!! जय श्री गिरिराज जी महाराज जय श्री राधे कृष्ण !!

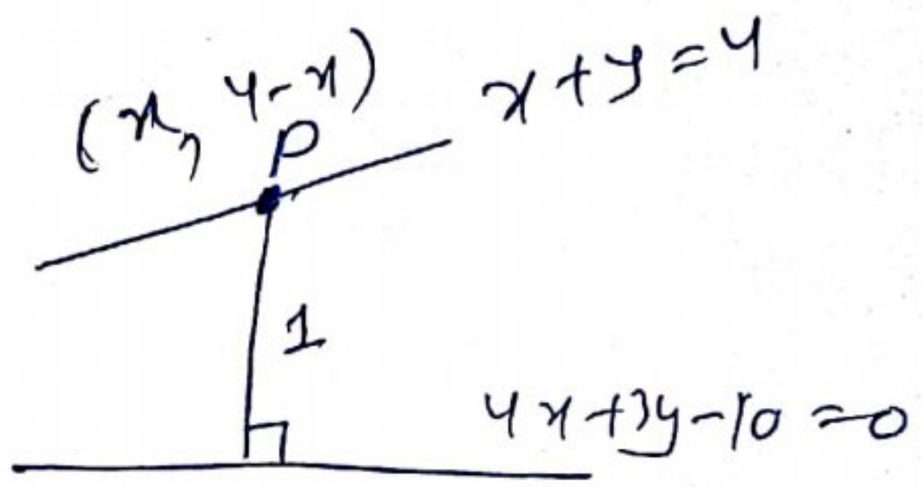
ULTIMATE MATHEMATICS= BY AJAY MITTAL

REVISION : STRAIGHT LINES

CLASS NO= 2

QNS= 1 → Find the points on the line $x+y=4$ which lie at a unit distance from the line $4x+3y=10$

Sol:



Let point of line is $P(x, 4-x)$

$$1 = \frac{|4x+12-3x-10|}{\sqrt{16+9}}$$

$$\Rightarrow 5 = |x+2|$$

$$\Rightarrow x+2 = \pm 5$$

$x+2=5$ | $x+2=-5$

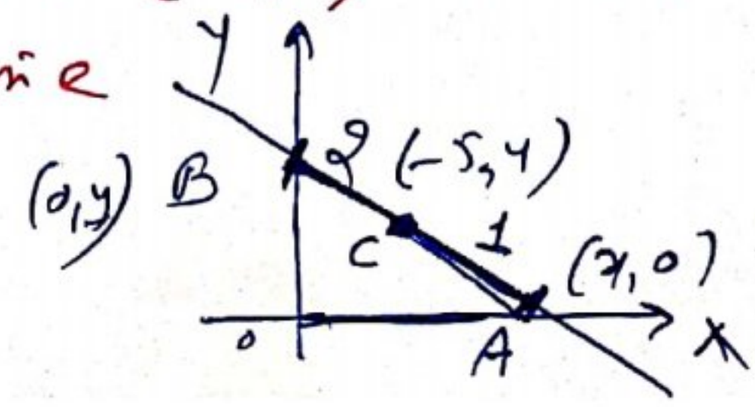
$x=3$ | $x=-7$

∴ points are $(3,1)$ or $(-7,11)$ Ans

QNS 2 → If the Intercept of a line between the coordinate axes is divided by the point $(-5,4)$ in the ratio 1:2. Find the equation of the line

Sol:

By Section formula



$-5 = \frac{0+2x}{1+2}$ | $4 = \frac{y+0}{1+2}$

$x = -\frac{15}{2}$ | $y = 12$

$\therefore A\left(-\frac{15}{2}, 0\right)$ $B(0,12)$ (OR) $x_{int} = -15/2$, $y_{int} = 12$

