|| जय श्री राधे कृष्णा ||

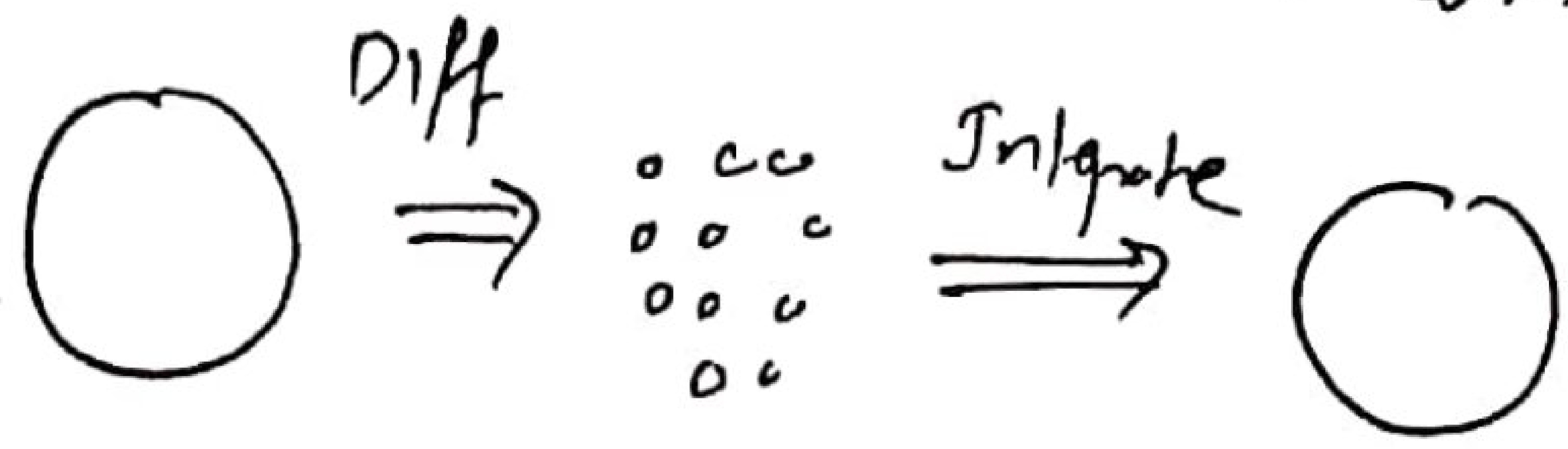
← ULTIMATE MATHEMATICS: BY AJAY MITTAL →

CHAPTER: INTEGRATION   CLASS NO: 1

(·) $\int f(x)\,dx = g(x) + C$

↳ Constant of Integration

(·)



(·)

$$\frac{d}{dx}(2x+3) = 2 \Rightarrow \int 2\,dx = 2x + C$$

$$\frac{d}{dx}(\tan x - 2) = \sec^2 x \Rightarrow \int \sec^2 x\,dx = \tan x + C$$

$$\frac{d}{dx}(\log x) = \frac{1}{x} \Rightarrow \int \frac{1}{x}\,dx = \log x + C$$

(·) Indefinite Integral $\int f(x)\,dx = g(x) + C$

(·) Definite Integrals $\int_a^b f(x)\,dx = \left(g(x)\right)_a^b$

$= g(b) - g(a)$

$=$ Definite value

FORMULAS

(1) $\int x^n\,dx = \frac{x^{n+1}}{n+1} + C$

(2) $\int e^x\,dx = e^x + C$

(3) $\int a^x\,dx = \frac{a^x}{\log a} + C$

$2x \rightarrow 2$

$2x \leftarrow 2$

(4) $\int \text{Constant}\,dx = x\,\text{Constant} + C$

(5) $\int \frac{1}{x}\,dx = \log|x| + C$

(6) $\int \sin x\, dx = -\cos x \cancel{dx} + c$

(7) $\int \cos x\, dx = \sin x \cancel{d} + c$

(8) $\int \tan x\, dx = \log|\sec x| + c$ (or) $-\log|\cos x| + c$

(9) $\int \cot x\, dx = \log|\sin x| + c$ (or) $-\log|\operatorname{cosec} x| + c$

(10) $\int \sec x\, dx = \log|\sec x + \tan x| + c$

(11) $\int \operatorname{cosec} x\, dx = \log|\operatorname{cosec} x - \cot x| + c$

(12) $\int \sec^2 x\, dx = \tan x + c$

(13) $\int \operatorname{cosec}^2 x\, dx = -\cot x + c$

} only two direct squares.

