← ULTIMATE MATHEMATICS →

(BY: AJAY MITTAL : 9891067390)

MATRICES : CLASS: 2 (M-2)

Properties of Multiplication

(1) $A^2 = AA$ $\quad$ $A \rightarrow$ must be a square matrix

(2) $A^3 = A^2 A = AA^2$

(3) $AB \neq BA$ (generally)

(4) $AI = A = IA$

(5) $II = I$

(6) $(A+B)^2 \neq A^2 + B^2 + 2AB$

$(A+B)^2 = (A+B)(A+B)$

$= A^2 + \underbrace{AB + BA}_{} + B^2$

(7) $ABC = (AB)C = A(BC)$

$\neq (AC)B$

$$\underset{A}{\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}} \underset{I}{\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = A$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$I \; I = I$

(·) Transpose of a Matrix

- denoted by A' or A^T
- $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}_{2\times 3}$

$A' = \begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix}_{3\times 2}$

(·) Properties of Transpose

(1) $(A')' = A$

(2) $(A+B)' = A' + B'$

(3) $(A-B)' = A' - B'$

(4) $(AB)' = B'A'$

(5) $(ABC)' = C'B'A'$

← ULTIMATE MATHEMATICS →

(BY: AJAY MITTAL: 9891067390)

(·) Symmetric Matrix

Matrix A is said to be "symmetric" if $\boxed{A' = A}$

eg $A = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$

$A' = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix} = A$

eg $A = \begin{bmatrix} 2 & 3 & 4 \\ 3 & 5 & 6 \\ 4 & 6 & 1 \end{bmatrix}$

$A' = \begin{bmatrix} 2 & 3 & 4 \\ 3 & 5 & 6 \\ 4 & 6 & 1 \end{bmatrix} = A$

$\boxed{a_{ij} = a_{ji}}$

(·) Skew-Symmetric Matrix

If $\boxed{A' = -A}$

eg $A = \begin{bmatrix} 0 & -2 \\ 2 & 0 \end{bmatrix}$

$A' = \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix}$

$A' = -\begin{bmatrix} 0 & -2 \\ 2 & 0 \end{bmatrix} = -A$

eg $A = \begin{bmatrix} 0 & 2 & -3 \\ -2 & 0 & 5 \\ 3 & -5 & 0 \end{bmatrix}$

$A' = \begin{bmatrix} 0 & -2 & 3 \\ 2 & 0 & -5 \\ -3 & 5 & 0 \end{bmatrix}$

$A' = -\begin{bmatrix} 0 & 2 & -3 \\ -2 & 0 & 5 \\ 3 & -5 & 0 \end{bmatrix} = -A$

$\boxed{a_{ij} = -a_{ji}}$

Qn 1 If $\underset{1\times3}{\begin{bmatrix} x & -5 & -1 \end{bmatrix}} \underset{3\times3}{\begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}} \begin{bmatrix} x \\ 4 \\ 1 \end{bmatrix} = 0$ find value of x

$\underset{1\times3}{\begin{bmatrix} x-2 & -10 & 2x-8 \end{bmatrix}} \underset{3\times1}{\begin{bmatrix} x \\ 4 \\ 1 \end{bmatrix}} = 0$

← ULTIMATE MATHEMATICS →

(By: AJAY MITTAL: 9891067390)

$$\Rightarrow \left[x^2 - \cancel{2x} - 40 + \cancel{2x} - 8\right] = 0$$

$$\Rightarrow \left[x^2 - 48\right] = \left[\,0\,\right]$$

$$\Rightarrow x^2 - 48 = 0$$

$$\Rightarrow x^2 = 48 \Rightarrow \boxed{x = \pm 4\sqrt{3}}$$ Ans

~~Qn-2 If $A = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$ then show that~~

~~$I + A = (I - A)\begin{bmatrix} \cos\alpha & - \\ -\sin\alpha & \end{bmatrix}$~~

Qn 2 If $A = \begin{bmatrix} 0 & -\tan(\alpha/2) \\ \tan(\alpha/2) & 0 \end{bmatrix}$ then show that

$$I + A = (I - A)\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$$

Soln $I + A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & -\tan(\alpha/2) \\ \tan(\alpha/2) & 0 \end{bmatrix} = \begin{bmatrix} 1 & -\tan(\alpha/2) \\ \tan(\alpha/2) & 1 \end{bmatrix}$ ✓

RHS $(I - A)\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$

$$\left(\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 0 & -\tan(\alpha/2) \\ \tan(\alpha/2) & 0 \end{bmatrix}\right)\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$$

$$= \begin{bmatrix} 1 & \tan(\alpha/2) \\ -\tan(\alpha/2) & 1 \end{bmatrix}\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$$

