← ULTIMATE MATHEMATICS →

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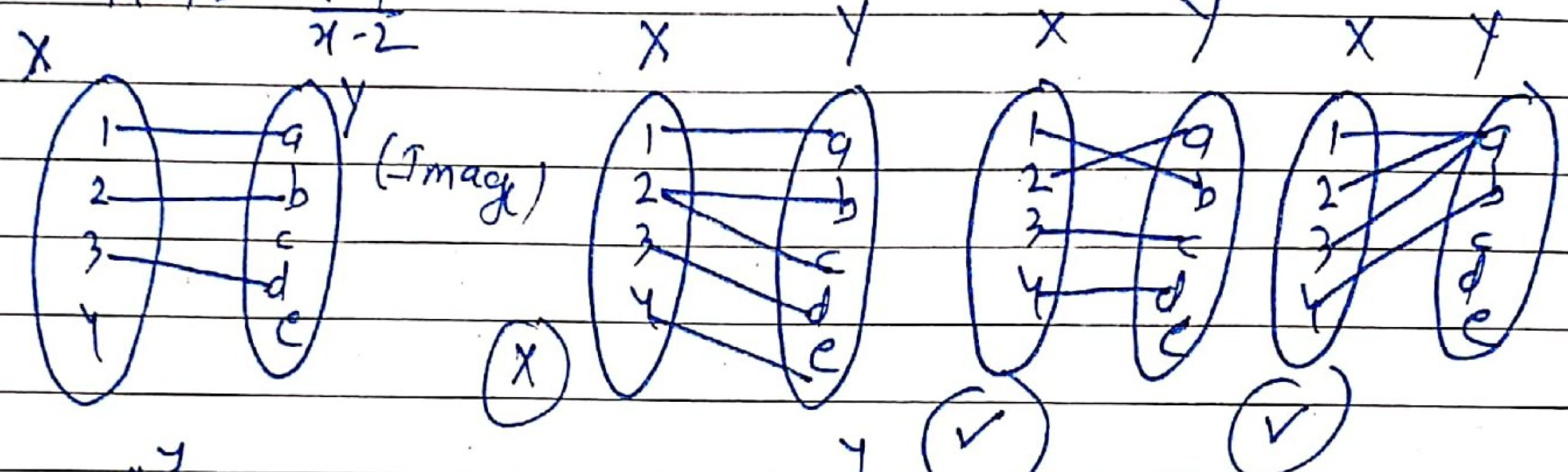
CHAPTER: RELATION FUNCTION

