!! जय श्री राधे कृष्णा। जय श्री गिरिराज जी महाराज !!

ULTIMATE MATHEMATICS : BY AJAY MITTAL

REVISION: SEQUENCE & SERIES | CLASS NO:- 2

QNS: 1 → Find three numbers in G.P, whose sum is 13, and the sum of whose ~~product~~ squares is 91

Sol: Let the three numbers in G.P are a, ar, ar^2

(·) $a + ar + ar^2 = 13 \Rightarrow a(1 + r + r^2) = 13$ --- (i)

(·) $a^2 + a^2r^2 + a^2r^4 = 91 \Rightarrow a^2(1 + r^2 + r^4) = 91$ --- (2)

(·) Square eq(i) $\Rightarrow a^2(1 + r + r^2)^2 = 169$ --- (3)

$\Rightarrow a^2(1 + r^2 + r^4 + 2r + 2r^3 + 2r^2) = 169$

$\Rightarrow a^2(1 + r^2 + r^4) + 2a^2r(1 + r^2 + r) = 169$

$\Rightarrow 91 + 2ar \cdot a(1 + r + r^2) = 169$ --- {from eq(2)}

$\Rightarrow 91 + 2ar(13) = 169$ --- {from eq(1)}

$\Rightarrow 26ar = 78$

$\Rightarrow \boxed{ar = 3}$

$a = \frac{3}{r}$ put in eq(i)

$\Rightarrow \frac{3}{r}(1 + r + r^2) = 13 \Rightarrow 3r + 3 + 3r^2 = 13r$

$\Rightarrow 3r^2 - 10r + 3 = 0$

$\Rightarrow 3r^2 - 9r - r + 3 = 0$

$\Rightarrow 3r(r-3) - 1(r-3) = 0$

$r = 3$ $r = 1/3$

we have $ar = 3$

when $r = 3 \Rightarrow a = 1$ | when $r = 1/3 \Rightarrow a = 9$

$\therefore$ Nos are $= 1, 3, 9$ (OR) $9, 3, 1$ Ans

Ou 2 → The sum of an infinite G.P is 15 and the sum of squares of these terms is 45. Find series.

Sol

(i) $a + ar + ar^2 + ar^3 + \dots \infty = 15$

$\Rightarrow \dfrac{a}{1-r} = 15 \quad --(1)$

(ii) $a^2 + a^2r^2 + a^2r^4 + a^2r^6 + \dots \infty = 45$

$\Rightarrow \dfrac{a^2}{1-r^2} = 45 \quad --(2)$

Squaring eq (1) $\Rightarrow \dfrac{a^2}{(1-r)^2} = 225 \quad --(3)$

(3) ÷ (2)

$$\frac{\dfrac{a^2}{(1-r)^2}}{\dfrac{a^2}{1-r^2}} = \frac{225}{45}$$

$\Rightarrow \dfrac{(1+r)\cancel{(1-r)}}{(1-r)^{\cancel{2}}} = 5 \Rightarrow \dfrac{1+r}{1-r} = 5$

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$\Rightarrow \quad 1+1 = 5-5r$

