← SOLUTIONS →

WORKSHEET No. 1 (A·O·D)

TANGENT & Normals

Ans 1 → Let the point of contact be (x_1, y_1)

equation of curve

$$y = (x-2)^2$$

Diff w.r.t x

$$\frac{dy}{dx} = 2(x-2)$$

Slope of Tangent at $(x_1, y_1) = 2(x_1 - 2)$

Slope of Chord $(2,0)$ & $(4,0) = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0}{2} = 0$

Since tangent is parallel to the chord

$$\Rightarrow 2(x_1 - 2) = 0$$

$$\Rightarrow x_1 = 2$$

also we have

$$y_1 = (x_1 - 2)^2 \quad \text{--- } \{\because (x_1, y_1) \text{ lies on the curve}\}$$

put $x_1 = 2$

$$y_1 = (2-2)^2 = 0$$

$\therefore$ Required point of Contact is $(2,0)$ Ans

Note- (there is a misprint in worksheet)

Ans 2 → Let point of contact be (x_1, y_1)

given equation of curve

$$y = x^3 - 3x^2 - 9x + 7$$

Diff w.r.t x

$$\frac{dy}{dx} = 3x^2 - 6x - 9$$

Slope of Tangent at $(x_1, y_1) = 3x_1^2 - 6x_1 - 9$

Slope of X-axis $= 0$

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Soln A.O.D. (W.S. 1) (2)

Since tangent is parallel to the X-axis

$\therefore\ 3x_1^2 - 6x_1 - 9 = 0$

$\Rightarrow x_1^2 - 2x_1 - 3 = 0$

$\Rightarrow (x_1 - 3)(x_1 + 1) = 0$

$\Rightarrow x_1 = 3;\quad x_1 = -1$

also we have,

$y_1 = x_1^3 - 3x_1^2 - 9x_1 + 7$

for $x_1 = 3$

$y_1 = \cancel{27} - \cancel{27} - 27 + 7 = -20$

for $x_1 = -1$ $\quad y_1 = -1 - 3 + 9 + 7 = 12$

$\therefore$ required points are $(3, -20)$ & $(-1, 12)$ Ans

Ans 3 → given equation of curve

$x = 1 - a\sin\theta, \quad y = b\cos^2\theta \quad ; \boxed{\theta = \pi/2}$

Point of contact = $\left(1 - a\sin\left(\frac{\pi}{2}\right),\ b\cos^2\left(\frac{\pi}{2}\right)\right)$ (No need)

$= (1-a,\ 0)$

Diff wrt θ

$\frac{dx}{d\theta} = -a\cos\theta$ & $\frac{dy}{d\theta} = 2b\cos\theta(-\sin\theta)$

$\frac{dy}{dx} = \frac{-2b\cos\theta\sin\theta}{-a\cos\theta} = \frac{2b\sin\theta}{a}$

Slope of tangent at $\theta = \pi/2 = \frac{2b\sin\left(\frac{\pi}{2}\right)}{a} = \frac{2b}{a}$

Slope of Normal at $\theta = \frac{\pi}{2} = \frac{-a}{2b}$ (-ve Reciprocal)

Ans

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Soln A.O.D (W.S 1)

Qns 4 → given: point of contact (0, 5)

Given: equation of curve

$$y = x^4 - 6x^3 + 13x^2 - 10x + 5$$

Diff wrt x

$$\frac{dy}{dx} = 4x^3 - 18x^2 + 26x - 10$$

Slope of Tangent at (0, 5)

$$\left(\frac{dy}{dx}\right)_{(0,5)} = 0 - 0 + 0 - 10 = -10$$

Slope of Normal = $\frac{1}{10}$ (-ve reciprocal)

Equation of Tangent at (0, 5)

$$y - 5 = -10(x - 0)$$

$$\Rightarrow y - 5 = -10x$$

$$\Rightarrow 10x + y = 5$$ Ans

Equation of Normal at (0, 5)

$$y - 5 = +\frac{1}{10}(x - 0)$$

$$10y - 50 = x$$

$$\Rightarrow x - 10y + 50 = 0$$ Ans ..

Qns 5 → given: point of contact $(at^2, 2at)$

given equation of curve

$$y^2 = 4ax$$

Diff wrt x

$$2y\frac{dy}{dx} = 4a$$

$$\Rightarrow \frac{dy}{dx} = \frac{4a}{2y} = \frac{2a}{y}$$

Slope of Tangent at $(at^2, 2at) = \frac{2a}{2at} = \frac{1}{t}$

Slope of Normal at $(at^2, 2at) = -t$ (-ve Reciprocal)

Equation of Tangent at $(at^2, 2at)$

$y - 2at = \frac{1}{t}(x - at^2)$

$\Rightarrow ty - 2at^2 = x - at^2$

$\Rightarrow x - ty + at^2 = 0$ Ans

Equation of Normal at $(at^2, 2at)$

$y - 2at = -t(x - at^2)$

$\Rightarrow y - 2at = -tx + at^3$

$\Rightarrow tx + ty = at^3 + 2at$ Ans.

