॥ जय श्री राधे कृष्णा जय श्री गिरिराज जी महाराज ॥ ①

ULTIMATE MATHEMATICS : BY AJAY MITTAL

$\boxed{\text{TRIGONOMETRY SPECIAL CLASS}}$

(•) $a\cos\theta \pm b\sin\theta$ (Trigo expression)

Maximum value $= \sqrt{a^2+b^2}$

Minimum value $= -\sqrt{a^2+b^2}$

(•) $a\cos\theta \pm b\sin\theta \longrightarrow$ we want in to a single Trigo function

eg $\cos\theta + \sin\theta$

here $a=1,\ b=1$

M & D by $\sqrt{a^2+b^2} = \sqrt{1+1} = \sqrt{2}$

$= \sqrt{2}\left[\frac{1}{\sqrt{2}}\cos\theta + \frac{1}{\sqrt{2}}\sin\theta\right]$

$= \sqrt{2}\left[\sin\left(\frac{\pi}{4}\right)\cos\theta + \cos\left(\frac{\pi}{4}\right)\sin\theta\right] = \sqrt{2}\sin\left(\frac{\pi}{4}+\theta\right)$

(•) $\sin\theta\cos\theta = \frac{1}{2}(2\sin\theta\cos\theta) = \frac{1}{2}\sin(2\theta)$

Que 1 Find the value of $\sqrt{3}\,\operatorname{cosec}(20^\circ) - \sec(20^\circ)$

Soln

$$\frac{\sqrt{3}}{\sin(20^\circ)} - \frac{1}{\cos(20^\circ)}$$

$$= \frac{\sqrt{3}\cos(20^\circ) - \sin(20^\circ)}{\sin(20^\circ)\cos(20^\circ)}$$

$$= \frac{2\left[\frac{\sqrt{3}}{2}\cos(20^\circ) - \frac{1}{2}\sin(20^\circ)\right]}{\frac{1}{2}\left(2\sin 20^\circ \cdot \cos(20^\circ)\right)}$$

$$= 4\frac{\left[\sin(60^\circ)\cos(20^\circ) - \cos(60^\circ)\sin(20^\circ)\right]}{\sin(40^\circ)} \quad \ldots \{2\sin\theta\cos\theta = \sin(2\theta)\}$$

$$= 4\frac{\sin(60^\circ - 20^\circ)}{\sin(40^\circ)} \quad \ldots \{\sin A\cos B - \cos A\sin B = \sin(A-B)\}$$

$$= \frac{4\cancel{\sin(40^\circ)}}{\cancel{\sin(40^\circ)}} = 4 \quad \underline{\text{Ans}}$$

Que 2 Find the value of $\tan 9^\circ - \tan(27^\circ) - \tan(63^\circ) + \tan(81^\circ)$

Soln

$$\tan(9^\circ) - \tan(27^\circ) - \tan(90^\circ - 27) + \tan(90^\circ - 9^\circ)$$

$$= \tan(9^\circ) - \tan(27^\circ) - \cot(27) + \cot(9^\circ)$$

$$= \tan(9^\circ) + \cot(9^\circ) - \left(\tan(27^\circ) + \cot(27^\circ)\right)$$

$$= \frac{\sin(9^\circ)}{\cos(9^\circ)} + \frac{\cos(9^\circ)}{\sin(9^\circ)} - \left\{\frac{\sin(27^\circ)}{\cos(27)} + \frac{\cos(27^\circ)}{\sin(27)}\right\}$$

$$= \frac{\sin^2(9^\circ)+\cos^2(9^\circ)}{\sin(9^\circ)\cos(9^\circ)} - \frac{\sin^2(27^\circ)+\cos^2(27^\circ)}{\sin(27^\circ)\cos(27^\circ)}$$

$$= \frac{1}{\sin 9^\circ \cos 9^\circ} - \frac{1}{\sin(27)\cos(27^\circ)}$$

$$= \frac{2}{2\sin 9^\circ \cos 9^\circ} - \frac{2}{2\sin(27)^\circ \cos(27^\circ)}$$

$$= \frac{2}{\sin(18^\circ)} - \frac{2}{\sin(54^\circ)}$$

$$\boxed{\sin(18^\circ) = \frac{\sqrt{5}-1}{4} \quad \cos(36^\circ) = \frac{\sqrt{5}+1}{4}}$$

$$= \frac{2}{\sin(18^\circ)} - \frac{2}{\cos(36^\circ)} \quad \cdots \{\sin\theta = \cos(90-\theta)\}$$

$$= \frac{8}{\sqrt{5}-1} - \frac{8}{\sqrt{5}+1}$$

$$= 8\left[\frac{\cancel{\sqrt{5}}+1-\cancel{\sqrt{5}}+1}{(\sqrt{5}-1)(\sqrt{5}+1)}\right]$$

$$= \frac{8(2)}{4}$$

$$= 4 \ \underline{\text{Ans}}$$

Scanned with CamScanner

Ques 3 If α and β are the roots of / solutions of equation $a\cos\theta + b\sin\theta = c$; then show that $\cos(\alpha+\beta) = \frac{a^2-b^2}{a^2+b^2}$ also find $\sin(\alpha+\beta)$

