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ULTIMATE MATHEMATICS: By - AJAY MITTAL

CHAPTER: PERMUTATION & COMBINATION (P&C

CLASS No: 1

(•) Factorial !

- $5! = 1\times2\times3\times4\times5 = 120$
- $5! = 5\times4\times3\times2\times1 = 120$
- $3! = 3\times2\times1 = 1\times2\times3 = 6$
- $\boxed{n! = 1\times2\times3 \cdots n = n(n-1)(n-2)\cdots 1}$
- $(-ve)!$
- $(\text{fraction})!$
- $n \in N$
- $\boxed{0! = 1}$ (Assumption)
- $1! = 1$
- $6! + 2! \neq 8!$
- $6! - 2! \neq 4!$
- $5! = 5\times4!$
  $= 5\times4\times3!$
- $\dfrac{6!}{3!} = \dfrac{6\times5\times4\times\cancel{3!}}{\cancel{3!}}$
  $= 120$

e.g (1) If $(n+1)! = 90(n-1)!$ find value of n

Sol. $\dfrac{(n+1)!}{(n-1)!} = 90 \Rightarrow \dfrac{(n+1)(n)\cancel{(n-1)!}}{\cancel{(n-1)!}} = 90 \Rightarrow n^2+n-90 = 0$

$(n+10)(n-9) = 0$

$n = -10$, $\boxed{n=9}$

(X)

ex:2 If $\frac{1}{9!} + \frac{1}{10!} = \frac{x}{11!}$ find value of x

Soln

$$\frac{1}{9!} + \frac{1}{10 \times 9!} = \frac{x}{11 \times 10 \times 9!}$$

$$\Rightarrow 1 + \frac{1}{10} = \frac{x}{110}$$

$$\Rightarrow \frac{11}{10} = \frac{x}{110}$$

$$\Rightarrow x = 121 \text{ Ans}$$

Note L.C.M of $8!$ & $10! = 10!$

✓ Permutation : Number of ways of "arrangement"

✓ Combination: Number of ways of "selection"

✓ $^nP_r = \frac{n!}{(n-r)!}$ n → No of items/objects available, r → No of " " to be arranged

✓ $^nC_r = \frac{n!}{r!\,(n-r)!}$ n → No of items/objects available, r → No " " to be selected

✓ $\boxed{^nP_r = r! \times {^nC_r}}$

✓ $r \leq n$

✓ $r \neq -ve$; $r \neq$ fraction

P&C (class No: 1)

✓ A, B, C (Permutation)

(·) arrange taking 2 at a time : AB, BA, BC, CB, AC, CA = 6 ways

(·) arrange taking 3 at a time: ABC, ACB, BCA, BAC, CAB, CBA = 6

(·) arrange taking 1 at a time: A, B, C = 3

(·) $n=3,\ r=2$ $\quad {}^3P_2 = \frac{3!}{(3-2)!} = \frac{3!}{1!} = \frac{6}{1} = 6$

(·) $n=3\ ; r=3$ $\quad {}^3P_3 = \frac{3!}{0!} = \frac{6}{1} = 6$

✓ A, B, C (Combination)

