SOLUTIONS : **CONIC SECTION** WORKSHEET NO: 3

Q.NO: 1 → Given Ellipse foci $(\pm 5, 0)$ and directrix $x = \frac{36}{5}$

Comparing foci with $(\pm ae, 0)$ and directrix with $x = \frac{a}{e}$ we get

$\boxed{ae = 5}$ and $\boxed{\frac{a}{e} = \frac{36}{5}}$

Multiply these equations

$$(ae)\left(\frac{a}{e}\right) = 5 \times \frac{36}{5}$$

$$\Rightarrow \boxed{a^2 = 36}$$

Now $e = \sqrt{1 - \frac{b^2}{a^2}}$

$$\Rightarrow ae = \sqrt{a^2 - b^2}$$

$$\Rightarrow 5 = \sqrt{36 - b^2}$$

$$\Rightarrow 25 = 36 - b^2$$

$$\Rightarrow \boxed{b^2 = 11}$$

equation $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$$\Rightarrow \boxed{\frac{x^2}{36} + \frac{y^2}{11} = 1}$$ Ans

Ques 2 →

Given lines $3x+y=14 \quad \times 5$

$2x+5y=18 \quad \times 1$

$\Rightarrow \quad 15x+5y=70$

$\underline{2x+5y=18}$

$13x = 52$

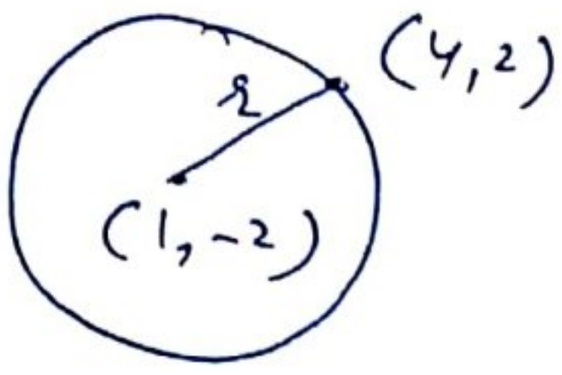
$x=4$ $\quad \therefore \quad 12+y=14 \Rightarrow y=2$

$\therefore$ circle passes through the point $(4,2)$

Centre of circle $= (1,-2)$ given

New radius $= \sqrt{(1-4)^2+(-2-2)^2}$

$= \sqrt{9+16}$



$r=5$

$\therefore$ equation of circle is : $(x-1)^2+(y+2)^2=25$

$\Rightarrow x^2+y^2-2x+4y+1+4=25$

$\Rightarrow \boxed{x^2+y^2-2x+4y-20=0}$ Ans

Ques 3 → let equation of ellipse is

$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

ellipse passes through the point $(-3,1)$

$\therefore \frac{9}{a^2}+\frac{1}{b^2}=1$

$\Rightarrow 9b^2+a^2=a^2b^2 \quad --(1)$

also $(2,-2)$ lies on the ellipse

$\therefore \frac{4}{a^2} + \frac{4}{b^2} = 1$

$\Rightarrow 4b^2 + 4a^2 = a^2b^2 \quad -(2)$

multiply eq(i) by 4

$\therefore \quad 36b^2 + 4a^2 = 4a^2b^2$

$4b^2 + 4a^2 = a^2b^2$

$32b^2 = 3a^2b^2$

$\Rightarrow \boxed{a^2 = \frac{32}{3}}$ put in eq (1)

$\therefore \quad 9b^2 + \frac{32}{3} = \frac{32}{3}b^2$

$\Rightarrow \quad \frac{32}{3} = \frac{32}{3}b^2 - 9b^2$

$\Rightarrow \quad \frac{32}{3} = \frac{5b^2}{3}$

$\Rightarrow \boxed{b^2 = \frac{32}{5}}$

$\therefore$ Equation of ellipse is

$\frac{x^2}{\frac{32}{3}} + \frac{y^2}{\frac{32}{5}} = 1$

$\Rightarrow \quad \frac{3x^2}{32} + \frac{5y^2}{32} = 1$

$\Rightarrow \boxed{3x^2 + 5y^2 = 32}$ Ans

Ques 4 → Hyperbola (I^st) (Transverse)

length of transverse axis = 7

$\Rightarrow 2a = 7 \Rightarrow \boxed{a = 7/2}$

Let equation of hyperbola is (Transverse hyperbola)

