॥ जय श्री राधे कृष्णा। जय श्री गिरिराज जी महाराज ॥.

ULTIMATE MATHEMATICS: BY AJAY MITTAL

CONIC SECTION MISCELLANOUS CLASS NO: 4

Ques: 1 → The focus of a parabolic mirror is at a distance of 5 cm from its vertex. If the mirror is 45 cm deep. Find its diameter

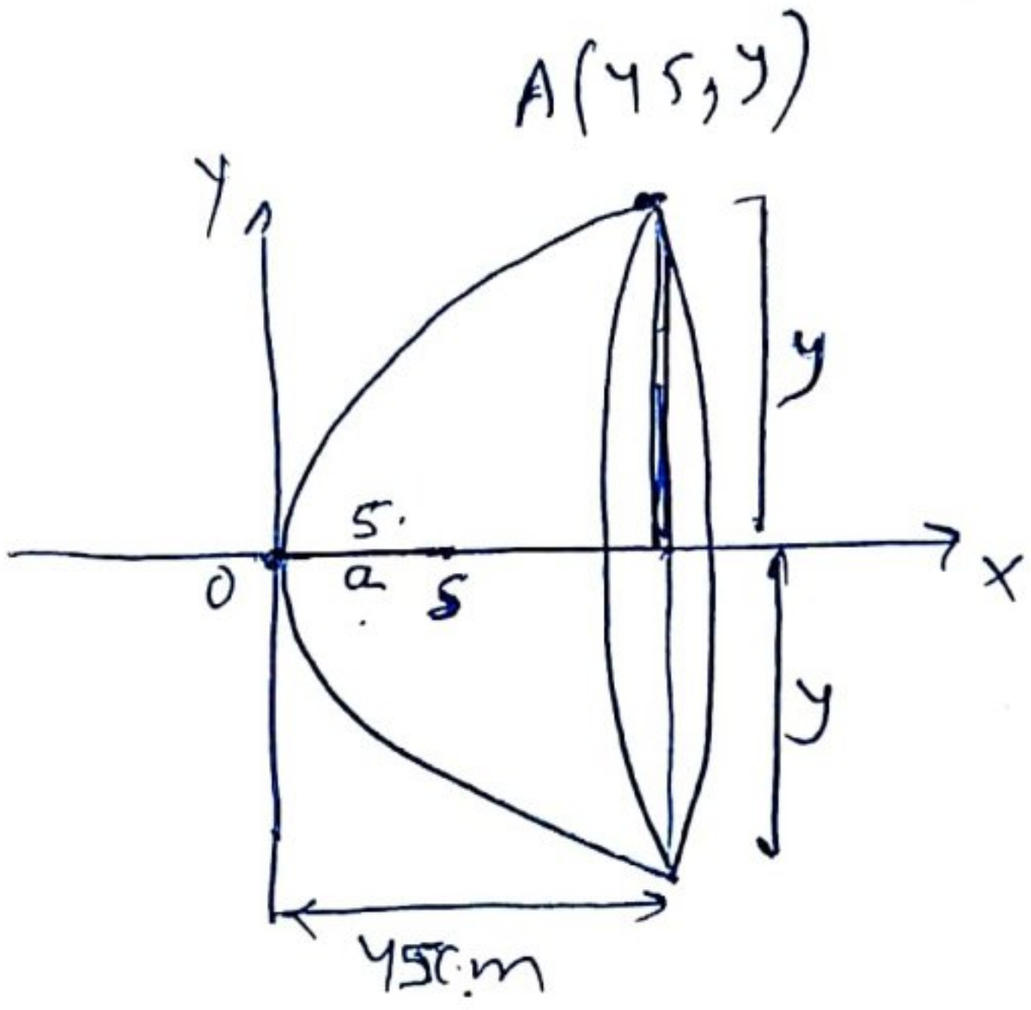
Soln Let equation of parabola is

$$y^2 = 4ax$$

we have $a = 5$

∴ equation becomes

$$y^2 = 20x$$



A(45, y) lies on it

$\Rightarrow y^2 = 900$

$\Rightarrow y = 30$

∴ Required diameter $= 2y = 60$ cm Ans

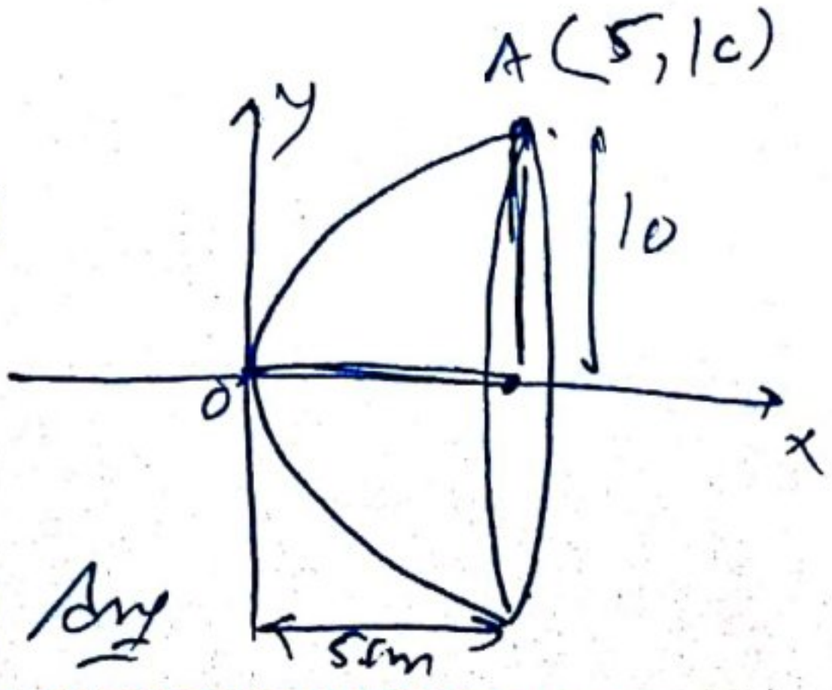
Ques 2 → If a parabolic reflector is 20 cm in diameter and 5 cm deep. Find the focus.

