ULTIMATE MATHEMATICS

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(T-3)

TRIGONOMETRY. CLASS NO: 3 (T-3)

Q16 we have $3A = 2A + A$

$\Rightarrow \tan(3A) = \tan(2A + A)$

$\Rightarrow \tan(3A) = \dfrac{\tan(2A) + \tan A}{1 - \tan(2A)\tan A}$

$\Rightarrow \tan(3A)[1 - \tan(2A)\tan A] = \tan(2A) + \tan A$

$\Rightarrow \tan(3A) - \tan(3A)\tan(2A)\tan A = \tan(2A) + \tan A$

$\Rightarrow \tan(3A) - \tan(2A) - \tan A = \tan(3A)\tan(2A)\tan A$

Proved

Q17 we have $3A = 2A + A$

$\Rightarrow \cot(3A) = \cot(2A + A)$

$\Rightarrow \cot(3A) = \dfrac{\cot(2A)\cot A - 1}{\cot(2A) + \cot A}$

$\Rightarrow \cot(3A)\cot(2A) + \cot(3A)\cot A = \cot(2A)\cot A - 1$

$\Rightarrow 1 = \cot(2A)\cot A - \cot(3A)\cot(2A) - \cot(3A)\cot A$

Ans

Q N20 To prove $\tan(70^\circ) = \tan(20^\circ) + 2\tan(50^\circ)$

we have $70^\circ = 50^\circ + 20^\circ$

$\Rightarrow \tan(70^\circ) = \tan(50^\circ + 20^\circ)$

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$$\tan(70^\circ) = \frac{\tan(50) + \tan(20^\circ)}{1 - \tan(50)\tan(20^\circ)}$$

$$\Rightarrow \tan(70^\circ) - \tan(70)\tan(50)\tan(20) = \tan(50) + \tan(20^\circ)$$

$$\Rightarrow \tan(70^\circ) - \tan(90-20)\tan(50)\tan(20) = \tan(50) + \tan(20)$$

$$\Rightarrow \tan(70^\circ) - \cot(20)\tan(50)\tan(20) = \tan(50) + \tan(20)$$

$$\Rightarrow \tan(70^\circ) - \tan(50^\circ) = \tan(50) + \tan(20)$$

$$\Rightarrow \tan(70) = 2\tan(50) + \tan(20^\circ)$$ Ans

SET-3

(1) $\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cdot\cos\left(\frac{A-B}{2}\right)$

(2) $\sin A - \sin B = 2\cos\left(\frac{A+B}{2}\right)\cdot\sin\left(\frac{A-B}{2}\right)$

(3) $\cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right)\cdot\cos\left(\frac{A-B}{2}\right)$

(4) $\cos A - \cos B = -2\sin\left(\frac{A+B}{2}\right)\cdot\sin\left(\frac{A-B}{2}\right)$

Qus 1 Simplify $\frac{\sin(9A)-\sin(3A)}{\cos(3A)+\cos(9A)}$

$$= \frac{2\cos(6A)\cdot\sin(3A)}{2\cos(6A)\cdot\cos(-3A)} = \frac{\sin(3A)}{\cos(3A)} = \tan(3A)$$

Qn 2
Simplify

$$\frac{\sin(7A) + \sin(4A) + \sin(3A) + \sin(6A)}{\cos(6A) + \cos(3A) + \cos(7A) + \cos(4A)}$$

$$= \frac{(\sin(7A) + \sin(3A)) + (\sin(6A) + \sin(4A))}{(\cos(6A) + \cos(4A)) + (\cos(7A) + \cos(3A))}$$

$$= \frac{2\sin(5A)\cdot\cos(2A) + 2\sin(5A)\cdot\cos(A)}{2\cos(5A)\cdot\cos(A) + 2\cos(5A)\cos(2A)}$$

$$= \frac{2\sin(5A)(\cos(2A) + \cos A)}{2\cos(5A)(\cos A + \cos(2A))} = \tan(5A)$$ Ans

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Qn 3
Simplify

$$\frac{\cos(4x) + \cos(2x) + \cos(3x)}{\sin(2x) + \sin(3x) + \sin(4x)}$$

$$= \frac{(\cos(4x) + \cos(2x)) + \cos(3x)}{(\sin(4x) + \sin(2x)) + \sin(3x)}$$

$$= \frac{2\cos(3x)\cos(x) + \cos(3x)}{2\sin(3x)\cos(x) + \sin(3x)}$$

(T-3)

$$= \frac{\cos(3x)\,(2\cos x + 1)}{\sin(3x)\,(2\cos x + 1)} = \cot(3x)$$

Q4 $(\cos\alpha - \cos\beta)^2 + (\sin\alpha - \sin\beta)^2$

Simplify

$$= \left(-2\sin\left(\frac{\alpha+\beta}{2}\right)\cdot\sin\left(\frac{\alpha-\beta}{2}\right)\right)^2 + \left(2\cos\left(\frac{\alpha+\beta}{2}\right)\cdot\sin\left(\frac{\alpha-\beta}{2}\right)\right)^2$$

$(abc)^2 = a^2b^2c^2$

$$= 4\sin^2\left(\frac{\alpha+\beta}{2}\right)\cdot\sin^2\left(\frac{\alpha-\beta}{2}\right) + 4\cos^2\left(\frac{\alpha+\beta}{2}\right)\cdot\sin^2\left(\frac{\alpha-\beta}{2}\right)$$

$$= 4\sin^2\left(\frac{\alpha-\beta}{2}\right)\left[\sin^2\left(\frac{\alpha+\beta}{2}\right) + \cos^2\left(\frac{\alpha+\beta}{2}\right)\right]$$

$$= 4\sin^2\left(\frac{\alpha-\beta}{2}\right)\times 1 = 4\sin^2\left(\frac{\alpha-\beta}{2}\right)$$ Ans.

Q5 Show $\sin\alpha + \sin\left(\alpha+\frac{2\pi}{3}\right) + \sin\left(\alpha+\frac{4\pi}{3}\right) = 0$

LHS $\sin\alpha + \sin(\alpha+120^\circ) + \sin(\alpha+240^\circ)$

$$= \left(\underset{A}{\sin(\alpha+240^\circ)} + \underset{B}{\sin\alpha}\right) + \sin(\alpha+120^\circ)$$

$$= 2\sin(\alpha+120^\circ)\cdot\cos(120^\circ) + \sin(\alpha+120^\circ)$$

$$= \sin(\alpha+120^\circ)\left[2\cos(120^\circ) + 1\right]$$

$120^\circ \to 180-60^\circ$

$$= \sin(\alpha+120^\circ)\left[-2\times\frac{1}{2} + 1\right]$$

$$= \sin(\alpha+120^\circ)\times 0 = 0$$ Ans