SOLUTIONS **WORKSHEET NO: 3** PROBABILITY (12th)

Ans: 1 → given p.d

x	0	1	2
$P(x)$	$3c^3$	$4c-10c^2$	$5c-1$

we have $\Sigma p_i = 1$

$\Rightarrow 3c^3 + 4c - 10c^2 + 5c - 1 = 1$

$\Rightarrow 3c^3 - 10c^2 + 9c - 2 = 0$

Hit & Trial Method

$c-1$ is the factor of this polynomial {$c=1$ satisfy the equation}

$$
\begin{array}{r}
3c^2 - 7c + 2 \\
c-1 \overline{) 3c^3 - 10c^2 + 9c - 2} \\
-(3c^3 - 3c^2) \\
\hline
-7c^2 + 9c - 2 \\
-(-7c^2 + 7c) \\
\hline
2c - 2 \\
2c - 2 \\
\hline
\times
\end{array}
$$

$\Rightarrow (c-1)(3c^2 - 7c + 2) = 0$

$\Rightarrow c - 1 = 0$ $\boxed{c=1}$ | $3c^2 - 7c + 2 = 0$

$(3c-1)(c-2) = 0$

$\boxed{c = 1/3}$ or $\boxed{c=2}$

$c=1$ & $c=2$ rejected

$\because P(x=0) = 3c^3$ will become more than 1 (not possible)

(i) $c = 1/3$

(ii) $P(x<2) = P(x=0) + P(x=1)$

$= 3c^3 + 4c - 10c^2$

$= 3\left(\frac{1}{3}\right)^3 + 4\left(\frac{1}{3}\right) - 10\left(\frac{1}{3}\right)^2$

$= \frac{1}{9} + \frac{4}{3} - \frac{10}{9}$

$= \frac{3}{9} = 1/3$ Ans

(iii) $P(1 < X \leq 2) = P(X=2)$

$= 5C - 1$

$= \frac{5}{3} - 1 = \frac{2}{3}$ Ans

Ans 2 →

Given $2P(X=x_1) = 3P(X=x_2) = P(X=x_3) = 5P(X=x_4)$

let $P(X=x_4) = p$

$\therefore P(X=x_1) = \frac{5p}{2}$; $P(X=x_2) = \frac{5p}{3}$; $P(X=x_3) = 5p$

we have $\Sigma p_i = 1$

$\Rightarrow P(X=x_1) + P(X=x_2) + P(X=x_3) + P(X=x_4) = 1$

$\Rightarrow \frac{5p}{2} + \frac{5p}{3} + 5p + p = 1$

$\Rightarrow \frac{15p + 10p + 36p}{6} = 1$

$\Rightarrow 61p = 6$

$\Rightarrow \boxed{p = \frac{6}{61}}$

Now $P(X=x_1) = \frac{30}{122}$; $P(X=x_2) = \frac{30}{183}$; $P(X=x_3) = \frac{30}{61}$

$P(X=x_4) = \frac{6}{61}$

∴ P.D

X	x_1	x_2	x_3	x_4
$P(X)$	$15/61$	$10/61$	$30/61$	$6/61$

Ans

Ans 3 → let X → denotes the number of face cards

X → can take values 0, 1, 2

$P(X=0) = P(\text{getting no face cards}) = \frac{{}^{40}C_2}{{}^{52}C_2}$

$$= \frac{40 \times 39}{52 \times 51} = \frac{130}{221}$$

$P(x=1) = P(\text{getting } 1 \text{ face card}) = \dfrac{40C_1 \times 12C_1}{52C_2}$

$$= \frac{40 \times 12}{\frac{52 \times 51}{2}} = \frac{40 \times 12 \times 2}{52 \times 51} = \frac{80}{221}$$

$P(x=2) = P(\text{getting } 2 \text{ face cards}) = \dfrac{12C_2}{52C_2}$

$$= \frac{12 \times 11}{52 \times 51} = \frac{11}{221}$$

$\therefore$ P.D

x	0	1	2
P(x)	$\frac{130}{221}$	$\frac{80}{221}$	$\frac{11}{221}$

Ans

Ans 4 → total bolts = 25
defective = 5 ; good = 20

let X → denotes the number of defective bolts
X → can take values 0, 1, 2, 3, 4

$P(x=0) = P(\text{no defective bolt}) = \dfrac{20C_4}{25C_4} = \dfrac{\frac{20 \times 19 \times 18 \times 17}{24}}{\frac{25 \times 24 \times 23 \times 22}{24}} = \dfrac{969}{2530}$

$P(x=1) = P(1 \text{ defective bolt}) = \dfrac{5C_1 \times 20C_3}{25C_4} = \dfrac{5 \times \frac{20 \times 19 \times 18}{6}}{\frac{25 \times 24 \times 23 \times 22}{24}} = \dfrac{1140}{2530}$

$P(x=2) = P(2 \text{ defective}) = \dfrac{5C_2 \times 20C_2}{25C_4} = \dfrac{\frac{5 \times 4}{2} \times \frac{20 \times 19}{2}}{\frac{25 \times 24 \times 23 \times 22}{24}} = \dfrac{380}{2530}$

