ANSWER KEY OF EXAM NO: 9

LIMITS & DERIVATIVES

SECTION: A

Ans: 1 given. $\lim_{x\to 3}\left(\frac{x^n-3^n}{x-3}\right) = 108$

$\Rightarrow n.3^{n-1} = 108$ --- $\left\{\lim_{x\to a}\left(\frac{x^n-a^n}{x-a}\right) = na^{n-1}\right\}$

$\Rightarrow n.3^{n-1} = 4\times 27$

$\Rightarrow n.3^{n-1} = 4\times 3^{4-1}$

comparing, we get $\boxed{n=4}$ Ans

Ans: 2 → $\lim_{x\to 0}\left[\frac{\sqrt{2+x}-\sqrt{2}}{x}\right]$

Rationalize

$= \lim_{x\to 0}\left[\frac{\cancel{2}+x-\cancel{2}}{x\left(\sqrt{2+x}+\sqrt{2}\right)}\right]$

$= \lim_{x\to 0}\left[\frac{1}{\sqrt{2+x}+\sqrt{2}}\right]$

$= \frac{1}{\sqrt{2}+\sqrt{2}} = \frac{1}{2\sqrt{2}}$ Ans

Ans 3 → given $f(x) = 1+x+\frac{x^2}{2}+\dots+\frac{x^{100}}{100}$

Differentiate w.r.t x

$f'(x) = 1+\frac{2x}{2}+\frac{3x^2}{3}+\dots+\frac{100x^{99}}{100}$

$\Rightarrow f'(x) = 1 + x + x^2 + \cdots x^{99}$

$f'(1) = 1+1+1+ \cdots +1$

put $x=1$

$= 100$

$\therefore \boxed{f'(1) = 100}$ ANS

Ans 4 → $\lim_{x\to 3^+}\left(\frac{x}{[x]}\right)$

put $x = 3+h$ & $h \to 0$

$= \lim_{h\to 0}\left(\frac{3+h}{[3+h]}\right)$

$= \lim_{h\to 0}\left(\frac{3+h}{3}\right) \quad \cdots \{[3+h] = [3.000001] = 3\}$

$= \frac{3}{3} = 1$ ANS

Ans. 5 → $\lim_{x\to \pi}\left(\frac{\sin x}{x-\pi}\right)$

put $x = \pi + h$ and $h \to 0$

$= \lim_{h\to 0}\left(\frac{\sin(\pi+h)}{\cancel{\pi}+h-\cancel{\pi}}\right)$

$= \lim_{h\to 0}\left(\frac{-\sin h}{h}\right) \quad \cdots \{\sin(180^\circ+\theta) = -\sin\theta\}$

$= -1$ ANS

Ans 6 → given $\lim_{x\to -a}\left(\frac{x^9+a^9}{x+a}\right) = 9$

$$\Rightarrow \lim_{x \to -a}\left(\frac{x^9-(-a)^9}{x-(-a)}\right)=9$$

$$\Rightarrow \quad 9(-a)^{9-1}=9 \quad \cdots \left\{\because \lim_{x\to a}\left(\frac{x^n-a^n}{x-a}\right)=na^{n-1}\right\}$$

$$\Rightarrow \quad 9a^8=9$$

$$\Rightarrow \quad a^8=1$$

$$\Rightarrow \quad \boxed{a=\pm 1} \quad \text{Ans}$$

SECTION: B

Ans 7 →

$$\lim_{x\to \frac{\pi}{6}}\left(\frac{2\sin^2 x+\sin x-1}{2\sin^2 x-3\sin x+1}\right)$$

$$=\lim_{x\to \frac{\pi}{6}}\left(\frac{2\sin^2 x+2\sin x-\sin x-1}{2\sin^2 x-2\sin x-\sin x+1}\right)$$

$$=\lim_{x\to \frac{\pi}{6}}\left(\frac{2\sin x(\sin x+1)-1(\sin x+1)}{2\sin x(\sin x-1)-1(\sin x-1)}\right)$$

$$=\lim_{x\to \frac{\pi}{6}}\left(\frac{(\sin x+1)\cancel{(2\sin x-1)}}{(\sin x-1)\cancel{(2\sin x-1)}}\right)$$

$$=\frac{\sin(\pi/6)+1}{\sin(\frac{\pi}{6})-1}$$

$$=\frac{1/2+1}{1/2-1}=\frac{3/2}{-1/2}=-3 \quad \text{Ans}$$

Ans 8 → $y=\dfrac{\cos x}{1+\sin x}$

Diff w.r.t. x (Quotient Rule)

$$\frac{dy}{dx} = \frac{(1+\sin x)\cdot\frac{d}{dx}(\cos x) - \cos x\cdot\frac{d}{dx}(1+\sin x)}{(1+\sin x)^2}$$

