SOLUTIONS **LIMITS & DERIVATIVES** (1)

WORKSHEET No: 4 (class - 5)

Ans 1 $f(x) = e^{\cos x}$

$$f'(x) = \lim_{h \to 0} \left[\frac{e^{\cos(x+h)} - e^{\cos x}}{h} \right]$$

$$= e^{\cos x} \cdot \lim_{h \to 0} \left(\frac{e^{\cos(x+h) - \cos x} - 1}{h} \right) \cdots \{ \text{take } e^{\cos x} \text{ common} \}$$

$$= e^{\cos x} \lim_{h \to 0} \left(\frac{e^{\cos(x+h) - \cos x} - 1}{\cos(x+h) - \cos x} \times \frac{(\cos(x+h) - \cos x)}{h} \right)$$

$$= e^{\cos x} \times 1 \times \lim_{h \to 0} \left(\frac{\cos(x+h) - \cos x}{h} \right) \cdots \left\{ \because \lim_{x \to 0} \left(\frac{e^x - 1}{x} \right) = 1 \right\}$$

$$= e^{\cos x} \times \lim_{h \to 0} \left(\frac{-2\sin\left(\frac{2x+h}{2}\right) \sin\left(\frac{h}{2}\right)}{\frac{h}{2} \times 2} \right)$$

$$= e^{\cos x} \times (-\cancel{2} \sin x) \times \frac{1}{\cancel{2}} \cdots \left\{ \because \lim_{x \to 0} \left(\frac{\sin x}{x} \right) = 1 \right\}$$

$$\Rightarrow \boxed{f'(x) = -\sin x \cdot e^{\cos x}} \text{ Ans}$$

Ans 2 $f(x) = e^{\sqrt{\tan x}}$

$$f'(x) = \lim_{h \to 0} \left[\frac{e^{\sqrt{\tan(x+h)}} - e^{\sqrt{\tan x}}}{h} \right]$$

take $e^{\sqrt{\tan x}}$ common

$$\Rightarrow f'(x) = e^{\sqrt{\tan x}} \times \lim_{h \to 0} \left(\frac{e^{\sqrt{\tan(x+h)} - \sqrt{\tan x}} - 1}{h} \right)$$

$$= e^{\sqrt{\tan x}} \times \lim_{h \to 0} \left[\frac{e^{\sqrt{\tan(x+h)} - \sqrt{\tan x}} - 1}{\left(\sqrt{\tan(x+h)} - \sqrt{\tan x}\right)} \times \frac{\left(\sqrt{\tan(x+h)} - \sqrt{\tan x}\right)}{h} \right]$$

$$= e^{\sqrt{\tan x}} \times 1 \times \lim_{h \to 0} \left(\frac{\sqrt{\tan(x+h)} - \sqrt{\tan x}}{h} \right) \quad \text{---} \left\{ \lim_{x \to 0} \left(\frac{e^x - 1}{x} \right) = 1 \right\}$$

Rationalize

$$= e^{\sqrt{\tan x}} \times \lim_{h \to 0} \left(\frac{\tan(x+h) - \tan x}{h\left(\sqrt{\tan(x+h)} + \sqrt{\tan x}\right)} \right)$$

$$= e^{\sqrt{\tan x}} \times \lim_{h \to 0} \left[\frac{\tan(h) \{1 + \tan(x+h)\tan x\}}{h\left(\sqrt{\tan(x+h)} + \sqrt{\tan x}\right)} \right]$$

$$= e^{\sqrt{\tan x}} \times 1 \times \frac{\left(1 + \tan^2 x\right)}{\left(\sqrt{\tan x} + \sqrt{\tan x}\right)} \quad \text{---} \left\{ \lim_{x \to 0} \left(\frac{\tan x}{x} \right) = 1 \right\}$$

$$\boxed{f'(x) \Rightarrow \frac{e^{\sqrt{\tan x}} \cdot \sec^2 x}{2\sqrt{\tan x}} \quad \text{Ans}}$$

