!! जय श्री राधे कृष्णा जय श्री विश्वेश्वर श्री महाराज !!

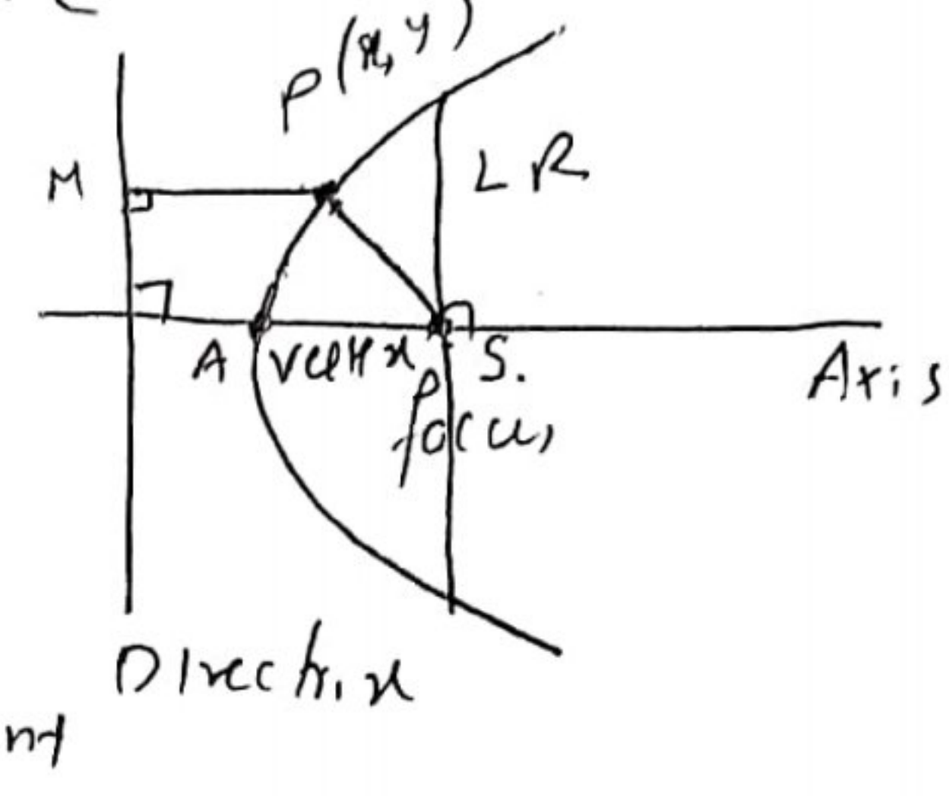
ULTIMATE MATHEMATICS: BY AJAY MITTAL

CHAPTER: CONIC SECTION | CLASS NO: 2

(*) focus → a fixed point

(*) Directrix: is a fixed st-line

(*) Axis: is a line passing through focus & ⊥r to directrix



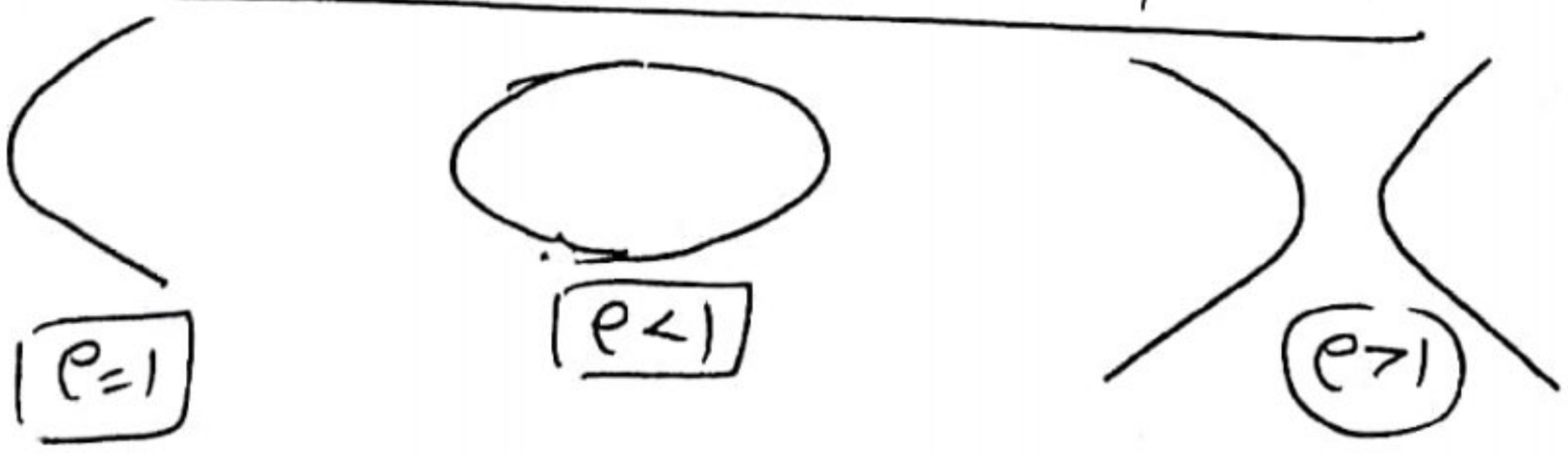
(*) vertex: is an Intersection pont of conic section with its Axis

