$\boxed{\text{TRIGONOMETRY}}$: REVISION CLASS No: 3

Show that $\cos^2 x + \cos^2\left(x+\frac{\pi}{3}\right) + \cos^2\left(x+\frac{2\pi}{3}\right) = \frac{3}{2}$

LHS $\cos^2 x + \cos^2(x+60^\circ) + \cos^2(x+120^\circ)$

$$= \frac{1+\cos(2x)}{2} + \frac{1+\cos(2x+120^\circ)}{2} + \frac{1+\cos(2x+240^\circ)}{2}$$

$$= \frac{1}{2}\left[3+\cos(2x) + \cos(2x\overset{(B)}{+}120^\circ) + \cos(2x\overset{(A)}{+}240^\circ)\right]$$

$$= \frac{1}{2}\left[3+\cos(2x) + 2\cos(2x\overset{\text{III}}{+}180^\circ)\cdot\cos(60^\circ)\right]$$

$$= \frac{1}{2}\left[3+\cos(2x) - \cancel{2}\cos(2x)\times\frac{1}{\cancel{2}}\right]$$

$$= \frac{1}{2}\left(3+\cancel{\cos(2x)} - \cancel{\cos(2x)}\right) = \frac{3}{2} \text{ Ans}$$

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Show that $\cos(2x)\cdot\cos\left(\frac{x}{2}\right) - \cos(3x)\cdot\cos\left(\frac{9x}{2}\right) = \sin(5x)\cdot\sin\left(\frac{5x}{2}\right)$

LHS $\cos(2x)\cdot\cos\left(\frac{x}{2}\right) - \cos(3x)\cos\left(\frac{9x}{2}\right)$

$= \frac{1}{2}\left[2\cos(2x)\cos\left(\frac{x}{2}\right) - 2\cos(3x)\cos\left(\frac{9x}{2}\right)\right]$

$= \frac{1}{2}\left[\cos\left(\frac{5x}{2}\right) + \cos\left(\frac{3x}{2}\right) - \left\{\cos\left(\frac{15x}{2}\right) + \cos\left(-\frac{3x}{2}\right)\right\}\right]$

$= \frac{1}{2}\left[\cos\left(\frac{5x}{2}\right) + \cancel{\cos\left(\frac{3x}{2}\right)} - \cos\left(\frac{15x}{2}\right) - \cancel{\cos\left(\frac{3x}{2}\right)}\right]$

$= \frac{1}{2}\left[\cos\left(\frac{5x}{2}\right) - \cos\left(\frac{15x}{2}\right)\right]$

$= \cancel{\frac{1}{2}}\left[-\cancel{2}\sin(5x)\sin\left(-\frac{5x}{2}\right)\right] = \sin(5x)\sin\left(\frac{5x}{2}\right)$ Ans

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Show that $2\cos\left(\frac{\pi}{13}\right)\cos\left(\frac{9\pi}{13}\right)+\cos\left(\frac{3\pi}{13}\right)+\cos\left(\frac{5\pi}{13}\right)=0$

Ans

$$2\cos\left(\frac{\pi}{13}\right)\cos\left(\frac{9\pi}{13}\right)+\cos\left(\frac{3\pi}{13}\right)+\cos\left(\frac{5\pi}{13}\right)$$

$$=\cos\left(\frac{10\pi}{13}\right)+\cos\left(\frac{8\pi}{13}\right)+\cos\left(\frac{3\pi}{13}\right)+\cos\left(\frac{5\pi}{13}\right)$$

link

$$=\cos\left(\overset{\text{II}}{\pi}-\frac{3\pi}{13}\right)+\cos\left(\overset{\text{II}}{\pi}-\frac{5\pi}{13}\right)+\cos\left(\frac{3\pi}{13}\right)+\cos\left(\frac{5\pi}{13}\right)$$

$$=\cancel{-\cos\left(\frac{3\pi}{13}\right)}\cancel{-\cos\left(\frac{5\pi}{13}\right)}+\cancel{\cos\left(\frac{3\pi}{13}\right)}+\cancel{\cos\left(\frac{5\pi}{13}\right)}$$

$$=0$$ Ans

TRIGONOMETRY : REVISION CLASS No: 3

Find the value of $\tan\left(\frac{\pi}{8}\right)$

Solution : we have $\tan(2\theta) = \frac{2\tan\theta}{1-\tan^2\theta}$

put $\theta = \frac{\pi}{8}$

$\Rightarrow \tan\left(\frac{\pi}{4}\right) = \frac{2\tan(\pi/8)}{1-\tan^2(\pi/8)}$

