← ULTIMATE MATHEMATICS →

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TRIGO: CLASS-5 (TT-5)

SET-5

(1) $\sin(2\theta) = 2\sin\theta\cdot\cos\theta$

✓ Proof $\sin(2\theta) = \sin(\overset{A}{\theta}+\overset{B}{\theta})$

$= \sin\theta\cdot\cos\theta + \cos\theta\sin\theta$

$= 2\sin\theta\cos\theta$

(2) $\cos(2\theta) = \cos^2\theta - \sin^2\theta$

$\cos(2\theta) = 2\cos^2\theta - 1$

$\cos(2\theta) = 1 - 2\sin^2\theta$

✓ Proof $\cos(2\theta) = \cos(\overset{A}{\theta}+\overset{B}{\theta})$

$= \cos\theta\cos\theta - \sin\theta\sin\theta$

$= \cos^2\theta - \sin^2\theta$

(3) $\tan(2\theta) = \dfrac{2\tan\theta}{1-\tan^2\theta}$

Proof $\tan(2\theta) = \tan(\theta+\theta)$

$= \dfrac{\tan\theta + \tan\theta}{1-\tan\theta\tan\theta} = \dfrac{2\tan\theta}{1-\tan^2\theta}$

(4) $\sin(3\theta) = 3\sin\theta - 4\sin^3\theta$

(5) $\cos(3\theta) = 4\cos^3\theta - 3\cos\theta$

(6) $\tan(3\theta) = \dfrac{3\tan\theta - \tan^3\theta}{1-3\tan^2\theta}$

(7) Main ✓ Imp

$1-\cos(2\theta) = 2\sin^2\theta$

$1+\cos(2\theta) = 2\cos^2(\theta)$

eg $1-\cos x = 2\sin^2\left(\frac{x}{2}\right)$

$1+\cos(4x) = 2\cos^2(2x)$

(8) $\sin^2\theta = \dfrac{1-\cos(2\theta)}{2}$

$\cos^2\theta = \dfrac{1+\cos(2\theta)}{2}$

Reason

$\sin^2\theta = \dfrac{1-\cos(2\theta)}{2}$ ($\checkmark$)

$\to 1-\cos^2\theta$ ($\times$)

Trigno set - 5
(T-5)

(9) $\sin(2\theta) = \dfrac{2\tan\theta}{1+\tan^2\theta}$

$\cos(2\theta) = \dfrac{1-\tan^2\theta}{1+\tan^2\theta}$

In the terms of $\tan\theta$

— x —

QUESTIONS:

Ans 1 → Show that $\cos^2(A) + \cos^2(A+\frac{\pi}{3}) + \cos^2(A-\frac{\pi}{3}) = \frac{3}{2}$

Sol: LHS $\cos^2 A + \cos^2(A+60°) + \cos^2(A-60°)$

$$\boxed{\cos^2\theta = \frac{1+\cos(2\theta)}{2}}$$

$$= \frac{1+\cos(2A)}{2} + \frac{1+\cos(2A+120°)}{2} + \frac{1+\cos(2A-120°)}{2}$$

$$= \frac{1}{2}\left[1+\cos(2A) + 1+\cos(2A+120°) + 1+\cos(2A-120°)\right]$$

$$= \frac{1}{2}\left[3 + \cos(2A) + \cos(2A+120°) + \cos(2A-120°)\right]$$

A B

$$= \frac{1}{2}\left[3 + \cos(2A) + 2\cos(2A)\cdot\cos(120°)\right]$$

$$= \frac{1}{2}\left[3 + \cos(2A) + 2\cos(2A)\cos(\underset{\text{II}}{180°-60°})\right]$$

$$= \frac{1}{2}\left[3 + \cos(2A) + \cancel{2}\cos(2A)\cdot\left(-\frac{1}{\cancel{2}}\right)\right]$$

$$= \frac{1}{2}\left[3 + \cancel{\cos(2A)} - \cancel{\cos(2A)}\right]$$

$$= 3/2 = \text{RHS}$$

Ans

— ULTIMATE MATHEMATICS →

(T-5)