$$= \frac{(1+\sin x)(-\sin x) - \cos x(\cos x)}{(1+\sin x)^2}$$

$$= \frac{-\sin x - \sin^2 x - \cos^2 x}{(1+\sin x)^2}$$

$$= \frac{-\sin x - (\sin^2 x + \cos^2 x)}{(1+\sin x)^2}$$

$$= \frac{-\sin x - 1}{(1+\sin x)^2}$$

$$= \frac{-\cancel{(1+\sin x)}}{(1+\sin x)^2}$$

$$\boxed{\frac{dy}{dx} = \frac{-1}{1+\sin x}} \quad \text{Ans}$$

Ans 10 →

given $\lim_{x\to 1}\left(\frac{x^4-1}{x-1}\right) = \lim_{x\to k}\left(\frac{x^3-k^3}{x^2-k^2}\right)$

$$\Rightarrow \lim_{x\to 1}\left(\frac{x^4-1^4}{x-1}\right) = \lim_{x\to k}\left(\frac{\frac{x^3-k^3}{x-k}}{\frac{x^2-k^2}{x-k}}\right)$$

$$\Rightarrow 4(1)^3 = \frac{3(k)^2}{2(k)^1} \quad \text{---} \left\{\because \lim_{x\to a}\left(\frac{x^n-a^n}{x-a}\right) = na^{n-1}\right\}$$

$$\Rightarrow 4 = \frac{3}{2}k$$

$$\Rightarrow \boxed{k = \frac{8}{3}} \quad \text{Ans}$$

Qn: 11 → given $y = \frac{\sin(x+9)}{\cos x}$

Diff w.r.t x (Quotient Rule)

$$\frac{dy}{dx} = \frac{\cos x \cdot \frac{d}{dx}\left(\sin(x+9)\right) - \sin(x+9) \cdot \frac{d}{dx}\left(\cos x\right)}{\cos^2 x}$$

$$= \frac{\cos x \cdot \cos(x+9) - \sin(x+9) \cdot \cos x}{\cos^2 x}$$

$$= \frac{\cos(x+9+x)}{\cos^2 x} \quad \text{---} \left\{ \cos A \cos B - \sin A \sin B = \cos(A+B) \right\}$$

$$\frac{dy}{dx} = \frac{\cos(2x+9)}{\cos^2 x}$$

$\frac{dy}{dx}$ at $x=0$ $= \frac{\cos(9)}{\cos^2 0}$

$$= \frac{\cos 9}{1} \quad \text{---} \left\{ \cos(0) = 1 \right\}$$

$$\Rightarrow \boxed{\left(\frac{dy}{dx}\right)_{x=0} = \cos 9} \quad \text{Ans}$$

Ans 12 → $\lim_{x \to 4} \left(\frac{|x-4|}{x-4} \right)$

$$LHL = \lim_{x \to 4^-} \left(\frac{|x-4|}{x-4} \right)$$

put $x = 4-h$ & $h \to 0$

$$LHL = \lim_{h \to 0} \left(\frac{|4-h-4|}{4-h-4} \right)$$

$$= \lim_{h \to 0} \left(\frac{|-h|}{-h} \right) = \lim_{h \to 0} \left(\frac{h}{-h} \right) = \lim_{h \to 0} (-1) \quad LHL = -1$$

$$RHL = \lim_{x \to 4^+} \left(\frac{|x-4|}{x-4} \right)$$

put $x = 4+h$ & $h \to 0$

$$RHL = \lim_{h \to 0} \left(\frac{|\cancel{4}+h-\cancel{4}|}{\cancel{4}+h-\cancel{4}} \right)$$

$$= \lim_{h \to 0} \left(\frac{\cancel{h}}{\cancel{h}} \right) = \lim_{h \to 0} (1)$$

$\therefore \boxed{RHL = 1}$

Since LHL $\neq$ RHL

$\therefore \lim_{x \to 4} (f(x))$ does not exists Ans

Ques 13 → $\lim_{x \to \infty} \left(\sqrt{x^2+x+1} - \sqrt{x^2+1} \right)$

Rationalize

$$= \lim_{x \to \infty} \left(\frac{\cancel{x^2}+x+\cancel{1}-\cancel{x^2}-\cancel{1}}{\sqrt{x^2+x+1} + \sqrt{x^2+1}} \right)$$

$$= \lim_{x \to \infty} \left(\frac{x}{\sqrt{x^2+x+1} + \sqrt{x^2+1}} \right)$$

Divide by x

$$= \lim_{x \to \infty} \left(\frac{1}{\sqrt{1+\frac{1}{x}+\frac{1}{x^2}} + \sqrt{1+\frac{1}{x^2}}} \right)$$

$$= \frac{1}{\sqrt{1+0+0} + \sqrt{1}}$$

$$= \frac{1}{2} \text{ Ans}$$

Ans 14 → $\lim_{x\to 1}\left(\dfrac{2^{x-1}-1}{\sin(\pi x)}\right)$

put $x = 1+h$ & $h\to 0$

$= \lim_{h\to 0}\left(\dfrac{2^h-1}{\sin(\pi(1+h))}\right)$

$= \lim_{h\to 0}\left(\dfrac{2^h-1}{\sin(\pi+\pi h)}\right)$

$= \lim_{h\to 0}\left(\dfrac{2^h-1}{-\sin(\pi h)}\right)$ ---- $\{\because \sin(180^\circ+\theta) = -\sin\theta\}$

