← ULTIMATE MATHEMATICS: BY: AJAY MITTAL →

CHAPTER: STRAIGHT LINES ← CLASS NO: 2 →

QN:1 → line through the points $(-2, 6)$ & $(4, 8)$ is perpendicular to the line through the points $(8, 12)$ & $(x, 24)$. Find the value of x

Soln Let m_1 → Slope of 1st line

$$m_1 = \frac{2}{6} = 1/3$$

Let m_2 → slope of 2nd line

$$m_2 = \frac{12}{x - 8}$$

Since lines are $\perp^r$

$$\therefore m_1 m_2 = -1$$

$$\Rightarrow \left(\frac{1}{3}\right) \left(\frac{12^{\,4}}{x - 8}\right) = -1$$

$$\Rightarrow \quad 4 = -x + 8$$

$$\Rightarrow \quad \boxed{x = 4} \text{ Ans}$$

QN:2 → If the angle between two lines is $\pi/4$ and Slope of one of the lines is $\frac{1}{2}$. find the slope of the other line.

Soln Given $\theta = \pi/4$; $m_1 = 1/2$

Let Slope of 2nd line $= m$

Now $\tan\theta = \left|\dfrac{m_1 - m_2}{1 + m_1 m_2}\right|$

$$\Rightarrow \tan(\pi/4) = \left|\dfrac{\frac{1}{2} - m}{1 + \frac{m}{2}}\right|$$

$$\Rightarrow 1 = \left|\dfrac{1 - 2m}{2 + m}\right|$$

$$\Rightarrow \dfrac{1 - 2m}{2 + m} = \pm 1$$

$\dfrac{1-2m}{2+m} = 1$ & $\dfrac{1-2m}{2+m} = -1$

$\Rightarrow 1-2m = 2+m$ & $1-2m = -2-m$

$\Rightarrow 3m = -1$ & $m = 3$

$\therefore \boxed{m = -1/3 \quad \text{(or)} \quad m = 3}$ Ans

Q.N. 3 → If three points $(h, 0)$, (a, b) & $(0, k)$ lie on a line. Show that $\dfrac{a}{h} + \dfrac{b}{k} = 1$.

