DIFFERENTIATION & CONTINUITY.

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SOLUTIONS OF WORKSHEET NO:-5

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Ans. 1 → $x = a\sin t - b\cos t$

$y = a\cos t + b\sin t$

Diff. wrt 't'

$\frac{dx}{dt} = a\cos t + b\sin t$ | $\frac{dy}{dt} = -a\sin t + b\cos t$

$\frac{dx}{dt} = y$ | $\frac{dy}{dt} = -x$

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$$

$$\frac{dy}{dx} = \frac{-x}{y} \quad ---- (i)$$

Diff. wrt. x

$$\frac{d^2y}{dx^2} = \frac{y(-1) - (-x)\frac{dy}{dx}}{y^2}$$

$$= \frac{-y + x\left(-\frac{x}{y}\right)}{y^2} \quad -- \{\text{from eq (i)}\}$$

$$= \frac{-y^2 - x^2}{y^3}$$

$$\frac{d^2y}{dx^2} = \frac{-(x^2+y^2)}{y^3} \quad \underline{\text{Ans}}$$

Ans 2 → $y = \log(1+\cos x)$

Diff. wrt x

$$\frac{dy}{dx} = \frac{1}{1+\cos x}(-\sin x) = \frac{-\sin x}{1+\cos x}$$

$$\frac{dy}{dx} = \frac{-\cancel{2}\sin(x/2)\cos(x/2)}{\cancel{2}\cos^2(x/2)}$$

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$$\frac{dy}{dx} = -\tan(x/2)$$

Diff again wrt x

$$\frac{d^2y}{dx^2} = -\sec^2\left(\frac{x}{2}\right)\cdot\frac{1}{2}$$

Diff again wrt x

$$\frac{d^3y}{dx^3} = -\frac{1}{\cancel{2}}\cdot\cancel{2}\sec\left(\frac{x}{2}\right)\cdot\sec\left(\frac{x}{2}\right)\tan\left(\frac{x}{2}\right)\cdot\frac{1}{2}$$

$$\frac{d^3y}{dx^3} = -\frac{1}{2}\sec^2\left(\frac{x}{2}\right)\tan\left(\frac{x}{2}\right)$$

taking LHS $\quad \frac{d^3y}{dx^3} + \frac{d^2y}{dx^2}\cdot\frac{dy}{dx}$

$$= -\frac{1}{2}\sec^2\left(\frac{x}{2}\right)\tan\left(\frac{x}{2}\right) + \left(-\frac{1}{2}\sec^2\frac{x}{2}\right)\cdot\left(-\tan\frac{x}{2}\right)$$

$$= \cancel{-\frac{1}{2}\sec^2\left(\frac{x}{2}\right)\tan\left(\frac{x}{2}\right)} + \cancel{\frac{1}{2}\sec^2\left(\frac{x}{2}\right)\tan\left(\frac{x}{2}\right)}$$

$= 0 =$ RHS **Ans**

Ans 3 → $y = \cos^{-1}x$

or $\quad x = \cos y$

Diff wrt 'y'

$$\frac{dx}{dy} = -\sin y$$

reciprocal $\quad \frac{dy}{dx} = \frac{-1}{\sin y} = -\operatorname{cosec} y \quad \cdots (1)$

Diff wrt x

$$\frac{d^2y}{dx^2} = -(-\operatorname{cosec} y\cdot\cot y)\cdot\frac{dy}{dx}$$

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$$\frac{d^2y}{dx^2} = \mathrm{cosec}^2 y \cdot \cot y \cdot (-\mathrm{cosec}\,y) \quad \text{---}\{\text{from eq no (1)}\}$$

$$\frac{d^2y}{dx^2} = -\mathrm{cosec}^2 y \cdot \cot y \qquad \underline{\text{Ans}}$$