Ans 3 → $\lim_{x \to \frac{\pi}{2}} \left[\frac{2^{-\cos x} - 1}{x(x-\frac{\pi}{2})} \right]$

put $x = \frac{\pi}{2} + h$ & $h \to 0$

$$= \lim_{h \to 0} \left[\frac{2^{-\cos(\frac{\pi}{2}+h)} - 1}{(\frac{\pi}{2}+h)(\cancel{\frac{\pi}{2}}+h-\cancel{\frac{\pi}{2}})} \right]$$

$$= \lim_{h \to 0} \left(\frac{2^{\sin h} - 1}{(\frac{\pi}{2}+h)(h)} \right) \quad \text{---} \left\{ \because \cos(\tfrac{\pi}{2}+\theta) = -\sin\theta \right\}$$

$$= \lim_{h \to 0} \left(\frac{2^{\sin h} - 1}{\sin h} \times \frac{\sin h}{(\frac{\pi}{2}+h)(h)} \right)$$

$$= \lim_{h \to 0} \left(\frac{2^{\sin h} - 1}{\sin h} \right) \times \lim_{h \to 0} \left(\frac{1}{\frac{\pi}{2}+h} \right) \times \lim_{h \to 0} \left(\frac{\sin h}{h} \right)$$

$$= \log 2 \times \frac{1}{\frac{\pi}{2}} \times 1 \quad \text{-----} \left\{ \because \lim_{x \to 0} \left(\frac{a^x - 1}{x} \right) = \log a \right.$$

and $\left. \lim_{x \to 0} \left(\frac{\sin x}{x} \right) = 1 \right\}$

$$= \frac{2}{\pi} \log 2 \quad \underline{\text{Ans}}$$

Ans 4 → $\lim_{x \to 5} \left(\frac{e^x - e^5}{x-5} \right)$

put $x = 5+h$ & $h \to 0$

$$= \lim_{h\to 0}\left[\frac{e^{5+h}-e^{5}}{5+h-\cancel{5}}\right]$$

Take e^5 common

$$= e^5 \lim_{h\to 0}\left(\frac{e^h-1}{h}\right)$$

$$= e^5 \times 1 \quad --- \left\{\because \lim_{x\to 0}\left(\frac{e^x-1}{x}\right)=1\right\}$$

$$= e^5 \quad \underline{\text{Ans}}$$

Ques. 5 → $\lim_{x\to 0}\left(\frac{a^x+b^x-c^x-d^x}{x}\right)$

$$= \lim_{x\to 0}\left(\frac{a^x+b^x-c^x-d^x-1-1+1+1}{x}\right)$$

$$= \lim_{x\to 0}\left(\frac{(a^x-1)}{x}+\frac{(b^x-1)}{x}-\frac{(c^x-1)}{x}-\frac{(d^x-1)}{x}\right)$$

$$= \log a + \log b - \log c - \log d \quad --- \left\{\lim_{x\to 0}\left(\frac{a^x-1}{x}\right)=\log a\right\}$$

$$= \log(ab) - \log(cd) \quad --- \{\log A + \log B = \log(AB)\}$$

$$= \log\left(\frac{ab}{cd}\right) \quad \underline{\text{Ans}}$$

Ques 6 → $\lim_{x\to 1}\left(\frac{x-1}{\log x}\right)$

put $x = 1+h$ & $h\to 0$

$$= \lim_{h\to 0}\left(\frac{\cancel{1}+h-\cancel{1}}{\log(1+h)}\right)$$

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$$= \lim_{h\to 0}\left(\frac{h}{\log(1+h)}\right)$$

Divide by h

$$= \lim_{h\to 0}\left(\frac{1}{\left(\frac{\log(1+h)}{h}\right)}\right)$$

$$= \frac{1}{1} \quad \cdots \left\{\because \lim_{x\to 0}\left(\frac{\log(1+x)}{x}\right) = 1\right\}$$

