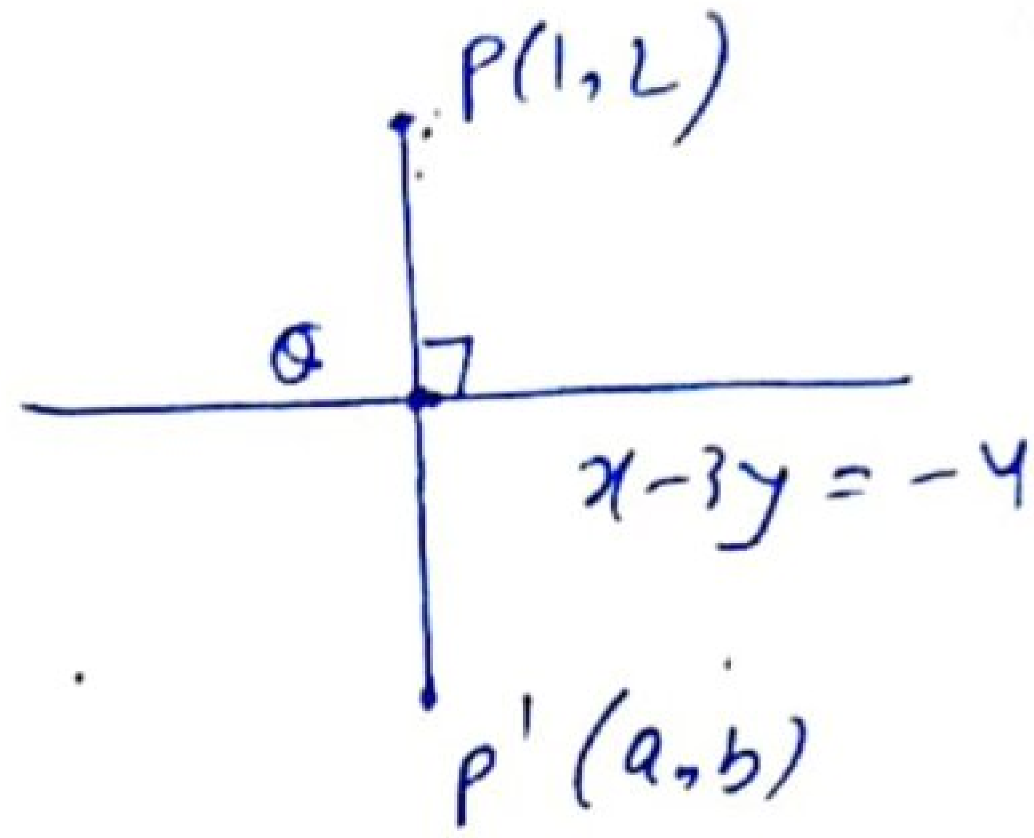
CHAPTER: STRAIGHT LINES $\boxed{\text{CLASS No: 3}}$

