$

Now $e = \sqrt{1 + \frac{b^2}{a^2}}$

$$\Rightarrow ae = \sqrt{a^2 + b^2}$$

$$\Rightarrow 3 = \sqrt{4 + b^2}$$

$$\Rightarrow 9 = 4 + b^2$$

$$\Rightarrow \boxed{b^2 = 5}$$

~~the~~ Equation of hyperbola is $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

$$\Rightarrow \frac{x^2}{4} - \frac{y^2}{5} = 1 \quad \underline{\underline{\text{Ans}}}$$

Q. No. 6 → HYPERBOLA

Given foci $(\pm 5, 0)$ & length of transverse axis = 8

Comp. foci with $(\pm ae, 0)$ & transverse axis with $2a$

we have $\boxed{ae=5}$ and $2a=8$

$$\Rightarrow \boxed{a=4}$$

Now, $e = \sqrt{1 + \frac{b^2}{a^2}}$

$$\Rightarrow ae = \sqrt{a^2 + b^2}$$

$$\Rightarrow 5 = \sqrt{16 + b^2}$$

$$\Rightarrow 25 = 16 + b^2$$

$$\Rightarrow \boxed{b^2 = 9}$$

Equation of hyperbola is

$$\frac{x^2}{16} - \frac{y^2}{9} = 1 \quad \underline{\underline{\text{Ans}}}$$

(5)

Ques 7 → HYPERBOLAGiven: foci $(\pm 3\sqrt{5}, 0)$ & LR = 8Compare foci ~~with~~ $(\pm 3\sqrt{5}, 0)$ with $(\pm ae, 0)$
& LR with $\frac{2b^2}{a}$ we have

$$ae = 3\sqrt{5}$$

$$\& \frac{2b^2}{a} = 8$$

$$\Rightarrow b^2 = 4a$$

$$\text{Now } e = \sqrt{1 + \frac{b^2}{a^2}}$$

$$\Rightarrow ae = \sqrt{a^2 + b^2}$$

$$\Rightarrow 3\sqrt{5} = \sqrt{a^2 + 4a}$$

$$\text{Squaring } 45 = a^2 + 4a$$

$$\Rightarrow a^2 + 4a - 45 = 0$$

$$\Rightarrow (a+9)(a-5) = 0$$

$$a = -9, \quad a = 5$$

$$\downarrow$$

$$b^2 = 4a$$

$$b^2 = -36$$

(Not possible)

$$\downarrow$$

$$b^2 = 4a$$

$$b^2 = 20$$

 \therefore required hyperbola is

$$\frac{x^2}{25} - \frac{y^2}{20} = 1 \quad \underline{\text{Ans}}$$

-a-