

← ULTIMATE MATHEMATICS →

(BY: AJAY MITTAL : 9891067390)

← TRIGONOMETRY ~ CLASS No: 2 (T.2) →

Formulae

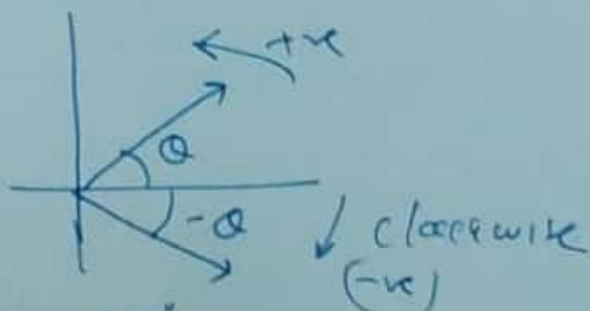
Set - I

$$(1) \sin(-\theta) = -\sin\theta \quad (3) \tan(-\theta) = -\tan\theta$$

$$(2) \csc(-\theta) = -\csc\theta \quad (4) \cot(-\theta) = -\cot\theta$$

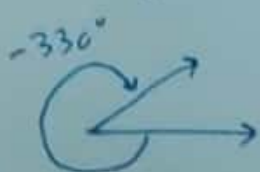
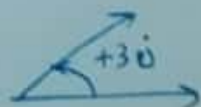
$$*(5) \boxed{\cos(-\theta) = \cos\theta} \quad (6) \sec(-\theta) = \sec\theta$$

-ve angle



(1)

✓



$$\boxed{30^\circ \approx -330^\circ}$$

$$60^\circ \approx -300^\circ ; 330^\circ \approx -30^\circ$$

eg $\sin(-30^\circ) = \sin(330^\circ) = \sin(360^\circ - 30^\circ) = -\sin(30^\circ)$

$$\cos(-30^\circ) = \cos(330^\circ) = \cos(360^\circ - 30^\circ) = \cos(30^\circ)$$

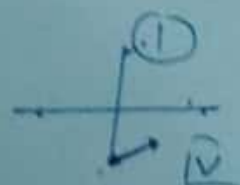
QNS find value of $\csc(-1410^\circ)$

$$= -\csc(1410^\circ)$$

$$= -\csc(15 \times 90^\circ + 60^\circ)$$

$$= + \sec(60^\circ) = 2 \quad \underline{\underline{\text{Ans}}}$$

$$\begin{array}{r} 15 \\ 90 \overline{) 1410} \\ \underline{90} \\ 510 \\ \underline{450} \\ 60 \end{array}$$



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SET-2

- (1) $\sin(A+B) = \sin A \cos B + \cos A \sin B$
- (2) $\sin(A-B) = \sin A \cos B - \cos A \sin B$
- (3) $\cos(A+B) = \cos A \cos B - \sin A \sin B$
- (4) $\cos(A-B) = \cos A \cos B + \sin A \sin B$
- (5) $\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$
- (6) $\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$
- (7) $\cot(A+B) = \frac{\cot A \cot B - 1}{\cot A + \cot B}$
- (8) $\cot(A-B) = \frac{\cot A \cot B + 1}{\cot B - \cot A}$

Ques 1 Find $\sin(15^\circ)$

$$= \sin(45^\circ - 30^\circ)$$

$$= \sin(45) \cos(30) - \cos(45) \sin(30)$$

$$= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} \times \frac{1}{2}$$

$$\sin(15^\circ) = \frac{\sqrt{3} - 1}{2\sqrt{2}} \quad \underline{\underline{\text{Ans}}}$$

Rationalize

$\sqrt{3}$	$- \sqrt{1}$	\checkmark
$\sqrt{3}$	$- \square$	\checkmark
\square	$- \sqrt{1}$	\checkmark

$\sqrt{\quad} \textcircled{x}$

Ques 2 $\tan(105^\circ) = \tan(60 + 45)$

$$= \frac{\tan(60) + \tan(45)}{1 - \tan(60) \tan(45)}$$

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$$= \frac{\sqrt{3} + 1}{1 - \sqrt{3}} \times \frac{1 + \sqrt{3}}{1 + \sqrt{3}}$$

$$= \frac{3 + 1 + 2\sqrt{3}}{1 - 3} = \frac{4 + 2\sqrt{3}}{-2} = -(2 + \sqrt{3})$$

Ques 3

$$\begin{aligned} & \tan\left(\frac{13\pi}{12}\right) \\ &= \tan\left(13 \times \frac{180^\circ}{12}\right) \\ &= \tan(13 \times 15^\circ) \\ &= \tan(195^\circ) \\ &= \tan\left(180^\circ + 15^\circ\right) \\ &= \tan(15^\circ) \\ &= \tan(45^\circ - 30^\circ) \end{aligned}$$

