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ULTIMATE MATHEMATICS (1)

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Chapter: PMI

— Class No: 2 —

Type: 2 Divisibility:

Qn 1 By PMI show that
$10^{2n-1}+1$ is divisible / multiple by 11

Sol: let $P(n)$: $10^{2n-1}+1$ is div by 11

$P(1)$: $10^1+1 = 11$ which is div by 11
$\therefore P(1)$ is true

let $P(k)$ be true
$P(k)$: $10^{2k-1}+1 = 11m$ --- $(m \in Z)$
TC prove $P(k+1)$ is true

$P(k+1)$: $10^{2(k+1)-1}+1$

$= 10^{2k+1}+1$

$\Rightarrow 10^{2k-1} \cdot 10^2 + 1$

$= (11m-1)100+1$ --- {from $P(k)$}

$= 1100m - 99$

$= 11(100m-9)$
which is div by 11
$\therefore P(k+1)$ is true
$\therefore$ By PMI, $P(n)$ is true for all $n \in N$

Ans

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PMI (Class No: 2)

Qn 2 → By PMI, Show that 3^{2n} when divided by 8, the remainder is always 1.

Soln let $P(n)$: 3^{2n} when divided by 8, the remainder is always 1

$P(1)$: $3^2 = 9 = 8 + 1$ Clearly $P(1)$ is true

let $P(k)$ be true

$P(k)$: $3^{2k} = 8m + 1$ .... $(m \in Z)$

$P(k+1)$: 3^{2k+2}

$\Rightarrow 3^{2k} \cdot 3^2$

$= (8m + 1) 9$

$= 72m + 9$

$= 72m + 8 + 1$

$= 8(9m + 1) + 1$

clearly when divide by 8, it leaves the remainder 1

$\therefore$ $P(k+1)$ is true

.... Ans

Qns 3 → By PMI, Show that $x^{2n} - y^{2n}$ is divisible by $x + y$

Soln let $P(n)$: $x^{2n} - y^{2n}$ is div by $(x+y)$

$P(1)$: $x^2 - y^2$

$= (x+y)(x-y)$ which is div by $(x+y)$

Clearly $P(1)$ is true

let $P(k)$ be true

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PMI (Class No:- 2) (3)

$P(k)$: $x^{2k} - y^{2k} = (x+y)m$ --- (m ≠ 2)

To prove $P(k+1)$ is true.

$P(k+1)$: $x^{2k+2} - y^{2k+2}$

$= x^{2k} \cdot x^2 - y^{2k} \cdot y^2$

$= ((x+y)m + y^{2k})x^2 - y^{2k} \cdot y^2$

$= (x+y)mx^2 + y^{2k} \cdot x^2 - y^{2k} \cdot y^2$

$= (x+y)mx^2 + y^{2k}(x^2 - y^2)$

$= (x+y)mx^2 + y^{2k}(x+y)(x-y)$

$= (x+y)\left[mx^2 + y^{2k} \cdot (x-y)\right]$

Clearly it is div by $(x+y)$

$\therefore$ $P(k+1)$ is true

$\therefore$ $P(n)$ --- -----.

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Qn 4 → By ~~proof~~ PMI show that

$10^n + 3.4^{n+2} + 5$ is divisible by 9

Sol let $P(n)$: $10^n + 3.4^{n+2} + 5$ is div by 9

$P(1)$: $10 + 3.4^3 + 5$

$= 10 + 192 + 5$

$= 207 = 9 \times 23$ which is div by 9

$\therefore P(1)$ is true

let $P(k)$ be true

$P(k)$: $10^k + 3.4^{k+2} + 5 = 9m$ ...($m \in Z$)

$P(k+1)$: $10^{k+1} + 3.4^{k+3} + 5$

$= 10^k.10 + 3.4^{k+2}.4 + 5$

$= (9m - 3.4^{k+2} - 5)10 + 12.4^{k+2} + 5$

$= 90m - 30.4^{k+2} - 50 + 12.4^{k+2} + 5$

$= 90m - 45 + 4^{k+2}(12 - 30)$

$= 90m - 45 - 18.4^{k+2}$

$= 9(10m - 5 - 2.4^{k+2})$ which is div by 9

$\therefore P(k+1)$ is true

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Qn 5 (IMP) Show that
$2.7^n + 3.5^n - 5$ is div by 24 for all $n \in N$

Sol $P(n)$: $2.7^n + 3.5^n - 5$ is div by 24
$P(1)$: $2.7 + 3.5 - 5$
$= 14 + 15 - 5$
$= 24$ which is div by 24
$\therefore P(1)$ is true
let $P(k)$ be true
$P(k)$: $2.7^k + 3.5^k - 5 = 24m$ --- $(m \in Z)$
To prove $P(k+1)$ is true
$P(k+1)$: $2.7^{k+1} + 3.5^{k+1} - 5$
$= 2.7^k .7 + 3.5^k.5 - 5$
$= (24m - 3.5^k + 5)7 + 15.5^k - 5$
$= 24m\times 7 - 21.5^k + 35 + 15.5^k - 5$