Ans 4 → $y = x^x$

taking log on both sides

$\log y = x \log x$

Diff w.r.t x

$$\frac{1}{y} \cdot \frac{dy}{dx} = x \cdot \frac{1}{x} + \log x \cdot 1$$

$$\Rightarrow \frac{dy}{dx} = y(1+\log x) \quad \text{----} (1)$$

Diff again w.r.t x

$$\frac{d^2y}{dx^2} = y \cdot \left(\frac{1}{x}\right) + (1+\log x)\frac{dy}{dx}$$

$$\frac{d^2y}{dx^2} = \frac{y}{x} + \left(\frac{dy}{dx}\right) \cdot \frac{1}{y}\frac{dy}{dx} \quad \text{...}\{\text{from eq. no (1)}\}$$

$$\frac{d^2y}{dx^2} - \frac{1}{y}\left(\frac{dy}{dx}\right)^2 - \frac{y}{x} = 0 \qquad \underline{\text{Ans}}$$

Ans 5 → $\cos y = x \cos(a+y)$

<u>Two Methods</u>

<u>Method - 1</u>

$$x = \frac{\cos y}{\cos(a+y)}$$

Diff w.r.t 'y'

$$\frac{dx}{dy} = \frac{\cos(a+y) \cdot (-\sin y) - \cos y \cdot (-\sin(a+y))}{\cos^2(a+y)}$$

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$$\frac{dx}{dy} = \frac{-\cos(a+y)\sin y + \sin(a+y)\cdot\cos a}{\cos^2(a+y)}$$

$$= \frac{\sin(\cancel{a}+y-\cancel{a})}{\cos^2(a+y)} \quad \cdots \left\{ \begin{array}{l} \sin A\cos B - \cos A\sin B \\ = \sin(A-B) \end{array} \right\}$$

$$\frac{dx}{dy} = \frac{\sin y}{\cos^2(a+y)}$$

Reciprocal

$$\frac{dy}{dx} = \frac{\cos^2(a+y)}{\sin y} \quad \text{Ans}$$

Method : II

$$\cos y = x\cos(a+y) \quad \cdots\cdots (1)$$

Diff wrt x

$$-\sin y\,\frac{dy}{dx} = x(-\sin(a+y))\frac{dy}{dx} + \cos(a+y)\cdot 1$$

$$\frac{dy}{dx}\left(x\sin(a+y) - \sin y\right) = \cos(a+y)$$

$$\frac{dy}{dx} = \frac{\cos(a+y)}{x\sin(a+y) - \sin y}$$

$$\frac{dy}{dx} = \frac{\cos(a+y)}{\frac{\cos y}{\cos(a+y)}\cdot\sin(a+y) - \sin y} \quad \cdots \{\text{from eq no. 1}\}$$

$$= \frac{\cos^2(a+y)}{\cos y\cdot\sin(a+y) - \sin y\cdot\cos(a+y)}$$

$$\frac{dy}{dx} = \frac{\cos^2(a+y)}{\sin(a+\cancel{y}-\cancel{y})} = \frac{\cos^2(a+y)}{\sin a} \quad \text{Ans}$$

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Q.6 → $x\sin(a+y) + \sin a \cdot \cos(a+y) = 0$

(here $\sin a$ is a constant : remember)

$\Rightarrow$ $$x = \frac{-\sin a \cdot \cos(a+y)}{\sin(a+y)}$$

Diff w.r.t y

$$\frac{dx}{dy} = \frac{\sin(a+y)\left[-\sin a \cdot (-\sin(a+y))\right] - (-\sin a \cdot \cos(a+y)) \cdot \cos(a+y)}{\sin^2(a+y)}$$