$= \underline{1}$ Ans

Ques:- 7 → $\lim_{x\to 0}\left(\frac{2^{3x}-3^{x}}{\sin(3x)}\right)$

$$= \lim_{x\to 0}\left(\frac{2^{3x}-3^{x}-1+1}{\sin(3x)}\right)$$

$$= \lim_{x\to 0}\left(\frac{(2^{3x}-1)-(3^{x}-1)}{\sin(3x)}\right)$$

$$= \lim_{x\to 0}\left(\frac{\frac{(2^{3x}-1)}{x}-\frac{(3^{x}-1)}{x}}{\frac{\sin(3x)}{x}}\right) \quad \cdots \{\text{divide N \& D by } x\}$$

$$= \frac{\lim_{x\to 0}\left(\frac{2^{3x}-1}{3x}\right)\times 3-\lim_{x\to 0}\left(\frac{3^{x}-1}{x}\right)}{\lim_{x\to 0}\left(\frac{\sin(3x)}{3x}\right)\times 3}$$

$$= \frac{(\log 2)\times 3-\log 3}{1\times 3} \quad \cdots \left\{\lim_{x\to 0}\left(\frac{a^{x}-1}{x}\right)=\log a\right\}$$

$$\left\{\lim_{x\to 0}\left(\frac{\sin x}{x}\right)=1\right\}$$

$= \log 2 - \frac{1}{3}\log 3$ Ans..

Qu. 8 → $\lim_{x\to 0}\left(\dfrac{12^x - 3^x - 4^x + 1}{\sin x}\right)$

$$= \lim_{x\to 0}\left(\frac{12^x - 3^x - 4^x + 1 - 1 + 1}{\sin x}\right)$$

$$= \lim_{x\to 0}\left(\frac{(12^x - 1) - (3^x - 1) - (4^x - 1)}{\sin x}\right)$$

divide N & D by x

$$= \lim_{x\to 0}\left(\frac{\left(\frac{12^x - 1}{x}\right) - \left(\frac{3^x - 1}{x}\right) - \left(\frac{4^x - 1}{x}\right)}{\frac{\sin x}{x}}\right)$$

$$= \frac{\log(12) - \log(3) - \log(4)}{1} \quad \text{- - -} \left\{\lim_{x\to 0}\left(\frac{a^x - 1}{x}\right) = \log a\right\}$$

$$= \log(12) - (\log 3 + \log 4)$$

$$= \log(\cancel{12}) - \log \cancel{12}$$

$$= 0 \quad \text{Ans}$$

Qu. 9 → $\lim_{x\to 0}\left(\dfrac{12^x - 3^x - 4^x + 1}{x \sin x}\right)$

$$= \lim_{x\to 0}\left(\frac{3^x(4^x - 1) - 1(4^x - 1)}{x \sin x}\right)$$

$$= \lim_{x\to 0}\left(\frac{(3^x - 1)(4^x - 1)}{x \sin x}\right)$$

$$= \lim_{x\to 0}\left(\frac{\left(\frac{3^x - 1}{x}\right) \cdot \left(\frac{4^x - 1}{x}\right)}{\frac{\sin x}{x}}\right) \quad \text{- - -} \left\{\text{divide by } x^2\right\}$$

$$=\frac{(\log 3)(\log 4)}{1} \quad \cdots \left\{ \lim_{x\to 0}\left(\frac{a^x-1}{x}\right) = \log a \right.$$

$$= (\log 3)(\log 4) \quad \underline{\text{Ans}} \qquad \left. \lim_{x\to 0}\left(\frac{\sin x}{x}\right) = 1 \right\}$$

Q.N. 10 → $\lim_{x\to\sqrt{2}}\left(\frac{x^4-4}{x^2+3\sqrt{2}x-8}\right)$

$$= \lim_{x\to\sqrt{2}}\left(\frac{(x^2+2)(x^2-2)}{x^2+4\sqrt{2}x-\sqrt{2}x-8}\right)$$

$$= \lim_{x\to\sqrt{2}}\left(\frac{(x^2+2)(x+\sqrt{2})\cancel{(x-\sqrt{2})}}{(x+4\sqrt{2})\cancel{(x-\sqrt{2})}}\right)$$

$$= \frac{(2+2)(\sqrt{2}+\sqrt{2})}{(\sqrt{2}+4\sqrt{2})} = \frac{(4)(2\cancel{\sqrt{2}})}{5\cancel{\sqrt{2}}} = \frac{8}{5} \quad \underline{\text{Ans}}$$

