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## SOLUTION of I - 1



S: 6 → $\sin^{-1}\left(\sin\left(\frac{3\pi}{4}\right)\right) + \cos^{-1}\left(\cos\frac{3\pi}{4}\right) + \tan^{-1}\left(\tan\frac{3\pi}{4}\right)$

$= \sin^{-1}\left(\sin\left(\pi-\frac{\pi}{4}\right)\right) + \frac{3\pi}{4} + \tan^{-1}\left(\tan\left(\pi-\frac{\pi}{4}\right)\right)$

$= \sin^{-1}\left(\sin\frac{\pi}{4}\right) + \frac{3\pi}{4} + \tan^{-1}\left(-\tan\frac{\pi}{4}\right)$

$= \frac{\pi}{4} + \frac{3\pi}{4} - \tan^{-1}\left(\tan\frac{\pi}{4}\right)$

$= \cancel{\frac{\pi}{4}} + \frac{3\pi}{4} \cancel{-\frac{\pi}{4}} = \frac{3\pi}{4}$ Ans..

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7 → $\tan^{-1}\left(\tan\frac{5\pi}{3}\right) + \cos^{-1}\left(\cos\frac{5\pi}{3}\right) + \sin^{-1}\left(\sin\frac{5\pi}{3}\right)$

$= \tan^{-1}\left(\tan\left(2\pi-\frac{\pi}{3}\right)\right) + \cos^{-1}\left(\cos\left(2\pi-\frac{\pi}{3}\right)\right) + \sin^{-1}\left(\sin\left(2\pi-\frac{\pi}{3}\right)\right)$

$= \tan^{-1}\left(-\tan\frac{\pi}{3}\right) + \cos^{-1}\left(\cos\frac{\pi}{3}\right) + \sin^{-1}\left(-\sin\frac{\pi}{3}\right)$

$= -\tan^{-1}\left(\tan\frac{\pi}{3}\right) + \frac{\pi}{3} - \sin^{-1}\left(\sin\frac{\pi}{3}\right)$

$= -\frac{\pi}{3} + \cancel{\frac{\pi}{3}} \cancel{-\frac{\pi}{3}} = -\frac{\pi}{3}$ Ans..

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→ $\sin^{-1}\left(\frac{-1}{\sqrt{2}}\right) + \cos^{-1}\left(\frac{-1}{\sqrt{2}}\right) + \tan^{-1}(-1)$

$= -\sin^{-1}\left(\frac{1}{\sqrt{2}}\right) + \pi - \cos^{-1}\left(\frac{1}{\sqrt{2}}\right) - \tan^{-1}(1)$

$= -\frac{\pi}{4} + \pi - \frac{\pi}{4} - \frac{\pi}{4}$

$= \pi - \frac{3\pi}{4} = \frac{\pi}{4}$ Ans.

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Ans: 9 → $\operatorname{cosec}^{-1}(-2) - \sec^{-1}(-2) + \cot^{-1}(-1)$

$= -\operatorname{cosec}^{-1}(2) - [\pi - \sec^{-1}(2)] + \pi - \cot^{-1}(1)$

$= -\sin^{-1}(\frac{1}{2}) - \pi + \cos^{-1}(\frac{1}{2}) + \pi - \tan^{-1}(1)$

$= -\frac{\pi}{6} \cancel{-\pi} + \frac{\pi}{3} \cancel{+\pi} - \frac{\pi}{4}$

$= \frac{-2\pi + 4\pi - 3\pi}{12} = \frac{4\pi - 5\pi}{12}$

$= -\frac{\pi}{12}$ Ans.

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Ans 10 → $\cot[\sec^{-1}(2) + \operatorname{cosec}^{-1}(x)] = 0$

We know that $\cot\theta = 0$ then $\theta = \frac{\pi}{2}$

$\Rightarrow \sec^{-1}(2) + \operatorname{cosec}^{-1}(x) = \frac{\pi}{2}$

By property $\sec^{-1}x + \operatorname{cosec}^{-1}x = \frac{\pi}{2}$

we get $\boxed{x = 2}$ Ans..

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Ans 11 → $\cos[\tan^{-1}(3x) + \cot^{-1}(5)] = 0$

We know that $\cos\theta = 0$ then $\theta = \frac{\pi}{2}$

$\Rightarrow \tan^{-1}(3x) + \cot^{-1}(5) = \frac{\pi}{2}$

By property

# Solution I - 1



$$\tan^{-1}(3x) + \cot^{-1}(5) = \frac{\pi}{2}$$

$$3x = 5 \Rightarrow \boxed{x = \frac{5}{3}}$$ Ans

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Q.12 → $\tan^{-1}\left(2\cos\left(2\sin^{-1}\frac{1}{2}\right)\right)$