← CLASS NO: 3 →

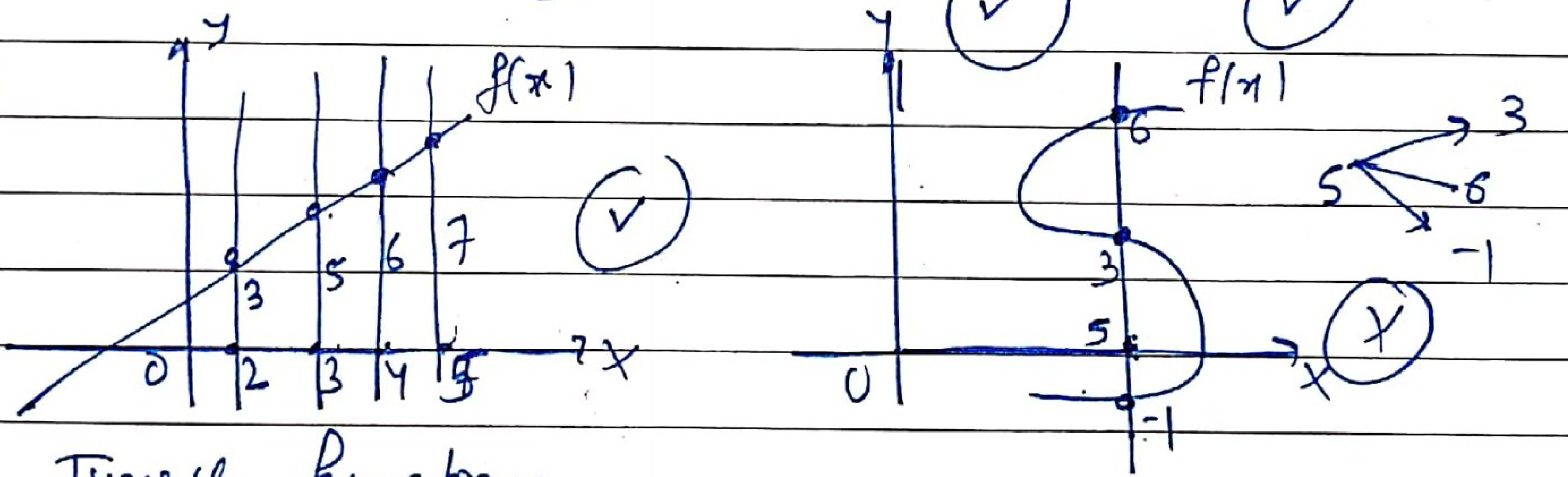
← FUNCTIONS →

XI (i) $f: X \to Y \to$ codomain
↳ domain

$f(x) = \frac{x-1}{x-2}$

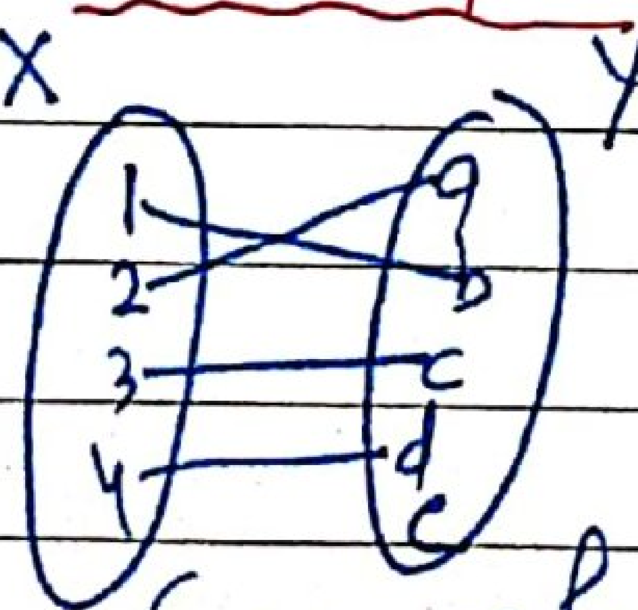


vertical line test

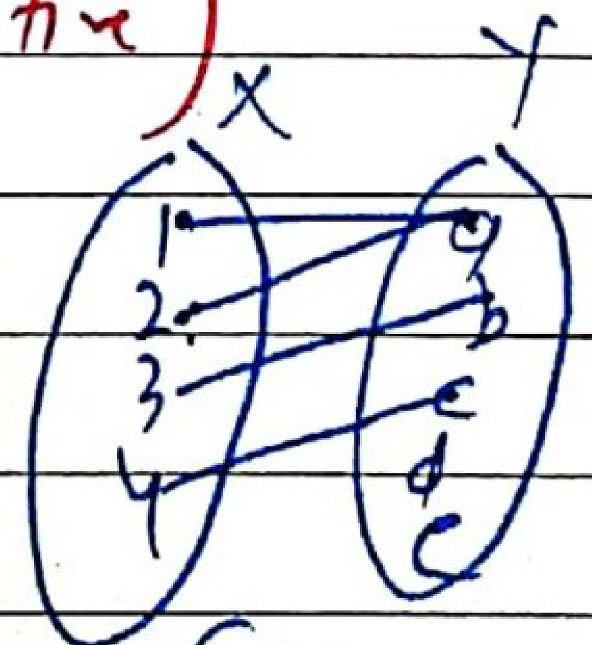


XII Types of functions

(i) one-one function (Injective)



