SOLUTION: [WORKSHEET : NO 4] (CLASS NO: 5)

STRAIGHT LINES

Q.NO: 1 given lines: $3x+y=2$ --(1)

$px+2y=3$ --(2)

$2x-y=3$ ---(3)

Solving (1) & (3)

$5x=5 \Rightarrow \boxed{x=1} \Rightarrow 3+y=2 \Rightarrow \boxed{y=-1}$

Since lines are concurrent / intersect at one point

$\therefore (1,-1)$ satisfies 2nd equation

$\Rightarrow p-2y=3$

$\Rightarrow \boxed{p=5}$ Ans

Q.NO: 2 ✓ given line $\frac{x}{4}+\frac{y}{6}=1$

✓ Slope of this line $= \frac{-\frac{1}{4}}{\frac{1}{6}} = -\frac{6}{4} = -\frac{3}{2}$

✓ Since required line is $\perp^r$ to given line

✓ $\therefore$ Slope of Required line $= \frac{2}{3}$ (-ve reciprocal)

✓ Required line passes through the point where given line $\frac{x}{4}+\frac{y}{6}=1$ meets the Y-axis

✓ $\Rightarrow$ Required line passes through $(0,6)$

✓ Now equation of Required line (point-slope form)

$y-6=\frac{2}{3}(x-0) \Rightarrow 3y-18=2x \Rightarrow \boxed{2x-3y+18=0}$ Ans

Qn. 3 → Given equation of line

$$x - \sqrt{3}y + 8 = 0$$

$$\Rightarrow \quad x - \sqrt{3}y = -8$$

$$\Rightarrow \quad -x + \sqrt{3}y = 8$$

divide both sides by $\sqrt{1+3} = 2$

$$\Rightarrow \quad \frac{-1}{2}x + \frac{\sqrt{3}}{2}y = 4$$

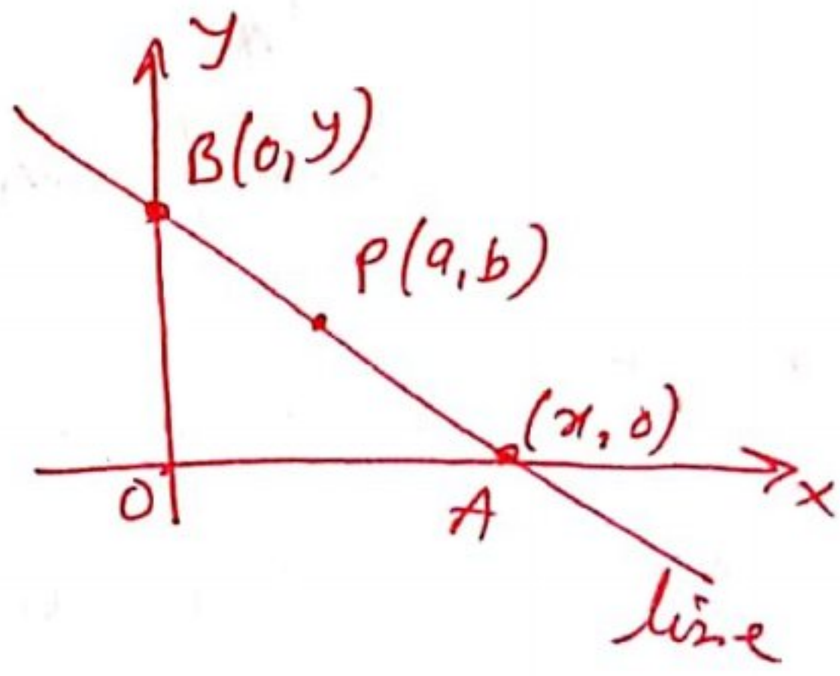
$$\Rightarrow \quad x\cos\left(\pi - \frac{\pi}{3}\right) + y\sin\left(\pi - \frac{\pi}{3}\right) = 4$$

$$\Rightarrow \quad x\cos\left(\frac{2\pi}{3}\right) + y\sin\left(\frac{2\pi}{3}\right) = 4$$

