॥ जय श्री गिरिराज जी महाराज ॥

Topic : .... Date. : .... Page No. : ....

ULTIMATE MATHEMATICS

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CHAPTER: SEQUENCE & SERIES

CLASS NO. 3

Topic: G.P (Question)

Ques 1 Find the value of 'n' so that $\dfrac{a^{n+1}+b^{n+1}}{a^n+b^n}$ may be the Geometric Mean between a & b.

Soln given $\dfrac{a^{n+1}+b^{n+1}}{a^n+b^n} = \sqrt{ab} = \sqrt{a}\,\sqrt{b}$

$$\Rightarrow \frac{a^{n+1}+b^{n+1}}{a^n+b^n} = a^{1/2}\cdot b^{1/2}$$

$$\Rightarrow a^{n+1}+b^{n+1} = a^{1/2}\cdot b^{1/2}\left(a^n+b^n\right)$$

$$\Rightarrow a^{n+1}+b^{n+1} = a^{n+\frac{1}{2}}\cdot b^{1/2} + a^{1/2}\cdot b^{n+\frac{1}{2}}$$

$$\Rightarrow a^{n+1} - a^{n+\frac{1}{2}}\cdot b^{1/2} = a^{1/2}\cdot b^{n+\frac{1}{2}} - b^{n+1}$$

$$\Rightarrow a^{n+\frac{1}{2}}\cancel{\left(a^{1/2}-b^{1/2}\right)} = b^{n+\frac{1}{2}}\cancel{\left(a^{1/2}-b^{1/2}\right)}$$

$$\Rightarrow \frac{a^{n+1/2}}{b^{n+\frac{1}{2}}} = 1$$

$$\Rightarrow \left(\frac{a}{b}\right)^{n+\frac{1}{2}} = 1$$

$$\Rightarrow \left(\frac{a}{b}\right)^{n+1/2} = \left(\frac{a}{b}\right)^0$$

$$\Rightarrow n+\frac{1}{2} = 0$$

$$\Rightarrow \boxed{n = -1/2} \text{ Ans}$$

Ques : 2 If a, b, c, d are in G.P Prove that $(a^n+b^n), (b^n+c^n), (c^n+d^n)$ are in G.P

Sol: given: a, b, c, d are in GP

let $a = a$, $b = ar$, $c = ar^2$, $d = ar^3$

T.P (a^n+b^n), (b^n+c^n), (c^n+d^n) are in GP

i.e $(b^n+c^n)^2 = (a^n+b^n)\cdot(c^n+d^n)$

LHS $(b^n+c^n)^2$

$= \left(a^n r^n + a^n r^{2n}\right)^2$

$= a^{2n} r^{2n}\left(1+r^n\right)^2$

RHS $(a^n+b^n)\cdot(c^n+d^n)$

$= (a^n + a^n r^n)\cdot(a^n r^{2n} + a^n r^{3n})$

$= a^n(1+r^n)\cdot a^n r^{2n}(1+r^n)$

$= a^{2n}\cdot r^{2n}(1+r^n)^2$

clearly LHS = RHS

$\therefore (a^n+b^n), (b^n+c^n), (c^n+d^n)$ are in GP proved

Ques 3 - If a function f, satisfying $f(x+y) = f(x)\cdot f(y)$ such that $f(1) = 3$ and $\sum_{x=1}^{n} f(x) = 120$. Find the value of n.

Sol: given $f(x+y) = f(x)\cdot f(y)$

$f(1) = 3$

$\sum_{x=1}^{n} f(x) = 120$

$f(1) + f(2) + f(3) + \cdots$ n terms $= 120$

S & S (Class No:- 3)

Now $f(2) = f(1+1) = f(1)\cdot f(1) = 3\times 3 = 9$

$f(3) = f(1+2) = f(1)\cdot f(2) = 3\times 9 = 27$

$\therefore$ equation becomes

$3 + 9 + 27 + \cdots n\text{ term} = 120$

GP $a = 3,\ r = 3,\ S_n = 120$

$$\frac{a(r^n - 1)}{r-1} = 120$$

$$\Rightarrow 3\frac{(3^n - 1)}{2} = 120$$

$$\Rightarrow 3^n - 1 = \frac{240}{3} = 80$$

$$\Rightarrow 3^n = 81$$

$$\Rightarrow 3^n = (3)^4$$

$$\Rightarrow \boxed{n = 4}\ \text{Ans}$$

Ques:- 4 → If a & b are the roots of $x^2 - 3x + p = 0$ and c, d are roots of $x^2 - 12x + q = 0$ where a, b, c, d form a G.P. prove that $(q+p) : (q-p) = 17 : 15$

