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ULTIMATE MATHEMATICS : BY AJAY MITTAL

CHAPTER: COMPLEX NUMBERS (CLASS NO: 3)

Ans 1 Solve $x^2 + x + \frac{1}{\sqrt{2}} = 0$

Sol

$$\sqrt{2}\,x^2 + \sqrt{2}\,x + 1 = 0$$

$$x = \frac{-\sqrt{2} \pm \sqrt{2 - 4\sqrt{2}}}{2\sqrt{2}}$$

e.g. $\sqrt{-5} = \sqrt{5}\,i$

$$x = \frac{-\sqrt{2} \pm \sqrt{-(4\sqrt{2} - 2)}}{2\sqrt{2}}$$

$$x = \frac{-\sqrt{2} \pm i\sqrt{4\sqrt{2} - 2}}{2\sqrt{2}}$$

$$x = \frac{-\sqrt{2} \pm \sqrt{2}\,i\sqrt{2\sqrt{2} - 1}}{2\sqrt{2}}$$

$$x = \frac{-1 \pm i\sqrt{2\sqrt{2} - 1}}{2} \quad \text{Ans}$$

Qn 2 Show that the complex number 'z' which satisfies the condition $\left|\frac{i+z}{i-z}\right| = 1$ lies on X-axis

Soln Let $z = x + iy$

$$\Rightarrow \left|\frac{i + x + iy}{i - x - iy}\right| = 1$$

$$\Rightarrow \left|\frac{x + i(y+1)}{-x + i(1-y)}\right| = 1$$

By **Prop** $\left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}$

$$\Rightarrow \frac{|x+i(y+1)|}{|-x+i(1-y)|} = 1$$

$$\Rightarrow |x+i(y+1)| = |-x+i(1-y)|$$

$$\Rightarrow \sqrt{x^2+(y+1)^2} = \sqrt{x^2+(1-y)^2}$$

Squaring both sides

$$\cancel{x^2}+\cancel{y^2}+\cancel{1}+2y = \cancel{x^2}+\cancel{1}+\cancel{y^2}-2y$$

$$\Rightarrow 4y=0$$

$$\Rightarrow y=0$$

i.e. z lies on X-axis **Ans**

Que 3 → If $f(z) = \frac{7-z}{1-z^2}$ when $z = 1+2i$ then find $|f(z)|$

Soln

$$|f(z)| = \left|\frac{7-z}{1-z^2}\right| = \frac{|7-z|}{|1-z^2|}$$

$$= \frac{|7-1-2i|}{|1-(1+2i)^2|}$$

$$= \frac{|6-2i|}{|\cancel{1}-\cancel{1}-4i^2-4i|} = \frac{|6-2i|}{|4-4i|} = \frac{\sqrt{36+4}}{\sqrt{16+16}}$$

$$\Rightarrow |f(z)| = \frac{\sqrt{40}}{\sqrt{32}} = \frac{2\sqrt{10}}{4\sqrt{2}} = \frac{\sqrt{10}}{2\sqrt{2}} = \frac{\sqrt{5}}{2} \text{ Ans}$$

Q.N. 4 → where does z lie if $\left|\frac{z-5i}{z+5i}\right| = 1$

Sol

Q.N. 5 → Solve the equation

$$z + \sqrt{2}\,|z+1| + i = 0$$

Sol let $z = x+iy$

$$\Rightarrow x+iy + \sqrt{2}\,|x+iy+1| + i = 0$$

$$\Rightarrow x+iy + \sqrt{2}\,|(x+1)+iy| + i = 0$$

$$\Rightarrow x+iy + \sqrt{2}\sqrt{(x+1)^2+y^2} + i = 0$$

$$\Rightarrow \left(x+\sqrt{2}\sqrt{(x+1)^2+y^2}\right) + i(y+1) = 0+0i$$

equating Real & Imaginary parts

$$x+\sqrt{2}\sqrt{(x+1)^2+y^2} = 0 \quad \text{and} \quad y+1=0$$

$$\Rightarrow y=-1$$

$$x+\sqrt{2}\sqrt{x^2+1+2x+1} = 0$$

$$x = -\sqrt{2}\sqrt{x^2+2x+2}$$

Squaring

$$x^2 = 2(x^2+2x+2)$$

$$\Rightarrow x^2+4x+4=0 \Rightarrow (x+2)^2=0$$

$$x=-2$$

$$\boxed{z=-2-i} \text{ Ans}$$

Qn. 6: If $|z_1| = 1$ and $z_2 = \frac{z_1 - 1}{z_1 + 1}$ then show that real part of z_2 is zero

Soln: let $z_1 = x + iy$

$\Rightarrow |z_1| = \sqrt{x^2+y^2} = 1$

$\Rightarrow \boxed{x^2+y^2=1}$

Now $z_2 = \frac{z_1 - 1}{z_1+1} = \frac{x+iy-1}{x+iy+1} = \frac{(x-1)+iy}{(x+1)+iy}$