$\underline{\mathbf{X1}}$

$$\sin(2\theta) = \frac{2\tan\theta}{1+\tan^2\theta}$$

$$\cos(2\theta) = \frac{1-\tan^2\theta}{1+\tan^2\theta}$$

$$= \begin{bmatrix} 1 & \tan(\alpha/2) \\ -\tan(\alpha/2) & 1 \end{bmatrix} \begin{bmatrix} \dfrac{1-\tan^2(\alpha/2)}{1+\tan^2(\alpha/2)} & -\dfrac{2\tan(\alpha/2)}{1+\tan^2(\alpha/2)} \\ \dfrac{2\tan(\alpha/2)}{1+\tan^2(\alpha/2)} & \dfrac{1-\tan^2(\alpha/2)}{1+\tan^2(\alpha/2)} \end{bmatrix}$$

$$= \frac{1}{1+\tan^2(\alpha/2)} \begin{bmatrix} 1 & \tan(\alpha/2) \\ -\tan(\alpha/2) & 1 \end{bmatrix} \begin{bmatrix} 1-\tan^2(\alpha/2) & -2\tan(\alpha/2) \\ 2\tan(\alpha/2) & 1-\tan^2(\alpha/2) \end{bmatrix}$$

$$= \frac{1}{1+\tan^2(\alpha/2)} \begin{bmatrix} 1-\tan^2(\alpha/2)+2\tan^2(\alpha/2) & -2\tan(\alpha/2)+\tan(\alpha/2)-\tan^3(\alpha/2) \\ \text{—} & \text{—} \end{bmatrix}$$

$$= \frac{1}{1+\tan^2(\alpha/2)} \begin{bmatrix} 1+\tan^2(\alpha/2) & -\tan(\alpha/2)-\tan^3(\alpha/2) \\ \text{—} & \text{—} \end{bmatrix}$$

$$= \frac{1}{1+\tan^2(\alpha/2)} \begin{bmatrix} 1+\tan^2(\alpha/2) & -\tan(\alpha/2)\left(1+\tan^2(\alpha/2)\right) \\ \tan(\alpha/2)\left(1+\tan^2(\alpha/2)\right) & 1+\tan^2(\alpha/2) \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -\tan(\alpha/2) \\ \tan(\alpha/2) & 1 \end{bmatrix} = \underline{\underline{\text{LHS}}}$$

→ ULTIMATE MATHEMATICS →

(BY: AJAY MITTAL : 9891067390)

Topic: finding unknown matrix in multiplication

Que find matrix X such that

$$X\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} = \begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{bmatrix}$$

Sol.

$$\begin{bmatrix} \quad \end{bmatrix}_{2\times 2}\begin{bmatrix} \quad \end{bmatrix}_{2\times 3} = \begin{bmatrix} \quad \end{bmatrix}_{2\times 3}$$

Same

Ans

Let $X = \begin{bmatrix} a & b \\ c & d \end{bmatrix}_{2\times 2}$

∴ given equation becomes

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} \quad \end{bmatrix} = \begin{bmatrix} \quad \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} \quad \end{bmatrix} = \begin{bmatrix} \quad \end{bmatrix}$$

$a =$, $b =$, $c =$, $d =$

$\therefore X = \begin{bmatrix} - & - \\ - & - \end{bmatrix}$ **Ans**

← ULTIMATE MATHEMATICS →

(By: AJAY MITTAL : 9891067390).

Qsn given $A = \begin{bmatrix} & & \end{bmatrix}_{3\times 3}$

and given polynomial $f(x) = x^2 + 3x + 2$

find $f(A)$

$$\boxed{f(A) = A^2 + 3A + 2I}$$

$$= \begin{bmatrix} & & \end{bmatrix}\begin{bmatrix} & & \end{bmatrix} + 3\begin{bmatrix} & & \end{bmatrix} + 2\begin{bmatrix} & & \end{bmatrix}$$

$$= \begin{bmatrix} & & \end{bmatrix} + \begin{bmatrix} & & \end{bmatrix} + \begin{bmatrix} & & \end{bmatrix}$$

$$= \begin{bmatrix} & & \end{bmatrix} + \begin{bmatrix} & & \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = O$$

Show that A is a root of this polynomial

To prove $f(A) = O$

!! जय श्री गिरिराज जी महाराज !!

Ons. 1 → If $A=\begin{bmatrix}3 & -2\\ 4 & -2\end{bmatrix}$, find 'k' so that $A^2 = kA - 2I$

Ons. 2 → If $A=\begin{bmatrix}1 & 0\\ -1 & 7\end{bmatrix}$, find 'k' so that $A^2 - 8A + kI = 0$

Ons. 3 → Find 'λ and μ' so that $A^2 = \lambda A + \mu I$ where $A=\begin{bmatrix}2 & 3\\ 1 & 2\end{bmatrix}$

Ons 4 → Let $f(x) = x^2 - 5x + 6$ Find $f(A)$; where $A=\begin{bmatrix}2 & 0 & 1\\ 2 & 1 & 3\\ 1 & -1 & 0\end{bmatrix}$

