!! जय श्री गिरिराज जी महाराज जय श्री राधे-कृष्णा !!

← ULTIMATE MATHEMATICS: BY AJAY MITTAL →

CHAPTER: INTEGRATION $\boxed{\text{CLASS NO: 9}}$

long formula

(1) $\int \sqrt{a^2-x^2}\,dx = \frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right) + C$

(2) $\int \sqrt{x^2+a^2}\,dx = \frac{x}{2}\sqrt{x^2+a^2} + \frac{a^2}{2}\log\left|x+\sqrt{x^2+a^2}\right| + C$

(3) $\int \sqrt{x^2-a^2}\,dx = \frac{x}{2}\sqrt{x^2-a^2} - \frac{a^2}{2}\log\left|x+\sqrt{x^2+a^2}\right| + C$

Qn. 1 $I = \int \sin^{-1}\sqrt{x}\,dx$

put $x = t^2$

$dx = 2t\,dt$

$\therefore I = 2\int \sin^{-1} t \cdot t\,dt$

$= 2\int \underset{II}{t}\cdot \underset{I}{\sin^{-1} t}\,dt$

$\sqrt{x-a}$

$= 2\left[\sin^{-1}t\cdot\frac{t^2}{2} - \frac{1}{2}\int \frac{1}{\sqrt{1-t^2}}\cdot t^2\,dt\right]$

$= 2\left[\sin^{-1}t\cdot\frac{t^2}{2} + \frac{1}{2}\int \frac{1-t^2-1}{\sqrt{1-t^2}}\,dt\right]$

$= t^2\cdot\sin^{-1}t + \int \sqrt{1-t^2} - \frac{1}{\sqrt{1-t^2}}\,dt$

$= t^2\cdot\sin^{-1}t + \frac{t}{2}\cdot\sqrt{1-t^2} + \frac{1}{2}\sin^{-1}t - \sin^{-1}t + C$ proceed

Q.No. 2 $I = \int \frac{\sin^{-1}\sqrt{x} - \cos^{-1}\sqrt{x}}{\sin^{-1}\sqrt{x} + \cos^{-1}\sqrt{x}} dx$

$$\boxed{\sin^{-1}\sqrt{x} + \cos^{-1}\sqrt{x} = \frac{\pi}{2}}$$

$$I = \int \frac{\sin^{-1}\sqrt{x} - \left(\frac{\pi}{2} - \sin^{-1}\sqrt{x}\right)}{\frac{\pi}{2}} dx$$

$$= \frac{2}{\pi}\int 2\sin^{-1}\sqrt{x} - \frac{\pi}{2}\, dx$$

$$= \frac{4}{\pi}\int \sin^{-1}\sqrt{x}\, dx - \int dx$$

$$I = \frac{4}{\pi} I_1 - x + c$$

where $I_1 = \int \sin^{-1}\sqrt{x}\, dx$ put $x = t^2$

(Proceed)

③ $I = \int \frac{x - \sin x}{1 - \cos x} dx$

$$I = \int \frac{x - 2\sin(x/2)\cos(x/2)}{2\sin^2(x/2)} dx$$

$$= \frac{1}{2}\int \underset{\mathrm{I}}{x}\, \underset{\mathrm{II}}{\operatorname{cosec}^2(x/2)}\, dx - \int \cot(x/2)\, dx$$

$$= \frac{1}{2}\left[-x\cot(x/2)\cdot 2 + 2\int \cot\left(\frac{x}{2}\right) dx\right] - \int \cot\left(\frac{x}{2}\right) dx$$

$$= -x\cot(x/2) + \cancel{\int \cot(x/2)\, dx} - \cancel{\int \cot\left(\frac{x}{2}\right) dx}$$

$$= -x\cot(x/2) + c \quad \text{Ans}$$

Qn. 4 → $I = \int \frac{\sqrt{x^2+1}\,\{\log(x^2+1) - 2\log x\}}{x^4}\,dx$

$$I = \int \frac{\sqrt{x^2+1}\,[\log(x^2+1) - \log x^2]}{x^4}\,dx$$

$$= \int \frac{\sqrt{x^2+1}\cdot\log\left(\frac{x^2+1}{x^2}\right)}{x^4}\,dx$$

Main step

$$= \int \frac{x\sqrt{1+\frac{1}{x^2}}\cdot\log\left(1+\frac{1}{x^2}\right)}{\cancel{x^4}\; x^3}\,dx$$

put $1+\frac{1}{x^2} = t$

$$-\frac{2}{x^3}\,dx = dt \Rightarrow \frac{dx}{x^3} = -\frac{dt}{2}$$

$$I = -\frac{1}{2}\int \underset{\text{II}}{\sqrt{t}}\cdot\underset{\text{I}}{\log t}\,dt$$