$\Rightarrow 1 = \frac{2x}{1-x^2}$

$\Rightarrow 1-x^2 = 2x$

$\Rightarrow x^2+2x-1=0$

$x = \frac{-2 \pm \sqrt{\cdots}}{2}$

$x = \frac{-2 \pm 2\sqrt{2}}{2}$

$x = -1 \pm \sqrt{2}$

$x = -1+\sqrt{2}$

$\boxed{\tan\left(\frac{\pi}{8}\right) = \sqrt{2}-1}$

Ans

$x = -1-\sqrt{2}$

$\tan(\pi/8) = -1-\sqrt{2}$

(Rejected)

$\frac{\pi}{8} \rightarrow$ Ist-Quad

TRIGONOMETRY : REVISION CLASS No: 3

ANS: 5 → given $\tan x = -\frac{4}{3}$; $\frac{\pi}{2} < x < \pi$. Find the value of $\sin\left(\frac{x}{2}\right)$, $\cos\left(\frac{x}{2}\right)$ and $\tan\left(\frac{x}{2}\right)$

Soln we have, $1 + \tan^2 x = \sec^2 x$

$$\Rightarrow 1 + \frac{16}{9} = \sec^2 x$$

$$\Rightarrow \sec^2 x = \frac{25}{9}$$

$$\Rightarrow \underbrace{\sec x}_{\text{IInd}} = \pm \frac{5}{3}$$

$$\Rightarrow \sec x = -\frac{5}{3}$$

$$\Rightarrow \boxed{\cos x = -\frac{3}{5}}$$

we have,

$$1 - \cos x = 2\sin^2\left(\frac{x}{2}\right)$$

$$\Rightarrow 1 + \frac{3}{5} = 2\sin^2\left(\frac{x}{2}\right)$$

$$\Rightarrow \frac{8}{5} = 2\sin^2(x/2)$$

$$\Rightarrow \frac{4}{5} = \sin^2(x/2)$$

$$\Rightarrow \underbrace{\sin\left(\frac{x}{2}\right)}_{\text{Ist Second}} = \pm \frac{2}{\sqrt{5}}$$

$$\Rightarrow \boxed{\sin(x/2) = \frac{2}{\sqrt{5}}} \quad \boxed{\cos(x/2) = \frac{1}{\sqrt{5}}}$$

$$\tan(x/2) = 2$$

$\boxed{\text{TRIGONOMETRY}}$: REVISION CLASS No: 3

QNS: 6 → (i) Show that $\sin(n+1)x \cdot \sin(n+2)x + \cos(n+1)x \cdot \cos(n+2)x = \cos x$

Lhs $\cos(n+1)x \cdot \cos(n+2)x + \sin(n+1)x \cdot \sin(n+2)x$

$= \cos(\cancel{nx} + x - \cancel{nx} - 2x)$ - - - $\{\cos A \cos B + \sin A \sin B = \cos(A-B)\}$

$= \cos(-x)$

$= \cos x$ Ans

(ii) Show that $\cos\left(\frac{\pi}{4}-x\right) \cdot \cos\left(\frac{\pi}{4}-y\right) - \sin\left(\frac{\pi}{4}-x\right) \cdot \sin\left(\frac{\pi}{4}-y\right) = \sin(x+y)$

Lhs

$\cos\left(\frac{\pi}{4} - x + \frac{\pi}{4} - y\right)$

$= \cos(90 - x - y)$

$= \cos(90 - (x+y)) = \sin(x+y)$

TRIGONOMETRY : REVISION CLASS No: 3

QNS: 7 Show that $\cot(4x)\left[\sin(5x)+\sin(3x)\right] = \cot x\left[\sin(5x)-\sin(3x)\right]$

LHS
$\cot(4x)\left[\sin(5x)+\sin(3x)\right]$

$= \frac{\cos(4x)}{\cancel{\sin(4x)}}\left[2\cancel{\sin(4x)}\cdot\cos(x)\right]$