(14) $\int \sec x \tan x\, dx = \sec x + c$

(15) $\int \operatorname{cosec} x \cdot \cot x\, dx = -\operatorname{cosec} x + c$

} two direct products

(16) $\int \frac{1}{\sqrt{x}}\, dx = 2\sqrt{x} + c$

(17) $\int \frac{1}{x^2}\, dx = -\frac{1}{x} + c$

(18) $\int \sqrt{x}\, dx = \frac{2}{3} x^{3/2} + c$

} shortcuts

SPECIAL Integrals

(1) $\int \frac{1}{\sqrt{a^2-x^2}}\, dx = \sin^{-1}\left(\frac{x}{a}\right) + c$

(2) $\int \frac{1}{\sqrt{x^2-a^2}}\, dx = \log|x + \sqrt{x^2-a^2}| + c$

(3) $\int \frac{1}{\sqrt{x^2+a^2}}\, dx = \log|x + \sqrt{x^2+a^2}| + c$

(4) $\int \frac{1}{x^2+a^2}\, dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + c$

(5) $\int \frac{1}{x^2-a^2}\, dx = \frac{1}{2a}\log\left|\frac{x-a}{x+a}\right| + c$

(6) $\int \frac{1}{a^2-x^2}\, dx = \frac{1}{2a}\log\left|\frac{a+x}{a-x}\right| + c$

Basics (Class No: 1)

(·) $\int \sin(3x)\,dx = -\frac{\cos(3x)}{3} + c$ LINEAR $ax+b$

(·) $\int e^{3-2x}\,dx = \frac{-e^{3-2x}}{2} + c$

(·) $\int 3^{4+2x}\,dx = \frac{3^{4+2x}}{2\log 3} + c$

(·) $\int \sqrt{2x+3}\,dx = \frac{2}{3}\frac{(2x+3)^{3/2}}{2} + c$

(·) $\int \frac{1}{1-2x}\,dx = -\frac{1}{2}\log|1-2x| + c$

(·) $\int \sec^2(3+4x)\,dx = \frac{1}{4}\tan(3+4x) + c$

(·) $\int \frac{1}{\sqrt{3+4x}}\,dx = \frac{2}{4}\sqrt{3+4x} + c$

(·) $\int \sin a\,dx = x\sin a + c$

(x) $\boxed{\int \sin(x^2)\,dx \neq -\frac{\cos(x^2)}{2x} + c}$ (x)

→ linear (x)

(·) $\int \frac{1}{\sqrt{x^2-4}}\,dx = \int \frac{1}{\sqrt{x^2-(2)^2}}\,dx = \log\left|x+\sqrt{x^2-4}\right| + c$

(·) $\int \frac{1}{2x^2-6}\,dx = \frac{1}{2}\int \frac{1}{x^2-(\sqrt{3})^2}\,dx = \frac{1}{2}\times\frac{1}{2\sqrt{3}}\log\left|\frac{x-\sqrt{3}}{x+\sqrt{3}}\right| + c$

(·) $\int \frac{1}{\sqrt{3-3x^2}}\,dx = \frac{1}{\sqrt{3}}\int \frac{1}{\sqrt{1-x^2}}\,dx = \frac{1}{\sqrt{3}}\sin^{-1}(x) + c$

SUBSTITUTION:

(·) $\int \sin(x^2) \cdot 2x \, dx$

put $x^2 = t$

Diff $2x\,dx = dt$

$= \int \sin t \, dt$

$= -\cos t + c$

$= -\cos(x^2) + c$

(·) $\int \frac{e^{2\tan^{-1}x}}{1+x^2} dx$

put $2\tan^{-1}x = t$

$\frac{2}{1+x^2} dx = dt$

$\frac{dx}{1+x^2} = \frac{dt}{2}$

$= \frac{1}{2}\int e^t \cdot dt$

$= \frac{1}{2} e^t + c$

$= \frac{1}{2} e^{2\tan^{-1}x} + c$ Ans

(·) $\int \sqrt{3-\sin x} \cdot \cos x \, dx$

put $3-\sin x = t$

$-\cos x\,dx = dt$

$\cos x\,dx = -dt$

$= -\int \sqrt{t} \, dt$

$= -\frac{2}{3}t^{3/2} + c$

$= -\frac{2}{3}(3-\sin x)^{3/2} + c$

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Q1 $\int (3x^2+5)^{5/2} \cdot x^3\, dx$

IMP Concept

$= \int (3x^2+5)^{3/2} \cdot x^2 \cdot x\,dx$

Put $3x^2+5 = t$
$6x\,dx = dt$
$x\,dx = \frac{dt}{6}$

go to

$= \frac{1}{6}\int t^{3/2} \cdot x^2\, dt$

$= \frac{1}{6}\int t^{3/2} \cdot \left(\frac{t-5}{3}\right) dt$

$= \frac{1}{18}\int t^{3/2} \cdot (t-5)\, dt$

$= \frac{1}{18}\int t^{5/2} - 5t^{3/2}\, dt$

$= \frac{1}{18}\left[\frac{2}{7}t^{7/2} - \frac{5\times 2}{5}t^{5/2}\right] + C$

$= \frac{1}{18}\left[\frac{2}{7}(3x^2+5)^{7/2} - 2(3x^2+5)^{5/2}\right] + C$ Ans