QNS:-1 Find the image of the point (1, 2) in the line mirror $x-3y+4=0$



Soln ✓ Slope of given line $= \frac{-1}{-3} = \frac{1}{3}$

✓ $PQ \perp$ given line

$\therefore$ Slope of $PQ = -3$

✓ equation of PQ

$$y-2 = -3(x-1)$$
$$y-2 = -3x+3$$
$$3x+y = 5$$

✓ solving equation of PQ & given line

$$x-3y=-4 \quad \times 3$$
$$3x+y=5$$

$$\cancel{3x}-9y=-12$$
$$\cancel{3x}+y=5$$
$$-10y=-17$$

$\Rightarrow \boxed{y = \frac{17}{10}} \Rightarrow 3x + \frac{17}{10} = 5$

$3x = 5 - \frac{17}{10}$

$3x = \frac{33}{10}$

$\boxed{x = \frac{11}{10}}$

✓ $\therefore Q\left(\frac{11}{10}, \frac{17}{10}\right)$

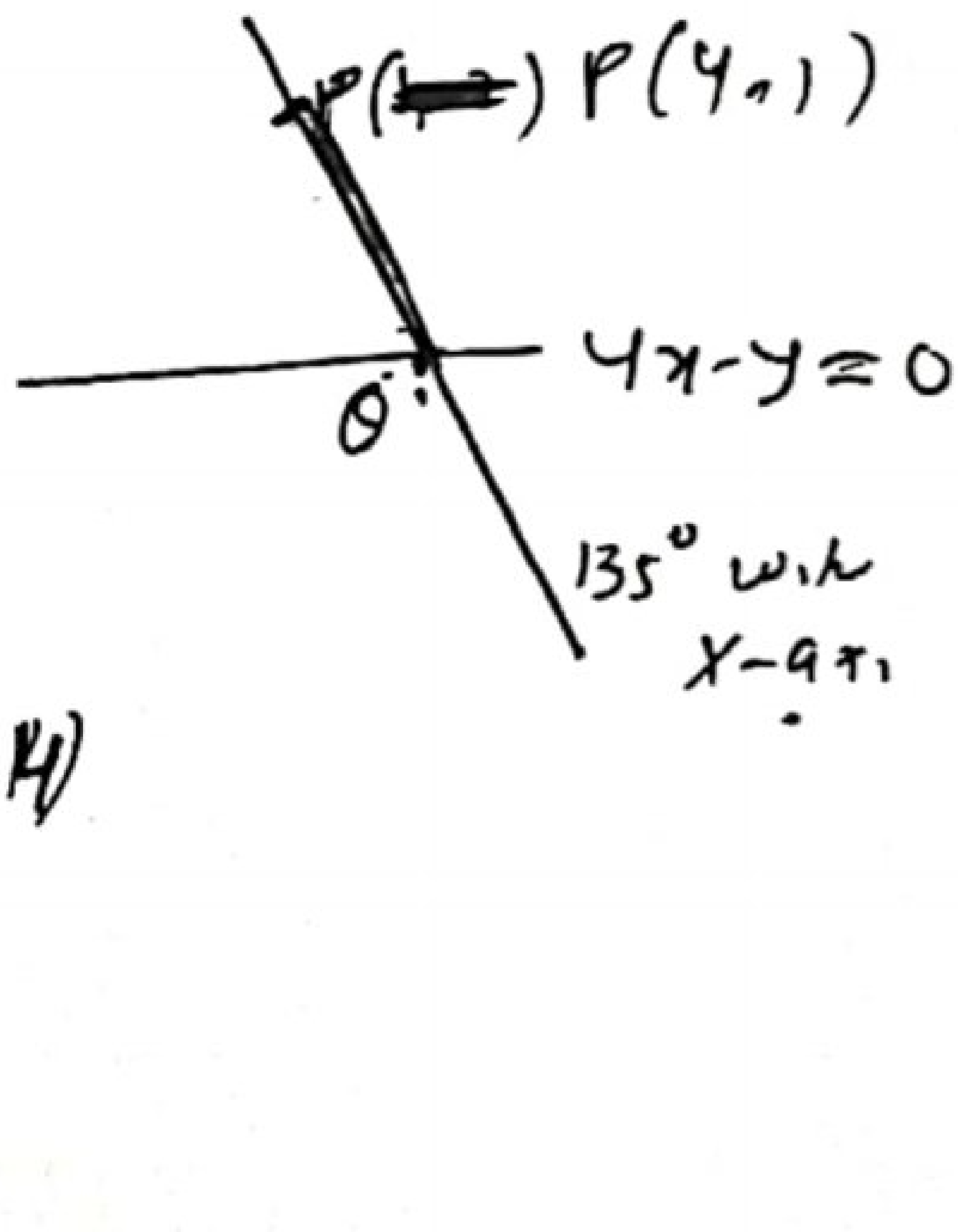
✓ Q is the Mid point of PP'
where P'(a,b) is the image of point P

✓ By Mid point formula

$\frac{11}{5} = \frac{1+a}{2}$	$\frac{17}{5} = \frac{2+b}{2}$
$11 = 5 + 5a$	$17 = 10 + 5b$
$a = \frac{6}{5}$	$b = 7/5$

$\therefore$ Image is $P'\left(\frac{6}{5}, 7/5\right)$ Ans

Que: 2 → Find the distance of the line $4x - y = 0$ from the point $P(4,1)$ measured along the line making an angle of 135° with the +ve X-axis