Soln: let the given points are

$A(h, 0)$ $B(a, b)$ & $C(0, k)$

Since points are collinear

$\therefore$ Slope of AB = Slope of BC

$$\Rightarrow \dfrac{b}{a-h} = \dfrac{k-b}{-a}$$

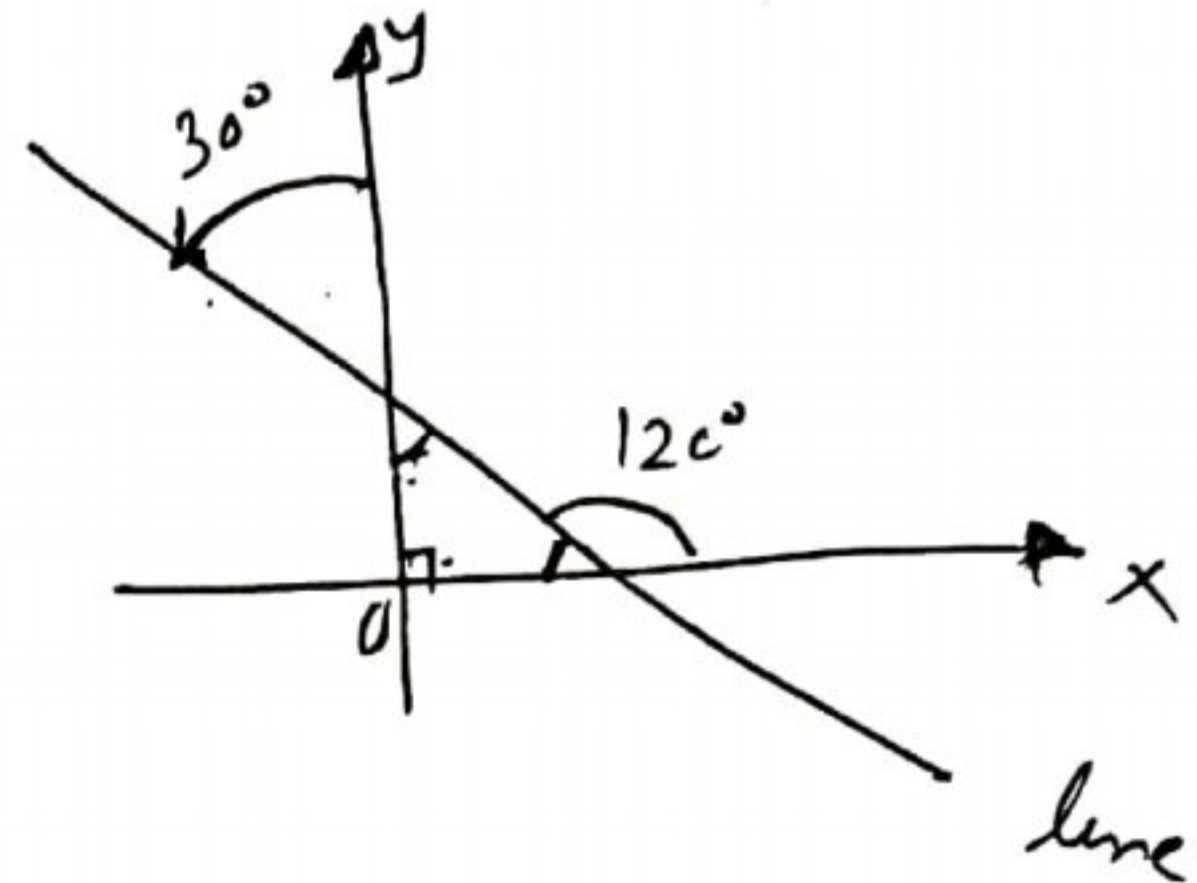$$\Rightarrow -ab = ak - ab - hk + bh$$

$$\Rightarrow ak + bh = hk$$

$$\Rightarrow \boxed{\dfrac{a}{h} + \dfrac{b}{k} = 1} \text{ Ans}$$

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Qns 4 → Find the slope of the line, which makes an angle of 30° with the +ve direction of y-axis measured anticlockwise

Son
= here $\theta = 120°$

$$m = \tan(120°)$$
$$= \tan(180 - 60)$$
$$= -\sqrt{3} \quad \text{Ans}$$

Qns- 5 → MEDIANS

$A(1,2)$ $B(2,-3)$ $C(4,5)$
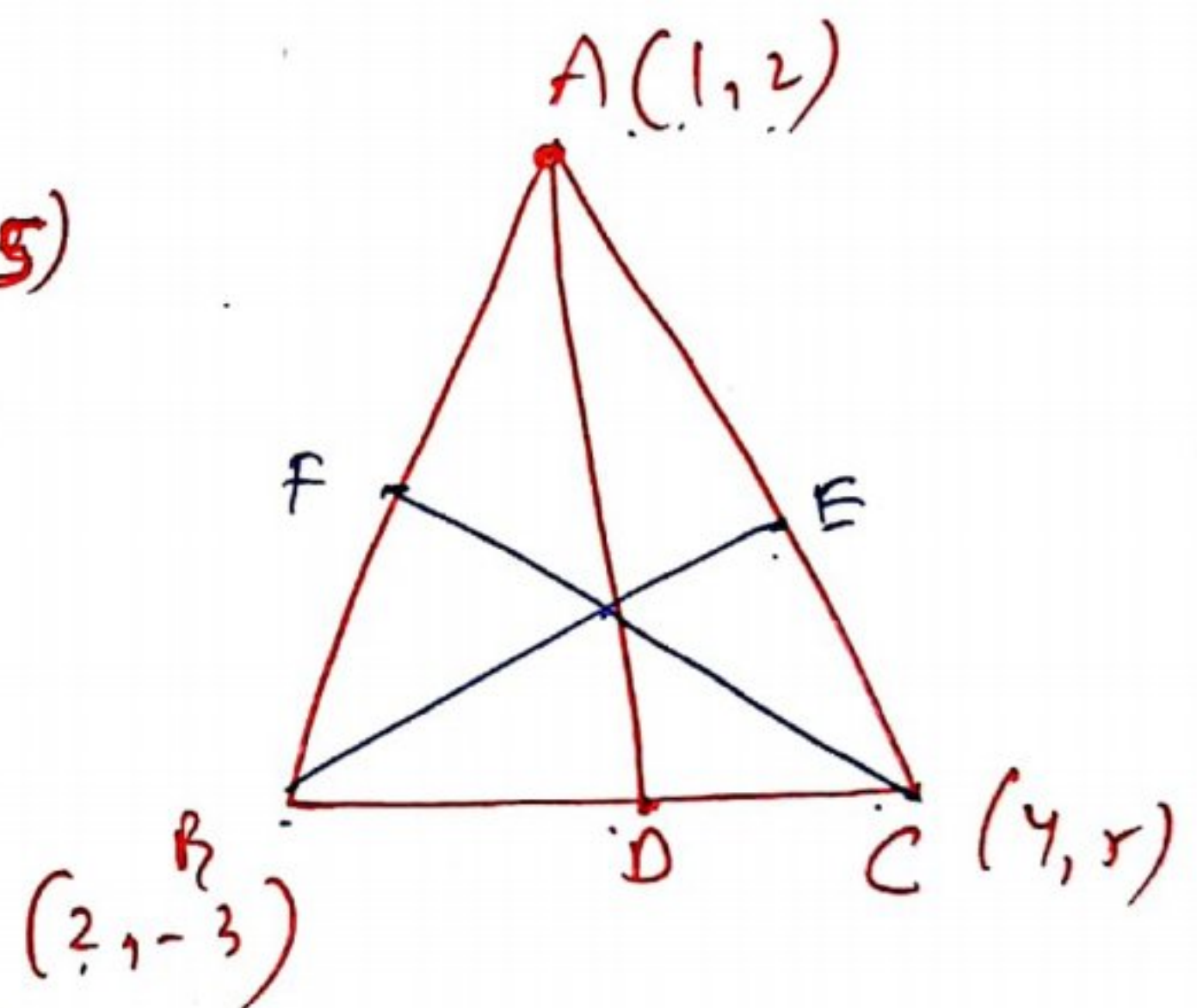
Find the equation of Medians

