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PROBABILITY | CLASS NO:- 2 | (12th class)

(*) $\boxed{P(A'/B) = 1 - P(A/B)}$

Proof $P(A'/B) = \frac{P(A' \cap B)}{P(B)}$

$= \frac{P(B) - P(A \cap B)}{P(B)}$

$= 1 - P(A/B)$ Hence

Multiplication theorem

$P(A/B) = \frac{P(A \cap B)}{P(B)}$

$\Rightarrow \boxed{P(A \cap B) = P(B) \cdot P(A/B)}$

$P(B/A) = \frac{P(A \cap B)}{P(A)}$

$\boxed{P(A \cap B) = P(A) \cdot P(B/A)}$

$\boxed{P(A \cap B \cap C) = P(A) \cdot P(B/A) \cdot P(C/A \cap B)}$

Qns 1 Three cards are drawn successively, without replacement from a pack of 52 cards. what is the probability that first two cards are kings and third card drawn is an Ace?

Soln Let A → 1st card drawn is a king

B → 2nd card drawn is also a king

C → 3rd card drawn is an ace.

$$P(A \cap B \cap C) = P(A) \cdot P(B/A) \cdot P(C/A \cap B)$$

$$P(A) = \frac{4}{52}$$

$$P(B/A) = \frac{3}{51}$$

$$P(C/A \cap B) = \frac{4}{50}$$

$$\therefore \text{Req prob} = \frac{\cancel{4}}{\cancel{52}\,13} \times \frac{3}{51} \times \frac{4^2}{\cancel{50}\,25} = \frac{2}{5525}$$ Ans

Independent events (Topic = 2)

Events A & B all Independent if

$$\boxed{P(A \cap B) = P(A) \cdot P(B)}$$

$$P(A \cap B \cap C) = P(A) \cdot P(B) \cdot P(C)$$

✗ $P(A \cap B) \neq P(A) \cdot P(B)$ (dependent events)

✓ $P(A/B) = P(A)$ & $P(B/A) = P(B)$

$$P(A \cap B) = P(A) \cdot P(B/A)$$

$$\boxed{P(A \cap B) = P(A)\,P(B)}$$

Qn 2 • Show that if A & B are Independent

(i) A & B' are also independent

(ii) A' & B' are also independent

Sol

Given A & B are Independent

$\therefore P(A \cap B) = P(A) \cdot P(B)$ --(i)

(1) **T.P** A & B' are also Independent.

i.e $P(A \cap B') = P(A) \cdot P(B')$

Pf $P(A \cap B') = P(A) - P(A \cap B)$

$= P(A) - P(A) \cdot P(B)$ --- (Given)

$= P(A)[1 - P(B)]$

$= P(A) \cdot P(B')$

Proved

(2) **TP** A' & B' are also Independent

i.e $P(A' \cap B') = P(A') \cdot P(B')$

Pf $P(A' \cap B') = 1 - P(A \cup B)$

$= 1 - [P(A) + P(B) - P(A \cap B)]$

$= 1 - [P(A) + P(B) - P(A) \cdot P(B)]$

$= 1 - [P(A) + P(B)(1 - P(A))]$

$$= 1 - [P(A) + P(B) \cdot P(A')]$$

$$= 1 - P(A) - P(B) \cdot P(A')$$

$$= P(A') - P(B) \cdot P(A')$$

$$= P(A')[1 - P(B)]$$

$$= P(A') \cdot P(B')$$ Proved

Qn 3 → Probability of solving a specific problem independently by A & B are $\frac{1}{2}$ and $\frac{1}{3}$ respectively. If both try to solve the problem independently. Find the prob. that

(i) the problem is solved

(ii) exactly one of them solve the ~~solved~~ problem.

Sol: Let A → A solve the problem

B → Student B solve the problem

Then $P(A) = 1/2$ & $P(B) = 1/3$

$P(A') = 1 - P(A) = 1/2$ & $P(B') = 1 - 1/3 = 2/3$

(i) $P(\text{problem is solved}) = P(A \cap B') + P(B \cap A') + P(A \cap B)$

$= P(A) \cdot P(B') + P(B) \cdot P(A') + P(A) \cdot P(B)$

$= \left(\frac{1}{2} \times \frac{2}{3}\right) + \left(\frac{1}{3} \times \frac{1}{2}\right) + \left(\frac{1}{2} \times \frac{1}{3}\right) = \boxed{\frac{3}{3}}$

(OR) $P(\text{problem solved}) = 1 - P(\text{problem not solved})$

$= 1 - P(A' \cap B')$

$= 1 - P(A') \cdot P(B')$

$= 1 - \frac{1}{2} \times \frac{2}{3}$

$= 1 - \frac{1}{3}$

$= 2/3$ Ans

(2) $P(\text{Exactly one will solve}) = P(A \cap B') + P(B \cap A')$

$= P(A) - P(B') + P(B) P(A')$

$= \square + \square = \square$ Ans

Qu) 4 → A die is tossed thrice. Find the probability of getting an odd number atleast once.