Soln we have $a\cos\theta + b\sin\theta = c$

$\Rightarrow \quad a\cos\theta = c - b\sin\theta$

Squaring

$a^2\cos^2\theta = c^2 + b^2\sin^2\theta - 2bc\sin\theta$

$\Rightarrow \quad a^2(1-\sin^2\theta) = c^2 + b^2\sin^2\theta - 2bc\sin\theta$

$\Rightarrow \quad a^2 - a^2\sin^2\theta = c^2 + b^2\sin^2\theta - 2bc\sin\theta$

$\Rightarrow \quad (a^2+b^2)\sin^2\theta - 2bc\sin\theta + (c^2-a^2) = 0$

$\sin\alpha$ and $\sin\beta$ are the roots of this equation

Product of roots : $\boxed{\sin\alpha \cdot \sin\beta = \frac{c^2-a^2}{a^2+b^2}}$

$x^2 - 3x + 2 = 0$
$x = 1$
$x = 2$

Again we have

$a\cos\theta + b\sin\theta = c$

$\Rightarrow \quad b\sin\theta = c - a\cos\theta$

Squaring

$b^2\sin^2\theta = c^2 + a^2\cos^2\theta - 2ac\cos\theta$

$\Rightarrow \quad b^2(1-\cos^2\theta) = c^2 + a^2\cos^2\theta - 2ac\cos\theta$

$\Rightarrow \quad b^2 - b^2\cos^2\theta = c^2 + a^2\cos^2\theta - 2ac\cos\theta$

$\Rightarrow \quad (a^2+b^2)\cos^2\theta - 2ac\cos\theta + (c^2-b^2) = 0$

$\cos\alpha$ & $\cos\beta$ are the roots of this equation

Now product of roots : $\boxed{\cos\alpha . \cos\beta = \frac{c^2-b^2}{a^2+b^2}}$

Now $\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

$$= \frac{c^2-b^2}{a^2+b^2} - \frac{c^2-a^2}{a^2+b^2}$$

$$= \frac{\cancel{c^2}-b^2-\cancel{c^2}+a^2}{a^2+b^2}$$

$$\boxed{\cos(\alpha+\beta) = \frac{a^2-b^2}{a^2+b^2}}$$ proved

we know $\sin^2(\alpha+\beta) + \cos^2(\alpha+\beta) = 1$

$$\sin(\alpha+\beta) = \sqrt{1-\cos^2(\alpha+\beta)}$$

$$= \sqrt{1-\left(\frac{a^2-b^2}{a^2+b^2}\right)^2}$$

$$= \sqrt{1-\frac{(a^4+b^4-2a^2b^2)}{a^4+b^4+2a^2b^2}}$$

$$= \frac{\sqrt{\cancel{a^4}+\cancel{b^4}+2a^2b^2-\cancel{a^4}-\cancel{b^4}+2a^2b^2}}{a^2+b^2}$$

$$\boxed{\sin(\alpha+\beta) = \frac{2ab}{a^2+b^2}}$$ Ans

(5)(B)

Qn 4 If α and β are the solutions of the equation $a\tan\theta + b\sec\theta = c$ then show that $\tan(\alpha+\beta) = \frac{2ac}{a^2-c^2}$

We have

$$a\tan\theta + b\sec\theta = c$$

$$\Rightarrow b\sec\theta = c - a\tan\theta$$

Squaring

$$b^2\sec^2\theta = c^2 + a^2\tan^2\theta - 2ac\tan\theta$$

$$\Rightarrow b^2(1+\tan^2\theta) = c^2 + a^2\tan^2\theta - 2ac\tan\theta$$

$$\Rightarrow b^2 + b^2\tan^2\theta = c^2 + a^2\tan^2\theta - 2ac\tan\theta$$

$$\Rightarrow (a^2-b^2)\tan^2\theta - 2ac\tan\theta + (c^2-b^2) = 0$$

$\tan\alpha$ & $\tan\beta$ are the roots of this equation

Sum of roots : $\tan\alpha + \tan\beta = \frac{2ac}{a^2-b^2}$

Product of roots : $\tan\alpha\tan\beta = \frac{c^2-b^2}{a^2-b^2}$

Also $\tan(\alpha+\beta) = \frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta} = \frac{\frac{2ac}{a^2-b^2}}{1-\frac{c^2-b^2}{a^2-b^2}}$

$$= \frac{2ac}{a^2-\cancel{b^2}-c^2+\cancel{b^2}}$$

$$\boxed{\tan(\alpha+\beta) = \frac{2ac}{a^2-c^2}}$$ Ans

Qn. 5 → If $x\cos\theta = y\cos\left(\theta+\frac{2\pi}{3}\right) = z\cos\left(\theta+\frac{4\pi}{3}\right)$

Find the value of $xy+yz+zx$

Soln let $x\cos\theta = y\cos\left(\theta+\frac{2\pi}{3}\right) = z\cos\left(\theta+\frac{4\pi}{3}\right) = k$ (constant)

$\Rightarrow x\cos\theta = k$; $y\cos\left(\theta+\frac{2\pi}{3}\right) = k$ & $z\cos\left(\theta+\frac{4\pi}{3}\right) = k$