Ons 5 → for what value of 'x' : $\begin{bmatrix}1 & 2 & 1\end{bmatrix}\begin{bmatrix}1 & 2 & 0\\ 2 & 0 & 1\\ 1 & 0 & 2\end{bmatrix}\begin{bmatrix}0\\ 2\\ x\end{bmatrix} = 0$

Ons 6 → Find 'x', if $\begin{bmatrix}x & -5 & -1\end{bmatrix}\begin{bmatrix}1 & 0 & 2\\ 0 & 2 & 1\\ 2 & 0 & 3\end{bmatrix}\begin{bmatrix}x\\ 4\\ 1\end{bmatrix} = 0$

Ons 7 → Show that matrix $A=\begin{bmatrix}3 & 1\\ -1 & 2\end{bmatrix}$ is a root of the polynomial $f(x) = x^2 - 5x + 7$

Ons 8 → If $A=\begin{bmatrix}1 & -1\\ 2 & -1\end{bmatrix}$ $B=\begin{bmatrix}a & 1\\ b & -1\end{bmatrix}$ and $(A+B)^2 = A^2 + B^2$

Find the value of a and b

Ons 9 → If $A=\begin{bmatrix}0 & -\tan(\alpha/2)\\ \tan(\alpha/2) & 0\end{bmatrix}$ show that

$$I + A = (I - A)\begin{bmatrix}\cos\alpha & -\sin\alpha\\ \sin\alpha & \cos\alpha\end{bmatrix}.$$

Ons 10 → prove that the product of matrices $\begin{bmatrix}\cos^2\theta & \cos\theta\sin\theta\\ \cos\theta\sin\theta & \sin^2\theta\end{bmatrix}$

x and $\begin{bmatrix}\cos^2\phi & \cos\phi\sin\phi\\ \cos\phi\sin\phi & \sin^2\phi\end{bmatrix}$ is the null matrix, when

θ and ϕ differ by an odd multiple of $\pi/2$

Ons 11 → Show that $F(x) \cdot F(y) = F(x+y)$ where

$$F(x) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Qns 12 → Find matrix X so that $X\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} = \begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{bmatrix}$

Qns 13 → If $\begin{bmatrix} 2 & -1 \\ 1 & 0 \\ -3 & 4 \end{bmatrix} A = \begin{bmatrix} -1 & -8 & -10 \\ 1 & -2 & -5 \\ 9 & 22 & 15 \end{bmatrix}$ Find matrix A

Qns 14 → Find matrix X for which $\begin{bmatrix} 1 & -4 \\ 3 & -2 \end{bmatrix} X = \begin{bmatrix} -16 & -6 \\ 7 & 2 \end{bmatrix}$

Qns 15 → Let $A = \begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 5 & 2 \\ 7 & 4 \end{bmatrix}$, $C = \begin{bmatrix} 2 & 5 \\ 3 & 8 \end{bmatrix}$ Find a matrix D such that $CD - AB = O$

Qns 16 → Find a matrix 'A' such that $2A - 3B + 5C = O$ where $B = \begin{bmatrix} -2 & 2 & 0 \\ 3 & 1 & 4 \end{bmatrix}$ and $C = \begin{bmatrix} 2 & 0 & -2 \\ 7 & 1 & 6 \end{bmatrix}$

Qns 17 → If $A = \begin{bmatrix} \alpha & 0 \\ 1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 \\ 5 & 1 \end{bmatrix}$ Find the values of 'α' for which $A^2 = B$

Qns 18 → If $\begin{bmatrix} x & 4 & 1 \end{bmatrix} \begin{bmatrix} 2 & 1 & 2 \\ 1 & 0 & 2 \\ 0 & 2 & -4 \end{bmatrix} \begin{bmatrix} x \\ 4 \\ 1 \end{bmatrix} = O$. Find x

← ANSWERS →

1). $k = 1$

2). $k = 7$

3). $\lambda = 4$, $\mu = -1$

4). $f(A) = \begin{bmatrix} 1 & -1 & -3 \\ -1 & -1 & -10 \\ -5 & 4 & 4 \end{bmatrix}$

5). $x = -1$

6). $x = \pm 4\sqrt{3}$

7). $f(A) = O$

8). $a = 1$, $b = 4$

12). $\begin{bmatrix} 1 & -2 \\ 2 & 0 \end{bmatrix}$

13). $\begin{bmatrix} 1 & -2 & -5 \\ 3 & 4 & 0 \end{bmatrix}$

(14). $\begin{bmatrix} 6 & 2 \\ 11/2 & 2 \end{bmatrix}$

(15). $\begin{bmatrix} -191 & -110 \\ 77 & 44 \end{bmatrix}$

(16). $A = \begin{bmatrix} -8 & 3 & 5 \\ -13 & -1 & -9 \end{bmatrix}$

(17) No value of α

(18). $x = -2$
$x = 3$