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← SOLUTIONS →

← PMI WORKSHEET No 2 →

← In all Qn do P(1) by yourself →

Qn 1 → P(n): $3^{2n+2}-8n-9$ is divisible by 8

Sol: P(k): $3^{2k+2}-8k-9 = 8m$ --- $(m \in Z)$

P(k+1): $3^{2k+4}-8(k+1)-9$

$= 3^{2k+2}\cdot 3^2 - 8k - 8 - 9$

$= (8m+8k+9)\cdot 9 - 8k - 17$ --- {from P(k)}

$= 72m + 72k + 81 - 8k - 17$

$= 72m + 64k + 64$

$= 8(9m + 8k + 8)$

which is div by 8

$\therefore$ P(k+1) is true

$\therefore$ By PMI, P(n) is true for all $n \in N$

Qn 2 → P(n): $41^n - 14^n$ is multiple of 27

Sol: P(k): $41^k - 14^k = 27m$ --- $(m \in Z)$

P(k+1): $41^{k+1} - 14^{k+1}$

$= 41^k \cdot 41 - 14^k \cdot 14$

$= (27m + 14^k)\cdot 41 - 14^k \cdot 14$

$= 27m \times 41 + 14^k \cdot 41 - 14^k \cdot 14$

$= 27m \times 41 + 14^k \cdot (41-14)$

$= 27m \times 41 + 14^k \cdot 27$

CLASSTIME

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Soln PMI (WS-2) (2)

$= 27(41m + 14^k)$ = which is multiply 27

$\therefore$ $P(k+1)$ is true

$\therefore$ By PMI, $P(n)$ is true for all $n \in N$ Ans

Ques 3 → $P(n)$: $n(n^2+5)$ is div by 6

Soln $P(k)$: $k(k^2+5) = 6m$ ... $(m \in z)$

or $k^3 + 5k = 6m$

$P(k+1)$: $(k+1)[(k+1)^2+5]$

$= (k+1)(k^2+2k+6)$

$= k^3 + 2k^2 + 6k + k^2 + 2k + 6$

$= 6m - 5\cancel{k} + 2k^2 + 6k + k^2 + 2k + 6$

$= 6m + 3k^2 + 3k + 6$

$= \cancel{3(2m + k^2 + k + 2)}$

$= 6(m+1) + 3(k^2+k)$ ... { (k^2+k) is always div by 2 for all $k \in N$ }

$= 6(m+1) + 3(2p)$

$= 6[m+1+p]$

which is div by 6

$\therefore$ $P(k+1)$ is true

$\therefore$ By PMI, $P(n)$ is true for all $n \in N$ Ans

Qn 4 → $P(n)$: $x^n - y^n$ is divisible by $x-y$

Soln $P(k)$: $x^k - y^k = (x-y)m$ ... $m \in z$

$P(k+1)$: $x^{k+1} - y^{k+1}$

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Soln PMI (W's 2) (3)

$= x^k \cdot x - y^k \cdot y$

$= [(x-y)m + y^k] \cdot x - y^k \cdot y$ --- {from $p(k)$}

$= (x-y)mx + y^k \cdot x - y^k \cdot y$

$= (x-y)mx + y^k(x-y)$

$= (x-y)[mx + y^k]$ which is div by $x-y$

$\therefore P(k+1)$ is true

$\therefore$ By PMI, $P(n)$ is true for all $n \in N$ **Ans**

Qn. 5 → $P(n)$: $7^n - 2^n$ is divisible by 5

Soln $P(k)$: $7^k - 2^k = 5m$ --- $(m \in z)$

$P(k+1)$: $7^{k+1} - 2^{k+1}$

$= 7^k \cdot 7 - 2^k \cdot 2$ --- (from $P(k)$)

