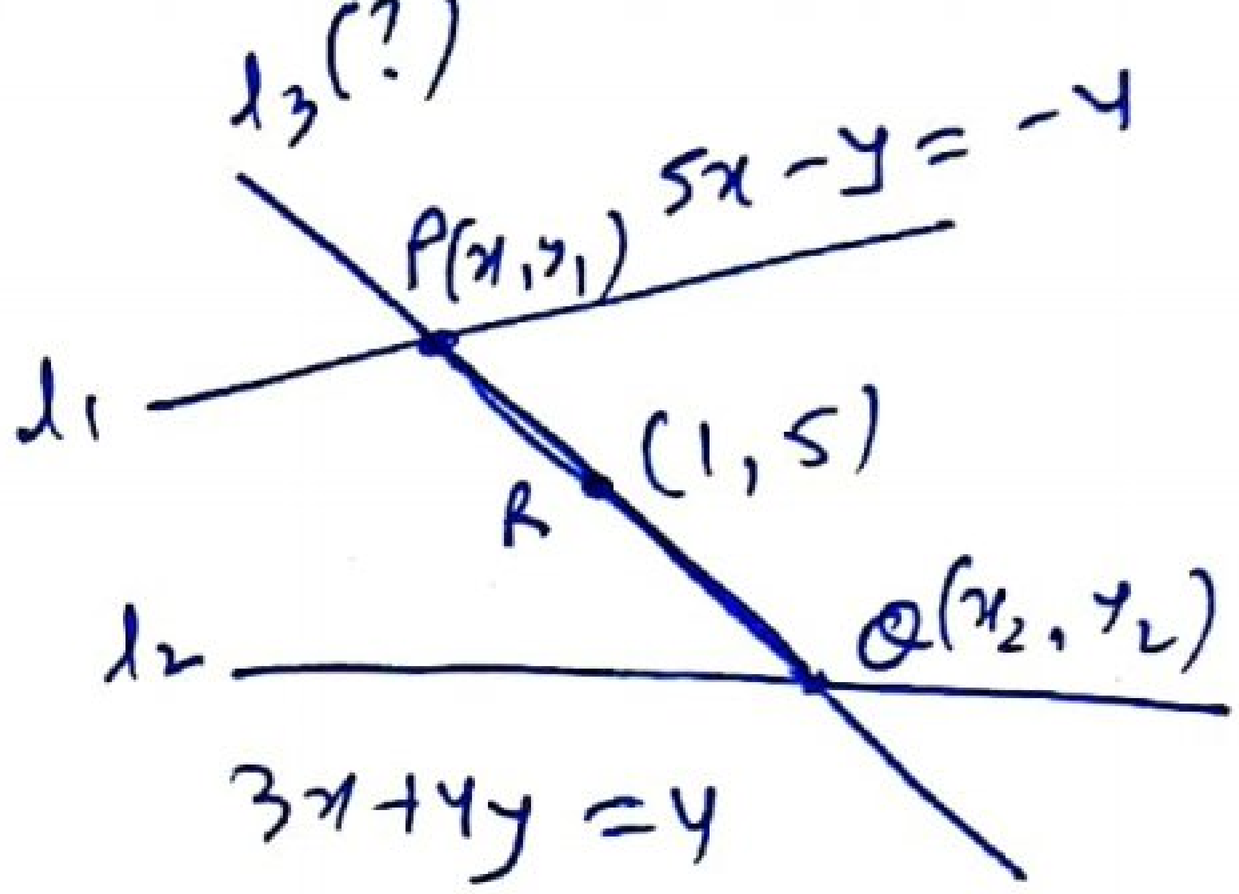
॥ जय श्री श्याम कृपा॥ जय श्री गिरिराज जी महाराज ॥

**ULTIMATE MATHEMATICS: BY AJAY MITTAL**

Chapter: STRAIGHT LINES $\boxed{\text{CLASS NO: 4}}$

**Ques 1** (Imp) A line is such that its segment between the lines $5x-y+4=0$ and $3x+4y-4=0$ is bisected at the point $(1,5)$. Obtain its equation.




