!! जय श्री गिरिराज जी महाराज !! जय श्री राधे कृष्णा !!

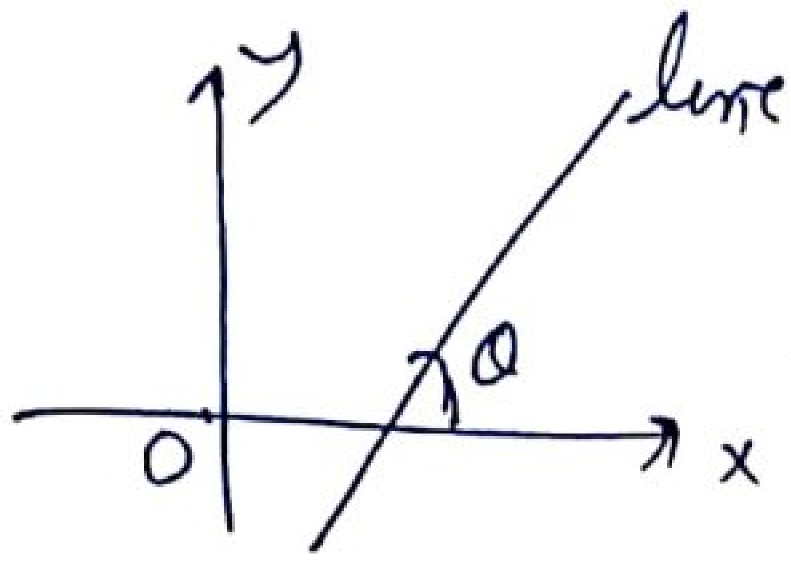
ULTIMATE MATHEMATICS: BY AJAY MITTAL

REVISION   Chapter: Straight lines   CLASS NO: 1

(·) general equation of line: (linear)

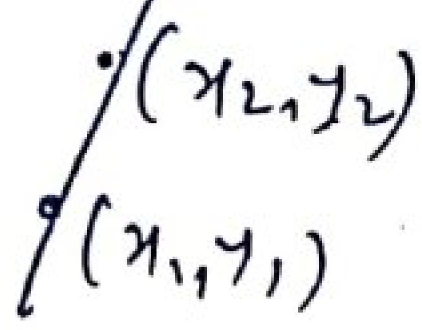
$$ax+by+c=0$$

(·) Slope of a line (m)



(·) $m = \tan\theta$

(·) $m = \dfrac{y_2-y_1}{x_2-x_1}$



(·) $m = \dfrac{-\text{coff of } x}{\text{coff of } y}$ (equation of line is given)

(·) ∥ condition $m_1 = m_2$

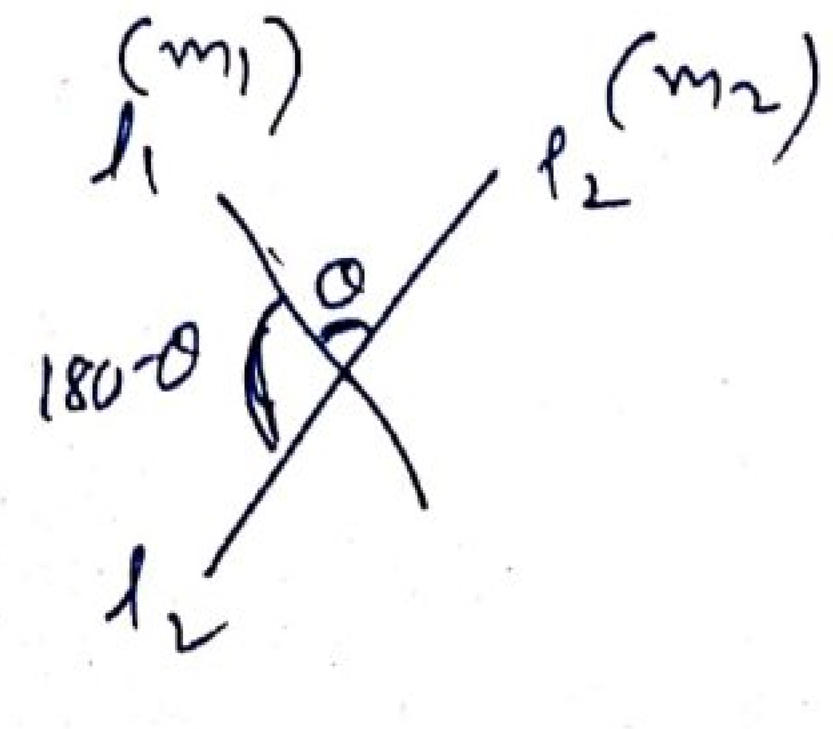
(·) ⊥ condition: $m_1 m_2 = -1$ (-ve reciprocal)

