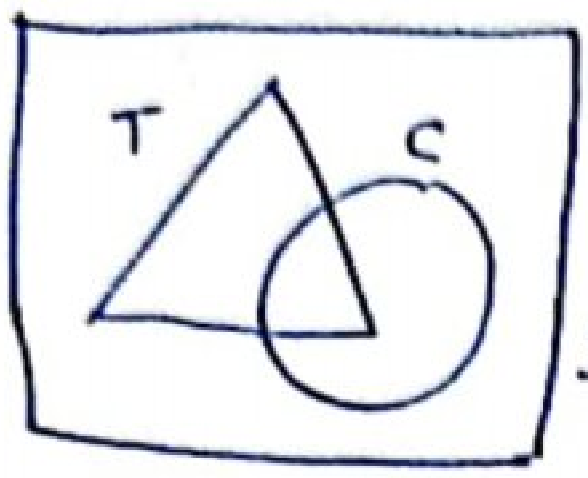
(ANSWER KEY) Solutions BIG TEST

SETS

Qns: 1 →



Clearly $S \cup T \cup C = S$

$\therefore$ $\boxed{\text{(C) part}}$ Ans

Qns: 2 → let $A \to$ set of people who watch 1st channel
$B \to$ set of people who watch another channel

let total No. of persons = 100

$\therefore \; n(U) = 100$

Given $n(A) = 63\%$ of $100 = 63$

$n(B) = 76\%$ of $100 = 76$

$n(A \cap B) = x\%$ of $100 = x$

we know that $\quad n(A \cup B) \le n(U)$

$\Rightarrow n(A) + n(B) - n(A \cap B) \le n(U)$

$\Rightarrow 63 + 76 - x \le 100$

$\Rightarrow -x \le 100 - 139$

$\Rightarrow -x \le -39$

$\Rightarrow x \ge 39 \quad --- (1)$

also $n(A \cap B) \le n(A)$ and $n(A \cap B) \le n(B)$

$\Rightarrow x \le 63$ and $x \le 76$

$\Rightarrow$ consider $x \le 63 \quad -- (2)$

from (1) & (2)

$\boxed{39 \le x \le 63}$ Ans

Qns: 3 → $A_1 = \{-----\}$

$A_2 = \{-----\}$

$\vdots$

$A_{30} = \{----\}$

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Given $S = A_1 \cup A_2 \cup A_3 \cdots A_{30}$

Maximum no. of elements in set S = $5 \times 30 = 150$

But each element of S belongs to exactly 10 of the A_i's.

$\therefore n(S) = \frac{150}{10} = 15$ --- (i)

Now $B_1 = \{ - - - \}$

$B_2 = \{ - - - \}$

$\vdots$

$B_n = \{ - - - \}$

Given $S = B_1 \cup B_2 \cup B_3 \cdots B_n$

Maximum number of elements is set $S = 3 \times n = 3n$

But each element of S belongs to exactly 9 of the B_j's

$\therefore n(S) = \frac{3n}{9} = \frac{n}{3}$ -- (2)

from (1) & (2)

$\frac{n}{3} = 15 \Rightarrow \boxed{n = 45}$ Ans..

Q.No. 4 → Let A and B are two sets

then given $n(A) = m$ and $n(B) = n$

No. of subsets of A set $= 2^m$

No. of subsets of B set $= 2^n$

Given that $2^m - 2^n = 112$

$\Rightarrow 2^m - 2^n = 128 - 16$

$\Rightarrow 2^m - 2^n = 2^7 - 2^4$

Comparing we get $\boxed{m = 7 \ \& \ n = 4}$ Ans

Ans. 5 → given $A \cap X = B \cap X = \phi$

and $A \cup X = B \cup X$

To prove $A = B$

we have $A \cup X = B \cup X$

$\Rightarrow A \cap (A \cup X) = A \cap (B \cup X)$

$\Rightarrow (A \cap A) \cup (A \cap X) = (A \cap B) \cup (A \cap X)$ --- {distributive prop.}

$\Rightarrow A \cup (A \cap X) = (A \cap B) \cup (A \cap X)$

$\Rightarrow A \cup \phi = (A \cap B) \cup \phi$ --- {given $A \cap X = \phi$}

$\Rightarrow A = (A \cap B)$ --- (1)

again consider $A \cup X = B \cup X$

$\Rightarrow B \cap (A \cup X) = B \cap (B \cup X)$

$\Rightarrow (B \cap A) \cup (B \cap X) = (B \cap B) \cup (B \cap X)$

$\Rightarrow (A \cap B) \cup \phi = B \cup \phi$ -- {$B \cap X = \phi$ given}

$\Rightarrow A \cap B = B$ -- (2)

from (1) & (2) $\boxed{A = B}$ PROVED

Ans 6 → A → let set of families who used newspaper A

B → " " " " " " B

C → " " " " " " C

given $n(U) = 10{,}000$

$n(A) = 40\%$ of $10000 = 4000 = a+b+e+d$

$n(B) = 20\%$ of $10000 = 2000 = b+c+e+f$

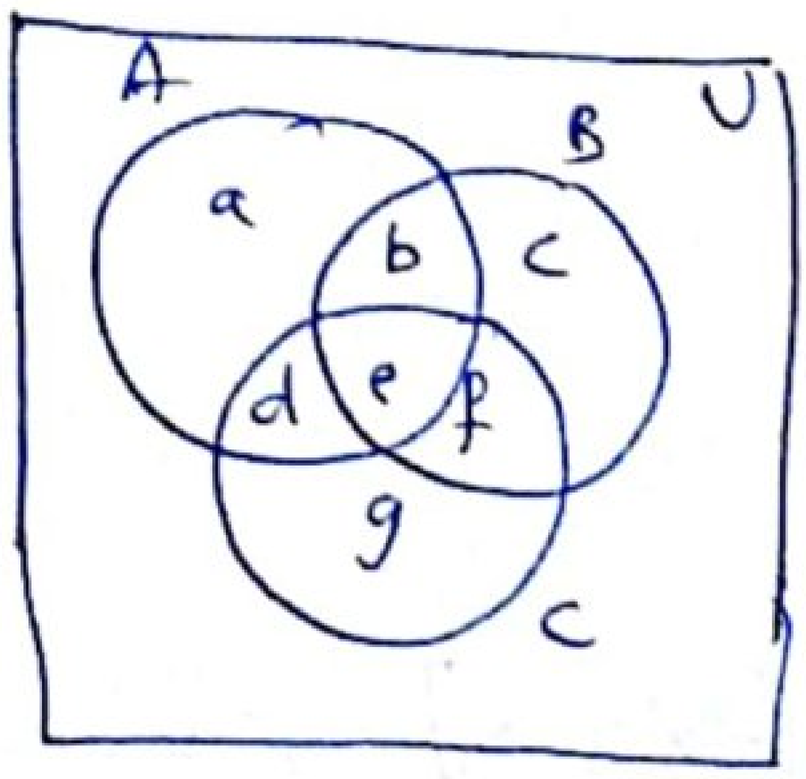
$n(C) = 10\%$ of $10000 = 1000 = d+e+f+g$

$n(A \cap B) = 5\%$ of $1000 = 500 = b+e$

$n(B \cap C) = 3\%$ of $10000 = 300 = e+f$

$n(A \cap C) = 4\%$ of $10000 = 400 = d+e$

$n(A \cap B \cap C) = 2\% = 200 = e$



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$\Rightarrow e = 200,\ d = 200,\ f = 100,\ b = 300,\ g = 500$
$c = 1400,\ a = 3300$

(i) No. of families which buy newspaper A only $= a = 3300$

(2) No. of families which buy none of A, B, C
$= 10000 - (a+b+c+d+e+f+g)$
$= 10000 - 6000$
$= 4000$

$\therefore$ $\boxed{\text{(i) } 3300 \quad \text{(ii) } 4000}$ Ans

—^—

CHAPTER : RELATION FUNCTION.

Ans. 1 → $A = \{2,4,6,9\}$ $B = \{4,6,18,27,54\}$; $a \in A,\ b \in B$
a is a factor of b & $a < b$
Required set $= \{(2,4)\ (2,6)\ (2,18)\ (2,54), (6,18), (6,54)\ (9,18)\ (9,27)\ (9,54)\}$ Ans

Ans 2 → Given: $g = \{(1,1)\ (2,3), (3,5), (4,7)\}$
$g(x) = \alpha x + \beta$

$(1,1) \in g \Rightarrow x = 1$ and $g(x) = 1 \Rightarrow 1 = \alpha + \beta$ --- (1)
$(2,3) \in g \Rightarrow x = 2,\ g(x) = 3 \Rightarrow 3 = 2\alpha + \beta$ --- (2)
Solving (1) & (2) we get $\alpha = 2,\ \beta = -1$
Now $\alpha^2 + \beta^2 = 4 + 1 = 5$
$\therefore \boxed{\alpha^2 + \beta^2 = 5}$ Ans

Ans 3 → $f(x) = x^3 - \frac{1}{x^3}$
$f\left(\frac{1}{x}\right) = \left(\frac{1}{x}\right)^3 - \frac{1}{\left(\frac{1}{x}\right)^3} = \frac{1}{x^3} - x^3$

Now $f(x) + f\left(\frac{1}{x}\right) = x^3 - \frac{1}{x^3} + \frac{1}{x^3} - x^3 = 0$

$\therefore \boxed{f(x) + f\left(\frac{1}{x}\right) = 0}$ Ans..

