Solutions of WORKSHEET No: 2

Chapter: COMPLEX NUMBERS

Qn 1 → given $(x-iy)(3+5i) = \overline{-6-24i}$

$\Rightarrow 3x + 5ix - 3iy - 5i^2y = -6+24i$

$\Rightarrow (3x+5y) + i(5x-3y) = -6+24i$

equating Real & Imaginary parts

$3x+5y = -6$ & $5x-3y = 24$

Solving these equations

$15x + 25y = -30$

$-(15x - 9y) = -(72)$

$34y = -102$

$\boxed{y=-3}$ put in eq (1)

we get $x=3$ $\therefore \boxed{x=3 \ \& \ y=3}$ Ans

Qn 2 → given $(a+ib)(c+id)(e+if)(g+ih) = A+iB$ ---(i)

taking conjugate on both sides

$(a-ib)(c-id)(e-if)(g-ih) = A-iB$ --(2)

(1) × (2)

$(a+ib)(a-ib)(c+id)(c-id)(e+if)(e-if)(g+ih)(g-ih) = (A+iB)(A-Bi)$

$\Rightarrow (a^2-i^2b^2)(c^2-i^2d^2)(e^2-i^2f^2)(g^2-i^2h^2) = A^2-i^2B^2$

$\Rightarrow (a^2+b^2)(c^2+d^2)(e^2+f^2)(g^2+h^2) = A^2+B^2$

Ans

Solution Complex No. (W-S 2)

Ans 3 → given $x - iy = \sqrt{\frac{a-ib}{c-id}}$ --- (1)

taking conjugate on both sides

$$x + iy = \sqrt{\frac{a+ib}{c+id}} \quad \text{--- (2)}$$

(1) × (2)

$$(x-iy)(x+iy) = \sqrt{\frac{a-ib}{c-id} \times \frac{a+ib}{c+id}}$$

$$\Rightarrow x^2 - i^2y^2 = \sqrt{\frac{a^2-i^2b^2}{c^2-i^2d^2}}$$

$$\Rightarrow x^2 + y^2 = \sqrt{\frac{a^2+b^2}{c^2+d^2}}$$

squaring both sides

$$(x^2+y^2)^2 = \frac{a^2+b^2}{c^2+d^2} \quad \text{Ans}$$

Ans 4 → given $\left(\frac{1+i}{1-i}\right)^3 - \left(\frac{1-i}{1+i}\right)^3 = x + iy$

$$\Rightarrow \left(\frac{1+i}{1-i} \times \frac{1+i}{1+i}\right)^3 - \left(\frac{1-i}{1+i} \times \frac{1-i}{1-i}\right)^3 = x + iy$$

$$\Rightarrow \left(\frac{(1+i)^2}{1-i^2}\right)^3 - \left(\frac{(1-i)^2}{1-i^2}\right)^3 = x + iy$$

$$\Rightarrow \left(\frac{\cancel{1}+\cancel{i^2}+2i}{1+1}\right)^3 - \left(\frac{\cancel{1}+\cancel{i^2}-2i}{1+1}\right)^3 = x + iy$$

$$\Rightarrow (i)^3 - (-i)^3 = x + iy$$

$$\Rightarrow i^3 + i^3 = x + iy$$

$$\Rightarrow -i - i = x + iy$$

$$\Rightarrow 0 - 2i = x + iy$$

$$\Rightarrow x = 0 \;\&\; y = -2 \quad \therefore (0, -2) \quad \text{Ans}$$

Qn. 5 → given $\left(\frac{1-i}{1+i}\right)^{100} = a+ib$

$\Rightarrow \left(\frac{1-i}{1+i} \times \frac{1-i}{1-i}\right)^{100} = a+ib$

$\Rightarrow \left(\frac{\cancel{1}+\cancel{i^2}-2i}{1-i^2}\right)^{100} = (a+ib)$

$\Rightarrow \left(\frac{-2i}{2}\right)^{100} = a+ib$

$\Rightarrow (-i)^{100} = a+ib$

$\Rightarrow i^{100} = a+ib$

$\Rightarrow (i^4)^{25} = a+ib$

$\Rightarrow 1 = a+ib$

$\therefore a = 1$ & $b = 0$ $(1, 0)$ Ans

Qn 6 → given $|z| = z+1+2i$

let $z = x+iy$

$\sqrt{x^2+y^2} = x+iy+1+2i$

$\Rightarrow \sqrt{x^2+y^2} = (x+1)+i(y+2)$

$\Rightarrow \sqrt{x^2+y^2}+0i = (x+1)+i(y+2)$

Equating Real & Imaginary parts

$x+1 = \sqrt{x^2+y^2}$ & $y+2 = 0$

$\Rightarrow y = -2$

$x+1 = \sqrt{x^2+4}$

Squaring

$\cancel{x^2}+1+2x = \cancel{x^2}+4$

$2x = 3 \Rightarrow x = 3/2$

$\therefore z = x+iy$

$z = \frac{3}{2} - 2i$ Ans

Ques 7

$$\frac{i^{4n+1} - i^{4n-1}}{2}$$

$$= \frac{i^{4n} \cdot i - i^{4n} \cdot i^{-1}}{2}$$

$$= \frac{i - \frac{1}{i}}{2} \quad \cdots \left\{ \because i^{4n} = (i^4)^n = (1)^n = 1 \right\}$$

$$= \frac{i + i}{2} \quad \cdots \left\{ \because \frac{1}{i} = -i \right\}$$

$$= \frac{2i}{2}$$

$$= i$$

Ans

Ans 8 → given $|z_1| = |z_2| = |z_3| = 1$

and $\left| \frac{1}{z_1} + \frac{1}{z_2} + \frac{1}{z_3} \right| = 1$

$$\Rightarrow \left| \frac{\bar{z}_1}{z_1 \bar{z}_1} + \frac{\bar{z}_2}{z_2 \bar{z}_2} + \frac{\bar{z}_3}{z_3 \bar{z}_3} \right| = 1$$

$$\Rightarrow \left| \frac{\bar{z}_1}{|z_1|^2} + \frac{\bar{z}_2}{|z_2|^2} + \frac{\bar{z}_3}{|z_3|^2} \right| = 1 \quad \cdots \left\{ \because z\bar{z} = |z|^2 \right\}$$

$$\Rightarrow \left| \bar{z}_1 + \bar{z}_2 + \bar{z}_3 \right| = 1 \quad \cdots \left\{ \because |z_1| = |z_2| = |z_3| = 1 \right\}$$

$$\Rightarrow \left| \overline{z_1 + z_2 + z_3} \right| = 1 \quad \cdots \left\{ \because \bar{z}_1 + \bar{z}_2 = \overline{z_1 + z_2} \right\}$$

$$\Rightarrow \left| z_1 + z_2 + z_3 \right| = 1 \quad \cdots \left\{ \because |\bar{z}| = |z| \right\}$$

Ans