(·) Select 2 at a time: AB, BC, CA = 3

(·) $n=3,\ r=2$ $\quad {}^3C_2 = \frac{3!}{2!\,1!} = \frac{6}{2} = 3$

(·) Select 1 at a time = A, B, C = 3

$${}^3C_1 = \frac{3!}{1!\,2!} = \frac{6}{2} = 3$$

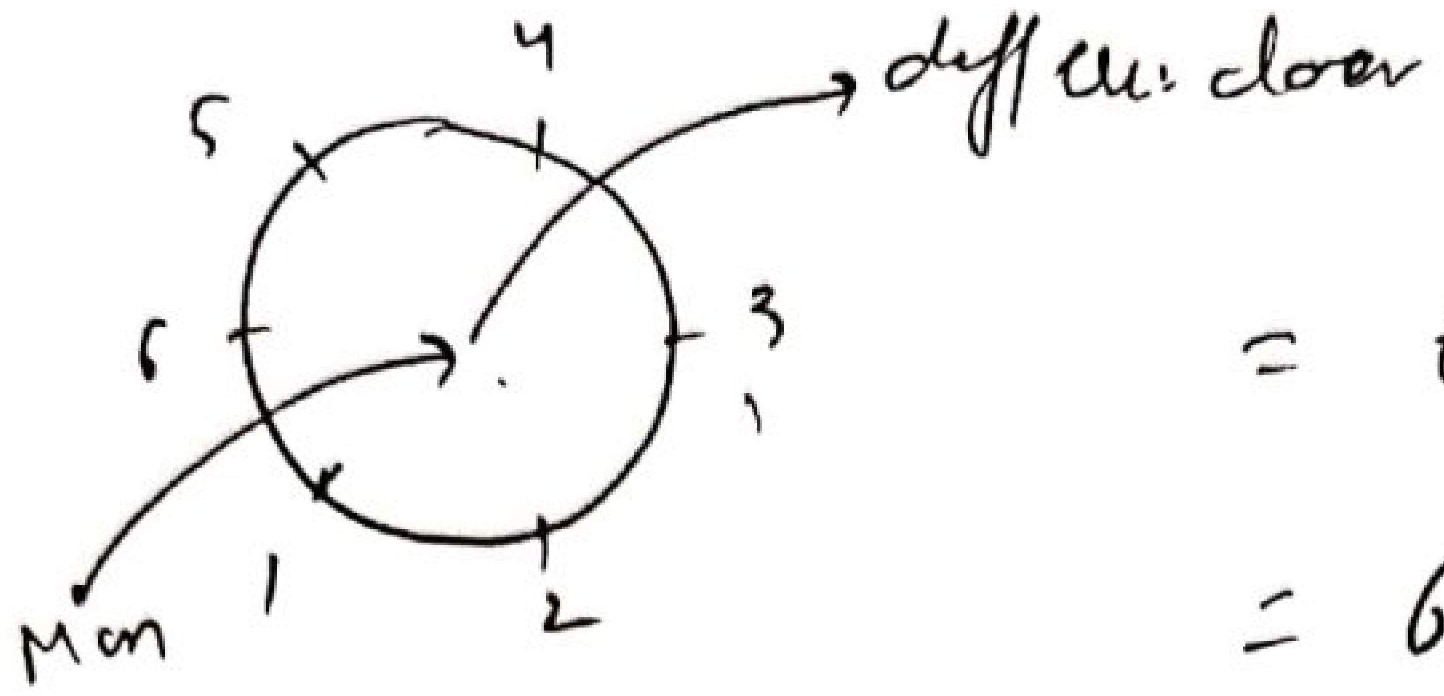
$$\boxed{{}^nP_1 = {}^nC_1 \quad \text{when } r=1}$$

(+) (×) → two operations

(+) → options, या, OR, cases

(×) → compulsion, and, ----

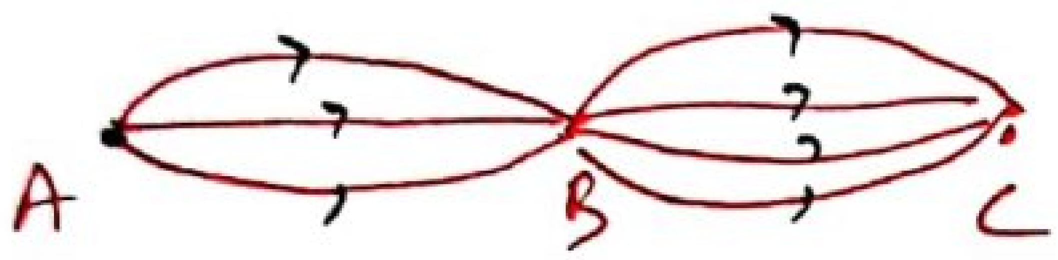
eg 1



$$= 6 \times \underbrace{5}_{\text{differ door}} = 30$$

$$= 6 \times \underbrace{1}_{\text{Same door}} = 6$$

eg 2



$$= 3 \times 4 = 12$$

eg 3

8 person

C	V.C
8	7

$$= 8 \times 7 = 56$$

eg 4

5 finger (distribute)