Soln ✓ Slope of $PQ = \tan(135^\circ)$

$= \tan(180-45)$

$= -1$

✓ equation of PQ : $y - 1 = -1(x-4)$

$y - 1 = -x + 4$

$x + y = 5$

✓ solve equation of PQ & given line

$5x = 5 \Rightarrow x = 1 \rightarrow y = 4$

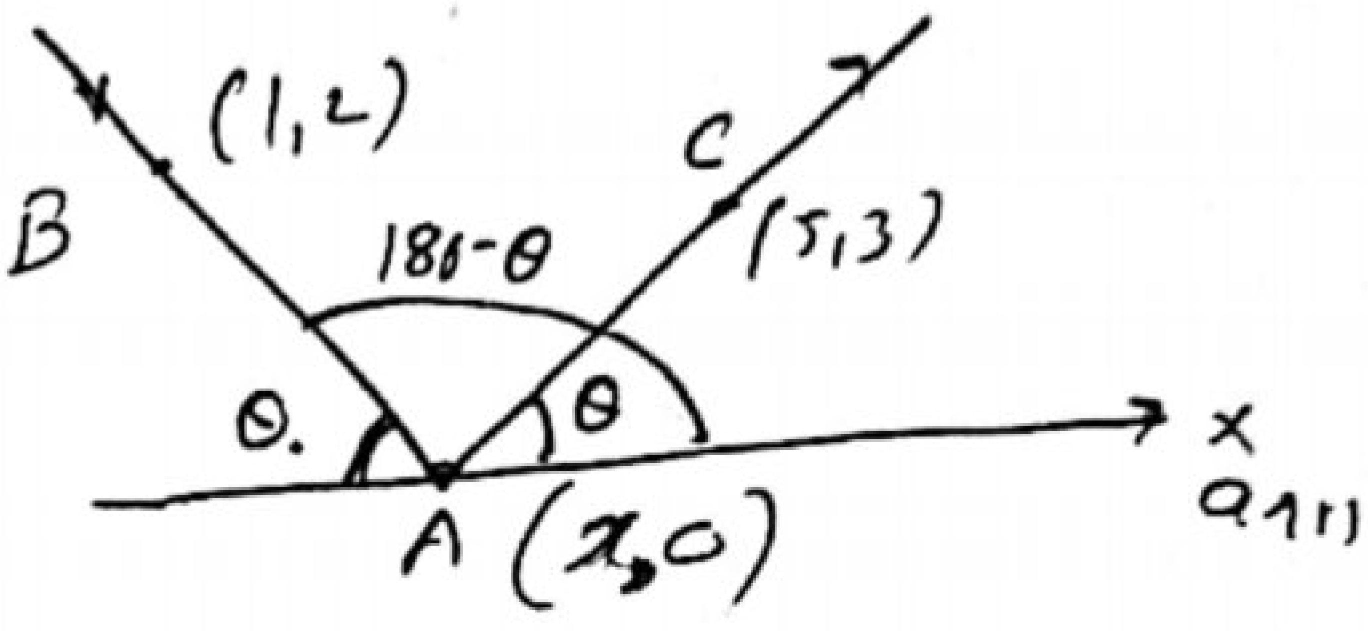
$\therefore Q(1,4)$

✓ Required distance $PQ = \sqrt{9+9} = \sqrt{18}$

$= 3\sqrt{2}$ units

Ans

Ques. 3 → A Ray of light passing through the point $(1,2)$ reflects on the X-axis at point A and the reflected ray passes through the point $(5,3)$. Find the coordinates of A



Sol^n ✓ slope of $AC = \frac{3}{5-x}$

also

✓ slope of $AC = \tan\theta$

$\Rightarrow \tan\theta = \frac{3}{5-x}$ ..... (i)

✓ Slope of $AB = \frac{2}{1-x}$

✓ also slope of $AB = \tan(180-\theta) = -\tan\theta$

$\Rightarrow -\tan\theta = \frac{2}{1-x}$

$\Rightarrow \tan\theta = \frac{-2}{1-x}$ --- (2)

from (1) & (2)