(•) D is the Mid point of BC

∴ $D(3, 1)$

(•) equation of AD (two point form)

$$y - 2 = \frac{-1}{2}(x-1)$$
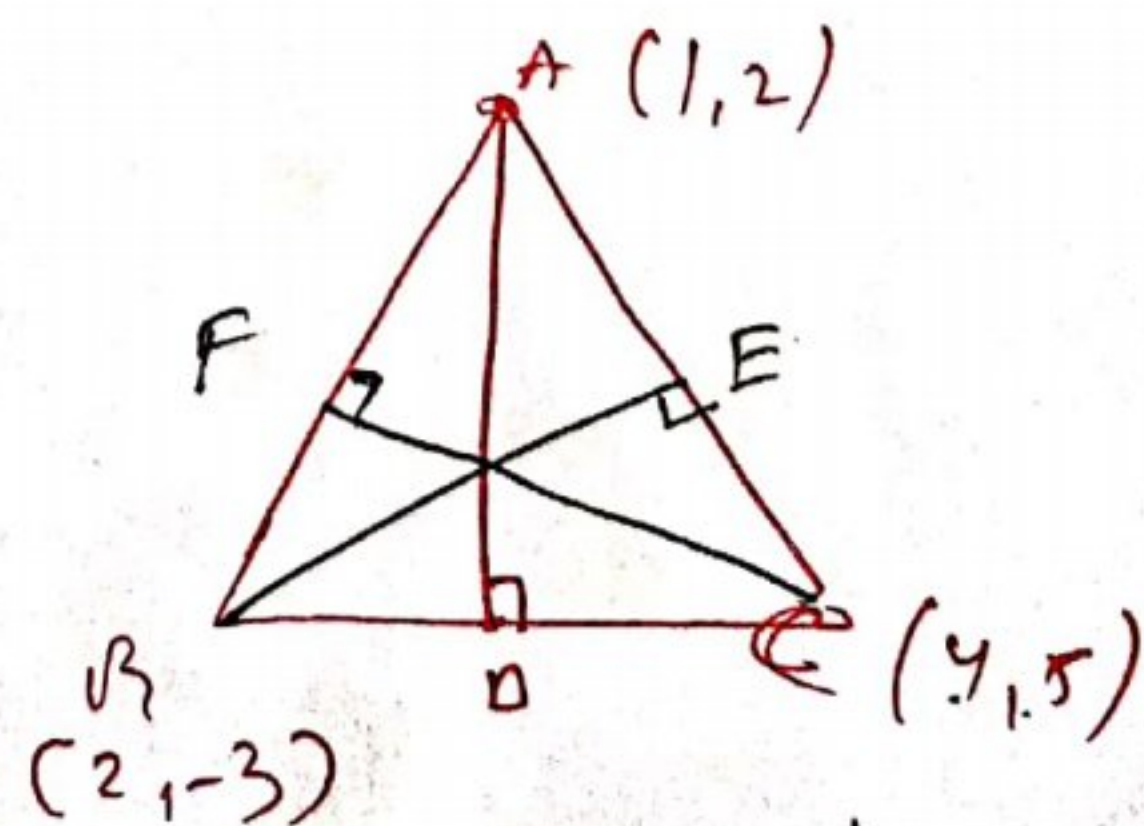$$\rightarrow 2y - 4 = -x + 1 \quad \Rightarrow \quad \boxed{x + 2y = 5} \quad \text{Ans}$$

A (1,2)