$= \lim_{h\to 0}\left(\dfrac{\dfrac{2^h-1}{h}\times \cancel{h}}{-\dfrac{\sin(\pi h)}{\pi h}\times \cancel{h}\pi}\right)$

$= \dfrac{\log 2}{-(1)(\pi)}$ -- $\left\{\because \lim_{x\to 0}\left(\dfrac{a^x-1}{x}\right) = \log a \quad \lim_{x\to 0}\left(\dfrac{\sin x}{x}\right) = 1\right\}$

$= -\dfrac{\log 2}{\pi}$ Ans

SECTION: C

Ans 15 → $\lim_{x\to 0}\left(\dfrac{10^x-2^x-5^x+1}{x\tan x}\right)$

$= \lim_{x\to 0}\left(\dfrac{2^x(5^x-1)-1(5^x-1)}{x\tan x}\right)$

$= \lim_{x\to 0}\left(\dfrac{(2^x-1)(5^x-1)}{x\tan x}\right)$

$$= \lim_{x\to 0}\left(\frac{\frac{(2^x-1)}{x}\times \cancel{x} \cdot \frac{(5^x-1)}{x}\times \cancel{x}}{\cancel{x}\,\frac{\tan x}{x}\times \cancel{x}} \right)$$

$$= \frac{\log 2 \cdot \log 5}{1} \quad \cdots \left\{ \because \lim_{x\to 0}\left(\frac{a^x-1}{x}\right) = \log a \right\}$$

$$= \log 2 \cdot \log 5 \quad \underline{\text{ANS}}$$

Ques 16 → $f(x) = \sqrt{\sin x}$

$$f'(x) = \lim_{h\to 0}\left(\frac{\sqrt{\sin(x+h)} - \sqrt{\sin x}}{h} \right)$$

$$= \lim_{h\to 0}\left(\frac{\sin(x+h) - \sin x}{h\left(\sqrt{\sin(x+h)} + \sqrt{\sin x}\right)} \right)$$

$$= \lim_{h\to 0}\left(\frac{2\cos\left(\frac{2x+h}{2}\right)\cdot \sin\left(\frac{h}{2}\right)}{h\left(\sqrt{\sin(x+h)} + \sqrt{\sin x}\right)} \right)$$

$$= \lim_{h\to 0}\left(\frac{\cancel{2}\cos\left(\frac{2x+h}{2}\right)\cdot \sin\left(\frac{h}{2}\right)}{\cancel{2}\times\frac{h}{2}\left(\sqrt{\sin(x+h)} + \sqrt{\sin x}\right)} \right)$$

$$= \frac{\cos(x)\times 1}{\sqrt{\sin x} + \sqrt{\sin x}} \quad \cdots \left\{ \because \lim_{x\to 0}\left(\frac{\sin x}{x}\right) = 1 \right\}$$

$$\boxed{f'(x) = \frac{\cos x}{2\sqrt{\sin x}}} \quad \underline{\text{ANS}}$$

Qn 17 → Given $f(x) = \lambda x^2 + \mu x + 12$

Diff wrt x

$$f'(x) = 2\lambda x + \mu$$

given $f'(4) = 15$ and $f'(2) = 11$

$\Rightarrow$ $8\lambda + \mu = 15$ and $4\lambda + \mu = 11$

Subtracting equations

$\Rightarrow$ $4\lambda = 4$

$\Rightarrow$ $\lambda = 1$ & $\mu = 7$ Ans

Qn. 18 $\rightarrow$ $\lim_{x \to \frac{\pi}{6}} \left(\frac{2 - \sqrt{3}\cos x - \sin x}{(6x - \pi)^2} \right)$

put $x = \frac{\pi}{6} + h$ & $h \to 0$

$$= \lim_{h \to 0} \left(\frac{2 - \sqrt{3}\cos\left(\frac{\pi}{6} + h\right) - \sin\left(\frac{\pi}{6} + h\right)}{\left(6\left(\frac{\pi}{6} + h\right) - \pi\right)^2} \right)$$

$$= \lim_{h \to 0} \left(\frac{2 - \sqrt{3}\left(\frac{\sqrt{3}}{2}\cos h - \frac{1}{2}\sin h\right) - \left(\frac{1}{2}\cos h + \frac{\sqrt{3}}{2}\sin h\right)}{(\cancel{\pi} + 6h - \cancel{\pi})^2} \right)$$

