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॥ जय श्री गिरिराज जी महाराज ॥ Page - 1

← ULTIMATE MATHEMATICS →

Chapter: Relation and Functions

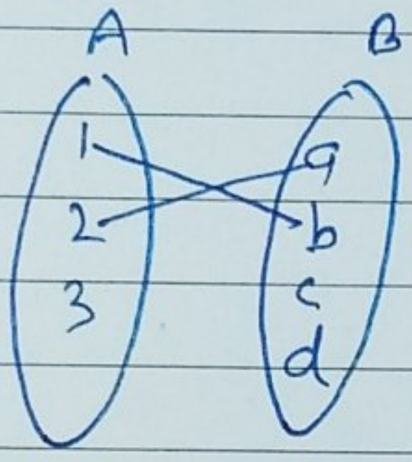
CLASS NO: 2

## Functions

(·) $f: A \to B$ function f from set A to B

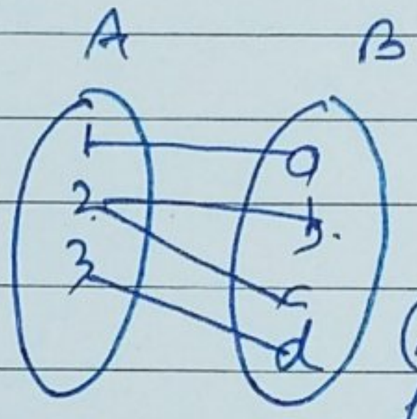
domain → A, codomain → B

(1) (Image)




(X)

(2)

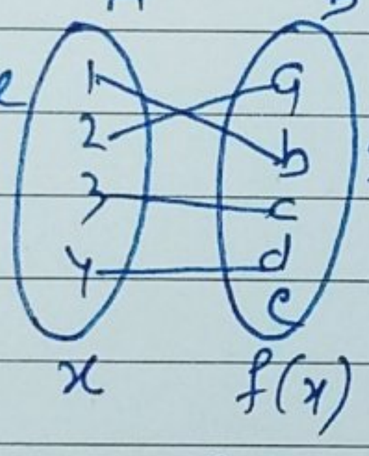



(X)

(✓)

f(2) = b ; f(2) = c

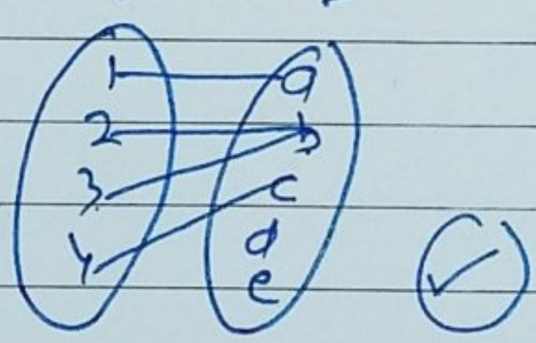
(3) Preimage




Image

(✓)

(4)




(✓)

(·) Every Relation is not a function but every function is a Relation.

$R \subset A \times B$ ; $f \subset R$

(·) $f(1) = b$ ; $f(2) = a$ Images

(·) Range : Set of all images.

Range $= \{a, b, c, d\}$

(·) Mapping or function

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R&F  Class No: 2  (2)

