

॥ जय श्री राधे कृष्ण ॥ : जय श्री गिरिराज जी महाराज ॥ (1)

ULTIMATE MATHEMATICS: BY AJAY MITTAL

CHAPTER: CONIC SECTIONS

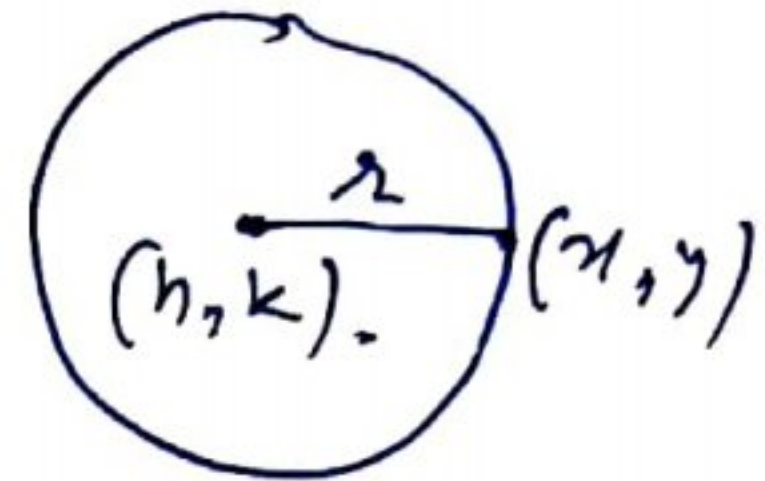
CLASS 11/0:1

TOPIC

CIRCLES

(1) Standard equation of circle

$$(x-h)^2 + (y-k)^2 = r^2$$



(2) General equation of circle

$$x^2 + y^2 + 2gx + 2fy + c = 0$$

Centre $(-g, -f)$ Radius $= \sqrt{g^2 + f^2 - c}$

eg (1) Given $(x+3)^2 + (y-1)^2 = 5$
here $h = -3, k = 1; r = \sqrt{5}$

Centre $(-3, 1)$ Radius $= \sqrt{5}$

(2) Given $x^2 + y^2 + 4x - 6y - 1 = 0$

Perfect square $(x^2 + 4x) + (y^2 - 6y) - 1 = 0$

$$\Rightarrow (x+2)^2 - 4 + (y-3)^2 - 9 - 1 = 0$$

$$(x+2)^2 + (y-3)^2 = 14$$

Centre $(-2, 3)$ Radius $= \sqrt{14}$

eg (3) $2x^2 + 2y^2 + 5x - 1 = 0$

$\Rightarrow x^2 + y^2 + \frac{5}{2}x - \frac{1}{2} = 0$

$\Rightarrow (x^2 + \frac{5}{2}x) + y^2 - \frac{1}{2} = 0$

$\Rightarrow (x + \frac{5}{4})^2 - \frac{25}{16} + y^2 - \frac{1}{2} = 0$

$\Rightarrow (x + \frac{5}{4})^2 + y^2 = \frac{25}{16} + \frac{1}{2} = \frac{33}{16}$

$\frac{25+8}{16}$

Centre $(-\frac{5}{4}, 0)$ $r = \frac{\sqrt{33}}{4}$

Q.4 Find the equation circle with Radius 5 and centre lies on x-axis and passes through the point (2,3)

