

RELATION & FUNCTION

①

Solution of Worksheet No: 3

Qn. 1 $\rightarrow f(x) = \frac{1}{\sqrt{x-5}}$

Domain: $x-5 > 0$
 $\Rightarrow x > 5$

\therefore Domain $x \in (5, \infty)$

Range: Let $y = f(x)$
 $\Rightarrow y = \frac{1}{\sqrt{x-5}} \quad \dots\dots (1)$

$\Rightarrow y^2 = \frac{1}{x-5}$

$\Rightarrow x-5 = \frac{1}{y^2}$

$\Rightarrow x = \frac{1}{y^2} + 5$

$\Rightarrow x = \frac{1+5y^2}{y^2}$

x is real for all value of y such that
 $y \in \mathbb{R} - \{0\}$

but from eq(1) cannot be -ve

\therefore Range = $(0, \infty)$ Ans

Qn. 2 $\rightarrow f(x) = \sqrt{16-x^2}$

Domain: $16-x^2 \geq 0$

$\Rightarrow x^2 - 16 \leq 0$

$\Rightarrow (x+4)(x-4) \leq 0$

$\begin{array}{c} + \quad - \quad + \\ -\infty \quad -4 \quad 4 \quad \infty \end{array} \quad \therefore \text{Domain} = [-4, 4]$

Range: Let $y = f(x)$
 $y = \sqrt{16-x^2} \quad \dots\dots (1)$

$$y^2 = 16 - x^2$$

$$\Rightarrow x^2 = 16 - y^2$$

$$\Rightarrow x = \pm \sqrt{16 - y^2}$$

x is real for all values of y such that

$$16 - y^2 \geq 0$$

$$y^2 - 16 \leq 0$$

$$(y+4)(y-4) \leq 0$$

$$\begin{array}{ccccccc} & + & & - & & + & \\ -\infty & & -4 & & 4 & & \infty \end{array}$$

$$y \in [-4, 4]$$

but from (1) y can't be -4

$$\therefore \text{Range} = [0, 4] \quad \underline{\text{Ans}}$$

Qn 3 $\rightarrow f(x) = \frac{3}{2-x^2}$

Domain $2 - x^2 \neq 0$

$$(x^2 - 2) \neq 0$$

$$(x + \sqrt{2})(x - \sqrt{2}) \neq 0$$

$$\therefore \text{Domain} = \mathbb{R} - \{-\sqrt{2}, \sqrt{2}\}$$

Range. let $y = f(x)$

$$y = \frac{3}{2-x^2}$$

$$\Rightarrow 2y - x^2y = 3$$

$$\Rightarrow x^2y = 2y - 3$$

$$\Rightarrow x^2 = \frac{2y-3}{y}$$

$$\Rightarrow x = \pm \sqrt{\frac{2y-3}{y}}$$

x is real for all values of y such that

$$\frac{2y-3}{y} \geq 0 \quad \text{and} \quad y \neq 0$$

$$\begin{array}{ccccccc} & + & & - & & + & \\ -\infty & & 0 & & 3/2 & & \infty \end{array}$$

$$; y \neq 0$$

$$\therefore \text{Range} = (-\infty, 0) \cup [3/2, \infty) \quad \underline{\text{Ans}}$$

Solution of Worksheet No: 3

(3)

Qn. 4 $\rightarrow f(x) = \frac{x}{1+x^2}$

Domain $x \in \mathbb{R}$

Range Let $y = f(x)$
 $y = \frac{x}{1+x^2}$

$$\Rightarrow y + x^2 y = x$$

$$\Rightarrow x^2 y - x + y = 0$$

Quadratic formula $x = \frac{1 \pm \sqrt{1-4y^2}}{2y}$

x is real for all values of y such that

$$1-4y^2 \geq 0 \quad \text{and} \quad 2y \neq 0$$

$$4y^2 - 1 \leq 0 \quad \text{and} \quad y \neq 0$$

$$(2y+1)(2y-1) \leq 0 \quad \text{and} \quad y \neq 0$$

$$\begin{array}{ccccccc} & + & & - & & + & \\ -\infty & & -\frac{1}{2} & & \frac{1}{2} & & \infty \end{array}$$

$$y \in \left[-\frac{1}{2}, \frac{1}{2}\right] \quad \& \quad y \neq 0$$

but when $x=0$ then $f(x)=0$

$$\therefore \text{Range} = \left[-\frac{1}{2}, \frac{1}{2}\right] \quad \underline{\text{Ans}}$$

