

## ← ULTIMATE MATHEMATICS →

Page ①

## ← RELATION FUNCTION →

CLASS NO. 3Q.1Find the domain of the function  $f(x) = \sqrt{9-x^2}$ Soln

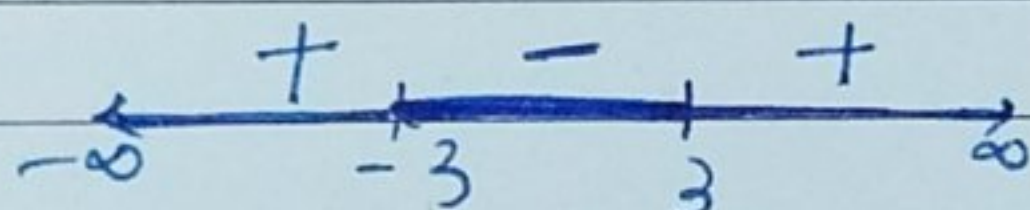
$$f(x) = \sqrt{9-x^2}$$

 $f(x)$  is real for all values of  $x$  such that

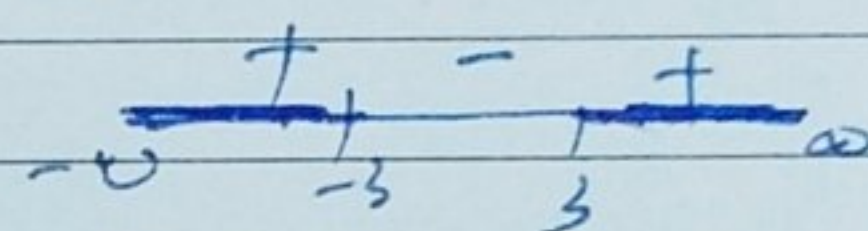
$$9-x^2 \geq 0$$

$$\Rightarrow x^2 - 9 \leq 0$$

$$\Rightarrow (x+3)(x-3) \leq 0$$

Domain  $x \in [-3, 3]$ 

$$(x+3)(x-3) \geq 0$$

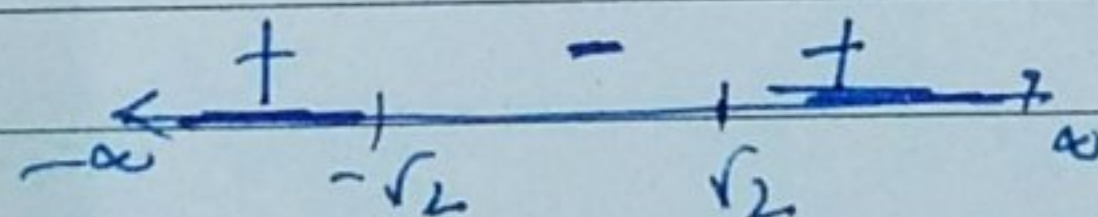


$$x \in (-\infty, -3] \cup [3, \infty)$$

Q.2Find the domain of  $f(x) = \frac{1}{\sqrt{x^2-2}}$ Soln $f(x)$  is real for all values of  $x$  such that

$$x^2 - 2 > 0$$

$$(x+\sqrt{2})(x-\sqrt{2}) > 0$$

 $\therefore$  Domain  $x \in (-\infty, -\sqrt{2}) \cup (\sqrt{2}, \infty)$  Ans



## Relation Function

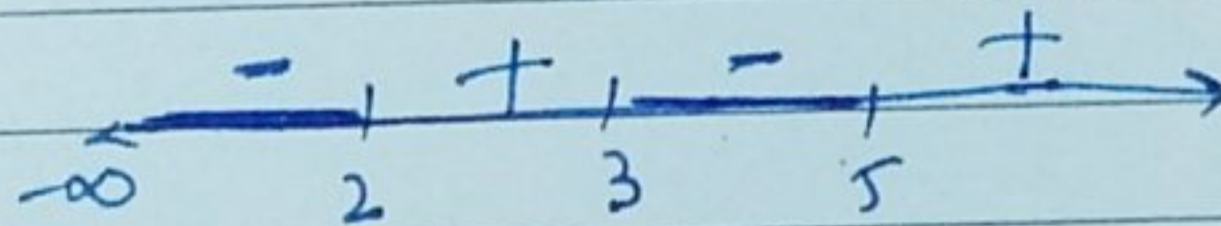
CLASS NO: 3

Q.3 Find the domain of the function  
 $f(x) = \frac{\sqrt{x-2}}{\sqrt{(3-x)(x-5)}}$