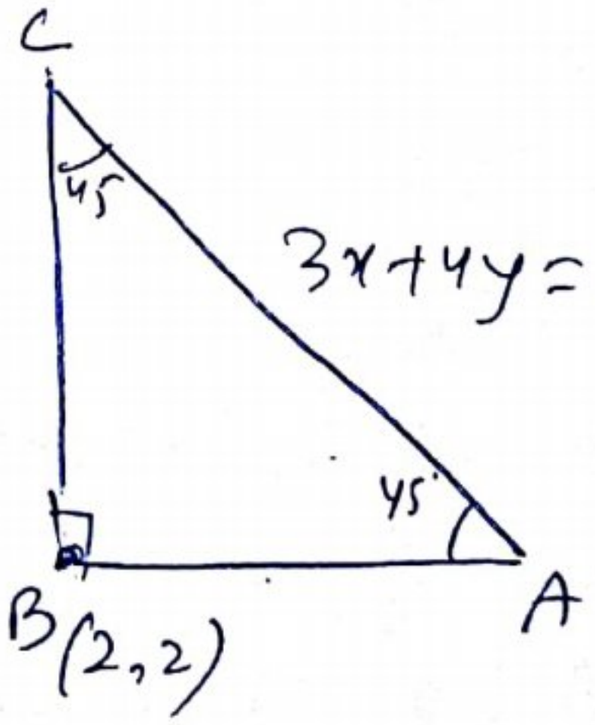
Intercept form

$$\frac{x}{\frac{-15}{2}} + \frac{y}{12} = 1$$

$$\Rightarrow \frac{-2x}{15} + \frac{y}{12} = 1$$

$$\Rightarrow \boxed{-8x + 5y = 60} \text{ Ans}$$

Ques 3 → Find the equation of one of the sides of an isosceles right angled triangle whose hypotenuse is given by $3x+4y=4$ and the opposite vertex of the hypotenuse is $(2,2)$



Soln (·) Slope of $AC = m_1 = -\frac{3}{4}$

(·) Let Slope of $AB = m_2 = m$

(·) angle b/w them $\theta = 45°$

$$\Rightarrow \tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$

$$\Rightarrow \tan(45°) = \left|\frac{-\frac{3}{4} - m}{1 + \frac{3m}{4}}\right| \Rightarrow 1 = \left|\frac{-3-4m}{4-3m}\right|$$

$\Rightarrow \pm 1 = \frac{-3-4m}{4-3m}$

$\Rightarrow 1 = \frac{-3-4m}{4-3m}$

$\Rightarrow 4-3m = -3-4m$

$m = -7$

$-1 = \frac{-3-4m}{4-3m}$

$-4+3m = -3-4m$

$7m = 1$

$m = 1/7$

Equation of AB slope $= -7$ & point $(2,2)$

$y-2 = -7(x-2)$

$\Rightarrow y-2 = -7x+14$

$\Rightarrow \boxed{7x+y-16=0}$ Ans

Qns 4 → If the sum of the distances of a moving point in a plane from the axes is 1. Then find the locus of the point.

