

$\leftarrow$  Solutions of worksheet no: 7  $\rightarrow$

①

### Differentiation & Continuity

Ques 1  $\rightarrow$   $f(x) = x - [x]$  let  $k$  is any integer

$$f(x) = \begin{cases} x - (k-1) & ; k-1 \leq x < k \\ x - k & ; k \leq x < k+1 \end{cases}$$

Main point

Continuity at  $x = k$

$$LHL = \lim_{x \rightarrow k^-} [x - (k-1)] \quad \text{put } x = k-h \quad \& h \rightarrow 0$$

$$LHL = \lim_{h \rightarrow 0} (k-h - (k-1)) = k - k + 1 = 1$$

$$RHL = \lim_{x \rightarrow k^+} (x - k) \quad \text{put } x = k+h \quad \& h \rightarrow 0$$

$$RHL = \lim_{h \rightarrow 0} (k+h - k) = \lim_{h \rightarrow 0} (h) = 0$$

Since  $LHL \neq RHL$

$\therefore f(x)$  is not continuous at  $x = k$

Since  $k \in \mathbb{Z}$

$\therefore f(x)$  is not continuous at all the integers Ans.

Ques 2  $\rightarrow$   $f(x) = \frac{\tan(\frac{\pi}{4} - x)}{\cot(2x)}$

then we have to find  $f\left(\frac{\pi}{4}\right)$

Since  $f(x)$  is continuous in  $[0, \frac{\pi}{2}]$   $\therefore f(x)$  is also continuous at  $x = \frac{\pi}{4}$

$\therefore$  By definition of continuity

$$\lim_{x \rightarrow \pi/4} (f(x)) = f\left(\frac{\pi}{4}\right)$$

Solution of Worksheet No 7 (Osc (2))

$$\Rightarrow \lim_{x \rightarrow \frac{\pi}{4}} \left[ \frac{\tan(\frac{\pi}{4} - x)}{\cot(2x)} \right] = f\left(\frac{\pi}{4}\right)$$

put  $x = \frac{\pi}{4} + h$  &  $h \rightarrow 0$

$$\Rightarrow \lim_{h \rightarrow 0} \left[ \frac{\tan\left(\frac{\pi}{4} - \frac{\pi}{4} - h\right)}{\cot(2(\frac{\pi}{4} + h))} \right] = f\left(\frac{\pi}{4}\right)$$

$$\Rightarrow \lim_{h \rightarrow 0} \left( \frac{\tan(-h)}{\cot(\frac{\pi}{2} + 2h)} \right) = f\left(\frac{\pi}{4}\right)$$

$$\Rightarrow \lim_{h \rightarrow 0} \left( \frac{f(-h)}{f(\frac{\pi}{2} + 2h)} \right) = f\left(\frac{\pi}{4}\right) \quad \dots \quad \begin{cases} \cot(\frac{\pi}{2} + \theta) \\ = -\tan \theta \end{cases}$$

$$\Rightarrow \lim_{h \rightarrow 0} \left( \frac{\frac{\tanh h}{h} \times h}{\frac{\tan(2h)}{2h} \times 2h} \right) = f\left(\frac{\pi}{4}\right)$$

$$\Rightarrow \frac{1}{1 \times 2} = f\left(\frac{\pi}{4}\right)$$

$$\therefore f\left(\frac{\pi}{4}\right) = \frac{1}{2} \quad \underline{\text{Ans.}}$$

$$\text{Ques 3} \rightarrow f(x) = \frac{(4^x - 1)^3}{\sin(\pi x/4) \cdot \log(1 + \frac{x^2}{3})}$$

Given  $f(x)$  is continuous at  $x=0$

$\therefore$  By definition of continuity

$$\lim_{x \rightarrow 0} (f(x)) = f(0)$$

$$\Rightarrow \lim_{x \rightarrow 0} \left( \frac{(4^x - 1)^3}{\sin(\frac{\pi x}{4}) \log(1 + \frac{x^2}{3})} \right) = f(0)$$

$$\Rightarrow \lim_{x \rightarrow 0} \left[ \frac{\left(\frac{4^x - 1}{x}\right)^3 \times x^3}{\frac{\sin(\frac{\pi x}{4})}{\frac{\pi x}{4}} \times \frac{\pi}{4} \cdot \frac{\log(1 + \frac{x^2}{3})}{\frac{x^2}{3}} \times \frac{x^2}{3}} \right] = f(0)$$

