SOLUTION : REVISION WORKSHEET NO: 2

SEQUENCE & SERIES

ANS: 1 → $\frac{1}{b+c}, \frac{1}{c+a}, \frac{1}{a+b}$ are in AP

if $\frac{1}{c+a} - \frac{1}{b+c} = \frac{1}{a+b} - \frac{1}{c+a}$

iff ⇒ $\frac{b+\cancel{c}-\cancel{c}-a}{\cancel{(c+a)}(b+c)} = \frac{c+\cancel{a}-\cancel{a}-b}{(a+b)\cancel{(c+a)}}$

iff ⇒ $(b-a)(a+b) = (c-b)(b+c)$

iff ⇒ $-(a^2-b^2) = -(b^2-c^2)$

iff $-a^2+b^2 = -b^2+c^2$

iff $2b^2 = a^2+c^2$

iff a^2, b^2, c^2 are in AP

which is true --- (given a^2, b^2, c^2 are in AP)

∴ $\frac{1}{b+c}, \frac{1}{c+a}, \frac{1}{a+b}$ are in A.P PROVED

ANS: 2 → given $a\left(\frac{1}{b}+\frac{1}{c}\right), b\left(\frac{1}{c}+\frac{1}{a}\right), c\left(\frac{1}{a}+\frac{1}{b}\right)$ are in A.P

⇒ $a\left(\frac{1}{b}+\frac{1}{c}\right)+1, b\left(\frac{1}{c}+\frac{1}{a}\right)+1, c\left(\frac{1}{a}+\frac{1}{b}\right)+1$ are in AP

⇒ $a\left(\frac{1}{b}+\frac{1}{c}+\frac{1}{a}\right), b\left(\frac{1}{c}+\frac{1}{a}+\frac{1}{b}\right), c\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)$ are in AP

divide by $\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)$

⇒ a, b, c are in AP Proved

ANS 3 → (i) given $\frac{b+c}{a}, \frac{c+a}{b}, \frac{a+b}{c}$ are in AP

$\Rightarrow \frac{b+c}{a}+1, \frac{c+a}{b}+1, \frac{a+b}{c}+1$ are in AP

$\Rightarrow \frac{b+c+a}{a}, \frac{c+a+b}{b}, \frac{a+b+c}{c}$ are in AP

divide by $(a+b+c)$

$\Rightarrow \frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in AP PROVED

(ii) From part (i) $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in AP

multiply by abc

$\Rightarrow bc, ac, ab$ are in AP PROVED

Ans 4 → given $\frac{b+c-a}{a}, \frac{c+a-b}{b}, \frac{a+b-c}{c}$ are in AP

adding 2

$\Rightarrow \frac{b+c-a}{a}+2, \frac{c+a-b}{b}+2, \frac{a+b-c}{c}+2$ are in AP

$\Rightarrow \frac{b+c+a}{a}, \frac{c+a+b}{b}, \frac{a+b+c}{c}$ are in AP

divide by $a+b+c$

$\Rightarrow \frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in AP PROVED

Ans 5 → let he saved Rs 'a' in 1st year

total amount saved = Rs 66,000

total time = 20 years

2nd year he saved = Rs $(a+200)$

3rd year he saved = Rs $(a+400)$

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AP 1st term = a

common difference = 200

Sum = 66000

$n = 20$

$66000 = \frac{20}{2}\left[2a + 19 \times 200\right]$

$\Rightarrow 66000 = 10(2a + 3800)$

$\Rightarrow 6600 = 2a + 3800$

$\Rightarrow 2800 = 2a$

$\Rightarrow a = 1400$

$\therefore$ In 1st year he saved Rs. 1400 Ans

Ans 6 → AM of p^{th} & q^{th} term $= \frac{p^{th} + q^{th}}{2}$

$= \frac{a + (p-1)d + a + (q-1)d}{2}$

$= \frac{2a + d(p+q-2)}{2}$

AM of r^{th} and s^{th} term $= \frac{r^{th} + s^{th}}{2}$

$= \frac{a + (r-1)d + a + (s-1)d}{2}$

$= \frac{2a + d(r+s-2)}{2}$

Given both AM's are equal

$\Rightarrow \frac{2a + d(p+q-2)}{\cancel{2}} = \frac{2a + d(r+s-2)}{\cancel{2}}$

$\Rightarrow \cancel{d}(p+q-2) = \cancel{d}(r+s-2)$

$\Rightarrow p+q = r+s$ Proved

Q. 17

Given $a_p = a$ & $a_q = b$

$\Rightarrow A + (p-1)d = a$ --- (1) & $A + (q-1)d = b$ --- (2)

