→ ULTIMATE MATHEMATICS →
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→ Solutions of T-3 →

Ans 1 → $\dfrac{\sin(5A) - \sin(3A)}{\cos(5A) + \cos(3A)}$

$= \dfrac{\cancel{2\cos(4A)}\cdot \sin(A)}{\cancel{2\cos(4A)}\cdot \cos(A)}$ --- { formula $\sin A - \sin B$ and $\cos A + \cos B$ }

$= \dfrac{\sin A}{\cos A} = \tan A = \text{Rhs}$ **Ans..**

Ans 2 → $\dfrac{\cos(7A) + \cos(5A)}{\sin(7A) - \sin(5A)}$ --- { $\cos A + \cos B$ & $\sin A - \sin B$ formula }

$= \dfrac{\cancel{2\cos(6A)}\cdot \cos(A)}{\cancel{2\cos(6A)}\sin(A)}$

$= \dfrac{\cos A}{\sin A} = \cot A = \text{Rhs}$ **Ans**

Ans 3 → $\dfrac{\cos(5x) - \cos(9x)}{\sin(3x) - \sin(17x)}$

$= \dfrac{-2\sin(7x)\cdot \sin(-2x)}{2\cos(10x)\cdot \sin(-7x)}$

$= \dfrac{\cancel{2}\cancel{\sin(7x)}\cdot \sin(2x)}{-\cancel{2}\cos(10x)\cdot \cancel{\sin(7x)}}$ --- { $\sin(-\theta) = -\sin\theta$ }

$= -\dfrac{\sin(2x)}{\cos(10x)} = \text{Rhs}$ **Ans.**

Ans 4 → $\dfrac{\sin A + \sin(3A) + \sin(5A) + \sin(7A)}{\cos A + \cos(3A) + \cos(5A) + \cos(7A)}$

pairing $= \dfrac{(\sin(7A) + \sin A) + (\sin(5A) + \sin(3A))}{(\cos(7A) + \cos A) + (\cos(5A) + \cos(3A))}$

$= \dfrac{2\sin(4A)\cos(3A) + 2\sin(4A)\cdot\cos(A)}{2\cos(4A)\cdot\cos(3A) + 2\cos(4A)\cdot\cos(A)}$

then take common

T3 Solutions

$$= \frac{\cancel{2}\sin(4A)\ \cancel{(\cos(3A)+\cos A)}}{\cancel{2}\cos(4A)\ \cancel{(\cos(3A)+\cos A)}}$$

$= \tan(4A) =$ RHS Ans

Ans. 5 → $\dfrac{\cos(3A)+\cos(5A)+\cos(7A)+\cos(9A)}{\sin(3A)+\sin(5A)+\sin(7A)+\sin(9A)}$

pairing $= \dfrac{(\cos(9A)+\cos(3A))+(\cos(7A)+\cos(5A))}{(\sin(9A)+\sin(3A))+(\sin(7A)+\sin(5A))}$

$$= \frac{2\cos(6A)\cdot\cos(3A)+2\cos(6A)\cdot\cos(A)}{2\sin(6A)\cos(3A)+2\sin(6A)\cdot\cos(A)}$$

Common

$$= \frac{\cancel{2}\cos(6A)\ \cancel{[\cos(3A)+\cos A]}}{\cancel{2}\sin(6A)\ \cancel{[\cos(3A)+\cos A]}}$$

$$= \frac{\cos(6A)}{\sin(6A)}$$

$= \cot(6A) =$ RHS Ans

Ans 6 → $\sin x+\sin(3x)+\sin(5x)+\sin(7x)$

pairing $= (\sin(7x)+\sin x)+(\sin(5x)+\sin(3x))$

$= 2\sin(4x)\cdot\cos(3x)+2\sin(4x)\cdot\cos(x)$

Common

$= 2\sin(4x)[\cos(3x)+\cos x]$

→ again Set-3 (cos A + cos B formula)