Qn. 6 → (i) let the point of contact be (x_1, y_1)

equation of curve

$y = x^2 - 2x + 7$

Diff wrt x

$\frac{dy}{dx} = 2x - 2$

Slope of Tangent at $(x_1, y_1) = 2x_1 - 2$

Slope of given line $(2x - y + 9 = 0) = \frac{-2}{-1} = 2$

Since Tangent is parallel to the given line

$\Rightarrow 2x_1 - 2 = 2$

$\Rightarrow \boxed{x_1 = 2}$

Soln A.O.D (W.S 1) (5)

also we have,

$y_1 = x_1^2 - 2x_1 + 7$ --- ($\because (x_1, y_1)$ lies on the curve)

put $x_1 = 2$

$y_1 = 4 - 4 + 7 = 7$

$\therefore$ point of contact $= (2, 7)$

Now equation of Tangent at $(2,7)$ is

$y - 7 = 2(x - 2)$

$y - 7 = 2x - 4$

$\Rightarrow 2x - y + 3 = 0$ Ans

(ii) Slope of given line $(5y - 15x = 13) = -\frac{(-15)}{5}$

$= 3$

Since Tangent is perpendicular to the given line

$\therefore m_1 m_2 = -1$

$\Rightarrow (2x_1 - 2)(3) = -1$

$\Rightarrow 6x_1 - 6 = -1$

$\Rightarrow x_1 = \frac{5}{6}$

also we have

$y_1 = x_1^2 - 2x_1 + 7$

put $x_1 = \frac{5}{6}$

$\therefore y_1 = \frac{25}{36} - \frac{10}{6} + 7$

$y_1 = \frac{25 - 60 + 252}{36} = \frac{217}{36}$

$\therefore$ point of contact is $\left(\frac{5}{6}, \frac{217}{36}\right)$

equation of Tangent at $\left(\frac{5}{6}, \frac{217}{36}\right)$ with

Slope $= -\frac{1}{3}$ (Since Tangent is $\perp$ to the line)

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Soln. A.O.D (W.S 1) (5)

$$y - \frac{217}{36} = \frac{-1}{3}\left(x - \frac{5}{6}\right)$$

$$\frac{36y - 217}{36} = \frac{-1}{3}\left(\frac{6x-5}{6}\right)$$

$$\frac{36y - 217}{2} = -(6x-5)$$

$$\Rightarrow 36y - 217 = -12x + 10$$

$$\Rightarrow 12x + 36y - 227 = 0$$ Ans

Qn 7 → let the ~~equation~~ point of contact is (x_1, y_1)

Equation of curve $y = \sqrt{3x-2}$

Diff w.r.t x $\quad \frac{dy}{dx} = \frac{1}{2\sqrt{3x-2}} \cdot (3)$

Slope of Tangent at $(x_1, y_1) = \frac{3}{2\sqrt{3x_1-2}}$

Slope of given line $(4x - 2y + 5 = 0) = \frac{-4}{-2} = 2$

Since tangent is parallel to the line

$$\Rightarrow \frac{3}{2\sqrt{3x_1-2}} = 2$$

$$\Rightarrow 3 = 4\sqrt{3x_1-2}$$

Squaring:

$$9 = 16(3x_1 - 2)$$

$$\Rightarrow 9 = 48x_1 - 32$$

$$\Rightarrow x_1 = \frac{41}{48}$$

above have

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A.O.D (W.S 1) Solution (7)

$y_1 = \sqrt{3x_1 - 2}$

put $x_1 = \frac{41}{48}$

$\Rightarrow y_1 = \sqrt{3\left(\frac{41}{48}\right) - 2}$

$\Rightarrow y_1 = \sqrt{\frac{41}{16} - 2}$

$\Rightarrow y_1 = \sqrt{\frac{9}{16}} = \frac{3}{4}$

Remember $\left\{ \text{not } \pm \frac{3}{4} \right\}$ square root never gives −ve value

$\therefore$ point of Contact is $\left(\frac{41}{48}, \frac{3}{4}\right)$

Now equation of Tangent at $\left(\frac{41}{48}, \frac{3}{4}\right)$ with Slope = 2 is

$y - \frac{3}{4} = 2\left(x - \frac{41}{48}\right)$

$\frac{4y-3}{\cancel{4}} = \cancel{2}\left(\frac{48x-41}{\cancel{48}_{\cancel{12}\,6}}\right)$

$\Rightarrow 24y - 18 = 48x - 41$

$\Rightarrow 48x - 24y - 23 = 0$ Ans.