(1) − (2)

$\Rightarrow d(p - \cancel{1} - q + \cancel{1}) = a - b$

$\Rightarrow d = \dfrac{a-b}{p-q}$ put in (i)

$\Rightarrow A + \dfrac{(p-1)(a-b)}{p-q} = a$

$\Rightarrow A = a - \dfrac{(p-1)(a-b)}{p-q}$

Now

$S_{p+q} = \dfrac{p+q}{2}\left[2A + (p+q-1)d\right]$

$= \dfrac{p+q}{2}\left[2a - \dfrac{2(p-1)(a-b)}{p-q} + (p+q-1)\left(\dfrac{a-b}{p-q}\right)\right]$

$= \dfrac{p+q}{2}\left[\text{open all the brackets}\right]$

$= \dfrac{p+q}{2}\left[\dfrac{ap + bp - aq - bq + a - b}{p-q}\right]$

$= \dfrac{p+q}{2}\left[\dfrac{a(p-q) + b(p-q) + a-b}{p-q}\right]$

$= \dfrac{p+q}{2}\left[\dfrac{(\cancel{p-q})(a+b)}{\cancel{p-q}} + \dfrac{(a-b)}{p-q}\right] = \dfrac{p+q}{2}\left[a+b + \dfrac{a-b}{p-q}\right]$ Proved

Ans 8 → Let the nos are a, ar, ar^2

Given Sum = 52

$\Rightarrow a + ar + ar^2 = 52$

$\Rightarrow a(1+r+r^2) = 52$ --- (1)

Given $a(ar) + (ar)(ar^2) + (ar^2)(a) = 624$

$\Rightarrow a^2 r(1 + r^2 + r) = 624$

$\Rightarrow ar. \boxed{a(1+r+r^2)} = 624$

$\Rightarrow ar(52) = 624$ --- {from eq (i)}

$\Rightarrow \boxed{ar = 12}$

$\Rightarrow a = \frac{12}{r}$ put in (i)

$\Rightarrow \frac{12}{r}(1+r+r^2) = 52$

$\Rightarrow 12 + 12r + 12r^2 = 52r$

$\Rightarrow 12r^2 - 40r + 12 = 0$

$\Rightarrow 3r^2 - 10r + 3 = 0$

$\Rightarrow 3r^2 - 9r - r + 3 = 0$

$\Rightarrow 3r(r-3) - 1(r-3) = 0$

$\Rightarrow r = 3$ or $r = 1/3$

put in $ar = 12$

$a = 4$ $a = 36$

∴ Nos are $4, 12, 36$ or $36, 12, 4$ Ans

Ans 9 → 1^{st} set = 4 mails

2^{nd} set = $4 \times 4 = 16$ mails

3^{rd} set = $16 \times 4 = 64$ mails

Clearly it form a G.P

$a = 4, \quad r = \frac{16}{4} = 4$

$n = 8$

Sum of all the mails when 8^{th} set is mailed

$$S_8 = \frac{a(r^8 - 1)}{r - 1}$$

$$= \frac{4(4^8 - 1)}{3}$$

$$= \frac{4(65535)}{3}$$

$$= 4 \times 21845$$

$$= 87380 \text{ mails}$$

Cost $= \frac{1}{2} \times 87380$

= Rs 43690 Ans.

Ans 10 → given $a_n = 2(a_{n+1} + a_{n+2} + \cdots)$

$\Rightarrow a_n = 2$ (sum of infinite GP whose 1^{st} term $= a_{n+1}$ common ratio r)

$$\Rightarrow a r^{n-1} = 2\left(\frac{a_{n+1}}{1 - r}\right)$$

$\Rightarrow a r^{n-1} = \dfrac{2 a r^n}{1-r}$

$\Rightarrow \dfrac{r^n}{r} = \dfrac{2 r^n}{1-r}$

$\Rightarrow 1-r = 2r$

$\Rightarrow 1 = 3r$

$\Rightarrow \boxed{r = 1/3}$ Ans

Ans 11 → (i) $0.\overline{231}$

$= 0.231231231\ldots\ldots$

$= 0.231 + 0.000231 + 0.000000231 + \ldots\ldots$

$= \dfrac{231}{10^3} + \dfrac{231}{10^6} + \dfrac{231}{10^9} + \ldots\ldots$

$= \dfrac{231}{10^3}\left(1 + \dfrac{1}{10^3} + \dfrac{1}{10^6} + \ldots\ldots\right)$