Soln ∵ $P(x_1,y_1)$ lies on line $l_1$

$5x_1-y_1=-4$ --(1)

∵ $Q(x_2,y_2)$ lies on line $l_2$

$3x_2+4y_2=4$ --(2)

∵ $R(1,5)$ is the Midpoint of P & Q

$1=\frac{x_1+x_2}{2}$ & $5=\frac{y_1+y_2}{2}$

$x_1+x_2=2$ & $y_1+y_2=10$

$\boxed{x_2=2-x_1}$ & $\boxed{y_2=10-y_1}$

put in eq (2)

$\Rightarrow 6-3x_1+40-4y_1=4$

$\Rightarrow 3x_1+4y_1=42$ --(3)

Solving (1) & (3)

$y_1=\frac{222}{23}$ & $x_1=\frac{26}{23}$

$\therefore P\left(\cancel{\frac{222}{23}},\cancel{\frac{26}{23}}\right)$ $P\left(\frac{26}{23},\frac{222}{23}\right)$

equation of $l_3$

(two point form)

$y-5=\frac{\left(\frac{222}{23}-5\right)}{\left(\frac{26}{23}-1\right)}(x-1)$

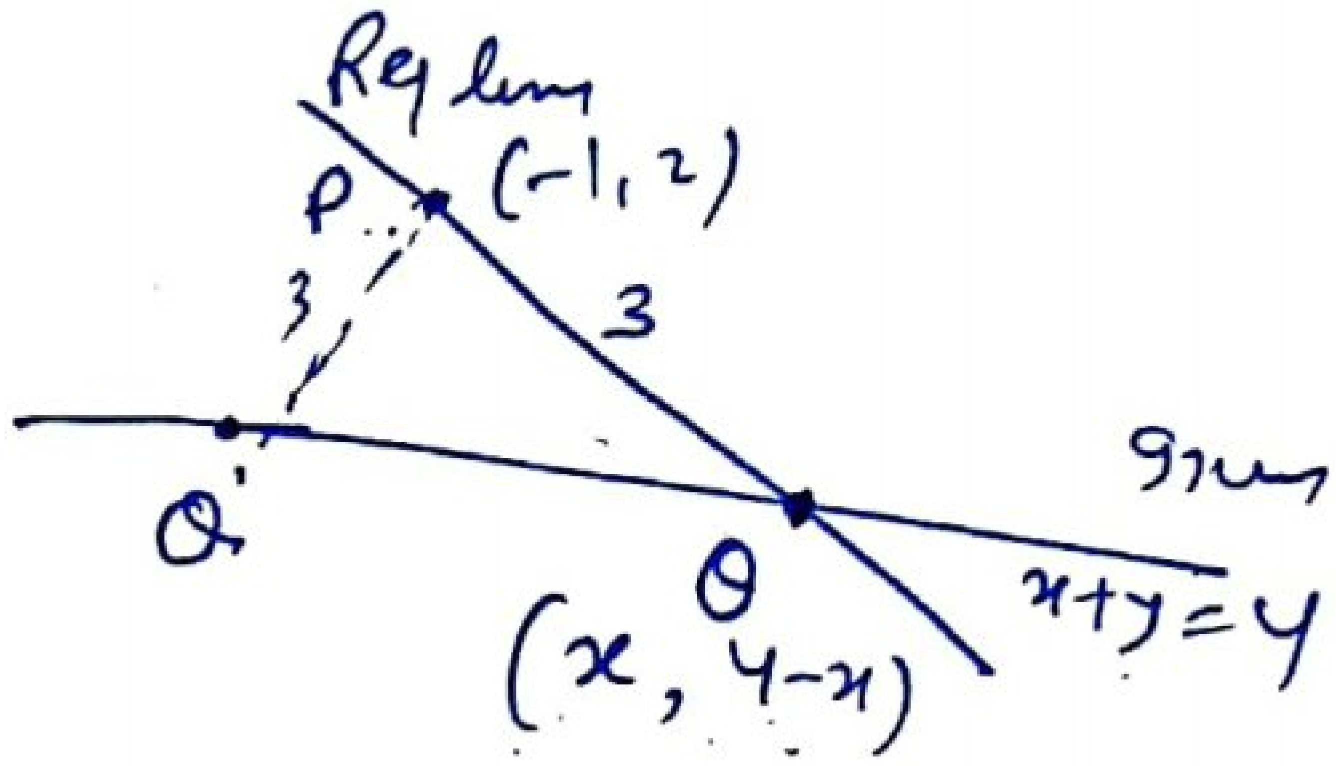
$y-5=\frac{107}{3}(x-1)$

$\Rightarrow 3y-15=107x-107$

$\Rightarrow \boxed{107x-3y-92=0}$

Ans

**Qn: 2 →** Find the direction in which a straight line must be drawn through the point $(-1, 2)$ so that its point of intersection with the line $x+y=4$ may be at a distance of 3 units from this point.




