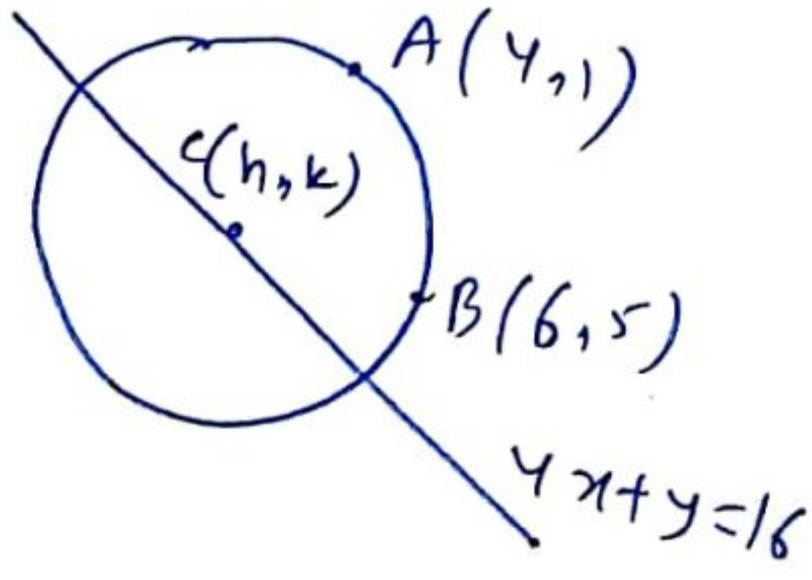
SOLUTION WORKSHEET: 1 (CIRCLES)

(CONIC SECTION)

QNS: 1 The equation of circle is

$$(x-h)^2 + (y-k)^2 = r^2$$



✓ A(4,1) lies on it

$$(4-h)^2 + (1-k)^2 = r^2$$

$$\Rightarrow h^2 + k^2 - 8h - 2k + 17 = r^2 \quad ---(1)$$

✓ B(6,5) lies on the circle

$$(6-h)^2 + (5-k)^2 = r^2$$

$$\Rightarrow h^2 + k^2 - 12h - 10k + 61 = r^2 \quad ---(2)$$

✓ C(h,k) lies on the line $4x+y=16$

$$4h + k = 16 \quad ---(3)$$

From (1) & (2)

$$h^2 + k^2 - 8h - 2k + 17 = h^2 + k^2 - 12h - 10k + 61$$

$$4h + 8k = 44 \quad ---(4)$$

Solving (3) & (4)

$$7k = 28$$

$$\boxed{k = 4} \quad \boxed{h = 3}$$ ∴ centre (3,4) put in eq (i)

