!! जय श्री राधे कृष्णा !!

Solution of TEST No: 5 (3hr test)

SECTION : A

QNS 1 : (3,1) should be added, to make it the smallest Equivalence relation.

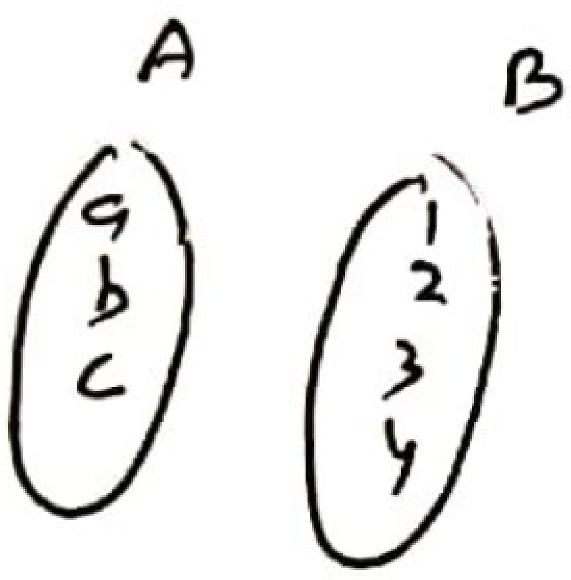
$R = \{(1,1), (2,2)\ (3,3)\ (1,3), (3,1)\}$

clearly It is reflexive, symmetric $(1,3) \in R$ & $(3,1) \in R$

It is transitive since $(1,3) \in R$, $(3,1) \in R$ and $(1,1) \in R$

$\therefore$ (3,1) **ANS**

QN: 2 →



a can be connected in 4 ways
b " " " " 3 ways
c " " " " 2 ways

$\therefore$ required No of one-one functions $= 4 \times 3 \times 2$
$= 24$

$\therefore$ (c) **ANS**

QNS 3 →

$2a + 3b = 30$; $a \in N$, $b \in N$

$a = \frac{30 - 3b}{2}$; $b = 2 \Rightarrow a = 12$
$b = 4 \Rightarrow a = 9$
$b = 6 \Rightarrow a = 6$
$b = 8 \Rightarrow a = 3$

$\therefore R = \{(12,2)\ (9,4)\ (6,6)\ (3,8)\}$ **ANS**

QN. 4 →

$\sin\left[2\cot^{-1}\left(-\frac{5}{12}\right)\right]$

$= \sin\left[2\left(\pi - \cot^{-1}\left(\frac{5}{12}\right)\right)\right]$

$= \sin\left[2\pi - 2\cot^{-1}\left(\frac{5}{12}\right)\right] \longrightarrow$ IVth quadrant

$= -\sin\left(2\cot^{-1}\left(\frac{5}{12}\right)\right)$

$= -\sin\left(2\tan^{-1}\left(\frac{12}{5}\right)\right)$

$= -\sin\left[\sin^{-1}\left(\frac{2\times\frac{12}{5}}{1+\frac{144}{25}}\right)\right]$

$= -\left(\frac{\frac{24}{5}}{\frac{169}{25}}\right) = \frac{-120}{169}$

$\therefore \frac{-120}{169}$ (C) ANS

QN-5 → Main Branch is $[0,\pi]-\{\pi/2\}$
other Principal Branch for $\sec^{-1}$ is
$[\pi,2\pi]-\{\frac{3\pi}{2}\}$ ANS

QN-6 → $\cos^{-1}[\cos(-680^\circ)]$

$= \cos^{-1}(\cos(680^\circ))$ --- $\{\cos(-\theta)=\cos\theta\}$

$= \cos^{-1}(\cos(8\times 90-40))$

$= \cos^{-1}(\cos(40^\circ))$ --- (IVth Quadrant)

$= 40^\circ$

$= 2\pi/9$ $\therefore$ (A) ANS

QN-7 → $\begin{bmatrix} 2x & 3 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ -3 & 0 \end{bmatrix}\begin{bmatrix} x \\ 8 \end{bmatrix}=0$

$\Rightarrow \begin{bmatrix} 2x-9 & 4x \end{bmatrix}\begin{bmatrix} x \\ 8 \end{bmatrix}=0$

$\Rightarrow 2x^2-9x+32x=0$

$\Rightarrow 2x^2+23x=0$

$\Rightarrow x(2x+23)=0$ $\Rightarrow x=0;\ x=\frac{-23}{2}$

$\therefore$ (D) ANS

QN. 8 → given $A' = -A$; $B' = -B$

let $P = AB$

$\Rightarrow P' = (AB)'$

$\Rightarrow P' = B'A'$

$\Rightarrow P' = (-B)(-A)$

$\Rightarrow P' = BA$

$\Rightarrow P' = P$ if $AB = BA$

$\therefore$ (B) ANS $(AB = BA)$

QN. 9 → $a_{ij} = 1$; $i \neq j$

$a_{ij} = 0$; $i = j$

let $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

$A^2 = AA = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$

$\therefore$ (C) ANS $= (I)$

QN. 10 → $\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+\sin\theta & 1 \\ 1+\cos\theta & 1 & 1 \end{vmatrix}$