**Sol:** Let $Q(x, 4-x)$

Given $PQ = 3$

$$\sqrt{(x+1)^2+(2-x)^2}=3$$

$$\Rightarrow x^2+2x+1+4+x^2-4x=9$$

$$\Rightarrow 2x^2-2x-4=0$$

$$\Rightarrow x^2-x-2=0$$

$$\Rightarrow (x-2)(x+1)=0$$

$$\Rightarrow x=2,\ x=-1$$

$$\therefore Q(2,2) \text{ or } Q(-1,5)$$

Equation of required line PQ (By two point form)

$(-1,2)$

$$y-2=\frac{0}{3}(x+1)$$

$$y-2=0$$

$$\Rightarrow \boxed{y=2}$$

$\Rightarrow$ line $\parallel$ to x-axis

$$y-2=\frac{3}{0}(x+1)$$

$$0=3(x+1)$$

$$\Rightarrow x+1=0$$

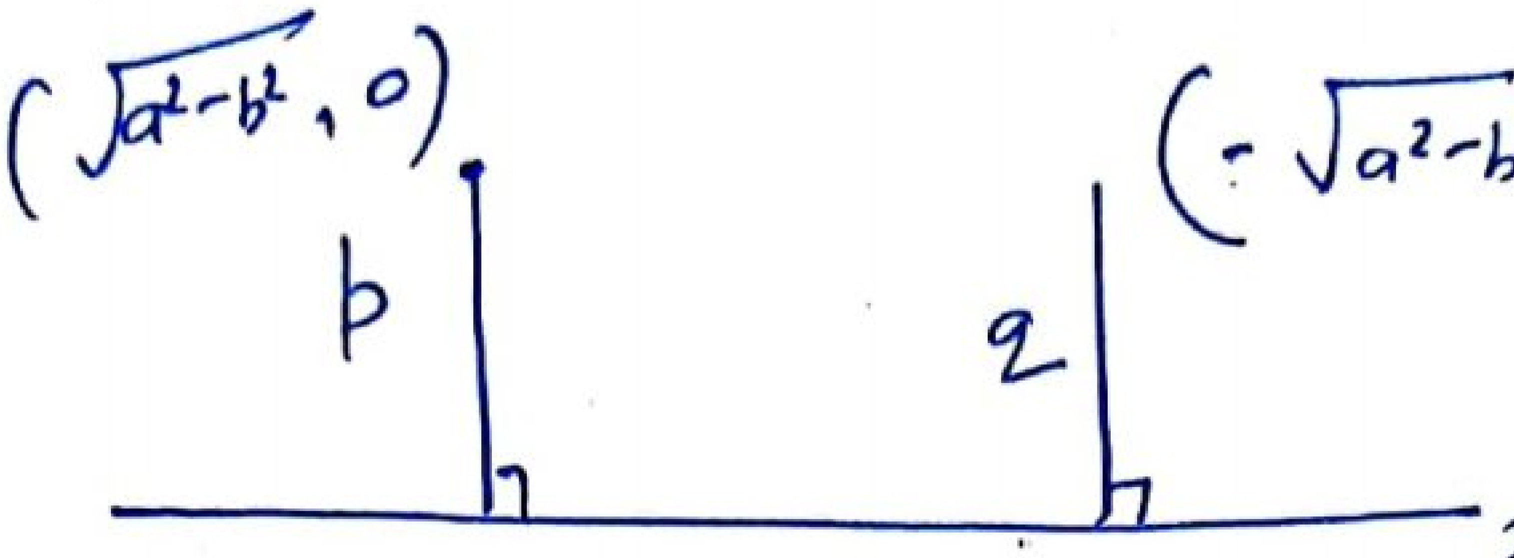
$$\Rightarrow x=-1$$

line $\parallel$ to y-axis.

direction line either parallel to x-axis or $\parallel$ to y-axis **Ans**

Ques. 3 ⇒ Prove that the product of the length of the perpendiculars drawn from the points $(\sqrt{a^2-b^2}, 0)$ and $(-\sqrt{a^2-b^2}, 0)$ to the line $\frac{x}{a}\frac{\cancel{\sin\theta}}{\cos\theta} + \frac{y}{b}\frac{\cancel{\sin\theta}}{\sin\theta} = 1$ is $b^2$.

