जय श्री राधे कृष्णा।। जय श्री गिरिराज जी महाराज !! ①

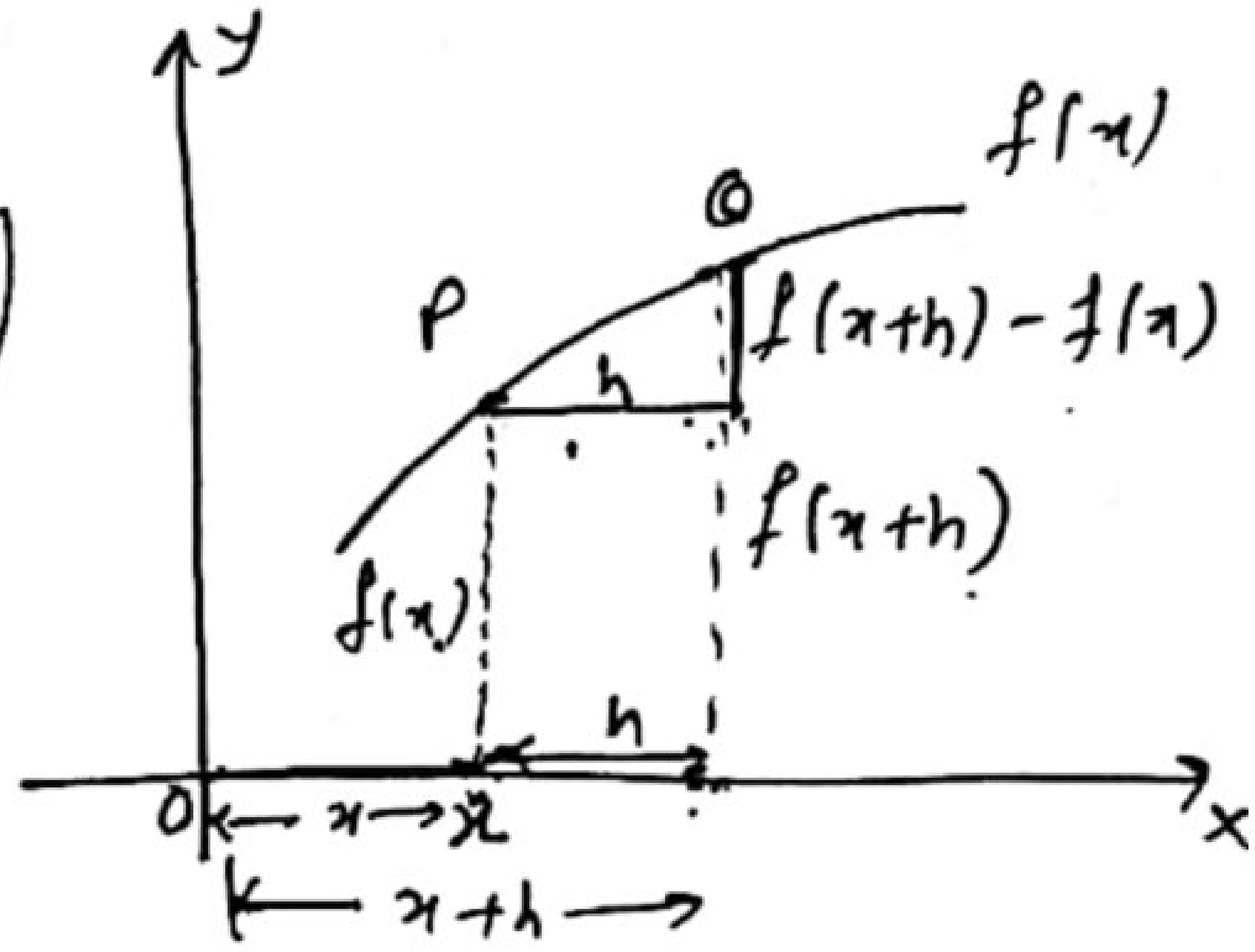
ULTIMATE MATHEMATICS : BY AJAY MITTAL

CHAPTER: Limits & Derivatives $\boxed{\text{CLASS NO: 3}}$

Derivative. First principle method (ab-initio Method)