$$9 + 16 - 24 - 8 + 17 = r^2$$

$$\boxed{r^2 = 10}$$

∴ equation of circle is

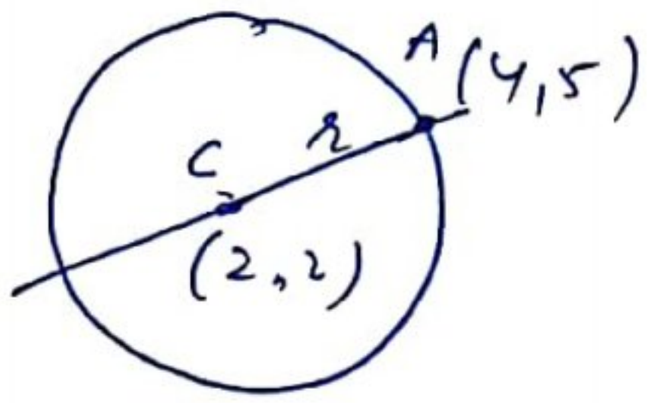
$$(x-3)^2 + (y-4)^2 = 10$$

$$\Rightarrow x^2+y^2-6x-8y+9+16 = 10$$

$$\Rightarrow \boxed{x^2+y^2-6x-8y+15=0}$$ Ans

Qns:2 →

Centre $C(2,2)$



$AC = r$

$$\sqrt{4+9} = r$$

$$\boxed{r^2 = 13}$$

equation circle $(x-2)^2 + (y-2)^2 = 13$

$$x^2+y^2-4x-4y+4+4=13$$

$$\Rightarrow \boxed{x^2+y^2-4x-4y=5}$$ Ans

Qns 3 →

Given $2x^2+2y^2-16x+20y-24=0$

divide by 2

$$x^2+y^2-8x+10y-12=0$$

$$\Rightarrow x^2-8x+y^2+10y-12=0$$

$$\Rightarrow (x-4)^2-16+(y+5)^2-25-12=0$$

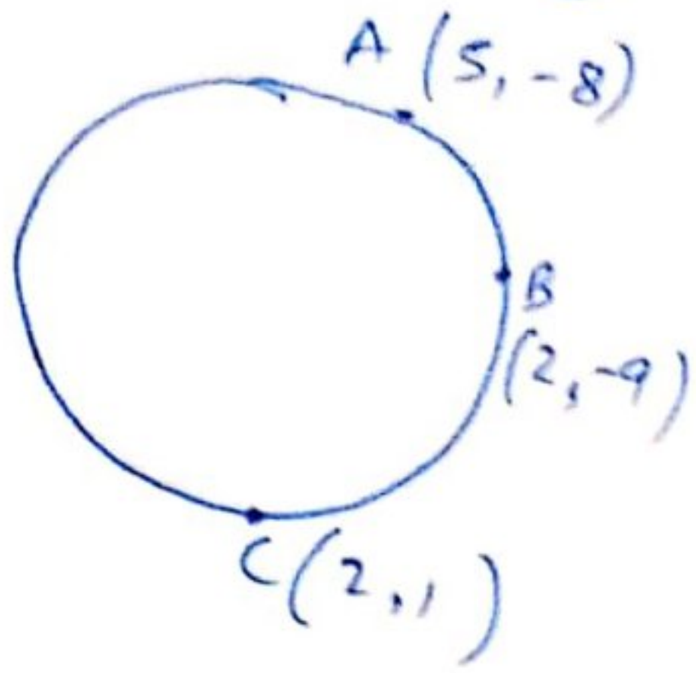
$$\Rightarrow (x-4)^2+(y+5)^2 = 53$$

Comp with $(x-h)^2+(y-k)^2=r^2$

$$\boxed{\text{Centre } (4,-5) \quad \text{Rad} = \sqrt{53}}$$ Ans

Q No 4 → The equation of circle is

$$(x-h)^2 + (y-k)^2 = r^2$$



✓ A(5, -8) lies on it

$$(5-h)^2 + (-8-k)^2 = r^2$$

$$\Rightarrow h^2 + k^2 - 10h + 16k + 89 = r^2 \quad \text{----(i)}$$

✓ B(2, -9) lies on it

$$(2-h)^2 + (-9-k)^2 = r^2$$

$$\Rightarrow h^2 + k^2 - 4h + 18k + 85 = r^2 \quad \text{---(2)}$$

✓ C(2, 1) lies on it

$$(2-h)^2 + (1-k)^2 = r^2$$

$$\Rightarrow h^2 + k^2 - 4h - 2k + 5 = r^2 \quad \text{---(3)}$$

From (1) & (2)

$$h^2 + k^2 - 10h + 16k + 89 = h^2 + k^2 - 4h + 18k + 85$$

$$\Rightarrow 6h + 2k = 4 \quad \text{---(4)}$$

From (2) & (3)

$$h^2 + k^2 - 4h + 18k + 85 = h^2 + k^2 - 4h - 2k + 5$$

$$\Rightarrow 20k = -80$$

$$\boxed{k = -4}$$ put in (4)

$$6h - 8 = 4 \rightarrow \boxed{h = 2}$$

put $h=2$ & $k=-4$ in eq (i)

