← ULTIMATE MATHEMATICS: BY: AJAY MITTAL →

CHAPTER: LINEAR INEQUALITIES

CLASS No: 3

Modulus

- $f(x) = |x| = \begin{cases} x & : x \geq 0 \\ -x & ; x < 0 \end{cases}$

- $|x-3| = \begin{cases} (x-3) & : x-3 \geq 0 & ; x \geq 3 \\ -(x-3) & ; x-3 < 0 & : x < 3 \end{cases}$

- $|x-2|$

  $\underset{2}{\downarrow}$ critical point (value of x)

- $|x-2|$

  Left $\longleftarrow \underset{-ve}{\underbrace{2}} \underset{+x}{\longrightarrow}$ Right

- $|x-2| = \begin{cases} +(x-2) & : x-2 \geq 0 & : x \geq 2 \\ -(x-2) & ; x-2 < 0 & : x \leq 2 \end{cases}$

FORMULA

(1) $|x| < a \implies -a < x < a$ $\quad \underset{-a \qquad a}{\longmapsto}$

(2) $|x| \leq a \implies -a \leq x \leq a$

(3) $|x| > a \implies x < -a \text{ (or) } x > a$

(4) $|x| \geq a \implies x \leq -a \text{ (or) } x \geq a$ $\quad \underset{-a \qquad a}{\longmapsto}$

Ques 1 Solve $|3x-2| \leq 5$

Sol)

$$-5 \leq 3x-2 \leq 5 \qquad \cdots \begin{cases} |x| \leq a \\ -a \leq x \leq a \end{cases}$$

$$\Rightarrow \quad -3 \leq 3x \leq 7$$

$$\Rightarrow \quad -1 \leq x \leq 7/3$$

$$\therefore \quad x \in [-1, 7/3]$$

Ques 2 Solve $|2-3x| > 4$

Sol

$$|x| > a \quad \Rightarrow \quad x < -a \quad (or) \quad x > a$$

$$\Rightarrow \quad 2-3x < -4 \quad (or) \quad 2-3x > 4$$

$$\Rightarrow \quad -3x < -6 \quad (or) \quad -3x > 2$$

$$\Rightarrow \quad x > 2 \quad (or) \quad x < \frac{-2}{3}$$

ⓐ $\quad x \in \left(-\infty, \frac{-2}{3}\right) \cup (2, \infty) \quad \underline{\underline{An}}$

Ques 3 Solve $1 \leq |x-2| \leq 3$

Sol

$$|x-2| \geq 1 \qquad and \qquad |x-2| \leq 3$$

$$\Rightarrow \quad x-2 \leq -1 \ (or) \ x-2 \geq 1 \qquad and \qquad -3 \leq x-2 \leq 3$$

$$\Rightarrow \quad x \leq 1 \ (or) \ x \geq 3 \qquad \boxed{and} \qquad -1 \leq x \leq 5$$