Solution Let equation of parabola is $y^2 = 4ax$

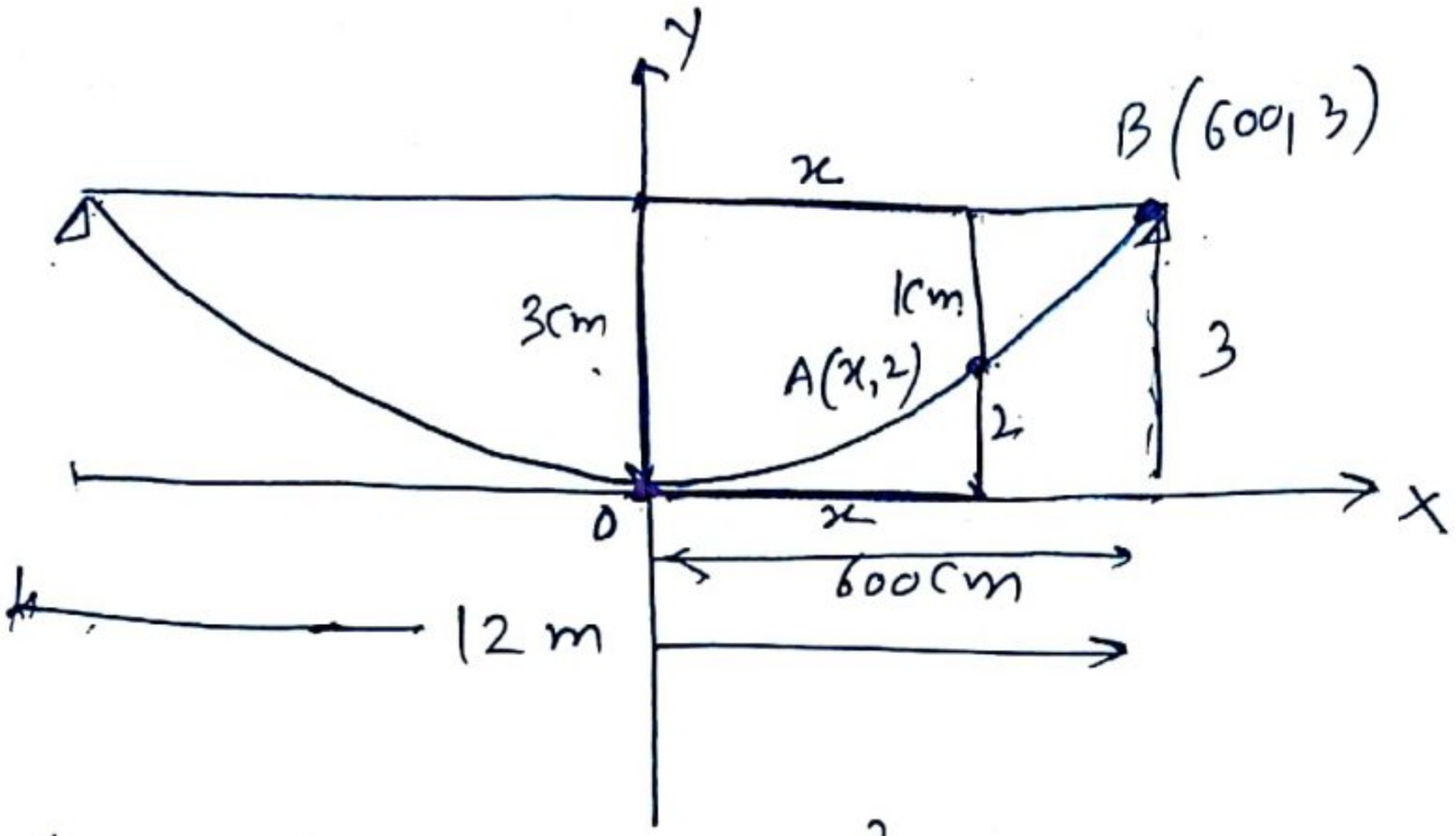
A(5, 10) lies on it

$100 = 20a$

$\Rightarrow \boxed{a = 5}$ ∴ focus $(a, 0) = (5, 0)$ Ans



Qn 3 → A beam is supported at its ends by supports which are 12 m apart. Since the load is concentrated at its centre, there is a deflection of 3cm at the centre and the deflected beam is in the shape of a parabola. How far from the centre is the deflection 1 cm?



Let equation of parabola is $x^2 = 4ay$

$B(600, 3)$ lies on it

$360000 = 12a$

$\Rightarrow \boxed{a = 30000}$