Qn 1 → $A = \{-2, -1, 0, 1\}$ and $f: A \to Z$ given by $f(x) = x^2 - 2x - 3$

(i) Find range of $f$

(ii) Find pre-image of $-3$ and $6$

(iii) Write $f$ as ordered pairs

Sol (i) $f(-2) = 4 + 4 - 3 = 5$

$f(-1) = 1 + 2 - 3 = 0$

$f(0) = 0 - 0 - 3 = -3$

$f(1) = 1 - 2 - 3 = -4$

Range of $f = \{5, 0, -3, -4\}$

(iii) $f = \{(-2, 5), (-1, 0), (0, -3), (1, -4)\}$

(ii) Here $f(x) = -3$

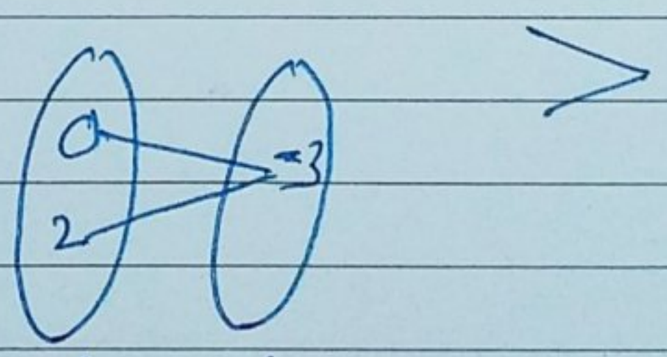
to find: $x$

$x^2 - 2x - 3 = -3$

$\Rightarrow x^2 - 2x = 0$

$\Rightarrow x(x-2) = 0$

$x = 0, \; x = 2$




$-3$ has two pre-images 0 & 2

Here $f(x) = 6$

$x^2 - 2x - 3 = 6$

$x^2 - 2x - 9 = 0$

get $x = 1 \pm \sqrt{10} \notin A$ (domain)

$\therefore$ 6 has no pre-image

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R&F  Class No: 2  (3)

Qn:2 → Let $f = \{(1,1), (2,3), (0,-1), (-1,-3)\}$ be a linear function from Z to Z. Find $f(x)$

Soln :-

Let $f(x) = ax + b$

Here $(1,1) \in f$

$x = 1, \quad f(x) = 1$

$\boxed{1 = a + b} \quad \cdots (1)$

also $(2,3) \in f$

Here $x = 2, \quad f(x) = 3$

$\boxed{3 = 2a + b} \quad \cdots (2)$

Solv, (1) & (2)

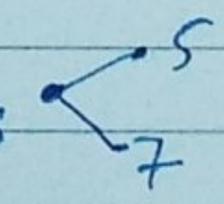
(2) − (1)

$\boxed{2 = a} \Rightarrow (b = -1)$

$\therefore f(x) = 2x - 1$ Ans

Qn:3 → $f: Z \to Z$

$f = \{(ab, a+b) : a, b \in Z\}$ Show that f is not a function




Sol :-

Let $a = 2, \quad b = 3$

$ab = 6$ and $a + b = 5$

$(6, 5) \in f$

Let $a = 6, \quad b = 1$

$ab = 6$ and $a + b = 7 \Rightarrow (6, 7) \in f$

Clearly it is not a function

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R&F  Class No: 2  (4)

Q.No 4 → $f(x) = \sqrt{x}$ and $g(x) = x+3$ be two functions

find (1) $(f+g)(x)$

(2) $(f-g)x$

(3) $(fg)x$

(4) $\left(\frac{f}{g}\right)x$

Soln (1) $(f+g)(x) = f(x) + g(x) = \sqrt{x} + x + 3$

(2) $(f-g)(x) = f(x) - g(x) = \sqrt{x} - x - 3$

(3) $(fg)(x) = f(x) \cdot g(x) = \sqrt{x}(x+3) = x^{3/2} + 3\sqrt{x}$

(4) $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{\sqrt{x}}{x+3}$ ; $x \neq -3$

Q.No 5 → Show that relation g defined as

$$g(x) = \begin{cases} x^2 & ; \ 0 \le x \le 2 \\ 3x & ; \ 2 \le x \le 10 \end{cases}$$ is not a function

Soln when $x = 2$ then $g(2) = (2)^2 = 4$

also when $x = 2$ then $g(2) = 3 \times 2 = 6$

Clearly element 2 has two different images. ∴ g is not a function

$-x-$

✓ $\frac{N}{D}$ ; $D \neq 0$  ✓ $\frac{1}{\sqrt{\square}}$ ; $\square > 0$

✓ $\sqrt{\square}$ ; $\square \geq 0$

✓ DOMAIN of a function : all the values of $x$ for which the given function exists / real / defined

