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ULTIMATE MATHEMATICS

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RELATION & FUNCTION

CLASS NO: 5:

Qu 1 → Let R be a relation from N to N defined by $R = \{(a,b) : a, b \in N \text{ and } a = b^2\}$ Are the following true?

(a) $(a,b) \in R$, implies $(b,a) \in R$

(b) $(a,b) \in R$, $(b,c) \in R$ implies $(a,c) \in R$

(c) $(a,a) \in R$ for all $a \in N$

Sol: (a) given $(a,b) \in R$

False

$(4,2) \in R$

since $4 = 2^2$

but $(2,4) \notin R$

since $2 \neq 4^2$

(b) $(16,4) \in R$ & $(4,2) \in R$

since $16 = 4^2$ & $4 = 2^2$

but $(16,2) \notin R$

since $16 \neq 2^2$

$\therefore$ false

(3) $3 \in N$

but $(3,3) \notin R$

since $3 \neq 3^2$

false

CLASSTIME

RQF Class No: 5

Qn: 2 → Let R be a relation from set Q to set Q defined by $R = \{(a,b): a, b \in Q$ and $a-b$ is an Integer. Show that

(a) $(a,b) \in R$ implies that $(b,a) \in R$

(b) $(a,b) \in R$ & $(b,c) \in R$ implies that $(a,c) \in R$

(c) $(a,a) \in R$ for all $a \in Q$

Sol → (1) given $(a,b) \in R$

$\Rightarrow$ $a-b$ is an integer

$\Rightarrow$ $a-b = \lambda \quad \text{---} \quad (\lambda \in Z)$

$\Rightarrow$ $b-a = -\lambda$ which is also an integer

$\Rightarrow$ $(b,a) \in R$

(2) given $(a,b) \in R$ & $(b,c) \in R$

$\Rightarrow$ $a-b = \lambda$ & $b-c = k \quad \text{---} \quad (\lambda, k \in Z)$

Now $a-c = (a-b) + (b-c)$

$\Rightarrow$ $a-c = \lambda + k$ which is also an integer

$\Rightarrow$ $(a,c) \in R$

(3) for all $a \in Q$

$a-a = 0$ which is an integer

$\Rightarrow$ $(a,a) \in R$

Qn 3 → The function f is defined by

$$f(x) = \begin{cases} 1-x & : x < 0 \\ 1 & : x = 0 \\ x+1 & : x > 0 \end{cases}$$