$$= -\frac{1}{2}\left[\frac{2}{3}\log t\cdot t^{3/2} - \frac{2}{3}\int \frac{1}{t}\cdot t^{3/2}\,dt\right]$$

$$= -\frac{1}{2}\left[\frac{2}{3}\cdot\log t\cdot t^{3/2} - \frac{2}{3}\int t^{1/2}\,dt\right]$$

$$= -\frac{1}{2}\left[\frac{2}{3}\log t\cdot t^{3/2} - \frac{4}{9}\cdot t^{3/2}\right] + C$$

$$= -\frac{1}{2}\times\frac{2}{3}\cdot t^{3/2}\left[\log t - \frac{2}{3}\right] + C$$

replace t Ans

(5) $I = \int \frac{\sin^{-1}x}{x^2}\,dx$

$I = \int \underset{\text{II}}{\frac{1}{x^2}} \cdot \underset{\text{I}}{\sin^{-1}x}\,dx$

$= \sin^{-1}x \cdot \left(-\frac{1}{x}\right) + \int \frac{1}{\sqrt{1-x^2}} \cdot \frac{1}{x}\,dx$

put $1 - x^2 = t^2$

$-\cancel{2}x\,dx = \cancel{2}t\,dt$

$dx = -\frac{t\,dt}{x}$

$\therefore I = -\frac{1}{x}\sin^{-1}x - \int \frac{\cancel{t}\,dt}{\cancel{t} \cdot x^2}$

$= -\frac{1}{x}\sin^{-1}x - \int \frac{1}{1-t^2}\,dt$

$= -\frac{1}{x}\sin^{-1}x - \frac{1}{2\times 1}\log\left|\frac{1+t}{1-t}\right| + c$

$= -\frac{1}{x}\sin^{-1}x - \frac{1}{2}\log\left|\frac{1+\sqrt{1-x^2}}{1-\sqrt{1-x^2}}\right| + c$ Ans

<u>Type</u> $I = \int e^x\left(f(x) + f'(x)\right)dx$

$= \int \underset{\text{II}}{e^x}\,\underset{\text{I}}{f(x)}\,dx + \int e^x \cdot f'(x)\,dx$

$= f(x) \cdot e^x - \cancel{\int f'(x) \cdot e^x\,dx} + \cancel{\int e^x \cdot f'(x)\,dx}$

$= e^x \cdot f(x) + c$ <u>Ans</u>

Qn 6 $I = \int e^x \left(\frac{1}{x} - \frac{1}{x^2} \right) dx$

$= \int e^x \cdot \frac{1}{x} dx - \int e^x \cdot \frac{1}{x^2} dx$
$\quad\ \ \text{II} \quad \text{I}$

$= \frac{1}{x} \cdot e^x + \cancel{\int \frac{1}{x^2} e^x dx} - \cancel{\int e^x \cdot \frac{1}{x^2} dx} + C$

$I = e^x \cdot \frac{1}{x} + C$ Ans

Qn 7 $I = \int e^x \left(\frac{1 - \sin x}{1 - \cos x} \right) dx$

$I = \int e^x \left(\frac{1 - 2\sin(x/2)\cos(x/2)}{2\sin^2(x/2)} \right) dx$

$= \int e^x \left(\frac{1}{2} \csc^2(x/2) - \cot(x/2) \right) dx$

$= -\int e^x \cdot \cot(x/2) dx + \frac{1}{2} \int e^x \cdot \csc^2(x/2) dx$
$\quad\ \ \text{II} \quad \text{I}$

$= -\left[\cot(x/2) \cdot e^x + \frac{1}{2} \int \csc^2(x/2) \cdot e^x dx \right] + \frac{1}{2} \int e^x \cdot \csc^2(x/2) dx$

$= -e^x \cdot \cot(x/2) - \frac{1}{2} \cancel{\int e^x \csc^2(x/2) dx} + \frac{1}{2} \cancel{\int e^x \cdot \csc^2(x/2) dx}$

