!! जय श्री राधे कृष्णा जय श्री बाँके बिहारी जी महाराज !!

ULTIMATE MATHEMATICS: BY AJAY MITTAL

CHAPTER: PROBABILITY (11th) | CLASS NO: 4

Ques: 1 Seven persons are to be seated in a row. Find the probability that two particular persons sit next to each other.

Solution
Total Number of ways of arrangement of 7 persons $= 7!$

fav. no. of ways (·) consider two particular persons as 1 $\bigcirc = 1$
(·) No of ways of arrangement (favourable) $= 6! \times 2!$

Required prob $= \dfrac{\text{favour ways}}{\text{total ways}}$

$= \dfrac{6! \times 2!}{7!}$

$= \dfrac{\cancel{6!} \times 2}{7 \times \cancel{6!}} = \dfrac{2}{7}$ Ans

Ques 2 → Four digits number are formed without repetition with the numbers 0, 2, 3, 5. Find the probability that the number is divisible by 5

Solution Total No of ways $=$ | 3 | 3 | 2 | 1 | $= 3 \times 3 \times 2 \times 1 = 18$

Fav. ways :- Two cases

(I) No. end with 0

3	2	1	1

$= 3 \times 2 \times 1 \times 1 = 6$

(II) No. end with 5

2	2	1	1

$= 2 \times 2 \times 1 \times 1 = 4$

$\therefore$ fav. no of ways $= 6 + 4 = 10$

Required prob $= \frac{10}{18} = \frac{5}{9}$ Ans

Qns 3 → A single letter is selected at random from the word PROBABILITY. Find the probability that its a vowel.

Solution PROBABILITY

total letters: 11

vowels = O, A, I, I

Consonants : P, R, B, B, L, T, Y

A → selecting a vowel.

$P(A) = \frac{4}{11}$ Ans

Qn. 4 → Three numbers are chosen from 1 to 20. Find the probability that they are not consecutive

Solution let $A \rightarrow$ they are not consecutive

$A' \rightarrow$ the three numbers are consecutive

total no. of ways of selecting 3 numbers out of 20 numbers $= {}^{20}C_3$

$A' = \{(1,2,3), (2,3,4), (3,4,5), \text{------} (18,19,20)\}$

fav. no of ways $= 18$

$$P(A') = \frac{18}{{}^{20}C_3}$$

Now Req. prob $= P(A) = 1 - P(A')$

$$= 1 - \frac{18}{{}^{20}C_3}$$

$$= 1 - \frac{\cancel{18}}{\frac{20 \times 19 \times \cancel{18}}{6}}$$

$$= 1 - \frac{6}{380}$$

$$= \frac{374}{380}$$

Req prob $= \dfrac{187}{190}$ Ans

Ou 5 → In a leap year, find the prob of having 53 tuesday or 53 wednesday.

Soln leap year $= 366$ days $= 52$ weeks $+ 2$ more days

These two days can be

$S = \{$ (Sun, Mon), (Mon, Tues), (Tues, wed), (wed, thus), (thus, Friday), (Friday, Sat.), (Sat., Sunday) $\}$

total No. of ways = 7

fav. no. of ways = $\{$ (Mon, Tues), (Tues, wed), (wed, thus) $\}$

= 3

Required prob = $\frac{3}{7}$ Ans

Ques 6 → If the letters of the word ASSASSINATION are arranged at random. Find the probability that

(a) Four S's come consecutively in the word

(b) Two I's & two N's come together

(c) All A's are not coming together

(d) No two A's are coming together

Solution ASSASSINATION

total letters = 13

A = 3 ; S = 4, I = 2, N = 2, T = 1, O = 1

total No. of ways = $\frac{13!}{3!\,4!\,2!\,2!}$

(a) Consider 4 S's as 1 (SSSS) = 1

fav. ways = $\frac{10!}{3!\,2!\,2!} \times 1 = \frac{10!}{3!\,2!\,2!}$

Req. prob $= \dfrac{\frac{10!}{3!\,2!\,2!}}{\frac{13!}{3!\,4!\,2!\,2!}} = \dfrac{10! \times 4!}{13!}$

$= \dfrac{\cancel{10!} \times \cancel{24}^{2}}{13 \times \cancel{12} \times 11 \times \cancel{10!}} = \dfrac{2}{143}$ Ans

(b) Two I's & two N's together

Consider 2 I's & 2 N's as one (I I N N) = 1

fav. ways $= \dfrac{10!}{\cancel{4!}\,3!} \times \dfrac{\cancel{4!}}{2!\,2!} = \dfrac{10!}{3!\,2!\,2!}$

Req. prob $= \dfrac{\frac{10!}{3!\,2!\,2!}}{\frac{13!}{3!\,4!\,2!\,2!}} = \dfrac{2}{143}$ Ans

(c) All A's not together.