$= 2\cos(4x)\cos x$

RHS
$\cot x\left[\sin(5x)-\sin(3x)\right]$

$= \frac{\cos x}{\cancel{\sin x}}\left[2\cos(4x)\cdot\cancel{\sin(x)}\right]$

$= 2\cos(4x)\cdot\cos x$ LHS = RHS

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QNS: 8 Show that $\tan(4x) = \dfrac{4\tan x(1-\tan^2 x)}{1-6\tan^2 x+\tan^4 x}$

LHS $\tan(4x) = \dfrac{2\tan(2x)}{1-\tan^2(2x)}$

$$= \frac{2 \times \dfrac{2\tan x}{1-\tan^2 x}}{1-\left(\dfrac{2\tan x}{1-\tan^2 x}\right)^2}$$

$$= \frac{\dfrac{4\tan x}{1-\tan^2 x}}{1-\dfrac{4\tan^2 x}{(1-\tan^2 x)^2}}$$

$$= \frac{\dfrac{4\tan x}{\cancel{1-\tan^2 x}}}{\dfrac{(1-\tan^2 x)^2-4\tan^2 x}{(1-\tan^2 x)^{\cancel{2}}}}$$

$$= \frac{4\tan x(1-\tan^2 x)}{1+\tan^4 x-6\tan^2 x}$$

= RHS

TRIGONOMETRY : REVISION CLASS No: 3

QNS=9 Show that $\sin(3x) + \sin(2x) - \sin x = 4\sin x \cdot \cos\left(\frac{x}{2}\right) \cdot \cos\left(\frac{3x}{2}\right)$

LHS

$$\left(\sin(3x) - \sin x\right) + \sin(2x)$$

$$= 2\cos(2x)\cdot\sin(x) + \sin(2x)$$

$$= 2\cos(2x)\sin x + 2\sin x\cos x$$

$$= 2\sin x\left(\cos(2x) + \cos x\right)$$

$$= 2\sin x\left(2\cos\left(\frac{3x}{2}\right)\cos\left(\frac{x}{2}\right)\right)$$

$$= 4\sin x\cdot\cos\left(\frac{3x}{2}\right)\cos\left(\frac{x}{2}\right)$$

$= RHS$

TRIGONOMETRY : REVISION CLASS No: 3

QNS=10 Show that $\dfrac{\sin(5x)-2\sin(3x)+\sin x}{\cos(5x)-\cos x}=\tan x$

Solution LHS

$$\frac{(\sin(5x)+\sin x)-2\sin(3x)}{\cos(5x)-\cos x}$$

$$=\frac{2\sin(3x)\cos(2x)-2\sin(3x)}{-2\sin(3x)\sin(2x)}$$

$$=\frac{\cancel{2\sin(3x)}[\cos(2x)-1]}{-\cancel{2\sin(3x)}\sin(2x)}$$

$$=\frac{1-\cos(2x)}{\sin(2x)}$$

$$=\frac{2\sin^2 x}{2\sin x\cos x}$$

$$=\tan x$$

$=$ RHS

TRIGONOMETRY : REVISION CLASS No: 3

QNS=11 Show $\cos(6x) = 32\cos^6 x - 48\cos^4 x + 18\cos^2 x - 1$

LHS $\cos(6x)$

$= 2\cos^2(3x) - 1 \quad \ldots \quad \{\cos(2\theta) = 2\cos^2\theta - 1\}$

$= 2(4\cos^3 x - 3\cos x)^2 - 1$

$= 2(16\cos^6 x + 9\cos^2 x - 24\cos^4 x) - 1$

$= 32\cos^6 x - 48\cos^4 x + 18\cos^2 x - 1$ Ans

QNS: 12 → Show that $\cot x \cdot \cot(2x) - \cot(3x)\cdot\cot(2x) - \cot(3x)\cot x = 1$

Soln. We have $3x = 2x + x$

$\Rightarrow \cot(3x) = \cot(2x + x)$

$\Rightarrow \cot(3x) = \dfrac{\cot(2x)\cdot\cot(x) - 1}{\cot(2x) + \cot x}$

$\Rightarrow \cot(3x)\cot(2x) + \cot(3x)\cot x = \cot(2x)\cot x - 1$

$\Rightarrow \cot(2x)\cot x - \cot(3x)\cot(2x) - \cot(3x)\cot x = 1$

Ans