$\boxed{\alpha = \frac{2\pi}{3}}$ $\boxed{p = 4}$ **Ans**

Qn. 4 → Let $A(x, 0)$ & $B(0, y)$

$P(a, b)$ is the Mid point of AB



$$\Rightarrow \quad a = \frac{x+0}{2} \qquad b = \frac{0+y}{2}$$

$$\Rightarrow \quad x = 2a \qquad y = 2b$$

$\therefore$ X-int $= 2a$ & Y-int $2b$

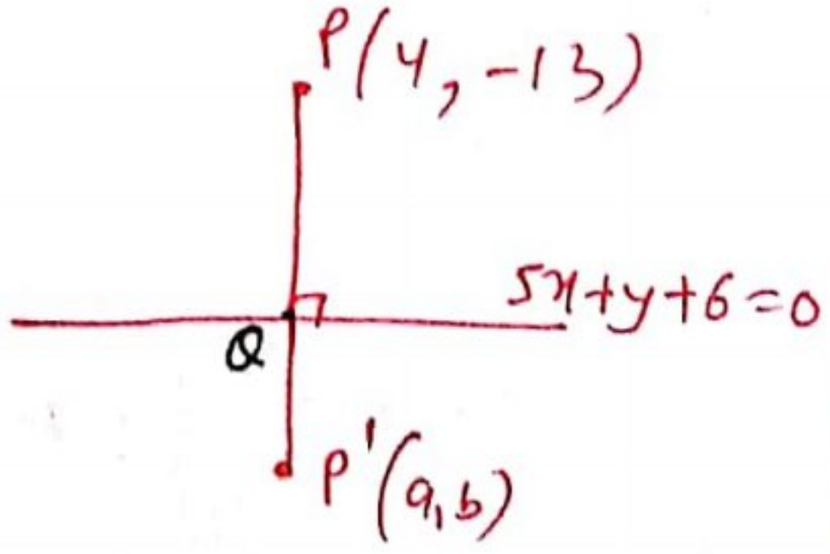
By Intercept form

$$\frac{x}{2a} + \frac{y}{2b} = 1$$

$$\Rightarrow \boxed{\frac{x}{a} + \frac{y}{b} = 2} \quad \text{Ans}$$

Qn. 5 → Let $P'(a, b)$ is the image

given line: $5x + y = -6$ ---(i)



Slope of given line = $\frac{-5}{1} = -5$

$PQ \perp$ given line

$\Rightarrow$ Slope of $PQ = \frac{1}{5}$ ... (-ve reciprocal)

Equation of PQ (point slope form)

$$y+13 = \frac{1}{5}(x-4)$$

$$\Rightarrow 5y + 65 = x - 4$$

$$\Rightarrow x - 5y = 69 \quad \text{---(2)}$$

Solving equation of given line & Equation of PQ

$$5x + y = -6$$
$$x - 25y = 345$$
$$\overline{26y = -351}$$

$$y = \frac{-351}{26} = \frac{-27}{2}$$

$\boxed{y = \frac{-27}{2}}$

put in eq(1)

$$5x - \frac{27}{2} = -6 \quad \Rightarrow 5x = -6 + \frac{27}{2}$$

$$\Rightarrow 5x = \frac{15}{2}$$

$$\Rightarrow \boxed{x = \frac{3}{2}}$$

$\therefore Q\left(\frac{3}{2}, \frac{-27}{2}\right)$

Now Q is the Mid point of P & P'

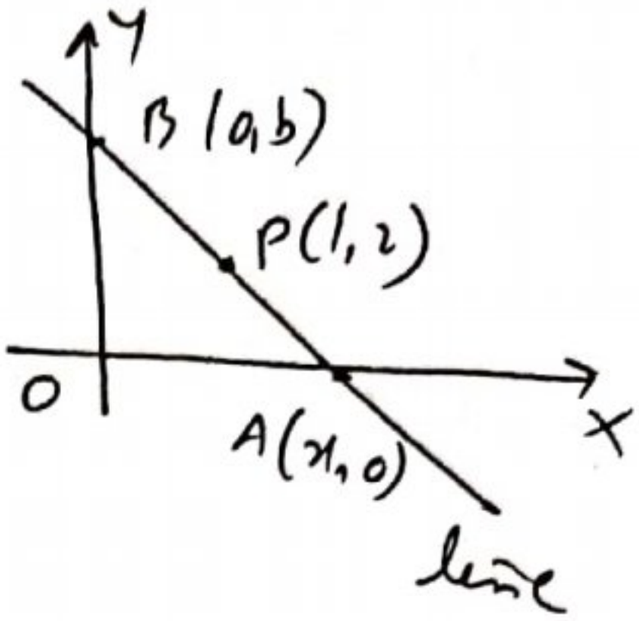
$\rightarrow \frac{3}{2} = \frac{4+a}{2}$ & $\frac{-27}{2} = \frac{-13+b}{2}$