$= 2\sin(4x)\cdot[2\cos(2x)\cdot\cos x]$

$= 4\sin(4x)\cdot\cos(2x)\cdot\cos x =$ RHS Ans

Ans 7 → $1+\cos(2x)+\cos(4x)+\cos(6x)$

(Main step)

$= \cos(0^\circ)+\cos(2x)+\cos(4x)+\cos(6x)$

(T-3) Solutions

Pairing

$$(\cos(6x)+\cos(0))+(\cos(4x)+\cos(2x))$$

$$=2\cos(3x)\cdot\cos(3x)+2\cos(3x)\cdot\cos x$$

$$=2\cos(3x)\left[\cos(3x)+\cos x\right]$$ again Set - 3

$$=2\cos(3x)\left[2\cos(2x)\cdot\cos(x)\right]$$

$$=4\cos x\cdot\cos(2x)\cdot\cos(3x)$$

- Rhs Ans

Ans 8 Taking L.H.S

$$\cot(4x)\left[\sin(5x)+\sin(3x)\right]$$

$$=\frac{\cos(4x)}{\sin(4x)}\left[2\sin(4x)\cdot\cos(x)\right]$$

$$=2\cos(4x)\cos x$$

Taking Rhs $\cot x\left[\sin(5x)-\sin(3x)\right]$

$$=\frac{\cos x}{\sin x}\left[2\cos(4x)\cdot\sin x\right]$$

$$=2\cos(4x)\cos x$$

Clearly Lhs = Rhs Proved

Ans 9 →

$$\frac{\cos(4x)+\cos(3x)+\cos(2x)}{\sin(4x)+\sin(3x)+\sin(2x)}$$

pairing

$$=\frac{(\cos(4x)+\cos(2x))+\cos(3x)}{(\sin(4x)+\sin(2x))+\sin(3x)}$$

$$=\frac{2\cos(3x)\cdot\cos x+\cos(3x)}{2\sin(3x)\cos x+\sin(3x)}$$

$$= \frac{\cos(3x)\,[2\cos x + 1]}{\sin(3x)\,[2\cos x + 1]}$$

$$= \cot(3x) = \text{RHS} \quad \text{Ans}$$

Ans 10 → $(\cos\alpha + \cos\beta)^2 + (\sin\alpha + \sin\beta)^2$

$$= \left[2\cos\left(\frac{\alpha+\beta}{2}\right)\cdot\cos\left(\frac{\alpha-\beta}{2}\right)\right]^2 + \left[2\sin\left(\frac{\alpha+\beta}{2}\right)\cdot\cos\left(\frac{\alpha-\beta}{2}\right)\right]^2$$

$$= 4\cos^2\left(\frac{\alpha+\beta}{2}\right)\cdot\cos^2\left(\frac{\alpha-\beta}{2}\right) + 4\sin^2\left(\frac{\alpha+\beta}{2}\right)\cdot\cos^2\left(\frac{\alpha-\beta}{2}\right)$$

$$= 4\cos^2\left(\frac{\alpha-\beta}{2}\right)\left[\cos^2\left(\frac{\alpha+\beta}{2}\right) + \sin^2\left(\frac{\alpha+\beta}{2}\right)\right]$$

$$= 4\cos^2\left(\frac{\alpha-\beta}{2}\right) \times 1 \quad \text{---} \quad \{\sin^2\theta + \cos^2\theta = 1\}$$

$$= 4\cos^2\left(\frac{\alpha-\beta}{2}\right) = \text{RHS} \quad \text{Ans}$$

Ans 11 → $(\cos\alpha - \cos\beta)^2 + (\sin\alpha - \sin\beta)^2$

$$= \left[-2\sin\left(\frac{\alpha+\beta}{2}\right)\cdot\sin\left(\frac{\alpha-\beta}{2}\right)\right]^2 + \left[2\cos\left(\frac{\alpha+\beta}{2}\right)\cdot\sin\left(\frac{\alpha-\beta}{2}\right)\right]^2$$