$\Delta = 1(\cancel{1}+\sin\theta-\cancel{1}) - 1(\cancel{1}-\cancel{1}-\cos\theta) + 1(\cancel{1}-\cancel{1}-\sin\theta-\cos\theta-\sin\theta\cos\theta)$

$\Delta = \cancel{\sin\theta} + \cancel{\cos\theta} - \cancel{\sin\theta} - \cancel{\cos\theta} + \sin\theta\cos\theta$

$\Delta = \sin\theta\cos\theta$

$\Delta = \frac{1}{2}(2\sin\theta\cos\theta)$

$\Delta = \frac{1}{2}\sin(2\theta)$

Max value of $\sin(2\theta) = 1$

$\therefore$ Maximum value of $\Delta = \frac{1}{2} \times 1 = 1/2$

(D) ANS $(1/2)$

QN 11 → given $|A| = 12$

to find: $|Adj\, A| = ?$

Note: If each element is replaced by its cofactor then it becomes Adj A

we know $|Adj\, A| = |A|^{n-1}$

$= (12)^{3-1} = (12)^2 = 144$

∴ (B) ANS (144)

QN 12 → $f(x) = |\cos x|$

To find $f'(3\pi/4)$

$3\pi/4 \to 2^{nd}$ quadrant $\to \cos\theta$ is $-ve$

$\Rightarrow |\cos x| = -\cos x$

$\therefore f(x) = -\cos x$

Differentiate $f'(x) = \sin x$

$f'(3\pi/4) = \sin(3\pi/4) = \sin(\pi - \pi/4) = \frac{1}{\sqrt{2}}$

$\therefore \frac{1}{\sqrt{2}}$ (ANS)

QN 13 → $f(x) = [x]$

we know that greatest Integer function is discontinuous at all the integers & 1.5 is not integer ∴ It is continuous at $x = 1.5$

∴ (D) ANS $x = 1.5$

Ans 14 → $f(x) = \sec(\tan^{-1}x)$

By conversion $\tan^{-1}x = \sec^{-1}\left(\frac{\sqrt{1+x^2}}{1}\right)$

$\therefore f(x) = \sec(\sec^{-1}\sqrt{1+x^2})$

$f(x) = \sqrt{1+x^2}$

Differential coefficient (or) derivative of $f(x)$ is

$f'(x) = \frac{1}{\cancel{2}\sqrt{1+x^2}} \cdot \cancel{2}x = \frac{x}{\sqrt{1+x^2}}$

$\therefore$ (A) Ans $\frac{x}{\sqrt{1+x^2}}$

On 15 → $f(x) = \frac{1}{\log|x|}$

$x = 0 \Rightarrow \log 0 \Rightarrow$ not defined

$x = \pm 1 \Rightarrow \log|\pm 1| = \log|1| = 0 \Rightarrow f(x)$ not defined

$\therefore f(x)$ is Dis-continuous at $x = 0, 1, -1$

i.e at three points

$\therefore$ (C) Ans (3 points)

On 16 → let $u = x^2$ & $v = x^3$

$\frac{du}{dx} = 2x$ & $\frac{dv}{dx} = 3x^2$

$\frac{du}{dv} = \frac{du/dx}{dv/dx} = \frac{2x}{3x^2} = \frac{2}{3x}$

$\therefore \frac{2}{3x}$ Ans

On 17 → $x = 1 - a\cos\theta$; $y = b\cos^2\theta$

Diff. w.r.t θ

$\frac{dx}{d\theta} = a\sin\theta$; $\frac{dy}{d\theta} = 2b\cos\theta \cdot (-\sin\theta)$

$$\frac{dy}{dx} = \frac{\cancel{+}2b\sin\theta\cancel{\cos\theta}}{\cancel{+}a\cancel{\cos\theta}} = \frac{2b\sin\theta}{a}$$

