Topic : ..... Date. : ..... Page No. : 1

Solution of worksheet x10: 4 (class 5)

Relation & Functions

Qu. 1 $A = \{2, 4, 6, 9\}$ $B = \{4, 6, 18, 27, 54\}$

a is a factor of b means a divides b & a < b

$R = \{(2,4)\ (2,6)\ (2,18)\ (2,54), (6,18)\ (6,54), (9,18)\ (9,27)\ (9,54)\}$ Ans

Qu. 2 → $R = \{(x, y): y = x + \frac{6}{x}\ ;\ x \in N,\ y \in N \text{ and } x < 6\}$

when $x = 1 \Rightarrow y = 1 + \frac{6}{1} = 7 \in N$

when $x = 2 \Rightarrow y = 2 + \frac{6}{2} = 5 \in N$

when $x = 3 \Rightarrow y = 3 + \frac{6}{3} = 5 \in N$

when $x = 4 \Rightarrow y = 4 + \frac{6}{4} = 5.5 \notin N$

when $x = 5 \Rightarrow y = 5 + \frac{6}{5} = 6.2 \notin N$

when $x = 6 \Rightarrow y = 6 + \frac{6}{1} = 7 \in N$

$\therefore R = \{(1,7)\ (2,5)\ (3,5)\ (6,7)\}$

Domain $= \{1, 2, 3, 6\}$.

Range $= \{5, 7\}$ Ans

Qu. 3 → $f(x) = |x-2| + |2+x|$; $-3 \le x \le 3$

(2) (-2)

$f(x) = |2+x| + |x-2|$

$-3 \leftarrow (-2) \leftarrow \cdot \rightarrow (2) \rightarrow 3$

$$f(x) = \begin{cases} -(2+x) - (x-2) & -3 \le x < -2 \\ (2+x) - (x-2) & -2 \le x < 2 \\ (2+x) + (x-2) & 2 \le x \le 3 \end{cases}$$

$$f(x) = \begin{cases} -2x & : -3 \le x < -2 \\ 4 & : -2 \le x < 2 \\ 2x & : 2 \le x \le 3 \end{cases}$$ Ans

CLASSTIME

Qn. 4 → $f(x) = 3x^2 - 1$; $g(x) = 3 + x$

given $f(x) = g(x)$

$\Rightarrow 3x^2 - 1 = 3 + x$

$\Rightarrow 3x^2 - x - 4 = 0$

$\Rightarrow 3x^2 - 4x + 3x - 4 = 0$

$\Rightarrow x(3x-4) + 1(3x-4) = 0$

$(x+1)(3x-4) = 0$

$x = -1, \quad x = 4/3$

$\therefore$ Domain $= \{-1, 4/3\}$ Ans

Qn. 5 → $f(x) = 2x + 3$; $g(x) = x^2 + 7$

given $g(f(x)) = 8$

$\Rightarrow g(2x+3) = 8$

$\Rightarrow (2x+3)^2 + 7 = 8$

$\Rightarrow 4x^2 + 9 + 12x + 7 = 8$

$\Rightarrow 4x^2 + 12x + 8 = 0$

$\Rightarrow x^2 + 3x + 2 = 0$

$\Rightarrow (x+1)(x+2) = 0$

$x = -1, \quad x = -2$ Ans

Qn. 6 → $f(x) = \frac{x+1}{x-1}$

$f(f(f(x))) = ?$

Formula $f(f(x)) = f\left(\frac{x+1}{x-1}\right) = \frac{\frac{x+1}{x-1} + 1}{\frac{x+1}{x-1} - 1} = \frac{x+1+x-1}{x+1-x+1}$

$f(f(x)) = \frac{2x}{2} = x$

$f(f(f(x))) = f(x) = \frac{x+1}{x-1}$

CLASSTIME

Now $f(f(f(2))) = \frac{2+1}{2-1} = \frac{3}{1} = 3$ Ans

Qu. 7 → $f(x) = y = \frac{ax-b}{cx-a}$

(01) $f(x) = \frac{ax-b}{cx-a}$ also $y = \frac{ax-b}{cx-a}$

Now $f(y) = f\left(\frac{ax-b}{cx-a}\right)$

$= \frac{a\left(\frac{ax-b}{cx-a}\right)-b}{c\left(\frac{ax-b}{cx-a}\right)-a}$

$= \frac{a^2x - ab - bcx + ab}{acx - bc - acx + a^2}$

$= \frac{a^2x - bcx}{a^2 - bc}$

$= \frac{x(a^2-bc)}{a^2-bc}$

$f(y) = x$ Ans

Qu. 8 → $f(x) = x + \frac{1}{x}$

T.P $[f(x)]^3 = f(x^3) + 3f\left(\frac{1}{x}\right)$

LHS $[f(x)]^3$

$= \left(x+\frac{1}{x}\right)^3$

$= x^3 + \frac{1}{x^3} + 3x + \frac{3}{x}$

RHS $f(x^3) + 3f\left(\frac{1}{x}\right)$

Ref= Solutions (Worksheet 4) (4)

$= x^3 + \frac{1}{x^3} + 3\left(\frac{1}{x} + x\right)$

$= x^3 + \frac{1}{x^3} + \frac{3}{x} + 3x$ = LHS

$\therefore$ LHS = RHS Proved

Qn. 9 → $A = \{1,2,3\}$ $R = \{(a,b) : |a^2 - b^2| \leq 5\}$

$R = \{(1,1)(1,2)(2,1)(2,2)(2,3)(3,2)(3,3)\}$ Ans

Qn 10 → $R = \{(x,y) : x-y$ is divisible by $n\}$

(1) Let $(x,y) \in R$

$\Rightarrow$ $x-y$ is divisible by n

$\Rightarrow$ $x-y = n\lambda$ ..... $(\lambda \in Z)$

↳ $n \times$ something

$\Rightarrow$ $y-x = -n\lambda$ which is div by n

$\Rightarrow$ $(y,x) \in R$

(2) Let $(x,y) \in R$ and $(y,z) \in Z$

$\Rightarrow$ $x-y = n\lambda$ and $y-z = nk$ --- $(\lambda, k \in Z)$

Now $x-z = (x-y) + (y-z)$

$x-z = n\lambda + nk$

$x-z = n(\lambda+k)$ which is divisible by n

$\Rightarrow$ $(x,y) \in R$

(3) for each $x \in Z$

$x-x = 0$ which is divisible by n

$\Rightarrow$ $(x,x) \in R$ Ans