TRIGONOMETRY CLASS NO: 6

Q.No 7 Show $\underbrace{\left(1+\cos\frac{\pi}{8}\right)\left(1+\cos\frac{3\pi}{8}\right)\left(1+\cos\frac{5\pi}{8}\right)\left(1+\cos\frac{7\pi}{8}\right)}_{\text{Link}} = \frac{1}{8}$

Sol:

$$= \left(1+\cos\frac{\pi}{8}\right)\left(1+\cos\frac{3\pi}{8}\right)\left(1+\cos\left(\pi-\frac{3\pi}{8}\right)\right)\left(1+\cos\left(\pi-\frac{\pi}{8}\right)\right)$$

$$= \left(1+\cos\frac{\pi}{8}\right)\left(1+\cos\frac{3\pi}{8}\right)\left(1-\cos\frac{3\pi}{8}\right)\left(1-\cos\frac{\pi}{8}\right)$$

$$= \left(1-\cos^2\frac{\pi}{8}\right)\left(1-\cos^2\frac{3\pi}{8}\right)$$

$$= \sin^2\left(\frac{\pi}{8}\right).\sin^2\left(\frac{3\pi}{8}\right)$$

$$= \left(\frac{1-\cos\left(\frac{\pi}{4}\right)}{2}\right)\left(\frac{1-\cos\frac{3\pi}{4}}{2}\right)$$

$$\boxed{\sin^2\theta = \frac{1-\cos(2\theta)}{2}}$$

$$= \left(\frac{1-\frac{1}{\sqrt{2}}}{2}\right)\left(\frac{1-\cos(180^\circ-45)}{2}\right)$$

$$= \left(\frac{1-\frac{1}{\sqrt{2}}}{2}\right)\left(\frac{1+\cos 45}{2}\right)$$

$$= \left(\frac{1-\frac{1}{\sqrt{2}}}{2}\right)\left(\frac{1+\frac{1}{\sqrt{2}}}{2}\right)$$

$$= \frac{1-\frac{1}{2}}{4} = \frac{1/2}{4} = \frac{1}{8}$$ Ans

Q.No 8 → Show that $\tan(4\theta) = \dfrac{4\tan\theta\,(1-\tan^2\theta)}{1-6\tan^2\theta+\tan^4\theta}$

Sol: $\tan(4\theta)$

= tan(4θ) Tajyo class No: 6 (T-6)

$$= \frac{2\tan(2\theta)}{1-\tan^2(2\theta)}$$

$$= \frac{2 \times \frac{2\tan\theta}{1-\tan^2\theta}}{1-\left(\frac{2\tan\theta}{1-\tan^2\theta}\right)^2}$$

$$= \frac{\frac{4\tan\theta}{1-\tan^2\theta}}{1-\frac{4\tan^2\theta}{(1-\tan^2\theta)^2}}$$

$$= \frac{\frac{4\tan\theta}{1-\tan^2\theta}}{\frac{(1-\tan^2\theta)^2-4\tan^2\theta}{(1-\tan^2\theta)^2}}$$

$$= \frac{4\tan\theta(1-\tan^2\theta)}{1+\tan^4\theta-2\tan^2\theta-4\tan^2\theta} = \frac{4\tan\theta(1-\tan^2\theta)}{1+\tan^4\theta-6\tan^2\theta}$$ Ans

Q.No.9 Show that

$$\cos(6A) = 32\cos^6 x - 48\cos^4 A + 18\cos^2 A - 1$$

Sol: LHS $\cos(6A)$

$$= 2\cos^2(3A) - 1$$

$$= 2(4\cos^3 A - 3\cos A)^2 - 1$$

$$= 2\left(4\cos^3 A - 3\cos A\right)^2 - 1$$

$$= 2\left(16\cos^6 A + 9\cos^2 A - 24\cos^4 A\right) - 1$$

$$= 32\cos^6 A + 18\cos^2 A - 48\cos^4 A - 1 \quad \underline{\text{Ans}}$$

Q.10 Show $\cos(4x) = 1 - 8\sin^2 x \cos^2 x$

SELF

Q.11 Given $\tan x = -\frac{4}{3}$; $x \to$ IVth quad

find the value of $\sin\frac{x}{2}, \cos\frac{x}{2}, \tan\frac{x}{2}$

Sol.

