॥ जय श्री राधे कृष्णा ॥

← ULTIMATE MATHEMATICS →

BY: AJAY MITTAL (9891067390)

Chapter: SEQUENCE & SERIES

← CLASS NO: 4 →

Que 1 → If a, b, c are in A.P ; b, c, d are in G.P and $\frac{1}{c}, \frac{1}{d}, \frac{1}{e}$ are in A.P , prove that a, c, e are in G.P

Soln (·) given $a, b, c \rightarrow AP$

$\Rightarrow \boxed{2b = a + c}$ ---(1)

(·) given: $b, c, d \rightarrow GP$

$\Rightarrow \boxed{c^2 = bd}$ --(2)

(·) given $\frac{1}{c}, \frac{1}{d}, \frac{1}{e}$ are in AP

$\Rightarrow \frac{2}{d} = \frac{1}{c} + \frac{1}{e}$

$\Rightarrow \frac{2}{d} = \frac{e+c}{ce}$

$\Rightarrow \frac{d}{2} = \frac{ce}{e+c}$

$\Rightarrow \boxed{d = \frac{2ce}{e+c}}$ --(3)

(·) T.P $a, c, e \rightarrow GP$

i.e $c^2 = ae$

& we have $c^2 = bd$ {from eq (2)}

$c^2 = \left(\frac{a+c}{\cancel{2}}\right)\left(\frac{\cancel{2}ce}{e+c}\right)$

$\cancel{c^2e} + c^3 = ace + \cancel{c^2e} \Rightarrow c^3 = ace$

$\Rightarrow c^2 = ae$ $\therefore a, c, e \rightarrow GP$

Ques 2 → A G.P consists of even number of terms. If the sum of all the terms is 5 times the sum of terms occupying odd places. Find the common ratio

Soln

let GP $a_1, a_2, a_3, a_4 ------ a_{2n}$

we have

$a_1 + a_2 + a_3 + --- (2n)$ term $= 5(a_1 + a_3 + a_5 + --- n$ term$)$

$a + ar + ar^2 + --- (2n)$ term $= 5(a + ar^2 + ar^4 + -- n$ term$)$

← GP : $I^{st} = a$ → ← GP : $I^{st} = a$ →

ratio $= r$ ratio $= r^2$

term $2n$ term $= n$

$$\Rightarrow a\frac{(r^{2n}-1)}{r-1} = 5a\frac{((r^2)^n - 1)}{r^2-1}$$

$$\Rightarrow \cancel{a}\frac{\cancel{(r^{2n}-1)}}{\cancel{r-1}} = 5\cancel{a}\frac{\cancel{(r^{2n}-1)}}{(r+1)\cancel{(r-1)}}$$

$$\Rightarrow 1 = \frac{5}{r+1}$$

$\Rightarrow 5 = r+1$

$\Rightarrow \boxed{r = 4}$ Ans

Ques 3 → Show that the ratio of the sum of first n terms of a GP to the sum of terms from $(n+1)^{th}$ to $(2n)^{th}$ term is $\frac{1}{r^n}$

Sol

$a_1, a_2 ------ a_{2n}$

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$\boxed{a_1, a_2 - - - a_n}$ $\boxed{a_{n+1}, - - - - . a_{2n}}$ (2, 4, 8) (16, 32, 64)

GP GP

I^{st} term $= a_1 = a$ I^{st} term $a_{n+1} = ar^n$

ratio $= r$ ratio $= r$

terms $= n$ terms $= n$

Sum $= S_n$ Sum $= S'_n$

T.P $\frac{S_n}{S'_n} = \frac{1}{r^n}$

L.H.S

$$\frac{S_n}{S'_n} = \frac{\cancel{a}\left(\frac{\cancel{r^n-1}}{\cancel{r-1}}\right)}{\cancel{a}r^n\left(\frac{\cancel{r^n-1}}{\cancel{r-1}}\right)}$$

$$\frac{S_n}{S'_n} = \frac{1}{r^n}$$ Ans

Qn 4 → If the p^{th}, q^{th}, r^{th} and s^{th} terms of an A.P are in G.P, then show that $(p-q), (q-r), (r-s)$ are also in G.P.

