॥ जय श्री राधे कृष्ण। जय श्री गिरिराज की महाराज !!

ULTIMATE MATHEMATICS: BY AJAY MITTAL

CONIC SECTIONS

CLASS NO: 3

Q.No: 1 Find the equation of ellipse whose Major axis on Y-axis and passes through the points (3,2) and (1,6)

Sol: Let the equation of ellipse is

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

(3,2) lies on it

$$\frac{9}{a^2} + \frac{4}{b^2} = 1$$

$$\Rightarrow 9b^2 + 4a^2 = a^2b^2 \quad --(1)$$

(1,6) lies on it

$$\frac{1}{a^2} + \frac{36}{b^2} = 1$$

$$b^2 + 36a^2 = a^2b^2 \quad --(2)$$

$$81b^2 + 36a^2 = 9a^2b^2$$
$$b^2 + 36a^2 = a^2b^2$$
$$80b^2 = 8a^2b^2$$

$a^2 = 10$ put in (2)

$$b^2 + 360 = 10b^2 \Rightarrow 9b^2 = 360 \Rightarrow b^2 = 40$$

∴ Equation of ellipse is

$$\frac{x^2}{10} + \frac{y^2}{40} = 1$$

Ans

Q.No: 2 → Find equation of hyperbola whose foci is $(\pm 4, 0)$ & $LR = 12$

Sol:

$$\text{foci} = (\pm 4, 0)$$

Comp with $(\pm ae, 0)$

$$\boxed{ae = 4}$$

$$LR = 12$$

$$\Rightarrow \frac{2b^2}{a} = 12$$

$$\Rightarrow \boxed{b^2 = 6a}$$

$$e = \sqrt{1 + \frac{b^2}{a^2}}$$

$$ae = \sqrt{a^2 + b^2}$$

$$4 = \sqrt{a^2 + 6a}$$

Squaring $16 = a^2 + 6a$

$$\rightarrow a^2 + 6a - 16 = 0$$

$$\rightarrow (a+8)(a-2) = 0$$

$$\Rightarrow a = -8 \quad | \quad a = 2$$

$b^2 = -48$ (X) | $b^2 = 12$

equation of hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

$$\frac{x^2}{4} - \frac{y^2}{12} = 1 \quad \text{Ans}$$

Ques 3 → Find equation of hyperbola whose foci is $(0, \pm\sqrt{10})$ and passes through the point $(2,3)$

Imp

Soln

$$\text{foci} = (0, \pm\sqrt{c})$$

Comp with $(0, \pm be)$

$$\boxed{be = \sqrt{10}}$$

Let equation of hyperbola is $\frac{-x^2}{a^2} + \frac{y^2}{b^2} = 1$

$(2,3)$ lies on it

$$\Rightarrow \frac{-4}{a^2} + \frac{9}{b^2} = 1$$

$$\Rightarrow \boxed{-4b^2 + 9a^2 = a^2b^2} \quad \text{---(1)}$$

$$e = \sqrt{1 + \frac{a^2}{b^2}}$$

$$be = \sqrt{b^2 + a^2}$$

$$\sqrt{10} = \sqrt{b^2 + a^2}$$

$$\boxed{b^2 + a^2 = 10} \Rightarrow b^2 = 10 - a^2 \text{ put in eq (1)}$$

$$\Rightarrow -4(10 - a^2) + 9a^2 = a^2(10 - a^2)$$

$$\Rightarrow -40 + 4a^2 + 9a^2 = 10a^2 - a^4$$

$$\Rightarrow a^4 + 3a^2 - 40 = 0$$

$$\Rightarrow a^4 + 8a^2 - 5a^2 - 40 = 0$$

$$\Rightarrow (a^2 + 8)(a^2 - 5) = 0$$

$a^2 = -8$ (x) | $a^2 = 5$, $b^2 = 5$

Equation of hyperbola

$$\frac{-x^2}{5} + \frac{y^2}{5} = 1$$

$$\Rightarrow \boxed{-x^2 + y^2 = 5} \quad \text{Ans}$$

Exemplar

Q.No. 4 → find the equation of ellipse which passes through the point $(-3,1)$ and has $e = \frac{\sqrt{2}}{5}$.