Infinite GP $a = 1$, $r = \dfrac{1}{1000}$

$= \dfrac{231}{1000}\left(\dfrac{1}{1-\frac{1}{1000}}\right)$

$= \dfrac{231}{\cancel{1000}}\left(\dfrac{\cancel{1000}}{999}\right)$

$= \dfrac{231}{999}$ Ans

(ii) $3.5\overline{2}$

$= 3.5222\ldots\ldots$

$= 3.5 + 0.02 + 0.002 + 0.0002 + \ldots\ldots$

$= \dfrac{35}{10} + \dfrac{2}{10^2} + \dfrac{2}{10^3} + \dfrac{2}{10^4} + \ldots\ldots$

$= \frac{35}{10} + \frac{2}{100}\left[1 + \frac{1}{10} + \frac{1}{100} + \cdots \infty\right]$

$\longrightarrow$ Infinite GP $a = 1,\ r = 1/10$

$= \frac{35}{10} + \frac{2}{100}\left(\frac{1}{1-\frac{1}{10}}\right)$

$= \frac{35}{10} + \frac{2}{100}\left(\frac{10}{9}\right)$

$= \frac{35}{10} + \frac{2}{90}$

$= \cancel{\frac{350+2}{90}}\ \frac{315+2}{90}$

$= \frac{317}{90}$ Ans

Qn 12 → Given $a, b, c \rightarrow GP$

$\Rightarrow b^2 = ac$

$x \rightarrow$ AM of a & b

$\Rightarrow x = \frac{a+b}{2}$

$y \rightarrow$ AM of b & c

$\Rightarrow y = \frac{b+c}{2}$

(i) Taking LHS $\frac{a}{x} + \frac{c}{y}$

$= \frac{a}{\frac{a+b}{2}} + \frac{c}{\frac{b+c}{2}}$

$= \frac{2a}{a+b} + \frac{2c}{b+c}$

$= \frac{2ab + 2ac + 2ac + 2bc}{ab + ac + b^2 + bc}$

$= \frac{2(ab + 2ac + bc)}{ab + 2ac + bc}$ --- $\{\because b^2 = ac\}$

$= 2 =$ RHS Proved

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(ii) LHS $\frac{1}{x}+\frac{1}{y}$

$$= \frac{1}{\frac{a+b}{2}} + \frac{1}{\frac{b+c}{2}}$$

$$= \frac{2}{a+b} + \frac{2}{b+c}$$

$$= \frac{2b+2c+2a+2b}{ab+ac+b^2+bc}$$

$$= \frac{4b+2c+2a}{ab+2b^2+bc} \quad --- \{\because ac=b^2\}$$

$$= \frac{2(\cancel{2b+c+a})}{b(\cancel{a+2b+c})}$$

$$= \frac{2}{b}$$ Ans

Qu. 13 → (i) given a, b, c, d are in GP

Let $a=a$, $b=ar$, $c=ar^2$

(i) T.P $(a^2+b^2+c^2)$, $(ab+bc+cd)$, $(b^2+c^2+d^2)$ are in GP

or T.P $(ab+bc+cd)^2 = (a^2+b^2+c^2)(b^2+c^2+d^2)$

LHS $(ab+bc+cd)^2$

$$= (a^2r + a^2r^3 + a^2r^5)^2$$

$$= [a^2r(1+r^2+r^4)]^2$$

$$= a^4r^2(1+r^2+r^4)^2$$

Rhs $(a^2+b^2+c^2)(b^2+c^2+d^2)$

$= (a^2+a^2r^2+a^2r^4)(a^2r^2+a^2r^4+a^2r^6)$

$= a^2(1+r^2+r^4)\ a^2r^2(1+r^2+r^4)$

$= a^4r^2(1+r^2+r^4)^2$

Lhs = Rhs

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(ii) TP

$\frac{1}{a^2+b^2}, \frac{1}{b^2+c^2}, \frac{1}{c^2+d^2}$ are in GP

i.e TP $\left(\frac{1}{b^2+c^2}\right)^2 = \left(\frac{1}{a^2+b^2}\right)\left(\frac{1}{c^2+d^2}\right)$

Lhs $\left(\frac{1}{b^2+c^2}\right)^2$

put value of b & c

$= \left(\frac{1}{a^2r^2+a^2r^4}\right)^2 = \frac{1}{(a^2r^2(1+r^2))^2}$

$= \frac{1}{a^4r^4(1+r^2)^2}$

Rhs

$\frac{1}{(a^2+b^2)} \cdot \frac{1}{(c^2+d^2)}$

$= \frac{1}{(a^2+a^2r^2)(a^2r^4+a^2r^6)}$

$= \frac{1}{a^2(1+r^2)\ a^2r^4(1+r^2)}$

$= \frac{1}{a^4r^4(1+r^2)^2}$ Lhs = Rhs Proved