(*) latusrectum: is a chord passing thru focus & ⊥r to ~~directrix~~ Axis

(*) Eccentricity (e): is a Constant Ratio

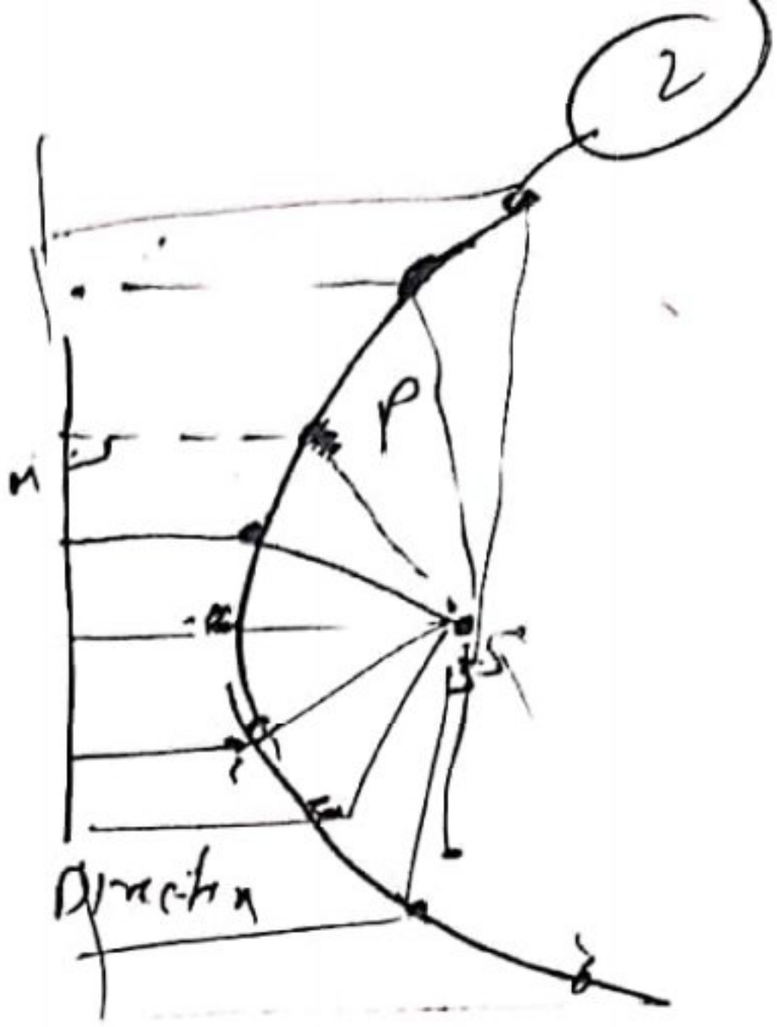
$$e = \frac{SP}{PM}$$

Parabola ; ellipse & hyperbola



PARABOLA

(·) $\boxed{e=1}$ $\quad e = \frac{SP}{PM} \rightarrow \boxed{SP = PM}$



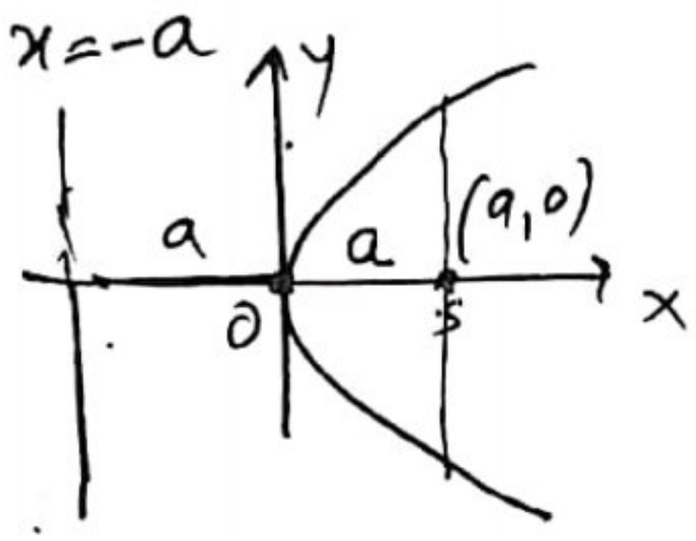
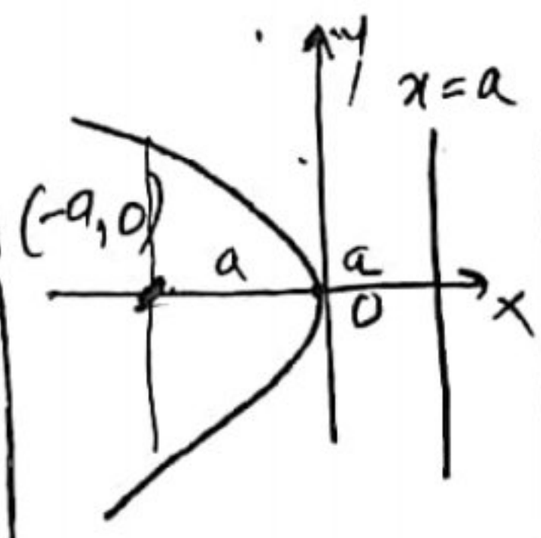
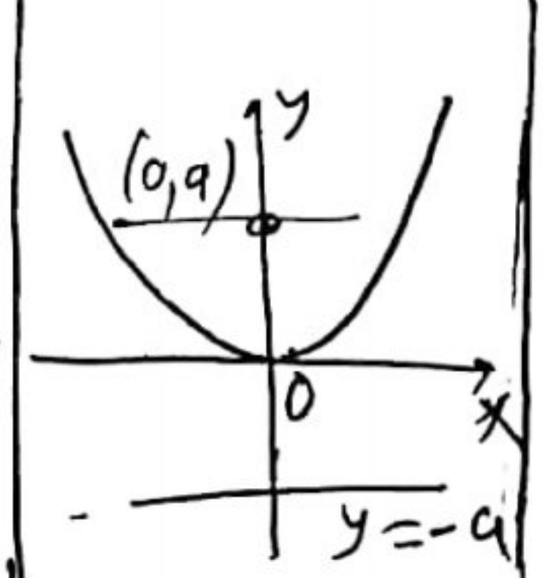
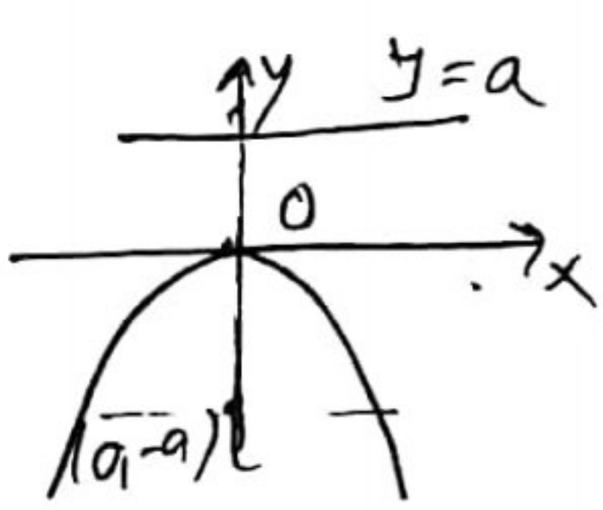
(·) Four types of parabola

(1) $y^2 = 4ax$

(2) $y^2 = -4ax$

(3) $x^2 = 4ay$

(4) $x^2 = -4ay$ (Standard parabolas)

(·) $y^2 = 4ax$	(·) $y^2 = -4ax$	$x^2 = 4ay$	$x^2 = -4ay$
			
