WORKSHEET NO: 2

Qn 1 → If $x^{16} \cdot y^9 = (x^2+y)^{17}$ show $\frac{dy}{dx} = \frac{2y}{x}$

Qn. 2 → If $x^{13} \cdot y^7 = (x+y)^{20}$ show $\frac{dy}{dx} = \frac{y}{x}$

Qn 3 → $y = \sqrt{x} \cdot \frac{(x+4)^{3/2}}{(4x-3)^{4/3}}$ find $\frac{dy}{dx}$

Ans $\frac{dy}{dx} = \frac{\sqrt{x} \cdot (x+4)^{3/2}}{(4x-3)^{4/3}} \left[\frac{1}{2x} + \frac{3}{2(x+4)} - \frac{16}{3(4x-3)} \right]$

Qn. 4 → If $xy \log(x+y) = 1$ show $\frac{dy}{dx} = \frac{-y(x^2y+x+y)}{x(xy^2+x+y)}$

Qn 5 → If $x = a\sec^3\theta$
$y = a\tan^3\theta$ show $\frac{dy}{dx}$ at $\theta = \pi/3$ $= \frac{\sqrt{3}}{2}$

Qn 6 → $x = a(\cos\theta + \theta\sin\theta)$
$y = a(\sin\theta - \theta\cos\theta)$ show $\frac{dy}{dx} = \tan\theta$

Qn. 7 → $x = ae^{\theta}(\sin\theta - \cos\theta)$
$y = ae^{\theta}(\sin\theta + \cos\theta)$ show $\frac{dy}{dx} = \cot\theta$

Hint: product rule

Qn. 8 → $x = e^{\cos(2t)}$
$y = e^{\sin(2t)}$ show $\frac{dy}{dx} = \frac{-y\log x}{x\log y}$

Hint: Take log on both sides

Qn. 9 → $x = \frac{1-t^2}{1+t^2}$, $y = \frac{2t}{1+t^2}$ find $\frac{dy}{dx}$

Ans $\frac{dy}{dx} = \frac{t^2-1}{2t}$ HINT Quotient Rule

Qn 10 → If $x = e^{\theta}\left(\theta + \frac{1}{\theta}\right)$
$y = e^{-\theta}\left(\theta - \frac{1}{\theta}\right)$ Show $\frac{dy}{dx} = e^{-2\theta}\left[\frac{\theta^2-\theta^3+\theta+1}{\theta^3+\theta^2+\theta-1}\right]$

Qn 11 → $x = \cos^{-1}\left(\frac{1}{\sqrt{1+t^2}}\right)$, $y = \sin^{-1}\left(\frac{t}{\sqrt{1+t^2}}\right)$

find $\frac{dy}{dx}$ HINT put $t = \tan\theta$

ANS $\frac{dy}{dx} = 1$

Qn. 12 → $x = \left(t + \frac{1}{t}\right)^a$, $y = a^{t+\frac{1}{t}}$

Show $\frac{dy}{dx} = \frac{a^{t+\frac{1}{t}} \cdot \log a}{a\left(t+\frac{1}{t}\right)^{a-1}}$

Qn 13 → $x = \sin^{-1}\left(\frac{2t}{1+t^2}\right)$, $y = \tan^{-1}\left(\frac{2t}{1-t^2}\right)$ Show $\frac{dy}{dx} = 1$

HINT put $t = \tan\theta$

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Chapter: Differentiation and continuity

← CLASS NO: 3 →

QNS 1 If $x^y = e^{x-y}$ show that $\frac{dy}{dx} = \frac{\log x}{(1+\log x)^2}$

Sol: taking log on both sides

$$y \log x = (x-y)\log e$$

$$\Rightarrow y\log x = x-y$$

$$\Rightarrow y(1+\log x) = x$$

$$\Rightarrow y = \frac{x}{1+\log x}$$

Diff

$$\frac{dy}{dx} = \frac{(1+\log x)\cdot(1) - x\cdot\left(\frac{1}{x}\right)}{(1+\log x)^2}$$

$$= \frac{\cancel{1}+\log x - \cancel{1}}{(1+\log x)^2}$$

$$\frac{dy}{dx} = \frac{\log x}{(1+\log x)^2} \quad \text{Ans}$$

(OR) $y\log x = x-y$

Diff $y\cdot\frac{1}{x} + \log x\cdot\frac{dy}{dx} = 1 - \frac{dy}{dx}$

$$\Rightarrow \frac{dy}{dx}(1+\log x) = 1 - \frac{y}{x}$$

$$\Rightarrow \frac{dy}{dx} = \frac{x-y}{x(1+\log x)}$$

$$= \frac{y\log x}{x(1+\log x)}$$

$$= \frac{\frac{x}{(1+\log x)}\cdot\log x}{x(1+\log x)}$$

$$= \frac{\log x}{(1+\log x)^2}$$

Ans

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Qn 2 → $y = \dfrac{\sqrt{1-x^2}\cdot(2x-3)^{5/2}}{(x^2+2)^{2/3}}$ find $\dfrac{dy}{dx}$

