→ ULTIMATE MATHEMATICS →

Page No. 1

Date :

T-4 Solutions : of T-4

Solutions

Qn 1 → LHS $\cos(20^\circ)\cos(40^\circ)\cos(60^\circ)\cos(80^\circ)$

$= \frac{1}{2}\cos(20^\circ)\cos(40^\circ)\cos(80^\circ)$

$= \frac{1}{4}\left[2\cos(20^\circ)\cos(40^\circ)\right]\cos(80^\circ)$

$= \frac{1}{4}\left[\cos(60^\circ)+\cos(-20^\circ)\right]\cos(80^\circ)$

$= \frac{1}{4}\left[\frac{1}{2}+\cos(20^\circ)\right]\cos(80^\circ)$

$= \frac{1}{4}\left[\frac{1}{2}\cos(80^\circ)+\cos(80^\circ)\cos(20^\circ)\right]$

$= \frac{1}{8}\left[\cos(80^\circ)+2\cos(80^\circ)\cos(20^\circ)\right]$

Link

$= \frac{1}{8}\left[\cos(80^\circ)+\cos(100^\circ)+\cos(60^\circ)\right]$

$= \frac{1}{8}\left[\cos(80^\circ)+\cos(\overset{\text{II}}{180^\circ}-80^\circ)+\frac{1}{2}\right]$

$= \frac{1}{8}\left[\cancel{\cos(80^\circ)}-\cancel{\cos(80^\circ)}+\frac{1}{2}\right]$

$= \frac{1}{16}$ Ans

Qn 2 → LHS $\sin(10^\circ)\sin(30^\circ)\sin(50)\sin(70)$

$= \frac{1}{2}\left[\sin(10^\circ)\sin(50)\sin(70^\circ)\right]$

$= \frac{1}{4}\left[2\sin(10^\circ)\sin(50)\right]\sin(70^\circ)$

$= \frac{1}{4}\left[\cos(-40^\circ)-\cos(60^\circ)\right]\sin(70^\circ)$

(T-4) Solution

$$= \frac{1}{4}\left[\cos(40^\circ) - \frac{1}{2}\right]\sin(70^\circ)$$

$$= \frac{1}{4}\left[\sin(70^\circ)\cos(40^\circ) - \frac{1}{2}\sin(70^\circ)\right]$$

$$= \frac{1}{8}\left(2\sin(70^\circ)\cos(40^\circ) - \sin(70^\circ)\right]$$

$$= \frac{1}{8}\left(\sin(110^\circ) + \sin(30^\circ) - \sin(70^\circ)\right]$$

Link

$$= \frac{1}{8}\left(\sin(180-70) + \frac{1}{2} - \sin(70)\right]$$

$$= \frac{1}{8}\left(\cancel{\sin(70)} + \frac{1}{2} - \cancel{\sin(70)}\right]$$

$$= \frac{1}{8} \times \frac{1}{2} = \frac{1}{16}$$ Ans

---

3. $\sin(20^\circ)\sin(40^\circ)\sin(60^\circ)\sin(80^\circ)$

$$= \frac{\sqrt{3}}{2}\sin(20^\circ)\cdot\sin(40^\circ)\cdot\sin(80^\circ)$$

$$= \frac{\sqrt{3}}{4}\left[2\sin(20^\circ)\sin(40^\circ)\right]\sin(80^\circ)$$

$$= \frac{\sqrt{3}}{4}\left[\cos(-20^\circ) - \cos(60^\circ)\right]\sin(80)$$

$$= \frac{\sqrt{3}}{4}\left(\cos(20^\circ) - \frac{1}{2}\right)\sin(80^\circ)$$

$$= \frac{\sqrt{3}}{4}\left(\sin(80)\cos(20^\circ) - \frac{1}{2}\sin(80^\circ)\right]$$

(T-4) Solutions

Page No.