Draw the graph of $f(x)$ & also find Domain & Range of $f(x)$

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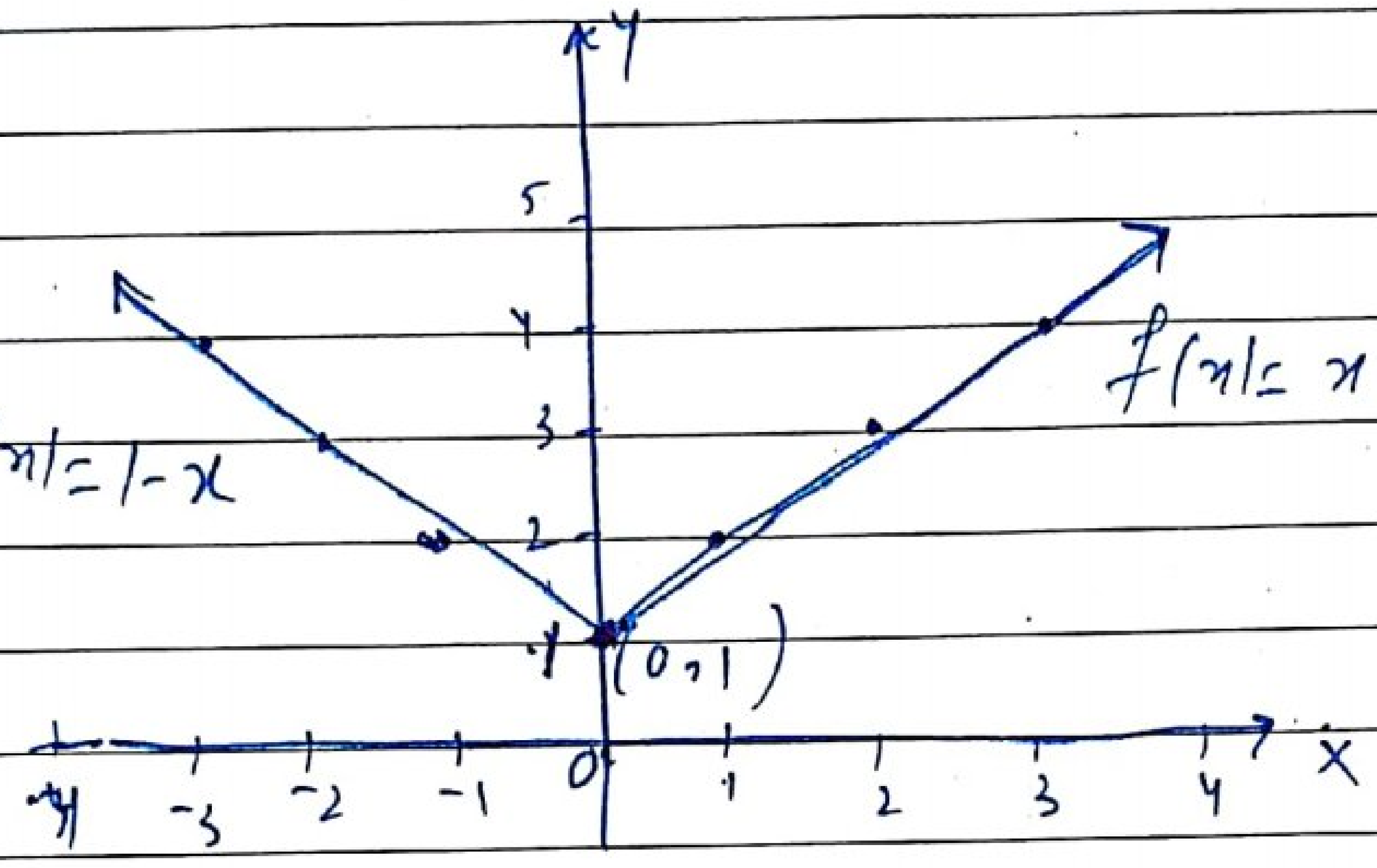
Ref Class No = 5

Sol $f(x) = \begin{cases} 1-x & : x<0 \\ 1 & : x=0 \\ x+1 & : x>0 \end{cases}$

pairs $(0,1)$, $(1,2)$ $(2,3)$ $(3,4)$ $(-1,2)$ $(-2,3)$ $(-3,4)$

graph



Domain : R

Range : $[1, \infty)$

Q.4 Let $f = \{(2,4), (5,6), (8,-1), (10,-3)\}$

$g = \{(2,5), (7,1), (8,4) (10,13), (11,5)\}$

(1) find $f-g$ (2) $f+g$, (3) $f \cdot g$ (4) f/g

Solution $D_f = \{2, 5, 8, 10\}$; $D_g = \{2, 7, 8, 10, 11\}$

$D_f \cap D_g = \{2, 8, 10\}$

(1) $f-g = \{(2,-1), (8,-5), (10,-16)\}$

(2) $f+g = \{(2,9), (8,3) (10, 10)\}$

(3) $f \cdot g = \{(2, 20), (8,-4) (10, -39)\}$

(4) $\frac{f}{g} = \{(2, 4/5), (8, -1/4), (10, -3/13)\}$ Ans

CLASSTIME

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Qn 5 If $[x]^2 - 5[x] + 6 = 0$, then find x

Sol $[x]^2 - 3[x] - 2[x] + 6 = 0$

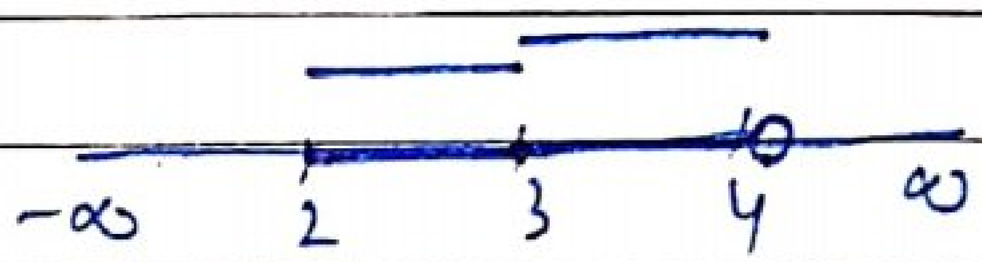
$\Rightarrow [x]\left([x]-3\right) - 2\left([x]-3\right) = 0$