Sol: taking log on both sides

$\log y = \log\left[\dfrac{\sqrt{1-x^2}\cdot(2x-3)^{5/2}}{(x^2+2)^{2/3}}\right]$

$\log y = \frac{1}{2}\log(1-x^2) + \frac{5}{2}\log(2x-3) - \frac{2}{3}\log(x^2+2)$

Diff w.r.t x

$\frac{1}{y}\cdot\frac{dy}{dx} = \frac{1}{2}\cdot\frac{1(-2x)}{1-x^2} + \frac{5}{2}\cdot\frac{1\cdot(2)}{2x-3} - \frac{2}{3}\frac{1\cdot(2x)}{x^2+2}$

$\frac{dy}{dx} = \frac{(\sqrt{1-x^2})(2x-3)^{5/2}}{(x^2+2)^{2/3}}\left[\frac{-x}{1-x^2} + \frac{5}{2x-3} - \frac{4x}{3(x^2+2)}\right]$ Ans

Qns 3 → If $x^m\cdot y^n = (x+y)^{m+n}$ Show $\frac{dy}{dx} = \frac{y}{x}$

Sol: taking log

$\log(x^m\cdot y^n) = \log(x+y)^{m+n}$

$\Rightarrow m\log x + n\log y = (m+n)\cdot\log(x+y)$

Diff $\frac{m}{x} + \frac{n}{y}\cdot\frac{dy}{dx} = (m+n)\cdot\frac{1}{x+y}\cdot\left(1+\frac{dy}{dx}\right)$

$\Rightarrow \frac{m}{x} + \frac{n}{y}\frac{dy}{dx} = \frac{m+n}{x+y} + \frac{m+n}{x+y}\cdot\frac{dy}{dx}$

$\Rightarrow \frac{dy}{dx}\left(\frac{n}{y} - \frac{m+n}{x+y}\right) = \frac{m+n}{x+y} - \frac{m}{x}$

$\Rightarrow \frac{dy}{dx}\left(\frac{mx+ny-my-ny}{y(x+y)}\right) = \frac{mx+nx-mx-my}{x(x+y)}$

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$$\frac{dy}{dx}\left(\frac{nx-my}{y}\right) = \frac{nx-my}{x}$$

$$\frac{dy}{dx} = \frac{y}{x} \quad \text{Ans}$$

Topic: 2   Parametric functions

$x = f(t) \rightarrow$ Diff w.r.t t $\frac{dx}{dt} =$

$y = g(t) \rightarrow$ Diff w.r.t t $\frac{dy}{dt} =$

Now $\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$

Qn 1 → $x = a\cos^3 t$, $y = a\sin^3 t$ find $\frac{dy}{dx}$ at $t = \pi/4$

Soln Diff w.r.t 't'

$\frac{dx}{dt} = 3a\cos^2 t.(-\sin t)$ | $\frac{dy}{dt} = 3a\sin^2 t.\cos t$

$$\frac{dy}{dx} = \frac{dy}{dt}\Big/\frac{dx}{dt}$$

$$= \frac{\cancel{3a}\sin^{\cancel{2}} t.\cancel{\cos t}}{-\cancel{3a}\cos^{\cancel{2}} t.\cancel{\sin t}}$$

$$\frac{dy}{dx} = -\tan t$$

$$\left(\frac{dy}{dx}\right)_{t=\pi/4} = -\tan\left(\frac{\pi}{4}\right) = -1 \quad \text{Ans}$$

Qn 2 → $x = 2\cos\theta - \cos(2\theta)$ show $\frac{dy}{dx} = \tan\left(\frac{3\theta}{2}\right)$

