॥ जय श्री राधे कृष्णा ॥ (1)

→ ULTIMATE MATHEMATICS →

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Chapter: SEQUENCE & SERIES | Class No: 5

Que 1 → Find the natural number 'a' for which $\sum_{k=1}^{n} f(a+k) = 16(2^n-1)$, where the function f satisfies $f(x+y) = f(x)\cdot f(y)$ and given $f(1)=2$

Solution Given $\sum_{k=1}^{n} f(a+k) = 16(2^n-1)$

$$\Rightarrow \sum_{k=1}^{n} f(a)\cdot f(k) = 16(2^n-1)$$

Imp $\Rightarrow f(a)\sum_{k=1}^{n} f(k) = 16(2^n-1)$

$$\Rightarrow f(a)\{f(1)+f(2)+f(3)+\dots n\ \text{terms}\} = 16(2^n-1)$$

$f(2) = f(1+1) = f(1)\cdot f(1) = 2\cdot 2 = 2^2$

$f(3) = f(1+2) = f(1)\cdot f(2) = 2\cdot 2^2 = 2^3$

∴ Equation becomes

$$f(a)\underbrace{[2+2^2+2^3 \dots n\ \text{term}]}_{GP:\ a=2,\ r=2} = 16(2^n-1)$$

$$\Rightarrow f(a)\cdot 2\frac{(\cancel{2^n-1})}{2-1} = 16(\cancel{2^n-1})$$

$\Rightarrow f(a)\cdot 2 = 16$

$\Rightarrow f(a) = 8$

$\Rightarrow 2^a = 2^3$

$\Rightarrow a=3$ Ans

Ques 2 → Find the minimum value of the expression $3^x + 3^{1-x}$

Soln

$\boxed{A.M \geq G.M}$ $\frac{a+b}{2} \geq \sqrt{ab}$

here $a = 3^x$ & $b = 3^{1-x}$

$\Rightarrow \frac{3^x + 3^{1-x}}{2} \geq \sqrt{3^x \cdot 3^{1-x}}$

$\Rightarrow \frac{3^x + 3^{1-x}}{2} \geq \sqrt{3}$

$\Rightarrow 3^x + 3^{1-x} \geq 2\sqrt{3}$

Min value of $3^x + 3^{1-x} = 2\sqrt{3}$ Ans

Ques 3 → Find the 10th common term between the series $3+7+11+ \cdots$ and $1+6+11+ \cdots$

Soln 1st series $3+7+11+15+19+23+27+31+35+39+43+47+51+55+ \cdots$

2nd series : $1+6+11+16+21+26+31+36+41+46+51+56+ \cdots$

Sequence of common terms

$11, 31, 51 \cdots$

It is also in AP with $a = 11$ & $d = 20$

we have to find 10th term of this sequence

$a_{10} = a + 9d = 11 + 180 = 191$ Ans

Method II ① $3+7+11+\dots$

② $1+6+11+\dots$

L.C.M of 4 & 5 = 20

$d=20;\ a=11$

$a_{10}=a+9d=11+180=191$ Ans

Qn. 4 → In a G.P of +ve terms, if any term is equal to the sum of the next two terms. Then show that common ratio of the G.P is $2\sin(18^\circ)$

Sol

Soln given $a_n = a_{n+1}+a_{n+2}$

$\Rightarrow ar^{n-1} = ar^{n} + ar^{n+1}$

$\Rightarrow r^{n-1} = r^{n}+r^{n+1}$

$\Rightarrow \cancel{r^{n-1}} = \cancel{r^{n-1}}(r+r^2)$

$\Rightarrow 1 = r+r^2$

$\Rightarrow r^2+r-1=0$

Quadratic formula $r=\dfrac{-1\pm\sqrt{1+4}}{2}$

$r=\dfrac{-1\pm\sqrt{5}}{2}$ $\qquad r=\dfrac{-1-\sqrt{5}}{2}$ ✗ (Rejected)

$r=\dfrac{\sqrt{5}-1}{2} \Rightarrow r=\left(\dfrac{\sqrt{5}-1}{4}\right)\times 2 = 2\sin(18^\circ)$ Ans

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Qn. 5 → If a, b, c, d, p are real numbers such that $(a^2+b^2+c^2)p^2 - 2(ab+bc+cd)p + (b^2+c^2+d^2) \leq 0$ then show that a, b, c, d are in G.P

Soln given $(a^2+b^2+c^2)p^2 - 2(ab+bc+cd)p + (b^2+c^2+d^2) \leq 0$

$\Rightarrow a^2p^2 + b^2p^2 + c^2p^2 - 2abp - 2bcp - 2cdp + b^2 + c^2 + d^2 \leq 0$

