ULTIMATE MATHEMATICS ①

Solution of worksheet No: 2

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Differentiation and Continuity

Ans 1 → $x^{16} \cdot y^9 = (x^2+y)^{17}$

taking log on both sides

$$\log(x^{16} \cdot y^9) = \log(x^2+y)^{17}$$

$$\Rightarrow 16\log x + 9\log y = 17\log(x^2+y)$$

Diff w.r.t x

$$16 \cdot \frac{1}{x} + 9 \cdot \frac{1}{y} \cdot \frac{dy}{dx} = 17 \cdot \frac{1}{x^2+y} \cdot \left(2x + \frac{dy}{dx}\right)$$

$$\frac{dy}{dx}\left(\frac{9}{y} - \frac{17}{x^2+y}\right) = \frac{34x}{x^2+y} - \frac{16}{x}$$

$$\Rightarrow \frac{dy}{dx}\left(\frac{9x^2+9y-17y}{\cancel{(x^2+y)}\,y}\right) = \frac{34x^2-16x^2-16y}{\cancel{(x^2+y)}\,x}$$

$$\Rightarrow \frac{dy}{dx}\left(\frac{9x^2-8y}{y}\right) = \frac{18x^2-16y}{x}$$

$$\Rightarrow \frac{dy}{dx}\left(\frac{\cancel{9x^2-8y}}{y}\right) = \frac{2\,\cancel{(9x^2-8y)}}{x}$$

$$\Rightarrow \frac{dy}{dx} = \frac{2y}{x} \quad \text{Ans}$$

Qn. 2 → $x^{13} \cdot y^7 = (x+y)^{20}$

Same as Qn: 1 (Do yourself)

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Qn 3 → $y = \sqrt{x} \cdot \dfrac{(x+4)^{3/2}}{(4x-3)^{4/3}}$

taking log on both sides

$$\log y = \log\left[\frac{x^{1/2} \cdot (x+4)^{3/2}}{(4x-3)^{4/3}}\right]$$

$$\log y = \frac{1}{2}\log x + \frac{3}{2}\log(x+4) - \frac{4}{3}\log(4x-3)$$

Diff wrt x

$$\frac{1}{y}\cdot\frac{dy}{dx} = \frac{1}{2}\cdot\frac{1}{x} + \frac{3}{2}\cdot\frac{1}{x+4} - \frac{4}{3}\cdot\frac{1}{4x-3}(4)$$

$$\frac{dy}{dx} = y\left\{\frac{1}{2x} + \frac{3}{2(x+4)} - \frac{16}{3(4x-3)}\right\}$$

$$\frac{dy}{dx} = \frac{\sqrt{x}\,(x+4)^{3/2}}{(4x-3)^{4/3}}\left[\frac{1}{2x} + \frac{3}{2(x+4)} - \frac{16}{3(4x-3)}\right] \quad \underline{\text{Ans}}$$

Qn 4 → $xy\log(x+y) = 1$

taking log on both sides

$$\log(xy\log(x+y)) = \log 1$$

$$\Rightarrow \log x + \log y + \log(\log(x+y)) = 0$$

Diff wrt x

$$\frac{1}{x} + \frac{1}{y}\cdot\frac{dy}{dx} + \frac{1}{\log(x+y)}\cdot\frac{1}{x+y}\cdot\left(1+\frac{dy}{dx}\right) = 0$$

$$\Rightarrow \frac{dy}{dx}\left(\frac{1}{y} - \frac{1}{(x+y)\log(x+y)}\right) = \frac{1}{(x+y)\log(x+y)} - \frac{1}{x}$$

$$\frac{dy}{dx}\left(\frac{(x+y)\log(x+y) - y}{y\cancel{(x+y)}\cancel{\log(x+y)}}\right) = \frac{x-(x+y)\log(x+y)}{\cancel{(x+y)}\,x\,\cancel{\log(x+y)}}$$

$$\Rightarrow \frac{dy}{dx}\left(\frac{(x+y)\frac{1}{xy} - y}{y}\right) = \frac{x-(x+y)\left(\frac{1}{xy}\right)}{x}$$

$$\cdots\left\{\because xy\log(x+y) = 1 \Rightarrow \log(x+y) = \frac{1}{xy}\right\}$$

$$\Rightarrow \frac{dy}{dx}\left(\frac{x+y-xy^2}{y\,\cancel{(xy)}}\right) = \frac{x^2y - x - y}{\cancel{(xy)}\,x}$$

$$\Rightarrow \frac{dy}{dx} = \frac{x^2y^2 - xy - y^2}{x^2+yx-x^2y^2} \quad \underline{\text{Ans}}$$

= [illegible] (worksheet me misprint hai)