$\rightarrow a = -1$ & $-27 = -13+b$

$b = -14$

$\therefore$ Image is $P'(-1, -14)$ Ans

Ans 6 → Let A(x, 0) & B(0, y)



Given: P(1,2) is the Mid point of A & B

$\therefore 1 = \frac{x+0}{2}$ | $2 = \frac{0+y}{2}$

$x = 2$ | & $y = 4$

∴ x-intercept = 2 & y-intercept = 4

By Intercept form $\frac{x}{2} + \frac{y}{4} = 1$

$\rightarrow \boxed{2x + y = 4}$ Ans

Ans 7 → Given $\boxed{b = 2a}$

By Intercept form

$\frac{x}{a} + \frac{y}{b} = 1$

$\rightarrow \frac{x}{a} + \frac{y}{2a} = 1$

$\rightarrow 2x + y = 2a$

Given that line passes through the point (1, 2)

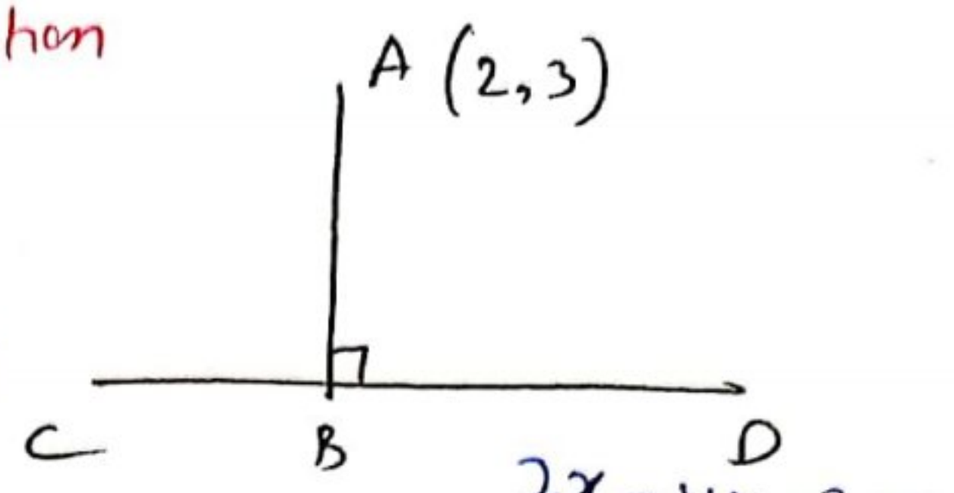
∴ $2 + 2 = 2a$

$\Rightarrow \boxed{a = 2}$ put in above Equation

∴ Equation of Req line

$\boxed{2x + y = 4}$ Ans

Qn 8 → (Note : In worksheet question equation of line is missing)



Equation of given line : $2x - 4y = 3$ ... (1)

Slope of this line = $\frac{-2}{-4} = \frac{1}{2}$

Since AB ⊥ given line

∴ Slope of AB = -2 (-ve reciprocal)

equation of AB (point slope form)

$y - 3 = -2(x-2)$

$\Rightarrow y - 3 = -2x + 4$

$\Rightarrow 2x + y = 7$ ... (2)

Solving (1) & (2) we get $y = \frac{4}{5}$ & $x = \frac{31}{10}$ ∴ foot of ⊥r is $B\left(\frac{31}{10}, \frac{4}{5}\right)$

Ans

Qns 9 → Given lines $ax + by = c$

$a'x + b'y = c'$

Slopes of these lines $m_1 = -\frac{a}{b}$ & $m_2 = -\frac{a'}{b'}$

Since lines are ⊥r

∴ $m_1 m_2 = -1$

$\Rightarrow \left(-\frac{a}{b}\right)\left(-\frac{a'}{b'}\right) = -1$

$\Rightarrow aa' = -bb'$

$\Rightarrow \boxed{aa' + bb' = 0}$ Ans

Qn 10 → Given lines $5x - 6y - 1 = 0$ & $3x + 2y = -5$

Solving these equations we get $x = -1$ & $y = -1$

~~Qno~~ Required line passes through this point $(-1,-1)$

Given 3rd line: $3x-5y+11=0$

Slope of this line $= \frac{-3}{-5} = \frac{3}{5}$

Since Required line is $\perp^r$ to this line

$\therefore$ Slope of Required line $= \frac{-5}{3}$ (-ve Reciprocal)