$$= 4\sin^2\left(\frac{\alpha+\beta}{2}\right)\cdot\sin^2\left(\frac{\alpha-\beta}{2}\right) + 4\cos^2\left(\frac{\alpha+\beta}{2}\right)\cdot\sin^2\left(\frac{\alpha-\beta}{2}\right)$$

$$= 4\sin^2\left(\frac{\alpha-\beta}{2}\right)\left[\sin^2\left(\frac{\alpha+\beta}{2}\right) + \cos^2\left(\frac{\alpha+\beta}{2}\right)\right]$$

$$= 4\sin^2\left(\frac{\alpha-\beta}{2}\right) \times 1$$

$$= 4\sin^2\left(\frac{\alpha-\beta}{2}\right) = \text{RHS} \quad \text{Ans}$$

Ans 12 → $\sin\alpha + \sin\left(\alpha + \frac{2\pi}{3}\right) + \sin\left(\alpha + \frac{4\pi}{3}\right)$

$$= \sin\alpha + \sin(\alpha + 120^\circ) + \sin(\alpha + 240^\circ)$$

pairing $= \left(\sin(\underset{A}{\alpha+240}) + \underset{B}{\sin\alpha}\right) + \sin(\alpha+120)$

$= 2\sin\left(\frac{\alpha+240+\alpha}{2}\right)\cdot\cos\left(\frac{\alpha+240-\alpha}{2}\right) + \sin(\alpha+120^\circ)$

$= 2\sin(\alpha+120^\circ)\cdot\cos(120^\circ) + \sin(\alpha+120^\circ)$

common

$= \sin(\alpha+120^\circ)\left[2\cos(120^\circ)+1\right]$

$= \sin(\alpha+120^\circ)\left[2\cos(180-60^\circ)+1\right]$

$= \sin(\alpha+120^\circ)\left[-2\cos(60^\circ)+1\right]$

$= \sin(\alpha+120^\circ)\left[-\cancel{2}\times\frac{1}{\cancel{2}}+1\right]$

$= \sin(\alpha+120^\circ)(-1+1)$

$= \sin(\alpha+120^\circ)\times 0$

$= 0$ Ans

Ans 13. LHS $\cos\alpha + \cos\beta + \cos\gamma + \cos(\alpha+\beta+\gamma)$

pairing $\left(\cos(\alpha+\beta+\gamma)+\cos\alpha\right) + \left(\cos\beta+\cos\gamma\right)$

$= 2\cos\left(\frac{2\alpha+\beta+\gamma}{2}\right)\cdot\cos\left(\frac{\beta+\gamma}{2}\right) + 2\cos\left(\frac{\beta+\gamma}{2}\right)\cdot\cos\left(\frac{\beta-\gamma}{2}\right)$

common

$= 2\cos\left(\frac{\beta+\gamma}{2}\right)\left[\cos\left(\underset{A}{\frac{2\alpha+\beta+\gamma}{2}}\right)+\cos\left(\underset{B}{\frac{\beta-\gamma}{2}}\right)\right]$

$= 2\cos\left(\frac{\beta+\gamma}{2}\right)\left(2\cos\left(\frac{\frac{2\alpha+\beta+\gamma}{2}+\frac{\beta-\gamma}{2}}{2}\right)\cdot\cos\left(\frac{\frac{2\alpha+\beta+\gamma}{2}-\frac{\beta-\gamma}{2}}{2}\right)\right)$

$= 2\cos\left(\frac{\beta+\gamma}{2}\right)\cdot\left[2\cos\left(\frac{2\alpha+2\beta}{4}\right)\cdot\cos\left(\frac{2\alpha+2\gamma}{4}\right)\right]$

$= 4\cos\left(\frac{\beta+\gamma}{2}\right)\cdot\cos\left(\frac{\alpha+\beta}{2}\right)\cdot\cos\left(\frac{\alpha+\gamma}{2}\right)$ Ans