A.O.D REVISION CLASS No. 2 →

1 → Prove that the line $\frac{x}{a}+\frac{y}{b}=2$ touches the curve $\left(\frac{x}{a}\right)^n+\left(\frac{y}{b}\right)^n=2$ at the point (a, b)

Soln we have $\left(\frac{x}{a}\right)^n+\left(\frac{y}{b}\right)^n=1$

Diff w.r.t x

$\Rightarrow n\left(\frac{x}{a}\right)^{n-1}\cdot\frac{1}{a}+n\left(\frac{y}{b}\right)^{n-1}\cdot\frac{1}{b}\cdot\frac{dy}{dx}=0$

Put $x=a$ & $y=b$

$\Rightarrow \frac{n}{a}+\frac{n}{b}\frac{dy}{dx}=0$

$\Rightarrow \boxed{\frac{dy}{dx}=-\frac{b}{a}}$ Slope of tangent

Equation of tangent at the point (a, b)

$y-b=-\frac{b}{a}(x-a)$

$\Rightarrow ay-ab=-bx+ab$

$\Rightarrow bx+ay=2ab$

$\Rightarrow \boxed{\frac{x}{a}+\frac{y}{b}=2}$

Clearly it touches the given curve Ans

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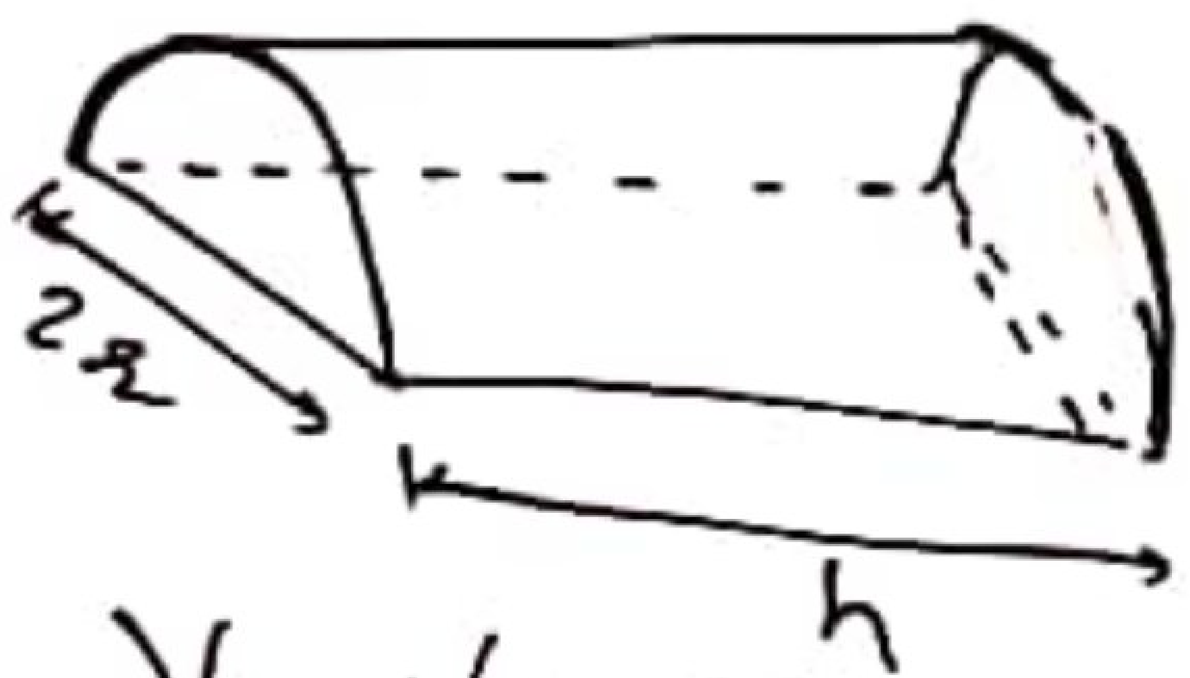
Que. 1 → A given Quantity of metal is to be cast in to a half cylinder with a rectangular base and semi-circular ends. Show that the total surface area may be minimum, the ratio of the length of the cylinder to the diameter of its semi-circular ends is $\pi : (\pi + 2)$