Qns 4 → $f(x) = \dfrac{1}{\sqrt{[x]^2 - [x] - 6}}$

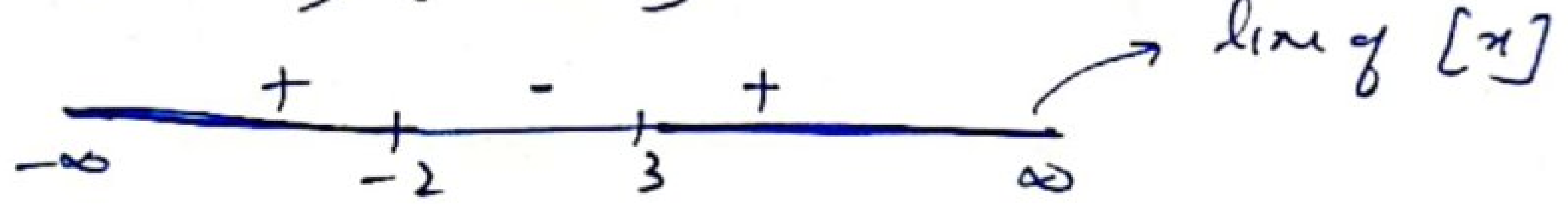
$f(x)$ exists for all value of x such that

$[x]^2 - [x] - 6 > 0$

$\Rightarrow [x]^2 - 3[x] + 2[x] - 6 > 0$

$\Rightarrow [x]([x]-3) + 2([x]-3) > 0$

$\Rightarrow ([x]-3)([x]+2) > 0$



$\Rightarrow [x] < -2$ (or) $[x] > 3$

$\Rightarrow [x] = -3, -4, \ldots -\infty$ (or) $[x] = 4, 5, 6, \ldots \infty$

$\Rightarrow x < -2$ (or) $x \geq 4$

domain $\boxed{x \in (-\infty, -2) \cup [4, \infty)}$ Ans

Qns: 5 → (Note: there is a Misprint in the Qns paper)

$f(x) = |x+2| + |x-2|$; $-3 \leq x \leq 3$

$-3 \leftarrow (-2) \leftrightarrow (2) \rightarrow 3$

Redefining the function

$$f(x) = \begin{cases} -(x+2) - (x-2) \; ; & -3 \leq x < -2 \\ (x+2) - (x-2) \; ; & -2 \leq x < 2 \\ (x+2) + (x-2) \; ; & 2 \leq x \leq 3 \end{cases}$$

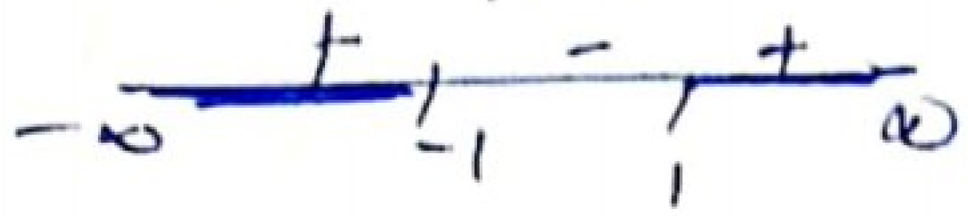
$$\therefore f(x) = \begin{cases} -2x & ; \quad -3 \le x < -2 \\ 4 & : \quad -2 \le x < 2 \\ 2x & ; \quad 2 \le x \le 3 \end{cases}$$ Ans

Qn: 6 ⇒ $f(x) = \sqrt{4-x} + \dfrac{1}{\sqrt{x^2-1}}$

$f(x)$ exists for all value of x such that

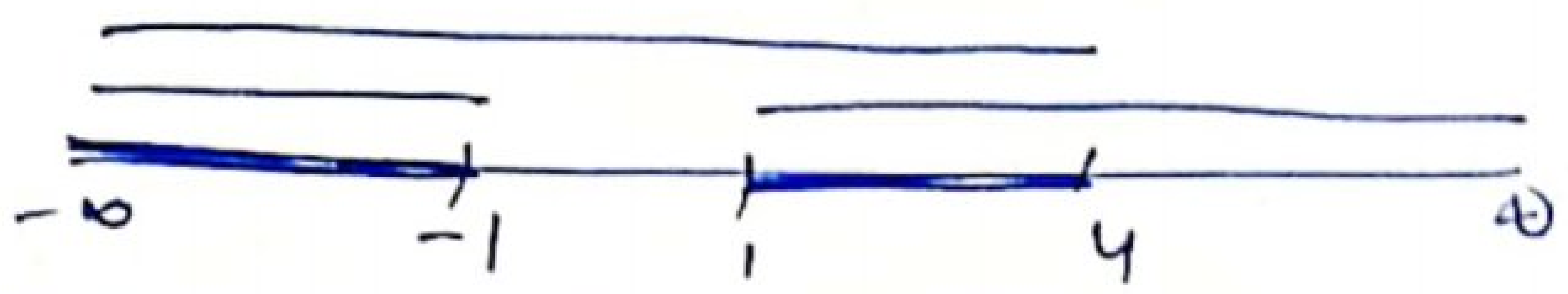
$4 - x \geq 0$ and $x^2 - 1 > 0$

$x \leq 4$ and $(x+1)(x-1) > 0$



$\Rightarrow x \in (-\infty, 4]$ and $x \in (-\infty, -1) \cup (1, \infty)$

taking common



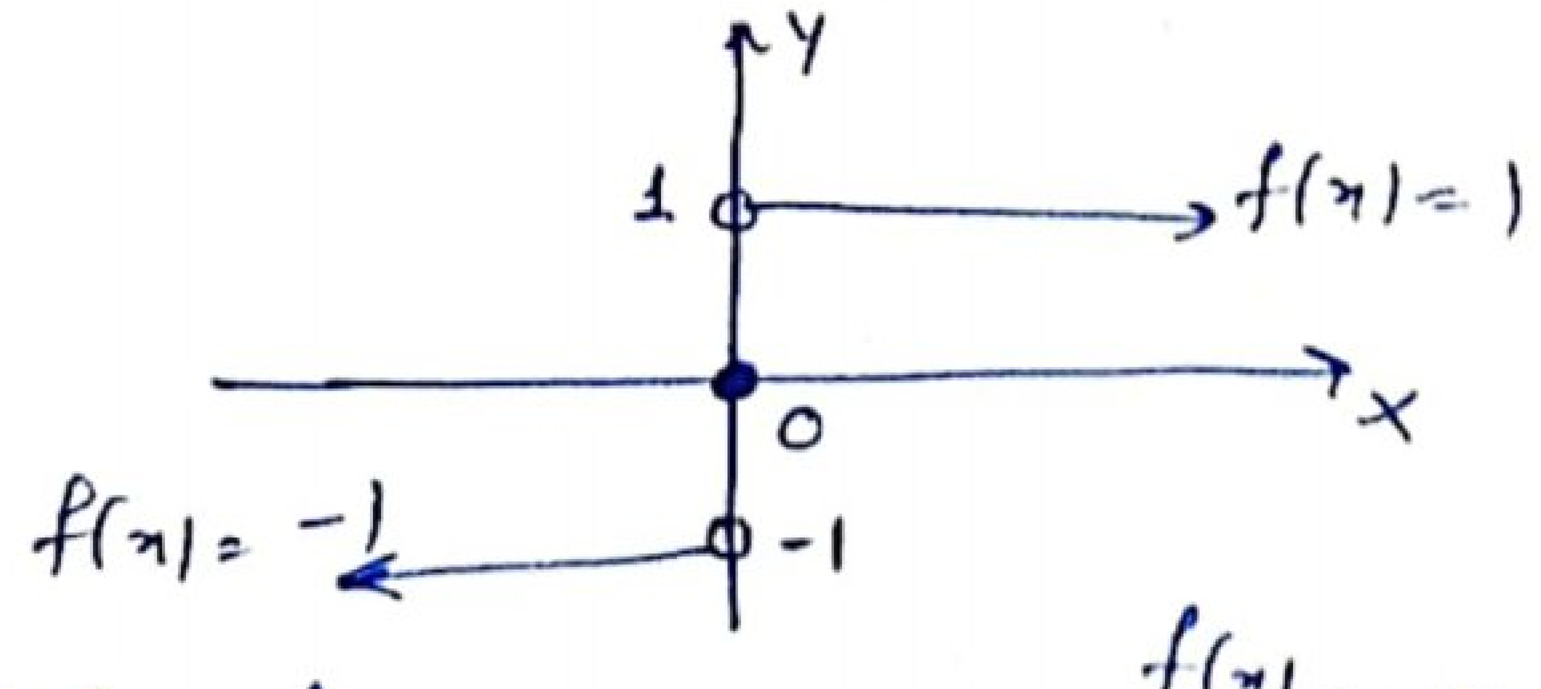
$\therefore$ $\boxed{\text{Domain } x \in (-\infty, -1) \cup (1, 4]}$ Ans

Qn: 7 ⇒ (1) Signum function

$$f(x) = \begin{cases} 1 & : \quad x > 0 \\ -1 & : \quad x < 0 \\ 0 & : \quad x = 0 \end{cases}$$