Slope of (T) at $\theta = \pi/2 = \frac{2b\sin(\pi/2)}{a} = \frac{2b}{a}$

Slope of (N) at $\theta = \pi/2) = \frac{-a}{2b}$

$\therefore$ (D) none of these ANS

Qn. 18 → $f(x) = -|x+1| + 3$

we have $|x+1| \geq 0$

$\Rightarrow -|x+1| \leq 0$

$\Rightarrow -|x+1| + 3 \leq 3$

$\Rightarrow f(x) \leq 3$

Clearly Max value of $f(x) = 3$

$\therefore$ Max value $= 3$ ANS

Qn 19 → let $y = \tan^{-1}\left(\frac{1+\sin x}{\cos x}\right)$

$y = \tan^{-1}\left(\frac{1+\cos(\pi/2 - x)}{\sin(\pi/2 - x)}\right)$

$y = \tan^{-1}\left(\frac{\cancel{2}\cos^2\left(\frac{\pi}{4}-\frac{x}{2}\right)}{\cancel{2}\sin\left(\frac{\pi}{4}-\frac{x}{2}\right)\cancel{\cos\left(\frac{\pi}{4}-\frac{x}{2}\right)}}\right)$

$y = \tan^{-1}\left(\cot\left(\frac{\pi}{4}-\frac{x}{2}\right)\right)$

$y = \tan^{-1}\left(\tan\left(\frac{\pi}{2}-\frac{\pi}{4}+\frac{x}{2}\right)\right)$

$y = \tan^{-1}\left(\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)\right)$

$y = \pi/4 + x/2$

Differentiate $\frac{dy}{dx} = 1/2$ $\therefore$ $1/2$ ANS

Qn 20 → let $P = B'AB$

$\Rightarrow P' = (B'AB)'$

$\Rightarrow P' = B'A'B$

$\Rightarrow P' = P$ if $A' = A$

$\therefore$ A must be a symmetric Matrix ANS

SECTION: B

Qn 21 → equation of curve

$x = t^2 + 3t - 8$ & $y = 2t^2 - 2t - 5$

Point of Contact $(2, -1)$

put $x = 2$ & $y = -1$

$2 = t^2 + 3t - 8$ and $-1 = 2t^2 - 2t - 5$

$\Rightarrow t^2 + 3t - 10 = 0$ and $2t^2 - 2t - 4 = 0$

$\Rightarrow (t+5)(t-2) = 0$ and $t^2 - t - 2 = 0$

$\Rightarrow t = -5$ (or) $t = 2$ and $(t-2)(t+1) = 0$

$t = 2$ or $t = -1$

take the common value of t $\boxed{t=2}$

Diff w.r.t t

$\frac{dx}{dt} = 2t + 3$ & $\frac{dy}{dt} = 4t - 2$

$$\frac{dy}{dx} = \frac{4t-2}{2t+3}$$

Slope of tangent $\left(\frac{dy}{dx}\right)_{t=2} = \frac{8-2}{4+3} = \frac{6}{7}$

$\therefore$ Slope of tangent $= \frac{6}{7}$ ANS

Qn. 22 → $f(x) = x^3 - 18x^2 + 96x \quad ; \quad x \in [0,9]$

$f'(x) = 3x^2 - 36x + 96$

$= 3(x^2 - 12x + 32)$

$f'(x) = 3(x-4)(x-8)$

for Max/Min put $f'(x) = 0$

$x = 4$ (or) $x = 8$

Here we have to find absolute Max value & Absolute Min value given $[0,9]$

$f(0) = 0$

$f(4) = 64 - 18(16) + 96(4) = 64 - 288 + 384 = 32$

$f(8) = 512 - 18(64) + 96(8) = 128$

$f(9) = 729 - 18(81) + 96(9) = 135$

Clearly smaller value of $f(x) = 0$

$\therefore$ Min value of $f(x) = 0$ ANS

Qn. 23 $x^y = e^{x-y}$

taking log on both sides

$y \log x = x - y \quad \text{---} \quad \{\log e = 1\}$

$y(1 + \log x) = x$

$\Rightarrow y = \dfrac{x}{1 + \log x}$

Diff wrt x

$\dfrac{dy}{dx} = \dfrac{(1+\log x)(1) - x\left(\frac{1}{x}\right)}{(1+\log x)^2}$

$= \dfrac{1 + \log x - 1}{(1+\log x)^2} \Rightarrow \dfrac{dy}{dx} = \dfrac{\log x}{(1+\log x)^2}$ ANS

QN 24 → $f(x) = x|x|$

$$f(x) = \begin{cases} x^2 ; & x \geq 0 \\ -x^2 ; & x < 0 \end{cases}$$

$$LHD = \lim_{x \to 0^-} \left(\frac{f(x) - f(0)}{x - 0} \right) = \lim_{x \to 0^-} \left(\frac{-x^2 - 0}{x - 0} \right)$$

$$= \lim_{x \to 0^-} (-x) \quad \text{put } x = 0 - h \ \& \ h \to 0$$

$$\therefore LHD = \lim_{h \to 0} (h) = 0 \qquad \underline{LHD = 0}$$

$$RHD = \lim_{x \to 0^+} \left(\frac{x^2 - 0}{x - 0} \right) = \lim_{x \to 0^+} (x) \quad \text{put } x = 0 + h \ \& \ h \to 0$$

$$RHD = \lim_{h \to 0} (h) = 0 \qquad \underline{RHD = 0}$$

$$LHD = RHD = 0$$

$\therefore f(x)$ is differentiable at $x = 0$ Ans.

