① ← ULTIMATE MATHEMATICS →

Solution of T-2 (upto Ans 15)

By: AJAY MITTAL

Ans 1 → given $\sin A = 3/5$, $\cos B = \frac{-12}{13}$

$A \to I^{st}$ quad, $B \to III^{rd}$ quad

$\sin^2 A + \cos^2 A = 1$	$\sin^2 B + \cos^2 B = 1$
$\cos^2 A = 1 - \sin^2 A$	$\sin^2 B = 1 - \cos^2 B$
$\cos^2 A = 1 - \frac{9}{25}$	$\sin^2 B = 1 - \frac{144}{169}$
$\cos^2 A = \frac{16}{25}$	$\sin^2 B = \frac{25}{169}$
$\cos A = \pm \frac{4}{5}$	$\sin B = \pm \frac{5}{13}$
$\cos A = \frac{4}{5} \cdots (A \to I^{st}$ quad$)$	$\sin B = -\frac{5}{13} \cdots \{B \to 2^{nd}$ quad

$\tan A = \frac{\sin A}{\cos A} = \frac{3/5}{4/5} = 3/4$

$\tan B = \frac{\sin B}{\cos B} = \frac{-5/13}{-12/13} = \frac{5}{12}$

(i) $\sin(A-B) = \sin A \cos B - \cos A \sin B$

$= \left(\frac{3}{5}\right)\left(\frac{-12}{13}\right) - \left(\frac{4}{5}\right)\left(\frac{-5}{13}\right)$

$= -\frac{36}{65} + \frac{20}{65} = \frac{-16}{65}$ Ans..

(ii) $\cos(A+B) = \cos A \cos B - \sin A \sin B$

$= \left(\frac{4}{5}\right)\left(\frac{-12}{13}\right) - \left(\frac{3}{5}\right)\left(\frac{-5}{13}\right)$

$= \frac{-48}{65} + \frac{15}{65} = \frac{-33}{65}$ Ans..

(iii) $\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$

$= \frac{\frac{3}{4} - \frac{5}{12}}{1 + \frac{3}{4} \times \frac{5}{12}} = \frac{36-20}{48+15} = \frac{16}{63}$ Ans..

(T-2) Solutions

Qns 2 → Given $\cos A = \frac{4}{5}$, $\cos B = \frac{12}{13}$

$A \to 4^{th}$ quad & $B \to 4^{th}$ quad

$\sin^2 A + \cos^2 A = 1$

$\sin^2 A = 1 - \cos^2 A$

$\sin^2 A = 1 - \frac{16}{25}$

$\sin^2 A = \frac{9}{25}$

$\sin A = \pm \frac{3}{5}$

$\sin A = \frac{-3}{5}$ (4^{th} quad)

$\sin^2 B + \cos^2 B = 1$

$\sin^2 B = 1 - \cos^2 B$

$\sin^2 B = 1 - \frac{144}{169}$

$\sin^2 B = \frac{25}{169}$

$\sin B = \pm \frac{5}{13}$

$\sin B = -\frac{5}{13}$ (4^{th} quad)

(i) $\cos(A+B) = \cos A \cos B - \sin A \sin B$

$= \left(\frac{4}{5}\right)\left(\frac{12}{13}\right) - \left(\frac{-3}{5}\right)\left(\frac{-5}{13}\right)$

$= \frac{48}{65} - \frac{15}{65} = \frac{33}{65}$ Ans

(ii) $\sin(A-B) = \sin A \cos B - \cos A \sin B$

$= \left(\frac{-3}{5}\right)\left(\frac{12}{13}\right) - \left(\frac{4}{5}\right)\left(\frac{-5}{13}\right)$

$= \frac{-36}{65} + \frac{20}{65} = \frac{-16}{65}$ Ans

Qns: 3 → Given $\cot \alpha = \frac{1}{2}$, $\sec \beta = \frac{-5}{3}$

$\alpha \to 3^{rd}$ quad, $\beta \to 2^{nd}$ quad

$\tan \alpha = \frac{1}{\cot \alpha} = \frac{1}{1/2} = 2$

$1 + \tan^2 \beta = \sec^2 \beta$

$\tan^2 \beta = \sec^2 \beta - 1$

$\tan^2 \beta = \frac{25}{9} - 1$

$\tan^2 \beta = \frac{16}{9}$

$\tan \beta = \pm \frac{4}{3}$

(T-2) Solutions

$\tan\beta = -\frac{4}{3}$

$\downarrow$ 2nd quad

Now $\tan(\alpha+\beta) = \frac{\tan\alpha + \tan\beta}{1-\tan\alpha\tan\beta}$

$= \frac{2 - \frac{4}{3}}{1-(2)(-4/3)} = \frac{\frac{2}{3}}{\frac{3+8}{3}} = \frac{2}{11}$ Ans.