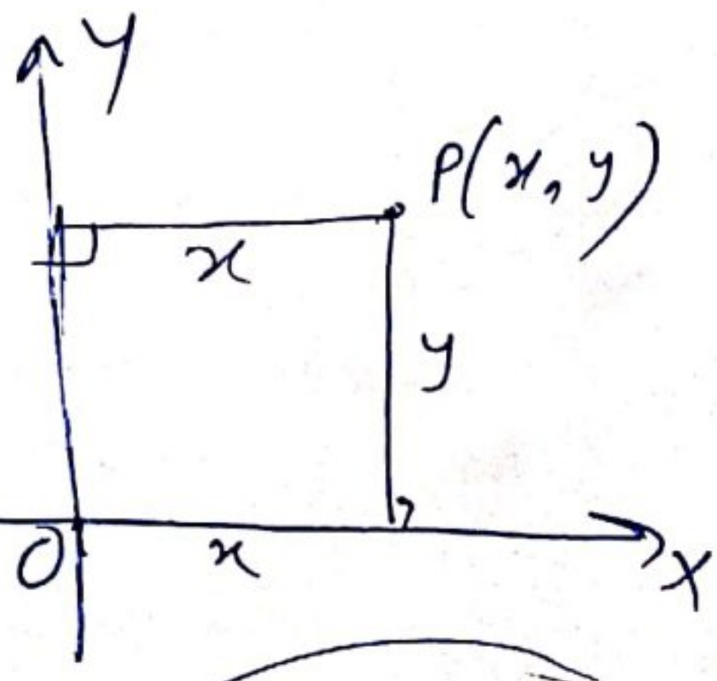
Sln $|x|+|y| = 1$

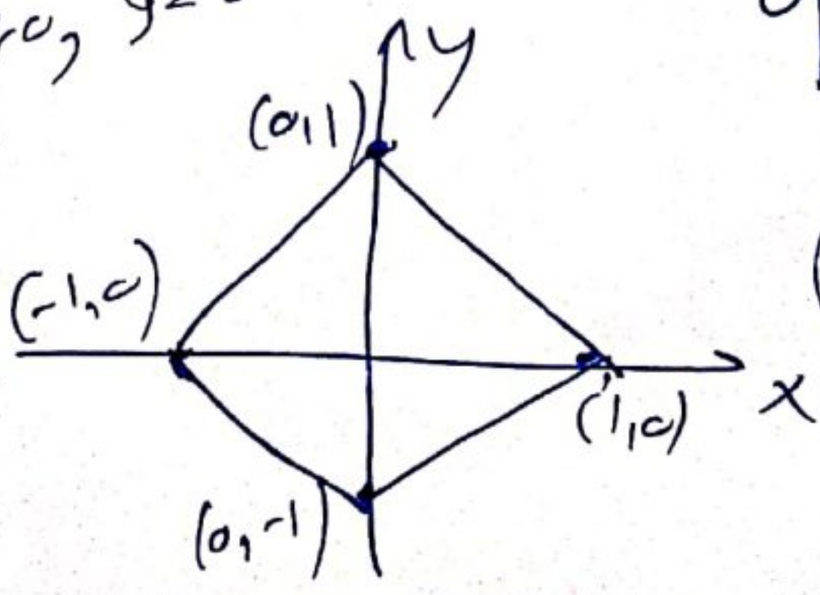
$x+y = 1 \; ; x \geq 0 \; ; y \geq 0$

$x-y = 1 \; ; x \geq 0, y < 0$

$-x+y = 1$

$-x-y = 1$





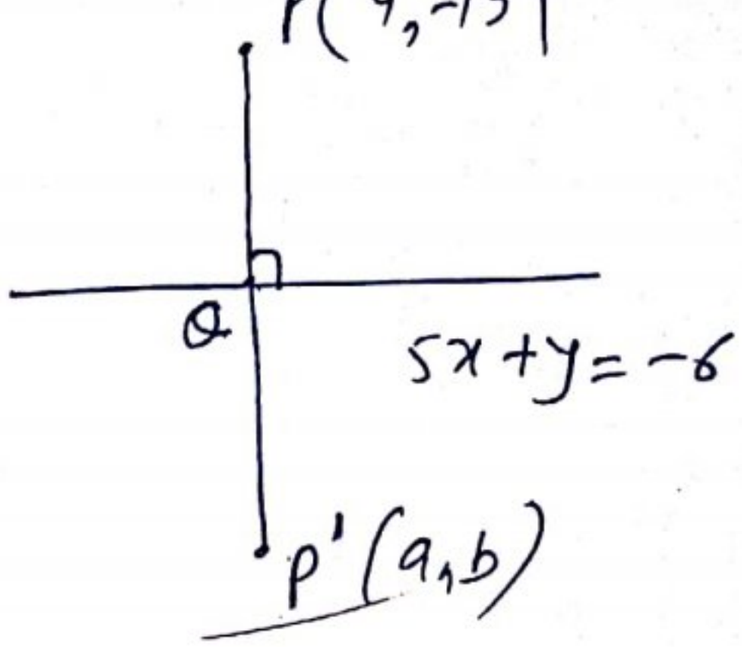
locus is a square

Qn. 5 → Find the reflection of the point $(4,-13)$ about the line $5x+y+6=0$

Soln (·) Slope of given line: $-\frac{5}{1} = -5$

(·) $PQ \perp$ line

$\therefore$ Slope of $PQ = 1/5$



(·) Equation of PQ

$$y+13 = \frac{1}{5}(x-4)$$

$$\Rightarrow 5y+65 = x-4$$

$$\Rightarrow \boxed{x-5y=69} \quad -(2)$$

$$\boxed{5x+y=-6} \quad --(1)$$

$$\Rightarrow 25x+5y=-30$$

Soln $26x = 39$

$\Rightarrow (x = 3/2)$ put in (1)