$\therefore$ Equation becomes $x^2 = 120000y$

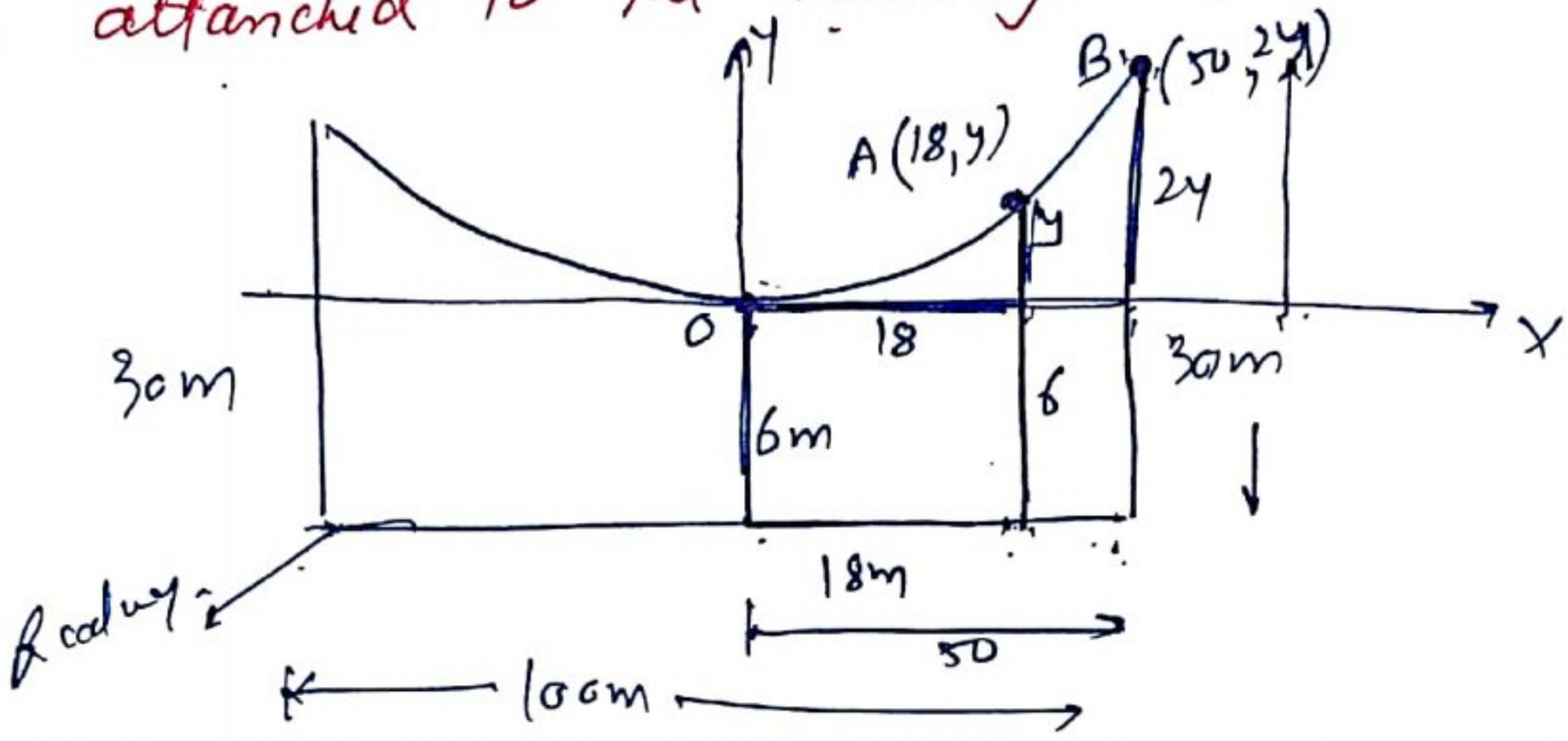
$A(x, 2)$ lies on it

$x^2 = 240000$

$x = 200\sqrt{6}$ cm

$\therefore$ Required distance $= 200\sqrt{6}$ cm Ans

Q.No. 4 → The cable of a uniformly loaded suspension bridge hangs in the form of a parabola. The roadway which is horizontal and 100 m long is supported by vertical wires attached to the cable, the longest wire being 30 m and the shortest being 6 m. Find the length of a supporting wire attached to the roadway 18 m from the middle.