$$\frac{3}{5-x} = \frac{-2}{1-x}$$

$$3-3x = -10+2x$$

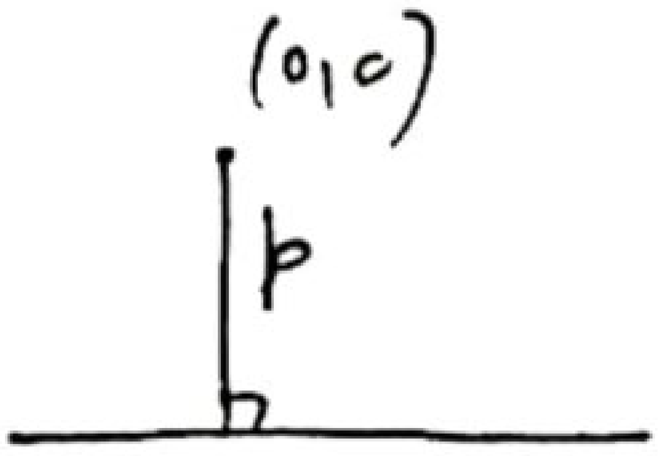
$$13 = 5x$$

$$\Rightarrow x = \frac{13}{5}$$

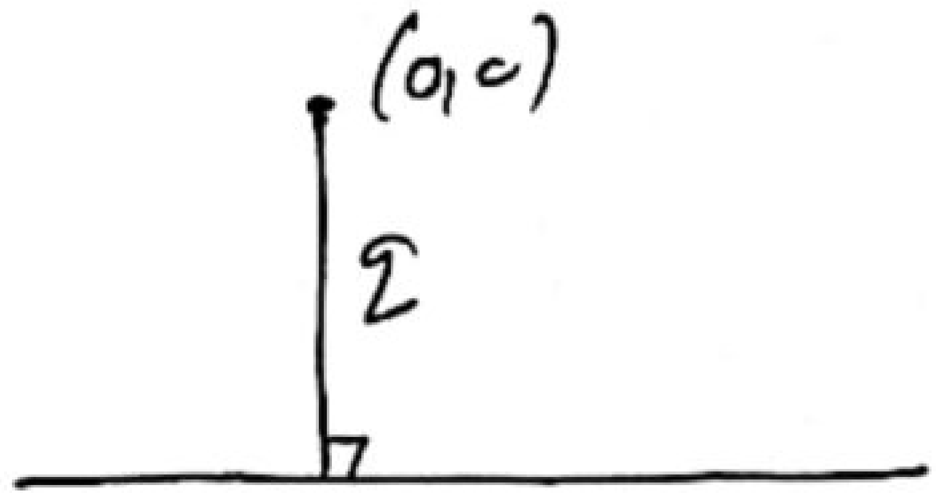
∴ Required point $A\left(\frac{13}{5}, 0\right)$ Ans

Qn. 4 → If p and q are the length of perpendiculars from the origin to the lines $x\cos\theta - y\sin\theta = k\cos(2\theta)$ and $x\sec\theta + y\operatorname{cosec}\theta = k$ respectively. Prove that $p^2 + 4q^2 = k^2$