$\Rightarrow \boxed{x = \frac{k}{\cos\theta}}$; $\boxed{y = \frac{k}{\cos\left(\theta+\frac{2\pi}{3}\right)}}$; $\boxed{z = \frac{k}{\cos\left(\theta+\frac{4\pi}{3}\right)}}$

$xy+yz+zx = xyz\left(\frac{1}{x}+\frac{1}{y}+\frac{1}{z}\right)$

Tricky

Now Consider $\frac{1}{x}+\frac{1}{y}+\frac{1}{z} = \frac{\cos\theta}{k} + \frac{\cos\left(\theta+\frac{2\pi}{3}\right)}{k} + \frac{\cos\left(\theta+\frac{4\pi}{3}\right)}{k}$

$= \frac{1}{k}\left[\cos\theta + \cos(\theta+120^\circ) + \cos(\theta+240^\circ)\right]$

B A

$= \frac{1}{k}\left[\cos\theta + 2\cos\left(\frac{2\theta+360^\circ}{2}\right)\cdot\cos(60^\circ)\right]$

III

$= \frac{1}{k}\left[\cos\theta + \cancel{2}\cos(180+\theta)\times\frac{1}{\cancel{2}}\right]$

$= \frac{1}{k}\left(\cancel{\cos\theta} - \cancel{\cos\theta}\right)$

$\frac{1}{x}+\frac{1}{y}+\frac{1}{z} = \frac{0}{k} = 0$

$xy+yz+zx = xyz(0) = 0$ Ans

Q.No. 6 → Find the value of $\cos\left(\frac{\pi}{5}\right)\cos\left(\frac{2\pi}{5}\right)\cos\left(\frac{4\pi}{5}\right)\cos\left(\frac{8\pi}{5}\right)$

Soln

$$= \cos\left(\frac{\pi}{5}\right)\cdot\cos\left(\frac{2\pi}{5}\right)\cdot\cos\left(\frac{4\pi}{5}\right)\cdot\cos\left(\frac{8\pi}{5}\right)$$

M & D by $2\sin\left(\frac{\pi}{5}\right)$

$$= \frac{1}{2\sin\left(\frac{\pi}{5}\right)}\left[2\sin\left(\frac{\pi}{5}\right)\cos\left(\frac{\pi}{5}\right)\cdot\cos\left(\frac{2\pi}{5}\right)\cdot\cos\left(\frac{4\pi}{5}\right)\cos\left(\frac{8\pi}{5}\right)\right]$$

$$= \frac{1}{2\sin\left(\frac{\pi}{5}\right)}\left[\sin\left(\frac{2\pi}{5}\right)\cos\left(\frac{2\pi}{5}\right)\cdot\cos\left(\frac{4\pi}{5}\right)\cdot\cos\left(\frac{8\pi}{5}\right)\right]$$

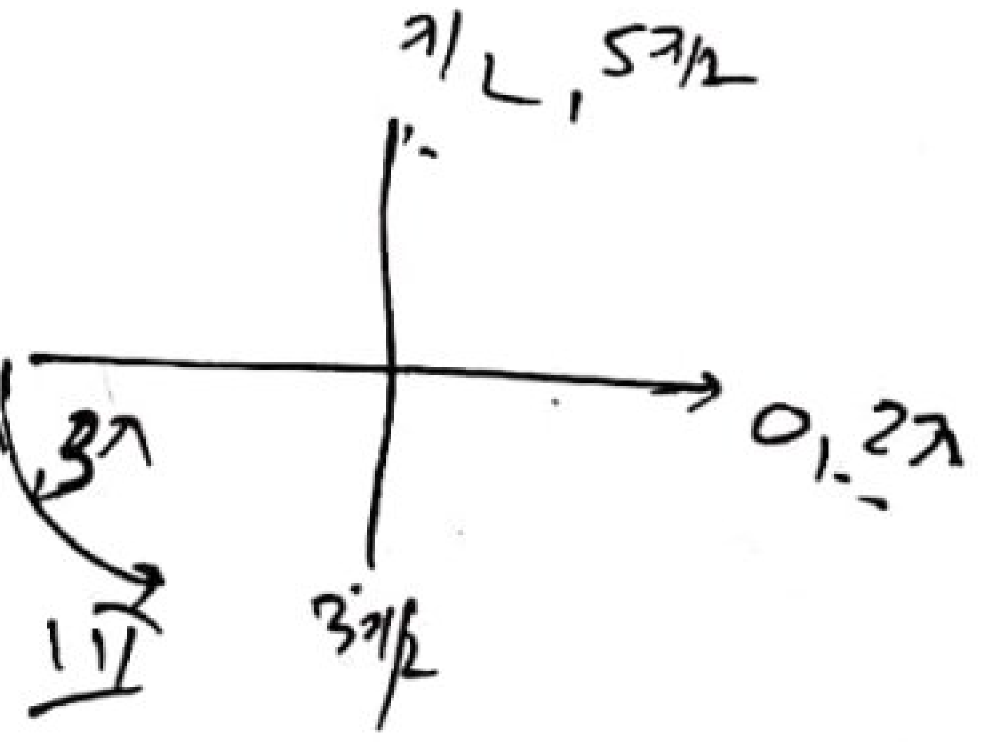
$\because \{2\sin\theta\cos\theta = \sin(2\theta)\}$