$$1 + \tan^2 x = \sec^2 x$$

$$\Rightarrow 1 + \frac{16}{9} = \sec^2 x$$

$$\Rightarrow \frac{25}{9} = \sec^2 x$$

$$\Rightarrow \sec x = \pm\frac{5}{3}$$

$$\Rightarrow \sec x = \frac{5}{3} \quad \text{-- } \{x \to 4^{th} \text{ quad}$$

$$\boxed{\cos x = \frac{3}{5}}$$

$$1 - \cos x = 2\sin^2\left(\frac{x}{2}\right)$$

$$1 - \frac{3}{5} = 2\sin^2(x/2)$$

$$\frac{2}{5} = 2\sin^2(x/2)$$

$$\sin^2(x/2) = \frac{1}{5}$$

$x \to$ I
$x/2 \to$ II

$$\sin(x/2) = \pm\frac{1}{\sqrt{5}} = +\frac{1}{\sqrt{5}}$$

II ↑

$$1 + \cos x = 2\cos^2(x/2)$$

$$1 + \frac{3}{5} = 2\cos^2(x/2)$$

$$\frac{8}{5} = 2\cos^2(x/2)$$

$$\frac{4}{5} = \cos^2(x/2)$$

$$\cos(x/2) = \pm\frac{2}{\sqrt{5}} = -\frac{2}{\sqrt{5}}$$

II ✓

Trigo class (T-6)

Q.12 → Given $\sin x = -\frac{1}{2}$; $x \to$ IV th quad

SELF find value of $\sin(x/2)$, $\cos(x/2)$, $\tan(x/2)$

ANS $\sin(x/2) \sqrt{\frac{2-\sqrt{3}}{2}}$, $\cos\frac{x}{2} = -\sqrt{\frac{2+\sqrt{3}}{2}}$, $\tan(x/2) = \sqrt{3} - 2$

Q.13 → Given $\cos x = -\frac{1}{3}$; $\pi < x < \frac{3\pi}{2}$

Sol) find $\sin(x/2)$, $\cos x/2$, $\tan x/2$

ANS $\sqrt{\frac{2}{3}}$, $-\frac{1}{\sqrt{3}}$, $-\sqrt{2}$

Q.M-14 → Show that $\frac{\sin(5x) - 2\sin(3x) + \sin x}{\cos(5x) - \cos x} = \tan x$

SELF

Q.M-15 → find value of $\tan(\pi/8)$ $\quad \frac{\pi}{8} = \frac{180^\circ}{8} = \frac{45^\circ}{2}$

Sol) we have $\tan(2\theta) = \frac{2\tan\theta}{1-\tan^2\theta}$

put $\theta = \pi/8$

$\Rightarrow \tan\left(\frac{\pi}{4}\right) = \frac{2\tan(\pi/8)}{1-\tan^2(\pi/8)}$ { let $\tan(\pi/8) = x$ }

$\Rightarrow 1 = \frac{2x}{1-x^2}$

$\Rightarrow 1 - x^2 = 2x$

$\Rightarrow x^2 + 2x - 1 = 0$

$x = \frac{-2 \pm \sqrt{4+4}}{2}$

$x = \frac{-2 \pm 2\sqrt{2}}{2}$

$x = -1 \pm \sqrt{2}$

$x = -1 + \sqrt{2}$

$\tan(\pi/8) = \sqrt{2} - 1$

Ans

$x = -1 - \sqrt{2}$

$\tan(\frac{\pi}{8}) = -1 - \sqrt{2}$

(X) Reject

Qn 17 Show that

(SELF $\tan A + \tan(60° + A) - \tan(60° - A) = 3\tan(3A)$

Qn 18 Show that

Self $\sin(4A) = 4\sin A \cos^3 A - 4\cos A \sin^3 A$