New equation of Required line (point - slope form)

$$y+1 = \frac{-5}{3}(x+1)$$

$$\Rightarrow 3y+3 = -5x-5$$

$$\Rightarrow \boxed{5x+3y+8=0}$$ Ans

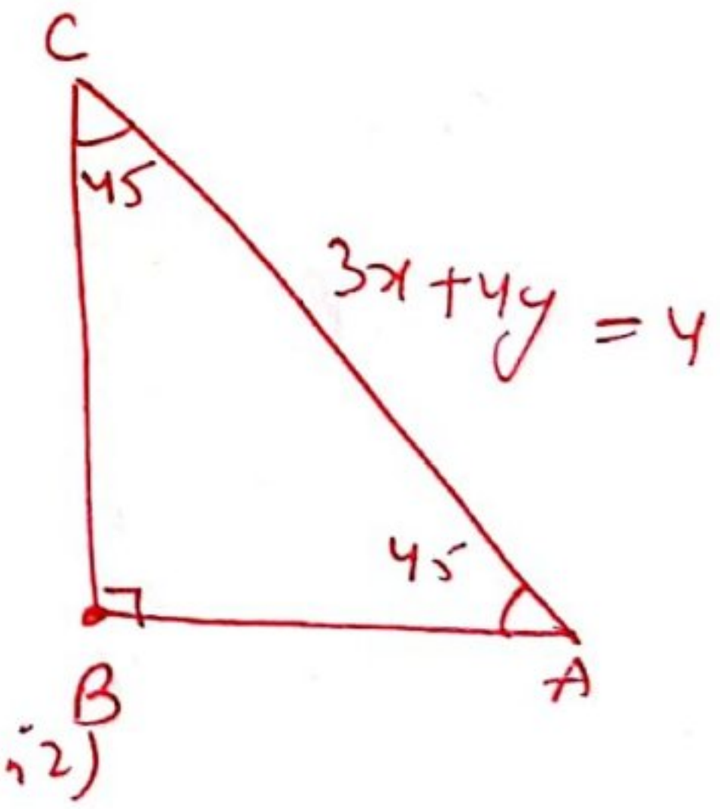
Qns = 11 →

Given: Equation of AC

$$3x+4y=4$$

Slope of AC: $m_1 = -\frac{3}{4}$

& Slope of AB: $m_2 = m$



angle b/w AB & AC = 45° --{∵ isosceles Right angle triangle}

$$\tan\theta = \left|\frac{m_1-m_2}{1+m_1m_2}\right|$$

$$1 = \left|\frac{-\frac{3}{4}-m}{1-\frac{3}{4}m}\right| \Rightarrow 1 = \left|\frac{-3-4m}{4-3m}\right|$$

$\Rightarrow \pm 1 = \frac{-3-4m}{4-3m}$

$\Rightarrow 1 = \frac{-3-4m}{4-3m}$ | $-1 = \frac{-3-4m}{4-3m}$

$\Rightarrow 4-3m = -3-4m$ | $-4+3m = -3-4m$

$\rightarrow \boxed{m = -7}$ | $7m = 1$

$\boxed{m = 1/7}$

Slope of $\overline{AB}$ (point slope form)