Adjustments

## Solution of Worksheet No. 7 (O&amp;C)

(3)

$$\Rightarrow \frac{(\log y)^3}{(1) \times \frac{1}{4} \times 1 \times \frac{1}{3}} = f(0) \quad \dots \quad \left\{ \begin{array}{l} \because \lim_{x \rightarrow 0} \left( \frac{a^x - 1}{x} \right) = \log a \\ \lim_{x \rightarrow 0} \left( \frac{\log(1+x)}{x} \right) = 1 \\ \lim_{x \rightarrow 0} \left( \frac{\sin x}{x} \right) = 1 \end{array} \right\}$$

$$\Rightarrow 12 \log(y)^3 = f(0)$$

$$\therefore f(0) = 12 \log(y)^3 \quad \text{Ans..}$$

Ques 4  $\rightarrow f(x) = \begin{cases} x-1 & : x < 2 \\ 2x-3 & : x \geq 2 \end{cases}$

$$\begin{aligned} LHD &= \lim_{x \rightarrow 2^-} \left( \frac{(x-1) - f(2)}{x-2} \right) & RHD &= \lim_{x \rightarrow 2^+} \left( \frac{2x-3 - f(2)}{x-2} \right) \\ &= \lim_{x \rightarrow 2^-} \left( \frac{(x-1) - (4-3)}{x-2} \right) & &= \lim_{x \rightarrow 2^+} \left( \frac{2x-3 - (4-3)}{x-2} \right) \\ &= \lim_{x \rightarrow 2^-} \left( \frac{x-2}{x-2} \right) & &= \lim_{x \rightarrow 2^+} \left( \frac{2x-4}{x-2} \right) \\ &= \lim_{x \rightarrow 2^-} (1) & &= \lim_{x \rightarrow 2^+} \left( \frac{2(x-2)}{x-2} \right) \\ \therefore LHD &= 1 & RHD &= 2 \end{aligned}$$

(Clearly  $LHD \neq RHD$ )  
 $\therefore f(x)$  is not differentiable at  $x=2$  Ans..

Ques 5  $\rightarrow f(x) = x^2$

for LHD & RHD we have to use same function (here)

$$\begin{aligned} LHD &= \lim_{x \rightarrow 1^-} \left( \frac{x^2 - f(1)}{x-1} \right) \\ &= \lim_{x \rightarrow 1^-} \left( \frac{x^2 - 1}{x-1} \right) \\ &= \lim_{x \rightarrow 1^-} \left( \frac{(x+1)(x-1)}{x-1} \right) \end{aligned}$$

Solution of Worksheet No. 7 (OBC)

(C)

$$\text{put } x = 1-h \text{ & } h \rightarrow 0$$

$$LHD = \lim_{h \rightarrow 0} (1-h+1)$$

$$LHD = 2$$

$$\text{Similarly } \underline{\text{do}} \quad RHD = 2$$

$\therefore f(x)$  is differentiable at  $x=1$  Ans ..