$\Rightarrow$

$(ap-b)^2 + (bp-c)^2 + (cp-d)^2 \leq 0$

But sum of squares can never be −ve

$\Rightarrow (ap-b)^2 + (bp-c)^2 + (cp-d)^2 = 0$

this is possible only when

$ap-b=0$; $bp-c=0$ and $cp-d=0$

$\Rightarrow p = \frac{b}{a}$; $p = \frac{c}{b}$ & $p = \frac{d}{c}$

$\Rightarrow \frac{b}{a} = \frac{c}{b} = \frac{d}{c}$

$\Rightarrow a, b, c, d$ are in GP

Qn. 6 → If A & G be A.M & G.M b/w two positive numbers, then prove that the numbers are $A \pm \sqrt{(A+G)(A-G)}$

Soln let the two numbers are a & b

$\Rightarrow A = \frac{a+b}{2}$ & $G = \sqrt{ab}$

$\Rightarrow a+b = 2A$ & $ab = G^2$

$\Rightarrow b = 2A - a$

$\Rightarrow a(2A-a) = G^2$

$\Rightarrow 2Aa - a^2 = G^2$

$\Rightarrow a^2 - 2Aa + G^2 = 0$

By quadratic formula

$$a = \frac{2A \pm \sqrt{4A^2 - 4G^2}}{2}$$

$$a = \frac{2A \pm 2\sqrt{(A+G)(A-G)}}{2}$$

$$a = A \pm \sqrt{(A+G)(A-G)}$$

Similarly $b = A \pm \sqrt{(A+G)(A-G)}$

$\therefore$ the numbers are $A \pm \sqrt{(A+G)(A-G)}$ Ans

Que 7 → If p^{th}, q^{th} and r^{th} terms of an AP and G.P are both a, b and c respectively. Show that

$$a^{b-c} \cdot b^{c-a} \cdot c^{a-b} = 1$$

Soln

GP	$a_p = a$ $\Rightarrow AR^{p-1} = a$	$a_q = b$ $AR^{q-1} = b$	$a_r = c$ $AR^{r-1} = c$
AP	$a_p = a$ $A + (p-1)d = a$	$a_q = b$ $A + (q-1)d = b$	$a_r = c$ $A + (r-1)d = c$

L.H.S $a^{b-c} \cdot b^{c-a} \cdot c^{a-b}$

$= \left(AR^{p-1}\right)^{d(q-r)} \cdot \left(AR^{q-1}\right)^{d(r-p)} \cdot \left(AR^{r-1}\right)^{d(p-q)}$

$= (A)^{d(q-r)} \cdot R^{d(p-1)(q-r)} \cdot A^{d(r-p)} \cdot R^{d(q-1)(r-p)} \cdot$

$A^{d(p-q)} \cdot R^{d(r-1)(p-q)}$

$= A^{d(\cancel{q}-\cancel{r}+\cancel{r}-\cancel{p}+\cancel{p}-\cancel{q})} \cdot R^{d(pq-r-q+r-\ldots)}$

$= A^{0} \cdot R^{0} = 1 \times 1 = 1$ Ans

Ques 8 → If $\theta_1, \theta_2, \theta_3 \ldots \theta_n$ are in AP with common difference 'd' show that

$\sec\theta_1 \cdot \sec\theta_2 + \sec\theta_2 \cdot \sec\theta_3 + \ldots \sec\theta_{n-1} \cdot \sec\theta_n$

$= \dfrac{\tan\theta_n - \tan\theta_1}{\sin d}$

Soln L.H.S

$$\frac{1}{\cos\theta_1 \cdot \cos\theta_2} + \frac{1}{\cos\theta_2 \cdot \cos\theta_3} + \ldots \frac{1}{\cos\theta_{n-1} \cdot \cos\theta_n}$$

M & D by $\sin d$

$$\frac{1}{\sin d}\left[\frac{\sin d}{\cos\theta_1 \cdot \cos\theta_2} + \frac{\sin d}{\cos\theta_2 \cdot \cos\theta_3} + \ldots \frac{\sin d}{\cos\theta_{n-1} \cdot \cos\theta_n}\right]$$

$$= \frac{1}{\sin d}\left[\frac{\sin(\theta_2-\theta_1)}{\cos\theta_1 \cdot \cos\theta_2} + \frac{\sin(\theta_3-\theta_2)}{\cos\theta_2 \cdot \cos\theta_3} + \ldots \frac{\sin(\theta_n-\theta_{n-1})}{\cos\theta_{n-1} \cdot \cos\theta_n}\right]$$

(Proceed)