Proceed

Ans

$$\boxed{2 - \sqrt{3}}$$

Ques 4

Given $\sin A = \frac{3}{5}$

& $\cos B = -\frac{12}{13}$

$0 < A < \frac{\pi}{2}$

$\pi < B < \frac{3\pi}{2}$

Find $\sin(A+B)$ and $\tan(A-B)$

Soln

$$\sin^2 A + \cos^2 A = 1$$

$$\cos^2 A = 1 - \sin^2 A$$

$$\cos^2 A = 1 - \frac{9}{25}$$

$$\cos^2 A = \frac{16}{25} \Rightarrow \cos A = \pm \frac{4}{5} \Rightarrow \boxed{\cos A = \frac{4}{5}}$$

\therefore Ist quadrant

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$$\sin^2 B + \cos^2 B = 1$$

$$\sin^2 B = 1 - \cos^2 B$$

$$= 1 - \left(-\frac{12}{13}\right)^2$$

$$= 1 - \frac{144}{169}$$

$$\sin^2 B = \frac{25}{169}$$

$$\sin B = \pm \frac{5}{13} \Rightarrow \boxed{\sin B = -\frac{5}{13}}$$

III.

$$(i) \sin(A+B) = \sin A \cos B + \cos A \sin B$$

$$= \left(\frac{3}{5}\right) \left(\frac{12}{13}\right) + \left(\frac{4}{5}\right) \left(-\frac{5}{13}\right)$$

$$= \frac{36}{65} - \frac{20}{65} = \frac{16}{65}$$

$$= -\frac{36}{65} - \frac{20}{65} = -\frac{56}{65} \quad \underline{\underline{Ans}}$$

$$(2) \tan(A-B)$$

$$\tan A = \frac{\sin A}{\cos A} = \frac{3/5}{4/5} = \frac{3}{4}$$

$$\tan B = \frac{\sin B}{\cos B} = \frac{-5/13}{12/13} = -\frac{5}{12}$$

$$\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

Ques 5 ✓ Show that $\tan(54^\circ) = \frac{\cos(9^\circ) + \sin(9^\circ)}{\cos 9^\circ - \sin 9^\circ}$

Hint $\tan(54^\circ) = \tan(45^\circ + 9^\circ) = \tan(A+B)$

Page 5

ULTIMATE MATHEMATICS

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Q116 Given $\tan \alpha = \frac{m}{m+1}$ & $\tan \beta = \frac{1}{2m+1}$

Show $\alpha + \beta = \frac{\pi}{4}$

We have $\tan(\alpha + \beta) = \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta}$

$$= \frac{\frac{m}{m+1} + \frac{1}{2m+1}}{1 - \frac{m}{m+1} \times \frac{1}{2m+1}}$$

L.C.M

$$\tan(\alpha + \beta) = 1$$

$$\Rightarrow \alpha + \beta = \frac{\pi}{4}$$

Q117 Show $\cos\left(\frac{\pi}{4} - A\right) \cdot \cos\left(\frac{\pi}{4} - B\right) - \sin\left(\frac{\pi}{4} - A\right) \cdot \sin\left(\frac{\pi}{4} - B\right) = \sin(A+B)$

Sol Ans $\cos(45^\circ - A) \cdot \cos(45^\circ - B) - \sin(45^\circ - A) \cdot \sin(45^\circ - B)$

$$= \cos(45^\circ - A + 45^\circ - B)$$
$$= \cos(90^\circ - A - B)$$
$$= \cos(90^\circ - (A+B))$$
$$= \sin(A+B)$$

Topic _____

Date _____

Q No. 1 \rightarrow If $\sin A = \frac{3}{5}$ and $\cos B = -\frac{12}{13}$, $0 < A < \frac{\pi}{2}$ and

$\pi < B < \frac{3\pi}{2}$. Find the value of the following:

(i) $\sin(A-B)$ (ii) $\cos(A+B)$ (iii) $\tan(A-B)$

Q No. 2 \rightarrow If $\cos A = \frac{4}{5}$, $\cos B = \frac{12}{13}$; $\frac{3\pi}{2} < A, B < 2\pi$. Find the value of the following

(i) $\cos(A+B)$ (ii) $\sin(A-B)$

Q No. 3 \rightarrow If $\cot \alpha = \frac{1}{2}$, $\sec \beta = -\frac{5}{3}$; $\pi < \alpha < \frac{3\pi}{2}$ and $\frac{\pi}{2} < \beta < \pi$. Find the value of $\tan(\alpha+\beta)$