$$\text{Ques 6} \rightarrow f(x) = x|x|$$

Redefine the function

$$f(x) = \begin{cases} x(x) & ; x \geq 0 \\ x(-x) & ; x < 0 \end{cases}$$

$$f(x) = \begin{cases} x^2 & ; x \geq 0 \\ -x^2 & ; x < 0 \end{cases}$$

Differentiability at  $x=0$

$$LHD = \lim_{x \rightarrow 0^-} \left( \frac{-x^2 - f(0)}{x - 0} \right)$$

$$LHD = \lim_{x \rightarrow 0^-} \left( \frac{-x^2 - 0}{x} \right)$$

$$LHD = \lim_{x \rightarrow 0^-} (-x^2)$$

$$\text{put } x=0-h = -h \text{ & } h \rightarrow 0$$

$$LHD = \lim_{h \rightarrow 0} (-h^2)$$

$$LHD = 0$$

$\therefore f(x)$  is differentiable at  $x=0$  Ans

$$RHD = \lim_{x \rightarrow 0^+} \left( \frac{x^2 - f(0)}{x - 0} \right)$$

$$= \lim_{x \rightarrow 0^+} \left( \frac{x^2 - 0}{x} \right)$$

$$= \lim_{x \rightarrow 0^+} (x)$$

$$\text{put } x=0+h=h \text{ & } h \rightarrow 0$$

$$RHD = \lim_{h \rightarrow 0} (h)$$

$$RHD = 0$$

Solutions of Worksheet No. 7 (D2C) (S)

Qn. 7  $\rightarrow$   $f(x) = |x-1| - |x-2|$   
 (1) (2)

Redefine the function

$$f(x) = \begin{cases} -(x-1) + (x-2) & : x < 1 \\ (x-1) + (x-2) & : 1 \leq x < 2 \\ (x-1) - (x-2) & : x \geq 2 \end{cases}$$

$$f(x) = \begin{cases} -1 & : x < 1 \\ 2x-3 & : 1 \leq x < 2 \\ 1 & : x \geq 2 \end{cases}$$

Differentiability at  $x=1$

$$\text{LHD} = \lim_{x \rightarrow 1^-} \left( \frac{-1 - f(1)}{x-1} \right)$$

$$\text{RHD} = \lim_{x \rightarrow 1^+} \left( \frac{2x-3 - f(1)}{x-1} \right)$$

$$\text{LHD} = \lim_{x \rightarrow 1^-} \left( \frac{-1 - (2-3)}{x-1} \right)$$

$$\begin{aligned} \text{RHD} &= \lim_{x \rightarrow 1^+} \left( \frac{2x-3 - (2-3)}{x-1} \right) \\ &= \lim_{x \rightarrow 1^+} \left( \frac{2x-2}{x-1} \right) \end{aligned}$$

$$\text{LHD} = \lim_{x \rightarrow 1^+} \left( \frac{0}{x-1} \right)$$

$$= \lim_{x \rightarrow 1^+} \left( \frac{2(1-1)}{x-1} \right)$$

$$\text{LHD} = 0$$

$$\text{RHD} = 2$$

Since  $\text{LHD} \neq \text{RHD}$   $\therefore f(x)$  is not differentiable at  $x=1$

Similarly (do yourself)  $f(x)$  is not differentiable at  $x=2$

Ans

Qn 8  $\rightarrow$   $f(x) = \begin{cases} x^2 & : x \leq 2 \\ ax+b & : x > 2 \end{cases}$

Given that  $f(x)$  is differentiable at  $x=2$   
 $\therefore f(x)$  must be continuous at  $x=2$  also

Solution of Worksheet No. 7 (DEC 6)

## Continuity at $x = 2$

$$LHL = RHL = f(2)$$

$$\Rightarrow \lim_{x \rightarrow 2^-} f(x) = \lim_{x \rightarrow 2^+} (ax+b) = f(2)$$

$$\text{put } x=2-h$$

$$\text{Put } x = 2 + h$$

$$\Rightarrow \lim_{h \rightarrow 0} ((g-h)^2) = \lim_{h \rightarrow 0} (g(2+h) + b) = (2)^2$$