Sol: (·) a & b roots of $x^2 - 3x + p = 0$

$a + b = 3$

$ab = p$

(·) c & d roots of $x^2 - 12x + q = 0$

$c + d = 12$

$cd = q$

(·) $a, b, c, d \rightarrow$ GP

let $a = a,\ b = ar,\ c = ar^2,\ d = ar^3$

T.P (·) $\frac{q+p}{q-p} = \frac{17}{15}$

$\text{Taking LHS} \quad \frac{q+p}{q-p}$

$= \frac{cd+ab}{cd-ab}$

$= \frac{(ar^2)(ar^3)+(a)(ar)}{(ar^2)(ar^3)-(a)(ar)}$

$= \frac{a^2r^5+a^2r}{a^2r^5-a^2r}$

$= \frac{\cancel{a^2r}(r^4+1)}{\cancel{a^2r}(r^4-1)}$

$\therefore \frac{q+p}{q-p} = \frac{r^4+1}{r^4-1}$

we have $\quad a+b=3 \quad \& \quad c+d=12$

$a+ar=3 \quad \& \quad ar^2+ar^3=12$

$a(1+r)=3 \quad \& \quad ar^2(1+r)=12$

divide these equation

$\frac{\cancel{a}r^2\cancel{(1+r)}}{\cancel{a}\cancel{(1+r)}} = \frac{12}{4}$

$r^2 = 4 \Rightarrow r = \pm 2$

$\therefore \frac{q+p}{q-p} = \frac{(\pm 2)^4+1}{(\pm 2)^4-1} = \frac{16+1}{16-1} = \frac{17}{15}$

$\therefore (q+p):(q-p) = 17:15$ Ans

Qn. 5 → Find the sum of the series up to n-terms
$8 + 88 + 888 + \dots$ n term.

Soln. let $S_n = 8 + 88 + 888 + \dots$ n term

$S_n = 8(1 + 11 + 111 + \dots \text{ n term})$

$= \frac{8}{9}[9 + 99 + 999 + \dots \text{ n term}]$

$= \frac{8}{9}[(10-1) + (10^2-1) + (10^3-1) + \dots \text{ n term}]$

$= \frac{8}{9}[(10 + 10^2 + 10^3 + \dots \text{ n term}) - (1 + 1 + 1 + \dots \text{ n term})]$
← GP : $a = 10$, $r = 10$

$= \frac{8}{9}\left[\frac{10(10^n - 1)}{9} - n\right]$

$S_n = \frac{8}{81}[10^{n+1} - 10 - 9n]$ Ans

Qn. 6 → Find the sum to n terms
$0.6 + 0.66 + 0.666 + \dots$ n term.

Soln. let $S_n = 0.6 + 0.66 + 0.666 + \dots$ n term

$S_n = 6[0.1 + 0.11 + 0.111 + \dots \text{ n term}]$

$= \frac{6}{9}(0.9 + 0.99 + 0.999 + \dots \text{ n term})$

$= \frac{2}{3}[(1 - 0.1) + (1 - .01) + (1 - .001) + \dots \text{ n term}]$

$= \frac{2}{3}[(1 + 1 + 1 \dots \text{ n term}) - (0.1 + 0.01 + 0.001 + \dots \text{ n term})]$

$= \frac{2}{3}\left[n - \left(\frac{1}{10} + \frac{1}{10^2} + \frac{1}{10^3} + \dots \text{ n term}\right)\right]$
← GP: $a = \frac{1}{10}$; $r = \frac{1}{10} < 1$

CLASSTIME

$$S_n = \frac{2}{3}\left[n - \frac{a(1-r^n)}{1-r}\right]$$

$$= \frac{2}{3}\left(n - \frac{\frac{1}{10}\left(1-\frac{1}{10^n}\right)}{1-\frac{1}{10}}\right)$$

$$= \frac{2}{3}\left(n - \frac{\cancel{\frac{1}{10}}\left[1-\frac{1}{10^n}\right]}{\frac{9}{\cancel{10}}}\right)$$

$$S_n = \frac{2}{27}\left[9n - 1 + \frac{1}{10^n}\right] \quad \underline{Ans}$$

Qn. 7 → The sum of two numbers is 6 times their geometric mean, show that the numbers are in the ratio $(3+2\sqrt{2}) : (3-2\sqrt{2})$