$$\frac{3}{2} - 5y = 69$$

$$\frac{3}{2} - 69 = 5y$$

$$\Rightarrow \frac{-135}{2} = 5y$$

$$\Rightarrow (y = -\frac{27}{2}) \quad \therefore Q(3/2, -27/2)$$

Q is the mid point of P & P'

$$\frac{3}{2} = \frac{4+a}{2}$$

$$\Rightarrow (a = -1)$$

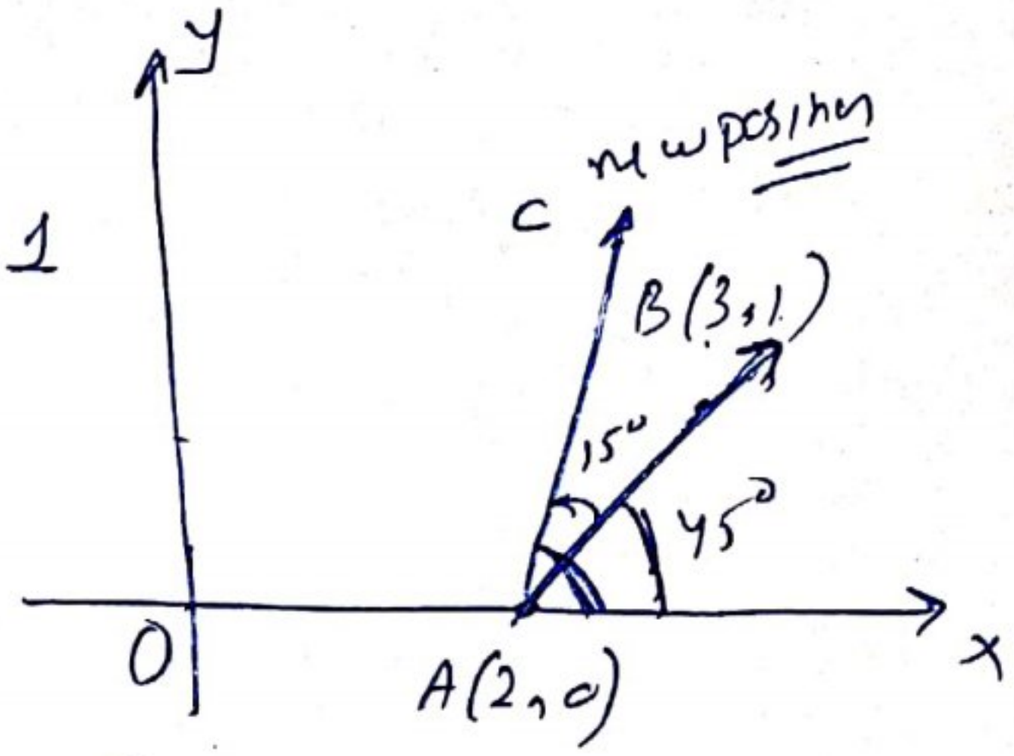
$$\frac{-27}{2} = \frac{-13+b}{2}$$

$$(b = -14)$$

$\therefore$ Image is $P'(-1,-14)$

Ans

Qn. 6 → If the line joining two points A(2,0) & B(3,1) is rotated about A in anticlockwise direction through an angle of 15°. Find the equation of the line in new position