$$= \frac{1}{4\sin\left(\frac{\pi}{5}\right)}\left[\sin\left(\frac{4\pi}{5}\right)\cos\left(\frac{4\pi}{5}\right)\cdot\cos\left(\frac{8\pi}{5}\right)\right]$$

$$= \frac{1}{8\sin\left(\frac{\pi}{5}\right)}\left[\sin\left(\frac{8\pi}{5}\right)\cdot\cos\left(\frac{8\pi}{5}\right)\right]$$

$$= \frac{1}{16\sin\left(\frac{\pi}{5}\right)}\left[\sin\left(\frac{16\pi}{5}\right)\right]$$

$$= \frac{1}{16\sin\left(\frac{\pi}{5}\right)}\cdot\sin\left(3\pi+\frac{\pi}{5}\right)$$



$$= \frac{1}{16\cancel{\sin\left(\frac{\pi}{5}\right)}}\left(-\cancel{\sin\left(\frac{\pi}{5}\right)}\right)$$

$$= -\frac{1}{16} \quad \text{Ans}$$

Qn. 7 Find $\cos\left(\frac{2\pi}{5}\right)\cdot\cos\left(\frac{4\pi}{5}\right)\cos\left(\frac{8\pi}{5}\right)\cdot\cos\left(\frac{14\pi}{5}\right)$

Sol.

$$\cos\left(\frac{2\pi}{5}\right)\cos\left(\frac{4\pi}{5}\right)\cos\left(\frac{8\pi}{5}\right)\cos\left(\frac{14\pi}{5}\right)$$

$$= \cos\left(\frac{2\pi}{5}\right)\cdot\cos\left(\frac{4\pi}{5}\right)\cdot\cos\left(\frac{8\pi}{5}\right)\cdot\cos\left(\overset{\text{II}}{3\pi-\frac{\pi}{5}}\right)$$

$$= \cos\left(\frac{2\pi}{5}\right)\cdot\cos\left(\frac{4\pi}{5}\right)\cdot\cos\left(\frac{8\pi}{5}\right)\left(-\cos\frac{\pi}{5}\right)$$

$$= -\cos\left(\frac{\pi}{5}\right)\cdot\cos\left(\frac{2\pi}{5}\right)\cdot\cos\left(\frac{4\pi}{5}\right)\cdot\cos\left(\frac{8\pi}{5}\right)$$

proceed

$$= \frac{1}{16} \text{ Ans}$$

Qn. 8 Find the least value of $2^{\sin\theta}+2^{\cos\theta}$

Soln

$$\boxed{A\cdot M \geq G\cdot M}$$

$$\frac{a+b}{2} \geq \sqrt{ab}$$

here $a = 2^{\sin\theta}$ & $b = 2^{\cos\theta}$

$$\Rightarrow \frac{2^{\sin\theta}+2^{\cos\theta}}{2} \geq \sqrt{2^{\sin\theta}\times 2^{\cos\theta}}$$

$$\Rightarrow \frac{2^{\sin\theta}+2^{\cos\theta}}{2} \geq \sqrt{2^{\sin\theta+\cos\theta}}$$

$$\Rightarrow \frac{2^{\sin\theta}+2^{\cos\theta}}{2} \geq 2^{\frac{1}{2}(\sin\theta+\cos\theta)}$$

$$\Rightarrow \frac{2^{\sin\theta} + 2^{\cos\theta}}{2} \geq 2^{\frac{\sqrt{2}}{2}\left(\frac{1}{\sqrt{2}}\cos\theta + \frac{1}{\sqrt{2}}\sin\theta\right)}$$

$$\Rightarrow \frac{2^{\sin\theta} + 2^{\cos\theta}}{2} \geq 2^{\frac{1}{\sqrt{2}}\sin\left(\frac{\pi}{4}+\theta\right)}$$

we know $-1 \leq \sin\left(\frac{\pi}{4}+\theta\right) \leq 1$

$$\Rightarrow \frac{2^{\sin\theta} + 2^{\cos\theta}}{2} \geq 2^{\frac{1}{\sqrt{2}}(-1)}$$

$$\Rightarrow 2^{\sin\theta} + 2^{\cos\theta} \geq 2 \cdot 2^{-\frac{1}{\sqrt{2}}}$$

$$\Rightarrow 2^{\sin\theta} + 2^{\cos\theta} \geq 2^{1-\frac{1}{\sqrt{2}}}$$

$\therefore$ Max/Min value of $2^{\sin\theta} + 2^{\cos\theta} = 2^{1-\frac{1}{\sqrt{2}}}$ Ans