$= (5m + 2^k) \cdot 7 - 2^k \cdot 2$

$= 5m \times 7 + 2^k \cdot 7 - 2^k \cdot 2$

$= 5m \times 7 + 2^k(7-2)$

$= 5m \times 7 + 2^k \cdot 5$

$= 5(7m + 2^k)$ which is div by 5

$\therefore P(k+1)$ is true

$\therefore$ By PMI, $P(n)$ is true for all $n \in N$ **Ans**

Qn 6 → $P(n)$: $n^3 - 7n + 3$ is div by 3

Soln $P(k)$: $k^3 - 7k + 3 = 3m$ --- $(m \in z)$

$P(k+1)$: $(k+1)^3 - 7(k+1) + 3$

$= k^3 + 1 + 3k^2 + 3k - 7k - 7 + 3$

$= k^3 + 3k^2 - 4k - 3$

$= (3m + 7k - 3) + 3k^2 - 4k - 3$ ---- {from P(k)}

$= 3m + 3k^2 + 3k - 6$

$= 3(m + k^2 + k - 2)$ which is div by 3

$\therefore P(k+1)$ is true

$\therefore$ By PMI, $P(n)$ is true for all $n \in N$ Ans

Qns 7 → $P(n)$: $3 \cdot 5^{2n+1} + 2^{3n+1}$ is div by 17

Soln $P(k)$: $3 \cdot 5^{2k+1} + 2^{3k+1} = 17m$ -- $(m \in z)$

$P(k+1)$: $3 \cdot 5^{2k+3} + 2^{3k+4}$

$= 3 \cdot 5^{2k+1} \cdot 5^2 + 2^{3k+1} \cdot 2^3$

$= (17m - 2^{3k+1}) \cdot 25 + 2^{3k+1} \cdot 8$

$= 17m \times 25 - 2^{3k+1} \cdot 25 + 2^{3k+1} \cdot 8$

$= 17m \times 25 - 17 \cdot 2^{3k+1}$

$= 17(25m - 2^{3k+1})$ which is div by 17

$\therefore P(k+1)$ is true

$\therefore$ By PMI, $P(n)$ is true for all $n \in N$.

Soln. PMI (W.S 2)

Qn. 8 → $P(n) : n^3 + 3n^2 + 5n + 3$ is div by 3

Soln $P(k) : k^3 + 3k^2 + 5k + 3 = 3m$ --- $(m \in z)$

$P(k+1) : (k+1)^3 + 3(k+1)^2 + 5(k+1) + 3$

$= k^3 + 1 + 3k^2 + 3k + 3k^2 + 3 + 6k + 5k + 5 + 3$

$= k^3 + 6k^2 + 14k + 12$

$= (3m - 3k^2 - 5k - 3) + 6k^2 + 14k + 12$ --- {from P(k)}

$= 3m + 3k^2 + 9k + 9$

$= 3(m + k^2 + 3k + 3)$ which is div by 3

$\therefore P(k+1)$ is true

$\therefore$ By PMI, $P(n)$ is true for all $n \in N$ Ans

Qn 9 → $P(n)$: Sum of cubes of three consecutive natural numbers is div by 9

or $P(n) : n^3 + (n+1)^3 + (n+2)^3$ is div by 9

Soln $P(k) : k^3 + (k+1)^3 + (k+2)^3 = 9m$ -- $(m \in z)$

(OR) $P(k) : k^3 + k^3 + 1 + 3k^2 + 3k + k^3 + 8 + 6k^2 + 12k = 9m$

$\Rightarrow 3k^3 + 9k^2 + 15k + 9 = 9m$

$P(k+1) : (k+1)^3 + (k+2)^3 + (k+3)^3$

$= k^3 + 1 + 3k^2 + 3k + k^3 + 8 + 6k^2 + 12k + k^3 + 27 + 9k^2 + 27k$

$= 3k^3 + 18k^2 + 42k + 36$

$= 9m - 9k^2 - 15k - 9 + 18k^2 + 42k + 36$

$9m + 9k^2 + 27k + 27$

$= 9(m + k^2 + 3k + 3)$

which is div by 9

$\therefore P(k+1)$ is true

$\therefore$ By PMI, $P(n)$ is true for all $n \in N$

Qn 10 $P(n)$: $5^{2n+2} - 24n - 25$ is div by 576

Soln $P(k)$: $5^{2k+2} - 24k - 25 = 576m$

$P(k+1)$: $5^{2k+4} - 24(k+1) - 25$

$= 5^{2k+2}\, 5^2 - 24k - 24 - 25$

$= (576m + 24k + 25) \cdot 25 - 24k - 49$

$= 576m \times 25 + 600k + 625 - 49 - 24k$

$= 576m \times 25 + \cancel{600k} \; 576k + 576$

$= 576(25m + k + 1)$

which is div by 576

$\therefore P(k+1)$ is true

$\therefore$ By PMI, $P(n)$ is true for all $n \in N$

Qn 11 → $P(n)$: $11^{n+2} + 12^{2n+1}$ is div by 133

Soln $P(k)$: $11^{k+2} + 12^{2k+1} = 133m$ -- (m ∈ z)

$P(k+1)$: $11^{k+3} + 12^{2k+3}$

$= 11^{k+2}\, 11 + 12^{2k+1} \cdot 12^2$

$= \left(133m - 12^{2k+1}\right) \cdot 11 + 12^{2k+1} \cdot 12^2$

$= 133m \times 11 - 12^{2k+1} \times 11 + 12^{2k+1} \times 144$

$= 133m \times 11 + 12^{2k+1}(144 - 11)$

$= 133m \times 11 + 12^{2k+1} \times 133$

$= 133\left(11m + 12^{2k+1}\right)$

which is div by 133

$\therefore P(k+1)$ is true

$\therefore$ By PMI, $P(n)$ is true for all $n \in \mathbb{N}$

Ans