Soln Slope of AB = $\frac{1}{1}$ = 1

$\Rightarrow \tan\theta = 1$

$\Rightarrow \theta = 45°$

∴ for AC the angle is 60°

∴ Slope of AC = $\tan(60°) = \sqrt{3}$

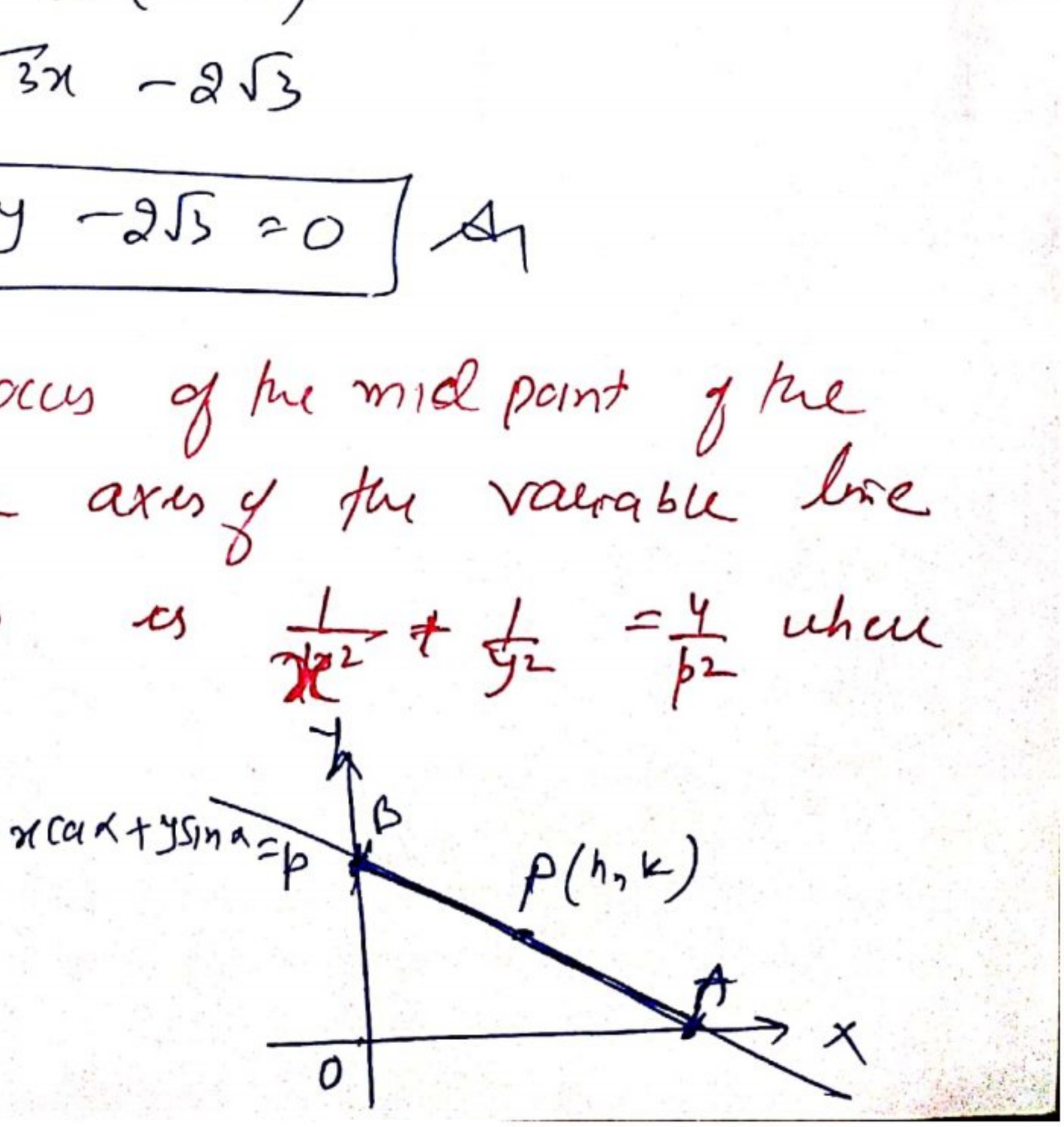
Equation of AC pass. A(2,0) Slope = $\sqrt{3}$

$$y - 0 = \sqrt{3}(x - 2)$$

$$\Rightarrow y = \sqrt{3}x - 2\sqrt{3}$$

$$\Rightarrow \boxed{\sqrt{3}x - y - 2\sqrt{3} = 0}$$ Ans

Qn. 7 → Show that the locus of the mid point of the distance between the axes of the variable line $x\cos\alpha + y\sin\alpha = p$ is $\frac{1}{x^2} + \frac{1}{y^2} = \frac{4}{p^2}$ where p is a constant



Soln Point on X-axis

∴ $A\left(\frac{p}{\cos\alpha}, 0\right)$

∴ $B\left(0, \frac{p}{\sin\alpha}\right)$

Now P is the Mid point of AB

$$h = \frac{\frac{p}{\cos\alpha} + 0}{2} \qquad \Big| \qquad k = \frac{0 + \frac{p}{\sin\alpha}}{2}$$

$$2h = \frac{p}{\cos\alpha} \qquad\qquad 2k = \frac{p}{\sin\alpha}$$

$$\Rightarrow \boxed{\cos\alpha = \frac{p}{2h}} \qquad \& \qquad \boxed{\sin\alpha = \frac{p}{2k}}$$