$$= \frac{\sin a \cdot \sin^2(a+y) + \sin a \cdot \cos^2(a+y)}{\sin^2(a+y)}$$

$$= \frac{\sin a \cdot \left(\sin^2(a+y) + \cos^2(a+y)\right)}{\sin^2(a+y)}$$

$$\frac{dx}{dy} = \frac{\sin a}{\sin^2(a+y)}$$

reciprocal

$$\frac{dy}{dx} = \frac{\sin^2(a+y)}{\sin a} \quad \text{ANS}$$

Q.7 → $\sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y)$

put $x = \sin A$ and $y = \sin B$

$$\sqrt{1-\sin^2 A} + \sqrt{1-\sin^2 B} = a(\sin A - \sin B)$$

$$\cos A + \cos B = a(\sin A - \sin B)$$

$\Rightarrow$ $$\cancel{2\cos\left(\frac{A+B}{2}\right)} \cos\left(\frac{A-B}{2}\right) = a \cdot \cancel{2\cos\left(\frac{A+B}{2}\right)} \sin\left(\frac{A-B}{2}\right)$$

$\Rightarrow$ $$\cot\left(\frac{A-B}{2}\right) = a$$

$\Rightarrow \frac{A-B}{2} = \cot^{-1} a$

$\Rightarrow A-B = 2\cot^{-1} a$

replace A and B

$\Rightarrow \sin^{-1} x - \sin^{-1} y = 2\cot^{-1} a$ Constant

Diff w.r.t x

$$\frac{1}{\sqrt{1-x^2}} - \frac{1}{\sqrt{1-y^2}} \cdot \frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}} \quad \text{Ans}$$

Qn 8 → $y\sqrt{1-x^2} + x\sqrt{1-y^2} = 1$

put $x = \sin A$ and $y = \sin B$

$\sin B \cdot \sqrt{1-\sin^2 A} + \sin A \cdot \sqrt{1-\sin^2 B} = 1$

$\sin B \cdot \cos A + \sin A \cos B = 1$

$\Rightarrow \sin(A+B) = 1$

$\Rightarrow A + B = \sin^{-1}(1)$

replace A & B

$\Rightarrow \sin^{-1} x + \sin^{-1} y = \frac{\pi}{2}$

Diff w.r.t x

$$\frac{1}{\sqrt{1-x^2}} + \frac{1}{\sqrt{1-y^2}} \cdot \frac{dy}{dx} = 0$$

$$\Rightarrow \frac{dy}{dx} = -\frac{\sqrt{1-y^2}}{\sqrt{1-x^2}} \quad \text{Ans}$$

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Qn. 9 → $x\sqrt{1+y} + y\sqrt{1+x} = 0$

$\Rightarrow x\sqrt{1+y} = -y\sqrt{1+x}$

Squaring

$$x^2(1+y) = y^2(1+x)$$

$$\Rightarrow x^2 + x^2y = y^2 + y^2x$$

$$\Rightarrow x^2 - y^2 = y^2x - x^2y$$

$$\Rightarrow (x+y)\cancel{(x-y)} = yx\overset{(-1)}{\cancel{(y-x)}}$$

$$\Rightarrow x+y = -yx$$

$$\Rightarrow y + yx = -x$$

$$\Rightarrow y(1+x) = -x$$

$$\Rightarrow y = \frac{-x}{1+x}$$

Diff w.r.t x

$$\frac{dy}{dx} = \frac{(1+x)(-1) - (-x)(1)}{(1+x)^2}$$

$$= \frac{-1 \cancel{-x} \cancel{+x}}{(1+x)^2}$$

$$\frac{dy}{dx} = \frac{-1}{(1+x)^2}$$

$$\Rightarrow (1+x)^2 \cdot \frac{dy}{dx} = -1$$

$$\Rightarrow (1+x)^2\frac{dy}{dx} + 1 = 0 \quad \underline{\text{Ans}}$$

Qn 10 → $y\sqrt{x^2+1} = \log\left(\sqrt{x^2+1} - x\right)$

Diff w.r.t x

$$y \cdot \frac{1}{\cancel{2}\sqrt{x^2+1}}(\cancel{2}x) + \sqrt{x^2+1}\cdot\frac{dy}{dx} = \frac{1}{(\sqrt{x^2+1}-x)}\cdot\left(\frac{\cancel{2}x}{\cancel{2}\sqrt{x^2+1}} - 1\right)$$