(·) Slope of X-axis = 0

(·) Slope of Y-axis = $\infty = \dfrac{1}{0}$

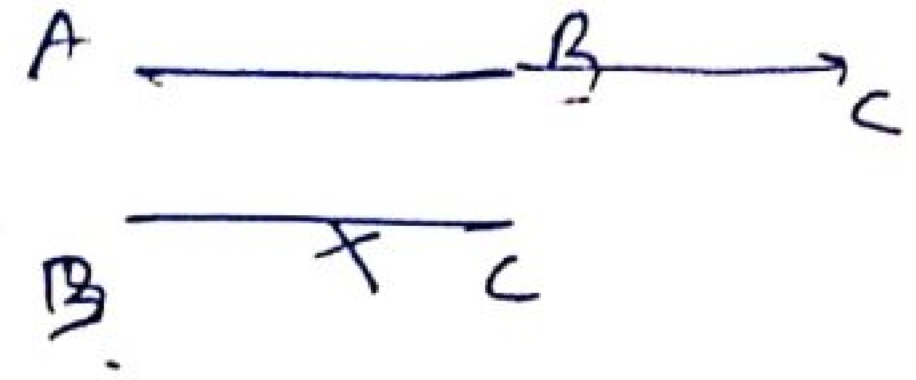
(·) Angle b/w two lines

$$\tan\theta = \left|\frac{m_1-m_2}{1+m_1m_2}\right|$$



(·) Using slopes : collinearity of three points
A( ), B( ), C( )

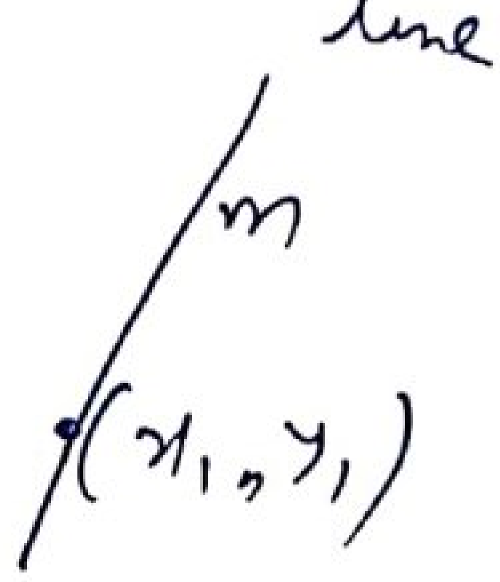
If slope of AB = slope of BC
⇒ AB ∥ BC
But point B is common



Different forms of equation of line

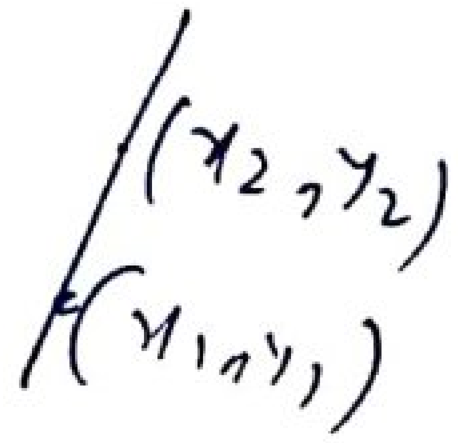
(·) point - slope form

$$\boxed{y - y_1 = m(x - x_1)}$$



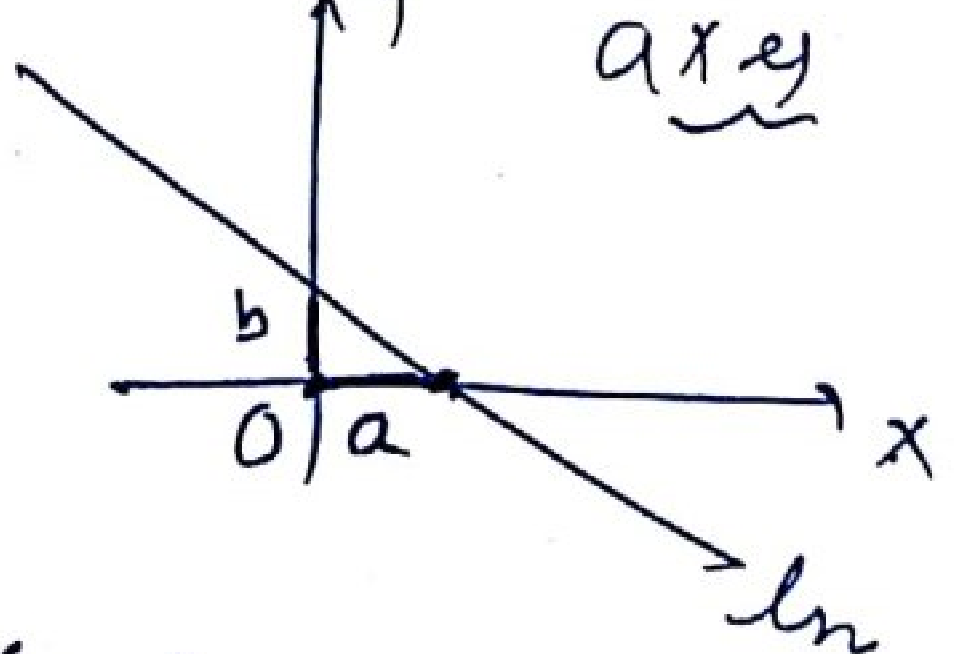
(·) Two point form

$$\boxed{y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)}$$



(·) Intercept form

$$\boxed{\frac{x}{a} + \frac{y}{b} = 1}$$



axes

eg X-int = 2
⇒ point on the line (2, 0)

