ULTIMATE MATHEMATICS: BY AJAY MITTAL

PROBABILITY $\boxed{\text{CLASS NO: 3}}$ (11th class)

__Que 1__ Show that

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A)$$
$$+ P(A \cap B \cap C)$$

__Sol__

LHS $P(A \cup B \cup C)$

$= P\Big((A \cup B) \cup C \Big)$

$= P(A \cup B) + P(C) - P\big((A \cup B) \cap C\big)$

$= P(A) + P(B) - P(A \cap B) + P(C) - P\Big((A \cap C) \cup (B \cap C)\Big)$

--- { distribution property }

$= P(A) + P(B) - P(A \cap B) + P(C) - \Big\{ P(A \cap C) + P(B \cap C)$
$\qquad\qquad\qquad\qquad\qquad\qquad - P\big((A \cap C) \cap (B \cap C)\big) \Big\}$

$= P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(A \cap C) + P(A \cap B \cap C)$

__Proved__

__Que 2__ →

Two students Anil and Ashima appeared in an examination. The probability that Anil will qualify the examination is 0.05 and that Ashima will qualify the examination is 0.10. The probability that both will qualify the examination

is 0·02. Find the prob that

(a) Both Anil and Ashima will not qualify the examination

(b) Atleast one of them will not qualify the examination

(c) only one of them will qualify the examination

Son let $A \to$ Anil will qualify the examination

$B \to$ Ashima " " " "

$$P(A) = 0·05 \qquad ; \quad P(B) = 0·10 \qquad ; \quad P(A \cap B) = 0·02$$

(a) Req prob

$$P(A' \cap B') = 1 - P(A \cup B)$$
$$= 1 - \left[P(A) + P(B) - P(A \cap B) \right]$$
$$= 1 - \left[0·05 + 0·10 - 0·02 \right]$$
$$= 1 - \left[0·13 \right] = 0·87 \; \text{Ans}$$

(b)
$$P\left(\underbrace{A' \cup B''}_{\text{Demorgan's law}} \right) = P(A \cap B)' = 1 - P(A \cap B)$$
$$= 1 - 0·02$$
$$= 0·98$$

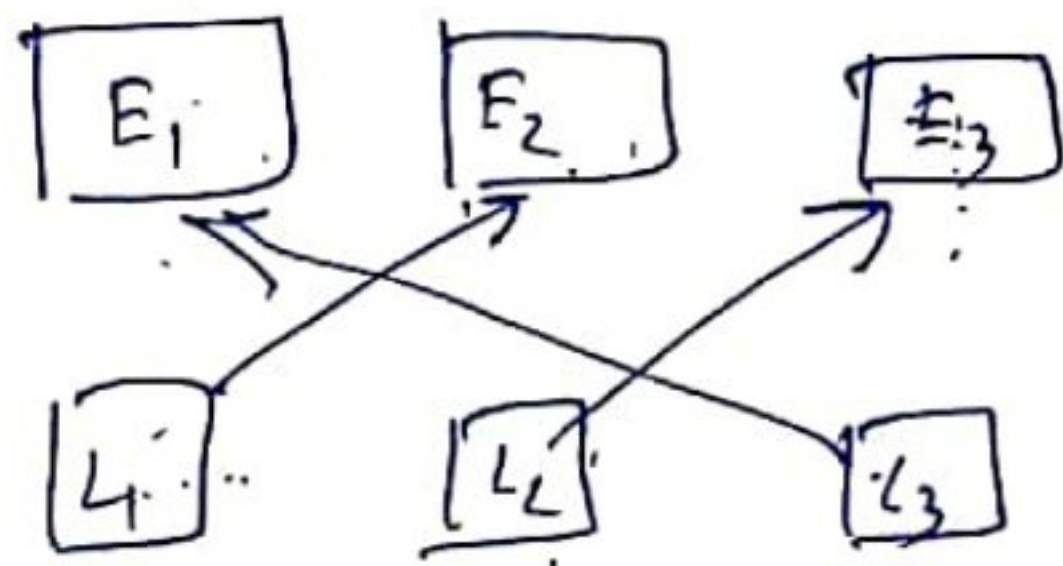(c) $P(A \cap B') + P(B \cap A') = P(A) - P(A \cap B) + P(B) - P(A \cap B)$
$$= P(A) + P(B) - 2P(A \cap B)$$
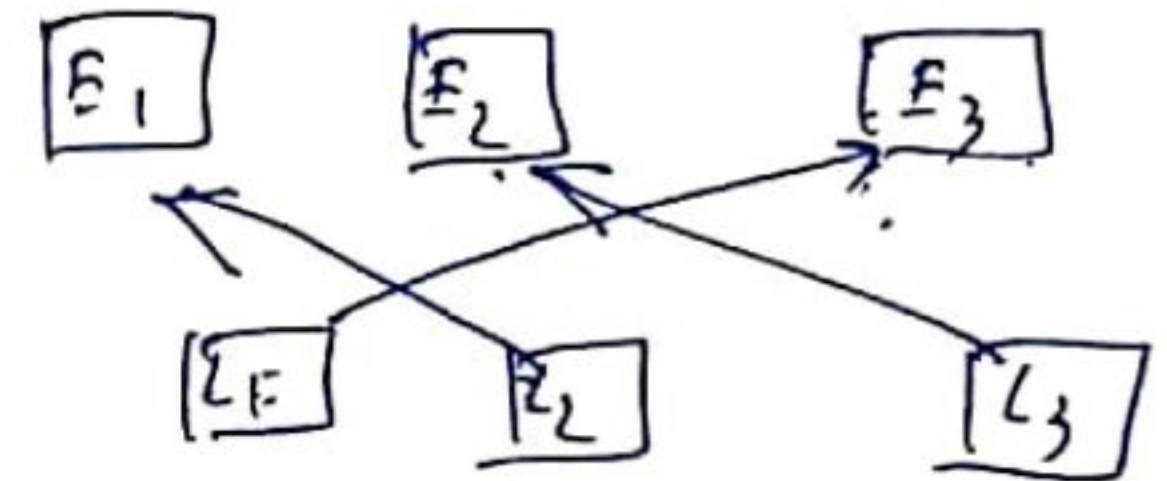$$= 0·05 + 0·10 - 0·04$$
$$= 0·11 \; \text{Ans}$$

