ULTIMATE MATHEMATICS: BY AJAY MITTAL

$\underline{\text{XI}}$ JEE MAINS :

$\boxed{\text{CLASS: NO: 3}}$

← INEAQUALITIES →

(·) $|x| < a \Rightarrow -a < x < a$

(·) $|x| \le a \Rightarrow -a \le x \le a$

(·) $|x| > a \Rightarrow x < -a$ (or) $x > a$

(·) $|x| \ge a \Rightarrow x \le -a$ (or) $x \ge a$

(·) $|x-a| \le r \Rightarrow a-r \le x \le a+r$

(·) $|x-a| \ge r \Rightarrow x \le a-r$ (or) $x \ge a+r$

(·) $a \le |x| \le b \Rightarrow x \in [-b, -a]$ (or) $x \in [a, b]$

(·) $a < |x-c| < b \Rightarrow x \in (-b+c, -a+c)$ (U) $x \in (a+c, b+c)$

(·) $ax^2+bx+c > 0$ for all $x \in R$

$a > 0$ and $b^2-4ac < 0$

(·) $ax^2+bx+c < 0$ for all $x \in R$

$a < 0$ and $b^2-4ac < 0$

(·) $|x-1|$

$\xleftarrow{\ -\ } 1 \xrightarrow{\ +\ }$

when $x < 1$

$|x-1| = -(x-1)$

when $x \ge 1$

$|x-1| = +(x-1)$

(·) $\frac{x-1}{x-2} < 1 \Rightarrow x-1 \not< x-2$ ✗

(·) $\frac{x-1}{|x-2|} < 1 \Rightarrow x-1 < |x-2|$ ✓

(·) $|a-b| = |b-a|$

(·) $|-a-b| = |a+b|$

(·) $|x|^2 = x^2 = |x^2|$

(·) $x^2 \leq a^2 \Rightarrow -a \leq x \leq a$

(·) $x^2 \geq a^2 \Rightarrow x \leq -a$ (or) $x \geq a$

(·) $-2 \leq x \leq 5 \Rightarrow 0 \leq x^2 \leq 25$

(·) $-5 \leq x \leq 2 \Rightarrow 0 \leq x^2 \leq 25$

(·) $-5 \leq x \leq -2 \Rightarrow 4 \leq x^2 \leq 25$

(·) $2 \leq x \leq 5 \Rightarrow 4 \leq x^2 \leq 25$

(·) $-2 \leq x \leq 5 \Rightarrow 0 \leq |x| \leq 5$

(·) If $a > b$

then $a+c > b+c$

also $a-c > b-c$

(·) If $a > b$

then $ac > bc$ ... $\{c > 0\}$ $\rightarrow \frac{a}{c} > \frac{b}{c}$

also $ac < bc$ ... $\{c < 0\}$ $\rightarrow \frac{a}{c} < \frac{b}{c}$

(·) If $a > b$ and $c > d$

then $a + c > b + d$

also $ac > bd$ --- { a, b, c, d +ve real numbers

(·) If a is +ve Real No.

then $\boxed{a + \frac{1}{a} \geq 2}$

If a is −ve Real No.

$\boxed{a + \frac{1}{a} \leq -2}$

(·) $\boxed{A.M \geq G.M}$ a & b are two +ve numbers

$\frac{a+b}{2} \geq \sqrt{ab}$

to find Minimum value of some function.