vertex (0,0)	(0,0)	(0,0)	(0,0)
focus (a,0)	(-a,0)	(0,a)	(0,-a)
directry $x = -a$	$x = a$	$y = -a$	$y = a$
LR $4a$	$4a$	$4a$	$4a$
Axis x-axis ($y=0$)	x-axis ($y=0$)	y-axis ($x=0$)	y-axis ($x=0$)

Q:1 Find equation of parabola $3y = -6x^2$

find : ael data

Sol:

$$6x^2 = -3y$$

$$x^2 = -\frac{1}{2}y$$

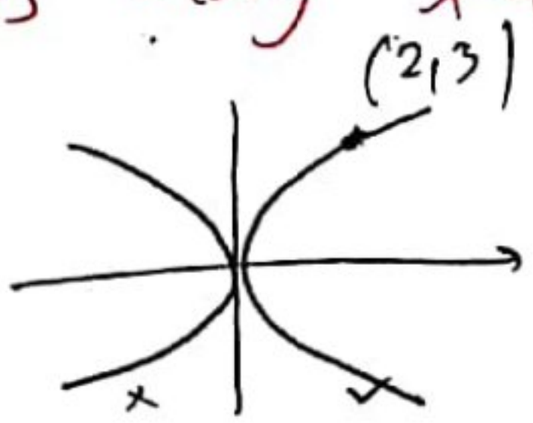
Compay with $x^2 = -4ay$

$$\Rightarrow 4a = \frac{1}{2} \Rightarrow \boxed{a = \frac{1}{8}}$$

- ✓ vertex $(0,0)$
- ✓ focus $(0, -a) = (0, -1/8)$
- ✓ directrix $y = a \Rightarrow y = 1/8$
- ✓ Axis y-axis $\Rightarrow x = 0$
- ✓ LR $= 4a = 4(\frac{1}{8}) = 1/2$

