SOLUTION

$\boxed{\text{COMPLEX NUMBER}}$ $\boxed{\text{CLASS No}=4}$

(Modulus & argument)

Q. 1 $z = \dfrac{1+i}{1-i} - \dfrac{1-i}{1+i}$

$\Rightarrow z = \dfrac{(1+i)^2 - (1-i)^2}{(1-i)(1+i)}$

$z = \dfrac{(\cancel{1} + \cancel{i^2} + 2i) - (\cancel{1} + \cancel{i^2} - 2i)}{1-i^2}$

$z = \dfrac{2i+2i}{1+1}$

$z = \dfrac{4i}{2} = 2i$

$z = 0 + 2i$

$|z| = \sqrt{0+4} = 2$ $\therefore$ $\boxed{|z| = 2}$ Ans

Q. 2 let $z = \dfrac{1+3i}{1-2i}$

$z = \dfrac{1+3i}{1-2i} \times \dfrac{1+2i}{1+2i}$

$z = \dfrac{1+2i+3i+6i^2}{1-4i^2}$

$z = \dfrac{-5+5i}{5}$

$z = -1+i$

here
$a = -1$, $b = 1$

$r = \sqrt{a^2+b^2} = \sqrt{1+1} = \sqrt{2}$

$\tan\alpha = \left|\dfrac{b}{a}\right| = \left|\dfrac{1}{-1}\right| = 1$

$\Rightarrow \alpha = \pi/4$

Since z is in 2^{nd} quad

$\therefore \theta = \pi - \alpha$

$\Rightarrow \theta = \pi - \pi/4 = 3\pi/4$

$\therefore$ argument/Amplitude $= 3\pi/4$ ANS

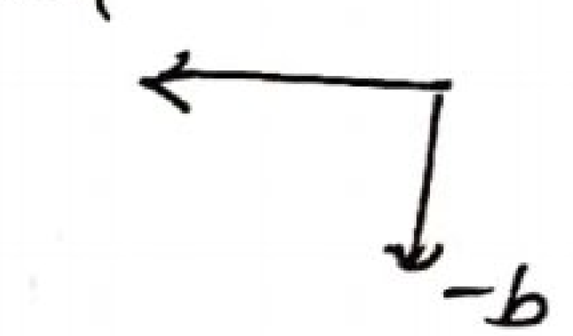
Ans 3 $\Rightarrow$ $z = -1-i$

here $a = -1$, $b = -1$

$r = \sqrt{a^2+b^2} = \sqrt{1+1} = \sqrt{2}$

$\tan\alpha = \left|\frac{b}{a}\right| = \left|\frac{-1}{-1}\right| = 1$

$\Rightarrow \alpha = \pi/4$



Since z is in 3rd quadrant

$\therefore \theta = -(\pi-\alpha)$

$\theta = -(\pi - \pi/4)$

$\theta = -3\pi/4$

polar form is given by

$z = r(\cos\theta + i\sin\theta)$

$z = \sqrt{2}\left(\cos\left(-\frac{3\pi}{4}\right) + i\sin\left(-\frac{3\pi}{4}\right)\right)$ **Ans**

Ans 4 $\rightarrow$ $z = \frac{-16}{1+i\sqrt{3}}$

$\Rightarrow z = \frac{-16}{1+i\sqrt{3}} \times \frac{1-i\sqrt{3}}{1-i\sqrt{3}}$

$\Rightarrow z = \frac{-16+16\sqrt{3}i}{1-3i^2}$

$$z = \frac{-16 + 16\sqrt{3}\,i}{4}$$

$$z = -4 + 4\sqrt{3}\,i$$

here $a = -4;\ b = 4\sqrt{3}$

$$r = \sqrt{a^2 + b^2} = \sqrt{16 + 48} = \sqrt{64}$$

$\therefore r = 8$

$$\tan\alpha = \left|\frac{b}{a}\right| = \left|\frac{4\sqrt{3}}{-4}\right| = |-\sqrt{3}| = \sqrt{3}$$

$\Rightarrow \boxed{\alpha = \pi/3}$

Since z is 2^{nd} quadrant

$\therefore \theta = \pi - \alpha$

$\theta = \pi - \pi/3$

$\boxed{\theta = 2\pi/3}$

$\therefore$ Amplitude / argument / principal argument $= \frac{2\pi}{3}$ Ans

Ans. 5 → let $z = (1 + i\sqrt{3})^2$

$$z = 1 + 3i^2 + 2i\sqrt{3}$$

$$z = -2 + 2i\sqrt{3}$$

here $a = -2$ & $b = 2\sqrt{3}$

$$\tan\alpha = \left|\frac{b}{a}\right| = \left|\frac{2\sqrt{3}}{-2}\right| = |-\sqrt{3}| = \sqrt{3}$$