$$= \lim_{h \to 0} \left(\frac{2 - \frac{3}{2}\cos h + \cancel{\frac{\sqrt{3}}{2}\sin h} - \frac{1}{2}\cos h - \cancel{\frac{\sqrt{3}}{2}\sin h}}{36h^2} \right)$$

$$= \lim_{h \to 0} \left(\frac{2 - 2\cos h}{36h^2} \right)$$

$$= \lim_{h \to 0} \left(\frac{1 - \cos h}{18h^2} \right)$$

$$= \lim_{h \to 0} \left(\frac{2\sin^2(h/2)}{18h^2} \right)$$

$$= \lim_{h \to 0} \left(\frac{\frac{2\sin^2\left(\frac{h}{2}\right)}{\frac{h^2}{4}} \times \frac{h^2}{4}}{18h^2} \right)$$

$= \dfrac{2 \times 1 \times \frac{1}{4}}{18}$ --- $\left\{ \because \lim_{x \to 0} \left(\frac{\sin x}{x} \right) = 1 \right\}$

$= \dfrac{1}{36}$ Ans

Qu. 19 → $f(x) = \begin{cases} a+bx & : x < 1 \\ 4 & : x = 1 \\ b-ax & : x > 1 \end{cases}$

given $f(1) = \lim_{x \to 1} (f(x))$

$\Rightarrow f(1) = LHL = RHL$ ... $\left\{ \lim_{x \to 1} (f(x)) \text{ exists} \right.$
$\left. \therefore LHL = RHL \right\}$

$\Rightarrow 4 = \lim_{x \to 1^-} (a+bx) = \lim_{x \to 1^+} (b-ax)$

put $x = 1-h$, $h \to 0$ put $x = 1+h$, $h \to 0$

$\Rightarrow 4 = \lim_{h \to 0} \left(a + (b)(1-h)\right) = \lim_{h \to 0} \left(b - a(1+h)\right)$

$\Rightarrow 4 = a+b = b-a$

$\Rightarrow a+b = 4$ and $b-a = 4$

adding equations

$\Rightarrow 2b = 8 \Rightarrow b = 4$ and $a = 0$ Ans

Qu. 20 → $f(x) = \begin{cases} |x|+1 & : x < 0 \\ 0 & : x = 0 \\ |x|-1 & : x > 0 \end{cases}$

$LHL = \lim_{x \to 0^-} (|x|+1)$

put $x = 0-h$ & $h \to 0$

$LHL = \lim_{h \to 0} (|-h|+1)$

$LHL = 0+1$

$LHL = 1$

$RHL = \lim_{x \to 0^+} (|x| - 1)$

put $x = 0+h$ & $h \to 0$

$RHL = \lim_{h \to 0} (|h| - 1)$

$RHL = -1$

$LHL \neq RHL$

$\therefore \lim_{x \to 0} f(x)$ does not exists

But given that $\lim_{x \to a} f(x)$ exists

$\therefore \quad a \neq 0$

$\therefore \boxed{a \in R - \{0\}}$ Ans

Q.No. 21 → $\lim_{x \to 0} \left(\dfrac{\cos(ax) - \cos(bx)}{\cos(cx) - 1} \right)$

$= \lim_{x \to 0} \left(\dfrac{\cancel{-}\cancel{2}\sin\left(\frac{a+b}{2}\right)x \cdot \sin\left(\frac{a-b}{2}\right)x}{\cancel{-}\cancel{2}\sin^2\left(\frac{cx}{2}\right)} \right) \cdots \left\{ \begin{array}{l} \cos A - \cos B \\ = -2\sin\left(\frac{A+B}{2}\right) \cdot \sin\left(\frac{A-B}{2}\right) \\ \& \ 1 - \cos\theta = 2\sin^2\frac{\theta}{2} \end{array} \right\}$

$= \lim_{x \to 0} \left(\dfrac{\dfrac{\sin\left(\frac{a+b}{2}\right)x}{\left(\frac{a+b}{2}\right)x} \times \left(\frac{a+b}{2}\right)\cancel{x} \cdot \dfrac{\sin\left(\frac{a-b}{2}\right)x}{\left(\frac{a-b}{2}\right)x} \cdot \left(\frac{a-b}{2}\right)\cancel{x}}{\dfrac{\sin^2\left(\frac{cx}{2}\right)}{\frac{c^2x^2}{4}} \times \dfrac{c^2\cancel{x^2}}{4}} \right)$