Date :

$$= \frac{\sqrt{3}}{8}\left[2\sin(80^\circ)\cos(20^\circ) - \sin(80^\circ)\right]$$

$$= \frac{\sqrt{3}}{8}\left[\sin(100^\circ) + \sin(60^\circ) - \sin(80^\circ)\right]$$

Link

$$= \frac{\sqrt{3}}{8}\left[\sin(\overset{\pi}{180} - 80) + \frac{\sqrt{3}}{2} - \sin(80)\right]$$

$$= \frac{\sqrt{3}}{8}\left[\cancel{\sin(80^\circ)} + \frac{\sqrt{3}}{2} - \cancel{\sin(80^\circ)}\right]$$

$$= \frac{\sqrt{3}}{8} \times \frac{\sqrt{3}}{2} = \frac{3}{16} \text{ Ans}$$

Q.No 4 → $\cos(10^\circ)\cos(30^\circ)\cos(50)\cos(70^\circ)$

Do yourself

Q.No 5 → LHS $\tan(20^\circ)\tan(40^\circ)\tan(60^\circ)\tan(80^\circ)$

$$= \sqrt{3}.\tan(20^\circ)\tan(40^\circ)\tan(80^\circ)$$

$$= \sqrt{3}.\frac{\sin(20)\sin(40)\sin(80^\circ)}{\cos(20).\cos(40^\circ)\cos(80^\circ)}$$

Multiply & divide by 2

$$= \sqrt{3}.\frac{(2\sin(20)\sin(40))\sin(80^\circ)}{(2\cos(20)\cos(40^\circ)).\cos(80^\circ)}$$

$$= \sqrt{3}.\frac{[\cos(-20^\circ) - \cos(60)]\sin(80)}{[\cos(60^\circ) + \cos(-20^\circ)]\cos(80^\circ)}$$

$$= \sqrt{3}\frac{[\cos(20) - 1/2]\sin(80^\circ)}{[1/2 + \cos(20^\circ)]\cos(80)}$$

(T-4) Solutions

$$= \sqrt{3}\,\frac{\left[\sin(80^\circ)\cos(20) - \frac{1}{2}\sin(80^\circ)\right]}{\left[\frac{1}{2}\cos(80^\circ) + \cos(80^\circ)\cdot\cos(20^\circ)\right]}$$

$$= \sqrt{3}\cdot\frac{\cancel{\frac{1}{2}}\left[2\sin(80^\circ)\cos(20) - \sin(80^\circ)\right]}{\cancel{\frac{1}{2}}\left[\cos(80^\circ) + 2\cos(80^\circ)\cos(20^\circ)\right]}$$

link

$$= \sqrt{3}\,\frac{\left[\sin(100^\circ) + \sin(60^\circ) - \sin(80^\circ)\right]}{\left[\cos(80^\circ) + \cos(100^\circ) + \cos(60^\circ)\right]}$$

$$= \sqrt{3}\,\frac{\left[\sin(180 - 80) + \frac{\sqrt{3}}{2} - \sin(80^\circ)\right]}{\left[\cos(80^\circ) + \cos(180^\circ - 80^\circ) + \frac{1}{2}\right]}$$

$$= \sqrt{3}\,\frac{\left[\cancel{\sin(80)} + \frac{\sqrt{3}}{2} - \cancel{\sin(80)}\right]}{\left[\cancel{\cos(80^\circ)} - \cancel{\cos(80^\circ)} + \frac{1}{2}\right]}$$

$$= \sqrt{3} \times \frac{\frac{\sqrt{3}}{\cancel{2}}}{\frac{1}{\cancel{2}}} = \sqrt{3} \times \sqrt{3} = 3 \text{ Ans}$$

→ Do yourself
(Same as Ans No. 5)

(T-4) Solution

Page No.