QN. 25 →

$$\begin{bmatrix} 2 & -1 \\ 1 & 0 \\ -3 & 4 \end{bmatrix}_{3\times 2} A = \begin{bmatrix} -1 & -8 & -10 \\ 1 & -2 & -5 \\ 9 & 22 & 15 \end{bmatrix}_{3\times 3}$$

let, $A = \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix}_{2\times 3}$

$$\therefore \begin{bmatrix} 2 & -1 \\ 1 & 0 \\ -3 & 0 \end{bmatrix} \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix} = \begin{bmatrix} -1 & -8 & -10 \\ 1 & -2 & -5 \\ 9 & 22 & 15 \end{bmatrix}$$

$\Rightarrow 2a - d = -1$	$2b - e = -8$	$2c - f = -10$
$a = 1$	$b = -2$	$c = -5$
$\Rightarrow d = 3$	$e = 4$	$f = 0$

$$\therefore A = \begin{bmatrix} 1 & -2 & -5 \\ 3 & 4 & 0 \end{bmatrix} \quad \text{Ans}$$

Q.N. 26 →

$f: R_+ \to [4, \infty)$

$f(x) = x^2 + 4$

one-one

let $x_1, x_2 \in R_+$

and $f(x_1) = f(x_2)$

$\Rightarrow x_1^2 + 4 = x_2^2 + 4$

$\Rightarrow x_1^2 = x_2^2$

$\Rightarrow x_1 = \pm x_2$ But $x_1 \neq -x_2$ $\{\because x_1, x_2 \in R_+\}$

$\therefore \boxed{x_1 = x_2}$

$\therefore f$ is one-one

on-to

let $y = f(x)$

$y = x^2 + 4$

$\Rightarrow x^2 = y - 4$

$\Rightarrow x = \pm\sqrt{y-4}$ But $x \neq -\sqrt{y-4}$ $\{\because x \in R_+\}$

$\Rightarrow x = \sqrt{y-4}$

Range : $y - 4 \geq 0$

$\Rightarrow y \geq 4$

$\therefore$ Range $= [4, \infty)$ which is equal to codomain

$\therefore$ Range = Codomain

$\therefore f(x)$ is on-to

Hence $f(x)$ is a Bijective function Ans

SECTION : C

Q.N. 27 → Equation of curve $x^2 = 4y$

Diff wrt x

$\Rightarrow 2x = 4\frac{dy}{dx}$

$\Rightarrow \frac{dy}{dx} = \frac{x}{2}$

Slope of (T) at $(x_1, y_1) = \frac{x_1}{2}$

{ remember (1,2) is NOT the point of contact }

Slope of (N) at $(x_1, y_1) = \frac{-2}{x_1}$

equation of Normal at (x_1, y_1) is

$y - y_1 = \frac{-2}{x_1}(x - x_1)$

→ this Normal "passes through" (1,2)

$\Rightarrow (2 - y_1) = \frac{-2}{x_1}(1 - x_1)$

$\Rightarrow \cancel{2x_1} - x_1 y_1 = -2 + \cancel{2x_1}$

$\Rightarrow \boxed{x_1 y_1 = 2}$ --- (1)

also we have

$x_1^2 = 4y_1$ --- { (x_1, y_1) also lies on the curve

$\Rightarrow x_1^2 = 4\left(\frac{2}{x_1}\right)$

$\Rightarrow x_1^3 = 8$

$\Rightarrow \boxed{x_1 = 2} \Rightarrow \boxed{y_1 = 1}$

$\therefore$ point of contact is $(2,1)$

Now equation of (N) at $(2,1)$

$y - 1 = \frac{-2}{2}(x - 2)$

$\Rightarrow y - 1 = -x + 2$

$\Rightarrow x + y = 3$ is the equation of Normal

ANS

Q.No 28 → $x = \sin t$; $y = \sin(pt)$

Diff w.r.t t

$$\frac{dx}{dt} = \cos t \quad ; \quad \frac{dy}{dt} = p\cos(pt)$$

$$\frac{dy}{dx} = \frac{p\cos(pt)}{\cos t}$$

$$\Rightarrow \cos t \cdot \frac{dy}{dx} = p\cos(pt)$$

Diff w.r.t x

$$\Rightarrow \cos t \cdot \frac{d^2y}{dx^2} + \frac{dy}{dx}\cdot(-\sin t)\frac{dt}{dx} = -p^2\sin(pt)\cdot\frac{dt}{dx}$$

$$\Rightarrow \cos t \cdot \frac{d^2y}{dx^2} - \sin t \cdot \frac{dy}{dx}\cdot\frac{1}{\cos t} = -p^2\sin(pt)\cdot\frac{1}{\cos t}$$