(i) $y = f(x)$

$$f'(x) = \lim_{h \to 0}\left(\frac{f(x+h)-f(x)}{h}\right)$$



(i) $\sin A - \sin B = 2\cos(\;)\cdot\sin(\;)$

(ii) $\cos A - \cos B = -2\sin(\;)\cdot\sin(\;)$

(iii) $\tan A - \tan B = \tan(A-B)(1+\tan A\tan B)$

Q.1 Using first principle method, find derivative of
$$f(x) = \cos(3x)$$

Solⁿ $f(x) = \cos(3x)$

$$f'(x) = \lim_{h \to 0}\left[\frac{f(x+h)-f(x)}{h}\right]$$

$$= \lim_{h \to 0}\left[\frac{\cos(3x+3h)-\cos(3x)}{h}\right]$$

$$= \lim_{h \to 0}\left[\frac{-2\sin\left(\frac{6x+3h}{2}\right)\cdot\sin\left(\frac{3h}{2}\right)}{h}\right]$$

$$= \lim_{h \to 0}\left[\frac{-2\sin\left(\frac{6x+3h}{2}\right)\cdot\sin\left(\frac{3h}{2}\right)}{\frac{3}{2}h}\times\frac{3}{2}\right]$$

$$= -\sin(3x)\times 1\times 3 \quad \cdots\cdots \left\{\because \lim_{x\to 0}\left(\frac{\sin x}{x}\right)=1\right\}$$

$$\therefore f'(x) = -3\sin(3x) \quad \text{Ans}$$

Que 2 $f(x) = \tan(2x+3)$

$$f'(x) = \lim_{h\to 0}\left(\frac{f(x+h)-f(x)}{h}\right)$$

$$= \lim_{h\to 0}\left[\frac{\tan(\overset{(A)}{2x+2h+3}) - \tan(\overset{(B)}{2x+3})}{h}\right]$$

$$= \lim_{h\to 0}\left[\frac{\tan(2h)\{1+\tan(2x+2h+3)\cdot\tan(2x+3)\}}{h}\right]$$

$$= \lim_{h\to 0}\left(\frac{\tan(2h)\{1+\tan(2x+2h+3)\cdot\tan(2x+3)}{2h}\times 2\right]$$

$$= 1\times\left(1+\tan^2(2x+3)\right)\times 2 \quad \cdots \left\{\because \lim_{x\to 0}\left(\frac{\tan x}{x}\right)=1\right\}$$

$$f'(x) = 2\sec^2(2x+3) \quad \text{Ans}$$

Que. 3 $f(x) = \sqrt{\sec(2x)}$

(Rough $\frac{1}{2\sqrt{\sec(2x)}} \cdot \sec(2x)\tan(2x)\cdot 2$

$$f'(x) = \lim_{h\to 0}\left[\frac{f(x+h)-f(x)}{h}\right]$$

$$= \lim_{h\to 0}\left(\frac{\sqrt{\sec(2x+2h)}-\sqrt{\sec(2x)}}{h}\right]$$

Rationalize

$$= \lim_{h\to 0}\left(\frac{\sec(2x+2h)-\sec(2x)}{h\left(\sqrt{\sec(2x+2h)}+\sqrt{\sec(2x)}\right)}\right)$$

$$= \lim_{h\to 0}\left[\frac{\frac{1}{\cos(2x+2h)}-\frac{1}{\cos(2x)}}{h\cdot\left(\sqrt{\sec(2x+2h)}+\sqrt{\sec(2x)}\right)}\right]$$