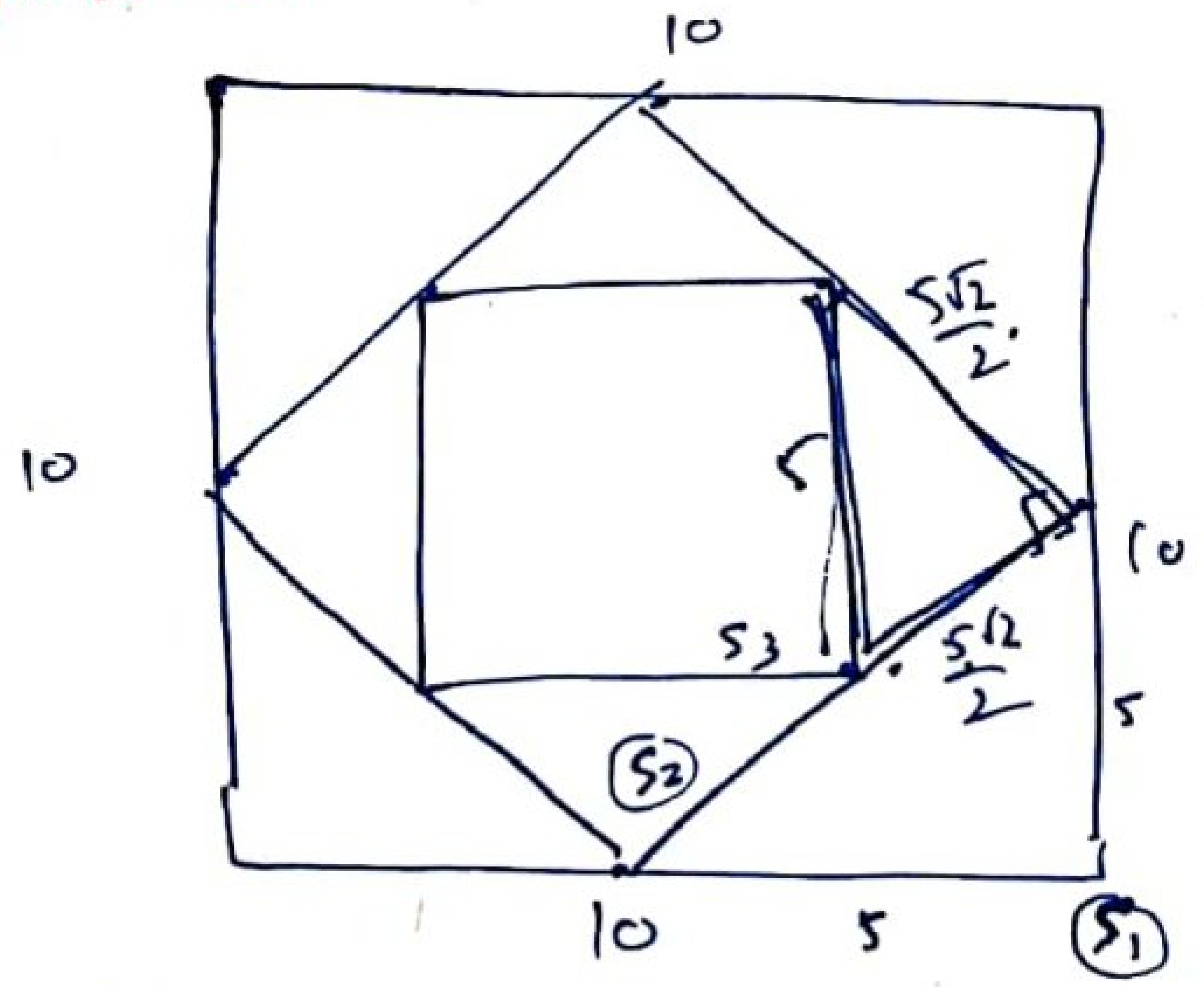
$\Rightarrow \quad 6r = 4$

$r = 2/3$ put in (1)

$$\frac{a}{1-\frac{2}{3}} = 15 \quad \Rightarrow \quad 3a = 15 \quad \Rightarrow \quad a = 5$$

GP $\quad 5 + \frac{10}{3} + \frac{20}{9}, \frac{40}{27}, \cdots \infty$ Ans

Ques 3 → A square is drawn by joining the mid points of a square. A third square is drawn inside the second square in the same way and the process is continued indefinitely. If the side of the square is 10cm. Find the sum of the areas of all the squares so formed.