Q.N. 11 → $\lim_{x\to 2}\left(\frac{x^3+3x^2-9x-2}{x^3-x-6}\right)$

$$\begin{array}{r|l} & x^2+5x+1 \\ \hline x-2 & x^3+3x^2-9x-2 \\ & -(x^3-2x^2) \\ \hline & 5x^2-9x \\ & -(5x^2-10x) \\ \hline & x-2 \\ & x-2 \\ \hline & \times \end{array} \qquad \begin{array}{r|l} & x^2+2x+3 \\ \hline x-2 & x^3-x-6 \\ & -(x^3-2x^2) \\ \hline & 2x^2-x-6 \\ & -(2x^2-4x) \\ \hline & 3x-6 \\ & 3x-6 \\ \hline & \times \end{array}$$

$$\therefore \lim_{x\to 2}\left(\frac{\cancel{(x-2)}(x^2+5x+1)}{\cancel{(x-2)}(x^2+2x+3)}\right)$$

$$= \frac{4+10+1}{4+4+3} = \frac{15}{11} \quad \underline{\text{Ans}}$$

Qn. 12 → $\lim_{x \to 1} \left(\frac{x-2}{x^2-x} - \frac{1}{x^3-3x^2+2x} \right)$

$$= \lim_{x \to 1} \left(\frac{x-2}{x(x-1)} - \frac{1}{x(x^2-3x+2)} \right)$$

$$= \lim_{x \to 1} \left[\frac{x-2}{x(x-1)} - \frac{1}{x(x-1)(x-2)} \right]$$

$$= \lim_{x \to 1} \left[\frac{(x-2)^2 - 1}{x(x-1)(x-2)} \right]$$

$$= \lim_{x \to 1} \left(\frac{x^2-4x+3}{x(x-1)(x-2)} \right)$$

$$= \lim_{x \to 1} \left(\frac{\cancel{(x-1)}(x-3)}{x\cancel{(x-1)}(x-2)} \right)$$

$$= \frac{(1-3)}{(1)(1-2)} = \frac{-2}{-1} = 2 \quad \text{Ans}$$

Qn. 13 → $\lim_{x \to 4} \left(\frac{3-\sqrt{5+x}}{1-\sqrt{5-x}} \right)$

Rationalize

$$\lim_{x \to 4} \left[\frac{(3-\sqrt{5+x})(3+\sqrt{5+x})(1+\sqrt{5-x})}{(1-\sqrt{5-x})(3+\sqrt{5+x})(1+\sqrt{5-x})} \right]$$

$$= \lim_{x \to 4} \left[\frac{(9-5-x)(1+\sqrt{5-x})}{(1-5+x)(3+\sqrt{5+x})} \right]$$

$$= \lim_{x \to 4} \left(\frac{(4-x)(1+\sqrt{5-x})}{(x-4)(3+\sqrt{5+x})} \right)$$

$$= \lim_{x\to 4}\left(\frac{-\cancel{(x-4)}\,(1+\sqrt{5-x})}{\cancel{(x-4)}\,(3+\sqrt{5+x})}\right]$$

$$= -\frac{(1+1)}{3+3} = = -\frac{2}{6} = -1/3 \text{ Ans}$$

Qns 14 → $\lim_{x\to 1}\left(\frac{(2x-3)(\sqrt{x}-1)}{2x^2+x-3}\right)$

$$= \lim_{x\to 1}\left[\frac{(2x-3)(\sqrt{x}-1)(\sqrt{x}+1)}{(2x^2+3x-2x-3)(\sqrt{x}+1)}\right]$$

$$= \lim_{x\to 1}\left(\frac{(2x-3)(x-1)}{x(2x+3)-1(2x+3)\,(\sqrt{x}+1)}\right]$$

$$= \lim_{x\to 1}\left(\frac{(2x-3)\,\cancel{(x-1)}}{(2x+3)\,\cancel{(x-1)}\,(\sqrt{x}+1)}\right)$$

$$= \frac{(2-3)}{(2+3)(1+1)} = \frac{-1}{(5)(2)}$$

$$= \frac{-1}{10} \text{ Ans}$$

Qns 15 → $\lim_{x\to 0}\left(\frac{\sqrt{1+x^2}-\sqrt{1+x}}{\sqrt{1+x^3}-\sqrt{1+x}}\right)$

