Solutions WORKSHEET NO: 4 (Class No: 5)

PROBABILITY

Ques: 1 Given P(atleast one of two events A & B) = $p$

$\Rightarrow P(A \cup B) = p$

Given P(exactly one of A & B occurs) = $q$

$\Rightarrow P(A \cap B') + P(B \cap A') = q$

$\Rightarrow P(A) - P(A \cap B) + P(B) - P(A \cap B) = q$

$\Rightarrow \left(P(A) + P(B) - P(A \cap B)\right) - P(A \cap B) = q$

$\Rightarrow P(A \cup B) - P(A \cap B) = q$

$\Rightarrow p - P(A \cap B) = q$

$\Rightarrow \boxed{P(A \cap B) = p - q}$

— x —

Now we have $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\Rightarrow p = \left(1 - P(A')\right) + \left(1 - P(B')\right) - (p - q)$

$\Rightarrow p = 2 - P(A') - P(B') - p + q$

$\Rightarrow P(A') + P(B') = 2 - p + q - p$

$\Rightarrow \boxed{P(A') + P(B') = 2 - 2p + q}$ PROVED

— x —

Ans 2 → let A → The tube picked is defective

$E_1 \rightarrow$ tube is produced by machine $E_1$

$E_2 \rightarrow$ tube is produced by Machine $E_2$

$E_3 \rightarrow$ tube is produced by machine $E_3$

Given $P(E_1) = \frac{50}{100}$ ; $P(E_2) = \frac{25}{100}$ ; $P(E_3) = \frac{25}{100}$

$P(A/E_1) = \frac{4}{100}$ ; $P(A/E_2) = \frac{4}{100}$ ; $P(A/E_3) = \frac{5}{100}$

By total law of probability

$$P(A) = P(E_1)\,P(A/E_1) + P(E_2)\,P(A/E_2) + P(E_3)\,P(A/E_3)$$

$$= \frac{50}{100} \times \frac{4}{100} + \frac{25}{100} \times \frac{4}{100} + \frac{25}{100} \times \frac{5}{100}$$

$$= \frac{200}{10000} + \frac{100}{10000} + \frac{125}{10000}$$

$$= \frac{425}{10000}$$

$$\boxed{P(A) = \frac{17}{400}}$$ Ans

— x —

Ans 3 → Given

$P(1) = P(2) = 0.2$

$P(3) = P(5) = P(6) = 0.1$

$P(4) = 0.3$

$A \rightarrow$ Same number on each dice

$A = \{(1,1), (2,2), (3,3), (4,4), (5,5), (6,6)\}$

$P(A) = (0.2 \times 0.2) + (0.2)(0.2) + (0.1)^2 + (0.3)^2 + (0.1)^2 + (0.1)^2$

$P(A) = 0.04 + 0.04 + 0.01 + 0.09 + 0.01 + 0.01$

$P(A) = 0.20$

$B \rightarrow$ total more than 9 (i.e 10, 11, 12)

$B = \{(4,6), (6,4), (5,5), (5,6), (6,5), (6,6)\}$

$P(B) = (0.3)(0.1) + (0.1)(0.3) + (0.1)^2 + (0.1)(0.1) + (0.1)(0.1) + (0.1)^2$

$P(B) = 0.03 + 0.03 + 0.01 + 0.01 + 0.01 + 0.01$

$P(B) = 0.10$

$A \cap B = \{(5,5), (6,6)\}$

$P(A \cap B) = (0.1)^2 + (0.1)^2 = 0.01 + 0.01$

$P(A \cap B) = 0.02$

Now $P(A) \cdot P(B) = (0.20)(0.10)$

$= 0.02$

$= P(A \cap B)$

$\therefore$ $\boxed{\text{A \& B are Independent Events}}$ Ans.