L.C.M

$$\Rightarrow \cos^2 t \cdot \frac{d^2y}{dx^2} - \sin t\frac{dy}{dx} = -p^2\sin(pt)$$

$$\Rightarrow (1-\sin^2 t)\frac{d^2y}{dx^2} - \sin t\cdot\frac{dy}{dx} + p^2\sin(pt) = 0$$

$$\Rightarrow (1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + p^2y = 0$$ proved **Ans**

--- {Using given equations}

Q.No 29 → $LHL = \lim_{x\to\pi/2}\left(\frac{1-\sin^3 x}{3\cos^2 x}\right)$

put $x = \frac{\pi}{2} - h$ & $h \to 0$

$$LHL = \lim_{h\to 0}\left(\frac{1-\sin^3\left(\frac{\pi}{2}-h\right)}{3\cos^2\left(\frac{\pi}{2}-h\right)}\right)$$

$$LHL = \lim_{h\to 0}\left(\frac{1-\cos^3 h}{3\sin^2 h}\right)$$

$$LHL = \lim_{h\to 0}\left(\frac{\cancel{(1-\cos h)}(1+\cos^2 h+\cos h)}{3(1+\cos h)\cancel{(1-\cos h)}}\right)$$

$$LHL = \frac{1+1+1}{3(1+1)} = 1/2 \qquad \therefore \boxed{LHL = 1/2}$$

$$RHL = \lim_{x\to \frac{\pi}{2}^+}\left(\frac{b(1-\sin x)}{(\pi-2x)^2}\right)$$

put $x = \frac{\pi}{2}+h$ & $h\to 0$

$$RHL = \lim_{h\to 0}\left[\frac{b\left(1-\sin\left(\frac{\pi}{2}+h\right)\right)}{\left(\pi-2\left(\frac{\pi}{2}+h\right)\right)^2}\right]$$

$$= \lim_{h\to 0}\left(\frac{b(1-\cos h)}{(\cancel{\pi}-\cancel{\pi}-2h)^2}\right)$$

$$= \lim_{h\to 0}\left(\frac{b\cdot 2\sin^2(h/2)}{4h^2}\right)$$

$$= \lim_{h\to 0}\left(\frac{2b\sin^2(h/2)}{4\frac{h^2}{4}\times 4}\right) = \frac{2b}{16} = \frac{b}{8}$$

$$\boxed{RHL = b/8}$$

$f(\pi/2) = a$

Since $f(x)$ is continuous at $x = \pi/2$

$\therefore LHL = RHL = f(\pi/2)$

$\Rightarrow \frac{1}{2} = \frac{b}{8} = a$

$\rightarrow \boxed{a = 1/2 \quad \& \quad b = 4}$ ANS

ANS : 30 → $A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$ & $B = \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$

$$AB = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}\begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix} = \begin{bmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{bmatrix}$$

$\Rightarrow AB = 6I$

$\Rightarrow A^{-1}AB = 6A^{-1}I$

$\Rightarrow IB = 6A^{-1}$

$\Rightarrow B = 6A^{-1}$

$\Rightarrow \boxed{A^{-1} = \frac{1}{6}B}$ we have to use this to solve the given equations

given equations $x - y = 3$; $2x + 3y + 4z = 17$;
$y + 2z = 7$

these equations can be written in the form

$AX = C \Rightarrow X = A^{-1}C$

$C = \begin{bmatrix} 3 \\ 17 \\ 7 \end{bmatrix}$; $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$

$\Rightarrow X = \left(\frac{1}{6}B\right)C$

$\Rightarrow X = \frac{1}{6}\begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}\begin{bmatrix} 3 \\ 17 \\ 7 \end{bmatrix}$

$\Rightarrow X = \frac{1}{6}\begin{bmatrix} 12 \\ -6 \\ 24 \end{bmatrix} \Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix}$

