ULTIMATE MATHEMATICS

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CHAPTER : PRINCIPLE OF MATHEMATICAL INDUCTION (PMI)

CLASS NO: 1

(·) $1+2+3+4 \cdots n = \frac{n(n+1)}{2}$ $n \in N$

let $P(n)$ be the statement given by

- ✓ to prove $P(1)$ is true
- ✓ let $P(k)$ be true
- ✓ to prove $P(k+1)$ is true (Main Step)

then Principle of Mathematical Induction, given statement $P(n)$ is true for all value of $n \in N$:

$\boxed{\text{TYPE SERIES}}$

Ques 1 Using PMI show that

$1\cdot2 + 2\cdot3 + 3\cdot4 + \cdots n(n+1) = \frac{n(n+1)(n+2)}{3}$

for all $n \in N$

Sol. let $P(n)$ be the statement given by

$P(n)$: $1\cdot2 + 2\cdot3 + 3\cdot4 + \cdots n(n+1) = \frac{n(n+1)(n+2)}{3}$

$P(1)$: $1\cdot2 = \frac{1(1+1)(1+2)}{3}$

$2 = 2$

Clearly $P(1)$ is true

let $P(k)$ be true

$P(k)$: $1\cdot2 + 2\cdot3 + 3\cdot4 + \cdots k(k+1) = \frac{k(k+1)(k+2)}{3}$

To prove $P(k+1)$ is true.

$P(k+1)$: $1\cdot2 + 2\cdot3 + 3\cdot4 + \dots\; k(k+1) + (k+1)\cdot(k+2) = \dfrac{(k+1)(k+2)(k+3)}{3}$

Taking LHS

$$\boxed{1\cdot2 + 2\cdot3 + 3\cdot4 + \dots\; k(k+1)} + (k+1)(k+2)$$

$$= \frac{k(k+1)(k+2)}{3} + (k+1)(k+2) \quad \text{--- \{from } p(k)\text{\}}$$

$$= (k+1)(k+2)\left[\frac{k}{3} + 1\right]$$

$$= \frac{(k+1)(k+2)(k+3)}{3} = \text{RHS}$$

$\therefore$ $p(k+1)$ is true

$\therefore$ By PMI, $P(n)$ is true for all values of $n \in N$ Ans.

Q.No. 2 Using PMI show that

$$\frac{1}{2\cdot5} + \frac{1}{5\cdot8} + \dots \dots \frac{1}{(3n-1)(3n+2)} = \frac{n}{6n+4}$$

Soln Let $P(n)$ be the statement given by

$$P(n): \frac{1}{2\cdot5} + \frac{1}{5\cdot8} + \dots \dots \frac{1}{(3n-1)(3n+2)} = \frac{n}{6n+4}$$

$$P(1): \frac{1}{2\cdot5} = \frac{1}{6+4}$$

$\frac{1}{10} = \frac{1}{10}$ clearly $P(1)$ is true

Let $p(k)$ be true

$$P(k): \frac{1}{2\cdot5} + \frac{1}{5\cdot8} + \dots \frac{1}{(3k-1)(3k+2)} = \frac{k}{6k+4}$$

To prove $P(k+1)$ is true

PMF class No = 1 $\boxed{3(k+1)-1}$ (3)

$P(k+1)$: $\frac{1}{2\cdot5}+\frac{1}{5\cdot8}+ \cdots \frac{1}{(3k-1)(3k+2)} + \frac{1}{(3k+2)(3k+5)}$

$$= \frac{k+1}{6k+10}$$

Take LHS

$$\boxed{\frac{1}{2\cdot5}+\frac{1}{5\cdot8}+ \cdots \frac{1}{(3k-1)(3k+2)}} + \frac{1}{(3k+2)(3k+5)}$$

$$= \frac{k}{6k+4} + \frac{1}{(3k+2)(3k+5)}$$

$$= \frac{k}{2(3k+2)} + \frac{1}{(3k+2)(3k+5)}$$

$$= \frac{3k^2+5k+2}{2(3k+2)(3k+5)}$$

$$= \frac{3k^2+3k+2k+2}{2(3k+2)(3k+5)}$$

$$= \frac{3k(k+1)+2(k+1)}{2(3k+2)(3k+5)}$$

$$= \frac{\cancel{(3k+2)}(k+1)}{2\cancel{(3k+2)}(3k+5)}$$

$$= \frac{k+1}{6k+10} = \text{RHS}$$

$\therefore$ $P(k+1)$ is true

By PMI, $P(n)$ is true for all values of $n \in N$. Ans

PMI Class No: 1 (4)