Q-6 **ALTITUDES**

(•) slope of BC = $\frac{8}{2} = 4$

(•) since AD ⊥ BC

∴ slope of AD = $-1/4$

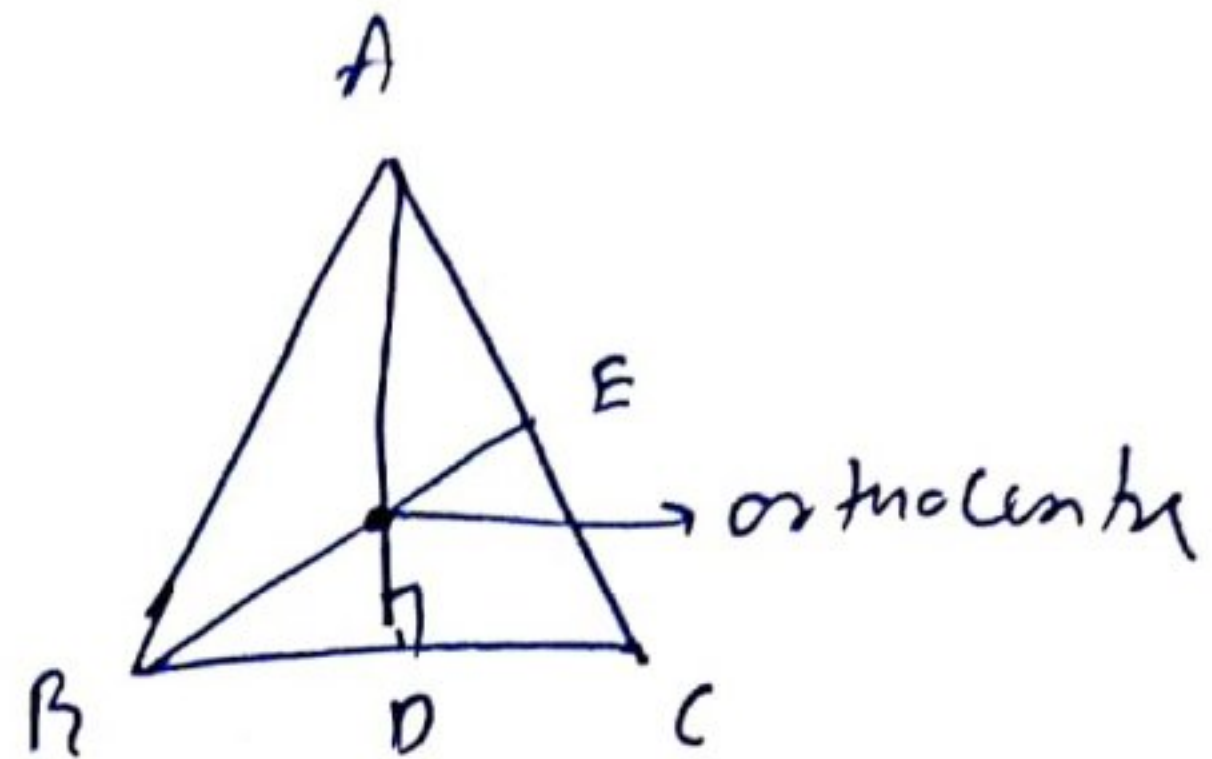$(2,-3)$

C (4,5)

(·) Equatio of AD (pent slope form)

