SOLUTION WORKSHEET NO: 2 (Class - 3)

LIMITS & DERIVATIVES

Ans 1 $f(x) = \sin(3x-4)$

$$f'(x) = \lim_{h\to 0}\left[\frac{\sin(3x+3h-4) - \sin(3x-4)}{h}\right]$$

$$= \lim_{h\to 0}\left[\frac{2\cos\left(\frac{6x-8+3h}{2}\right)\cdot\sin\left(\frac{3h}{2}\right)}{h}\right]$$

$$= \lim_{h\to 0}\left[\frac{2\cos\left(\frac{6x+3h-8}{2}\right)\sin\left(\frac{3h}{2}\right)}{\frac{3h}{2}}\times\frac{3}{2}\right]$$

$$= \cancel{2}\cos\left(\frac{6x-8}{2}\right)\times 1\times\frac{3}{\cancel{2}} \quad -- \quad \left\{\lim_{x\to 0}\left(\frac{\sin x}{x}\right)=1\right\}$$

$$f'(x) = 3\cos(3x-4) \quad \text{Ans}$$

Ans 2 → $f(x) = \sec(2x-3)$

$$f'(x) = \lim_{h\to 0}\left[\frac{\sec(2x+2h-3) - \sec(2x-3)}{h}\right]$$

$$= \lim_{h\to 0}\left[\frac{\cos(2x-3) - \cos(2x+2h-3)}{h\cdot\cos(2x+2h-3)\cos(2x-3)}\right]$$

$$= \lim_{h\to 0}\left[\frac{-2\sin\left(\frac{4x+2h-6}{2}\right)\cdot\sin\left(-\frac{2h}{2}\right)}{h\cos(2x+2h-3)\cos(2x-3)}\right]$$

$$= \lim_{h\to 0}\left[\frac{2\sin\left(\frac{4x+2h-6}{2}\right)\cdot\sin(h)}{h\cdot\cos(2x+2h-3)\cos(2x-3)}\right]$$

$$= \lim_{h\to 0}\left(\frac{\sin h}{h}\right) \times \lim_{h\to 0}\left[\frac{2\sin(2x+h-3)}{\cos(2x+2h-3)\cos(2x-3)}\right]$$

$$= 1 \times \frac{2\sin(2x-3)}{\cos(2x-3)\cdot\cos(2x-3)} \quad \text{---} \left\{\lim_{x\to 0}\left(\frac{\sin x}{x}\right)=1\right\}$$

$$f'(x) = 2\sec(2x-3)\tan(2x-3) \quad \underline{\text{Ans}}$$

Ques 3 → $f(x) = \sqrt{\cos(3x)}$

$$f'(x) = \lim_{h\to 0}\left(\frac{\sqrt{\cos(3x+3h)} - \sqrt{\cos(3x)}}{h}\right)$$

Rationalize

$$= \lim_{h\to 0}\left(\frac{\cos(3x+3h) - \cos(3x)}{h\left(\sqrt{\cos(3x+3h)} + \sqrt{\cos(3x)}\right)}\right)$$

$$= \lim_{h\to 0}\left[\frac{-2\sin\left(\frac{6x+3h}{2}\right)\cdot\sin\left(\frac{3h}{2}\right)}{h\left(\sqrt{\cos(3x+3h)} + \sqrt{\cos(3x)}\right)}\right]$$

$$= \lim_{h\to 0}\left(\frac{-\cancel{2}\sin\left(\frac{6x+3h}{2}\right)\cdot\sin\left(\frac{3h}{2}\right) \times \frac{3}{\cancel{2}}}{\frac{3}{2}h\left(\sqrt{\cos(3x+3h)} + \sqrt{\cos(3x)}\right)}\right)$$

$$= \lim_{h\to 0}\left(\frac{\sin(3h/2)}{3h/2}\right) \times \lim_{h\to 0}\left(\frac{-\sin\left(\frac{6x+3h}{2}\right) \cancel{\times} 3}{\sqrt{\cos(3x+3h)} + \sqrt{\cos(3x)}}\right)$$

$$= 1 \times \left(\frac{-3\sin(3x)}{2\sqrt{\cos(3x)}}\right) \quad \text{---} \left\{\lim_{x\to 0}\left(\frac{\sin x}{x}\right) = 1\right\}$$

$$f'(x) = \frac{1}{2\sqrt{\cos(3x)}}\cdot(-3\sin(3x))$$ Ans { Misprint in worksheet Ans }

Ans 4 → $f(x) = \sqrt{\tan(2x)}$

$$f'(x) = \lim_{h\to 0}\left(\frac{\sqrt{\tan(2x+2h)} - \sqrt{\tan(2x)}}{h}\right)$$

