!! जय श्री गिरिराज जी महाराज जय श्री राधे कृष्णा !! (1)

ULTIMATE MATHEMATICS: BY AJAY MITTAL

REVISION: LIMITS & DERIVATIVES $\boxed{\text{CLASS No:3}}$

**Ans 1** evaluate $\lim\limits_{x\to a}\left[\dfrac{(x+2)^{5/2}-(a+2)^{5/2}}{x-a}\right]$

Soln put $x+2=y$

when $x\to a$ then $y\to a+2$

$$\therefore \lim_{y\to(a+2)}\left[\frac{y^{5/2}-(a+2)^{5/2}}{y-2-a}\right]$$

$$=\lim_{y\to(a+2)}\left[\frac{y^{5/2}-(a+2)^{5/2}}{y-(a+2)}\right]$$

$$=\frac{5}{2}(a+2)^{5/2-1} \quad \text{---}\left\{\lim_{x\to a}\left(\frac{x^n-a^n}{x-a}\right)=na^{n-1}\right\}$$

$$=\frac{5}{2}(a+2)^{3/2} \quad \text{Ans}$$

**Ans. 2** evaluate $\lim\limits_{y\to 0}\left[\dfrac{(x+y)\sec(x+y)-x\sec x}{y}\right]$

Solution: $$\lim_{y\to 0}\left[\frac{x\sec(x+y)+y\sec(x+y)-x\sec x}{y}\right]$$

$$=\lim_{y\to 0}\left[\frac{x\left(\sec(x+y)-\sec x\right)}{y}+\frac{\cancel{y}\sec(x+y)}{\cancel{y}}\right]$$

$$=\lim_{y\to 0}\left[\frac{x\left(\cos x-\cos(x+y)\right)}{y\cos(x+y)\cos x}+\sec(x+y)\right]$$

$$= \lim_{y \to 0} \left( \frac{-x \cdot 2\sin\left(\frac{2x+y}{2}\right) \cdot \sin\left(\frac{-y}{2}\right)}{y \cos(x+y) \cos x} + \sec(x+y) \right)$$

$$= \lim_{y \to 0} \left( \frac{\cancel{2} x \sin\left(\frac{2x+y}{2}\right) \cdot \sin\left(\frac{y}{2}\right)}{\frac{y}{2} \times \cancel{2} \cdot \cos(x+y) \cos x} + \sec(x+y) \right)$$

$$= \frac{x \sin x \times 1}{\cos^2 x} + \sec x \quad \cdots \left\{ \lim_{x \to 0} \left( \frac{\sin x}{x} \right) = 1 \right\}$$

$$= x \tan x \cdot \sec x + \sec x \quad \text{Ans}$$

Ques 3 → Evaluate $\lim_{x \to \frac{\pi}{6}} \left( \frac{\cot^2 x - 3}{\operatorname{cosec} x - 2} \right)$

Soln =

$$\lim_{x \to \frac{\pi}{6}} \left( \frac{\operatorname{cosec}^2 x - 1 - 3}{\operatorname{cosec} x - 2} \right)$$

$$= \lim_{x \to \frac{\pi}{6}} \left( \frac{\operatorname{cosec}^2 x - 4}{\operatorname{cosec} x - 2} \right)$$

$$= \lim_{x \to \frac{\pi}{6}} \left( \frac{(\operatorname{cosec} x + 2)\cancel{(\operatorname{cosec} x - 2)}}{\cancel{\operatorname{cosec} x - 2}} \right)$$

$$= \operatorname{cosec}\left(\frac{\pi}{6}\right) + 2$$

$$= 2 + 2$$

$$= 4 \quad \text{Ans}$$

Qu. 4 → Evaluate $\lim_{x \to \frac{\pi}{4}} \left( \frac{\tan^3 x - \tan x}{\cos(x + \frac{\pi}{4})} \right)$

