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SOLUTIONS : WORKSHEET NO: 3 (class-4)

INTEGRATION (WORKSHEET NO: 3)

Ans: 1 $I = \int 4x^3 \sqrt{5-x^2}\, dx$

$$I = 4\int x^2 \sqrt{5-x^2} \cdot x\, dx$$

put $5-x^2 = t$

$$-2x\,dx = dt \Rightarrow x\,dx = -\frac{dt}{2}$$

$$\therefore I = -\frac{4}{2}\int (5-t)\sqrt{t}\, dt$$

$$= -2\int 5\sqrt{t} - t^{3/2}\, dt$$

$$= -2\left[\frac{2}{3}\times 5\, t^{3/2} - \frac{2}{5} t^{5/2}\right] + c$$

$$I = -\frac{20}{3}(5-x^2)^{3/2} + \frac{4}{5}(5-x^2)^{5/2} + c \quad \underline{\text{Ans}}$$

Ans: 2 $I = \int \frac{1}{x^2(x^4+1)^{3/4}}\, dx$ {Tip: take common and then put $= t$}

$$I = \int \frac{1}{x^2 \cdot x^3\left(1+\frac{1}{x^4}\right)^{3/4}}\, dx$$

$$= \int \frac{1}{x^5\left(1+\frac{1}{x^4}\right)^{3/4}}\, dx$$

put $1+\frac{1}{x^4} = t$

$$-\frac{4}{x^5}\, dx = dt \Rightarrow \frac{dx}{x^5} = -\frac{dt}{4}$$

$$\therefore I = -\frac{1}{4}\int \frac{dt}{t^{3/4}}$$

$$= -\frac{1}{4}\int t^{-3/4}\, dt$$

$$I = -\tfrac{1}{4} \times 4\, t^{1/4} + c$$

$$= -t^{1/4} + c$$

$$I = -\left(1 + \frac{1}{x^4}\right)^{1/4} + c \quad \text{Ans}$$

Qns:- 3 → $I = \int \frac{x^5}{\sqrt{1+x^3}}\,dx$

$$= \int \frac{x^3}{\sqrt{1+x^3}} \cdot x^2\,dx$$

put $1 + x^3 = t$

$\Rightarrow 3x^2\,dx = dt$

$\Rightarrow x^2\,dx = \frac{dt}{3}$

$$\therefore I = \frac{1}{3}\int \frac{(t-1)\,dt}{\sqrt{t}}$$

$$= \frac{1}{3}\int \sqrt{t} - \frac{1}{\sqrt{t}}\,dt$$

$$= \frac{1}{3}\left[\frac{2}{3}t^{3/2} - 2\sqrt{t}\right] + c$$

$$I = \frac{2}{9}(1+x^3)^{3/2} - \frac{2}{3}\sqrt{1+x^3} + c \quad \text{Ans}$$

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Qns 4 → $I = \int \frac{1}{\sqrt{x} + x}\,dx$

$$I = \int \frac{1}{\sqrt{x}(1+\sqrt{x})}\,dx$$

put $1 + \sqrt{x} = t$

$\frac{1}{2\sqrt{x}}\,dx = dt \Rightarrow \frac{1}{\sqrt{x}}\,dx = 2\,dt$

$$\therefore I = 2\int \frac{dt}{t}$$

$$= 2\log|t| + c$$

$$I = 2\log|1+\sqrt{x}| + c \quad \text{ANS}$$

Ans. 5 → $I = \int \frac{e^{\sqrt{x}} \cdot \cos(e^{\sqrt{x}})}{\sqrt{x}}\, dx$

put $e^{\sqrt{x}} = t$

$$\Rightarrow e^{\sqrt{x}} \cdot \frac{1}{2\sqrt{x}}\, dx = dt$$

$$\Rightarrow \frac{e^{\sqrt{x}}}{\sqrt{x}}\, dx = 2dt$$

$$I = 2\int \cos t\, dt$$

$$I = 2\sin(e^{\sqrt{x}}) + C \quad \text{ANS}$$

Ans 6 → $I = \int 5^{5^{5^x}} \cdot 5^{5^x} \cdot 5^x\, dx$

put $5^{5^{5^x}} = t$

$$\Rightarrow 5^{5^{5^x}} \cdot \log 5 \cdot 5^{5^x} \cdot \log 5 \cdot 5^x \cdot \log 5\, dx = dt$$

