Solutions

WORKSHEET NO: 2 (CLASS NO: 3)

SEQUENCE & SERIES

Ans 8

Given $a_4 = x$

$a_{10} = y$

and $a_{16} = z$

{ Note. Ans No: 1 (later) after the 7th question }

$\Rightarrow a r^3 = x$

$\Rightarrow a r^9 = y$

& $a r^{15} = z$

Now $y^2 = a^2 r^{18}$

$y^2 = (a r^3)(a r^{15})$

$y^2 = x z$

Clearly x, y, z are in GP **PROVED**

Ans 2 → Let the numbers are

$\frac{a}{r}, a, ar$

Given product $= -1$

$\Rightarrow \left(\frac{a}{r}\right)(a)(ar) = -1$

$\Rightarrow a^3 = -1$

$\Rightarrow \boxed{a = -1}$

Given sum $= \frac{13}{12}$

$\Rightarrow \frac{a}{r} + a + ar = \frac{13}{12}$

$$a\left(\frac{1}{r}+1+r\right)=\frac{13}{12}$$

put $a=-1$

$$\Rightarrow -1\left(\frac{1+r+r^2}{r}\right)=\frac{13}{12}$$

$$\Rightarrow -12-12r-12r^2=13r$$

$$\Rightarrow 12r^2+25r+12=0$$

$$\Rightarrow 12r^2+16r+9r+12=0$$

$$\Rightarrow 4r(3r+4)+3(3r+4)$$

$$\Rightarrow (3r+4)(4r+3)=0$$

$$\Rightarrow r=\frac{-4}{3}\ ;\ r=\frac{-3}{4}$$

$\therefore$ for $a=-1$ & $r=\frac{-4}{3}$

numbers are

$\frac{3}{4}, -1, \frac{4}{3}$ Ans

for $a=-1$ & $r=\frac{-3}{4}$

numbers are

$\frac{4}{3}, -1, \frac{3}{4}$ Ans

Ans 3 →

let $S_n = 7+77+777+ \text{---} n\,\text{term}$

$S_n = 7(1+11+111+ \text{---} n\,\text{term})$

Multiply & divide by 9

$$S_n=\frac{7}{9}\left[9+99+999+\text{---}\ n\,\text{term}\right]$$

$$=\frac{7}{9}\left[(10-1)+(10^2-1)+(10^3-1)+\text{----}\ n\,\text{term}\right]$$

$$= \frac{7}{9}\left[(10+10^2+10^3+ \cdots n \text{ term}) - (1+1+1 \cdots n \text{ term}) \right]$$

$\leftarrow$ GP: $a = 10;\ r = 10 \rightarrow$

$$= \frac{7}{9}\left[10\left(\frac{10^n - 1}{10-1} \right) - n \right]$$

$$= \frac{7}{9}\left(\frac{10^{n+1} - 10 - 9n}{9} \right)$$

$$\therefore S_n = \frac{7}{81}\left[10^{n+1} - 10 - 9n \right] \quad \text{Ans}$$

