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ULTIMATE MATHEMATICS

D&C SOLUTION of WORKSHEET No-4 Class no 5 ①

Solutions of W-4

Qn. 1 → $y = \sin^{-1}x$

Diff wrt x

$$\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}}$$

$$\Rightarrow \sqrt{1-x^2}\,\frac{dy}{dx} = 1$$

Diff again wrt x

$$\sqrt{1-x^2}\cdot\frac{d^2y}{dx^2} + \frac{dy}{dx}\cdot\frac{1}{\cancel{2}\sqrt{1-x^2}}(-\cancel{2}x) = 0$$

LCM

$$(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = 0 \qquad \text{Ans}$$

Qn. 2 → $y = \log\left(x + \sqrt{x^2+a^2}\right)$

Diff wrt x

$$\frac{dy}{dx} = \frac{1}{x+\sqrt{x^2+a^2}}\cdot\left(1 + \frac{\cancel{2}x}{\cancel{2}\sqrt{x^2+a^2}}\right)$$

$$\frac{dy}{dx} = \frac{1}{\cancel{x+\sqrt{x^2+a^2}}}\cdot\left(\frac{\cancel{\sqrt{x^2+a^2}+x}}{\sqrt{x^2+a^2}}\right)$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{\sqrt{x^2+a^2}}$$

$$\Rightarrow \sqrt{x^2+a^2}\,\frac{dy}{dx} = 1$$

Diff again wrt x

$$\sqrt{x^2+a^2}\cdot\frac{d^2y}{dx^2} + \frac{dy}{dx}\cdot\frac{1}{\cancel{2}\sqrt{x^2+a^2}}(\cancel{2}x) = 0$$

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Solution worksheet = 4 (2)

L.C.M

$$(x^2+a^2)\frac{d^2y}{dx^2} + x\frac{dy}{dx} = 0 \quad \text{Ans}$$

Ans 3 →

$$y = \left(x + \sqrt{x^2+1}\right)^m \quad \text{----(1)}$$

taking log on both sides

$$\log y = m\log\left(x + \sqrt{x^2+1}\right)$$

Diff w.r.t x

$$\frac{1}{y}\cdot\frac{dy}{dx} = m\cdot\frac{1}{x+\sqrt{x^2+1}}\cdot\left(1 + \frac{\cancel{2}x}{\cancel{2}\sqrt{x^2+1}}\right)$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{m}{\cancel{x+\sqrt{x^2+1}}}\cdot\left(\frac{\cancel{\sqrt{x^2+1}+x}}{\sqrt{x^2+1}}\right)$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{m}{\sqrt{x^2+1}}$$

$$\Rightarrow \sqrt{x^2+1}\frac{dy}{dx} = my \quad \text{---- (2)}$$

Diff again w.r.t x

$$\sqrt{x^2+1}\cdot\frac{d^2y}{dx^2} + \frac{dy}{dx}\cdot\frac{1}{\cancel{2}\sqrt{x^2+1}}\cdot(\cancel{2}x) = m\frac{dy}{dx}$$

L.C.M

$$(x^2+1)\frac{d^2y}{dx^2} + x\frac{dy}{dx} = m\sqrt{x^2+1}\frac{dy}{dy}$$

$$\Rightarrow (x^2+1)\frac{d^2y}{dx^2} + x\frac{dy}{dx} = m(my) \quad \text{---} \{\text{from eq (2)}\}$$

$$\Rightarrow (x^2+1)\frac{d^2y}{dx^2} + x\frac{dy}{dx} - m^2y = 0 \quad \text{Ans}$$

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Solutions of worksheet - 4

Qn. 4 → $x = \tan\left(\frac{1}{a}\log y\right)$

$\Rightarrow \tan^{-1}x = \frac{1}{a}\log y$

$\Rightarrow \log y = a\tan^{-1}x \quad \text{---(1)}$

Diff wrt x

$\frac{1}{y}\cdot\frac{dy}{dx} = \frac{a}{1+x^2}$

$\Rightarrow (1+x^2)\frac{dy}{dx} = ay \quad \text{---(2)}$

Diff wrt x (again)

$(1+x^2)\frac{d^2y}{dx^2} + \frac{dy}{dx}(2x) = a\frac{dy}{dx}$

$\Rightarrow (1+x^2)\frac{d^2y}{dx^2} + 2x\frac{dy}{dx} - a\frac{dy}{dx} = 0$

$\Rightarrow (1+x^2)\frac{d^2y}{dx^2} + (2x-a)\frac{dy}{dx} = 0$ Ans

Qn. 5 → $y = 500e^{7x} + 600e^{-7x}$

Diff wrt x

$\frac{dy}{dx} = 500 \times 7e^{7x} + 600 \times (-7)e^{-7x}$

Diff again wrt x

$\frac{d^2y}{dx^2} = 500 \times (49)e^{7x} + 600(-7)^2e^{-7x}$

$= 49(500e^{7x} + 600e^{-7x})$

$= 49y$ Ans

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Solution of worksheet No. 4

Ans 6 → $y = 3e^{2x} + 2e^{3x}$

Diff wrt x

$$\frac{dy}{dx} = 6e^{2x} + 6e^{3x}$$

Diff again wrt x

$$\frac{d^2y}{dx^2} = 12e^{2x} + 18e^{3x}$$

Taking $\underbrace{\text{L.H.S}}$

$$\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y$$

Substituting value of $\frac{d^2y}{dx^2}$, $\frac{dy}{dx}$ & y

$$\Rightarrow 12e^{2x} + 18e^{3x} - 5(6e^{2x} + 6e^{3x}) + 6(3e^{2x} + 2e^{3x})$$

