SOLUTIONS : INTEGRATION

WORKSHEET No: 7 Class No: 9

Qn: 1 $I = \int \tan^{-1}\sqrt{x}\, dx$

put $x = t^2$

$dx = 2t\,dt$

$I = 2\int \underset{I}{\tan^{-1}(t)} \cdot \underset{II}{t}\, dt$

$= 2\left[\tan^{-1}t \cdot \frac{t^2}{2} - \frac{1}{2}\int \frac{1}{1+t^2} \cdot t^2\, dt\right]$

$= \tan^{-1}t \cdot t^2 - \int \frac{1+t^2-1}{1+t^2}\, dt$

$= t^2 \cdot \tan^{-1}t - \int 1 - \frac{1}{1+t^2}\, dt$

$I = t^2 \cdot \tan^{-1}t - (t - \tan^{-1}t) + C$

$I = x\tan^{-1}\sqrt{x} - \sqrt{x} + \tan^{-1}\sqrt{x} + C$

$I = (x+1)\tan^{-1}\sqrt{x} - \sqrt{x} + C$ **Ans**

Qn: 2 ⇒ $I = \int \frac{x + \sin x}{1+\cos x}\, dx$

$I = \int \frac{x + 2\sin\frac{x}{2}\cos\frac{x}{2}}{2\cos^2 x/2}\, dx$

$= \int \frac{1}{2} x \sec^2(x/2) + \tan(x/2)\, dx$

$= \frac{1}{2}\int \underset{I}{x}\, \underset{II}{\sec^2(x/2)}\, dx + \int \tan(x/2)\, dx$

$I = \frac{1}{2}\left[2x \cdot \tan(x/2) - 2\int \tan(x/2)\, dx\right] + \int \tan(x/2)\, dx$

$\Rightarrow I = x\tan(x/2) - \int \cancel{\tan(x/2)}\,dx + \int \cancel{\tan(x/2)}\,dx$

$I = x\tan\left(\frac{x}{2}\right) + c$ Ans

Que. 3 → $I = \int \frac{\log x}{x^2}\,dx$

$I = \int \underset{I}{\log x} \cdot \underset{II}{\frac{1}{x^2}}\,dx$

$I = \log x \cdot \left(-\frac{1}{x}\right) - \int \frac{1}{x} \cdot \left(-\frac{1}{x}\right) dx$

$I = -\frac{\log x}{x} + \int \frac{1}{x^2}\,dx$

$I = \frac{-\log x}{x} - \frac{1}{x} + C$

$I = -\frac{1}{x}(1+\log x) + C$ Ans

Que 4 → $I = \int e^{\sqrt{x}}\,dx$

put $x = t^2$

$dx = 2t\,dt$

$I = 2\int \underset{II}{e^t} \cdot \underset{I}{t}\,dt$

$= 2\left[t \cdot e^t - \int e^t\,dt\right]$

$= 2\left[te^t - e^t\right] + c$

$= 2e^t(t-1) + c$

$I = 2e^{\sqrt{x}}(\sqrt{x} - 1) + C$ Ans

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(Scrub. w.s 7)

Ans: 6 → $I = \int \sin^{-1}\left(\frac{2x}{1+x^2}\right) dx$

put $x = \tan\theta \Rightarrow dx = \sec^2\theta \, d\theta$

$$I = \int \sin^{-1}\left(\frac{2\tan\theta}{1+\tan^2\theta}\right) \cdot \sec^2\theta \, d\theta$$