Soln Let $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$(-3,1)$ lies on it

$$\frac{9}{a^2} + \frac{1}{b^2} = 1$$

$$\Rightarrow \boxed{9b^2 + a^2 = a^2b^2} \quad \text{---} (1)$$

$$e = \sqrt{1 - \frac{b^2}{a^2}}$$

$$\frac{\sqrt{2}}{5} = \sqrt{\frac{a^2 - b^2}{a^2}}$$

$$\frac{2}{25} = \frac{a^2 - b^2}{a^2}$$

$$\Rightarrow 2a^2 = 25a^2 - 25b^2$$

$$\Rightarrow 25b^2 = 23a^2$$

$$\Rightarrow \boxed{b^2 = \frac{23a^2}{25}} \quad \text{---} (2)$$

pw. in (1)

$$9\left(\frac{23a^2}{25}\right) + a^2 = a^2\left(\frac{23a^2}{25}\right)$$

$$207a^2 + 25a^2 = 23a^4$$

$$\Rightarrow 23a^4 = 232a^2$$

$$\Rightarrow 23a^2 = 232$$

$$\Rightarrow \boxed{a^2 = \frac{232}{23}} \quad \text{pw. (2)}$$

$$b^2 = \left(\frac{23}{25}\right)\left(\frac{232}{23}\right)$$

$$\boxed{b^2 = \frac{232}{25}}$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$\frac{x^2}{\frac{232}{23}} + \frac{y^2}{\frac{232}{25}} = 1$$

$$\boxed{23x^2 + 25y^2 = 232}$$

Ans

Q.N 5 → find equation of hyperbola whose vertices are $(\pm 6, 0)$ and one of the directrices is $x = 4$

Sol =

Vertices $(\pm 6, 0)$

Comp with $(\pm a, 0)$

$a = 6$

directrix $x = \frac{a}{e} = 4$

$\Rightarrow \frac{6}{e} = 4$

$\Rightarrow e = \frac{3}{2}$

$e = \sqrt{1 + \frac{b^2}{a^2}}$

$\frac{3}{2} = \sqrt{1 + \frac{b^2}{36}}$

$\frac{9}{4} = 1 + \frac{b^2}{36}$

$\frac{b^2}{36} = \frac{9}{4} - 1$

$\frac{b^2}{36} = \frac{5}{4}$

$b^2 = \frac{5 \times 36}{4}^{9}$

$b^2 = 45$

∴ equ of hyp $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

$\frac{x^2}{36} - \frac{y^2}{45} = 1$ Ans

Q.No. 6 → Find the eccentricity of the hyperbola

Sol $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ which passes through the points

$(3, 0)$ and $(3\sqrt{2}, 2)$

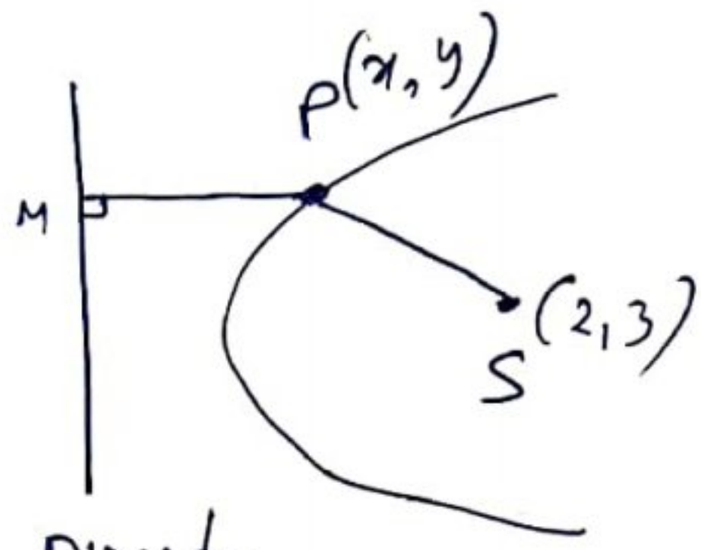
Q.n. 7 Find the equation of parabola whose focus is the point (2,3) and directrix is the line $x - 4y + 3 = 0$