(·) $[x]$

$[x] > 5$

$[x] = 6, 7, 8, 9 --- \infty$

$x \geq 6$

$[x] = 2$

$x \in [2, 3)$

$[x] = -2$

$x \in [-2, -1)$

$[x] = -5$

$x \in [-5, -4)$

(·) $\log_a x$ $a \neq 1$ $a < 0$

$0 < a < 1$ or $a > 1$

(·) If $a > 1$

and $x > y$ then $\log_a x > \log_a y$

and $x > y$ then $a^x > a^y$

(·) If $0 < a < 1$

$x > y$ then

$\log_a x < \log_a y$

$x > y$ then

$a^x < a^y$

Que 1 Solution set of inequations

$$\frac{3}{|x|+2} \geq 1 \text{ is}$$

(a) $[-1, 1]$ (b) $(-1, 1)$ (c) $(-\infty, 1]$ (d) $[1, \infty)$

Sol $3 \geq |x|+2$

$\Rightarrow 1 \geq |x|$

$\Rightarrow |x| \leq 1 \Rightarrow -1 \leq x \leq 1$

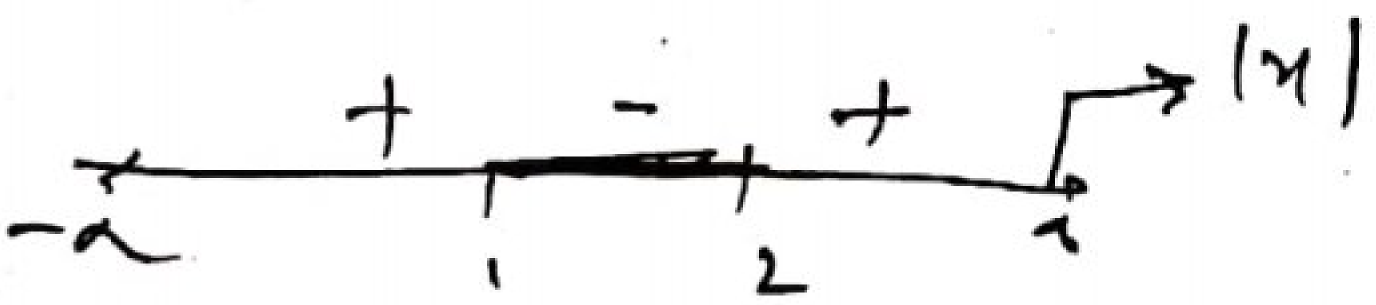
$\Rightarrow x \in [-1, 1]$ (a) Ans

Que 2 → If $\frac{|x|-1}{|x|-2} \leq 0$, then x lies in the interval

(a) $[-1, 2]$ (b) $(-2, 2)$ (c) $(-2, -1] \cup [1, 2)$ (d) $[-1, 1]$

Sol $\frac{|x|-1}{|x|-2} \leq 0$ $\quad |x|-2 \neq 0$

$|x| \neq 2$

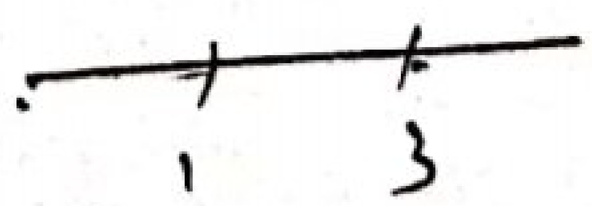


$1 \leq |x| < 2$

$\Rightarrow (-2, -1] \cup [1, 2)$ Ans (c)

Que 3 → The solution set of inequation $|x-1| \geq |x-3|$ is

(a) $(-\infty, 2]$ (b) $[2, \infty)$ (c) $[1, 3)$ / none of these



Sol case I $x < 1$

$-(x-1) \geq -(x-3)$

$\Rightarrow -x+1 \geq -x+3$

$\Rightarrow -2 \geq 0$ (absurd)

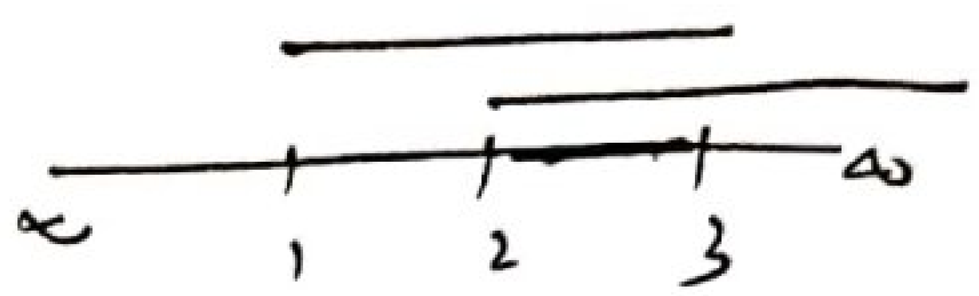
$x \in \phi$ ...(1)

<u>Case II</u> $1 \leq x < 3$

$(x-1) \geq -x+3$

$\Rightarrow 2x \geq 4$

$\Rightarrow x \geq 2$



$x \in [2, 3)$ ...(2)