Date :

Qns 7 → $4\cos(12°)\cos(48°)\cos(72°)$

$= 2\left[2\cos(12°)\cos(48°)\right]\cos(72°)$

$= 2\left[\cos(60) + \cos(-36)\right]\cos(72°)$

$= 2\left[\frac{1}{2} + \cos(36°)\right]\cos(72°)$

$= 2\left[\frac{1}{2}\cos(72°) + \cos(72°)\cos(36°)\right]$

$= \frac{\cancel{2}}{\cancel{2}}\left[\cos(72°) + 2\cos(72°)\cos(36°)\right]$

Link

$= \cos(72°) + \cos(108°) + \cos(36°)$

$= \cos(72°) + \cos(180° - 72) + \cos(36)$

$= \cancel{\cos(72°)} - \cancel{\cos(72°)} + \cos(36)$

$= \cos(36°)$ **Ans**

Qns 8 → $\underline{LHS}$ $\sin A \cdot \sin(60°-A)\sin(60°+A)$

$= \frac{1}{2}\left[2\sin(\overset{A}{60°-A})\sin(\overset{B}{60°+A})\right]\sin A$

$= \frac{1}{2}\left[\cos(-2A) - \cos(120°)\right]\sin A$

$= \frac{1}{2}\left[\cos(2A) - \cos(\overset{II}{180°} - 60)\right]\sin A$

(T-4) Solutions

Page No.

Date :

$$= \frac{1}{2}\left[\cos(2A) + \cos(60^\circ)\right]\sin A$$

$$= \frac{1}{2}\left[\cos(2A) + \frac{1}{2}\right]\sin A$$

$$= \frac{1}{2}\left[\sin A \cdot \cos(2A) + \frac{1}{2}\sin A\right]$$

$$= \frac{1}{4}\left[2\sin A \cdot \cos(2A) + \sin A\right]$$

$$= \frac{1}{4}\left[\sin(3A) + \sin(-A) + \sin A\right]$$

$$= \frac{1}{4}\left[\sin(3A) - \cancel{\sin A} + \cancel{\sin A}\right]$$

$$= \frac{1}{4}\sin(3A) \quad \underline{\underline{\text{Ans}}}$$

$\underline{9} \rightarrow$ $\underline{\text{LHS}}$ $4\sin\theta \cdot \sin(60^\circ + \theta) \cdot \sin(120^\circ + \theta)$

$$= 2\left[2\sin(\underset{A}{60^\circ + \theta}) \cdot \sin(\underset{B}{120^\circ + \theta})\right] \cdot \sin\theta$$

III

$$= 2\left[\cos(-60^\circ) - \cos(180^\circ + 2\theta)\right]\sin\theta$$

$$= 2\left[\cos(60^\circ) + \cos(2\theta)\right]\sin\theta$$

$$= 2\left[\frac{1}{2} + \cos(2\theta)\right]\sin\theta$$

$$= 2\left[\frac{1}{2}\sin\theta + \sin\theta \cdot \cos(2\theta)\right]$$

$$= \frac{\cancel{2}}{\cancel{2}}\left[\sin\theta + 2\sin\theta \cdot \cos(2\theta)\right]$$

(T-4) Solutions

Page No.

Date :

$$= \sin\theta + \sin(3\theta) + \sin(-\theta)$$

$$= \cancel{\sin\theta} + \sin(3\theta) - \cancel{\sin\theta}$$

$$= \sin(3\theta)$$

= Ans

Qn 10 → LHS $2\cos\left(\overset{A}{\frac{\pi}{13}}\right)\cos\left(\overset{B}{\frac{9\pi}{13}}\right) + \cos\left(\frac{3\pi}{13}\right) + \cos\left(\frac{5\pi}{13}\right)$

$$= \cos\left(\frac{\pi}{13} + \frac{9\pi}{13}\right) + \cos\left(\frac{\pi}{13} - \frac{9\pi}{13}\right) + \cos\left(\frac{3\pi}{13}\right) + \cos\left(\frac{5\pi}{13}\right)$$