Soln



$$V = \frac{1}{2}\left(\pi r^2 h\right) \quad ---(i)$$

$$S = \pi r h + \frac{\pi r^2}{2} + \frac{\pi r^2}{2} + 2rh$$

$$S = \pi r h + \pi r^2 + 2rh$$

$$S = rh(\pi + 2) + \pi r^2 \quad ---(\text{to be Min})$$

$$S = \cancel{r}\left(\frac{2V}{\pi r \cancel{r}}\right)(\pi + 2) + \pi r^2$$

$$S = \frac{2V(\pi+2)}{\pi} \cdot \frac{1}{r} + \pi r^2$$

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Diff w.r.t r

$$\frac{ds}{dr} = -\frac{2v(\pi+2)}{\pi r^2} + 2\pi r = 0$$

$$\Rightarrow \frac{\cancel{2}v(\pi+2)}{\pi r^2} = \cancel{2}\pi r$$

$$\Rightarrow \boxed{v = \frac{\pi^2 r^3}{\pi+2}}$$

Diff

$$\frac{d^2s}{dr^2} = \frac{4v(\pi+2)}{\pi r^3} + 2\pi > 0$$

∴ T.S.A of half cylinder is Minimum

Put eq (i)

$$v = \frac{1}{2}\pi r^2 h$$

$$\frac{\pi^{\cancel{2}} r^{\cancel{3}}}{\pi+2} = \frac{1}{2}\cancel{\pi}\cancel{r^2} h$$

$$\Rightarrow \frac{\pi r}{\pi+2} = \frac{1}{2}h$$

$$\Rightarrow \frac{\pi}{\pi+2} = \frac{h}{2r}$$

$$h : 2r = \pi : (\pi+2) \text{ Ans}$$

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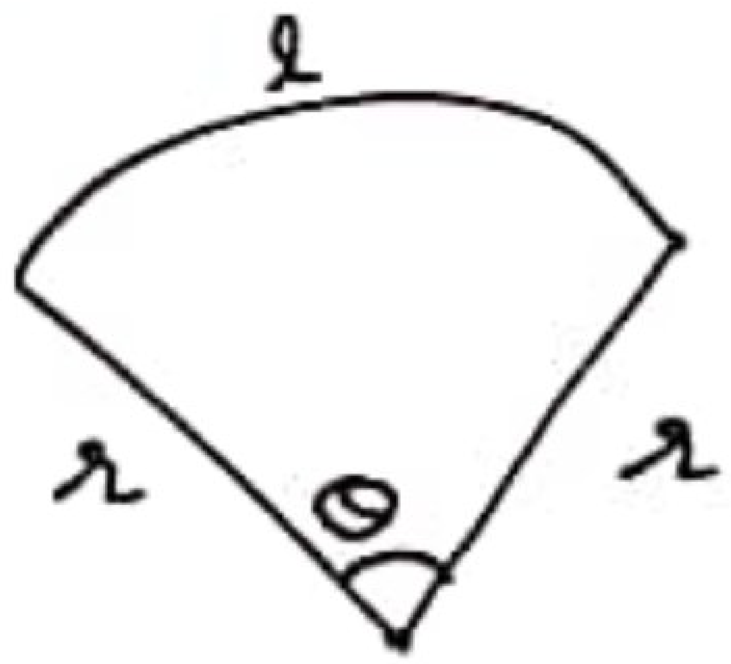
3 → If the perimeter of a sector of a circle of radius r is constant, show that the sectorial angle θ form for maximum area of sector is 2 radians.

Soln



$P = 2r + l$

We know that $l = r\theta$

$P = 2r + r\theta \quad \text{----}(1)$

$A = \frac{\theta}{360} \times \pi r^2 \quad \text{---} (\text{to be Max})$

$A = \left(\frac{P - 2r}{r}\right) \times \frac{1}{360} \times \pi r^2$

$A = \frac{\pi}{360}(Pr - 2r^2)$

$\frac{dA}{dr} = \frac{\pi}{360}(P - 4r) = 0$

$P = 4r$

$\frac{d^2A}{dr^2} = \frac{\pi}{360}(-4) < 0$

from eq (1)

$4r = 2r + r\theta$

$2\cancel{r} = \cancel{r}\theta$

$\theta = 2$ Radian

Ans

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Qns. 4 → If $y = \frac{ax-b}{(x-1)(x-4)}$ has a turning point $P(2,-1)$. Find the value of a and b and show t y is maximum at P.