— x —

Ans 4 → Red = 5 ; Black = 3 total = 8 marbles

Given that Ist ball is red and we have to find that there is atleast one is black

$\therefore$ favourable/possible ways = RBR, RBB, RRB

Required Probability $= \left(\frac{5}{8} \times \frac{3}{7} \times \frac{4}{6}\right) + \left(\frac{5}{8} \times \frac{3}{7} \times \frac{2}{6}\right) + \left(\frac{5}{8} \times \frac{4}{7} \times \frac{3}{6}\right)$

$= \frac{60 + 30 + 60}{8 \times 7 \times 6} = \frac{150}{8 \times 7 \times 6} = \frac{25}{56}$ Ans

NB (Misprint in worksheet Ans)

**Qn. 5 →** A → getting 3 two's (required event)

B → Sum is 6 (given event)

$A = \{(2,2,2)\}$

$B = \{(1,2,3), (3,2,1), (1,3,2), (2,1,3), (2,3,1), (3,1,2),$
$(1,1,4), (1,4,1), (4,1,1), (2,2,2)\}$

$A \cap B = \{(2,2,2)\}$

$P(A \cap B) = \frac{1}{216}$

$P(B) = \frac{10}{216}$

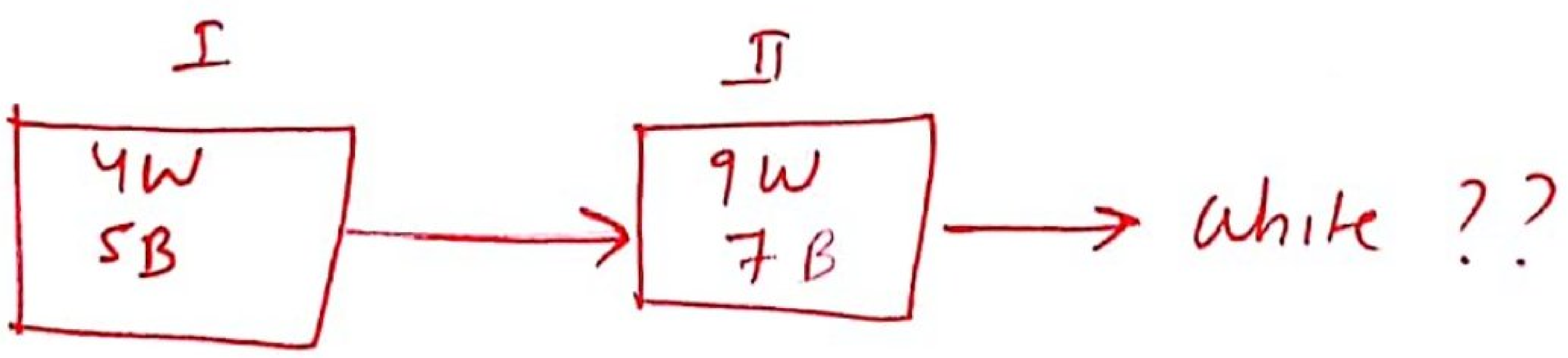
Required prob $P(A/B) = \frac{P(A \cap B)}{P(B)}$

$= \frac{\frac{1}{2\cancel{16}}}{\frac{10}{\cancel{216}}}$

$\therefore \boxed{\text{Req. prob} = \frac{1}{10}}$ Ans

— x —

**Qn. 6 →**




A → getting white ball from the $2^{nd}$ bag

$E_1$ → white ball is transferred from Bag I to Bag II

$E_2$ → Black ball is transferred from Bag I to bag II

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$P(E_1) = \frac{4}{9}$

$P(E_2) = \frac{5}{9}$

$P(A/E_1) = \frac{10}{17}$

$P(A/E_2) = \frac{9}{17}$

By total law of probability

$$P(A) = P(E_1)\,P(A/E_1) + P(E_2)\,P(A/E_2)$$

$$= \left(\frac{4}{9} \times \frac{10}{17}\right) + \left(\frac{5}{9} \times \frac{9}{17}\right)$$

$$= \frac{40 + 45}{153}$$

$\therefore$ $\boxed{\text{Req prob} = \frac{85}{153}}$ Ans

— x —

Ans 7 → $A \to$ a left handed person is selected

$E_1 \to$ person having blood group O

$E_2 \to$ person having blood group other than O

$P(E_1) = \frac{30}{100}$ ; $P(E_2) = 1 - \frac{30}{100} = \frac{70}{100}$

$P(A/E_1) = \frac{6}{100}$ ; $P(A/E_2) = \frac{10}{100}$

By Bayes's theorem

Required prob = $P(E_1/A) = \dfrac{P(E_1) \cdot P(A/E_1)}{P(E_1) \cdot P(A/E_1) + P(E_2)\,P(A/E_2)}$