Rationalize

$$= \lim_{x\to 0}\left(\frac{(\sqrt{1+x^2}-\sqrt{1+x})(\sqrt{1+x^2}+\sqrt{1+x})(\sqrt{1+x^3}+\sqrt{1+x})}{(\sqrt{1+x^3}-\sqrt{1+x})(\sqrt{1+x^2}+\sqrt{1+x})(\sqrt{1+x^3}+\sqrt{1+x})}\right)$$

$$= \lim_{x\to 0}\left[\frac{(\cancel{1}+x^2-\cancel{1}-x)\left(\sqrt{1+x^3}+\sqrt{1+x}\right)}{(\cancel{1}+x^3-\cancel{1}-x)\left(\sqrt{1+x^2}+\sqrt{1+x}\right)}\right]$$

$$= \lim_{x\to 0}\left(\frac{\cancel{x}(\cancel{x-1})\left(\sqrt{1+x^3}+\sqrt{1+x}\right)}{\cancel{x}(x+1)(\cancel{x-1})\left(\sqrt{1+x^2}+\sqrt{1+x}\right)}\right)$$

$$= \frac{1+1}{(1)(1+1)} = \frac{2}{2}$$

$$= 1 \quad \text{Ans}$$

Qns 16 → Given $\lim_{x\to 2}\left(\frac{x^n-2^n}{x-2}\right) = 80$

$$\Rightarrow n(2)^{n-1} = 80$$

$$\Rightarrow n(2)^{n-1} = 5\times 16$$

$$\Rightarrow n(2)^{n-1} = 5\times(2)^{5-1}$$

Comparing $\boxed{n=5}$ Ans

Qns 17 → Given $\lim_{x\to 1}\left(\frac{x^4-1}{x-1}\right) = \lim_{x\to k}\left(\frac{x^3-k^3}{x^2-k^2}\right)$

$$\Rightarrow \lim_{x\to 1}\left(\frac{x^4-1^4}{x-1}\right) = \lim_{x\to k}\left(\frac{\frac{x^3-k^3}{x-k}}{\frac{x^2-k^2}{x-k}}\right)$$

$$\Rightarrow 4(1)^{4-1} = \frac{3(k)^2}{2(k)^1} \quad \cdots \left\{\lim_{x\to a}\left(\frac{x^n-a^n}{x-a}\right) = na^{n-1}\right\}$$

$$\Rightarrow 4 = \frac{3k}{2}$$

$$\Rightarrow \boxed{k = 8/3} \quad \text{Ans}$$

Qn. 18 → $\lim_{x\to 0}\left(\frac{(1-x)^n-1}{x}\right)$

let $1-x=y$ | when $x\to 0$

$\Rightarrow x=1-y$ | then $y\to 1$

$\therefore \lim_{y\to 1}\left(\frac{y^n-1}{1-y}\right)$

$= -\lim_{y\to 1}\left(\frac{y^n-1^n}{y-1}\right)$

$= -n(1)^{n-1}$

$= -n$ Ans

Qn 19 → $\lim_{x\to 0}\left(\frac{(1+x)^6-1}{(1+x)^2-1}\right)$

let $1+x=y$

when $x\to 0$

then $y\to 1$

$\therefore \lim_{y\to 1}\left(\frac{y^6-1}{y^2-1}\right)$

$\Rightarrow \lim_{y\to 1}\left(\frac{\frac{y^6-1^6}{y-1}}{\frac{y^2-1^2}{y-1}}\right)$ ... {divide by $y-1$}

$= \frac{6(1)^5}{2(1)^1}$

$= \frac{6}{2} = 3$ Ans

Ans: 20 → $f(x) = x^{1/3}$

$$\Rightarrow f'(x) = \lim_{h \to 0} \left(\frac{(x+h)^{1/3} - x^{1/3}}{h} \right)$$

let $x + h = y \Rightarrow h = y - x$

when $h \to 0$

then $y \to x$

$$\therefore f'(x) = \lim_{y \to x} \left(\frac{y^{1/3} - x^{1/3}}{y - x} \right)$$

this is in the form

$$\lim_{x \to a} \left(\frac{x^n - a^n}{x - a} \right) = n a^{n-1}$$

$$f'(x) = \frac{1}{3} (x)^{\frac{1}{3} - 1}$$

$$f'(x) = \frac{1}{3} (x)^{-2/3}$$ Ans

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