Qn 5 $\quad x = a\sec^3\theta$

$y = a\tan^3\theta$

Diff wrt θ

$$\frac{dx}{d\theta} = 3a\sec^2\theta\cdot\sec\theta\tan\theta$$

$$\frac{dy}{d\theta} = 3a\tan^2\theta\cdot\sec^2\theta$$

$$\frac{dy}{dx} = \frac{\cancel{3a}\tan^{\cancel{2}}\theta\,\cancel{\sec^2\theta}}{\cancel{3a}\cancel{\sec^2\theta}\sec\theta\cancel{\tan\theta}} = \frac{\tan\theta}{\sec\theta}$$

$$\frac{dy}{dx} = \sin\theta \Rightarrow \left(\frac{dy}{dx}\right) = \sqrt{3} \quad \text{Ans}$$

$\left(\frac{d^2y}{dx^2}\right)_{\theta=\pi/3}$

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Q N. 6 → $x = a(\cos\theta + \theta\sin\theta)$

$y = a(\sin\theta - \theta\cos\theta)$

Diff wrt θ

$\frac{dx}{d\theta} = a(-\cancel{\sin\theta} + \theta\cos\theta + \cancel{\sin\theta})$

$\frac{dx}{d\theta} = a\theta\cos\theta$

$\frac{dy}{d\theta} = a(\cos\theta - (-\theta\sin\theta + \cos\theta))$

$\frac{dy}{d\theta} = a\theta\sin\theta$

$\frac{dy}{dx} = \frac{\cancel{a\theta}\sin\theta}{\cancel{a\theta}\cos\theta} = \tan\theta$

Qn 7 → $x = ae^{\theta}(\sin\theta - \cos\theta)$

$y = ae^{\theta}(\sin\theta + \cos\theta)$

Diff wrt θ

$\frac{dx}{d\theta} = a[e^{\theta}(\cos\theta + \sin\theta) + (\sin\theta - \cos\theta)e^{\theta}]$

$\frac{dx}{d\theta} = a(2e^{\theta}\sin\theta)$

$\frac{dx}{d\theta} = 2ae^{\theta}\sin\theta$

$\frac{dy}{d\theta} = a[e^{\theta}(\cos\theta - \sin\theta) + (\sin\theta + \cos\theta)\cdot e^{\theta}]$

$$\frac{dy}{d\theta} = 2ae^{\theta}\cos\theta$$

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$$\frac{dy}{dx} = \frac{\cancel{2ae^{\theta}}\cos\theta}{\cancel{2ae^{\theta}}\sin\theta} = \cot\theta$$

Q. 8 → $x = e^{\cos(2t)}$
$y = e^{\sin(2t)}$

taking log on both sides

$\log x = \cos(2t)\log e$

$\log x = \cos(2t)$ --- $\{\log e = 1\}$

$\log y = \sin(2t)$

Diff w.r.t t

$$\frac{1}{x}\frac{dx}{dt} = -\sin(2t)\cdot 2$$

$$\frac{dx}{dt} = -2x\sin(2t)$$

$$\frac{dx}{dt} = -2x\log y \quad -- \left\{\because \log y = \sin(2t)\right\}$$

$$\frac{1}{y}\frac{dy}{dt} = 2\cos(2t)$$

$$\frac{dy}{dt} = 2y\cos(2t)$$

$$\frac{dy}{dt} = 2y\log x$$

Now $\frac{dy}{dx} = \frac{\cancel{2}y\log x}{\ldots} = -y\log x$ Ans

$$= \frac{dy}{dx} \quad \frac{\quad}{-\cancel{2}x \log y} \qquad \frac{\quad}{x \log y}$$

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Qn. 9 → $x = \frac{1-t^2}{1+t^2}$, $y = \frac{2t}{1+t^2}$

Diff wrt t (Quotient rule)

$$\frac{dx}{dt} = \frac{(1+t^2)(-2t) - (1-t^2)(2t)}{(1+t^2)^2}$$

$$= \frac{-2t - \cancel{2t^3} - 2t + \cancel{2t^3}}{(1+t^2)^2}$$

$$\frac{dx}{dt} = \frac{-4t}{(1+t^2)^2}$$

$$\frac{dy}{dt} = \frac{(1+t^2)(2) - (2t)(2t)}{(1+t^2)^2}$$

$$= \frac{2 + 2t^2 - 4t^2}{(1+t^2)^2}$$

$$\frac{dy}{dt} = \frac{2-2t^2}{(1+t^2)^2}$$

$$\frac{dy}{dx} = \frac{\frac{2(1-t^2)}{\cancel{(1+t^2)^2}}}{\frac{-4t}{\cancel{(1+t^2)^2}}}$$

$$= -\frac{(1-t^2)}{2t}$$

$$\frac{dy}{dx} = \frac{t^2-1}{2t} \quad \underline{\text{Ans}}$$

Solution W.S 2

Qn. 10 → $x = e^{\theta}\left(\theta + \frac{1}{\theta}\right)$

$y = e^{-\theta}\left(\theta - \frac{1}{\theta}\right)$

Diff wrt

$$\frac{dx}{d\theta} = e^{\theta}\left(1 - \frac{1}{\theta^2}\right) + \left(\theta + \frac{1}{\theta}\right)e^{\theta}$$

