SOLUTIONS Solutions (WORKSHEET No: 1) Integration (1)

(Class No: 2)

Qns 1 $I = \int \cos^2 x \, dx$

$= \frac{1}{2}\int 1 + \cos(2x) \, dx$

$= \frac{1}{2}\left[x + \frac{\sin(2x)}{2}\right] + C$ Ans

Qns 2 $I = \int \cot^2(2x-3) \, dx$

$= \int \operatorname{cosec}^2(2x-3) - 1 \, dx$

$= -\frac{\cot(2x-3)}{2} - x + C$ Ans

Qns 3 $I = \int \cos^3 x \, dx$

$= \frac{1}{4}\int 3\cos x + \cos(3x) \, dx$

$= \frac{1}{4}\left[3\sin x + \frac{\sin(3x)}{3}\right] + C$ Ans

Qns 4 $I = \int \sin^3(2x) \, dx$

$= \frac{1}{4}\int 3\sin(2x) - \sin(6x) \, dx$

$= \frac{1}{4}\left[-\frac{3\cos(2x)}{2} + \frac{\cos(6x)}{6}\right] + C$ Ans

Qns 5 $I = \int \sin^2(4x+3) \, dx$

$= \frac{1}{2}\int 1 - \cos(8x+6) \, dx$

$= \frac{1}{2}\left[x - \frac{\sin(8x+6)}{8}\right] + C$ Ans

Qns 6 $I = \int \cos^4(4x) \, dx$

$= \int \left(\cos^2(4x)\right)^2 dx$

$$= \int \left(\frac{1+\cos(8x)}{2}\right)^2 dx$$

$$= \frac{1}{4}\int 1+\cos^2(8x)+2\cos(8x)\,dx$$

$$= \frac{1}{4}\int 1+\frac{1+\cos(16x)}{2}+2\cos(8x)\,dx$$

$$= \frac{1}{8}\int 3+\cos(16x)+4\cos(8x)\,dx$$

$$= \frac{1}{8}\left[3x+\frac{\sin(16x)}{16}+\frac{\sin(8x)}{2}\right]+C \quad \text{Ans}$$

Qn 7 $\quad I = \int \sin^4(3x-2)\,dx$

$$= \int \left(\sin^2(3x-2)\right)^2 dx$$

$$= \int \left(\frac{1-\cos(6x-4)}{2}\right)^2 dx$$

$$= \frac{1}{4}\int 1+\cos^2(6x-4)-2\cos(6x-4)\,dx$$

$$= \frac{1}{4}\int 1+\frac{1+\cos(12x-8)}{2}-2\cos(6x-4)\,dx$$

$$= \frac{1}{8}\int 3+\cos(12x-8)-4\cos(6x-4)\,dx$$

$$= \frac{1}{8}\left[3x+\frac{\sin(12x-8)}{12}-\frac{4\sin(6x-4)}{6}\right]+C \quad \text{Ans}$$

Qn 8 → $\quad I = \int \operatorname{cosec}^4 x\,dx$

$$= \int \operatorname{cosec}^2 x \cdot \operatorname{cosec}^2 x\,dx$$

$$= \int (1+\cot^2 x)\cdot \operatorname{cosec}^2 x\,dx$$

$$= \int \operatorname{cosec}^2 x\,dx + \int \cot^2 x \cdot \operatorname{cosec}^2 x\,dx$$

put $\cot x = t$ in 2^{nd} Integrals

$$-\csc^2 x \, dx = dt$$
$$\Rightarrow \csc^2 x \, dx = -dt$$

$$\therefore I = -\cot x \quad - \int t^2 \, dt$$
$$= -\cot x \quad - \frac{t^3}{3} + C$$
$$I = -\cot x \quad - \frac{\cot^3 x}{3} + C \quad \underline{\text{Ans}}$$