$$4+16-20-64+89=r^2$$

$$\Rightarrow \boxed{r^2=25}$$

∴ Equation of circle is

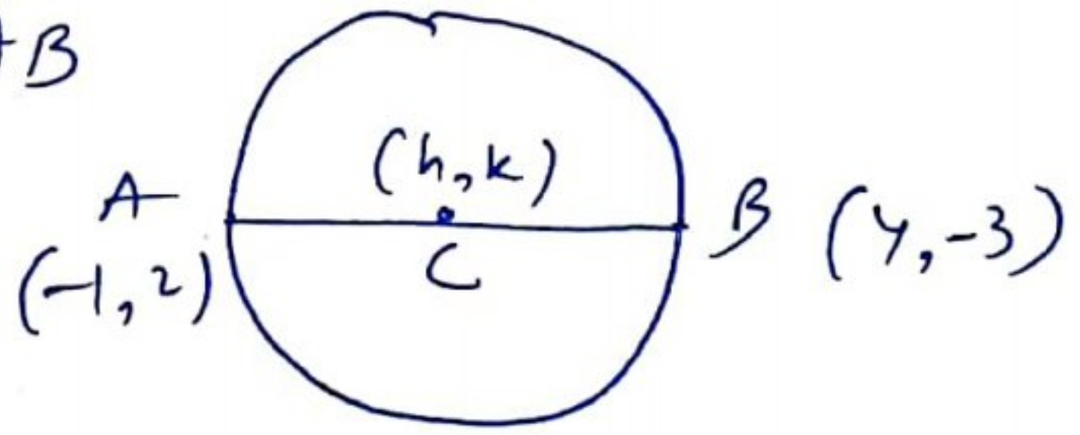
$$(x-2)^2+(y+4)^2=25$$

$$\Rightarrow x^2+y^2-4x+8y+4+16=25$$

$$\Rightarrow \boxed{x^2+y^2-4x+8y-5=0}$$ Ans

Ans

C is the Mid point of AB



∴ Coordinate of C is

$$h=\frac{-1+4}{2} \quad | \quad k=\frac{2-3}{2}$$

$$h=\frac{3}{2} \quad | \quad k=\frac{-1}{2}$$

∴ Centre $\left(\frac{3}{2},-\frac{1}{2}\right)$

Radius $=\frac{1}{2}AB$

Radius $=\frac{1}{2}\sqrt{(4+1)^2+(-3-2)^2}$

$$=\frac{1}{2}\sqrt{25+25}$$

$$=\frac{1}{2}\times 5\sqrt{2}$$

$$\boxed{r=\frac{5}{\sqrt{2}}}$$

$\therefore$ Equation of circle is

$$\left(x-\frac{3}{2}\right)^2+\left(y+\frac{1}{2}\right)^2=\frac{25}{2}$$

$$\Rightarrow x^2+y^2-3x+y+\frac{9}{4}+\frac{1}{4}=\frac{25}{2}$$

$$\Rightarrow x^2+y^2-3x+y=\frac{25}{2}-\frac{10}{4}$$

$$\Rightarrow \boxed{x^2+y^2-3x+y=10} \quad \text{Ans}$$

Qn.6 → Given Circle

$$x^2+y^2-6x+12y+15=0$$

$$\Rightarrow x^2-6x+y^2+12y+15=0$$

$$\Rightarrow (x-3)^2-9+(y+6)^2-36+15=0$$

$$\Rightarrow (x-3)^2+(y+6)^2=30$$

(centre $(3,-6)$ Rad $=\sqrt{30}$

Required circle is "Concentric" with given circle that Mean "Centre is same"