$\Rightarrow \left([x]-2\right)\left([x]-3\right) = 0$

$\Rightarrow [x] = 2$ (or) $[x] = 3$

$x \in [2, 3)$ | $x \in [3, 4)$

Taking union $[2, 4)$



Qn 6 (1) $f(x) = 1 + 3\cos(2x)$

(2) $f(x) = 1 - |x-2|$

Find the Range & Domain

Sol (1) $f(x) = 1 + 3\cos(2x)$

Domain $x \in R$

Range: We know that

$-1 \le \cos(2x) \le 1$

$\Rightarrow -3 \le 3\cos(2x) \le 3$

$\Rightarrow -3+1 \le 1 + 3\cos(2x) \le 3+1$

$\Rightarrow -2 \le f(x) \le 4$

$\therefore$ Range $[-2, 4]$ Ans

(2) $f(x) = 1 - |x-2|$

Domain R

Range $= (-\infty, 1]$ Ans

$\cancel{(-\infty, 0]}$

CLASSTIME

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Q1.7 If $f(x) = x^3 - \frac{1}{x^3}$, then find $f(x) + f\left(\frac{1}{x}\right)$

Sol $f(x) = x^3 - \frac{1}{x^3}$

$f\left(\frac{1}{x}\right) = \frac{1}{x^3} - \frac{1}{\frac{1}{x^3}} = \frac{1}{x^3} - x^3$

Now $f(x) + f\left(\frac{1}{x}\right) = \cancel{x^3} - \cancel{\frac{1}{x^3}} + \cancel{\frac{1}{x^3}} - \cancel{x^3} = 0$ Ans

Q1.8 $f(x) = \frac{x-1}{x+1}$ show that $f(f(x)) = \frac{-1}{x}$

Sol $f(f(x)) = \frac{f(x) - 1}{f(x) + 1}$

$= \frac{\frac{x-1}{x+1} - 1}{\frac{x-1}{x+1} + 1} = \frac{\cancel{x} - 1 - \cancel{x} - 1}{x - \cancel{1} + x + \cancel{1}} = \frac{-2}{2x}$

$= \frac{-1}{x}$ Ans

Q1.9 Redefine the function $f(x) = |x-1| + |1+x|$ $\quad -2 \le x \le 2$

Sol $f(x) = |x-1| + |x+1|$

$\quad\quad (1) \quad\quad (-1)$

$f(x) = |x+1| + |x-1|$

$-2 \leftarrow (-1) \leftrightarrow (1) \rightarrow 2$

$$f(x) = \begin{cases} -(x+1) - (x-1) & : -2 \le x < -1 \\ (x+1) - (x-1) & : -1 \le x < 1 \\ (x+1) + (x-1) & : 1 \le x \le 2 \end{cases}$$

Qn. 10 → Find the domain of $f(x) = \dfrac{1}{\sqrt{[x]^2 - [x] - 6}}$

Sol: $f(x) = \dfrac{1}{\sqrt{[x]^2 - [x] - 6}}$

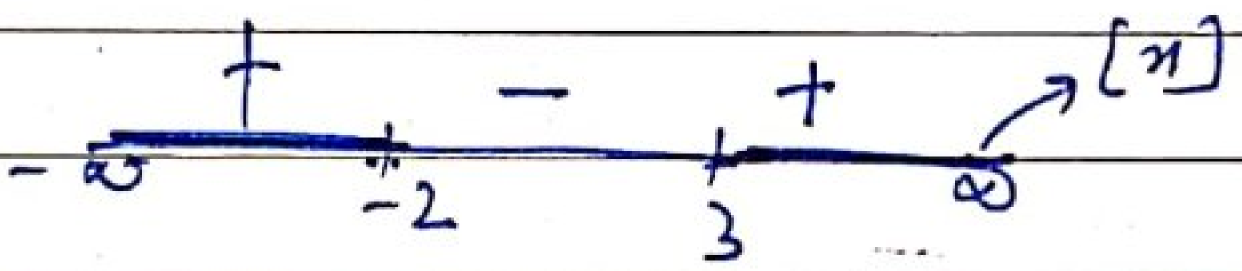
$f(x)$ is real for all values of x such that