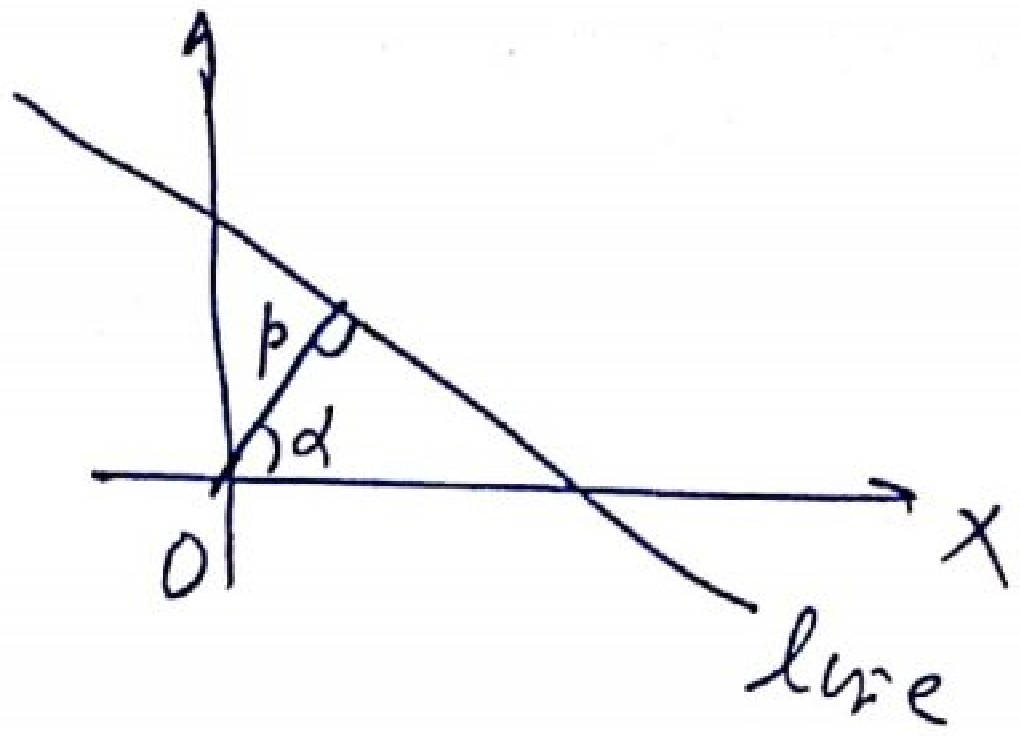
(·) Slope - Intercept form
(y-int)

$$\boxed{y = mx + c}$$ c → y-int

$$2y = x + 3$$
$$y = \frac{x}{2} + \frac{3}{2}$$
$m = 1/2$

(·) **Normal form**

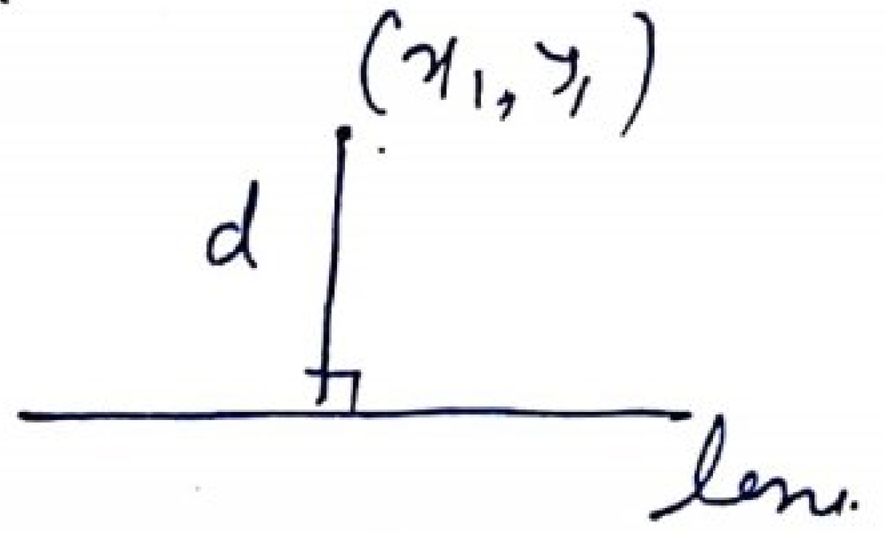
$$\boxed{x\cos\alpha + y\sin\alpha = p}$$