Qn. 5 $\rightarrow f(x) = \frac{|x-2|}{2-x}$

Domain $2-x \neq 0$

$$x \neq 2$$

\therefore Domain $\mathbb{R} - \{2\}$

Range $f(x) = \frac{|x-2|}{2-x}$

$f(x)$ can take only two values $-1, 1$

$$\therefore \text{Range} = \{-1, 1\} \quad \underline{\text{Ans}}$$

Q1.6 $\rightarrow f(x) = \frac{1}{1-x^2}$

Domain $1-x^2 \neq 0$

$(1+x)(1-x) \neq 0$

$x \neq -1, x \neq 1$

$\therefore \text{Domain} = x \in \mathbb{R} - \{-1, 1\}$

Range let $y = f(x)$

$y = \frac{1}{1-x^2}$

$\Rightarrow y - x^2 y = 1$

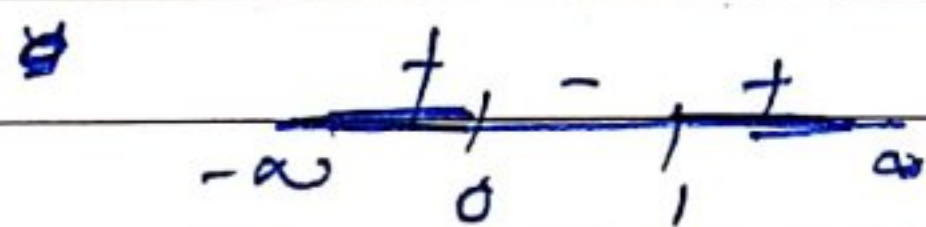
$\Rightarrow x^2 y = y - 1$

$\Rightarrow x^2 = \frac{y-1}{y}$

$\Rightarrow x = \pm \sqrt{\frac{y-1}{y}}$

x is real for all values of y such that

$\frac{y-1}{y} \geq 0 \quad \& \quad y \neq 0$

$\#$ 

Range $(-\infty, 0) \cup (1, \infty)$ Ans

Q1.7 $\rightarrow f(x) = \frac{ax-b}{cx-d}$

Domain $cx-d \neq 0$

$x \neq \frac{d}{c}$

$\therefore \text{Domain} = \mathbb{R} - \left\{ \frac{d}{c} \right\}$

Range let $y = f(x)$

$\Rightarrow y = \frac{ax-b}{cx-d}$

$\Rightarrow cxy - dy = ax - b$

$\Rightarrow x(cy - a) = dy - b$

Solution of Worksheet No. 3

(5)

$$x = \frac{dy-b}{cy-a}$$

x is real for all values of y such that
 $cy-a \neq 0$

$$\Rightarrow y \neq \frac{a}{c}$$

$$\therefore \text{Range} = \mathbb{R} - \left\{ \frac{a}{c} \right\} \quad \underline{\text{Ans}}$$

