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ULTIMATE MATHEMATICS

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CHAPTER: RELATION FUNCTION

CLASS No: 1

Relations

(·) Relation from set A to set B

(·) $R = \{(a,b) : aRb;\ a \in A,\ b \in B\}$

(·) Relation from set A to set A ✓ XII

(·) $R = \{(a,b) : aRb;\ a \in A,\ b \in A\}$

(·) Relation in set A ✓ XII

(·) Types of Relations:

(1) Symmetric relation

for each $(a,b) \in R$, $(b,a) \in R$

(2) Transitive Relation

If $(a,b) \in R$ & $(b,c) \in R$ then $(a,c) \in R$

(3) ~~Ref~~ Reflexive Relation

for each $a \in A$, $(a,a) \in R$

(·) If all three, then it is called an "Equivalence Relation"

(·) 'Equivalence class'

Qn 1 Relation on $A = \{1, 2, 3, 4, 5, \ldots 9\}$

$R = \{(a,b) : a-b$ is divisible by 2 (or) even$\}$

Show that R is an Equivalence Relation

Also find equivalence class [3]

Soln

CLASSTIME

Relation & Function Class No-1 ②

(1) Symmetric Relation

let $(a,b) \in R$

$\Rightarrow$ $a-b$ is divisble by 2

$\Rightarrow$ $a-b = 2\lambda$ ... $(\lambda \in Z)$

$\Rightarrow$ $b-a = -2\lambda$ (which is div by 2)

$\Rightarrow$ $(b,a) \in R$

$\therefore$ R is symmetric relation

(2) Transitive Relation

let $(a,b) \in R$ & $(b,c) \in R$

$\Rightarrow$ $a-b = 2\lambda$ & $(b-c) = 2k$; $(\lambda, k \in Z)$

$\Rightarrow$ $a-c = (a-b) + (b-c)$

$\Rightarrow$ $a-c = 2\lambda + 2k$

$\Rightarrow$ $a-c = 2(\lambda + k)$

$\Rightarrow$ $a-c$ is div by 2

$\Rightarrow$ $(a,c) \in R$

$\therefore$ R is transitive relation.

(3) Reflexive relation

for each $a \in A$

$\Rightarrow$ $a-a = 0$ which is divisble by 2

$\Rightarrow$ $(a,a) \in R$

$\therefore$ R is reflexive relation.

Since R is symmetric, reflexive & transitive

$\therefore$ R is an equivalence relation.

(·) Equivalency class $[3] = \{1,3,5,7,9\}$

$3 \to a$

→ is a set of all values of b which are related to the given element [a]

Relations Function CLASS No-1

Qn.2 → Let L be the set of all lines in xy plane and R be the relation in L defined as $R = \{(L_1, L_2) : L_1 \text{ is parallel to } L_2\}$. Show that R is an equivalence relation. Find the set of all lines related to the line $y = 2x + 4$.

(1) Symmetric Relation

let $(L_1, L_2) \in R$

$\Rightarrow L_1 \parallel^{l} L_2$

$\Rightarrow L_2 \parallel^{l} L_1$

$\Rightarrow (L_2, L_1) \in R$

$\therefore$ R is Symmetric Relation

(2) Transitive relation

let $(L_1, L_2) \in R$ & $(L_2, L_3) \in R$

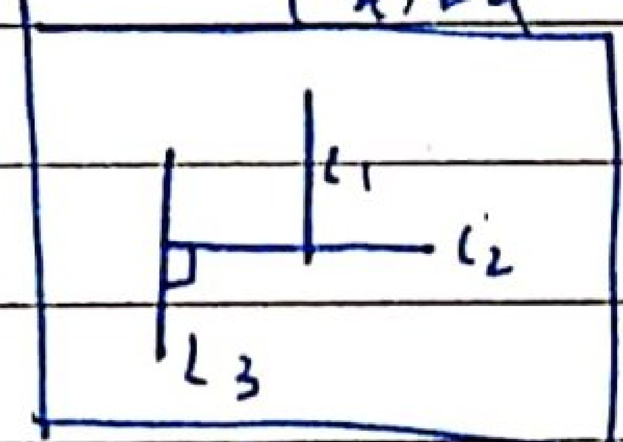
$\Rightarrow L_1 \parallel L_2$ & $L_2 \parallel L_3$