— x —

(·) Distance b/w point and line

$$d = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2+b^2}}$$



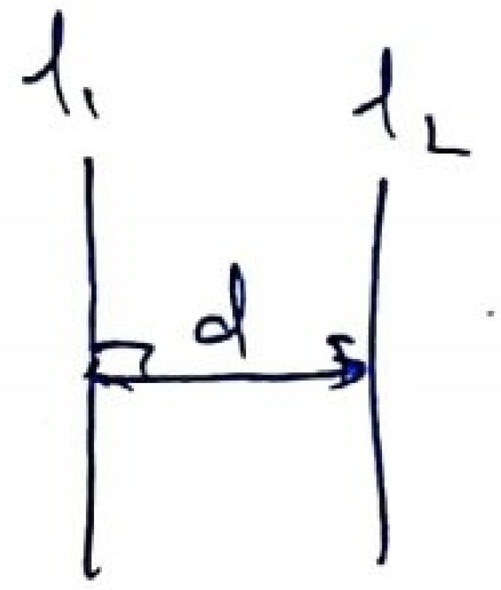
$ax + by + c = 0$

(·) Distance b/w two parallel lines

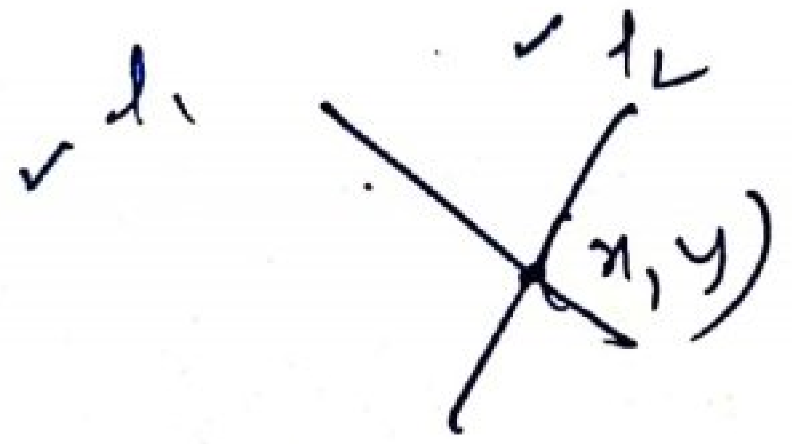
$ax + by + c_1 = 0$

$ax + by + c_2 = 0$

$$d = \frac{|c_2 - c_1|}{\sqrt{a^2+b^2}}$$



(·)



Intersect point of two lines
(Solve the two equations)

