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RELATION & FUNCTION

Solution of Worksheet No:- 3

Qn 1 → $f(x) = \dfrac{1}{\sqrt{x-5}}$

Domain: $x-5 > 0$

$\Rightarrow x > 5$

$\therefore$ Domain $x \in (5, \infty)$

Range: let $y = f(x)$

$\Rightarrow y = \dfrac{1}{\sqrt{x-5}}$ ------(1)

$\Rightarrow y^2 = \dfrac{1}{x-5}$

$\Rightarrow x-5 = \dfrac{1}{y^2}$

$\Rightarrow x = \dfrac{1}{y^2} + 5$

$\Rightarrow x = \dfrac{1+5y^2}{y^2}$

x is real for all value of y such that $y \in R - \{0\}$

but from eq(1) cannot be -ve

$\therefore$ Range $= (0, \infty)$ Ans

Qn-2 → $f(x) = \sqrt{16 - x^2}$

Domain: $16 - x^2 \geq 0$

$\Rightarrow x^2 - 16 \leq 0$

$\Rightarrow (x+4)(x-4) \leq 0$

$$\underset{-\infty}{\quad} \overset{+}{\longrightarrow} \underset{-4}{|} \overset{-}{\longrightarrow} \underset{4}{|} \overset{+}{\longrightarrow} \underset{\infty}{\quad}$$

$\therefore$ Domain $= [-4, 4]$

Range: let $y = f(x)$

$y = \sqrt{16 - x^2}$ ---(1)

CLASSTIME

Solution Worksheet No: 3 (2)

$y^2 = 16 - x^2$

$\Rightarrow x^2 = 16 - y^2$

$\Rightarrow x = \pm\sqrt{16 - y^2}$

x is real for all value of y such that

$16 - y^2 \geq 0$

$y^2 - 16 \leq 0$

$(y+4)(y-4) \leq 0$

$-\infty \quad + \quad -4 \quad - \quad 4 \quad + \quad \infty$ $\qquad y \in [-4, 4]$

but from eq (1) y can't be $-ve$

$\therefore$ Range $= [0, 4]$ Ans

Qs 3 → $f(x) = \dfrac{3}{2 - x^2}$

Domain $\quad 2 - x^2 \neq 0$

$(x^2 - 2) \neq 0$

$(x + \sqrt{2})(x - \sqrt{2}) \neq 0$

$\therefore$ Domain $= R - \{-\sqrt{2}, \sqrt{2}\}$

Range. let $y = f(x)$

$y = \dfrac{3}{2 - x^2}$

$\Rightarrow 2y - x^2 y = 3$

$\Rightarrow x^2 y = 2y - 3$

$\Rightarrow x^2 = \dfrac{2y - 3}{y}$

$\Rightarrow x = \pm\sqrt{\dfrac{2y - 3}{y}}$

x is real for all values of y such that

$\dfrac{2y - 3}{y} \geq 0$ and $y \neq 0$

$-\infty \quad + \quad 0 \quad - \quad 3/2 \quad + \quad \infty$ $\qquad ; y \neq 0$

$\therefore$ Range $(-\infty, 0) \cup [3/2, \infty)$ Ans

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Solution of Worksheet No: 3 (3)

Qn. 4 → $f(x) = \frac{x}{1+x^2}$

Domain $x \in R$

Range let $y = f(x)$

$y = \frac{x}{1+x^2}$

$\Rightarrow y + x^2 y = x$

$\Rightarrow x^2 y - x + y = 0$

Quadratic formula $x = \frac{1 \pm \sqrt{1-4y^2}}{2y}$

x is real for all value of y such that

$1-4y^2 \geq 0$ and $2y \neq 0$

$4y^2 - 1 \leq 0$ and $y \neq 0$

$(2y+1)(2y-1) \leq 0$ and $y \neq 0$

$-\infty \quad + \quad -1/2 \quad - \quad 1/2 \quad + \quad \infty$

$y \in [-\frac{1}{2}, \frac{1}{2}]$ & $y \neq 0$

but when $x = 0$ then $f(x) = 0$

$\therefore$ Range $= [-1/2, 1/2]$ Ans

Qn. 5 → $f(x) = \frac{|x-2|}{2-x}$

Domain $2 - x \neq 0$

$x \neq 2$ $\therefore$ Domain $R - \{2\}$

Range $f(x) = \frac{|x-2|}{2-x}$

$f(x)$ can take only two values $-1, 1$

$\therefore$ Range $= \{-1, 1\}$ Ans

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Solution of Worksheet No: 3 (4)