$$= \cos\left(\frac{10\pi}{13}\right) + \cos\left(-\frac{8\pi}{13}\right) + \cos\left(\frac{3\pi}{13}\right) + \cos\left(\frac{5\pi}{13}\right)$$

$$= \cos\left(\frac{10\pi}{13}\right) + \cos\left(\frac{8\pi}{13}\right) + \cos\left(\frac{3\pi}{13}\right) + \cos\left(\frac{5\pi}{13}\right)$$

link link

$$= \cos\left(\overset{\text{II}}{\pi - \frac{3\pi}{13}}\right) + \cos\left(\overset{\text{II}}{\pi - \frac{5\pi}{13}}\right) + \cos\left(\frac{3\pi}{13}\right) + \cos\left(\frac{5\pi}{13}\right)$$

$$= \cancel{-\cos\left(\frac{3\pi}{13}\right)} \cancel{- \cos\left(\frac{5\pi}{13}\right)} + \cancel{\cos\left(\frac{3\pi}{13}\right)} + \cancel{\cos\left(\frac{5\pi}{13}\right)}$$

$$= 0$$ Ans

Qns 11 → LHS $\cos(2\theta)\cos\frac{\theta}{2} - \cos(3\theta)\cos\left(\frac{9\theta}{2}\right)$

$$= \frac{1}{2}\left[2\cos(\overset{A}{2\theta})\cos\overset{B}{\frac{\theta}{2}} - 2\cos(\overset{A}{3\theta})\cos\left(\overset{B}{\frac{9\theta}{2}}\right)\right]$$

(T-4) Solutions

$$= \frac{1}{2}\left[\cos\left(2\theta+\frac{\theta}{2}\right)+\cos\left(2\theta-\frac{\theta}{2}\right)-\left\{\cos\left(3\theta+\frac{9\theta}{2}\right)+\cos\left(3\theta-\frac{9\theta}{2}\right)\right\}\right]$$

$$= \frac{1}{2}\left[\cos\left(\frac{5\theta}{2}\right)+\cos\left(\frac{3\theta}{2}\right)-\cos\left(\frac{15\theta}{2}\right)-\cos\left(\frac{-3\theta}{2}\right)\right]$$

$$= \frac{1}{2}\left[\cos\left(\frac{5\theta}{2}\right)+\cancel{\cos\left(\frac{3\theta}{2}\right)}-\cos\left(\frac{15\theta}{2}\right)-\cancel{\cos\left(\frac{3\theta}{2}\right)}\right]$$

$$= \frac{1}{2}\left[\cos\left(\overset{A}{\frac{5\theta}{2}}\right)-\cos\left(\overset{B}{\frac{15\theta}{2}}\right)\right]$$

$$= \frac{1}{2}\left(-2\sin\left(\frac{\frac{5\theta}{2}+\frac{15\theta}{2}}{2}\right)\cdot\sin\left(\frac{\frac{5\theta}{2}-\frac{15\theta}{2}}{2}\right)\right)$$

$$= \frac{1}{\cancel{2}}\left(-\cancel{2}\sin\left(\frac{20\theta}{4}\right)\cdot\sin\left(\frac{-10\theta}{4}\right)\right)$$

$$= -\sin(5\theta)\cdot\sin\left(-\frac{5\theta}{2}\right)$$

$$= +\sin(5\theta)\cdot\sin\left(\frac{5\theta}{2}\right) \quad \text{--- } \{\sin(-\theta) = -\sin$$

$$= \sin(5\theta)\cdot\sin\left(\frac{5\theta}{2}\right) \quad \text{Ans}$$

Ans 12 → DO yourself

(Same as Qn 11)

