!! जय श्री राधे कृष्णा।। जय श्री गिरिराज जी महाराज !!

ULTIMATE MATHEMATICS: BY AJAY MITTAL

REVISION: LIMITS & DERIVATIVES

(CLASS No: 1)

(·) Derivatives

$y = f(x)$

Diff both sides w.r.t x

(·) $\frac{dy}{dx}$ or $f'(x)$

(·) formula

① $\frac{d}{dx}(x^n) = nx^{n-1}$

$n \to$ constant

② (·) $\frac{d}{dx}(\text{constant}) = 0$

(·) $\frac{d}{dx}(\sin a) = 0$

③ (·) $\frac{d}{dx}(\log x) = \frac{1}{x}$

(·) $\frac{d}{dx}(\log_e x) = \frac{1}{x}$

ⓧ $\frac{d}{dx}(\log_a x) \neq \frac{1}{x}$

Change of Base property $\log_a b = \frac{\log_e b}{\log_e a}$

(·) $\frac{d}{dx}(\log_2 x) = \frac{d}{dx}\left(\frac{\log x}{\log 2}\right)$

$= \frac{1}{x \log 2}$ Ans

④ (·) $\frac{d}{dx}(e^x) = e^x$

⑤ (·) $\frac{d}{dx}(a^x) = a^x \log a$

$\{a > 0 \ \& \ a \neq 1\}$

(·) $\frac{d}{dx}(3^x) = 3^x \cdot \log 3$

Shortcut

⑥ $\frac{d}{dx}\left(\frac{1}{x}\right) = -\frac{1}{x^2}$

$\frac{1}{x} \to \frac{-1}{x^2} \to \frac{2}{x^3} \to -\frac{6}{x^4} \to \frac{24}{x^5}$

⑦ $\frac{d}{dx}(\sqrt{x}) = \frac{1}{2\sqrt{x}}$

(8) (·) $\frac{d}{dx}(\sin x) = \cos x$

(9) (·) $\frac{d}{dx}(\cos x) = -\sin x$

(10) (·) $\frac{d}{dx}(\tan x) = \sec^2 x$

(11) (·) $\frac{d}{dx}(\cot x) = -\operatorname{cosec}^2 x$

(12) (·) $\frac{d}{dx}(\sec x) = \sec x \tan x$

(13) (·) $\frac{d}{dx}(\operatorname{cosec} x) = -\operatorname{cosec} x \cdot \cot x$

Rules

(·) $\frac{d}{dx}(k\, f(x)) = k \cdot \frac{d}{dx} f(x)$

$\frac{d}{dx}\left(\frac{f(x)}{k}\right) = \frac{1}{k} \cdot \frac{d}{dx}(f(x))$

(·) $\frac{d}{dx}(f(x) \pm g(x)) = \frac{d}{dx}(f(x)) \pm \frac{d}{dx}(g(x))$

(·) Product rule

$y = f(x)\, g(x)$

$\frac{dy}{dx} = f(x) \cdot g'(x) + g(x)\, f'(x)$

$y = f(x) \cdot g(x) \cdot u(x) = \mathrm{I} \cdot \mathrm{II} \cdot \mathrm{III}$

$\frac{dy}{dx} = f'(x) \cdot \mathrm{II} \cdot \mathrm{III} + g'(x) \cdot \mathrm{I} \cdot \mathrm{III} + u'(x) \cdot \mathrm{I} \cdot \mathrm{II}$

(·) Quotient Rule

$y = \frac{f(x)}{g(x)} = \frac{N}{D} \Rightarrow \frac{dy}{dx} = \frac{D \cdot \frac{d}{dx}(N) - N \cdot \frac{d}{dx}(D)}{D^2}$