$$\hookrightarrow \text{Common / intersection}$$

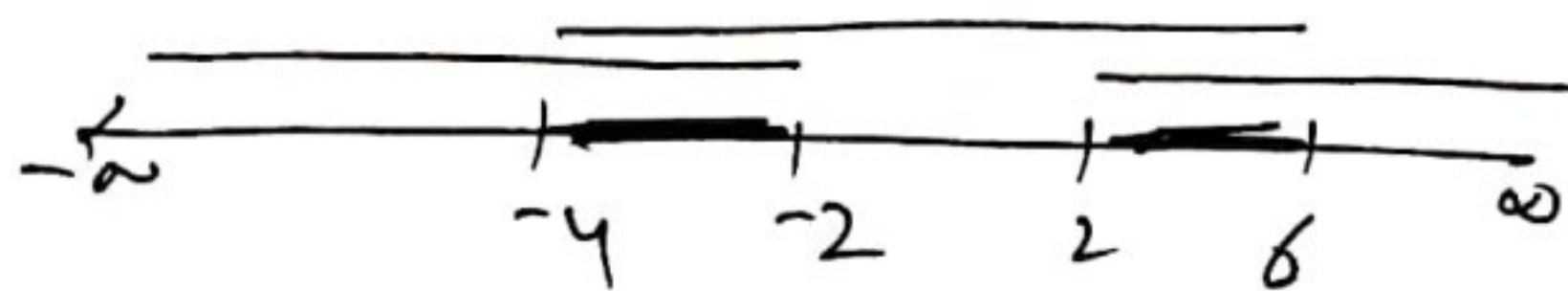$$\therefore \ x \in [-1, 1] \cup [3, 5]$$
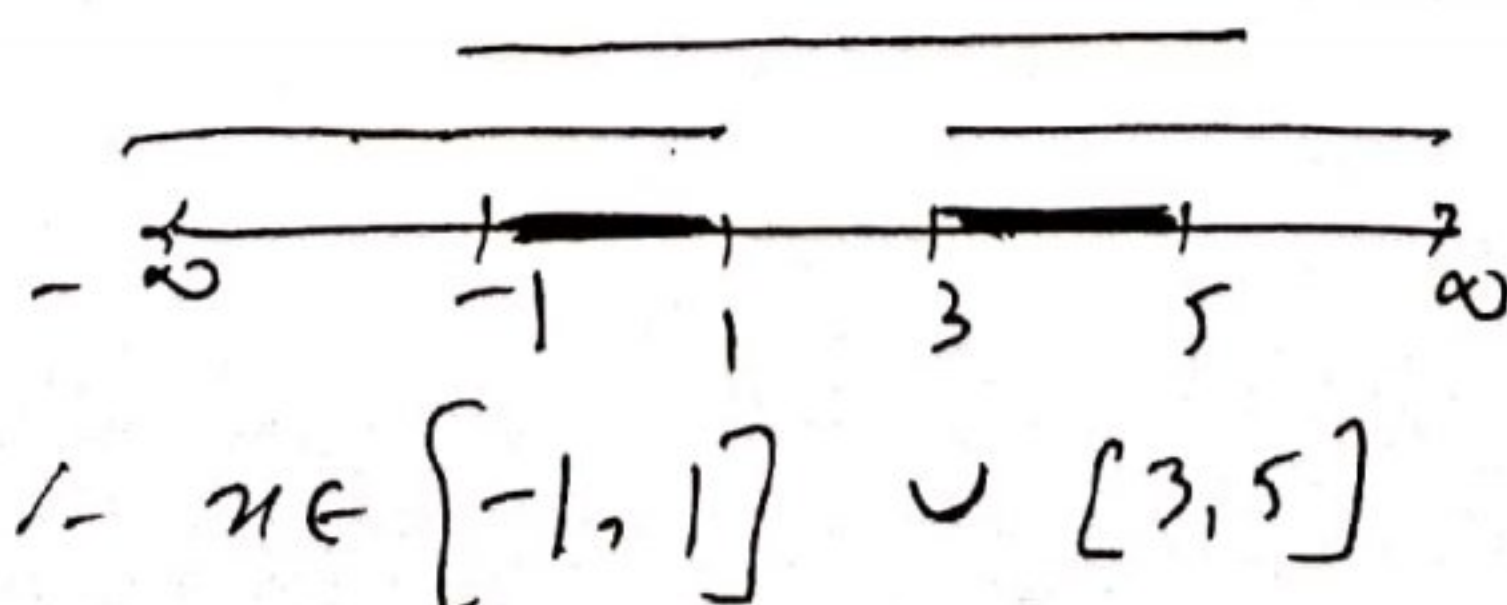
Qn. y   Solve the following system y Inequalities
$$|x-1| \le 5 \quad ; \quad |x| \ge 2$$

