॥ ॐ श्री राधे कृष्णा ॥

← ULTIMATE MATHEMATICS : BY AJAY MITTAL

XI JEE MAINS: CLASS NO: 2

Topic Logarithmic functions:

Qn 1 If $0 \le a \le x$, then the minimum value of $\log_a x + \log_x x$ is

(a) 1 (b) 2 (c) 0 (d) none of these

Solution when $x \geq a$; $\log_a x \geq 1$

$\Rightarrow \log_a x \geq 1$

$\Rightarrow \log_a x + \log_x x \geq 1 + \log_x x$

$\Rightarrow \log_a x + \log_x x \geq 2$

Minimum value $= 2$ Ans (b)

Qn: 2 If $y = a^{\frac{1}{1-\log_a x}}$; $z = a^{\frac{1}{1-\log_a y}}$ and $x = a^k$

then $k =$

(a) $\frac{1}{a^{1-\log_a z}}$ (b) $\frac{1}{1-\log_a z}$ (c) $\frac{1}{1+\log_z a}$ (d) $\frac{1}{1-\log_z a}$

Sol $y = a^{\frac{1}{1-\log_a x}}$

$\Rightarrow \log_a y = \frac{1}{1-\log_a x}$

$\Rightarrow 1-\log_a x = \frac{1}{\log_a y}$

$\Rightarrow \boxed{\log_a x = 1 - \frac{1}{\log_a y}}$

$z = a^{\frac{1}{1-\log_a y}}$

$\log_a z = \frac{1}{1-\log_a y}$

$\Rightarrow 1-\log_a y = \frac{1}{\log_a z}$

$\Rightarrow \log_a y = 1 - \frac{1}{\log_a z}$

$\log_a x = 1 - \frac{1}{1-\frac{1}{\log_a z}}$

$\log_a x = 1 - \frac{\log_a z}{\log_a z - 1}$

$\log_a x = \frac{-1}{\log_a z - 1}$

$\log_a x = \frac{1}{1-\log_a z}$

$$\log_a x = \frac{1}{1-\log_a z}$$

$$\Rightarrow x = a^{\frac{1}{1-\log_a z}}$$

Given $x = a^k$

$$\Rightarrow k = \frac{1}{1-\log_a z} \quad \text{(b) Ans}$$

Ques 3 → If $4^x + 2^{2x-1} = 3^{x+\frac{1}{2}} + 3^{x-\frac{1}{2}}$ then $x =$

(a) $\frac{1}{2}$ (b) $\frac{3}{2}$ (c) $\frac{5}{2}$ (d) 1

Sol

$$4^x + \frac{2^{2x}}{2} = 3^x \cdot 3^{1/2} + \frac{3^x}{3^{1/2}}$$

$\boxed{2^{2x} = (2^2)^x}$

$$\Rightarrow 4^x + \frac{4^x}{2} = \frac{3 \cdot 3^x + 3^x}{3^{1/2}}$$

$$\Rightarrow \frac{2 \cdot 4^x + 4^x}{2} = \frac{4 \cdot 3^x}{3^{1/2}}$$

$$\Rightarrow \frac{3 \cdot 4^x}{2} = \frac{4 \cdot 3^x}{3^{1/2}}$$

$$\Rightarrow \frac{3 \cdot 3^{1/2}}{8} = \frac{3^x}{4^x}$$

$$\Rightarrow \frac{3^{3/2}}{8} = \left(\frac{3}{4}\right)^x$$

$4^{3/2} = (2)^3 = 8$

$$\Rightarrow \left(\frac{3}{4}\right)^{3/2} = \left(\frac{3}{4}\right)^x \Rightarrow x = 3/2 \quad \text{(b)}$$

Ques 4 → If $\frac{\log x}{2a+3b-5c} = \frac{\log y}{2b+3c-5a} = \frac{\log z}{2c+3a-5b}$ then

$xyz =$

(a) 2 (b) 1 (c) 0 (d) −1

Soln Let $\frac{\log x}{2a+3b-5c} = \frac{\log y}{2b+3c-5a} = \frac{\log z}{2c+3a-5b} = k$

$\Rightarrow \frac{\log x}{2a+3b-5c} = k$ | $\frac{\log y}{2b+3c-5a} = k$ | $\frac{\log z}{2c+3a-5b} = k$

$\Rightarrow \log x = k(2a+3b-5c)$ | $\log y = k(2b+3c-5a)$ | $\log z = k(2c+3a-5b)$