Sol) Let A → 1st die shows odd no.

B → 2nd " " " ,

C → 3rd " " odd ,

$P(A) = \frac{3}{6} = \frac{1}{2}$	$P(A') = 1 - 1/2 = 1/2$
$P(B) = 1/2$	$P(B') = 1/2$
$P(C) = 1/2$	$P(C') = 1/2$

✓ X X
X ✓ X
X X ✓
✓ ✓ X
✓ X ✓
X ✓ ✓
✓ ✓ ✓
X X X

$P(\text{atleast one die shows odd}) = 1 - P(\text{none of the Die shows odd no.})$

$= 1 - P(A' \cap B' \cap C')$

$= 1 - P(A') P(B') P(C')$

$= 1 - \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$

$= 1 - \frac{1}{8}$

$= 7/8$ Ans

Qn 5 → Three coins are tossed simultaneously. Consider the Events:

E: three heads or three tails

F: atleast two heads

G: atmost two heads

Check the Independency of the events

So $E = \{ HHH, TTT \}$ $P(E) = 2/8$

$F = \{ HHT, HTH, THH, HHH \}$ $P(F) = 4/8$

$G = \{ TTT, HTT, THT, TTH, HHT, HTH, THH \}$ $P(G) = 7/8$

~~E or F~~ $E \cap F = \{ HHH \}$ $P(E \cap F) = 1/8$

$F \cap G = \{ HHT, HTH, THH \}$ $P(F \cap G) = 3/8$

$E \cap G = \{ TTT \}$ $P(E \cap G) = 1/8$

$E \cap F \cap G = \{ \}$ $P(E \cap F \cap G) = 0$

Now $P(E) \cdot P(F) = \frac{1}{8} = P(E \cap F)$

$\therefore$ E & F are Indep events

Topic: 3 PROBABILITY DISTRIBUTION (P.D)

Random variable

X	0	1	2	----
P(x)	-	-	-	. = 1

✓ $\sum p_i = 1$

Qn. 6 → Find the P.D of "number of kings" when two cards are drawn without replacement.

Sol^n Let X → denotes the number of kings

X → can take values 0, 1, 2

$P(X=0) = P(\text{getting No king}) = \frac{\cancel{12}^{4}}{13} \times \frac{47}{\cancel{52}\ 17} = \frac{\cancel{48}\ 188}{221}$

$P(X=1) = P(\text{getting 1 king}) = \left(\frac{4}{52} \times \frac{48}{51}\right) + \left(\frac{48}{52} \times \frac{4}{51}\right)$

$= \left(\frac{1}{13} \times \frac{16}{17}\right) \times 2 = \frac{32}{221}$

$P(X=2) = P(\text{getting 2 king}) = \frac{\cancel{4}}{13} \times \frac{3}{17} = \frac{1}{221}$

X	0	1	2
P(x)	$\frac{188}{221}$	$\frac{32}{221}$	$\frac{1}{221}$

Ans

Ans 7 A coin is biased so that head is 3 times as likely to occur as tail. If the coin is tossed 4 times. Find the P.D of number of tails

Sol: Let $X \rightarrow$ denotes the number of tails

$X \rightarrow$ can take values $0, 1, 2, 3, 4$

Given $P(H) = 3P(T)$

We know $P(H) + P(T) = 1$

$3P(T) + P(T) = 1$

$P(T) = 1/4$

$\therefore P(H) = 3/4$

$P(X=0) = P(\text{getting no tail}) = \frac{3}{4} \times \frac{3}{4} \times \frac{3}{4} \times \frac{3}{4} = \frac{81}{256}$

$P(X=1) = P(\text{getting 1 tail}) = \left(\frac{1}{4} \times \frac{3}{4} \times \frac{3}{4} \times \frac{3}{4}\right) \times {}^4C_1$ ← No. of cases

$= \frac{108}{256}$

$P(X=2) = P(\text{getting 2 tails}) = \left(\frac{1}{4} \times \frac{1}{4} \times \frac{3}{4} \times \frac{3}{4}\right) \times {}^4C_2$

$= \frac{54}{256}$

$P(X=3) = \left(\frac{1}{4} \times \frac{1}{4} \times \frac{1}{4} \times \frac{3}{4}\right) \times {}^4C_3 = \frac{12}{256}$

$P(X=4) = \frac{1}{4} \times \frac{1}{4} \times \frac{1}{4} \times \frac{1}{4} = \frac{1}{256}$

P.D table

X	0	1	2	3	4
$P(X)$	$\frac{81}{256}$	$\frac{108}{256}$	$\frac{54}{256}$	$\frac{12}{256}$	$\frac{1}{256}$

Qn 8 → In a game, a man wins a rupee for a six and loses a rupee for any other number when a fair die is thrown. The man decided to throw a die thrice, but to quit as and when he gets a six. Find the P.D of amount he can win or lose @.