Qn 4 → $\tan A = \frac{3}{4}$, $\cos B = \frac{9}{41}$

$A \to$ 3rd quad & $B \to$ Ist quad

$\sin^2 B + \cos^2 B = 1$

$\sin^2 B = 1 - \cos^2 B$

$\sin^2 B = 1 - \frac{81}{1681} = \frac{1600}{1681}$

$\sin B = \pm\frac{40}{41}$

$\downarrow$ Ist quad

$\sin B = \frac{40}{41}$

$\tan B = \frac{\sin B}{\cos B} = \frac{40/41}{9/41} = \frac{40}{9}$

Now $\tan(A+B) = \frac{\tan A + \tan B}{1-\tan A\tan B}$

$= \frac{\frac{3}{4} + \frac{40}{9}}{1 - \frac{3}{4}\times\frac{40}{9}}$

$= \frac{27+160}{36-120} = \frac{187}{-84}$ Ans..

(T-2) Solutions

4

Qn 5 (1) $\sin(75^\circ) = \sin(45^\circ + 30^\circ)$

$= \sin(45^\circ)\cos(30^\circ) + \cos(45^\circ)\sin(30^\circ)$

$= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \times \frac{1}{2}$

$= \frac{\sqrt{3}+1}{2\sqrt{2}}$ Ans

(2) $\sin(15^\circ) = \sin(45 - 30^\circ)$

$= \sin(45^\circ)\cos(30^\circ) - \cos(45^\circ)\sin(30^\circ)$

$= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} \times \frac{1}{2}$

$= \frac{\sqrt{3}-1}{2\sqrt{2}}$ Ans

(3) $\tan(15^\circ) = \tan(45^\circ - 30^\circ)$

$= \frac{\tan(45^\circ) - \tan(30^\circ)}{1 + \tan(45^\circ)\tan(30^\circ)}$

$= \frac{1 - \frac{1}{\sqrt{3}}}{1 + \frac{1}{\sqrt{3}}} = \frac{\sqrt{3}-1}{\sqrt{3}+1}$

Rationalize

$= \frac{\sqrt{3}-1}{\sqrt{3}+1} \times \frac{\sqrt{3}-1}{\sqrt{3}-1}$

$= \frac{3+1-2\sqrt{3}}{3-1} = \frac{4-2\sqrt{3}}{2} = 2-\sqrt{3}$ Ans

(4) $\tan(75^\circ) = \tan(45^\circ + 30^\circ)$

Proceed as above

$=$ Ans $2+\sqrt{3}$

(5) $\tan(105^\circ) = \tan(60^\circ + 45^\circ)$

$= \frac{\tan(60^\circ) + \tan(45^\circ)}{1 - \tan(60^\circ)\tan(45^\circ)}$

$= \frac{\sqrt{3}+1}{1-\sqrt{3}}$ Rationalize

(T-2) Solutions

$$= \frac{\sqrt{3}+1}{1-\sqrt{3}} \times \frac{1+\sqrt{3}}{1+\sqrt{3}}$$

$$= \frac{3+1+2\sqrt{3}}{1-3}$$

$$= \frac{4+2\sqrt{3}}{-2} = -(2+\sqrt{3})$$ Ans..

(6) $\tan\left(\frac{13\pi}{12}\right)$

$$= \tan\left(13 \times \frac{180^\circ}{12}\right) = \tan(13 \times 15^\circ) = \tan(195^\circ)$$

$$= \tan(180^\circ + 15^\circ)$$

$$= \tan(15^\circ)$$

$= \tan(45^\circ - 30^\circ)$ proceed $\tan(A-B)$ formula

Ans $2-\sqrt{3}$

(7) $\cos(105^\circ) + \cos(15^\circ)$

$$= \cos(60^\circ + 45^\circ) + \cos(45^\circ - 30^\circ)$$

$$= \cos(60^\circ)\cos(45^\circ) - \sin(60^\circ)\sin(45^\circ) + \cos(45)\cos(30^\circ) + \sin(45)\sin(30^\circ)$$

$$= \frac{1}{2} \times \frac{1}{\sqrt{2}} - \frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \times \frac{1}{2}$$

$$= \frac{1 - \cancel{\sqrt{3}} + \cancel{\sqrt{3}} + 1}{2\sqrt{2}} = \frac{2}{2\sqrt{2}} = \frac{1}{\sqrt{2}}$$ Ans..

