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CHAPTER: PROBABILITY (12M) | CLASS NO: 5

Q.No: 1 → A and B throw a die alternatively till one of them gets a '6' and wins the game. Find their respective probabilities of winning, If A starts first.

Solution

If A starts first,

A will get chances 1st, 3rd, 5th, 7th & so-on

Prob of winning of A in his 1st chance $= \frac{1}{6}$

" " " A " " 3rd chance $= \frac{5}{6} \times \frac{5}{6} \times \frac{1}{6} = \frac{25}{216}$

" " " A " " 5th chance $= \left(\frac{5}{6}\right)^2 \times \frac{1}{6}$

and so-on $= \left(\frac{5}{6}\right)^4 \times \frac{1}{6}$

$$P(\text{winning of } A) = \frac{1}{6} + \left(\frac{5}{6}\right)^2 \times \frac{1}{6} + \left(\frac{5}{6}\right)^4 \times \frac{1}{6} + \cdots \infty$$

← G.P: $a = \frac{1}{6}$; $r = \frac{25}{36}$

$$P(A) = \frac{a}{1-r} = \frac{\frac{1}{6}}{1-\frac{25}{36}} = \frac{\frac{1}{6}}{\frac{11}{36}} = \frac{6}{11}$$

$\therefore \boxed{P(A) = \frac{6}{11}}$

We know $P(A) + P(B) = 1$

$P(B) = 1 - \frac{6}{11} = \frac{5}{11}$ $\therefore \boxed{P(B) = \frac{5}{11}}$ Ans

Qn 2 → If each element of a second order determinant is either zero or one, what is the probability that the value of the determinant is positive?

Soln = Total No of ways = $\begin{vmatrix} 0/1 & 0/1 \\ 0/1 & 0/1 \end{vmatrix}$

$= 2 \times 2 \times 2 \times 2 = 16$

fav ways $\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 1\ (+ve)$

$\begin{vmatrix} 1 & 0 \\ 1 & 1 \end{vmatrix} = 1\ (+ve)$

$\begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix} = 1\ (+ve)$

fav ways = 3

Req prob = $\frac{3}{16}$ Ans

Qn 3 → A & B are two candidates seeking admission in a college. The probability that A is selected is 0.7 and the probability that "exactly one of them" is selected is 0.6. Find the probability that B is selected.

Soln = let A → person A is selected

B → " B " "

Given $P(A) = 0.7$

$P(\text{exactly one selected}) = 0.6$

To find: $P(B) = ?$

$\Rightarrow P(A \cap B') + P(B \cap A') = 0.6$

IMP $\Rightarrow P(A) \cdot P(B') + P(B) \cdot P(A') = 0.6$ --- { ∵ A & B are Independent events ∴ A & B' & B & A' are also Independently

$\Rightarrow 0.7[1 - P(B)] + P(B)(0.3) = 0.6$

$\Rightarrow 0.7 - 0.7P(B) + 0.3P(B) = 0.6$

$\Rightarrow 0.1 = 0.4P(B)$

$\Rightarrow P(B) = \frac{0.1}{0.4} = \frac{1}{4} = 0.25$ Ans

Qn. 4 → A committee of 4 students is selected at random from a group consisting 8 boys and 4 girls. Given that there is atleast one girl on the committee, calculate the probability that there are exactly 2 girls on the committee.

Soln. Let A → Select exactly 2 girls

B → there is atleast one girl

$A \cap B = A$

$P(A \cap B) = P(A)$ → 2G, 2B

$P(A \cap B) = P(A) = \frac{{}^4C_2 \times {}^8C_2}{{}^{12}C_4} = \frac{6 \times 28 \times 24}{12 \times 11 \times 10 \times 9} = \square$

$P(B) = 1 - P(\text{none})$ → none of girl

$P(B) = 1 - \frac{{}^8C_4}{{}^{12}C_4} = 1 - \left(\frac{8 \times 7 \times 6 \times 5}{12 \times 11 \times 10 \times 9}\right) = \square$

$P(A|B) = \frac{P(A \cap B)}{P(B)}$

$= \frac{168}{421}$ Ans

Ans 5 → An urn contains 'm' white and 'n' black balls. A ball is drawn at random and is put back in to the urn along with 'k' additional balls of the same colour as that of the ball drawn. A ball is again drawn at random. Show that the probability of drawing a white ball now does not depend on 'k'.

Sol:

$A \rightarrow$ gets white ball (2^{nd} time)

$E_1 \rightarrow$ I^{st} ball was white

$E_2 \rightarrow$ I^{st} ball was black.

$$P(E_1) = \frac{m}{m+n} \quad ; \quad P(E_2) = \frac{n}{m+n}$$

$$P(A/E_1) = \frac{m+k}{m+n+k} \quad ; \quad P(A/E_2) = \frac{m}{m+n+k}$$

By total law of prob

$$P(A) = P(E_1) \cdot P(A/E_1) + P(E_2)\, P(A/E_2)$$

$$= \frac{m}{m+n} \times \frac{m+k}{m+n+k} + \frac{n}{m+n} \times \frac{m}{m+n+k}$$

$$= \frac{m(m+k) + mn}{(m+n)(m+n+k)}$$

$$= \frac{m(\cancel{m+k+n})}{(m+n)(\cancel{m+n+k})}$$

$P(A) = \frac{m}{m+n}$ which is clearly independent of k

Any

Qn 6 → A Shopkeeper sells three types of flower seeds A_1, A_2 & A_3. They are sold as a mixture where the proportions are 4:4:2 respectively. The germination rates of the three types of seeds are 45%, 60% and 35%. Calculate the probability

(i) of a randomly chosen seed to germinate.