Imp step
$= 24m\times 7 - 6.5^k + 30$
$= 24m\times 7 - 6(5^k - 5)$
$= 24m\times 7 - 6(4p)$ --- $\{\because (5^k - 5)$ is div by 4 for all $k \in N$

$5^k - 5$	
$k=1$	0
$k=2$	20
$k=3$	120
$k=4$	620
⋮	

$= 24(7m - p)$
which is div by 24
$\therefore P(k+1)$ is true

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PMF (Ex. No. - 2) (r)

Qn. 6 Show $n(n+1)(n+5)$ is multiple of 3 for all $n \in N$

Sol $P(n)$: $n(n+1)(n+5)$ is multiple of 3

$P(1)$: $1(1+1)(1+5)$

$= 12$ which is multiple of 3

$\therefore P(1)$ is true

$P(k)$: $k(k+1)(k+5) = 3m$ -- $(m \in Z)$

$P(k)$: $k^3 + 6k^2 + 5k = 3m$

$P(k+1)$: $(k+1)(k+2)(k+6)$

$= (k+1)(k^2 + 8k + 12)$

$= k^3 + 8k^2 + 12k + k^2 + 8k + 12$

$= (3m - 6k^2 - 5k) + 8k^2 + 12k + k^2 + 8k + 12$

$= 3m + 3k^2 + 15k + 12$

$= 3(m + k^2 + 5k + 4)$

which is div by 3

$\therefore P(k+1)$ is true

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Qn. 7 Show $n^3 - n$ is div by 6 for each natural number ≥ 2

Sol $P(n)$: $n^3 - n$ is div by 6 ; $n \geq 2$.

$P(2)$: $8 - 2 = 6$ which is div by 6

$\therefore P(1)$ is true

$P(k)$: $k^3 - k = 6m$

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PMI (Case No 2)

$P(k+1): \quad (k+1)^3 - (k+1)$

$= k^3 + 1 + 3k^2 + 3k - k - 1$

$= 6m + k + \cancel{1} + 3k^2 + \cancel{3}2k - \cancel{1}$

$= 6m + 3k^2 + 3k$

$= 6m + 3(k^2+k) \quad \ldots \{ k^2+k$ is always div by 2 $\}$

$= 6m + 3(2p) \quad$ for all $k \geq 2 \}$

$= 6(m+p)$

= which is div by 6

$\therefore P(k+1)$ is true

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Q No 8 If $P(n): \; 2.4^{2n+1} + 3^{3n+1}$ is divisible by λ, then find value of λ

Soln

$n=1 \quad P(1): \quad 2.4^3 + 3^4$

$= 128 + 81 = 209$

$n=2 \quad P(2): \quad 2.4^5 + 3^7$

$= 2 \times 1024 + 2187$

$= 2048 + 2187$

$= 4285$

H.C.F of 209 & 4285 is 11

Main part $\therefore P(n)$ is div by 11

$(\lambda = 11)$ Ans

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PMI class No = 2

Q.No 9 → If $P(n): 49^n + 16^n + k$ is divisible by 64 for all $n \in N$, then find the least −ve integral value of k

Sol: $P(n): 49^n + 16^n + k$

$P(1): 49 + 16 + k$

$P(1) = 65 + k$

$\therefore$ ~~64k + 65+k~~

$\therefore$ least −ve Integral value of $k = -1$ Ans

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← WORKSHEET No:- 2 →

Topic:- PMI

Qn 1 → Using PMI show that $3^{2n+2} - 8n - 9$ is divisible by 8

Qn 2 → Show that $41^n - 14^n$ is multiple of 27

Qn 3 → Show $n(n^2+5)$ is divisible by 6 for all $n \in N$

Qn 4 → Show $x^n - y^n$ is divisible by $x-y$ for all $n \in N$

Qn 5 → Show that $7^n - 2^n$ is div by 5 for all $n \in N$

Qn 6 → Show $n^3 - 7n + 3$ is div by 3 for all $n \in N$

Qn 7 → Show $3.5^{2n+1} + 2^{3n+1}$ is div by 17 for all $n \in N$

Qn 8 → Show that $n^3 + 3n^2 + 5n + 3$ is div by 3 for all $n \in N$

Qn 9 → Show that the sum of the cubes of three consecutive natural numbers is divisible by 9

<u>Hint</u> take $P(n): n^3 + (n+1)^3 + (n+2)^3$ is div by 9

Qn 10 → Show $5^{2n+2} - 24n - 25$ is divisible by 576 for all $n \in N$

Qn 11 → Show $11^{n+2} + 12^{2n+1}$ is div by 133 for all $n \in N$

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