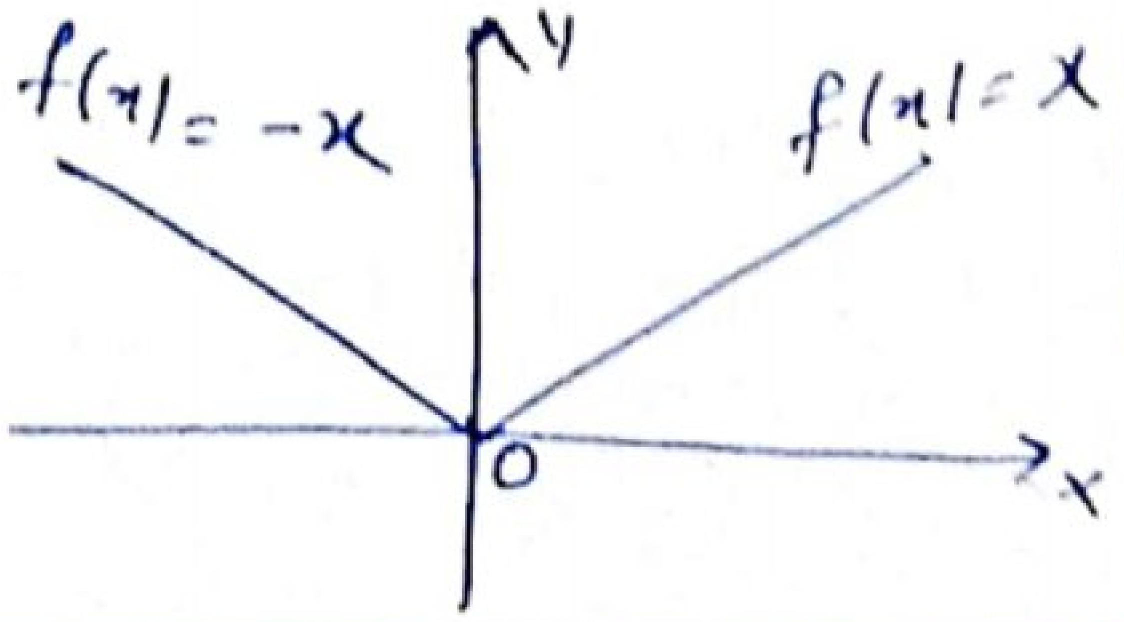
Domain = R
Range = $\{-1, 0, 1\}$

graph



(2) Modulus function

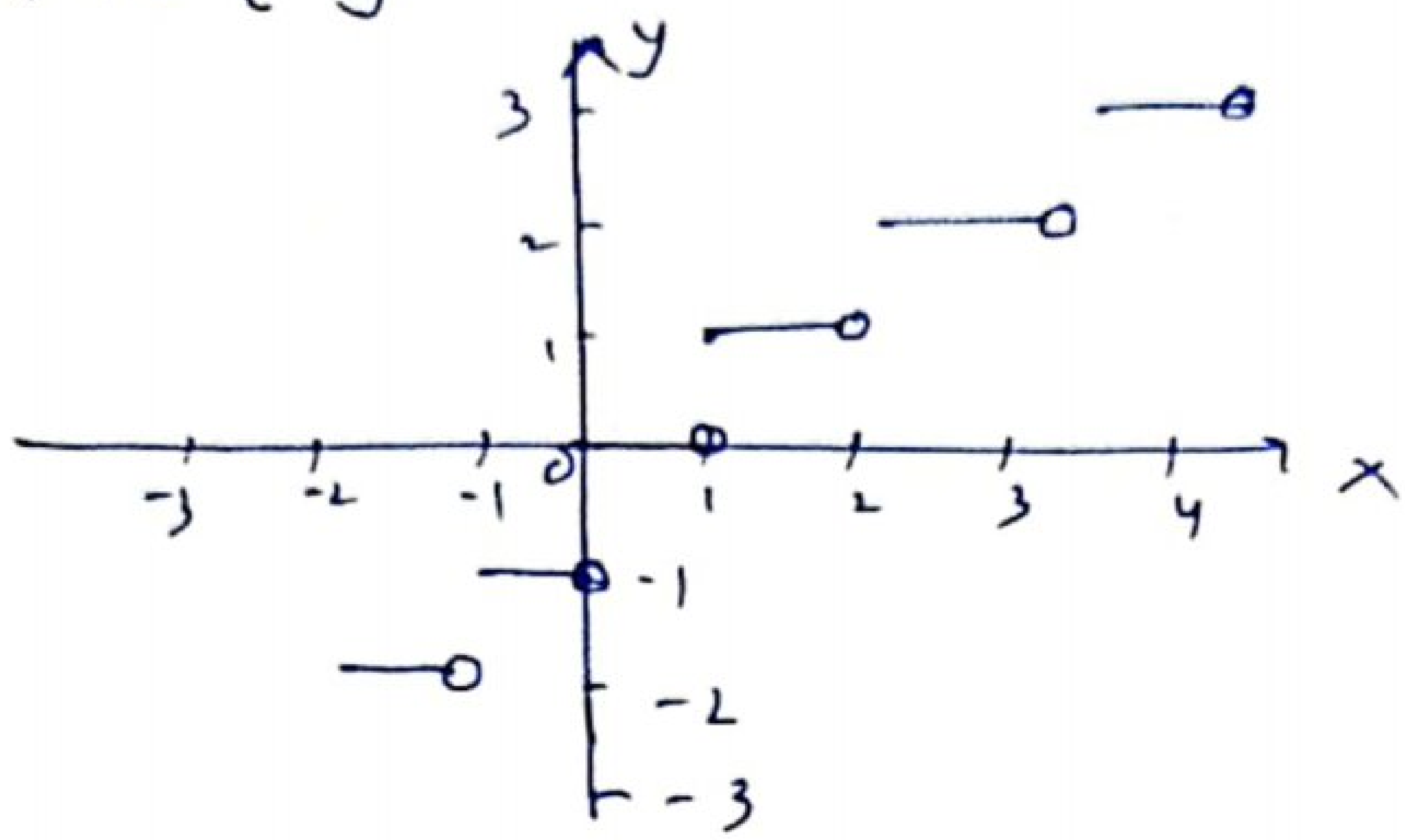
$$f(x) = |x| = \begin{cases} x & : \quad x \geq 0 \\ -x & : \quad x < 0 \end{cases}$$



Domain R
Range = $[0, \infty)$

(3) Greatest Integer function

$f(x) = [x]$



Domain = R
Range = Z

Ans 8 → $R = \{(a,b) : a, b, \in Q \text{ and } a-b \in Z\}$

(i) for each $a \in Q$

$a - a = 0$ which is an Integer

$\Rightarrow (a,a) \in R$

(2) $(a,b) \in R$

$\Rightarrow a-b$ is an Integer

$\Rightarrow a - b = \lambda$ --- $(\lambda \in Z)$

$\Rightarrow (b-a) = -\lambda$ which is also an Integer

$\Rightarrow (b,a) \in R$

(3) $(a,b) \in R$ and $(b,c) \in R$

$\Rightarrow a-b = \lambda$ and $b-c = k$ --- $(\lambda \in Z; k \in Z)$

$\Rightarrow a-c = (a-b) + (b-c)$

$\Rightarrow a-c = \lambda + k$ = which is also an Integer

$\Rightarrow (a,c) \in R$

PROVED

Ans 9 → $f(x) = \frac{x^2}{1+x^2}$

Clearly $f(x)$ exists for all values of x such that

$x \in R$

$\therefore \boxed{\text{Domain} = R}$

Range let $f(x) = y$

$\Rightarrow y = \dfrac{x^2}{1+x^2}$

$\Rightarrow x^2 y + y = x^2$

$\Rightarrow x^2(y-1) = -y$

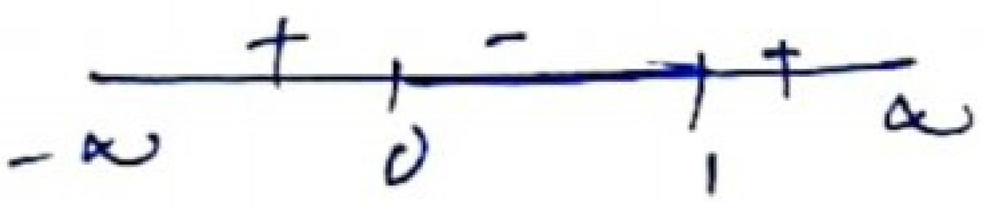
$\Rightarrow x^2 = \dfrac{-y}{y-1}$

$\Rightarrow x = \pm\sqrt{\dfrac{-y}{y-1}}$

x exists for all value of y such that

$\dfrac{-y}{y-1} \geq 0$ and $y-1 \neq 0$

$\Rightarrow \dfrac{y}{y-1} \leq 0$ and $y \neq 1$



and $y \neq 1$

$\therefore y \in [0, 1)$ $\{\because y \neq 1\}$

$\boxed{\text{Range} = [0, 1)}$ Ans

— x —

CHAPTER : TRIGONOMETRY

Ans. 1 → degree measure $= \left(\dfrac{180}{\pi} \times 4\right)^\circ$

$= \left(\dfrac{180}{22} \times 7 \times 4\right)^\circ$

$= \left(\dfrac{2520}{11}\right)^\circ$

$= \left(229\dfrac{1}{11}\right)^\circ$

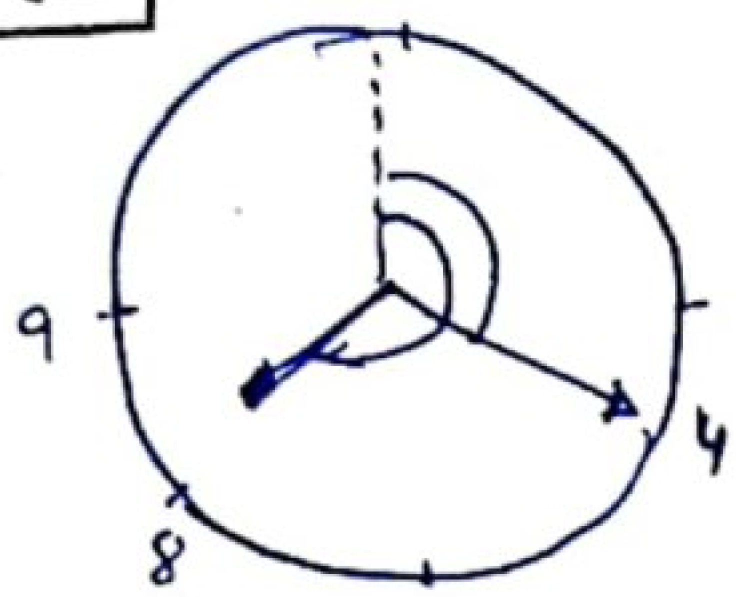
$= 229^\circ \left(\frac{1}{11} \times 60\right)'$

$= 229^\circ \left(5 \frac{5}{11}\right)'$

$= 229^\circ 5' \left(\frac{5}{11} \times 60\right)''$

$= 229^\circ 5' 27''$ (Approx) **Ans**

Ans. 2 →



Time: $8:20$ hrs

$8:20 \text{ hrs} = \left(8 \frac{20}{60}\right) \text{hr} = \left(\frac{25}{3}\right) \text{hr}$

Minute hand	Hour hand
$60 \text{ Min} \rightarrow 360^\circ$	$12 \text{ hr} \rightarrow 360^\circ$
$1 \text{ Min} \rightarrow \frac{360}{60} = 6^\circ$	$1 \text{ hr} = \frac{360}{12} = 30^\circ$
$20 \text{ Min} \rightarrow 20 \times 6 = 120^\circ$	$\frac{25}{3} \text{ hr} = \frac{25}{3} \times 30 = 250^\circ$

Required angle $= 250^\circ - 120^\circ = \boxed{130^\circ}$ **Ans**

Ans 3 → we have $3x = 2x + x$

$\Rightarrow \cot(3x) = \cot(2x + x)$

$\Rightarrow \cot(3x) = \dfrac{\cot(2x)\cot x - 1}{\cot(2x) + \cot x}$

$\Rightarrow \cot(3x)\cot(2x) + \cot(3x)\cot x = \cot(2x)\cot x - 1$

$\Rightarrow \cot(2x)\cot x - \cot(2x)\cot(3x) - \cot(3x)\cot x = 1$ **Ans**

Ans 4 → we have $\operatorname{cosec} x = -2$

$\Rightarrow \sin x = -1/2$

III quadrant	IVth quadrant
$\sin x = \sin(\pi + \pi/6)$	$\sin x = \sin(2\pi - \pi/6)$
$\Rightarrow x = 7\pi/6$	$x = 11\pi/6$

$\therefore \boxed{\frac{7\pi}{6}, \frac{11\pi}{6}}$ **Ans**

Qn: 5 → RHS $\frac{\tan(8\theta)}{\tan(2\theta)}$; LHS $\frac{\sec(8\theta)-1}{\sec(4\theta)-1}$