Q1. 6 → $f(x) = \frac{1}{1-x^2}$

Domain $1-x^2 \neq 0$

$(1+x)(1-x) \neq 0 \qquad x \neq -1, \quad x \neq 1$

$\therefore$ Domain $x \in R - \{-1, 1\}$

Range let $y = f(x)$

$y = \frac{1}{1-x^2}$

$\Rightarrow y - x^2 y = 1$

$\Rightarrow x^2 y = y - 1$

$\Rightarrow x^2 = \frac{y-1}{y}$

$\Rightarrow x = \pm\sqrt{\frac{y-1}{y}}$

x is real for all value of y such that

$\frac{y-1}{y} \geq 0 \quad \& \quad y \neq 0$

$-\infty \quad + \quad 0 \quad - \quad 1 \quad + \quad \infty$

Range $(-\infty, 0) \cup [1, \infty)$ Ans

Q1. 7 → $f(x) = \frac{ax-b}{cx-d}$

Domain $cx - d \neq 0$

$x \neq \frac{d}{c}$

$\therefore$ Domain $R - \{\frac{d}{c}\}$

Range let $y = f(x)$

$\Rightarrow y = \frac{ax-b}{cx-d}$

$\Rightarrow cxy - dy = ax - b$

$\Rightarrow x(cy - a) = dy - b$

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Solution of Worksheet No: 3 (5)

$$x = \frac{dy-b}{cy-a}$$

x is real for all values of y such that

$cy - a \neq 0$

$\Rightarrow y \neq \frac{a}{c}$

$\therefore$ Range $= R - \left\{\frac{a}{c}\right\}$ Ans

Qn. 8 $\rightarrow$ $f(x) = \sqrt{4x - x^2}$

Domain $4x - x^2 \geq 0$

$\Rightarrow x^2 - 4x \leq 0$

$\Rightarrow x(x-4) \leq 0$

$+ \quad - \quad +$

$-\infty \quad 0 \quad 4 \quad \infty$

$\therefore$ Domain $= [0, 4]$ Ans

Qn. 9 $\rightarrow$ $f(x) = \sqrt{\frac{x-2}{x+2}} + \sqrt{\frac{1-x}{1+x}}$

$\frac{x-2}{x+2} \geq 0 \; ; \; x+2 \neq 0$

$\frac{x-2}{x+2} \geq 0$ & $x \neq -2$

$+ \quad - \quad +$

$-\infty \quad -2 \quad 2 \quad \infty$

$x \in (-\infty, -2) \cup [2, \infty)$

$\frac{1-x}{1+x} \geq 0$ & $1 + x \neq 0$

$\frac{x-1}{1+x} \leq 0$ & $x \neq -1$

$+ \quad - \quad +$

$-\infty \quad -1 \quad 1 \quad \infty$

$x \in (-1, 1]$

taking common

$-\infty \quad -2 \quad -1 \quad 1 \quad 2 \quad \infty$

No common part

$\therefore$ Domain $= \phi$ Ans

CLASSTIME

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Solution of Worksheet No: 3 (6)

Q.10 → $f(x) = \sqrt{\dfrac{x+3}{(2-x)(x-5)}}$

$\dfrac{x+3}{(2-x)(x-5)} \geq 0$ and $(2-x)(x-5) \neq 0$

$\dfrac{x+3}{(x-2)(x-5)} \leq 0$ and $x \neq 2$ & $x \neq 5$

$-\infty$ — -3 + 2 − 5 + ∞

∴ Domain $(-\infty, -3] \cup (2, 5)$ Ans

Q.11 → $f(x) = \dfrac{x^2 - x}{x^2 + 2x}$

Range let $y = f(x)$

$\Rightarrow y = \dfrac{x^2 - x}{x^2 + 2x}$

$\Rightarrow x^2 y + 2xy = x^2 - x$

$\Rightarrow x^2 y - x^2 + 2xy + x = 0$

$\Rightarrow x^2(y-1) + x(2y+1) = 0$

$\Rightarrow x\left[x(y-1) + (2y+1)\right] = 0$

$\Rightarrow x = 0$ (or) $x(y-1) + (2y+1) = 0$

$\Rightarrow x = 0$ (or) $x = \dfrac{-(y-1)}{2y+1}$

x is real for all values y such that $2y+1 \neq 0 \Rightarrow y \neq -1/2$

Imp part ⟹ { when $y = 1$, then $x = 0$
{ but when $x = 0$, $f(x)$ d not defined

CLASSTIME

Scanned with CamScanner

$\therefore y = 1$ cannot be included

$\therefore$ Range $= R - \{ -\frac{1}{2}, 1 \}$ Ans

Qn. 12 → $f(x) = \sqrt{x-1} + \sqrt{3-x}$

$x-1 \geq 0$ and $3-x \geq 0$

$x \geq 1$ and $x-3 \leq 0$

$x \geq 1$ and $x \leq 3$

$x \in [1, \infty)$ and $x \in (-\infty, 3)$

taking common

$\therefore x \in [1, 3]$ Ans.

— x —