$$= 12e^{2x} + 18e^{3x} - 30e^{2x} - 30e^{3x} + 18e^{2x} + 12e^{3x}$$

$$= 30\cancel{e^{2x}} - 30\cancel{e^{2x}} + 30\cancel{e^{3x}} - 30\cancel{e^{3x}}$$

$$= 0 = \text{R.H.S} \quad \text{Ans}$$

Ans 7 → $y = (\tan^{-1}x)^2$

Diff wrt x

$$\frac{dy}{dx} = 2\tan^{-1}x \cdot \frac{1}{1+x^2}$$

$$\Rightarrow (1+x^2)\frac{dy}{dx} = 2\tan^{-1}x$$

Diff again wrt x

$$(1+x^2)\frac{d^2y}{dx^2} + \frac{dy}{dx} \cdot (2x) = \frac{2}{1+x^2}$$

$$\Rightarrow (1+x^2)^2 \cdot \frac{d^2y}{dx^2} + 2x(1+x^2)\frac{dy}{dx} = 2 \quad \text{Ans}$$

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Solution of worksheet No= 4 (5)

Qn 8 → $y = (\sin^{-1}x)^2$

Diff wrt x

$$\frac{dy}{dx} = \frac{2\sin^{-1}x}{\sqrt{1-x^2}}$$

$$\Rightarrow \sqrt{1-x^2}\,\frac{dy}{dx} = 2\sin^{-1}x$$

Diff wrt x

$$\Rightarrow \sqrt{1-x^2}\cdot\frac{d^2y}{dx^2} + \frac{dy}{dx}\cdot\frac{1(-\cancel{2}x)}{\cancel{2}\sqrt{1-x^2}} = \frac{2}{\sqrt{1-x^2}}$$

L.C.M

$$(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = 2 \quad \underline{\text{Ans}}$$

Qn 9 → $y = \operatorname{cosec}^{-1}x$

Diff wrt x

$$\frac{dy}{dx} = \frac{-1}{x\sqrt{x^2-1}}$$

$$\Rightarrow x\sqrt{x^2-1}\,\frac{dy}{dx} = -1$$

Diff again wrt x

$$\Rightarrow (1)\cdot\sqrt{x^2-1}\,\frac{dy}{dx} + x\cdot\left(\frac{1(\cancel{2}x)}{\cancel{2}\sqrt{x^2-1}}\right)\frac{dy}{dx} + x\sqrt{x^2-1}\cdot\frac{d^2y}{dx^2} = 0$$

L.C.M

$$(x^2-1)\left(\frac{dy}{dx}\right) + x^2\frac{dy}{dx} + x(x^2-1)\frac{d^2y}{dx^2} = 0$$

$$\Rightarrow x(x^2-1)\frac{d^2y}{dx^2} + (2x^2-1)\frac{dy}{dx} = 0 \quad \underline{\text{Ans}}$$

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Solution of Worksheet No: 4 (6)

Qn/10 → $y = 2\sin t - \sin(2t)$

$x = 2\cos t - \cos(2t)$

Diff w.r.t t

$$\frac{dy}{dt} = 2\cos t - 2\cos(2t)$$

$$\frac{dx}{dt} = -2\sin t + 2\sin(2t)$$

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{2\cos t - 2\cos(2t)}{2\sin(2t) - 2\sin t}$$

$$\frac{dy}{dx} = \frac{\cos t - \cos(2t)}{\sin(2t) - \sin t}$$

$$= \frac{-\cancel{2}\sin\left(\frac{3t}{2}\right)\cdot\sin\left(-\frac{t}{2}\right)}{\cancel{2}\cos\left(\frac{3t}{2}\right)\cdot\sin\left(\frac{t}{2}\right)}$$

$$= \frac{\sin(3t/2)\cdot\cancel{\sin(t/2)}}{\cos\left(\frac{3t}{2}\right)\cdot\cancel{\sin(t/2)}} \quad \cdots \left\{\sin(-\theta) = -\sin\theta\right\}$$

$$\frac{dy}{dx} = \tan\left(\frac{3t}{2}\right)$$

Diff again w.r.t x

$$\frac{d^2y}{dx^2} = \sec^2\left(\frac{3t}{2}\right)\cdot\frac{3}{2}\cdot\frac{dt}{dx}$$

$$\frac{d^2y}{dx^2} = \frac{3}{2}\sec^2\left(\frac{3t}{2}\right)\cdot\frac{1}{(-2\sin t + 2\sin(2t))}$$

put $t = \pi/2$

$$\left(\frac{d^2y}{dx^2}\right)_{t=\pi/2} = \frac{3}{2}\sec^2\left(\frac{3\pi}{4}\right)\cdot\frac{1}{\left(-2\sin\left(\frac{\pi}{2}\right) + 2\sin(\pi)\right)}$$