Solution   Consider $|x-1| \le 5$ and $|x| \ge 2$

$$-5 \le x-1 \le 5 \quad \text{and} \quad x \le -2 \text{ (or) } x \ge 2$$

$$\Rightarrow -4 \le x \le 6 \quad \text{and} \quad x \le -2 \text{ (or } | x \ge 2$$

$$x \in [-4, -2] \cup [2, 6) \quad \text{Ans}$$
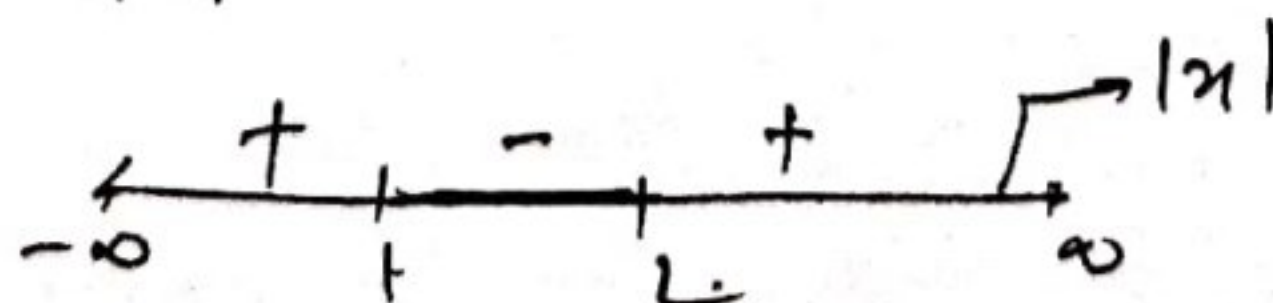
Qns 5 →   Solu
$$\frac{-1}{|x|-2} \ge 1$$

Solution
$$\frac{-1}{|x|-2} - 1 \ge 0$$

$$\Rightarrow \frac{-1 - |x| + 2}{|x| - 2} \ge 0$$
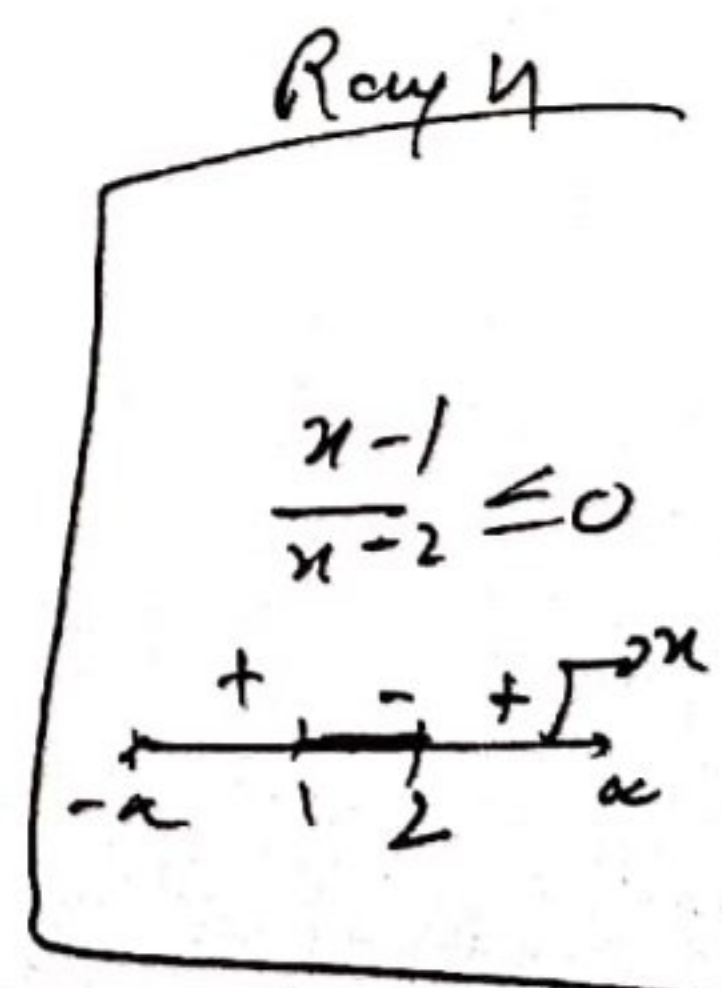
$$\Rightarrow \frac{-|x| + 1}{|x| - 2} \ge 0$$

$$\Rightarrow \frac{|x| - 1}{|x| - 2} \le 0 \quad (\text{remember})$$

Ray 4
$$\frac{x-1}{x-2} \le 0$$

$$1 \le |x| < 2$$

$$|x| \ge 1 \quad \text{and} \quad |x| < 2$$

$$\Rightarrow x \le -1 \text{ (or) } x \ge 1 \quad \text{and} \quad -2 < x < 2 \qquad x \in (-2, -1] \cup [1, 2)$$