Sol^n Let equation of parabola is $x^2 = 4ay$

$B(50, 24)$ lies on it

$2500 = 96a$

$\Rightarrow \boxed{a = \frac{2500}{96}}$

∴ equation becomes

$x^2 = 4\left(\frac{2500}{96}\right)y$

$x^2 = \frac{2500}{24}y$

$A(18, y)$ lies on it

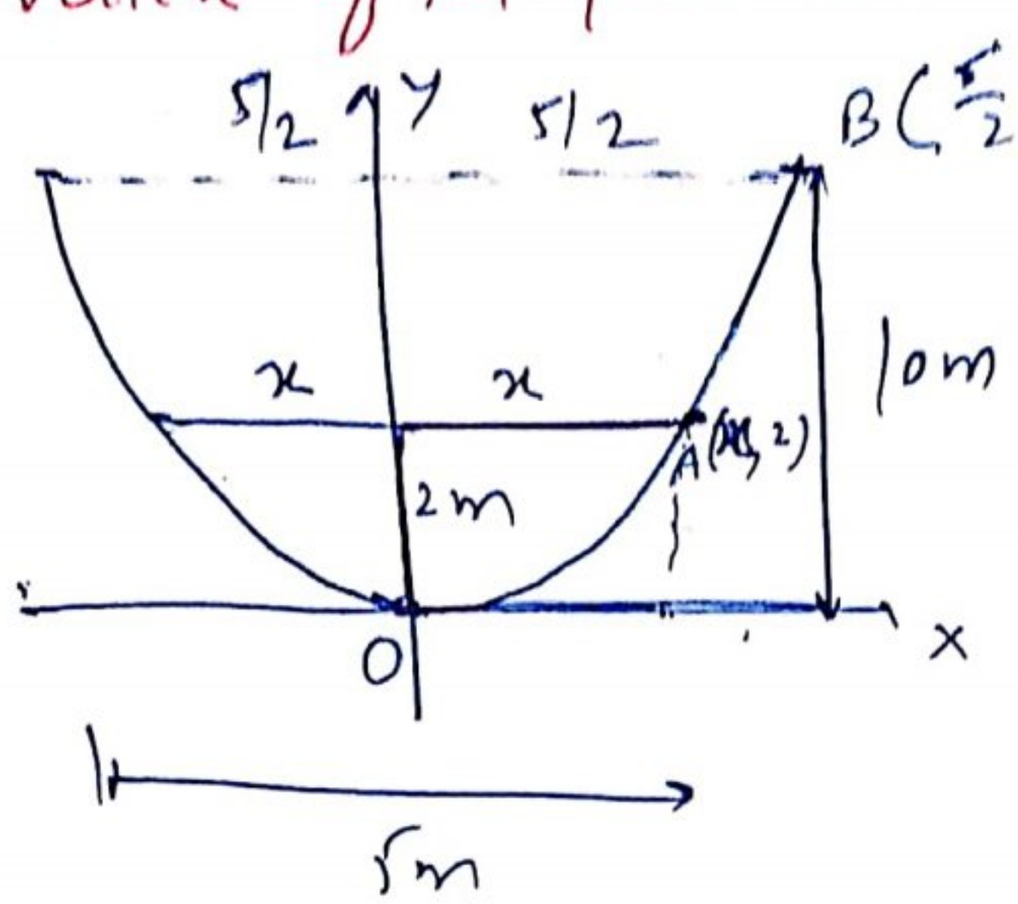
$324 = \frac{2500}{24}y$

$\Rightarrow \boxed{y = 3.1}$

Required length of wire
$= 6 + y$
$= 6 + 3.1$
$= 9.1$ m (Approx)

Ch. 5 → An arch is in the form of parabola with its Axis vertical. The arch is 10m high and 5 m wide at the base. How wide is it 2m from the vertex of the parabola?