Sol

$$\lim_{x \to \frac{\pi}{4}} \left( \frac{\tan x (\tan x + 1)(\tan x - 1)}{\cos(x + \frac{\pi}{4})} \right)$$

$$= \lim_{x \to \frac{\pi}{4}} \left( \tan x \cdot (\tan x + 1) \right) \times \lim_{x \to \frac{\pi}{4}} \left( \frac{\tan x - 1}{\cos(x + \frac{\pi}{4})} \right)$$

$$= 2 \times \lim_{x \to \frac{\pi}{4}} \left( \frac{\tan x - 1}{\cos(x + \frac{\pi}{4})} \right)$$

put $x = \frac{\pi}{4} + h$ & $h \to 0$

$$= 2 \lim_{h \to 0} \left( \frac{\tan(\frac{\pi}{4} + h) - 1}{\cos(\frac{\pi}{4} + h + \frac{\pi}{4})} \right)$$

$$= 2 \lim_{h \to 0} \left[ \frac{\frac{1 + \tan h}{1 - \tan h} - 1}{-\sin h} \right]$$

$$= 2 \lim_{h \to 0} \left( \frac{\cancel{1} + \tan h - \cancel{1} + \tan h}{-\sin h \cdot (1 - \tan h)} \right)$$

$$= -4 \lim_{h \to 0} \left( \frac{\tan h}{\sin h \, (1 - \tan h)} \right)$$

$$= -4 \lim_{h \to 0} \left( \frac{\frac{\tan h}{h}}{\frac{\sin h}{h} \cdot (1 - \tan h)} \right)$$

$$= -4 \times \frac{1}{1} = -4 \text{ Ans}$$

Qn. 5 → Using first principle, find derivative of $f(x) = \sin x - \cos x$

Soln

$$f(x) = \sin x - \cos x$$

$$f'(x) = \lim_{h \to 0} \left[ \frac{(\sin(x+h) - \cos(x+h)) - (\sin x - \cos x)}{h} \right]$$

$$= \lim_{h \to 0} \left[ \frac{(\sin(x+h) - \sin x) - (\cos(x+h) - \cos x)}{h} \right]$$

$$= \lim_{h \to 0} \left[ \frac{2\cos\left(\frac{2x+h}{2}\right) \cdot \sin\left(\frac{h}{2}\right) + 2\sin\left(\frac{2x+h}{2}\right) \cdot \sin\left(\frac{h}{2}\right)}{h} \right]$$

$$= \lim_{h \to 0} \left[ \frac{\cancel{2}\sin\left(\frac{h}{2}\right) \left\{ \cos\left(\frac{2x+h}{2}\right) + \sin\left(\frac{2x+h}{2}\right) \right\}}{\cancel{2} \times \frac{h}{2}} \right]$$

$$= 1 \times (\cos x + \sin x) \quad \cdots \left\{ \lim_{x \to 0} \left( \frac{\sin x}{x} \right) = 1 \right\}$$

$$\boxed{f'(x) = \cos x + \sin x}$$ Ans

Qn. 6 → $f(x) = 2x^2 + 3x - 5$. Using first principle method, show that $3f'(-1) + f'(0) = 0$

Soln

$$f'(-1) = \lim_{h \to 0} \left[ \frac{f(-1+h) - f(-1)}{h} \right]$$

$$= \lim_{h \to 0} \left( \frac{(2(-1+h)^2 + 3(-1+h) - 5) - (2 - 3 - 5)}{h} \right)$$

