Solutions (WORKSHEET No: 5)

Chapter SETS

Q.N 1 (i) $2^x - 1$ is always an odd number for all x less than 10

$\therefore A = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ Ans

(ii) $t^3 = t$

$\Rightarrow t^3 - t = 0$

$\Rightarrow t(t^2 - 1) = 0$

$\Rightarrow t = 0, \quad t = \pm 1$

$\therefore A = \{0, 1, -1\}$ Ans

(iii) $\frac{x+5}{x-7} - 5 = \frac{4x-40}{13-x}$

$\Rightarrow \frac{x+5-5x+35}{x-7} = \frac{4x-40}{13-x}$

$\Rightarrow (40-4x)(13-x) = (4x-40)(x-7)$

$\Rightarrow (40-4x)(13-x) - (4x-40)(x-7) = 0$

$\Rightarrow (40-4x)[13-x+x-7] = 0$

$\Rightarrow (40-4x)(6) = 0$

$\Rightarrow 4x = 40$

$\Rightarrow x = 10$

$\therefore A = \{10\}$ Ans

(iv) $x^4 - 5x^2 + 6 = 0$

$x^4 - 3x^2 - 2x^2 + 6 = 0$

$(x^2 - 3)(x^2 - 2) = 0$

$x^2 = 3$ or $x^2 = 2$

$x = \pm\sqrt{3}$ or $x = \pm\sqrt{2}$

$\therefore A = \{-\sqrt{3}, \sqrt{3}, -\sqrt{2}, \sqrt{2}\}$ Ans

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Solution Worksheet No. 5 (SETS)

Qn 2 → (i) Let y be any arbitary element of set Y

Let $y \in Y$

$\Rightarrow y \in X \cup Y$

$\Rightarrow Y \subset (X \cup Y)$ Proved

(ii) Let x be any arbitary element of set $X \cap Y$.

Let $x \in (X \cap Y)$

$\Rightarrow x \in X$ and $x \in Y$

$\Rightarrow (X \cap Y) \subset X$ proved

Qns 3 → LHS $(A-B) \cap (C-B)$

$= (A \cap B') \cap (C \cap B')$ --- {$\because A-B = A \cap B'$}

$= (A \cap C) \cap B'$ -- {Distributive law}

$= (A \cap C) - B$ Proved

Qn 4 → LHS $(A-B) \cap (C-B)$ → Misprint in worksheet

$= (A \cap B') \cap (C \cap B')$

NOTE "LHS" should be $(A-B) \cap (A-C)$

$= (A \cap B') \cap (A \cap C')$

$= A \cap (B' \cap C')$ --- {distributive property}

$= A \cap (B \cup C)'$ --- {De-morgan's law}

$= A - (B \cup C)$ --- {$A \cap B' = A - B$}

$= RHS$

Proved

Solution (Worksheet No: 5) SETS (3)

Q4. 5 (Note) Most point Question should be

~~A-(B-C)~~ $A-(B-C) \neq (A-B)-C$

LHS $A-(B-C)$

$= A-(B \cap C')$

$= A \cap (B \cap C')'$ --- {$\because A-B = A \cap B'$}

$= A \cap (B' \cup C)$ --- {demorgans laws}

$= (A \cap B') \cup (A \cap C)$

RHS $(A-B)-C$

$= (A \cap B')-C$

$= (A \cap B') \cap C'$

$= (A \cap B') \cap (A \cap C')$

Clearly LHS $\neq$ RHS Ans

Q5. 6 → T.P $A-(A-B) = A \cap B$

LHS $A-(A-B)$

$= A-(A \cap B')$ --- {$\because A-B = A \cap B'$}

$= A \cap (A \cap B')'$

$= A \cap (A' \cup B)$ --- {demorgan's laws}

$= (A \cap A') \cup (A \cap B)$ --- {distributive laws}

$= \phi \cup (A \cap B)$

$= A \cap B$ --- {$\phi \cup A = A$}

$=$ RHS Ans

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Solution SETS (W.S - 5) (4)

Qn 7 → T.P $X \cap (X \cup Y')' = \phi$

LHS $X \cap (X \cup Y)'$

$= X \cap (X' \cap Y')$ --- { demorgan's law

$= (X \cap X') \cap (X \cap Y')$ --- { distributive law.

$= \phi \cap (X \cap Y')$

$= \phi$ --- { $\because \phi \cap A = \phi$ }

$= R.H.S$ Any

Qn 8 → T.P $[(A' \cup B') - A]' = A$

LHS $[(A' \cup B') - A]'$

$= [(A' \cup B') \cap A']'$ --- { $A - B = A \cap B'$ }

$= (A' \cup B')' \cup A$ --- { demorgan's law

$= (A \cap B) \cup A$ --- { demorgan law

$= A$ -- { $\because A \cap B \subset A$ $\therefore (A \cap B) \cup A = A$ }

Ans Proved

Qn. 9 → T.P $[B' \cup (B' - A)]' = B$

LHS $[B' \cup (B' - A)]'$

$= [B \cap (B' - A)']$ --- { demorgan's law }

$= B \cap (B' \cap A')'$ --- { $A - B = A \cap B'$ }

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Solution (SET) (VV.S - 5) (5)

$= B \cap (B \cup A)$ --- {Demorgan's law}

$= (B \cap B) \cup (B \cap A)$

$= B \cup (A \cap B)$

$= B$ --- {$\because A \cap B \subset B \Rightarrow (A \cap B) \cup B = B$}

= Ry Am

Q.N. 10 → $X_1 = \{ - - - - - \}$

$X_2 = \{ - - - - \}$

$\vdots$

$X_{20} = \{ - - - - \}$

$S = X_1 \cup X_2 \cup X_3 \cup \cdots X_{20}$

Max. number of elements in set $S = 5 \times 20 = 100$

But each element of 'S' belongs exactly 10 of A's sets

$\therefore$ no. of elements in set $S \Rightarrow n(S) = \frac{100}{10} = 10$ - (1)

$Y_1 = \{ - - \}$

$Y_2 = \{ - - \}$

$\vdots$

$Y_n = \{ - - \}$

also $S = Y_1 \cup Y_2 \cup Y_3 \cup \cdots Y_n$

$\therefore \max\ n(S) = 2 \times n = 2n$

But every element of 'S' belongs to exactly 4 of the Y's set

$\therefore n(S) = \frac{2n}{4} = \frac{n}{2}$ --- (2)

from (1) & (2)

$\frac{n}{2} = 10 \Rightarrow \boxed{n = 20}$ Ans

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