$$= e^{\theta}\left(1 - \frac{1}{\theta^2} + \theta + \frac{1}{\theta}\right)$$

$$= e^{\theta}\left(\frac{\theta^2 - 1 + \theta^3 + \theta}{\theta^2}\right)$$

$$\frac{dy}{d\theta} = e^{-\theta}\left(1 + \frac{1}{\theta^2}\right) + \left(\theta - \frac{1}{\theta}\right)e^{-\theta}(-1)$$

$$= e^{-\theta}\left(1 + \frac{1}{\theta^2} - \theta + \frac{1}{\theta}\right)$$

$$= e^{-\theta}\left(\frac{\theta^2 + 1 - \theta^3 + \theta}{\theta^2}\right)$$

$$\frac{dy}{dx} = \frac{e^{-\theta}\left(-\theta^3 + \theta^2 + \theta + 1\right)}{e^{\theta}\left(\theta^2 - 1 + \theta^3 + \theta\right)}$$

$$\frac{dy}{dx} = e^{-2\theta}\left(\frac{\theta^2 - \theta^3 + \theta + 1}{\theta^3 + \theta^2 + \theta - 1}\right) \quad \underline{\text{Ans}}$$

Q. 11 → $x = \cos^{-1}\left(\frac{1}{\sqrt{1+t^2}}\right)$ $y = \sin^{-1}\left(\frac{t}{\sqrt{1+t^2}}\right)$

put $t = \tan\theta$

$x = \cos^{-1}\left(\frac{1}{\sqrt{1+\tan^2\theta}}\right)$	$y = \sin^{-1}\left(\frac{\tan\theta}{\sqrt{1+\tan^2\theta}}\right)$
$x = \cos^{-1}\left(\frac{1}{\sec\theta}\right)$	$y = \sin^{-1}\left(\frac{\tan\theta}{\sin\theta}\right)$
$x = \cos^{-1}(\cos\theta)$	$y = \sin^{-1}(\sin\theta)$
$x = \theta$	$y = \theta$
replace θ	
$x = \tan^{-1} t$	$y = \tan^{-1} t$

Diff wrt t

$\frac{dx}{dt} = \frac{1}{1+t^2}$ $\frac{dy}{dt} = \frac{1}{1+t^2}$

$\frac{dy}{dx} = \frac{\frac{1}{1+t^2}}{\frac{1}{1+t^2}} = 1$ Ans

Q. 12 $x = \left(t + \frac{1}{t}\right)^a$, $y = a^{t+\frac{1}{t}}$

Diff wrt. t

$\frac{dx}{dt} = a\left(t+\frac{1}{t}\right)^{a-1} \cdot \left(1 - \frac{1}{t^2}\right)$

$\frac{dx}{dt} = a\left(t+\frac{1}{t}\right)^{a-1} \cdot \left(\frac{t^2-1}{\dots}\right)$

$$\frac{dy}{dt} = a^{t+\frac{1}{t}} \cdot \log a \left(1 - \frac{1}{t^2}\right) \quad \cdots \left\{ \frac{d}{dx}(a^x) = a^x \log a \right)$$

$$\frac{dy}{dx} = \frac{a^{t+\frac{1}{t}} \cdot \log a \cancel{\left(\frac{t^2-1}{t^2}\right)}}{a\left(t+\frac{1}{t}\right)^{a-1} \cancel{\left(\frac{t^2-1}{t^2}\right)}}$$

$$\frac{dy}{dx} = \frac{a^{t+\frac{1}{t}} \cdot \log a}{a\left(t+\frac{1}{t}\right)^{a-1}}$$ **ANS**..

Q13 + $x = \sin^{-1}\left(\frac{2t}{1+t^2}\right)$; $y = \tan^{-1}\left(\frac{2t}{1-t^2}\right)$

Put $t = \tan\theta$

$x = \sin^{-1}\left(\frac{2\tan\theta}{1+\tan^2\theta}\right)$; $y = \tan^{-1}\left(\frac{2\tan\theta}{1-\tan^2\theta}\right)$

$x = \sin^{-1}(\sin(2\theta))$; $y = \tan^{-1}(\tan(2\theta))$

$x = 2\theta$; $y = 2\theta$

Replace θ

$x = 2\tan^{-1} t$; $y = 2\tan^{-1} t$

Diff wrt t

$\frac{dx}{dt} = \frac{2}{1+t^2}$ $\qquad \frac{dy}{dt} = \frac{2}{1+t^2}$

$$\frac{dy}{dx} = \frac{\frac{\cancel{2}}{\cancel{1+t^2}}}{\frac{\cancel{2}}{\cancel{1+t^2}}} = 1$$ **Ans**