$$= \lim_{h \to 0} \left( \frac{2(1+h^2-2h) - 3 + 3h - 5 + 6}{h} \right)$$

$$= \lim_{h \to 0} \left( \frac{2h^2 - h}{h} \right)$$

$$= \lim_{h \to 0} \left( 2h - 1 \right)$$

$$\boxed{f'(-1) = -1}$$

Now $f'(0) = \lim_{h \to 0} \left( \frac{f(0+h) - f(0)}{h} \right)$

$$= \lim_{h \to 0} \left( \frac{(2h^2 + 3h - 5) - (-5)}{h} \right)$$

$$= \lim_{h \to 0} \left( \frac{2h^2 + 3h}{h} \right)$$

$$= \lim_{h \to 0} \left( 2h + 3 \right)$$

$$\boxed{f'(0) = 3}$$

Ans $\quad 3f'(-1) + f'(0)$

$$= 3(-1) + 3$$

$$= 0 \quad \text{Ans}$$

Ques. 7 → If $\lim_{x \to 0} \left( \sin(mx) \cdot \cot\left(\frac{x}{\sqrt{3}}\right) \right) = 2$. Find value of 'm'

Soln $\quad \lim_{x \to 0} \left( \frac{\sin(mx)}{\tan\left(\frac{x}{\sqrt{3}}\right)} \right) = 2$

⑥

$$\Rightarrow \lim_{x\to 0}\left(\frac{\frac{\sin(mx)}{mx}\times m\cancel{x}}{\frac{\tan\left(\frac{x}{\sqrt{3}}\right)}{\frac{x}{\sqrt{3}}}\times\frac{\cancel{x}}{\sqrt{3}}}\right)=2$$

$$\Rightarrow \frac{1\times m}{1\times\frac{1}{\sqrt{3}}}=2 \quad \cdots \left\{\lim_{x\to 0}\left(\frac{\sin x}{x}\right)=1\right\}$$

$$\Rightarrow m\sqrt{3}=2$$

$$\Rightarrow \boxed{m=\frac{2}{\sqrt{3}}} \quad \underline{\text{Ans}}$$

**Qn. 8 → evaluate** $\lim_{x\to 0}\left[\frac{\sin(\alpha+\beta)x+\sin(\alpha-\beta)x+\sin(2\alpha x)}{\cos(2\beta x)-\cos(2\alpha x)}\right].x$

(A) (B) (A) (C)

Soln =

$$\lim_{x\to 0}\left[\frac{2\sin(\alpha x).\cos(\beta x)+2\sin(\alpha x).\cos(\alpha x)}{-2\sin(\alpha+\beta)x\,.\sin(\beta-\alpha)x}\right].x$$

$$=\lim_{x\to 0}\left[\frac{\cancel{2}\sin(\alpha x)\{\cos(\beta x)+\cos(\alpha x)\}}{-\cancel{2}\sin(\alpha+\beta)x.\sin(\beta-\alpha)x}\right]x$$

$$=\lim_{x\to 0}\left(\frac{\frac{\sin(\alpha x)}{\alpha x}\alpha\cancel{x}\,.\{\cos(\beta x)+\cos(\alpha x)\}.\cancel{x}}{-\frac{\sin(\alpha+\beta)x}{(\alpha+\beta)x}(\alpha+\beta)\cancel{x}\,.\frac{\sin(\beta-\alpha)x}{(\beta-\alpha)x}(\beta-\alpha)\cancel{x}}\right)$$

$$=\frac{(1)(\alpha)(1+1)}{-(1)(\alpha+\beta)(\beta-\alpha)}$$

$$= \frac{2\alpha}{\alpha^2-\beta^2} \quad \text{Ans}$$

Ques : 9 → If $f(x) = \begin{cases} |x|+1 & ; x<0 \\ 0 & ; x=0 \\ |x|-1 & ; x>0 \end{cases}$

for what value of 'a' does $\lim_{x\to a} f(x)$ exists ?