Sol Let Centre is $(h, 0)$

Given $AB = 5$

$\sqrt{(h-2)^2 + 9} = 5$

$h^2 + 4 - 4h + 9 = 25$

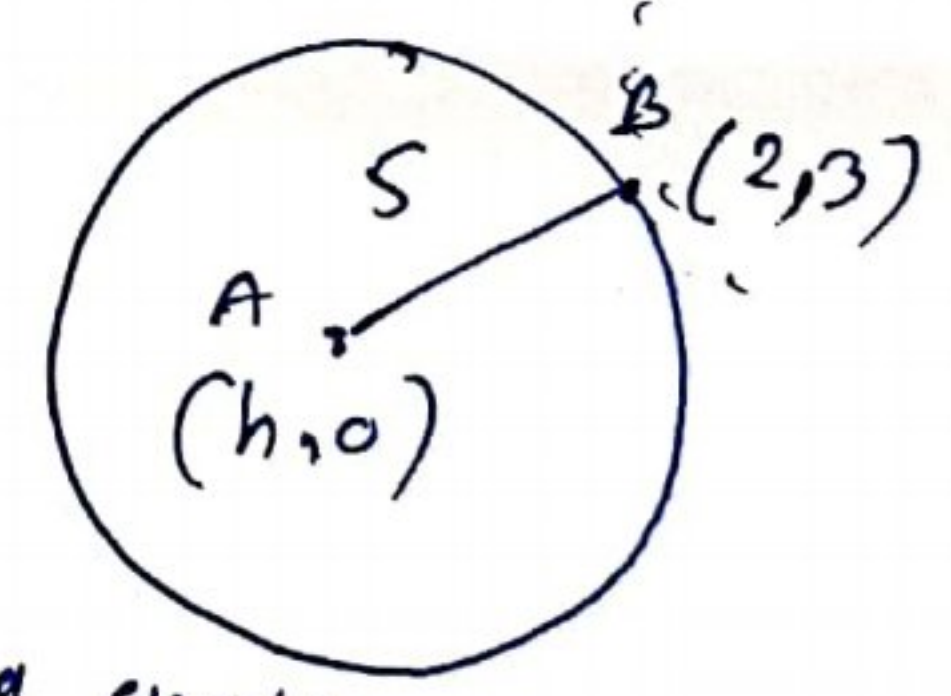
$h^2 - 4h - 12 = 0$

$(h-6)(h+2) = 0$

$h = 6, h = -2$

Centre $(6, 0)$ & $(-2, 0)$

Rad = 5



(1) equation circle
 $(x-6)^2 + y^2 = 25$

(2) equation circle
 $(x+2)^2 + y^2 = 25$
Ans

Q.5 Find the equation of the circle passing through the points $(2,3)$ and $(-1,1)$ whose centre lies on the line $x-3y-11=0$ (3)

