॥ जय श्री गिरिराज जी महाराज ॥

ULTIMATE MATHEMATICS

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CLASS NO=6 (S-6)

Q.No.1 → From 50 students taking examinations in Maths, physics and chemistry, each of the student has passed in atleast one of the subject, 37 passed mathematics, 24 physics and 43 chemistry. Atmost 19 passed mathematics and physics, atmost 29 Maths and chemistry and atmost 20 physics and chemistry. what is the largest possible number that could have passed all three examination?

Sol, A → Maths B → physics ; C → Chemistry

= Given $n(A \cup B \cup C) = 50$

= $n(A) = 37$, $n(B) = 24$; $n(C) = 43$

$n(A \cap B) \leq 19$; $n(B \cap C) \leq 20$; $n(A \cap C) \leq 29$

to find $n(A \cap B \cap C)$

=

$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(C \cap A) + n(A \cap B \cap C)$

$\Rightarrow 37 + 24 + 43 - n(A \cap B) - n(B \cap C) - n(C \cap A) + n(A \cap B \cap C) = 50$

$\Rightarrow 37 + 24 + 43 - 19 - 20 - 29 + n(A \cap B \cap C) \leq 50$

$\Rightarrow n(A \cap B \cap C) \leq 14$ ∴ largest no = 14 Ans

Distributive $\wedge$ $A \wedge (B \cup C) = (A \cap B) \cup (A \cap C)$

Dually $(A \cap B) \cup (A \cap C) = A \cap (B \cup C)$

Q.N 2 Show that

$$[(A \cup B \cup C) \cap (A \cap B' \cap C')'] \cap C' = B \cap C'$$

Solution

$$[(A \cup B \cup C) \cap (A \cap B' \cap C')'] \cap C'$$

$= [(A \cup B \cup C) \cap (A' \cup B \cup C)] \cap C'$ --- {De-morgan's law}

$= [(B \cup C) \cup (A \cap A')] \cap C'$ --- {distributive prop}

$= [(B \cup C) \cup \phi] \cap C'$ --- {$A \cup \phi = A$}

$= [B \cup C] \cap C'$

$= (B \cap C') \cup (C \cap C')$ --- {distributive prop}

$= (B \cap C') \cup \phi$

$= B \cap C'$ Ans

Q.N-3 → $A = \{ x : x$ is a positive factor of the number $2^{p-1} \cdot (2^p - 1)$ where $(2^p - 1)$ is a prime number$\}$

write Roster form:

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eg $2^5 \cdot 7 \rightarrow 1, 7$ $= \{1, 7, 2^1, 2^2, \ldots 2^5\}$

$1, 2^1, 2^2, 2^3, 2^4, 2^5$

$A = \{ x : 2^{p-1} \cdot (2^p - 1)$

$A = \{ 1, (2^p - 1), 2^1, 2^2, 2^3, \ldots 2^{p-1} \}$

Remarks

Q.1 $X = \{ 8^n - 7n - 1 : n \in N \}$

$Y = \{ n : n \in N \text{ and } 49n - 49 \}$

Prove (i) $X \subset Y$ (2) $Y \subset X$ (3) $X = Y$ (4) $X \cap Y = \phi$

Sol $X = \{ 8^n - 7n - 1 ; n \in N \}$

$\checkmark$ $X = \{0, 49, 490, \ldots\}$

$n=1$
$8 - 7 - 1 = 0$
$n=2$
$64 - 14 - 1$
$n=3$ $512 - 21 - 1$

$Y = \{ 49n - 49 : n \in N \}$

$\checkmark$ $Y = \{0, 49, 98, \ldots\}$

$n=1$
$49 - 49 = 0$
$n=2$ $98 - 49 = 49$
$147 - 49$

$\Rightarrow X \subset Y$ Ans

SETS

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WORKSHEET NO:5 (Class 5-6)

Q.No 1 Write in Roster form

(1) $A = \{x : x$ is a +ve integer less than 10 and $2^x - 1$ is an odd number$\}$

(2) $A = \{t : t^3 = t \ , \ t \in R\}$

(3) $A = \left\{x : \frac{x+5}{x-7} - 5 = \frac{4x-40}{13-x}\right\}$

(4) $A = \{x : x^4 - 5x^2 + 6 = 0 \ ; \ x \in R\}$

Q.No 2 If X and Y are two sets then show that

(i) $Y \subset (X \cup Y)$

(2) $(X \cap Y) \subset X$

Q.No 3 → Show that $(A - B) \cap (C - B) = (A \cap C) - B$

Q.No 4 Show that $(A - B) \cap (C - B) = A - (B \cup C)$

Q.No 5 Show that $A - (B - C) = (A - B) - C$

Q.No 6 Show that $A - (A - B) = A \cap B$

Q.No 7 Show $X \cap (X \cup Y)' = \phi$

Q.No 8 Show $[(A' \cup B') - A]' = A$

Q.No 9 Show $[B' \cup (B' - A)]' = B$

worksheet = 5 (class 5-6)

Q.10 → Each set X_i contains 5 elements and each set Y_i contains 2 elements and $\bigcup_{i=1}^{20} X_i = \bigcup_{i=1}^{n} Y_i = S$. If each element of S belong to exactly 10 of the X_i's and to exactly 4 of the Y_i's, then find value of n

ANSWERS

(1) (i) $A = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

(ii) $A = \{0, -1, 1\}$

(iii) $A = \{10\}$

(iv) $A = \{-\sqrt{2}, \sqrt{2}, -\sqrt{3}, \sqrt{3}\}$

(10) $n = 20$