॥ जय श्री राधे कृष्ण॥ जय श्री गिरिराज की महाराज ॥ (1)

ULTIMATE MATHEMATICS: BY AJAY MITTAL

COMPLEX NUMBERS: CLASS 10:4

POLAR FORM of a Complex Number

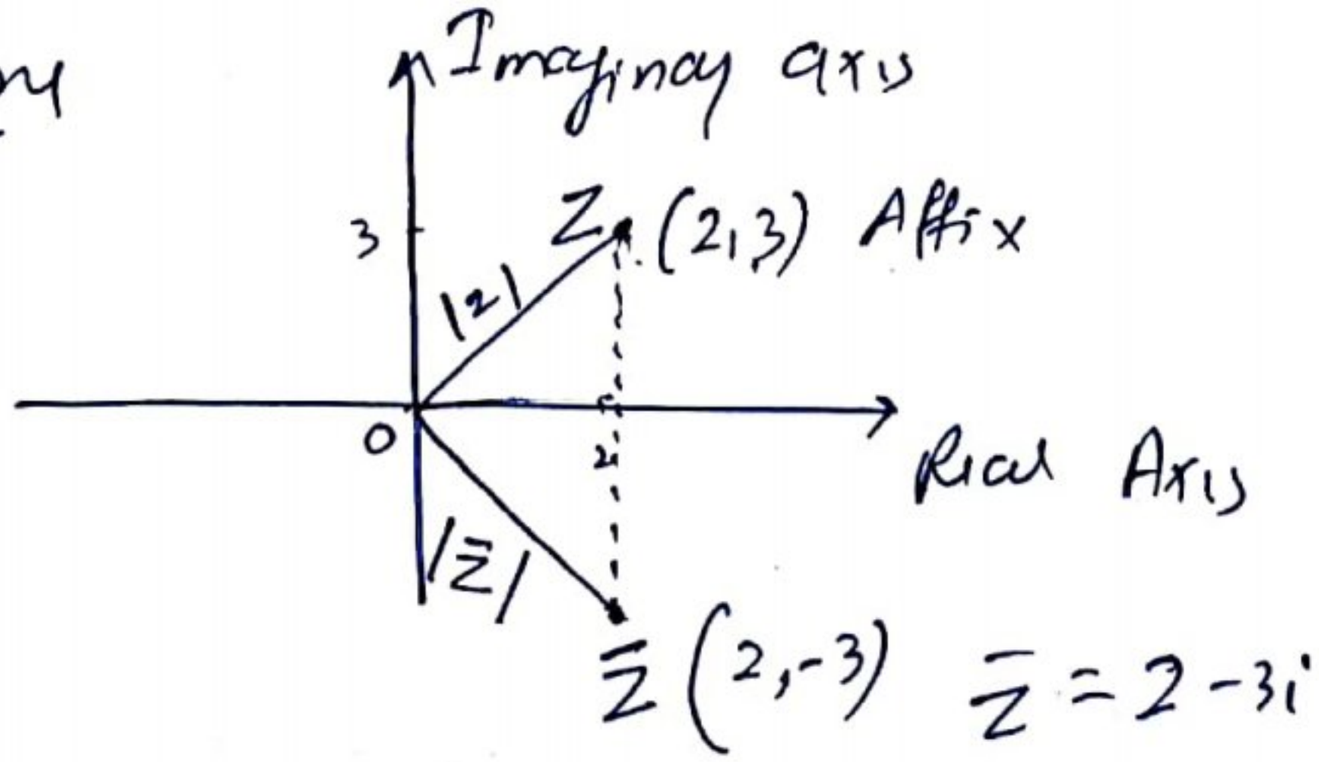
$z = a+ib$ (Standard form) (Cartesian form)

$\boxed{z = r(\cos\theta + i\sin\theta)}$ (polar form) (Trigo. form)