Taking LHS $\frac{\sec(8\theta)-1}{\sec(4\theta)-1}$

$$= \frac{\frac{1}{\cos(8\theta)} - 1}{\frac{1}{\cos(4\theta)} - 1}$$

$$= \frac{1-\cos(8\theta)}{1-\cos(4\theta)} \cdot \frac{\cos(4\theta)}{\cos(8\theta)}$$

$$= \frac{2\sin^2(4\theta)\cdot\cos(4\theta)}{2\sin^2(2\theta)\ \cos(8\theta)}$$

$$= \frac{[2\sin(4\theta)\cos(4\theta)]\cdot\sin(4\theta)}{2\sin^2(2\theta)\cos(8\theta)}$$

$$= \frac{\sin(8\theta)\cdot \cancel{2}\cancel{\sin(2\theta)}\cos(2\theta)}{\cancel{2}\sin^{\cancel{2}}(2\theta)\cos(8\theta)} \quad \text{---- } \left\{ 2\sin\theta\cos\theta = \sin(2\theta) \right\}$$

$$= \frac{\tan(8\theta)}{\tan(2\theta)}$$

Qn: 6 → given $m\sin\theta = n\sin(\theta+2\alpha)$

$$\Rightarrow \frac{m}{n} = \frac{\sin(\theta+2\alpha)}{\sin\theta}$$

Apply Componendo & dividendo

$$\Rightarrow \frac{m+n}{m-n} = \frac{\sin(\theta+2\alpha)+\sin\theta}{\sin(\theta+2\alpha)-\sin\theta}$$

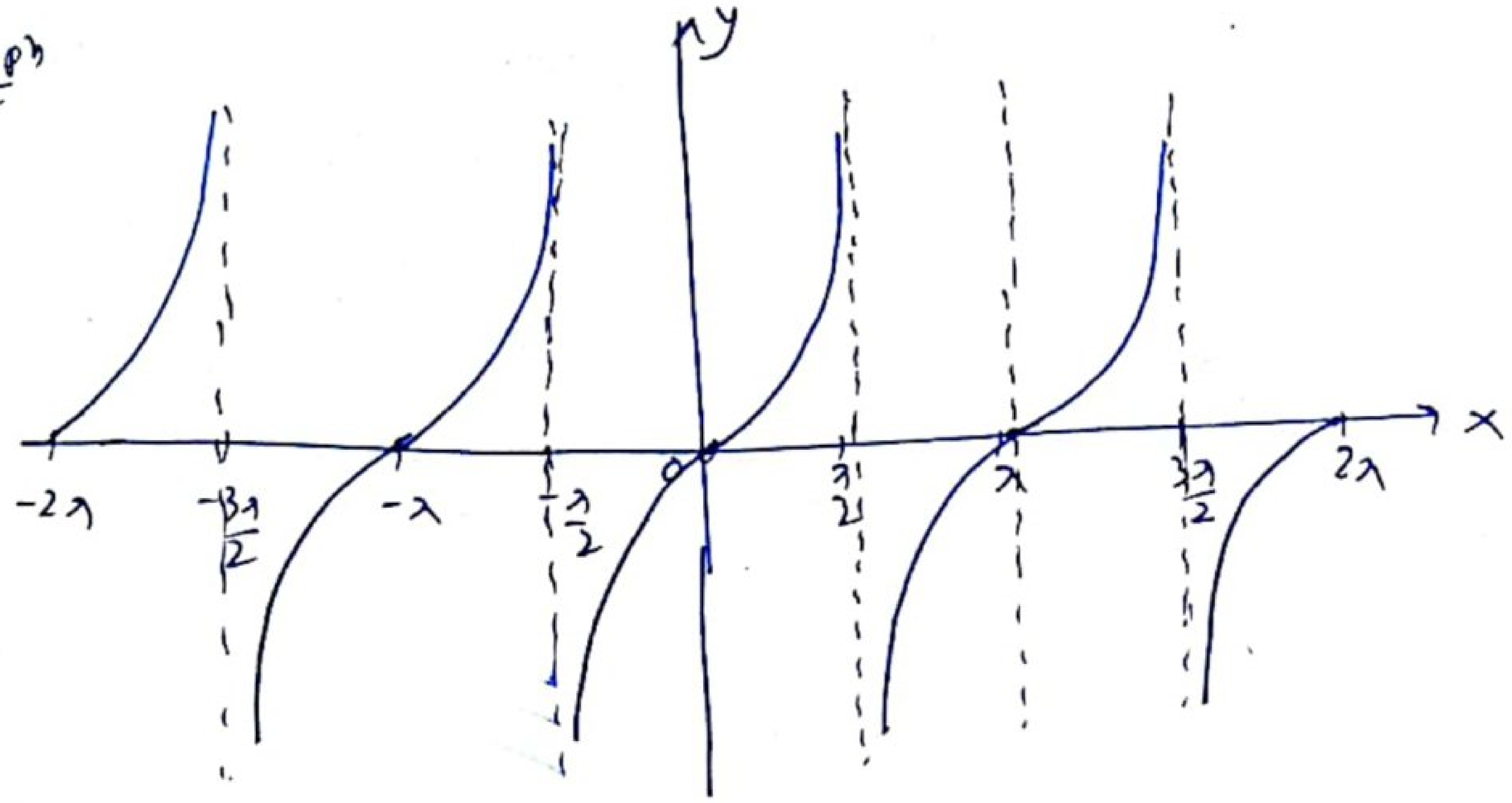
$$\Rightarrow \frac{m+n}{m-n} = \frac{\cancel{2}\sin(\theta+\alpha)\cdot\cos(\alpha)}{\cancel{2}\cos(\theta+\alpha)\cdot\sin(\alpha)} \quad \text{---} \left\{ \begin{array}{l}\sin A+\sin B \ \& \\ \sin A - \sin B \text{ formulas}\end{array}\right.$$

$$\Rightarrow \frac{m+n}{m-n} = \tan(\theta+\alpha)\cdot\cot\alpha \qquad \text{PROVED}$$