$$\Rightarrow 4 = 2a + b = 4$$

$$\Rightarrow \boxed{2a+b=4} \quad \dots \textcircled{1}$$

# Differentiability at $x=2$

$$LHD = \lim_{x \rightarrow 2^-} \left( \frac{x^2 - f(2)}{x - 2} \right)$$

$$\lim_{n \rightarrow \infty} \left( \frac{x^2 - 4}{n - 2} \right)$$

$$\text{LHD} = \frac{D_1}{n+2} - \left( \frac{(x+2)(x-2)}{n-2} \right)$$

plus  $x = a - h$  as  $h \rightarrow 0$

$$LHD = \lim_{h \rightarrow 0} (2-h+i)$$

$$LHD = y$$

$$RnD = \lim_{x \rightarrow 2^+} \left( \frac{ax+b - f(2)}{x-2} \right)$$

$$\frac{Rn - b}{n+2} \left( \frac{an+b-c}{n-2} \right) \text{ from } 29(1)$$

$$R_{50} = \lim_{x \rightarrow 2^+} \left( \frac{ax + (4 - 2a) - 4}{x - 2} \right)$$

$$\Rightarrow f(x) = \ln \left( \frac{ax - 2a}{x - 2} \right)$$

$$RHD = \lim_{x \rightarrow 2^+} \left( \frac{g(x-h)}{x-2} \right)$$

$$R \neq a$$

Since  $LHD = RHD$  (Given)

$\Rightarrow$   $a=4$  put in equation (1)

$$8+b=4$$

$$\Rightarrow b = -y \quad \underline{\text{Ans.}}$$

**Ques**  $\rightarrow f(x) = \sin^2 x ; 0 \leq x \leq \pi$

Soln (1) Sine function is everywhere continuous function and product of two continuous functions is also continuous.  $\therefore f(x)$  is continuous in  $[0, \pi]$

(2) Diff w.r.t  $x$   $f'(x) = 2 \sin x \cos x = \sin(2x)$

(Clearly  $f'(x)$  exists for all  $x \in (0, \pi)$ )

$\therefore f(x)$  is differentiable in open interval  $(0, \pi)$

(3)  $f(0) = \sin^2(0) = 0$

$f(\pi) = \sin^2(\pi) = 0$

$f(0) = f(\pi)$

The three conditions of Rolle's theorem are satisfied  
then there exists a value  $c \in (0, \pi)$  such that

$f'(c) = 0$

$\Rightarrow \sin(2c) = 0$

$\Rightarrow 2c = 0 \quad | \quad 2c = \pi \quad | \quad 2c = 2\pi$

$c=0$   
 $\times$

$c=\frac{\pi}{2}$

$c=\pi$   
 $\times$

$c=0 \notin (0, \pi)$

$\therefore c = \frac{\pi}{2} \in (0, \pi)$

Hence Rolle's theorem is verified. Ans

**Ques**  $\rightarrow f(x) = x^3 - 6x^2 + 11x - 6 ; x \in [1, 3]$

Soln (1) Given  $f(x)$  is a polynomial function, which is everywhere continuous  $\therefore f(x)$  is continuous in  $[1, 3]$

(2) Diff w.r.t  $x$   $f'(x) = 3x^2 - 12x + 11$

Clearly  $f'(x)$  exists for all  $x \in (1, 3)$

# Solutions of Worksheet No. 7 (OEC ⑧)

$\therefore f(x)$  is differentiable in  $(1, 3)$

$$(3) f(1) = 1 - 6 + 11 - 6 = 0$$

$$f(3) = 27 - 54 + 33 - 6 = 0$$

$$f(1) = f(3)$$

then there exists a value  $c \in (1, 3)$  such that  $f'(c) = 0$

$$\Rightarrow 3c^2 - 12c + 11 = 0$$

$$c = \frac{12 \pm \sqrt{144 - 132}}{2 \times 3}$$

$$c = \frac{12 \pm \sqrt{12}}{6}$$

$$c = \frac{12 \pm 2\sqrt{3}}{6}$$

$$c = 2 \pm \frac{\sqrt{3}}{3}$$

Clearly  $c = 2 \pm \frac{\sqrt{3}}{3}$  (both values)  $\in (1, 3)$

Hence Rolle's theorem is verified Ans

Ques 11  $\rightarrow f(x) = e^x(\sin x - \cos x)$ ;  $x \in [\frac{\pi}{4}, \frac{5\pi}{4}]$

Soln (1) Sine function, cosine function & exponential function are everywhere continuous & differentiable and multiplication of two continuous functions is also continuous.