Soln $y = \frac{ax-b}{x^2-5x+4}$

Diff $\frac{dy}{dx} = \frac{(x^2-5x+4)(a) - (ax-b)(2x-5)}{(x^2-5x+4)^2}$

P(2,-1) is the turning point

$\Rightarrow \left(\frac{dy}{dx}\right)_{(2,-1)} = 0$

$\Rightarrow \frac{(-2)(a) - (2a-b)(-1)}{4} = 0$

$-2a + 2a - b = 0$

$b = 0$

P(2,-1) also satisfy the equation of curve

$-1 = \frac{2a}{(1)(-2)}$

$2 = 2a$

$a = 1$

$\therefore a = 1,\ b = 0$

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NS. 5 → Find the local Maximum value and local minimum value of

$f(x) = \sin^4 x + \cos^4 x \quad ; 0 < x < \frac{\pi}{2}$

Soln $f(x) = \sin^4 x + \cos^4 x$

$f'(x) = 4\sin^3 x . \cos x - 4\cos^3 x . \sin x$

$f'(x) = 4\sin x \cos x (\sin^2 x - \cos^2 x)$

$f'(x) = -2\sin(2x) . \cos(2x)$

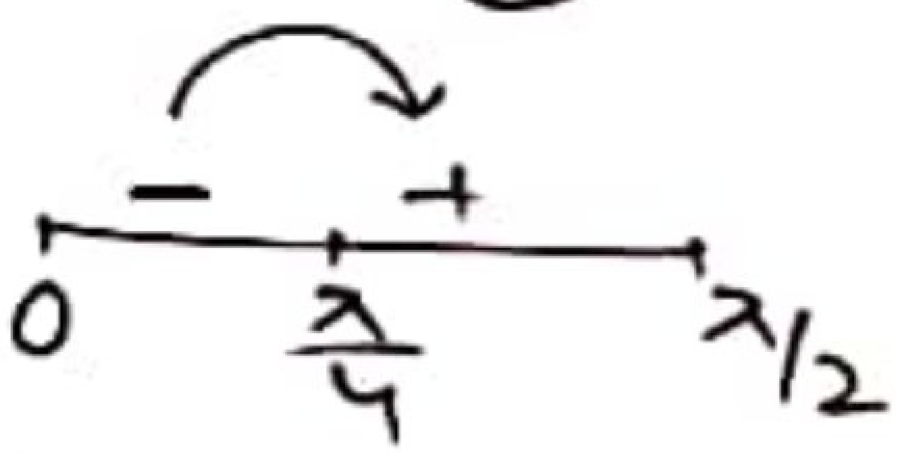
$f'(x) = -\sin(4x)$

Put $f'(x) = 0$

$-\sin(4x) = 0$

$\sin(4x) = 0$

$4x = 0$	$4x = \pi$	$4x = 2\pi$
$x = 0$	$x = \frac{\pi}{4}$	$x = \frac{\pi}{2}$
⊗	✓	⊗



$x = \frac{\pi}{4}$ is the point of local Minima

local Min Value $= f(\frac{\pi}{4})$

$= \frac{1}{4} + \frac{1}{4} = 1/2$ Ans

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QNS. 6 → If $f(x) = 2x^3 - 9mx^2 + 12m^2x + 1$, where $m > 0$ attains its maximum and minimum at p & q respectively such that $p^2 = q$, then find value of m)

Soln Diff $f'(x) = 6x^2 - 18mx + 12m^2$

put $f'(x) = 0$

$$6x^2 - 18mx + 12m^2 = 0$$

$$\Rightarrow x^2 - 3mx + 2m^2 = 0$$

$$\Rightarrow x^2 - 2mx - mx + 2m^2 = 0$$

$$\Rightarrow (x-2m)(x-m) = 0$$

$x = 2m$
$x = m$

$f''(x) = 12x - 18m$

$f''(2m) = 24m - 18m$
$= 6m > 0$

$\therefore x = 2m$ $f(x)$ is Min

$\therefore q = 2m$

Similary $p = m$

$p^2 = q$

$m^2 = 2m \Rightarrow m = 2$

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Qs. 7 → Find the values of 'k' for which $f(x) = kx^3 - 9kx^2 + 9x + 3$ strictly increasing on R