(·) (3,5) line $x + y = 8$ check

(·) (3,5) line $2x + y + a = 0$

$a = -11$

(·) given line $ax+by+c=0$

✓ any line parallel to it $\Rightarrow ax+by+\lambda=0$

✓ any line $\perp^r$ to it $\Rightarrow bx-ay+\lambda=0$

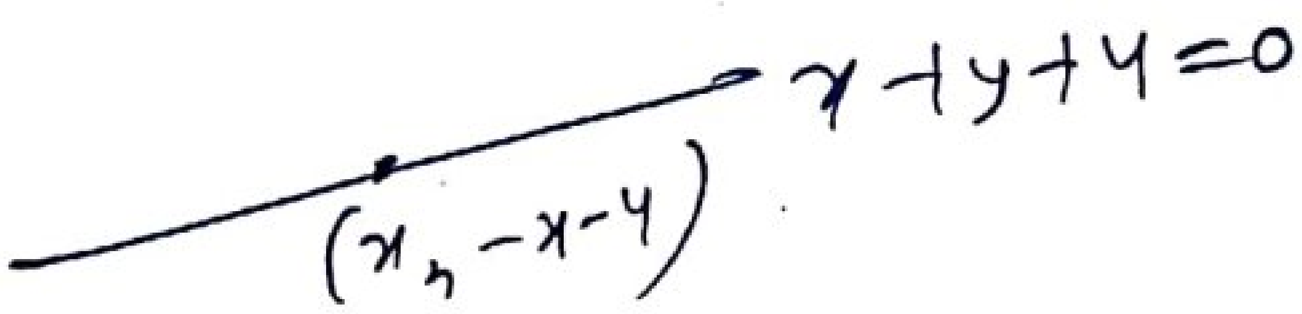
(·) $|-2|=2$

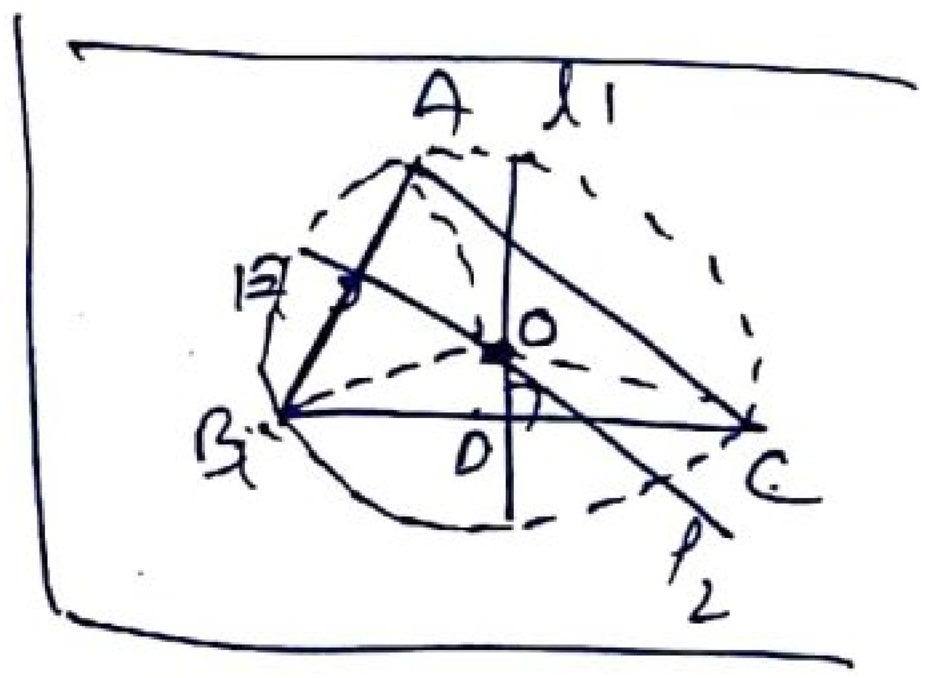
$|x^2|=x^2$ $\qquad |a||b|=|ab|$

$|x|^2=x^2$ $\qquad |a^2+b^2|=a^2+b^2$

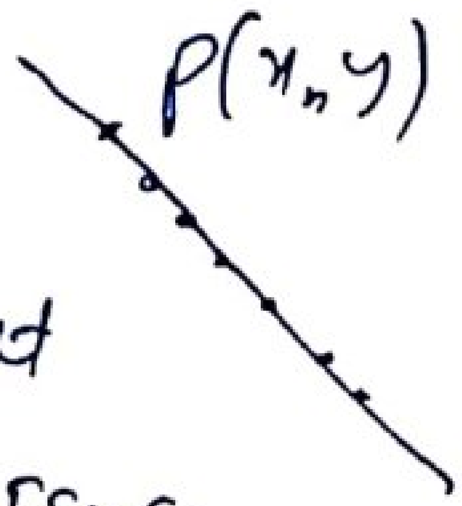
$|-a|=|a|$ $\qquad |-a-b|=|a+b|$

(·)

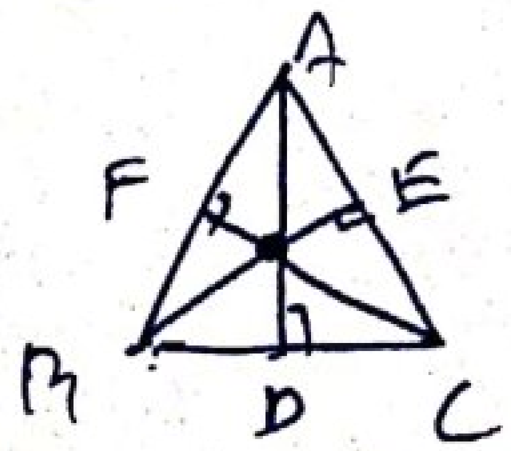




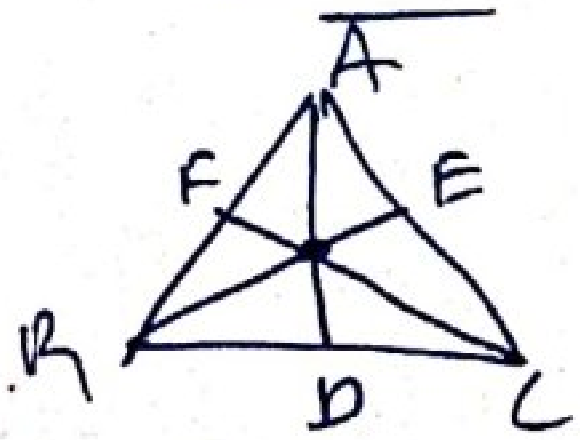
(·) ~~Lot~~ Locus → is a path traced by the moving point under some given condition



(·) orthocentre → Int. point of altitudes



(·) Centroid → Medians Meet



Equilat / Isosc.

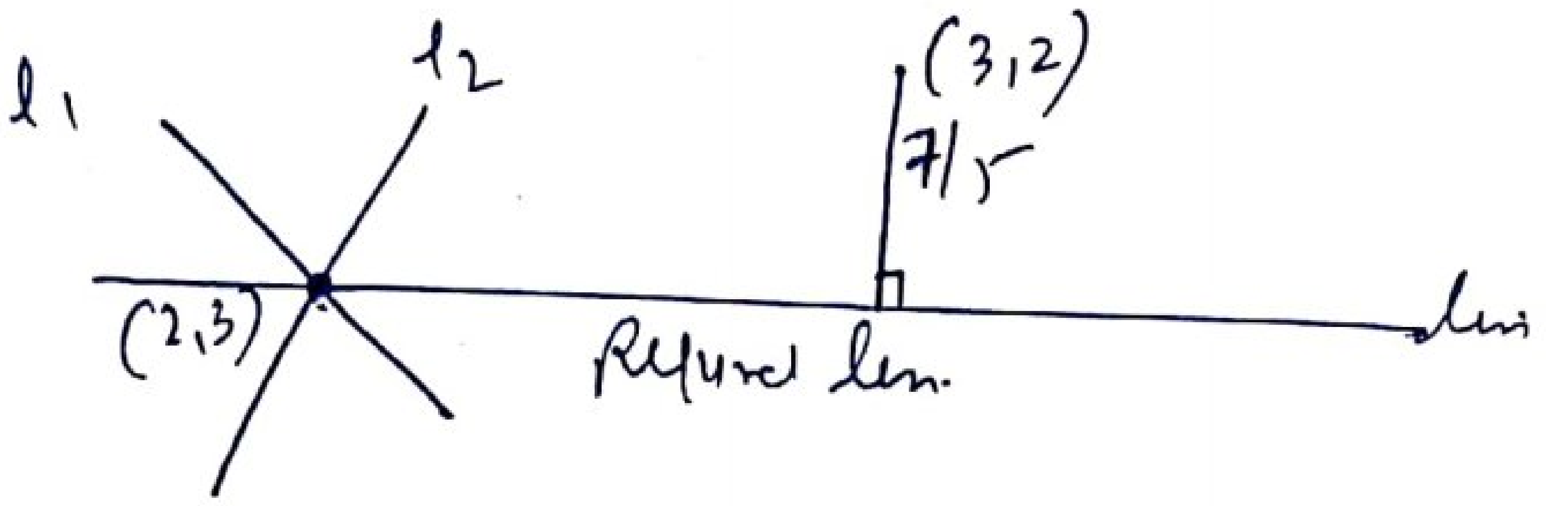
Median = altitude = $\perp^r$ bisector

(·) Circumcentre → $\perp^r$ bisectors of side meet

QUESTIONS

Ques 1 Find the equations of the lines through the point of Intersection of the lines $x-y+1=0$ and $2x-3y+5=0$ and whose distance from the point $(3,2)$ is $\frac{7}{5}$.