} Ans

Q:2 Find equation of parabola with vertex $(0,0)$, passing through $(2,3)$ and axis is along x-axis



Sol: Since axis is along x-axis

∴ parabola consists of the form

$y^2 = 4ax$ (or) $y^2 = -4ax$

But it passes through $(2,3)$ which is in I.st quadrant

∴ parabola must be of the type $\boxed{y^2 = 4ax}$

Now $(2,3)$ lies on it

$$\Rightarrow 9 = 8a \Rightarrow \boxed{a = \frac{9}{8}}$$

∴ equation becomes $\boxed{y^2 = \frac{9}{2}x}$

Q.N 3 Find equation of parabola with focus $(-6,0)$ and directrix $x=6$ (4)

Soln Parabola must be of the form $\boxed{y^2 = -4ax}$

Comp focus $(-6,0)$ with $(-a,0)$ and directrix $x=6$ with $x=a$

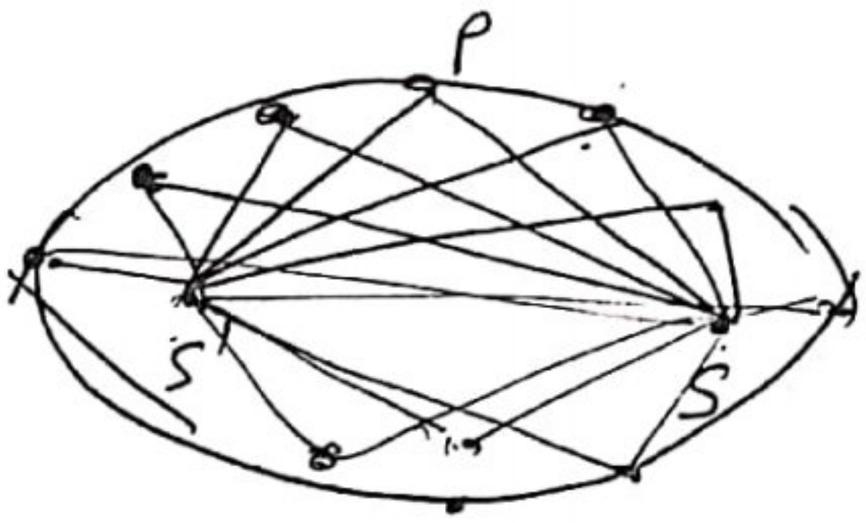
we get $(a=6)$

$\therefore$ equation of parabola becomes $\boxed{y^2 = -24x}$ Ans

ELLIPSE

- $\boxed{e<1}$

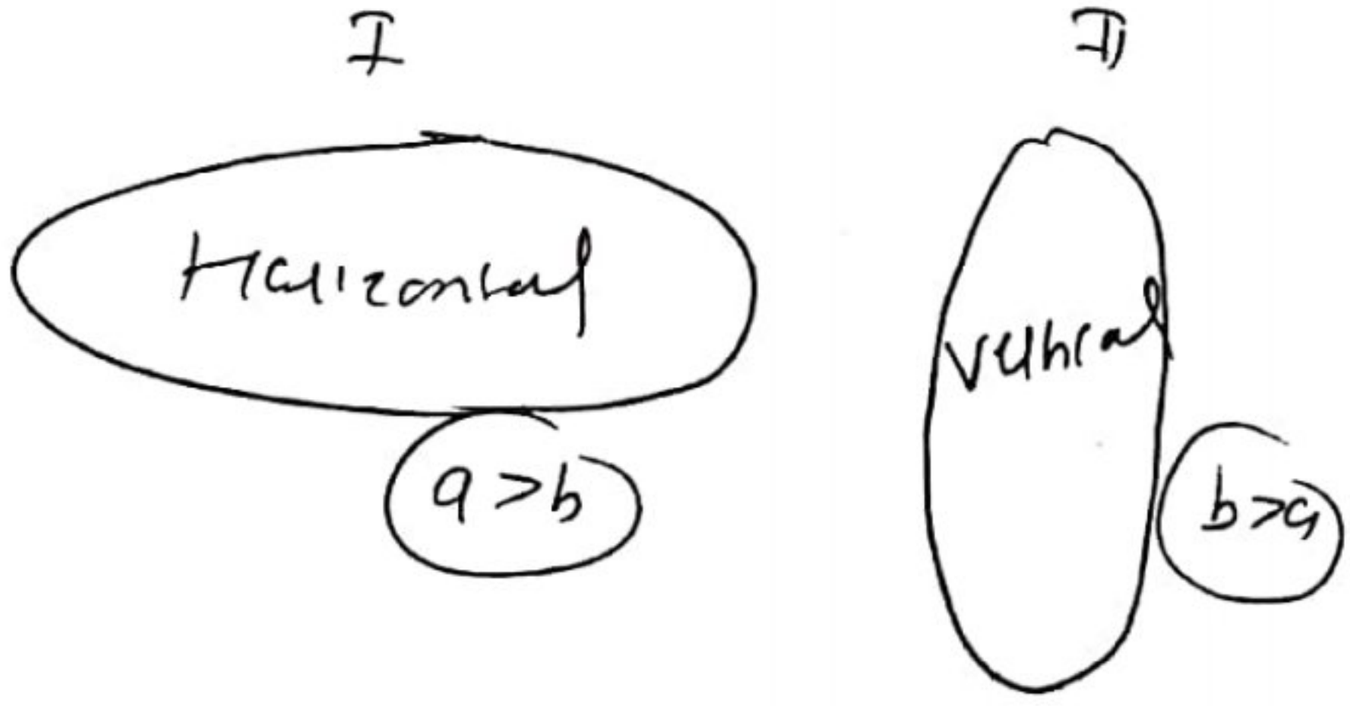
- Two types of Ellipse



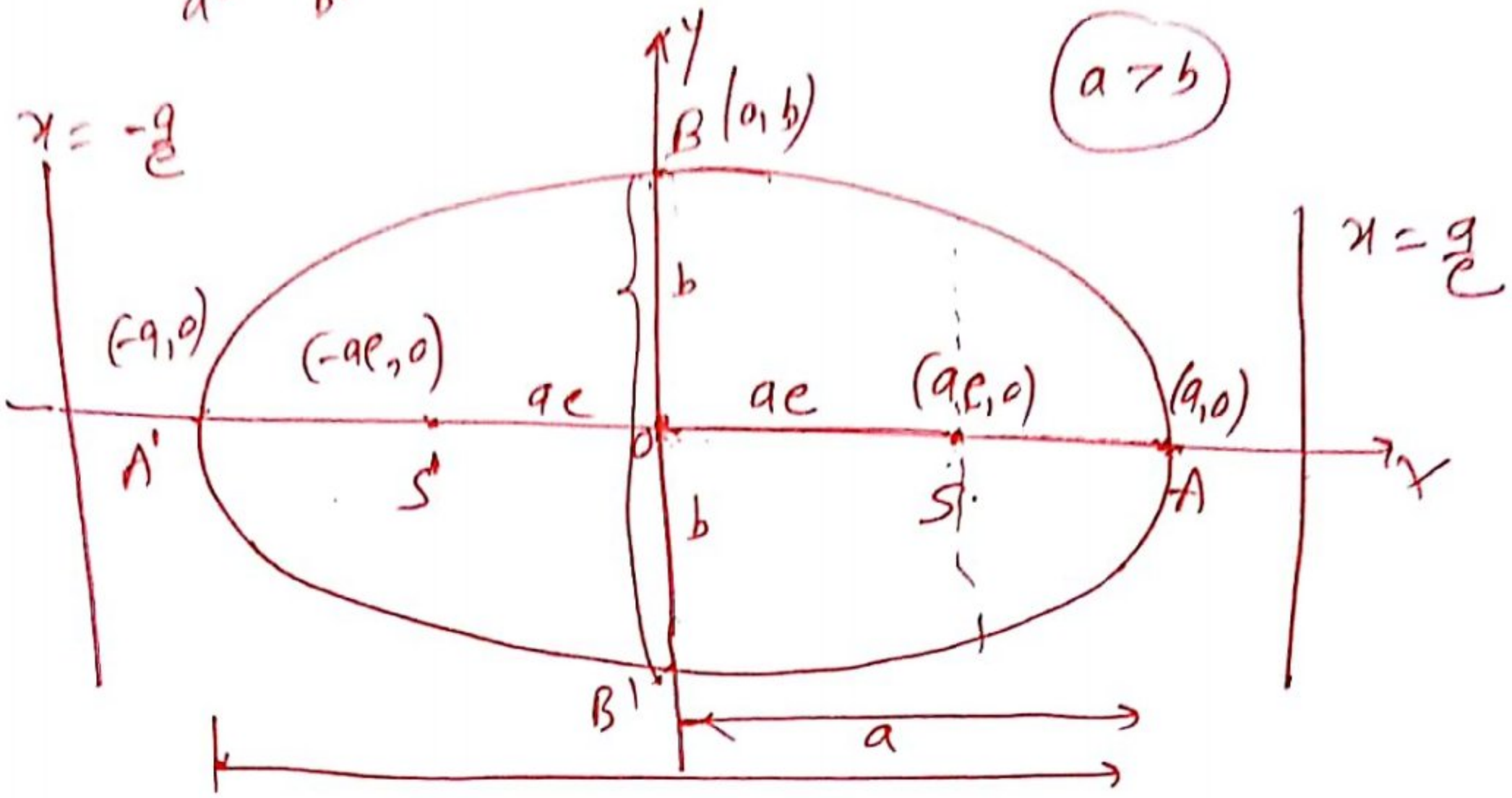
I $\boxed{\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1}$