$$I = \int \sin^{-1}(\sin(2\theta)) \cdot \sec^2\theta \, d\theta$$

$$I = 2\int \underset{I}{\theta} \cdot \underset{II}{\sec^2\theta} \, d\theta$$

$$I = 2\left[\theta \cdot \tan\theta - \int \tan\theta \, d\theta\right]$$

$$I = 2\left[\theta \tan\theta - \log|\sec\theta|\right] + c$$

$$I = 2\left[\tan^{-1}x \cdot x - \log\left|\sqrt{1+x^2}\right|\right] + c$$

$$= 2\left[x\tan^{-1}x - \frac{1}{2}\log|1+x^2|\right] + c$$

$$I = 2x\tan^{-1}x - \log|1+x^2| + c \quad \text{Ans}$$

Ans: 7 → $I = \int e^x\left(\frac{2+\sin(2x)}{1+\cos(2x)}\right) dx$

$$I = \int e^x\left(\frac{2 + 2\sin x\cos x}{2\cos^2 x}\right) dx$$

$$I = \int e^x(\sec^2 x + \tan x)\, dx$$

$$= \int e^x \underset{II\ \ I}{\tan x} \, dx + \int e^x \sec^2 x \, dx$$

$$I = \tan x \cdot e^x - \cancel{\int \sec^2 x \cdot e^x \, dx} + \cancel{\int e^x \sec^2 x \, dx}$$

$$I = e^x \tan x + C \quad \text{Ans}$$

Ans = 8 → $I = \int \sin(\log x) + \cos(\log x)\, dx$

put $\log x = t$

$x = e^t$

$\Rightarrow dx = e^t dt$

$$I = \int e^t (\sin t + \cos t)\, dt$$

$$= \int \underset{\mathrm{II}}{e^t} . \underset{\mathrm{I}}{\sin t}\, dt + \int e^t . \cos t\, dt$$

$$= \sin t . e^t - \int \cancel{\cos t . e^t\, dt} + \int \cancel{e^t . \cos t\, dt}$$

$$I = e^t . \sin t + c$$

$$I = x . \sin(\log x) + c$$ Ans

Ans 9 → $I = \int e^x \left(\frac{x-3}{(x-1)^3} \right) dx$

$$I = \int e^x \left(\frac{x-1-2}{(x-1)^3} \right) dx$$

$$= \int e^x \left(\frac{1}{(x-1)^2} - \frac{2}{(x-1)^3} \right) dx$$

$$= \int \underset{\mathrm{II}}{e^x} . \underset{\mathrm{I}}{\frac{1}{(x-1)^2}}\, dx - 2 \int e^x . \frac{1}{(x-1)^3}\, dx$$

$$= \frac{1}{(x-1)^2} . e^x - \int \frac{-2}{(x-1)^3} . e^x\, dx - 2 \int e^x . \frac{1}{(x-1)^3}\, dx$$

$$= \frac{e^x}{(x-1)^2} + 2 \int \cancel{\frac{e^x}{(x-1)^3}\, dx} - 2 \int \cancel{e^x . \frac{1}{(x-1)^3}\, dx}$$

$$I = \frac{e^x}{(x-1)^2} + c$$ Ans

Ans: 10 → $I = \int \frac{2-x}{(1-x)^2} \cdot e^x \, dx$

$$I = \int e^x \left(\frac{1-x+1}{(1-x)^2} \right) dx$$

$$= \int e^x \left(\frac{1}{1-x} + \frac{1}{(1-x)^2} \right) dx$$

$$= \int \underset{II}{e^x} \cdot \underset{I}{\frac{1}{1-x}} \, dx + \int e^x \cdot \frac{1}{(1-x)^2} \, dx$$

$$= \frac{1}{1-x} \cdot e^x - \int \frac{-1}{(1-x)^2} \cdot (-1) \cdot e^x \, dx + \int e^x \cdot \frac{1}{(1-x)^2} \, dx$$

$$I = \frac{e^x}{1-x} - \cancel{\int e^x \cdot \frac{1}{(1-x)^2} \, dx} + \cancel{\int e^x \cdot \frac{1}{(1-x)^2} \, dx}$$

$$I = \frac{e^x}{1-x} + C \quad \underline{\text{Ans}}$$

Ans 11 → $I = \int \frac{1}{\log x} - \frac{1}{(\log x)^2} \, dx$

put $\log x = t$

$x = e^t$

$dx = e^t \, dt$

$$I = \int e^t \cdot \left(\frac{1}{t} - \frac{1}{t^2} \right) dt$$

$$= \int \underset{II}{e^t} \cdot \underset{I}{\frac{1}{t}} \, dt - \int e^t \cdot \frac{1}{t^2} \, dt$$

