# ← ULTIMATE MATHEMATICS →

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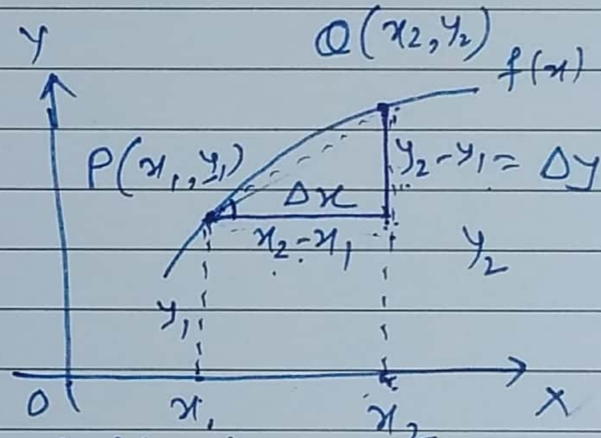
## CHAPTER: "DIFFERENTIATION & CONTINUITY"

(·) derivatives: Rate of change of one variable w.r.t another variable.

(·) $y = f(x)$

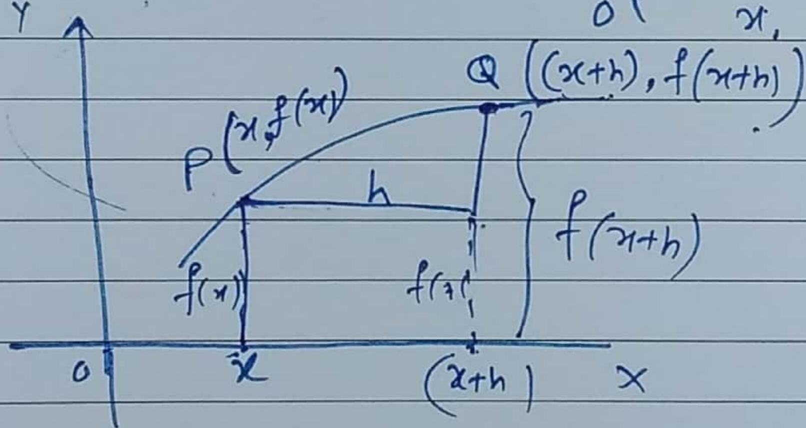
Differentiate both sides w.r.t $x$

$$\frac{dy}{dx} = f'(x)$$




(·) $$\frac{dy}{dx} = \lim_{\Delta x \to 0}\left(\frac{\Delta y}{\Delta x}\right)$$

(·)




$$\frac{dy}{dx} = \lim_{h \to 0}\left(\frac{f(x+h) - f(x)}{h}\right)$$

Application = Slope of tangent

$$\boxed{m = \frac{dy}{dx}}$$

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FORMULAE

(1) $\frac{d}{dx}(x^n) = n x^{n-1}$

(2) $\frac{d}{dx}(\text{constant}) = 0$

(3) $\frac{d}{dx}(\log x) = \frac{1}{x}$

(4) $\frac{d}{dx}(e^x) = e^x$

(5) $\frac{d}{dx}(a^x) = a^x \cdot \log a$

(6) $\frac{d}{dx}(\sin x) = \cos x$

(7) $\frac{d}{dx}(\cos x) = -\sin x$

(8) $\frac{d}{dx}(\tan x) = \sec^2 x$

(9) $\frac{d}{dx}(\cot x) = -\operatorname{cosec}^2 x$

(10) $\frac{d}{dx}(\sec x) = \sec x \tan x$

(11) $\frac{d}{dx}(\operatorname{cosec} x) = -\operatorname{cosec} x \cdot \cot x$

(12) Shortcuts $\frac{d}{dx}(\sqrt{x}) = \frac{1}{2\sqrt{x}}$

(13) $\frac{d}{dx}\left(\frac{1}{x}\right) = \frac{-1}{x^2}$

(14) $\frac{1}{x} \rightarrow \frac{-1}{x^2} \rightarrow \frac{2}{x^3} \rightarrow \frac{-6}{x^4} \rightarrow \frac{24}{x^5} \rightarrow \frac{-120}{x^6}$

(·) (15) $\frac{d}{dx}(\sin^{-1}x) = \frac{1}{\sqrt{1-x^2}}$

(·) (16) $\frac{d}{dx}(\cos^{-1}x) = \frac{-1}{\sqrt{1-x^2}}$

(17). $\frac{d}{dx}(\tan^{-1}x) = \frac{1}{1+x^2}$

(18) $\frac{d}{dx}(\cot^{-1}x) = \frac{-1}{1+x^2}$

(19) $\frac{d}{dx}(\sec^{-1}x) = \frac{1}{|x|\sqrt{x^2-1}}$

(·) (20) $\frac{d}{dx}(\operatorname{cosec}^{-1}x) = \frac{-1}{|x|\sqrt{x^2-1}}$

1. Rules

(1) $\frac{d}{dx}\left(f(x) \pm g(x)\right) = \frac{d}{dx}\left(f(x)\right) \pm \frac{d}{dx}\left(g(x)\right)$

(2) Product Rule $y = f(x)\cdot g(x)$

$$\frac{dy}{dx} = f(x)\cdot\frac{d}{dx}\left(g(x)\right) + g(x)\cdot\frac{d}{dx}\left(f(x)\right)$$

