Solutions LIMIT & DERIVATIVES

CLASS NO: 4 WORKSHEET NO: 3

Ans: 1 $f(x) = x\tan x$

$$f'(x) = \lim_{h\to 0}\left[\frac{(x+h)\tan(x+h) - x\tan x}{h}\right]$$

$$= \lim_{h\to 0}\left[\frac{x\tan(x+h) + h\tan(x+h) - x\tan x}{h}\right]$$

$$= \lim_{h\to 0}\left[\frac{x(\tan(x+h) - \tan x)}{h} + \frac{\cancel{h}\tan(x+h)}{\cancel{h}}\right]$$

$$= \lim_{h\to 0}\left[\frac{x\{\tan(h)(1+\tan(x+h)\tan x)\}}{h} + \tan(x+h)\right]$$

$$= \lim_{h\to 0}\left[\left(\frac{\tan h}{h}\right)\cdot x(1+\tan(x+h)\tan x) + \tan(x+h)\right]$$

$$= 1\times x(1+\tan x\cdot\tan x) + \tan x \;\cdots\cdots \left\{\lim_{x\to 0}\left(\frac{\tan x}{x}\right) = 1\right\}$$

$$\boxed{f'(x) = x\sec^2 x + \tan x} \quad \text{Ans.}$$

Ans: 2 → $f(x) = x^2\sin x$

$$f'(x) = \lim_{h\to 0}\left(\frac{(x+h)^2\sin(x+h) - x^2\sin x}{h}\right)$$

$$= \lim_{h\to 0}\left(\frac{x^2\sin(x+h) + (h^2+2hx)\sin(x+h) - x^2\sin x}{h}\right)$$

$$= \lim_{h\to 0}\left[\frac{x^2(\sin(x+h) - \sin x)}{h} + \frac{\cancel{h}(h+2x)\sin(x+h)}{\cancel{h}}\right]$$

$$= \lim_{h\to 0}\left[\frac{x^2\cdot 2\cos\left(\frac{2x+h}{2}\right)\sin\left(\frac{h}{2}\right)}{h} + (h+2x)\sin(x+h)\right]$$

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$$f'(x) = \lim_{h\to 0}\left(\frac{x^2 \cdot \cancel{2}\cos\left(\frac{2x+h}{2}\right)\sin\left(\frac{h}{2}\right)}{\frac{h}{2}\times \cancel{2}} + (h+2x)\sin(x+h)\right)$$

$$= x^2\cos x \times 1 \quad + (2x)\sin x \quad \cdots \left\{ \lim_{x\to 0}\left(\frac{\sin x}{x}\right)=1\right\}$$

$$\boxed{f'(x) = x^2\cos x + 2x\sin x}$$ Ans.

Ans: 3 → $f(x) = \cot(x^2)$

$$f'(x) = \lim_{h\to 0}\left[\frac{\cot(x+h)^2 - \cot(x^2)}{h}\right]$$

$$= \lim_{h\to 0}\left[\frac{\frac{1}{\tan(x+h)^2} - \frac{1}{\tan x^2}}{h}\right]$$

$$= \lim_{h\to 0}\left[\frac{\tan(x^2) - \tan(x+h)^2}{h\tan(x+h)^2\tan(x^2)}\right]$$

$$= \lim_{h\to 0}\left[\frac{\tan(-h^2-2hx)\left\{1+\tan(x^2)\tan(x+h)^2\right\}}{h\tan(x+h)^2\tan(x^2)}\right]$$

$$= \lim_{h\to 0}\left[\frac{-\tan(h^2+2hx)\left\{1+\tan(x^2)\tan(x+h)^2\right\}}{h\tan(x+h)^2\tan(x^2)}\right]$$

$$= \lim_{h\to 0}\left(\frac{-\tan(h^2+2hx)\left\{1+\tan(x^2)\tan(x+h)^2\right\}(\cancel{h^2}+2\cancel{h}x)}{(h^2+2hx)\cdot\cancel{h}\tan(x+h)^2\tan(x^2)}\right)$$

$$= \frac{-1\times\left\{1+\tan x^2\cdot\tan x^2\right\}\cdot(0+2x)}{\tan x^2\cdot\tan x^2} \quad \cdots \left\{\lim_{x\to 0}\left(\frac{\tan x}{x}\right)=1\right\}$$

