In all Qns P(1) do yourself

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Solution of WORKSHEET NO: 1

PMI

Qn 1 - $P(n): 1^2+2^2+3^2 ---- n^2 = \frac{n(n+1)(2n+1)}{6}$

Soln

$P(k): 1^2+2^2+3^2+--- k^2 = \frac{k(k+1)(2k+1)}{6}$

$P(k+1): 1^2+2^2+3^2 --- k^2+(k+1)^2 = \frac{(k+1)(k+2)(2k+3)}{6}$

Lhs $1^2+2^2+3^2 --- k^2+(k+1)^2$

$= \frac{k(k+1)(2k+1)}{6} + (k+1)^2$ --- { from P(k) }

$= (k+1)\left[\frac{k(2k+1)}{6} + (k+1)\right]$

$= (k+1)\left[\frac{2k^2+k+6k+6}{6}\right]$

$= \frac{1}{6}(k+1)(2k^2+7k+6)$

$= \frac{1}{6}(k+1)[2k^2+4k+3k+6]$

$= \frac{1}{6}(k+1)[2k(k+2)+3(k+2)]$

$= \frac{(k+1)(k+2)(2k+3)}{6}$ = Rhs

$\therefore$ P(k+1) is true

$\therefore$ By PMI, P(n) is true for all values of $n \in N$.

Qn. 2 - $P(n): 1^3+2^3+3^3 --- n^3 = \left[\frac{n(n+1)}{2}\right]^2$

Soln $P(k): 1^2+2^3+3^3 --- k^3 = \left[\frac{k(k+1)}{2}\right]^2$

$P(k+1): 1^3+2^3+3^3 --- k^3+(k+1)^3 = \left[\frac{(k+1)(k+2)}{2}\right]^2$

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solution PMI (W.S. 1)

$\underline{Lhs}$ $1^3+2^3+3^3 - - . k^3 + (k+1)^3$

$= \left[\frac{k(k+1)}{2}\right]^2 + (k+1)^3$ - - . {from P(k)}

$= \frac{k^2(k+1)^2}{4} + (k+1)^3$

$= (k+1)^2\left[\frac{k^2}{4} + (k+1)\right]$

$= (k+1)^2\left[\frac{k^2+4k+4}{4}\right]$

$= \frac{(k+1)^2 (k+2)^2}{4} = Rhs$

$\therefore P(k+1)$ is true

$\therefore$ By PMI, $P(n)$ is true for all $n \in N$ Ans

Qn 3 → $P(n): \frac{1}{1\cdot3} + \frac{1}{5\cdot7} + - - - \frac{1}{7\cdot9} + \frac{1}{(2n+1)(2n+3)} = \frac{n}{3(2n+3)}$

$\underline{Sol}$ $P(k): \frac{1}{1\cdot3} + \frac{1}{5\cdot7} + - - - - - \frac{1}{(2k+1)(2k+3)} = \frac{k}{3(2k+3)}$

$P(k+1) = \frac{1}{1\cdot3} + \frac{1}{5\cdot7} + - - - \frac{1}{(2k+1)(2k+3)} + \frac{1}{(2k+3)(2k+5)} = \frac{k+1}{3(2k+5)}$

$\underline{Lhs}$ $\frac{1}{1\cdot3} + \frac{1}{5\cdot7} + - - - - \frac{1}{(2k+1)(2k+3)} + \frac{1}{(2k+3)(2k+5)}$

$= \frac{k}{3(2k+3)} + \frac{1}{(2k+3)(2k+5)}$ - - - {from P(k)}

$= \frac{k(2k+5) + 3}{3(2k+3)(2k+5)}$

$= \frac{2k^2+5k+3}{3(2k+3)(2k+5)}$

$= \frac{2k^2+2k+3k+3}{3(2k+3)(2k+5)}$

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Solution (PMI W.S 1)

$$= \frac{2k(k+1)+3(k+1)}{3(2k+3)(2k+5)}$$

$$= \frac{(2k+3)(k+1)}{3(\cancel{2k+3})(2k+5)}$$

$$= \frac{k+1}{3(2k+5)} = \text{RHS}$$

$\therefore P(k+1)$ is true

$\therefore$ By PMI, $P(n)$ is true for all $n \in N$ Ans

Qn 4 → $P(n)$: $\frac{1}{1\cdot 4}+\frac{1}{4\cdot 7}+ \cdots \frac{1}{(3n-2)(3n+1)} = \frac{n}{3n+1}$

