← ULTIMATE MATHEMATICS →

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TRIGONOMETRY CLASS NO: 4 - (T-4)

Set-4

(1) $2\sin A\cos B = \sin(A+B) + \sin(A-B)$

(2) $2\cos A\sin B = \sin(A+B) - \sin(A-B)$

(3) $2\cos A\cos B = \cos(A+B) + \cos(A-B)$

*(4) $2\sin A\sin B = \cos(A-B) - \cos(A+B)$

Ques 1 Show that $\sin(10^\circ)\sin(30^\circ)\sin(50^\circ)\sin(70^\circ) = \frac{1}{16}$

Sol. LHS $\sin(10^\circ)\sin(30^\circ)\sin(50^\circ)\sin(70^\circ)$

$= \frac{1}{2}\sin(10^\circ)\sin(50^\circ)\sin(70^\circ)$

$= \frac{1}{4}\left[2\sin(\overset{A}{10})\sin(\overset{B}{50})\right]\cdot\sin(70^\circ)$

$= \frac{1}{4}\left[\cos(-40^\circ) - \cos(60^\circ)\right]\cdot\sin(70^\circ)$

$= \frac{1}{4}\left[\cos(40^\circ) - \frac{1}{2}\right]\cdot\sin(70^\circ)$

$= \frac{1}{4}\left[\sin(70^\circ)\cos(40^\circ) - \frac{1}{2}\sin(70^\circ)\right]$

$= \frac{1}{8}\left[2\sin(\overset{A}{70^\circ})\cos(\overset{B}{40^\circ}) - \sin(70)\right]$

$= \frac{1}{8}\left[\sin(110^\circ) + \sin(30^\circ) - \sin(70^\circ)\right]$

$= \frac{1}{8}\left[\sin(\overset{A}{110}) + \frac{1}{2} - \sin(\overset{B}{70})\right]$ (link)

ULTIMATE MATHEMATICS

$$= \frac{1}{8}\left[2\cos\left(\frac{110^\circ + 70^\circ}{2}\right)\cdot \sin\left(\frac{110^\circ - 70^\circ}{2}\right) + \frac{1}{2}\right]$$

$$= \frac{1}{8}\left(2\cos(90^\circ)\cdot \sin(20^\circ) + \frac{1}{2}\right)$$

$$= \frac{1}{8}\left[0 + \frac{1}{2}\right] \quad \ldots \{\cos(90^\circ) = 0\}$$

$$= \frac{1}{16} \quad \underline{\text{Ans}}$$

(OR) $\frac{1}{8}\left[\sin(110^\circ) + \frac{1}{2} - \sin(70^\circ)\right]$

Link (Sum = 180°)

$$= \frac{1}{8}\left[\sin(\overset{\text{II}}{180 - 70^\circ}) + \frac{1}{2} - \sin(70^\circ)\right]$$

$$= \frac{1}{8}\left[\sin(\cancel{70^\circ}) + \frac{1}{2} - \sin(\cancel{70})\right]$$

$$= \frac{1}{8} \times \frac{1}{2} = \frac{1}{16} \quad \underline{\text{Ans}}$$

Qns 2 Show that $4\sin\theta\cdot\sin\left(\frac{\pi}{3}+\theta\right)\cdot\sin\left(\frac{2\pi}{3}+\theta\right) = \sin(3\theta)$

Ans $4\sin\theta\cdot\sin(60^\circ+\theta)\cdot\sin(120^\circ+\theta)$

$$= 2\left[2\sin(\overset{A}{60+\theta})\cdot\sin(\overset{B}{120+\theta})\right]\cdot\sin\theta$$

$$= 2\left[\cos(-60^\circ) - \cos(\underset{\text{III}}{180+2\theta})\right]\cdot\sin\theta$$

180 → III

$$= 2\left[\frac{1}{2} + \cos(2\theta)\right]\sin\theta$$

$$= 2\left\{ \frac{1}{2}\sin\theta + \sin\theta \cdot \cos(2\theta) \right\}$$

$$= \cancel{2}\left\{ \frac{\sin\theta + 2\overset{A}{\sin\theta}\,\overset{B}{\cos(2\theta)}}{\cancel{2}} \right\}$$

$$= \sin\theta + \sin(3\theta) + \sin(-\theta)$$

$$= \sin\theta + \sin(3\theta) - \cancel{\sin\theta}$$

$$= \sin(3\theta) \quad \underline{\text{Ans}}$$

Qn 10 : Show that $2\cos\left(\frac{\pi}{13}\right)\cos\left(\frac{9\pi}{13}\right) + \cos\left(\frac{3\pi}{13}\right) + \cos\left(\frac{5\pi}{13}\right) = 0$