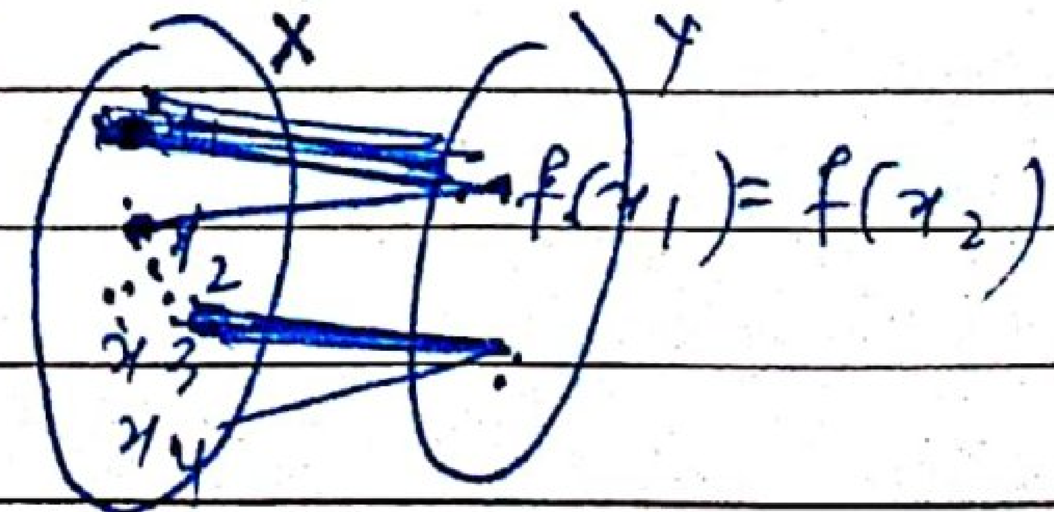
(one-one function)



(Many - one)

Accept { let $x_1, x_2 \in X$ (domain)
and $f(x_1) = f(x_2)$

$\boxed{x_1 = x_2}$ then f is one-one



CLASSTIME

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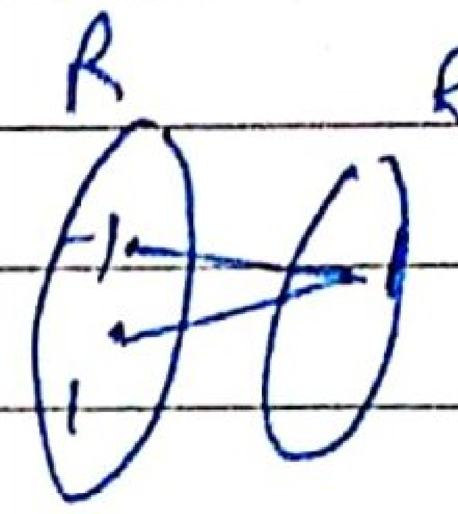
R & F (class No: 3)



Reject $f: R \to R$

$f(x) = |x|$

$f(-1) = |-1| = 1$

$f(1) = |1| = 1$

$\therefore$ Since two different elements -1 & 1 in domain has same image 1 in codomain $\therefore$

f is not one-one / many one-function

(or) let $x_1, x_2 \in R$ (domain

and $f(x_1) = f(x_2)$

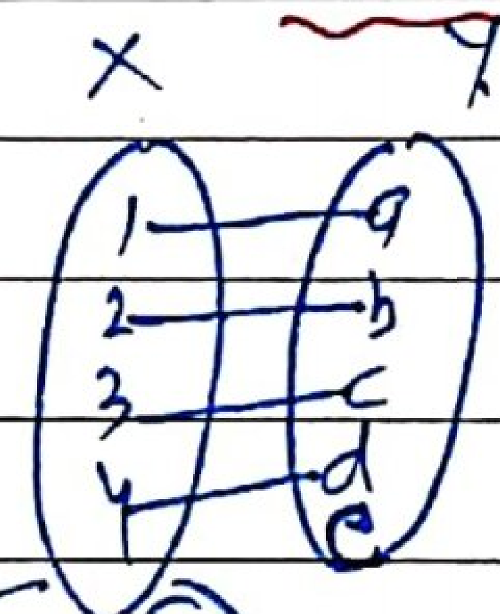
$\Rightarrow |x_1| = |x_2|$

$\Rightarrow x_1 = \pm x_2$

$x_1 = x_2, \quad x_1 = -x_2$

$\therefore f$ is ~~not~~ not one-one functi

(2) on-to function (Surjective)

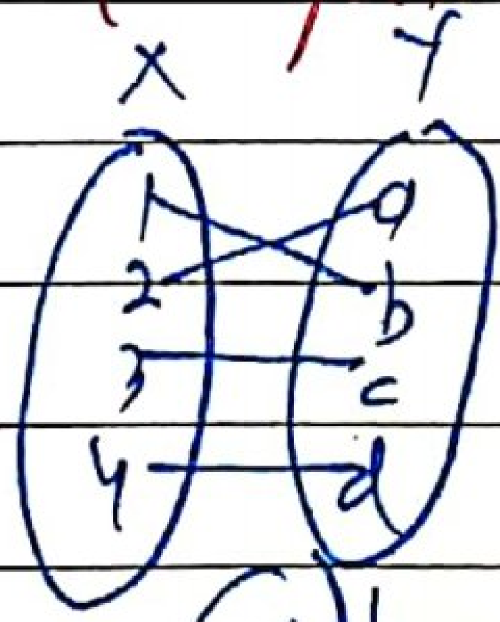


(X)