Sol$^n$




$\frac{x}{a}\frac{\cos\theta}{\cancel{\sin\theta}} + \frac{y}{b}\frac{\sin\theta}{\cancel{\cos\theta}} = 1$

To prove $\boxed{pq = b^2}$

eq. of line

$$bx\cos\theta + ay\sin\theta - ab = 0$$

Sol$^n$

$$pq = \frac{|b\sqrt{a^2-b^2}\cos\theta - ab|}{\sqrt{b^2\cos^2\theta + a^2\sin^2\theta}} \times \frac{|-b\sqrt{a^2-b^2}\cos\theta - ab|}{\sqrt{b^2\cos^2\theta + a^2\sin^2\theta}}$$

$$= \frac{|b\sqrt{a^2-b^2}\cos\theta - ab| \times |b\sqrt{a^2-b^2}\cos\theta + ab|}{b^2\cos^2\theta + a^2\sin^2\theta} \quad \cdots \{|-a-b| = |a+b|\}$$

$$= \frac{|b^2(a^2-b^2)\cos\theta - a^2b^2|}{b^2\cos^2\theta + a^2\sin^2\theta} \quad \cdots \{|a||b| = |ab|\}$$

$$= b^2\frac{|a^2\cos^2\theta - b^2\cos^2\theta - a^2|}{b^2\cos^2\theta + a^2\sin^2\theta}$$

$$= b^2\frac{|a^2(\cos^2\theta - 1) - b^2\cos^2\theta|}{b^2\cos^2\theta + a^2\sin^2\theta}$$

$$= b^2\frac{|-a^2\sin^2\theta - b^2\cos^2\theta|}{b^2\cos^2\theta + a^2\sin^2\theta}$$

$$= b^2\frac{|a^2\sin^2\theta + b^2\cos^2\theta|}{a^2\sin^2\theta + b^2\cos^2\theta}$$

$$= b^2\frac{\cancel{(a^2\sin^2\theta + b^2\cos^2\theta)}}{\cancel{a^2\sin^2\theta + b^2\cos^2\theta}} \quad \cdots \{|x^2| = x^2\}$$