Ans

$$2\cos\overset{A}{\left(\frac{\pi}{13}\right)} \cdot \cos\overset{B}{\left(\frac{9\pi}{13}\right)} + \cos\left(\frac{3\pi}{13}\right) + \cos\left(\frac{5\pi}{13}\right)$$

$$= \cos\left(\frac{\pi}{13} + \frac{9\pi}{13}\right) + \cos\left(\frac{\pi}{13} - \frac{9\pi}{13}\right) + \cos\left(\frac{3\pi}{13}\right) + \cos\left(\frac{5\pi}{13}\right)$$

$$= \cos\left(\frac{10\pi}{13}\right) + \cos\left(-\frac{8\pi}{13}\right) + \cos\left(\frac{3\pi}{13}\right) + \cos\left(\frac{5\pi}{13}\right)$$

$$= \cos\left(\frac{10\pi}{13}\right) + \cos\left(\frac{8\pi}{13}\right) + \cos\left(\frac{3\pi}{13}\right) + \cos\left(\frac{5\pi}{13}\right)$$

II link   II link

$$= \cos\left(\pi - \frac{3\pi}{13}\right) + \cos\left(\pi - \frac{5\pi}{13}\right) + \cos\left(\frac{3\pi}{13}\right) + \cos\left(\frac{5\pi}{13}\right)$$

$$= \cancel{-\cos\left(\frac{3\pi}{13}\right)} \cancel{-\cos\left(\frac{5\pi}{13}\right)} + \cancel{\cos\left(\frac{3\pi}{13}\right)} + \cancel{\cos\left(\frac{5\pi}{13}\right)}$$

$$= 0 \quad \underline{\text{Ans}}$$

← ULTIMATE MATHEMATICS →

(T-4)

Qn. 1) → Show that

$$\cos(2\theta)\cdot\cos\frac{\theta}{2} - \cos(3\theta)\cdot\cos\left(\frac{9\theta}{2}\right) = \sin(5\theta)\cdot\sin\left(\frac{5\theta}{2}\right)$$

Ans $\cos(2\theta)\cdot\cos\frac{\theta}{2} - \cos(3\theta)\cdot\cos\left(\frac{9\theta}{2}\right)$

$$= \frac{1}{2}\left[2\cos(\overset{A}{2\theta})\cdot\cos\overset{B}{\frac{\theta}{2}} - 2\cos(\overset{A}{3\theta})\cdot\cos\left(\overset{B}{\frac{9\theta}{2}}\right)\right]$$

$$= \frac{1}{2}\left[\cos\left(2\theta+\frac{\theta}{2}\right) + \cos\left(2\theta-\frac{\theta}{2}\right) - \left\{\cos\left(3\theta+\frac{9\theta}{2}\right) + \cos\left(3\theta-\frac{9\theta}{2}\right)\right\}\right]$$

$$= \frac{1}{2}\left[\cos\left(\frac{5\theta}{2}\right) + \cos\left(\frac{3\theta}{2}\right) - \cos\left(\frac{15\theta}{2}\right) - \cos\left(-\frac{3\theta}{2}\right)\right]$$

$$= \frac{1}{2}\left(\cos\left(\frac{5\theta}{2}\right) + \cancel{\cos\left(\frac{3\theta}{2}\right)} - \cos\left(\frac{15\theta}{2}\right) - \cancel{\cos\left(\frac{3\theta}{2}\right)}\right)$$

$$= \frac{1}{2}\left[\cos\left(\overset{(A)}{\frac{5\theta}{2}}\right) - \cos\left(\overset{(B)}{\frac{15\theta}{2}}\right)\right]$$

$$= \frac{1}{\cancel{2}}\left[-\cancel{2}\sin\left(\frac{\frac{5\theta}{2}+\frac{15\theta}{2}}{2}\right)\cdot\sin\left(\frac{\frac{5\theta}{2}-\frac{15\theta}{2}}{2}\right)\right]$$

$$= -\sin\left(\frac{20\theta}{4}\right)\cdot\sin\left(\frac{-10\theta}{4}\right)$$

$$= -\sin(5\theta)\cdot\sin\left(-\frac{5\theta}{2}\right)$$

$$= \sin(5\theta)\cdot\sin\left(\frac{5\theta}{2}\right) = \underline{\text{Ans}}$$