Ques 3 → Three lettes are dictated to three person and an envelope is addressed to each of them, the lettes are inserted in to the envelope at random so that each envelope conteins exactly one letter. find the probability that atleast one letter is in its proper envelope.

Soln

Case I

Case II

$A \rightarrow$ atleast one letter in its proper envelope

$A' \rightarrow$ none of the letter in its proper envelope

$$P(A') = P(case I) + P(case II)$$

$$= \left(\frac{1}{3} \times \frac{1}{2} \times \frac{1}{1}\right) + \left(\frac{1}{3} \times \frac{1}{2} \times 1\right)$$

$$= \frac{1}{8} + \frac{1}{8} = \frac{2}{6} = \frac{1}{3}$$

By prop

$$P(A) = 1 - P(A')$$

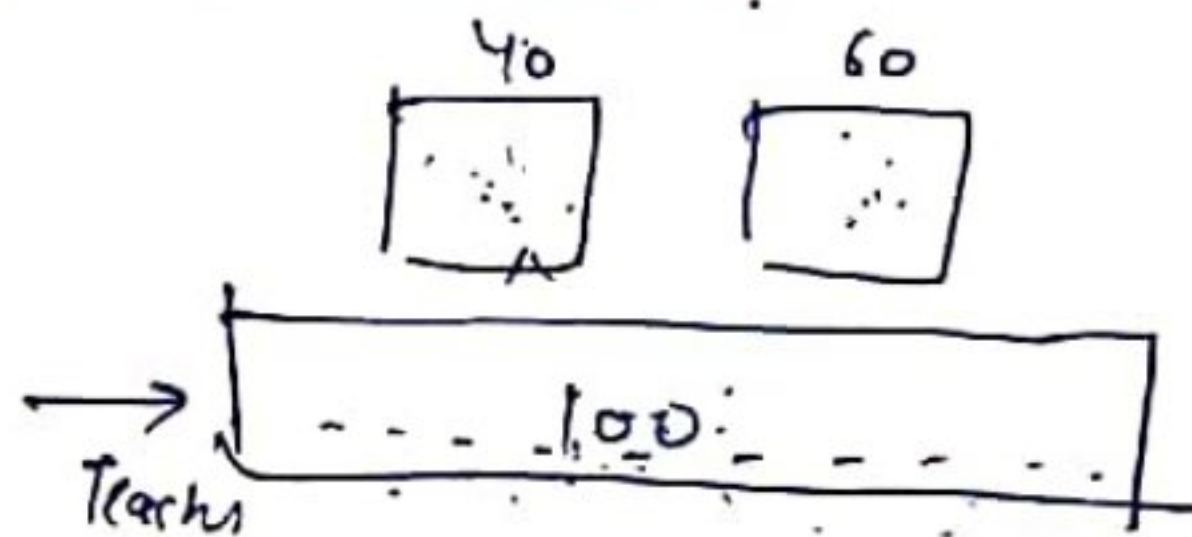$$= 1 - \frac{1}{3} = \frac{2}{3} \quad \text{Ans}$$

Qn 4 → out of 100 students, two sections of 40 & 60 are formed. If you and your friend are among the 100 students, what is the probability that

(a) You both enter the same section?