Side $S_1 = 10$

Side $S_2 = \sqrt{5^2+5^2} = 5\sqrt{2}$

Side $S_3 = \sqrt{\frac{25}{2} + \frac{25}{2}} = \sqrt{25} = 5$

let $S \rightarrow$ sum of their areas

$S = 100 + 50 + 25 + \cdots \infty$

$$S = \frac{100}{1-\frac{1}{2}}$$

$S = 200 \text{ cm}^2$ Ans

Ans 4 →

Prove that $2^{1/4} \cdot 4^{1/8} \cdot 8^{1/16} \cdot 16^{1/32} \cdots \infty = 2$

Soln

L.H.S $\quad 2^{1/4} \cdot 2^{2/8} \cdot 2^{3/16} \cdot 2^{4/32} \cdots \infty$

$= 2^{\frac{1}{4} + \frac{2}{8} + \frac{3}{16} + \frac{4}{32} + \cdots \infty}$

power $\quad \frac{1}{4} + \frac{2}{8} + \frac{3}{16} + \frac{4}{32} + \cdots \infty$

$= \left(\frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} + \cdots \infty\right) + \left(\frac{1}{8} + \frac{1}{16} + \frac{1}{32} + \cdots \infty\right) + \left(\frac{1}{16} + \frac{1}{32} + \cdots \infty\right) + \cdots$

$= \dfrac{\frac{1}{4}}{1-\frac{1}{2}} + \dfrac{\frac{1}{8}}{1-\frac{1}{2}} + \dfrac{\frac{1}{16}}{1-\frac{1}{2}} + \cdots \infty$

$= \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots \infty$

$= \dfrac{\frac{1}{2}}{1-\frac{1}{2}} = \dfrac{1/2}{1/2} = 1 =$ power

$\therefore 2^1 = 2$ Ans

Ans 5 → If a, b, c are in G.P., then show that $\log a^n, \log b^n, \log c^n$ are in A.P.

Soln Given $a, b, c \to GP$

$\Rightarrow \boxed{b^2 = ac}$

T.P $\log a^n, \log b^n, \log c^n$ are in A.P

i.e $2\log b^n = \log a^n + \log c^n$

LHS $2\log b^n$

$= 2n \log b$ $\boxed{\log m^n = n\log m}$

RHS $\log a^n + \log c^n$ $\boxed{\log(AB) = \log A + \log B}$

$= \log(a^n \cdot c^n)$

$= \log(ac)^n$

$= \log(b^2)^n$ ... $\{$ given $ac = b^2 \}$

$= \log(b^{2n})$

$= 2n \cdot \log b$ -- $\boxed{\log m^n = n\log m}$

LHS = RHS

$\Rightarrow \log a^n, \log b^n, \log c^n$ are in A.P Proved

Q.No.6 $\Rightarrow$ Find two positive numbers whose difference is 12 and whose A.M exceeds the G.M by 2

Soln let the two nos are a & b

let $a > b$

$\Rightarrow \boxed{a-b=12} \quad \cdots (1)$

given $\quad A.M - G.M = 2$

$\Rightarrow \quad \frac{a+b}{2} - \sqrt{ab} = 2$

$\Rightarrow \quad a+b-2\sqrt{ab} = 4$

$\Rightarrow \quad (\sqrt{a}-\sqrt{b})^2 = 4$

$\Rightarrow \quad \sqrt{a}-\sqrt{b} = \pm 2$

$\Rightarrow \quad \boxed{\sqrt{a}-\sqrt{b} = 2} - (2) \quad -- \left\{ \begin{array}{l} \because a>b \\ \sqrt{a} > \sqrt{b} \end{array} \right\}$

from (1) $\quad a-b=12$

$(\sqrt{a}+\sqrt{b})(\sqrt{a}-\sqrt{b}) = 12$

$\Rightarrow \boxed{\sqrt{a}+\sqrt{b} = 6} -- (3)$

(2) + (3)

$2\sqrt{a} = 8$

$\sqrt{a} = 4$

$a = 16 \quad b = 4$ Ans

Qn. 7 ⇒ Let S be the sum, P the product and R be the sum of the reciprocals of 3 terms of a GP Then find $P^2R^3 : S^3$