$$= \frac{1}{t} \cdot e^t - \cancel{\int \frac{-1}{t^2} e^t \, dt} - \cancel{\int e^t \cdot \frac{1}{t^2} \, dt}$$

$$I = \frac{1}{t} \cdot e^t + C \Rightarrow \frac{x}{\log x} + C \quad \underline{\text{Ans}}$$

Ans. 12 → $I = \int \frac{e^x}{x} \cdot \left(x(\log x)^2 + 2\log x \right) dx$

$I = \int e^x \cdot \left((\log x)^2 + \frac{2\log x}{x} \right) dx$

$= \underset{\text{II}}{\int e^x} \cdot \underset{\text{I}}{(\log x)^2}\, dx + \int e^x \cdot \frac{2\log x}{x}\, dx$

$= (\log x)^2 \cdot e^x - \cancel{2\int \frac{\log x}{x} \cdot e^x dx} + \cancel{\int e^x \cdot \frac{2\log x}{x} dx}$

$I = e^x \cdot (\log x)^2 + C$ Ans

Ans. 13 → $I = \int e^x \left(\frac{\sin(4x) - 4}{1 - \cos(4x)} \right) dx$

$I = \int e^x \left(\frac{2\sin(2x)\cos(2x) - 4}{2\sin^2(2x)} \right) dx$

Separate

$= \int e^x \left(\cot(2x) - 2\operatorname{cosec}^2(2x) \right) dx$

$I = \int \underset{\text{II}}{e^x} \cdot \underset{\text{I}}{\cot(2x)}\, dx - 2\int e^x \cdot \operatorname{cosec}^2(2x)\, dx$

$I = \cot(2x) \cdot e^x + \cancel{2\int \operatorname{cosec}^2(2x) \cdot e^x dx} - \cancel{2\int e^x \cdot \operatorname{cosec}^2(2x) dx}$

$I = e^x \cdot \cot(2x) + C$ Ans

Ans. 14 → $I = \int e^{2x} \left(\frac{1 + \sin(2x)}{1 + \cos(2x)} \right) dx$

$I = \int e^{2x} \cdot \left(\frac{1 + \sin(2x)}{2\cos^2 x} \right)$

$$I = \int e^{2x} \cdot \left(\frac{1}{2} \sec^2 x + \frac{2 \sin x \cos x}{2 \cos^2 x} \right) dx$$

$$= \int e^{2x} \left(\frac{1}{2} \sec^2 x + \tan x \right) dx$$

$$I = \int \underset{II}{e^{2x}} \cdot \underset{I}{\tan x}\, dx + \frac{1}{2} \int e^{2x} \cdot \sec^2 x\, dx$$

$$= \cancel{e \tan x \cdot \frac{e^{2x}}{2} \ldots}$$

$$= \tan x \cdot \frac{e^{2x}}{2} - \cancel{\int \sec^2 x \cdot \frac{e^{2x}}{2} dx} + \cancel{\frac{1}{2} \int e^{2x} \sec^2 x\, dx}$$

$$I = \frac{e^{2x}}{2} \cdot \tan x + C \quad \underline{\text{Ans}}$$

Ans. 15 → $I = \int \underset{II}{e^{2x}} \cdot \underset{I}{\sin(3x)}\, dx$

$$I = \sin(3x) \cdot \frac{e^{2x}}{2} - \frac{3}{2} \int \underset{I}{\cos(3x)} \cdot \underset{II}{e^{2x}}\, dx$$

$$I = \frac{e^{2x}}{2} \cdot \sin(3x) - \frac{3}{2} \left[\cos(3x) \cdot \frac{e^{2x}}{2} + \frac{3}{2} \int \sin(3x) \cdot e^{2x} dx \right]$$