Sol Let equation of circle is
 $(x-h)^2 + (y-k)^2 = r^2$

✓ $A(2,3)$ lies on the circle

$$(2-h)^2 + (3-k)^2 = r^2$$

$$\Rightarrow \boxed{h^2 + k^2 - 4h - 6k + 13 = r^2} \quad \dots (i)$$

✓ $B(-1,1)$ lies on the circle

$$(-1-h)^2 + (1-k)^2 = r^2$$

$$\Rightarrow \boxed{h^2 + k^2 + 2h - 2k + 2 = r^2} \quad \dots (2)$$

✓ $C(h,k)$ lies on the line $x-3y=11$

$$\Rightarrow \boxed{h-3k=11} \quad \dots (3)$$

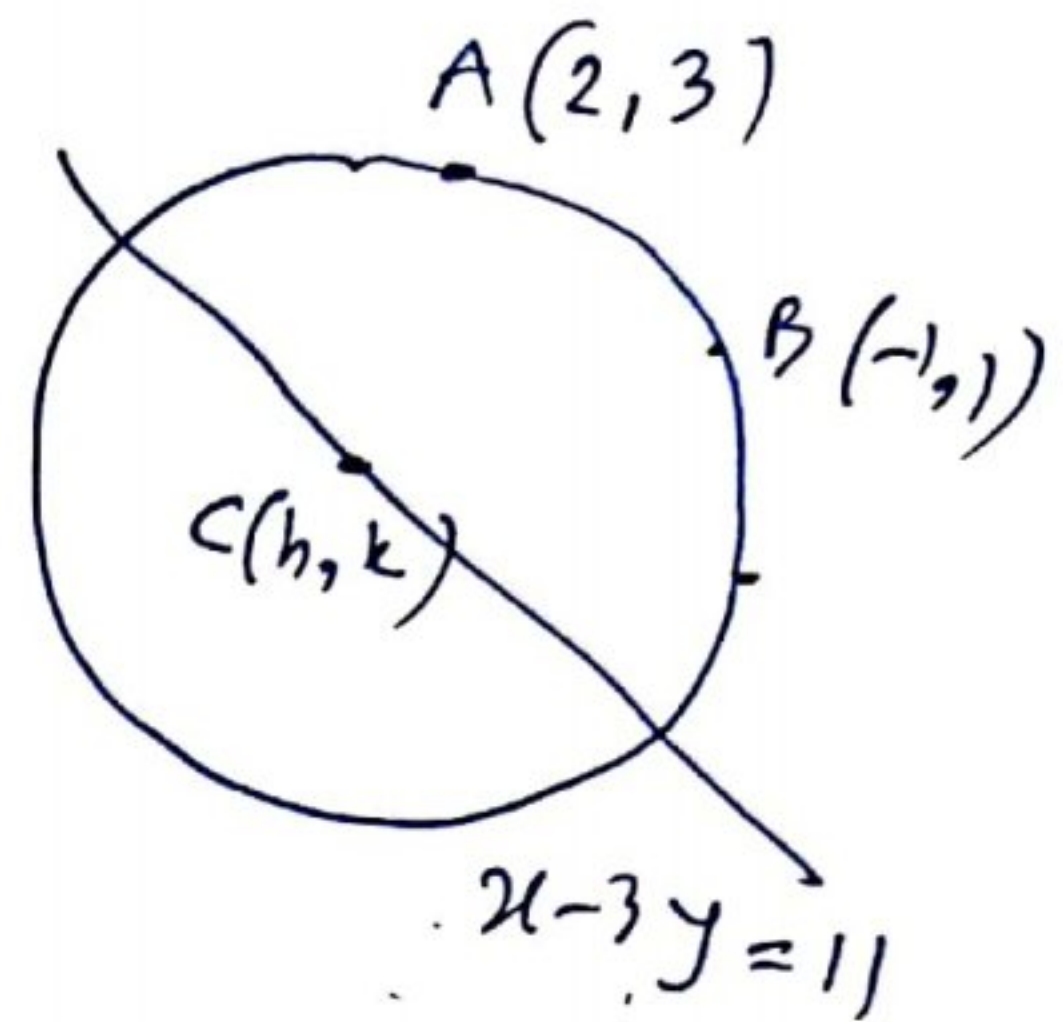
From (1) & (2)

$$\cancel{h^2+k^2} + \cancel{h^2+k^2} - 4h - 6k + 13 = \cancel{h^2+k^2} + \cancel{h^2+k^2} + 2h - 2k + 2$$

$$\Rightarrow -6h - 4k = -11$$

$$\Rightarrow \boxed{6h + 4k = 11} \quad \dots (4)$$

Sol, (3) & (4)



(4)

$$\begin{array}{r} 6h - 18k = 66 \\ 6h + 4k = 11 \\ \hline -22k = 55 \end{array}$$

$$k = \frac{-55}{22} = -\frac{5}{2}$$

$$6h - 10 = 11$$

$$6h = 21$$

$$h = \frac{21}{6} = \frac{7}{2}$$

$$\therefore \text{center } \left(\frac{7}{2}, -\frac{5}{2} \right)$$

put in $h^2 + k^2 + 2h - 2k + 2 = 12$

$$\Rightarrow \frac{49}{4} + \frac{25}{4} + 7 + 5 + 2 = 12$$

$$\Rightarrow \frac{74 + 56}{4} = 12$$

$$\Rightarrow \frac{130}{4} = 12$$

$$12 = \frac{65}{2}$$

equation of circle $(x - \frac{7}{2})^2 + (y + \frac{5}{2})^2 = \frac{65}{2}$ Ans