(8) $\tan(75^\circ) + \cot(75^\circ)$

$\tan(75^\circ) = \tan(45^\circ + 30^\circ)$
and get $2+\sqrt{3}$

$$\cot(75^\circ) = \frac{1}{\tan(75)} = \frac{1}{2+\sqrt{3}}$$

$$\tan(75) + \cot(75^\circ) = 2+\sqrt{3} + \frac{1}{2+\sqrt{3}}$$

(T-2) Solutions

$$= \frac{4+3+4\sqrt{3}+1}{2+\sqrt{3}}$$

$$= \frac{8+4\sqrt{3}}{2+\sqrt{3}}$$

$$= 4\left(\frac{\cancel{2+\sqrt{3}}}{\cancel{2+\sqrt{3}}}\right)$$

$$= 4 \quad \underline{\text{Ans.}}$$

(9) $\cos\left(\frac{13\pi}{12}\right) = \cos\left(13 \times \frac{180^\circ}{12}\right) = \cos(13 \times 15^\circ)$

$$= \cos(195^\circ)$$

$$= \cos(\overset{III}{180+15})$$

$$= -\cos(15)$$

$$= -\cos(45^\circ - 30^\circ)$$

$$= -\left[\cos 45^\circ \cos 30^\circ + \sin 45^\circ \sin 30^\circ\right]$$

$$= -\left[\frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \times \frac{1}{2}\right]$$

$$= -\left(\frac{\sqrt{3}+1}{2\sqrt{2}}\right) \quad \underline{\underline{\text{Ans}}}$$

(10) $\sin\left(\frac{13\pi}{12}\right) = \sin\left(13 \times \frac{180^\circ}{12}\right) = \sin(195^\circ)$

$$= \sin(\overset{III}{180+15^\circ})$$

$$= -\sin(15^\circ)$$

$$= -\sin(45^\circ - 30^\circ)$$

$$= -\left(\sin(45)\cos(30^\circ) - \cos(45)\sin(30)\right)$$

$$= -\left[\frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} \cdot \frac{1}{2}\right]$$

$$= -\left(\frac{\sqrt{3}-1}{2\sqrt{2}}\right) \quad \underline{\text{Ans}}$$

(T-2) Solutions

Ans 6 → $\underline{Lhs}$ $\overset{(A)}{\sin(n+1)A} \cdot \overset{(B)}{\sin(n+2)A} + \overset{(A)}{\cos(n+1)A} \cdot \overset{(B)}{\cos(n+2)A}$

compare with $(\sin A \sin B + \cos A \cos B) \longrightarrow \cos(A-B)$

$= \cos((n+1)A - (n+2)A)$

$= \cos(\cancel{n}A + A - \cancel{n}A - 2A)$

$= \cos(-A)$

$= \cos A$ --- $\{\because \cos(-\theta) = \cos\theta\}$

$= \underline{Rhs}$ $\underline{\text{Proved}}$

Ans 7 → $\underline{Lhs}$ $\cos(\frac{\pi}{4} - A)\cos(\frac{\pi}{4} - B) - \sin(\frac{\pi}{4} - A)\cdot\sin(\frac{\pi}{4} - B)$

Compare with $(\cos A \cos B - \sin A \sin B) \longrightarrow \cos(A+B)$

$= \cos(\frac{\pi}{4} - A + \frac{\pi}{4} - B)$

$= \cos(\frac{\pi}{2} - (A+B))$

$= \cos(90 - (A+B))$

$= \sin(A+B) = Rhs$ $\underline{\text{Ans}}$

Ans 8 → $\underline{Lhs}$ $\dfrac{\sin(B-C)}{\cos B \cos C} + \dfrac{\sin(C-A)}{\cos C \cdot \cos A} + \dfrac{\sin(A-B)}{\cos A \cos B}$

$= \dfrac{\sin B \cos C - \cos B \cdot \sin C}{\cos B \cos C} + \dfrac{\sin C \cdot \cos A - \cos C \cdot \sin A}{\cos C \cdot \cos A} +$

$\dfrac{\sin A \cos B - \cos A \sin B}{\cos A \cdot \cos B}$

(Separate) $= \dfrac{\sin B \cancel{\cos C}}{\cos B \cancel{\cos C}} - \dfrac{\cancel{\cos B}\sin C}{\cancel{\cos B}\cos C} + \dfrac{\sin C \cancel{\cos A}}{\cos C \cancel{\cos A}} - \dfrac{\cancel{\cos C}\sin A}{\cancel{\cos C}\cdot\cos A} + \dfrac{\sin A \cancel{\cos B}}{\cos A \cancel{\cos B}}$