$$\Rightarrow 5^{5^{5^x}} \cdot 5^{5^x} \cdot 5^x\, dx = \frac{dt}{(\log 5)^3}$$

$$\therefore I = \frac{1}{(\log 5)^3} \cdot \int dt$$

$$= \frac{1}{(\log 5)^3} \cdot t + c$$

$$I = \frac{1}{(\log 5)^3} \cdot 5^{5^{5^x}} + c \quad \text{ANS}$$

Ans. 7 →
$$I = \int \frac{\sin(2x)}{(a+b\cos x)^2}\,dx$$

$$I = 2\int \frac{\sin x \cos x}{(a+b\cos x)^2}\,dx \quad \cdots \left\{ (\;)^n \to \text{If not linear, then put} = t \right\}$$

put $a + b\cos x = t$

$-b\sin x\,dx = dt$

$\sin x\,dx = -\frac{dt}{b}$

$$\therefore I = -\frac{2}{b}\int \frac{\left(\frac{t-a}{b}\right)dt}{t^2} \quad \cdots \left\{ \cos x = \frac{t-a}{b} \right\}$$

$$= -\frac{2}{b^2}\int \frac{t-a}{t^2}\,dt$$

$$= -\frac{2}{b^2}\int \frac{1}{t} - \frac{a}{t^2}\,dt$$

$$= -\frac{2}{b^2}\left[\log|t| + \frac{a}{t}\right] + c$$

$$I = -\frac{2}{b^2}\left[\log|a+b\cos x| + \frac{a}{a+b\cos x}\right] + C \quad \text{Ans}$$

Ans. 8 →
$$I = \int \frac{(x^4 - x)^{1/4}}{x^5}\,dx$$

Type = take common and then put t

$$I = \int \frac{x\left(1-\frac{1}{x^3}\right)^{1/4}}{x^5}\,dx$$

$$= \int \frac{\left(1-\frac{1}{x^3}\right)^{1/4}}{x^4}\,dx$$

put $1 - \frac{1}{x^3} = t \Rightarrow \frac{3}{x^4}\,dx = dt \Rightarrow \frac{dx}{x^4} = \frac{dt}{3}$