Sol

$a_p = a + (p-1)d$

$a_q = a + (q-1)d$

$a_r = a + (r-1)d$

$a_s = a + (s-1)d$

these terms are in GP

$\therefore \frac{a_q}{a_p} = \frac{a_r}{a_q} = \frac{a_s}{a_r}$

T.P $(q-r)^2 = (p-q)(r-s)$

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Consider

$$\frac{a_q}{a_p} = \frac{a_r}{a_q} \quad \cdots \left\{ \text{If } \frac{a}{b} = \frac{c}{d} = \frac{a+c}{b+d} = \frac{a-c}{b-d} \right\}$$

$$\frac{a_q}{a_p} = \frac{a_r}{a_q} = \frac{a_q - a_r}{a_p - a_q}$$

$$= \frac{(\cancel{a}+(q-1)d) - (\cancel{a}+(r-1)d)}{(\cancel{a}+(p-1)d) - (\cancel{a}+(q-1)d)}$$

$$= \frac{\cancel{d}(q-\cancel{1}-r+\cancel{1})}{\cancel{d}(p-\cancel{1}-q+\cancel{1})}$$

$$\frac{a_q}{a_p} = \frac{a_r}{a_q} = \cancel{\frac{p-q}{}} \; \frac{q-r}{p-q} \quad \cdots (1)$$

Consider

$$\frac{a_r}{a_q} = \frac{a_s}{a_r} = \frac{a_r - a_s}{a_q - a_r}$$

$$= \frac{(\cancel{a}+(r-1)d) - (\cancel{a}+(s-1)d)}{(\cancel{a}+(q-1)d) - (\cancel{a}+(r-1)d)}$$

$$= \frac{\cancel{d}(r-\cancel{1}-s+\cancel{1})}{\cancel{d}(q-\cancel{1}-r+\cancel{1})}$$

$$\frac{a_r}{a_q} = \frac{a_s}{a_r} = \frac{r-s}{q-r} \quad - (2)$$

from (1) & (2)

$$\frac{q-r}{p-q} = \frac{r-s}{q-r}$$

$$\Rightarrow (q-r)^2 = (p-q)(r-s)$$

$\therefore (p-q), (q-r), (r-s)$ are in GP Ans

CLASSTIME

Qn. 5 → If p, q, r are in G.P and the equations $px^2+2qx+r=0$ and $dx^2+2ex+f=0$ have common root, then show that $\frac{d}{p}, \frac{e}{q}, \frac{f}{r}$ are in A.P

Sol: given: $p, q, r \rightarrow GP$

$$\boxed{q^2=pr}$$

T.P $\frac{d}{p}, \frac{e}{q}, \frac{f}{r}$ are in AP

i.e $\frac{2e}{q} = \frac{d}{p}+\frac{f}{r}$

Consider the equation $px^2+2qx+r=0$

$$x = \frac{-2q \pm \sqrt{4q^2-4pr}}{2p}$$

$$x = \frac{-2q \pm \sqrt{4pr-4pr}}{2p} \quad \text{--- } \{q^2=pr\}$$

$$x = \frac{-2q}{2p} = \frac{-q}{p}$$

$$x = \frac{-q}{p}$$

this is also the root of $dx^2+2ex+f=0$

$$\Rightarrow \frac{dq^2}{p^2} - \frac{2eq}{p} + f = 0$$

$$\Rightarrow \frac{dpr}{p^2} - \frac{2eq}{p} + f = 0 \quad \text{--- } \{\because q^2=pr\}$$

$$\Rightarrow \frac{dr}{p} - \frac{2eq}{p} + f = 0$$

$\Rightarrow dr - 2eq + fp = 0$

$\Rightarrow dr + fp = 2eq$

divide by q^2

$$\frac{dr}{q^2} + \frac{fp}{q^2} = \frac{2e}{q}$$

$$\frac{dr}{pr} + \frac{fp}{pr} = \frac{2e}{q} \quad \text{---} \{ \because q^2 = pr \}$$

$$\Rightarrow \frac{d}{p} + \frac{f}{r} = \frac{2e}{q}$$

$\therefore \frac{d}{p}, \frac{e}{q}, \frac{f}{r}$ are in AP

Qn 6 → If a, b, c are in GP and $a^{1/x} = b^{1/y} = c^{1/z}$

prove that x, y, z are in A.P

Soln (i) $a, b, c \rightarrow GP$

$\Rightarrow \boxed{b^2 = ac}$

(ii) T.P $x, y, z \rightarrow AP$

i.e $\boxed{2y = x + z}$

let $a^{1/x} = b^{1/y} = c^{1/z} = k$ (constant)