Range = {a, b, c, d}

Codomain {a, b, c, d, e}

Range $\neq$ Codomain

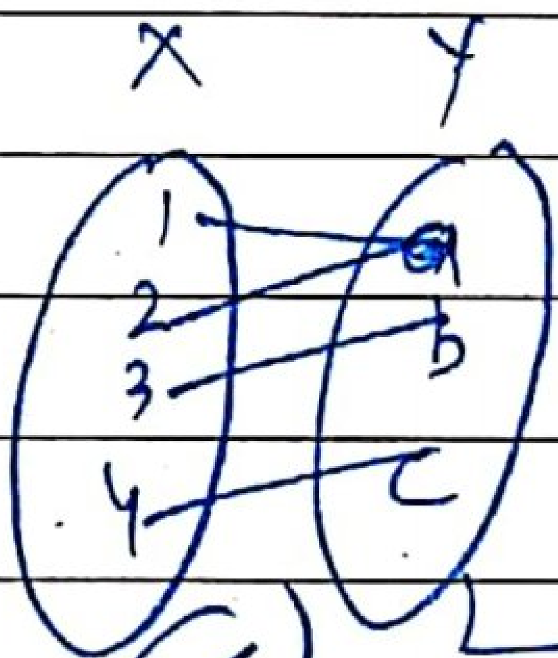


(✓)

Range = {a, b, c, d}

Codomain {a, b, c, d}

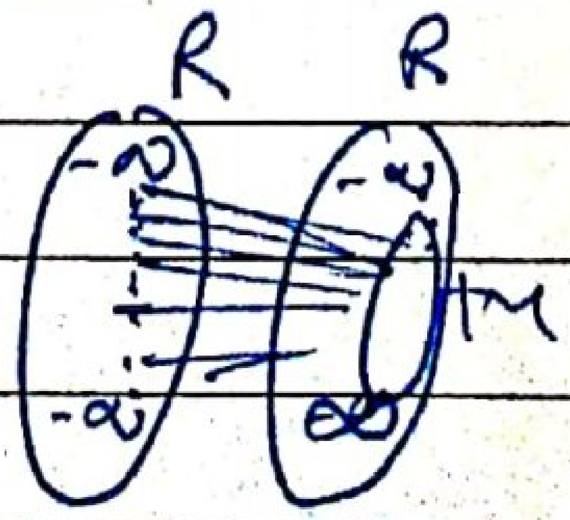
$\boxed{\text{Range} = \text{Codomain}}$