$= \dfrac{1 \times \left(\frac{a+b}{2}\right) \times 1 \times \left(\frac{a-b}{2}\right)}{\frac{c^2}{4}} \cdots \left\{ \because \lim_{x \to 0} \left(\frac{\sin x}{x}\right) = 1 \right\}$

$$= \frac{\frac{a^2-b^2}{4}}{\frac{c^2}{4}}$$

$$= \frac{a^2-b^2}{c^2} \quad \underline{\text{Ans}}$$

Ans 22 → $f(x) = x\sin x$

$$f'(x) = \lim_{h\to 0}\left(\frac{(x+h)\sin(x+h) - x\sin x}{h}\right)$$

$$= \lim_{h\to 0}\left(\frac{x\sin(x+h) + h\sin(x+h) - x\sin x}{h}\right)$$

$$= \lim_{h\to 0}\left(\frac{x\left(\sin(x+h) - \sin x\right)}{h} + \frac{\cancel{h}\sin(x+h)}{\cancel{h}}\right)$$

$$= \lim_{h\to 0}\left[\frac{x\cdot \cancel{2}\cos\left(\frac{2x+h}{2}\right)\cdot\sin\left(\frac{h}{2}\right)}{\frac{h}{2}\times\cancel{2}} + \sin(x+h)\right]$$

$$= x\cos x \times 1 + \sin x \quad \cdots \left\{\because \lim_{x\to 0}\left(\frac{\sin x}{x}\right) = 1\right\}$$

$$\boxed{f'(x) = x\cos x + \sin x} \quad \underline{\text{Ans}}$$

SECTION: D

Ans. 23 → $\lim_{x\to \frac{\pi}{4}}\left(\frac{\tan^3 x - \tan x}{\cos(x+\frac{\pi}{4})}\right)$

$$= \lim_{x\to\frac{\pi}{4}}\left(\frac{\tan x(\tan x+1)(\tan x-1)}{\cos(x+\pi/4)}\right)$$

$$= \lim_{x\to\frac{\pi}{4}}\left(\tan x\cdot(\tan x+1)\right) \times \lim_{x\to\frac{\pi}{4}}\left(\frac{\tan x - 1}{\cos(x+\pi/4)}\right)$$

put $x = \frac{\pi}{4} + h$ & $h \to 0$ in 2^{nd} limit

$$= 1(1+1) \times \lim_{h \to 0} \left(\frac{\tan\left(\frac{\pi}{4}+h\right) - 1}{\cos\left(\frac{\pi}{4}+h+\frac{\pi}{4}\right)} \right)$$

$$= 2 \times \lim_{h \to 0} \left(\frac{\frac{1+\tan h}{1-\tan h} - 1}{\cos\left(\frac{\pi}{2}+h\right)} \right)$$

$$= 2 \lim_{h \to 0} \left(\frac{\cancel{1}+\tan h - \cancel{1}+\tan h}{(1-\tan h)\,(-\sin h)} \right) \quad \cdots \left\{ \because \cos(90+\theta) = -\sin\theta \right\}$$

$$= 2 \lim_{h \to 0} \left(\frac{2\tan h}{(1-\tan h)(-\sin h)} \right)$$

$$= 2 \lim_{h \to 0} \left(\frac{2\frac{\tan h}{h} \times \cancel{h}}{(1-\tan h)\left(-\frac{\sin h}{h}\right) \times \cancel{h}} \right)$$

$$= \frac{2 \times 2 \times 1}{(1-0)(-1)} \quad \cdots \left\{ \lim_{x \to 0} \left(\frac{\tan x}{x} \right) = 1 \right\}$$

$$= -4 \quad \underline{\text{Ans}}$$

Qn 24 →
$$\lim_{x \to 0} \left[\frac{\overset{(A)}{\sin(\alpha+\beta)x} + \overset{(B)}{\sin(\alpha-\beta)x} + \sin(2\alpha x)}{\cos(2\beta x) - \cos(2\alpha x)} \right] \cdot x$$

$$= \lim_{x \to 0} \left[\frac{2\sin(\alpha x)\cdot\cos(\beta x) + 2\sin(\alpha x)\cdot\cos(\alpha x)}{-2\sin(\beta+\alpha)x \cdot \sin(\beta-\alpha)x} \right] \cdot x$$

$$= \lim_{x \to 0} \left[\frac{\cancel{2}\sin(\alpha x)\cdot\left(\cos(\beta x) + \cos(\alpha x)\right)}{-\cancel{2}\sin(\beta+\alpha)x \cdot \sin(\beta-\alpha)x} \right] \cdot x$$

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$$= \lim_{x \to 0} \left(\frac{\frac{\sin(\alpha x)}{\alpha x} \cdot \alpha \cancel{x} \cdot \left(\cos(\beta x) + \cos(\alpha x) \right) \cdot \cancel{x}}{- \frac{\sin(\beta+\alpha)x}{(\beta+\alpha)x} \times (\beta+\alpha)\cancel{x} \cdot \frac{\sin(\beta-\alpha)x}{(\beta-\alpha)x} \cdot (\beta-\alpha)\cancel{x}} \right)$$