Q No. 4 \rightarrow If $\tan A = \frac{3}{4}$, $\cos B = \frac{9}{41}$, where $\pi < A < \frac{3\pi}{2}$ and $0 < B < \frac{\pi}{2}$. Find the value of $\tan(A+B)$

Q No. 5 \rightarrow Find the value of (1). $\sin(75^\circ)$ (2). $\sin(15^\circ)$ (3). $\tan(15^\circ)$ (4). $\tan(75^\circ)$ (5). $\tan(105^\circ)$ (6). $\tan(135^\circ)$ (7). $\cos(105^\circ) + \cos(15^\circ)$ (8). $\tan(75^\circ) + \cot(75^\circ)$ (9). $\cos(135^\circ)$ (10). $\sin(135^\circ)$

Q No. 6 \rightarrow Show that $\sin(n+1)A \cdot \sin(n+2)A + \cos(n+1)A \cdot \cos(n+2)A = \cos A$

Q No. 7 \rightarrow Show that $\cos(\frac{\pi}{4}-A) \cos(\frac{\pi}{4}-B) - \sin(\frac{\pi}{4}-A) \sin(\frac{\pi}{4}-B) = \sin(A+B)$

Q No. 8 \rightarrow Show that $\frac{\sin(B-C)}{\cos B \cos C} + \frac{\sin(C-A)}{\cos C \cos A} + \frac{\sin(A-B)}{\cos A \cos B} = 0$

Q No. 9 \rightarrow Show that $\frac{\tan(\frac{\pi}{4}+x)}{\tan(\frac{\pi}{4}-x)} = \left(\frac{1+\tan x}{1-\tan x} \right)^2$

Q No. 10 \rightarrow Show that $\frac{\sin(A-B)}{\cos A \cos B} + \frac{\sin(B-C)}{\sin B \sin C} + \frac{\sin(C-A)}{\sin C \sin A} = 0$

Qn. 11 → Show that $\tan(56^\circ) = \frac{\cos(11^\circ) + \sin(11^\circ)}{\cos(11^\circ) - \sin(11^\circ)}$

Qn. 12 → Show that $\frac{\cos(9^\circ) + \sin(9^\circ)}{\cos(9^\circ) - \sin(9^\circ)} = \tan(54^\circ)$

Qn. 13 → Show that $\frac{\cos(8^\circ) - \sin(8^\circ)}{\cos(8^\circ) + \sin(8^\circ)} = \tan(37^\circ)$

Qn. 14 → If $\tan A = 5/6$ and $\tan B = 1/11$, show that $A+B = \frac{\pi}{4}$

Qn. 15 → If $\tan \alpha = \frac{m}{m+1}$ and $\tan \beta = \frac{1}{2m+1}$, show that $\alpha + \beta = \frac{\pi}{4}$

Qn. 16 → Show that $\tan(3A) \tan(2A) \tan A = \tan(3A) - \tan(2A) - \tan A$

Qn. 17 → Show that $\cot A \cdot \cot(2A) - \cot(2A) \cot(3A) - \cot(3A) \cot A = 1$

Qn. 18 → Show that $\tan(130^\circ) - \tan(90^\circ) - \tan(40^\circ) = \frac{\tan(130^\circ) \tan(90^\circ)}{\tan(40^\circ)}$

Qn. 19 → Show that $\tan(36^\circ) + \tan 9^\circ + \tan(36^\circ) \tan(9^\circ) = 1$

Qn. 20 → Show that $\tan(70^\circ) = \tan(40^\circ) + 2 \tan(50^\circ)$

Qn. 21 → If $A+B = \pi/4$, show that

(i) $(1+\tan A)(1+\tan B) = 2$ (ii) $(\cot A - 1)(\cot B - 1) = 2$

→ ANSWERS →

(1) (i) $-\frac{16}{65}$ (ii) $-\frac{33}{65}$ (iii) $\frac{16}{63}$ (2) (i) $\frac{33}{65}$ (ii) $-\frac{16}{65}$ (3) $\frac{2}{11}$ (4) $-\frac{187}{84}$

(5) (i) $\frac{\sqrt{3}+1}{2}$ (2) $\frac{\sqrt{3}-1}{2\sqrt{2}}$ (3) $2-\sqrt{3}$ (4) $2+\sqrt{3}$ (5) $-(2+\sqrt{3})$ (6) $2-\sqrt{3}$

(7) $1/\sqrt{2}$ (8) $1/4$ (9) $-(\frac{\sqrt{3}+1}{2\sqrt{2}})$ (10) $\frac{1-\sqrt{3}}{2\sqrt{2}}$ — x —