SOLUTIONS VECTORS | WORKSHEET No: 1

Ans 1 →

Point is $(12, n)$

$\therefore \vec{a} = 12\hat{i} + n\hat{j}$

$|\vec{a}| = \sqrt{144 + n^2} = 13$ (given)

$\Rightarrow 144 + n^2 = 169$

$\Rightarrow n^2 = 25$

$\Rightarrow \boxed{n = \pm 5}$ Ans

Ans 2 →

Given points $(1, -1)$ & $(-2, m)$

$\therefore \vec{a} = \hat{i} - \hat{j}$ and $\vec{b} = -2\hat{i} + m\hat{j}$

Given: $\vec{a}$ & $\vec{b}$ are collinear

$\therefore$ their corresponding components are in same ratio

$\Rightarrow \frac{1}{-2} = \frac{-1}{m} \Rightarrow \boxed{m = 2}$ Ans

Ans 3 →

Given $P(1, 2, 3)$ & $Q(4, 5, 6)$

$\vec{QP} = \vec{OP} - \vec{OQ}$

$\Rightarrow \vec{QP} = -(4\hat{i} + 5\hat{j} + 6\hat{k}) + (\hat{i} + 2\hat{j} + 3\hat{k})$

$\Rightarrow \vec{QP} = -3\hat{i} - 3\hat{j} - 3\hat{k}$

$|\vec{QP}| = \sqrt{9 + 9 + 9} = \sqrt{27} = 3\sqrt{3}$

Now $\hat{QP} = \frac{\vec{QP}}{|\vec{QP}|} = \frac{-3\hat{i} - 3\hat{j} - 3\hat{k}}{3\sqrt{3}}$

$\boxed{\hat{QP} = -\frac{1}{\sqrt{3}}(\hat{i} + \hat{j} + \hat{k})}$ Ans

Ans 4 → given points $A(-2,3,5)$ $B(1,2,3)$ $C(7,0,-1)$

$\vec{AB} = 3\hat{i} - \hat{j} - 2\hat{k} \Rightarrow |\vec{AB}| = \sqrt{9+1+4} = \sqrt{14}$

$\vec{BC} = 6\hat{i} - 2\hat{j} - 4\hat{k} \Rightarrow |\vec{BC}| = \sqrt{36+4+16} = \sqrt{56} = 2\sqrt{14}$

$\vec{CA} = -9\hat{i} + 3\hat{j} + 6\hat{k} \Rightarrow |\vec{CA}| = \sqrt{81+9+36} = \sqrt{126} = 3\sqrt{14}$

clearly $|\vec{CA}| = |\vec{BC}| + |\vec{AB}|$

$\therefore$ points A, B, C are collinear **Ans**

Ans 5 → Given vectors $\vec{a} = 2\hat{i} + 3\hat{j} - \hat{k}$

$\vec{b} = \hat{i} - 2\hat{j} + \hat{k}$

Resultant $= \vec{a} + \vec{b} = 3\hat{i} + \hat{j} + 0\hat{k} = \vec{c}$ (let)

$|\vec{c}| = \sqrt{9+1} = \sqrt{10}$

$\hat{c} = \dfrac{\vec{c}}{|\vec{c}|} = \dfrac{1}{\sqrt{10}}(3\hat{i} + \hat{j})$

Required vector $= 5\hat{c} = \dfrac{5}{\sqrt{10}}(3\hat{i} + \hat{j})$

$= \sqrt{\dfrac{25}{10}}(3\hat{i} + \hat{j})$

$= \sqrt{\dfrac{5}{2}}(3\hat{i} + \hat{j})$ **Ans**

Ans 6 →

given $\vec{a} = \hat{i} + \hat{j} + \hat{k}$; $\vec{b} = 4\hat{i} - 2\hat{j} + 3\hat{k}$

$\vec{c} = \hat{i} - 2\hat{j} + \hat{k}$

let $\vec{d} = 2\vec{a} - \vec{b} + 3\vec{c}$

$\vec{d} = 2\hat{i} + 2\hat{j} + 2\hat{k} - 4\hat{i} + 2\hat{j} - 3\hat{k} + 3\hat{i} - 6\hat{j} + 3\hat{k}$

$\vec{d} = \hat{i} - 2\hat{j} + 2\hat{k}$

$|\vec{d}| = \sqrt{1+4+4} = 3$

$\hat{d} = \frac{\vec{d}}{|\vec{d}|} = \frac{1}{3}(\hat{i} - 2\hat{j} + 2\hat{k})$