Q.No.9 If $3\tan(\theta-15) = \tan(\theta+15)$; $\boxed{0 < \theta < 90}$ Find θ

Sol

$$\frac{\tan(\theta-15)}{\tan(\theta+15)} = \frac{1}{3}$$

$$\Rightarrow \frac{\frac{\sin(\theta-15)}{\cos(\theta-15)}}{\frac{\sin(\theta+15)}{\cos(\theta+15)}} = \frac{1}{3}$$

$$\Rightarrow \frac{\sin(\theta-15)\cdot\cos(\theta+15)}{\sin(\theta+15)\cdot\cos(\theta-15)} = \frac{1}{3}$$

$$\Rightarrow \frac{2\sin(\theta-15)\cos(\theta+15)}{2\sin(\theta+15)\cos(\theta-15)} = \frac{1}{3}$$

$$\Rightarrow \frac{\sin(2\theta)+\sin(-30^\circ)}{\sin(2\theta)+\sin(30)} = \frac{1}{3}$$

$$\Rightarrow \frac{\sin(2\theta)-\frac{1}{2}}{\sin(2\theta)+\frac{1}{2}} = \frac{1}{3}$$

$$\Rightarrow \frac{2\sin(2\theta)-1}{2\sin(2\theta)+1} = \frac{1}{3}$$

$$\Rightarrow 6\sin(2\theta)-3 = 2\sin(2\theta)+1$$

$$\Rightarrow 4\sin(2\theta) = 4$$

$$\Rightarrow \sin(2\theta) = 1$$

$$\Rightarrow 2\theta = \frac{\pi}{2}$$

$$\Rightarrow \boxed{\theta = \frac{\pi}{4}}$$ Ans

Qu. 10 → If $a\cos(2\theta) + b\sin(2\theta) = c$ has α and β as its roots, then prove that

$$\tan\alpha + \tan\beta = \frac{2b}{a+c}$$

Sol

$$a\frac{(1-\tan^2\theta)}{1+\tan^2\theta} + b\cdot\frac{2\tan\theta}{1+\tan^2\theta} = c$$

$$\Rightarrow a - a\tan^2\theta + 2b\tan\theta = c + c\tan^2\theta$$

$\Rightarrow (a+c)\tan^2\theta - 2b\tan\alpha + (c-a) = 0$

$\tan\alpha$ and $\tan\beta$ are roots of this equation

$\therefore$ Sum of roots $\tan\alpha + \tan\beta = \dfrac{2b}{a+c}$ Ans

Que. 11 $\rightarrow$ Find the value of

$$3\left[\sin^4\left(\frac{3\pi}{2}-\alpha\right) + \sin^4(3\pi+\alpha)\right] - 2\left[\sin^6\left(\frac{\pi}{2}+\alpha\right) + \sin^6(5\pi-\alpha)\right]$$

Soln $3\left[\cos^4\alpha + \sin^4\alpha\right] - 2\left[\cos^6\alpha + \sin^6\alpha\right]$

$a^2+b^2 = (a+b)^2 - 2ab$	$a^3+b^3 = (a+b)(a^2+b^2-ab)$

$= 3\left[(\cos^2\alpha + \sin^2\alpha)^2 - 2\cos^2\alpha\sin^2\alpha\right] - 2\left[(\cos^2\alpha+\sin^2\alpha)(\cos^4\alpha + \sin^4\alpha - \cos^2\alpha\sin^2\alpha)\right]$

$= 3\left[1 - 2\cos^2\alpha\sin^2\alpha\right] - 2\left[\cos^4 + \sin^4\alpha - \cos^2\alpha\sin^2\alpha\right]$

$= 3 - 6\cos^2\alpha\sin^2\alpha - 2\left[(\cos^2\alpha + \sin^2\alpha)^2 - 2\cos^2\alpha\sin^2\alpha - \cos^2\alpha\sin^2\alpha\right]$

$= 3 - 6\cos^2\alpha\sin^2\alpha - 2\left[1 - 3\cos^2\alpha\sin^2\alpha\right]$

$= 3 - \cancel{6\cos^2\alpha\sin^2\alpha} - 2 + \cancel{6\cos^2\alpha\sin^2\alpha}$

$= 1$ Ans

Qn. 12 If $\tan(\frac{\theta}{2}) = \sqrt{\frac{1-e}{1+e}} \cdot \tan(\frac{\phi}{2})$ (12)

Show that $\cos\phi = \dfrac{\cos\theta - e}{1 - e\cos\theta}$

Ans $\cos\phi = \dfrac{1-\tan^2(\phi/2)}{1+\tan^2(\phi/2)}$

given $\tan\frac{\theta}{2} = \sqrt{\frac{1-e}{1+e}} \cdot \tan(\phi/2)$

Squaring $\tan^2(\frac{\theta}{2}) = \frac{1-e}{1+e} \cdot \tan^2(\phi/2)$

$\Rightarrow \tan^2(\phi/2) = \left(\frac{1+e}{1-e}\right)\tan^2\frac{\theta}{2}$

Ans $\cos\phi = \dfrac{1-\tan^2(\phi/2)}{1+\tan^2(\phi/2)}$

$$= \frac{1 - \left(\frac{1+e}{1-e}\right)\tan^2\frac{\theta}{2}}{1 + \left(\frac{1+e}{1-e}\right)\tan^2\frac{\theta}{2}}$$

$$= \frac{1-e - \tan^2\frac{\theta}{2} - e\tan^2\frac{\theta}{2}}{1-e + \tan^2\frac{\theta}{2} + e\tan^2\frac{\theta}{2}}$$

$$= \frac{\left(1-\tan^2\frac{\theta}{2}\right) - e\left(1+\tan^2\frac{\theta}{2}\right)}{\left(1+\tan^2\frac{\theta}{2}\right) - e\left(1-\tan^2\frac{\theta}{2}\right)}$$