4 rings

$$= 5 \times 4 \times 3 \times 2 = 120$$

$$= 5 \times 5 \times 5 \times 5 = 5^4 = 625$$

Ques 1 If $P(5, r) = 2 \cdot P(6, r-1)$ find r $\boxed{{}^nP_r = P(n,r)}$

Sol

$${}^5P_r = 2 \cdot {}^6P_{r-1}$$

$$\Rightarrow \frac{{}^5P_r}{{}^6P_{r-1}} = 2$$

$$\Rightarrow \frac{\frac{5!}{(5-r)!}}{\frac{6!}{(7-r)!}} = 2$$

$$\Rightarrow \frac{5!\,(7-r)!}{(5-r)!\,6!} = 2$$

$$\Rightarrow \frac{\cancel{5!}\,(7-r)(6-r)\cancel{(5-r)!}}{\cancel{(5-r)!}\,6\times \cancel{5!}} = 2$$

$$\Rightarrow 42 - 7r - 6r + r^2 = 12$$

$$\Rightarrow r^2 - 13r + 30 = 0$$

$$\Rightarrow (r-10)(r-3) = 0$$

$r = 10$, $\boxed{r=3}$ Ans

Q No. 2 → If $^{2n}C_3 : {}^{n}C_3 = 11:1$ Find value of n

Sol

$$\frac{^{2n}C_3}{^{n}C_3} = \frac{11}{1}$$

$$\Rightarrow \frac{\dfrac{(2n)!}{\cancel{3!}(2n-3)!}}{\dfrac{n!}{\cancel{3!}(n-3)!}} = 11$$

$$\Rightarrow \frac{(2n)!\,(n-3)!}{(2n-3)!\,n!} = 11$$

$$\Rightarrow \frac{(2n)(2n-1)(2n-2)\cancel{(2n-3)!}\,\cancel{(n-3)!}}{\cancel{(2n-3)!}\,n(n-1)(n-2)\cancel{(n-3)!}} = 11$$

$$\Rightarrow \frac{2\times 2\,\cancel{(n-1)}\,(2n-1)}{\cancel{(n-1)}\,(n-2)} = 11 \Rightarrow 8n - 4 = 11n - 22$$

$3n = 18$ $n = 6$ Ans

Ques 3 If ${}^nP_r = {}^nP_{r+1}$ & ${}^nC_r = {}^nC_{r-1}$ find n & r

Sol, $\dfrac{{}^nP_r}{{}^nP_{r+1}} = 1$

$\Rightarrow \dfrac{\dfrac{\cancel{n!}}{(n-r)!}}{\dfrac{\cancel{n!}}{(n-r-1)!}} = 1$

$\Rightarrow \dfrac{(n-r-1)!}{(n-r)!} = 1$

$\Rightarrow \dfrac{\cancel{(n-r-1)!}}{(n-r)\cancel{(n-r-1)!}} = 1$

$\Rightarrow \boxed{1 = n-r}$

$\dfrac{{}^nC_r}{{}^nC_{r-1}} = 1$

$\dfrac{\dfrac{\cancel{n!}}{r!(n-r)!}}{\dfrac{\cancel{n!}}{(r-1)!(n-r+1)!}} = 1$

$\Rightarrow \dfrac{(r-1)!(n-r+1)!}{r!\ (n-r)!} = 1$

$\Rightarrow \dfrac{\cancel{(r-1)!}(n-r+1)\cancel{(n-r)!}}{r\cancel{(r-1)!}\ \cancel{(n-r)!}} = 1$

$\Rightarrow n-r+1 = r$

$\Rightarrow \boxed{n-2r = -1}$

Solve then equal,

$\boxed{n=3 \quad \& \ r=2}$ Ans

(·) 10 chairs $n=10$, 6 Boys $r=6$, no. of ways of arrangem $= {}^{10}P_6$

$= {}^{10}C_6 \times 6! = {}^{10}P_6$

(·) 10 chair $n=10$, 10 Boys $r=10$, no. of ways $= {}^{10}P_{10} = \dfrac{10!}{0!} = 10!$

✓ If n different items are to be arranged then no. of ways $= n!$ (Imp)