Sol let the three term of GP are : a, ar, ar^2

$S = a + ar + ar^2 \Rightarrow \boxed{S = a(1+r+r^2)}$

$P = a \cdot ar \cdot ar^2$

$\Rightarrow \boxed{P = a^3 r^3}$

$R = \frac{1}{a} + \frac{1}{ar} + \frac{1}{ar^2}$

$R = \frac{1}{a}\left(1 + \frac{1}{r} + \frac{1}{r^2}\right)$

$\boxed{R = \frac{1}{a}\left(\frac{r^2 + r + 1}{r^2}\right)}$

Taking $\frac{P^2 R^3}{S^3} = \frac{a^6 r^6 \cdot \frac{1}{a^3}\left(\frac{r^2+r+1}{r^2}\right)^3}{a^3(1+r+r^2)^3}$

$= 1$

$\therefore \; P^2 R^3 : S^3 = 1 : 1$ Ans

Qn. 8 → At the end of each year, the value of a certain machine has depreciated by 20% of its initial value at the beginning of that year. If its initial value was Rs 1250. Find the value at the end of 5 years

Soln

Initial value of Machine = Rs 1250

value of Machine at the end of 1st year $= \frac{8}{10} \times 1250$

$= 1000$

value of machine at the 2nd year = $\frac{80}{100} \times 1000 = 800$

" " " 3rd year $= \frac{80}{100} \times 800 = 640$

Hence

$1250, 1000, 800, \ldots$

It is a G.P $\quad a = \cancel{100}\ 1250$

$r = \frac{1000}{1250} = \frac{4}{5} \ ; \ \frac{800}{1000} = \frac{4}{5}$

value of Machine at the end of 5th year = 6th term of this G.P

$= ar^5$

$= 1250\left(\frac{4}{5}\right)^5$

$= 1250 \times \frac{1024}{3125} = \boxed{Rs\ \ \ }$ Ans

Qn 9 → which is the rational number having the decimal expansion $0.3\overline{56}$

Soln

$0.3\overline{56} = 0.3565656\ldots$

$= 0.3 + 0.056 + 0.00056 + 0.0000056 + \ldots \infty$

$= \frac{3}{10} + \frac{56}{10^3} + \frac{56}{10^5} + \frac{56}{10^7} + \ldots \infty$

$= \frac{3}{10} + \frac{56}{10^3}\left(1 + \frac{1}{10^2} + \frac{1}{10^4} + \ldots \infty\right)$

Infinite G.P $\quad a = 1, \ r = \frac{1}{100}$

$$= \frac{3}{10} + \frac{56}{10^3}\left(\frac{1}{1-\frac{1}{100}}\right)$$

$$= \frac{3}{10} + \frac{56}{1000}\left(\frac{100}{99}\right)$$

$$= \frac{3}{10} + \frac{56}{990}$$

$$= \frac{297+56}{990}$$

$$= \frac{353}{990} \text{ Ans}$$

Qn 10 → 150 workers were engaged to finish a piece of work in a certain number of days. Four workers dropped the second day, four more workers dropped the 3rd day and so on. It takes 8 more days to finish the work now. Find the number of days in which the work was completed.

Soln let the initial no of days = n

total No of workers required = $150n$

New condition $150, 146, 142, \ldots\ldots (n+8)$ term

AP $a = 150,\ d = -4$; no of term $= n+8$

$$S_{(n)} = \frac{n+8}{2}\left[300 + (n+7)(-4)\right]$$

$$= \left(\frac{n+8}{2}\right)\left[272 - 4n\right] = (n+8)(136-2n)$$

Since the work is same

$$150n = (n+8)(136-2n)$$

Proceed

$$n = 17$$

$\therefore$ Required No. of days $= 17+8 = 25$ days Ans

Q.N. 11 → divide 32 in to four parts which are in A.P such that the product of extremes is to the product of means is 7:15