Required vector $= 6\hat{d}$

$= 2(\hat{i} - 2\hat{j} + 2\hat{k})$

$= 2\hat{i} - 4\hat{j} + 4\hat{k}$ Ans

Qns 7 →

given $\vec{OA} = 2\hat{i} - \hat{j} + \hat{k}$

$\vec{OB} = \hat{i} - 3\hat{j} - 5\hat{k}$

$\vec{OC} = 3\hat{i} - 4\hat{j} - 4\hat{k}$

$\vec{AB} = \vec{OB} - \vec{OA} = -\hat{i} - 2\hat{j} - 6\hat{k}$

$\vec{BC} = \vec{OC} - \vec{OB} = 2\hat{i} - \hat{j} + \hat{k}$

$\vec{CA} = \vec{OA} - \vec{OC} = -\hat{i} + 3\hat{j} + 5\hat{k}$

$|\vec{AB}| = \sqrt{1+4+36} = \sqrt{41} \Rightarrow |\vec{AB}|^2 = 41$

$|\vec{BC}| = \sqrt{4+1+1} = \sqrt{6} \Rightarrow |\vec{BC}|^2 = 6$

$|\vec{CA}| = \sqrt{1+9+25} = \sqrt{35} \Rightarrow |\vec{CA}|^2 = 35$

clearly $|\vec{AB}|^2 = |\vec{BC}|^2 + |\vec{CA}|^2$

$\therefore$ A, B, C are the vertices of a right angled triangle

Proved

Qns: 8 →

we know that

$l^2 + m^2 + n^2 = 1$

$\Rightarrow \cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

$\Rightarrow 1 - \sin^2\alpha + 1 - \sin^2\beta + 1 - \sin^2\gamma = 1$

$\Rightarrow \sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2$ Proved

Ans: 9 → vector makes $\frac{\pi}{3}$ with $\hat{i}$ means with X-axis

$\therefore \alpha = \pi/3$

vector makes $\pi/4$ with $\hat{j}$ means with Y-axis

$\therefore \beta = \pi/4$

vector make θ with $\hat{k}$ means with Z-axis

$\therefore \gamma = \theta$ (acute here)

We have $l^2 + m^2 + n^2 = 1$

$\Rightarrow \cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

$\Rightarrow \cos^2(\pi/3) + \cos^2(\pi/4) + \cos^2\theta = 1$

$\Rightarrow \frac{1}{4} + \frac{1}{2} + \cos^2\theta = 1$

$\Rightarrow \cos^2\theta = 1 - \frac{1}{2} - \frac{1}{4}$

$\Rightarrow \cos^2\theta = \frac{1}{4}$

$\Rightarrow \cos\theta = \pm 1/2$

But θ is acute (means Ist quadrant)

$\therefore \cos\theta = 1/2$

$\Rightarrow \boxed{\theta = \pi/3}$ Ans

Ans 10 → given $\vec{a} = 3\hat{i} - \hat{j} - 4\hat{k}$; $\vec{b} = -2\hat{i} + 4\hat{j} - 3\hat{k}$

and $\vec{c} = \hat{i} + 2\hat{j} - \hat{k}$

let $\vec{d} = 3\vec{a} - 2\vec{b} + 4\vec{c}$

$\vec{d} = 9\hat{i} - 3\hat{j} - 12\hat{k} + 4\hat{i} - 8\hat{j} + 6\hat{k} + 4\hat{i} + 8\hat{j} - 4\hat{k}$

$\vec{d} = 17\hat{i} - 3\hat{j} - 10\hat{k}$

Now $|\vec{d}| = \sqrt{289 + 9 + 100} = \sqrt{398}$ Ans

Ans 11 → Given $\beta = \pi/3$

$\gamma = \pi/2$

$\alpha = \theta$ (acute)

we have $l^2 + m^2 + n^2 = 1$

$\Rightarrow \cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

$\Rightarrow \cos^2\theta + \cos^2(\pi/3) + \cos^2(\pi/2) = 1$

$\Rightarrow \cos^2\theta + \frac{1}{4} + 0 = 1$

$\Rightarrow \cos^2\theta = 3/4$

$\Rightarrow \cos\theta = \pm\frac{\sqrt{3}}{2}$

Since θ is acute (I^{st} quadrant)

$\therefore \cos\theta = \frac{\sqrt{3}}{2}$

$\boxed{\theta = \pi/6}$ Ans (Note: Misprint in worksheet Answer)

Ans 12 → given $\vec{a} = 6\hat{i} - 2\hat{j} + 3\hat{k}$

here Direction Ratios are

$a = 6, \quad b = -2, \quad c = 3$

$$l = \frac{a}{\sqrt{a^2+b^2+c^2}} = \frac{6}{\sqrt{36+4+9}} = \frac{6}{7}$$

$$m = \frac{b}{\sqrt{a^2+b^2+c^2}} = \frac{-2}{7}$$

$$n = \frac{c}{\sqrt{a^2+b^2+c^2}} = \frac{3}{7}$$

$\therefore$ D.C's are $\frac{6}{7}, -\frac{2}{7}, \frac{3}{7}$ Ans

Ans 13 → let $\vec{a} = x\hat{i} + 2\hat{j} - z\hat{k}$

and $\vec{b} = 3\hat{i} - y\hat{j} + \hat{k}$

given $\vec{a} = \vec{b}$

$\Rightarrow x\hat{i} + 2\hat{j} - z\hat{k} = 3\hat{i} - y\hat{j} + \hat{k}$

$\Rightarrow x = 3, \quad y = -2, \quad z = -1$

$\therefore x + y + z = 3 - 2 - 1 = 0$ Ans