Sol

For parabola $(e = 1)$

$$e = \frac{SP}{PM}$$

$$\Rightarrow \boxed{SP = PM}$$



$$\Rightarrow \sqrt{(x-2)^2 + (y-3)^2} = \frac{|x - 4y + 3|}{\sqrt{1 + 16}}$$

Squaring,

$$x^2 + 4 - 4x + y^2 + 9 - 6y = \frac{x^2 + 16y^2 + 9 - 8xy - 24y + 6x}{17}$$

$$\Rightarrow 17x^2 + 17y^2 - 68x - 102y + 221 = x^2 + 16y^2 + 9 - 8xy - 24y + 6x$$

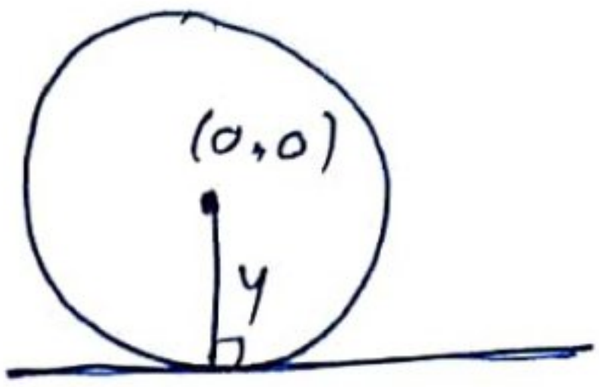
$$= \boxed{16x^2 + y^2 + 8xy - 74x - 78y + 212 = 0}$$ Ans

Qn. 8 → Find the value of 'k' if the line $y = \sqrt{3}x + k$ touches the circle $x^2 + y^2 = 16$

Sol:- $x^2 + y^2 = 16$

Comp with $(x-h)^2 + (y-k)^2 = r^2$

Centre $(0,0)$ rad $= 4$



$\sqrt{3}x - y + k = 0$

eqn of line $\sqrt{3}x - y + k = 0$

$$4 = \frac{|k|}{\sqrt{3+1}}$$

$$8 = |k|$$

$k = \pm 8$ Ans

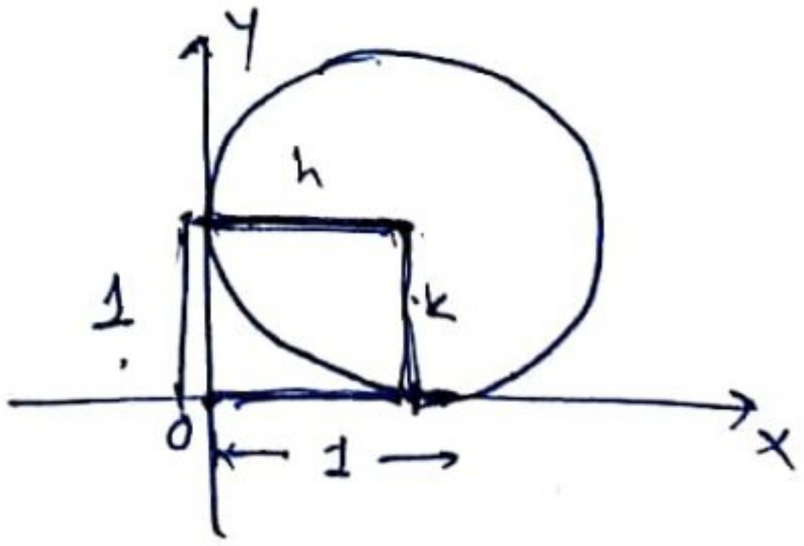
Qn. 9 → Find the equation of the circle in the first quadrant touching each coordinate axis at a distance of one unit from the origin. $(h = k = r)$