Soln

let the No. are $a-3d,\ \overbrace{a-d,\ a+d}^{\text{mean}},\ a+3d$ (extremes: $a-3d$, $a+3d$)

$$\frac{(a-3d)(a+3d)}{(a-d)(a+d)} = \frac{7}{15}$$

also

$$a-\cancel{3d}+a-\cancel{d}+a+\cancel{d}+a+\cancel{3d} = 32$$

$$\Rightarrow 4a = 32 \Rightarrow a = 8$$

$$\Rightarrow \frac{a^2-9d^2}{a^2-d^2} = \frac{7}{15}$$

$$\Rightarrow \frac{64-9d^2}{64-d^2} = \frac{7}{15}$$

$$\Rightarrow 960 - 135d^2 = 448 - 7d^2$$

$$512 = 128d^2 \Rightarrow d^2 = 4 \Rightarrow d = \pm 2$$

$a = 8,\ d = 2$

$2, 6, 10, 14$ Ans

OR

$a = 8,\ d = -2$

$14, 10, 6, 2$ Ans

Q.No. 12 → The first, second and the last terms of an A.P are a, b, c respectively. Prove that the sum is $\frac{(a+c)(b+c-2a)}{2(b-a)}$

Soln

1^{st} term $= a$; 2^{nd} term $= b$

$\therefore d = b-a$

Let total No. of term $= n$

$\therefore \quad a_n = c$

$\Rightarrow \quad a + (n-1)(b-a) = c$

$\Rightarrow \quad (n-1)(b-a) = c-a$

$\Rightarrow \quad n-1 = \frac{c-a}{b-a}$

$\Rightarrow \quad n = \frac{c-a}{b-a} + 1$

$\Rightarrow \quad \boxed{n = \frac{b+c-2a}{b-a}}$

Sum $= \frac{n}{2}[a+c]$

$= \frac{(b+c-2a)}{2(b-a)}(a+c)$ = Rhs Proved.

Q. 13 → If there are $(2n+1)$ terms in an AP, then Prove that the ratio of the sum of odd terms and the sum of even terms is $(n+1):n$

Sol:

A.P $a_1, a_2, a_3, a_4 \ldots\ldots a_{2n+1}$

odd term $a_1, a_3, a_5, \ldots\ldots a_{2n+1}$

AP
- $I^{st} = a_1 = a$
- diff $= 2d$
- no. term $= n+1$

even Term $a_2, a_4, a_6 \ldots a_{2n}$

AP
- I^{st} term $a_2 = a+d$
- diff $= 2d$
- no. of term $= n$

Sum of odd term $= \frac{n+1}{2}\left[2a + (n)2d\right]$

$S_o = (n+1)(a+nd)$

Sum of even term $= \frac{n}{2}\left(2a+2d+(n-1)(2d)\right)$

$= \frac{n}{2}(2a+2nd)$

$S_E = n(a+nd)$

$\frac{S_o}{S_E} = \frac{(n+1)\cancel{(a+nd)}}{n\cancel{(a+nd)}} = n+1 : n$ Ans

Qn. 14 → Show that (x^2+xy+y^2), (z^2+zx+x^2), (y^2+yz+z^2) are in AP if x, y, z are in AP

Soln. (x^2+xy+y^2), (z^2+zx+x^2), (y^2+yz+z^2) will be in AP if

$$(z^2+zx+x^2)-(x^2+xy+y^2)=(y^2+yz+z^2)-(z^2+zx+x^2)$$

i.e

$$z^2+zx+\cancel{x^2}-xy-y^2=y^2+yz-zx-x^2$$

$$\Rightarrow x^2+z^2+2zx-y^2=y^2+yz+xy$$

$$\Rightarrow (x+z)^2-y^2=y(y+z+x)$$

$$\Rightarrow \cancel{(x+z+y)}(x+z-y)=y\cancel{(y+z+x)}$$

$$\Rightarrow x+z-y=y$$

$$\Rightarrow x+z=2y$$

$\Rightarrow$ x, y, z are in AP

which is true

Qn. 15 → If a, b, c are in AP, then show that

(1) $\frac{1}{bc}, \frac{1}{ca}, \frac{1}{ab}$ are in AP

(2) $a\left(\frac{1}{b}+\frac{1}{c}\right), b\left(\frac{1}{c}+\frac{1}{a}\right), c\left(\frac{1}{a}+\frac{1}{b}\right)$ are in AP