$\therefore$ Centre of Required circle is also $(3,-6)$

Let Radius of Required Circle $= R$

Ar of given Circle $= \pi(\sqrt{30})^2 = 30\pi$

Ar of Required Circle $= \pi r^2$

Given $\pi r^2 = 2(30\pi)$

$\Rightarrow \boxed{r^2 = 60}$

Now equation required circle is

$$(x-3)^2 + (y+6)^2 = 60$$

$\Rightarrow x^2+y^2-6x+12y+9+36 = 60$

$\Rightarrow \boxed{x^2+y^2-6x+12y-15=0}$ Ans

Ques:= 7 →

Given $OA = 3$ & $OB = 4$

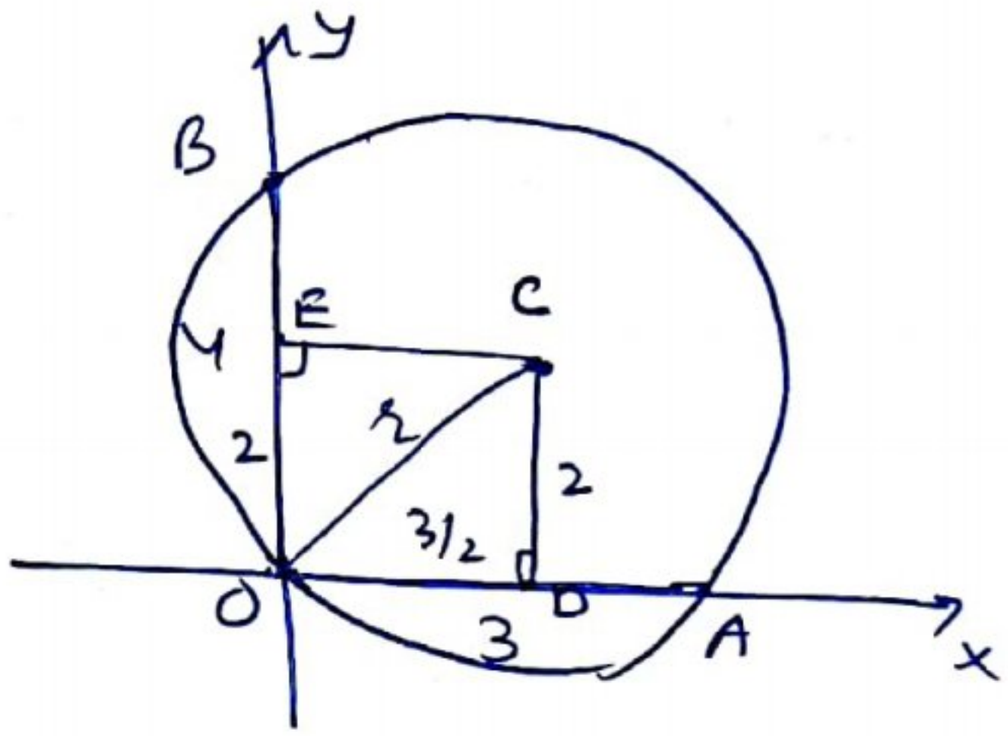
$CD \perp OA \Rightarrow OD = 3/2$

$\Rightarrow h = 3/2$

$CE \perp OB \Rightarrow OE = 2$

$\Rightarrow k = 2$

$\therefore$ Centre $(\frac{3}{2}, 2)$



Now $OC^2 = OD^2 + DE^2$

$\Rightarrow r^2 = \frac{9}{4} + 4$

$\Rightarrow r^2 = \frac{25}{4}$

$\Rightarrow \boxed{r = 5/2}$

Now equation of circle is given by

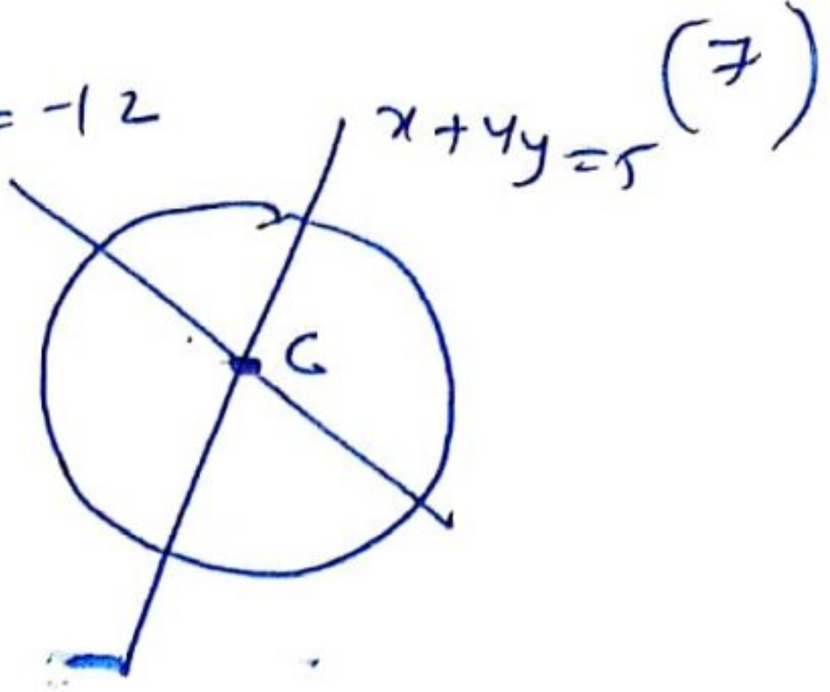
$$(x-\tfrac{3}{2})^2 + (y-2)^2 = \frac{25}{4}$$

$\Rightarrow x^2+y^2-3x-4y+\frac{9}{4}+4 = \frac{25}{4}$

$\Rightarrow x^2+y^2-3x-4y = \frac{25}{4} - \frac{9}{4} - 4$