Sol:- Centre (h, k)

here $h = 1$ & $k = 1$

$\therefore$ Centre $(1, 1)$

rad $= 1$ $\therefore (x-1)^2 + (y-1)^2 = 1$ Ans



Qn. 8 → Find the value of 'k' if the line $y = \sqrt{3}x + k$ touches the circle $x^2 + y^2 = 16$

Soln $x^2 + y^2 = 16$

Comp with $(x-h)^2 + (y-k)^2 = r^2$

Centre $(0,0)$ rad = 4

$\sqrt{3}x - y + k = 0$

equation of line $\sqrt{3}x - y + k = 0$

$$4 = \frac{|k|}{\sqrt{3+1}}$$

$$8 = |k|$$

$k = \pm 8$ Ans

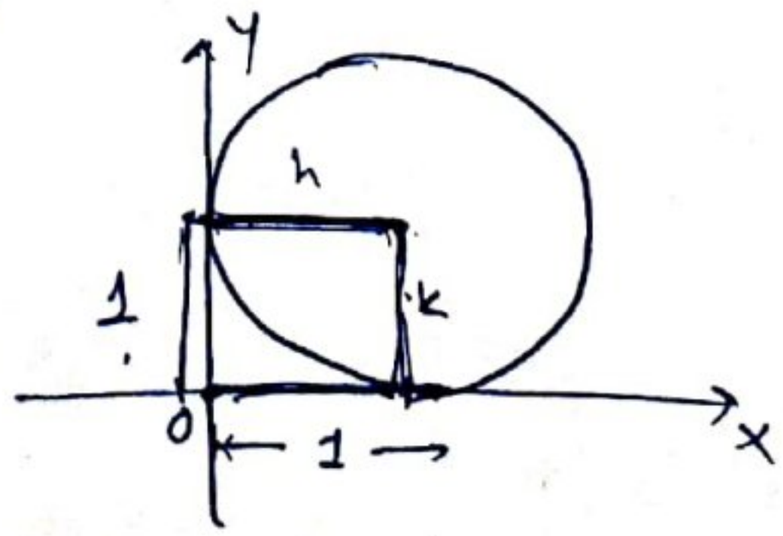
Qn. 9 → Find the equation of the circle in the first quadrant touching each coordinate axis at a distance of one unit from the origin. $(h = k = 1)$

Soln = Centre (h, k)