Soln

Let the numbers are a & b

given. $a+b = 6\sqrt{ab}$

$$\Rightarrow \frac{a+b}{2\sqrt{ab}} = \frac{3}{1}$$

Now *. Apply Componendo & Dividendo (C&D) $\left(\frac{N+D}{N-D}\right)$

$$\Rightarrow \frac{a+b+2\sqrt{ab}}{a+b-2\sqrt{ab}} = \frac{3+1}{3-1}$$

$$\Rightarrow \frac{(\sqrt{a})^2+(\sqrt{b})^2+2\sqrt{a}\sqrt{b}}{(\sqrt{a})^2+(\sqrt{b})^2-2\sqrt{a}\sqrt{b}} = 2$$

$$\Rightarrow \frac{(\sqrt{a}+\sqrt{b})^2}{(\sqrt{a}-\sqrt{b})^2} = 2$$

Sol (class No: 3) (7)

$$\Rightarrow \frac{\sqrt{a}+\sqrt{b}}{\sqrt{a}-\sqrt{b}} = \pm\frac{\sqrt{2}}{1}$$

again (C & D)

$$\Rightarrow \frac{(\cancel{\sqrt{a}}+\sqrt{b}) + (\cancel{\sqrt{a}}-\sqrt{b})}{(\cancel{\sqrt{a}}+\sqrt{b}) - (\cancel{\sqrt{a}}-\sqrt{b})} = \pm\left(\frac{\sqrt{2}+1}{\sqrt{2}-1}\right)$$

$$\Rightarrow \frac{\cancel{2}\sqrt{a}}{\cancel{2}\sqrt{b}} = \pm\frac{(\sqrt{2}+1)}{(\sqrt{2}-1)}$$

Squaring both sides

$$\frac{a}{b} = \frac{2+1+2\sqrt{2}}{2+1-2\sqrt{2}}$$

$$\therefore a:b = (3+2\sqrt{2}):(3-2\sqrt{2}) \quad \text{Ans}$$

Ques 8 → If the first and the n^{th} term of a GP are a and b respectively, and if P is the product of n terms, prove that $P^2 = (ab)^n$

Sol

given $a_1 = a$

$a_n = b$

$\Rightarrow ar^{n-1} = b$

$\Rightarrow r^{n-1} = \frac{b}{a}$

$\Rightarrow \boxed{r = \left(\frac{b}{a}\right)^{\frac{1}{n-1}}}$

$P = a \cdot ar \cdot ar^2 \cdots\cdots ar^{n-1}$

$P = a^n \cdot r^{1+2+3\cdots(n-1)}$ ... $\left\{\begin{array}{l} 1+2+3+\cdots n = \frac{n(n+1)}{2} \\ 1+2+3\cdots(n-1) = \frac{(n-1)(n)}{2} \end{array}\right.$

$P = a^n \cdot r^{\frac{n(n-1)}{2}}$

Topic : .... Date. : .... Page No. : ....

(Ses) (Class No: 3) (8)

$$P = a^n \cdot \left[\left(\frac{b}{a}\right)^{\frac{1}{n-1}}\right]^{\frac{n(n-1)}{2}}$$

$$P = a^n \cdot \left(\frac{b}{a}\right)^{\frac{n}{2}}$$

$$P = a^n \cdot \frac{b^{n/2}}{a^{n/2}}$$

$$P = a^{n-\frac{n}{2}} \cdot b^{n/2}$$

$$P = a^{n/2} \cdot b^{n/2}$$

$$P = (ab)^{n/2}$$

Square $P^2 = (ab)^n$ Prove

Qn. 9 → Let S be the sum, P the product and R the sum of reciprocals of n terms in a GP. Prove that $P^2 R^n = S^n$

Soln (·) $S = a + ar + ar^2 + \cdots ar^{n-1}$

$$S = a\left(\frac{r^n - 1}{r - 1}\right) \; ; \; \boxed{r > 1}$$

(·) $P = a \cdot ar \cdot ar^2 \cdots ar^{n-1}$

$$P = a^n \cdot r^{1+2+3 \cdots (n-1)}$$

$$P = a^n \cdot r^{\frac{n(n-1)}{2}}$$

2, 4, 8, 16, 32, 64 → GP
$\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \cdots$

