ULTIMATE MATHEMATICS

Maths: By AJAY MITTAL: 8910067390

Chapter: SEQUENCE & SERIES

← CLASS NO: 2 →

A.P

Q No 1 → If the Sum of n terms of an A.P is $np + \frac{1}{2}n(n-1)Q$, where P & Q are constants find the common difference

Soln Given $S_n = np + \frac{1}{2}n(n-1)Q$

$n=1$ $\quad S_1 = P = a_1$

$n=2$ $\quad S_2 = 2P + Q = a_1 + a_2$

$\Rightarrow 2P + Q = P + a_2$

$\Rightarrow a_2 = P + Q$

Now $d = a_2 - a_1 = P + Q - P = Q$

$\therefore$ Common difference $= Q$ Ans

Q No 2 → The Sums of n terms of two A.P's are in the ratio $5n+4 : 9n+6$. Find the ratio of their 18^{th} terms

Soln

	1st A.P	2nd A.P
	a	a'
	d	d
	S_n	S'_n
	a_{18}	a'_{18}

Given $\frac{S_n}{S'_n} = \frac{5n+4}{9n+6}$ To find $\frac{a_{18}}{a'_{18}} = \frac{a+17d}{a'+17d'}$

we have

$$\frac{S_n}{S'_n} = \frac{5n+4}{9n+6}$$

$$\Rightarrow \frac{\frac{n}{2}[2a+(n-1)d]}{\frac{n'}{2}[2a'+(n-1)d']} = \frac{5n+4}{9n+6}$$

put $n = 35$

$$\Rightarrow \frac{2a+34d}{2a'+34d'} = \frac{35\times5+4}{35\times9+6}$$

$$\Rightarrow \frac{a+17d}{a'+17d'} = \frac{179}{321}$$

$\therefore$ Required Ratio: 179 : 321 Ans

Qn 3 → Sum of the first p, q and r terms of an A.P are a, b and c respectively prove that

$$\frac{a}{p}(q-r) + \frac{b}{q}(r-p) + \frac{c}{r}(p-q) = 0$$

Soln

$$S_p = a = \frac{p}{2}[2A+(p-1)d]$$

$$S_q = b = \frac{q}{2}[2A+(q-1)d]$$

$$S_r = c = \frac{r}{2}[2A+(r-1)d]$$

L.H.S $\frac{a}{p}(q-r) + \frac{b}{q}(r-p) + \frac{c}{r}(p-q)$

$= \frac{1}{2}[2A+(p-1)d](q-r) + \frac{1}{2}[2A+(q-1)d](r-p) +$

$\frac{1}{2}[2A+(r-1)d](p-q)$

$$= \frac{1}{2}\left[2A(q-1) + (p-1)d(q-1) + 2A(1-p) + (q-1)d(1-p) + 2A(p-q) + (1-1)d(p-q)\right]$$

$$= \frac{1}{2}\left[2Aq - 2A - 2Ap + 2A + dpq - dq - dp + d + 2Ap - 2A - dq + d + 2Aq - 2Ap - 2A + dp - d\right]$$

$$= \frac{1}{2}\left[2A \times 0 + d \times 0\right]$$

$$= 0 \quad \underline{\text{Ans}}$$

Qu 4 → Insert 5 numbers between 8 and 26 such that the resulting sequence is an A.P

Solution then $a = 8$ & $b = 26$

$n = 5$

$$d = \frac{b-a}{n+1} = \frac{26-8}{5+1} = \frac{18}{6} = 3$$

the 5 numbers to be inserted are

A_1, A_2, A_3, A_4, A_5

$A_1 = a + d = 8 + 3 = 11$

$A_2 = a + 2d = 8 + 6 = 14$

$A_3 = a + 3d = 8 + 9 = 17$

$A_4 = 20$

$A_5 = 23$

Qu 5 → Between 1 and 31, m numbers have been inserted in such a way that the resulting sequence is an A.P and the ratio of

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7th and $(m-1)^{th}$ number is 5:9
Find the value of m.