Soln

$$f(x) = \begin{cases} -x+1 & ; x<0 \\ 0 & ; x=0 \\ x-1 & ; x>0 \end{cases}$$

$LHL = \lim_{x\to 0^-} (-x+1)$ put $x = 0-h = -h$ & $h\to 0$

$LHL = \lim_{h\to 0} (h+1)$

$LHL = 1$

$RHL = \lim_{x\to 0^+} (x-1)$ put $x = 0+h = h$ & $h\to 0$

$RHL = \lim_{h\to 0} (h-1)$

$RHL = -1$

Since $LHL \neq RHL$

$\therefore \lim_{x\to 0} f(x)$ does not exists

But given that $\lim_{x\to a} f(x)$ exists

$\Rightarrow a \neq 0 \quad \therefore \boxed{a \in R-\{0\}}$ Ans

Qn. 10 → Evaluate $\lim_{x \to 5} ([x] - 5)$

Soln. LHL $= \lim_{x \to 5^-} ([x] - 5)$ put $x = 5 - h$ & $h \to 0$

LHL $= \lim_{h \to 0} ([5-h] - 5)$

$= 4 - 5$

$\boxed{\text{LHL} = -1}$

RHL $= \lim_{x \to 5^+} ([x] - 5)$ put $x = 5 + h$ & $h \to 0$

RHL $= \lim_{h \to 0} ([5+h] - 5)$

$= 5 - 5$

RHL $= 0$

LHL $\neq$ RHL $\therefore \lim_{x \to 5} f(x)$ does not exists

Qn. 11 → Evaluate $\lim_{x \to \frac{\pi}{6}} \left( \dfrac{\sqrt{3} \sin x - \cos x}{x - \frac{\pi}{6}} \right)$

Soln. put $x = \frac{\pi}{6} + h$ & $h \to 0$

$$= \lim_{h \to 0} \left[ \frac{\sqrt{3} \sin\left(\frac{\pi}{6} + h\right) - \cos\left(\frac{\pi}{6} + h\right)}{\cancel{\frac{\pi}{6}} + h - \cancel{\frac{\pi}{6}}} \right]$$

$$= \lim_{h \to 0} \left[ \frac{\sqrt{3} \cdot \left( \frac{1}{2} \cos h + \frac{\sqrt{3}}{2} \sinh \right) - \left( \frac{\sqrt{3}}{2} \cos h - \frac{1}{2} \sinh \right)}{h} \right]$$

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$$= \lim_{h\to 0}\left(\frac{\cancel{\frac{\sqrt{3}}{2}\cos h} + \frac{3}{2}\sin h - \cancel{\frac{\sqrt{3}}{2}\cos h} + \frac{1}{2}\sin h}{h}\right)$$

$$= \lim_{h\to 0}\left(\frac{2\sin h}{h}\right)$$

$$= 2\times 1$$

$$= 2 \quad \text{Ans}$$

Qn. 12 → evaluate $\lim_{x\to\pi}\left(\frac{1-\sin\left(\frac{x}{2}\right)}{\cos\frac{x}{2}\left(\cos\frac{x}{4}-\sin\frac{x}{4}\right)}\right)$

Soln

$$\lim_{x\to\pi}\left(\frac{1-\sin(x/2)}{\cos\left(\frac{x}{2}\right)\left(\cos\frac{x}{4}-\sin\frac{x}{4}\right)}\cdot\frac{\left(\cos\left(\frac{x}{4}\right)+\sin\left(\frac{x}{4}\right)\right)}{\left(\cos\frac{x}{4}+\sin\frac{x}{4}\right)}\right)$$

$$= \lim_{x\to\pi}\left(\frac{(1-\sin(x/2))\cdot\left(\cos\frac{x}{4}+\sin\frac{x}{4}\right)}{\cos\frac{x}{2}\cdot\left(\cos^2\frac{x}{4}-\sin^2\frac{x}{4}\right)}\right)$$

$$= \lim_{x\to\pi}\left[\frac{\left(1-\sin\left(\frac{x}{2}\right)\right)\left(\cos\frac{x}{4}+\sin\frac{x}{4}\right)}{\cos\frac{x}{2}\cdot\cos(x/2)}\right]$$

$$= \lim_{x\to\pi}\left(\frac{(1-\sin(x/2))\left(\cos\frac{x}{4}+\sin\frac{x}{4}\right)}{\left(1-\sin^2\left(\frac{x}{2}\right)\right)}\right)$$

$$= \lim_{x\to\pi}\left(\frac{\cos\frac{x}{4}+\sin\frac{x}{4}}{1+\sin\frac{x}{2}}\right)$$

$$= \frac{\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}}{1+1} = \frac{\frac{2}{\sqrt{2}}}{2} = \frac{1}{\sqrt{2}} \quad \text{Ans}$$