$$= \frac{\frac{30}{100} \times \frac{6}{100}}{\frac{30}{100} \times \frac{6}{100} + \frac{70}{100} \times \frac{10}{100}}$$

$$= \frac{180}{180 + 700}$$

$$= \frac{180}{880}$$

$\boxed{\text{Req. prob} = \frac{9}{44}}$ Ans

— x —

Ans 8 → A → A person selected is diagnosed to be have T.B

$E_1$ → Person has T.B

$E_2$ → the person does not have T.B

$P(E_1) = \frac{1}{1000}$

$P(E_2) = 1 - \frac{1}{1000} = \frac{999}{1000}$

$P(A|E_1) = 0.99 = \frac{99}{100}$

$P(A|E_2) = .001 = \frac{1}{1000}$

By Bayes theorem

Required prob. $P(E_1|A) = \frac{P(E_1)\,P(A|E_1)}{P(E_1)\,P(A|E_1) + P(E_2)\,P(A|E_2)}$

$$= \frac{\frac{1}{1000} \times \frac{99}{100}}{\left(\frac{1}{1000} \times \frac{99}{100}\right) + \left(\frac{999}{1000} \times \frac{1}{1000}\right)}$$

$$= \frac{\frac{99}{100000}}{\frac{99}{100000} + \frac{999}{1000000}}$$

$$= \frac{990}{990 + 999}$$

$$= \frac{990}{1989}$$

$$\boxed{\text{Req prob} = \frac{110}{221}}$$ **Ans**

— x —

**Ans 9 →** total coins $= (2n+1)$

two headed coins $= n$

fair coins $= (2n+1) - n = n+1$

$A \rightarrow$ the coin results in head

$E_1 \rightarrow$ two headed coin is selected

$E_2 \rightarrow$ fair coin is selected

$P(E_1) = \frac{n}{2n+1}$ | $P(A/E_1) = 1$

$P(E_2) = \frac{n+1}{2n+1}$ | $P(A/E_2) = 1/2$

By total law of prob

$$P(A) = P(E_1)\,P(A/E_1) + P(E_2)\,P(A/E_2)$$

$$\frac{31}{42} = \frac{n}{2n+1} \times 1 + \frac{n+1}{2n+1} \times \frac{1}{2}$$

given

$$\Rightarrow \frac{31}{42} = \frac{2n + n + 1}{2(2n+1)}$$

$\Rightarrow \frac{31}{21} = \frac{3n+1}{2(2n+1)}$

$\Rightarrow$

$31(2n+1) = 21(3n+1)$

$\Rightarrow 62n + 31 = 63n + 21$

$\Rightarrow \boxed{n = 10}$ Ans

— x —

Ans 10 →

(•) Prob of getting sum 6 $= \frac{5}{36}$ ; not getting $= \frac{31}{36}$

$\{(1,5)(5,1)(2,4)(4,2)(3,3)\}$

(•) Prob of getting sum 7 $= \frac{6}{36} = \frac{1}{6}$ ; not getting $\frac{5}{6}$

$\{(1,6)(6,1)(2,5)(5,2)(3,4)(4,3)\}$

If A start first,

A will get chances $I^{st}$, $3^{rd}$, $5^{th}$, ----

Probability of winning of A in $I^{st}$ chance $= \frac{5}{36}$

" " " A " $3^{rd}$ chance $= \left(\frac{31}{36} \times \frac{5}{6}\right) \times \frac{5}{36}$

" " " A " $5^{th}$ chance $= \left(\frac{31}{36} \times \frac{5}{6} \times \frac{31}{36} \times \frac{5}{6}\right) \times \frac{5}{36}$

and so-on

$\therefore P(A) = \frac{5}{36} + \left(\frac{31}{36} \times \frac{5}{6}\right) \times \frac{5}{36} + \left(\frac{31}{36} \times \frac{5}{6}\right)^2 \times \frac{5}{36} + \cdots \infty$

Infinite G.P with $a = \frac{5}{36}$ & $r = \frac{31}{36} \times \frac{5}{6} = \frac{155}{216}$