Qu. 8 → let the point of Contact be (x_1, y_1)

given equation of curve

$$\frac{x^2}{9} + \frac{y^2}{16} = 1$$

Diff w.r.t x

$$\frac{2x}{9} + \frac{2y\frac{dy}{dx}}{16} = 0$$

Solution A.OD. (W.S 1)

$$\frac{y \frac{dy}{dx}}{16} = -\frac{x}{9}$$

$$\frac{dy}{dx} = -\frac{16x}{9y}$$

Slope of Tangent at (x_1, y_1) is $= -\frac{16x_1}{9y_1}$

(i) Slope of X-axis $= 0$

Since tangent is parallel to X-axis

$\therefore$ $\frac{-16x_1}{9y_1} = 0$

$\Rightarrow x_1 = 0$

also we have

$$\frac{x_1^2}{9} + \frac{y_1^2}{16} = 1$$

put $x_1 = 0$

$\frac{y_1^2}{16} = 1 \Rightarrow y_1^2 = 16 \Rightarrow y_1 = \pm 4$

$\therefore$ Point of Contacts are $(0, \pm 4)$ Ans

(ii) Slope of Y-axis $= \frac{1}{0}$ (∞)

Since tangent is parallel to Y-axis

$\therefore$ $\frac{-16x_1}{9y_1} = \frac{1}{0}$

$\Rightarrow 0 = 9y_1$

$\Rightarrow y_1 = 0$

put in $\frac{x_1^2}{9} + \frac{y_1^2}{16} = 1$

$\frac{x_1^2}{9} = 1 \Rightarrow x_1^2 = 9 \Rightarrow x_1 = \pm 3$

$\therefore$ Required points are $(\pm 3, 0)$ Ans

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Soln A.O.D (W.S-1)

Qus 9 → Let the point of contact be (x_1, y_1)

given: Equation of curve
$y = x^3$
Diff w.r.t x
$$\frac{dy}{dx} = 3x^2$$

Slope of tangent at $(x_1, y_1) = 3x_1^2$

also Slope of Tangent $= y_1$ --- (given)

$\Rightarrow \quad y_1 = 3x_1^2$ --- (i)

also we have
$y_1 = x_1^3$ --- {∵ (x_1, y_1) lies on the curve} --- (2)

from (1) & (2)
$$3x_1^2 = x_1^3$$
$$\Rightarrow x_1^3 - 3x_1^2 = 0$$
$$\Rightarrow x_1^2(x_1 - 3) = 0$$
$$\Rightarrow x_1 = 0, \quad x = 3$$ put in eq (2)

$x_1 = 0 \qquad y_1 = 0$

$x_1 = 3 \qquad y_1 = (3)^3 = 27$

∴ Required points are $(0, 0)$ & $(3, 27)$ Ans

Qu. 10 → Let the points of contact is (x_1, y_1)

given: Equation of curve
$$x^2 + y^2 - 2x - 3 = 0$$

Diff w.r.t x $\quad 2x + 2y\frac{dy}{dx} - 2 = 0$

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Soln A.D (WS 1)

$\Rightarrow x + y\frac{dy}{dx} - 1 = 0$

$\Rightarrow \frac{dy}{dx} = \frac{1-x}{y}$

Slope of Tangent at $(x_1, y_1) = \frac{1-x_1}{y_1}$

equation of tangent at (x_1, y_1)

$y - y_1 = \frac{1-x_1}{y_1}(x - x_1)$

the tangent pass through $(0,0)$

$-y_1 = \frac{1-x_1}{y_1}(0 - x_1)$

$\Rightarrow -y_1^2 = -x_1 + x_1^2$

$\Rightarrow x_1^2 + y_1^2 = x_1$ --(i)

also we have $x_1^2 + y_1^2 - 2x_1 - 3 = 0$ --(2)

Solv (1) & (2)

$x_1 - 2x_1 - 3 = 0$

$x_1 = -3$ $\quad$ ~~$x_1 = 1$~~ $\quad x_1 = -3$ put in (2)

$9 + y_1^2 + 6 - 3 = 0$

$y_1^2 = -12$

$y_1 = \pm 2\sqrt{3}\,i$ (Imginary roots)

$\therefore$ ~~Required point~~

NO real points

$\therefore$ there ~~no~~ there no such points

(Note there is a misprint in worksheet)

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