Qns 2 → (SELF) Show $\cos^2 A + \cos^2\left(A + \frac{2\pi}{3}\right) + \cos^2\left(A - \frac{2\pi}{3}\right) = 3/2$

Qns 3 → Show that $\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+2\cos(16\theta)}}}} = 2\cos\theta$

Lhs

$$\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2(1+\cos(16\theta))}}}}$$

$$= \sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2\times 2\cos^2 8\theta}}}} \quad \text{---} \left\{ \begin{array}{l} 1+\cos(2\theta) \\ = 2\cos^2\theta \end{array} \right\}$$

$$= \sqrt{2+\sqrt{2+\sqrt{2+2\cos(8\theta)}}}$$

$$= \sqrt{2+\sqrt{2+\sqrt{2(1+\cos(8\theta))}}}$$

$$= \sqrt{2+\sqrt{2+\sqrt{2\times 2\cos^2(4\theta)}}}$$

$$= \sqrt{2+\sqrt{2+2\cos(4\theta)}}$$

$$= \sqrt{2+\sqrt{2(1+\cos(4\theta))}}$$

$$= \sqrt{2+\sqrt{2\times 2\cos^2(2\theta)}}$$

$$= \sqrt{2+2\cos(2\theta)}$$

$$= \sqrt{2(1+\cos(2\theta))} = \sqrt{2\times 2\cos^2\theta} = 2\cos\theta$$

Ans

Q.N 4 → Show that $\sqrt{2+\sqrt{2+\sqrt{2+2\cos(8\theta)}}} = 2\cos\theta$

(SELF)

Q.N 5 → Show that $\sin^4\left(\frac{\pi}{8}\right) + \sin^4\left(\frac{3\pi}{8}\right) + \sin^4\left(\frac{5\pi}{8}\right) + \sin^4\left(\frac{7\pi}{8}\right) = \frac{3}{2}$

link

link

Ans

$= \sin^4(\pi/8) + \sin^4\left(\frac{3\pi}{8}\right) + \sin^4\left(\overset{\pi}{\pi} - \frac{3\pi}{8}\right) + \sin^4\left(\pi - \frac{\pi}{8}\right)$

$= \sin^4\left(\frac{\pi}{8}\right) + \sin^4\left(\frac{3\pi}{8}\right) + \sin^4\left(\frac{3\pi}{8}\right) + \sin^4\left(\frac{\pi}{8}\right)$

$= 2\left[\sin^4\left(\frac{\pi}{8}\right) + \sin^4\left(\frac{3\pi}{8}\right)\right]$

$= 2\left[\left(\sin^2\left(\frac{\pi}{8}\right)\right)^2 + \left(\sin^2\left(\frac{3\pi}{8}\right)\right)^2\right]$

$= 2\left[\left(\frac{1-\cos\left(\frac{\pi}{4}\right)}{2}\right)^2 + \left(\frac{1-\cos\left(\overset{135^\circ}{\frac{3\pi}{4}}\right)}{2}\right)^2\right]$

$= 2\left[\left(\frac{1-\frac{1}{\sqrt{2}}}{2}\right)^2 + \left(\frac{1-\cos\left(\overset{\pi}{180}-45\right)}{2}\right)^2\right]$

$= 2\left[\frac{\left(1-\frac{1}{\sqrt{2}}\right)^2}{2} + \left(\frac{1+\frac{1}{\sqrt{2}}}{2}\right)^2\right]$

$= 2\left[\frac{1+\frac{1}{2}-\cancel{\frac{2}{\sqrt{2}}}}{4} + \frac{1+\frac{1}{2}+\cancel{\frac{2}{\sqrt{2}}}}{4}\right]$

$= 2\left[\frac{3}{4}\right]$

$= 3/2$ Ans

Q.N 6 Show $\cos^4(\pi/8) + \cos^4(3\pi/8) + \cos^4\left(\frac{5\pi}{8}\right) + \cos^4\left(\frac{7\pi}{8}\right) = 3/2$

SELF