Sol =



Let $x^2 = 4ay$

$A\left(\frac{5}{2}, 4c\right)$ lies on it

$\frac{25}{4} = 4a$

$\Rightarrow a = \frac{25}{160}$

$\therefore$ eqn becomes

$x^2 = 4\left(\frac{25}{160}\right)y$

$x^2 = \left(\frac{25}{40}\right)y$

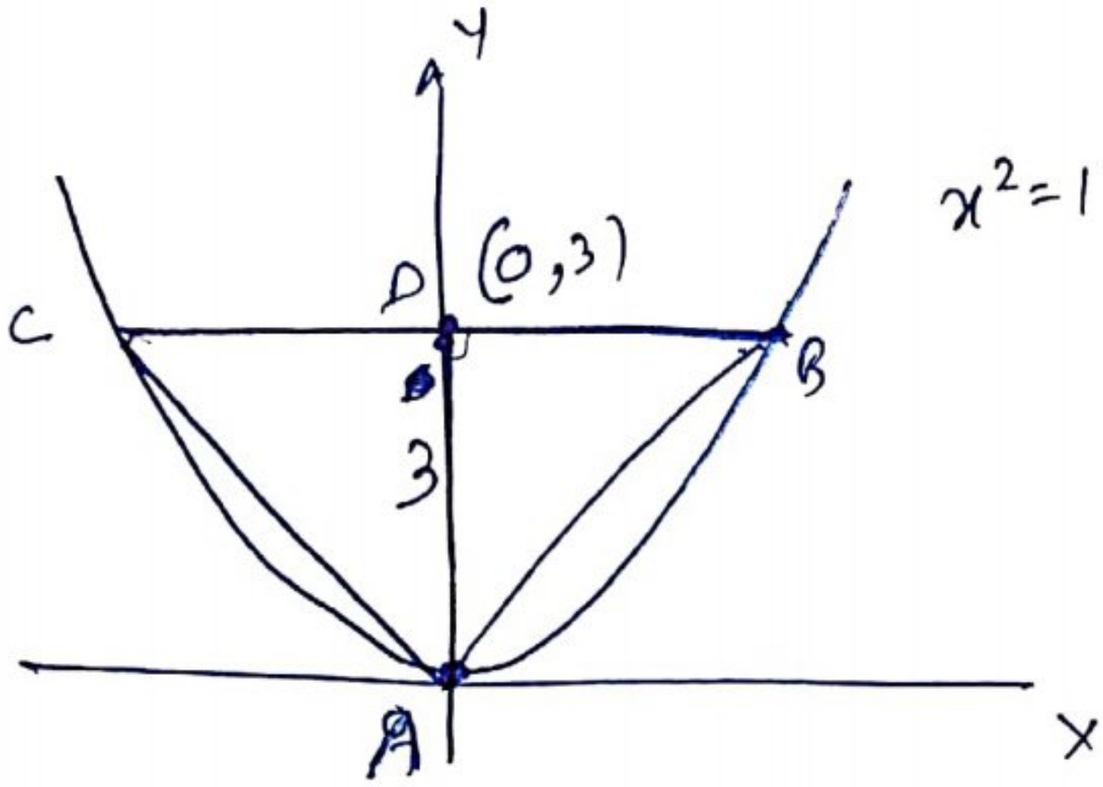
$x^2 = \frac{5}{8}y$

A (x, 2) lies on it

$x^2 = \frac{5}{4} \Rightarrow x = \frac{\sqrt{5}}{2}$

$\therefore$ Req. width $= 2x = \sqrt{5}$ m

Q#6 → Find the area of the triangle formed by the lines joining the vertex of the parabola $x^2=12y$ to the ends of its latus rectum.