(13) LHS =

$$\left(\frac{\cos A+\cos B}{\sin A-\sin B}\right)^n+\left(\frac{\sin A+\sin B}{\cos A-\cos B}\right)^n$$

$$=\left[\frac{\cancel{2}\cancel{\cos\left(\frac{A+B}{2}\right)}\cdot\cos\left(\frac{A-B}{2}\right)}{\cancel{2}\cancel{\cos\left(\frac{A+B}{2}\right)}\cdot\sin\left(\frac{A-B}{2}\right)}\right]^n+\left[\frac{\cancel{2\sin\left(\frac{A+B}{2}\right)}\cdot\cos\left(\frac{A-B}{2}\right)}{-\cancel{2\sin\left(\frac{A+B}{2}\right)}\cdot\sin\left(\frac{A-B}{2}\right)}\right]^n$$

$$=\left(\cot\left(\frac{A-B}{2}\right)\right)^n+\left(-\cot\left(\frac{A-B}{2}\right)\right)^n$$

Case I $n \to$ even

$$\left(\cot\left(\frac{A-B}{2}\right)\right)^n+\left(\cot\left(\frac{A-B}{2}\right)\right)^n$$

$$=2\cot^n\left(\frac{A-B}{2}\right)$$

$\boxed{(\cot\theta)^2=\cot^2\theta}$

Case II $n \to$ odd

$$\cancel{\left(\cot\left(\frac{A-B}{2}\right)\right)^n}-\cancel{\left(\cot\left(\frac{A-B}{2}\right)\right)^n}$$

$$=0$$

TRIGONOMETRY Class XI

WORKSHEET NO: 4

Topic ______ Date ______

Qn. 1 → Show that $\cos(20^\circ)\cos(40^\circ)\cos(60^\circ)\cos(80^\circ) = \frac{1}{16}$

Qn. 2 → Show that $\sin(10^\circ)\sin(30^\circ)\sin(50^\circ)\sin(70^\circ) = \frac{1}{16}$

Qn. 3 → Show that $\sin(20^\circ)\sin(40^\circ)\sin(60^\circ)\sin(80^\circ) = \frac{3}{16}$

Qn. 4 → Show that $\cos(10^\circ)\cos(30^\circ)\cos(50^\circ)\cos(70^\circ) = \frac{3}{16}$

Qn. 5 → Show that $\tan(20^\circ)\tan(40^\circ)\tan(60^\circ)\tan(80^\circ) = 3$

Qn. 6 → Show that $\tan(20^\circ)\tan(30^\circ)\tan(40^\circ)\tan(80^\circ) = 1$

Qn. 7 → Show that $4\cos(12^\circ)\cos(48^\circ)\cos(72^\circ) = \cos(36^\circ)$

Qn. 8 → Show that $\sin A \cdot \sin(60^\circ - A) \cdot \sin(60^\circ + A) = \frac{1}{4}\sin(3A)$

Qn. 9 → Show that $4\sin\theta \sin\left(\frac{\pi}{3}+\theta\right)\sin\left(\frac{2\pi}{3}+\theta\right) = \sin(3\theta)$

Qn. 10 → Show that $2\cos\left(\frac{\pi}{13}\right)\cos\left(\frac{9\pi}{13}\right) + \cos\left(\frac{3\pi}{13}\right) + \cos\left(\frac{5\pi}{13}\right) = 0$

Qn. 11 → Show that $\cos(2\theta)\cos\frac{\theta}{2} - \cos(3\theta)\cos\left(\frac{9\theta}{2}\right) = \sin(5\theta)\cdot\sin\left(\frac{5\theta}{2}\right)$

Qn. 12 → Show that $\cos\theta \cdot \cos\frac{\theta}{2} - \cos(3\theta)\cos\left(\frac{9\theta}{2}\right) = \sin\left(\frac{7\theta}{2}\right)\sin(4\theta)$

Qn. 13 → Show that $\left(\frac{\cos A + \cos B}{\sin A - \sin B}\right)^n + \left(\frac{\sin A + \sin B}{\cos A - \cos B}\right)^n = \begin{cases} 2\cot^n\left(\frac{A-B}{2}\right); & \text{when } n \text{ is even} \\ 0; & \text{when } n \text{ is odd} \end{cases}$

Qn. 14 → If $\cos(\alpha-\beta) + \cos(\beta-\gamma) + \cos(\gamma-\alpha) = -3/2$ show that $\cos\alpha + \cos\beta + \cos\gamma = \sin\alpha + \sin\beta + \sin\gamma = 0$