SOLUTION WORKSHEET No: 2 PROBABILITY

Ans 1 → let $A \to$ student passes the first examination
$B \to$ student passes the 2nd examination

given $P(A) = 0.8$
$P(B) = 0.7$
$P(\text{atleast one of them}) = P(A \cup B) = 0.95$

to find $P(\text{passing both}) = P(A \cap B) = ?$

we have
$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\Rightarrow 0.95 = 0.8 + 0.7 - P(A \cap B)$
$\Rightarrow P(A \cap B) = 1.5 - 0.95$
$\Rightarrow \boxed{P(A \cap B) = 0.55}$ ANS

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Ans 2 →
let $A \to$ student passes English examination
$B \to$ student passes Hindi examination

given
$P(A \cap B) = 0.5$
$P(\text{neither}) = P(A' \cap B') = 0.1$
$P(A) = 0.75$

to find $P(\text{Hindi}) = P(B) = ?$

we have $P(A' \cap B') = 1 - P(A \cup B)$

$\Rightarrow 0.1 = 1 - P(A \cup B)$

$\Rightarrow P(A \cup B) = 1 - 0.1$

$\Rightarrow P(A \cup B) = 0.9$

also we have $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\Rightarrow 0.9 = 0.75 + P(B) - 0.5$

$\Rightarrow P(B) = 0.9 - 0.25$

$\Rightarrow \boxed{P(B) = 0.65}$ Ans

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Ans 3 →

(a) Given $P(A) = \frac{3}{5}$

$P(B) = 1/5$

A & B are mutually exclusive events

$\Rightarrow P(A \cap B) = 0$

we have

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\Rightarrow P(A \cup B) = \frac{3}{5} + \frac{1}{5} - 0$

$\Rightarrow \boxed{P(A \cup B) = \frac{4}{5}}$ Ans

(b) Given $P(\text{not } E \text{ or not } F) = 0.25$

$\Rightarrow P(E' \cup F') = 0.25$

By demorgan's law

$\Rightarrow P(E \cap F)' = 0.25$

$\Rightarrow 1 - P(E \cap F) = 0.25$

$\Rightarrow P(E \cap F) = 1 - 0.25 = 0.75 \neq 0$

$\therefore$ E & F are not mutually exclusive events Ans

Ans 4 → Given $P(A) = 0.54$

$P(B) = 0.69$

$P(A \cap B) = 0.35$

(i) $P(A' \cap B') = 1 - P(A \cup B)$

$= 1 - [P(A) + P(B) - P(A \cap B)]$

$= 1 - [0.54 + 0.69 - 0.35]$

$= 1 - 0.88$

$\boxed{P(A' \cap B') = 0.12}$ Ans

(2) $P(B \cap A') = P(B) - P(A \cap B)$

$= 0.69 - 0.35$

$\boxed{P(B \cap A') = 0.34}$ Ans

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Ans 5 →

total cards = 52

kings = 4

non-kings = 48

total No of ways of drawing 7 cards from 52 cards $= {}^{52}C_7$

(i) all kings : that means 4 king & 3 non-kings

$= {}^4C_4 \times {}^{48}C_3$

Required prob $= \dfrac{{}^4C_4 \times {}^{48}C_3}{{}^{52}C_7} = \dfrac{1 \times \dfrac{\cancel{48} \times \cancel{47} \times \cancel{46}}{\cancel{6}}}{\dfrac{52 \times 51 \times 50 \times 49 \times \cancel{48} \times \cancel{47} \times \cancel{46}}{7 \times \cancel{6} \times 5 \times 4 \times 3 \times 2 \times 1}}$

$= \dfrac{\cancel{7} \times \cancel{5} \times \cancel{4} \times \cancel{3} \times \cancel{2} \times 1}{\underset{13}{\cancel{52}} \times \underset{17}{\cancel{51}} \times \underset{5}{\cancel{10}} \times \underset{7}{\cancel{49}}}$

$\therefore$ Req prob $= \frac{1}{13\times17\times5\times7} = \frac{1}{7735}$ Ans