Rationalize

$$= \lim_{h\to 0}\left(\frac{\tan(2x+2h) - \tan(2x)}{h\left(\sqrt{\tan(2x+2h)} + \sqrt{\tan(2x)}\right)}\right)$$

$$= \lim_{h\to 0}\left(\frac{\tan(2h)\{1+\tan(2x+2h)\tan(2x)\}}{h\left(\sqrt{\tan(2x+2h)} + \sqrt{\tan(2x)}\right)}\right)$$

$$= \lim_{h\to 0}\left[\frac{\tan(2h)\{1+\tan(2x+2h)\tan(2x)\}}{2h\left(\sqrt{\tan(2x+2h)} + \sqrt{\tan(2x)}\right)}\right]\times 2$$

$$= 1\times\frac{(1+\tan^2(2x))}{\cancel{2}\sqrt{\tan(2x)}}\times\cancel{2}$$

$$f'(x) = \frac{\sec^2(2x)}{\sqrt{\tan(2x)}}$$ Ans — { Misprint in worksheet Ans

Ans 5 → $f(x) = \csc^2(2x-4)$

$$f'(x) = \lim_{h\to 0}\left[\frac{\csc^2(2x+2h-4) - \csc^2(2x-4)}{h}\right]$$

$$f'(x) = \lim_{h\to 0}\left(\frac{\sin^2(2x-4) - \sin^2(2x+2h-4)}{h\cdot \sin^2(2x+2h-4)\sin^2(2x-4)}\right)$$

$$= \lim_{h\to 0}\left(\frac{\{\sin(2x-4)+\sin(2x+2h-4)\}\{\sin(2x-4)-\sin(2x+2h-4)\}}{h\cdot \sin^2(2x+2h-4)\cdot\sin^2(2x-4)}\right)$$

$$= \lim_{h\to 0}\left[\frac{\sin(2x-4)+\sin(2x+2h-4)}{\sin^2(2x+2h-4)\cdot\sin^2(2x-4)}\right] \times \lim_{h\to 0}\left(\frac{\sin(2x-4)-\sin(2x+2h-4)}{h}\right)$$

$$= \frac{2\sin(2x-4)}{\sin^2(2x-4)\cdot\sin^2(2x-4)} \times \lim_{h\to 0}\left(\frac{2\cos\left(\frac{4x+2h-8}{2}\right)\cdot\sin(-h)}{h}\right)$$

$$= \frac{2}{\sin^2(2x-4)\sin(2x-4)} \times \left(-\lim_{h\to 0}\frac{\sin h}{h}\right) \times \lim_{h\to 0}\left(2\cos(2x+h-4)\right)$$

$$= \frac{2}{\sin^2(2x-4)\cdot\sin(2x-4)} \times (-1) \times 2\cos(2x-4)$$

$$f'(x) = -4\,\mathrm{cosec}^2(2x-4)\cdot\cot(2x-4) \quad \text{Ans}$$

Qu 6 → ~~f~~ $f(x) = \cot^2(5x)$

$$f'(x) = \lim_{h\to 0}\left(\frac{\cot^2(5x+5h) - \cot^2(5x)}{h}\right)$$

$$f'(x) = \lim_{h\to 0}\left(\frac{\tan^2(5x) - \tan^2(5x+5h)}{h\tan^2(5x+5h)\tan^2(5x)}\right)$$

$$= \lim_{h\to 0}\left[\frac{\{\tan(5x)+\tan(5x+5h)\}\{\tan(5x)-\tan(5x+5h)\}}{h\cdot\tan^2(5x+5h)\tan^2(5x)}\right]$$

$$= \lim_{h\to 0}\left[\frac{\tan(5x)+\tan(5x+5h)}{\tan^2(5x+5h)\tan^2(5x)}\right]\times\lim_{h\to 0}\left[\frac{\tan(5x)-\tan(5x+5h)}{h}\right]$$

$$= \frac{2\tan(5x)}{\tan^2(5x)\tan^2(5x)}\times\lim_{h\to 0}\left(\frac{\tan(-5h)\{1+\tan(5x)\tan(5x+5h)\}}{h}\right)$$

$$= \frac{2}{\tan^2(5x)\cdot\tan(5x)}\times\lim_{h\to 0}\left(1+\tan(5x)\tan(5x+5h)\right)\times 5\lim_{h\to 0}\left(\frac{-\tan 5h}{5h}\right)$$

$$= \frac{2}{\tan^2(5x)\tan(5x)}\times\left(1+\tan^2(5x)\right)\times 5\times(-1)\quad\cdots\left\{\lim_{x\to 0}\left(\frac{\tan x}{x}\right)=1\right\}$$

$$= \frac{-10\sec^2(5x)}{\tan^2(5x)\tan(5x)} = -10\times\frac{\frac{1}{\cancel{\cos^2(5x)}}}{\frac{\sin^2(5x)}{\cancel{\cos^2(5x)}}\times\tan(5x)}$$

$$f'(x) = -10\csc^2(5x)\cdot\cot(5x)$$ Ans

Ans 7 $f(x) = \sec(\sqrt{x})$

$$f'(x) = \lim_{h\to 0}\left(\frac{\sec(\sqrt{x+h}) - \sec\sqrt{x}}{h}\right)$$

$$= \lim_{h\to 0}\left(\frac{\cos(\sqrt{x}) - \cos(\sqrt{x+h})}{h\cdot\cos(\sqrt{x+h})\cos\sqrt{x}}\right)$$