Soln



$x\cos\theta - y\sin\theta - k\cos(2\theta) = 0$



$x\sec\theta + y\operatorname{cosec}\theta - k = 0$

$$p = \frac{|-k\cos(2\theta)|}{\sqrt{\cos^2\theta + \sin^2\theta}}$$

$$\boxed{p = |k\cos(2\theta)|}$$

$$q = \frac{|-k|}{\sqrt{\sec^2\theta + \operatorname{cosec}^2\theta}}$$

$$q = \frac{|k|}{\sqrt{\frac{1}{\cos^2\theta} + \frac{1}{\sin^2\theta}}}$$

$$\boxed{q = |k|\sin\theta\cos\theta}$$

$|-x| = |x|$
$|-2| = 2$
$|x|^2 = x^2$

Lhs $p^2 + 4q^2$

$$= k^2\cos^2(2\theta) + 4k^2\sin^2\theta\cos^2\theta$$

$$= k^2\left[\cos^2(2\theta) + 4\sin^2\theta\cos^2\theta\right]$$

$$= k^2\left[\cos^2(2\theta) + (2\sin\theta\cos\theta)^2\right]$$

$$= k^2\left[\cos^2(2\theta) + \sin^2(2\theta)\right]$$

$$= k^2 \times 1 = k^2 = \text{Rhs}$$

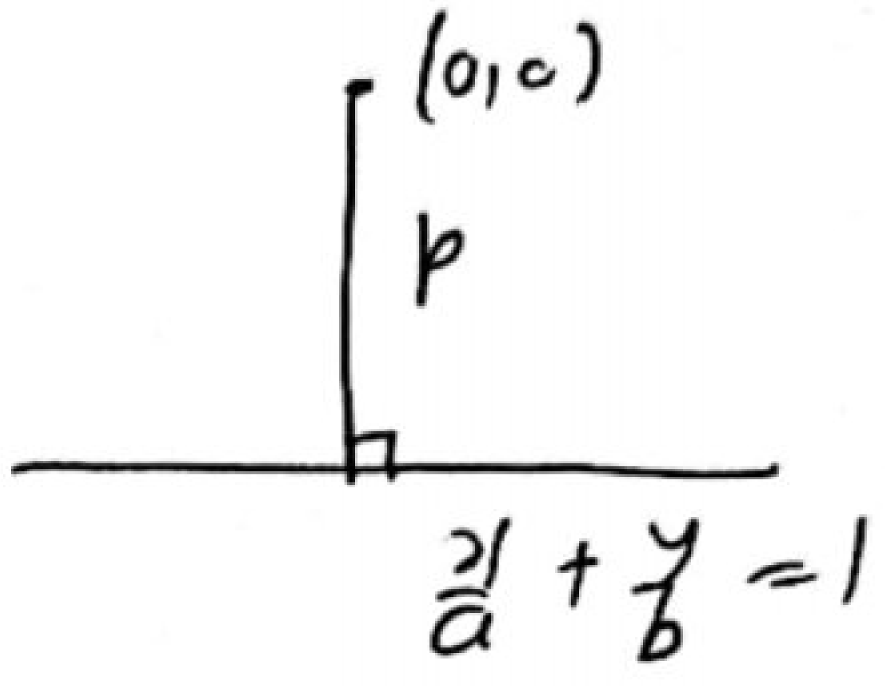
Ans

Scanned with CamScanner

Qn. 5 → If p is the length of perpendicular from the origin to the line whose intercepts on the axes are a and b, then show that

