ULTIMATE MATHEMATICS (1) →

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Solution T-5 & T-6

(Solutions)

2 → $\underline{\text{Lhs}}$ $\cos^2 A + \cos^2\left(A + \frac{2\pi}{3}\right) + \cos^2\left(A - \frac{2\pi}{3}\right)$

$= \cos^2 A + \cos^2(A + 120^\circ) + \cos^2(A - 120^\circ)$

~~1 + cos cos~~

$= \left[\frac{1 + \cos(2A)}{2}\right] + \left[\frac{1 + \cos(2A + 240^\circ)}{2}\right] + \left[\frac{1 + \cos(2A - 240^\circ)}{2}\right]$

$\frac{1}{2}\left[3 + \cos(2A) + \cos(\underbrace{2A + 240^\circ}_{A}) + \cos(\underbrace{2A - 240^\circ}_{B})\right]$

$\frac{1}{2}\left[3 + \cos(2A) + 2\cos(2A)\cdot\cos(240^\circ)\right]$

$\ldots \left\{\cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)\right\}$

$= \frac{1}{2}\left[3 + \cos(2A) + 2\cos(2A)\cdot\cos(\underbrace{180^\circ + 60^\circ}_{\text{III}})\right]$

$= \frac{1}{2}\left[3 + \cos(2A) + 2\cos(2A)\cdot\left(-\cos(60^\circ)\right)\right]$

$= \frac{1}{2}\left[3 + \cos(2A) + \cancel{2}\cos(2A) \times \left(\frac{-1}{\cancel{2}}\right)\right]$

$= \frac{1}{2}\left[3 + \cancel{\cos(2A)} - \cancel{\cos(2A)}\right]$

$= \frac{3}{2}$ $\underline{\text{Ans}}$

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4 → $\sqrt{2 + \sqrt{2 + \sqrt{2 + 2\cos(8\theta)}}}$

$= \sqrt{2 + \sqrt{2 + \sqrt{2(1 + \cos(8\theta))}}}$

Solution T-5 & T-6

$$= \sqrt{2+\sqrt{2+\sqrt{2\times 2\cos^2(4\theta)}}}$$

$$= \sqrt{2+\sqrt{2+2\cos(4\theta)}}$$

$$= \sqrt{2+\sqrt{2(1+\cos(4\theta))}}$$

$$= \sqrt{2+\sqrt{2\times 2\cos^2(2\theta)}}$$

$$= \sqrt{2+2\cos(2\theta)}$$

$$= \sqrt{2(1+\cos(2\theta))}$$

$$= \sqrt{2\times 2\cos^2\theta}$$

$$= 2\cos\theta \quad \underline{\text{Ans}}$$

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6 → LHS $= \cos^4\left(\frac{\pi}{8}\right) + \cos^4\left(\frac{3\pi}{8}\right) + \cos^4\left(\frac{5\pi}{8}\right) + \cos^4\left(\frac{7\pi}{8}\right)$

$$= \cos^4\left(\frac{\pi}{8}\right) + \cos^4\left(\frac{3\pi}{8}\right) + \cos^4\left(\pi-\frac{3\pi}{8}\right) + \cos^4\left(\pi-\frac{\pi}{8}\right)$$

$$= \cos^4\left(\frac{\pi}{8}\right) + \cos^4\left(\frac{3\pi}{8}\right) + \cos^4\left(\frac{3\pi}{8}\right) + \cos^4\left(\frac{\pi}{8}\right)$$

$$= 2\left[\cos^4\left(\frac{\pi}{8}\right) + \cos^4\left(\frac{3\pi}{8}\right)\right]$$

$$= 2\left[\left(\cos^2\left(\frac{\pi}{8}\right)\right)^2 + \left(\cos^2\left(\frac{3\pi}{8}\right)\right)^2\right]$$

$(180^\circ-45^\circ)$

$$= 2\left[\left(\frac{1+\cos\left(\frac{\pi}{4}\right)}{2}\right)^2 + \left(\frac{1+\cos\left(\frac{3\pi}{4}\right)}{2}\right)^2\right]$$

$$= 2\left[\left(\frac{1+\frac{1}{\sqrt{2}}}{2}\right)^2 + \left(\frac{1-\frac{1}{\sqrt{2}}}{2}\right)^2\right]$$

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$$= 2\left[\frac{1+\frac{1}{2}+\frac{2}{\sqrt{2}}}{4} + \frac{1+\frac{1}{2}-\frac{2}{\sqrt{2}}}{4}\right]$$

$$= 2\left[\frac{3}{4}\right]$$

$$= 3/2 \quad \underline{\text{Ans..}}$$

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0 → $\underline{\text{LHS}} \quad \cos(4x)$

$= \cos(2\times 2x)$

$= 1-2\sin^2(2x) \quad \ldots \{\cos(2\theta) = 1-2\sin^2\theta\}$

$= 1-2[\sin(2x)]^2$

$= 1-2[2\sin x \cos x]^2$

$= 1-2(4\sin^2 x\cos^2 x)$

$= 1-8\sin^2 x\cos^2 x \quad \underline{\text{Ans}}$

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12 → given $\sin x = \frac{-1}{2}$; $x \to 4^{th}$ quad

we have $\sin^2 x + \cos^2 x = 1$

$\Rightarrow \frac{1}{4} + \cos^2 x = 1$

$\Rightarrow \cos^2 x = 1 - \frac{1}{4}$

$\Rightarrow \cos^2 x = 3/4$

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$\Rightarrow \quad \cos x = \pm \frac{\sqrt{3}}{2}$