$\boxed{\text{Ans. 7} \rightarrow}$ (1) $y = \tan x$

Domain $= R - \{(2n+1)\frac{\pi}{2}, n \in Z\}$ & Range $= R$

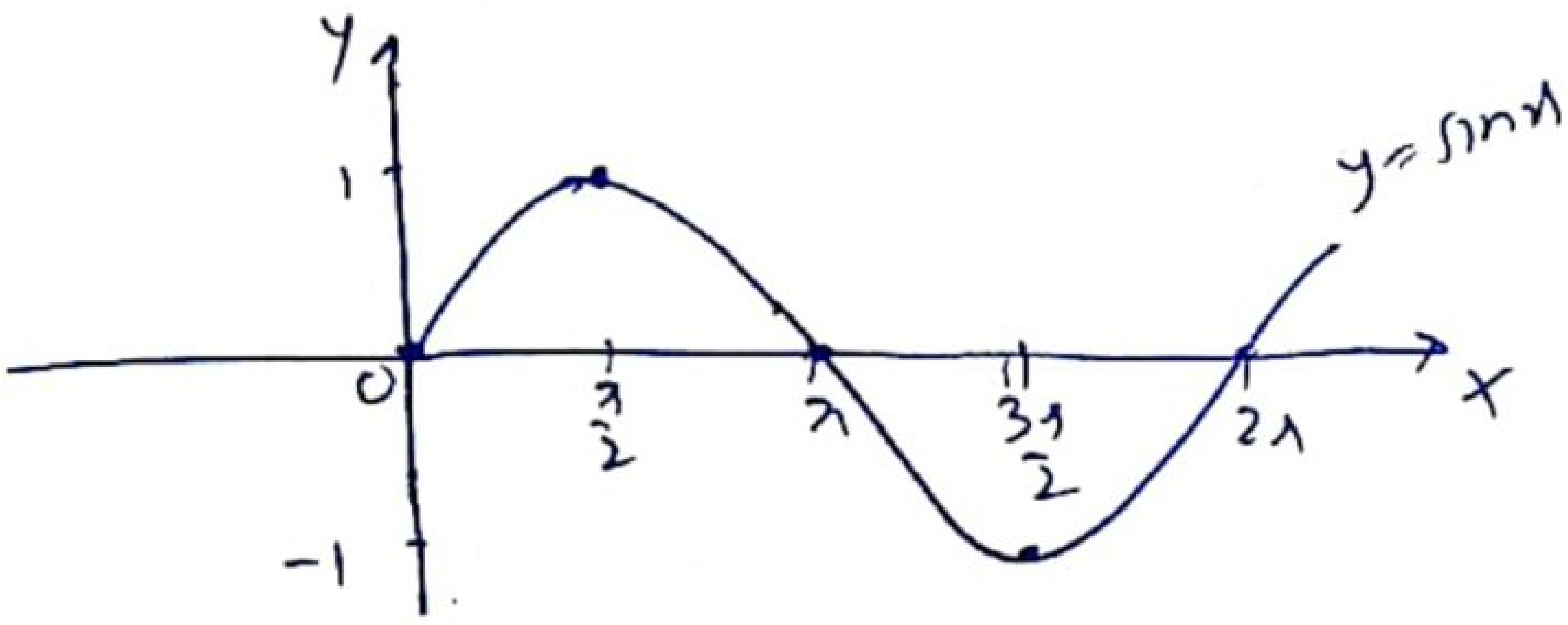
Graph



(2) $y = \sin x$

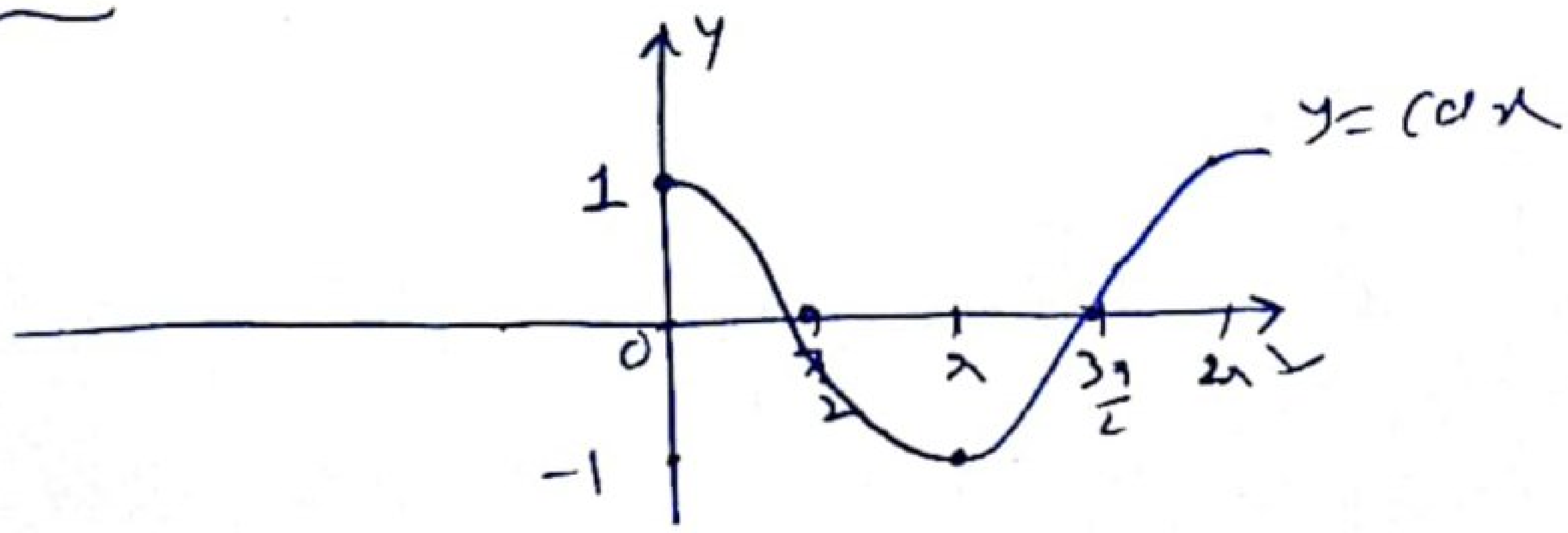
domain $= R$ Range $= [-1, 1]$

Graph



(3) $y = \cos x$

domain $= R$ Range $= [-1, 1]$



Qn. 8 → L.H.S $\cos(2x)\cos\left(\frac{x}{2}\right) - \cos(3x)\cos\left(\frac{9x}{2}\right)$

$$= \frac{1}{2}\left[2\cos(2x)\cos\left(\frac{x}{2}\right) - 2\cos(3x)\cos\left(\frac{9x}{2}\right)\right]$$

$$= \frac{1}{2}\left[\cos\left(2x+\frac{x}{2}\right) + \cos\left(2x-\frac{x}{2}\right) - \left\{\cos\left(3x+\frac{9x}{2}\right) + \cos\left(3x-\frac{9x}{2}\right)\right\}\right]$$

$$= \frac{1}{2}\left[\cos\left(\frac{5x}{2}\right) + \cancel{\cos\left(\frac{3x}{2}\right)} - \cos\left(\frac{15x}{2}\right) - \cancel{\cos\left(\frac{3x}{2}\right)}\right]$$

$$= \frac{1}{2}\left[\cos\left(\frac{5x}{2}\right) - \cos\left(\frac{15x}{2}\right)\right] \quad \text{--- } \left\{\cos A - \cos B = -2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)\right\}$$

$$= \cancel{\frac{1}{2}}\left[-\cancel{2}\sin\left(\frac{\frac{5x}{2}+\frac{15x}{2}}{2}\right)\cdot\sin\left(\frac{\frac{5x}{2}-\frac{15x}{2}}{2}\right)\right]$$

$$= -\sin(5x)\cdot\sin\left(-\frac{5x}{2}\right) \quad \text{--- } \{\because \sin(-\theta) = -\sin\theta\}$$

$$= \sin(5x)\sin\left(\frac{5x}{2}\right) = \text{RHS} \quad \text{Ans}$$

Qns. 9 → Given $\sin x = -\frac{1}{4}$; $x \to 3^{rd}$ quadrant

we have $\sin^2 x + \cos^2 x = 1$

$$\Rightarrow \cos^2 x = 1 - \sin^2 x$$

$$\Rightarrow \cos^2 x = 1 - \frac{1}{16} = \frac{15}{16}$$

$$\Rightarrow \cos x = \pm\frac{\sqrt{15}}{4}$$

$$\Rightarrow \boxed{\cos x = -\frac{\sqrt{15}}{4}} \quad \text{--- } \{x \to 3^{rd} \text{ quadrant}\}$$

we have $1 - \cos x = 2\sin^2(x/2)$

$$\Rightarrow 1 + \frac{\sqrt{15}}{4} = 2\sin^2(x/2)$$

$$\Rightarrow \frac{4+\sqrt{15}}{8} = \sin^2(x/2)$$

$$\Rightarrow \sin(x/2) = \pm\sqrt{\frac{4+\sqrt{15}}{8}}$$

$1 + \cos x = 2\cos^2(x/2)$

$$1 - \frac{\sqrt{15}}{4} = 2\cos^2(x/2)$$

$$\frac{4-\sqrt{15}}{8} = \cos^2(x/2)$$

$$\cos(x/2) = \pm\sqrt{\frac{4-\sqrt{15}}{8}}$$

$x \to 3^{rd}$ quadrant

$\because \frac{x}{2} \to 2^{nd}$ quadrant

$\therefore \sin(\frac{x}{2}) = \sqrt{\frac{4+\sqrt{15}}{8}}$ and $\cos(\frac{x}{2}) = -\sqrt{\frac{4-\sqrt{15}}{8}}$

Now $\tan(x/2) = \frac{\sin(x/2)}{\cos(x/2)} = -\sqrt{\frac{4+\sqrt{15}}{4-\sqrt{15}}}$

$= -\sqrt{\frac{(4+\sqrt{15})(4+\sqrt{15})}{16-15}}$

$= -\sqrt{(4+\sqrt{15})^2}$

$\tan(x/2) = -(4+\sqrt{15})$ Ans

$\therefore \sin(x/2) = \sqrt{\frac{4+\sqrt{15}}{8}}$; $\cos(\frac{x}{2}) = -\sqrt{\frac{4-\sqrt{15}}{8}}$; $\tan\frac{x}{2} = -(4+\sqrt{15})$

Ans

Qn. 10 →

LHS $\cos^2 x + \cos^2(x+\pi/3) + \cos^2(x-\pi/3)$

$= \cos^2 x + \cos^2(x+60°) + \cos^2(x-60°)$

$= \frac{1+\cos(2x)}{2} + \frac{1+\cos(2x+120)}{2} + \frac{1+\cos(2x-120)}{2}$ ... $\left\{\cos^2\theta = \frac{1+\cos 2\theta}{2}\right\}$

$= \frac{1}{2}\left[3 + \cos(2x) + \cos(\overset{A}{2x+120°}) + \cos(\overset{B}{2x-120})\right]$

$= \frac{1}{2}\left[3 + \cos(2x) + 2\cos(2x)\cdot\cos(120°)\right]$

... $\left\{\cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right)\cdot\cos\left(\frac{A-B}{2}\right)\right\}$

$= \frac{1}{2}\left(3 + \cos(2x) + 2\cos(2x)\cos(180-60)\right)$

$= \frac{1}{2}\left[3 + \cos(2x) + \cancel{2}\cos(2x)\left(-\frac{1}{\cancel{2}}\right)\right]$

$= \frac{1}{2}\left[3 + \cancel{\cos(2x)} - \cancel{\cos(2x)}\right]$

$= 3/2 =$ RHS PROVED

— x —

CHAPTER : COMPLEX NUMBERS

Ans: 1 → given $(x+iy)^{1/3} = a+ib$

Cubing both sides

$\Rightarrow x+iy = (a+ib)^3$

$\Rightarrow x+iy = a^3 + i^3b^3 + 3a^2ib + 3ai^2b^2$

$\Rightarrow x+iy = a^3 - ib^3 + 3a^2ib - 3ab^2$

$\Rightarrow x+iy = (a^3-3ab^2) + i(3a^2b-b^3)$

equating real and imaginary parts

$x = a^3-3ab^2$ & $y = 3a^2b-b^3$

Taking LHS $\frac{x}{a} - \frac{y}{b}$

put value of x & y

$= \frac{a^3-3ab^2}{a} - \frac{3a^2b-b^3}{b}$

$= a^2-3b^2 - (3a^2-b^2)$

$= a^2-3b^2-3a^2+b^2$

$= -2a^2-2b^2$

$= -2(a^2+b^2) =$ RHS **Ans**

Ans: 2 → given $z^2 = \bar{z}$

let $z = x+iy$

$\Rightarrow (x+iy)^2 = x-iy$

$\Rightarrow x^2+i^2y^2+2ixy = x-iy$

$\Rightarrow (x^2-y^2)+2ixy = x-iy$

equating Real & Imaginary part

$x^2 - y^2 = x$ and $2xy = -y$

(don't cancel y)

$\Rightarrow 2xy + y = 0$

$\Rightarrow y(2x+1) = 0$

$\Rightarrow y = 0$ or $x = -1/2$

<u>Case I</u> when $y = 0$

then $x^2 - 0 = x$

$\Rightarrow x^2 - x = 0$

$\Rightarrow x(x-1) = 0$

$\Rightarrow x = 0 \; ; \; x = 1$

$\therefore z = 0 + 0i$ & $z = 1 + 0i$

<u>Case II</u> $x = -1/2$

then $\frac{1}{4} - y^2 = \frac{-1}{2}$

$\Rightarrow y^2 = \frac{1}{4} + \frac{1}{2} = \frac{3}{4}$

$\Rightarrow y = \pm\frac{\sqrt{3}}{2}$

$\therefore z = \frac{-1}{2} + \frac{\sqrt{3}}{2} i \; ; \; z = \frac{-1}{2} - \frac{\sqrt{3}}{2} i$

Hence $z = 0+0i, \; 1+0i; \; \frac{-1}{2} + \frac{\sqrt{3}}{2} i, \; -\frac{1}{2} - \frac{\sqrt{3}}{2} i$ <u>Ans.</u>

$\boxed{\text{Ans:-3}}$ Given $|\beta| = 1$

To find: $\left| \frac{\beta - \alpha}{1 - \bar{\alpha}\beta} \right| = ?$

By property $|z|^2 = z\bar{z}$

we have $\left| \frac{\beta - \alpha}{1 - \bar{\alpha}\beta} \right|^2 = \left(\frac{\beta - \alpha}{1 - \bar{\alpha}\beta} \right) \left(\overline{\frac{\beta - \alpha}{1 - \bar{\alpha}\beta}} \right)$

$$\Rightarrow \left(\frac{\beta-\alpha}{1-\bar{\alpha}\beta}\right)\left(\frac{\bar{\beta}-\bar{\alpha}}{1-\alpha\bar{\beta}}\right)$$

$$= \frac{\beta\bar{\beta}-\beta\bar{\alpha}-\alpha\bar{\beta}+\alpha\bar{\alpha}}{1-\alpha\bar{\beta}-\bar{\alpha}\beta+\alpha\bar{\alpha}\beta\bar{\beta}}$$

$$= \frac{|\beta|^2-\beta\bar{\alpha}-\alpha\bar{\beta}+|\alpha|^2}{1-\alpha\bar{\beta}-\bar{\alpha}\beta+|\alpha|^2|\beta|^2} \quad --- \{z\bar{z}=|z|^2\}$$

$$= \frac{1-\beta\bar{\alpha}-\alpha\bar{\beta}+|\alpha|^2}{1-\alpha\bar{\beta}-\bar{\alpha}\beta+|\alpha|^2} \quad --- \{\because |\beta|=1\}$$

$$\left|\frac{\beta-\alpha}{1-\bar{\alpha}\beta}\right|^2 = 1$$

$$\Rightarrow \left|\frac{\beta-\alpha}{1-\bar{\alpha}\beta}\right| = 1 \quad \underline{\text{ANS}}$$