$= \tan^{-1}\left(2\cos\left(2 \times \frac{\pi}{6}\right)\right)$

$= \tan^{-1}\left(2\cos\frac{\pi}{3}\right)$

$= \tan^{-1}\left(2 \times \frac{1}{2}\right)$

$= \tan^{-1}(1)$

$= \pi/4$ Ans

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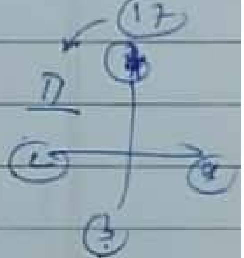
Q.13 → $\cos^{-1}(\cos 1540^\circ)$

$$\begin{array}{r} 17 \\ 90\overline{)1540} \\ \underline{90} \\ 640 \\ \underline{630} \\ \underline{10^\circ} \end{array}$$




change

$= \cos^{-1}(\cos(17 \times 90^\circ + 10^\circ))$

II quad

$= \cos^{-1}(-\sin(10^\circ))$

$=$

$= \pi - \cos^{-1}(\sin(10^\circ))$

$= 180^\circ - \cos^{-1}(\cos(90 - 10))$

$= 180^\circ - \cos^{-1}(\cos 80^\circ)$

$= 180^\circ - 80^\circ$

$= 100^\circ$ Ans

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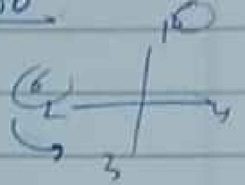
Q.14 $\sin^{-1}(\sin(-600^\circ))$

$= \sin^{-1}\left(-\sin(600^\circ)\right)$ --- $\{\sin(-\theta) = -\sin\theta\}$

$= -\sin^{-1}\left(\sin(600^\circ)\right)$

$$\begin{array}{r} 6 \\ 90\,)\,600^\circ \\ 540^\circ \\ \hline 60^\circ \end{array}$$

$= -\sin^{-1}\left(\sin(\underbrace{6 \times 90^\circ}_{\text{no change}} + 60^\circ)\right)$ III



$= -\sin^{-1}\left(-\sin(60^\circ)\right)$

$= \sin^{-1}\left(\sin(60^\circ)\right)$

$= 60^\circ$ **Ans**

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Qn 15 → given $4\sin^{-1}x + \cos^{-1}x = \pi$ --- (1)

we know that $\sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}$ (2)

(1) − (2)

$3\sin^{-1}x = \pi - \frac{\pi}{2}$

$3\sin^{-1}x = \pi/2$

$\sin^{-1}x = \pi/6$

$x = \sin(\pi/6)$

$x = 1/2$ **Ans**

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Qn 16 → given $\sin^{-1}x - \cos^{-1}x = \pi/6$ --- (1)

we know that $\sin^{-1}x + \cos^{-1}x = \pi/2$ --- (2)

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① + ②

$2\sin^{-1}x = \frac{\pi}{6} + \frac{\pi}{2}$

$2\sin^{-1}x = \frac{2\pi}{3}$

$\sin^{-1}x = \frac{\pi}{3}$

$x = \sin(\pi/3)$

$\boxed{x = \frac{\sqrt{3}}{2}}$ Ans

Q.17 → $\cos\left[\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right) + \frac{\pi}{4}\right]$

$= \cos^{-1}\left[\pi - \cos^{-1}\left(\frac{\sqrt{3}}{2}\right) + \frac{\pi}{4}\right]$

$= \cos^{-1}\left(\pi - \frac{\pi}{6} + \frac{\pi}{4}\right)$

$= \cos^{-1}(180^\circ - 30^\circ + 45^\circ)$

$= \cos^{-1}(195^\circ)$

III rd quad

$= \cos^{-1}(180^\circ + 15)$

$= -\cos(15^\circ)$

$= -\cos(45^\circ - 30^\circ)$

$= -\left[\cos(45)\cos(30^\circ) + \sin(45)\sin 30^\circ\right]$

$= -\left[\frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \times \frac{1}{2}\right] = -\left(\frac{\sqrt{3}+1}{2\sqrt{2}}\right)$ Ans

# Solution I - 1

Qn 18 → $\tan\left(\sec^{-1}a + \csc^{-1}a\right)$

By property $\sec^{-1}x + \csc^{-1}x = \frac{\pi}{2}$

$= \tan\left(\frac{\pi}{2}\right)$

$= \infty$ **Ans**

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Qn 19 → given $\sin^{-1}x + \sin^{-1}y + \sin^{-1}z = \frac{3\pi}{2}$

we know that $-\frac{\pi}{2} \le \sin^{-1}x \le \pi/2$.

$\therefore$ Max value of $\sin^{-1}x = \pi/2$

$\Rightarrow \sin^{-1}x = \frac{\pi}{2}$ | $\sin^{-1}y = \frac{\pi}{2}$ | $\sin^{-1}z = \frac{\pi}{2}$

$\Rightarrow x = \sin\frac{\pi}{2}$ | $y = \sin\frac{\pi}{2}$ | $z = \sin\frac{\pi}{2}$

$\boxed{x = 1}$ | $\boxed{y = 1}$ | $\boxed{z = 1}$

$\therefore x + y + z = 1 + 1 + 1 = 3$ **Ans.**

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Qn 20 → given $\cos^{-1}\alpha + \cos^{-1}\beta + \cos^{-1}\gamma = 3\pi$

we know that $0 \le \cos^{-1}x \le \pi$

$\therefore$ Max value of $\cos^{-1}x = \pi$

$\Rightarrow \cos^{-1}\alpha = \pi$ | $\cos^{-1}\beta = \pi$ | $\cos^{-1}\gamma = \pi$

$\Rightarrow \alpha = \cos\pi$ | $\beta = \cos\pi$ | $\gamma = \cos\pi$

$\alpha = -1$ | $\beta = -1$ | $\gamma = -1$

$\therefore \alpha^2 + \beta^2 + \gamma^2 = 1 + 1 + 1 = 3$ **Ans**