$\Rightarrow x = 10^{k(2a+3b-5c)}$

$y = 10^{k(2b+3c-5a)}$

$z = 10^{k(2c+3a-5b)}$

Now $x \cdot y \cdot z = 10^{k(0)} = 1$ (b) Ans

Ques 5 → If $9^{\log_3(\log_2 x)} = \log_2 x - (\log_2 x)^2 + 1$ then $x =$

(a) 1 (b) 1/2 (c) 3 (d) none of them

Sol. $3^{2\log_3(\log_2 x)} = \log_2 x - (\log_2 x)^2 + 1$

$\boxed{\log m^n = n \log m}$

$\Rightarrow 3^{\log_3(\log_2 x)^2} = \log_2 x - (\log_2 x)^2 + 1$

$\boxed{a^{\log_a x} = x}$

$$\Rightarrow (\log_2 x)^2 = \log_2 x - (\log_2 x)^2 + 1$$

$$\Rightarrow 2(\log_2 x)^2 - \log_2 x - 1 = 0$$

let $y = \log_2 x$

$$\Rightarrow 2y^2 - y - 1 = 0$$

$$\Rightarrow 2y^2 - 2y + y - 1 = 0$$

$$\Rightarrow 2y(y-1) + 1(y-1) = 0$$

$$\Rightarrow (y-1)(2y+1) = 0$$

$y = 1$	$y = -\frac{1}{2}$
$\log_2 x = 1$	$\log_2 x = -1/2$
$\Rightarrow x = 2^1$	$x = 2^{-1/2}$

$x = 2, \frac{1}{\sqrt{2}}$ (d) none of these

Qn. 6 → If $x > 1$, then the least value of the expression

$2\log_{10} x - \log_x(0.01)$ is

(a) 1 (b) 2 (c) 4 (d) none of these

Sol. $2 \cdot \frac{1}{\log_x 10} - \log_x\left(\frac{1}{100}\right)$

$= \frac{2}{\log_x 10} - [\log_x \cancel{1} - \log_x 100]$

$= \frac{2}{\log_x 10} + \log_x (10)^2$

$= \frac{2}{\log_x 10} + 2\log_x 10$

$= 2\left(\frac{1}{\log_x 10} + \log_x 10\right)$

$= 2(\geq 2)$

≥ 4 (Min value $= 4$)

$y \geq 1$

$\boxed{y + \frac{1}{y} \geq 2}$

Qn. 7 → The number of solutions of the equation

$$x^{\log_{\sqrt{x}}(2x)} = 4 \text{ is}$$

(a) 0 (b) 1 (c) 2 (d) Infinite many

Soln here $x > 0$ and $x \neq 1$

$$\Rightarrow x^{\log_{x^{1/2}}(2x)^1} = 4$$

$$\Rightarrow x^{2\log_x(2x)} = 4$$

$$\Rightarrow x^{\log_x(2x)^2} = 4$$

$$\Rightarrow (2x)^2 = 4$$

$$\Rightarrow 4x^2 = 4$$

$$\Rightarrow x^2 = 1$$

$$\Rightarrow x = \pm 1$$

But $x > 0$ $\therefore x \neq -1$ | also $x \neq 1$

$\therefore$ No solution (a) Ans

Qn. 8 → If $\log_{(0.04)}(x-1) \geq \log_{(0.2)}(x-1)$ then $x \in$

(a) $(1,2]$ (b) $(-\infty, 2]$ (c) $[2,\infty)$ (d) none of these

Soln $\log_{(0.2)^2}(x-1)^1 \geq \log_{0.2}(x-1)$

xi] JEE (class No: 2 (6)