<u>Case III</u> $x \geq 3$

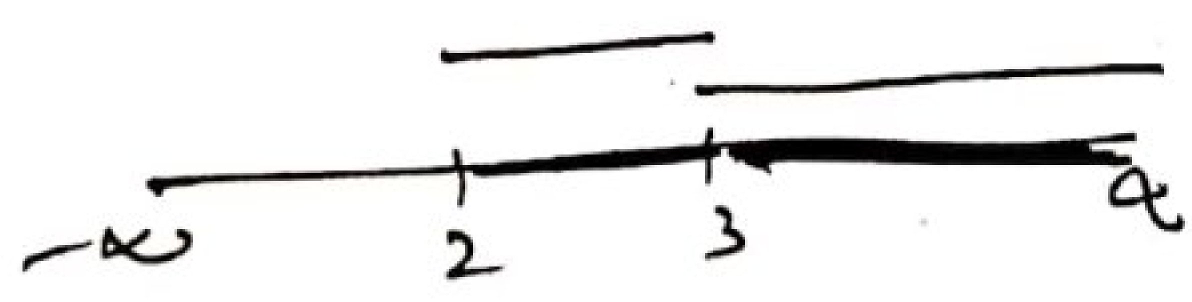
$(x-1) \geq x-3$

$\Rightarrow -1 \geq -3$

$\Rightarrow 2 \geq 0$ → True

$x \geq 3$ ...(3)

<u>Union</u> I, (2), (3)



$\therefore x \in [2, \infty)$ (b) Ans

<u>Query</u> → The solution set of the inequation $\left||x|-1\right| < |1-x|$ is

(a) $(-1, 1)$ (b) $(0, \infty)$ (c) $(-1, \infty)$ (d) none of these

Sol^n

$$||x|-1| < |1-x|$$

(·) when $x \geq 0$

$$|x| = x$$

$$\therefore \; |x-1| < |1-x|$$

$2 < 2$ ✗

$2 \leq 2$ ✓

$$\Rightarrow |x-1| < |x-1|$$

$$\therefore \; x \in \phi$$

(·) when $\boxed{x < 0}$

$$|x| = -x$$

$$\therefore \; |-x-1| < |1-x|$$

$$\Rightarrow |x+1| < |x-1|$$

$(-1) \quad (1)$

(I) when $x < -1$

$$-(x+1) < -(x-1)$$

$$-x-1 < -x+1$$

$-2 < 0$ (which is always true)

$$\therefore \; x \in (-\infty, -1) \quad \text{--- (1)}$$

(II) $-1 \leq x < 0$

$$(x+1) < -x+1$$

$$2x < 0$$

$$x < 0$$

$\therefore \; x \in [-1, 0)$ --- (2)

(I) ∪ (II) $\quad x \in (-\infty, 0)$ Ans

Qn 5 The least integral value of k for which $(k-2)x^2 + 8x + k+4 \geq 0$ is

(a) 5 (b) 4 (c) 3 (d) none of them

Sol

$$(k-2)x^2 + 8x + (k+4) \geq 0$$

$$k-2 > 0 \quad \text{and} \quad 64 - 4(k-2)(k+4) \leq 0$$

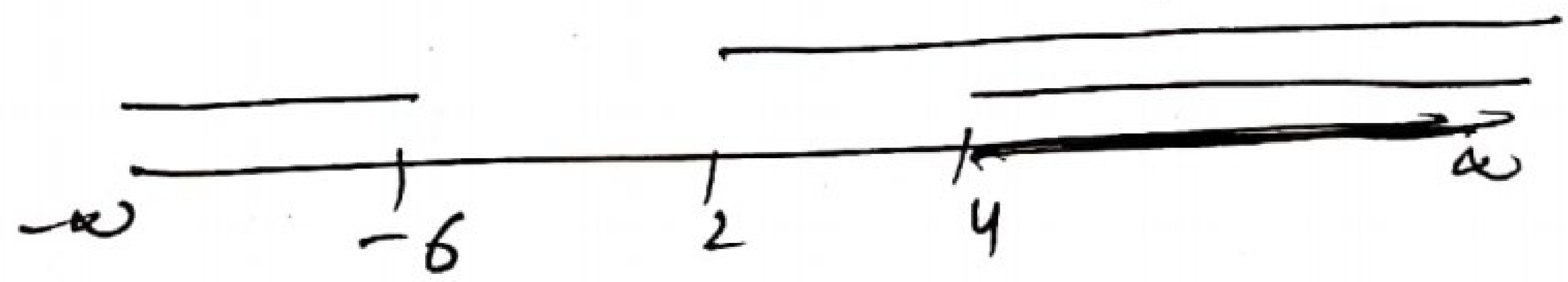
$$k > 2 \quad \text{and} \quad 64 - 4k^2 - 8k + 32 \leq 0$$