Consider All A's as one (A A A) = 1

no of ways $= \dfrac{11!}{4!\,2!\,2!} \times 1$

P(all A's together) $= \dfrac{\frac{11!}{\cancel{4!}\,\cancel{2!}\,\cancel{2!}}}{\frac{13!}{3!\,\cancel{4!}\,\cancel{2!}\,\cancel{2!}}} = \dfrac{11! \times 3!}{13!}$

$= \dfrac{\cancel{11!} \times \cancel{6}}{13 \times \cancel{12}_{2} \times \cancel{11!}} = \dfrac{1}{26}$

P(all A's not together) $= 1 - \dfrac{1}{26} = \dfrac{25}{26}$ Ans

(d) No two A's are together

$\_S\_S\_S\_S\_I\_N\_T\_I\_O\_N\_$

Remaining 10 letters can be arranged in $= \frac{10!}{4!\,2!\,2!}$ ways

3 A's can arranged in $= \frac{^{11}P_3}{3!} = \frac{11!}{8!\,3!}$

$\therefore$ fav no. of ways $= \frac{10!}{4!\,2!\,2!} \times \frac{11!}{8!\,3!}$

Req. prob $= \dfrac{\frac{10!}{\cancel{4!}\,\cancel{2!}\,\cancel{2!}} \times \frac{11!}{8!\,\cancel{3!}}}{\frac{13!}{\cancel{4!}\,\cancel{3!}\,\cancel{2!}\,\cancel{2!}}} = \frac{10! \times 11!}{13! \times 8!}$

$= \frac{10! \times \cancel{11!}}{13 \times 12 \times \cancel{11!} \times 8!}$

$= \frac{10 \times 9 \times \cancel{8!}}{13 \times 12 \times \cancel{8!}}$

$= \frac{90}{156}$

biased $= \frac{15}{26}$ Ans

Ques 7 + A die is $\boxed{\text{loaded}}$ in such a way that each odd number is twice as likely to occur as each even number. Find $P(G)$ where G is the event that a number greater than 3 occurs on a single roll of the die.

Solution

given $P(\text{each odd no}) = 2P(\text{each even no})$

let $P(\text{each even no}) = p$

$\therefore P(\text{each odd no}) = 2p$

we have

$$P(1) + P(2) + P(3) + P(4) + P(5) + P(6) = 1$$

$$2p + p + 2p + p + 2p + p = 1$$

$$\Rightarrow 9p = 1$$

$$\Rightarrow p = 1/9$$

$\therefore P(\text{each odd No}) = \frac{2}{9}$

$P(\text{each even No}) = \frac{1}{9}$

$G \rightarrow$ getting more than 3

$G = \{4, 5, 6\}$

$P(G) = \frac{1}{9} + \frac{2}{9} + \frac{1}{9} = \frac{4}{9}$ Ans

Qn. 8 → Suppose an integer from 1 through 1000 is chosen. find the prob that the integer chosen is neither multiple of 2 nor multiple of 9