$\{\because$ Modulus cannot be -ve$\}$

$\boxed{\text{Qn. 4} \rightarrow}$ given $\left(\frac{1+i}{1-i}\right)^m = 1$

$$\Rightarrow \left(\frac{1+i}{1-i}\times\frac{1+i}{1+i}\right)^m = 1$$

$$\Rightarrow \left(\frac{\cancel{1}+\cancel{i^2}+2i}{1-i^2}\right)^m = 1$$

$$\Rightarrow \left(\frac{2i}{2}\right)^m = 1$$

$$\Rightarrow i^m = 1$$

the least +ve Integral value of $\boxed{m=4}$ $\underline{\text{ANS}}$

$\{\because i^4 = 1\}$

Ans 5 → $Z = \dfrac{1+i\cos\theta}{1-2i\cos\theta}$

$$z = \frac{1+i\cos\theta}{1-2i\cos\theta} \times \frac{1+2i\cos\theta}{1+2i\cos\theta}$$

$$Z = \frac{1+2i\cos\theta + i\cos\theta - 2\cos\theta}{1+4\cos^2\theta}$$

$$z = \frac{(1-2\cos\theta)}{1+4\cos^2\theta} + \frac{3i\cos\theta}{1+4\cos^2\theta}$$

Given that Z is purely Real

$\therefore Im(z) = 0$

$\Rightarrow \dfrac{3\cos\theta}{1+4\cos^2\theta} = 0$

$\Rightarrow \cos\theta = 0$

$\Rightarrow \boxed{\theta = 90^\circ}$ (or) $\theta = (2n+1)\frac{\pi}{2}$; $n \in Z$ Ans

Ans 6 → given $\left|\dfrac{i+z}{i-z}\right| = 1$

let $z = x+iy$

$\Rightarrow \dfrac{|i+z|}{|i-z|} = 1$ ---- $\left(\left|\dfrac{z_1}{z_2}\right| = \dfrac{|z_1|}{|z_2|} \right)$

$\Rightarrow$

$|i+z| = |i-z|$

put $z = x+iy$

$\Rightarrow |i+x+iy| = |i-x-iy|$

$\Rightarrow |x + i(1+y)| = |-x + i(1-y)|$

$\Rightarrow \sqrt{x^2+(1+y)^2} = \sqrt{x^2+(1-y)^2}$

Squaring

$\cancel{x^2} + \cancel{1}+\cancel{y^2}+2y = \cancel{x^2}+\cancel{1}+\cancel{y^2}-2y$

$\Rightarrow 4y = 0$

$\Rightarrow y = 0$

Clearly z lies on X-axis **ANS**

Ans. 7 → given $|z_1| = |z_2| = |z_3| \cdots |z_n| = 1$

RHS $\left| \frac{1}{z_1} + \frac{1}{z_2} + \cdots \frac{1}{z_n} \right|$

$= \left| \frac{\bar{z}_1}{z_1\bar{z}_1} + \frac{\bar{z}_2}{z_2\bar{z}_2} \cdots \frac{\bar{z}_n}{z_n\bar{z}_n} \right|$

$= \left| \frac{\bar{z}_1}{|z_1|^2} + \frac{\bar{z}_2}{|z_2|^2} + \cdots \frac{\bar{z}_n}{|z_n|^2} \right| \cdots \{\because z\bar{z} = |z|^2\}$

$= |\bar{z}_1 + \bar{z}_2 + \cdots \bar{z}_n| \cdots \{\because |z_1| = |z_2| \cdots |z_n| = 1\}$

$= |\overline{z_1+z_2+\cdots z_n}| \cdots \{\because \bar{z}_1+\bar{z}_2 = \overline{z_1+z_2}\}$

$= |z_1+z_2+\cdots z_n| = \cdots \{|\bar{z}| = |z|\}$

$=$ LHS **PROVED**

Ans 8 → given $\left(\frac{1+i}{1-i}\right)^3 - \left(\frac{1-i}{1+i}\right)^3 = x+iy$

$\Rightarrow \left(\frac{1+i}{1-i} \times \frac{1+i}{1+i}\right)^3 - \left(\frac{1-i}{1+i} \times \frac{1-i}{1-i}\right)^3 = x+iy$

$$\Rightarrow \left(\frac{1+i^2+2i}{1-i^2}\right)^3 - \left(\frac{1+i^2-2i}{1-i^2}\right)^3 = x+iy$$

$$\Rightarrow \left(\frac{2i}{2}\right)^3 - \left(\frac{-2i}{2}\right)^3 = x+iy$$

$$\Rightarrow i^3 - (-i)^3 = x+iy$$

$$\Rightarrow -i + (\cdot i^3) = x+iy$$

$$= -i - i = x+iy$$

$$\Rightarrow x=0, \quad y=-2 \qquad \therefore \boxed{(0,-2)} \text{ Ans}$$

$\boxed{\text{Ans 9} \rightarrow}$ Given $(1+2i)(1+3i)(1+4i) \text{ ---- } (1+ni) = A-iB$ --- ①

taking conjugate on both sides

$$\Rightarrow (1-2i)(1-3i)(1-4i) \text{ ------ } (1-ni) = A+iB \text{ -- ②}$$

① × ②

$$(1+2i)(1-2i)(1+3i)(1-3i)(1+4i)(1-4i) \text{ ---- } (1+ni)(1-ni) = (A+iB)(A+iB)$$

$$\Rightarrow (1+4)(1+9)(1+16) \text{ ---- } (1+n^2) = A^2+B^2 \quad \text{---} \{ (a+ib)(a-ib) = a^2+b^2 \}$$

$$= (5)(10)(17) \text{ --- } (1+n^2) = A^2+B^2$$

Ans

CHAPTER: LINEAR INEQUALITIES

$\boxed{\text{Ans: 1} \rightarrow}$

640 l 8%	+	x l 2%	=	Mixture (640+x) l 4% < Acid < 6%

Let the required quantity be x litre

we have

$$\left(\frac{8}{100}\times 640\right) + \left(\frac{2}{100}\times x\right) > \frac{4}{100}\times(640+x)$$

$\Rightarrow$ $5120 + 2x > 2560 + 4x$

$\Rightarrow$ $2x - 4x > 2560 - 5120$

$\Rightarrow$ $-2x > -2560$

$\Rightarrow$ $x < 1280$ ----(1)

again we have

$$\left(\frac{8}{100}\times 640\right) + \left(\frac{2}{100}\times x\right) < \frac{6}{100}(640+x)$$

$\Rightarrow$ $5120 + 2x < \cancel{2560}\ 3840 + 6x$

$\Rightarrow$ $-4x < \cancel{-2560} - 1280$

$\Rightarrow$ $x > 320$ -- (2)

from (1) & (2)

$320 < x < 1280$

$\therefore$ 2% boric acid solution should be more than 320 l & less than 1280 litre Ans

$\boxed{\text{Ans 2}}$ $\rightarrow$ let two consecutive even +ve Integers are x and $x+2$

we have $x > 5$ and $x+2 > 5$

$\Rightarrow$ $x > 5$ and $x > 3$

Consider $x > 5$ ---(1)

we have $x + (x+2) < 23$

$\Rightarrow$ $2x < 21$

$\Rightarrow$ $x < 10.5$ -- (2)

from (1) & (2)

$5 < x < 10.5$

Since x is even

$\therefore$ Required pairs are $(x, x+2)$: $(6,8)$, $(8,10)$ $(10,12)$ Ans

Ans 3 →

(1) $x - 2y \leq 3$

points $(0, -\frac{3}{2})$ $(3, 0)$

soln $0 \leq 3$ (Towards)

(2) $3x + 4y \geq 12$

points $(0,3)$ $(4,0)$

soln $0 \geq 12$ (away)

(3) $y \geq 1$

line parallel to x-axis

soln $0 \geq 1$ (away)