$$\frac{xy}{\sqrt{x^2+1}} + \sqrt{x^2+1}\cdot\frac{dy}{dx} = \frac{1}{\cancel{\sqrt{x^2+1}-x}} \cdot {-1}\left(\frac{\cancel{x-\sqrt{x^2+1}}}{\sqrt{x^2+1}}\right)$$

$$\Rightarrow \frac{xy}{\sqrt{x^2+1}} + \sqrt{x^2+1}\,\frac{dy}{dx} = \frac{-1}{\sqrt{x^2+1}}$$

L.C.M

$$\Rightarrow xy + (x^2+1)\frac{dy}{dx} = -1$$

$$\Rightarrow (x^2+1)\frac{dy}{dx} + xy + 1 = 0$$ Ans

Qn. 11 → $e^x + e^y = e^{x+y}$ ---- (1)

Diff w.r.t x

$$e^x + e^y\cdot\frac{dy}{dx} = e^{x+y}\cdot\left(1+\frac{dy}{dx}\right)$$

$$e^x + e^y\cdot\frac{dy}{dx} = e^{x+y} + e^{x+y}\cdot\frac{dy}{dx}$$

$$\Rightarrow \frac{dy}{dx}\left(e^y - e^{x+y}\right) = e^{x+y} - e^x$$

$$\Rightarrow \frac{dy}{dx} = \frac{e^{x+y}-e^x}{e^y-e^{x+y}}$$

$$= \frac{e^x(e^y-1)}{e^y(1-e^x)}$$

$$\frac{dy}{dx} = \frac{-e^x(e^y-1)}{e^y(e^x-1)}$$ Ans

Qn. 12 → $\sqrt{y+x} + \sqrt{y-x} = c$

Diff w.r.t x

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$$\frac{1}{2\sqrt{y+x}}\cdot\left(\frac{dy}{dx}+1\right)+\frac{1}{2\sqrt{y-x}}\cdot\left(\frac{dy}{dx}-1\right)=0$$

$$\frac{dy}{dx}\left(\frac{1}{\cancel{2}\sqrt{y+x}}+\frac{1}{\cancel{2}\sqrt{y-x}}\right)=\frac{1}{\cancel{2}\sqrt{y-x}}-\frac{1}{\cancel{2}\sqrt{y+x}}$$

$$\frac{dy}{dx}\left(\frac{\sqrt{y-x}+\sqrt{y+x}}{\cancel{\sqrt{y-x^2}}}\right)=\frac{\sqrt{y+x}-\sqrt{y-x}}{\cancel{\sqrt{y^2-x^2}}}$$

$$\frac{dy}{dx}=\frac{\sqrt{y+x}-\sqrt{y-x}}{\sqrt{y-x}+\sqrt{y+x}}$$

Rationalize

$$\frac{dy}{dx}=\frac{(\sqrt{y+x}-\sqrt{y-x})(\sqrt{y-x}-\sqrt{y+x})}{(\sqrt{y-x}+\sqrt{y+x})(\sqrt{y-x}-\sqrt{y+x})}$$

$$=\frac{-(y+\cancel{x}+y-\cancel{x}-2\sqrt{y^2-x^2})}{\cancel{y}-x-\cancel{y}-x}$$

$$=\frac{-2y+2\sqrt{y^2-x^2}}{-2x}$$

$$=\frac{y}{x}-\frac{\sqrt{y^2-x^2}}{x}$$

$$=\frac{y}{x}-\sqrt{\frac{y^2-x^2}{x^2}}$$

$$\frac{dy}{dx}=\frac{y}{x}-\sqrt{\frac{y^2}{x^2}-1}$$ Ans

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