here $h = 1$ & $k = 1$

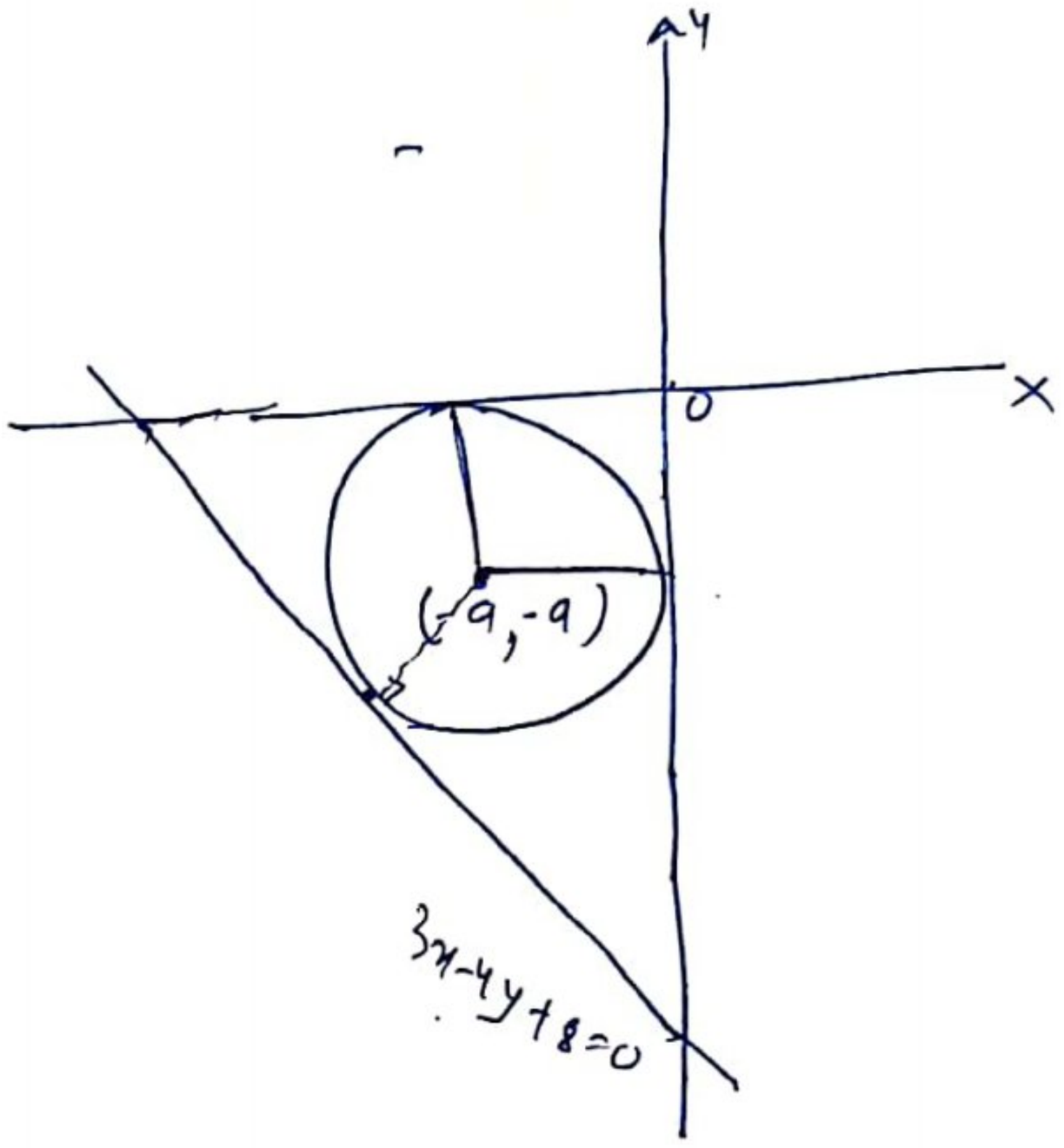
∴ Centre $(1, 1)$

rad = 1



∴ $(x-1)^2 + (y-1)^2 = 1$ Ans

Q.No. 10 → Find the equation of the circle which touches both the axes and line $3x-4y+8=0$ and lies in the third quadrant