$\therefore \boxed{x = 2, \quad y = -1, \quad z = 4}$ ANS

Ans : 31) → let the Investments are Rs x & Rs $30000-x$

let A → denotes the Investments in two types of bonds

$$A = \begin{array}{cc} \mathrm{I} & \mathrm{II} \\ \left[\begin{matrix} x & 30000-x \end{matrix}\right]_{1\times 2} \end{array}$$

let B → denotes the rate of interest from two types of Bonds

$$B = \begin{array}{c} \mathrm{I} \\ \mathrm{II} \end{array} \left[\begin{matrix} 5/100 \\ 7/100 \end{matrix}\right]_{2\times 1}$$

$$AB = \left[\begin{matrix} x & 30000-x \end{matrix}\right] \left[\begin{matrix} 5/100 \\ 7/100 \end{matrix}\right]$$

$$= \left[\frac{5x}{100} + \frac{210000-7x}{100}\right]$$

$$[2000] = \left[\frac{210000-2x}{100}\right]$$

$$\Rightarrow 2000 = \frac{210000-2x}{100}$$

$$\Rightarrow 200000 = 210000 - 2x$$

$$\Rightarrow 2x = 10000$$

$$\Rightarrow x = 5000$$

∴ the two Investments are Rs 5000, & Rs 25000 **Ans**.

Ans 32 → T.P

$$\tan\left(\frac{\pi}{4} + \frac{1}{2}\cos^{-1}\frac{a}{b}\right) + \tan\left(\frac{\pi}{4} - \frac{1}{2}\cos^{-1}\frac{a}{b}\right) = \frac{2b}{a}$$

let $\frac{1}{2}\cos^{-1}\left(\frac{a}{b}\right) = x$

$$\Rightarrow \tan\left(\frac{\pi}{4}+x\right) + \tan\left(\frac{\pi}{4}-x\right)$$

$= \cancel{\tan x + 1}$

$$= \frac{1+\tan x}{1-\tan x} + \frac{1-\tan x}{1+\tan x}$$

$$= \frac{1+\tan^2 x + 2\cancel{\tan x} + 1 + \tan^2 x - 2\cancel{\tan x}}{1-\tan^2 x}$$

$$= \frac{2(1+\tan^2 x)}{1-\tan^2 x}$$

$$= \frac{2}{\cos(2x)}$$

$$= \frac{2}{\cos\left(\cancel{2} \times \frac{1}{\cancel{2}} \cos^{-1}\frac{a}{b}\right)} = \frac{2}{\frac{a}{b}} = \frac{2b}{a} = \text{RHS}$$

Ans

SECTION: D

Ans 33 →

$(x, y)\ R\ (u, v)$

$\Rightarrow xv = yu$

~~Reflexivity~~ Symmetric

let $(x, y)\ R\ (u, v)$

$\Rightarrow xv = yu$

$\Rightarrow vx = uy$

$\Rightarrow uy = vx$

$\Rightarrow (u, v)\ R\ (x, y)$

$\left\{\begin{array}{l}\text{Rough} \\ (u,v)\ R\ (x,y) \\ uy = vx\end{array}\right.$

$\therefore$ R is ~~reflexive~~ symmetric

Transitive

let $(x, y)\ R\ (u, v)$ & $(u, v)\ R\ (a, b)$

$\Rightarrow xv = yu$ & $ub = va$

$\Rightarrow x\left(\frac{ub}{a}\right) = yu$

$\left\{\begin{array}{l}\text{Rough} \\ (x,y)\ R\ (a,b) \\ xb = ya\end{array}\right.$

$\Rightarrow xb = ya$

$\Rightarrow (x, y)\, R\, (a, b)$

$\therefore$ R is transitive

Reflexive: for each $(x, y) \in A$

$\Rightarrow xy = yx$

$\Rightarrow (x, y)\, R\, (x, y)$

$\therefore$ R is reflexive

$\therefore$ R is an equivalence relation

(b) $A = \{1, 2, 3\}$

Equivalence relations containing $(1, 2)$ are

$R_1 = \{(1,1)\ (2,2)\ (3,3)\ (1,2)\ (2,1)\}$

$R_2 = \{(1,1)\ (2,2)\ (3,3)\ (1,2)\ (2,1)\ (2,3)\ (3,2)\ (1,3)\ (3,1)\}$

No. = 2 Ans

Ans 34 → $A = \begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix}$

LHS $A^2 - 4A + 7I$

$= \begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix} - 4\begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix} + 7\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 1 & 12 \\ -4 & 1 \end{bmatrix} - \begin{bmatrix} 8 & 12 \\ -4 & 8 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$

$= \begin{bmatrix} -7 & 0 \\ 0 & -7 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$