$\Rightarrow a^{1/x} = k \; ; \; b^{1/y} = k \; ; \; c^{1/z} = k$

$\Rightarrow a = k^x \; ; \; b = k^y \; ; \; c = k^z$

we have $b^2 = ac$

$(k^y)^2 = (k^x) \cdot (k^z)$

$k^{2y} = k^{x+z} \Rightarrow 2y = x + z \; \therefore x, y, z \rightarrow AP$

CLASSTIME

Qn. 7 → If a, b, c are three consecutive terms of an AP and x, y, z are three consecutive terms of a GP. Then prove that

$$x^{b-c} \cdot y^{c-a} \cdot z^{a-b} = 1$$

Sol (·) $a, b, c \rightarrow AP$

$2b = a + c$

(·) $x, y, z \rightarrow GP$

$y^2 = xz$

LHS $x^{b-c} \cdot y^{c-a} \cdot z^{a-b}$

$= x^{b-c} \cdot (\sqrt{xz})^{c-a} \cdot z^{a-b}$

$= x^{b-c} \cdot (xz)^{\frac{c-a}{2}} \cdot z^{a-b}$

$= x^{b-c} \cdot x^{\frac{c-a}{2}} \cdot z^{\frac{c-a}{2}} \cdot z^{a-b}$

$= x^{b-c+\frac{c-a}{2}} \cdot z^{\frac{c-a}{2}+a-b}$

$= x^{\frac{2b-2c+c-a}{2}} \cdot z^{\frac{c-a+2a-2b}{2}}$

$= x^{\frac{2b-c-a}{2}} \cdot z^{\frac{c+a-2b}{2}}$

$= x^{\frac{\cancel{a+c}-\cancel{c-a}}{2}} \cdot z^{\frac{\cancel{2b}-2b}{2}}$

$= x^0 \cdot z^0$

$= 1 \times 1 = 1 = $ RHS

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Qn 8 → If A is the A.M and G_1, G_2 be two geometric means b/w any two +ve numbers then prove that

$$2A = \frac{G_1^2}{G_2} + \frac{G_2^2}{G_1}$$

Soln let the two nos are a & b

a, A, b

$\Rightarrow A = \frac{a+b}{2}$

a, G_1, G_2, b ; here $n=2$

$r = \left(\frac{b}{a}\right)^{\frac{1}{n+1}}$

$r = \left(\frac{b}{a}\right)^{1/3}$

$$G_1 = ar = a\left(\frac{b}{a}\right)^{1/3}$$

$$G_2 = ar^2 = a\left(\frac{b}{a}\right)^{2/3}$$

RHS $\frac{G_1^2}{G_2} + \frac{G_2^2}{G_1}$

$$= \frac{a^2\left(\frac{b}{a}\right)^{2/3}}{a\left(\frac{b}{a}\right)^{2/3}} + \frac{a^2\left(\frac{b}{a}\right)^{4/3}}{a\left(\frac{b}{a}\right)^{1/3}}$$

$$= a + a\left(\frac{b}{a}\right)^{\frac{4}{3}-\frac{1}{3}}$$

$$= a + \cancel{a}\left(\frac{b}{\cancel{a}}\right)$$

$$= a+b$$

$$= 2A = \text{LHS}$$

Proved

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Qns 9 If $a_1, a_2, a_3, \ldots a_n$ are in AP with common difference 'd' then show that

$$\sin d\left(\operatorname{cosec} a_1 \cdot \operatorname{cosec} a_2 + \operatorname{cosec} a_2 \cdot \operatorname{cosec} a_3 + \cdots \operatorname{cosec}(a_{n-1}) \cdot \operatorname{cosec} a_n\right) = \cot a_1 - \cot a_n$$