Q. 6 Find the equation of the circle passing through (0,0) and making Intercepts a & b on the coordinate axes

Sol Let cent (h, k)

$$OD \perp AC$$

$$\Rightarrow OC = a/2$$

$$\Rightarrow \boxed{h = a/2}$$

$$OE \perp BC$$

$$\Rightarrow EC = OD = b/2$$

$$\therefore \boxed{k = b/2}$$

△ODC

$$\boxed{r^2 = \frac{a^2}{4} + \frac{b^2}{4}}$$

Equation of Circle

$$(x - \frac{a}{2})^2 + (y - \frac{b}{2})^2 = \frac{a^2}{4} + \frac{b^2}{4}$$

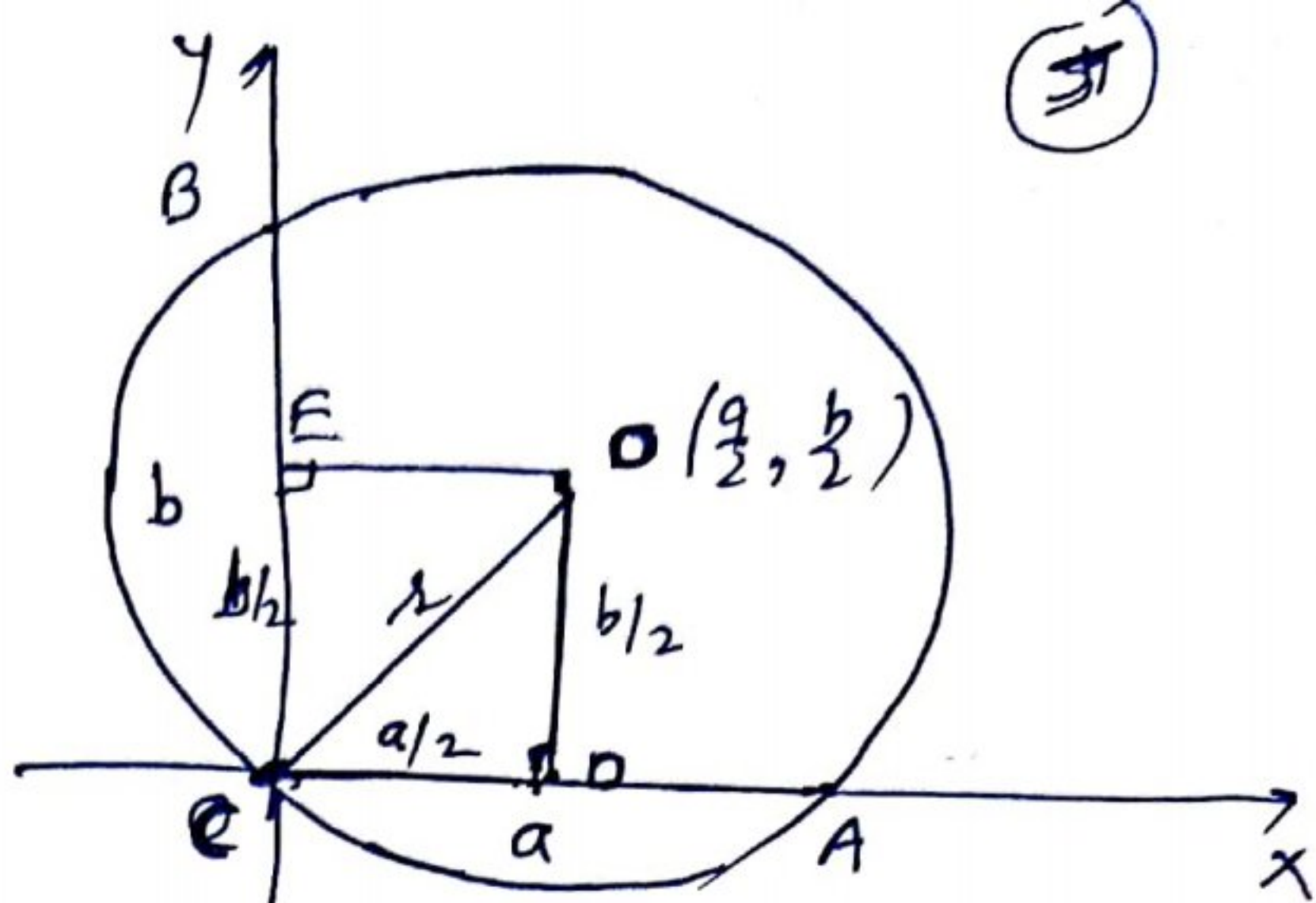
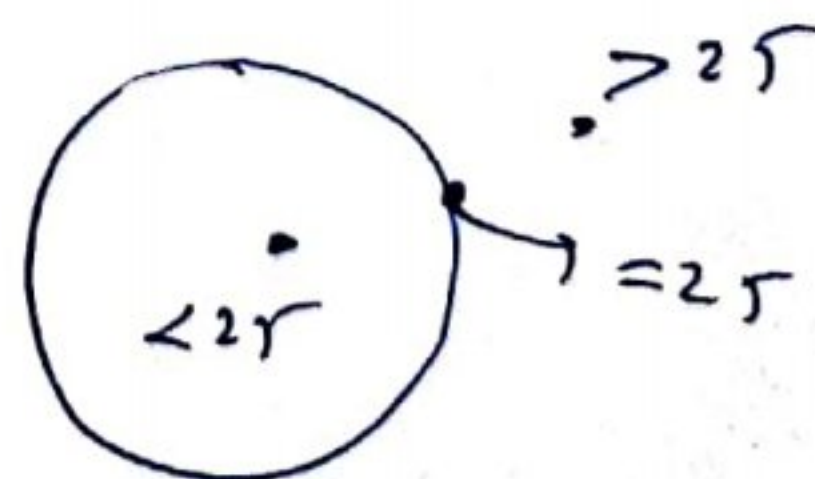
$$\Rightarrow x^2 + \frac{a^2}{4} - ax + y^2 + \frac{b^2}{4} - by = \frac{a^2}{4} + \frac{b^2}{4}$$