Soln let $A \rightarrow$ no. is multiple of 2

$B \rightarrow$ " " " of 9

$A = \{2, 4, 6, \ldots 1000\}$

$B = \{9, 18, 27, \ldots 999\}$

$A \cap B = \{18, 36, 54, \ldots 990\}$

$$P(A) = \frac{500}{1000}$$

$$P(B) = \frac{111}{1000}$$

$$P(A \cap B) = \frac{55}{1000}$$

$$P(A \cup B) = \frac{500}{1000} + \frac{111}{1000} - \frac{55}{1000}$$

$$= \frac{556}{1000}$$

Req. prob $P(A' \cap B') = 1 - P(A \cup B)$

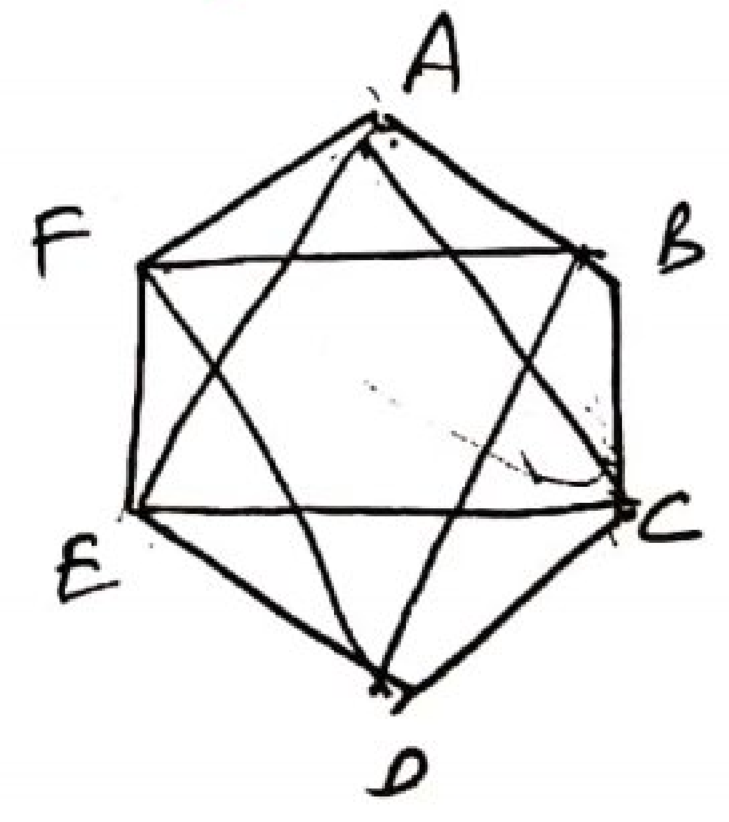
$$= 1 - \frac{556}{1000}$$

$$= \frac{444}{1000} = 0.444 \quad \underline{\text{Ans}}$$

Qn. 9 → three of the six vertices of a regular hexagon are chosen at random. what is the probability that the triangle with these vertices is equilateral?

Solution total: 6 vertices

total No of triangles that can be formed $= {}^6C_3$



favourable No of ways: only two equilateral triangle possible $\triangle AEC$ & $\triangle BDF$

$\therefore$ fav ways $= 2$

Req. prob $= \frac{2}{{}^6C_3} = \frac{2}{20} = \frac{1}{10}$ Ans

Qu-10 → Three squares of chess board are selected at random. Find the probability of getting 2 squares of one colour and other of a different colour.

Solution: total squares = 64

32 Black squares & 32 white squares

total No. of ways of selecting 3 squares = $^{64}C_3$

Fav ways. Two cases

Case I 2 white & 1 black square = $^{32}C_2 \times {}^{32}C_1$

Case II. 2 black & 1 white square = $^{32}C_2 \times {}^{32}C_1$

Req. prob $= \dfrac{(^{32}C_2 \times {}^{32}C_1) + (^{32}C_2 \times {}^{32}C_1)}{^{64}C_3}$

$= \dfrac{\left(\frac{32\times31}{\cancel{2}} \times 32\right) \times \cancel{2}}{\frac{64 \times 63 \times 62}{6}}$

$= \dfrac{\cancel{32} \times \cancel{31} \times \cancel{32}^{16} \times \cancel{6}\,\cancel{3}}{\cancel{64}_{\cancel{2}} \times \cancel{63}_{21} \times \cancel{62}_{\cancel{2}}}$

