Solutions of M-3 (Matrices)

Ans 1 → $A = \begin{bmatrix} -1 & 2 & 3 \\ 5 & 7 & 9 \\ -2 & 1 & 1 \end{bmatrix}$, $B = \begin{bmatrix} -4 & 1 & -5 \\ 1 & 2 & 0 \\ 1 & 3 & 1 \end{bmatrix}$

(ii) $A - B = \begin{bmatrix} 3 & 1 & 8 \\ 4 & 5 & 9 \\ -3 & -2 & 0 \end{bmatrix}$

$(A-B)' = \begin{bmatrix} 3 & 4 & -3 \\ 1 & 5 & -2 \\ 8 & 9 & 0 \end{bmatrix}$

$A' = \begin{bmatrix} -1 & 5 & -2 \\ 2 & 7 & 1 \\ 3 & 9 & 1 \end{bmatrix}$

$B' = \begin{bmatrix} -4 & 1 & 1 \\ 1 & 2 & 3 \\ -5 & 0 & 1 \end{bmatrix}$

$A' - B' = \begin{bmatrix} 3 & 4 & -3 \\ 1 & 5 & -2 \\ 8 & 9 & 0 \end{bmatrix}$ Clearly $(A-B)' = A' - B'$

(i) Do yourself

(iii) $AB = \begin{bmatrix} -1 & 2 & 3 \\ 5 & 7 & 9 \\ -2 & 1 & 1 \end{bmatrix} \begin{bmatrix} -4 & 1 & -5 \\ 1 & 2 & 0 \\ 1 & 3 & 1 \end{bmatrix}$

$= \begin{bmatrix} 9 & 12 & 8 \\ -4 & 46 & -16 \\ 10 & 3 & 11 \end{bmatrix}$

$(AB)' = \begin{bmatrix} 9 & -4 & 10 \\ 12 & 46 & 3 \\ 8 & -16 & 11 \end{bmatrix}$

$B' = \begin{bmatrix} -4 & 1 & 1 \\ 1 & 2 & 3 \\ -5 & 0 & 1 \end{bmatrix}$ & $A' = \begin{bmatrix} -1 & 5 & -2 \\ 2 & 7 & 1 \\ 3 & 9 & 1 \end{bmatrix}$

$$B'A' = \begin{bmatrix} -4 & 1 & 1 \\ 1 & 2 & 3 \\ -5 & 0 & 1 \end{bmatrix} \begin{bmatrix} -1 & 5 & -2 \\ 2 & 7 & 1 \\ 3 & 9 & 1 \end{bmatrix}$$

$$B'A' = \begin{bmatrix} 9 & -4 & 10 \\ 12 & 46 & 3 \\ 8 & -16 & 11 \end{bmatrix}$$

Clearly $(AB)' = B'A'$

Ans 2 → $A = \begin{bmatrix} -1 \\ 4 \\ 3 \end{bmatrix}$ & $B = \begin{bmatrix} -1 & 2 & 1 \end{bmatrix}$

$$AB = \begin{bmatrix} -1 \\ 4 \\ 3 \end{bmatrix}_{3\times 1} \begin{bmatrix} -1 & 2 & 1 \end{bmatrix}_{1\times 3}$$

$$AB = \begin{bmatrix} 1 & -2 & -1 \\ -4 & 8 & 4 \\ -3 & 6 & 3 \end{bmatrix}$$

$$(AB)' = \begin{bmatrix} 1 & -4 & -3 \\ -2 & 8 & 6 \\ -1 & 4 & 3 \end{bmatrix}$$

Now $B' = \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}$ & $A' = \begin{bmatrix} -1 & 4 & 3 \end{bmatrix}$

$$B'A' = \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}_{3\times 1} \begin{bmatrix} -1 & 4 & 3 \end{bmatrix}_{1\times 3}$$

$$B'A' = \begin{bmatrix} 1 & -4 & -3 \\ -2 & 8 & 6 \\ -3 & 4 & 3 \end{bmatrix}$$

Clearly $(AB)' = B'A'$ Proved

(M-3) Solutions

Qns 3 → given $A = \begin{bmatrix} 1 & 5 \\ 6 & 7 \end{bmatrix}$

$A' = \begin{bmatrix} 1 & 6 \\ 5 & 7 \end{bmatrix}$

(i) let $P = A + A'$

$P = \begin{bmatrix} 2 & 11 \\ 11 & 14 \end{bmatrix}$

$P' = \begin{bmatrix} 2 & 11 \\ 11 & 14 \end{bmatrix} = P$

$\therefore$ $A + A'$ is a symmetric Matrix

(ii) let $Q = A - A'$

$Q = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

$Q' = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$

$Q' = -\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} = -Q$

$\therefore$ $A - A'$ is a skew symmetric Matrix

Qns 4 → $A = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$

$A' = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$

given $A + A' = I$

$\begin{bmatrix} 2\cos\alpha & 0 \\ 0 & 2\cos\alpha \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\Rightarrow$ $2\cos\alpha = 1$