$\Rightarrow L_1 \parallel L_3$

$\Rightarrow (L_1, L_3) \in R$

$\therefore$ R is transitive relation

extra



($\perp^r$ case)

(3) Reflexive

for each $L_1 \in L$

$(L_1, L_1) \in R$ since each line is parallel to itself

$\therefore$ R is ~~s~~ reflexive relation

$\therefore$ Since all three $\therefore$ R is an equivalence Relation.

(.) Required set is the set of all the lines having slope 2

(OR) required set is the set of all the line having form $y = 2x + c$

Relation & Function class No:-1

Qu 3 Relation in $A = \{1, 2, 3, 4, \ldots 15\}$

$R = \{(x, y) : |x - y|$ is multiple of $4\}$. Show that R is an equivalence relation

∴ Also find equivalence class [2]

Sol (1) Symmetric relation

let $(x, y) \in R$

$\Rightarrow |x - y| = 4\lambda \quad \ldots (\lambda \in z)$

$\Rightarrow |y - x| = 4\lambda$ (which is multiple of 4)

$\Rightarrow (y, x) \in R$

∴ R is Symm. Relation

(2) Transitive Relation

let $(x, y) \in R$ & $(y, z) \in R$

$\Rightarrow |x - y| = 4\lambda$ & $|y - z| = 4k \quad \ldots (\lambda, k \in z)$

$\Rightarrow x - y = \pm 4\lambda$ & $y - z = \pm 4k$

Now

$x - z = (x - y) + (y - z)$

$x - z = \pm 4\lambda \pm 4k$

$\Rightarrow x - z = \pm 4(\lambda + k)$

$\Rightarrow |x - z| = 4|\lambda + k|$ which is multiple of 4

∴ $(x, z) \in R$

∴ R is transitive Relation

(3) Reflexive relation

for each $x \in A$

$\Rightarrow |x - x| = 0$ which is multiple of 4

$\Rightarrow (x, x) \in R$

∴ R is a Reflexive relation

∴ R is an equivalence relation.

(4) equivalence class $[2] = \{2, 6, 10, 14\}$ Ans

Relation & Function class No: 1

Q. 4 Relation in set R defined as
$R = \{(a, b) : a \leq b^3\}$
Show R is not symmetric, not transitive, not reflexive.

Sol. $R = \{(a, b) : a \leq b^3\}$

(1) Symmetric

$(1, 2) \in R$
since $1 \leq 2^3$
but $(2, 1) \notin R$
since $2 \nleq 1^3$
$\therefore$ R is not symmetric

(2) Transitive

$(16, 4) \in R$ & $(4, 2) \in R$
$\Rightarrow 16 \leq 4^3$ & $4 \leq 2^3$
but $(16, 2) \notin R$
since $16 \nleq 2^3$
$\therefore$ R is not transitive

(3) Reflexive relation

$\frac{1}{2} \in R$ (Real numbers)
but $\left(\frac{1}{2}, \frac{1}{2}\right) \notin R$ (Relation)
since $\frac{1}{2} \nleq \left(\frac{1}{2}\right)^3$
$\therefore$ R is not reflexive relation

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Qn 5 Let R be the relation on the set {1, 2, 3, 4}
given by R = {(1,2), (2,2), (1,1), (4,4) (1,3), (3,3) (3,2)}
~~other~~ check whether R is symmetric,
reflexive or transitive?

Sol (1) reflexive

for each $a \in A$ (given set)

$(a,a) \in R$

$\therefore$ R is reflexive relation.

(2) Symmetric Relation.

$(1,2) \in R$

but $(2,1) \notin R$

$\therefore$ R is not symmetric.

(3) Transitive relation

$R = \{(1,1)(2,2)(3,3)(4,4)(1,2)(1,3),(3,2),\}$

$\therefore$ R is transitive relation

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