$$\therefore \quad I = \frac{1}{3}\int t^{1/4}\,dt$$

$$= \frac{1}{3}\times\frac{4}{5}(t)^{5/4} + c$$

$$I = \frac{4}{15}\left(1-\frac{1}{x^3}\right)^{5/4} + C \quad \text{Ans}$$

Q.No 9 → $I = \int \frac{\sin x}{\sqrt{3+2\cos x}}\,dx$

put $3+2\cos x = t$

$-2\sin x\,dx = dt \Rightarrow \sin x\,dx = -\frac{dt}{2}$

$$I = -\frac{1}{2}\int \frac{dt}{\sqrt{t}}$$

$$= -\frac{1}{\cancel{2}}\times \cancel{2}\sqrt{t} + c$$

$$I = -\sqrt{3+2\cos x} + C \quad \text{Ans}$$

Q.No 10 → $I = \int \frac{\sec x}{\sec(2x)}\,dx$

$$I = \int \frac{\frac{1}{\cos x}}{\frac{1}{\cos(2x)}}\,dx$$

$$= \int \frac{\cos(2x)}{\cos x}\,dx$$

$$= \int \frac{2\cos^2 x - 1}{\cos x}\,dx$$

$$= \int 2\cos x - \sec x\,dx$$

$$I = 2\sin x - \log|\sec x + \tan x| + C \quad \text{Ans}$$

Qns 11 → $I = \int \frac{10x^9 + 10^x \log 10 \, dx}{10^x + x^{10}}$

put $10^x + x^{10} = t$

$10^x \cdot \log 10 + 10x^9 \, dx = dt$

$\therefore I = \int \frac{dt}{t}$

$= \log|t| + c$

$I = \log|10^x + x^{10}| + c$ Ans

Qns. 12 → $I = \int \frac{1}{x \log x \cdot \log(\log x)} dx$

put $\log(\log x) = t$

$\frac{1}{\log x} \cdot \frac{1}{x} \cdot dx = dt$

$\therefore I = \int \frac{dt}{t}$

$= \log|t| + c$

$I = \log|\log(\log x)| + c$ Ans

Qn. 13 → $I = \int \tan(2x) \tan(3x) \tan(5x) \, dx$

XI class we have $5x = 3x + 2x$

$\Rightarrow \tan(5x) = \tan(3x + 2x)$

$\Rightarrow \tan(5x) = \frac{\tan(3x) + \tan(2x)}{1 - \tan(3x)\tan(2x)}$

$$\Rightarrow \tan(5x) - \tan(5x)\tan(3x)\tan(2x) = \tan(3x) + \tan(2x)$$

$$\Rightarrow \tan(5x)\tan(3x)\tan(2x) = \tan(5x) - \tan(3x) - \tan(2x)$$

$$\therefore I = \int \tan(5x) - \tan(3x) - \tan(2x)\, dx$$

$$I = \frac{1}{5}\log|\sec(5x)| - \frac{1}{3}\log|\sec(3x)| - \frac{1}{2}\log|\sec(2x)| + C$$

ANS

Ans. 14 → $I = \int \sqrt{\dfrac{1-\sin(2x)}{1+\sin(2x)}}\, dx$

$$I = \int \sqrt{\frac{1-\cos(\frac{\pi}{2}-2x)}{1+\cos(\frac{\pi}{2}-2x)}}\, dx$$

$$= \int \sqrt{\frac{\cancel{2}\sin^2(\frac{\pi}{4}-x)}{\cancel{2}\cos^2(\frac{\pi}{4}-x)}}\, dx$$

$$= \int \tan(\tfrac{\pi}{4}-x)\, dx$$

$$= -\log|\sec(\tfrac{\pi}{4}-x)| + C$$

(or) $\log\left|\dfrac{1}{\sec(\frac{\pi}{4}-x)}\right| + C$ --- $\left\{\because \log\left(\frac{A}{B}\right) = -\log\frac{B}{A}\right\}$

$$I = \log|\cos(\tfrac{\pi}{4}-x)| + C$$ ANS

{Note: there are many methods to solve this question}

Ans. 15 → $I = \int \dfrac{\sin(2x)}{a^2\sin^2 x + b^2\cos^2 x}\, dx$

put $a^2\sin^2 x + b^2\cos^2 x = t$

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$$\Rightarrow \quad a^2 \cdot 2\sin x \cos x \quad + b^2 \cdot 2\cos x(-\sin x)\, dx = dt$$

$$\Rightarrow \quad \sin(2x) \cdot (a^2 - b^2)\, dx = dt$$

$$\Rightarrow \quad \sin(2x)\, dx = \frac{dt}{a^2-b^2}$$

$$\therefore I = \frac{1}{a^2-b^2} \int \frac{dt}{t}$$

$$= \frac{1}{a^2-b^2} \cdot \log|t| + c$$

$$\therefore I = \frac{1}{a^2-b^2} \log\left|a^2\sin^2 x + b^2\cos^2 x\right| + c \quad \text{Ans}$$

Ans:- 16 $\Rightarrow$ $I = \int \frac{\tan x}{a + b\tan^2 x}\, dx$

$$I = \int \frac{\frac{\sin x}{\cos x}}{a + b\frac{\sin^2 x}{\cos^2 x}}\, dx$$

$$= \int \frac{\sin x \cdot \cos x \; dx}{a\cos^2 x + b\sin^2 x}$$

$$= \frac{1}{2} \int \frac{\sin(2x)\, dx}{a\cos^2 x + b\sin^2 x}$$

put- $a\cos^2 x + b\sin^2 x = t$

$$\Rightarrow \quad -a\sin(2x) + b\sin(2x)\, dx = dt$$

$$\Rightarrow \quad \sin(2x) \cdot (b - a)\, dx = dt$$

$$\Rightarrow \quad \sin(2x)\, dx = \frac{dt}{b-a}$$

$$\therefore I = \frac{1}{2(b-a)}\int \frac{dt}{t}$$

$$I = \frac{1}{2(b-a)} \log|t| + C$$

$$I = \frac{1}{2(b-a)} \log\left| a\cos^2 x + b\sin^2 x \right| + C \quad \underline{\text{Ans}}$$