$\therefore f(x)$  is continuous in  $[\frac{\pi}{4}, \frac{5\pi}{4}]$

(2) Diff w.r.t  $x$

$$f'(x) = e^x(\cos x + \sin x) + (\sin x - \cos x)e^x$$

$$f'(x) = 2e^x \sin x$$

Clearly  $f'(x)$  exists for all  $x \in (\frac{\pi}{4}, \frac{5\pi}{4})$

Solution of Worksheet No. 7 (OEC) (9)

$\therefore f(x)$  is differentiable in  $(\frac{\pi}{4}, \frac{5\pi}{4})$

$$(1) f(\frac{\pi}{4}) = e^{\frac{\pi}{4}} (\sin \frac{\pi}{4} - \cos \frac{\pi}{4}) = e^{\frac{\pi}{4}} \left( \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} \right) = 0$$

$$f(\frac{5\pi}{4}) = e^{\frac{5\pi}{4}} \left( \sin \left( \frac{5\pi}{4} \right) - \cos \left( \frac{5\pi}{4} \right) \right) = e^{\frac{5\pi}{4}} \left( \sin \left( \pi + \frac{\pi}{4} \right) - \cos \left( \pi + \frac{\pi}{4} \right) \right) \\ = e^{\frac{5\pi}{4}} \left( -\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} \right) = 0$$

Then there exists a value  $c \in (\frac{\pi}{4}, \frac{5\pi}{4})$  such that  $f'(c) = 0$

$$\Rightarrow 2e^c \sin c = 0$$

$$\Rightarrow e^c \sin c = 0$$

$\Rightarrow \sin c = 0$  ---- { $\because e^x \neq 0$  for all  $x \in \mathbb{R}$ }

$$\Rightarrow c = 0 \quad | \quad c = \pi$$

$$(X) \quad c = \pi \in (\frac{\pi}{4}, \frac{5\pi}{4})$$

Hence Rolle's theorem is verified Ans

Ques 12  $\Rightarrow f(x) = x(x+3) e^{-x/2}; x \in [-3, 0]$

(a)  $f(x) = (x^2 + 3x) e^{-x/2}$

Sol 12 (1) Since polynomial function & exponential function are everywhere continuous & product of two continuous functions is also continuous  
 $\therefore f(x)$  is continuous in  $[-3, 0]$

(2) Diff. w.r.t.  $f'(x) = (x^2 + 3x)e^{-x/2} \cdot (-\frac{1}{2}) + e^{-x/2} \cdot (2x+3)$

$$f'(x) = e^{-x/2} \left( \frac{-(x^2 + 3x)}{2} + 2x + 3 \right)$$

$$f'(x) = e^{-x/2} \left( \frac{-x^2 + x + 6}{2} \right)$$

Solution of Worksheet No 7 (DEC 10)

$$f'(x) = -\frac{e^{-x/2}}{2} (x^2 - x - 6)$$

Clearly  $f'(x)$  exists for all  $x \in (-3, 0)$

$$(3) f(-3) = (9-9)e^{3/2} = 0$$

$$f(0) = 0$$

$$\therefore f(-3) = f(0)$$

then there exists a value  $c \in (-3, 0)$  such that

$$f'(c) = 0 \\ \Rightarrow -\frac{e^{-c/2}}{2} (c^2 - c - 6) = 0$$

$$\Rightarrow e^{-c/2} (c-3)(c+2) = 0$$

$$\Rightarrow c = 3 \text{ or } c = -2 \quad \left\{ \begin{array}{l} e^{-c/2} \neq 0 \text{ for} \\ \text{any value of } c \end{array} \right.$$

$c = 3 \notin (-3, 0)$  &  $c = -2 \in (-3, 0)$

$\therefore$  Rolle's theorem is verified Ans

Qn 13  $\Rightarrow f(x) = \sin^4 x + \cos^4 x ; x \in [0, \frac{\pi}{2}]$

Soln (1) Sine function & Cosine function are everywhere continuous & product & addition of two continuous functions is also continuous.  
 $\therefore f(x)$  is continuous in  $[0, \frac{\pi}{2}]$