Comp $x^2=12y$ with $x^2=4ay$

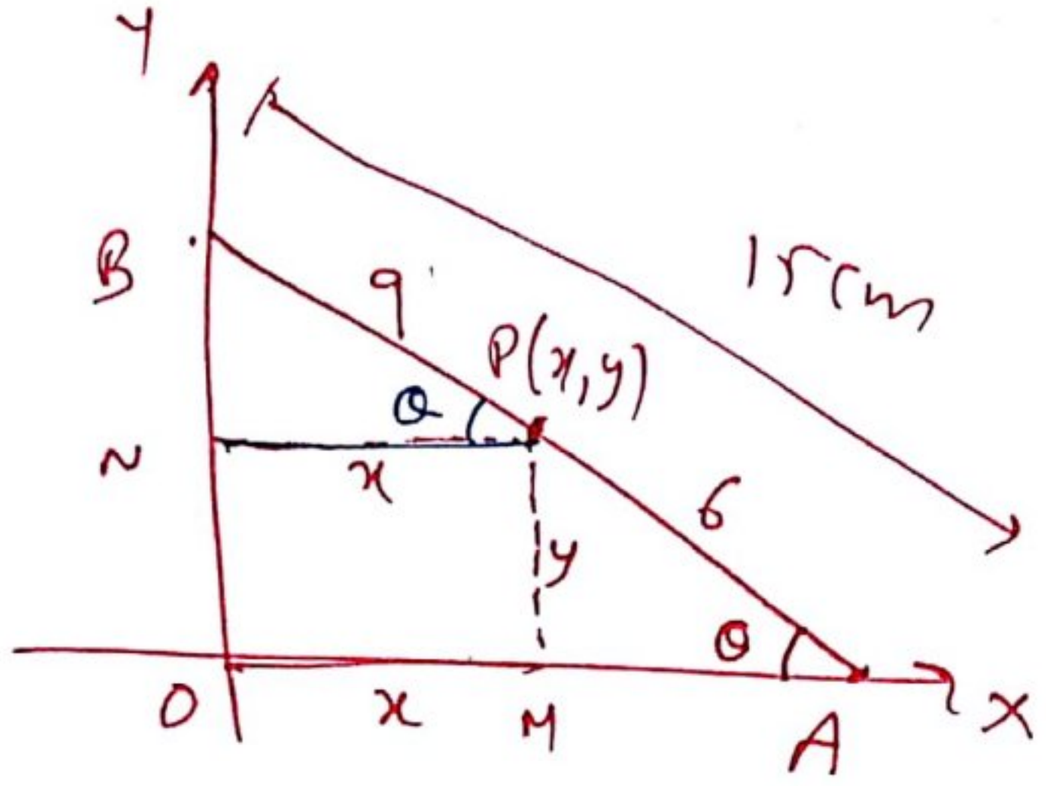
we get $(a=3)$

Length of latus rectum $= 4a = 4\times3 = 12$ (Base)

altitude $= a = 3$

Area $= \frac{1}{2} \times$ base $\times$ altitude $= \frac{1}{2}(12)(3) = 18$ sq. units

Q#7 → A rod AB of length 15cm rests in between two coordinates axes in such a way that the end point A lies on x-axis and end point B lies on y-axis. A point $P(x,y)$ is taken on the rod in such a way that $AP = 6cm$ show that the locus of point P is an ellipse



In Δ PNB

$\cos\theta = \frac{x}{9}$

$\cos^2\theta = \frac{x^2}{81}$

In Δ PAM

$\sin\theta = \frac{y}{6}$

$\sin^2\theta = \frac{y^2}{36}$

Adding

$\cos^2\theta + \sin^2\theta = \frac{x^2}{81} + \frac{y^2}{36}$

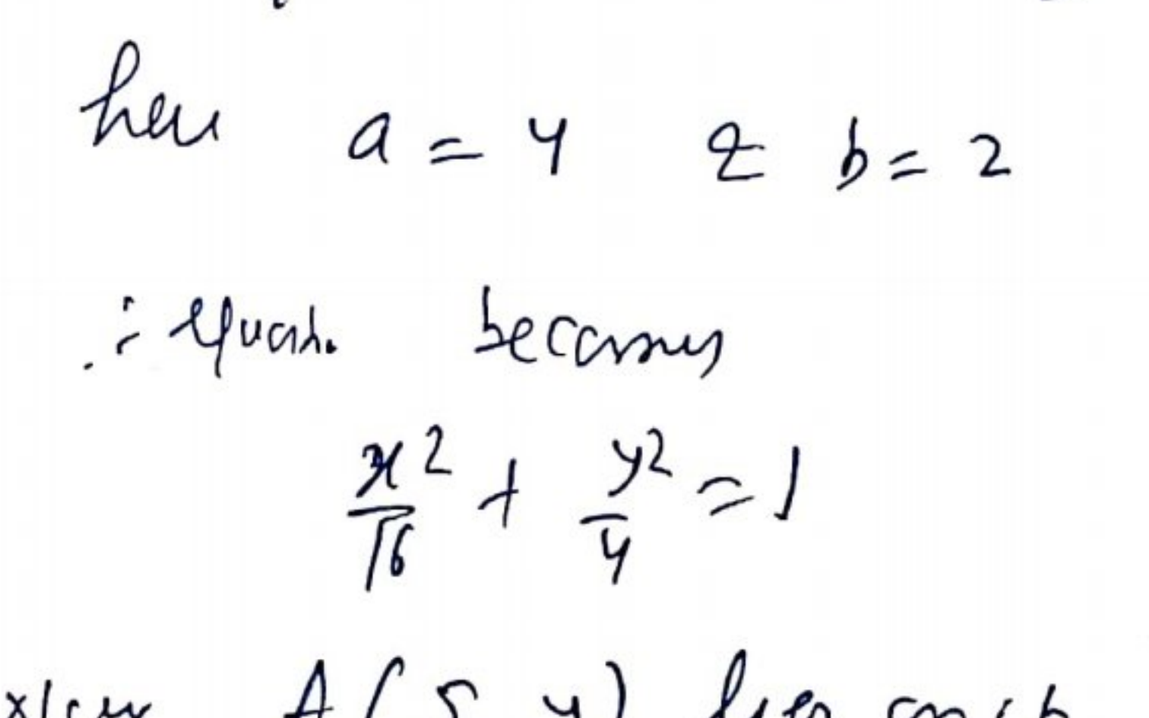
$\Rightarrow \frac{x^2}{81} + \frac{y^2}{36} = 1$

Clearly it represents the equation of ellipse

$\therefore$ locus is an ellipse

Qn 8 → An arch is in the form of a semi-ellipse. It is 8m wide and 2 m high at the centre. Find the height of the arch at a point 1.5 m from one end.