Ques: 3 Using PMI, show that

$$\frac{1}{1\cdot2\cdot3}+\frac{1}{2\cdot3\cdot4}+\cdots\cdots\frac{1}{n(n+1)(n+2)}=\frac{n(n+3)}{4(n+1)(n+2)}$$

Sol: let $P(n): \frac{1}{1\cdot2\cdot3}+\frac{1}{2\cdot3\cdot4}+\cdots\frac{1}{n(n+1)(n+2)}=\frac{n(n+3)}{4(n+1)(n+2)}$

$$P(1): \frac{1}{1\cdot2\cdot3}=\frac{1(1+3)}{4(1+1)(1+2)}$$

$$\frac{1}{6}=\frac{\cancel{4}}{\cancel{4}(2)(3)}=\frac{1}{6} \quad \therefore \quad P(1) \text{ is true}$$

let $P(k)$ be true

$$P(k): \frac{1}{1\cdot2\cdot3}+\frac{1}{2\cdot3\cdot4}+\cdots\frac{1}{k(k+1)(k+2)}=\frac{k(k+3)}{4(k+1)(k+2)}$$

To prove $P(k+1)$ is true

$$P(k+1): \frac{1}{1\cdot2\cdot3}+\frac{1}{2\cdot3\cdot4}+\cdots\frac{1}{k(k+1)(k+2)}+\frac{1}{(k+1)(k+2)(k+3)}=\frac{(k+1)(k+4)}{4(k+2)(k+3)}$$

LHS $\boxed{\frac{1}{1\cdot2\cdot3}+\frac{1}{2\cdot3\cdot4}+\cdots\frac{1}{k(k+1)(k+2)}}+\frac{1}{(k+1)(k+2)(k+3)}$

$$=\frac{k(k+3)}{4(k+1)(k+2)}+\frac{1}{(k+1)(k+2)(k+3)}$$

$$=\frac{k(k+3)^2+4}{4(k+1)(k+2)(k+3)}$$

$$=\frac{k^3+6k^2+9k+4}{4(k+1)(k+2)(k+3)}$$

Hit & Trial Method

k^3+6k^2+9k+4

$k=-1$

$-1+6-9+4=0$

factor $(k+1)$

Divide.

Scanned with CamScanner

$$
\begin{array}{r|l}
 & k^2 + 5k + 4 \\
\hline
k+1 & \cancel{k^3} + 6k^2 + 9k + 4 \\
 & -(\cancel{k^3} + k^2) \\
\hline
 & \cancel{5k^2} + 9k + 4 \\
 & -(\cancel{5k^2} + 5k) \\
\hline
 & 4k + 4 \\
 & 4k + 4 \\
\hline
 & \times
\end{array}
$$

$$= \frac{(k+1)(k^2+5k+4)}{4(k+1)(k+2)(k+3)}$$

$$= \frac{\cancel{(k+1)}(k+1)(k+4)}{4\cancel{(k+1)}(k+2)(k+3)}$$

$$= \frac{(k+1)(k+4)}{4(k+2)(k+3)} = \text{RHS}$$

$\therefore P(k+1)$ is true

$\therefore$ By PMI, $P(n)$ is true for all values of $n \in N$

Ans

Ques 4 → Show that (using PMI)

$$1\cdot3 + 2\cdot3^2 + 3\cdot3^3 + \cdots n\cdot3^n = \frac{(2n-1)\cdot3^{n+1}+3}{4}$$

Soln: let $P(n)$: $1\cdot3 + 2\cdot3^2 + 3\cdot3^3 + \cdots n\cdot3^n = \frac{(2n-1)\cdot3^{n+1}+3}{4}$

$P(1)$: $1\cdot3 = \frac{(2-1)\cdot3^2+3}{4}$

$3 = \frac{9+3}{4} = 3$

$\therefore P(1)$ is true.

PMI Class No: 1

$P(k): 1\cdot 3 + 2\cdot 3^2 + 3\cdot 3^3 + \cdots k\cdot 3^k = \frac{(2k-1)\cdot 3^{k+1}+3}{4}$

to $p(k+1)$ is true

$\frac{(2k+1)\cdot 3^{k+2}+3}{4}$

$P(k+1): 1\cdot 3 + 2\cdot 3^2 + 3\cdot 3^3 + \cdots k\cdot 3^k + (k+1)\cdot 3^{k+1} =$

$\overset{(n)}{=} \boxed{1\cdot 3 + 2\cdot 3^2 + 3\cdot 3^3 + \cdots k\cdot 3^k} + (k+1)\cdot 3^{k+1}$