$$y - 2 = \frac{-1}{4}(x-1)$$

$$=: \quad 4y - 8 = -x + 1$$

$$=: \quad \boxed{x + 4y = 9} \text{ Ans}$$

Q:7 | orthocentre |

→ Intersection point of altitudes

→ orthocentre

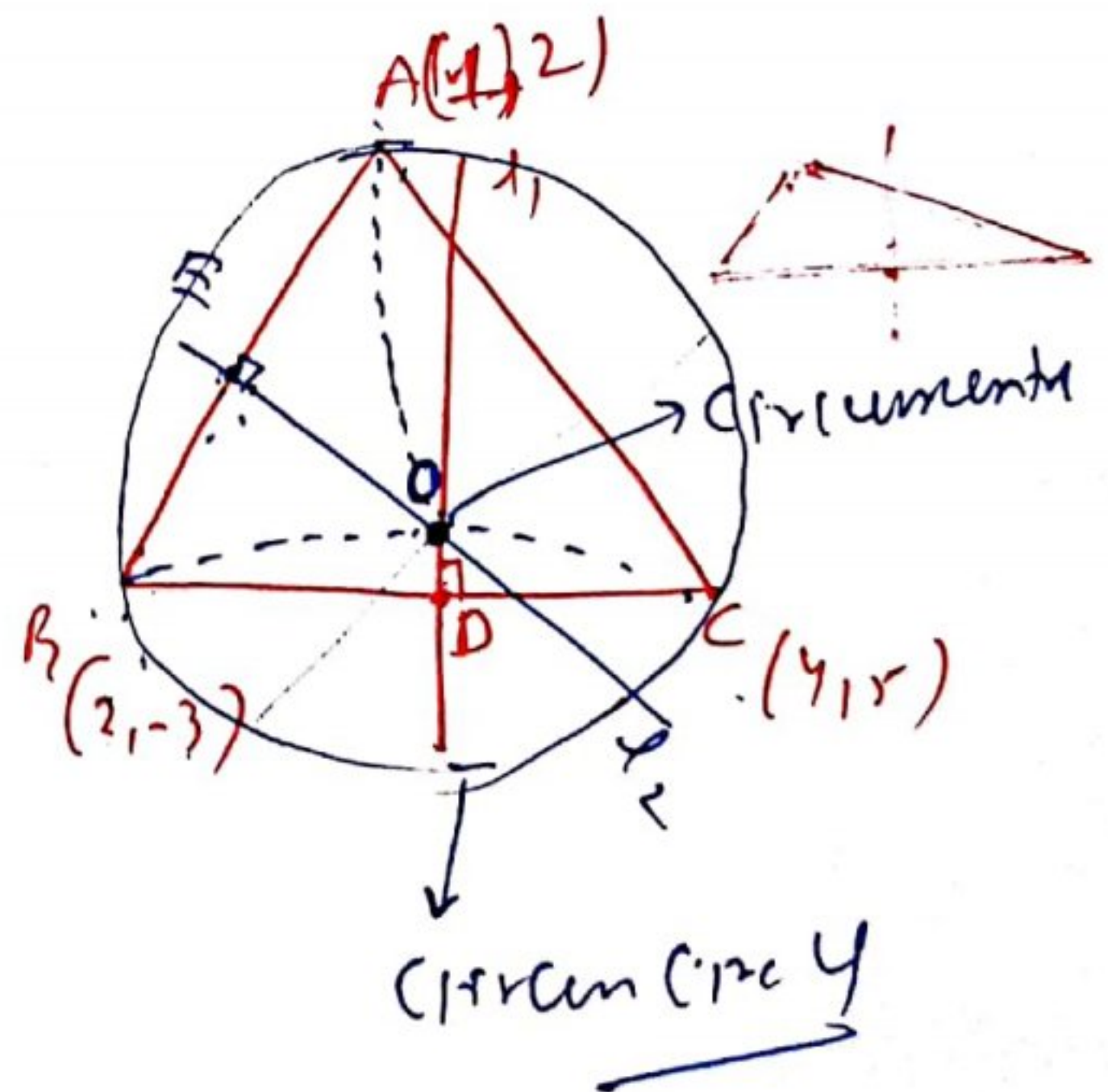✓ prepare any two altitudes (equation)

✓ solu then two equation & get $x =$
$y =$

✓ (x, y) orthocentre

Q:8 | Circumcentre |

→ Is the Intersection point of $\perp^r$ bisectors of the sides

→ circumcentre

circum circle

(·) slope of $BC = \frac{8}{2} = 4$

(·) $l_1 \perp BC$

∴ slope of $l_1 = \frac{-1}{4}$

(·) Mid point of BC is $D. (3,1)$

(·) equal of l_1 (By pent slope form)

⇒ $\quad y - 1 = -\frac{1}{4}(x - 3)$

⇒ $\quad 4y - 4 = -x + 3$

⇒ $\quad \boxed{x + 4y = 7}$ --- ①

Similarly find equation of l_2

solve equation of l_1 & l_2 & get Circum centre <u>Ans</u>

ON: 9 → Find the equation of line passing through the point $(2, 2)$ and cutting off intercepts on the <u>axes</u> whose sum is 9.

Soln) given $\boxed{a + b = 9}$

let equation of <u>line</u> (Intercept form)

$$\frac{x}{a} + \frac{y}{b} = 1$$

⇒ $\quad \dfrac{x}{a} + \dfrac{y}{9-a} = 1$

$(2, 2)$ lies on this line

⇒ $\quad \dfrac{2}{a} + \dfrac{2}{9-a} = 1$

⇒ $\quad 18 - 2a + 2a = 9a - a^2$

⇒ $\quad a^2 - 9a + 18 = 0$

⇒ $\quad (a-3)(a-6) = 0$

⇒ $\quad a = 3 \quad | \quad a = 6$
$\qquad b = 6 \quad | \quad b = 3$

equations of line

$\dfrac{x}{3} + \dfrac{y}{6} = 1$ <u>An</u>

(or) $\dfrac{x}{6} + \dfrac{y}{3} = 1$ <u>Ans</u>

Que. 10 → Find the point on the X-axis, where distances from the line $\frac{x}{3} + \frac{y}{4} = 1$ are 4 units?