Soln



$$x-y=-1$$
$$2x-3y=-5$$
$$2x-2y=-2$$
$$-y=-3$$

$y=3$ $x=2$

(:) let Slope of req-line $= m$

(:) equation of req line (point-slope slope)

$$y-3 = m(x-2)$$
$$\Rightarrow y-3 = mx-2m$$
$$\Rightarrow \boxed{mx-y-2m+3=0}$$

(:) given distance b/w this line & $(3,2) = 7/5$

$$\frac{7}{5} = \frac{|3m-2-2m+3|}{\sqrt{m^2+1}}$$

$$\frac{7}{5} = \frac{|m+1|}{\sqrt{m^2+1}}$$

$$\Rightarrow \frac{49}{25} = \frac{m^2+2m+1}{m^2+1} \Rightarrow 49m^2+49 = 25m^2+50m+25$$
$$\Rightarrow 24m^2-50m+24=0$$

$\Rightarrow \quad 12m^2 - 25m + 12 = 0$

$\Rightarrow \quad 12m^2 - 16m - 9m + 12 = 0$

$\Rightarrow \quad 4m(3m-4) - 3(3m-4) = 0$

$\Rightarrow \quad m = 4/3 \qquad m = 3/4$

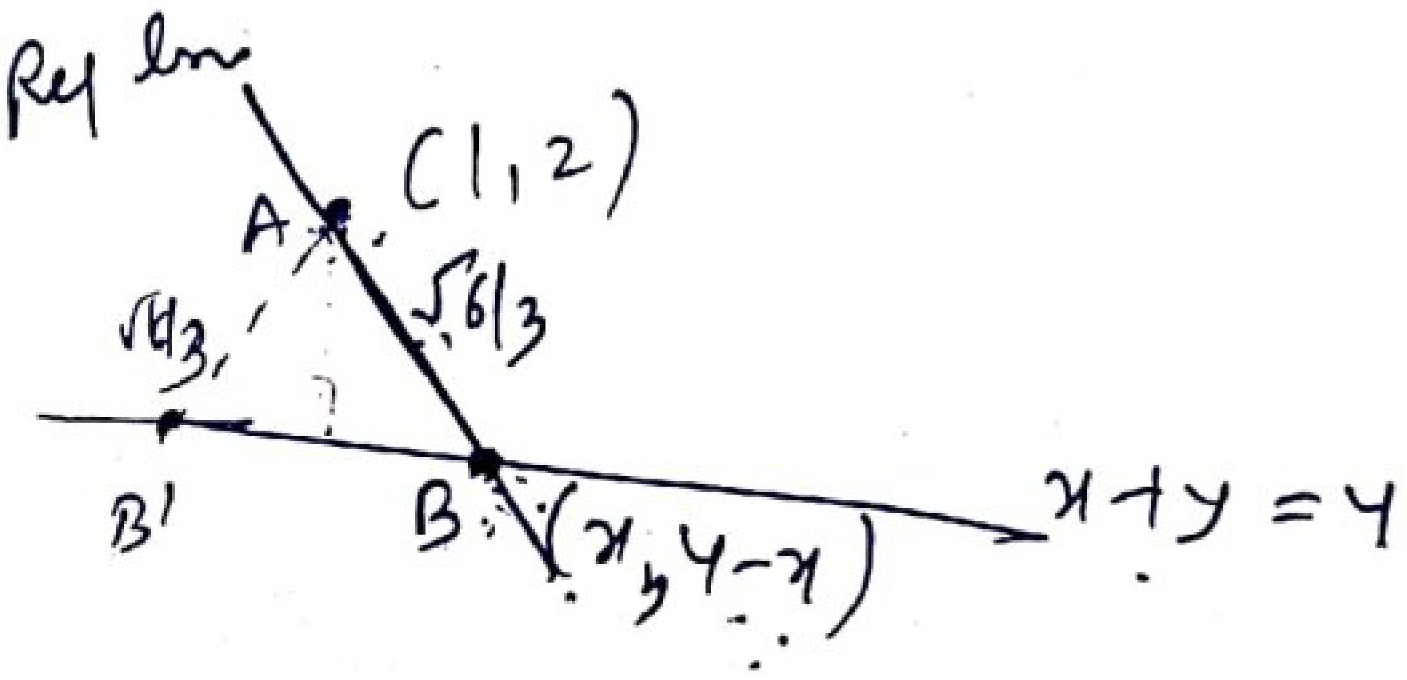
$\therefore$ eqn of lines

$\frac{4}{3}x - y - 2\left(\frac{4}{3}\right) + 3 = 0$ | $\frac{3}{4}x - y - 2\left(\frac{3}{4}\right) + 3 = 0$

Ans | Ans

Qn-2 → In what direction should a line be drawn through the point (1,2) so that its point of intersection with the line $x+y=4$ is at a distance of $\frac{\sqrt{6}}{3}$ from the given point