Sol: $P(k)$: $\frac{1}{1\cdot 4}+\frac{1}{4\cdot 7}+\cdots \frac{1}{(3k-2)(3k+1)} = \frac{k}{3k+1}$

$P(k+1)$: $\frac{1}{1\cdot 4}+\frac{1}{4\cdot 7}+\cdots \frac{1}{(3k-2)(3k+1)}+\frac{1}{(3k+1)(3k+4)} = \frac{k+1}{3k+4}$

LHS $\frac{1}{1\cdot 4}+\frac{1}{4\cdot 7}+\cdots \frac{1}{(3k-2)(3k+1)}+\frac{1}{(3k+1)(3k+4)}$

$$= \frac{k}{3k+1}+\frac{1}{(3k+1)(3k+4)} \quad \text{--- \{from } P(k)\}$$

$$= \frac{k(3k+4)+1}{(3k+1)(3k+4)}$$

$$= \frac{3k^2+4k+1}{(3k+1)(3k+4)}$$

$$= \frac{3k^2+3k+k+1}{(3k+1)(3k+4)}$$

$$= \frac{3k(k+1)+1(k+1)}{(3k+1)(3k+4)}$$

$$= \frac{(\cancel{3k+1})(k+1)}{(\cancel{3k+1})(3k+4)} = \frac{k+1}{3k+4} = \text{RHS}$$

$\therefore P(k+1)$ is true

$\therefore$ By PMI, $P(n)$ is true for all $n \in N$ Ans

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Solution PMI (W.S 1) (3)

Qn. 5 → $P(n): (1+\frac{1}{1})(1+\frac{1}{2}) \cdots\cdots (1+\frac{1}{n}) = (n+1)$

Soln $P(k): (1+\frac{1}{1})(1+\frac{1}{2}) \cdots (1+\frac{1}{k}) = (k+1)$

$P(k+1): (1+\frac{1}{1})(1+\frac{1}{2}) \cdots (1+\frac{1}{k})(1+\frac{1}{k+1}) = (k+2)$

LHS $(1+\frac{1}{1})(1+\frac{1}{2}) \cdots\cdots (1+\frac{1}{k})(1+\frac{1}{k+1})$

$= (k+1)(1+\frac{1}{k+1})$ --- {from P(k)}

$= \cancel{(k+1)}\left(\frac{k+1+1}{\cancel{k+1}}\right)$

$= (k+2)$ = Rhs

∴ P(k+1) is true

∴ By PMI, P(n) is true for all $n \in N$ Ans

Qn. 6 → $P(n): 1.3 + 3.5 + \cdots\cdots (2n-1)(2n+1) = \frac{n(4n^2+6n-1)}{3}$

Soln $P(k): 1.3 + 3.5 + \cdots (2k-1)(2k+1) = \frac{k(4k^2+6k-1)}{3}$

$P(k+1): 1.3 + 3.5 + \cdots (2k-1)(2k+1) + (2k+1)(2k+3)$

$= (k+1)\left[\frac{4(k+1)^2 + 6(k+1) - 1}{6}\right]$

LHS $1.3 + 3.5 + \cdots (2k-1)(2k+1) + (2k+1)(2k+3)$

$= \frac{k(4k^2+6k-1)}{3} + (2k+1)(2k+3)$ ---- {from P(k)}

$= \frac{4k^3 + 6k^2 - k + (4k^2 + 6k + 2k + 3)3}{3}$

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Sachin PMI (WS 1) (5)

$= \frac{4k^3 + 18k^2 + 23k + 9}{3}$

$= \frac{(k+1)(4k^2+14k+9)}{3}$

$= \frac{(k+1)\left[(4k^2+8k+4) + 6k+5\right]}{3}$

$= \frac{(k+1)\left[4(k+1)^2 + 6(k+1) - 1\right]}{3}$

= Rhs

$\therefore$ P(k+1) is true

Hit & Trial method

when $k = -1$

$4k^3 + 18k^2 + 23k + 9 = 0$

$\therefore (k+1)$ is the factor

$$\begin{array}{r|l} & 4k^2 + 14k + 9 \\ k+1 & 4k^3 + 18k^2 + 23k + 9 \\ & -(4k^3 + 4k^2) \\ & 14k^2 + 23k + 9 \\ & -(14k^2 + 14k) \\ & 9k + 9 \\ & 9k + 9 \\ & \times \end{array}$$

