Topic : ..... Date. : ..... Page No. : .....

← ULTIMATE MATHEMATICS →
BY: AJAY MITTAL: 9891067390

CLASS NO: 5 (5-5)

Q No 1 Proof of Demorgan's law

Solution (i) $(A \cup B)' = A' \cap B'$

Let $x \in (A \cup B)'$

$\Rightarrow x \notin (A \cup B)$ simp

$\Rightarrow x \notin A$ and $x \notin B$

$\Rightarrow x \in A'$ and $x \in B'$

$\Rightarrow x \in A' \cap B'$

$\Rightarrow (A \cup B)' \subset A' \cap B'$ ---(1)

Let $y \in A' \cap B'$

$\Rightarrow y \in A'$ and $y \in B'$

$\Rightarrow y \notin A$ and $y \notin B$

$\Rightarrow y \notin (A \cup B)$

$\Rightarrow y \in (A \cup B)'$

$\Rightarrow A' \cap B' \subset (A \cup B)'$ -(2)

from (1) & (2)

$(A \cup B)' = A' \cap B'$ Proved

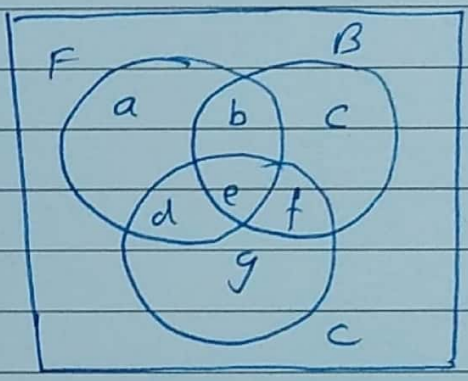
(ii) $(A \cap B)' = A' \cup B'$

Do Yourself

Q.No 2 → A college awarded 38 medals in football 15 in basketball and 20 to Cricket If these medals went to a total of 58 men and only three men got medals in all the three sports, how many recieved medals in exactly two of the three sports?

Solution Let F → No of persons receiving medals in Football
B → " " " " " Basketball
C → " " " " " Cricket

Given $n(F) = 38$
$n(B) = 15$
$n(C) = 20$
$n(F \cup B \cup C) = 58$
$n(F \cap B \cap C) = 3 = e$



We know that

$n(F \cup B \cup C) = n(F) + n(B) + n(C) - n(F \cap B) - n(B \cap C) - n(F \cap C) + n(F \cap B \cap C)$

$\Rightarrow 58 = 38 + 15 + 20 - n(F \cap B) - n(B \cap C) - n(F \cap C) + 3$

$\Rightarrow n(F \cap B) + n(B \cap C) + n(F \cap C) = 76 - 58$

$\Rightarrow b + e + e + f + d + e = 18$

$\Rightarrow (b + d + f) + 3e = 18$

$\Rightarrow b + d + f + 3 \times 3 = 18$ --- $\{\because n(F \cap B \cap C) = e = 3\}$

$\Rightarrow b + d + f = 18 - 9 = 9$

$\therefore$ 9 persons received medals exactly in two sports.

Ans

Que 3 → Suppose $A_1, A_2, A_3, \ldots A_{30}$ are thirty sets each with five elements and $B_1, B_2, \ldots B_n$ are n sets each with three elements.

Let $\bigcup_{i=1}^{30} A_i = S$ also $\bigcup_{j=1}^{n} B_j = S$

Assume that each element of S belongs to exactly ten of the A_i's and exactly 9 of B_j's. Find value of n.

Soln: $A_1 = \{ - - - - - \}$

$A_2 = \{ - - - - - \}$

⋮

$A_{30} = \{ - - - - - \}$

If all elements of all A sets are different

then $n(A_1 \cup A_2 \cup A_3 \cup \ldots A_{30}) = 150$

Given $S = \bigcup_{i=1}^{30} A_i$

$S = A_1 \cup A_2 \cup A_3 \cup A_4 \ldots A_{30}$

$\Rightarrow n(S) = 150$

But each element of S belongs to exactly ten element of A_i's

$\Rightarrow n(S) = \frac{150}{10} = 15 \quad \ldots (i)$

Similarly $B_1 = \{ - - - \}$

$B_2 = \{ - - - \}$

$\vdots$

$B_n = \{ - - - \}$

Given $S = \bigcup_{j=1}^{n} B_j$

$S = B_1 \cup B_2 \cup B_3 \cdots B_n$

$\Rightarrow n(S) = 3n$

But each element of S belongs to exactly 9 element of B_j's

$\Rightarrow n(S) = \frac{3n}{9} = \frac{n}{3}$ --(2)

$\Rightarrow 15 = \frac{n}{3}$

$\Rightarrow \boxed{n = 45}$ Ans

Qn. 4 → A survey shows that 63% of the Americans like cheese whereas 76% like apples. If x% of the Americans like both cheese and Apples. Find value of x

Solution Let the total number of Americans = 100

$\therefore n(U) = 100$

Given $n(A) = 63\%$ of $100 = 63$

$n(B) = 76\%$ of $100 = 76$

and $n(A \cap B) = x\%$ of $1000 = x$

IMP points

- $n(A \cup B) \leq n(U)$
- $n(A \cap B) \leq n(A)$
- $n(A \cap B) \leq n(B)$

(·) $n(A \cup B) \leq n(U)$

$\Rightarrow n(A \cup B) \leq 100$

$\Rightarrow n(A) + n(B) - n(A \cap B) \leq 100$

$\Rightarrow 63 + 76 - x \leq 100$

$\Rightarrow 139 - x \leq 100$

$\Rightarrow 139 - 100 \leq x$

$\Rightarrow 39 \leq x$

(or) $x \geq 39$ ---(1)

(·) $n(A \cap B) \leq n(A)$ (·) $n(A \cap B) \leq n(B)$

$\Rightarrow x \leq 63$ and $x \leq 76$

consider $x \leq 63$ --- (2)

from (1) & (2)

$39 \leq x \leq 63$ Ans

Qns 5 (SELF) → A survey shows that 76% of the Indians like oranges whereas 62% like Bananas.

what percentage of the indians like both oranges and bananas?

Ans 38% to 62%

INTERVALS (Inequalities)

(1) Open Interval : $<$, $>$, ( ), value not Included

(2) Closed Interval : $\leq$, $\geq$, [ ], value Included

e.g. $3 \leq x \leq 5 \Rightarrow x \in [3, 5]$

$3 < x < 5 \Rightarrow x \in (3, 5)$

$3 \leq x < 5 \Rightarrow x \in [3, 5)$

$3 < x \leq 5 \Rightarrow x \in (3, 5]$

$x \geq 3 \Rightarrow x \in [3, \infty)$

$x \leq 3 \Rightarrow x \in (-\infty, 3]$

$x > -2 \Rightarrow x \in (-2, \infty)$

Remember: x is a Real number

Q Write the following sets as intervals

(1) $A = \{x : x \in R \quad -4 < x \leq 6\}$

(2) $B = \{x : x \in R \quad -12 < x < -10\}$

Ans (i) $x \in (-4, 6]$ (ii) $x \in (-12, -10)$

Q Write in Set Builder form (i) $(-7, 0)$ (2) $[6, 12]$

(i) $A = \{x : x \in R, \; -7 < x < 0\}$

(2) $A = \{x : x \in R ; \; 6 < x \leq 12\}$ Ans