$$I = \frac{e^{2x}}{2} \cdot \sin(3x) - \frac{3}{4} e^{2x} \cdot \cos(3x) - \frac{9}{4} I$$

$$I + \frac{9}{4} I = \frac{e^{2x}}{4} \left(2 \sin(3x) - 3 \cos(3x) \right)$$

$$\Rightarrow \frac{13 I}{\cancel{4}} = \frac{e^{2x}}{\cancel{4}} \left(2 \sin(3x) - 3 \cos(3x) \right)$$

$$I = \frac{e^{2x}}{13} \left(2 \sin(3x) - 3 \cos(3x) \right) + C \quad \underline{\text{Ans}}$$

Qns No: 16 $I = \int \underset{II}{e^{ax}} \cdot \underset{I}{\cos(bx+c)}\, dx$

$I = \cos(bx+c) \cdot \frac{e^{ax}}{a} - \int -\sin(bx+c) \cdot b \cdot \frac{e^{ax}}{a}\, dx$

$= \frac{e^{ax}}{a} \cdot \cos(bx+c) + \frac{b}{a} \int \underset{II}{e^{ax}} \cdot \underset{I}{\sin(bx+c)}\, dx$

$= \frac{e^{ax}}{a} \cdot \cos(bx+c) + \frac{b}{a} \left[\sin(bx+c) \cdot \frac{e^{ax}}{a} - \frac{b}{a} \int \cos(bx+c) \cdot e^{ax}\, dx \right]$

$I = \frac{e^{ax}}{a} \cdot \cos(bx+c) + \frac{b}{a^2} \cdot e^{ax} \cdot \sin(bx+c) - \frac{b^2}{a^2} I$

$I + \frac{b^2}{a^2} I = \frac{e^{ax}}{a^2} \left(a\cos(bx+c) + b\sin(bx+c) \right)$

$\left(\frac{a^2+b^2}{\cancel{a^2}} \right) I = \frac{e^{ax}}{\cancel{a^2}} \left(a\cos(bx+c) + b\sin(bx+c) \right)$

$I = \frac{e^{ax}}{a^2+b^2} \left(a\cos(bx+c) + b\sin(bx+c) \right) + C$ Ans

Qns : 17 → $I = \int e^{x} \cdot \cos^2 x\, dx$

$I = \int e^{x} \cdot \left(\frac{1+\cos(2x)}{2} \right) dx$

$I = \frac{1}{2} \int e^{x} dx + \frac{1}{2} \int e^{x} \cdot \cos(2x)\, dx$

$I = \frac{1}{2} e^{x} + \frac{1}{2} I_1 + c$

when $I_1 = \int \underset{II}{e^{x}} \underset{I}{\cos(2x)}\, dx$

(Solution WS. 7)

$$I_1 = \cos(2x)\cdot e^x + 2\int \underset{I}{\sin(2x)}\cdot \underset{II}{e^x}\,dx$$

$$I_1 = e^x\cdot\cos(2x) + 2\left[\sin(2x)\cdot e^x - 2\int\cos(2x)\cdot e^x\,dx\right]$$

$$I_1 = e^x\cdot\cos(2x) + 2e^x\sin(2x) - 4I_1$$

$$5I_1 = e^x\left(\cos(2x) + 2\sin(2x)\right)$$

$$I_1 = \frac{e^x}{5}\left[\cos(2x) + 2\sin(2x)\right]$$

$$\therefore I = \frac{1}{2}e^x + \frac{1}{2}\left(\frac{e^x}{5}\left(\cos(2x) + 2\sin(2x)\right)\right) + C$$