Sol:



Let Equation of ellipse is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

Then $a = 4$ & $b = 2$

$\therefore$ Equation becomes

$$\frac{x^2}{16} + \frac{y^2}{4} = 1$$

Also $A\left(\frac{5}{2}, y\right)$ lies on it

$$\frac{25}{4 \times 16} + \frac{y^2}{4} = 1$$

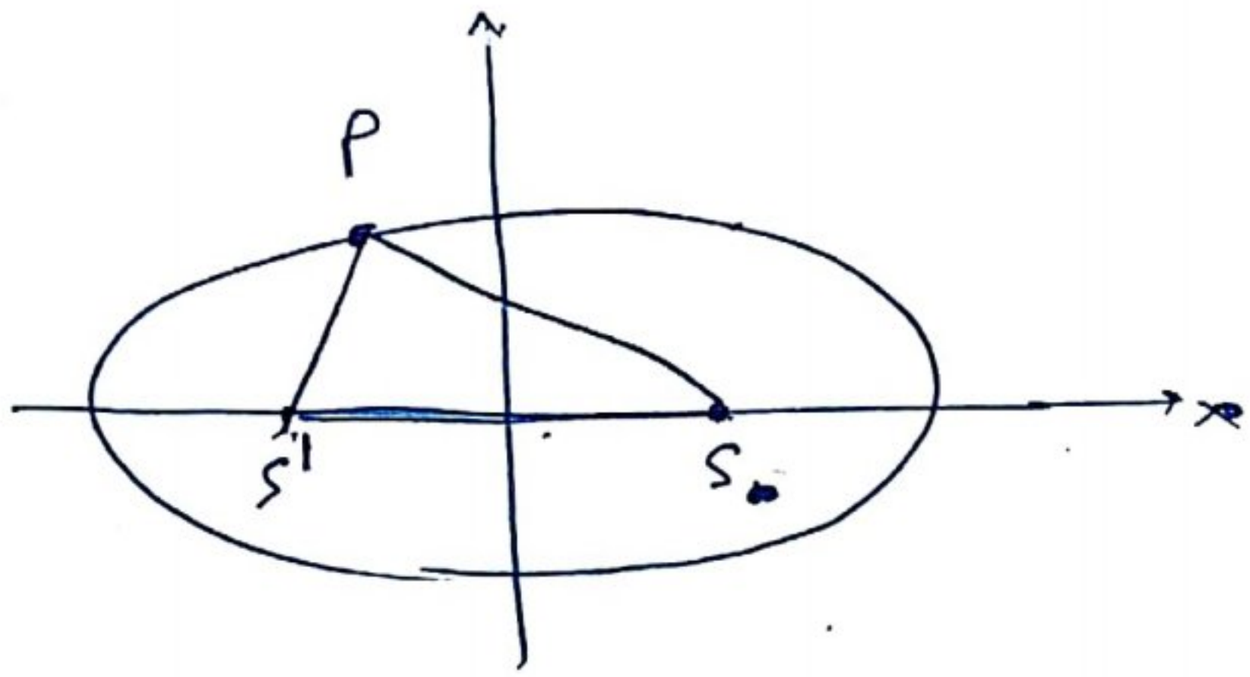
$$\frac{y^2}{4} = 1 - \frac{25}{64}$$

$$\frac{y^2}{4} = \frac{64 - 25}{64}$$

$$y^2 = \frac{39}{16}$$

$$\Rightarrow y = \frac{\sqrt{39}}{4} \text{ m}$$

Qn. 9 → A man running a race course notes the sum of the distances from the two flag posts from him is always 10m and the distance ~~from the~~ between the flag posts is 8m. Find the equation of the path traced by the man