Sol:-

(·) let the point on X-axis be $(x, 0)$

(·) equation of line $\frac{x}{3} + \frac{y}{4} = 1$

$$\Rightarrow 4x + 3y - 12 = 0$$

(·) distance = 4

$(x, 0)$

4

$4x - 3y - 12 = 0$

(·) $4 = \left| \dfrac{4x - 12}{\sqrt{16 + 9}} \right|$

$\Rightarrow 20 = |4x - 12|$

$\Rightarrow 4x - 12 = \pm 20$

$4x - 12 = 20$	$4x - 12 = -20$
$4x = 32$	$4x = -8$
$x = 8$	$x = -2$

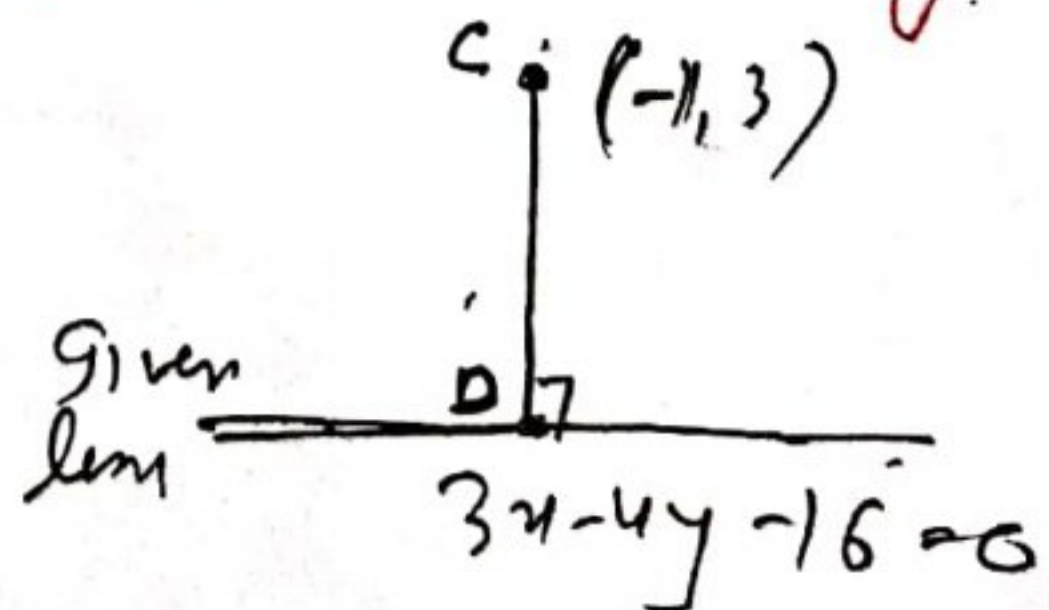∴ Required points are $(8, 0)$ or $(-2, 0)$ **Ans**
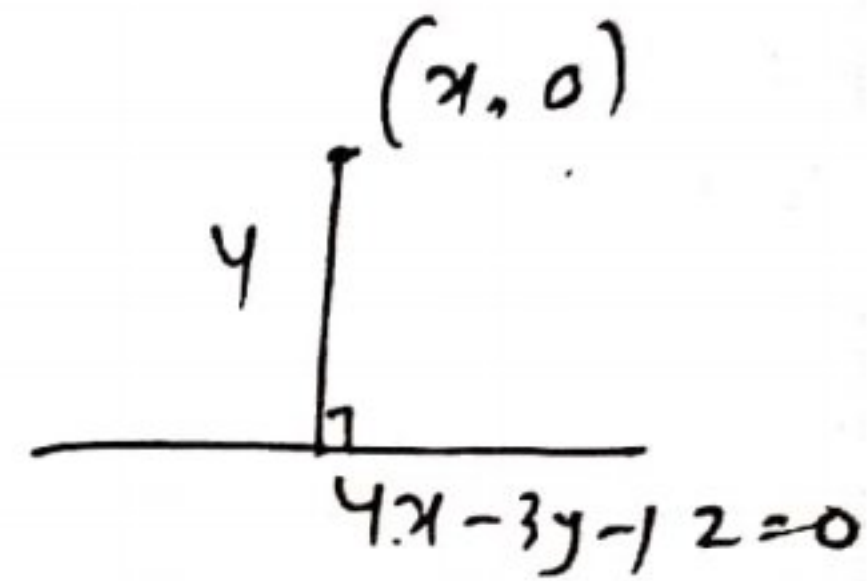
Qu. 11 → Find the coordinates of the foot of perpendicular from the point $(-1, 3)$ to the line $3x - 4y - 16 = 0$