$$I = \frac{1}{2}e^x + \frac{e^x}{10}\left(\cos(2x) + 2\sin(2x)\right) + C$$

Ans: 18 → $I = \int \sin(\log x)\,dx$

put $\log x = t$

$x = e^t$

$dx = e^t\,dt$

$$I = \int \underset{II}{e^t}\cdot\underset{I}{\sin t}\,dt$$

$$= \sin t\cdot e^t - \int \underset{I}{\cos t}\cdot\underset{II}{e^t}\,dt$$

$$= e^t\cdot\sin t - \left\{\cos t\cdot e^t + \int \sin t\cdot e^t\,dt\right\}$$

$$I = e^t\sin t - e^t\cos t - I$$

$$2I = e^t(\sin t - \cos t)$$

$$I = \frac{e^t}{2}(\sin t - \cos t) + C$$

$$\Rightarrow I = \frac{x}{2}\left[\sin(\log x) - \cos(\log x)\right] + c \quad \underline{\text{Ans}}$$

Q.No. 19 →

$$I = \int e^{2x} \cdot \sin x \cdot \cos x \, dx$$

$$I = \int e^{2x} \cdot \frac{\sin(2x)}{2} \, dx$$

$$I = \frac{1}{2}\int \underset{\text{II}}{e^{2x}} \cdot \underset{\text{I}}{\sin(2x)} \, dx$$

Proceed

Q.No. 20 →

$$I = \int \frac{\sin^{-1}\sqrt{x} - \cos^{-1}\sqrt{x}}{\sin^{-1}\sqrt{x} + \cos^{-1}\sqrt{x}} \, dx$$

we know that $\sin^{-1}\sqrt{x} + \cos^{-1}\sqrt{x} = \pi/2$

$$\therefore I = \int \frac{\sin^{-1}\sqrt{x} - \left(\frac{\pi}{2} - \sin^{-1}\sqrt{x}\right)}{\pi/2} \, dx$$

$$I = \frac{2}{\pi}\int 2\sin^{-1}\sqrt{x} - \frac{\pi}{2} \, dx$$

$$I = \frac{4}{\pi}\int \sin^{-1}\sqrt{x} \, dx - \int dx$$

~~So~~ $I = \frac{4}{\pi} I_1 - x + c$

where $I_1 = \int \sin^{-1}\sqrt{x} \, dx$

put $x = t^2$

$dx = 2t\,dt$

$$I_1 = 2\int \underset{\text{II}}{t} \cdot \underset{\text{I}}{\sin^{-1} t} \, dt$$

$$I_1 = 2\left[\sin^{-1}t\cdot\frac{t^2}{2} - \frac{1}{2}\int \frac{1}{\sqrt{1-t^2}}\cdot t^2\,dt\right]$$

$$I_1 = t^2\cdot\sin^{-1}t - \int \frac{t^2}{\sqrt{1-t^2}}\,dt$$

$$I_1 = t^2\sin^{-1}t + \int \frac{1-t^2-1}{\sqrt{1-t^2}}\,dt$$

$$I_1 = t^2\sin^{-1}t + \int \sqrt{1-t^2} - \frac{1}{\sqrt{1-t^2}}\,dt$$

$$I_1 = t^2\sin^{-1}t + \frac{t}{2}\sqrt{1-t^2} + \frac{1}{2}\sin^{-1}t - \sin^{-1}t$$

$$I_1 = t^2\sin^{-1}t + \frac{t}{2}\sqrt{1-t^2} - \frac{1}{2}\sin^{-1}t$$

$$I_1 = x\cdot\sin^{-1}\sqrt{x} + \frac{\sqrt{x}\sqrt{1-x}}{2} - \frac{1}{2}\sin^{-1}\sqrt{x}$$

$$I_1 = \sin^{-1}\sqrt{x}\left(\frac{2x-1}{2}\right) + \frac{\sqrt{x-x^2}}{2}$$

$$\therefore I = \frac{4}{\lambda}\left[\left(\frac{2x-1}{2}\right)\cdot\sin^{-1}\sqrt{x} + \frac{\sqrt{x-x^2}}{2}\right] - x + C$$

$$I = \frac{2}{\lambda}\left[(2x-1)\sin^{-1}\sqrt{x} + \sqrt{x-x^2}\right] - x + C$$ **Ans**