$\Rightarrow \boxed{x^2+y^2-3x-4y=0}$ Ans

Qn. 8 →



Centre is the Intersection point of diameters

$2x-3y=-12$

$x+4y=5$

$\Rightarrow$

$2x-3y=-12$

$7x+8y=10$

$-11y=-22$ $\quad (y=2)$ $\Rightarrow 2x-6=-12$

$2x=-6$

$(x=-3)$

$\therefore$ Centre $(-3,2)$

Given Area circle $= 154$

$\Rightarrow \pi r^2 = 154$

$\Rightarrow \frac{22}{7} r^2 = 154$

$\Rightarrow r^2 = \frac{154 \times 7}{22}$

$\Rightarrow (r=7)$

$\therefore$ Equation circle is given by

$$\boxed{(x+3)^2+(y-2)^2=49}$$ Ans

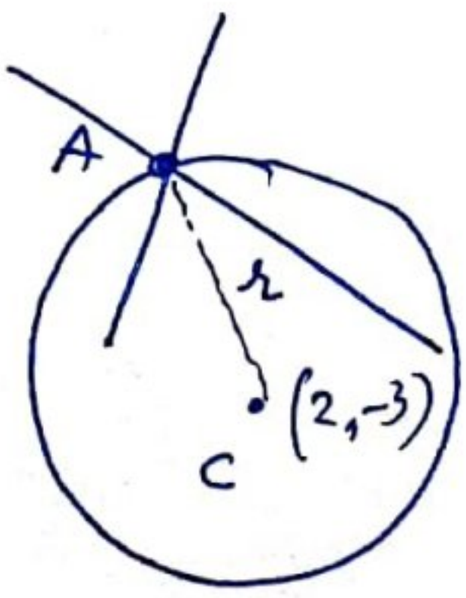
Qn 9 →

Given lines

$3x-2y=1$

$4x+y=27$

Solving these equations we get $x=5$; $y=7$



$\therefore$ Coordinate of A is $(5, 7)$

Given Centre $C(2, -3)$

Radius = AC

$\Rightarrow$ $r = \sqrt{(5-2)^2 + (7+3)^2}$

$r = \sqrt{9 + 100}$

$\boxed{r^2 = 109}$

Equation of circle is

$(x-2)^2 + (y+3)^2 = 109$

$\Rightarrow$ $x^2 + y^2 - 4x + 6y + 4 + 9 = 109$

$\Rightarrow$ $\boxed{x^2 + y^2 - 4x + 6y - 96 = 0}$ Ans

— x —