$r = |z| = \sqrt{a^2+b^2}$

θ = argument / amplitude of z = $\arg(z)$ $\boxed{-\pi \le \theta \le \pi}$

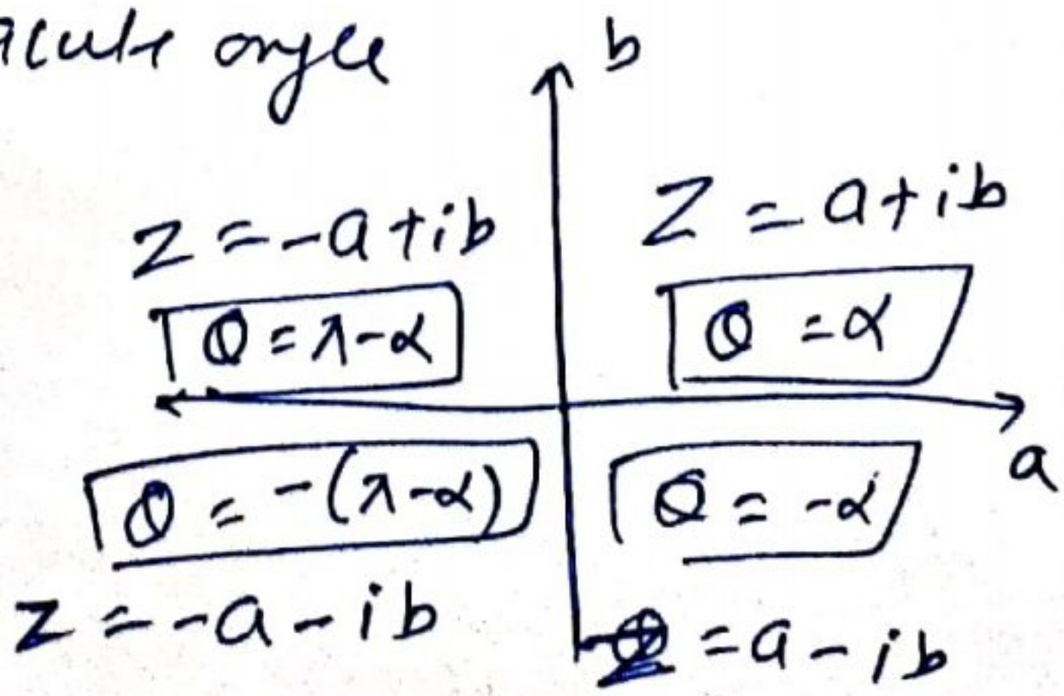
Argand plane



e.g $z = 2+3i$

$\bar{z} = 2-3i$

- $\tan\alpha = \left|\frac{b}{a}\right|$; $\alpha \rightarrow$ acute angle



Q.4) Convert $z = \frac{1+7i}{(2-i)^2}$ in to polar form

Sol:

$$z = \frac{1+7i}{(2-i)^2}$$

$$z = \frac{1+7i}{4+i^2-4i}$$

$$z = \frac{1+7i}{3-4i} \times \frac{3+4i}{3+4i}$$

$$z = \frac{3+4i+21i+28i^2}{9-16i^2}$$

$$z = \frac{-25}{25} + \frac{25i}{25}$$

$$\boxed{z = -1+i.} \quad \text{here } a=-1,\ b=1$$

$$r = \sqrt{a^2+b^2} = \sqrt{1+1} = \sqrt{2}$$

$$\tan\alpha = \left|\frac{b}{a}\right| = \left|\frac{1}{-1}\right| = 1 \Rightarrow \boxed{\alpha = \pi/4}$$

z is in 2nd quadrant

$$\therefore \theta = \pi - \alpha$$

$$\Rightarrow \theta = \pi - \pi/4 = 3\pi/4$$

$$\therefore z = r(\cos\theta + i\sin\theta)$$

Polar form $z = \sqrt{2}\left[\cos(3\pi/4) + i\sin(3\pi/4)\right]$ Ans

Qn. 2

Imp

$$z = \frac{i-1}{\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}}$$

$$z = \frac{-1+i}{\frac{1}{2} + i\frac{\sqrt{3}}{2}}$$

$$z = \frac{-2+2i}{1+i\sqrt{3}} \times \frac{1-i\sqrt{3}}{1-i\sqrt{3}}$$

$$z = \frac{-2+2\sqrt{3}i+2i-2\sqrt{3}i^2}{1-3i^2}$$

$$z = \frac{(2\sqrt{3}-2)}{4} + i\frac{(2\sqrt{3}+2)}{4}$$

$$\boxed{z = \frac{\sqrt{3}-1}{2} + i\left(\frac{\sqrt{3}+1}{2}\right)}$$

then $a = \frac{\sqrt{3}-1}{2}$; $b = \frac{\sqrt{3}+1}{2}$

$$r = \sqrt{\left(\frac{\sqrt{3}-1}{2}\right)^2 + \left(\frac{\sqrt{3}+1}{2}\right)^2}$$

$$r = \sqrt{\frac{3+1-2\sqrt{3}}{4} + \frac{3+1+2\sqrt{3}}{4}}$$

$$r = \sqrt{1+1} = \sqrt{2}$$

$$\boxed{r = \sqrt{2}}$$

$$\tan(\alpha) = \left|\frac{\frac{\sqrt{3}+1}{2}}{\frac{\sqrt{3}-1}{2}}\right|$$