Soln  $f(x)$  ----- that

$$\frac{x-2}{(3-x)(x-5)} \geq 0 \quad \text{and} \quad (3-x)(x-5) \neq 0$$

$$\Rightarrow \frac{x-2}{(x-3)(x-5)} \leq 0 \quad \text{and} \quad x \neq 3 \text{ \& } x \neq 5$$



$$x \in (-\infty, 2] \cup (3, 5) \quad \underline{\text{Ans}}$$

Q.4 Find the domain of the function  
 $f(x) = \sqrt{4-x} + \frac{1}{\sqrt{x^2-1}}$

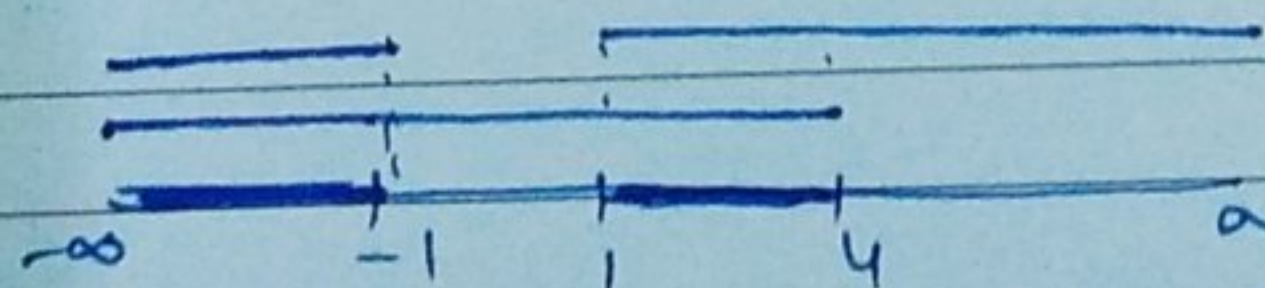
Soln  $4-x \geq 0$  and  $x^2-1 > 0$

$$x-4 \leq 0 \quad \text{and} \quad (x+1)(x-1) > 0$$

$$x \leq 4 \quad \text{and} \quad \begin{array}{c} + \quad - \quad + \\ -\infty \quad -1 \quad 1 \quad \infty \end{array}$$

$$x \in (-\infty, 4] \quad \text{and} \quad x \in (-\infty, -1) \cup (1, \infty)$$

Common



$\therefore$  domain

$$x \in (-\infty, -1) \cup (1, 4] \quad \underline{\text{Ans}}$$



## Relation function

Page (3)

CLASS NO: 3

Q. 5  $\rightarrow$  (1)  $f(x) = \frac{1}{\sqrt{x-|x|}}$

find the domain

(2)  $f(x) = \frac{1}{\sqrt{x+|x|}}$

Sol: (1)  $f(x) = \frac{1}{\sqrt{x-|x|}}$

$f(x)$  ----- that

$$x - |x| > 0$$

$$x > |x|$$

there is no value of  $x$   
for which  $x > |x|$

$$\therefore x \in \phi$$

Domain =  $\phi$

(2)  $f(x) = \frac{1}{\sqrt{x+|x|}}$

$f(x)$  ----- that

$$x + |x| > 0$$

$$\Rightarrow x \in (0, \infty)$$

Domain =  $(0, \infty)$

Q. X

$$-1 + 1 = 0 \quad (x)$$

$$-2 + 2 = 0 \quad (x)$$

$$1 + 1 = 2 > 0$$

$$2 + 2 = 4 > 0$$

RANGE:

$$f(x) =$$

Range all the values of  $f(x)$



# Relation Function

CIAA 110: 3

Qn. 6 Range  $f(x) = \frac{x+2}{x-1}$   
Find

Sol

$$\text{let } y = f(x)$$

$$y = \frac{x+2}{x-1}$$

get  $x$  in terms of  $y$

$$\Rightarrow xy - y = x + 2$$

$$\Rightarrow xy - x = y + 2$$

$$\Rightarrow x(y-1) = y+2$$

$$\Rightarrow x = \frac{y+2}{y-1}$$

$x$  is real for all values of  $y$  such that  
 $y-1 \neq 0$   
 $y \neq 1$

$$\therefore \text{Range} = \mathbb{R} - \{1\} \quad \underline{\text{Ans}}$$

Qn. 7  $f(x) = \sqrt{4-x^2}$  find the range

Sol

$$\text{let } y = \sqrt{4-x^2} \quad \text{--- (1)}$$

$$y^2 = 4-x^2$$

$$x^2 = 4-y^2$$

$$x = \pm \sqrt{4-y^2}$$

$x$  is real for all values of  $y$  such that  
 $4-y^2 \geq 0$



Topic : Relation

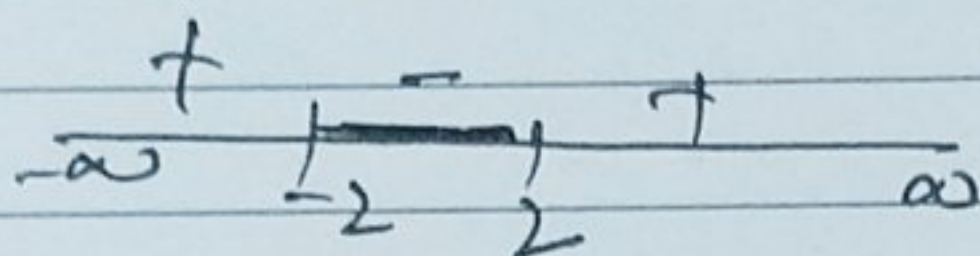
Date : .....

Page No. : 5Relation Function

CLASS No: 3

$$\Rightarrow y^2 - 4 \leq 0$$

$$\Rightarrow (y+2)(y-2) \leq 0$$



$$y \in [-2, 2]$$

Range  $[0, 2]$  --- { Since  $y$  can't be  
-ve from (i) }

Q1: 8  $\rightarrow f(x) = \frac{1}{\sqrt{x-4}}$  find the Range

Sol: let  $y = \frac{1}{\sqrt{x-4}} \quad \text{--- (1)}$

$$\Rightarrow y^2 = \frac{1}{x-4}$$

$$\Rightarrow x-4 = \frac{1}{y^2}$$

$$\Rightarrow x = \frac{1}{y^2} + 4$$

$$\Rightarrow x = \frac{1 + 4y^2}{y^2}$$

$x$  is real for all values of  $y$  such that  
 $y^2 \neq 0$   
 $y \neq 0$

$$y \in \mathbb{R} - \{0\}$$

Range  $(0, \infty)$  Ans



Topic : .....

Date : .....

Page No. : .....

Relation function  
Class No: 3

Page (6)

Q. 9 →  $f(x) = \frac{x^2}{1+x^2}$  Find the Range

Sol. let  $y = f(x)$

$$y = \frac{x^2}{1+x^2}$$

$$\Rightarrow y + x^2 y = x^2$$

$$\Rightarrow x^2 y - x^2 = -y$$

$$\Rightarrow x^2 (y-1) = -y$$

$$\Rightarrow x^2 = \frac{-y}{y-1}$$

$$\Rightarrow x = \pm \sqrt{\frac{-y}{y-1}}$$

$x$  is real for all values of  $y$  such that  
 $\frac{-y}{y-1} \geq 0$  and  $y-1 \neq 0$

$$\Rightarrow \frac{y}{y-1} \leq 0 \quad \text{and} \quad y \neq 1$$

$$\Rightarrow \begin{array}{c} + \quad - \quad + \\ \leftarrow \quad 0 \quad 1 \quad \rightarrow \end{array}$$

$$y \in [0, 1)$$

$$\text{Range } [0, 1)$$

ex hq

$$\begin{array}{l} y \in [0, 1] ; y \neq \frac{1}{2} \\ y \in [0, 1] - \{\frac{1}{2}\} \end{array}$$



Topic : .....