Rationalize

$$z_2 = \frac{(x-1)+iy}{(x+1)+iy} \times \frac{(x+1)-iy}{(x+1)-iy}$$

$$z_2 = \frac{(x^2-1) - i\cancel{xy} + iy + i\cancel{xy} + iy - i^2y^2}{(x+1)^2 - i^2y^2}$$

$$z_2 = \frac{(x^2+y^2-1) + 2iy}{(x+1)^2+y^2}$$

$$z_2 = \frac{0 + 2iy}{(x+1)^2+y^2} \quad \text{--- } \{\because x^2+y^2=1\}$$

$$z_2 = \frac{i2y}{(x+1)^2+y^2}$$

$\therefore$ Real part of z_2 is zero Ans

Qn 7 what does the equation $\left|\frac{z+1-i}{z-1+i}\right| = 1$ represents?

Self

Ans Straight line

Qn 8 → If the imaginary part of $\frac{2z+1}{iz+1}$ is -2 then show that the path/curve traced by the moving point z (locus) is a Straight line.

Sol let $z = x+iy$

$$\Rightarrow \frac{2z+1}{iz+1} = \frac{2(x+iy)+1}{i(x+iy)+1}$$

$$= \frac{(2x+1)+i2y}{(1-y)+ix} \times \frac{(1-y)-ix}{(1-y)-ix}$$

$$= \frac{2x - \cancel{2xy} + 1 - y - i2x^2 - ix + i2y - 2iy^2 - \cancel{2i^2xy}}{(1-y)^2 - i^2x^2}$$

$$\frac{2z+1}{iz+1} = \frac{(2x+1-y) + i(-2x^2 - x + 2y - 2y^2)}{(1-y)^2 + x^2}$$

$$\text{Im}\left(\frac{2z+1}{iz+1}\right) = \frac{-2x^2 - x + 2y - 2y^2}{(1-y)^2 + x^2} = -2 \quad \text{-- (given)}$$

$$\Rightarrow -2x^2 - x + 2y - 2y^2 = -2\left[1 + y^2 - 2y + x^2\right]$$

$$\Rightarrow \cancel{-2x^2} - x + 2y \cancel{-2y^2} = -2 \cancel{-2y^2} + 4y \cancel{-2x^2}$$

$$\boxed{-x - 6y + 2 = 0} \text{ St. line}$$

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Ans 9 If $x = -5 + 2\sqrt{-4}$, find the value of
$x^4 + 9x^3 + 35x^2 - x + 4$

(Finding the value of polynomial)

Sol: given $x = -5 + 4i$ --- $\{\sqrt{-4} = 2i\}$

$\Rightarrow x + 5 = 4i$

Squaring: $x^2 + 25 + 10x = 16i^2$

$\Rightarrow \boxed{x^2 + 10x + 41 = 0}$

$$
\begin{array}{r|l}
 & x^2 - x + 4 \\
x^2 + 10x + 41 & x^4 + 9x^3 + 35x^2 - x + 4 \\
 & -(x^4 + 10x^3 + 41x^2) \\
\hline
 & -x^3 - 6x^2 - x + 4 \\
 & -(-x^3 - 10x^2 - 41x) \\
\hline
 & 4x^2 + 40x + 4 \\
 & -(4x^2 + 40x + 164) \\
\hline
 & -160
\end{array}
$$

Now $x^4 + 9x^2 + 35x^2 - x + 4 = (x^2 + 10x + 41)(x^2 - x + 4) - 160$

$= 0(x^2 - x + 4) - 160$

$= -160$ Ans

Complex (Class No: 3)

Qn 10 → (self) If $x = -1 + \sqrt{-2}$ find value of
$x^4 + 4x^3 + 6x^2 + 4x + 9$ Ans = 12

Qn 11 → self If $x = 1 + \sqrt{-4}$ find value of
$x^3 + 7x^2 - x + 16$ Ans $-17 + 24i$

Qn 12 self If $z = x + iy$ and $z_2 = \dfrac{1 - iz}{z - i}$
If $|z_2| = 1$ show that z is purely real

Hint use $\left|\dfrac{z_1}{z_2}\right| = \dfrac{|z_1|}{|z_2|}$ & show $y = 0$

Qn 13 self If $a + ib = \dfrac{c + i}{c - i}$ show that $\dfrac{b}{a} = \dfrac{2c}{c^2 - 1}$

and $a^2 + b^2 = 1$

Qn 14 → Solve the equation
(self) $25x^2 - 30x + 11 = 0$

Ans $x = \dfrac{3}{5} \pm \dfrac{\sqrt{2}}{5} i$

Qn 15 Solve $\sqrt{5}x^2 + x + \sqrt{5} = 0$
self Ans $x = \dfrac{-1 \pm \sqrt{19}\, i}{2\sqrt{5}}$

— x —