$$\Rightarrow \boxed{x^2 + y^2 - ax - by = 0} \quad \underline{\text{Ans}}$$

Q. 14 = 7 → Does the point $(-2.5, 3.5)$ lie inside, outside or on the circle $x^2 + y^2 = 25$?

$$x = -\frac{5}{2}$$

$$y = \frac{7}{2}$$



WORKSHEET No = 1 (CIRCLES)

Q₁ Find the equation of the circle passing through the points (4,1) & (6,5) and whose centre lies on the line $4x + y = 16$ Ans $x^2 + y^2 - 6x - 8y + 15 = 0$

Q₂ Find equation of circle with centre (2,2) and passes through the point (4,5) Ans $x^2 + y^2 - 4x - 4y = 5$

Q₃ Find Centre and radius (by ~~C~~ Perfect Square) of the circle

$$2x^2 + 2y^2 - 16x + 20y - 24 = 0 \quad \text{Ans } (4, -5); \text{ radius} = \sqrt{53}$$

Q₄ Find the equation of the circle passes through the points (5, -8), (2, -9) & (2, 1)

$$\text{Ans } x^2 + y^2 - 4x + 8y - 5 = 0$$

Q₅ Find the equation of circle, the coordinates of the end points of whose diameter are (-1, 2) and (4, -3) Ans $x^2 + y^2 - 3x + y - 10 = 0$

Q₆ Find the equation of the circle "concentric" with the circle $x^2 + y^2 - 6x + 12y + 15 = 0$ and double of its area Ans $x^2 + y^2 - 6x + 12y - 15 = 0$

Qn. 7 Find the equation of the circle which passes through the origin and cuts off intercepts 3 and 4 on the coordinate axes

Ans $(x-\frac{3}{2})^2 + (y-2)^2 = \frac{25}{4}$

Qn. 8 Find the equation of a circle whose diameters are $2x-3y+12=0$ and $x+4y-5=0$ and area is 154 square units

Ans $(x+3)^2 + (y-2)^2 = 49$

Qn. 9 Find the equation of the circle which passes through the line of intersection of the lines $3x-2y-1=0$ and $4x+y-27=0$ and whose centre is $(2, -3)$

Ans $x^2 + y^2 - 4x + 6y - 96 = 0$

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