$\Rightarrow$ $\cos\alpha = 1/2$

$\Rightarrow$ $\alpha = 2n\pi \pm \pi/3$; $n \in Z$ ANS

Qns 5 given $A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$

(M-3) Solutions

$$A' = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$

$$A + A' = \begin{bmatrix} 12 & -4 & 4 \\ -4 & 6 & -2 \\ 4 & -2 & 6 \end{bmatrix}$$

$$A - A' = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

let $P = \frac{1}{2}(A + A')$

$$P = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$

$$P' = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix} = P$$

$\therefore$ P is a symm matrix

let $Q = \frac{1}{2}(A - A')$

$$Q = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$Q' = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$Q' = -\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} = -Q$$

$\therefore$ Q is a Skew-symm Matrix

Now $P + Q = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix} + \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

$$P + Q = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix} = A$$

$\therefore$ A can be expressed as the sum of symm and skew symm. matrix.

(M-3) Solutions

Qn 6 → Do yourself (Same as Qn No 5)

Qn 7 → given $A' = A$ and $B' = B$

(i) let $P = AB + BA$

$P' = (AB + BA)'$

$\Rightarrow P' = (AB)' + (BA)'$

$\Rightarrow P' = B'A' + A'B'$

$\Rightarrow P' = BA + AB$ --- {given}

$\Rightarrow P' = AB + BA$

$\Rightarrow P' = P$

$\therefore P$ is a Symm. Matrix

(ii) let $Q = AB - BA$

$\Rightarrow Q' = (AB - BA)'$

$\Rightarrow Q' = (AB)' - (BA)'$

$\Rightarrow Q' = B'A' - A'B'$

$\Rightarrow Q' = BA - AB$ --- (given)

$\Rightarrow Q' = -(AB - BA)$

$\Rightarrow Q' = -Q$

$\therefore Q$ is a Skew-Symm Matrix

Qn. 8 → Case I let $A \rightarrow$ Symm. Matrix

$\Rightarrow A' = A$

let $P = B'AB$

$\Rightarrow P' = (B'AB)'$

$\Rightarrow P' = B'A'B$

$\Rightarrow P' = B'AB$ --- (given: $A' = A$)

$\Rightarrow P' = P$

$\therefore P \rightarrow$ Symm Matrix

Case II let $A \rightarrow$ Skew symm Matrix

$\Rightarrow A' = -A$

(M-3) Solutions

Let $Q = B'AB$

$\Rightarrow Q' = (B'AB)'$

$\Rightarrow Q' = B'A'B$

$\Rightarrow Q' = B'(-A)B$ .... (given $A' = -A$)

$\Rightarrow Q' = -B'AB$

$\Rightarrow Q' = -Q$

$\therefore$ Q is a skew symm Matrix Ans

Qn 9 $\Rightarrow$ given $A = \begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{bmatrix}$

$A' = \begin{bmatrix} 0 & x & x \\ 2y & y & -y \\ z & -z & z \end{bmatrix}$

given $A'A = I$

$\Rightarrow \begin{bmatrix} 0 & x & x \\ 2y & y & -y \\ z & -z & z \end{bmatrix} \begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 2x^2 & 0 & 0 \\ 0 & 6y^2 & 0 \\ 0 & 0 & 3z^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$\Rightarrow 2x^2 = 1 \Rightarrow x^2 = \frac{1}{2} \Rightarrow x = \pm 1/\sqrt{2}$

$6y^2 = 1 \Rightarrow y^2 = 1/6 \Rightarrow y = \pm \frac{1}{\sqrt{6}}$

$3z^2 = 1 \Rightarrow z^2 = 1/3 \Rightarrow z = \pm 1/\sqrt{3}$ Ans

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Qn 10 $\Rightarrow$ given $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ a & 2 & b \end{bmatrix}$