$$\frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}$$

Soln



✓ let equation of line is

$$\frac{x}{a} + \frac{y}{b} = 1$$

$$\Rightarrow \quad bx + ay = ab$$

$$\Rightarrow \quad bx + ay - ab = 0$$

✓ distance $\quad p = \left| \frac{-ab}{\sqrt{b^2+a^2}} \right|$

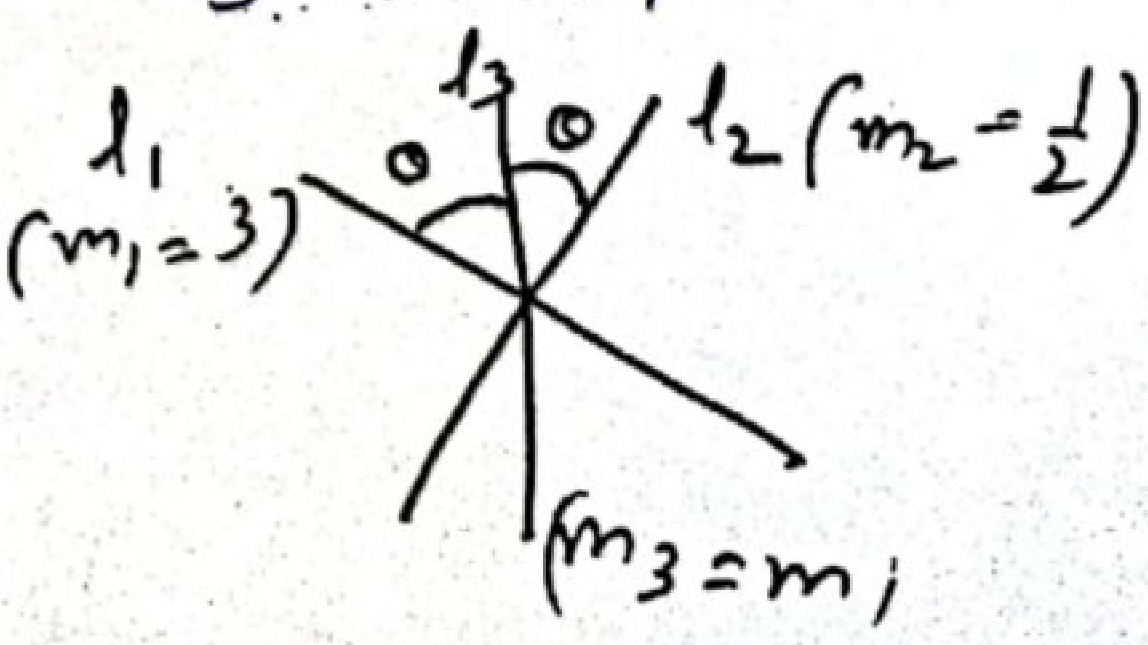
$$p = \frac{|ab|}{\sqrt{b^2+a^2}}$$

$$\Rightarrow \quad p^2 = \frac{a^2b^2}{b^2+a^2}$$

$$\Rightarrow \quad \frac{1}{p^2} = \frac{b^2+a^2}{a^2b^2} = \frac{1}{a^2} + \frac{1}{b^2} = \text{RHS}$$

proved

Qn. 6 → If the lines $3x - y + 1 = 0$ and $2y = x + 3$ are equally inclined to the line $y = mx + 4$. Find the value of m.



Soln slope of l_1 : $m_1 = 3$

slope of l_2 : $m_2 = 1/2$

slope of l_3 : $m_3 = m$

angle b/w l_1 and l_3

$$\tan\theta = \left|\frac{3-m}{1+3m}\right| \quad \cdots (1)$$

angle b/w l_2 & l_3

$$\tan\theta = \left|\frac{\frac{1}{2}-m}{1+\frac{m}{2}}\right| = \left|\frac{1-2m}{2+m}\right| \quad -(2)$$

from (1) & (2)

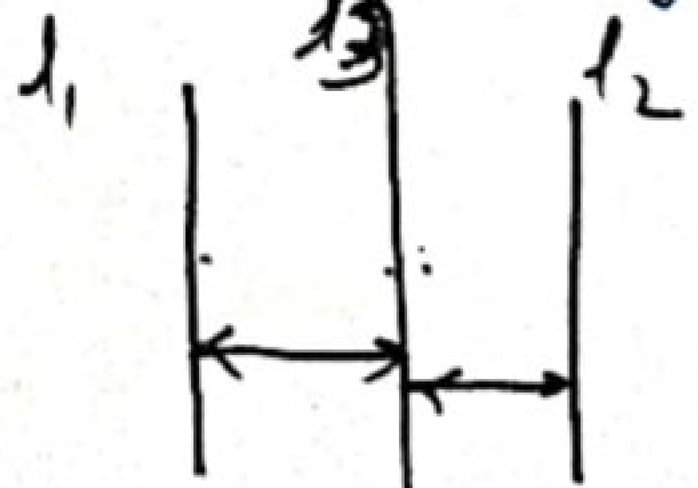
$$\left|\frac{3-m}{1+3m}\right| = \left|\frac{1-2m}{2+m}\right| \quad \cdots \{ \text{if } |x| = |y| \Rightarrow x = \pm y \}$$

$$\Rightarrow \frac{3-m}{1+3m} = \pm\left(\frac{1-2m}{2+m}\right)$$

$$\Rightarrow \frac{3-m}{1+3m} = \frac{1-2m}{2+m} \quad \Big| \quad \frac{3-m}{1+3m} = -\left(\frac{1-2m}{2+m}\right)$$

Proceed Ans

Qu. 7 → find equation of the line which is equidistant from parallel lines $9x+6y-7=0$ and $3x+2y+6=0$



Soln equation of l_1: $3x+2y-7/3=0$

Equat of l_2: $3x+2y+6=0$

$\boxed{\text{line parallel to } ax+by+c=0 \text{ is } ax+by+\lambda=0}$

let equation of l_3: $3x+2y+\lambda=0$

Now distance b/w l_1 & l_3 = Distance b/w l_2 & l_3

$$\Rightarrow \frac{|-7/3-\lambda|}{\cancel{\sqrt{9+4}}} = \frac{|6-\lambda|}{\cancel{\sqrt{9+4}}}$$

$$\Rightarrow -\frac{7}{3}-\lambda = \pm(6-\lambda)$$

$$\Rightarrow -\frac{7}{3}-\cancel{\lambda} = 6-\cancel{\lambda} \qquad \Bigg| \qquad -\frac{7}{3}-\lambda = -6+\lambda$$