Date : .....

Page No. : .....

7

Relation function class no. 3

Q. 10 →  $f(x) = \frac{|x-3|}{x-3}$  Find the range.

Range =  $\{-1, 1\}$

~~(-1, 1)~~

Q. 11 →  $f(x) = \frac{x}{1+x^2}$  Find the range.

Q. 1

Let  $y = \frac{x}{1+x^2}$

⇒  $y + x^2y = x$

⇒  $x^2y - x + y = 0$

$x = \frac{1 \pm \sqrt{1-4y^2}}{2y}$

$x = \dots \dots \dots$  that  
 $1-4y^2 \geq 0$  and  $2y \neq 0$

$4y^2 - 1 \leq 0$  and  $y \neq 0$

$(2y+1)(2y-1) \leq 0$  &  $y \neq 0$

$\begin{array}{c} + \quad - \quad + \\ \hline -\infty \quad -\frac{1}{2} \quad \frac{1}{2} \quad \infty \end{array}$

$y \in \left[-\frac{1}{2}, \frac{1}{2}\right] - \{0\}$

~~Range~~  $\left[-\frac{1}{2}, \frac{1}{2}\right] - \{0\}$

Range  $\left[-\frac{1}{2}, \frac{1}{2}\right]$  Ans

for  $x=0$   
 $f(x)=0$



Topic : .....

Date : .....

Page No. : .....

## RELATION &amp; FUNCTION

Page (1)

WORKSHEET NO. 3Q.1.1  $\rightarrow f(x) = \frac{1}{\sqrt{x-5}}$  Find domain and RangeAns. Domain :  $(5, \infty)$  Range =  $(0, \infty)$ Q.1.2  $\rightarrow f(x) = \sqrt{16-x^2}$  Find domain and Range  
Ans Domain  $[-4, 4]$  Range  $[0, 4]$ Q.1.3  $\rightarrow f(x) = \frac{3}{2-x^2}$  Find domain & RangeAns Domain  $\mathbb{R} - \{-\sqrt{2}, \sqrt{2}\}$  Range =  $(-\infty, 0) \cup [\frac{3}{2}, \infty)$ Q.1.4  $\rightarrow f(x) = \frac{x}{1+x^2}$  Find domain & Range.Ans Domain =  $\mathbb{R}$  and Range  $[-\frac{1}{2}, \frac{1}{2}]$ Q.1.5  $\rightarrow$  Find domain and Range of  $f(x) = \frac{|x-2|}{2-x}$ Ans Domain =  $\mathbb{R} - \{2\}$  Range =  $\{-1, 1\}$ Q.1.6  $\rightarrow f(x) = \frac{1}{1-x^2}$  Find domain and RangeAns Domain =  $\mathbb{R} - \{-1, 1\}$  Range  $(-\infty, 0) \cup [1, \infty)$ Q.1.7  $\rightarrow f(x) = \frac{ax-b}{cx-d}$  Find domain & RangeAns Domain  $\mathbb{R} - \{\frac{d}{c}\}$  ; Range =  $\mathbb{R} - \{\frac{a}{c}\}$ Q.1.8  $\rightarrow$  Find the domain of  $f(x) = \sqrt{4x-x^2}$



Ans Domain  $[0, 4]$

Topic : .....

Date : .....

Page No. : .....

Page 2

Qns 9 Find the domain of  $f(x) = \sqrt{\frac{x-2}{x+2}} + \sqrt{\frac{1-x}{1+x}}$

Ans Domain =  $\phi$

Qn 10  $\rightarrow$   $f(x) = \sqrt{\frac{x+3}{(2-x)(x-5)}}$  find the domain

Ans Domain  $(-\infty, -3] \cup [2, 5)$

Qn 11  $\rightarrow$  Find the range of  $f(x) = \frac{x^2 - x}{x^2 + 2x}$

Ans Range  $R - \{-\frac{1}{2}, 1\}$

Qn 12  $\rightarrow$  Find domain of  $f(x) = \sqrt{x-1} + \sqrt{3-x}$

Ans Domain  $[1, 3]$