$y = 2\sin\theta - \sin(2\theta)$

Soln Diff w.r.t θ

$$\frac{dx}{d\theta} = -2\sin\theta + 2\sin(2\theta)$$

$$\frac{dy}{d\theta} = 2\cos\theta - 2\cos(2\theta)$$

$$\frac{dy}{dx} = \frac{2\cos\theta - 2\cos(2\theta)}{2\sin(2\theta) - 2\sin\theta}$$

$$\frac{dy}{dx} = \frac{\cos\theta - \cos(2\theta)}{\sin(2\theta) - \sin\theta}$$

$$\frac{dy}{dx} = \frac{-\cancel{2}\sin\left(\frac{3\theta}{2}\right)\cdot\sin\left(\frac{-\theta}{2}\right)}{\cancel{2}\cos\left(\frac{3\theta}{2}\right)\cdot\sin\left(\frac{\theta}{2}\right)}$$

$$= \frac{\sin(3\theta/2)}{\cos(3\theta/2)} = \tan\left(\frac{3\theta}{2}\right) \quad \underline{\text{Ans}}$$

Ques 3 → $x = a\left[\cos t + \log\left(\tan\frac{t}{2}\right)\right]$; $y = a\sin t$ find $\frac{dy}{dx}$

Soln Diff w.r.t t

$$\frac{dx}{dt} = a\left[-\sin t + \frac{1}{\tan\left(\frac{t}{2}\right)}\cdot\sec^2\left(\frac{t}{2}\right)\cdot\frac{1}{2}\right] \quad ; \frac{dy}{dt} = a\cos t$$

$$= a\left[-\sin t + \frac{\frac{1}{\cos^{\cancel{2}}(t/2)}}{\frac{\sin(t/2)}{\cancel{\cos(t/2)}}}\cdot\frac{1}{2}\right]$$

$$= a\left[-\sin t + \frac{1}{2\sin(t/2)\cos(t/2)}\right]$$

$$= a\left(-\sin t + \frac{1}{\sin t}\right)$$

$$= a\left(\frac{-\sin^2 t + 1}{\sin t}\right) = \boxed{\frac{a\cos^2 t}{\sin t} = \frac{dx}{dt}}$$

$$\frac{dx}{dt} = \frac{a\cos^2 t}{\sin t}, \quad \frac{dy}{dt} = a\cos t$$

$$\frac{dy}{dx} = \frac{\cancel{a}\cos t}{\frac{\cancel{a}\cos^2 t}{\sin t}} = \frac{\sin t \cdot \cos t}{\cos^2 t}$$

$$= \tan t \quad \underline{\text{Ans}}$$

Ques 4 → $x = \sqrt{a^{\sin^{-1} t}}$, $y = \sqrt{a^{\cos^{-1} t}}$ show $\frac{dy}{dx} = \frac{-y}{x}$

Sol. Diff wrt t

$$\frac{dx}{dt} = \frac{1}{2\sqrt{a^{\sin^{-1} t}}} \cdot a^{\sin^{-1} t} \cdot \log a \cdot \frac{1}{\sqrt{1-t^2}}$$

$$\frac{2}{\sqrt{2}} = \sqrt{2}$$

$$\frac{dx}{dt} = \frac{\sqrt{a^{\sin^{-1} t}} \cdot \log a}{2 \cdot \sqrt{1-t^2}} = \frac{x \log a}{2\sqrt{1-t^2}}$$

$$\frac{dy}{dt} = \frac{1}{2\sqrt{a^{\cos^{-1} t}}} \cdot a^{\cos^{-1} t} \cdot \log a \cdot \left(\frac{-1}{\sqrt{1-t^2}}\right)$$

$$= \frac{-\sqrt{a^{\cos^{-1} t}} \log a}{2\sqrt{1-t^2}} = \frac{-y \log a}{2\sqrt{1-t^2}}$$

Now $\frac{dy}{dx}$

$$= \frac{\frac{-y \cancel{\log a}}{\cancel{2\sqrt{1-t^2}}}}{\frac{x \cancel{\log a}}{\cancel{2\sqrt{1-t^2}}}} = \frac{-y}{x} \quad \underline{\text{Ans}}$$

(or) $x = \sqrt{a^{\sin^{-1}t}}$, $y = \sqrt{a^{\cos^{-1}t}}$

taking log

$\log x = \log\left(a^{\sin^{-1}t}\right)^{1/2}$ | $\log y = \log\left(a^{\cos^{-1}t}\right)^{1/2}$

$\Rightarrow \log x = \frac{1}{2}\log\left(a^{\sin^{-1}t}\right)$, $\log y = \frac{1}{2}\log\left(a^{\cos^{-1}t}\right)$

$\Rightarrow \log x = \frac{\sin^{-1}t}{2}\log a$, $\log y = \frac{1}{2}\cos^{-1}t \cdot \log a$

Diff. w.r.t. t

$\frac{1}{x}\cdot\frac{dx}{dt} = \frac{1}{2}\log a \cdot \frac{1}{\sqrt{1-t^2}}$; $\frac{1}{y}\frac{dy}{dt} = \frac{1}{2}\log a\left(\frac{-1}{\sqrt{1-t^2}}\right)$