New point

$$SP + S'P = 2a$$

Sum of focal distance = 2a

$$\Rightarrow 2a = 10 \text{ (given)}$$

$$a = 5$$

$$SS' = 2ae$$

$$2ae = 8 \text{ (given)}$$

$$ae = 4$$

$$e = \sqrt{1 - \frac{b^2}{a^2}}$$

$$ae = \sqrt{a^2 - b^2}$$

$$4 = \sqrt{25 - b^2}$$

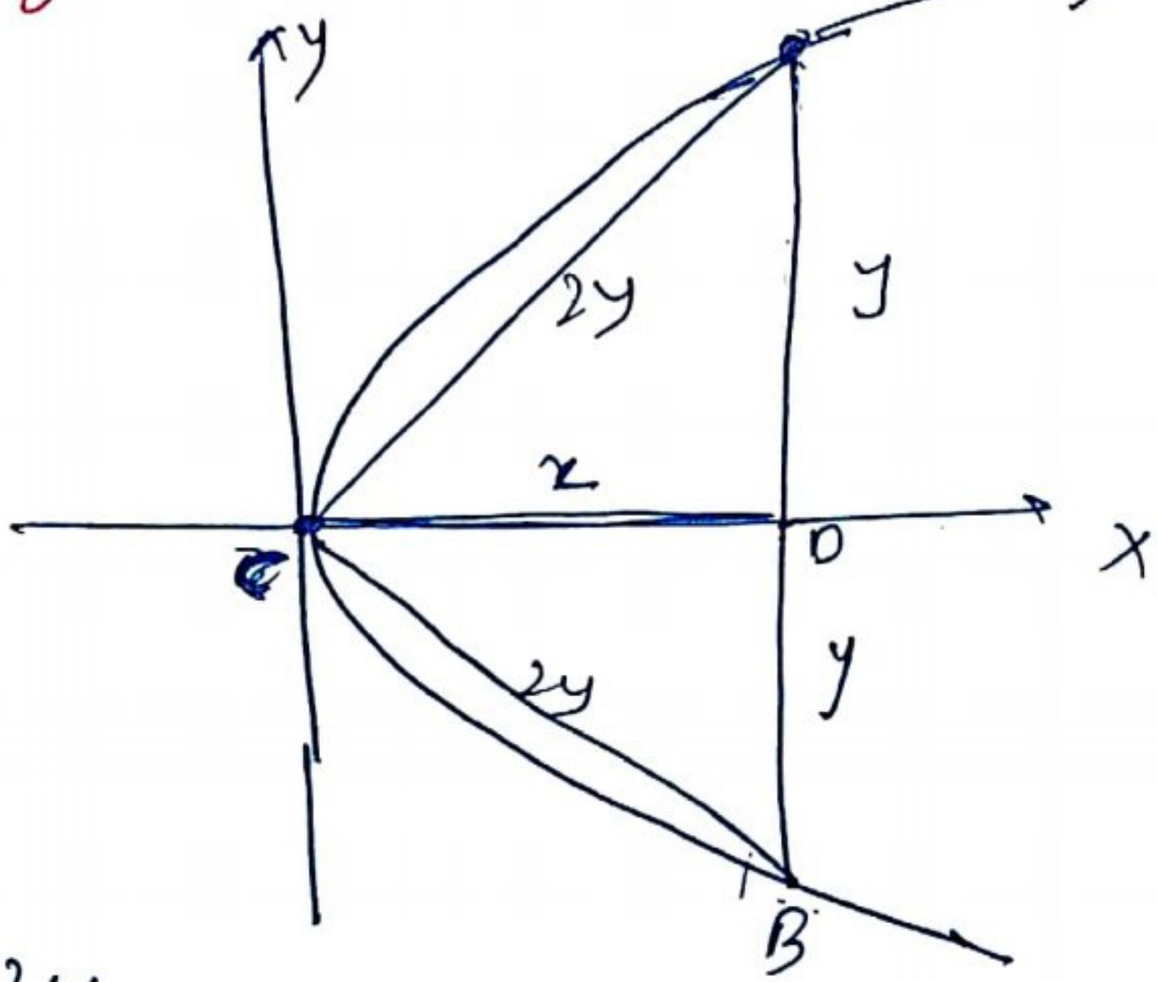
$$16 = 25 - b^2$$

$$\Rightarrow b^2 = 9$$

∴ equation of path / ellipse

$$\frac{x^2}{25} + \frac{y^2}{9} = 1 \text{ Ans}$$

Ans 10 → An equilateral triangle is inscribed in the parabola $y^2 = 4ax$, where one vertex is at the vertex of the parabola. Find the length of the side of the parabola



Let side $= 2y$

$\triangle ADC$ $\quad 4y^2 = x^2 + y^2$

$$3y^2 = x^2$$

We have eq. of parabola $y^2 = 4ax$

$$3y^2 = \left(\frac{y^2}{4a}\right)^2$$

$$\Rightarrow 3y^2 = \frac{y^4}{16a^2}$$

$$\Rightarrow y^4 = 48a^2y^2$$

$$\Rightarrow y^2 = 48a^2$$

$$\Rightarrow y = \sqrt{48a^2} \Rightarrow y = 4\sqrt{3}a$$

Side $= 2y$

Side $= 8\sqrt{3}a$ unit