$$-4k^2 - 8k + 96 \leq 0$$

$$k^2 + 2k - 24 \geq 0$$

$$(k+6)(k-4) \geq 0$$

$$k > 2 \quad \text{and} \quad x \in (-\infty, -6] \cup [4, \infty)$$



$$\therefore k \in [4, \infty)$$

$\therefore$ least integral value of $k = 4$

$\therefore$ (b) Ans

Qn 6 → The minimum value of $9^x + 9^{1-x}$ is

(a) 2 (b) 3 (c) 4 (d) none of these

Sol

$AM \geq GM$

$$\frac{a+b}{2} \geq \sqrt{ab}$$

let, $a = 9^x$

$b = 9^{1-x}$

$$\Rightarrow \frac{9^x + 9^{1-x}}{2} \geq \sqrt{9^x \cdot 9^{1-x}}$$

$$\Rightarrow \frac{9^x + 9^{1-x}}{2} \geq \sqrt{9}$$

$$\Rightarrow 9^x + 9^{1-x} \geq 6$$

$\therefore$ Min value = 6 (d) Ans

Qn 7 → If a, b, c are distinct +ve Real numbers then

(a) $a^2+b^2+c^2 > ab+bc+ca$ (b) $a^2+b^2+c^2 < ab+bc+ca$

(c) $a^2+b^2+c^2 \geq ab+bc+ca$ (d) $a^2+b^2+c^2 = ab+bc+ca$

Sol

✓ AM ≥ GM

✓ $\boxed{AM > GM}$

a, b

$(a=b)$ $\frac{a+b}{2} = \frac{2a}{2} = a$

$\sqrt{ab} = \sqrt{a^2} = a$

✓ $\frac{a^2+b^2}{2} > \sqrt{a^2b^2}$

$a^2+b^2 > 2ab$ --(1)

✓ $\frac{b^2+c^2}{2} > \sqrt{b^2c^2}$

$\Rightarrow b^2+c^2 > 2bc$ --(2)

✓ $c^2+a^2 > 2ac$ --(3)

add (1), (2), (3)

$2(a^2+b^2+c^2) > 2(ab+bc+ca)$

$\Rightarrow a^2+b^2+c^2 > ab+bc+ca$ (a) Ans

Q. 8 Solution set of Inequality

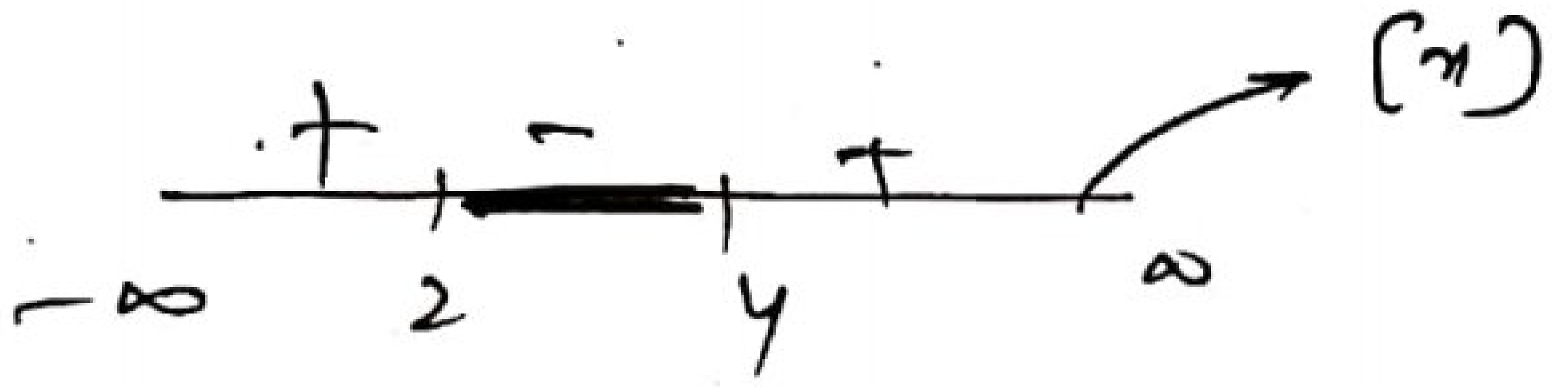
$$\frac{[x]-2}{4-[x]} > 0 \quad \text{is}$$