(2) 3 kings $\Rightarrow$ 3 kings & 4 non-kings

$= {}^4C_3 \times {}^{48}C_4$

Required prob $= \frac{{}^4C_3 \times {}^{48}C_4}{{}^{52}C_7}$

$= \frac{\cancel{4} \times \frac{\cancel{48}\times\cancel{47}\times\cancel{46}\times\cancel{45}}{\cancel{4}\times3\times\cancel{2}\times1}}{\frac{52\times51\times50\times49\times\cancel{48}\times\cancel{47}\times\cancel{46}}{7\times6\times5\times4\times\cancel{3}\times\cancel{2}\times1}}$

$= \frac{\cancel{7}\times6\times\cancel{5}\times\cancel{45}\,9\times4}{52\times51\times\underset{\underset{2}{10}}{\cancel{50}}\times\underset{7}{\cancel{49}}}$

$= \frac{9\times6\,\cancel{3}}{\underset{26}{\cancel{52}}\times\underset{17}{\cancel{51}}\times2\times7}$

$= \frac{9\times4}{26\times17\times14}$

$\boxed{\text{Req prob} = \frac{9}{1547}}$ Ans

(3) atleast 3 kings : Two cases

3 king & 4 non kings $= {}^4C_3 \times {}^{48}C_4$

or 4 king & 3 non-king $= {}^4C_4 \times {}^{48}C_3$

Req prob = prob of (i) part + prob of (ii) part

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$$\text{Req prob} = \frac{1}{7735} + \frac{9}{1547}$$

$$= \frac{1 + 45}{7735}$$

$$\boxed{\text{Req prob} = \frac{46}{7735}} \quad \text{ANS}$$

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Q.No 6 → total people = 6000

females = 2000

over 50 year old = 1200

females over 50 year old = 30% of 2000

$$= \frac{30}{100} \times 2000 = 600$$

let A → person chosen is a female

B → person chosen is over 50 years old

$A \cap B$ → female with over 50 years old

$$P(A) = \frac{2000}{6000} \; ; \; P(B) = \frac{1200}{6000}$$

$$P(A \cap B) = \frac{600}{6000}$$

Required prob: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$= \frac{2000}{6000} + \frac{1200}{6000} - \frac{600}{6000}$$

$$= \frac{2600}{6000}$$

$$\boxed{\text{Req prob} = \frac{13}{30}} \quad \text{ANS}$$

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Qn 7 → 3 coins

$S = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$

let A → getting atmost two tails

B → getting ~~atmost~~ atleast two heads

$A = \{HHH, HHT, HTH, HTT, THH, THT, TTH\}$

$B = \{HHT, HTH, THH, HHH\}$

$A \cap B = \{HHT, HTH, THH, HHH\}$

$P(A) = \frac{7}{8}$

$P(B) = \frac{4}{8}$

$P(A \cap B) = \frac{4}{8}$

Required probability = $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= \frac{7}{8} + \cancel{\frac{4}{8}} - \cancel{\frac{4}{8}}$

$\boxed{\text{Req prob} = \frac{7}{8}}$ Ans

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Qn. 8 → let A → both the cards are Red

B → both the cards are kings

$A \cap B$ → both are Red kings

$P(A) = \frac{26C_2}{52C_2}$; $P(B) = \frac{4C_2}{52C_2}$; $P(A \cap B) = \frac{2C_2}{52C_2}$

Required probability

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= \frac{^{26}C_2}{^{52}C_2} + \frac{^{4}C_2}{^{52}C_2} - \frac{^{2}C_2}{^{52}C_2}$$

$$= \frac{^{26}C_2 + {}^{4}C_2 - {}^{2}C_2}{^{52}C_2}$$

$$= \frac{325 + 6 - 1}{1326} \quad \text{---} \left\{ {}^{n}C_2 = \frac{n(n-1)}{2} \right\}$$

$$= \frac{330}{1326}$$

$$\boxed{\text{Req prob} = \frac{55}{221}} \text{ ANS}$$

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Qn 9 →

let $A \rightarrow$ sum of two numbers on dice is divisible by 3 {i.e sum = 3, 6, 9, 12}

$B \rightarrow$ Sum of two numbers on the dice is divisible by 4 {i.e Sum = 4, 8, 12}