Basics

(i) (1) $\frac{d}{dx}\left(\sin(x^2)\right)$

$= \cos(x^2)\cdot\frac{d}{dx}(x^2)$

$= 2x\cdot\cos(x^2)$

(ii) (2) $\frac{d}{dx}\left(\sqrt{\tan x}\right)$

$= \frac{1}{2\sqrt{\tan x}}\cdot\frac{d}{dx}(\tan x)$

$= \sec^2 x\cdot\frac{1}{2\sqrt{\tan x}}$

(iii) (3) $\frac{d}{dx}\left(3^{\sqrt{x}}\right)$

$= 3^{\sqrt{x}}\cdot\log 3\,\frac{d}{dx}(\sqrt{x})$

$= \frac{1}{2\sqrt{x}}\cdot 3^{\sqrt{x}}\cdot\log 3$

(iv) (4) $\frac{d}{dx}\left(e^{1/x}\right)$

$= e^{1/x}\cdot\frac{d}{dx}\left(\frac{1}{x}\right)$

$= \frac{-1}{x^2}\cdot e^{1/x}$

(v) (5) $\frac{d}{dx}\left(x^3+2x^2-5x+6\right)^{5/2}$

$= \frac{5}{2}\left(x^3+2x^2-5x+6\right)^{3/2}\cdot\frac{d}{dx}\left(x^3+2x^2-5x+6\right)$

$= \frac{5}{2}\left(x^3+2x^2-5x+6\right)^{3/2}\cdot\left(3x^2+4x-5\right)$

(vi) (6) $\frac{d}{dx}\left(\frac{1}{\sqrt{3x+5}}\right)$

$= \frac{d}{dx}(3x+5)^{-1/2}$

$= -\frac{1}{2}(3x+5)^{-3/2}\cdot\frac{d}{dx}(3x+5)$

$= -\frac{1}{2}\cdot\frac{1}{(3x+5)^{3/2}}\times 3$

(vii) (7) $\frac{d}{dx}\left(\frac{3}{(4-3x^2)^{7/2}}\right)$

$= \frac{d}{dx}\left(3\,(4-3x^2)^{-7/2}\right)$

$= 3\times -\frac{7}{2}(4-3x^2)^{-9/2}\cdot\frac{d}{dx}(4-3x^2)$

$= -\frac{21}{2}\times\frac{1}{(4-3x^2)^{9/2}}\cdot(-6x)$

(viii) (8) $\frac{d}{dx}(\sin^4 x)$

$= \frac{d}{dx}(\sin x)^4$

$= 4\sin^3 x\cdot\cos x$

(ix) $\frac{d}{dx}(\sec^{99} x)$

$= 99\sec^{98} x\cdot\sec x\tan x$

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(·) $\frac{d}{dx}\left(\log\left(\sin\left(\sqrt{\tan x^2}\right)\right)\right)$

$$= \frac{1}{\sin\left(\sqrt{\tan x^2}\right)} \cdot \frac{d}{dx}\left(\sin\left(\sqrt{\tan(x^2)}\right)\right)$$

$$= \frac{1}{\sin\left(\sqrt{\tan x^2}\right)} \cdot \cos\left(\sqrt{\tan(x^2)}\right) \cdot \frac{d}{dx}\left(\sqrt{\tan x^2}\right)$$

$$= \frac{1}{\sin\left(\sqrt{\tan x^2}\right)} \cdot \cos\left(\sqrt{\tan x^2}\right) \cdot \frac{1}{2\sqrt{\tan x^2}} \cdot \frac{d}{dx}\left(\tan(x^2)\right)$$

$$= \frac{1}{\sin\left(\sqrt{\tan(x^2)}\right)} \cdot \cos\left(\sqrt{\tan x^2}\right) \cdot \frac{1}{2\sqrt{\tan(x^2)}} \cdot \sec^2(x^2) \cdot 2x$$

(·) $\frac{d}{dx}\left(x^3 \cdot \sin(x^2)\right)$

$$= x^3 \cdot \frac{d}{dx}\left(\sin(x^2)\right) + \sin(x^2) \cdot \frac{d}{dx}\left(x^3\right)$$

$$= x^3 \cdot \cos(x^2) \cdot 2x + \sin(x^2) \cdot 3x^2$$

(·) $\frac{d}{dx}\left(\frac{x\cos x}{x^2 + \sin x}\right)$

$$= \frac{(x^2+\sin x) \cdot \frac{d}{dx}(x\cos x) - (x\cos x) \cdot \frac{d}{dx}(x^2+\sin x)}{(x^2+\sin x)^2}$$

$$= \frac{(x^2+\sin x) \cdot (-x\sin x + \cos x) - x\cos x \cdot (2x + \cos x)}{(x^2+\sin x)^2}$$

First principle Method (ab-initio method)