$- \dfrac{\cancel{\cos A}\sin B}{\cancel{\cos A}\cos B}$

$= \cancel{\tan B} - \cancel{\tan C} + \cancel{\tan C} - \cancel{\tan A} + \cancel{\tan A} - \cancel{\tan B}$

$= 0 = RHS$ Proved

Ans 9 → LHS $\dfrac{\tan\left(\frac{\pi}{4}+x\right)}{\tan\left(\frac{\pi}{4}-x\right)}$

$= \dfrac{\tan(45^\circ + x)}{\tan(45^\circ - x)}$

$= \dfrac{\dfrac{\tan 45 + \tan x}{1 - \tan 45 \tan x}}{\dfrac{\tan(45) - \tan x}{1 + \tan 45 \tan x}} = \dfrac{\dfrac{1+\tan x}{1-\tan x}}{\dfrac{1-\tan x}{1+\tan x}}$

$= \dfrac{(1+\tan x)^2}{(1-\tan x)^2} = \left(\dfrac{1+\tan x}{1-\tan x}\right)^2$ Ans

Ans 10 → LHS $\dfrac{\sin(A-B)}{\sin A \sin B} + \dfrac{\sin(B-C)}{\sin B \sin C} + \dfrac{\sin(C-A)}{\sin C \sin A}$

Misprint in worksheet

$= \dfrac{\sin A \cos B - \cos A \sin B}{\sin A \sin B} + \dfrac{\sin B \cos C - \cos B \sin C}{\sin B \sin C} + \dfrac{\sin C \cdot \cos A - \cos C \sin A}{\sin C \sin A}$

Separate

$= \cancel{\cot B} - \cot A + \cot C - \cot B + \cot A - \cot C$

$= 0$ Ans

(7-2) Solutions (9)

Qn 11 → LHS

$\tan(56^\circ)$

$= \tan(45^\circ + 11^\circ)$

$= \dfrac{\tan(45^\circ) + \tan(11^\circ)}{1 - \tan(45^\circ)\tan(11^\circ)}$

$= \dfrac{1 + \tan(11^\circ)}{1 - \tan(11^\circ)}$

$= \dfrac{1 + \frac{\sin(11^\circ)}{\cos(11^\circ)}}{1 - \frac{\sin(11^\circ)}{\cos(11^\circ)}}$

LCM

$= \dfrac{\cos(11^\circ) + \sin(11^\circ)}{\cos(11^\circ) - \sin(11^\circ)} = RHS$ Proved

---

Qn 12 → LHS $\tan(54^\circ)$

$= \tan(45 + 9)$

= proceed as above

Qn 13 → LHS $\tan(37^\circ)$

$= \tan(45^\circ - 8)$

$= \dfrac{\tan(45^\circ) - \tan(8^\circ)}{1 + \tan(45^\circ)\tan(8^\circ)}$

$= \dfrac{1 - \tan(8^\circ)}{1 + \tan(8^\circ)}$

$= \dfrac{1 - \frac{\sin 8^\circ}{\cos 8^\circ}}{1 + \frac{\sin 8^\circ}{\cos 8^\circ}}$

(T-2) Solutions

$= LHS \quad = \dfrac{\cos 8^\circ - \sin 8^\circ}{\cos 8^\circ + \sin 8^\circ}$ RHS proved

Ans 15 given $\tan\alpha = \dfrac{m}{m+1}$ & $\tan\beta = \dfrac{1}{2m+1}$

Now $\tan(\alpha+\beta) = \dfrac{\tan\alpha + \tan\beta}{1 - \tan\alpha \tan\beta}$

$$= \dfrac{\dfrac{m}{m+1} + \dfrac{1}{2m+1}}{1 - \dfrac{m}{m+1} \times \dfrac{1}{2m+1}}$$

$$= \dfrac{\dfrac{2m^2+m+m+1}{\cancel{(m+1)(2m+1)}}}{\dfrac{(m+1)(2m+1) - m}{\cancel{(m+1)(2m+1)}}}$$

$$= \dfrac{2m^2+2m+1}{2m^2+3m+1-m}$$

$$= \dfrac{\cancel{2m^2+2m+1}}{\cancel{2m^2+2m+1}}$$

$\tan(\alpha+\beta) = 1$

$\Rightarrow \tan(\alpha+\beta) = \tan(45^\circ) = \tan(\pi/4)$

$\Rightarrow \boxed{\alpha+\beta = \pi/4}$ proved

Ans 14 → Same as Ans 15

— x —