Sol: then $a=1$, $b=31$, $n=m$

$$d = \frac{b-a}{n+1} = \frac{30}{m+1}$$

Given

$$\frac{A_7}{A_{m-1}} = \frac{5}{9}$$

$$\Rightarrow \frac{a+7d}{a+(m-1)d} = \frac{5}{9}$$

$$\Rightarrow \frac{1+\frac{7\times 30}{m+1}}{1+\frac{(m-1)\times 30}{m+1}} = \frac{5}{9}$$

$$\Rightarrow \frac{m+1+210}{m+1+30m-30} = \frac{5}{9}$$

$$\Rightarrow \frac{m+211}{31m-29} = \frac{5}{9}$$

$$\Rightarrow 9m + 1899 = 155m - 145$$

$$\Rightarrow 155m - 9m = 1899 + 145$$

$$\Rightarrow 146m = 2044$$

$$\Rightarrow m = \frac{2044}{146}$$

$$\Rightarrow \boxed{m = 14} \text{ Ans}$$

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Q.No. 6 → If $\frac{a^n+b^n}{a^{n-1}+b^{n-1}}$ is the A.M between a & b
Find the value of n

Soln $\underline{\text{Given}}$ $\frac{a^n+b^n}{a^{n-1}+b^{n-1}} = \frac{a+b}{2}$ --- $\because AM = \frac{a+b}{2}$

$\Rightarrow 2a^n + 2b^n = (a+b)(a^{n-1}+b^{n-1})$

$\Rightarrow 2a^n + 2b^n = a^n + ab^{n-1} + ba^{n-1} + b^n$

$\Rightarrow a^n + b^n = ab^{n-1} + ba^{n-1}$

Shifting

$\Rightarrow a^n - ba^{n-1} = ab^{n-1} - b^n$

$\Rightarrow a^{n-1}(a-b) = b^{n-1}(a-b)$

$\Rightarrow a^{n-1} = b^{n-1}$

$\Rightarrow \frac{a^{n-1}}{b^{n-1}} = 1$

$\Rightarrow \left(\frac{a}{b}\right)^{n-1} = 1$

$\Rightarrow \left(\frac{a}{b}\right)^{n-1} = \left(\frac{a}{b}\right)^0$

$\Rightarrow n-1 = 0$

$\Rightarrow (n=1)$ Ans

Q.No. 7 → Find the sum of integers from 1 to 100 which are divisible by 2 or 5

Soln (2, 4, 6, 8, 10, 12 ---- 100), (5, 15, 25, --- 95)

Let $S = (2+4+6+8+---+100) + (5+15+25+---+95)$

AP: $a=2, d=2, n=50$ AP: $a=5, d=10, n=10$

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$$S = \frac{50}{2}[4 + (49)2] + \frac{10}{2}[10 + (9)10]$$

$$= 25(102) + 5(100)$$

$$= 2550 + 500$$

$$= 3050 \quad \underline{\text{Ans}}$$

Qu 8 → If $a\left(\frac{1}{b}+\frac{1}{c}\right), b\left(\frac{1}{c}+\frac{1}{a}\right), c\left(\frac{1}{a}+\frac{1}{b}\right)$ are in AP
Sood
Show that a, b, c are in AP

Sol

$a\left(\frac{1}{b}+\frac{1}{c}\right), b\left(\frac{1}{c}+\frac{1}{a}\right), c\left(\frac{1}{a}+\frac{1}{b}\right)$ are in AP

$\Rightarrow a\left(\frac{1}{b}+\frac{1}{c}\right)+1, b\left(\frac{1}{c}+\frac{1}{a}\right)+1, c\left(\frac{1}{a}+\frac{1}{b}\right)+1$ are also in AP

$\Rightarrow a\left(\frac{1}{b}+\frac{1}{c}+\frac{1}{a}\right), b\left(\frac{1}{c}+\frac{1}{a}+\frac{1}{b}\right), c\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)$ are in AP

$\Rightarrow$ divide by $\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)$

$\Rightarrow$ a, b, c are in AP $\underline{\text{Any}}$

Qu 9 → The difference between any two consecutive interior angles of a polygon is 5°. If the smallest angle is 120°. Find the number of the sides of the polygon.