T-4 Solution

Ans 13 → $\left(\frac{\cos A + \cos B}{\sin A - \sin B}\right)^n + \left(\frac{\sin A + \sin B}{\cos A - \cos B}\right)^n$

$$= \left[\frac{\cancel{2}\cancel{\cos\left(\frac{A+B}{2}\right)}\cdot\cos\left(\frac{A-B}{2}\right)}{\cancel{2}\cancel{\cos\left(\frac{A+B}{2}\right)}\cdot\sin\left(\frac{A-B}{2}\right)}\right]^n + \left[\frac{\cancel{2}\cancel{\sin\left(\frac{A+B}{2}\right)}\cdot\cos\left(\frac{A-B}{2}\right)}{-\cancel{2}\cancel{\sin\left(\frac{A+B}{2}\right)}\cdot\sin\left(\frac{A-B}{2}\right)}\right]^n$$

$$= \left(\cot\left(\frac{A-B}{2}\right)\right)^n + \left(-\cot\left(\frac{A-B}{2}\right)\right)^n$$

$$= \cot^n\left(\frac{A-B}{2}\right) + (-1)^n\cdot\cot^n\left(\frac{A-B}{2}\right)$$

Case I when $n \rightarrow$ even

we know that $(-1)^n = 1$

$$= \cot^n\left(\frac{A-B}{2}\right) + \cot^n\left(\frac{A-B}{2}\right)$$

$$= 2\cot^n\left(\frac{A-B}{2}\right)$$

Case II when $n \rightarrow$ odd

we know that $(-1)^n = -1$

$$= \cancel{\cot^n\left(\frac{A-B}{2}\right)} - \cancel{\cot^n\left(\frac{A-B}{2}\right)}$$

$= 0$ Ans

Page No.

Date :

(T-4) Solutions

Ans 1(4) given $\cos(\alpha-\beta)+\cos(\beta-\gamma)+\cos(\gamma-\alpha)=-\frac{3}{2}$

$\cos\alpha\cos\beta+\sin\alpha\sin\beta+\cos\beta\cos\gamma+\sin\beta\sin\gamma+\cos\gamma\cos\alpha$
$+\sin\gamma\sin\alpha=-\frac{3}{2}$

$2\cos\alpha\cos\beta+2\sin\alpha\sin\beta+2\cos\beta\cos\gamma+2\sin\beta\sin\gamma$
$+2\cos\gamma\cos\alpha+2\sin\gamma\sin\alpha=-3$

$2\cos\alpha\cos\beta+2\sin\alpha\sin\beta+2\cos\beta\cos\gamma+2\sin\beta\sin\gamma$
$+2\cos\gamma\cos\alpha+2\sin\gamma\sin\alpha+1+1+1=0$

$2\cos\alpha\cos\beta+2\sin\alpha\sin\beta+2\cos\beta\cos\gamma+2\sin\beta\sin\gamma$
$+2\cos\gamma\cos\alpha+2\sin\gamma\sin\alpha+(\sin^2\alpha+\cos^2\alpha)+$
$(\sin^2\beta+\cos^2\beta)+(\sin^2\gamma+\cos^2\gamma)=0$

$(\cos^2\alpha+\cos^2\beta+\cos^2\gamma+2\cos\alpha\cos\beta+2\cos\beta\cos\gamma+$
$\cos\gamma\cos\alpha)+(\sin^2\alpha+\sin^2\beta+\sin^2\gamma+2\sin\alpha\sin\beta$
$+2\sin\beta\sin\gamma+2\sin\alpha\sin\gamma)=0$

$(\cos\alpha+\cos\beta+\cos\gamma)^2+(\sin\alpha+\sin\beta+\sin\gamma)^2=0$

$\{\because a^2+b^2+c^2+2ab+2bc+2ca=(a+b+c)^2\}$

this is possible only when

$\cos\alpha + \cos\beta + \cos\gamma = 0$ and

$\sin\alpha + \sin\beta + \sin\gamma = 0$

{∵ If $a^2 + b^2 = 0$
then $a = 0$ and $b = 0$}

$\Rightarrow \cos\alpha + \cos\beta + \cos\gamma = \sin\alpha + \sin\beta + \sin\gamma = 0$

proved