Ans

Qn. 6 :— Solv

spend

$$\frac{|x+3|+x}{x+2} > 1$$

Soln

$$\frac{|x+3|+x}{x+2} - 1 > 0$$

$$\Rightarrow \quad \frac{|x+3|+x-x-2}{x+2} > 0$$

$$\Rightarrow \quad \frac{|x+3|-2}{x+2} > 0$$

There will be two cases:-

Case I $\quad |x+3| = (x+3)$; $x+3 \geq 0 \Rightarrow \boxed{x \geq -3}$

$$\Rightarrow \quad \frac{(x+3)-2}{x+2} > 0 \quad \text{and} \quad x \geq -3$$

$$\Rightarrow \quad \frac{x+1}{x+2} > 0 \quad \text{and} \quad x \geq -3$$

$$\begin{array}{c} + \quad - \quad + \\ \overline{-\infty \quad -2 \quad -1 \quad \infty} \end{array} \quad \text{and} \quad x \geq -3$$

$$\overline{-\infty \quad -3 \quad -2 \quad -1 \quad \infty}$$

$$x \in [-3, -2) \cup (-1, \infty) \quad \cdots (i)$$

Case II $\quad |x+3| = -(x+3)$; $x+3 < 0 \Rightarrow \boxed{x < -3}$

$$\therefore \quad \frac{-(x+3)-2}{x+2} > 0$$

$$\Rightarrow \quad \frac{-x-5}{x+2} > 0$$

$$\Rightarrow \quad \frac{-x-5}{x+2} > 0 \qquad \text{and} \qquad \boxed{x < -3}$$

$$\Rightarrow \quad \frac{x+5}{x+2} < 0 \qquad \text{and} \qquad x < -3$$

$$\text{and} \qquad x < -3$$

$$\therefore \quad x \in (-5, -3) \quad - \; -\boxed{2}$$

$$\boxed{Case\ I \;\textcircled{$\cup$}\; Case\ 2}$$

$$x \in [-3, -2) \cup (-1, \infty) \qquad \longrightarrow \quad x \in (-5, -3)$$

$$x \in (-5, -2) \cup (-1, \infty) \quad \underline{\underline{Ans}}$$

Qn. 7 Solve $|x-1| + |x-2| + |x-3| \geqslant 6$

Spural $\boxed{f(x) \geqslant 6}$

Soln   Let $f(x) = |x-1| + |x-2| + |x-3|$

$\boxed{Redefine}$

$$f(x) = \begin{cases} -(x-1) - (x-2) - (x-3) & ; \quad x < 1 \\ (x-1) - (x-2) - (x-3) & ; \quad 1 \leq x < 2 \\ (x-1) + (x-2) - (x-3) & ; \quad 2 \leq x < 3 \\ (x-1) + (x-2) + (x-3) & ; \quad x \geqslant 3 \end{cases}$$

$\Rightarrow f(x) = \begin{cases} -3x + 6 & : \quad x < 1 \\ -x + 4 & : \quad 1 \leq x < 2 \\ x & \quad 2 \leq x < 3 \\ 3x - 6 & : \quad x \geq 3 \end{cases}$