$y-2 = -7(x-2)$ | $y-2 = \frac{1}{7}(x-2)$

$\Rightarrow y-2 = -7x+14$ | $7y-14 = x-2$

$\rightarrow \boxed{7x+y=16}$ | $\Rightarrow \boxed{x-7y+12=0}$

Ans

Qn. 12 →

Let $A(x,0)$ & $B(0,y)$

$P(-5,4)$ divides AB in the ratio $1:2$

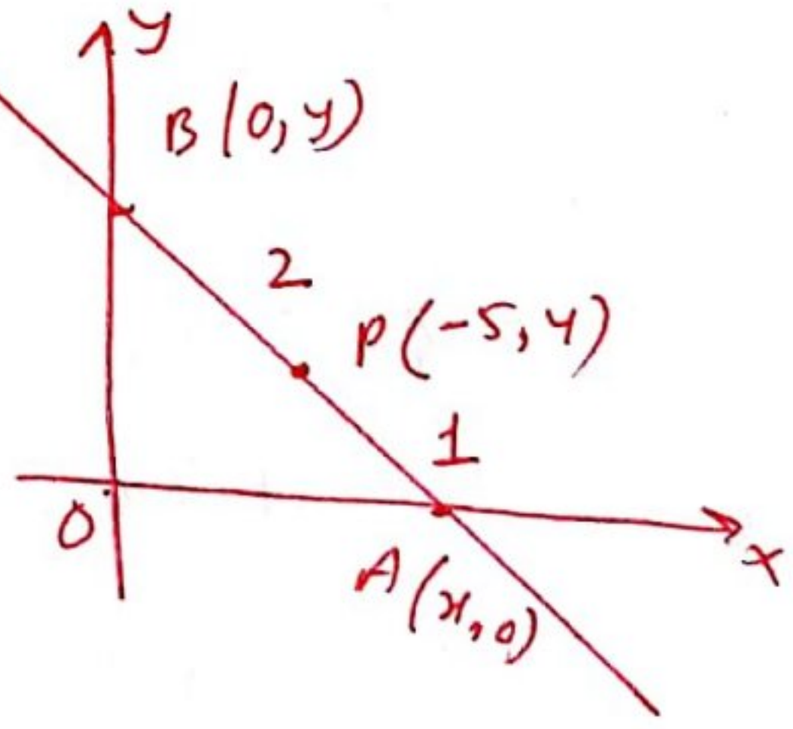
By Section formula

$-5 = \frac{0+2x}{2+1}$ | $4 = \frac{y+0}{2+1}$

$\Rightarrow x = \frac{-15}{2}$ | $y = 12$

$\therefore$ x-intercept $= \frac{-15}{2}$ & y-int $= 12$

By Intercept form



$$\frac{x}{-\frac{15}{2}} + \frac{y}{12} = 1 \quad \text{--- } \{ \frac{x}{a} + \frac{y}{b} = 1 \} \qquad (8)$$