(a) $(2,3]$ (b) $[3,4)$ (c) $[2,3]$ (d) $[3,4]$

Sol.

$$\frac{[x]-2}{4-[x]} > 0$$

$$\frac{[x]-2}{[x]-4} < 0$$



$2 < [x] < 4$.

$\Rightarrow$ $[x] = 3$.

$\Rightarrow$ $x \in [3,4)$ (b) Ans

Q. 9 → The equation $||x-1|+a| = 4$ can have real solutions for x if 'a' belongs to the Interval

(a) $(-\infty,4]$ (b) $(-\infty,-4]$ (c) $(4,\infty)$ (d) $[-4,4]$

Sol. Basic: if $\boxed{|x| = a \Rightarrow x = \pm a}$

(×) $|x| = -2$ ⇒ $|x| = a$ ⇒ $a \geq 0$

$(a = -2)$ (×)

Sol. $||x-1|+a| = 4$

$\Rightarrow |x-1| + a = \pm 4$

$\Rightarrow \quad |x-1| + a = \pm 4$

$\Rightarrow \quad |x-1| = \pm 4 - a$

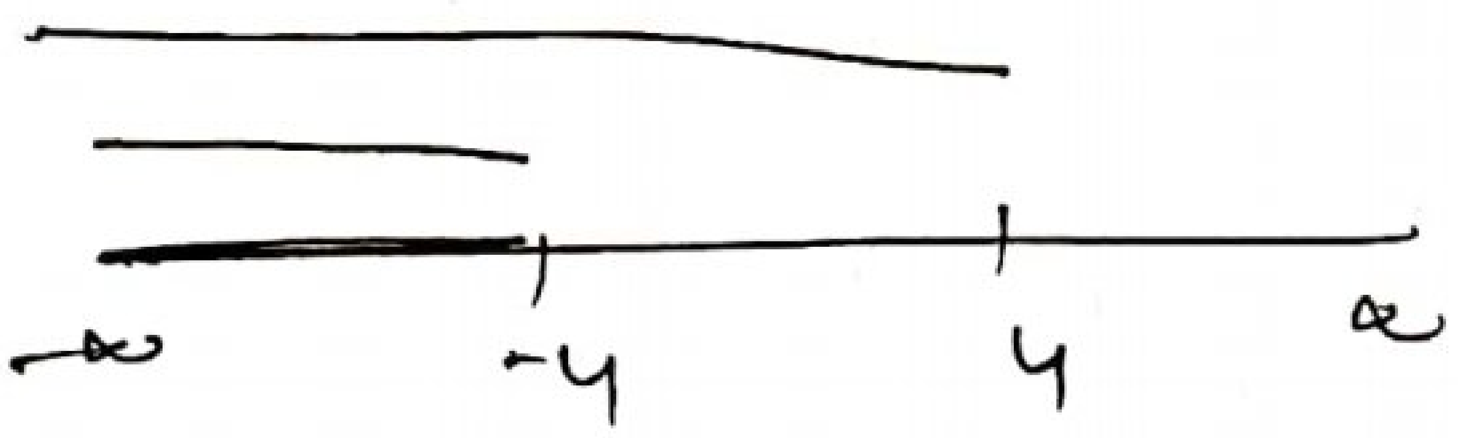
for real soln.

$$\pm 4 - a \geq 0$$

$4 - a \geq 0 \quad$ and $\quad -4 - a \geq 0$

$-a \geq -4 \quad$ and $\quad -a \geq 4$

$a \leq 4 \quad$ and $\quad a \leq -4$



$\therefore a \in (-\infty, -4]$ (5) Ans