Let Radius = a

Centre $(-a,-a)$

Radius = $\perp^r$ distance b/w centre & tangent

$$a = \frac{|-3a+4a+8|}{\sqrt{9+16}}$$

$$5a = |a+8|$$

$$\Rightarrow \quad 5a = a+8 \quad \Big| \quad -5a = a+8$$

$$4a = 8 \quad \Big| \quad -6a = 8$$

$$(a=2) \quad \Big| \quad a = -\frac{4}{3}$$ (X) Radius can't be -ve

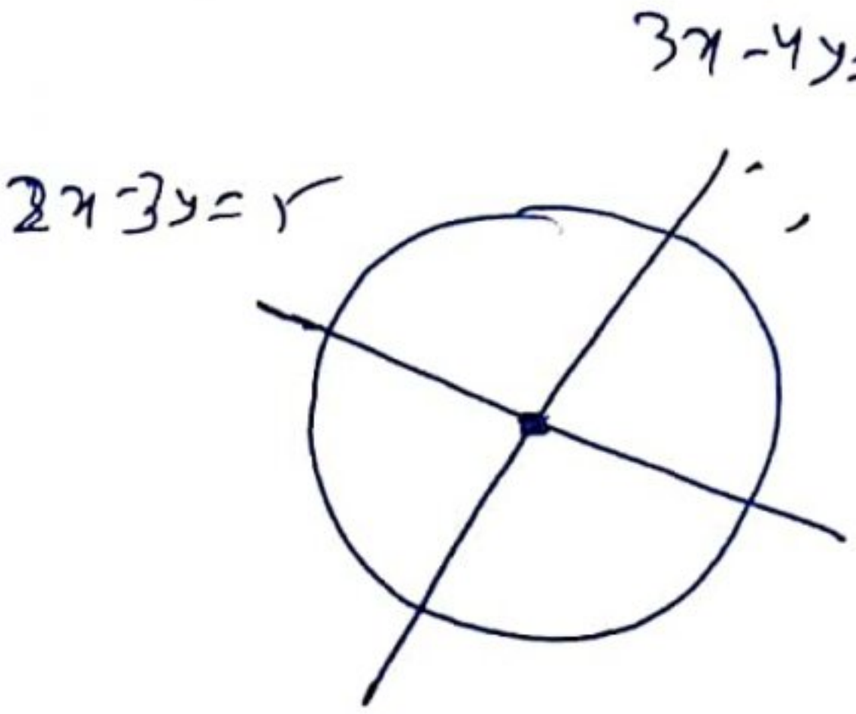
$\therefore$ Equation Circle $(x-h)^2 + (y-k)^2 = r^2$

$$(x+a)^2 + (y+a)^2 = a^2$$

$$(x+2)^2 + (y+2)^2 = 4$$

$$\Rightarrow x^2+y^2+4x+4y+4=0 \quad \underline{\text{Ans}}$$

Qno 11 → If the lines $2x-3y=5$ and $3x-4y=7$ are the diameters of a circle of area 154 sq.units. Find the equation of circle