(4) $x \geq y$

points $(0,0)$ $(1,1)$

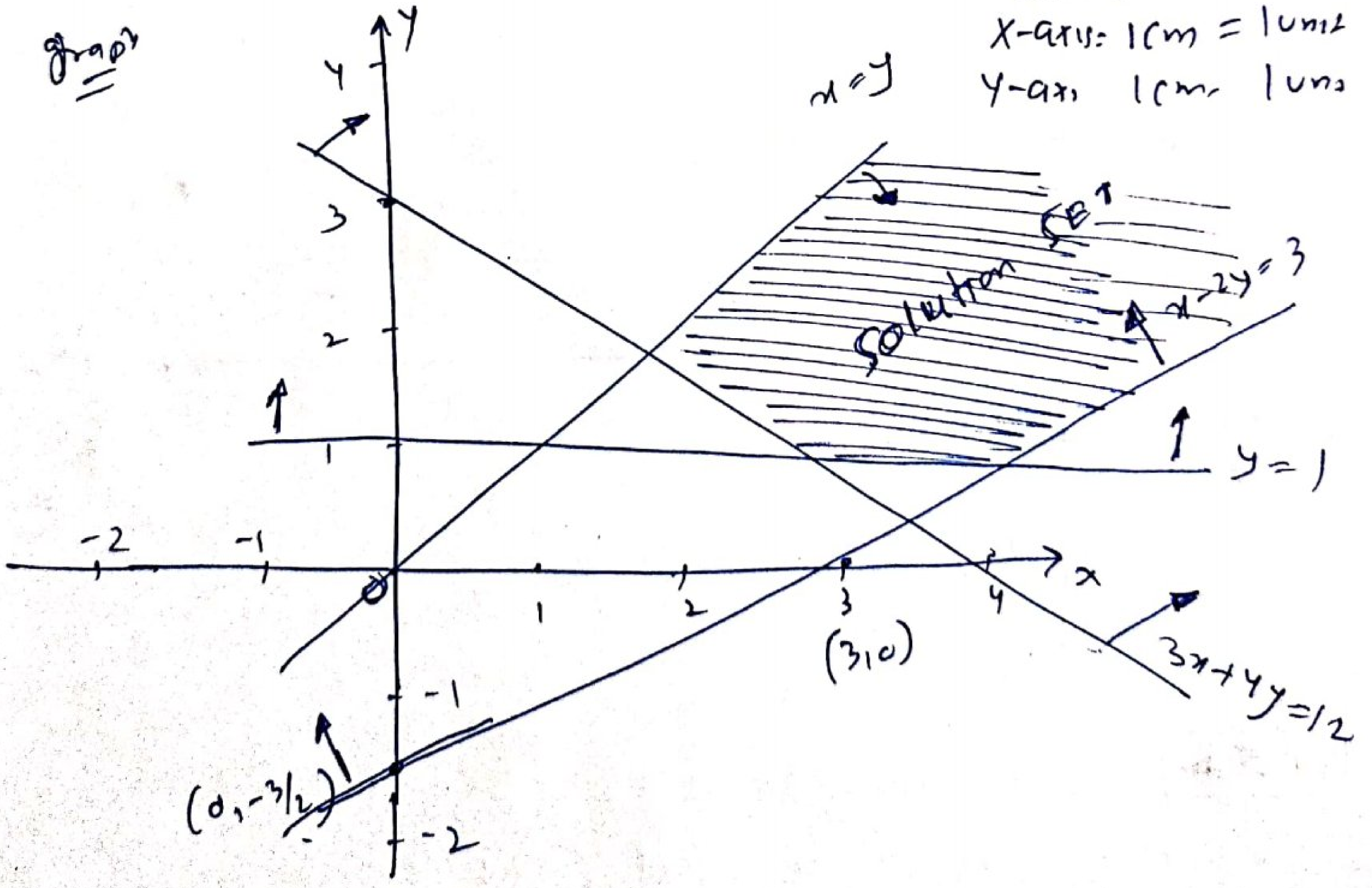
soln towards X-axis

(5) $x, y \geq 0$

soln. is in Ist quadrant

graph

Scale
X-axis: 1cm = 1 unit
Y-axis 1cm = 1 unit



On. 4 → (1) $2x+3y \geq 3$ points $(0,1)$ $(\frac{3}{2},0)$

Solution $0 \geq 3$ (~~towards~~ away origin)

(2) $3x+4y \leq 18$ points $(0,\frac{9}{2})$ $(6,0)$

Solut $0 \leq 18$ (Soln toward)

(3) $-7x+4y \leq 14$ point $(0, 7/2)$ $(-2, 0)$

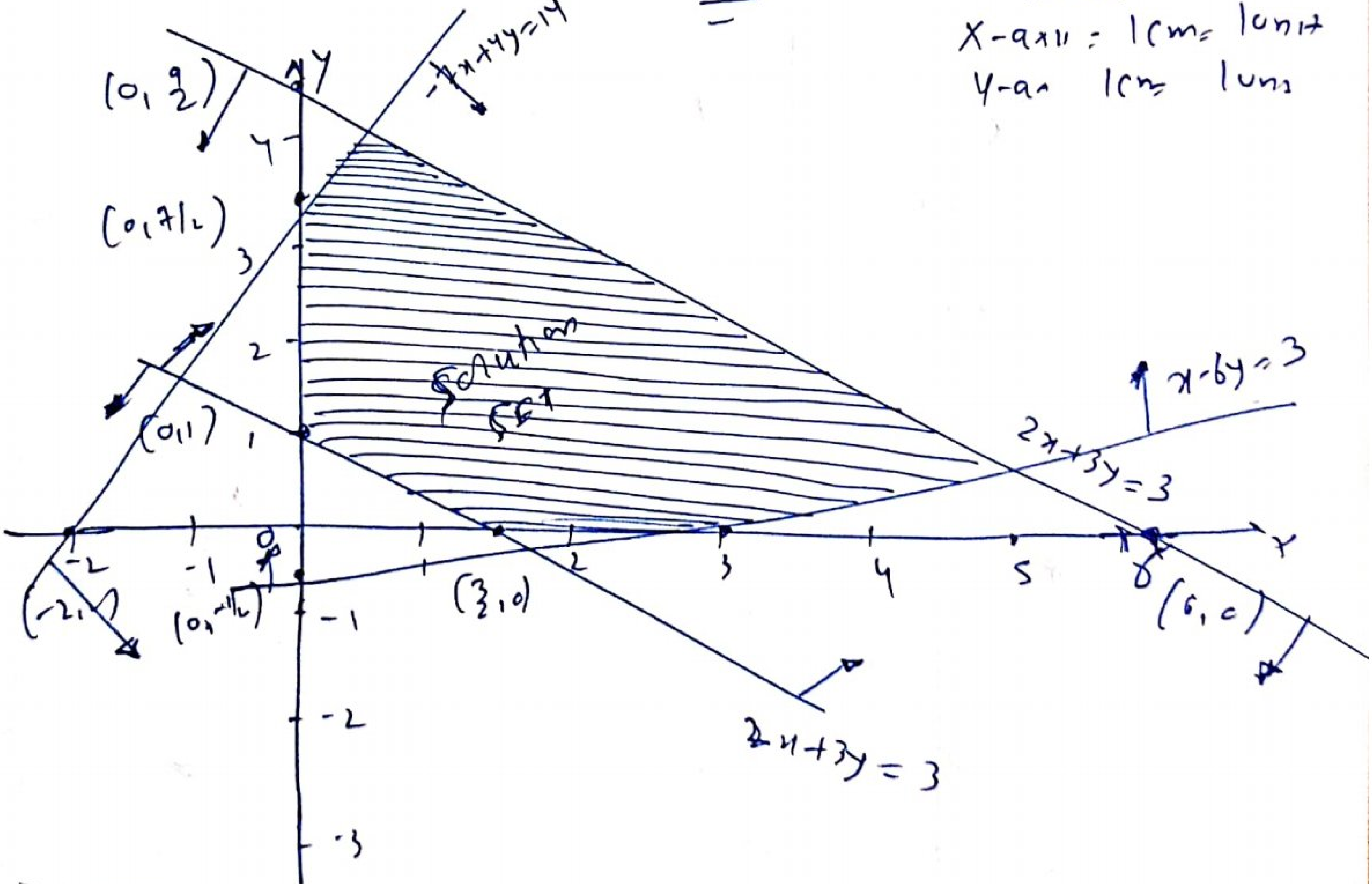
Soln $0 \leq 14$ (Toward)

(4) $x-6y \leq 3$ point $(0,-\frac{1}{2})$ $(3,0)$

Soln $0 \leq 3$ (towards)

(5) $x, y \geq 0$; Ist quadrant

Scale
X-axis : 1cm = 1unit
Y-axis 1cm = 1unit



On. 5 → $5(2x-7)-3(2x+3) \leq 0$ & $2x+19 \leq 6x+47$

$\Rightarrow 10x-35-6x-9 \leq 0$ and $2x-6x \leq 47-19$

$\Rightarrow 4x \leq 44$ and $-4x \leq -28$

$\Rightarrow \quad x \leq 11 \quad$ and $\quad x \geq -7$

$\therefore \quad x \in [-7, 11]$ **Ans**

$\boxed{\text{Ques 6} \rightarrow}$ given $\quad |x-1| + |x-2| \geq 4$

$\leftarrow (1) \longleftrightarrow (2) \rightarrow$

Case I $\quad$ when $x < 1$

$-(x-1) - (x-2) \geq 4$

$\Rightarrow \quad -2x + 3 \geq 4$

$\Rightarrow \quad -2x \geq 1$

$\Rightarrow \quad x \leq -\frac{1}{2} \quad$ and $\quad x < 1$

$-\infty \qquad -1/2 \qquad 1 \qquad \infty$

$\therefore x \in (-\infty, -\frac{1}{2}] \quad ---(i)$

Case II $\quad$ when $1 \leq x < 2$

$(x-1) - (x-2) \geq 4$

$\Rightarrow \quad 1 \geq 4 \quad$ (not possible)

$\therefore x \in \phi \quad --(2)$

Case III $\quad$ when $x \geq 2$

$(x-1) + (x-2) \geq 4$

$\Rightarrow \quad 2x - 3 \geq 4$

$\Rightarrow \quad 2x \geq 7$

$\Rightarrow \quad x \geq 7/2 \quad$ and $\quad x \geq 2$

$-\infty \qquad 2 \qquad 7/2 \qquad \infty \qquad \therefore x \in [7/2, \infty) \quad --(3)$

from (1), (2), (3)

$\therefore \quad x \in (-\infty, -\frac{1}{2}] \cup [7/2, \infty)$ **Ans**

— x —

CHAPTER: P & C

Ans 1 → given $^4P_r = 6 \cdot {}^5P_{r-1}$

$$\Rightarrow \frac{^4P_r}{^5P_{r-1}} = 6$$

$$\Rightarrow \frac{\frac{4!}{(4-r)!}}{\frac{5!}{(6-r)!}} = 6$$

$$\Rightarrow \frac{4!\,(6-r)!}{5!\,(4-r)!} = 6$$

$$\Rightarrow \frac{\cancel{4!}\,(6-r)(5-r)\cancel{(4-r)!}}{5 \times \cancel{4!}\,\cancel{(4-r)!}} = 6$$

$$\Rightarrow 30 - 11r + r^2 = 6$$

$$\Rightarrow r^2 - 11r - 24 = 0$$

$$\Rightarrow (r-8)(r+3) = 0$$

$$\Rightarrow r = 8, \quad r = -3$$

{ Both are rejected since r cannot be −ve
also when $r = 8$ then Ans becomes
$^4P_8$ (not possible) 'r' cannot greater than n }

Imp

$\therefore$ No value of r **Ans**

Qn. 2 → word DAUGHTER

total letters = 8

vowels 3 : A, U, E

Consonants = 5, D, G, H, T, R

(i) (∵) Consider 3 vowels as 1 letter = (A, U, E) = 1

(∵) Now we have to arrange 5+1 = 6 letters

(∵) which they can be arranged in = $6!$ ways

(∵) 3 vowels can mutually arranged in = $3!$ ways

(∵) No of words in which all vowels occur together

$= 6! \times 3!$

$= 720 \times 6 = 4320$

(∵) total No. of words = $8! = 40320$

(∵) ∴ No of words in which all vowels <u>never</u> occur together = $40320 - 4320 = \boxed{36000}$ <u>Ans</u>

(2)

