Topic ____________________ Date ______

ULTIMATE MATHEMATICS
(BY: AJAY MITTAL: 9891067390)

$\boxed{\text{MATRICES}}$ & Determinants : total = 10 Marks
(3) (4)

Matrices

(·) Symbol = [ ]

(·) denoted by A, B, C, ---

eg $A = \begin{bmatrix} 2 & -1 & 0 \\ 1 & 3 & 5 \end{bmatrix}$ ← elements

eg $A = \begin{bmatrix} x & \sin x & i \\ 2 & -1 & a \end{bmatrix}$

(·) Rows ⇉ columns ↓↓

(·) ORDER of a Matrix : Rows × columns

(·) eg $A = \begin{bmatrix} 2 & -1 & 0 \\ 1 & 3 & 4 \end{bmatrix}_{2\times 3}$

(1M) A matrix has 15 elements. What are the possible orders it can have?

Solution: 1×15, 15×1, 3×5, 5×3

(1M) A matrix of order 2×2. Each element is either 0 or 1. How many possible number of matrices?

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← ULTIMATE MATHEMATICS →

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Sol,

$$A = \begin{bmatrix} 0/1 & 0/1 \\ 0/1 & 0/1 \end{bmatrix}_{2\times 2}$$

$= 2 \times 2 \times 2 \times 2 = 2^4 = 16$

general matrix

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}_{3\times 3}$$

Topic (1M) Construction of a matrix

✓ given: order of the Matrix

✓ given: formula / relation

(1M) Construct a matrix of order 2x2 whose elements are given by $a_{ij} = \frac{(2i-j)^2}{2}$

Sol.

let $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}_{2\times 2}$

$a_{11} = 1/2$ $\quad a_{12} = 0$

$a_{21} = 9/2$ $\quad a_{22} = 2$

$\therefore A = \begin{bmatrix} 1/2 & 0 \\ 9/2 & 2 \end{bmatrix}_{2\times 2}$ Ans

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Types of Matrices:

Square matrix : Rows = columns

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}_{2\times 2}$$

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}_{3\times 3} \qquad A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$$

→ Principal diagonal

$$A = \begin{bmatrix} 3 \end{bmatrix}_{1\times 1}$$

Identity Matrix (Unit Matrix) (I)

$$I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Scalar Matrix

$$A = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} \to \text{same}$$

$$B = \begin{bmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{bmatrix}$$

Diagonal Matrix

$$A = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix} \qquad A = \begin{bmatrix} 3 & 0 & 0 \\ 0 & i & 0 \\ 0 & 0 & x \end{bmatrix} \to \text{(OK)}$$

$A = \text{diag}(3, i, x)$

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Raw Matrix

$$A = \begin{bmatrix} 2 & -1 & 3 & 4 \end{bmatrix}_{1\times 4}$$

Column Matrix

$$A = \begin{bmatrix} 3 \\ 1 \\ 2 \\ 4 \end{bmatrix}_{4\times 1}$$

$$\begin{bmatrix} 4 \end{bmatrix}_{1\times 1}$$

Zero Matrix / Null Matrix

$$O = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Any

$$\boxed{A - A = O}$$

$$O = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$O = \begin{bmatrix} 0 \end{bmatrix}$$

—x—

OPERATION ON MATRICES

Addition / Subtraction of two Matrices

Rule: order must be same

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & -3 & 4 \end{bmatrix}_{2\times 3} \quad \& \quad B = \begin{bmatrix} 3 & -1 & 0 \\ 1 & 2 & 5 \end{bmatrix}_{2\times 3}$$

Find $5A + 2B$

$$\begin{bmatrix} 5 & 10 & 15 \\ 10 & -15 & 20 \end{bmatrix} + \begin{bmatrix} 6 & -2 & 0 \\ 2 & 4 & 10 \end{bmatrix} = \begin{bmatrix} 11 & 8 & 15 \\ 12 & -11 & 30 \end{bmatrix}$$

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(2) Multiplication:

Rule

$$\underset{2\times 2}{[\ \ ]}\ \underset{2\times 3}{[\ \ ]} = [\ \ ]_{2\times 3}$$

Same

Ans

eg $AB = \begin{bmatrix} 2 & 1 & 3 \\ 2 & 1 & 2 \\ -3 & 2 & 0 \end{bmatrix}_{3\times 3} \begin{bmatrix} 1 & 2 & 3 \\ 3 & 1 & -1 \\ 1 & 2 & 4 \end{bmatrix}_{3\times 3}$

$= \begin{bmatrix} 2+3+3 & 11 & \cancel{17} \\ 7 & 9 & 13 \\ 3 & -4 & -11 \end{bmatrix}$

(i) $A^2 = AA$ — $A \to$ must be a square Matrix

(ii) $A^3 = A^2 . A$

(3) equality of two Matrices

$$[\ \ ]_{2\times 2} = [\ \ ]_{2\times 2}$$

ans (1) If $\begin{bmatrix} x+2y & -3 \\ 5 & 2x-y \end{bmatrix} = \begin{bmatrix} 5 & -3 \\ 5 & 10 \end{bmatrix}$

find value of x & y

$x + 2y = 5$
$2x - y = 10$ } Solve

17 $A^2 = \begin{bmatrix} \; \end{bmatrix}\begin{bmatrix} \; \end{bmatrix} = \begin{bmatrix} \; \end{bmatrix}$