Q.No.1 $f(x) = \frac{x-3}{x-2}$

$f(x)$ exists for all values of $x$ such that

$x-2 \neq 0$

$\Rightarrow x \neq 2$

Domain $= R - \{2\}$

Q.2 $f(x) = \frac{x-1}{x^2+5x+6}$

$f(x)$ exists for all values of $x$ such that

$x^2+5x+6 \neq 0$

$(x+2)(x+3) \neq 0$

$x \neq -2$ or $x \neq -3$

$\therefore$ Domain $= R - \{-2, -3\}$

Q.3 $f(x) = \frac{x+1}{x^2-4}$

Q.4 $f(x) = \frac{1}{x^2+1}$

Domain = R

CLASSTIME

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RLF  Class No: 2  (6)

Q-5  $f(x) = \sqrt{x-3}$

$f(x)$ exists for all real values of $x$ such that

$x-3 \geq 0$

$\Rightarrow x \geq 3$

$\therefore x \in [3, \infty)$

Q-6  $f(x) = \frac{1}{\sqrt{2-x}}$

$f(x)$ exists for all values of $x$ such that

$2-x > 0$

$x-2 < 0$  (sign change)

$x < 2$

$x \in (-\infty, 2)$

$2-x > 0$

$2 > x$

$x < 2$

Q-7  $f(x) = \frac{1}{\sqrt{3+x}}$

$3+x > 0$

$x > -3$

$x \in (-3, \infty)$

(8)  $f(x) = \frac{1}{2-\sin(3x)}$

But $\sin(3x) \neq 2$

$\sin\theta \in [-1, 1]$

$\cos\theta \in [-1, 1]$

$\therefore$ Domain $= R$

CLASSTIME

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# ← WORKSHEET NO: 2 →

## RELATIONS & FUNCTION

Q.No 1 → $f : A \to R$ ; $f(x) = x^2 + 1$ where $A = \{-1, 0, 2, 4\}$
Find Range of $f$ and write $f$ as set of ordered pairs also find preimage of 3

Q.No 2 → Find the domain (or values of $x$) for which the functions $f(x) = 2x^2 - 1$ and $g(x) = 1 - 3x$ are equal

Q.No 3 → $f(x) = x^2 - 1$ ; $g(x) = 2x + 3$ be two functions
find $(f+g)(x)$, $(f-g)(x)$, $(fg)(x)$, $\left(\frac{f}{g}\right)(x)$

Q.No 4 → If $g = \{(3,5), (2,3), (1,1), (4,7)\}$ is a function given by $g(x) = \alpha x + \beta$. Find values of $\alpha$ and $\beta$

Q.No. 5 → Write the following relations as set of ordered pairs and find which of them are functions

(1) $\{(x, y) : y = 3x \text{ , } x \in \{1, 2, 3\} \text{ , } y \in \{3, 6, 9, 12\}\}$

(2) $\{(x, y) : y > x + 1 \text{ , } x = 1, 2 \text{ and } y = 2, 4, 6\}$

(3) $\{(x, y) : x + y = 3 \text{ , } x \& y \in \{0, 1, 2, 3\}\}$

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Worksheet No: 2

Qn. 6 $A = \{12, 13, 14, 15, 16, 17\}$ and $f : A \to Z$ be a function given by

$f(x)$ = highest prime factor of $x$

Find Range of $f$

Qn. 7 → $f(x) = x^2$

find $\dfrac{f(1.1) - f(1)}{1.1 - 1}$

Qn. 8 → Find domain of given function

(1) $f(x) = \dfrac{x-1}{x+2}$

(2) $f(x) = \dfrac{x^2+3x+5}{x^2-5x+4}$

(3) $f(x) = \dfrac{3x-1}{x^2-2}$

(4) $f(x) = \dfrac{1}{x^2+2}$

(5) $f(x) = \sqrt{4x-3}$

(6) $f(x) = \dfrac{1}{\sqrt{3x-2}}$

(7) $f(x) = 1$

(8) $f(x) = |x-2|$

(10) $f(x) = \dfrac{1}{|x-3|}$

(11) $f(x) = \dfrac{1}{|x|-2}$

(12) $f(x) = \dfrac{1}{1-2\sin x}$

$\sqrt{3-2x}$

(g) $f(x) = \sqrt{1-\sin(3x)}$