Sol

(·) Let the number of sides be 'n'

(·) Sum of Interior angles

$120°, 125°, 130°, \ldots\ldots$ n terms

(·) AP $a = 120, \quad d = 5°$

Sequence (Class No. 2) (7)

(.) Sum of all these Interior angles $= \frac{n}{2}(240 + (n-1)5)$

$= \frac{n}{2}(235 + 5n)$ --(1)

Sol^n (.) also Sum of all the r angles in any polygon with n sides $=$ $\underbrace{(n-2)\times 180^\circ}$ --(2)
Interior

From (1) & (2)

$\frac{n}{2}[235 + 5n] = (n-2)\times 180$

$\Rightarrow 235n + 5n^2 = 360n - 720$

$\Rightarrow 5n^2 - 125n + 720 = 0$

$\Rightarrow n^2 - 25n + 144 = 0$

$\Rightarrow (n-9)(n-16) = 0$

$\Rightarrow n = 9, \quad n = 16$

$a_9 = a + 8d = 120 + 40 = 160^\circ < 180^\circ$

$a_{16} = a + 15d = 120 + 15\times 5 = 120 + 75 = 195^\circ > 180^\circ$

$\therefore n = 16$ (Rejected)

$\therefore (n = 9)$ Ans

Q.No 10 → A farmer buys a used tractor for Rs 12,000. He pays Rs 6000 cash and agrees to pay the balance in annual instalments of Rs 500 plus 12% Interest on the unpaid amount. How much will the tractor cost him?

Sol^n no. of Installments $= \frac{6000}{500} = 12$

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Sequence (Class No: 2)

1st Installment = $500 + \frac{12}{100}(6000)$

$= 500 + 720 = 1220$

2nd Installment = $500 + \frac{12}{100}(5500)$

$= 500 + 660 = 1160$

3rd Install:- $500 + \frac{12}{100} \times (5000)$

$= 500 + 600 = 1100$

Sequence of Installment

1220, 1160, 1100, ---- 12 term

Sn AP $a = 1220, \quad d = -60, \quad n = 12$

Sum $S_{12} = \frac{12}{2}[2440 + (11)(-60)]$

$= 6[2440 - 660]$

$= 6(1780)$

$= 10680$

∴ Cost of tractor = $6000 + 10680$

$= ₹ 16680$ Ans

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Qu. 1 If the Sum of n terms of an A.P is $(2n + pn^2)$ where p and q are constants. Find the common difference Ans: 2p

Qu. 2 → The Sum of n terms of two A.P's are in the ratio $3n+8 : 7n+15$. Find the ratio of their 12^{th} terms Ans 7:16

Qu. 3 → The ratio of the Sums of m and n terms of an A.P is $m^2 : n^2$. Show that the ratio of m^{th} and n^{th} term is $(2m-1) : (2n-1)$

Qu. 4 → Insert ~~5~~ ~~n~~ numbers b/w 3 and 24 such that the resulting sequence is an A.P

Ans 6, 9, 12, 15, 18, 21

Qu. 5 → If the Sum of n terms of an AP is $3n^2 + 5n$ and its m^{th} term is 164, find the value of m Ans m = 27

Qu. 6 → If the Sum of first p terms of an AP is equal to the sum of the first q terms, then find the sum of the first (p+q) terms

Ans = 0

Qu. 7 → The sum of the first four terms of an AP is 56. The sum of the last four terms is 112. If its first term is 11. Find the

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number of terms

Qn. 8 → Shamshad Ali buys a scooter for Rs 22000 He pays Rs 4000 cash and agrees to pay the balance in annual installment of Rs 1000 plus 10% interest on the unpaid amount. How much will the scooter cost him?

Ans Rs 39100

Qn 9 → The p^{th}, q^{th} and s^{th} terms of an AP are a, b, c respectively. Show that

$$(q-r)a + (r-p)b + (p-q)c = 0$$

Qn. 10

If the sum of three numbers in AP is 24 and their product is 440. Find the numbers

Hint: use $a-d, a, a+d$

Ans 5, 8, 11

Qn 11 → Let the Sum of n, $2n$, $3n$ terms of an A.P be S_1, S_2 and S_3 respectively, Show that

$$S_3 = 3(S_2 - S_1)$$