$P(A) = \frac{a}{1-r}$

$$P(A) = \frac{\frac{5}{36}}{1 - \frac{155}{216}}$$

$$= \frac{\frac{5}{\cancel{36}}}{\frac{216-155}{\cancel{216}\ 6}}$$

$$= \frac{30}{61}$$

$$\therefore \boxed{P(A) = \frac{30}{61}}$$

Now $P(A) + P(B) = 1$

$$\Rightarrow P(B) = 1 - \frac{30}{61} = \frac{31}{61}$$

$$\therefore \boxed{P(A) : P(B) = 30:31}$$ Ans

— x —

Ans 11 → Given P(Simultaneous occurrence of A & B) $= \frac{1}{8}$

$$\Rightarrow P(A \cap B) = \frac{1}{8}$$

$$\Rightarrow \boxed{P(A)\, P(B) = \frac{1}{8}} \quad \text{---(i)} \quad \left\{\because A \ \& \ B \text{ are Independent events}\right\}$$

Given P(neither occurs) $= \frac{3}{8}$

$$\Rightarrow P(A' \cap B') = \frac{3}{8}$$

$$\Rightarrow P(A') \cdot P(B') = \frac{3}{8} \quad \left\{\text{If } A \ \& \ B \text{ are Independent then } A' \ \& \ B' \text{ are also independent events}\right\}$$

$$\Rightarrow \boxed{(1 - P(A)) \cdot (1 - P(B)) = \frac{3}{8}} \quad \text{---(ii)}$$

let $P(A) = x$ and $P(B) = y$

$\therefore$ equation (i) & (ii) becomes

$xy = \frac{1}{8}$ and $(1-x)(1-y) = \frac{3}{8}$

$\Rightarrow 1 - y - x + xy = \frac{3}{8}$

$\Rightarrow 1 - \frac{1}{8x} - x + \frac{1}{8} = \frac{3}{8}$ ... $\{\because xy = \frac{1}{8}\}$

$\Rightarrow \frac{8x - 1 - 8x^2 + x}{\cancel{8}x} = \frac{3}{\cancel{8}}$

$\Rightarrow -8x^2 + 9x - 1 = 3x$

$\Rightarrow 8x^2 - 6x + 1 = 0$

$\Rightarrow 8x^2 - 4x - 2x + 1 = 0$

$\Rightarrow 4x(2x-1) - 1(2x-1) = 0$

$\Rightarrow (2x-1)(4x-1) = 0$

$\Rightarrow x = \frac{1}{2}$ or $x = \frac{1}{4}$

$\Rightarrow y = \frac{1}{4}$ or $y = \frac{1}{2}$

$\therefore P(A) = \frac{1}{2}$ and $P(B) = \frac{1}{4}$

OR $P(A) = \frac{1}{4}$ and $P(B) = \frac{1}{2}$

Ans

— x —

Qn. 12 →  Prob of getting 5 on dice $= \frac{1}{6}$

Prob of not getting 5 on dice $= \frac{5}{6}$

If A starts first,

A will get chances $1^{st}, 4^{th}, 7^{th}$ ......

Prob of winning of A in $I^{st}$ chance $= \frac{1}{6}$

" " " A " $4^{th}$ chance $= \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} \times \frac{1}{6}$

" " " A " $7^{th}$ chance $= \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} \times \frac{1}{6}$

and so-on

$\therefore P(A) = \frac{1}{6} + \left(\frac{125}{216}\right) \times \frac{1}{6} + \left(\frac{125}{216}\right)^2 \times \frac{1}{6} + \cdots \infty$

Infinite GP $\quad a = \frac{1}{6} \quad \& \quad r = \frac{125}{216}$

$P(A) = \frac{a}{1-r}$

$P(A) = \frac{\frac{1}{6}}{1 - \frac{125}{216}}$

$= \frac{\cancel{\frac{1}{6}}}{\frac{216-125}{\cancel{216}\ 36}}$

$\boxed{P(A) = \frac{36}{91}}$

Now B will get chances $2^{nd}$, $5^{th}$, $8^{th}$ ....