INFINITE G.P

$$\boxed{S_\infty = \frac{a}{1-r}} \quad |r|<1$$

Qn. 9 → Find the sum of the Infinite G.P
6, 1.2, .24, -----

Sol: here $a = 6$

$$r = \frac{1.2}{6} = 0.2$$

$$S_\infty = \frac{a}{1-r} = \frac{6}{1-0.2} = \frac{6}{0.8} = \frac{60}{8} = 7.5 \text{ Ans}$$

Qn. 10 Show $3^{1/2} \times 3^{1/4} \times 3^{1/8} \times --- = 3$

Soln LHS $3^{\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+--\infty}$

here $a = 1/2$; $r = 1/2$

$$= 3^{\frac{\frac{1}{2}}{1-\frac{1}{2}}}$$

$$= 3^{\cancel{1/2}/\cancel{1/2}} = 3^1 = 3 \text{ Ans}$$

Qn. 11 → let $x = 1+a+a^2 ---- \infty$ where $|a|<1$
$y = 1+b+b^2 --- \infty$ and $|b|<1$

Show that $1+ab+a^2b^2+ --- \infty = \frac{xy}{x+y-1}$

Soln $x = 1+a+a^2+ --- \infty$ ⇒ GP First term = 1
Common Ratio = a

$$x = \frac{1}{1-a}$$

$y = 1+b+b^2 + -- \infty$ ⇒ GP first term = 1
ratio = b

$$y = \frac{1}{1-b}$$

LHS $1+ab+a^2b^2+\dots\infty$

$\Rightarrow$ GP Ist term = 1

ratio = ab

$$\boxed{\text{LHS} = \frac{1}{1-ab}}$$

$$\text{RHS} = \frac{xy}{x+y-1}$$

put value of x & y

$$= \frac{\left(\frac{1}{1-a}\right)\left(\frac{1}{1-b}\right)}{\left(\frac{1}{1-a}\right)+\left(\frac{1}{1-b}\right)-1}$$

$$= \frac{1}{1-b+1-a-1(1-a)(1-b)}$$

$$= \frac{1}{\cancel{1}-\cancel{b}+1-\cancel{a}-\cancel{1}+\cancel{b}+\cancel{a}-ab}$$

$$= \frac{1}{1-ab}$$

= LHS Proved

Qn 12 : The sum of an Infinite G.P is 8 and its second term is 2. Find the Ist term

Soln given $\boxed{\frac{a}{1-r}=8}$ given $\boxed{ar=2}$ $\Rightarrow r=\frac{2}{a}$

$\Rightarrow$ $\cancel{\frac{a}{1-a}=8}$ $\Rightarrow$ (a = 4) Ans

Proved

(Worksheet No: 3) chapter: Sequence - series (class No: 5)

Qn: 1 The lengths of three unequal edges of a rectangular solid block are in G.P. The volume of the block is $216\ cm^3$ and the total surface area is $252\ cm^2$. Then length of the longest edge is

(A) 12 cm (B) 6 cm (C) 18 cm (D) 3 cm

Ans: (A)

Qn 2 → Find the minimum value of $4^x + 4^{1-x}$

Ans = 4

Ans 3 → If in AP ; $S_n = 2n^2$ and $S_m = 2m^2$ find S_2

Ans = 2^3

Qn 4 → If $x, 2y, 3z$ are in AP and x, y, z are in G.P find the common ratio

Ans = $1/3$

Qn 5 → If $\theta_1, \theta_2, \theta_3 \ldots \theta_n$ are in AP whose common difference is 'd', then show that

$$\sec\theta_1 \cdot \sec\theta_2 + \sec\theta_2 \cdot \sec\theta_3 + \cdots \sec\theta_{n-1} \cdot \sec\theta_n = \frac{\tan\theta_n - \tan\theta_1}{\sin d}$$

Qn 6 → Let S be the sum, P be the product and R be the sum of the reciprocals of 3 terms of a G.P. Then find $P^2R^3 : S^3$

Ans 1 : 1

Qn 7 → If a, b, c, d are in G.P, prove that a^2-b^2, b^2-c^2, c^2-d^2 are also in G.P

Sequence Series (Worksheet No:3) Class: 5

Q No. 8 → The product of three numbers in A.P is 224 and the largest number is 7 times the smallest. Find the numbers

Ans 2, 8, 14

Q No. 9 → If there are $(2n+1)$ terms in an A.P, then prove that the ratio of the sum of odd terms and the sum of even terms is $(n+1):n$

Q No. 10 → The p^{th} term of an A.P is 'a' and q^{th} term is 'b'. Prove that the sum of its $(p+q)$ terms is $\frac{p+q}{2}\left[a+b+\frac{a-b}{p-q}\right]$

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