$$= -\frac{(\sec^2(x^2)) \cdot 2x}{\tan^2 x^2}$$

$$= -\frac{\frac{1}{\cos^2(x^2)}}{\frac{\sin^2(x^2)}{\cos^2(x^2)}} \cdot 2x$$

$$\boxed{f'(x) = -2x \cdot \operatorname{cosec}^2(x^2)}$$ Ans

Ans : 4 → $f(x) = \operatorname{cosec}(x^2)$

$$f'(x) = \lim_{h\to 0}\left[\frac{\operatorname{cosec}(x+h)^2 - \operatorname{cosec}(x^2)}{h}\right]$$

$$= \lim_{h\to 0}\left(\frac{\sin(x^2) - \sin(x+h)^2}{h \cdot \sin(x+h)^2 \sin(x^2)}\right)$$

$$= \lim_{h\to 0}\left(\frac{\sin(x^2) - \sin(x^2+h^2+2hx)}{h \sin(x+h)^2 \cdot \sin(x^2)}\right)$$

$$= \lim_{h\to 0}\left(\frac{2\cos\left(\frac{2x^2+2hx+h^2}{2}\right) \cdot \sin\left(\frac{-h^2-2hx}{2}\right)}{h \cdot \sin(x+h)^2 \cdot \sin(x^2)}\right)$$

$$= -\lim_{h\to 0}\left[\frac{2\cos\left(\frac{2x^2+2hx+h^2}{2}\right) \cdot \sin\left(\frac{h^2+2hx}{2}\right) \times \left(\frac{h^2+2hx}{2}\right)}{\left(\frac{h^2+2hx}{2}\right) \cdot h \cdot \sin(x+h)^2 \cdot \sin(x^2)}\right]$$

$$= -\frac{2\cos\left(\frac{2x^2}{2}\right) \times 1 \times \left(\frac{0+2x}{2}\right)}{\sin(x^2)\sin(x^2)} \quad \cdots \left\{\lim_{x\to 0}\left(\frac{\sin x}{x}\right) = 1\right\}$$

$$\boxed{f'(x) = -2x \cdot \operatorname{cosec}(x^2) \cdot \cot(x^2)}$$ Ans.

Ans: 5 → $f(x) = \dfrac{3x-5}{2x+4}$

$$f'(x) = \lim_{h\to 0}\left(\frac{\dfrac{3x+3h-5}{2x+2h+4} - \dfrac{3x-5}{2x+4}}{h}\right)$$

$$= \lim_{h\to 0}\left(\frac{(3x+3h-5)(2x+4) - (3x-5)(2x+2h+4)}{h(2x+2h+4)(2x+4)}\right)$$

$$= \lim_{h\to 0}\left(\frac{\cancel{6x^2} + \cancel{12x} + \cancel{6hx} + 12h - \cancel{10x} - \cancel{20} - \cancel{6x^2} - \cancel{6hx} - \cancel{12x} + \cancel{10x} + 10h + \cancel{20}}{h(2x+2h+4)(2x+4)}\right)$$

$$= \lim_{h\to 0}\left(\frac{22\cancel{h}}{\cancel{h}(2x+2h+4)(2x+4)}\right)$$

$$\boxed{f'(x) = \frac{22}{(2x+4)^2}}$$ Ans

Ans: 6 → $f(x) = \dfrac{1}{\sqrt{2x+3}}$

$$f'(x) = \lim_{h\to 0}\left(\frac{\dfrac{1}{\sqrt{2x+2h+3}} - \dfrac{1}{\sqrt{2x+3}}}{h}\right)$$

$$= \lim_{h\to 0}\left(\frac{\sqrt{2x+3} - \sqrt{2x+2h+3}}{h\sqrt{2x+2h+3}\cdot\sqrt{2x+3}}\right)$$

Rationalise

$$= \lim_{h\to 0}\left(\frac{\cancel{2x}+\cancel{3} - \cancel{2x} - 2h - \cancel{3}}{h\sqrt{2x+2h+3}\,\sqrt{2x+3}\left(\sqrt{2x+2h+3}+\sqrt{2x+3}\right)}\right)$$

$$f'(x) = \lim_{h\to 0}\left[\frac{-2\cancel{h}}{\cancel{h}\sqrt{2x+2h+3}\,\sqrt{2x+3}\left(\sqrt{2x+2h+3}+\sqrt{2x+3}\right)}\right]$$

$$= \frac{-2}{\sqrt{2x+3}\cdot\sqrt{2x+3}\cdot\left(\sqrt{2x+3}+\sqrt{2x+3}\right)}$$

$$= \frac{-\cancel{2}}{(2x+3)\cdot\cancel{2}\sqrt{2x+3}}$$

$$\boxed{f'(x) = \frac{-1}{(2x+3)^{3/2}}}$$ Ans.