$\frac{dx}{dt} = \frac{x\log a}{2\sqrt{1-t^2}}$ | $\frac{dy}{dt} = \frac{-y\log a}{2\sqrt{1-t^2}}$

now $\frac{dy}{dx} = \frac{\frac{-y\cancel{\log a}}{\cancel{2\sqrt{1-t^2}}}}{\frac{x\cancel{\log a}}{\cancel{2\sqrt{1-t^2}}}} = \frac{-y}{x}$ Ans

(or) $x = \sqrt{a^{\sin^{-1}t}}$, $y = \sqrt{a^{\cos^{-1}t}}$

$\Rightarrow xy = \sqrt{a^{\sin^{-1}t}}\cdot\sqrt{a^{\cos^{-1}t}}$

$\sqrt{a}\sqrt{b} = \sqrt{ab}$

$xy = \sqrt{a^{\sin^{-1}t+\cos^{-1}t}}$

$xy = \sqrt{a^{\pi/2}}$

$$xy = \sqrt{a^{\pi/2}}$$

Diff w.r.t x

$$x\frac{dy}{dx} + y.1 = 0$$

$$\frac{dy}{dx} = \frac{-y}{x} \quad \underline{\text{Ans}}$$

Qn 5 → $x = \frac{\sin^3 t}{\sqrt{\cos(2t)}}$; $y = \frac{\cos^3 t}{\sqrt{\cos(2t)}}$ show $\frac{dy}{dx} = -\cot(3t)$

Soln Taking log

$$\log x = \log\left(\frac{\sin^3 t}{\sqrt{\cos 2t}}\right)$$

$$\log x = 3\log(\sin t) - \frac{1}{2}\log(\cos(2t))$$

Diff w.r.t t

$$\frac{1}{x}\frac{dx}{dt} = 3.\frac{1}{\sin t}.\cos t - \frac{1}{\cancel{2}}\frac{1}{\cos(2t)}(-\sin 2t).\cancel{2}$$

$$\frac{dx}{dt} = \frac{\sin^3 t}{\sqrt{\cos(2t)}}\left[\frac{3}{\tan t} + \tan(2t)\right]$$

$$\log y = \log\left(\frac{\cos^3 t}{\sqrt{\cos(2t)}}\right)$$

$$\log y = 3\log(\cos t) - \frac{1}{2}\log(\cos(2t))$$

Diff w.r.t t

$$\frac{1}{y}\frac{dy}{dt} = \frac{3}{\cos t}.(-\sin t) - \frac{1}{\cancel{2}}.\frac{1}{\cos(2t)}(-\sin(2t)).\cancel{2}$$

$$\frac{dy}{dt} = \frac{\cos^3 t}{\sqrt{\cos(2t)}}\left[-3\tan t + \tan(2t)\right]$$

Now $\frac{dy}{dx} = \frac{dy}{dt} \Big/ \frac{dx}{dt}$

$$= \frac{\frac{\cos^3 t}{\sqrt{\cos(2t)}}\left[-3\tan t + \tan(2t)\right]}{\frac{\sin^3 t}{\sqrt{\cancel{\cos(2t)}}}\left[\frac{3}{\tan t} + \tan(2t)\right]}$$

$$= \frac{1}{\tan^3 t}\left[\frac{-3\tan t + \frac{2\tan t}{1-\tan^2 t}}{\frac{3}{\tan t} + \frac{2\tan t}{1-\tan^2 t}}\right]$$

$$= \frac{1}{\tan^3 t}\left[\frac{\frac{-3\tan t + 3\tan^3 t + 2\tan t}{\cancel{1-\tan^2 t}}}{\frac{3 - 3\tan^2 t + 2\tan^2 t}{\tan t\,(\cancel{1-\tan^2 t})}}\right]$$

$$= \frac{\tan t}{\tan^3 t}\left[\frac{-\tan t + 3\tan^3 t}{3 - \tan^2 t}\right]$$

$$= \frac{\tan^2 t}{\tan^3 t}\left(\frac{-1 + 3\tan^2 t}{3 - \tan^2 t}\right)$$

$$= \frac{1}{\tan t}\left(\frac{-1 + 3\tan^2 t}{3 - \tan^2 t}\right)$$

$$= \frac{-1 + 3\tan^2 t}{3\tan t - \tan^3 t} = -\left(\frac{1 - 3\tan^2 t}{3\tan t - \tan^3 t}\right)$$

$$= -\frac{1}{\tan(3t)} = -\cot(3t)$$