(ii) that it will not germinate given that the seed is of type A_3

(iii) that it is of type A_2, given that a randomly chosen seed does not germinate.

Soln (i) $A \rightarrow$ Seed will germinate (Req'd)

$E_1 \rightarrow$ Seed is of type A_1

$E_2 \rightarrow$ " " " type A_2

$E_3 \rightarrow$ " " " type A_3

$P(E_1) = \frac{4}{10}$; $P(E_2) = \frac{4}{10}$; $P(E_3) = \frac{2}{10}$

$P(A|E_1) = \frac{45}{100}$; $P(A|E_2) = \frac{60}{100}$; $P(A|E_3) = \frac{35}{100}$

Req'd (By total law of prob)

$P(A) = P(E_1) \cdot P(A|E_1) + P(E_2) \cdot P(A|E_2) + P(E_3) \cdot P(A|E_3)$

$= \frac{180}{1000} + \frac{240}{1000} + \frac{70}{1000} = \frac{490}{1000} = \frac{49}{100}$

(ii) Req'd $P(A'|E_3) = 1 - P(A|E_3) = 1 - \frac{35}{100} = \frac{65}{100}$

(iii) Bayes theorm

$P(A'/B) = 1 - P(A/B)$

$P(A/B') \neq 1 - P(A/B)$

Given: $A' \rightarrow$ seed not germinate

Ref fact $P(E_2/A') = \dfrac{P(E_2) \cdot P(A'/E_2)}{P(E_1)\cdot P(A'/E_1) + P(E_2)\,P(A'/E_2) + P(E_3)\cdot P(A'/E_3)}$

$P(A'/E_1) = 1 - P(A/E_1) = 1 - \frac{45}{100} = \frac{55}{100}$

$P(A'/E_2) = 1 - P(A/E_2) = 1 - \frac{60}{100} = \frac{40}{100}$

$P(A'/E_3) = 1 - \frac{3}{100} = \frac{97}{100}$

value put (Ans)

Qn.7 ⇒ A and B are two students. Their chances of solving a problem correctly are $\frac{1}{3}$ and $\frac{1}{4}$ respectively. If the probability of their making a common error is $\frac{1}{20}$, and they obtain the same answer. Find the probability that their answer is correct.

Sol $A \rightarrow$ they obtain the same answer

$E_1 \rightarrow$ their answer is correct (both correct)

(Bayes theorem) $E_2 \rightarrow$ one of them is correct

$E_3 \rightarrow$ both are incorrect

$P(E_1) = \frac{1}{3} \times \frac{1}{4} = \frac{1}{12}$

$P(E_2) = \left(\frac{1}{3} \times \frac{3}{4}\right) + \left(\frac{2}{3} \times \frac{1}{4}\right) = \frac{5}{12}$

$P(E_3) = \frac{2}{3} \times \frac{3}{4} = \frac{6}{12}$

$P(A/E_1) = 1$

$P(A/E_2) = 0.$

$P(A/E_3) = \frac{1}{20}$

$P(E_1/A) = \dfrac{\frac{1}{12} \times 1}{\left(\frac{1}{12} \times 1\right) + 0 + \left(\frac{6}{12} \times \frac{1}{20}\right)}$

$= \dfrac{1}{1 + \frac{6}{20}} = \dfrac{20}{26} = \frac{10}{13}$ Ans

Qn 8 → Find the probability distribution of the number of success in two tosses of a die. where a success is defined as

(i) number greater than 4

(ii) Six appears on atleast one dice

Soln (i) Success → No more than 4 (Single dice)

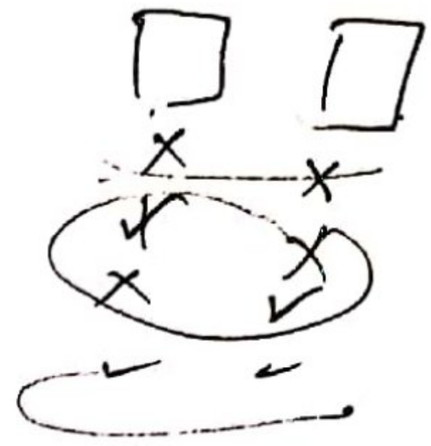
outcomes = {5, 6}

$P(\text{success}) = \frac{2}{6} = 1/3$

$P(\text{not success}) = \frac{4}{6} = 2/3$

let X → denotes the number of success

X → Can take value 0, 1, 2



$P(X=0) = P(\text{no success}) = \frac{2}{3} \times \frac{2}{3} = \frac{4}{9}$

$P(X=1) = P(1 \text{ success}) = \left(\frac{1}{3} \times \frac{2}{3}\right) + \left(\frac{2}{3} \times \frac{1}{3}\right) = \frac{4}{9}$

