ULTMATE MATHEMATICS

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CHAPTER : SETS CLASS No: 4 (S-4)

PROPERTIES

(1) $A \cup A = A$

(2) $A \cap A = A$

(3) $A \cup \phi = A$

(4) $A \cap \phi = \phi$

(5) $A \cup U = U$

(6) $A \cap U = A$

(7) $\phi' = U$

(8) $U' = \phi$

(9) $A \cup A' = U$

(10) $A \cap A' = \phi$

(11) $A - B = A \cap B'$

(12) Distributive law

(·) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

(·) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

(13) Demorgan's law

(·) $(A \cup B)' = A' \cap B'$

(·) $(A \cap B)' = A' \cup B'$

(14) If $x \in A$ (or) $x \in B$ then $x \in (A \cup B)$

(15) If $x \in (A \cup B)$ then $x \in A$ (or) $x \in B$

(16) If $x \in A$ and $x \in B$ then $x \in (A \cap B)$

(17) If $x \in (A \cap B)$ then $x \in A$ and $x \in B$

(18) If $x \in A$ then $x \notin A'$

(19) If $x \in A'$ then $x \notin A$

(20) If $A \subset B$ and $B \subset A$

then $A = B$

Idea

let $x \in A$	let $y \in B$
⋮	⋮
$x \in B$	$y \in A$
then $A \subset B$ -- (1)	$B \subset B$ - (2)

from (1) & (2) $A = B$

(21) ~~let~~ If $X \subset A$ then $X \in P(A)$

$x \subset A$ (×)

$x \in A$ (✓)

(22) If $X \in P(A)$

then $X \subset A$

(23) Associative prop.

$A \cup (B \cup C) = (A \cup B) \cup C$

$A \cap (B \cap C) = (A \cap B) \cap C$

(24) If $A \subset B$ then $A \cup B = B$

then $A \cap B = A$

-x-

~~Ques~~

QUESTION: 1 Show that $A \cup B = A \cap B$ implies $A = B$

Sol: given $A \cup B = A \cap B$

T.P. $A = B$

let $x \in A$

$\Rightarrow x \in A \cup B$

$\Rightarrow x \in A \cap B$ --- (given)

$\Rightarrow x \in A$ and $x \in B$

$\Rightarrow A \subset B$ --- (1)

let $y \in B$

$\Rightarrow y \in A \cup B$

$\Rightarrow y \in A \cap B$ --- (given)

$\Rightarrow y \in A$ and $y \in B$

$\Rightarrow B \subset A$ --- (2)

From (1) & (2)

$A = B$ proved

Ques 2 → Show that

$P(A \cap B) = P(A) \cap P(B)$

Property

$\boxed{\begin{matrix} X \in P(A) \\ X \subset A \end{matrix}}$

Sol: let $X \in P(A \cap B)$

$\Rightarrow X \subset A \cap B$

$\Rightarrow X \subset A$ and $X \subset B$

$\Rightarrow X \in P(A)$ and $X \in P(B)$

$\Rightarrow X \in P(A) \cap P(B)$

$\Rightarrow P(A \cap B) \subset P(A) \cap P(B)$ -- (1)

let $Y \in P(A) \cap P(B)$

$\Rightarrow Y \in P(A)$ and $Y \in P(B)$

$\Rightarrow Y \subset A$ and $Y \subset B$

$\Rightarrow Y \subset (A \cap B)$

$\Rightarrow Y \in P(A \cap B)$ --

$\Rightarrow P(A) \cap P(B) \subset P(A \cap B)$ -- (2)

from (1) & (2)

$P(A \cap B) = P(A) \cap P(B)$

Ans 3 → Assume that $P(A) = P(B)$, Show that $A = B$

Sol → let $x \in A$ x is any arbitrary element of A

$\Rightarrow X \subset A$

$\Rightarrow X \in P(A)$

$\Rightarrow X \in P(B)$ --- (given)

$\Rightarrow X \subset B$

$\Rightarrow x \in B$

$\Rightarrow A \subset B$ --- (1)

let $y \in B$

$\Rightarrow Y \subset B$

$\Rightarrow Y \in P(B)$

$\Rightarrow Y \in P(A)$ --- (given)

$\Rightarrow Y \subset A$

$\Rightarrow y \in A$

$\Rightarrow B \subset A$ - (2)

from (1) & (2) $A = B$

CLASS 5-Y

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Ques 4 → show that $P(A \cup B) \neq P(A) \cup P(B)$

Sol: let $A = \{1, 2\}$

$B = \{2, 3\}$

$A \cup B = \{1, 2, 3\}$

✓ $P(A) = \{\{1\}, \{2\}, \{1,2\}, \phi\}$

✓ $P(B) = \{\{2\}, \{3\}, \{2,3\}, \phi\}$

$P(A \cup B) = \{\{1\}, \{2\}, \{3\}, \{1,2\}, \{2,3\}, \{1,3\}, \{1,2,3\}, \phi\}$

RHS $P(A) \cup P(B) = \{\{1\}, \{2\}, \{3\}, \{1,2\}, \{2,3\}, \phi\}$

Clearly $P(A \cup B) \neq P(A) \cup P(B)$ Ans

Ques 5 → show that if $\boxed{A \subset B}$ then $(C-B) \subset (C-A)$

Sol: let $x \in (C-B)$

$x \in (C \cap B')$

$\Rightarrow x \in C$ and $x \in B'$

$\Rightarrow x \in C$ and $x \notin B$.

$\Rightarrow x \in C$ and $x \notin A$ -- {if $A \subset B$ and $x \notin B$ then $x \notin A$}

$\Rightarrow x \in C$ and $x \in A'$

$\Rightarrow x \in (C \cap A')$

$\Rightarrow x \in (C-A)$

$\Rightarrow (C-B) \subset (C-A)$

Qn.6 → If $A \cup B = A \cup C$ and $A \cap B = A \cap C$ then show that $B = C$

Sol: We have $A \cup B = A \cup C$

$\Rightarrow B \cap (A \cup B) = B \cap (A \cup C)$

$\Rightarrow (B \cap A) \cup (B \cap B) = (B \cap A) \cup (B \cap C)$ --- {distributive property

$\Rightarrow (A \cap B) \cup B = (A \cap B) \cup (B \cap C)$

$\Rightarrow B = (A \cap B) \cup (B \cap C)$ — (1)

Again Similarly $A \cup B = A \cup C$

$\Rightarrow C \cap (A \cup B) = C \cap (A \cup C)$

$\Rightarrow (C \cap A) \cup (C \cap B) = (C \cap A) \cup (C \cap C)$

$\Rightarrow (A \cap C) \cup (B \cap C) = (A \cap C) \cup C$

$\Rightarrow (A \cap C) \cup (B \cap C) = C$

$\Rightarrow (A \cap B) \cup (B \cap C) = C$ -- (2) — {from given}

from (1) & (2)

$\Rightarrow B = C$ Proved