$$\tan\alpha = \frac{\sqrt{3}+1}{\sqrt{3}-1}$$

$$\boxed{\tan(A+B) = \frac{\tan A + \tan B}{1-\tan A \tan B}}$$

Divide N & D by $\sqrt{3}$

$$\tan\alpha = \frac{1+\frac{1}{\sqrt{3}}}{1-\frac{1}{\sqrt{3}}\times 1}$$

$$\tan\alpha = \frac{\tan(\pi/4) + \tan(\pi/6)}{1-\tan(\pi/4)\tan(\pi/6)}$$

$$= \tan\left(\frac{\pi}{4}+\frac{\pi}{6}\right)$$

$$\tan\alpha = \tan\left(\frac{5\pi}{12}\right)$$

$$\alpha = \frac{5\pi}{12}$$

z is in I^{st} quadrant

$\theta = \alpha$

$\Rightarrow \theta = 5\pi/12$

polar form

$$z = \sqrt{2}\left[\cos\left(\frac{5\pi}{12}\right) + i\sin\left(\frac{5\pi}{12}\right)\right]$$

Any

(Complex x class=4) (4)

Ques: 3 Find the amplitude / argument of

$$z = \sin\left(\frac{\pi}{5}\right) + i\left(1 - \cos\frac{\pi}{5}\right)$$

Sol: Here $a = \sin(\pi/5)$ & $b = 1 - \cos(\pi/5)$

$$\tan(\alpha) = \left|\frac{1-\cos(\pi/5)}{\sin(\pi/5)}\right|$$

$$= \frac{2\sin^2(\pi/10)}{2\sin(\pi/10)\cos(\pi/10)} \quad \cdots \begin{cases} 1-\cos\theta = 2\sin^2\theta_2 \\ \sin\theta = 2\sin\theta_2\cos\theta_2 \end{cases}$$

$$\tan\alpha = \tan\left(\frac{\pi}{10}\right)$$

$$\Rightarrow \boxed{\alpha = \pi/10}$$

$z \rightarrow$ Ist Quadrant

$\therefore \theta = \alpha$ $\Rightarrow$ $\theta = \frac{\pi}{10}$ Ans

Qn. 4 Find amplitude of $-i$ also find polar form

Soln Let $z = 0 - i$

then $a = 0$, $b = -1$

$$r = \sqrt{0+1} = 1$$

$$\tan\alpha = \left|\frac{-1}{0}\right| = \infty$$

$$\alpha = \pi/2$$

$z \rightarrow$ 4th quad

$\theta = -\alpha$

$\Rightarrow \boxed{\theta = -\frac{\pi}{2}}$

$$z = 1\left(\cos\left(-\frac{\pi}{2}\right) + i\sin\left(-\frac{\pi}{2}\right)\right)$$

Qn.5 what is the locus of z, if amplitude of $z-2-3i$ is $\pi/4$?

Soln Let $z = x+iy$

$\therefore \quad z-2-3i = x+iy-2-3i$

$= (x-2) + i(y-3)$

$\tan\left(\frac{\pi}{4}\right) = \frac{y-3}{x-2}$

$1 = \frac{y-3}{x-2}$

$\Rightarrow x-2 = y-3$

$\Rightarrow \boxed{x-y=1}$ locus of z is a straight line.

WORKSHEET

Qns 1 Find the modulus of $z = \frac{1+i}{1-i} - \frac{1-i}{1+i}$ Ans = 2

Qn. 2 Find the principal argument / amplitude of

$\frac{1+3i}{1-2i}$ Ans $3\pi/4$

Qn. 3 → convert in to polar form $z = -1-i$

Ans $\sqrt{2}\left(\cos\left(-\frac{3\pi}{4}\right) + i\sin\left(-\frac{3\pi}{4}\right)\right)$

Qn 4 → Find the Modulus and amplitude / principal argument of $z = \frac{-16}{1+i\sqrt{3}}$ Ans 8 ; $2\pi/3$

Qn. 5 → Find the principal argument of $(1+i\sqrt{3})^2$

Ans = $2\pi/3$

Qn. 6 → Write $z = \frac{1-i}{\cos\frac{\pi}{3} + i\sin\pi/3}$ in to polar form

Ans $\sqrt{2}\left(\cos\frac{5\pi}{12} + i\sin\frac{5\pi}{12}\right)$

Qn 7 Find the Modulus and argument of $(i^{25})^3$

Ans 1 ; $-\frac{\pi}{2}$

— x —