$$\Rightarrow -7/3 = 6 \qquad\qquad 2\lambda = -7/3+6$$

$\therefore$ No value of λ $\qquad 2\lambda = \frac{11}{3}$

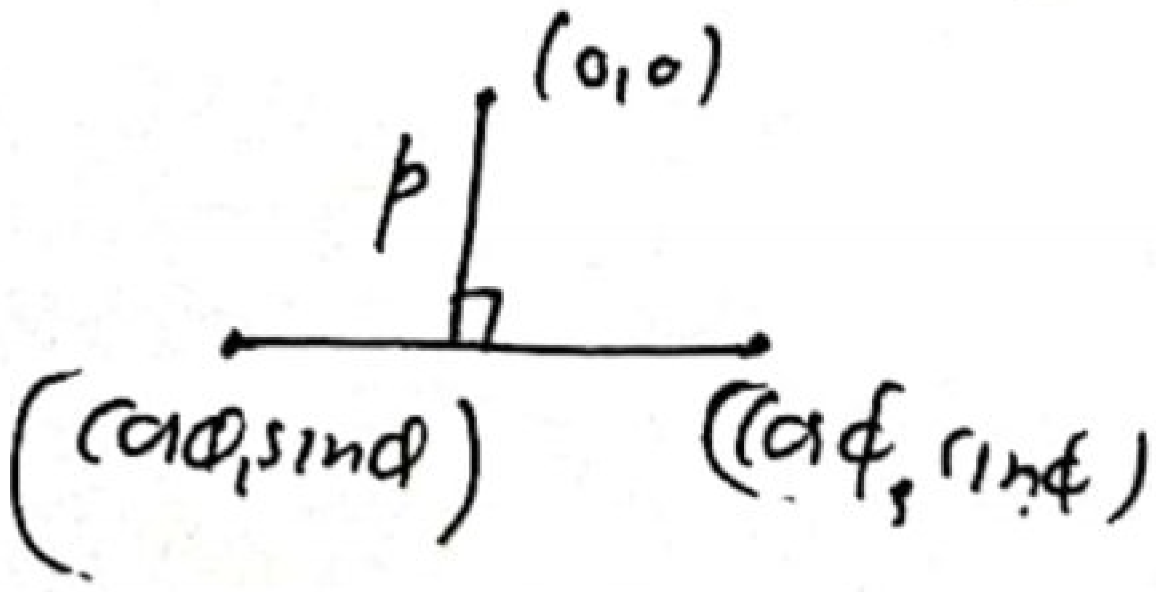
$$\lambda = \frac{11}{6}$$

$\therefore$ equation of required line $3x+2y+\frac{11}{6}=0$

$$\Rightarrow 18x+12y+11=0 \quad \text{Ans}$$

Qn 8 → find the perpendicular distance from the origin to the line joining the points $(\cos\theta, \sin\theta)$ and $(\cos\phi, \sin\phi)$.

Soln equation of line (two point form)



$$y-\sin\theta = \frac{\sin\phi-\sin\theta}{\cos\phi-\cos\theta}(x-\cos\theta)$$

$$\Rightarrow y-\sin\theta = \frac{\cancel{2}\cos\left(\frac{\phi+\theta}{2}\right)\cdot\cancel{\sin\left(\frac{\phi-\theta}{2}\right)}}{-\cancel{2}\sin\left(\frac{\phi+\theta}{2}\right)\cdot\cancel{\sin\left(\frac{\phi-\theta}{2}\right)}}(x-\cos\theta)$$

$$\Rightarrow -y\sin\left(\frac{\phi+\theta}{2}\right) + \sin\theta\cdot\sin\left(\frac{\phi+\theta}{2}\right) = x\cos\left(\frac{\phi+\theta}{2}\right) - \cos\theta\cdot\cos\left(\frac{\phi+\theta}{2}\right)$$

$$\Rightarrow x\cos\left(\frac{\phi+\theta}{2}\right) + y\sin\left(\frac{\phi+\theta}{2}\right) - \cos\theta\cdot\cos\left(\frac{\phi+\theta}{2}\right) - \sin\theta\cdot\sin\left(\frac{\phi+\theta}{2}\right) = 0$$

$$\Rightarrow x\cos\left(\frac{\phi+\theta}{2}\right) + y\sin\left(\frac{\phi+\theta}{2}\right) - \left\{\cos\theta\cdot\cos\left(\frac{\phi+\theta}{2}\right) + \sin\theta\cdot\sin\left(\frac{\phi+\theta}{2}\right)\right\} = 0$$