$I = -e^x \cdot \cot(x/2) + C$ Ans

Q.No. 8 → $I = \int e^x \cdot \frac{x}{(x+1)^2}\, dx$

$= \int e^x \left(\frac{x+1-1}{(x+1)^2} \right) dx$

$= \int e^x \left(\frac{1}{x+1} - \frac{1}{(x+1)^2} \right) dx$

Proceed Ans $e^x \cdot \frac{1}{x+1} + C$

Q.No. 9 $I = \int e^x \cdot \left(\frac{x-4}{(x-2)^3} \right) dx$

$= \int e^x \left(\frac{x-2-2}{(x-2)^3} \right) dx$

$= \int e^x \left(\frac{1}{(x-2)^2} - \frac{2}{(x-2)^3} \right) dx$

Proceed $I = e^x \cdot \frac{1}{(x-2)^2} + C$

Q.No. 10 → $I = \int e^x \cdot \left(\frac{x^2+1}{(x+1)^2} \right) dx$

$I = \int e^x \left(\frac{x^2+1+2x-2x}{(x+1)^2} \right) dx$

$= \int e^x \left(1 - \frac{2x}{(x+1)^2} \right) dx$

$= \int e^x dx - 2 \int e^x \cdot \frac{x}{(x+1)^2}\, dx$

$= e^x - 2 \int e^x \left(\frac{x+1-1}{(x+1)^2} \right) dx$

$= e^x - 2 \int e^x \left(\frac{1}{x+1} - \frac{1}{(x+1)^2} \right) dx$

$= e^x - 2 \left[\int \underset{II}{e^x} \cdot \underset{I}{\frac{1}{x+1}}\, dx - \int e^x \cdot \frac{1}{(x+1)^2}\, dx \right]$

$= e^x - 2 e^x \cdot \frac{1}{x+1} + C$

$= e^x \left(1 - \frac{2}{x+1} \right) + C$

Q. 11 $I = \int \frac{\log x}{(1+\log x)^2} dx$

put $\log x = t$

$\Rightarrow x = e^t$

$\Rightarrow dx = e^t dt$

$I = \int e^t \cdot \frac{t}{(1+t)^2} dt$

$= \int e^t \cdot \left(\frac{t+1-1}{(1+t)^2} \right) dt$

$= \int e^t \left(\frac{1}{t+1} - \frac{1}{(t+1)^2} \right) dt$

$= \int e^t \cdot \underset{I}{\frac{1}{t+1}} \underset{II}{} dt - \int e^t \cdot \frac{1}{(t+1)^2} dt$

$= \frac{1}{t+1} \cdot e^t + \int \cancel{\frac{1}{(t+1)^2} \cdot e^t} - \cancel{\int e^t \cdot \frac{1}{(t+1)^2} dt}$

$= \frac{e^t}{t+1} + C$

$= \frac{x}{\log x + 1} + C$ Ans

Qn. 12 → $I = \int \log(\log x) + \frac{1}{(\log x)^2} dx$

put $\log x = t$

$x = e^t$

$dx = e^t dt$

$$I = \int e^t\left(\log t + \frac{1}{t^2}\right) dt$$

$$= \int e^t\left(\log t + \frac{1}{t} - \frac{1}{t} + \frac{1}{t^2}\right) dt$$

$$= \int e^t\left(\log t + \frac{1}{t}\right) dt - \int e^t\left(\frac{1}{t} - \frac{1}{t^2}\right) dt$$

$$= \int \underset{II}{e^t} \cdot \underset{I}{\log t}\, dt + \int e^t \cdot \frac{1}{t} dt - \left[\int \underset{II}{e^t} \cdot \underset{I}{\frac{1}{t}} dt - \int e^t \cdot \frac{1}{t^2} dt\right]$$

$$= \log t \cdot e^t - \int \cancel{\frac{1}{t} \cdot e^t} + \cancel{\int e^t \cdot \frac{1}{t} dt} - \left[\frac{1}{t} \cdot e^t + \cancel{\int \frac{1}{t^2} \cdot e^t dt} - \cancel{\int e^t \cdot \frac{1}{t^2} dt}\right]$$

$$= e^t \cdot \log t - \frac{1}{t} \cdot e^t + C$$

$$= e^t\left(\log t - \frac{1}{t}\right) + C$$

$$= x\left[\log(\log x) - \frac{1}{\log x}\right] + C \quad \underline{\text{Ans}}$$

Type: $I = \int e^{ax} \cdot \sin(bx)\, dx$

$I = \int e^{ax} \cdot \cos(bx)\, dx$

'I' repeats after two times By parts

Qn-13 $I = \int \underset{II}{e^{2x}} \cdot \underset{I}{\cos(3x)}\, dx$

$= \cos(3x) \cdot \frac{e^{2x}}{2} + \frac{3}{2}\int \underset{I}{\sin(3x)} \cdot \underset{II}{e^{2x}}\, dx$

$= \cos(3x) \frac{e^{2x}}{2} + \frac{3}{2}\left[\sin(3x) \cdot \frac{e^{2x}}{2} - \frac{3}{2}\int \cos(3x) \cdot e^{2x} dx\right]$

$I = \cos(3x) \cdot \frac{e^{2x}}{2} + \frac{3}{4} e^{2x} \cdot \sin(3x) - \frac{9}{4} I$