(·) $y = f(x)$

$$f'(x) = \lim_{h \to 0}\left(\frac{f(x+h) - f(x)}{h}\right)$$ (remember this)

$$f'(a) = \lim_{x \to a}\left(\frac{f(x) - f(a)}{x - a}\right)$$

generally

put $x = a+h$ & $h \to 0$

$$f'(a) = \lim_{h \to 0}\left(\frac{f(a+h) - f(a)}{h}\right)$$

option put $x = a-h$ & $h \to 0$

$$f'(a) = \lim_{h \to 0}\left(\frac{f(a-h) - f(a)}{-h}\right)$$

$$\boxed{\tan A - \tan B = \tan(A-B)\left(1 + \tan A \tan B\right)}$$

(·) Rate of change of one variable with another variable

(·) Application : Slope of tangent $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{dy}{dx}$

(·) $$\boxed{\frac{dy}{dx} = \lim_{\Delta x \to 0}\left(\frac{\Delta y}{\Delta x}\right)}$$

LIMITS

$$\lim_{x \to a} (f(x))$$

$$\lim_{x \to 2} (x+3) = 5$$

$\boxed{\text{when } x \to 2 \text{ then } f(x) \to 5}$

LHL → 2 ← RHL

1.9999 2.0001

$x < a$

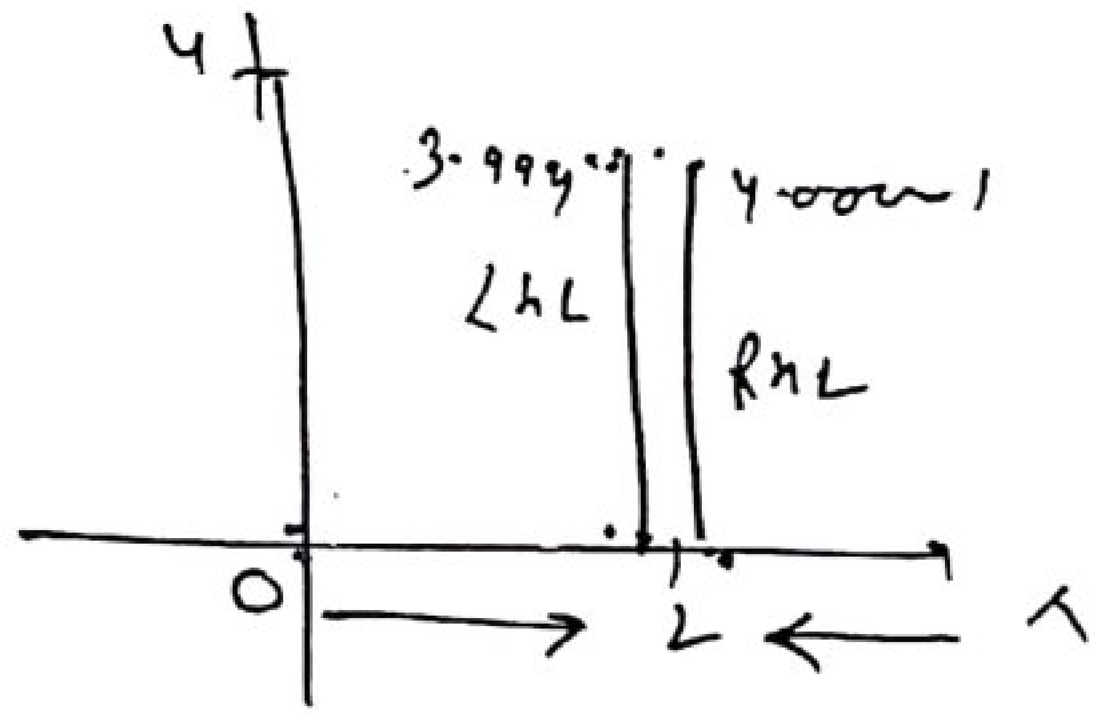
$LHL = \lim_{x \to a^-} (f(x))$ put $x = a-h$ & $h \to 0$

$RHL = \lim_{x \to a^+} (f(x))$ put $x = a+h$ & $h \to 0$

$x > a$

If $\boxed{LHL = RHL}$

then $\lim_{x \to a} (f(x))$ exists



(·) $|x| = \begin{cases} x & : x \geq 0 \\ -x & ; x < 0 \end{cases}$

(Break)