Ans: 7 → $f(x) = \begin{cases} \dfrac{x}{|x|} ; & x \neq 0 \\ 0 ; & x = 0 \end{cases}$

$LHL = \lim_{x\to 0^-}\left(\frac{x}{|x|}\right)$ put $x = 0-h = -h$ & $h\to 0$

$\therefore LHL = \lim_{h\to 0}\left(\frac{-h}{|-h|}\right)$

$= \lim_{h\to 0}\left(\frac{-h}{h}\right)$

$= \lim_{h\to 0}(-1)$

$\therefore \boxed{LHL = -1}$

$RHL = \lim_{x\to 0^+}\left(\frac{x}{|x|}\right)$

put $x = 0+h = h$ & $h\to 0$

$RHL = \lim_{h\to 0}\left(\frac{h}{|h|}\right)$

$= \lim_{h\to 0}\left(\frac{h}{h}\right) = 1$ $\therefore \boxed{RHL = 1}$

Since $LHL \neq RHL$

$\therefore \lim_{x\to 0} f(x)$ doesnot exist Ans

Qu: 8 → $f(x) = \begin{cases} 4x-5 & : x \le 2 \\ x-\lambda & : x > 2 \end{cases}$

given that $\lim_{x \to 2} f(x)$ exists

⇒ LHL = RHL

$\Rightarrow \lim_{x \to 2^-} (4x-5) = \lim_{x \to 2^+} (x-\lambda)$

put $x = 2-h$, $h \to 0$ put $x = 2+h$, $h \to 0$

$\Rightarrow \lim_{h \to 0} (4(2-h)-5) = \lim_{h \to 0} (2+h-\lambda)$

$\Rightarrow 8-5 = 2-\lambda$

$\Rightarrow 3 = 2-\lambda$

$\Rightarrow \boxed{\lambda = -1}$ Ans

Qu. 9 → $f(x) = \begin{cases} \dfrac{3x}{|x|+2x} & : x \neq 0 \\ 0 & : x = 0 \end{cases}$

$LHL = \lim_{x \to 0^-} \left(\frac{3x}{|x|+2x}\right)$

put $x = 0-h = -h$ & $h \to 0$

$= \lim_{h \to 0} \left(\frac{-3h}{|-h|+2(-h)}\right)$

$= \lim_{h \to 0} \left(\frac{-3h}{h-2h}\right)$

$= \lim_{h \to 0} \left(\frac{-3h}{-h}\right)$

$\boxed{LHL = 3}$

$RHL = \lim_{x \to 0^+} \left(\frac{3x}{|x|+2x}\right)$

put $x = 0+h = h$ & $h \to 0$

$RHL = \lim_{h \to 0} \left(\frac{3h}{|h|+2h}\right)$

$= \lim_{h \to 0} \left(\frac{3h}{h+2h}\right)$

$= \lim_{h \to 0} \left(\frac{\cancel{3h}}{\cancel{3h}}\right)$

$= \lim_{h \to 0} (1)$

$\boxed{RHL = 1}$

LHL $\neq$ RHL

$\therefore \lim_{x \to 0} f(x)$ does not exist **Ans**

Ans. 10 → $f(x) = \begin{cases} x - [x] & ; \ x < 2 \\ 4 & : \ x = 2 \\ 3x - 5 & , \ x > 2 \end{cases}$

$LHL = \lim_{x \to 2^-} \left(x - [x] \right)$

put $x = 2 - h$ & $h \to 0$

$LHL = \lim_{h \to 0} \left((2-h) - [2-h] \right)$

$= \lim_{h \to 0} \left(2-h - 1 \right)$ --- {Since $2-h$ is almost 1.9999 & $[1.99999] = 1$}

IMP point

$= 2 - 1$

$\boxed{LHL = 1}$

$RHL = \lim_{x \to 2^+} (3x - 5)$

put $x = 2 + h$ & $h \to 0$

$RHL = \lim_{h \to 0} \left(3(2+h) - 5 \right)$

$= 6 - 5$

$\boxed{RHL = 1}$

Since $LHL = RHL = 1$

$\therefore \lim_{x \to 2} f(x)$ exists

and $\lim_{x \to 2} f(x) = 1$ Ans

— x —