Sol:) ~~Sol:~~

Centre → Intersection point of two diameters

Solving equations of diameters

$$6x-9y = 15$$
$$6x-8y = 14$$
$$\overline{\quad\quad\quad\quad}$$
$$-y = 1$$

$y=-1$

$6x+9 = 15$

$6x = 6$

$x = 1$

$\therefore$ Centre $(1,-1)$

Area $= 154$

$\pi r^2 = 154$

$\frac{22}{7} \times r^2 = 154$

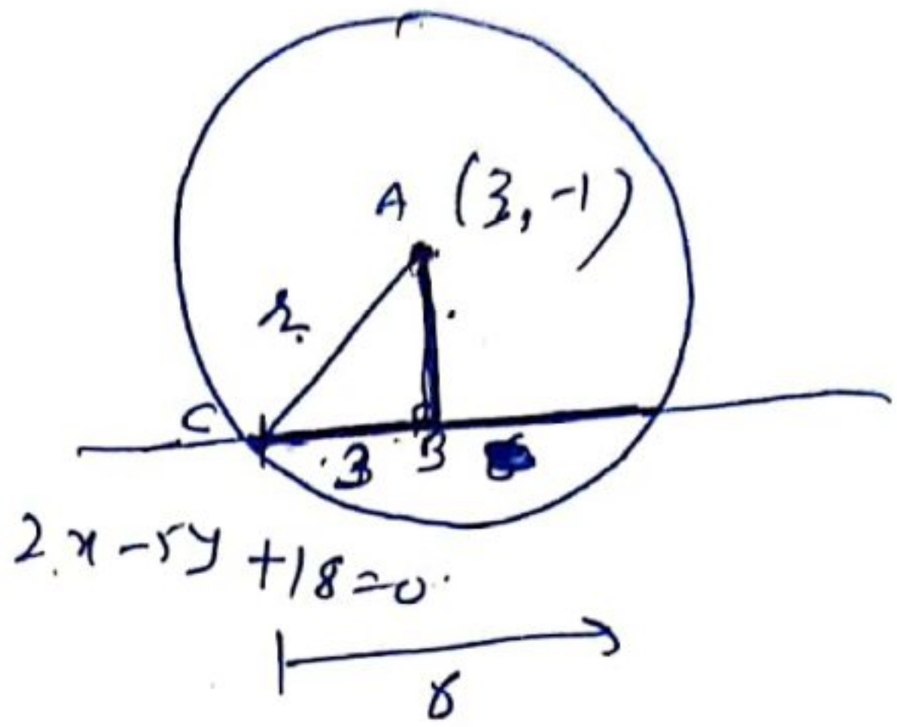
$r^2 = \frac{154 \times 7}{22}$

$r = 7$

$\therefore (x-1)^2 + (y+1)^2 = 49$ Ans

Qn. 12 → Find the equation of a circle whose centre is $(3,-1)$ and which cuts off a chord of length 6 units on the line $2x-5y+18=0$

Sol: Centre $(3,-1)$



AB is $\perp^r$ distance b/w the point $(3,-1)$ & the line

$$2x-5y+18=0$$

$$AB = \frac{|6+5+18|}{\sqrt{4+25}}$$

$$AB = \frac{29}{\sqrt{29}} = \sqrt{29}$$

$$r^2 = BC^2 + AB^2$$

$$r^2 = 9 + 29$$

$$r^2 = 38$$

$\therefore$ eqn of circle $\boxed{(x-3)^2 + (y+1)^2 = 38}$

Ans

WORKSHEET No: 3 (CONIC SECTION)

Qus: 1 → Find the equation of ellipse with foci $(\pm 5, 0)$ and $x = \frac{36}{5}$ as one of the directrices **Ans** $\frac{x^2}{36} + \frac{y^2}{11} = 1$

Qus 2 → Find the equation of the circle having centre $(1, -2)$ and passing through the point of intersection of the lines $3x + y = 14$ and $2x + 5y = 18$

Ans $x^2 + y^2 - 2x + 4y - 20 = 0$

Qus 3 → Find the equation of the ellipse whose centre is at origin and x-axis as Major axis which passes through the points $(-3, 1)$ & $(2, -2)$ **Ans** $3x^2 + 5y^2 = 32$

Qu. 4 → The length of the transverse axis along x-axis with centre at origin of a hyperbola is 7 and it passes through the point $(5, -2)$. Find the equation of hyperbola **Ans** $\frac{4x^2}{49} - \frac{51y^2}{196} = 1$

Qu. 5 → A circle has radius 3 units and its centre lies on the line $y = x - 1$. If it passes through the point $(7, 3)$. Find its equation **Ans** $x^2 + y^2 - 14x - 12y + 76 = 0$

Qu. 6 → Find the eccentricity of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ which passes through the points $(3, 0)$ & $(3\sqrt{2}, 2)$

Ans $e = \frac{\sqrt{13}}{3}$

Qn. 7 → If the distance between the foci of a hyperbola is 16 and its eccentricity is $\sqrt{2}$. Obtain the equation of hyperbola Ans $x^2-y^2=32$

Qn. 8 → If the $e=\frac{5}{8}$ and distance between foci of an ellipse is 10. find L.R of ellipse Ans $\frac{39}{4}$

Qn 9 → Find the equation of a circle of radius 5 which is touching another circle $x^2+y^2-2x-4y-20=0$ at (5,5) Ans $x^2+y^2-18x-16y+120=0$

Qn. 10 → Find the equation of the circle passing through the point (7,3) having Radii 3 units and whose centre lies on the line $y=x-1$

Ans $x^2+y-8x-6y+16=0$

-x-