(·) $R = \frac{1}{a} + \frac{1}{ar} + \frac{1}{ar^2} + \cdots \frac{1}{ar^{n-1}}$

← GP 1st term $= \frac{1}{a}$, ratio $= \frac{1}{r} < 1$

$$R = \frac{1}{a}\left(\frac{1 - \frac{1}{r^n}}{1 - \frac{1}{r}}\right) = \frac{1}{a}\left(\frac{r^n - 1}{r - 1}\right) \cdot \frac{r}{r^n}$$

CLASSTIME

Taking LHS $\quad P^2 \cdot R^n$

$$= \left[a^n \cdot r^{\frac{n(n-1)}{2}}\right]^2 \cdot \left[\frac{1}{a}\left(\frac{r^n-1}{r-1}\right) \cdot \frac{r}{r^n}\right]^n$$

$$= \left[a^{2n} \cdot r^{n(n-1)} \cdot \frac{1}{a^n}\left(\frac{r^n-1}{r-1}\right)^n \cdot \frac{r^n}{r^{n^2}}\right]$$

$$= a^n \cdot r^{\cancel{n^2}-\cancel{n}+\cancel{n}-\cancel{n^2}} \cdot \left(\frac{r^n-1}{r-1}\right)^n$$

$$= a^n \cdot \left(\frac{r^n-1}{r-1}\right)^n$$

$$= \left(a \cdot \left(\frac{r^n-1}{r-1}\right)\right)^n$$

$$= S^n$$

$$= RHS \quad \text{proved}$$

C'/

WORKSHEET No:-3.

← Sequence & Series →

Qn. 1 → How many terms of the GP $3, \frac{3}{2}, \frac{3}{4}, \ldots$ are needed to give the sum $\frac{3069}{512}$? **Ans** $n = 10$

Qn. 2 → The sum of first three terms of a GP is $\frac{13}{12}$ and their product is -1. Find the common ratio and the terms

Ans $r = -\frac{3}{4}, -\frac{4}{3}$ **terms** $\frac{4}{3}, -1, \frac{3}{4}$ & $\frac{3}{4}, -1, \frac{4}{3}$

Hint use selection of terms $\frac{a}{r}, a, ar$

Qn. 3 → Find the sum of sequence $7, 77, 777, \ldots$ n terms

Ans $S_n = \frac{7}{81}\left[10^{n+1} - 10 - 9n\right]$

Qn. 4 → Insert three numbers between 1 and 256, so that the resulting sequence is a GP

Ans 4, 16, 64

Qn. 5 If A.M and G.M of two +ve numbers a & b are 10 and 8 respectively. Find the numbers

Qn. 6 → Find four numbers forming a G.P in which the third term is greater than the first term by 9, and the second term is greater than the 4th term by 18.

Ans 3, −6, 12, −24

Qn. 7 → If a, b, c, d are in GP show that

$$(a^2+b^2+c^2)(b^2+c^2+d^2) = (ab+bc+cd)^2$$

Qn 8 → If the 4th, 10th & 16th terms of a GP are x, y and z respectively show that x, y, z are in GP

Qn 9 → The sum of first three terms of a GP is $\frac{39}{10}$ and their product is 1. Find the common ratio and the terms

ANS $\frac{2}{5}, 1, \frac{5}{2}$ (or) $\frac{5}{2}, 1, \frac{2}{5}$

Qn 10 → The sum of three terms in GP is 56. If we subtract 1, 7, 21 from these numbers in that order, we obtain an A.P. Find the numbers

ANS 8, 16, 32

Qn 11 → The ratio of A.M and G.M of two positive numbers a and b is $m:n$.

Show that $a:b = \left(m+\sqrt{m^2-n^2}\right) : \left(m-\sqrt{m^2-n^2}\right)$

HINT Use C&D (Componendo) & dividendo (two times)

Qn 12 → If $\frac{a+bx}{a-bx} = \frac{b+cx}{b-cx} = \frac{c+dx}{c-dx}$, then

Show that a, b, c & d are in GP

HINT : Consider equality one by one & then do cross multiply

— x —

CLASSTIME