$\therefore$ By PMI, P(n) is true for all $n \in N$ Ans

Ans 7 → $P(n): 1.2 + 2.2^2 + 3.2^2 + \ldots n.2^n = (n-1).2^{n+1} + 2$

Sol) $P(k): 1.2 + 2.2^2 + 3.2^2 \ldots \cancel{k}.2^{\cancel{k}} = (k-1).2^{k+1} + 2$

$P(k+1): 1.2 + 2.2^2 + 3.2^2 + \ldots k.2^k + (k+1).2^{k+1} = (k).2^{k+2} + 2$

Lhs) $1.2 + 2.2^2 + 3.2^2 + \ldots k.2^k + (k+1).2^{k+1}$

$= (k-1).2^{k+1} + 2 + (k+1).2^{k+1}$

common

$= 2^{k+1}.(k - \cancel{1} + k + \cancel{1}) + 2$

$= 2^{k+1}.(2k) + 2$

$= 2^{k+1}.2k + 2$

$= 2^{k+2}.k + 2 = $ Rhs

$\therefore$ P(k+1) is true

$\therefore$ By PMI, P(n) is true for all $n \in N$. Ans

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PMI Solution (WS 1) (8)

Qn 8 → $P(n) : 1\cdot2\cdot3 + 2\cdot3\cdot4 \dots n(n+1)(n+2) = \frac{n(n+1)(n+2)(n+3)}{4}$

Soln $P(k): 1\cdot2\cdot3 + 2\cdot3\cdot4 + \dots k(k+1)(k+2) = \frac{k(k+1)(k+2)(k+3)}{4}$

$P(k+1): 1\cdot2\cdot3 + 2\cdot3\cdot4 \dots k(k+1)(k+2) + (k+1)(k+2)(k+3) = \frac{(k+1)(k+2)(k+3)(k+4)}{4}$

Taking L.H.S

$1\cdot2\cdot3 + 2\cdot3\cdot4 + \dots k(k+1)(k+2) + (k+1)(k+2)(k+3)$

$= \frac{k(k+1)(k+2)(k+3)}{4} + (k+1)(k+2)(k+3)$

$= (k+1)(k+2)\left[\frac{k(k+3)}{4} + (k+3)\right]$

$= (k+1)(k+2)(k+3)\left[\frac{k}{4}+1\right]$

$= \frac{(k+1)(k+2)(k+3)(k+4)}{4} = \text{RHS}$

$\therefore P(k+1)$ is true

$\therefore$ By PMI, $P(n)$ is true for all $n \in N$ Ans

Qn 9 → $P(n): a + ar + ar^2 + \dots ar^{n-1} = a\left(\frac{r^n-1}{r-1}\right)$

Soln $P(k): a + ar + ar^2 + \dots ar^{k-1} = a\left(\frac{r^k-1}{r-1}\right)$

$P(k+1): a + ar + ar^2 + \dots ar^{k-1} + ar^k = a\left(\frac{r^{k+1}-1}{r-1}\right)$

$= a\left(\frac{r^k-1}{r-1}\right) + ar^k$ ---- {from $P(k)$}

$= a\left[\frac{r^k-1}{r-1} + r^k\right]$

$= a\left[\frac{r^k-1+r^k(r-1)}{r-1}\right]$

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PMI Soln (W.S. 1) (7)

$$= a\left(\frac{r^{k}-1}{r-1} + \frac{r^{k+1} - r^{k}}{r-1}\right)$$

$$= a\left(\frac{r^{k+1}-1}{r-1}\right) = RHS$$

$\therefore P(k+1)$ is true

$\therefore$ By PMI, $P(n)$ is true for all $n \in N$ Ans

Qn 10 → $P(n): \frac{1}{2} + \frac{1}{4} + \cdots \frac{1}{2^n} = 1 - \frac{1}{2^n}$