$$= \frac{(1)(\alpha)(1+1)}{-(1)(\beta+\alpha)(1)(\beta-\alpha)}$$

$$= \frac{2\alpha}{-(\beta^2-\alpha^2)}$$

$$= \frac{2\alpha}{\alpha^2-\beta^2} \quad \underline{\underline{Ans}}$$

Q.N. 25 → $\lim_{x \to \frac{\pi}{2}} \left(\frac{\cot x - \cos x}{(\pi - 2x)^3} \right)$

put $x = \frac{\pi}{2} + h$ & $h \to 0$

$$= \lim_{h \to 0} \left(\frac{\cot(\frac{\pi}{2}+h) - \cos(\frac{\pi}{2}+h)}{(\pi - 2(\frac{\pi}{2}+h))^3} \right)$$

$$= \lim_{h \to 0} \left(\frac{-\tan h + \sin h}{(\pi - \pi - 2h)^3} \right)$$

$$= \lim_{h \to 0} \left(\cancel{+} \frac{(\tan h - \sin h)}{\cancel{-}8h^3} \right)$$

$$= \frac{1}{8} \lim_{h \to 0} \left[\frac{\frac{\sin h}{\cos h} - \sin h}{h^3} \right]$$

$$= \frac{1}{8} \lim_{h \to 0} \left(\frac{\sin h - \sin h \cdot \cos h}{h^3 \cdot \cos h} \right)$$

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$$= \frac{1}{8} \lim_{h \to 0} \left(\frac{\sin h . (1 - \cos h)}{h^3 . \cos h} \right)$$

$$= \frac{1}{8} \lim_{h \to 0} \left(\frac{\sin h . 2\sin^2(h/2)}{h^3 . \cos h} \right)$$

$$= \frac{1}{8} \lim_{h \to 0} \left(\frac{\sin h}{h} . \frac{2 \sin^2(h/2)}{\frac{h^2}{4} \times 4} \times \frac{1}{\cos h} \right)$$

$$= \frac{1}{8} \times 1 \times \frac{2}{4} \times \frac{1}{1} \quad \cdots \left\{ \lim_{x \to 0} \left(\frac{\sin x}{x} \right) = 1 \right\}$$

$$= \frac{1}{16} \text{ Ans}$$

Ans 26 → $f(x) = \cot \sqrt{x}$

$$f'(x) = \lim_{h \to 0} \left(\frac{\cot \sqrt{x+h} - \cot \sqrt{x}}{h} \right)$$

$$= \lim_{h \to 0} \left(\frac{\tan \sqrt{x} - \tan \sqrt{x+h}}{h . \tan \sqrt{x+h} . \tan \sqrt{x}} \right)$$

$$= \lim_{h \to 0} \left(\frac{\tan(\sqrt{x} - \sqrt{x+h}) \, (1 + \tan\sqrt{x} \tan\sqrt{x+h})}{h \tan(\sqrt{x+h}) \tan\sqrt{x}} \right)$$

$$= \lim_{h \to 0} \left(\frac{\tan(\sqrt{x} - \sqrt{x+h}) \, (1 + \tan\sqrt{x} \tan\sqrt{x+h}) \times (\sqrt{x} - \sqrt{x+h})}{(\sqrt{x} - \sqrt{x+h}) \; h \tan(\sqrt{x+h}) \tan\sqrt{x}} \right)$$

$$= \lim_{h \to 0} \left(\frac{\tan(\sqrt{x} - \sqrt{x+h})}{\sqrt{x} - \sqrt{x+h}} \right) \times \lim_{h \to 0} \left(\frac{1 + \tan\sqrt{x} \tan\sqrt{x+h}}{\tan\sqrt{x+h} \tan\sqrt{x}} \right) \times \lim_{h \to 0} \left(\frac{\sqrt{x} - \sqrt{x+h}}{\cancel{h}} \right)$$

$$= 1 \times \frac{(1 + \tan^2\sqrt{x})}{\tan^2\sqrt{x}} \times \lim_{h \to 0} \left(\frac{\cancel{x} - \cancel{x} - h}{\cancel{h}(\sqrt{x} + \sqrt{x+h})} \right) \quad \cdots \left\{ \lim_{x \to 0} \left(\frac{\tan x}{x} \right) = 1 \right\}$$

$$= \frac{\sec^2\sqrt{x}}{\tan^2\sqrt{x}} \times \lim_{h\to 0}\left(\frac{-\cancel{h}}{\cancel{h}(\sqrt{x}+\sqrt{x+h})}\right)$$

$$= \frac{\frac{1}{\cancel{\cos^2\sqrt{x}}}}{\frac{\sin^2\sqrt{x}}{\cancel{\cos^2\sqrt{x}}}} \times \frac{-1}{\sqrt{x}+\sqrt{x}}$$

$$\boxed{f'(x) = -\frac{1}{2\sqrt{x}}\cdot \operatorname{cosec}^2\sqrt{x}} \quad \text{Ans}$$