$= b^2 \times 1$

$= b^2$ Ans

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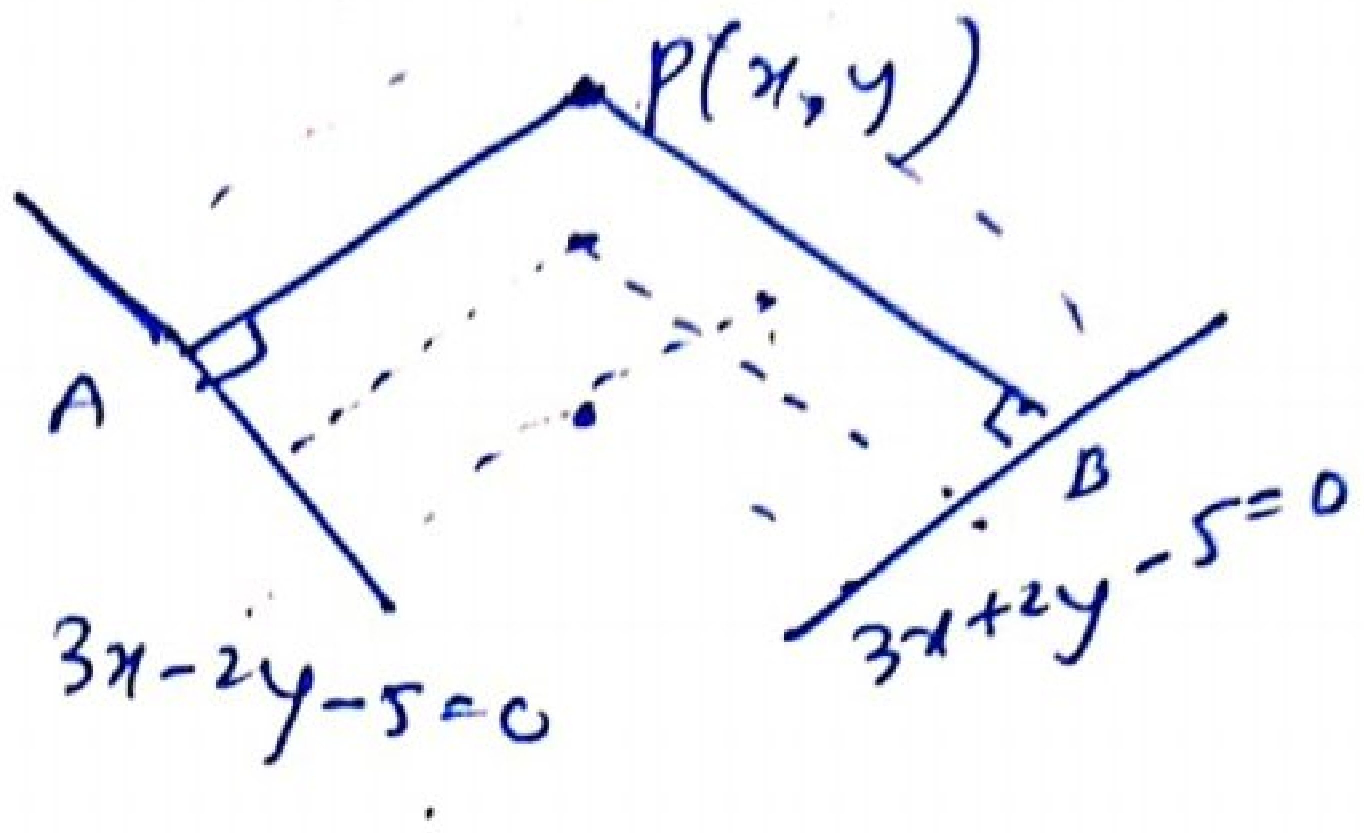
Qn 4 → Show that the path (locus) of a moving point such that its distances from two straight lines $3x-2y=5$ and $3x+2y=5$ are equal is a straight line