$P(X=3) = P(3 \text{ defective bolts}) = \dfrac{{}^5C_3 \times {}^{20}C_1}{{}^{25}C_4} = \dfrac{\frac{5\times4\times3}{6}\times 20}{\frac{25\times24\times23\times22}{24}} = \dfrac{40}{2530}$

$P(X=4) = P(4 \text{ defective bolts}) = \dfrac{{}^5C_4}{{}^{25}C_4} = \dfrac{\frac{5\times4\times3\times2}{24}}{\frac{25\times24\times23\times22}{24}} = \dfrac{1}{2530}$

P.D

X	0	1	2	3	4
$P(X)$	$\frac{969}{2530}$	$\frac{1140}{2530}$	$\frac{380}{2530}$	$\frac{40}{2530}$	$\frac{1}{2530}$

Ans

Qn. 5 → let X → denotes the minimum number

X → can take values 1, 2, 3, 4, 5, 6

$P(X=1) = P\{(1,1), (1,2) (2,1) (1,3) (3,1) (1,4) (4,1) (1,5) (5,1) (1,6) (6,1)\}$
$= \frac{11}{36}$

$P(X=2) = P\{(2,2), (2,3) (3,2), (4,2), (2,4) (5,2), (2,5), (6,2) (2,6)\} = \frac{9}{36}$

$P(X=3) = P\{(3,3) (3,4) (4,3) (5,3) (3,5), (6,3) (3,6)\} = \frac{7}{36}$

$P(X=4) = P\{(4,4) (4,5) (5,4) (6,4) (4,6)\} = \frac{5}{36}$

$P(X=5) = P\{(5,5) (5,6) (6,5)\} = \frac{3}{36}$

$P(X=6) = P\{(6,6)\} = \frac{1}{36}$

X	1	2	3	4	5	6
$P(X)$	$11/36$	$9/36$	$7/36$	$5/36$	$3/36$	$1/36$

Ans

Ans 6 →

A → the coin drawn is gold

E_1 → Box I is chosen

E_2 → Box II is chosen

E_3 → box III is chosen

$P(E_1) = P(E_2) = P(E_3) = 1/3$

$P(A|E_1) = \frac{2}{2} = 1$

$P(A|E_2) = \frac{0}{2} = 0$

$P(A|E_3) = \frac{1}{2}$

Required probability : $P(E_1/A) = \dfrac{P(E_1)\,P(A|E_1)}{P(E_1)\,P(A|E_1) + P(E_2)\,P(A|E_2) + P(E_3)\,P(A|E_3)}$

$$= \frac{\frac{1}{3}\times 1}{\left(\frac{1}{3}\times 1\right) + 0 + \left(\frac{1}{3}\times\frac{1}{2}\right)}$$

$$= \frac{1}{1+\frac{1}{2}} = \frac{1}{\frac{3}{2}} = \frac{2}{3} \text{ Ans}$$

Ans 7 →

A → New product is introduced

E_1 → I^{st} group wins

E_2 → 2^{nd} group wins

$P(E_1) = 0.6$ & $P(E_2) = 0.4$

$P(A|E_1) = 0.7$ & $P(A|E_2) = 0.3$

Required prob : $P(E_2/A) = \dfrac{P(E_2)\,P(A|E_2)}{P(E_1)\,P(A|E_1) + P(E_2)\cdot P(A|E_2)}$

$$= \frac{0.4 \times 0.3}{(0.6 \times 0.7) + (0.4 \times 0.3)}$$

$$= \frac{0.12}{0.42 + 0.12} = \frac{0.12}{0.54} = \frac{12}{54} = \frac{2}{9} \quad \underline{\text{Ans}}$$

Qn 8 → A → Student chosen has an A grade

E_1 → the Student is a hostler

E_2 → the Student is a day Scholar

$P(E_1) = \frac{60}{100}$ & $P(E_2) = \frac{40}{100}$

$P(A|E_1) = \frac{30}{100}$ & $P(A|E_2) = \frac{20}{100}$

Required prob $P(E_1/A) = \dfrac{P(E_1) \cdot P(A/E_1)}{P(E_1) \cdot P(A/E_1) + P(E_2) P(A/E_2)}$

$$= \frac{\frac{60}{\cancel{100}} \times \frac{30}{\cancel{100}}}{\left(\frac{60}{\cancel{100}} \times \frac{30}{\cancel{100}}\right) + \left(\frac{40}{\cancel{100}} \times \frac{20}{\cancel{100}}\right)}$$

$$= \frac{1800}{1800 + 800} = \frac{18}{26} = \frac{9}{13}$$

Qn-9 → A → She obtained exactly one head

E_1 → She threw 1, 2, 3, 4 on the dice

E_2 → She threw 5 or 6 on the dice

$P(E_1) = \frac{4}{6}$; $P(E_2) = 2/6$

$P(A/E_1) = \frac{1}{2}$

$P(A/E_2) = \frac{3}{8}$

Req prob $P(E_1/A) = \dfrac{P(E_1)\,P(A/E_1)}{P(E_1)\,P(A/E_1) + P(E_2)\,P(A/E_2)}$