Soln



Ans 15° or 75°

tn(15°) = $\sqrt{2} - 1$

tn(75°) = $\sqrt{2} + 1$

let point B is $(x, 4-x)$

distance $AB = \frac{\sqrt{6}}{3}$

$$\sqrt{(x-1)^2 + (2-x)^2} = \frac{\sqrt{6}}{3}$$

$\Rightarrow \quad x^2 - 2x + 1 + 4 + x^2 - 4x = \frac{2}{3}$

$\Rightarrow \quad 2x^2 - 6x + 5 = 2/3$

$6x^2 - 18x + 15 = 2$

$6x^2 - 18x + 13 = 0$

$\cancel{6x^2 \ldots + 13 = 0}$

$x =$

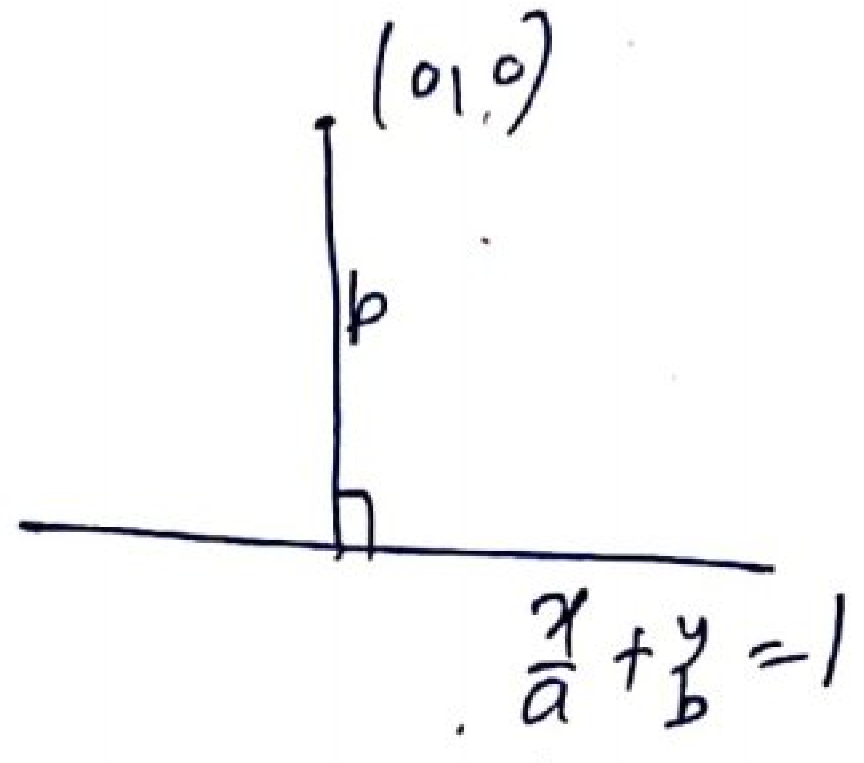
$x =$

Quadratic formula

Q.3 → If p is the length of perpendicular from the origin on the line $\frac{x}{a} + \frac{y}{b} = 1$ and a^2, p^2, b^2 are in A.P, then show that $a^4 + b^4 = 0$

Soln

Given line : $bx + ay - ab = 0$



$$p = \frac{|0 + 0 - ab|}{\sqrt{b^2 + a^2}}$$

$$p^2 = \frac{a^2b^2}{b^2 + a^2}$$

Given a^2, p^2, b^2 are in AP

$\Rightarrow 2p^2 = a^2 + b^2$

$\Rightarrow \frac{2a^2b^2}{b^2 + a^2} = a^2 + b^2$

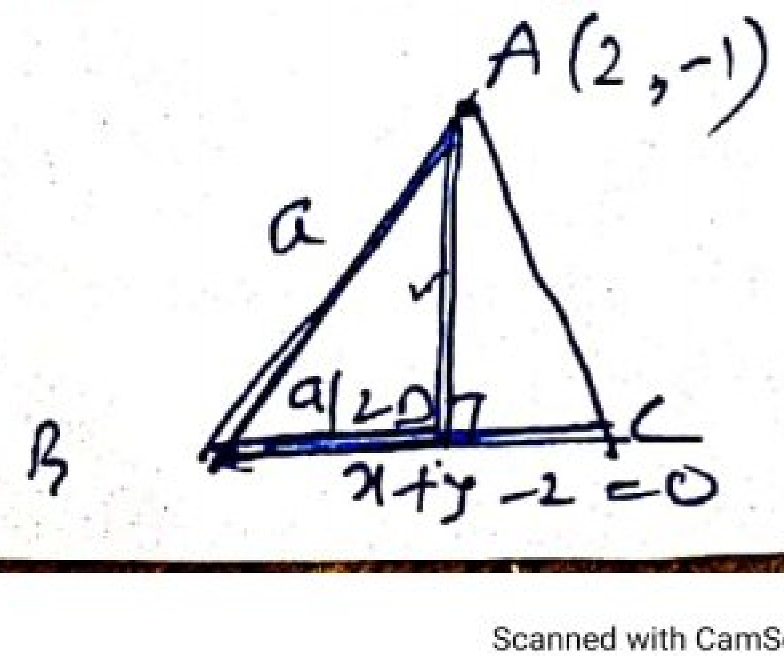
$\Rightarrow 2a^2b^2 = (a^2 + b^2)^2$

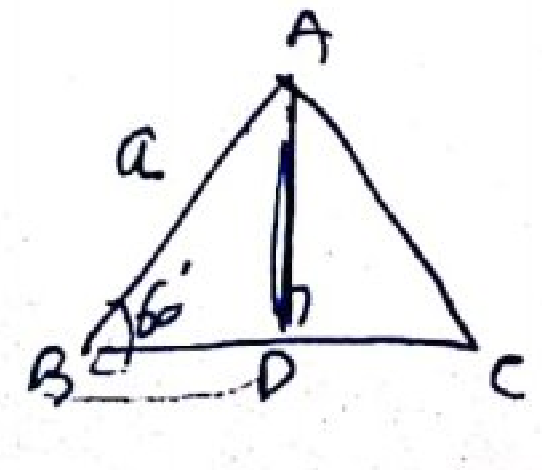
$\Rightarrow 2a^2b^2 = a^4 + b^4 + 2a^2b^2$