(·) $\boxed{\lim_{x \to 2} ([x]) \to \text{doesn't exist}}$

$LHL = \lim_{x \to 2^-} ([x]) = \lim_{h \to 0} ([2-h]) = 1$

put $x = 2-h$
$h \to 0$

$RHL = \lim_{x \to 2^+} ([x])$

put $x = 2+h$ & $h \to 0$

$= \lim_{h \to 0} ([2+h]) = 2$

(·) $f(x) = \begin{cases} 3x-2 & : x \le 3 \\ 1-2x^2 & : x > 3 \end{cases}$

(·) $f(x) = \begin{cases} 2x^2-1 & : x \ne 2 \\ 0 & : x = 2 \end{cases}$

(·) $f(x) = |x-2| + 5 \qquad \lim_{x \to 2} f(x)$??

$f(x) = \begin{cases} (x-2)+5 & ; x-2 \ge 0 \\ -(x-2)+5 & ; x-2 < 0 \end{cases}$

$f(x) = \begin{cases} x+3 & : x \ge 2 \\ -x+7, & x < 2 \end{cases}$

(·) Indeterminate form $\boxed{\frac{0}{0}, \ \frac{\infty}{\infty}, \ \infty - \infty}$

(·) Factorization $\lim_{x \to 2} \left(f(x) \right)$

$\to (x-2)$ is the factor of $f(x)$

(·) Rationalization : $\sqrt{\ } - \sqrt{\ }$, $\sqrt{\ } - \square$, $\square - \sqrt{\ }$

$\sqrt{\ }$ ⓧ

(·) formula $\lim_{x \to a} \left(\frac{x^n - a^n}{x - a} \right) = n a^{n-1}$

(power wali type)

(·) Trigo limits

(·) $\lim_{x\to 0}\left(\frac{\sin x}{x}\right) = 1$ & $\lim_{x\to 0}\left(\frac{x}{\sin x}\right) = 1$

(·) $\lim_{x\to 0}\left(\frac{\tan x}{x}\right) = 1$

(·) $\lim_{x\to 0}\left(\cos x\right) = 1$

(·) $\lim_{x\to 0}\left(\frac{\sin^3 x}{x^3}\right) = 1$

form $\lim_{x\to a}\left(f(x)\right)$

put $x = a+h$ & $h\to 0$ (option put $x = a-h$, $h\to 0$)

(·) $\lim_{x\to 0}\left(\frac{e^x-1}{x}\right) = 1$

(·) $\lim_{x\to 0}\left(\frac{a^x-1}{x}\right) = \log a$

(·) $\lim_{x\to 0}\left(\frac{\log(1+x)}{x}\right) = 1$

form $\lim_{x\to a}\left(f(x)\right)$

put $x = a+h$ & $h\to 0$

(·) $\lim_{x \to \infty} (f(x))$

(·) $f(x) \to$ must be in fraction

(·) divide N & D by highest power of x

(·) $\frac{1}{\infty} = 0$

eg $\lim_{x \to \infty} \left(\frac{2x^2 + 3x - 1}{3x^2 + 4x + 2} \right)$

Divide by x^2

$= \lim_{x \to \infty} \left(\frac{2 + \frac{3}{x} - \frac{1}{x^2}}{3 + \frac{4}{x} + \frac{2}{x^2}} \right)$

$= \frac{2+0+0}{3+0+0} = 2/3$

Log

(1) $\log(AB) = \log A + \log B$

$\log(ABC) = \log A + \log B + \log C$

(2) $\log\left(\frac{A}{B}\right) = \log A - \log B$

(3) $\log(m)^n = n \log m$

(4) $\log\left(\frac{A^{3/2} \cdot \sqrt{B}}{CD^2}\right) = \frac{3}{2}\log A + \frac{1}{2}\log B - \log C - 2\log D$

(5) $\boxed{\log 1 = 0 \quad \log e = 1}$

(·) Change of Base

$\log_a b = \frac{\log b}{\log a}$

(·) $\boxed{e^{\log_e x} = x}$

$a^{\log_a x} = x$

Vimp (·) If $\log x = y$ then $x = e^y$