Q1.8 $\rightarrow f(x) = \sqrt{4x-x^2}$

Domain $4x-x^2 \geq 0$

$$\Rightarrow x^2-4x \leq 0$$

$$\Rightarrow x(x-4) \leq 0$$

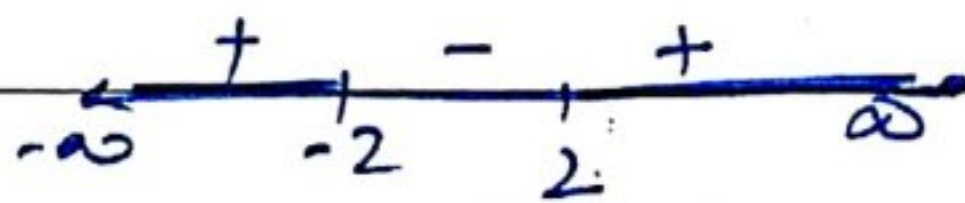


$$\therefore \text{Domain} = [0, 4] \quad \underline{\text{Ans}}$$

Q1.9 $\rightarrow f(x) = \sqrt{\frac{x-2}{x+2}} + \sqrt{\frac{1-x}{1+x}}$

$$\frac{x-2}{x+2} \geq 0 ; x+2 \neq 0$$

$$\frac{x-2}{x+2} \geq 0 \quad \& \quad x \neq -2$$

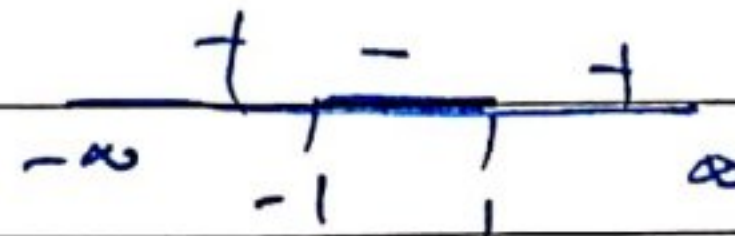


$$x \in (-\infty, -2) \cup [2, \infty)$$

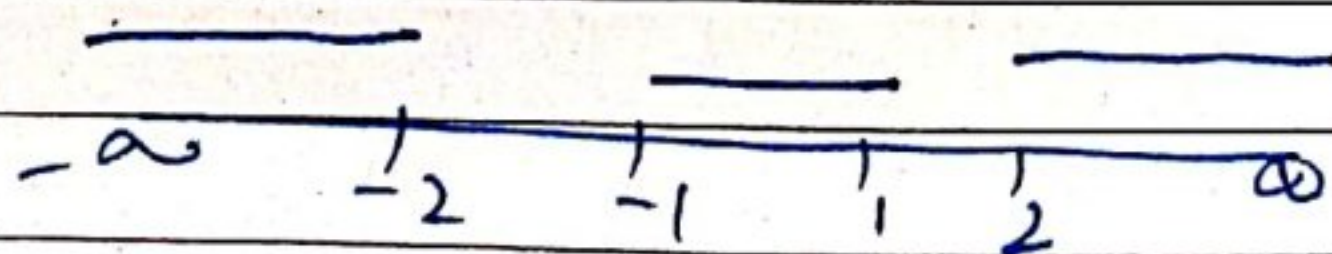
taking common

$$\frac{1-x}{1+x} \geq 0 \quad \& \quad 1+x \neq 0$$

$$\frac{x-1}{1+x} \leq 0 \quad \& \quad x \neq -1$$



$$x \in (-1, 1]$$



No common part

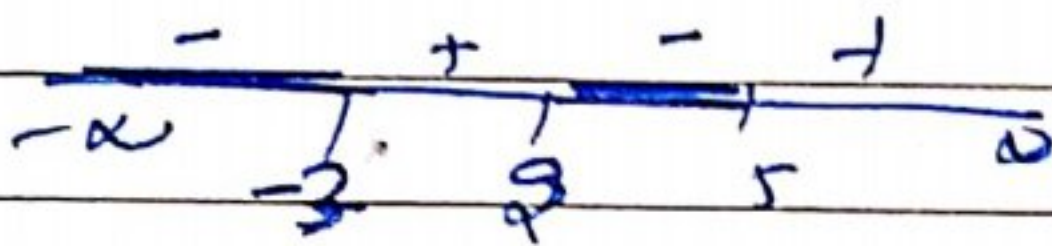
$$\therefore \text{Domain} = \phi \quad \underline{\text{Ans}}$$

Solution of Worksheet No. 3 (6)

$$Qn/0 \rightarrow f(x) = \sqrt{\frac{x+3}{(2-x)(x-5)}}$$

$$\frac{x+3}{(2-x)(x-5)} \geq 0 \quad \text{and} \quad (2-x)(x-5) \neq 0$$

$$\frac{x+3}{(x-2)(x-5)} \leq 0 \quad \text{and} \quad x \neq 2 \text{ \& } x \neq 5$$



$$\therefore \text{Domain } (-\infty, -3] \cup (2, 5) \quad \underline{\text{Ans}}$$

$$Qn/1 \rightarrow f(x) = \frac{x^2 - x}{x^2 + 2x}$$

$$\text{Range let } y = f(x)$$

$$\Rightarrow y = \frac{x^2 - x}{x^2 + 2x}$$

$$\Rightarrow x^2 y + 2xy = x^2 - x$$

$$\Rightarrow x^2 y - x^2 + 2xy + x = 0$$

$$\Rightarrow x^2(y-1) + x(2y+1) = 0$$

$$\Rightarrow x [x(y-1) + (2y+1)] = 0$$

$$\Rightarrow x = 0 \quad (\text{or}) \quad x(y-1) + (2y+1) = 0$$

$$\Rightarrow x = 0 \quad (\text{or}) \quad x = \frac{-(y-1)}{2y+1}$$

x is real for all values y such that $2y+1 \neq 0 \Rightarrow y \neq -1/2$

Imp part $\left\{ \begin{array}{l} \text{when } y=1, \text{ then } x=0 \\ \text{but when } x=0, f(x) \text{ is not defined} \end{array} \right.$

Soln

walkthrough no. 3

(7)

$\therefore y=1$ cannot be included

$$\therefore \text{Range} = R - \left\{ \frac{-1}{2}, 1 \right\} \quad \underline{\text{Ans}}$$

Qn. 12 \rightarrow

$$f(x) = \sqrt{x-1} + \sqrt{3-x}$$

$$x-1 \geq 0 \quad \text{and} \quad 3-x \geq 0$$

$$x \geq 1 \quad \text{and} \quad x-3 \leq 0$$

$$x \geq 1 \quad \text{and} \quad x \leq 3$$

$$x \in [1, \infty) \quad \text{and} \quad x \in (-\infty, 3] \quad x \in (-\infty, 3]$$

taking common



$$\therefore x \in [1, 3] \quad \underline{\text{Ans}}$$

-x-