Soln

(·) Slope of given line

$$= \frac{-3}{-4} = 3/4$$

$C \bullet (-1, 3)$

given line

D

$3x - 4y - 16 = 0$

(·) CD ⊥ given line

∴ slope of CD $= -4/3$

(·) equation of CD (point slope form)

$\Rightarrow$ $y - 3 = -\frac{4}{3}(x+1)$

$\rightarrow$ $3y - 9 = -4x - 4$

$\rightarrow$ $4x + 3y = 5$ $\cdots$ ②

equal y given line $3x - 4y = 16$ $\cdots$ ①

Soln (1) & (2) (ref)

$x =$

$y =$

$\therefore D(\quad)$ __Ans__

__Ans 12__ $\rightarrow$ A person standing at the junction of two straight paths represented by the equations $2x - 3y + 4 = 0$ and $3x + 4y - 5 = 0$ wants to reach the path whose equation is $6x - 7y + 8 = 0$ in the least time. find equation of the path that he should follow.
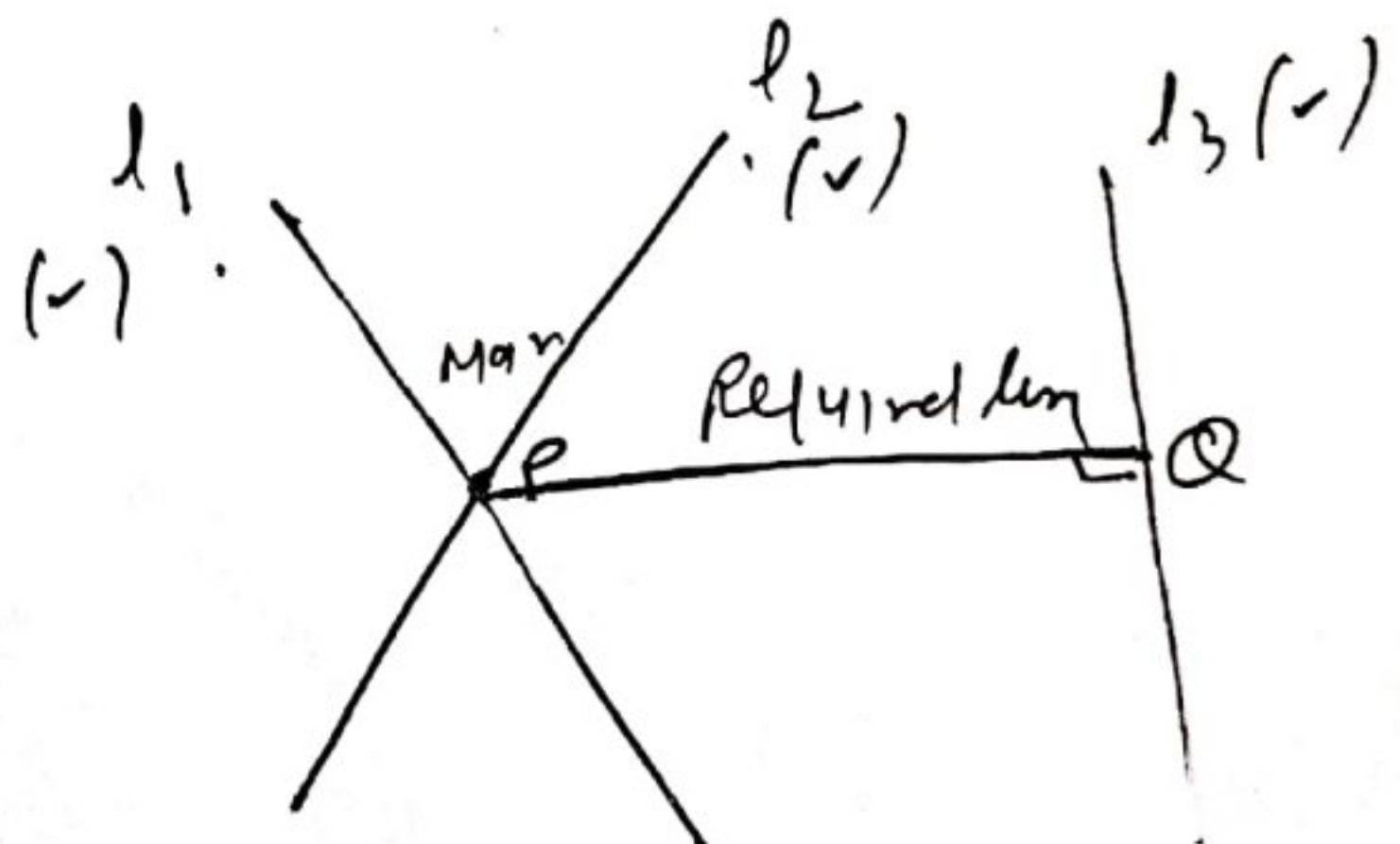
Soln
=