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Solution of worksheet no-4

$$= \frac{3}{2}(-\sqrt{2})^2 \cdot \frac{1}{(-2\times 1 + 2\times 0)}$$ --- {since $\sin(\pi) = 0$

$$= \frac{3}{2}(2) \cdot \left(\frac{1}{-2}\right)$$ --- {$\sec^2\left(\frac{3\pi}{4}\right) = 2$}

$$= -\frac{3}{2}$$ Ans

Qn 11 → $y = \frac{\sin^{-1}x}{\sqrt{1-x^2}}$

$\Rightarrow \sqrt{1-x^2} \cdot y = \sin^{-1}x$

Diff wrt x

$$\sqrt{1-x^2} \cdot \frac{dy}{dx} + y \cdot \frac{1}{2\sqrt{1-x^2}}(-2x) = \frac{1}{\sqrt{1-x^2}}$$

L.C.M

$$(1-x^2)\frac{dy}{dx} - xy = 1$$

Diff again wrt x

$$(1-x^2)\frac{d^2y}{dx^2} + \frac{dy}{dx}(-2x) - \left(x\frac{dy}{dx} + y\right) = 0$$

$$\Rightarrow (1-x^2)\frac{d^2y}{dx^2} - 3x\frac{dy}{dx} - y = 0$$ Ans

Qn 12 → $y = 3\cos(\log x) + 4\sin(\log x)$

Diff wrt x $\quad \frac{dy}{dx} = \frac{-3\sin(\log x)}{x} + \frac{4\cos(\log x)}{x}$

$$\Rightarrow x\frac{dy}{dx} = -3\sin(\log x) + 4\cos(\log x)$$

solution

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Diff again wrt x

$$\Rightarrow x\frac{d^2y}{dx^2} + \frac{dy}{dx}\cdot 1 = \frac{-3\cos(\log x)}{x} - \frac{4\sin(\log x)}{x}$$

$$\Rightarrow x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} = -\left(3\cos(\log x) + 4\sin(\log x)\right)$$

$$\Rightarrow x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} = -y \quad \underline{\text{Ans.}}$$

Qn. 13 → $x = a(\theta + \sin\theta)$

$y = a(1+\cos\theta)$

Diff wrt θ

$$\frac{dx}{d\theta} = a(1+\cos\theta)$$

$$\frac{dy}{d\theta} = a(-\sin\theta)$$

$$\frac{dy}{dx} = \frac{-\cancel{a}\sin\theta}{\cancel{a}(1+\cos\theta)} = -\frac{\sin\theta}{1+\cos\theta}$$

Diff again wrt x

$$\frac{d^2y}{dx^2} = -\left[\frac{(1+\cos\theta)\cos\theta - \sin\theta(-\sin\theta)}{(1+\cos\theta)^2}\right]\cdot\frac{d\theta}{dx}$$

$$= -\frac{\left[\cos\theta + \cos^2\theta + \sin^2\theta\right]}{(1+\cos\theta)^2}\cdot\frac{1}{a(1+\cos\theta)}$$

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$$= -\frac{(\cancel{1+\cos\theta})}{(1+\cos\theta)^2}\cdot\frac{1}{a(\cancel{1+\cos\theta})}$$

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Solution of worksheet No: 4

$$\frac{d^2y}{dx^2} = -\frac{1}{a(1+\cos\theta)^2}$$

$$= \frac{-a}{a^2(1+\cos\theta)^2} \quad \text{.... (M \& D by a)}$$

$$= \frac{-a}{(a(1+\cos\theta))^2}$$

$$\frac{d^2y}{dx^2} = \frac{-a}{y^2} \quad \text{Ans}$$

Qn. 14 → $y = e^x(\sin x + \cos x)$

Diff w.r.t x

$$\frac{dy}{dx} = e^x(\cos x - \sin x) + (\sin x + \cos x) \cdot e^x$$

$$\frac{dy}{dx} = e^x(\cos x - \cancel{\sin x} + \cancel{\sin x} + \cos x)$$

$$\frac{dy}{dx} = 2e^x \cos x$$

Diff again w.r.t x

$$\frac{d^2y}{dx^2} = 2\left[e^x(-\sin x) + e^x \cdot \cos x\right]$$

Taking LHS

$y_2 - 2y_1 + 2y$

putting the value of y_2, y_1 & y

$$-\cancel{2e^x \sin x} + \cancel{2e^x \cos x} - \cancel{4e^x \cos x} + \cancel{2e^x \sin x} + \cancel{2e^x \cos x}$$

$= 0 = \text{RHS}$ Ans