Soln

given $PA = PB$




$$\frac{|3x-2y-5|}{\sqrt{9+4}} = \frac{|3x+2y-5|}{\sqrt{9+4}}$$

$\Rightarrow |3x-2y-5| = |3x+2y-5|$

$\Rightarrow 3x-2y-5 = \pm(3x+2y-5)$

$3x-2y-5 = 3x+2y-5$

$-4y = 0$

$\Rightarrow \boxed{y=0}$

equation X-axis

$3x-2y-5 = -3x-2y+5$

$6x = 10$

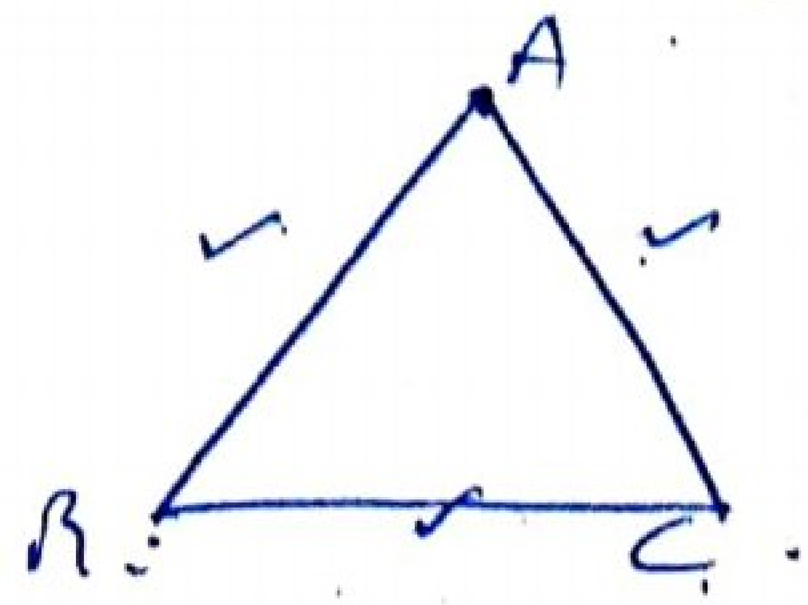
$\boxed{x = 5/3}$

line $\parallel$ to Y-axis

∴ clearly path of a moving point is a straight line

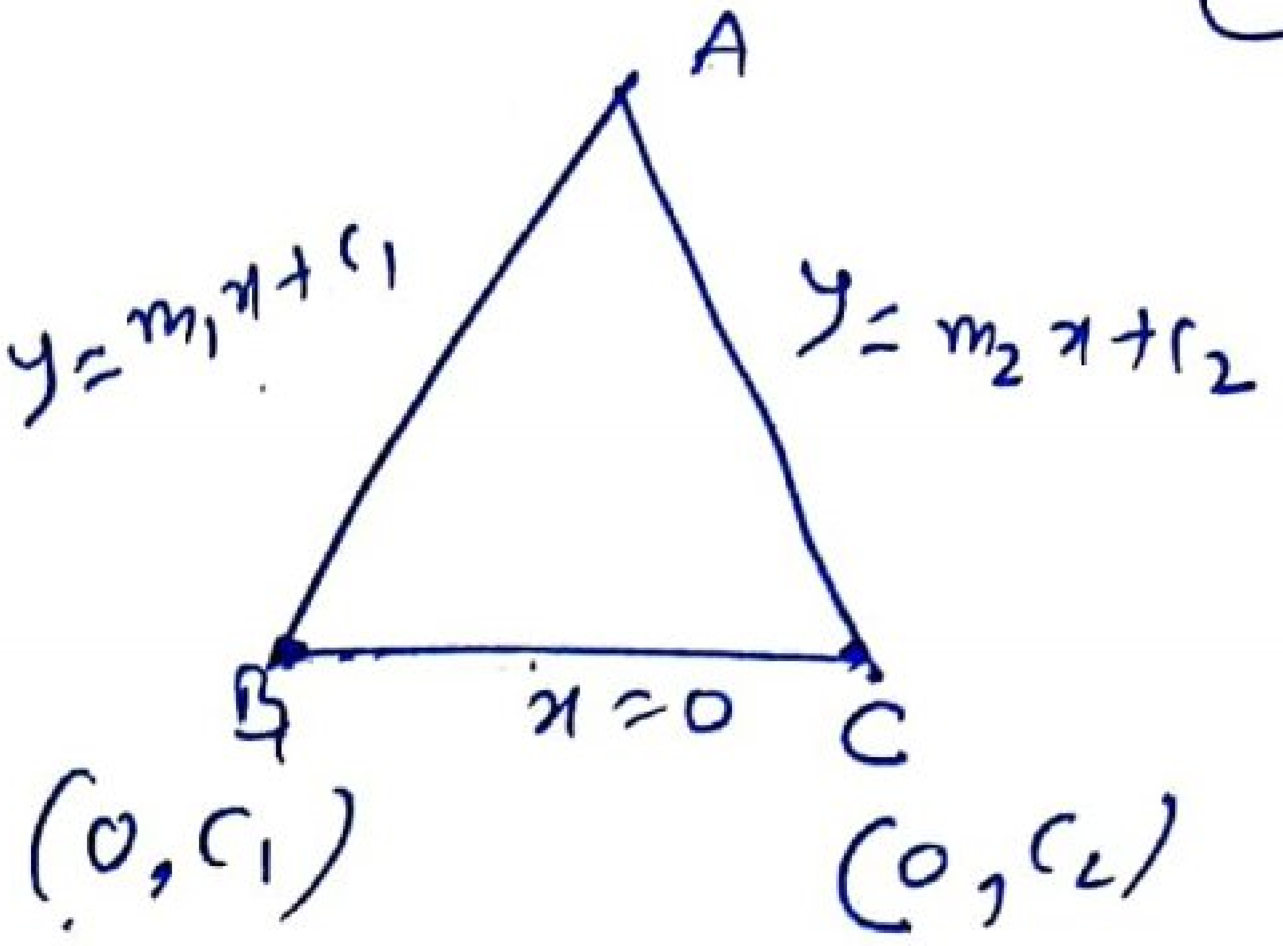
Ans

Qn 5 → Show that the area of the triangle formed by the lines $y = m_1x + c_1$ & $y = m_2x + c_2$ & $x = 0$ is $\frac{(c_1-c_2)^2}{2|m_1-m_2|}$