Ans 9 → $I = \int \tan^4(5x) \, dx$

$$= \int \tan^2(5x) \cdot \tan^2(5x) \, dx$$
$$= \int \tan^2(5x) \cdot (\sec^2(5x) - 1) \, dx$$
$$= \int \tan^2(5x) \cdot \sec^2(5x) \, dx - \int \tan^2(5x) \, dx$$

put $\tan(5x) = t$ in I^{st} Integral

$$\sec^2(5x) \cdot 5 \, dx = dt$$
$$\sec^2(5x) \, dx = \frac{dt}{5}$$

$$\therefore I = \frac{1}{5} \int t^2 \cdot dt \quad - \int \sec^2(5x) - 1 \, dx$$
$$= \frac{1}{5} \cdot \frac{t^3}{3} \quad - \left(\frac{\tan(5x)}{5} - x \right) + C$$
$$= \frac{\tan^3(5x)}{15} - \frac{\tan(5x)}{5} + x + C \quad \underline{\text{Ans}}$$

Ans 10 → $I = \int \cot^3(3x) \, dx$

$$I = \int \cot(3x) \cdot \cot^2(3x) \, dx$$
$$= \int \cot(3x) \cdot (\csc^2(3x) - 1) \, dx$$

$$= \int \cot(3x)\cdot \operatorname{cosec}^2(3x)\,dx - \int \cot(3x)\,dx$$

put $\cot(3x) = t$

$$-\operatorname{cosec}^2(3x)\cdot 3\,dx = dt$$

$$\operatorname{cosec}^2(3x)\,dx = \frac{-dt}{3}$$

$$\therefore I = \frac{-1}{3}\int t\cdot dt - \frac{1}{3}\log|\sin(3x)|$$

$$= -\frac{1}{3}\cdot\frac{t^2}{2} - \frac{1}{3}\log|\sin(3x)| + C$$

$$= -\frac{\cot^2(3x)}{6} - \frac{1}{3}\log|\sin(3x)| + C \quad \underline{\text{Ans}}$$

Qn 11) $I = \int \cos^5 x\,dx$

$$= \int \cos^4 x\cdot \cos x\,dx$$

$$= \int (\cos^2 x)^2\cdot \cos x\,dx$$

$$= \int (1-\sin^2 x)^2\cdot \cos x\,dx$$

put $\sin x = t$

$\cos x\,dx = dt$

$$I = \int (1-t^2)^2\cdot dt$$

$$= \int (1+t^4-2t^2)\,dt$$

$$= t + \frac{t^5}{5} - 2\frac{t^3}{3} + C$$

$$= \sin x + \frac{\sin^5 x}{5} - \frac{2\sin^3 x}{3} + C \quad \underline{\text{Ans}}$$

Qn 12 → $I = \int \sin^9 x\,dx$

$$I = \int \sin^8 x\cdot \sin x\,dx$$

$$= \int (1-\cos^2 x)^4 \cdot \sin x \, dx$$

put $\cos x = t$

$-\sin x \, dx = dt \Rightarrow \sin x \, dx = -dt$

$$I = -\int (1-t^2)^4 \, dt$$

$$= -\int \left((1-t^2)^2\right)^2 dt$$

$$= -\int (1+t^4-2t^2)^2 \, dt$$

add

$$= -\int (1+t^8+4t^4+2t^4-4t^6-4t^2) \, dt$$

$$\cdots \{ (a+b+c)^2 = a^2+b^2+c^2+2ab+2bc+2ca \}$$

$$= -\left[t + \frac{t^9}{9} + \frac{6t^5}{5} - \frac{4t^7}{7} - \frac{4t^3}{3} \right] + C$$

$$= -\left[\cos x + \frac{\cos^9 x}{9} + \frac{6\cos^5 x}{5} - \frac{4\cos^7 x}{7} - \frac{4\cos^3 x}{3} \right] + C$$

Ans

Qn 13 $I = \int \cot^5 x \, dx$

$$= \int \cot^3 x \cdot \cot^2 x \, dx$$

$$= \int \cot^3 x \cdot (\csc^2 x - 1) \, dx$$

$$= \int \cot^3 x \cdot \csc^2 x \, dx - \int \cot^3 x \, dx$$

$$= \int \cot^3 x \cdot \csc^2 x \, dx - \int \cot x \cdot \cot^2 x \, dx$$

$$= \int \cot^3 x \cdot \csc^2 x \, dx - \int \cot x \cdot (\csc^2 x - 1) \, dx$$

$$= \int \cot^3 x \cdot \csc^2 x \, dx \quad - \int \cot x \cdot \csc^2 x \, dx \ + \int \cot x \, dx$$