$\Rightarrow \frac{1}{2}\log_{0.2}(x-1) \geq \log_{0.2}(x-1)$

$\Rightarrow \log_{0.2}(x-1) \geq 2\log_{0.2}(x-1)$

$\Rightarrow \log_{0.2}(x-1) \geq \log_{0.2}(x-1)^2$

$\Rightarrow (x-1) \leq (x-1)^2$

$\Rightarrow 1 \leq (x-1)$

$\Rightarrow 1+1 \leq x$

$\Rightarrow x \geq 2$

$x \in [2, \infty)$

$a < 1$
$\log x > \log y$
$\Rightarrow x < y$

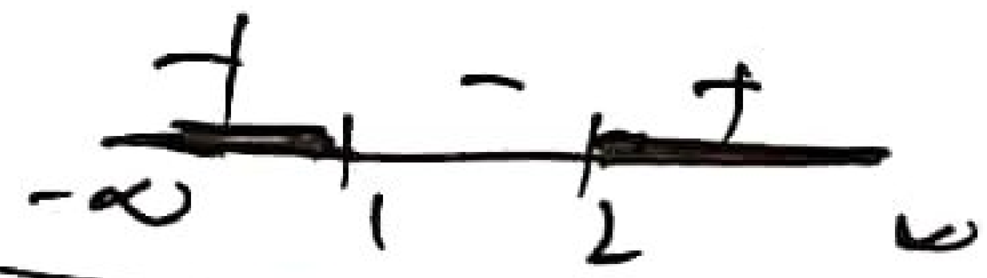
OR

$0 \leq (x-1)^2 - (x-1)$

$0 \leq (x-1)(x-1-1)$

$0 \leq (x-1)(x-2)$

$(x-1)(x-2) \geq 0$



Qno. 9 →

Quadratic ~~equation~~ expression

ax^2+bx+c

(1) If $D<0$ and $a>0$

then $ax^2+bx+c>0$ for all $x \in R$

$2x^2-x+1$

$D = 1-8 = -7 < 0$

$a>0$

then $2x^2-x+1>0$

(2) If $D<0$ and $a<0$

then $ax^2+bx+c<0$ for all $x \in R$

$-2x^2+x-1$

$D = 1-8 = -7 < 0$

$a = -2 < 0$

$-2x^2+x-1<0$

(·) $\log_a x \geq y$

If $x \geq a^y$

(a>1)

$\log_a x \geq y$

(a<1)

$x \leq a^y$

Qn 9 → The set of real values of x satisfying $\log_{1/2}(x^2-6x+12) \geq -2$ is

(a) $(-\infty, 2]$ (b) $[2,4]$ (c) $[4,\infty)$ (d) none of these

Soln $\quad x^2-6x+12>0 \qquad a=1>0$

$D = 36-48 = -12 < 0$

$\therefore x^2-6x+12>0$ for all $x \in R$

$\Rightarrow \quad x^2-6x+12 \leq \left(\frac{1}{2}\right)^{-2}$

$\Rightarrow x^2-6x+12 \leq 4$

$\Rightarrow x^2-6x+8 \leq 0$

$\Rightarrow (x-4)(x-2) \leq 0$

$x \in [2,4]$

$\therefore$ Soln. $[2,4]$ (b) Ans

Qn 10 → The number of values of x satisfying $1+\log_5(x^2+1) \geq \log_5(x^2+4x+1)$ is

(a) 1 (b) 2 (c) 3 (d) Infinite many

Sol: $\log_5 5 + \log_5(x^2+1) \geq \log_5(x^2+4x+1)$

$\Rightarrow \log_5(5\cdot(x^2+1)) \geq \log_5(x^2+4x+1)$

$\Rightarrow 5x^2+5 \geq x^2+4x+1$

$\Rightarrow 4x^2-4x+4 \geq 0$

$\Rightarrow x^2-x+1 \geq 0$ $\qquad a=1>0$

$D=1-4=-3<0$

$\therefore x^2-x+1 \geq 0$ for all $x \in R$

also

$x^2+1>0$

$x \in R$

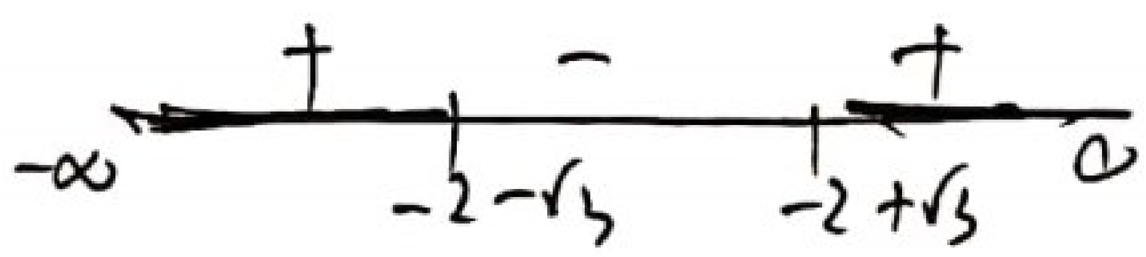
$x^2+4x+1>0$ $\qquad D=16-4=12$

eg $x^2+4x+1=0$

$x=\dfrac{-4 \pm \sqrt{16-4}}{2}$

$x=\dfrac{-4 \pm 2\sqrt{3}}{2}$

$x=-2 \pm \sqrt{3}$ $\qquad$ $x=-2+\sqrt{3}$, $x=-2-\sqrt{3}$



$x \in (-\infty, -2-\sqrt{3}) \cup (-2+\sqrt{3}, \infty)$

$\therefore x \in (-\infty, -2-\sqrt{3}) \cup (-2+\sqrt{3}, \infty)$

$\therefore$ Infinite many solution (d)