$= \frac{(2k-1)\cdot 3^{k+1}+3}{4} + (k+1)\cdot 3^{k+1}$

$= \frac{(2k-1)\cdot 3^{k+1}+3 + (4k+4)\cdot 3^{k+1}}{4}$

$= \frac{3^{k+1}(2k-1+4k+4)+3}{4}$

$= \frac{3^{k+1}(6k+3)+3}{4}$

$= \frac{3^{k+1}\cdot 3(2k+1)+3}{4}$

$= \frac{3^{k+2}\cdot(2k+1)+3}{4} = RHS$

$\therefore$ $P(k+1)$ is true

$\therefore$ By PMI $P(n)$ is true for all values $n \in \mathbb{N}$

Q No. 5 Using PMI show that

$$\left(1+\frac{3}{1}\right)\cdot\left(1+\frac{5}{4}\right)\cdots\left(1+\frac{(2n+1)}{n^2}\right)=(n+1)^2$$

Sol: let $P(n)$: $\left(1+\frac{3}{1}\right)\left(1+\frac{5}{4}\right)\cdots\left(1+\frac{(2n+1)}{n^2}\right)=(n+1)^2$

$P(k)$: $\left(\left(1+\frac{3}{1}\right)\left(1+\frac{5}{4}\right)\cdots\left(1+\frac{(2k+1)}{k^2}\right)\right)=(k+1)^2$

$P(k+1)$: $\left(1+\frac{3}{1}\right)\left(1+\frac{5}{4}\right)\cdots\left(1+\frac{(2k+1)}{k^2}\right)\cdot\left(1+\frac{2k+3}{(k+1)^2}\right)=(k+2)^2$

LHS $\boxed{\left(1+\frac{3}{1}\right)\left(1+\frac{5}{4}\right)\cdots\left(1+\frac{2k+1}{k^2}\right)}\left(1+\frac{2k+3}{(k+1)^2}\right)$

$$=(k+1)^2\left(1+\frac{(2k+3)}{(k+1)^2}\right)$$

$$=\frac{(k+1)^2\left((k+1)^2+2k+3\right)}{(k+1)^2}$$

$$=\frac{\cancel{(k+1)^2}\left(k^2+1+2k+2k+3\right)}{\cancel{(k+1)^2}}$$

$$=k^2+4k+4$$

$$=(k+2)^2$$

$=$ RHS

$\therefore P(k+1)$ is true

$\therefore$ By PMI, $P(n)$ is true for all value of $n\in N$

PRINCIPLE OF MATHEMATICAL INDUCTION

← WORKSHEET No: 1 →

Using PMI show that

Qn 1 $1^2+2^2+3^2+\dots n^2 = \dfrac{n(n+1)(2n+1)}{6}$

Qn 2 $1^3+2^3+3^3+\dots n^3 = \left(\dfrac{n(n+1)}{2}\right)^2$

Qn 3 → $\dfrac{1}{3\cdot 5}+\dfrac{1}{5\cdot 7}+\dfrac{1}{7\cdot 9}+\dots \dfrac{1}{(2n+1)(2n+3)} = \dfrac{n}{3(2n+3)}$

Qn 4 → $\dfrac{1}{1\cdot 4}+\dfrac{1}{4\cdot 7}+\dots \dfrac{1}{(3n-2)(3n+1)} = \dfrac{n}{3n+1}$

Qn 5 → $\left(1+\frac{1}{1}\right)\left(1+\frac{1}{2}\right)\left(1+\frac{1}{3}\right)\dots\left(1+\frac{1}{n}\right) = (n+1)$

Qn 6 → $1\cdot 3+3\cdot 5+\dots (2n-1)(2n+1) = \dfrac{n(4n^2+6n-1)}{3}$

Qn 7 → $1\cdot 2+2\cdot 2^2+3\cdot 2^2+\dots n\cdot 2^n = (n-1)\cdot 2^{n+1}+2$

Qn 8 → $1\cdot 2\cdot 3+2\cdot 3\cdot 4+\dots n(n+1)(n+2) = \dfrac{n(n+1)(n+2)(n+3)}{4}$

Qn 9 → $a+ar+ar^2+\dots ar^{n-1} = \dfrac{a(r^n-1)}{r-1}$

Qn 10 → $\dfrac{1}{2}+\dfrac{1}{4}+\dfrac{1}{8}+\dots \dfrac{1}{2^n} = 1-\dfrac{1}{2^n}$

Qn 11 → $1+3+3^2+\dots 3^{n-1} = \dfrac{3^n-1}{2}$

Qn 12 → Show that

$$\left(1-\frac{1}{2^2}\right)\left(1-\frac{1}{3^2}\right)\left(1-\frac{1}{4^2}\right)\dots\left(1-\frac{1}{n^2}\right) = \frac{n+1}{2n}$$

for all natural numbers $n \geq 2$

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