Prob of winning of B in $2^{nd}$ chance $= \frac{5}{6} \times \frac{1}{6}$

" " " B " $5^{th}$ " $= \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} \times \frac{1}{6}$

" " " B " $8^{th}$ " $= \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} \times \frac{1}{6}$

and so-on

$P(B) = \frac{5}{6} \times \frac{1}{6} + \left(\frac{5}{6} \times \frac{5}{6} \times \frac{5}{6}\right) \times \frac{5}{6} \times \frac{1}{6} + \left(\frac{5}{6} \times \frac{5}{6} \times \frac{5}{6}\right)^2 \times \frac{5}{6} \times \frac{1}{6} + \cdots \infty$

Infinite GP

with $a = \frac{5}{6} \times \frac{1}{6} = \frac{5}{36}$

and $r = \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} = \frac{125}{216}$

$P(B) = \frac{a}{1-r}$

$P(B) = \dfrac{\frac{5}{36}}{1 - \frac{125}{216}}$

$= \dfrac{\frac{5}{\cancel{36}}}{\frac{216-125}{\cancel{216}}}$

$\boxed{P(B) = \frac{30}{91}}$

Now we have $P(A) + P(B) + P(C) = 1$

$\Rightarrow P(C) = 1 - (P(A) + P(B))$

$P(C) = 1 - \left(\frac{36}{91} + \frac{30}{91}\right)$

$\boxed{P(C) = \frac{25}{91}}$ Ans

— x —

Qn. 13 → $S = \{1, 2, 3, 4, \ldots 11\}$ → 6 odd & 5 even numbers

A → both the numbers selected are odd

B → sum is even

$A = \{6\text{ odd}\}$ ; $B = \{6\text{ odd or } 5\text{ even}\}$

$A \cap B = \{6\text{ odd}\}$

$$P(B) = \frac{{}^6C_2 + {}^5C_2}{{}^{11}C_2}$$

$$P(A \cap B) = \frac{{}^6C_2}{{}^{11}C_2}$$

By Conditional probability

Req. prob $P(A|B) = \dfrac{P(A \cap B)}{P(B)}$

$$= \frac{\frac{{}^6C_2}{\cancel{{}^{11}C_2}}}{\frac{{}^6C_2 + {}^5C_2}{\cancel{{}^{11}C_2}}}$$

$$= \frac{15}{15+10} \quad \text{---} \quad \left\{ {}^nC_2 = \frac{n(n-1)}{2} \right\}$$

$$= \frac{15}{25}$$

$\therefore \boxed{\text{Req prob} = \frac{3}{5}}$ Ans

— x —

Qn. 14 →

A → Student chosen gets first class marks

$E_1$ → the student chosen is a boy

$E_2$ → the student chosen is a girl

$P(E_1) = \frac{2}{3}$

$P(E_2) = 1 - \frac{2}{3} = 1/3$

$P(A|E_1) = 0.28$

$P(A|E_2) = 0.25$

By total law of probability

Required Prob: $P(A) = P(E_1)P(A/E_1) + P(E_2)P(A/E_2)$

$= \frac{2}{3} \times 0.28 + \frac{1}{3} \times 0.25$

$= \frac{0.56}{3} + \frac{0.25}{3}$

$= \frac{0.81}{3}$

$\boxed{\text{Req prob} = 0.27}$ Ans

Qu: 15 →

A → the two balls drawn are both white

$E_1$ → ~~can~~ Bag contains 2 white & 2 non white balls

$E_2$ → bag contain 3 white & 1 non-white balls

$E_3$ → bag contains 4 white balls

$P(E_1) = 1/3$ $\quad$ $\cancel{P(A/E_2)}$ $P(E_2) = 1/3$ ; $P(E_3) = 1/3$

$P(A/E_1) = \frac{2C_2}{4C_2}$ ; $P(A/E_2) = \frac{3C_2}{4C_2}$ ; $P(A/E_3) = \frac{4C_2}{4C_2}$

By Bayes theorem

Req prob

$$P(E_3/A) = \frac{P(E_3) \cdot P(A/E_3)}{P(E_1) \cdot P(A/E_1) + P(E_2)P(A/E_2) + P(E_3)P(A/E_3)}$$

$$= \frac{\cancel{\frac{1}{3}} \times \frac{4C_2}{\cancel{4C_2}}}{\left(\cancel{\frac{1}{3}} \times \frac{2C_2}{\cancel{4C_2}}\right) + \left(\cancel{\frac{1}{3}} \times \frac{3C_2}{\cancel{4C_2}}\right) + \left(\cancel{\frac{1}{3}} \times \frac{4C_2}{\cancel{4C_2}}\right)} = \frac{6}{1+3+6} = \frac{3}{5} \text{ Ans}$$

—✕—