$$= \lim_{h\to 0}\left(\frac{-2\sin\left(\frac{\sqrt{x}+\sqrt{x+h}}{2}\right)\cdot\sin\left(\frac{\sqrt{x}-\sqrt{x+h}}{2}\right)}{h\cos(\sqrt{x+h})\cdot\cos\sqrt{x}}\right)$$

$$= \lim_{h\to 0}\left(\frac{-2\sin\left(\frac{\sqrt{x}+\sqrt{x+h}}{2}\right)\cdot\sin\left(\frac{\sqrt{x}-\sqrt{x+h}}{2}\right)\times\left(\frac{\sqrt{x}-\sqrt{x+h}}{2}\right)}{h\cos(\sqrt{x+h})\cdot\cos\sqrt{x}\cdot\left(\frac{\sqrt{x}-\sqrt{x+h}}{2}\right)}\right)$$

$$= \lim_{h\to 0}\left(\frac{\sin\left(\frac{\sqrt{x}-\sqrt{x+h}}{2}\right)}{\frac{\sqrt{x}-\sqrt{x+h}}{2}}\right)\times\lim_{h\to 0}\left(\frac{-\cancel{2}\sin\left(\frac{\sqrt{x}+\sqrt{x+h}}{2}\right)}{\cos(\sqrt{x+h})\cdot\cos\sqrt{x}}\right)\times\lim_{h\to 0}\left(\frac{\sqrt{x}-\sqrt{x+h}}{\cancel{2}h}\right)$$

$$= 1\times\left(\frac{-\sin(\sqrt{x})}{\cos\sqrt{x}\cdot\cos\sqrt{x}}\right)\times\lim_{h\to 0}\left(\frac{\cancel{x}-\cancel{x}-h}{h(\sqrt{x}+\sqrt{x+h})}\right)$$

$$= -\frac{\sin(\sqrt{x})}{\cos\sqrt{x}\cos\sqrt{x}}\times\lim_{h\to 0}\left(\frac{-1}{\sqrt{x}+\sqrt{x+h}}\right)$$

$$= \frac{\sin\sqrt{x}}{\cos\sqrt{x}\cdot\cos\sqrt{x}}\times\frac{1}{2\sqrt{x}}$$

$$f'(x) = \frac{1}{2\sqrt{x}}\sec\sqrt{x}\cdot\tan\sqrt{x}$$ ANS

Ans 8 → $f(x) = \cot\sqrt{x}$

$$f'(x) = \lim_{h\to 0}\left(\frac{\cot(\sqrt{x+h}) - \cot\sqrt{x}}{h}\right)$$

$$= \lim_{h\to 0}\left(\frac{\tan\sqrt{x} - \tan\sqrt{x+h}}{h\cdot\tan\sqrt{x+h}\cdot\tan\sqrt{x}}\right)$$

$$= \lim_{h\to 0}\left[\frac{\tan(\sqrt{x} - \sqrt{x+h})\{1+\tan\sqrt{x}\tan\sqrt{x+h}\}}{h\tan(\sqrt{x+h})\tan\sqrt{x}}\right]$$

$$= \lim_{h\to 0}\left(\frac{\tan(\sqrt{x} - \sqrt{x+h})\{1+\tan\sqrt{x}\cdot\tan\sqrt{x+h}\}\times(\sqrt{x} - \sqrt{x+h})}{(\sqrt{x} - \sqrt{x+h})\,h\,\tan(\sqrt{x+h})\tan\sqrt{x}}\right]$$

$$= \lim_{h\to 0}\left(\frac{\tan(\sqrt{x} - \sqrt{x+h})}{\sqrt{x} - \sqrt{x+h}}\right)\times\lim_{h\to 0}\left(\frac{1+\tan\sqrt{x}\tan\sqrt{x+h}}{\tan\sqrt{x+h}\tan\sqrt{x}}\right)\times\lim_{h\to 0}\left(\frac{\sqrt{x} - \sqrt{x+h}}{h}\right)$$

$$= 1\times\left(\frac{1+\tan^2(\sqrt{x})}{\tan\sqrt{x}\cdot\tan\sqrt{x}}\right)\times\lim_{h\to 0}\left(\frac{\cancel{x} - \cancel{x} - h}{h(\sqrt{x} + \sqrt{x+h})}\right)$$

$$= \frac{\sec^2(\sqrt{x})}{\tan^2(\sqrt{x})}\times\lim_{h\to 0}\left(\frac{-1}{(\sqrt{x} + \sqrt{x+h})}\right)$$

$$= \frac{\frac{1}{\cancel{\cos^2(\sqrt{x})}}}{\frac{\sin^2\sqrt{x}}{\cancel{\cos^2\sqrt{x}}}}\times\left(-\frac{1}{2\sqrt{x}}\right)$$

$$f'(x) = -\frac{1}{2\sqrt{x}}\operatorname{cosec}^2(\sqrt{x})$$ Ans

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