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$$= \lim_{h\to 0}\left(\frac{\cos(2x)-\cos(2x+2h)}{h\cdot\cos(2x+2h)\cdot\cos(2x)\left(\sqrt{\sec(2x+2h)}+\sqrt{\sec(2x)}\right)}\right]$$

$$= \lim_{h\to 0}\left(\frac{-2\sin\left(\frac{4x+2h}{2}\right)\cdot\sin(-h)}{h\ \cos(2x+2h)\cos(2x)\left(\sqrt{\sec(2x+2h)}+\sqrt{\sec(2x)}\right)}\right]$$

$$= \frac{2\sin(2x)\times 1}{\cos(2x)\cdot\cos(2x)\times 2\sqrt{\sec(2x)}} \quad \cdots\left\{\lim_{x\to 0}\left(\frac{\sin x}{x}\right)=1\right\}$$

$$f'(x) = \frac{\sec(2x)\tan(2x)}{\sqrt{\sec(2x)}} \quad \text{Ans}$$

Q. 4 $f(x) = \sqrt{\cot(3x-2)}$

$\left[\frac{-1}{2\sqrt{\cot(3x-2)}}\cdot\operatorname{cosec}^2(3x-2)\cdot 3\right.$ Rough

$$f'(x) = \lim_{h\to 0}\left[\frac{f(x+h)-f(x)}{h}\right]$$

$$= \lim_{h\to 0}\left[\frac{\sqrt{\cot(3x+3h-2)}-\sqrt{\cot(3x-2)}}{h}\right]$$

Rationalise

$$= \lim_{h\to 0}\left(\frac{\cot(3x+3h-2)-\cot(3x-2)}{h\left(\sqrt{\cot(3x+3h-2)}+\sqrt{\cot(3x-2)}\right)}\right]$$

$$= \lim_{h\to 0}\left(\frac{\tan\overset{(A)}{(3x-2)}-\tan\overset{(B)}{(3x+3h-2)}}{h\cdot\tan(3x+3h-2)\tan(3x-2)\left(\sqrt{\cot(3x+3h-2)}+\sqrt{\cot(3x-2)}\right)}\right]$$

$$= \lim_{h \to 0} \left(\frac{\tan(-3h) \{ 1 + \tan(3x-2) \tan(3x+3h-2) \}}{h \tan(3x+3h-2) \tan(3x-2) \left(\sqrt{\cot(3x+3h-2)} + \sqrt{\cot(3x-2)} \right)} \right)$$

$$= -\lim_{h \to 0} \left(\frac{\tan(3h) \{ 1 + \tan(3x-2) \tan(3x+3h-2) \} \times 3}{3h \tan(3x+3h-2) \tan(3x-2) \left(\sqrt{\cot(3x+3h-2)} + \sqrt{\cot(3x-2)} \right)} \right)$$

$$= -1 \times \frac{\left(1 + \tan^2(3x-2) \right) \times 3}{\tan^2(3x-2) \times 2\sqrt{\cot(3x-2)}} \quad \ldots \left\{ \lim_{x \to 0} \left(\frac{\tan x}{x} = 1 \right) \right\}$$

$$= \frac{-3 \sec^2(3x-2)}{2 \tan^2(3x-2) \sqrt{\cot(3x-2)}}$$

$$= \frac{-3}{2} \cdot \frac{\frac{1}{\cancel{\cos^2(3x-2)}}}{\frac{\sin^2(3x-2)}{\cancel{\cos^2(3x-2)}} \sqrt{\cot(3x-2)}}$$

$$f'(x) = \frac{-3 \operatorname{cosec}^2(3x-2)}{2\sqrt{\cot(3x-2)}} \quad \text{Ans}$$

Qn. 5 $f(x) = \sin^2 x$

$\left\{ \begin{array}{l} \text{Rolu} \\ 2\sin x \cdot \cos x \end{array} \right.$

$$f'(x) = \lim_{h \to 0} \left(\frac{f(x+h) - f(x)}{h} \right)$$

$$f'(x) = \lim_{h \to 0} \left(\frac{\sin^2(x+h) - \sin^2 x}{h} \right)$$

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$$f'(x) = \lim_{h\to 0}\left[\frac{(\sin(x+h)+\sin x)(\sin(x+h)-\sin x)}{h}\right]$$

$$= \lim_{h\to 0}\left(\frac{(\sin(x+h)+\sin x)\cdot \cancel{2}\cos\left(\frac{2x+h}{2}\right)\cdot\sin\left(\frac{h}{2}\right)}{\frac{h}{2}\times\cancel{2}}\right)$$