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

It passes through the point (5, -2)

$$\Rightarrow \frac{25}{a^2} - \frac{4}{b^2} = 1$$

$$\Rightarrow \frac{25}{\frac{49}{4}} - \frac{4}{b^2} = 1 \quad \text{--- } \{\because a = 7/2\}$$

$$\Rightarrow \frac{100}{49} - 1 = \frac{4}{b^2}$$

$$\Rightarrow \frac{51}{49} = \frac{4}{b^2}$$

$$\Rightarrow \frac{51}{196} = \frac{1}{b^2}$$

$$\Rightarrow \boxed{b^2 = \frac{196}{51}}$$

∴ equation of hyperbola is

$$\frac{x^2}{\frac{49}{4}} - \frac{y^2}{\frac{196}{51}} = 1$$

$$\Rightarrow \boxed{\frac{4x^2}{49} - \frac{51y^2}{196} = 1}$$ Ans

Q.No 5 → CIRCLE

Given Radius $(r) = 3$

Its centre is (h, k)

Centre lies on line $y = x - 1$

$\Rightarrow k = h - 1 \quad \cdots (1)$

Let equation of circle is

$$(x-h)^2 + (y-k)^2 = r^2$$

$$\Rightarrow (x-h)^2 + (y-k)^2 = 9$$

It passes through the point $(7, 3)$

$$\Rightarrow (7-h)^2 + (3-k)^2 = 9$$

$$\Rightarrow h^2 - 14h + 49 + k^2 - 6k + 9 = 9$$

$$\Rightarrow h^2 + k^2 - 14h - 6k + 49 = 0$$

put $k = h - 1$

$$\Rightarrow h^2 + (h-1)^2 - 14h - 6(h-1) + 49 = 0$$

$$\Rightarrow h^2 + h^2 - 2h + 1 - 14h - 6h + 6 + 49 = 0$$

$$\Rightarrow 2h^2 - 22h + 56 = 0$$

$$\Rightarrow h^2 - 11h + 28 = 0$$

$$\Rightarrow (h-4)(h-7) = 0$$

$$\Rightarrow h = 4 \; ; \; h = 7$$

$$\Rightarrow k = 3 \; ; \; k = 6$$

$\therefore$ Centre can be $(4, 3)$ and $(7, 6)$

and Radius $= 3$

∴ Equation of circles

$(x-4)^2 + (y-3)^2 = 9$ (or) $(x-7)^2 + (y-6)^2 = 9$

$\Rightarrow x^2+y^2-8x-6y+16=0$ (or) $x^2+y^2-14x-6y+76=0$

(both are Answers) Ans

Qn. 6 → Hyperbola

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

It passes through the point $(3,0)$

$\therefore \frac{9}{a^2} - 0 = 1$

$\Rightarrow \frac{9}{a^2} = 1$

$\Rightarrow \boxed{a^2 = 9}$

also hyperbola passes through the point $(3\sqrt{2}, 2)$

$\frac{18}{a^2} - \frac{4}{b^2} = 1$

$\Rightarrow \frac{18}{9} - \frac{4}{b^2} = 1$

$\Rightarrow 2 - \frac{4}{b^2} = 1 \Rightarrow 1 = \frac{4}{b^2} \Rightarrow \boxed{b^2 = 4}$