Solution:

$$f(x) = kx^3 - 9kx^2 + 9x + 3$$

$$f'(x) = 3kx^2 - 18kx + 9$$

$$f'(x) = 3(kx^2 - 6kx + 3)$$

given $f(x)$ is strictly increasing on R

$$f'(x) > 0$$

$$\Rightarrow 3(kx^2 - 6kx + 3) > 0$$

$$\Rightarrow kx^2 - 6kx + 3 > 0$$

CONCEPT:

If $ax^2 + bx + c > 0$

then $a > 0$

and $b^2 - 4ac < 0$

$\Rightarrow k > 0$ and

$36k^2 - 12k < 0$

$\Rightarrow k > 0$ and $12k(3k-1) < 0$

$\Rightarrow k > 0$ and $\underset{0 \quad\quad 1/3}{+ \quad - \quad +}$

$\Rightarrow k > 0$ and $k \in (0, 1/3)$

$\therefore \boxed{k \in (0, 1/3)}$ Ans

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8 Find the values of 'a' for which the function $f(x) = (a+2)x^3 - 3ax^2 + 9ax - 1$ is strictly decreasing for all $x \in R$

Soln: $f(x) = (a+2)x^3 - 3ax^2 + 9ax - 1$

$f'(x) = 3(a+2)x^2 - 6ax + 9a$

$f'(x) = 3\{(a+2)x^2 - 2ax + 3a\}$

given $f'(x) < 0$

$(a+2)x^2 - 2ax + 3a < 0$

$$\boxed{\begin{array}{l} ax^2+bx+c<0 \\ a<0 \ \& \ b^2-4ac<0 \end{array}}$$

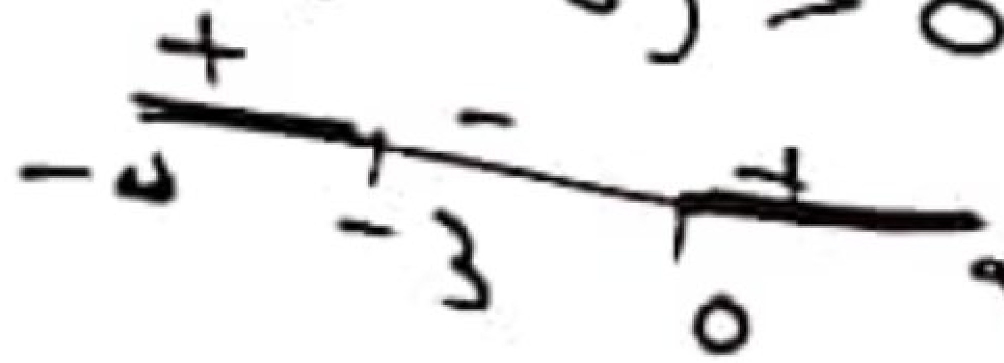
$a+2<0$ and $4a^2 - 4(a+2)(3a) < 0$

$a < -2$ and $4a^2 - 12a^2 - 24a < 0$

$a < -2$ and $-8a^2 - 24a < 0$

$a^2 + 3a > 0$

$a(a+3) > 0$



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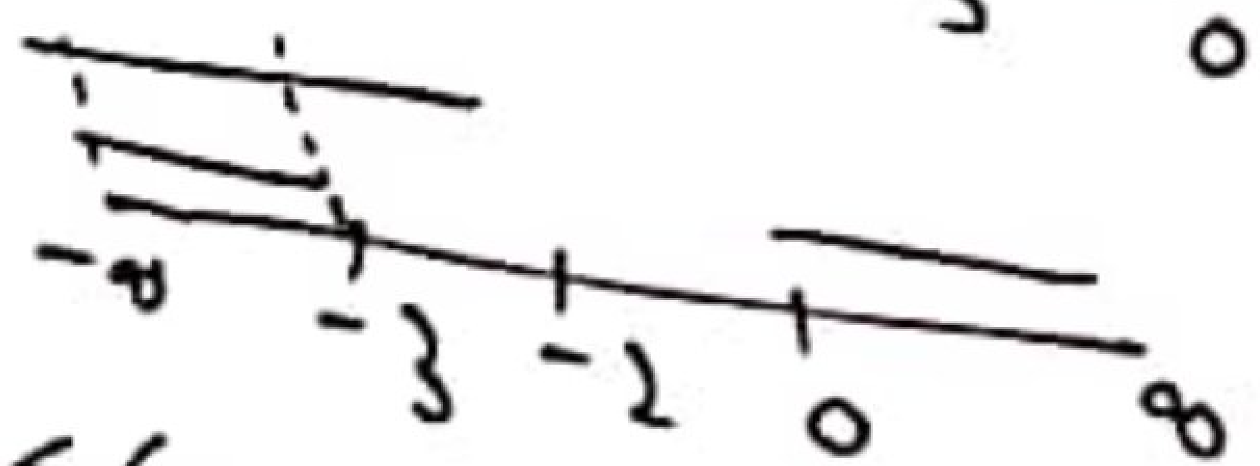