$$= (2\sin x)\cdot\cos x \times 1 \quad \cdots \left\{\lim_{x\to 0}\left(\frac{\sin x}{x}\right)=1\right\}$$

$$f'(x) = 2\sin x\cos x \quad \underline{\text{Ans}}$$

Q^n 6 $\quad f(x) = \cot^2(3x)$ $\quad$ {Rough: $-2\cot(3x)\operatorname{cosec}^2(3x)\cdot 3$

$$f'(x) = \lim_{h\to 0}\left(\frac{f(x+h)-f(x)}{h}\right)$$

$$= \lim_{h\to 0}\left[\frac{\cot^2(3x+3h)-\cot^2(3x)}{h}\right]$$

$$= \lim_{h\to 0}\left[\frac{(\cot(3x+3h)+\cot(3x))\cdot(\cot(3x+3h)-\cot(3x))}{h}\right]$$

$$= \lim_{h\to 0}\left[\frac{(\cot(3x+3h)+\cot(3x))\cdot(\tan(3x)-\tan(3x+3h))}{h\cdot\tan(3x+3h)\tan(3x)}\right]$$

$$= \lim_{h\to 0}\left[\frac{(\cot(3x+3h)+\cot(3x))\cdot(\tan(-3h)\{1+\tan(3x)\tan(3x+3h)\})}{h\tan(3x+3h)\tan(3x)}\right]$$

$$= -\lim_{h\to 0}\left[\frac{(\cot(3x+3h)+\cot(3x))\cdot 3\tan(3h)(1+\tan(3x)\tan(3x+3h))}{3h\tan(3x+3h)\tan(3x)}\right]$$

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$$= -\frac{2\cot(3x)\times 3 \times \sec^2(3x)}{\tan^2(3x)}$$

$$= -6\cot(3x)\times\frac{\frac{1}{\cos^2(3x)}}{\frac{\sin^2(3x)}{\cos^2(3x)}}$$

$$f'(x) = -6\cot(3x)\,\csc^2(3x) \quad \underline{\text{Ans}}$$

Q.7 $f(x) = \cos\sqrt{x}$

$\left\{\begin{array}{l}\text{Rough} \\ -\sin(\sqrt{x})\cdot\frac{1}{2\sqrt{x}}\end{array}\right.$

$$f'(x) = \lim_{h\to 0}\left(\frac{f(x+h)-f(x)}{h}\right)$$

$$f'(x) = \lim_{h\to 0}\left(\frac{\cos\overset{A}{\sqrt{x+h}} - \cos\overset{B}{\sqrt{x}}}{h}\right)$$

$$f'(x) = \lim_{h\to 0}\left[\frac{-2\sin\left(\frac{\sqrt{x+h}+\sqrt{x}}{2}\right)\cdot\sin\left(\frac{\sqrt{x+h}-\sqrt{x}}{2}\right)}{h}\right]$$

$$= \lim_{h\to 0}\left[\frac{-\cancel{2}\sin\left(\frac{\sqrt{x+h}+\sqrt{x}}{2}\right)\cdot\sin\left(\frac{\sqrt{x+h}-\sqrt{x}}{2}\right)\times\left(\frac{\sqrt{x+h}-\sqrt{x}}{\cancel{2}}\right)}{h\left(\frac{\sqrt{x+h}-\sqrt{x}}{2}\right)}\right]$$

$$= \lim_{h\to 0}\left[\frac{-\sin\left(\frac{\sqrt{x+h}+\sqrt{x}}{2}\right)\cdot\sin\left(\frac{\sqrt{x+h}-\sqrt{x}}{2}\right)\cdot(\cancel{x}+h-\cancel{x})}{h\left(\frac{\sqrt{x+h}-\sqrt{x}}{2}\right)\times\left(\sqrt{x+h}+\sqrt{x}\right)}\right]$$

$$f'(x) = -\sin(\sqrt{x})\times 1\times\frac{1}{2\sqrt{x}} = \frac{-\sin(\sqrt{x})}{2\sqrt{x}} \quad \text{Ans}$$