$A^3 = \begin{bmatrix} \; \end{bmatrix}\begin{bmatrix} \; \end{bmatrix} = \begin{bmatrix} \; \end{bmatrix}$

Lhs $= A^3 - 6A^2 + 7A + 2I$

$$= \begin{bmatrix} \; \end{bmatrix} - 6\begin{bmatrix} \; \end{bmatrix} + 7\begin{bmatrix} \; \end{bmatrix} + 2\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \; \end{bmatrix} + \begin{bmatrix} \; \end{bmatrix} + \begin{bmatrix} \; \end{bmatrix} + \begin{bmatrix} \; \end{bmatrix}$$

$$= \begin{bmatrix} \; \end{bmatrix} + \begin{bmatrix} \; \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = O$$

॥ जय श्री गिरिराज जी महाराज ॥

Qns. 1 → Find the number of all possible matrices of order 3×3 with each entry 0 or 1

Qns. 2 → Find the number of all possible matrices of order 3×2 with each entry $2, -1, 0$

Qns. 3 → Construct a matrix of order 2×2, $A = [a_{ij}]$ where elements are given by $a_{ij} = \frac{(i+2j)^2}{2}$

Qns. 4 → Construct a 3×4 matrix whose elements are given by $a_{ij} = \frac{1}{2}|-3i+j|$

Qns. 5 → Construct a matrix of order 3×2 whose elements are given by

$$a_{ij} = \begin{cases} (i-j)^2 ; & \text{when } i \neq j \\ \frac{i+j}{2} ; & \text{when } i = j \end{cases}$$

Qns. 6 → Construct a matrix of order 2×2 whose elements are given by

$$a_{ij} = \begin{cases} i+j ; & i \geq j \\ \frac{1}{2}(i-j) ; & i < j \end{cases}$$

Qns. 7 → Construct a matrix of order 2×2 whose elements are given by $a_{ij} = e^{2ix} \cdot \sin(jx)$

Qns. 8 → If a matrix has 26 elements, what are the possible orders it can have?

Qns. 9 → If a matrix has 15 elements, what are the possible orders it can have?

Qns. 10 → Find the values of x, y, z if $\begin{bmatrix} x+y & 2 \\ 5+z & xy \end{bmatrix} = \begin{bmatrix} 6 & 2 \\ 5 & 8 \end{bmatrix}$

Qn. 11 → Find the value of a, b, c, d if $\begin{bmatrix} a-b & 2a+c \\ 2a-b & 3c+d \end{bmatrix} = \begin{bmatrix} -1 & 5 \\ 0 & 13 \end{bmatrix}$

Qn. 12 → Find the value of x and y which makes the following pair of matrices equal $\begin{bmatrix} 3x+7 & 5 \\ y+1 & 2-3x \end{bmatrix} = \begin{bmatrix} 0 & y-2 \\ 8 & 4 \end{bmatrix}$

Qn. 13 → Find the value of x and y so that $\begin{bmatrix} 2x+1 & 3y \\ 0 & y^2-5y \end{bmatrix} = \begin{bmatrix} x+3 & y^2+2 \\ 0 & -6 \end{bmatrix}$

Qn. 14 → Find value of a and b so that $\begin{bmatrix} 2b & 0 \\ 2a+1 & b^2-5b \end{bmatrix} = \begin{bmatrix} b^2+1 & 0 \\ a+3 & -6 \end{bmatrix}$

Qn 15 → Find matrices X and Y if

(1) $X+Y = \begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix}$ and $X-Y = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$ ANS: $X = \begin{bmatrix} 5 & 0 \\ 1 & 4 \end{bmatrix}$ $Y = \begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix}$

(2) $2X+3Y = \begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix}$ and $3X+2Y = \begin{bmatrix} 2 & -2 \\ -1 & 5 \end{bmatrix}$

Qn. 16 → If $A = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$ Find $A^2 - 5A + 6I$; ANS: $\begin{bmatrix} 1 & -1 & -3 \\ -1 & -1 & -10 \\ -5 & 4 & 4 \end{bmatrix}$

Qn. 17 → If $A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$ Show that $A^3 - 6A^2 + 7A + 2I = 0$

← ANSWERS →

(1). 512

(2). 729

(3). $\begin{bmatrix} 9/2 & 25/2 \\ 8 & 18 \end{bmatrix}$

(4). $\begin{bmatrix} 1 & 1/2 & 0 & 1/2 \\ 5/2 & 2 & 3/2 & 1 \\ 4 & 7/2 & 3 & 5/2 \end{bmatrix}$

(5). $\begin{bmatrix} 1 & 1 \\ 1 & 2 \\ 4 & 1 \end{bmatrix}$

(6). $\begin{bmatrix} 2 & -1/2 \\ 3 & 4 \end{bmatrix}$

(7). $\begin{bmatrix} e^{2x}\sin x & e^{2x}\sin 2x \\ e^{4x}\sin x & e^{4x}\sin 2x \end{bmatrix}$

(8). 1x26, 26x1, 2x13, 13x2

(9). 1x15, 15x1, 3x5, 5x3

(10). x=4, y=2, z=0

(OR) x=2, y=4, z=0

(11). a=1, b=2, c=3, d=4

(12). Not possible to find

(13). x=2, y=2

(14). No values of a & b

(15)(ii) $X = \begin{bmatrix} 2/5 & -12/5 \\ -11/5 & 3 \end{bmatrix}$

$Y = \begin{bmatrix} 2/5 & 13/5 \\ 14/5 & -2 \end{bmatrix}$