Qn 4 → here $a = 1$ & $b = 256$

& $n = 3$

let the three numbers are G_1, G_2, G_3

$$r = \left(\frac{b}{a}\right)^{\frac{1}{n+1}}$$

$$r = (256)^{\frac{1}{4}}$$

$$\boxed{r = 4}$$

$G_1 = ar^1 = 1(4) = 4$

$G_2 = ar^2 = 1(4)^2 = 16$

$G_3 = ar^3 = 1(4)^3 = 64$

$\therefore$ the three GM's are 4, 16, 64 Ans

Qns 5 → let the two numbers are a & b

Given $A \cdot M = 10$

& $G \cdot M = 8$

$\Rightarrow \frac{a+b}{2} = 10$ & $\sqrt{ab} = 8$

$\Rightarrow a+b = 20$ & $ab = 64$

$\Rightarrow a(20-a) = 64$

$\Rightarrow 20a - a^2 = 64$

$\Rightarrow a^2 - 20a + 64 = 0$

$\Rightarrow (a-16)(a-4) = 0$

$\Rightarrow a = 16$ (or) $a = 4$

$b = 4$ $b = 16$

$\therefore$ the two numbers are 16 & 4 (or) 4 & 16 Ans

Ans 6 →

Given $a_3 = a_1 + 9$ & $a_2 = a_4 + 18$

$\Rightarrow ar^2 = a + 9$ and $ar = ar^3 + 18$

$\Rightarrow a(r^2 - 1) = 9$ & $ar(1 - r^2) = 18$

divide these equations

$\Rightarrow \frac{\cancel{a}\cancel{(r^2-1)}}{\cancel{a}r\cancel{(1-r^2)}} = \frac{9}{18}$ (−1)

$\Rightarrow -\frac{1}{r} = \frac{1}{2}$

$\Rightarrow r = -2$ put in (i)

$\therefore a(4-1) = 9$

$\Rightarrow a = 3$ $\therefore$ the four numbers are $3, -6, 12, -24$ Ans

Ans 7 →

given. a, b, c, d are in GP

let $a = a$, $b = ar$, $c = ar^2$, $d = ar^3$

T.P $(a^2+b^2+c^2)(b^2+c^2+d^2) = (ab+bc+cd)^2$

LHS $(a^2+b^2+c^2)(b^2+c^2+d^2)$

$= (a^2+a^2r^2+a^2r^4)(a^2r^2+a^2r^4+a^2r^6)$

$= a^2(1+r^2+r^4) \cdot a^2r^2(1+r^2+r^4)$

$= a^4r^2(1+r^2+r^4)^2$

RHS $(ab+bc+cd)^2$

$= (a^2r+a^2r^3+a^2r^5)^2$

$= [a^2r(1+r^2+r^4)]^2$

$= a^4r^2(1+r^2+r^4)^2$

Clearly LHS = RHS PROVED

Ans 8 →

given GP $3, \frac{3}{2}, \frac{3}{4}, ----$

here $a=3$ & $r = 1/2$

given $S_n = \frac{3069}{512}$

Since $r<1$

$\therefore S_n = \frac{a(1-r^n)}{1-r}$

$$\Rightarrow \frac{3069}{512} = 3\left[\frac{1-\left(\frac{1}{2}\right)^n}{1-\frac{1}{2}}\right]$$

$$\frac{3069}{512} = 3\,\frac{\left[1-\left(\frac{1}{2}\right)^n\right]}{\frac{1}{2}}$$

$$\frac{3069}{512} = 6\left[1-\left(\frac{1}{2}\right)^n\right]$$

$$\Rightarrow \frac{3069}{512\times 6} = 1-\left(\frac{1}{2}\right)^n$$

$$\Rightarrow \left(\frac{1}{2}\right)^n = 1-\frac{3069}{3072}$$

$$\Rightarrow \left(\frac{1}{2}\right)^n = \frac{3}{3072}$$

$$\Rightarrow \left(\frac{1}{2}\right)^n = \frac{1}{1024}$$

$$\Rightarrow \left(\frac{1}{2}\right)^n = \left(\frac{1}{2}\right)^{10}$$

$\Rightarrow \boxed{n=10}$ Ans

Ques 9 → Same as Ques No: 2 (Do yourself)

Ques 10 → Let the three terms in GP are

$a,\ ar,\ ar^2$

Given Sum = 56

$$a + ar + ar^2 = 56$$

$$a(1+r+r^2) = 56 \quad \text{---} ①$$

According to Ques $a-1,\ ar-7,\ ar^2-21$ are in A.P

$\Rightarrow 2(ar-7) = (a-1)+(ar^2-21)$ --- $\{2b=(a+c)\}$

$\Rightarrow 2ar-14 = a+ar^2-22$

$\Rightarrow ar^2-2ar+a=8$

$\Rightarrow a(r^2-2r+1)=8$ --- (2)

(1) $\div$ (2)

$$\frac{\cancel{a}(1+r+r^2)}{\cancel{a}(r^2-2r+1)} = \frac{\cancel{56}\,7}{\cancel{8}}$$