Soln: $P(k): \frac{1}{2} + \frac{1}{4} + \cdots \frac{1}{2^k} = 1 - \frac{1}{2^k}$

$P(k+1): \frac{1}{2} + \frac{1}{4} + \cdots \frac{1}{2^k} + \frac{1}{2^{k+1}} = 1 - \frac{1}{2^{k+1}}$

LHS $\frac{1}{2} + \frac{1}{4} + \cdots \frac{1}{2^k} + \frac{1}{2^{k+1}}$

$$= 1 - \frac{1}{2^k} + \frac{1}{2^{k+1}} \quad \text{--- \{from } P(k)\}$$

$$= 1 - \left(\frac{1}{2^k} - \frac{1}{2^{k+1}}\right)$$

$$= 1 - \left[\frac{2-1}{2^{k+1}}\right]$$

$$= 1 - \frac{1}{2^{k+1}} = RHS$$

$\therefore P(k+1)$ is true

$\therefore$ By PMI, $P(n)$ is true for all $n \in N$. Ans

Qn 11 → $P(n): 1 + 3 + 3^2 + \cdots 3^{n-1} = \frac{3^n - 1}{2}$

Soln: $P(k): 1 + 3 + 3^2 + \cdots 3^{k-1} = \frac{3^k - 1}{2}$

$P(k+1): 1 + 3 + 3^2 \cdots 3^{k-1} + 3^k = \frac{3^{k+1} - 1}{2}$

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PMI Solution (W.S. 1) (8)

LHS $P(k+1)$: $1+3+3^2 --- 3^{k-1} + 3^k$

$= \frac{3^k-1}{2} + 3^k$ --- {from $P(k)$}

common

$= \frac{3^k-1+2\cdot 3^k}{2}$

$= \frac{3^k(1+2)-1}{2}$

$= \frac{3^k\cdot 3-1}{2}$

$= \frac{3^{k+1}-1}{2} = $ RHS

$\therefore P(k+1)$ is true

$\therefore$ By PMI, $P(n)$ is true for all $n \in N$ Ans

Qn 12 → $P(n)$: $\left(1-\frac{1}{2^2}\right)\left(1-\frac{1}{3^2}\right) ---- \left(1-\frac{1}{n^2}\right) = \frac{n+1}{2n}$ $n \geq 2$

Soln $P(2)$: $\left(1-\frac{1}{2^2}\right)\left(\cancel{\quad}\right) = \frac{2+1}{2(2)}$

$= \left(1-\frac{1}{4}\right)\left(\cancel{\quad}\right) = \frac{3}{4}$

$= \left(\frac{3}{4}\right)\left(\cancel{\quad}\right) = \frac{3}{4}$

$= \frac{3}{4} = \frac{3}{4}$ $\therefore P(2)$ is true

$P(k)$: $\left(1-\frac{1}{2^2}\right)\left(1-\frac{1}{3^2}\right) ---- \left(1-\frac{1}{k^2}\right) = \frac{k+1}{2k}$

$P(k+1)$: $\left(1-\frac{1}{2^2}\right)\left(1-\frac{1}{3^2}\right) ---- \left(1-\frac{1}{k^2}\right)\left(1-\frac{1}{(k+1)^2}\right) = \frac{k+2}{2k+2}$

LHS $\left(1-\frac{1}{2^2}\right)\left(1-\frac{1}{3^2}\right) ---- \left(1-\frac{1}{k^2}\right)\left(1-\frac{1}{(k+1)^2}\right)$

$= \left(\frac{k+1}{2k}\right) * \left(1-\frac{1}{(k+1)^2}\right)$

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PMI Solution (W.S 1) 9

$$= \left(\frac{k+1}{2k}\right)\left(\frac{(k+1)^2-1}{(k+1)^2}\right)$$

$$= \frac{(k+1)}{2k}\left(\frac{k^2+2k+\cancel{1}-\cancel{1}}{(k+1)^2}\right)$$

$$= \frac{\cancel{(k+1)}\,\cancel{k}(k+2)}{2\cancel{k}(k+1)^{\cancel{2}}}$$

$$= \frac{k+2}{2k+2} = \text{RHS}$$

$\therefore$ $P(k+1)$ is true

$\therefore$ By PMI, $P(n)$ is true for all $n \in N$