Qn. 8 $f(x) = \operatorname{cosec}(\sqrt{x})$

{ Rough $\dfrac{-\operatorname{cosec}(\sqrt{x})\cot(\sqrt{x})}{2\sqrt{x}}$

$$f'(x) = \lim_{h\to 0}\left(\frac{f(x+h)-f(x)}{h}\right)$$

$$= \lim_{h\to 0}\left(\frac{\operatorname{cosec}(\sqrt{x+h}) - \operatorname{cosec}\sqrt{x}}{h}\right)$$

$$= \lim_{h\to 0}\left(\frac{\sin\sqrt{x} - \sin(\sqrt{x+h})}{h\sin(\sqrt{x+h})\sin\sqrt{x}}\right)$$

$$= \lim_{h\to 0}\left[\frac{2\cos\left(\frac{\sqrt{x}+\sqrt{x+h}}{2}\right)\cdot\sin\left(\frac{\sqrt{x}-\sqrt{x+h}}{2}\right)}{h\sin(\sqrt{x+h})\sin(\sqrt{x})}\right]$$

$$= \lim_{h\to 0}\left[\frac{\cancel{2}\cos\left(\frac{\sqrt{x}+\sqrt{x+h}}{2}\right)\cdot\sin\left(\frac{\sqrt{x}-\sqrt{x+h}}{2}\right)\times\left(\frac{\sqrt{x}-\sqrt{x+h}}{\cancel{2}}\right)}{h\left(\frac{\sqrt{x}-\sqrt{x+h}}{2}\right)\sin(\sqrt{x+h})\sin\sqrt{x}}\right]$$

$$= \lim_{h\to 0}\left(\frac{\cos\left(\frac{\sqrt{x}+\sqrt{x+h}}{2}\right)\sin\left(\frac{\sqrt{x}-\sqrt{x+h}}{2}\right)\cdot(\cancel{x}-\cancel{x}-h)}{h\left(\frac{\sqrt{x}-\sqrt{x+h}}{2}\right)\sin(\sqrt{x+h})\sin\sqrt{x}\,(\sqrt{x}+\sqrt{x+h})}\right)$$

$$= \frac{-\cos(\sqrt{x})\times 1}{\sin\sqrt{x}\cdot\sin\sqrt{x}\times 2\sqrt{x}} \quad \cdots\left\{\lim_{x\to 0}\left(\frac{\sin x}{x}\right)=1\right\}$$

$$f'(x) = \frac{-\operatorname{cosec}(\sqrt{x})\cot(\sqrt{x})}{2\sqrt{x}}$$ Ans

LIMITS & DERIVATIVES (WORKSHEET No: 2)

(Class - 3)

Differentiate using First Principle method:

Qn: 1 $f(x) = \sin(3x-4)$ Ans $f'(x) = 3\cos(3x-4)$

Qn 2 $f(x) = \sec(2x-3)$ Ans $f'(x) = 2\sec(2x-3)\tan(2x-3)$

Qn 3. $f(x) = \sqrt{\cos(3x)}$ Ans $\frac{1}{2\sqrt{\cot(3x)}} \cdot 3$

Qn 4 $f(x) = \sqrt{\tan(2x)}$ Ans $f(x) = \frac{1}{2\sqrt{\tan(2x)}} \cdot \cancel{2}$

Qn 5 $f(x) = \operatorname{cosec}^2(2x-4)$ Ans $-4\operatorname{cosec}^2(2x-4) \cdot \cot(2x-4)$

Qn. 6 $f(x) = \cot^2(5x)$ Ans $-10\cot(5x) \cdot \operatorname{cosec}^2(5x)$

Qn. 7 $f(x) = \sec\sqrt{x}$ Ans $\frac{\sec\sqrt{x}\ \tan\sqrt{x}}{2\sqrt{x}}$

Qn 8 $f(x) = \cot\sqrt{x}$ Ans $-\frac{\operatorname{cosec}^2\sqrt{x}}{2\sqrt{x}}$

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