(2) Diff wrt x

$$f'(x) = 4\sin^3 x \cdot (\cos x) + 4(\cos^3 x) \cdot (-\sin x)$$

$$f'(x) = 4\sin x \cos x (-4\sin^2 x - \cos^2 x)$$

$$f'(x) = -8\sin(2x) (\cos^2 x - \sin^2 x)$$

sdarsh & Walkerne No: 7 (02C)

(11)

$$f'(x) = -2\sin(2x) \cos(2x) \quad \dots \quad \begin{cases} \because 2\sin(2x)\cos(2x) = \sin(4x) \\ (\cos^2 x - \sin^2 x) = \cos(2x) \end{cases}$$

$$f'(x) = -\sin(4x)$$

clearly  $f'(x)$  exists for all  $x \in (0, \frac{\pi}{2})$

$\therefore f(x)$  is differentiable in  $(0, \frac{\pi}{2})$

$$(3) \quad f(0) = \sin^4(0) + \cos^4(0) = 0 + 1 = 1$$

$$f(\frac{\pi}{4}) = \sin^4(\frac{\pi}{4}) + \cos^4(\frac{\pi}{4}) = 1 + 0 = 1$$

$$\therefore f(0) = f(\frac{\pi}{4})$$

then there exists a value  $c \in (0, \frac{\pi}{4})$  such that

$$f'(c) = 0$$

$$\Rightarrow -\sin(4c) = 0$$

$$\Rightarrow \sin(4c) = 0$$

$$\Rightarrow 4c = 0$$

$$c=0$$

$\times$

$$4c = \pi$$

$$c = \frac{\pi}{4}$$

$$4c = 2\pi$$

$$c = \frac{\pi}{2}$$

$\times$

$$4c = 3\pi$$

$$c = \frac{3\pi}{4}$$

$\times$

clearly  $c = \frac{\pi}{4} \in (0, \frac{\pi}{4})$

Hence Rolle's theorem is verified Ans

$$\text{Ques 14} \rightarrow f(x) = 12(x+1)(x-2) ; x \in [-1, 2]$$

$$\text{Sol} \quad f(x) = 12(x^2 - x - 2)$$

- polynomial function which is everywhere continuous  
 $\therefore f(x)$  is continuous in  $[-1, 2]$

-  $f'(x) = 12(2x-1)$  clearly  $f'(x)$  exists for all  $x \in (-1, 2)$

$\therefore f(x)$  is differentiable in  $(-1, 2)$

## Solutions of Question No 7 (OEC) (12)

$$f(-1) = 12(1+1-2) = 0$$

$$f(2) = 12(4-2-2) = 0$$

$$\therefore f(-1) = f(2)$$

then there exists a value  $c \in (-1, 2)$  such that  
 $f'(c) = 0$

$$\Rightarrow 12(2c-1) = 0$$

$$\Rightarrow c = \frac{1}{2} \in (-1, 2)$$

$\therefore$  R-T is verified

Now  $f(c) = f(\frac{1}{2}) = 12\left(\frac{1}{2}+1\right)\left(\frac{1}{2}-2\right)$

$$= 12\left(\frac{3}{2}\right)\left(-\frac{3}{2}\right) = -27$$

$(c, f(c))$  is the point on the curve where tangent is parallel to  $x$ -axis

$$\therefore \left(\frac{1}{2}, -27\right) \quad \underline{\text{Ans}}$$

Qn-15  $\rightarrow f(x) = x(x-1)(x-2); x \in [0, 1]$

Soln  $f(x) = x(x^2 - 3x + 2) = x^3 - 3x^2 + 2x$

(i)  $f(x)$  is a polynomial function which is everywhere continuous  $\therefore f(x)$  is continuous in  $[0, \frac{1}{2}]$