$\Rightarrow 1+r+r^2 = 7r^2-14r+7$

$\Rightarrow 6r^2-15r+6=0$

$\Rightarrow 2r^2-5r+2=0$

$\Rightarrow 2r^2-4r-r+2=0$

$\Rightarrow 2r(r-2)-1(r-2)=0$

$\Rightarrow (r-2)(2r-1)=0$

$\Rightarrow r=2$ (or) $r=1/2$

Put $r=2$ in eq(1)	put $r=1/2$ in eq(1)
$a(1+2+4)=56$	$a(1+\frac{1}{2}+\frac{1}{4})=56$
$7a=56$	$a(\frac{7}{4})=56$
$a=8$	$a=32$

$\therefore$ Nos are 8, 16, 32 (OR) Nos are 32, 16, 8 Ans

Ans 11 → Given $\frac{A\cdot M}{GM} = \frac{m}{n}$

$$\Rightarrow \frac{\frac{a+b}{2}}{\sqrt{ab}} = \frac{m}{n}$$

$$\Rightarrow \frac{a+b}{2\sqrt{ab}} = \frac{m}{n}$$

Apply Componendo & dividendo

$$\frac{a+b+2\sqrt{ab}}{a+b-2\sqrt{ab}} = \frac{m+n}{m-n}$$

$$\Rightarrow \frac{(\sqrt{a}+\sqrt{b})^2}{(\sqrt{a}-\sqrt{b})^2} = \frac{m+n}{m-n}$$

$$\Rightarrow \frac{\sqrt{a}+\sqrt{b}}{\sqrt{a}-\sqrt{b}} = \pm\frac{\sqrt{m+n}}{\sqrt{m-n}}$$

again apply C&D

$$\frac{(\sqrt{a}+\cancel{\sqrt{b}}) + (\sqrt{a}-\cancel{\sqrt{b}})}{(\cancel{\sqrt{a}}+\sqrt{b}) - (\cancel{\sqrt{a}}-\sqrt{b})} = \pm\left(\frac{\sqrt{m+n}+\sqrt{m-n}}{\sqrt{m+n}-\sqrt{m-n}}\right)$$

$$\Rightarrow \frac{\cancel{2}\sqrt{a}}{\cancel{2}\sqrt{b}} = \pm\left(\frac{\sqrt{m+n}+\sqrt{m-n}}{\sqrt{m+n}-\sqrt{m-n}}\right)$$

Squaring both sides

$$\Rightarrow \frac{a}{b} = \frac{(m+\cancel{n}) + (m-\cancel{n}) + 2\sqrt{m^2-n^2}}{(m+\cancel{n}) + (m-\cancel{n}) - 2\sqrt{m^2-n^2}}$$

$$\Rightarrow \frac{a}{b} = \frac{2m+2\sqrt{m^2-n^2}}{2m-2\sqrt{m^2-n^2}}$$

$\Rightarrow a:b = \left(m+\sqrt{m^2-n^2}\right) : \left(m-\sqrt{m^2-n^2}\right)$ Proved

Ans 12 →

given $\dfrac{a+bx}{a-bx} = \dfrac{b+cx}{b-cx} = \dfrac{c+dx}{c-dx}$

Consider

$$\frac{a+bx}{a-bx} = \frac{b+cx}{b-cx}$$

$\Rightarrow \cancel{ab} - \cancel{acx} + b^2x - \cancel{bcx^2} = \cancel{ab} + \cancel{acx} - b^2x - \cancel{bcx^2}$

$\Rightarrow$

$2b^2x = 2acx$

$\Rightarrow b^2 = ac$

$\Rightarrow a, b, c$ are in GP -- (1)

Consider $\dfrac{b+cx}{b-cx} = \dfrac{c+dx}{c-dx}$

$\Rightarrow \cancel{bc} - bdx + c^2x - \cancel{cdx^2} = \cancel{bc} + bdx - c^2x - \cancel{cdx^2}$

$\Rightarrow 2c^2x = 2bdx$

$\Rightarrow c^2 = bd$

$\therefore b, c, d$ are in GP -- (2)

from (1) & (2)

a, b, c, d are in GP Proved

—x—