Qn. 27 → $f(x) = \dfrac{\sin x - x\cos x}{x\sin x + \cos x}$

Diff wrt x (Quotient Rule)

$$\Rightarrow f'(x) = \frac{(x\sin x+\cos x)\cdot\frac{d}{dx}(\sin x - x\cos x) - (\sin x - x\cos x)\cdot\frac{d}{dx}(x\sin x+\cos x)}{(x\sin x+\cos x)^2}$$

$$= \frac{(x\sin x+\cos x)(\cancel{\cos x} - (-x\sin x + \cancel{\cos x})) - (\sin x - x\cos x)(x\cos x + \cancel{\sin x} - \cancel{\sin x})}{(x\sin x+\cos x)^2}$$

$$= \frac{(x\sin x+\cos x)(x\sin x) - (\sin x - x\cos x)(x\cos x)}{(x\sin x+\cos x)^2}$$

$$= \frac{x^2\sin^2 x + \cancel{x\sin x\cos x} - \cancel{x\sin x\cos x} + x^2\cos^2 x}{(x\sin x+\cos x)^2}$$

$$= \frac{x^2(\sin^2 x+\cos^2 x)}{(x\sin x+\cos x)^2}$$

$$f'(x) = \frac{x^2}{(x\sin x+\cos x)^2}$$

$$f'(\pi/2) = \frac{\frac{\pi}{4}^2}{(\frac{\pi}{2}\sin\frac{\pi}{2}+\cos\frac{\pi}{2})^2}$$

$$= \frac{\frac{\pi^2}{4}}{\left(\frac{\pi}{2}+0\right)^2}$$

$$= \frac{\frac{\pi^2}{4}}{\frac{\pi^2}{4}}$$

$$\boxed{f'(\pi/2) = 1}$$ Ans

Q.N. 28 → (1) $\lim_{y \to 0} \left(\frac{(x+y)\sec(x+y) - x\sec x}{y} \right)$

$$= \lim_{y \to 0} \left[\frac{x\sec(x+y) + y\sec(x+y) - x\sec x}{y} \right]$$

$$= \lim_{y \to 0} \left[\frac{x\left(\sec(x+y) - \sec x\right)}{y} + \frac{\cancel{y}\sec(x+y)}{\cancel{y}} \right]$$

$$= \lim_{y \to 0} \left[\frac{x\left(\cos x - \cos(x+y)\right)}{y \cdot \cos(x+y)\cos x} + \sec(x+y) \right]$$

$$= \lim_{y \to 0} \left[\frac{-x \cdot 2\sin\left(\frac{2x+y}{2}\right) \cdot \sin\left(\frac{-y}{2}\right)}{y \cdot \cos(x+y) \cdot \cos x} + \sec(x+y) \right]$$

$$= \lim_{h \to 0} \left[\frac{\cancel{2}x \cdot \sin\left(\frac{2x+y}{2}\right) \cdot \sin\left(\frac{y}{2}\right)}{\cancel{2} \times \frac{y}{2} \times \cos(x+y) \cdot \cos x} + \sec(x+y) \right]$$

$$= \frac{x \cdot \sin x \times 1}{\cos x \cdot \cos x} + \sec x \quad \cdots \left\{ \lim_{x \to 0} \left(\frac{\sin x}{x} \right) = 1 \right\}$$

$$= x\tan x \cdot \sec x + \sec x$$ Ans

(ii) $\lim_{x \to a} \left(\frac{\sqrt{a+2x} - \sqrt{3x}}{\sqrt{3a+x} - 2\sqrt{x}} \right)$

Rationalize

$$= \lim_{x \to a} \left(\frac{(a+2x-3x)\,(\sqrt{3a+x} + 2\sqrt{x})}{(3a+x-4x)\,(\sqrt{a+2x} + \sqrt{3x})} \right)$$

$$= \lim_{x \to a} \left(\frac{\cancel{(a-x)}}{3\cancel{(a-x)}} \cdot \frac{(\sqrt{3a+x} + 2\sqrt{x})}{(\sqrt{a+2x} + \sqrt{3x})} \right)$$

$$= \frac{\sqrt{3a+a} + 2\sqrt{a}}{3(\sqrt{a+2a} + \sqrt{3a})}$$

$$= \frac{2\sqrt{a} + 2\sqrt{a}}{3(\sqrt{3}\sqrt{a} + \sqrt{3}\sqrt{a})}$$

$$= \frac{4\cancel{\sqrt{a}}}{3(2\sqrt{3}\cancel{\sqrt{a}})}$$

$$= \frac{2}{3\sqrt{3}} \quad \text{Ans}$$

Qn. 29 → $\lim_{x \to 0} \left(\frac{1 - \cos x \cdot \sqrt{\cos(2x)}}{x^2} \right)$

