!! जय श्री राधा कृष्ण। जय श्री गिरिराज जी महाराज !!

ULTIMATE MATHEMATICS: BY AJAY MITTAL

→ CHAPTER: LIMITS & DERIVATIVES ←

CLASS NO: 1

Topic Derivatives

(·) $y = f(x)$

(·) Differentiate both sides w.r.t x

(·) $\frac{dy}{dx}$ (or) $f'(x)$

(·) Meaning: Rate of change of one variable w.r.t another variable

Formula.

(·) Constant (or) variable

① $\frac{d}{dx}(x^n) = nx^{n-1}$

eg $\frac{d}{dx}(x^3) = 3x^2$

$\frac{d}{dx}(x^{-5/2}) = -\frac{5}{2}(x)^{-7/2}$

② $\frac{d}{dx}(\text{constant}) = 0$

eg $\frac{d}{dx}(a) = 0$

③ $\frac{d}{dx}(\log x) = \frac{1}{x}$

④ $\frac{d}{dx}(e^x) = e^x \cdot \log e = e^x$

⑤ $\frac{d}{dx}(a^x) = a^x \cdot \log a$ $\boxed{a>0}$

eg $\frac{d}{dx}(3^x) = 3^x \cdot \log 3$

$\boxed{e \approx 2.7}$ $\boxed{\log e = 1}$ $\boxed{\log 1 = 0}$

⑥ $\frac{d}{dx}(\sin x) = \cos x$.

⑦ $\frac{d}{dx}(\cos x) = -\sin x$

⑧ $\frac{d}{dx}(\tan x) = \sec^2 x$

⑨ $\frac{d}{dx}(\cot x) = -\operatorname{cosec}^2 x$

⑩ $\frac{d}{dx}(\sec x) = \sec x \cdot \tan x$

⑪ $\frac{d}{dx}(\operatorname{cosec} x) = -\operatorname{cosec} x \cdot \cot x$

Shortcuts

(12) $\frac{d}{dx}(\sqrt{x}) = \frac{1}{2\sqrt{x}}$

Reason $\sqrt{x} \to x^{1/2} \to \frac{1}{2}x^{\frac{1}{2}-1} \to \frac{1}{2}x^{-1/2} \to \frac{1}{2x^{1/2}} = \frac{1}{2\sqrt{x}}$

(13) $\frac{d}{dx}\left(\frac{1}{x}\right) = -\frac{1}{x^2}$

$\frac{1}{x} \to x^{-1} \to (-1)x^{-2} \to \frac{-1}{x^2}$

(14) $\frac{1}{x} \to -\frac{1}{x^2} \to +\frac{2}{x^3} \to -\frac{6}{x^4} \to \frac{24}{x^5} \to -\frac{120}{x^6} \to \frac{720}{x^7}$

Rules

(1) $\frac{d}{dx}(f(x) \pm g(x)) = \frac{d}{dx}(f(x)) \pm \frac{d}{dx}(g(x))$

eg $\frac{d}{dx}(\sin x + x^{7/2}) = \cos x + \frac{7}{2}x^{5/2}$

(2) $\frac{d}{dx}(kf(x)) = k\frac{d}{dx}(f(x))$

eg $\frac{d}{dx}(3\log x) = 3\cdot\frac{d}{dx}(\log x) = \frac{3}{x}$

eg $\frac{d}{dx}\left(\frac{x^3}{4}\right) = \frac{3x^2}{4}$

(3) Product rule $y = f(x)\cdot g(x)$

$$\frac{dy}{dx} = f(x)\cdot\frac{d}{dx}(g(x)) + g(x)\cdot\frac{d}{dx}(f(x))$$

eg $\frac{d}{dx}(\sin x \cdot 3^x) = \sin x\cdot\frac{d}{dx}(3^x) + 3^x\cdot\frac{d}{dx}(\sin x)$

$= \sin x \cdot 3^x \cdot \log 3 + 3^x \cdot \cos x$

* $y = u\cdot v\cdot w$

$$\frac{dy}{dx} = \frac{du}{dx}\cdot v\cdot w + \frac{dv}{dx}\cdot u\cdot w + \frac{dw}{dx}\cdot u\cdot v$$

(4) Quotient Rule. $y = \frac{f(x)}{g(x)} = \frac{N}{D}$

$$\frac{dy}{dx} = \frac{D \cdot \frac{d}{dx}(N) - N \cdot \frac{d}{dx}(D)}{D^2}$$

Eg $y = \frac{\log x}{\tan x}$

$$\frac{dy}{dx} = \frac{\tan x \cdot \frac{d}{dx}(\log x) - \log x \cdot \frac{d}{dx}(\tan x)}{\tan^2 x}$$

$$\frac{dy}{dx} = \frac{\tan x \cdot \frac{1}{x} - \log x \cdot \sec^2 x}{\tan^2 x}$$

* $\frac{d}{dx}(3\sin x) = 3\cos x$

↳ No Product rule (X)