(✓) Range = Codomain

Reject $f: R \to R$

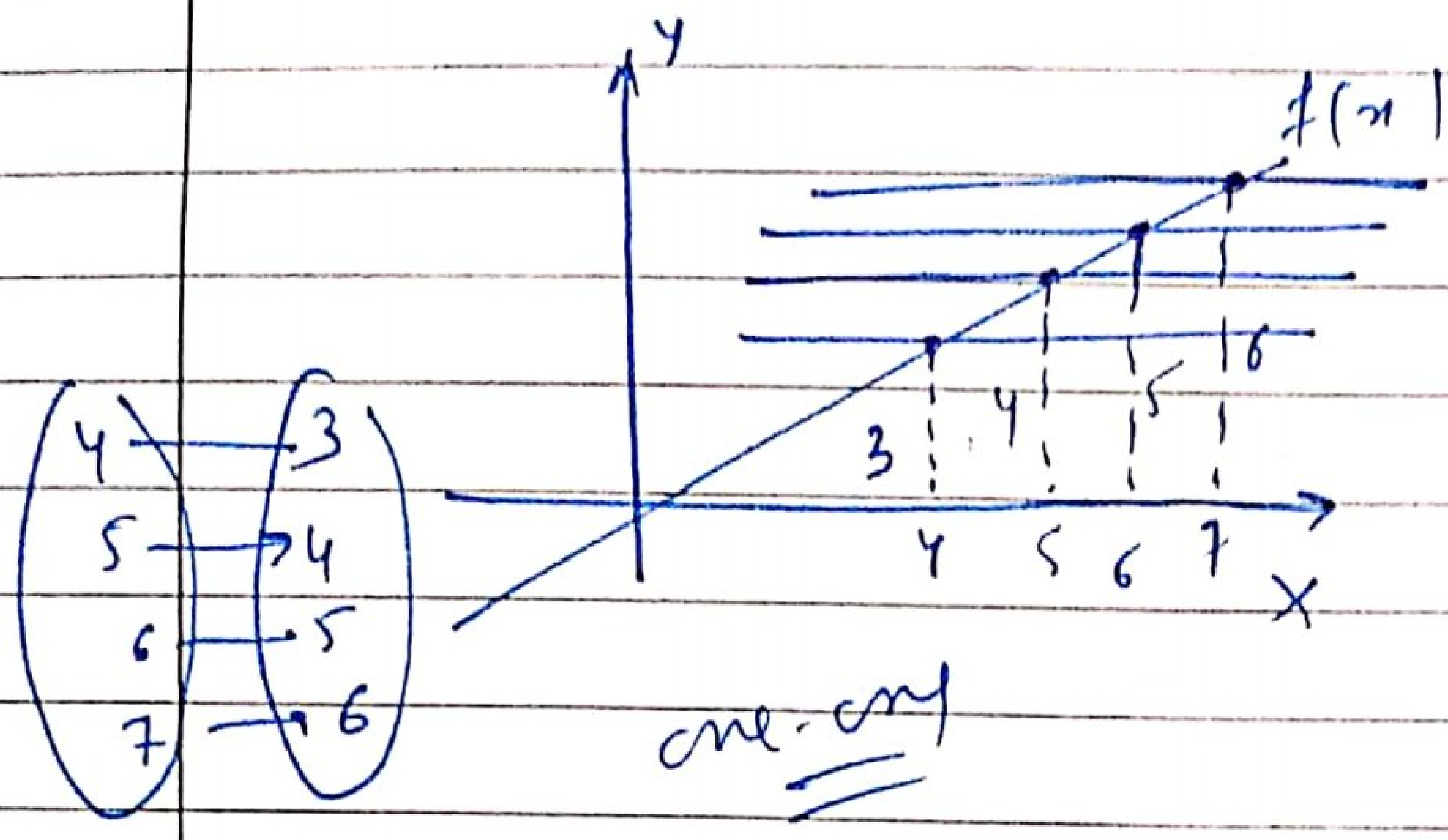
$f(x) = |x|$

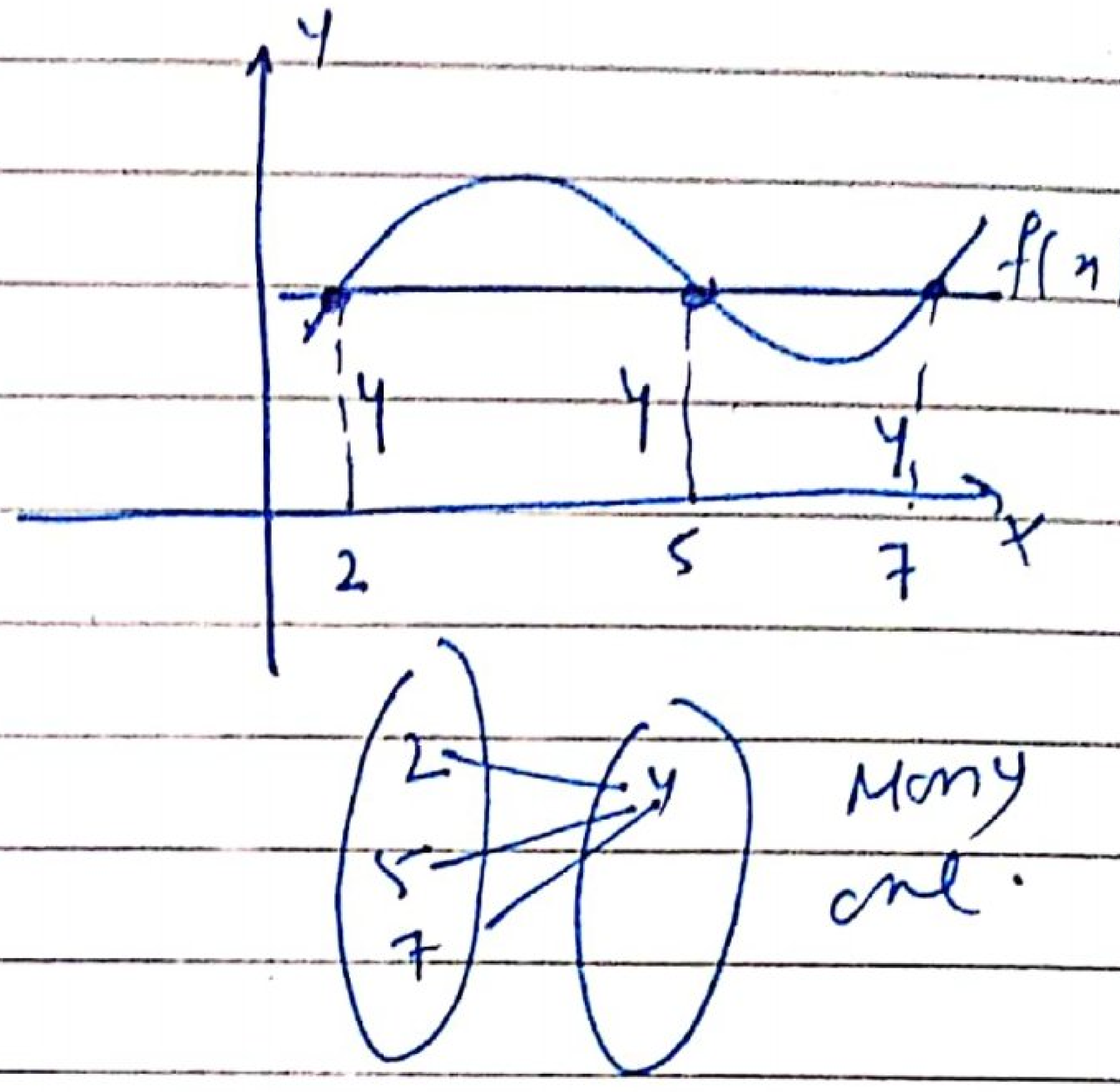


$-1 \in R$ (codomain) But there does not exists any element x in domain such that $f(x) = |x| = -1$

R & F (class No: 3) (3)

for one-one function (HORIZONTAL LINE TEST)





Qn 1 $f: R \to R$

$f(x) = [x]$

one-one $f(1.1) = [1.1] = 1$

$f(1.2) = [1.2] = 1$