$\Rightarrow \boxed{\alpha = \pi/3}$

Since z in 2^{nd} quadrant

$$-a \quad +b$$

$\therefore \theta = \pi - \alpha$

$\Rightarrow \theta = \pi - \pi/3$

$\Rightarrow \theta = 2\pi/3$

$\therefore$ Principal argument $= 2\pi/3$ Ans

Ans 6 →

$$z = \frac{1-i}{\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}}$$

$$z = \frac{1-i}{\frac{1}{2} + i\frac{\sqrt{3}}{2}}$$

$$z = \frac{2-2i}{1+i\sqrt{3}}$$

$$z = \frac{2-2i}{1+i\sqrt{3}} \times \frac{1-i\sqrt{3}}{1-i\sqrt{3}}$$

$$z = \frac{2-2\sqrt{3}i - 2i + 2\sqrt{3}i^2}{1-3i^2}$$

$$z = \frac{(2-2\sqrt{3})}{4} - i\frac{(2+2\sqrt{3})}{4}$$

$$z = \left(\frac{1-\sqrt{3}}{2}\right) - \left(\frac{1+\sqrt{3}}{2}\right)i$$

here $a = \frac{1-\sqrt{3}}{2}$ & $b = -\left(\frac{1+\sqrt{3}}{2}\right)$

$$r = \sqrt{a^2+b^2} = \sqrt{\left(\frac{1-\sqrt{3}}{2}\right)^2 + \left(\frac{1+\sqrt{3}}{2}\right)^2}$$

$$r = \sqrt{\frac{1+3-2\sqrt{3}}{4} + \frac{1+3+2\sqrt{3}}{4}}$$

$$r = \sqrt{\frac{8}{4}}$$

$$r = \sqrt{2}$$

$$\tan\alpha = \left|\frac{b}{a}\right| = \left|\frac{-\left(\frac{1+\sqrt{3}}{\cancel{2}}\right)}{\frac{1-\sqrt{3}}{\cancel{2}}}\right|$$

$$= \left|\frac{\cancel{-}(1+\sqrt{3})}{\cancel{-}(\sqrt{3}-1)}\right|$$

$$\tan a = \frac{\sqrt{3}+1}{\sqrt{3}-1}$$

Divide by $\sqrt{3}$

$$\tan\alpha = \frac{1+\frac{1}{\sqrt{3}}}{1-\frac{1}{\sqrt{3}}}$$

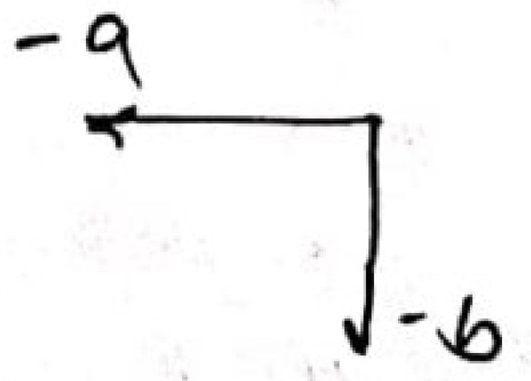
$$= \frac{\tan\left(\frac{\pi}{4}\right)+\tan\left(\frac{\pi}{6}\right)}{1-\tan\frac{\pi}{4}\times\tan\frac{\pi}{6}}$$

$$= \tan\left(\frac{\pi}{4}+\frac{\pi}{6}\right)$$

$$\tan\alpha = \tan\left(\frac{5\pi}{12}\right)$$

$$\boxed{\alpha = \frac{5\pi}{12}}$$

Since z in 3^{rd} quadrant



$\therefore \theta = -(\pi - \alpha)$

$\theta = -\left(\pi - \frac{5\pi}{12}\right)$

$\boxed{\theta = -\frac{7\pi}{12}}$

$\therefore$ polar form is given by

$z = \sqrt{2}\left[\cos\left(-\frac{7\pi}{12}\right) + i\sin\left(-\frac{7\pi}{12}\right)\right]$ ANS

(Note : Misprint in worksheet Ans because Misprint in Exemplar book)

Qn. 7 let $z = (i^{25})^3$

$z = (i)^3$ --- $\{ i^{25} = i^{24} \cdot i = 1 \times i = i \}$

$z = -i$ -- $\{\because i^3 = -i\}$

$z = 0 - i$

here $a = 0$, $b = -1$

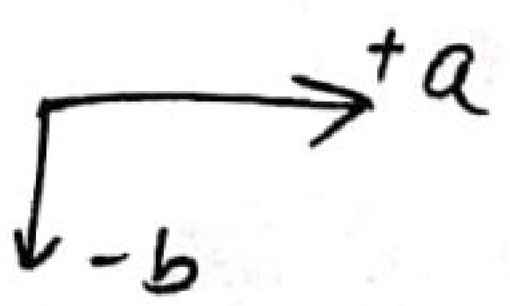
$r = \sqrt{a^2+b^2} = \sqrt{0+1} = 1$

$\boxed{r = 1}$

$\tan\alpha = \left|\frac{b}{a}\right| = \left|\frac{-1}{0}\right| = \left|\frac{1}{0}\right| = \infty$

$\Rightarrow \boxed{\alpha = \frac{\pi}{2}}$

z in 4th quadrant



$\therefore \theta = -\alpha$

$\boxed{\theta = -\pi/2}$ Ans

—x—