✓ Soln $l_1 \& l_2$

  get point $P(\sim)$

✓ Slope l_3

  $PQ \perp l_3$

✓ Slope PQ (-ve reciprocal)

✓ eq. of PQ (By point slope form)

l_1 (-) l_2 (✓) l_3 (-)

Man

P Required line Q

Qu.13 Find the equation of the lines through the Point (3,2) which make an angle of $45°$ with the line $x - 2y = 3$

Sol aple b/w two two line : $\theta = 45°$

Slope of given line $(m_1) = \dfrac{-1}{-2} = \dfrac{1}{2}$

let slope of required line $= m$

$$\tan(45) = \left| \dfrac{\frac{1}{2} - m}{1 + \frac{m}{2}} \right|$$

$$1 = \left| \dfrac{1 - 2m}{2 + m} \right|$$

$\Rightarrow \dfrac{1 - 2m}{2 + m} = 1$ | $\dfrac{1 - 2m}{2 + m} = -1$

$\Rightarrow 1 - 2m = 2 + m$ | $1 - 2m = -2 - m$

$\Rightarrow 3m = -1$ | $m = 3$

$\boxed{m = -\tfrac{1}{3}}$ (or) $\boxed{m = 3}$

Equation of lines $y - 2 = -\tfrac{1}{3}(x - 3)$ $\Big\}$ two Ans

also $y - 2 = 3(x - 3)$

WORKSHEET NO: 1
(STRAIGHT LINES)

Que: 1 Without using pythagoras theorem, show that the points $(4,4)$ $(3,5)$ & $(-1,-1)$ are the vertices of a right angled triangle.

Que:2 → Find the value of x for which the points $(x,-1)$, $(2,1)$ & $(4,5)$ are collinear. **ANS** $x=1$

Que 3 → The slope of a line is double the slope of another line. If tangent of the angle between them is $1/3$. Find the slopes of the lines

Ans 1 and 2 (or) $\frac{1}{2}$ and 1 (or) -1 & -2 (or) $-\frac{1}{2}$ & -1

Que 4 → Find the equation of a line that cuts off equal intercepts on the coordinate axes and passes through the point $(2,3)$ **ANS** $x+y=5$

Que 5 → The perpendicular from the origin to a line meets it at the point $(-2,9)$. Find the equation of the line **Ans** $2x-9y+85=0$

Que: 6 → The vertices of a $\triangle PQR$ are $P(2,1)$ $Q(-2,3)$ & $R(4,5)$. Find the equation of the median through the vertex R **ANS** $3x-4y+8=0$

Que 7 → Find the equation of the line passing through $(-3,5)$ and perpendicular to the line through the

Points $(2,5)$ and $(-3,6)$ ANS $5x - y + 200 = 0$

Qn.8 → Find the distance of the point $(-1,1)$ from the line $12(x+6) = 5(y-2)$ Ans 5 units

Qn. 9 → In the $\triangle ABC$ with vertices $A(2,3)$ $B(4,-1)$ & $C(1,2)$. Find the equation and length of altitude from the vertex A   Ans $x - y + 1 = 0$
and length $= \sqrt{2}$

Qn-10 → The line through the points $(h,3)$ and $(4,1)$ intersects the line $7x - 9y - 19 = 0$ at right angle. Find the value of h Ans $h = \dfrac{22}{9}$

Qn 11 → what are the points on Y-axis whose distance from the line $\dfrac{x}{3} + \dfrac{y}{4} = 1$ is 4 units
          ANS $\left(0, -\dfrac{8}{3}\right)$, $\left(0, \dfrac{32}{3}\right)$

Qn 12 → Find the equation of the lines which cut off intercepts on the axes whose sum and product are 1 and -6 respectively   ANS $2x - 3y = 6$;
$3x - 2y = -6$

Qn. 13 → Find the values of k for which the line
$(k-3)x - (4-k^2)y + (k^2 - 7k + 6) = 0$ is
(a) parallel to x-axis   ANS 3
(b) parallel to Y-axis   An ± 2
(c) passing through the origin   An 6 or 1

Qn. 14 Find orthocentre of $\triangle ABC$ $A(10,4)$ $B(-4,9)$ $C(-2,-1)$ ANS $\left(-1, \dfrac{9}{5}\right)$