clearly two different elements in domain has same image in codomain. $\therefore f(x)$ is not one-one.

on-to $0.1 \in R$ (codomain) but there does not exists any element x in domain such that $f(x) = [x] = 0.1$ $\therefore f(x)$ is not on-to.

Note If $f(x)$ is both one-one (injective) and on-to (surjective) then it is called "Bijective function"

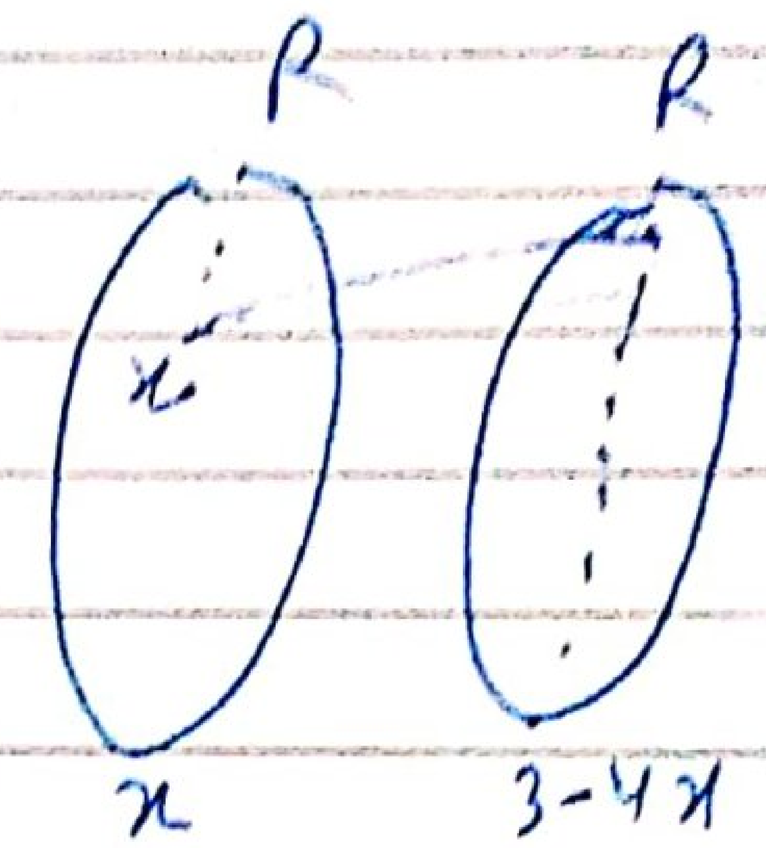
Note If $f(x)$ is bijective function then $f(x)$ is 'invertible'