Ans 17 → $I = \int \frac{1+\cos(4x)}{\cot x - \tan x}\, dx$

$$I = \int \frac{2\cos^2(2x)}{\frac{\cos x}{\sin x} - \frac{\sin x}{\cos x}}\, dx$$

$$= \int \frac{2\cos^2(2x)}{\cos^2 x - \sin^2 x} \cdot \sin x \cdot \cos x \, dx$$

$$= \int \frac{\cancel{2}\cos^{\cancel{2}}(2x) \cdot \frac{\sin(2x)}{\cancel{2}}}{\cancel{\cos(2x)}}\, dx$$

$$= \int \cos(2x) \cdot \sin(2x)\, dx$$

$$= \frac{1}{2}\int \sin(4x)\, dx$$

$$= \frac{1}{2}\left[-\frac{\cos(4x)}{4}\right] + C$$

$$I = -\frac{1}{8}\cos(4x) + C \quad \underline{\text{Ans}}$$

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Q.N. 18 → $I = \int \frac{1}{\sqrt{x+3} - \sqrt{x+2}} dx$

Rationalize

$$I = \int \frac{1}{\sqrt{x+3} - \sqrt{x+2}} \times \frac{(\sqrt{x+3} + \sqrt{x+2})}{(\sqrt{x+3} + \sqrt{x+2})} dx$$

$$= \int \frac{\sqrt{x+3} + \sqrt{x+2}}{\cancel{x}+3-\cancel{x}-2} dx$$

$$= \int \sqrt{x+3} + \sqrt{x+2}\, dx$$

$$I = \frac{2}{3}(x+3)^{3/2} + \frac{2}{3}(x+2)^{3/2} + C \quad \text{Ans}$$

Q.N. 19 → $I = \int \frac{1}{\sqrt{1-2x} + \sqrt{3-2x}} dx$

Rationalize

$$I = \int \frac{\sqrt{1-2x} - \sqrt{3-2x}}{(1-2x)-(3-2x)} dx$$

$$= \int \frac{\sqrt{1-2x} - \sqrt{3-2x}}{-2} dx$$

$$= -\frac{1}{2}\int \sqrt{1-2x} - \sqrt{3-2x}\, dx$$

$$= -\frac{1}{2}\left[\frac{2}{3}\frac{(1-2x)^{3/2}}{(-2)} - \frac{2}{3}\frac{(3-2x)^{3/2}}{(-2)}\right] + C$$

Don't forget

$$I = \frac{1}{6}(1-2x)^{3/2} - \frac{1}{6}(3-2x)^{3/2} + C \quad \text{Ans}$$

Qn. 20 → $f'(x) = a\sin x + b\cos x$

$f'(0) = 4$; $f(0) = 3$; $f(\pi/2) = 5$

we know that

$$f(x) = \int f'(x)\,dx$$

$$\Rightarrow f(x) = \int a\sin x + b\cos x \,dx$$

$$f(x) = -a\cos x + b\sin x + C$$

given $f(0) = 3$ & $f(\pi/2) = 5$

$$\Rightarrow 3 = -a\cos(0) + b\sin(0) + C$$

$$\Rightarrow \boxed{3 = -a + C}$$

$$5 = -a\cos(\pi/2) + b\sin(\pi/2) + C$$

$$\Rightarrow \boxed{5 = b + C}$$

given $f'(0) = 4$ → {go to $f'(x)$}

$$\Rightarrow 4 = a\sin(0) + b\cos(0)$$

$$\Rightarrow \boxed{4 = b}$$

$$\Rightarrow C = 1$$

$$\Rightarrow a = -2$$

$\therefore f(x) = 2\cos x + 4\sin x + 1$ Ans

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