(ii)  $f'(x) = 3x^2 - 6x + 2$

Clearly  $f'(x)$  exist for all  $x \in (0, \frac{1}{2})$

$\therefore f(x)$  is differentiable in  $(0, \frac{1}{2})$

two conditions of LMV are satisfied, then  
 there exists a value  $c \in (0, \frac{1}{2})$  such that

$$f'(c) = \frac{f(\frac{1}{2}) - f(0)}{\frac{1}{2} - 0}$$

## Solutions of Worksheet No. 7 (OEC) (13)

$$3c^2 - 6c + 2 = \frac{\left(\frac{1}{8} - \frac{3}{4} + 1\right) - 1}{2}$$

$$\Rightarrow 3c^2 - 6c + 2 = \frac{1 - 6 + 8}{4}$$

$$\Rightarrow 3c^2 - 6c + 2 = \frac{3}{4}$$

$$\Rightarrow 12c^2 - 24c + 5 = 0$$

$$c = \frac{24 \pm \sqrt{576 - 240}}{24}$$

$$c = \frac{24 \pm \sqrt{336}}{24}$$

$$c = \frac{24 \pm \sqrt{16 \times 21}}{24}$$

$$c = \frac{24 \pm 4\sqrt{21}}{24}$$

$$c = 1 \pm \frac{\sqrt{21}}{6}$$

$$c = 1 + \frac{\sqrt{21}}{6} > \frac{1}{2} \therefore c \notin (0, \frac{1}{2})$$

$$c = 1 - \frac{\sqrt{21}}{6} \in (0, \frac{1}{2}) \therefore \text{LMV Verified } \underline{\text{Ans}}$$

Qn. 16  $\rightarrow f(x) = (x-3)^2$  chord joining  $(3, 0) \in C$ ,  $(4, 1)$

Sol: here  $a = 3$ ,  $b = 4$   $\therefore x \in [3, 4]$

do you see very LMV

$$\text{find } c = \frac{7}{2}$$

$$\therefore f(c) = f(2) = 4/4$$

$\therefore (7/2, 4/4)$  is the point on the curve where tangent is parallel to the chord Ans

## solution worksheet No. 7

Qn. 17  $\rightarrow f(x) = x^3 + bx^2 + ax ; x \in [1, 3]$

$$c = 2 + \frac{1}{\sqrt{3}}$$

SOLN

Given ROLLE theorem holds

$$\therefore f(1) = f(3)$$

$$\Rightarrow 1 + b + a = 27 + 9b + 3a$$

$$\Rightarrow 2a + 8b = -26$$

$$\Rightarrow \boxed{a + 4b = -13} \quad \dots \textcircled{1}$$

Differentiate  $f'(x) = 3x^2 + 2bx + a$

$$f'(c) = 3c^2 + 2bc + a$$

Given ROLLE theorem holds

$$\therefore f'(c) = 0$$

$$\Rightarrow 3c^2 + 2bc + a = 0$$

$$\text{put } c = 2 + \frac{1}{\sqrt{3}} \quad (\text{given})$$

$$\Rightarrow 3\left(2 + \frac{1}{\sqrt{3}}\right)^2 + 2b\left(2 + \frac{1}{\sqrt{3}}\right) + a = 0$$

$$\Rightarrow 3\left(4 + \frac{1}{3} + \frac{4}{\sqrt{3}}\right) + 4b + \frac{2b}{\sqrt{3}} + a = 0$$

$$\Rightarrow 3\left(\frac{13\sqrt{3} + \sqrt{3} + 12}{3\sqrt{3}}\right) + \frac{2b}{\sqrt{3}} + (a + 4b) = 0$$

$$\Rightarrow \frac{13\sqrt{3} + 12}{\sqrt{3}} + \frac{2b}{\sqrt{3}} + (-13) = 0 - \{ \text{from } \textcircled{1} \}$$

$$\Rightarrow 13\cancel{\sqrt{3}} + 12 + 2b - 13\cancel{\sqrt{3}} = 0$$

$$\Rightarrow 2b + 12 = 0$$

$$\Rightarrow b = -6 \quad \text{put in } \textcircled{1}$$

$$a - 24 = -43$$

$$\boxed{a = 17}$$

Ans