Q.No. 13 → Using first principle method find derivative of $f(x) = \sqrt[3]{\sin x}$

Soln $f(x) = (\sin x)^{1/3}$

$$f'(x) = \lim_{h \to 0} \left( \frac{(\sin(x+h))^{1/3} - (\sin x)^{1/3}}{h} \right)$$

$$= \lim_{h \to 0} \left[ \frac{(\sin(x+h))^{1/3} - (\sin x)^{1/3}}{(\sin(x+h) - \sin x)} \times \frac{\sin(x+h) - \sin x}{h} \right]$$

$$= \lim_{h \to 0} \left[ \frac{(\sin(x+h))^{1/3} - (\sin x)^{1/3}}{\sin(x+h) - \sin x} \right] \times \lim_{h \to 0} \left( \frac{\sin(\overset{A}{x+h}) - \sin \overset{B}{x}}{h} \right)$$

$\boxed{\text{when } h \to 0. \quad \sin(x+h) \to \sin x}$

$$= \frac{1}{3} (\sin x)^{\frac{1}{3} - 1} \times \lim_{h \to 0} \left( \frac{\cancel{2} \cos\left(\frac{2x+h}{2}\right) \cdot \sin\left(\frac{h}{2}\right)}{\frac{h}{2} \times \cancel{2}} \right)$$

$$\cdots \left\{ \lim_{x \to a} \left( \frac{x^n - a^n}{x - a} \right) = n a^{n-1} \right\}$$

$$= \frac{1}{3} (\sin x)^{-2/3} \times \cos x$$

$$f'(x) = \frac{1}{3} (\sin x)^{-2/3} \cdot \cos x$$

# LIMITS & DERIVATIVES (REVISION)

## WORKSHEE NO: 1

**Ans 1** $f(x) = \begin{cases} 4x-5 ; & x \le 2 \\ x-\lambda ; & x > 2 \end{cases}$ . Find value of $\lambda$

if $\lim_{x \to 2} (f(x))$ exists  **Ans** $\lambda = -1$

**Qn. 2 →** If $f(x) = \begin{cases} mx^2+n ; & x<0 \\ nx+m : & 0 \le x \le 1 \\ nx^3+m ; & x>1 \end{cases}$

For what integers $m$ & $n$ does the limits $\lim_{x\to 0} f(x)$ and $\lim_{x \to 1} f(x)$ exists? **Ans** $m$ & $n \in Z$ such that $m=n$

**Ans 3 →** $f(x) = \begin{cases} a+bx ; & x<1 \\ 4 ; & x=1 \\ b-ax ; & x>1 \end{cases}$

and if $\lim_{x\to 1} (f(x)) = f(1)$ . what are the possible values of $a$ & $b$? **Ans** $a=0, b=4$

**Qn 4 →** evaluate $\lim_{x\to 1} \left( \frac{1}{x^2+x-2} - \frac{x}{x^3-1} \right)$ **Ans** $-\frac{1}{9}$

**Ans 5 →** evaluate $\lim_{x\to 1} \left( \frac{x^4-3x^3+2}{x^3-5x^2+3x+1} \right)$ **Ans** $= \frac{5}{4}$

**Ans 6 →** evaluate $\lim_{x\to 1} \left( \frac{(2x-3)(\sqrt{x}-1)}{2x^2+x-3} \right)$ **Ans** $= -\frac{1}{10}$

Qn. 7 → evaluate $\lim_{x \to a} \left( \dfrac{\sqrt{a+2x} - \sqrt{3x}}{\sqrt{3a+x} - 2\sqrt{x}} \right)$ Ans $\dfrac{2}{3\sqrt{3}}$