Now $e = \sqrt{1 + \frac{b^2}{a^2}}$

$e = \sqrt{1 + \frac{4}{9}}$

$e = \sqrt{\frac{13}{9}}$

$\boxed{e = \frac{\sqrt{13}}{3}}$ Ans

Qns 7 →

foci = $(\pm ae, 0)$

New distance b/w foci = $2ae$

Given $2ae = 16$

$\Rightarrow ae = 8$

Given $e = \sqrt{2}$

$\Rightarrow a(\sqrt{2}) = 8$

$\Rightarrow a = \frac{8}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$

$a = 4\sqrt{2}$

New $e = \sqrt{1 + \frac{b^2}{a^2}}$

$\Rightarrow ae = \sqrt{a^2 + b^2}$

$\Rightarrow 8 = \sqrt{32 + b^2}$

$\Rightarrow 64 = 32 + b^2$

$\Rightarrow b^2 = 32$

∴ equation of hyperbola

$\frac{x^2}{(4\sqrt{2})^2} - \frac{y^2}{32} = 1$

$\Rightarrow \frac{x^2}{32} - \frac{y^2}{32} = 1$

$\Rightarrow x^2 - y^2 = 32$ Ans

Qns 8 → ELLIPSE $e = \frac{5}{8}$

distance b/w foci = $2ae = 10$

$\Rightarrow \boxed{ae = 5}$

$\Rightarrow a\left(\frac{5}{8}\right) = 5$

$\Rightarrow \boxed{a = 8}$

Also $e = \sqrt{1 - \frac{b^2}{a^2}}$

$\Rightarrow ae = \sqrt{a^2 - b^2}$

$\Rightarrow 5 = \sqrt{64 - b^2}$

$\Rightarrow 25 = 64 - b^2$

$\Rightarrow \boxed{b^2 = 39}$

Latus rectum $LR = \frac{2b^2}{a}$

$= \frac{2(39)}{8}$

$\boxed{LR = \frac{39}{4} \text{ units}}$ Ans

Q.N. 9 * CIRCLE

Given Radius = 5

Given Circle: $x^2 + y^2 - 2x - 4y - 20 = 0$

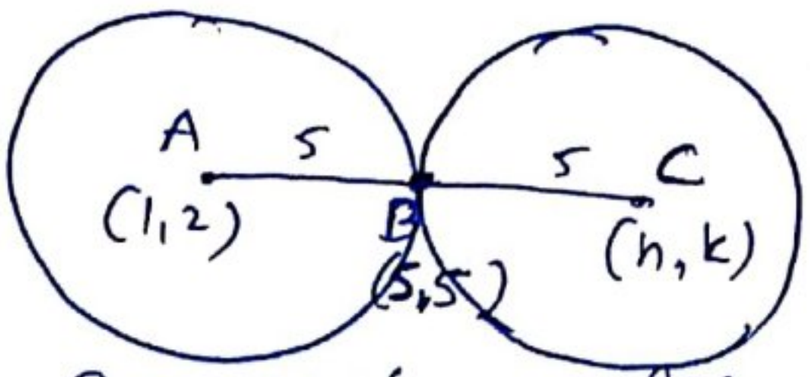
$\Rightarrow x^2 - 2x + y^2 - 4y - 20 = 0$

$\Rightarrow (x-1)^2 - 1 + (y-2)^2 - 4 - 20 = 0$

$\rightarrow (x-1)^2 + (y-2)^2 = 25$

Centre $(1, 2)$ Radius $= 5$

Let centre of Required circle is (h, k)



From the figure

Clearly $B(5,5)$ is the mid point of $A(1,2)$ & $C(h,k)$

$$\Rightarrow \quad 5 = \frac{1+h}{2} \quad \text{and} \quad 5 = \frac{2+k}{2}$$

$$\Rightarrow \quad h = 9 \quad \& \quad k = 8$$

$\therefore$ Centre of Required circle is $(9,8)$

Now equation of Required circle is

$$(x-9)^2 + (y-8)^2 = 25$$

$$\Rightarrow \quad x^2 + y^2 - 18x - 16y + 81 + 64 = 25$$

$$\Rightarrow \quad \boxed{x^2 + y^2 - 18x - 16y + 120 = 0} \quad \text{Ans}$$

Qn. 10 → Same as Qn No: 5 (In fact Repeated)

— x —