↳ $3\frac{d}{dx}(\sin x) + \sin x \cdot \frac{d}{dx}(3) = 3\cos x + 0 = 3\cos x$

* $\frac{d}{dx}\left(\frac{x^3}{4}\right) \rightarrow \frac{3x^2}{4}$

↳ No Quotient rule

* $\frac{d}{dx}\left(3\sin x + \frac{\tan x}{2} + 4\right) = 3\cos x + \frac{\sec^2 x}{2} + 0$

* $\frac{d}{dx}(\sin a) = 0$

* $\frac{d}{dx}(3\sin x \cdot \log x) = 3\left(\sin x \cdot \frac{1}{x} + \log x \cdot \cos x\right)$

BASICS

(·) $\frac{d}{dx}(x) = 1$ (·) $\frac{d}{dx}(2x) = 2$ (·) $\frac{d}{dx}(bx) = b$

① $y = \sin(3x)$

$\frac{dy}{dx} = \cos(3x) \cdot \frac{d}{dx}(3x)$

$= 3\cos(3x)$.

② $y = \tan(\sqrt{x})$

$\frac{dy}{dx} = \sec^2(\sqrt{x}) \cdot \frac{d}{dx}(\sqrt{x})$

$= \frac{1}{2\sqrt{x}} \cdot \sec^2(\sqrt{x})$

③ $y = e^{\sin x}$

$\frac{dy}{dx} = e^{\sin x} \cdot \frac{d}{dx}(\sin x)$

$= \cos x \cdot e^{\sin x}$

④ $y = 3^{\sqrt{x}}$

$\frac{dy}{dx} = 3^{\sqrt{x}} \cdot \log 3 \cdot \frac{1}{2\sqrt{x}}$

⑤ $y = \sqrt{\tan x}$

$\frac{dy}{dx} = \frac{1}{2\sqrt{\tan x}} \cdot \frac{d}{dx}(\tan x)$

$= \frac{\sec^2 x}{2\sqrt{\tan x}}$

(6) $y = \sqrt{\log x}$

$\frac{dy}{dx} = \frac{1}{2\sqrt{\log x}} \cdot \frac{1}{x}$

(7) $y = \log(\sin x)$

$\frac{dy}{dx} = \frac{1}{\sin x} \cdot \frac{d}{dx}(\sin x)$

$= \frac{1}{\sin x} \cdot \cos x = \cot x$

(8) $y = \log(4^x)$

$\frac{dy}{dx} = \frac{1}{4^x} \cdot 4^x \cdot \log 4 = \log 4$

(9) $y = \sin(\sqrt{\log x})$

$\frac{dy}{dx} = \cos(\sqrt{\log x}) \cdot \frac{1}{2\sqrt{\log x}} \cdot \frac{1}{x}$

(10) $y = \log(\tan(\sqrt{x}))$

$\frac{dy}{dx} = \frac{1}{\tan\sqrt{x}} \cdot \sec^2(\sqrt{x}) \cdot \frac{1}{2\sqrt{x}}$

(11) $y = \sin^4 x$

$y = (\sin x)^4$

$\frac{dy}{dx} = 4(\sin x)^3 \cdot \frac{d}{dx}(\sin x)$

$\frac{dy}{dx} = 4\sin^3 x \cdot \cos x$

(12) $y = \sec^5 x \Rightarrow \frac{dy}{dx} = 5\sec^4 x \cdot \sec x \tan x$

(13) $y = \frac{1}{\sqrt{3-4x^2}}$

$y = (3-4x^2)^{-1/2}$

$\frac{dy}{dx} = \frac{-1}{2}(3-4x^2)^{-3/2} \cdot \frac{d}{dx}(3-4x^2)$

$= -\frac{1}{2}(3-4x^2)^{-3/2} \cdot (-8x)$

$= \frac{4x}{(3-4x^2)^{3/2}}$

(14) $y = \frac{1}{(\sin x - \cos x)^{3/2}}$

$y = (\sin x - \cos x)^{-3/2}$

$\frac{dy}{dx} = -\frac{3}{2}(\sin x - \cos x)^{-5/2} \cdot (\cos x + \sin x)$

(15) $y = \underline{\sin^3(x^2)} \cdot \underline{\cos^4(x^3)}$

$\frac{dy}{dx} = \sin^3(x^2) \cdot \frac{d}{dx}(\cos^4(x^3)) + \cos^4(x^3) \cdot \frac{d}{dx}(\sin^3(x^2))$

$= \sin^3(x^2) \cdot 4\cos^3(x^3) \cdot (-\sin(x^3)) \cdot 3x^2 + \cos^4(x^3) \cdot 3\sin^2(x^2) \cdot \cos(x^2) \cdot 2x$

(16) $y = \frac{\sin x - \cos x}{\sin x + x\cos x}$

$\frac{dy}{dx} = \frac{(\sin x + x\cos x) \cdot \frac{d}{dx}(\sin x - \cos x) - (\sin x - \cos x)\frac{d}{dx}(\sin x + x\cos x)}{(\sin x + x\cos x)^2}$

$\frac{dy}{dx} = \frac{(\sin x + x\cos x) \cdot (\cos x + \sin x) - (\sin x - \cos x) \cdot (\cos x + x\sin x + \cos x \cdot 1)}{(\sin x + x\cos x)^2}$