Squaring & adding

$$\cos^2\alpha + \sin^2\alpha = \frac{p^2}{4h^2} + \frac{p^2}{4k^2}$$

$$\Rightarrow 1 = \frac{p^2}{4h^2} + \frac{p^2}{4k^2}$$

$$\Rightarrow \frac{1}{4} = \frac{p^2}{h^2} + \frac{p^2}{k^2}$$

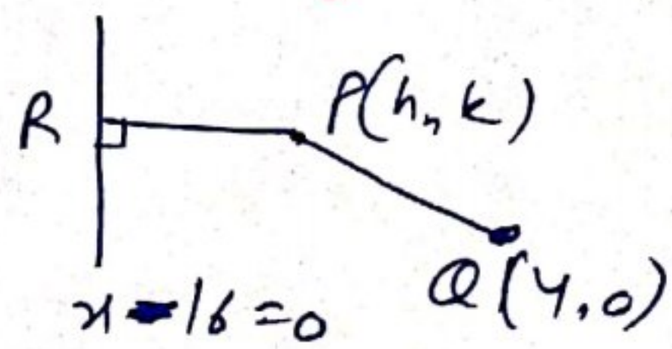
$$\Rightarrow \frac{4}{p^2} = \frac{1}{h^2} + \frac{1}{k^2}$$

$\therefore$ locus equation $\boxed{\frac{1}{x^2} + \frac{1}{y^2} = \frac{4}{p^2}}$

Q.N. 8 → A point moves such that its distance from the point $(4, 0)$ is half that of its distance from the line $x = 16$. Find the locus of the point

Sol^n = Let Moving point is $P(h, k)$

Given = $PQ = \frac{1}{2} PR$



$$\Rightarrow \sqrt{(h-4)^2+k^2} = \frac{1}{2}\left|\frac{h-16}{\sqrt{1+0}}\right|$$

Squaf

$$h^2+16-8h+k^2 = \frac{1}{4}(h^2+256-32h)$$

$$\Rightarrow 4h^2+4k^2-32k+64 = h^2+256-32h$$

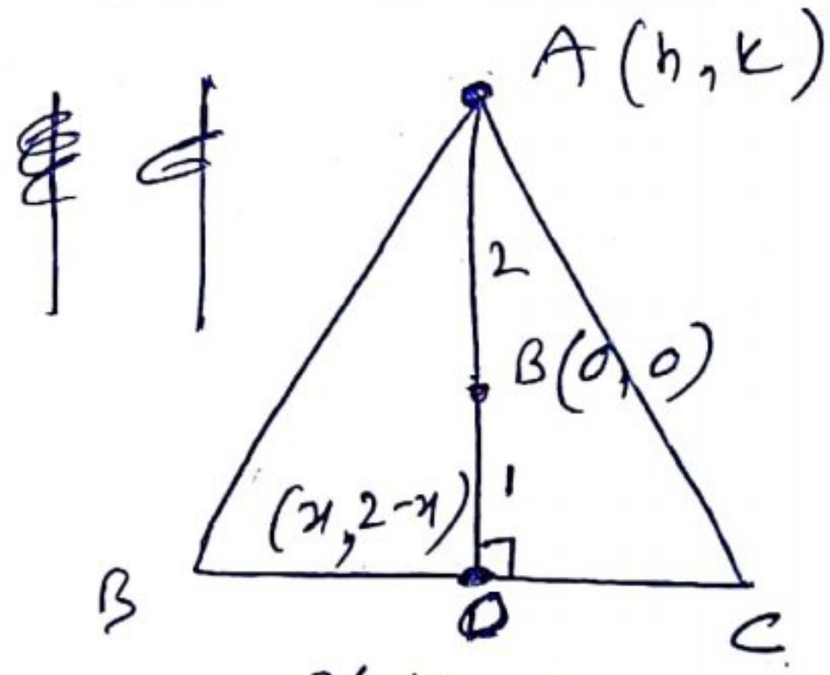
$$\Rightarrow 3h^2+4k^2 = 192$$

$$\Rightarrow \boxed{3x^2+4y^2=192}$$ locus of the point (ellipse)