Divide N & D by $(1+\tan^2\theta/2)$

Scanned with CamScanner

$$= \frac{\frac{1-\tan^2 \theta/2}{1+\tan^2 \theta/2} - e}{1 - e\frac{(1-\tan^2\frac{\theta}{2})}{1+\tan^2\frac{\theta}{2}}}$$

$$\boxed{\cos\phi = \frac{\cos\theta - e}{1 - e\cos\theta}}$$ Ans

Q.No. 13 If $\tan\frac{\theta}{2} = \sqrt{\frac{a-b}{a+b}}\tan(\phi/2)$

self Show that $\cos\theta = \frac{a\cos\phi + b}{a + b\cos\phi}$

Q.No. 14 Show that

$$\frac{\sin x}{\cos(3x)} + \frac{\sin(3x)}{\cos(9x)} + \frac{\sin(9x)}{\cos(27x)} = \frac{1}{2}(\tan(27x) - \tan x)$$

LHS $$\frac{\sin x}{\cos(3x)} + \frac{\sin(3x)}{\cos(9x)} + \frac{\sin(9x)}{\cos(27x)}$$

Tricky $$\frac{2\sin x\cos x}{2\cos(3x)\cos x} + \frac{2\sin(3x)\cos(3x)}{2\cos(9x)\cos(3x)} + \frac{2\sin(9x)\cos(9x)}{2\cos(27x)\cos(9x)}$$

$$= \frac{\sin(2x)}{2\cos(3x)\cos x} + \frac{\sin(6x)}{2\cos(9x)\cos(3x)} + \frac{\sin(18x)}{2\cos(27x)\cos(9x)}$$

$$= \frac{1}{2}\left[\frac{\sin(3x-x)}{\cos(3x)\cos x} + \frac{\sin(9x-3x)}{\cos(9x)\cos(3x)} + \frac{\sin(27x-9x)}{\cos(27x)\cos(9x)}\right]$$

$$= \frac{1}{2}\left[\frac{\sin(3x)\cos x - \cos(3x)\sin x}{\cos(3x)\cdot\cos x} + \frac{\sin(9x)\cos(3x) - \cos(9x)\sin(3x)}{\cos(9x)\cos(3x)} + \frac{\sin(27x)\cos(9x) - \cos(27x)\sin(9x)}{\cos(27x)\cdot\cos(9x)}\right]$$

$$= \frac{1}{2}\left(\cancel{\tan(3x)} - \tan x + \cancel{\tan(9x)} - \cancel{\tan(3x)} + \tan(27x) - \cancel{\tan(9x)}\right)$$

$$= \frac{1}{2}\left[\tan(27x) - \tan x\right] = \text{R.H.S.}\quad \text{Ans}$$

Qn 15 → Show that

$$2\sin^2\beta + 4\cos(\alpha+\beta)\cdot\sin\alpha\sin\beta + \cos(2\alpha+2\beta) = \cos(2\alpha)$$

Soln

$$= 2\sin^2\beta + 4(\cos\alpha\cos\beta - \sin\alpha\sin\beta)\sin\alpha\cdot\sin\beta + \cos(2\alpha)\cos(2\beta) - \sin(2\alpha)\sin(2\beta)$$

$$= 2\sin^2\beta + \boxed{4\cos\alpha\cos\beta\sin\alpha\sin\beta} - 4\sin^2\alpha\sin^2\beta + \cos(2\alpha)\cos(2\beta) - \sin(2\alpha)\sin(2\beta)$$

$2\sin\theta\cos\theta = \sin(2\theta)$

$$= 2\sin^2\beta + \cancel{\sin(2\alpha)\sin(2\beta)} - 4\sin^2\alpha\sin^2\beta + \cos(2\alpha)\cos(2\beta) - \cancel{\sin(2\alpha)\sin(2\beta)}$$

$$= 2\sin^2\beta - 4\sin^2\alpha\cdot\sin^2\beta + \cos(2\alpha)\cos(2\beta)$$

$$= \cancel{2}\left(\frac{1-\cos(2\beta)}{\cancel{2}}\right) - \cancel{4}\left(\frac{1-\cos(2\alpha)}{\cancel{2}}\right)\cdot\left(\frac{1-\cos(2\beta)}{\cancel{2}}\right) + \cos(2\alpha)\cos(2\beta)$$

$$= \cancel{1} - \cancel{\cos(2\beta)} - \cancel{1} + \cancel{\cos(2\beta)} + \cos(2\alpha) - \cancel{\cos(2\alpha)\cos(2\beta)} + \cancel{\cos(2\alpha)\cos(2\beta)}$$

$$= \cos(2\alpha)\quad \text{Ans}$$

Scanned with CamScanner

Qn. 16 - If $\sin\alpha + \sin\beta = a$ and $\cos\alpha + \cos\beta = b$ then show that

(i) $\cos(\alpha+\beta) = \dfrac{b^2-a^2}{b^2+a^2}$ (2) $\sin(\alpha+\beta) = \dfrac{2ab}{a^2+b^2}$