$A' = \begin{bmatrix} 1 & 2 & a \\ 2 & 1 & 2 \\ 2 & -2 & b \end{bmatrix}$

given $AA' = 9I$

$$\Rightarrow \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ a & 2 & b \end{bmatrix} \begin{bmatrix} 1 & 2 & a \\ 2 & 1 & 2 \\ 2 & -2 & b \end{bmatrix} = 9 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 9 & 0 & a+4+2b \\ 0 & 9 & 2a+2-2b \\ a+4+2b & 2a+2-2b & a^2+4+b^2 \end{bmatrix} = \begin{bmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{bmatrix}$$

$\Rightarrow a+4+2b=0$ | $2a+2-2b=0$ | $a+4+2b=0$ | $a^2+b^2+4=9$

$2b = -a-4$

$\Rightarrow 2a+2-(-a-4)=0$

$2a+2+a+4=0$

$3a=-6$

$\boxed{a=-2}$

$\Rightarrow 2b = 2-4$

$\boxed{b=-1}$

Clearly $a=-2$, $b=-1$ satisfy the equation $a^2+b^2+4=9$

$\therefore \boxed{a=-2}$ $\boxed{b=-1}$ Ans

Qn 11. for Skew-symm Matrix

$a_{ij} = -a_{ji}$

$\Rightarrow a_{ij} + a_{ji} = 0$

for diagonal elements

$i=j$

$\Rightarrow a_{ii} + a_{ii} = 0$

$\Rightarrow 2a_{ii} = 0$

$\Rightarrow a_{ii} = 0$

put $i=1$ $\Rightarrow a_{11}=0$

put $i=2$ $\Rightarrow a_{22}=0$

put $i=3$ $\Rightarrow a_{33}=0$

Clearly all diagonal elements of a Skew-symm Matrix are always zero

(M-3) Solution

Qn 12 → given $A' = A$ and $B' = B$

case I given $AB = BA$

To prove $AB \rightarrow$ Symm Matrix

let $P = AB$

$\Rightarrow P' = (AB)'$

$\Rightarrow P' = B'A'$

$\Rightarrow P' = BA$ --- (given)

$\Rightarrow P' = AB$ --- (given)

$\Rightarrow P' = P$

$\therefore P \rightarrow$ Symm Matrix

Conversely given; $AB \rightarrow$ Symm Matrix

T.P $AB = BA$

we have AB is a symm Matrix

$\Rightarrow (AB)' = AB$

$\Rightarrow B'A' = AB$

$\Rightarrow BA = AB$ --- (given)

$\Rightarrow$ A and B commute proved

Qn 13 → (i) given $A \rightarrow$ Symm Matrix

$\Rightarrow A' = A$ --- (i)

also given $A \rightarrow$ Skew symm Matrix

$\Rightarrow A' = -A$ -- (ii)

from (1) & (2)

$A = -A$

$\Rightarrow A + A = 0$

$\Rightarrow 2A = 0$

$\Rightarrow \boxed{A = 0}$ i.e. A is a Null matrix Ans

(ii) given $A^2 = A$

$(I+A)^3 - 7A$

$= (I+A)(I+A)(I+A) - 7A$

$= (I + A + A + A^2)(I+A) - 7A$

$= (I + A + A + A)(I+A) - 7A$ --- (given $A^2 = A$)

$\Rightarrow (I+3A)(I+A) - 7A$

$= I + A + 3A + 3A^2 - 7A$

$= I + A + 3A + 3A - 7A$ --- (given $A^2 = A$)

$= I + 7A - 7A$

$= I + 0$

$= I$ **Ans**

Qn. 14 → given $A^2 = I$

$(A-I)^3 + (A+I)^3 - 7A$

$= (A-I)(A-I)(A-I) + (A+I)(A+I)(A+I) - 7A$

$= (A^2 - A - A + I)(A-I) + (A^2 + A + A + I)(A+I) - 7A$

$= (I - 2A + I)(A-I) + (I + 2A + I)(A+I) - 7A$ --- (given $A^2 = I$)

$= (2I - 2A)(A-I) + (2I + 2A)(A+I) - 7A$

$= 2A - 2I - 2A^2 + 2A + 2A + 2I + 2A^2 + 2A - 7A$

$= 2A - 2I - 2I + 2A + 2A + 2I + 2I + 2A - 7A$ ----- (given $A^2 = I$)

$= 8A + 0 - 7A$

$= A$ **Ans**

Qn 15 → let $A = \begin{bmatrix} 0 & a & 3 \\ 2 & b & -1 \\ c & 1 & 0 \end{bmatrix}$