Rationalize

$$= \lim_{x \to 0} \left[\frac{1 - \cos^2 x \cdot \cos(2x)}{x^2 \left(1 + \cos x \sqrt{\cos(2x)}\right)} \right]$$

$$= \lim_{x \to 0} \left[\frac{1 - \cos^2 x \cdot (2\cos^2 x - 1)}{x^2 \left(1 + \cos x \cdot \sqrt{\cos(2x)}\right)} \right]$$

$$= \lim_{x \to 0} \left(\frac{1 - 2\cos^4 x + \cos^2 x}{x^2 \left(1 + \cos x \sqrt{\cos(2x)}\right)} \right)$$

$$= -\lim_{x\to 0}\left(\frac{2\cos^4 x - \cos^2 x - 1}{x^2\left(1+\cos x\cdot\sqrt{\cos 2x}\right)}\right)$$

$$= -\lim_{x\to 0}\left[\frac{2\cos^4 x - 2\cos^2 x + \cos^2 x - 1}{x^2\left(1+\cos x\sqrt{\cos(2x)}\right)}\right]$$

$$= -\lim_{x\to 0}\left[\frac{2\cos^2 x\left(\cos^2 x - 1\right) + 1\left(\cos^2 x - 1\right)}{x^2\left(1+\cos x\sqrt{\cos(2x)}\right)}\right]$$

$$= -\lim_{x\to 0}\left(\frac{\left(2\cos^2 x + 1\right)\left(\cos^2 x - 1\right)}{x^2\left(1+\cos x\sqrt{\cos(2x)}\right)}\right)$$

$$= \lim_{x\to 0}\left(\frac{\left(2\cos^2 x + 1\right)\cdot\sin^2 x}{x^2\left(1+\cos x\sqrt{\cos(2x)}\right)}\right)$$

$$= \frac{2+1}{1+(1)(1)} \quad \cdots \quad \left\{\lim_{x\to 0}\left(\frac{\sin x}{x}\right) = 1, \ \& \ \cos(0) = 1\right\}$$

$$= \frac{3}{2} \quad \text{Ans}$$

Ans 30 (i) $f(x) = \dfrac{ax+b}{cx+d}$

$$f'(x) = \lim_{h\to 0}\frac{\left(\dfrac{ax+ah+b}{cx+ch+d} - \dfrac{ax+b}{cx+d}\right)}{h}$$

$$= \lim_{h\to 0}\left[\frac{(ax+ah+b)(cx+d) - (ax+b)(cx+ch+d)}{h(cx+ch+d)(cx+d)}\right]$$

$$= \lim_{h\to 0}\left[\frac{\cancel{acx^2} + \cancel{adx} - \cancel{achx} + adh + \cancel{bcx} + \cancel{bd} - \cancel{acx^2} + \cancel{achx} - \cancel{adx} - \cancel{bcx} - bch - \cancel{bd}}{h(cx+ch+d)(cx+d)}\right]$$

$$= \lim_{h \to 0} \left[\frac{adh - bch}{h\left(cx + ch + d\right)(cx+d)} \right]$$

$$= \lim_{h \to 0} \left[\frac{\cancel{h}(ad - bc)}{\cancel{h}(cx + ch + d)\ (cx+d)} \right]$$

$$= \frac{ad - bc}{(cx+d)\ (cx+d)}$$

$$\therefore \boxed{f'(x) = \frac{ad - bc}{(cx+d)^2}}$$ Ans

(ii) $f(x) = \begin{cases} \dfrac{k\cos x}{\pi - 2x} & ; x \neq \pi/2 \\ 3 & : x = \pi/2 \end{cases}$

given $\lim_{x \to \frac{\pi}{2}} (f(x)) = f(\pi/2)$

$\Rightarrow LHL = RHL = 3$

$$\Rightarrow \lim_{x \to \frac{\pi}{2}^-} \left(\frac{k\cos x}{\pi - 2x} \right) = \lim_{x \to \frac{\pi}{2}^+} \left(\frac{k\cos x}{\pi - 2x} \right) = 3$$

put $x = \frac{\pi}{2} - h$ & $h \to 0$ put $x = \frac{\pi}{2} + h$ & $h \to 0$

$$\Rightarrow \lim_{h \to 0} \left[\frac{k\cos\left(\frac{\pi}{2} - h\right)}{\pi - 2\left(\frac{\pi}{2} - h\right)} \right] = \lim_{h \to 0} \left[\frac{k\cos\left(\frac{\pi}{2} + h\right)}{\pi - 2\left(\frac{\pi}{2} + h\right)} \right] = 3$$

$$\rightarrow \lim_{h \to 0} \left(\frac{k \sin h}{2h} \right) = \lim_{h \to 0} \left(\frac{\cancel{-}k\sin h}{\cancel{-}2h} \right) = 3$$

$\Rightarrow \frac{k}{2} \times 1 = \frac{k}{2} \times 1 = 3$ --- $\left\{ \because \lim_{x \to 0} \left(\frac{\sin x}{x} \right) = 1 \right\}$

$\Rightarrow \frac{k}{2} = 3$

$\Rightarrow \boxed{k = 6}$ Ans

—x—