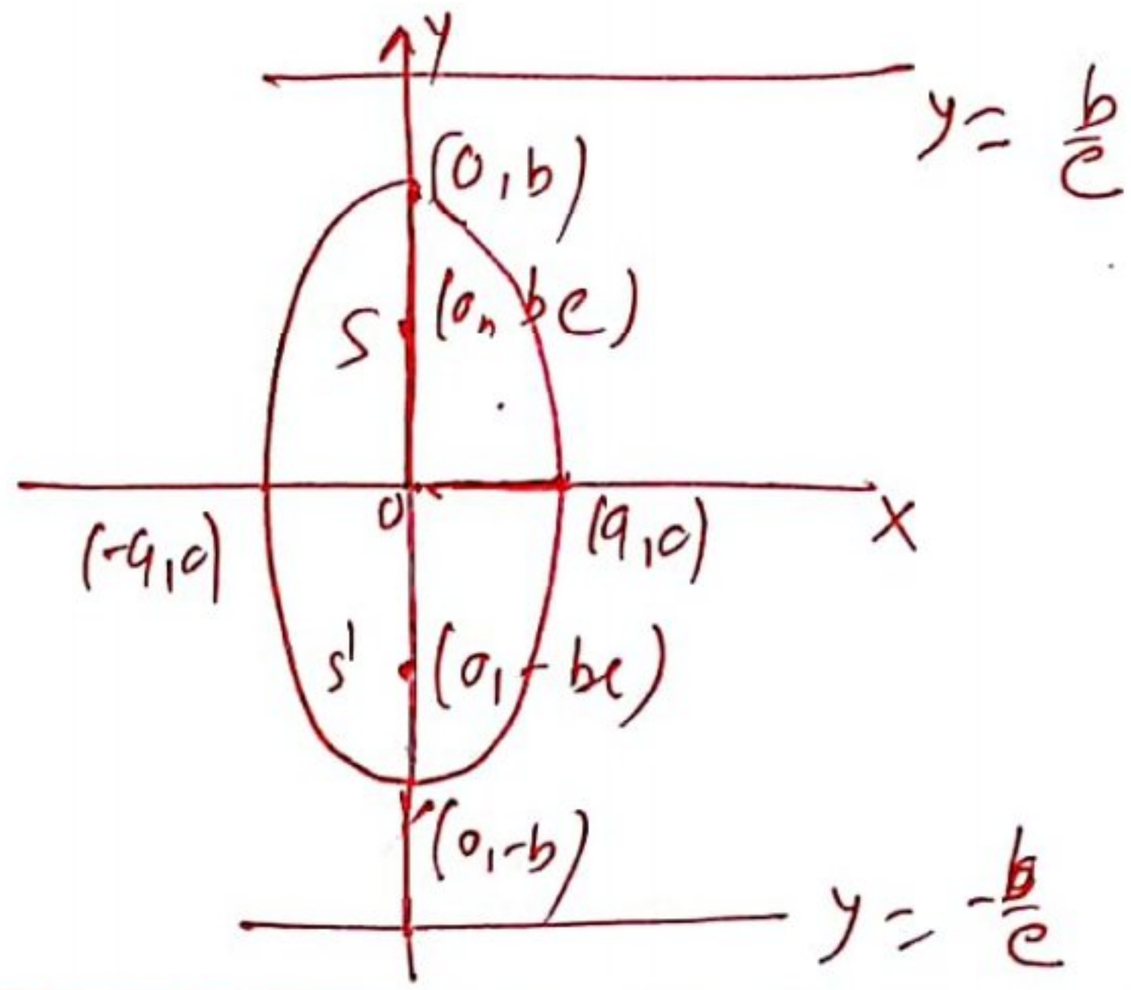
Condition $\boxed{SP + SP' = \text{Constant}}$