$= \dfrac{16}{21}$ Ans

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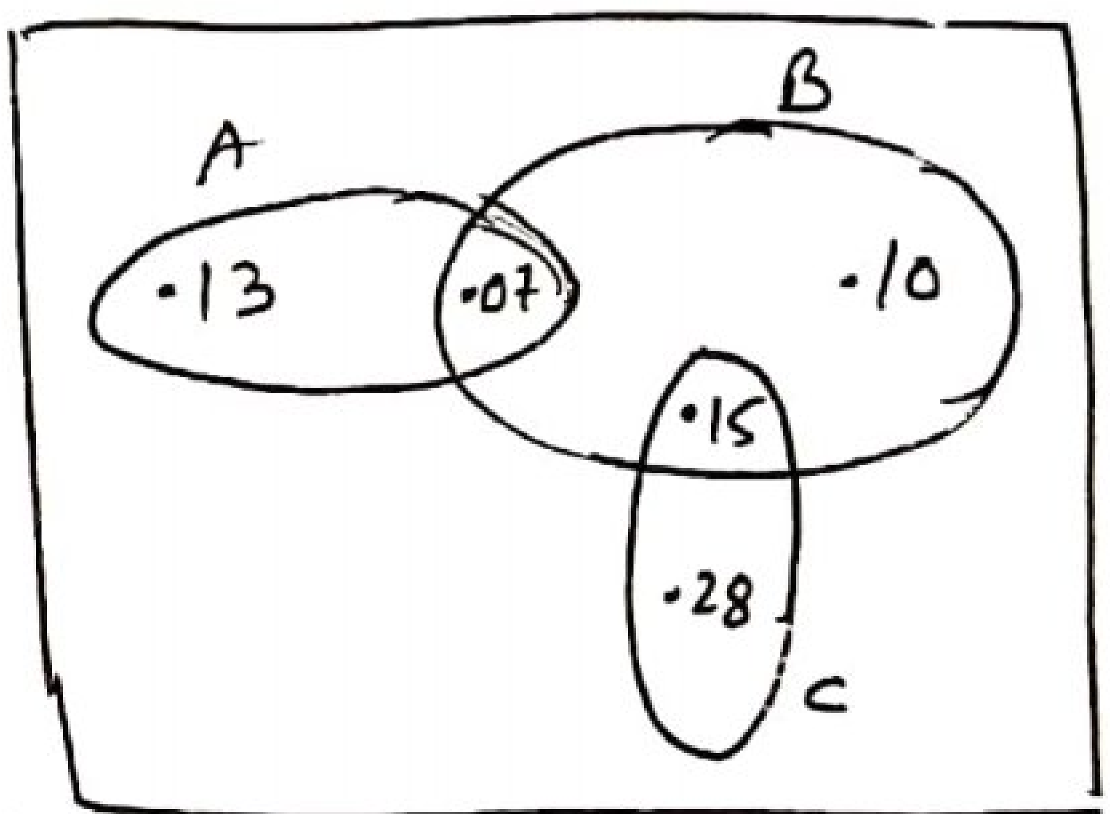
WORKSHEET No: 3 (class No: 4)

PROBABILITY (11th class)

Ques 1 6 boys and 6 girls sit in a row at random. Find the probability that all the girls sit together **Ans** = $\frac{1}{132}$

Ques 2 → While shuffling a pack of 52 playing cards, 2 cards are accidentally dropped. Find the probability that the missing cards to be of different colours **Ans** = $\frac{26}{51}$

Ques 3 → Given venn diagram (showing probabilities)



Find (a) $P(A)$

(b) $P(B \cap C')$

(c) $P(A \cup B)$

(d) $P(A \cap B')$

(e) $P(B \cap C)$

(f) probability of exactly one of the three occurs

Ans 0·20, 0·17, 0·45, 0·13, 0·15, 0·51

Ques 4 → If the letters of the word ALGORITHM are arranged at random in a row what is the probability the letters GOR must remain together as a unit?

Ans $\frac{1}{72}$

Ques 5 → If A and B are two events having $P(A \cup B) = \frac{1}{2}$ and $P(A') = \frac{2}{3}$. Find probability of $A' \cap B$

Ans = $\frac{1}{6}$

Ques 6 → If A, B, C are mutually exclusive and exhaustive events such that

$3P(A) = 2P(B) = P(C)$, then find $P(A)$ Ans $= \frac{2}{11}$

Qn 7 → A, B, C are events such that

$P(A) = 0.3$; $P(B) = 0.5$; $P(C) = 0.7$; $P(A \cap B) = 0.09$

$P(A \cap C) = 0.27$; $P(A \cap B \cap C) = 0.08$

If $P(A \cup B \cup C) \geq 0.8$ then show that $P(B \cap C)$ lies in the interval $[0.22, 0.42]$

Qn. 8 → Two cards are drawn at random from a pack of 52 cards. Find the probability that both the cards are of red colour or they are queen Ans $\frac{55}{221}$

Qn 9 → 12 balls are distributed among 4 boxes. Find the probability that the first box contains 3 balls

Ans $\frac{{}^{12}C_3 \times 3^9}{4^{12}}$

Qn 10 → A bag contains 3 Red, 4 Black & 3 green balls. 4 Balls are drawn at random. Find the probability of getting atleast 1 ball of each colour

Hint 3 cases

2R 1B 1G
1R 2B 1G
1R 1B 2G

Ans: self

—x—

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