Qns.9 → find one vertex of the equilateral triangle with centroid at the origin and one side as $x+y-2=0$

Sol:

A (h,k)
B $(0,0)$
D $(x, 2-x)$



B divides AD in Ratio 2:1

By section formula

$$0 = \frac{2x+h}{3} \qquad 0 = \frac{y-2x+k}{3}$$

$$2x+h=0 \quad -(1) \qquad y-2x+k=0 \quad -(2)$$

$$\text{Slope of } BC = \frac{-1}{1} = -1$$

$$\text{Slop of } AD = \frac{k-0}{h-0} = \frac{k}{h}$$

$BC \perp AD$

$\therefore \quad (-1)\left(\frac{k}{h}\right) = -1$

$\rightarrow \boxed{k = h}$

$\therefore$ eqn 2 becom

$y - 2k + h = 0 \quad -(3)$

Soln (1) & (3)

$y - x\left(-\frac{h}{2}\right) + h = 0$

$y + \frac{x}{2} + h = 0$

$\boxed{h = -2} \quad \boxed{k = -2}$

$\therefore A(-2, -2)$ Ans

Q No. 10 → Find the ratio in which the line $3x + 4y + 2 = 0$ divides the distance between the lines

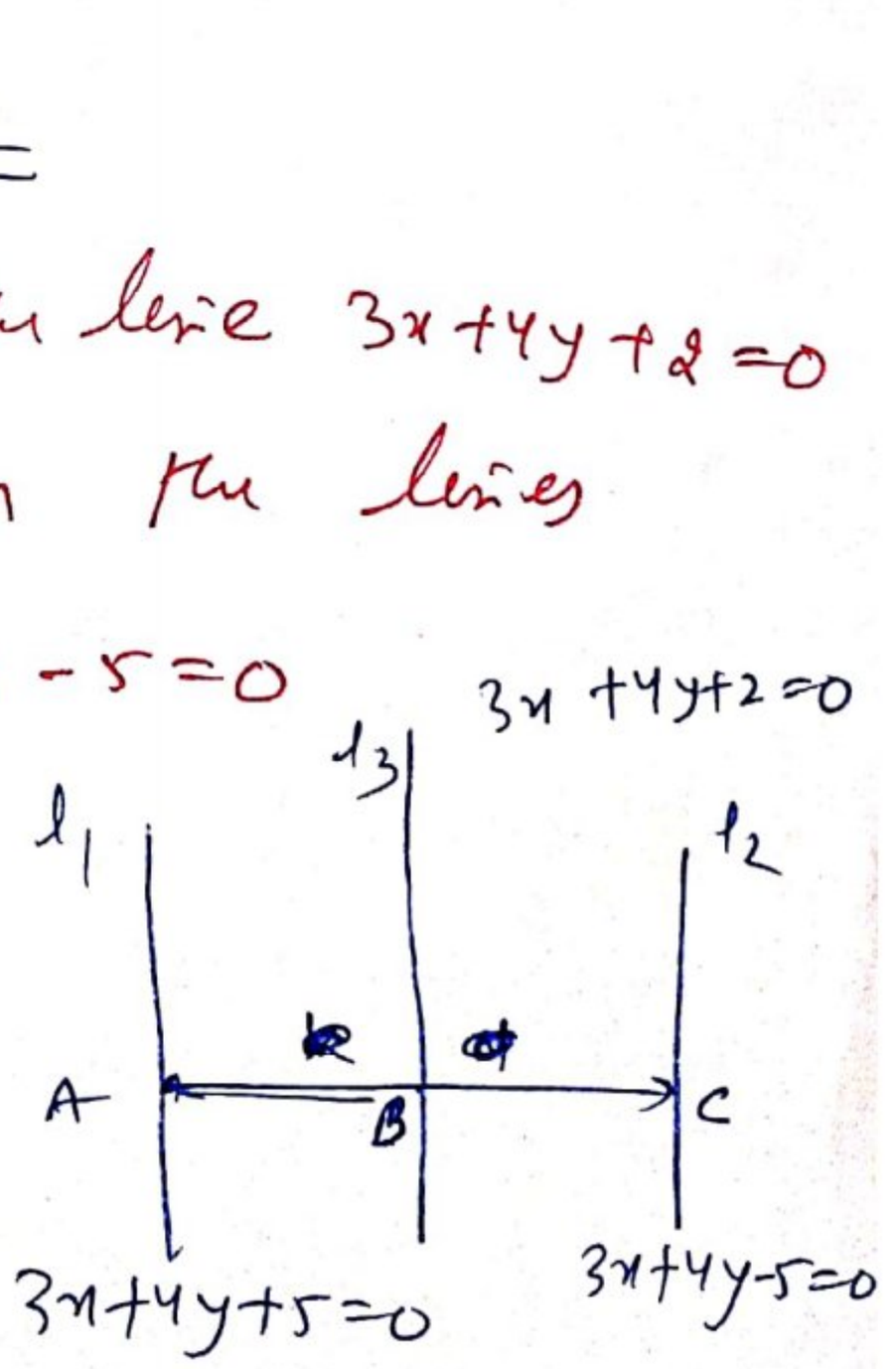
$3x + 4y + 5 = 0$ and $3x + 4y - 5 = 0$

Soln Distance b/w l_1 & l_3

$AB = \frac{|5 - 2|}{\sqrt{9 + 16}} = \frac{3}{5}$

Distance b/w l_2 & l_3

$BC = \frac{|-5 - 2|}{\sqrt{9 + 16}} = \frac{7}{5}$



$\therefore$ Required Ratio

$AB : BC = 3 : 7$ Ans

REVISION STRAIGHT LINES WORKSHEET NO:-1

QNS: 1 → find the distance b/w the lines

$3x+4y=9$ and $6x+8y=15$ ANS= $\frac{3}{10}$ units

QNS 2 → If the slope of a line through the point $A(3,2)$ is $\frac{3}{4}$, then find points on the line which are 5 units away from the point A

ANS $(-1,-1)$ or $(7,5$

QNS 3 → Find the equation of the line passing through the point of Intersection of the lines $5x-6y=1$ and $3x+2y+5=0$ and perpendicular to the line $3x-5y+11=0$ ANS $5x+3y+8=0$