$I + \frac{9}{4} I = \frac{e^{2x}}{4}\left[2\cos(3x) + 3\sin(3x)\right] + C$

$\frac{13}{4} I = \frac{e^{2x}}{4}\left[2\cos(3x) + 3\sin(3x)\right] + C$

$I = \frac{e^{2x}}{13}\left(2\cos(3x) + 3\sin(3x)\right) + C$ Ans

Qn-14 $I = \int e^{x} \cdot \sin^2 x\, dx$

$I = \int e^{x} \cdot \left(\frac{1 - \cos(2x)}{2}\right) dx$

$= \frac{1}{2}\int e^{x}\left(1 - \cos(2x)\right) dx$

$I = \frac{1}{2}\int e^{x} dx - \frac{1}{2}\int e^{x} \cdot \cos(2x)\, dx$

$I = \frac{1}{2} e^{x} - \frac{1}{2} I_1 + c$

where $I_1 = \int e^{x} \cdot \cos(2x)\, dx$ (Recall)

Scanned with CamScanner

← INTEGRATION →

WORKSHEET NO: 7 (Class No: 9)

Qn: 1 $I = \int \tan^{-1}\sqrt{x}\,dx$ Ans $(x+1)\tan^{-1}\sqrt{x} - \sqrt{x} + C$

Qn: 2 → $I = \int \frac{x+\sin x}{1+\cos x}\,dx$ Ans $x\tan\frac{x}{2} + C$

Qn. 3 → $I = \int \frac{\log x}{x^2}\,dx$ Ans $-\frac{1}{x}(1+\log x) + C$

Qn 4 $I = \int x \cdot \log(x+1)$ Ans $\frac{x^2}{2}\log(x+1) - \frac{1}{2}\left[\frac{x^2}{2} - x + \log|x+1|\right] + C$

Qn. 5 → $I = \int e^{\sqrt{x}}\,dx$ Ans $2e^{\sqrt{x}}(\sqrt{x}-1) + C$

HINT put $x = t^2$

Qn 6 → $I = \int \sin^{-1}\left(\frac{2x}{1+x^2}\right)dx$ Ans $2x\tan^{-1}x - \log|1+x^2| + C$

Qn 7 → $\int e^x\left(\frac{2+\sin(2x)}{1+\cos(2x)}\right)dx$ Ans $e^x\tan x + C$

Qn 8 → $\int(\sin(\log x) + \cos(\log x))\,dx$ Ans $x\sin(\log x) + C$

Qn 9 → $\int e^x\left(\frac{x-3}{(x-1)^3}\right)dx$ Ans $\frac{e^x}{(x-1)^2} + C$

Qn 10 → $\int \frac{2-x}{(1-x)^2}\cdot e^x\,dx$ Ans $\frac{e^x}{1-x} + C$

Qn 11 → $\int \frac{1}{\log x} - \frac{1}{(\log x)^2}\,dx$ Ans $\frac{x}{\log x} + C$

Qn 12 → $\int \frac{e^x}{x}\left(x(\log x)^2 + 2\log x\right)dx$ Ans $e^x(\log x)^2 + C$

Qn 13 $\int e^x\left(\frac{\sin(4x)-4}{1-\cos(4x)}\right)dx$ Ans $e^x\cdot\cot(2x) + C$

Qn 14 → $\int e^{2x}\left(\frac{1+\sin(2x)}{1+\cos(2x)}\right)dx$ Ans $\frac{1}{2}e^{2x}\cdot\tan x + c$

Qn 15 → $\int e^{2x}\cdot\sin(3x)dx$ Ans $\frac{e^{2x}}{13}\left(2\sin(3x) - 3\cos(3x)\right) + c$

Qn 16 → $\int e^{ax}\cdot\cos(bx+c)dx$ Ans $\frac{e^{ax}}{a^2+b^2}\left[a\cos(bx+c) + b\sin(bx+c)\right] + C_1$

Qn 17 → $\int e^{x}\cdot\cos^2 x\, dx$ Ans $\frac{1}{2}e^{x} + \frac{e^{x}}{10}\left(\cos(2x) + 2\sin(2x)\right) + c$

Qn 18 → $\int \sin(\log x)dx$ Ans $\frac{x}{2}\left[\sin(\log x) - \cos(\log x)\right] + c$

Hint put $\log x = t \Rightarrow x = e^t \Rightarrow dx = e^t.dt$

Qn 19 → $\int e^{2x}\cdot\sin x\cos x\, dx$ Ans $\frac{e^{2x}}{8}\left[\sin(2x) - \cos(2x)\right] + c$

Qn 20 → $\int \frac{\sin^{-1}\sqrt{x} - \cos^{-1}\sqrt{x}}{\sin^{-1}\sqrt{x} + \cos^{-1}\sqrt{x}}dx$ Ans $\frac{2}{\pi}\left[\sqrt{x-x^2} - (1-2x)\cdot\sin^{-1}\sqrt{x}\right] - x + c$

— x —