Soln

Solving equation of AB & AC




$\Rightarrow m_1 x + c_1 = m_2 x + c_2$

$\Rightarrow x(m_1 - m_2) = c_2 - c_1$

$\Rightarrow x = \dfrac{c_2 - c_1}{m_1 - m_2}$ put in eq AB

$$y = \frac{m_1 c_2 - m_1 c_1}{m_1 - m_2} + c_1$$

$$y = \frac{m_1 c_2 - \cancel{m_1 c_1} + \cancel{m_1 c_1} - m_2 c_1}{m_1 - m_2}$$

$$y = \frac{m_1 c_2 - m_2 c_1}{m_1 - m_2}$$

$\therefore A\left(\dfrac{c_2 - c_1}{m_1 - m_2}, \dfrac{m_1 c_2 - m_2 c_1}{m_1 - m_2}\right)$ ; $B(0, c_1)$ & $C(0, c_2)$

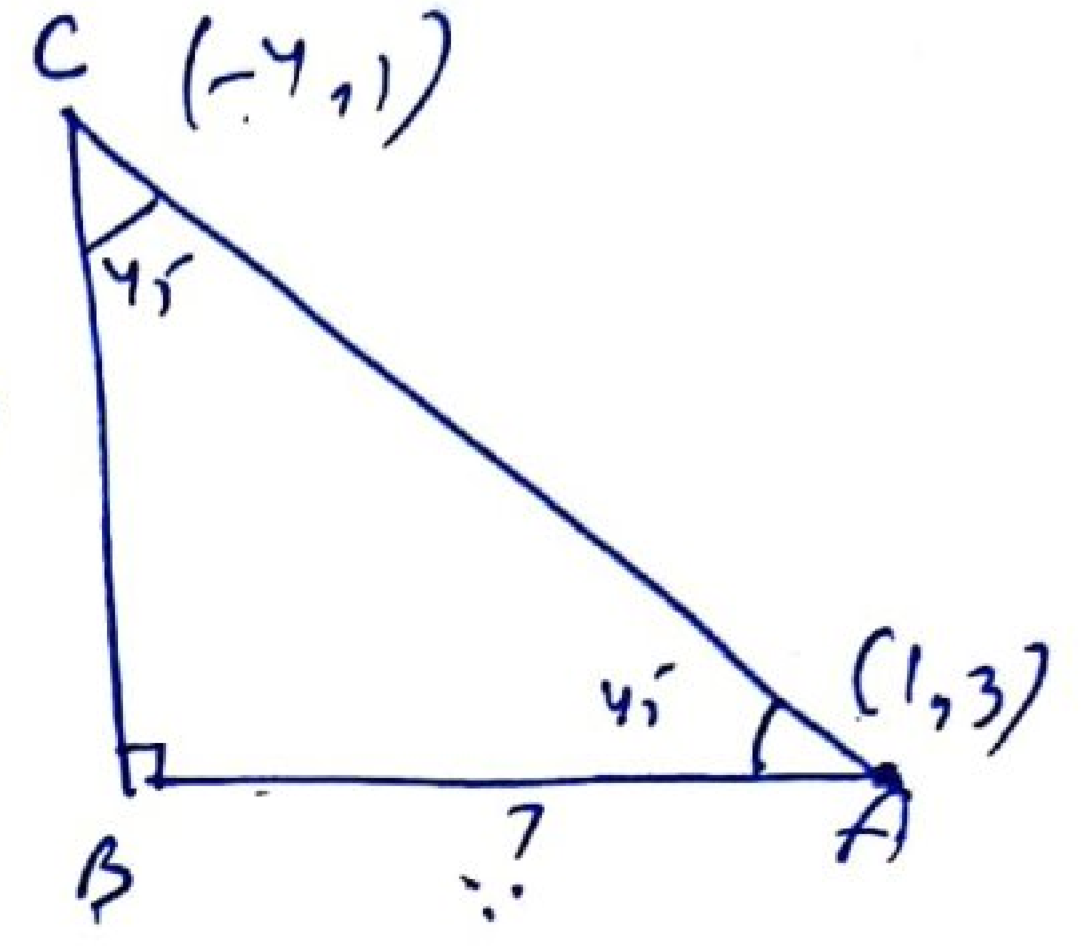
Now Area of $\Delta ABC$

$$= \frac{1}{2}\left| \frac{c_2 - c_1}{m_1 - m_2}(c_1 - c_2) + 0 + 0 \right|$$

$$= \frac{1}{2}\left| -\frac{(c_2 - c_1)^2}{m_1 - m_2} \right|$$

$$= \frac{1}{2}\frac{(c_2 - c_1)^2}{|m_1 - m_2|}$$ Ans. $\quad \ldots \{|x^2| = x^2\}$

Q.No. 6 → The hypotenuse of a right isosceles triangle has its ends at the point (1, 3) & (−4, 1). Find the equation of its legs.




Soln

(·) Slope of AC $= \frac{-2}{-5} = \frac{2}{5}$

$m_1 = 2/5$

(·) let slope of AB $= m_2 = m$

(·) angle b/w them $= 45^\circ$

(·) $\tan(45) = \left| \frac{\frac{2}{5} - m}{1 + \frac{2}{5}m} \right|$

$\Rightarrow 1 = \left| \frac{2 - 5m}{5 + 2m} \right|$

$\Rightarrow \frac{2-5m}{5+2m} = 1$ & $\frac{2-5m}{5+2m} = -1$

$\Rightarrow 2 - 5m = 5 + 2m$ & $2 - 5m = -5 - 2m$

$\Rightarrow 7m = -3$ $\qquad \Rightarrow 3m = 7$

$\Rightarrow m = -3/7$ $\qquad m = 7/3$

∴ Slope of AB are $-3/7$ & $7/3$

When slope of AB $= -3/7$ then slope of BC $= 7/3$

When slope of AB $= 7/3$ then slope of BC $= -3/7$

| Equⁿ of AB | eq of BC |
|---|---|
| ✓ $y - 3 = \frac{-3}{7}(x - 1)$ & | $y - 1 = 7/3(x + 4)$ |
| (OR) ✓ $y - 3 = \frac{7}{3}(x - 1)$ & | $y - 1 = \frac{-3}{7}(x + 4)$ |