(b) You both enter the different sections?

Soln

(a) total No of ways

$$= {}^{100}C_{40} \times {}^{60}C_{60}$$

$$\boxed{{}^{100}C_{60} = {}^{100}C_{40}}$$

fav ways $= \left({}^{2}C_{2} \times {}^{98}C_{38} \times {}^{60}C_{60} \right) + \left({}^{2}C_{2} \times {}^{98}C_{58} \times {}^{40}C_{40} \right)$

Prob

$$= \frac{{}^{98}C_{38} + {}^{98}C_{58}}{{}^{100}C_{40}}$$

$$= \frac{\dfrac{98!}{38! \, 60!} + \dfrac{98!}{58! \, 40!}}{\dfrac{100!}{40! \, 60!}}$$

$$= \frac{98! \left(\dfrac{40! \, 60!}{38! \, 60!} + \dfrac{40! \times 60!}{58! \times 40!} \right)}{100!}$$

$$= \frac{40 \times 39 + 60 \times 59}{100 \times 99} = \frac{510}{990} = \frac{17}{33}$$

(b) P(different Section) $= 1 - P(\text{same Section}) = \dfrac{16}{33}$ **Ans**
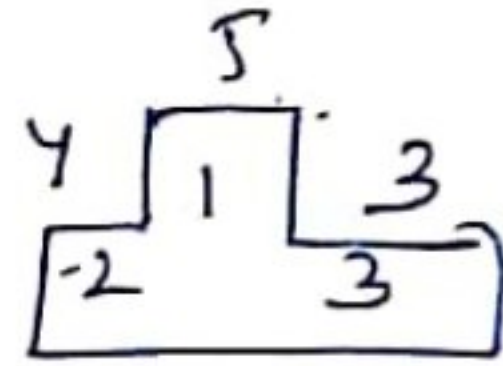
Ans 5 → In a relay race, there are five teams A, B, C, D, E

(a) what is the probability that A, B & C finish first second and third respectively.

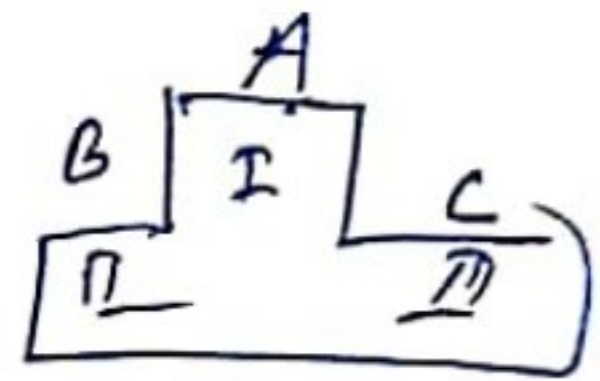(b) what is the probability that A, B & C are first three to finish.

Sun

(a) total No of ways $= 5 \times 4 \times 3$
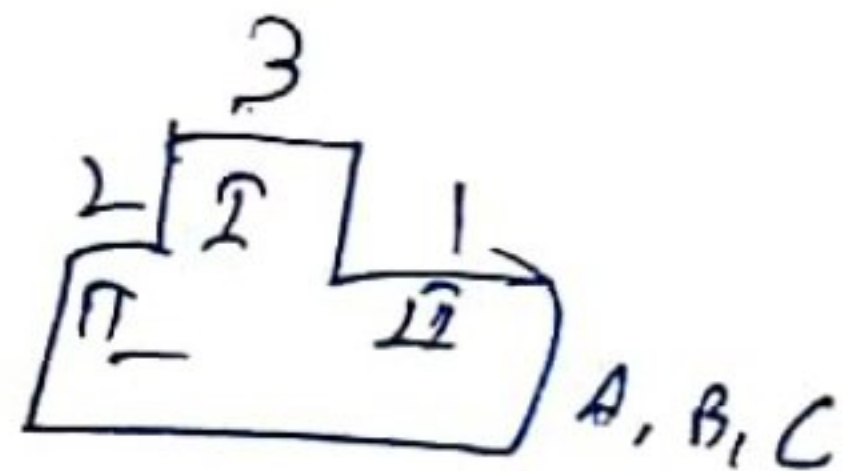
$= 60$

fav. no of ways $= 1 \times 1 \times 1 = 1$

By prob $= \dfrac{1}{60}$

(b) total N of ways $= 5 \times 4 \times 3 = 60$

fav. way $= 3 \times 2 \times 1 = 6$

By prob $= \dfrac{6}{60} = \dfrac{1}{10}$ **An**

$-x-$