$a \in (-\infty, -3)$ Ans

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QNS: 9 Determine the value of x for which $f(x) = x^x$; $x > 0$ is Increasing or decre.

Soln

$$f(x) = x^x$$

$$\log(f(x)) = x \lg x$$

Diff

$$\frac{1}{f(x)} \cdot f'(x) = x \times \frac{1}{x} + \lg x$$

$$\Rightarrow f'(x) = x^x(1 + \lg x)$$

Put $f'(x) = 0$

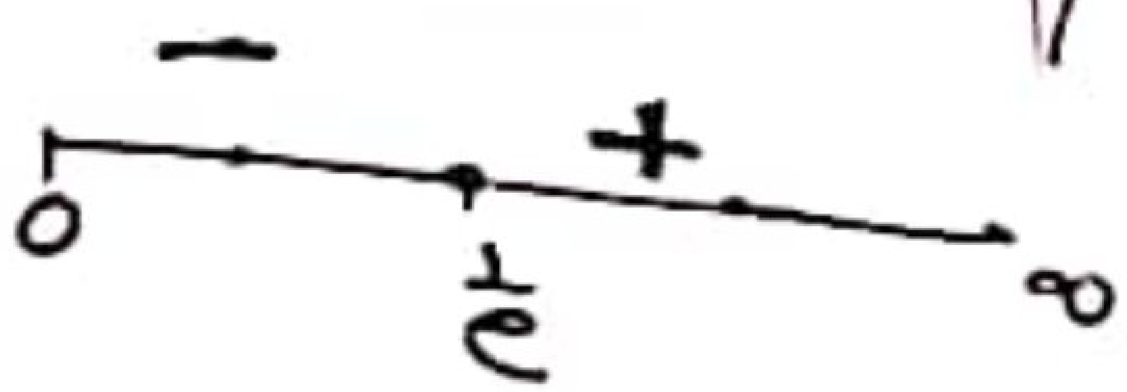
$$\Rightarrow x^x(1 + \log x) = 0$$

$$\Rightarrow 1 + \lg x = 0$$

$$\Rightarrow \lg x = -1$$

$$\Rightarrow x = e^{-1}$$

$$\Rightarrow x = 1/e$$



$x = \frac{1}{e}$

$\log x = -1$

$\therefore f(x) \downarrow$ in $\left(0, \frac{1}{e}\right]$

$f(x) \uparrow$ in $\left[\frac{1}{e}, \infty\right)$

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QNS: 10 → Find the point on the curve $y = \frac{x}{1+x^2}$ where tangent to the curve has the "greatest slope"

Soln

$$m = \frac{dy}{dx} = \frac{(1+x^2)(1) - (x)(2x)}{(1+x^2)^2}$$

$$m = \frac{1-x^2}{(1+x^2)^2}$$

$$\frac{dm}{dx} = 0 \cdot R$$

$$\frac{dm}{dx} = 0$$

$$\frac{d^2m}{dx^2} =$$

$$\left(\frac{d^2m}{dx^2}\right)_{x=} < 0$$

$\therefore$ Max slope

$x =$ put in eq. of curve

$y =$

$\therefore (\quad , \quad)$