we have $\qquad f(x) \geq 6$

Case I

$\qquad -3x + 6 \geq 6 \qquad$ and $\qquad x < 1$

$\Rightarrow \qquad -3x \geq 0 \qquad$ and $\qquad x < 1$

$\Rightarrow \qquad x \leq 0 \qquad$ and $\qquad x < 1$
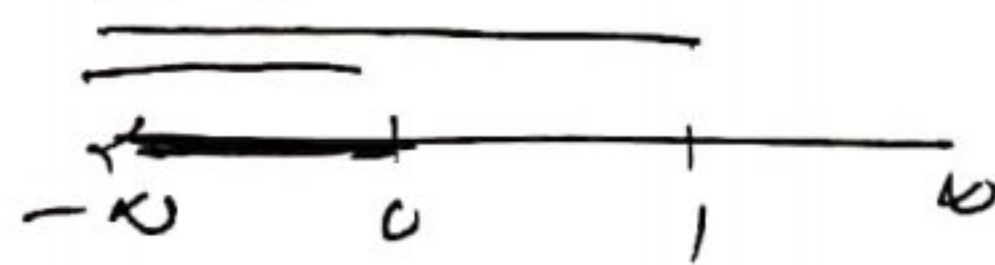
$\qquad x \in (-\infty, 0] \quad \text{- - } \textcircled{1}$

Case II

$\qquad -x + 4 \geq 6 \qquad$ and $\qquad 1 \leq x < 2$

$\Rightarrow \quad -x \geq -10 \qquad$ and $\qquad 1 \leq x < 2$

$\Rightarrow \qquad x \leq 10 \qquad$ and $\qquad 1 \leq x < 2$
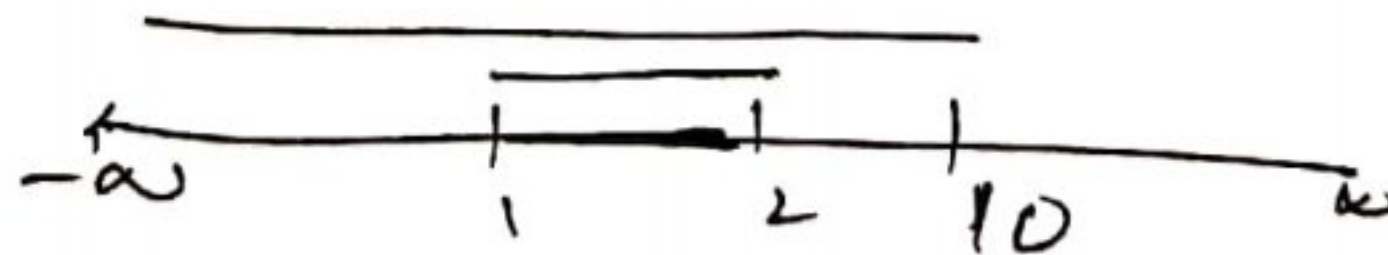
$\qquad x \in [1, 2) \quad \text{-} \textcircled{2}$

Case III

$\qquad x \geq 6 \qquad$ and $\qquad 2 \leq x < 3$
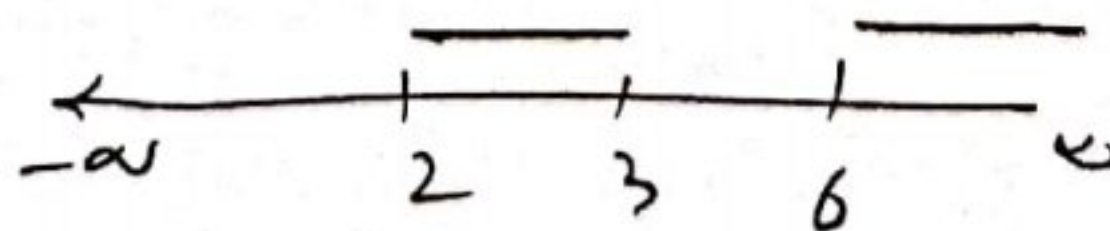
$\qquad x \in \phi \quad \text{- - } \textcircled{3}$