$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = O$ Proved

~~Remember~~ Remember we have to find A^{-1} and A^4 using this equation

for A^{-1}

$$A^2 - 4A + 7I = 0$$

$$A^{-1}A^2 - 4A^{-1}A + 7A^{-1}I = 0$$

$$\Rightarrow A^{-1}AA - 4I + 7A^{-1} = 0$$

$$\Rightarrow A - 4I + 7A^{-1} = 0$$

$$\Rightarrow 7A^{-1} = 4I - A$$

$$\Rightarrow 7A^{-1} = 4\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix}$$

$$A^{-1} = \frac{1}{7}\begin{bmatrix} 2 & -3 \\ 1 & 2 \end{bmatrix}$$ Ans

for A^4

we have $A^2 - 4A + 7I = 0$

$$\Rightarrow A^2 = 4A - 7I$$

$$\Rightarrow A^3 = 4A^2 - 7A$$

$$\Rightarrow A^3 = 4(4A - 7I) - 7A$$

$$\Rightarrow A^3 = 16A - 28I - 7A$$

$$\Rightarrow A^3 = 9A - 28I$$

$$\Rightarrow A^4 = 9A^2 - 28A$$

$$\Rightarrow A^4 = 9(4A - 7I) - 28A$$

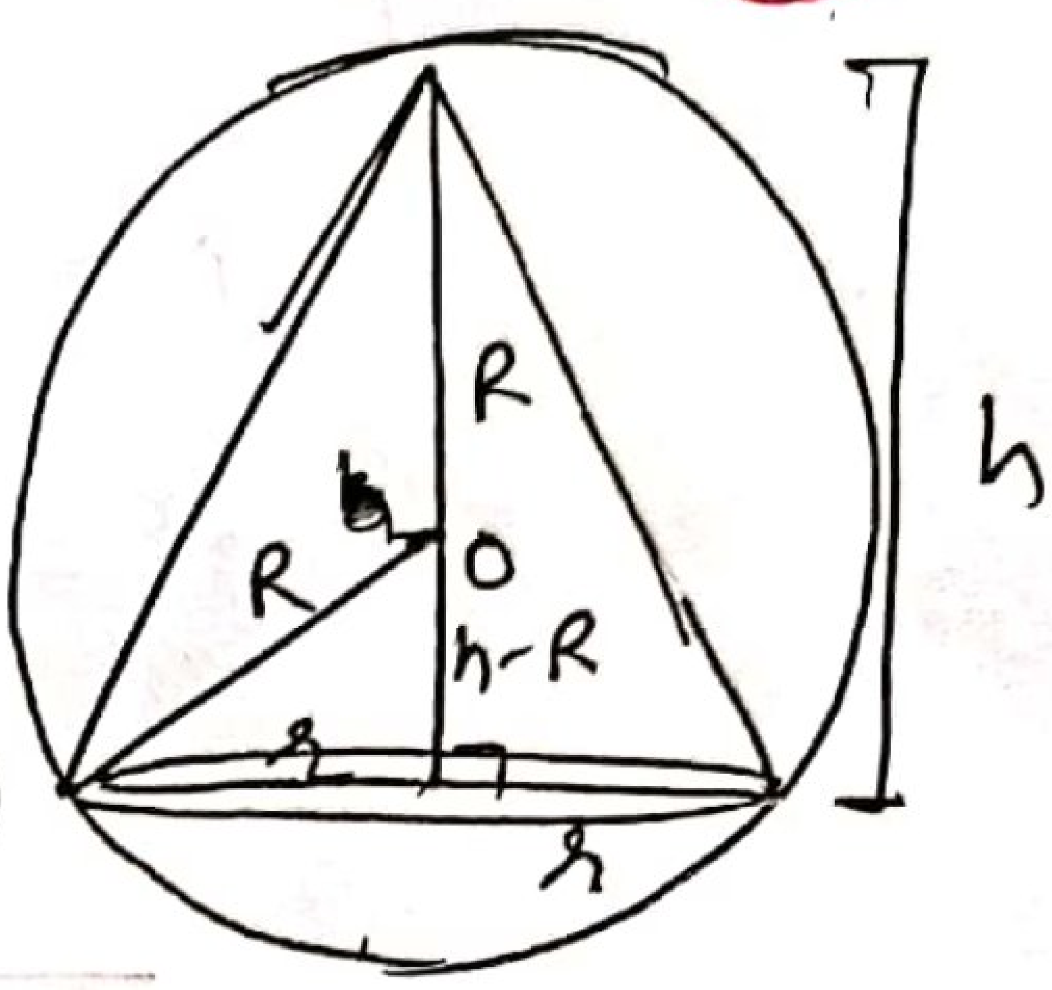
$$\Rightarrow A^4 = 8A - 63I$$

$$\Rightarrow A^4 = 8\begin{bmatrix} 2 & +3 \\ -1 & 2 \end{bmatrix} - 63\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 16 & 24 \\ -8 & 16 \end{bmatrix} - \begin{bmatrix} 63 & 0 \\ 0 & 63 \end{bmatrix}$$

$$\therefore A^4 = \begin{bmatrix} -47 & 24 \\ -8 & -47 \end{bmatrix}$$ Ans

Qn: 35 → let $h \to$ height of cone
$r \to$ Radius of cone
(constant) $R \to$ Radius of sphere