$$\Rightarrow x\cos\left(\frac{\phi+\theta}{2}\right) + y\sin\left(\frac{\phi+\theta}{2}\right) - \cos\left(\theta - \frac{\phi+\theta}{2}\right) = 0$$

$$\Rightarrow \boxed{x\cos\left(\frac{\phi+\theta}{2}\right) + y\sin\left(\frac{\phi+\theta}{2}\right) - \cos\left(\frac{\theta-\phi}{2}\right) = 0}$$

distance $$p = \frac{\left|-\cos\left(\frac{\theta-\phi}{2}\right)\right|}{\sqrt{\cos^2\left(\frac{\theta+\phi}{2}\right) + \sin^2\left(\frac{\theta+\phi}{2}\right)}}$$

$$p = \left|\cos\left(\frac{\theta-\phi}{2}\right)\right| \text{ units}$$ Ans

Ques 9 → Show that the equation of the line passing through the origin and making an angle θ with the line $y = mx + c$ is $\frac{y}{x} = \frac{m \pm \tan\theta}{1 \mp m\tan\theta}$

Sol: slope of given line : $m_1 = m$

let slope of required line : $m_2 = M$

angle b/w them $= \theta$.

Now $\tan\theta = \left|\frac{m-M}{1+mM}\right|$

$$\Rightarrow \frac{m-M}{1+mM} = \pm\tan\theta$$

$$\Rightarrow \frac{m-M}{1+mM} = \tan\theta$$

$$\Rightarrow m - M = \tan\theta + mM\tan\theta$$

$$\Rightarrow M + mM\tan\theta = m - \tan\theta$$

$$\Rightarrow M(1+m\tan\theta) = m - \tan\theta$$

$$\boxed{M = \frac{m-\tan\theta}{1+m\tan\theta}}$$

$$\frac{m-M}{1+mM} = -\tan\theta$$

get

$$\boxed{M = \frac{m+\tan\theta}{1-m\tan\theta}}$$

✓ $M = \dfrac{m \pm \tan\theta}{1 \mp m\tan\theta}$

✓ point $(0, c)$

✓ equation of Required line

$$y - 0 = \frac{m \pm \tan\theta}{1 \mp m\tan\theta}(x - c)$$

$$\Rightarrow \boxed{\frac{y}{x} = \frac{m \pm \tan\theta}{1 \mp m\tan\theta}}$$ Ans

Straight lines | WORKSHEET No: 2
(class 3)

Qn 1 → Find the image of the point $(3,8)$ with respect to the line $x+3y=7$. ANS $(-1,-4)$

Qn 2 → Find the distance of the line $4x+7y+5=0$ from the point $(1,2)$ along the line $2x-y=0$

ANS $\frac{23\sqrt{5}}{18}$ units

Qn 3 → In what ratio, the line joining $(-1,1)$ and $(5,7)$ is divided by the line $x+y=4$ ANS $1:2$

Qn 4 → Find the distance of the point $(2,3)$ from the line $2x-3y+9=0$ measured along the line $x-y+1=0$

ANS $4\sqrt{2}$ units

Qn 5 → Find the equation of the line which passes through the point $(3,4)$ and the sum of its intercepts on the axes is 14 ANS $4x+3y=24$; $x+y=7$

Qn 6 → (Special) The equations of two sides of a triangle are $3x-2y+6=0$ and $4x+5y-20=0$ and the orthocentre is $(1,1)$. Find the equation of the third side ANS $26x-122y-1675=0$

Qn 7 → Find the equation of a line which is equidistant from two parallel lines $5x-2y-9=0$ and $5x-2y+7=0$ ANS $5x-2y-1=0$

Qn 8 → Prove that $2x+3y=6$ is "Mid-parallel" to the lines $2x+3y=19$ and $2x+3y+7=0$

Qn: 9 → The equations of two sides of a square are $5x-12y-65=0$ and $5x-12y+26=0$. Find the area of the square Ans 49 square units

Qn 10 → Find the equations of the two straight lines through $(7, 9)$ and making an angle of 60° with the line $x-\sqrt{3}y-2\sqrt{3}=0$

Ans $x=7$ and $x+\sqrt{3}y=7+9\sqrt{3}$

—⋏—