(3) $\frac{1}{\sqrt{b}+\sqrt{c}}, \frac{1}{\sqrt{c}+\sqrt{a}}, \frac{1}{\sqrt{a}+\sqrt{b}}$ are in AP

Soln (i) $\frac{1}{bc}, \frac{1}{ca}, \frac{1}{ab}$ will be in AP if

$$\frac{1}{ca} - \frac{1}{bc} = \frac{1}{ab} - \frac{1}{ca}$$

i.e $$\frac{b-a}{ab\cancel{c}} = \frac{c-b}{ab\cancel{c}}$$

i.e $$b-a = c-b$$

i.e $$2b = a+c$$

which is true --- {given a, b, c in AP}

$\therefore \frac{1}{bc}, \frac{1}{ca}, \frac{1}{ab}$ are in AP

(2) $a\left(\frac{1}{b}+\frac{1}{c}\right), b\left(\frac{1}{c}+\frac{1}{a}\right), c\left(\frac{1}{a}+\frac{1}{b}\right)$ will be in AP if

$a\left(\frac{1}{b}+\frac{1}{c}\right)+1, b\left(\frac{1}{c}+\frac{1}{a}\right)+1, c\left(\frac{1}{a}+\frac{1}{b}\right)+1$ in AP

ie $a\left(\frac{1}{b}+\frac{1}{c}+\frac{1}{a}\right), b\left(\frac{1}{c}+\frac{1}{a}+\frac{1}{b}\right), c\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)$, AP

if a, b, c are in AP -- { divided by $\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)$

which is true

(3) $\frac{1}{\sqrt{b}+\sqrt{c}}, \frac{1}{\sqrt{c}+\sqrt{a}}, \frac{1}{\sqrt{a}+\sqrt{b}}$ will be in AP if

$$\frac{1}{\sqrt{c}+\sqrt{a}} - \frac{1}{\sqrt{b}+\sqrt{c}} = \frac{1}{\sqrt{a}+\sqrt{b}} - \frac{1}{\sqrt{c}+\sqrt{a}}$$

i.e $$\frac{\sqrt{b}+\cancel{\sqrt{c}}-\cancel{\sqrt{c}}-\sqrt{a}}{\cancel{(\sqrt{c}+\sqrt{a})}(\sqrt{b}+\sqrt{c})} = \frac{\sqrt{c}+\cancel{\sqrt{a}}-\cancel{\sqrt{a}}-\sqrt{b}}{(\sqrt{a}+\sqrt{b})\cancel{(\sqrt{c}+\sqrt{a})}}$$

or $$\frac{\sqrt{b}-\sqrt{a}}{\sqrt{b}+\sqrt{c}} = \frac{\sqrt{c}-\sqrt{b}}{\sqrt{a}+\sqrt{b}}$$