$$R^2 = r^2 + (h-R)^2 \text{ --- given } (1)$$

$V \to$ volume of cone

$$V = \frac{1}{3}\pi r^2 h \text{ --- (to be Max)}$$

$$V = \frac{1}{3}\pi\left[R^2 - (h-R)^2\right]h \text{ --- \{from eq (1)\}}$$

$$V = \frac{1}{3}\pi\left[\cancel{R^2} - h^2 - \cancel{R^2} + 2hR\right]h$$

$$V = \frac{1}{3}\pi\left(-h^3 + 2h^2R\right)$$

$$\frac{dV}{dh} = \frac{1}{3}\pi\left(-3h^2 + 4hR\right)$$

for Max / Min put $\frac{dV}{dh} = 0$

$$3h^2 = 4hR$$

$$\Rightarrow \boxed{h = \frac{4R}{3}}$$

Diff again $\frac{d^2V}{dh^2} = \frac{1}{3}\pi\left(-6h + 4R\right)$

$$\left(\frac{d^2V}{dh^2}\right)_{h=\frac{4R}{3}} = \frac{1}{3}\pi\left(-6\left(\frac{4R}{3}\right) + 4R\right)$$

$$= \frac{1}{3}\pi\left(-4R\right) < 0$$

$\therefore$ volume of cone is Maximum at $h = 4R/3$

put $h = \frac{4R}{3}$ in eq (1)

$$R^2 = r^2 + \left(\frac{4R}{3} - R\right)^2$$

$$R^2 = r^2 + \frac{R^2}{9}$$

$$\Rightarrow \boxed{r^2 = \frac{8R^2}{9}}$$

$$V_{max} = \frac{1}{3}\pi r^2 h$$

$$= \frac{1}{3}\pi\left(\frac{8R^2}{9}\right)\left(\frac{4R}{3}\right)$$

$$= \frac{8}{27} \times \frac{4}{3}\pi R^3$$

vol of cone $= \frac{8}{27} \times$ volume of sphere Proved

Ans 36 →

(a) $y\sqrt{1-x^2} + x\sqrt{1-y^2} = 1$ (Misprint in Ans paper)

put $x = \sin A$ and $y = \sin B$

$$\Rightarrow \sin B\sqrt{1-\sin^2 A} + \sin A\sqrt{1-\sin^2 B} = 1$$

$$\Rightarrow \sin B \cos A + \sin A \cos B = 1$$

$$\Rightarrow \sin(A+B) = 1$$

$$\Rightarrow A + B = \sin^{-1}(1)$$

replace A & B

$$\Rightarrow \sin^{-1}x + \sin^{-1}y = \sin^{-1}(1)$$

Diff wrt x

$$\Rightarrow \frac{1}{\sqrt{1-x^2}} + \frac{1}{\sqrt{1-y^2}}\frac{dy}{dx} = 0$$

$$\Rightarrow \frac{dy}{dx} = -\frac{\sqrt{1-y^2}}{\sqrt{1-x^2}}$$ ANS

(b) let, $y = \cot^{-1}\left(\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right)$

$$y = \tan^{-1}\left(\frac{\sqrt{1+\sin x}-\sqrt{1-\sin x}}{\sqrt{1+\sin x}+\sqrt{1-\sin x}}\right) \text{ --- } \left\{\cot^{-1}x = \tan^{-1}\left(\frac{1}{x}\right)\right\}$$

divide by $\sqrt{1+\sin x}$

$$y = \tan^{-1}\left(\frac{1-\sqrt{\frac{1-\sin x}{1+\sin x}}}{1+\sqrt{\frac{1-\sin x}{1+\sin x}}}\right)$$

$$y = \tan^{-1}(1) - \tan^{-1}\left(\sqrt{\frac{1-\sin x}{1+\sin x}}\right)$$

$$y = \frac{\pi}{4} - \tan^{-1}\left(\sqrt{\frac{1-\cos(\pi/2-x)}{1+\cos(\frac{\pi}{2}-x)}}\right)$$

$$y = \frac{\pi}{4} - \tan^{-1}\left(\sqrt{\frac{\cancel{2}\sin^2(\frac{\pi}{4}-\frac{x}{2})}{\cancel{2}\cos^2(\frac{\pi}{4}-\frac{x}{2})}}\right)$$

$$y = \frac{\pi}{4} - \tan^{-1}\left(\tan\left(\frac{\pi}{4}-\frac{x}{2}\right)\right)$$

$$y = \cancel{\frac{\pi}{4}} - \cancel{\frac{\pi}{4}} + \frac{x}{2}$$

$$y = x/2$$

Diff wrt x

$$\frac{dy}{dx} = \frac{1}{2}$$ (which is independent of x) ANS

— x —