$\Rightarrow \boxed{a^4 + b^4 = 0}$ proved

Q.4 → If the equation of the base of an equilateral triangle is $x + y = 2$ and the vertex is $(2, -1)$. Find the length of side of triangle.

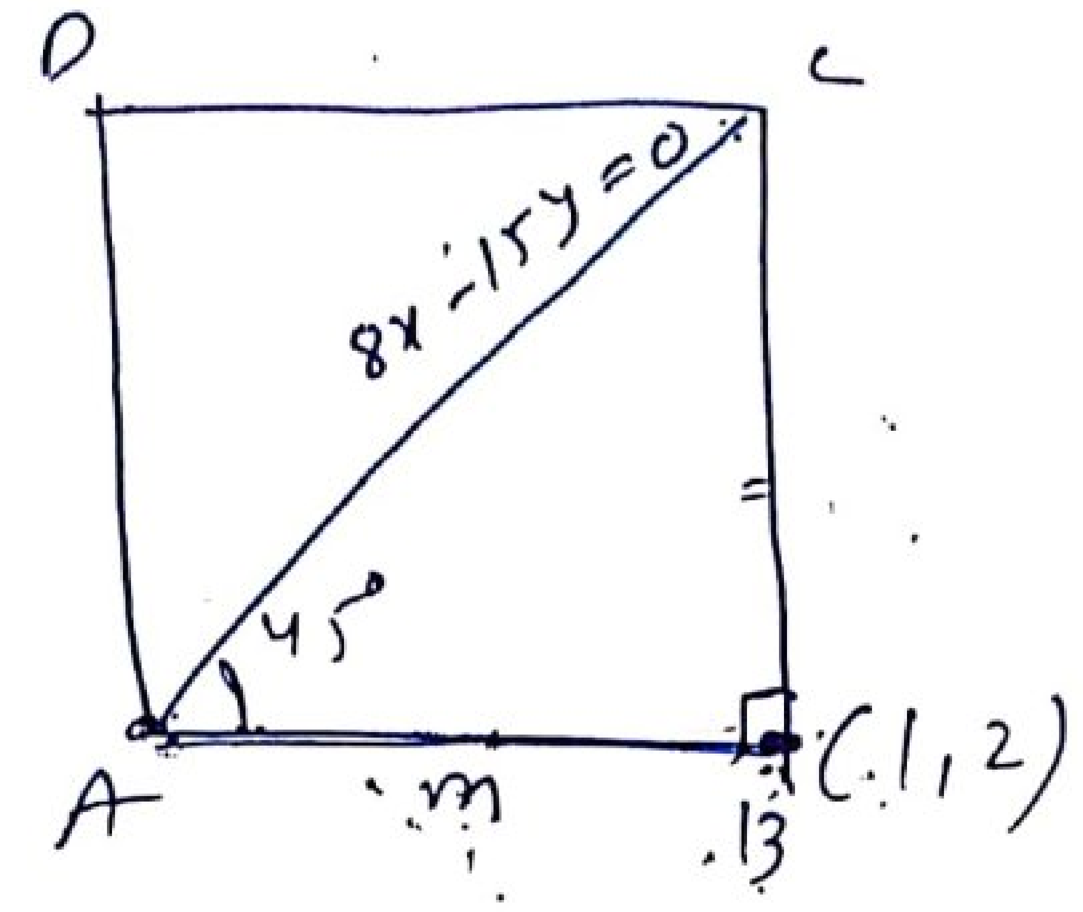
Soln.





Q.No. 5 → If one diagonal of a square is along $8x-15y=0$ and one of its vertex is $(1,2)$. Then find the equation of sides of the square passing through this vertex.

Solution



Slope of AC $\quad m_1 = \frac{-8}{-15} = \frac{8}{15}$

let slope of AB $\quad m_2 = m$

$\theta = 45°$

$$\tan(45) = \left| \frac{\frac{8}{15} - m}{1 + \frac{8m}{15}} \right|$$

$$\Rightarrow 1 = \left| \frac{8-15m}{15+8m} \right|$$

$\Rightarrow 1 = \frac{8-15m}{15+8m}$	$-1 = \frac{8-15m}{15+8m}$
$\Rightarrow 15+8m = 8-15m$	$-15-8m = 8-15m$
$\Rightarrow 23m = -7$	$7m = 23$
$m = -7/23$	$m = \frac{23}{7}$

equ of AB $\quad m = -\cancel{7}/2\cancel{3}$ & point $(1,\cancel{2})$

$$y - \cancel{2} = \frac{-7}{23}(x-1)$$

eq of BC slope $= 23/7$ point $(1,2)$ (proceed)