Topic : .... Date. : .... Page No. : .... ①

Solutions of WORKSHEET NO. 1

Complex Numbers

Ans 1 →

$$Z = \frac{(3-2i)(2+3i)}{(1+2i)(2-i)}$$

$$Z = \frac{6+9i-4i-6i^2}{2-i+4i-2i^2}$$

$$Z = \frac{12+5i}{4+3i} \times \frac{4-3i}{4-3i}$$

$$Z = \frac{48-36i+20i-15i^2}{16-9i^2}$$

$$Z = \frac{63-16i}{25}$$

$$\Rightarrow Z = \frac{63}{25} - \frac{16i}{25} \quad \text{Ans}$$

Ans 2 →

$$Z = \left(\frac{1}{1-2i} + \frac{3}{1+i}\right)\left(\frac{3+4i}{2-4i}\right)$$

$$Z = \left(\frac{1+i+3-6i}{(1-2i)(1+i)}\right)\left(\frac{3+4i}{2-4i}\right)$$

$$Z = \left(\frac{(4-5i)}{(1+i-2i-2i^2)}\right)\left(\frac{3+4i}{2-4i}\right)$$

$$Z = \frac{(4-5i)}{(3-i)}\frac{(3+4i)}{(2-4i)}$$

$$Z = \frac{12+16i-15i-20i^2}{6-12i-2i+4i^2}$$

$$Z = \frac{32+i}{2-14i} \times \frac{2+14i}{2+14i}$$

$$Z = \frac{64+448i+2i+14i^2}{4-196i^2}$$

$$Z = \frac{50+450i}{200}$$

$$Z = \frac{50}{200} + \frac{450i}{200} \Rightarrow \boxed{Z = \frac{1}{4} + \frac{9i}{4}}$$

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$|z| = \sqrt{\frac{1}{16} + \frac{81}{16}} = \frac{\sqrt{82}}{4}$ Ans..

Qn 3 → given $(1-i)x + (1+i)y = 1-3i$

$\Rightarrow x - ix + y + iy = 1-3i$

$\Rightarrow (x+y) + i(-x+y) = 1-3i$

$\Rightarrow x+y = 1$ & $-x+y = -3$

add (1) & (2)

$2y = -2 \Rightarrow y = -1$

$\therefore x = 2$ Ans..

Qn 4 → given $\frac{x-1}{3+i} + \frac{y-1}{3-i} = i$

$\Rightarrow \frac{(x-1)(3-i) + (y-1)(3+i)}{9-i^2} = i$

$\Rightarrow \frac{3x - ix - 3 + \cancel{i} + 3y + iy - 3 - \cancel{i}}{10} = i$

$\Rightarrow (3x+3y-6) + i(-x+y) = 0 + 10i$

equating real and imaginary parts

$\Rightarrow 3x+3y-6 = 0$ and $-x+y = 10$

$\Rightarrow 3y = -3x+6$ put in eq (2)

$y = -x+2$

Adding (1) & (2)

$2y = 12 \Rightarrow y = 6$

$\Rightarrow x = -4$

$\therefore x = -4$ & $y = 6$ Ans

Qn. 5 → let $z = \frac{1+i\cos\theta}{1-2i\cos\theta}$

$z = \frac{1+i\cos\theta}{1-2i\cos\theta} \times \frac{1+2i\cos\theta}{1+2i\cos\theta}$

$z = \frac{1+2i\cos\theta + i\cos\theta + 2i^2\cos^2\theta}{1-4i^2\cos^2\theta}$

Solution complex No. (W.S 1) (3)

$$z = \frac{(1-2\cos^2\theta)}{1+4\cos^2\theta} + \frac{3i\cos\theta}{1+4\cos^2\theta}$$

Since z is purely real (given)

$\therefore \ \mathrm{Im}(z) = 0$

$\Rightarrow \ \dfrac{3\cos\theta}{1+4\cos^2\theta} = 0$

$\Rightarrow \ 3\cos\theta = 0$

$\Rightarrow \ \cos\theta = 0 \ \Rightarrow \ \theta = (2n+1)\frac{\pi}{2} \quad n \in z$ Ans.

Q.No. 6 → $z = (1+i\sqrt{3})^2$

$z = 1+3i^2+2\sqrt{3}i$

$z = -2+2\sqrt{3}i$

Multiplicative Inverse

$$\frac{1}{z} = \frac{1}{-2+2\sqrt{3}i} \times \frac{-2-2\sqrt{3}i}{-2-\sqrt{3}i}$$

$$\frac{1}{z} = \frac{-2-2\sqrt{3}i}{4-12i^2}$$

$$\frac{1}{z} = \frac{-2}{16} - \frac{2\sqrt{3}i}{16}$$

$$\frac{1}{z} = \frac{-1}{8} - \frac{i\sqrt{3}}{8}$$ Ans

Q.No. 7 → $z_1 = 2-i, \quad z_2 = 1+i$

$$\left|\frac{z_1+z_2+1}{z_1-z_2+i}\right| = \left|\frac{2-\cancel{i}+1+\cancel{i}+1}{2-i-1-\cancel{i}+\cancel{i}}\right|$$

$$= \left|\frac{4}{1-i}\right|$$

$$= \left|\frac{4}{1-i} \times \frac{1+i}{1+i}\right| = \left|\frac{4+4i}{1+1}\right| = |2+2i| = \sqrt{4+4}$$

$$= 2\sqrt{2}$$ Ans

Topic : .... Date. : .... Page No. : ....

Solution Complex No. (W.S. 1) (4)

Q.No. 8 → $z_1 = 2-i$; $z_2 = -2+i$

(i) $Re\left(\frac{z_1 z_2}{\bar{z}_1}\right)$

$= Re\left(\frac{(2-i)(-2+i)}{2+i}\right)$

$= Re\left(\frac{-4+2i+2i-i^2}{2+i}\right)$

$= Re\left(\frac{-3+4i}{2+i} \times \frac{2-i}{2-i}\right)$

$= Re\left(\frac{-6+3i+8i-4i^2}{4-i^2}\right)$

$= Re\left(\frac{-2}{5} + \frac{11i}{5}\right)$

$= \frac{-2}{5}$ Ans

(ii) $Im\left(\frac{1}{z_1 \bar{z}_1}\right)$

$= Im\left(\frac{1}{(2-i)(2+i)}\right)$

$= Im\left(\frac{1}{4-i^2}\right)$

$= Im\left(\frac{1}{5}\right)$

$= 0$ Ans. — x —

CLASSTIME

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