put $\cot x = t$

$$-\csc^2 x \, dx = dt \quad \Rightarrow \quad \csc^2 x \, dx = -dt$$

$$I = -\int t^3 \, dt \quad + \int t \, dt \quad + \log|\sin x|$$

$$= -\frac{t^4}{4} + \frac{t^2}{2} + \log|\sin x| + C$$

$$= -\frac{\cot^4 x}{4} + \frac{\cot^2 x}{2} + \log|\sin x| + C$$ Ans

Q. 14 $I = \int \cos^6 (2x) \, dx$

$$= \int \left(\cos^3(2x)\right)^2 dx$$

$$= \int \left(\frac{3\cos(2x) + \cos(6x)}{4}\right)^2 dx$$

$$= \frac{1}{16}\int 9\cos^2(2x) + \cos^2(6x) + 6\cos(2x)\cos(6x) \, dx$$

$$= \frac{1}{16}\int 9\frac{(1+\cos(4x))}{2} + \frac{1+\cos(12x)}{2} + 3\left(\cos(8x) + \cos(4x)\right) dx$$

$$= \frac{1}{32}\int 10 + 9\cos(4x) + \cos(12x) + 6\cos(8x) + 6\cos(4x) \, dx$$

$$= \frac{1}{32}\left[10x + \frac{9\sin(4x)}{4} + \frac{\sin(12x)}{12} + \frac{3\sin(8x)}{4} + \frac{3\sin(4x)}{2}\right] + C$$

Ans

Qn. 15 → $I = \int \sin(2x) \cdot \sin(5x)\, dx$

$$I = \frac{1}{2}\int 2\sin(2x) \cdot \sin(5x)\, dx$$

$$= \frac{1}{2}\int \cos(3x) - \cos(7x)\, dx$$

$$= \frac{1}{2}\left[\frac{\sin(3x)}{3} - \frac{\sin(7x)}{7} \right] + C \quad \text{Ans}$$

Qn. 16 $I = \int \cos(2x) \cdot \cos(4x) \cdot \cos(6x)\, dx$

$$= \frac{1}{2}\int \left(2\cos(2x) \cdot \cos(4x)\right) \cdot \cos(6x)\, dx$$

$$= \frac{1}{2}\int \left(\cos(6x) + \cos(2x)\right) \cdot \cos(6x)\, dx$$

$$= \frac{1}{2}\int \cos^2(6x) + \cos(2x) \cdot \cos(6x)\, dx$$

$$= \frac{1}{2}\int \frac{1+\cos(12x)}{2} + \cos(2x) \cdot \cos(6x)\, dx$$

$$= \frac{1}{4}\int 1 + \cos(12x) + 2\cos(2x) \cdot \cos(6x)\, dx$$

$$= \frac{1}{4}\int 1 + \cos(12x) + \cos(8x) + \cos(4x)\, dx$$

$$= \frac{1}{4}\left[x + \frac{\sin(12x)}{12} + \frac{\sin(8x)}{8} + \frac{\sin(4x)}{4} \right] + C$$

Ans

Qn. 17 $I = \int \sin^5 x \cdot \cos^5 x\, dx$

$$= \int (\sin x \cdot \cos x)^5\, dx$$

$$= \int \left(\frac{\sin(2x)}{2}\right)^5 dx$$

$$= \frac{1}{32}\int \sin^5(2x)\, dx$$

$$= \frac{1}{32}\int \sin^4(2x)\cdot \sin(2x)\, dx$$

$$= \frac{1}{32}\int \left(\sin^2(2x)\right)^2\cdot \sin(2x)\, dx$$

$$= \frac{1}{32}\int \left(1-\cos^2(2x)\right)^2\cdot \sin(2x)\, dx$$

$$= \frac{1}{32}$$

put $\cos(2x) = t$

$$-\sin(2x)\cdot 2\, dx = dt$$

$$\sin(2x)\, dx = -\frac{dt}{2}$$

$$I = -\frac{1}{64}\int (1-t^2)^2\, dt$$

$$= -\frac{1}{64}\int (1+t^4-2t^2)\, dt$$

$$= -\frac{1}{64}\left[t + \frac{t^5}{5} - \frac{2t^3}{3}\right] + C$$

$$I = -\frac{1}{64}\left[\cos(2x) + \frac{\cos^5(2x)}{5} - \frac{2\cos^3(2x)}{3}\right] + C \quad \text{Ans}$$