LHS

$$\sin d\left(\frac{1}{\sin a_1 \cdot \sin a_2} + \frac{1}{\sin a_2 \cdot \sin a_3} + \cdots \frac{1}{\sin(a_{n-1}) \cdot \sin a_n}\right)$$

$$= \frac{\sin d}{\sin a_1 \cdot \sin a_2} + \frac{\sin d}{\sin a_2 \cdot \sin a_3} + \cdots \frac{\sin d}{\sin(a_{n-1}) \cdot \sin a_n}$$

$$= \frac{\sin(a_2 - a_1)}{\sin a_1 \cdot \sin a_2} + \frac{\sin(a_3 - a_2)}{\sin a_2 \cdot \sin a_3} + \cdots \frac{\sin(a_n - a_{n-1})}{\sin(a_{n-1}) \cdot \sin a_n}$$

$$= \frac{\sin a_2 \cdot \cos a_1 - \cos a_2 \cdot \sin a_1}{\sin a_1 \cdot \sin a_2} + \frac{\sin a_3 \cos a_2 - \cos a_3 \sin a_2}{\sin a_2 \cdot \sin a_3} + \cdots \frac{\sin(a_n)\cos(a_{n-1}) - \cos(a_n)\sin(a_{n-1})}{\sin(a_{n-1}) \cdot \sin(a_n)}$$

$$= \cot(a_1) - \cancel{\cot(a_2)} + \cancel{\cot(a_2)} - \cancel{\cot(a_3)} + \cdots + \cancel{\cot(a_{n-1})} - \cot(a_n)$$

$$= \cot(a_1) - \cot(a_n)$$

$= \text{RHS}$ Ans

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Qu. 10 → If $a_1, a_2, a_3 \ldots a_n$ are in A.P show that

$$\frac{1}{\sqrt{a_1}+\sqrt{a_2}} + \frac{1}{\sqrt{a_2}+\sqrt{a_3}} + \cdots \frac{1}{\sqrt{a_{n-1}}+\sqrt{a_n}} = \frac{n-1}{\sqrt{a_1}+\sqrt{a_n}}$$

L.H.S =

$$\frac{1}{\sqrt{a_1}+\sqrt{a_2}} + \frac{1}{\sqrt{a_2}+\sqrt{a_3}} + \cdots \frac{1}{\sqrt{a_{n-1}}+\sqrt{a_n}}$$

$$= \frac{\sqrt{a_1}-\sqrt{a_2}}{a_1-a_2} + \frac{\sqrt{a_2}-\sqrt{a_3}}{a_2-a_3} + \cdots \frac{\sqrt{a_{n-1}}-\sqrt{a_n}}{a_{n-1}-a_n}$$

$$= \frac{\sqrt{a_2}-\sqrt{a_1}}{d} + \frac{\sqrt{a_3}-\sqrt{a_2}}{d} + \cdots \frac{\sqrt{a_n}-\sqrt{a_{n-1}}}{d}$$

$$= \frac{1}{d}\left(\cancel{\sqrt{a_2}} - \sqrt{a_1} + \sqrt{a_3} - \cancel{\sqrt{a_2}} + \cdots + \sqrt{a_n} - \sqrt{a_{n-1}}\right)$$

$$= \frac{1}{d}\left(-\sqrt{a_1} + \sqrt{a_n}\right)$$

$$= \frac{1}{d}\left(\sqrt{a_n}-\sqrt{a_1}\right)$$

Rationalize $\frac{1}{d}\left(\frac{a_n-a_1}{\sqrt{a_n}+\sqrt{a_1}}\right)$

$$= \frac{1}{d}\left(\frac{\cancel{a}+(n-1)d-\cancel{a}}{\sqrt{a_n}+\sqrt{a_1}}\right)$$

$$= \frac{\frac{1}{\cancel{d}}((n-1)\cancel{d})}{\sqrt{a_n}+\sqrt{a_1}}$$

$$= \frac{n-1}{\sqrt{a_n}+\sqrt{a_1}} = R.H.S$$

Proved

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