Qn. 8 → If $\lim_{x \to 2} \left( \dfrac{x^n - 2^n}{x-2} \right) = 80$ find 'n' Ans $n = 5$

Qn. 9 → evaluate $\lim_{x \to 0} \left( \dfrac{(1-x)^n - 1}{x} \right)$ Ans $-n$

Qn. 10 → If $\lim_{x \to -a} \left( \dfrac{x^9 + a^9}{x+a} \right) = 9$ find value of a
Ans $a = \pm 1$

Qn. 11 → evaluate $\lim_{x \to 0} \left( \dfrac{\tan x - \sin x}{x^3} \right)$ Ans $1/2$

Qn. 12 → evaluate $\lim_{x \to 0} \left( \dfrac{\sec(4x) - \sec(2x)}{\sec(3x) - \sec x} \right)$ Ans $3/2$

Qn. 13 → evaluate $\lim_{x \to \frac{\pi}{6}} \left( \dfrac{2\sin^2 x + \sin x - 1}{2\sin^2 x - 3\sin x + 1} \right)$ Ans $-3$

Qn. 14 → If $\lim_{x \to 0} \left( kx \operatorname{cosec}(x) \right) = \lim_{x \to 0} \left( x \operatorname{cosec}(kx) \right)$
Find value of k Ans $k = \pm 1$

Qn. 15 → evaluate $\lim_{h \to 0} \left( \dfrac{(a+h)^2 \sin(a+h) - a^2 \sin a}{h} \right)$
Ans $2a\sin a + a^2 \cos a$

Qn. 16 → evaluate $\lim_{x \to \frac{\pi}{6}} \left( \dfrac{2 - \sqrt{3}\cos x - \sin x}{(6x - \pi)^2} \right)$ Ans $= \dfrac{1}{36}$

Qns 17 → evaluate $\lim_{x \to \frac{\pi}{2}} \left( \frac{\cot x - \cos x}{(\pi - 2x)^3} \right)$ Ans = $\frac{1}{16}$

Qn 18 → evaluate $\lim_{x \to \frac{\pi}{2}} \left( \frac{\tan(2x)}{x - \frac{\pi}{2}} \right)$ Ans = 2

Qn 19 → evaluate $\lim_{x \to \frac{\pi}{2}} \left( \frac{\sqrt{2} - \sqrt{1 + \sin x}}{\cos^2 x} \right)$ Ans = 2

Qn 20 → evaluate $\lim_{x \to 0} \left( \frac{3^{2x} - 2^{3x}}{x} \right)$ Ans $\log\left(\frac{9}{8}\right)$

Qn 21 → evaluate $\lim_{x \to \frac{\pi}{2}} \left( \frac{2^{-\cos x} - 1}{x\left(x - \frac{\pi}{2}\right)} \right)$ Ans $\frac{2}{\pi} \log 2$

Qn 22 → evaluate $\lim_{x \to 0} \left( \frac{3^x + 3^{-x} - 2}{x^2} \right)$ Ans $(\log 3)^2$

Qn 23 → evaluate $\lim_{x \to 0} \left( \frac{e^{3+x} - \sin x - e^3}{x} \right)$ Ans $e^3 - 1$

Qn 24 → $f(x) = \sqrt{\tan x}$ find $f'(x)$ (using first principle)

Ans $\frac{1}{2\sqrt{\tan x}} \cdot \sec^2 x$

Qn 25 → $f(x) = \sec\sqrt{x}$. Find $f'(x)$ using first principle

Ans $\sec\sqrt{x} \cdot \tan\sqrt{x} \cdot \frac{1}{2\sqrt{x}}$

Qn 26 $f(x) = x^2 \sin x$ find $f'(x)$ using first principle

Ans $x^2 \cos x + 2x \sin x$

Qn 27 $f(x) = \frac{3x - 1}{2x + 3}$ find $f'(x)$ using first principle

Ans $\frac{11}{(2x+3)^2}$

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