Sol Let X → denotes the amount he can win/lose

X → can take values = 1, 0, -1, -3

✓ = 1

× ✓ = 0

× × ✓ = -1

× × × = -3

$P(X=1) = P(\text{Rs } 1 \text{ gain}) = \frac{1}{6}$

$P(X=0) = P(\text{No Gain/Loss}) = \frac{5}{6} \times \frac{1}{6} = \frac{5}{36}$

$P(X=-1) = P(\text{Rs } 1 \text{ Gain}) = \frac{5}{6} \times \frac{5}{6} \times \frac{1}{6} = \frac{25}{216}$

$P(X=-3) = P(\text{Rs } 3 \text{ Gain}) = \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} = \frac{125}{216}$

X	1	0	-1	-3
P(X)	-	-	-	-

P.D table

WORKSHEET No: 2 (PROBABILITY) (12th class)

Qus 1 → If A and B are two Independent events, then the probability of occurrence of atleast one of A and B is given by $1 - P(A')P(B')$

Qus 2 → Given that the events A and B are such that $P(A) = 1/2$; $P(A \cup B) = 3/5$ and $P(B) = p$. Find 'p' if they are (i) Independent (2) Mutually exclusive

Ans (1) 1/5 (2) 1/10

Qus 3 → Events A and B are such that $P(A) = 1/2$; $P(B) = \frac{7}{12}$ and $P(\text{not } A \text{ or not } B) = \frac{1}{4}$. State whether events A and B are independent? Ans NO

Qus 4 → Let A and B be independent events with $P(A) = 0.3$ & $P(B) = 0.4$ Find

(i) $P(A \cap B)$ (2) $P(A \cup B)$ (3) $P(A/B)$ (4) $P(B/A)$

Ans 0.12, 0.58, 0.3, 0.4

Qus 5 → Two balls are drawn with replacement from a box containing 10 black and 8 red balls. Find the probability that

(i) first ball is black and second is red

(2) one of them is black and other is red.

Ans $\frac{20}{81}$, $\frac{40}{81}$

Qns 6 → A box of oranges is inspected by examining three randomly selected oranges drawn without replacement. If all the three oranges are good, the box is approved for sale, otherwise it is rejected. Find the probability that a box containing 15 oranges out of which 12 are good and 3 are bad ones will be approved for sale. Ans $\frac{44}{91}$

Qns 7 The probability of obtaining an even prime number on each die, when a pair of die is ~~rolled~~ rolled as

(A) 0 (B) $1/3$ (C) $1/12$ (D) $\frac{1}{36}$

Qns 8 → Two cards are drawn with replacement from a pack of 52 cards. Find the prob distribution of number of Aces. Ans

X	0	1	2
P(x)	$\frac{144}{169}$	$\frac{24}{169}$	$\frac{1}{169}$

Qns 9 → Find the prob. distribution of number of doubles in three throws of a pair of dice

Ans.

X	0	1	2	3
P(x)	$\frac{125}{216}$	$\frac{75}{216}$	$\frac{15}{216}$	$\frac{1}{216}$

Qns 10 → Give P.D table

X	0	1	2	3	4	5	6	7
P(x)	0	k	2k	2k	3k	k^2	$2k^2$	$7k^2+k$

(i) Find value of k
(ii) $P(0 < x < 3)$

Ans (1) $k = \frac{1}{10}$

(2) $\frac{3}{10}$

Q.No. 11 → From a lot of 30 bulbs which include 6 defectives, a sample of 4 bulbs is drawn at random with replacement. Find the prob distribution of number of defective bulbs

Ans:-

x	0	1	2	3	4
p(x)	$\frac{256}{625}$	$\frac{256}{625}$	$\frac{96}{625}$	$\frac{16}{625}$	$\frac{1}{625}$

Q.No 12 → Find the p.d of the number obtained on throwing a die having written 1 on three faces, 2 on two faces and 5 on one face

Q.No. 13 → Two dice are thrown simultaneously. Find the p.d of number of 1's seen.

Ans

x	0	1	2
P(x)	$\frac{25}{36}$	$\frac{10}{36}$	$\frac{1}{36}$

— x —