II $\boxed{\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1}$



$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$





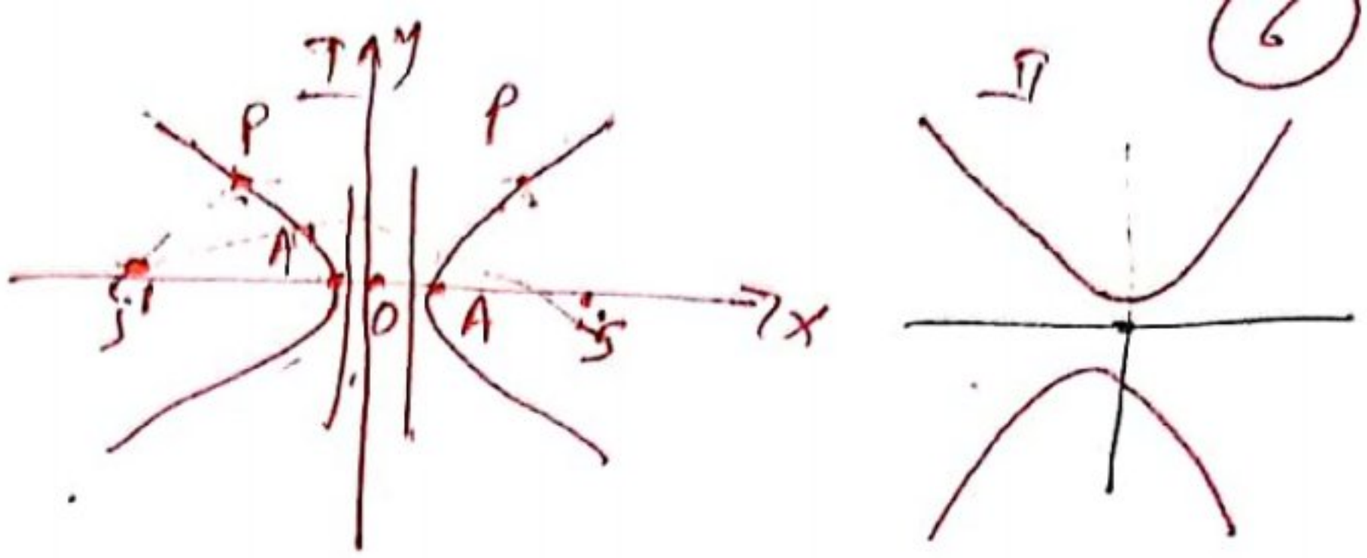
$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

b > a

Data	$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$; $a > b$ (I)	$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$; $b > a$ (II)
Centre	$(0, c)$	$(0, c)$
foci	$(\pm ae, c)$	$(0, \pm be)$
vertices	$(\pm a, c)$	$(0, \pm b)$
equation of directrices	$x = \pm \frac{a}{e}$	$y = \pm \frac{b}{e}$
e	$e = \sqrt{1 - \frac{b^2}{a^2}}$	$e = \sqrt{1 - \frac{a^2}{b^2}}$
LR	$\frac{2b^2}{a}$	$\frac{2a^2}{b}$
Major Axis = Minor Axis	$2a$ / $2b$	$2b$ / $2a$

HYPERBOLA

(·) $e > 1$



(·) Two types of hyperbola.

(·) $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ (I) (Transverse Hyperbola)

(·) $-\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ (II) (Conjugate hyperbola)

$$|SP - SP'| = \text{constant}$$

	I	II
data	$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$	$-\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$
	Transverse	Conjugate
(1) centre	$(0,0)$	$(0,0)$
(2) foci	$(\pm ae, 0)$	$(0, \pm be)$
(3) vertices	$(\pm a, 0)$	$(0, \pm b)$
(4) directrices	$x = \pm \frac{a}{e}$	$y = \pm \frac{b}{e}$
(5) e	$\sqrt{1 + \frac{b^2}{a^2}}$	$\sqrt{1 + \frac{a^2}{b^2}}$
(6) 2R	$\frac{2b^2}{a}$	$\frac{2a^2}{b}$
(7) length of Transverse axis	$2a$	$2b$
(8) length of Conjugate axis	$2b$	$2a$

Q.4 given equation of hyperbola

$5y^2 - 9x^2 = 36$ Find all data

Sol:

$$-9x^2 + 5y^2 = 36$$

$$\Rightarrow -\frac{x^2}{4} + \frac{5y^2}{36} = 1$$

$$\Rightarrow -\frac{x^2}{4} + \frac{y^2}{\frac{36}{5}} = 1$$

$$\Rightarrow -\frac{x^2}{(2)^2} + \frac{y^2}{\left(\frac{6}{\sqrt{5}}\right)^2} = 1$$