Case IV $3x - 6 \geqslant 6$ and $x \geqslant 3$

$\Rightarrow \quad 3x \geqslant 12$ and $x \geqslant 3$

$\Rightarrow \quad x \geqslant 4$ and $x \geqslant 3$

$$x \in [4, \infty) \quad \text{---} \quad \textcircled{4}$$
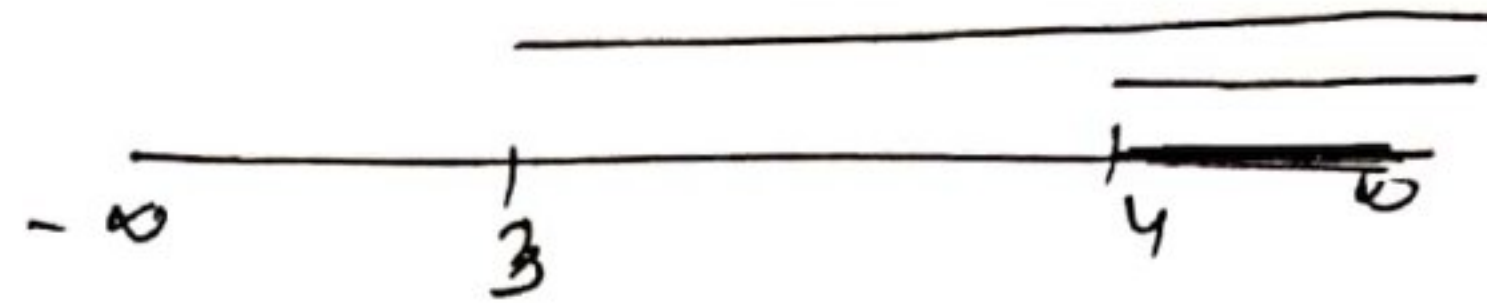
Case I (∪) Case (II) ∪ Case III ∪ Case IV
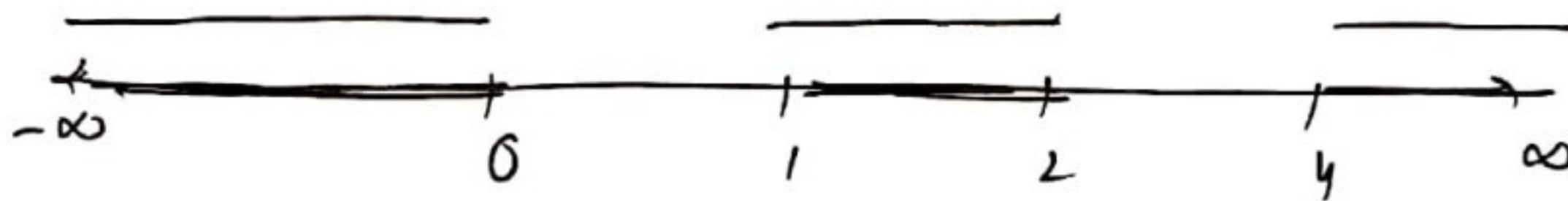
$$\therefore \quad x \in (-\infty, 0] \cup [1, 2] \cup [4, \infty) \quad \underline{An}$$

Linear Inequalities

Qn. 1 Solve $\dfrac{|x|-1}{|x|-2} \geq 0$ **Ans** $x \in [-1, 1] \cup (-\infty, -2) \cup (2, \infty)$

Qn. 2 Solve $\dfrac{|x-2|-1}{|x-2|-2} \leq 0$ **Ans** $x \in [0, 1] \cup [3, 4)$

Qn. 3 → Solve $\dfrac{1}{|x|-3} < \dfrac{1}{2}$ **Ans** $x \in (-\infty, -5) \cup (-3, 3) \cup (5, \infty)$

Qn. 4 → Solve $|x-1| + |x-2| \geq 4$ **Ans** $x \in (-\infty, -1/2] \cup [7/2, \infty)$

Qn. 5 → Solve $\dfrac{|x-1|}{x+2} < 1$ **Ans** $x \in (-\infty, -2) \cup \left(-\dfrac{1}{2}, \infty\right)$

Qn. 6 → Solve $\left|\dfrac{3x-4}{2}\right| \leq \dfrac{5}{12}$ **Ans** $x \in \left[\dfrac{19}{18}, \dfrac{29}{18}\right]$

Qn. 7 → Solve $\dfrac{|x+2|-x}{x} < 2$ **Ans** $x \in (-\infty, 0) \cup (1, \infty)$

Qn. 8 → Solve $\left|\dfrac{2}{x-4}\right| > 1 \; ; \; x \neq 4$ **Ans** $(2, 4) \cup (4, 6)$
       or $(2, 6) - \{4\}$

$-\times-$