$- C_1 - C_2 - C_3 - C_4 - C_5 -$

(∵) there are 6 places available for 3 vowels

(∵) which they can be arranged in $^6P_3$ ways

(∵) 5 consonants can mutually arranged in $= 5!$ ways

(∵) ∴ No of words in which <u>No two</u> vowels together $= {^6P_3} \times 5! = \frac{6!}{3!} \times 5! = \boxed{14400}$

<u>Ans</u>

Que 3 → digits available 1, 2, 0, 2, 4, 2, 4

(i) Numbers starting with 1 = $1 \times \frac{6!}{3!\,2!} = \frac{720}{6 \times 2} = 60$

(ii) Numbers starting with 2 = $1 \times \frac{6!}{2!\,2!} = \frac{720}{4} = 180$

(iii) Numbers starting with 4 = $1 \times \frac{6!}{3!} = \frac{720}{6} = 120$

∴ Required Number of words = $60 + 180 + 120 = \boxed{360}$ Ans

Que. 4 → RANDOM

total words = $5! = 720$

A	D	M	N	O	R
A - - - - - $= 5!$ $= 120$	D - - - - - $= 5!$ $= 120$	M - - - - - $= 5!$ $= 120$	N - - - - - $= 5!$ $= 120$	O - - - - - $= 5!$ $= 120$	

RAD _ _ _ $= 3! = 6$

RAM _ _ _ $= 3! = 6$

RANDMO $= 1$

RANDOM $= = 1$

∴ Rank = $120 + 120 + 120 + 120 + 120 + 6 + 6 + 1 + 1$

Rank = $\boxed{614}$ Ans

Qn. 5 → digits 2, 3, 4, 5

total Numbers = $4! = \cancel{120}\ 24$

(9) TH	(9) H	(8) T	U
84	84	84	84
93	2	2	4

$2 \times 6 = 12$
$3 \times 6 = 18$
$4 \times 6 = 24$
$5 \times 6 = 30$
$\overline{84}$

each digit repeat 6 times in each place

$\therefore$ Sum of digits in each place = 84

$\therefore$ total Sum = $84(1000 + 100 + 10 + 1) = 84(1111)$

$= 93224$

Ans

Qn 6 → Word MATHEMATICS

total letters = 1

M = 2
A = 2
T = 2
H = 1
I = 1
E = 1
C = 1
S = 1

Total ~~F~~ types cases

(1) all are distinct letters

$= {}^8C_4 \times 4!$

$= \dfrac{8 \times 7 \times 6 \times 5 \times \cancel{4}}{24} \times 24 = 1680$

(2) 1 pair, 2 distinct

$= {}^3C_1 \times {}^7C_2 \times \dfrac{4!}{2!} = 3 \times 21 \times 12 = 756$

Case III 2 pairs

$$= {}^3C_2 \times \frac{4!}{2!\,2!}$$

$$= 3 \times \frac{24}{4} = 18$$

$\therefore$ Required No of words $= 1680 + 756 + 18$

$= \boxed{2454}$ Ans

Que 7 +

(6) A	(7) B
4	6
5	5
6	4

required $= 10$

three cases ① selecting 4 from A & 6 from B

$$= {}^6C_4 \times {}^7C_6 = 15 \times 7 = 105$$

② selecting 5 from A & 5 from B.

$$= {}^6C_5 \times {}^7C_5 = 6 \times 21 = 126$$

③ selecting 6 from A & 4 from B

$$= {}^6C_6 \times {}^7C_4 = 1 \times 35 = 35$$

$\therefore$ Required No of ways $= 105 + 126 + 35$

$= \boxed{266}$ Ans

—x—

CHAPTER: SEQUENCE SERIES

Qus. 1 → here $a = 1$, $b = 31$ and $n = m$

$$d = \frac{b-a}{n+1} = \frac{31-1}{m+1} = \frac{30}{m+1}$$

Given

$$\frac{A_7}{A_{m-1}} = \frac{5}{9}$$

$$\Rightarrow \frac{a+7d}{a+(m-1)d} = \frac{5}{9}$$

$$\Rightarrow \frac{1+7\left(\frac{30}{m+1}\right)}{1+(m-1)\frac{30}{m+1}} = \frac{5}{9}$$

$$\Rightarrow \frac{m+1+210}{m+1+30m-30} = \frac{5}{9}$$

$$\Rightarrow \frac{m+211}{31m-29} = \frac{5}{9}$$

$$\Rightarrow 9m + 1899 = 155m - 145$$

$$\Rightarrow 146m = 2044$$

$$\Rightarrow m = \frac{2044}{146}$$

$$\Rightarrow \boxed{m = 14}$$ Ans

Qus. 2 → let $S_n = 0.6 + 0.66 + 0.666 + \text{---} n \text{ term}$

$$S_n = 6(0.1 + 0.11 + 0.111 + \text{---} n \text{ term})$$

$\Rightarrow S_n = \frac{6}{9}[0.9 + 0.99 + 0.999 + \dots n\text{ term}]$

$= \frac{2}{3}[(1-0.1) + (1-0.01) + (1-.001) + \dots n\text{ term}]$

$= \frac{2}{3}[(1+1+\dots n\text{ term}) - (0.1 + .01 + .001 + \dots n\text{ term})]$

$= \frac{2}{3}[n - (\frac{1}{10} + \frac{1}{10^2} + \frac{1}{10^3} + \dots n\text{ term})]$

$= \frac{2}{3}[n - \frac{1}{10}(\frac{1-\frac{1}{10^n}}{1-\frac{1}{10}})]$

$= \frac{2}{3}[n - \frac{1}{9}(1-\frac{1}{10^n})]$

$S_n = \frac{2}{27}[9n - 1 + \frac{1}{10^n}]$ Ans

Qn:3 → Given $\frac{a^{n+1}+b^{n+1}}{a^n+b^n} = GM$

$\Rightarrow \frac{a^{n+1}+b^{n+1}}{a^n+b^n} = \sqrt{ab}$

$\Rightarrow a^{n+1}+b^{n+1} = a^{1/2}b^{1/2}(a^n+b^n)$

$\Rightarrow a^{n+1}+b^{n+1} = a^{n+\frac{1}{2}}.b^{1/2} + a^{1/2}b^{n+1/2}$

$\Rightarrow a^{n+1} - a^{n+\frac{1}{2}}.b^{1/2} = a^{1/2}b^{n+1/2} - b^{n+1}$

$\Rightarrow a^{n+1/2}\cancel{(a^{1/2}-b^{1/2})} = b^{n+1/2}\cancel{(a^{1/2}-b^{1/2})}$

$\Rightarrow a^{n+1/2} = b^{n+1/2}$

$\Rightarrow \left(\frac{a}{b}\right)^{n+1/2} = 1$

$\Rightarrow \left(\frac{a}{b}\right)^{n+1/2} = \left(\frac{a}{b}\right)^{0}$

$\Rightarrow n + 1/2 = 0$

$\Rightarrow \boxed{n = -1/2}$ Ans

Que 4 let the two numbers are a & b

given that $a+b = 6(GM)$

$\Rightarrow a+b = 6\sqrt{ab}$

$\Rightarrow \frac{a+b}{2\sqrt{ab}} = \frac{3}{1}$

Apply Componendo & dividendo

$$\Rightarrow \frac{a+b+2\sqrt{ab}}{a+b-2\sqrt{ab}} = \frac{3+1}{3-1}$$

$$\Rightarrow \frac{(\sqrt{a}+\sqrt{b})^2}{(\sqrt{a}-\sqrt{b})^2} = 2$$

$$\Rightarrow \frac{\sqrt{a}+\sqrt{b}}{\sqrt{a}-\sqrt{b}} = \pm\frac{\sqrt{2}}{1}$$

Again C&D

$$\Rightarrow \frac{(\sqrt{a}+\sqrt{b})+(\sqrt{a}-\sqrt{b})}{(\sqrt{a}+\sqrt{b})-(\sqrt{a}-\sqrt{b})} = \pm\left(\frac{\sqrt{2}+1}{\sqrt{2}-1}\right)$$

$$\Rightarrow \frac{\cancel{2}\sqrt{a}}{\cancel{2}\sqrt{b}} = \pm\frac{(\sqrt{2}+1)}{\sqrt{2}-1}$$

Squaring both sides

$\Rightarrow \frac{a}{b} = \frac{2+1+2\sqrt{2}}{2+1-2\sqrt{2}}$

$\Rightarrow \frac{a}{b} = \frac{3+2\sqrt{2}}{3-2\sqrt{2}}$

$\therefore \boxed{a:b = 3+2\sqrt{2} : 3-2\sqrt{2}}$ Ans

Qn: 5 → T.P $(a^n+b^n), (b^n+c^n), (c^n+d^n)$ are in GP

i.e to prove $(b^n+c^n)^2 = (a^n+b^n)(c^n+d^n)$

Given a, b, c, d are in GP

let $a=a$, $b=ar$, $c=ar^2$, $d=ar^3$

LHS $(b^n+c^n)^2$

$= (a^n r^n + a^n r^{2n})^2$

$= a^{2n} r^{2n} (1+r^n)^2$

RHS $(a^n+b^n)(c^n+d^n)$

$= (a^n + a^n r^n)(a^n r^{2n} + a^n r^{3n})$

$= a^n(1+r^n) \cdot a^n r^{2n}(1+r^n)$

$= a^{2n} \cdot r^{2n}(1+r^n)^2$

Hence $(b^n+c^n)^2 = (a^n+b^n)(c^n+d^n)$

$\Rightarrow (a^n+b^n), (b^n+c^n), (c^n+d^n)$ are in GP Ans

Ans: 6 → given $a_1 = a$

and $a_n = b$

$\Rightarrow ar^{n-1} = b$

$\Rightarrow r^{n-1} = \frac{b}{a}$

$\Rightarrow r = \left(\frac{b}{a}\right)^{\frac{1}{n-1}}$

given $P = a \cdot ar \cdot ar^2 \cdots ar^{n-1}$

$\Rightarrow P = a^n r^{1+2+3 \cdots (n-1)}$

$\Rightarrow P = a^n \cdot r^{\frac{n(n-1)}{2}}$

$\Rightarrow P = a^n \left[\left(\frac{b}{a}\right)^{\frac{1}{\cancel{n-1}}}\right]^{\frac{n\cancel{(n-1)}}{2}}$

$\Rightarrow P = a^n \left(\frac{b}{a}\right)^{n/2}$

$\Rightarrow P = \frac{a^n \cdot b^{n/2}}{a^{n/2}}$

$\Rightarrow P = a^{n/2} \cdot b^{n/2}$

$\Rightarrow P = (ab)^{n/2}$

Squaring

$\boxed{P^2 = (ab)^n}$ Ans

—x—