$\Rightarrow \cancel{t}(a-b) = \cancel{t}(b-c)$

$\Rightarrow a-b = b-c$

$\Rightarrow 2b = a+c$

which is true $\therefore$ Proved

Que 15 $\Rightarrow$ If a^2, b^2, c^2 are in AP then show that

$\frac{a}{b+c}, \frac{b}{c+a}, \frac{c}{a+b}$ are in AP

Soln

$\frac{a}{b+c}, \frac{b}{c+a}, \frac{c}{a+b}$ will be in AP if

$\frac{a}{b+c}+1, \frac{b}{c+a}+1, \frac{c}{a+b}+1$ will be in AP if

$\frac{a+b+c}{b+c}, \frac{a+b+c}{c+a}, \frac{a+b+c}{a+b}$ will be in AP if

divided by $(a+b+c)$ $\frac{1}{b+c}, \frac{1}{c+a}, \frac{1}{a+b}$ will be AP if

$$\frac{1}{c+a} - \frac{1}{b+c} = \frac{1}{a+b} - \frac{1}{c+a}$$

ie $$\frac{b+\cancel{c}-\cancel{c}-a}{(\cancel{c+a})(b+c)} = \frac{c+\cancel{a}-\cancel{a}-b}{(a+b)(\cancel{c+a})}$$

$$\Rightarrow \frac{b-a}{b+c} = \frac{c-b}{a+b}$$

$$\Rightarrow b^2-a^2 = c^2-b^2$$

$$\Rightarrow 2b^2 = a^2+c^2$$

which is true Proved

WORKSHEET NO: 2 REVISION SEQUENCE SERIES

Qn: 1 If a^2, b^2, c^2 are in AP, then show that $\frac{1}{b+c}, \frac{1}{c+a}, \frac{1}{a+b}$ are in AP

Qn. 2 If $a\left(\frac{1}{b}+\frac{1}{c}\right), b\left(\frac{1}{c}+\frac{1}{a}\right), c\left(\frac{1}{a}+\frac{1}{b}\right)$ are in AP then show that a, b, c are in AP

Qn 3 If $\frac{b+c}{a}, \frac{c+a}{b}, \frac{a+b}{c}$ are in AP, show that

(i) $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in AP

(ii) bc, ca, ab are in AP

Qn 4 If $\frac{b+c-a}{a}, \frac{c+a-b}{b}, \frac{a+b-c}{c}$ are in AP, then show that $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are also in AP

Hint adding 2

Qn 5 A man saved Rs 66000 in 20 years. In each succeeding year after the first year he saved Rs 200 more than what he saved in the previous year. How much did he save in the first year? Ans Rs 1400

Qn 6 If the A.M between p^{th} and q^{th} terms of an AP be equal to the A.M between r^{th} and the s^{th} terms of the A.P, then show that $p+q = r+s$

Qn. 7 The p^{th} term of an A.P is a and the q^{th} term is b. Prove that the sum of its $(p+q)$ term is $\frac{p+q}{2}\left[a+b+\frac{a-b}{p-q}\right]$

Qn. 8 → Find the three numbers in GP whose sum is 52 and the sum of whose products in pairs 624 Ans 4, 12, 36 (or) 36, 12, 4

Qn 9 → A person writes a letter to four of his friends. He asks each one of them to copy the letter and mail to four different persons with instruction that they move the chain similarly. Assuming that the chain is not broken and that it costs 50 paise to mail one letter. Find the amount spent on the postage when 8th set of letter is mailed

Ans Rs 43690

Qn 10 → If each term of an infinite GP is twice the sum of the terms following it, then find the common ratio of the GP Ans $r = 1/3$

Qn 11 → Find the ration number having decimal expansion (i) $0.\overline{231}$ (ii) $3.5\overline{2}$ Ans (i) $\frac{231}{999}$ (ii) $\frac{317}{90}$

Qn 12 → If a, b, c are in GP and x, y are the Arithmetic means of a, b and b, c respectively. Then show that $\frac{a}{x} + \frac{c}{y} = 2$ and $\frac{1}{x} + \frac{1}{y} = \frac{2}{b}$

Qn 13 → a, b, c, d are in GP then show that

(i) $(a^2+b^2+c^2), (ab+bc+cd), (b^2+c^2+d^2)$ are in GP

(ii) $\frac{1}{a^2+b^2}, \frac{1}{b^2+c^2}, \frac{1}{c^2+d^2}$ are in GP

— x —