Solution = given $\sin\alpha + \sin\beta = a$ & $\cos\alpha + \cos\beta = b$

$\Rightarrow 2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right) = a$ and $2\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right) = b$

divide these equations

$$\Rightarrow \frac{\cancel{2}\sin\left(\frac{\alpha+\beta}{2}\right)\cancel{\cos\left(\frac{\alpha-\beta}{2}\right)}}{\cancel{2}\cos\left(\frac{\alpha+\beta}{2}\right)\cancel{\cos\left(\frac{\alpha-\beta}{2}\right)}} = \frac{a}{b}$$

$$\Rightarrow \boxed{\tan\left(\frac{\alpha+\beta}{2}\right) = \frac{a}{b}}$$

(i) RHS $\dfrac{b^2-a^2}{b^2+a^2}$

(we have to use value of $\frac{a}{b}$)

so divide N & D by b^2

$$= \frac{1-\left(\frac{a}{b}\right)^2}{1+\left(\frac{a}{b}\right)^2}$$

$$= \frac{1-\tan^2\left(\frac{\alpha+\beta}{2}\right)}{1+\tan^2\left(\frac{\alpha+\beta}{2}\right)}$$

$$= \cos\left(\cancel{2}\left(\frac{\alpha+\beta}{\cancel{2}}\right)\right) \quad --- \left\{\because \frac{1-\tan^2\theta}{1+\tan^2\theta} = \cos(2\theta)\right\}$$

$= \cos(\alpha+\beta)$ proved

(ii) RHS $\frac{2ab}{a^2+b^2}$

again Divide N & D by b^2

$$= \frac{2\frac{a}{b}}{(\frac{a}{b})^2+1}$$

$$= \frac{2\tan(\frac{\alpha+\beta}{2})}{\tan^2(\frac{\alpha+\beta}{2})+1}$$

$$= \sin\left(\cancel{2}\left(\frac{\alpha+\beta}{\cancel{2}}\right)\right) \quad --- \left\{ \frac{2\tan\theta}{1+\tan^2\theta} = \sin(2\theta) \right\}$$

$$= \sin(\alpha+\beta)$$

= RHS Proved

Ans 17 → prove that

$$\frac{\cos(6\theta)+6\cos(4\theta)+15\cos(2\theta)+10}{\cos(5\theta)+5\cos(3\theta)+10\cos\theta} = 2\cos\theta$$

Sol LHS

$$\frac{\cos(6\theta)+6\cos(4\theta)+15\cos(2\theta)+10}{\cos(5\theta)+5\cos(3\theta)+10\cos\theta}$$

$$= \frac{\cos(6\theta)+\cos(4\theta)+5\cos(4\theta)+5\cos(2\theta)+10\cos(2\theta)+10}{\cos(5\theta)+5\cos(3\theta)+10\cos\theta}$$

$$= \frac{(\cos(6\theta)+\cos(4\theta))+5(\cos(4\theta)+\cos(2\theta))+10(\cos(2\theta)+1)}{\cos(5\theta)+5\cos(3\theta)+10\cos\theta}$$

$$= \frac{2\cos(5\theta)\cos\theta+5\times2\cos(3\theta)\cos\theta+10\times2\cos^2\theta}{\cos(5\theta)+5\cos(3\theta)+10\cos\theta}$$

$$= \frac{2\cos\theta\left[\cancel{\cos(5\theta) + 5\cos(3\theta) + 10\cos\theta}\right]}{\cancel{\cos(5\theta) + 5\cos(3\theta) + 10\cos\theta}}$$

$= 2\cos\theta \quad = \text{RHS} \qquad$ PROVED

Qn. 18 → If $\sin\alpha + \sin\beta = a$ and $\cos\alpha + \cos\beta = b$
Show that (1) $\cos(\alpha - \beta) = \frac{a^2+b^2-2}{2}$

(2) $\tan\left(\frac{\alpha-\beta}{2}\right) = \sqrt{\frac{4-a^2-b^2}{a^2+b^2}}$

Solution
Given $\sin\alpha + \sin\beta = a$ & $\cos\alpha + \cos\beta = b$

Note (here don't apply formula $\sin A + \sin B$ & $\cos A + \cos B$) we will not get the answer

(1) Take RHS $\quad \frac{a^2+b^2-2}{2}$

put value of a & b

$$= \frac{(\sin\alpha + \sin\beta)^2 + (\cos\alpha + \cos\beta)^2 - 2}{2}$$

$$= \frac{\sin^2\alpha + \sin^2\beta + 2\sin\alpha\sin\beta + \cos^2\alpha + \cos^2\beta + 2\cos\alpha\cos\beta - 2}{2}$$

$$= \frac{(\sin^2\alpha + \cos^2\alpha) + (\sin^2\beta + \cos^2\beta) + 2(\cos\alpha\cos\beta + \sin\alpha\sin\beta) - 2}{2}$$

$$= \frac{\cancel{1+1} + 2\cos(\alpha-\beta) - \cancel{2}}{2} = \frac{2\cos(\alpha-\beta)}{2} = \boxed{\cos(\alpha-\beta)}$$

