

!! जय श्री राधे कृष्ण जय श्री गिरिराज श्री महाराज !!

EXAM No: 9

LIMITS & DERIVATIVES

Note: You can leave 1 QNS from each section (REVISION TEST)

SECTION: A (ONE MARKS EACH)

Max. MARKS: 75

Time: 9 hrs
30 Min

QNS 1 Find the +ve Integer 'n' so that

$$\lim_{x \rightarrow 3} \left(\frac{x^n - 3^n}{x - 3} \right) = 108$$

(150 Min)

QNS 2 → Evaluate $\lim_{x \rightarrow 0} \left(\frac{\sqrt{2+x} - \sqrt{2}}{x} \right)$

QNS 3 → If $f(x) = 1 + x + \frac{x^2}{2} + \dots + \frac{x^{100}}{100}$

then $f'(1)$ is equal to

(A) $\frac{1}{100}$ (B) 100 (C) does not exist (D) 0

QNS 4 → Evaluate $\lim_{x \rightarrow 3^+} \left(\frac{x}{[x]} \right)$

QNS 5 → Evaluate $\lim_{x \rightarrow \pi} \left(\frac{\sin x}{x - \pi} \right)$

QNS 6 → If $\lim_{x \rightarrow -a} \left(\frac{x^9 + a^9}{x + a} \right) = 9$, then value of a is

(A) 1 (B) -1 (C) ± 1 (D) 3

SECTION: B (TWO MARKS EACH)

QNS 7 → Evaluate $\lim_{x \rightarrow 2} \left[\frac{1}{x-2} - \frac{2(2x-3)}{x^3 - 3x^2 + 2x} \right]$

Q_{ns} 8 → evaluate $\lim_{x \rightarrow \frac{\pi}{8}} \left[\frac{2\sin^2 x + \sin x - 1}{2\sin^2 x - 3\sin x + 1} \right]$

Q_{ns} 9 → If $y = \frac{\cos x}{1 + \sin x}$ find $\frac{dy}{dx}$

Q_{ns} 10 → If $\lim_{x \rightarrow 1} \left(\frac{x^4 - 1}{x - 1} \right) = \lim_{x \rightarrow k} \left(\frac{x^3 - k^3}{x^2 - k^2} \right)$. Find value of k

Q_{ns} 11 → If $y = \frac{\sin(x+9)}{\cos x}$; then find $\frac{dy}{dx}$ at $x=0$

Q_{ns} 12 → show that $\lim_{x \rightarrow 4} \left(\frac{|x-4|}{x-4} \right)$ does not exist

Q_{ns} 13 → evaluate $\lim_{x \rightarrow \infty} \left(\sqrt{x^2 + x + 1} - \sqrt{x^2 + 1} \right)$

Q_{ns} 14 → evaluate $\lim_{x \rightarrow 1} \left(\frac{2^{x-1} - 1}{\sin(\pi x)} \right)$

SECTION: C : 3 MARKS EACH

Q_{ns} 15 → evaluate $\lim_{x \rightarrow 0} \left(\frac{10^x - 2^x - 5^x + 1}{x \tan x} \right)$

Q_{ns} 16 → $f(x) = \sqrt{\sin x}$; find $f'(x)$ using first principle method

Q_{ns} 17 → $f(x) = \lambda x^2 + \mu x + 12$
 $f'(4) = 15$ and $f'(2) = 11$. Find λ & μ

Q_{ns} 18 → evaluate $\lim_{x \rightarrow \frac{\pi}{8}} \left(\frac{2 - \sqrt{3} \cos x - \sin x}{(\cos x - 1)^2} \right)$

Q_N 19 → $f(x) = \begin{cases} a+bx & ; x < 1 \\ y & ; x = 1 \\ b-ax & ; x > 1 \end{cases}$

If $\lim_{x \rightarrow 1} (f(x)) = f(1)$. Find the values of a & b

Q_N 20 → $f(x) = \begin{cases} |x|+1 & ; x < 0 \\ 0 & ; x = 0 \\ |x|-1 & ; x > 0 \end{cases}$

For what values of a does $\lim_{x \rightarrow a} (f(x))$ exist?

Q_N 21 → Evaluate $\lim_{x \rightarrow 0} \left(\frac{\cos(ax) - \cos(bx)}{\cos(cx) - 1} \right)$

Q_N 22 → $f(x) = x \sin x$; differentiate using first principle method

SECTION : D (FIVE MARKS EACH)

Q_N 23 → Evaluate $\lim_{x \rightarrow \frac{\pi}{4}} \left(\frac{\tan^3 x - \tan x}{\cos(x + \pi/4)} \right)$

Q_N 24 → Evaluate

$$\lim_{x \rightarrow 0} \left[\frac{\sin(\alpha + \beta)x + \sin(\alpha - \beta)x + \sin(2\alpha x)}{\cos(2\beta x) - \cos(2\alpha x)} \right] \cdot x$$

Q_N 25 → Evaluate $\lim_{x \rightarrow \frac{\pi}{2}} \left(\frac{\cot x - \cos x}{(\pi - 2x)^3} \right)$

Q. 26 → $f(x) = \cot \sqrt{x}$ differentiate using first principle method

Q. 27 → $f(x) = \frac{\sin x - x \cos x}{x \sin x + \cos x}$
~~Pl~~ find $f'(x)$. Hence find $f'(\pi/2)$

Q. 28 → evaluate
(i) $\lim_{y \rightarrow 0} \left(\frac{(x+y) \sec(x+y) - x \sec x}{y} \right)$

(ii) evaluate $\lim_{x \rightarrow a} \left[\frac{\sqrt{a+2x} - \sqrt{3x}}{\sqrt{3a+x} - 2\sqrt{x}} \right]$

Q. 29 → Evaluate $\lim_{x \rightarrow 0} \left(\frac{1 - \cos x \sqrt{\cos(2x)}}{x^2} \right)$

Q. 30 → (i) $f(x) = \frac{ax+b}{cx+d}$ find $f'(x)$ using first principle method

(ii) $f(x) = \begin{cases} \frac{k \cos x}{x-2x} & ; \text{ when } x \neq \pi/2 \\ 3 & ; x = \pi/2 \end{cases}$

and if $\lim_{x \rightarrow \pi/2} (f(x)) = f(\pi/2)$

find value of k

- x -