$\Rightarrow \quad \boxed{\cos x = \frac{\sqrt{3}}{2}}$ --- { $x \to$ 4^{th} Quadrant }

We know that

$1-\cos x = 2\sin^2\left(\frac{x}{2}\right)$ | $1+\cos x = 2\cos^2\left(\frac{x}{2}\right)$

$1-\frac{\sqrt{3}}{2} = 2\sin^2\left(\frac{x}{2}\right)$ | $1+\frac{\sqrt{3}}{2} = 2\cos^2\left(\frac{x}{2}\right)$

$\frac{2-\sqrt{3}}{2} = 2\sin^2\left(\frac{x}{2}\right)$ | $\frac{2+\sqrt{3}}{2} = 2\cos^2\left(\frac{x}{2}\right)$

$\sin^2\left(\frac{x}{2}\right) = \frac{2-\sqrt{3}}{4}$ | $\cos^2\left(\frac{x}{2}\right) = \frac{2+\sqrt{3}}{4}$

$\sin\left(\frac{x}{2}\right) = \pm\sqrt{\frac{2-\sqrt{3}}{4}}$ | $\cos\left(\frac{x}{2}\right) = \pm\sqrt{\frac{2+\sqrt{3}}{4}}$

$x \to IV^{th}$

$x/2 \to 2^{nd}$ Quad

$\sin\left(\frac{x}{2}\right) = \sqrt{\frac{2-\sqrt{3}}{4}}$ | $\cos(x/2) = -\sqrt{\frac{2+\sqrt{3}}{4}}$

$\tan(x/2) = \frac{\sin(x/2)}{\cos(x/2)} = \frac{\sqrt{\frac{2-\sqrt{3}}{4}}}{-\sqrt{\frac{2+\sqrt{3}}{4}}}$

$= -\sqrt{\frac{2-\sqrt{3}}{2+\sqrt{3}}}$

Rationalising

$= -\sqrt{\frac{2-\sqrt{3}}{2+\sqrt{3}} \times \frac{2-\sqrt{3}}{2-\sqrt{3}}}$

$= -\frac{(2-\sqrt{3})}{4-3} = -(2-\sqrt{3})$ Ans

Solution T-5 & T-6

13 → Do yourself

14 →

LHS $\dfrac{\sin(5x) - 2\sin(3x) + \sin x}{\cos(5x) - \cos x}$

$= \dfrac{[\overset{A}{\sin(5x)} + \overset{B}{\sin x}] - 2\sin(3x)}{\cos(5x) - \cos x}$

$= \dfrac{2\sin(3x).\cos(2x) - 2\sin(3x)}{-2\sin(3x)\cdot\sin(2x)}$

$= \dfrac{\cancel{2\sin(3x)}\,[\cos(2x) - 1]}{-\cancel{2\sin(3x)}\,\sin(2x)}$

$= \dfrac{\cos(2x) - 1}{-\sin(2x)}$

$= \dfrac{\cancel{-}(1 - \cos(2x))}{\cancel{-}\sin(2x)}$

$= \dfrac{\cancel{2}\sin^2 x}{\cancel{2}\cancel{\sin x}\cos x}$

$= \tan x$ Ans

17 → LHS $\tan A + \tan(60^\circ + A) - \tan(60^\circ - A)$

$= \tan A + \dfrac{\tan(60^\circ) + \tan A}{1 - \tan(60^\circ)\tan A} - \dfrac{\tan(60^\circ) - \tan A}{1 + \tan(60^\circ)\tan A}$

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$$= \tan A + \frac{\sqrt{3} + \tan A}{1 - \sqrt{3}\tan A} - \frac{\sqrt{3} - \tan A}{1 + \sqrt{3}\tan A}$$

$$= \tan A + \frac{(\sqrt{3} + \tan A)(1 + \sqrt{3}\tan A) - (\sqrt{3} - \tan A)(1 - \sqrt{3}\tan A)}{1 - 3\tan^2 A}$$

$$- \sqrt{3}\tan^2 A$$

$$= \tan A + \frac{\sqrt{3} + 3\tan A + \tan A + \sqrt{3}\tan^2 A - \sqrt{3} + 3\tan A + \tan A}{1 - 3\tan^2 A}$$

$$= \tan A + \frac{8\tan A}{1 - 3\tan^2 A}$$

$$= \frac{\tan A - 3\tan^3 A + 8\tan A}{1 - 3\tan^2 A}$$

$$= \frac{9\tan A - 3\tan^3 A}{1 - 3\tan^2 A}$$

$$= \frac{3(3\tan A - \tan^3 A)}{1 - 3\tan^2 A}$$

$$= 3\tan(3A)$$ Ans

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1. LHS $\sin(4A)$

$$= \sin(2 \times 2A)$$

$$= 2\sin(2A)\cos(2A) \quad \text{---} \quad \{\sin(2\theta) = 2\sin\theta\cos\theta\}$$

$$= 2[2\sin A\cos A][\cos^2 A - \sin^2 A]$$

$$= 2\{2\sin A\cos^3 A - 2\sin^3 A\cos A\}$$

$$= 4\sin A\cos^3 A - 4\sin^3 A\cos A$$ Ans