$[x]^2 - [x] - 6 > 0$

$[x]^2 - 3[x] + 2[x] - 6 > 0$

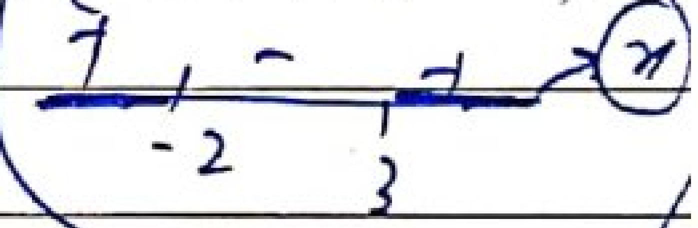
$[x]([x]-3) + 2([x]-3) > 0$

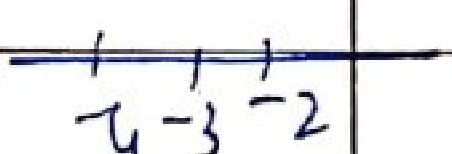
$([x]+2)([x]-3) > 0$



Rough

$(x+2)(x-3) > 0$

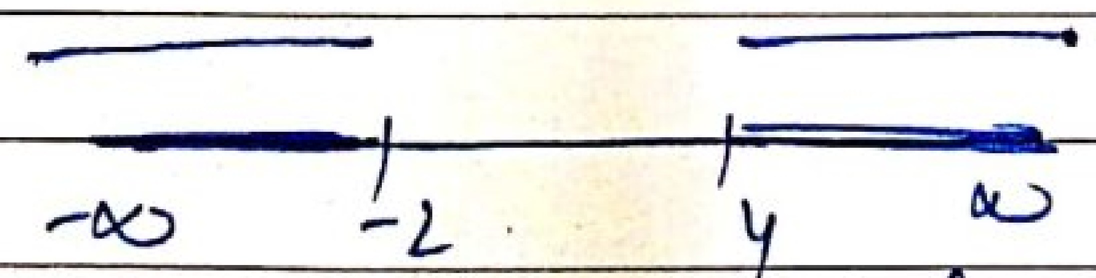




$[x] < -2$ (or) $[x] \geq 3$

$[x] = -3, -4, -5, \ldots -\infty$ | $[x] = 4, 5, 6, 7, \ldots \infty$

$x < -2$ | $x \geq 4$



$x \in (-\infty, -2) \cup [4, \infty)$ Ans

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Relation & FUNCTION

1

WORKSHEET NO: 4 (Class No: 5)

Q.1 If $A = \{2, 4, 6, 9\}$ and $B = \{4, 6, 18, 27, 54\}$ $a \in A$, $b \in B$ find the set of ordered pairs such that "a" is a factor of "b" and $a < b$

Q.2 Find the domain and Range of the relation R given by $R = \{(x, y): y = x + \frac{6}{x}$; where $x \in N$, $y \in N$ and $x < 6\}$

Q.3 Redefine the function $f(x) = |x-2| + |2+x|$; $-3 \le x \le 3$

Q.4 Find the domain for which the functions $f(x) = 3x^2 - 1$ and $g(x) = 3 + x$ are equal.

Q.5 $f(x) = 2x+3$; $g(x) = x^2 + 7$. find the values of x for which $g(f(x)) = 8$

Ans $x = -1, -2$

Q.6 $f(x) = \frac{x+1}{x-1}$ find $f(f(f(2)))$ Ans = 3

Q.7 If $f(x) = y = \frac{ax-b}{cx-a}$, then prove that $f(y) = x$

Q.8 $f(x) = x + \frac{1}{x}$ prove that

$$[f(x)]^3 = f(x^3) + 3f(\tfrac{1}{x})$$

Q.9 Let $A = \{1, 2, 3\}$ & $R = \{(a, b): |a^2 - b^2| \le 5$; $a, b \in A\}$ write R as a set of ordered pairs.

Q.10 Let R be a relation on the set Z (integers) defined by $R = \{(x, y): x - y$ is divisible by $n\}$

Show that (i) $(x, y) \in R$ implies $(y, x) \in R$

(2) $(x, y) \in R$ & $(y, z) \in R$ implies $(x, z) \in R$

(3) $(x, x) \in R$ for all $x \in Z$

— x —