$$\Rightarrow \frac{-2x}{15} + \frac{y}{12} = 1$$

L.C.M

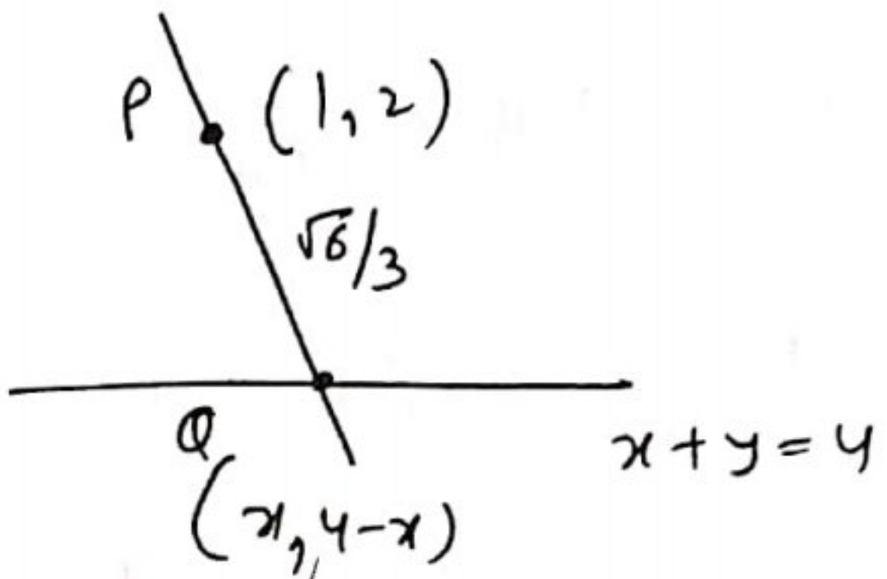
$$\Rightarrow -8x + 5y = 60$$

$$\Rightarrow \boxed{8x - 5y + 60 = 0} \quad \text{Ans}$$

Q.No. 13 →

Given $P(1,2)$

Let $Q(x, 4-x)$ --- $\{\because x+y=4 \; \therefore y = 4-x\}$



Given $PQ = \frac{\sqrt{6}}{3}$

$$\rightarrow \sqrt{(x-1)^2 + (2-x)^2} = \frac{\sqrt{6}}{3}$$

Squaring $x^2 - 2x + 1 + x^2 - 4x + 4 = \frac{6}{9} \; \frac{2}{3}$

$$\Rightarrow 2x^2 - 6x + 5 = \frac{2}{3}$$

$$\Rightarrow 6x^2 - 18x + 15 = 2$$

$$\Rightarrow 6x^2 - 18x + 13 = 0$$

$$\Rightarrow x = \frac{18 \pm \sqrt{324 - 312}}{12}$$

$$x = \frac{18 \pm \sqrt{12}}{12}$$

$$x = \frac{18 \pm 2\sqrt{3}}{12}$$

$$x = \frac{9 \pm \sqrt{3}}{6}$$

$$x = \frac{9+\sqrt{3}}{6} \quad (\text{or}) \quad x = \frac{9-\sqrt{3}}{6}$$

$$4 - x = 4 - \left(\frac{9+\sqrt{3}}{6}\right) = \frac{15-\sqrt{3}}{6}$$

& $4 - x = 4 - \left(\frac{9-\sqrt{3}}{6}\right) = \frac{15+\sqrt{3}}{6}$

$\therefore Q\left(\frac{9+\sqrt{3}}{6}, \frac{15-\sqrt{3}}{6}\right)$ or $Q\left(\frac{9-\sqrt{3}}{6}, \frac{15+\sqrt{3}}{6}\right)$

New equation of PQ

$$y-2 = \left(\frac{\frac{15-\sqrt{3}}{6}-2}{\frac{9+\sqrt{3}}{6}-1}\right)(x-1)$$

$$\Rightarrow y-2 = \frac{3-\sqrt{3}}{3+\sqrt{3}}(x-1)$$

$$\Rightarrow y-2 = \frac{(3-\sqrt{3})^2}{9-3}(x-1)$$

$$\Rightarrow y-2 = \frac{9+3-6\sqrt{3}}{6}(x-1)$$

$$y-2 = (2-\sqrt{3})(x-1)$$

then $m = 2-\sqrt{3}$ & $m = \tan\theta$

We know that $\tan(15^\circ) = 2-\sqrt{3}$

$\therefore \theta = 15^\circ$

another equation of PQ

$$y-2 = \frac{\left(\frac{15+\sqrt{3}}{6}-2\right)}{\left(\frac{9-\sqrt{3}}{6}-1\right)}(x-1)$$

$$y-2 = \left(\frac{3+\sqrt{3}}{3-\sqrt{3}}\right)(x-1)$$

$$y-2 = \left(\frac{9+3+6\sqrt{3}}{9-3}\right)(x-1)$$

$y - 2 = (2+\sqrt{3})(x-1)$

then $m = 2+\sqrt{3} = \tan\theta$

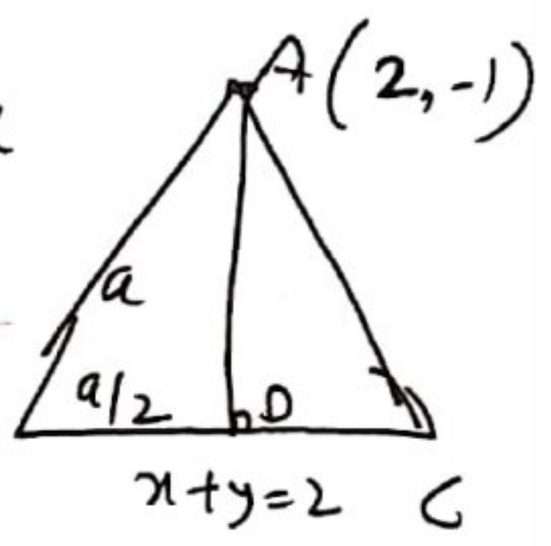
we know that $\tan(75^\circ) = 2+\sqrt{3}$

$\therefore \theta = 75^\circ$

$\therefore$ direction in which the line must be drawn either makes 15° or 75° with the +ve direction X-axis Ans

Q~14→ ΔABC is an equilateral triangle

$\therefore AD \perp BC$ & D is Mid point of BC



AD = $\perp^r$ distance b/w point A & line BC

$\Rightarrow AD = \dfrac{|x-1-x|}{\sqrt{1+1}}$ ---{ $d = \dfrac{|ax_1+by_1+c|}{\sqrt{a^2+b^2}}$ }

$AD = \dfrac{1}{\sqrt{2}} = \dfrac{1}{\sqrt{2}}$

let side of $\Delta ABC = a$

In ΔABD

$AB^2 = AD^2 + BD^2$

$a^2 = \frac{1}{2} + \frac{a^2}{4}$

$\Rightarrow a^2 - \frac{a^2}{4} = \frac{1}{2}$

$\Rightarrow \frac{3a^2}{4} = \frac{1}{2} \Rightarrow a^2 = \frac{2}{3} \Rightarrow \boxed{a = \sqrt{\frac{2}{3}}}$ Ans