QN. 4 → Find the coordinates of the foot of perpendicular from the point $(2,3)$ on the line $x+y-11=0$

ANS $(5,6)$

QN. 5 → A line passes through point $P(1,2)$. Such that its intercept between the axes is bisected at point P. Find the equation of the line

ANS $2x+y-4=0$

QNS 6 → Find the equation of the lines which ~~cut~~ passes through the point $(3,4)$ and cuts off intercepts from the coordinate axes such that their sum is 14

ANS $x+y=7$ (or) $\frac{x}{6}+\frac{y}{8}=1$

Qn. 7 → Find the equation of the line passing through the intersection of $2x+y=5$ and $x+3y+8=0$ and parallel to the line $3x+4y=7$

Ans $3x+4y+3=0$

Qn. 8 → Find the distance of the point of Intersection of the lines $2x-3y+5=0$ and $3x+4y=0$ from the line $5x-2y=0$ Ans $\frac{130}{17\sqrt{29}}$

Qn. 9 → Find the equations of the lines passing through the point $(1,0)$ and at a distance of $\frac{\sqrt{3}}{2}$ from the origin Ans $\sqrt{3}x+y-\sqrt{3}=0$ or $\sqrt{3}x-y-\sqrt{3}=$

Qn 10 → If the line $\frac{x}{a}+\frac{y}{5}=1$ passes through the points $(2,-3)$ and $(4,-5)$, then value of (a,b) is ? Ans $(-1,-1)$

Qn 11 → Find the equations of the lines which passes through the points $(3,-2)$ and are inclined at $60°$ to the line $\sqrt{3}x+y=1$

Ans $y+2=0$; $\sqrt{3}x-y-2-3\sqrt{3}=0$

Qn. 12 → Find the equations of the lines passes through the point of Intersection of the lines $x-y+1=0$ and $2x-3y+5=0$ and whose distance from the point $(3,2)$ is $7/5$ Ans $3x-4y+6=0$ and $4x-3y+1=0$