$= \dfrac{\frac{4}{8} \times \frac{1}{2}}{\left(\frac{4}{8} \times \frac{1}{2}\right) + \left(\frac{2}{8} \times \frac{3}{8}\right)}$

$= \dfrac{4}{4 + \frac{3}{2}} = \dfrac{8}{11}$ Ans

Qn 10 → $A \rightarrow$ A grey haired person is chosen

$E_1 \rightarrow$ the person selected is a male

$E_2 \rightarrow$ the person selected is a female

$P(E_1) = 1/2$ & $P(E_2) = 1/2$

$P(A/E_1) = \frac{5}{100}$ & $P(A/E_2) = \frac{0.25}{100}$

Req prob $P(E_1/A) = \dfrac{\cancel{\frac{1}{2}} \times \frac{5}{\cancel{100}}}{\left(\cancel{\frac{1}{2}} \times \frac{5}{\cancel{100}}\right) + \left(\cancel{\frac{1}{2}} \times \frac{0.25}{\cancel{100}}\right)}$

$= \dfrac{5}{5.25} = \dfrac{5}{\frac{21}{4}} = \dfrac{20}{21}$ Ans

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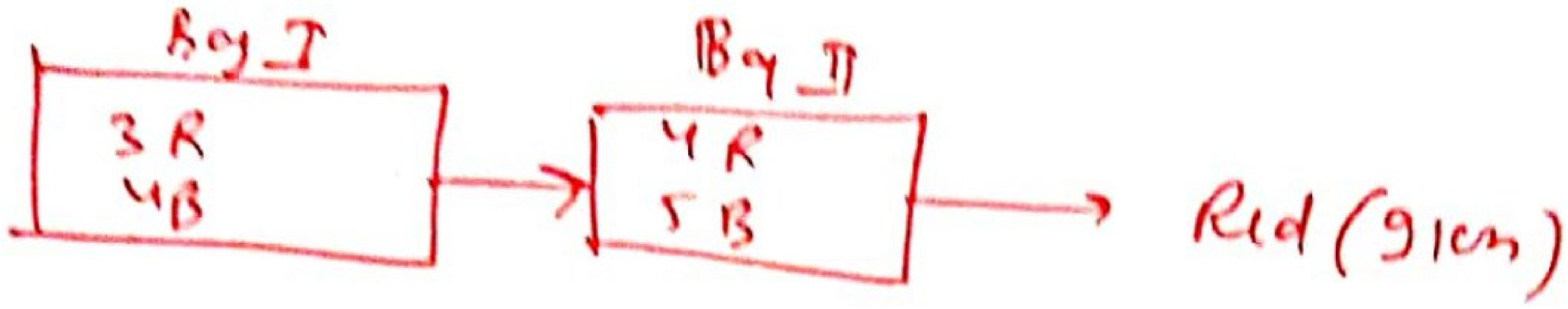
Qn 11 →



Let A → ball drawn from Bag II is found to be Red

E_1 → Red ball is transferred from Bag I to Bag II

E_2 → Black ball is transferred from I to II

$P(E_1) = 3/7$; $P(E_2) = \frac{4}{7}$

$P(A/E_1) = \frac{5}{10}$ & $P(A/E_2) = \frac{4}{10}$

Required $P(E_2/A) = \dfrac{P(E_2)\cdot P(A/E_2)}{P(E_1)\cdot P(A/E_1) + P(E_2)P(A/E_2)}$

$$= \frac{\frac{4}{7} \times \frac{4}{10}}{\left(\frac{3}{7} \times \frac{5}{10}\right) + \left(\frac{4}{7} \times \frac{4}{10}\right)}$$

$$= \frac{16}{15+16} = \frac{16}{31}$$ **Ans**

Qn 12 → A → two cards drawn from remaining cards are both diamonds

E_1 → the card lost is a diamond

E_2 → the card lost is not a diamond

$P(E_1) = \frac{13}{52}$

$P(E_2) = \frac{39}{52}$

$P(A/E_1) = \frac{^{12}C_2}{^{51}C_2}$

$P(A/E_2) = \frac{^{13}C_2}{^{51}C_2}$

Req prob $P(E_1/A) = \dfrac{\frac{13}{\cancel{52}} \times \frac{^{12}C_2}{\cancel{^{51}C_2}}}{\left(\frac{13}{\cancel{52}} \times \frac{^{12}C_2}{\cancel{^{51}C_2}}\right) + \left(\frac{39}{\cancel{52}} \times \frac{^{13}C_2}{\cancel{^{51}C_2}}\right)}$

$= \dfrac{\cancel{13} \times \frac{\cancel{12} \times 11}{\cancel{2}}}{\left(\cancel{13} \times \frac{\cancel{12} \times 11}{\cancel{2}}\right) + \left(39 \times \frac{\cancel{13} \times \cancel{12}}{\cancel{2}}\right)}$

$= \dfrac{11}{11 + 39}$

$= \dfrac{11}{50}$ Ans

— x —