Ans

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# STRAIGHT LINES (WORKSHEET NO: 3)

(Class No: 4)

**Qns 1** Find the equation of the line passing through the point of Intersection of the lines $4x+7y-3=0$ and $2x-3y+1=0$ that has equal Intercepts on the axes

Ans $13x+13y=6$

**Qn. 2** Find the area of the triangle formed by the lines

$y-x=0$, $x+y=0$, $x-k=0$ Ans. $k^2$ sq. units

**Qn 3** Find the equation of the line parallel to Y-axis and drawn through the point of Intersection of the lines $x-7y+5=0$ and $3x+y=0$ Ans $22x+5=0$

**Qn 4** The perpendicular from the origin to the line $y=mx+c$ meets it at the point $(-1,2)$. Find the value of m and c Ans $m=1/2$, $c=5/2$

**Qn. 5** Find the equation of the right bisector of the line segment joining the points $(3,4)$ & $(-1,2)$

Ans $2x+y=5$

**Qn. 6** Find the equation of the line perpendicular to the line $x-7y+5=0$ and having X-intercept 3

Ans $7x+y=21$

**Qn. 7** A line perpendicular to the line segment joining the points $(1,0)$ and $(2,3)$ divides it in the ratio $1:n$. Find the equation of the line

Ans $(1+n)x+3(1+n)y=n+11$

Qn. 8 → Find the equation of the line through the point (0, 2) making an angle $\frac{2\pi}{3}$ with the x-axis. Also find the equation of line parallel to it and crossing the y-axis at a distance of 2 units below the origin

Ans $\sqrt{3}x + y - 2 = 0$ and
$\sqrt{3}x + y + 2 = 0$

Qn. 9 → By using concept of equation of a line show that the points (3, 0), (−2, −2) and (8, 2) are collinear

Qn. 10 → Find the angle between the two lines
$\sqrt{3}x + y = 1$ & $\sqrt{3}y + x = 1$

Ans $30^\circ$ & $150^\circ$

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