$P(X=2) = P(2 \text{ success}) = \frac{1}{3} \times \frac{1}{3} = \frac{1}{9}$

P.D

X	0	1	2
P(x)	4/9	4/9	1/9

Ans

Special

(ii) Success → 6 apears on atleast one dice

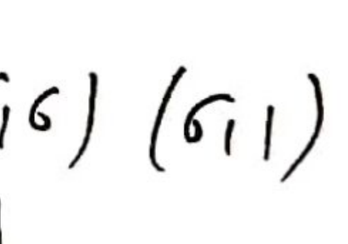
Sample space = {(1,6) (2,6) (3,6) (4,6) (5,6) (6,6) (6,1) (6,2) (6,3) (6,4) (6,5)}

$P(\text{success}) = \frac{11}{36}$

$P(\text{not success}) = \frac{25}{36}$

X → denotes the no. of success

X → can take value 0, 1



X

$P(X=0) = \frac{25}{36}$

$P(X=1) = \frac{11}{36}$

X	0	1
P(x)	25/36	11/36

Ans

WORKSHEET No: 4 (class No: 5)

PROBABILITY (12th)

Qn: 1 → The probability of simultaneous occurence of atleast one of the two events A & B is p. If the probability that exactly one of A & B occurs is q, then prove that $P(A') + P(B') = 2 - 2p + q$

Qn. 2 → Three machines E_1, E_2, E_3 in a certain factory produce 50%, 25% and 25% respectively of the total daily output of electric tubes. It is known that 4% of the tubes produced one each of machines E_1 and E_2 are defective and that 5% of those produced on E_3 are defective. If one tube is picked up at random from a day's production. Calculate the probability that it is defective

Ans $\frac{17}{400}$

Qn: 3 → For a loaded die;

$P(1) = P(2) = 0.2$

$P(3) = P(5) = P(6) = 0.1$

$P(4) = 0.3$

The die is thrown two times. Let A & B be the events "same number on each dice" and a "total score is more than 9" respectively. Determine whether A & B are Independent or not?

Qn 4 → A bag contains 5 red marbles and 3 black marbles. Three marbles are drawn one by one without replacement. What is the probability that atleast one of the three marbles drawn be black, if the first marble is red? Ans $\frac{25}{36}$

Qn. 5 → Three dice are thrown at the same time. Find the probability of getting three two's, if it is known that the sum of the numbers on the dice was Six Ans $\frac{1}{10}$

Qn. 6 → A bag Contains 4 W & 5 B balls. Another bag Contains 9 W & 7 B balls. A ball is transferred from the first bag to the second and then a ball is drawn at random from the second bag. Find the probability that the ball drawn is white Ans $\frac{85}{153}$

Qn. 7 → Suppose that 6% of the people with blood group O are left handed and 10% of those with other blood groups are left handed. 30% of the people have blood group O. If a left handed person is selected at random, what is the probability that he/she will have blood group O? Ans $\frac{9}{44}$

Qn. 8 → By examining a chest X-ray, the probability that T.B is detected when a person is actually suffering is 0.99. The probability of an healthy person diagnosed to have T.B is 0.001. In a certain city, 1 in 1000 people suffers from T.B. A person is selected at random and is diagnosed to have T.B. what is the probability that he actually has T.B ?

Ans $\frac{110}{221}$

Qn. 9 → A bag contains (2n+1) coins. If it is known that 'n' of these coins have a head on both sides whereas the rest coins are fair. A coin is picked up at random from the bag and is tossed. If the probability that the toss results in a head is $\frac{31}{42}$. Determine the value of 'n'

Ans $n = 10$

Qn 10 → A and B throw a pair of dice alternately. A wins the game if he gets a total of 6 and B wins if he gets a total of 7. If A starts first. Find the ratio of respective winning probabilities

Ans 30:31

Qn 11 → Let A and B are independent events. The probability of there simultaneous occurence is $\frac{1}{8}$ and the probability that neither occurs is $\frac{3}{8}$. Find P(A) and P(B)

Ans $\frac{1}{2}$ and $\frac{1}{4}$ (or) $\frac{1}{4}$ and $\frac{1}{2}$

Qn 12 → Three persons A, B, C throw a dice in succession till one gets a 'five' and wins the game. Find their respective probabilities of winning

If A starts first Ans $\frac{36}{91}$, $\frac{30}{91}$, $\frac{25}{91}$

Qn 13 → Two Integers are selected at random from Integers 1 to 11. If the sum is even find the probabilities that both the numbers are odd. Ans = $\frac{3}{5}$

Qn 14 → Two thirds of the students in a class are boys and the rest are girls. It is known that the probability of a girl getting a first class is 0.25 and that of a boy getting a first class is 0.28. Find the probability that a student chosen at random will get first class marks in the subject

Ans 0.27

Qn 15 → A bag contains 4 balls. Two balls are drawn at random and are found to be white. What is the probability that all the balls are white? Ans = $\frac{3}{5}$

HINT : Bayes theorem

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