$A = \{(1,2), (2,1), (1,5), (5,1), (2,4), (4,2)\ (3,3), (3,6), (6,3), (4,5), (5,4), (6,6)\}$

$B = \{(1,3), (3,1)\ (2,2), (2,6), (6,2), (3,5), (5,3), (4,4), (6,6)\}$

$A \cap B = \{(6,6)\}$

$P(A) = \frac{12}{36}$; $P(B) = \frac{9}{36}$; $P(A \cap B) = \frac{1}{36}$

Required Probability

P(neither nor)

$\Rightarrow \quad P(A' \cap B') = 1 - P(A \cup B)$

$= 1 - [P(A) + P(B) - P(A \cap B)]$

$= 1 - \left[\frac{12}{36} + \frac{9}{36} - \frac{1}{36}\right]$

$= 1 - \frac{20}{36}$

$= \frac{16}{36}$

$\boxed{\text{Req prob} = \frac{4}{9}}$ Ans

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Ans. 10 → total bolts = 30

total nuts = 40

total items = 30+40 = 70

Rusted bolts = 15

Rusted bolts = 20

total Rusted items = 35

let A → both are Rusted items

B → both items are bolts

$A \cap B$ → both the rusted bolts

$P(A) = \frac{{}^{35}C_2}{{}^{70}C_2}$; $P(B) = \frac{{}^{30}C_2}{{}^{70}C_2}$; $P(A \cap B) = \frac{{}^{15}C_2}{{}^{70}C_2}$

Required probability

$$P(A\cup B) = P(A) + P(B) - P(A\cap B)$$

$$= \frac{{}^{35}C_2}{{}^{70}C_2} + \frac{{}^{30}C_2}{{}^{70}C_2} - \frac{{}^{15}C_2}{{}^{70}C_2}$$

$$= \frac{{}^{35}C_2 + {}^{30}C_2 - {}^{15}C_2}{{}^{70}C_2}$$

$$= \frac{\frac{35\times 34}{\cancel{2}} + \frac{30\times 29}{\cancel{2}} - \frac{15\times 14}{\cancel{2}}}{\frac{70\times 69}{\cancel{2}}}$$

$$= \frac{1190 + 870 - 210}{4830}$$

$$= \frac{185\cancel{0}}{483\cancel{0}}$$

$$\therefore \boxed{\text{Required probability} = \frac{185}{483}}$$ Ans

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Qn. 11 → Given $P(B) = \frac{3}{2} P(A)$

$P(C) = \frac{1}{2} P(B)$

given A, B, C are mutually exclusive & exhaustive events

$\therefore P(A) + P(B) + P(C) = 1$

$\Rightarrow P(A) + \frac{3}{2} P(A) + \frac{1}{2}\left(\frac{3}{2} P(A)\right) = 1$

$\Rightarrow P(A)\left(1 + \frac{3}{2} + \frac{3}{4}\right) = 1$

$\Rightarrow P(A)\left(\frac{13}{4}\right) = 1$

$\Rightarrow \boxed{P(A) = \frac{4}{13}}$ Ans

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Ans. 12 ⇒

A → Card selected is a king

B → Card selected is a heart

C → Card selected is Red

$P(A) = \frac{4}{52}$; $P(B) = \frac{13}{52}$; $P(C) = \frac{26}{52}$

$P(A \cap B) = P(\text{king of heart}) = \frac{1}{52}$

$P(B \cap C) = P(\text{heart of Red}) = \frac{13}{52}$

$P(C \cap A) = P(\text{Red king}) = \frac{2}{52}$

$P(A \cap B \cap C) = P(\text{Red king heart}) = \frac{1}{52}$

Required Prob

$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) + P(A \cap B \cap C)$

$= \frac{4}{52} + \frac{13}{52} + \frac{26}{52} - \frac{1}{52} - \frac{13}{52} - \frac{2}{52} + \frac{1}{52}$

$= \frac{28}{52}$

$\therefore \boxed{\text{Required Prob} = \frac{7}{13}}$ Ans

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Qn. 13 →

(·) P(atleast one of A & B occur) = 0·6

$\Rightarrow P(A \cup B) = 0.6$

(·) P(simultaneously occur) = 0·2

$\Rightarrow P(A \cap B) = 0.2$

we have $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$0.6 = 1 - P(A') + 1 - P(B') - 0.2$

$\Rightarrow P(A') + P(B') = 2 - 0.2 - 0.6$

$\Rightarrow \boxed{P(A') + P(B') = 1.2}$ Ans

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