R&F (Class No: 3) 4

Accept (on-to) $f: R \to R$

$f(x) = 3-4x$

Show $f(x)$ is on-to



Sol: let $y = f(x)$

$\Rightarrow y = 3-4x$

$\Rightarrow 4x = 3-y$

$\Rightarrow \boxed{x = \frac{3-y}{4}}$

{ for each $y \in R$ (codomain) there exists an element x in R (domain) such that

$f(x) = f\left(\frac{3-y}{4}\right) = 3-\cancel{4}\left(\frac{3-y}{\cancel{4}}\right)$

$= 3-3+y$

$= y$

$\therefore f$ is onto

(OR) Codomain = R

Range let $y = f(x)$

$y = 3-4x$

$x = \frac{3-y}{4}$

$y \in R$ Range = R = codoma..

$\therefore f$ is onto

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R&F (class No: 3) (5)

Q NCERT (CBSE) $f : R_+ \to [-5, \infty)$ & $f(x) = 9x^2 + 6x - 5$
Show that f is "invertible" and also find the inverse.

Sol $f(x) = 9x^2 + 6x - 5$

one-one let $x_1, x_2 \in R_+$ (domain)
& $f(x_1) = f(x_2)$

$\Rightarrow 9x_1^2 + 6x_1 \cancel{-5} = 9x_2^2 + 6x_2 \cancel{-5}$

$\Rightarrow 9x_1^2 - 9x_2^2 + 6x_1 - 6x_2 = 0$

$\Rightarrow 9(x_1 + x_2)(x_1 - x_2) + 6(x_1 - x_2) = 0$

$\Rightarrow (x_1 - x_2)(9x_1 + 9x_2 + 6) = 0$

$\boxed{x_1 = x_2}$ or $9x_1 + 9x_2 + 6 = 0$ (not valid)
since $x_1, x_2 \in R_+$

$\therefore$ f is one-one

onto let $y = 9x^2 + 6x - 5$

$\Rightarrow 9x^2 + 6x - (5 + y) = 0$

By quadratic formula

$$x = \frac{-6 \pm \sqrt{36 + 36(5+y)}}{18}$$

$$x = \frac{-6 \pm 6\sqrt{1+5+y}}{18}$$

$$x = \frac{-1 \pm \sqrt{6+y}}{3}$$

$\boxed{x = \frac{-1 + \sqrt{6+y}}{3}}$ but $x = \frac{-1 - \sqrt{6+y}}{3}$ (Reject)
since $x \in R_+$

CLASSTIME

R8 1= (class No= 3)

for each $y \in [-5, \infty)$ then exists an element x in R_+ such that

$$f(x) = f\left(\frac{-1+\sqrt{6+y}}{3}\right)$$

$$= 9\left(\frac{-1+\sqrt{6+y}}{3}\right)^2 + 6\left(\frac{-1+\sqrt{6+y}}{3}\right) - 5$$

$$= \cancel{1} + \cancel{6} + y - \cancel{2\sqrt{6+y}} - \cancel{2} + \cancel{2\sqrt{6+y}} - \cancel{5}$$

$$= y$$

$\therefore f$ is onto

Since f is one one & on-to

$\therefore f$ is bijective

$\therefore f$ is invertible

$$\begin{cases} f^{-1}(y) = \dfrac{-1+\sqrt{6+y}}{3} \checkmark \\ f^{-1}(x) = \dfrac{-1+\sqrt{6+x}}{3} \end{cases}$$