Ans

(ii) <u>Imp step</u>

$$\boxed{\tan\frac{\theta}{2} = \sqrt{\frac{1-\cos\theta}{1+\cos\theta}}}$$

Reason :

$$\sqrt{\frac{1-\cos\theta}{1+\cos\theta}} = \sqrt{\frac{\cancel{2}\sin^2\frac{\theta}{2}}{\cancel{2}\cos^2\frac{\theta}{2}}}$$

$$= \sqrt{\tan^2\frac{\theta}{2}} = \tan(\theta/2)$$

<u>Now</u>

$$\tan\left(\frac{\alpha-\beta}{2}\right) = \sqrt{\frac{1-\cos(\alpha-\beta)}{1+\cos(\alpha-\beta)}}$$

Now use value of $\cos(\alpha-\beta)$ from part(i)

$$\tan\left(\frac{\alpha-\beta}{2}\right) = \sqrt{\frac{1-\frac{(a^2+b^2-2)}{2}}{1+\frac{a^2+b^2-2}{2}}}$$

$$= \sqrt{\frac{2-a^2-b^2+2}{\cancel{2}+a^2+b^2-\cancel{2}}}$$

$$\tan\left(\frac{\alpha-\beta}{2}\right) = \sqrt{\frac{4-a^2-b^2}{a^2+b^2}}$$

<u>PROVED</u>

<u>Ans. 19</u> → (i) Find the Maximum value of $3\cos x + 4\sin x + 8$

(2) If $3\cos\theta = 2\sin^2\theta$; $0 \le \theta \le 2\pi$ Find θ

<u>Solution</u> (1) $3\cos x + 4\sin x + 8$

<u>Consider</u> $3\cos x + 4\sin x$

that is in the form $a\cos\theta + b\sin\theta$

and Maximum value of this expression is $= \sqrt{a^2+b^2}$

$\therefore$ Max. value of $3\cos x + 4\sin x = \sqrt{9+16} = 5$

$\therefore$ Max. value of $3\cos x + 4\sin x + 8 = 5 + 8 = 13$ Ans

(2) Given $3\cos\theta = 2\sin^2\theta$; $0 \le \theta \le 2\pi$

$\Rightarrow 3\cos\theta = 2(1-\cos^2\theta)$

$\Rightarrow 3\cos\theta = 2 - 2\cos^2\theta$

$\Rightarrow 2\cos^2\theta + 3\cos\theta - 2 = 0$

$\Rightarrow 2\cos^2\theta + 4\cos\theta - \cos\theta - 2 = 0$

$\Rightarrow 2\cos\theta(\cos\theta + 2) - 1(\cos\theta + 2) = 0$

$\Rightarrow (2\cos\theta - 1)(\cos\theta + 2) = 0$

$\Rightarrow \boxed{\cos\theta = \frac{1}{2}}$ | $\cos\theta = -2$ (Not possible) $-1 \le \cos\theta \le 1$

Ist: $\theta = \frac{\pi}{3}$

IV: $\theta = 2\pi - \frac{\pi}{3}$

$\theta = \frac{5\pi}{3}$

$\therefore \boxed{\theta = \frac{\pi}{3}, \frac{5\pi}{3}}$ Ans

Qn 20 If $\sin(\theta+\alpha) = a$ and $\sin(\theta+\beta) = b$

Show that $\cos 2(\alpha-\beta) - 4ab\cos(\alpha-\beta) = 1 - 2a^2 - 2b^2$

Solution Given $\sin(\theta+\alpha)=a$ and $\sin(\theta+\beta)=b$

$\boxed{\text{we know that } \cos\theta=\sqrt{1-\sin^2\theta}}$ (10th class)

$\therefore \cos(\theta+\alpha)=\sqrt{1-\sin^2(\theta+\alpha)} = \sqrt{1-a^2}$

and $\cos(\theta+\beta)=\sqrt{1-\sin^2(\theta+\beta)}=\sqrt{1-b^2}$

Now $\cos(\alpha-\beta)= \cos[\underset{A}{(\theta+\alpha)}-\underset{B}{(\theta+\beta)}]$

$= \cos(\theta+\alpha)\cos(\theta+\beta) + \sin(\theta+\alpha)\sin(\theta+\beta)$

$\cos(\alpha-\beta) = \sqrt{1-a^2}\sqrt{1-b^2} + ab$

$\cos(2(\alpha-\beta)) = 2\cos^2(\alpha-\beta)-1$ --- formula $\{\cos(2\theta) = 2\cos^2\theta-1\}$

$=2\left[\sqrt{1-a^2}\sqrt{1-b^2}+ab\right]^2-1$

$=2\left((1-a^2)(1-b^2)+a^2b^2+2ab\sqrt{1-a^2}\sqrt{1-b^2}\right)-1$

$=2\left(1-b^2-a^2+a^2b^2+a^2b^2+2ab\sqrt{1-a^2}\sqrt{1-b^2}\right)-1$

$\cos(2(\alpha-\beta))= 1+4a^2b^2-2a^2-2b^2+4ab\sqrt{1-a^2}\sqrt{1-b^2}$

Now Taking LHS

$\cos(2(\alpha-\beta)) -4ab\cos(\alpha-\beta)$

$= 1+\cancel{4a^2b^2}-2a^2-2b^2+\cancel{4ab\sqrt{1-a^2}\sqrt{1-b^2}} -4ab(\cancel{\sqrt{1-a^2}\sqrt{1-b^2}}+\cancel{ab})$

$= 1-2a^2-2b^2$ Ans

—x—