This is Conjugate hyperbola (II)

with $a = 2$ & $b = \frac{6}{\sqrt{5}}$

(1) $e = \sqrt{1 + \frac{a^2}{b^2}} = \sqrt{1 + \frac{4 \times 5}{36}} = \sqrt{1 + \frac{5}{9}} = \frac{\sqrt{14}}{3}$

(2) Centre $(0,0)$

(3) Vertices $(0, \pm b) = \left(0, \pm \frac{6}{\sqrt{5}}\right)$

(4) Foci $(0, \pm be) = \left(0, \pm \frac{6\sqrt{14}}{3\sqrt{5}}\right)$

(5) eq-y directrices $y = \pm \frac{b}{e} \Rightarrow y = \pm \frac{\frac{6}{\sqrt{5}}}{\frac{\sqrt{14}}{3}} = \pm \frac{18}{\sqrt{70}}$

(6) $LR = \frac{2a^2}{b} = \frac{2 \times 4}{\frac{6}{\sqrt{5}}} = \frac{8\sqrt{5}}{6} = \frac{4\sqrt{5}}{3}$

(7) Transverse axis $= 2b = 2\left(\frac{6}{\sqrt{5}}\right) = \frac{12}{\sqrt{5}}$

(8) Conjugate axis $= 2a = 4$ Ans

Qn. 5 Find equation of ellipse whose vertices $(\pm 5, 0)$ and foci $(\pm 4, 0)$

Soln = Comparing vertices with $(\pm a, 0)$

we get $\boxed{a = 5}$

Comparing foci with $(\pm ae, 0)$

we get $\boxed{ae = 4}$

$$e = \sqrt{1 - \frac{b^2}{a^2}}$$

$$\Rightarrow ae = \sqrt{a^2 - b^2}$$

$$\Rightarrow 4 = \sqrt{25 - b^2}$$

$$\Rightarrow 16 = 25 - b^2$$

$$\Rightarrow \boxed{b^2 = 9}$$

Now equation of ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$$\Rightarrow \boxed{\frac{x^2}{25} + \frac{y^2}{9} = 1}$$ Ans

Qn. 6 → Find equation of hyperbola whose ~~Conj~~ conjugate axis is of length 24 & foci $(0, \pm 13)$

Soln Hyperbola is of the form

$$-\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

Comp. Conjugate axis length with $2a \Rightarrow 2a = 24$

$\Rightarrow a = 12$

Comp foci with $(0, \pm be)$

$\boxed{be = 13}$

$$e = \sqrt{1 + \frac{a^2}{b^2}}$$

$$be = \sqrt{b^2 + a^2}$$

$$13 = \sqrt{b^2 + 144}$$

$$169 = b^2 + 144$$

$$b^2 = 25$$

$\therefore$ Eqn of hyperbola is $-\frac{x^2}{144} + \frac{y^2}{25} = 1$ Ans

WORKSHEET: 2

Q.1 Find foci, vertices, Major axis, minor axis, e, LR

(a)(i) $36x^2 + 4y^2 = 144$

(2) $4x^2 + 9y^2 = 36$

Ans (i) $(0, \pm 4\sqrt{2})$, $(0, \pm 6)$, 12, 4, $\frac{2\sqrt{2}}{3}$, $\frac{4}{3}$

(2) $(\pm\sqrt{5}, 0)$, $(\pm 3, 0)$, 6, 4, $\frac{\sqrt{5}}{3}$

Q.2 Find foci, vertices, e & LR of hyperbola

(i) $16x^2 - 9y^2 = 576$

(2) $49y^2 - 16x^2 = 784$

Ans (1) $(\pm 10, 0), (\pm 6, 0)$, $e = \frac{5}{3}$, $LR = \frac{64}{3}$

(2) $(0, \pm\sqrt{65})$, $(0, \pm 4)$, $e = \frac{\sqrt{65}}{4}$; $LR = \frac{49}{2}$

Q.3 Find equation ellipse whose Vertices $(0, \pm 13)$ & foci $(0, \pm 5)$ Ans $\frac{x^2}{144} + \frac{y^2}{169} = 1$

Q.4 Find equation ellipse Major axis is 26 & foci $(\pm 5, 0)$

Ans $\frac{x^2}{169} + \frac{y^2}{144} = 1$

Q.5 Find equation of hyperbola whose vertices $(\pm 2, 0)$ and foci $(\pm 3, 0)$ Ans $\frac{x^2}{4} - \frac{y^2}{5} = 1$

Q.6 Find equation of hyperbola foci $(\pm 5, 0)$ & transverse axis length is 8 Ans $\frac{x^2}{16} - \frac{y^2}{9} = 1$

Q.7 Find equation of hyperbola whose foci $(\pm 3\sqrt{5}, 0)$ and latus rectum length is 8 Ans $\frac{x^2}{25} - \frac{y^2}{20} = 1$

—x—