(3) Quotient Rule $y = \frac{f(x)}{g(x)} = \frac{N}{D}$

$$\frac{dy}{dx} = \frac{D\cdot\frac{d}{dx}(N) - N\cdot\frac{d}{dx}(D)}{D^2}$$

(4) $\frac{d}{dx}\left(kf(x)\right) = k\cdot\frac{d}{dx}\left(f(x)\right)$

— x —

Basics $\frac{d}{dx}(x) = 1$ $\quad x^1 \to (1)x^{1-1} = 1$

$\frac{d}{dx}(2x) = 2$

(1) $y = \sin(3x)$

$\frac{dy}{dx} = \cos(3x)\cdot\frac{d}{dx}(3x)$

$= \cos(3x)\cdot 3 = 3\cos(3x)$

(2) $y = \tan\sqrt{x}$

$\frac{dy}{dx} = \sec^2(\sqrt{x})\cdot\frac{1}{2\sqrt{x}}$

(3) $y = \log(\tan x)$

$\frac{dy}{dx} = \frac{1}{\tan x}\cdot\frac{d}{dx}(\tan x)$

$= \frac{1}{\tan x}\cdot\sec^2 x$

(4) $y = \log(x^3)$

$\frac{dy}{dx} = \frac{1}{x^3}\cdot 3x^2$

(5) $y = e^{\cot x}$

$\frac{dy}{dx} = e^{\cot x} \cdot \frac{d}{dx}(\cot x)$

$= -\operatorname{cosec}^2 x \cdot e^{\cot x}$

(6) $y = 3^{\sin x}$

$\frac{dy}{dx} = 3^{\sin x} \cdot \lg 3 \cdot \frac{d}{dx}(\sin x)$

$= \cos x \cdot 3^{\sin x} \cdot \lg 3$

(7) $y = \sqrt{\cos x}$

$\frac{dy}{dx} = \frac{1}{2\sqrt{\cos x}} \cdot \frac{d}{dx}(\cos x)$

$= \frac{-\sin x}{2\sqrt{\cos x}}$

(8) $y = \sqrt{\log x}$

$\frac{dy}{dx} = \frac{1}{2\sqrt{\lg x}} \cdot \frac{1}{x}$

(9) $y = (3x-2)^{5/3}$

$\frac{dy}{dx} = \frac{5}{3}(3x-2)^{5/3-1} \cdot \frac{d}{dx}(3x-2)$

$= \frac{5}{\cancel{3}}(3x-2)^{2/3} \cdot (\cancel{3})$

(10) $y = \frac{1}{(5x-3)^{2/3}}$

$y = (5x-3)^{-2/3}$

$\frac{dy}{dx} = \frac{-2}{3}(5x-3)^{-\frac{5}{3}} \cdot (5)$

(11) $y = \sin^4 x$

$y = (\sin x)^4$

$\frac{dy}{dx} = 4(\sin x)^3 \cdot \frac{d}{dx}(\sin x)$

$\frac{dy}{dx} = 4\sin^3 x \cdot \cos x$

(12) $y = \sin^{-1}(x^3)$

$\frac{dy}{dx} = \frac{1}{\sqrt{1-x^6}} \cdot \frac{d}{dx}(x^3)$

$= \frac{3x^2}{\sqrt{1-x^6}}$

(13) $y = \cot^{-1}(\sqrt{x})$

$\frac{dy}{dx} = \frac{-1}{1+x} \cdot \frac{1}{2\sqrt{x}}$

(14) $y = \sqrt{\log(\tan(3^{x^3}))}$

Diff w.r.t $x$

$$\frac{dy}{dx} = \frac{1}{2\sqrt{\log(\tan(3^{x^3}))}} \cdot \frac{1}{\tan(3^{x^3})} \cdot \sec^2(3^{x^3}) \cdot 3^{x^3} \cdot \log 3 \cdot 3x^2$$

(15) $y = \sin\left(e^{\sqrt{\tan^{-1}x^2}}\right)$

$$\frac{dy}{dx} = \cos\left(e^{\sqrt{\tan^{-1}x^2}}\right) \cdot e^{\sqrt{\tan^{-1}x^2}} \cdot \frac{1}{2\sqrt{\tan^{-1}x^2}} \cdot \frac{1}{1+x^4} \cdot 2x$$

(16) $y = \sin(x^3) \cdot \sqrt{\cos x}$

Product rule

$$\frac{dy}{dx} = \sin(x^3) \cdot \frac{d}{dx}(\sqrt{\cos x}) + \sqrt{\cos x} \cdot \frac{d}{dx}(\sin(x^3))$$

$$= \sin(x^3) \cdot \frac{1}{2\sqrt{\cos x}} \cdot (-\sin x) + \sqrt{\cos x} \cdot \cos(x^3) \cdot 3x^2$$

(17) $y = \cos^3(x^2) \cdot \sin^4(x^3)$

Product rule

$$\frac{dy}{dx} = \cos^3(x^2) \cdot \frac{d}{dx}(\sin^4(x^3)) + \sin^4(x^3) \cdot \frac{d}{dx}(\cos^3(x^2))$$

$$= \cos^3(x^2) \cdot 4\sin^3(x^3) \cdot \frac{d}{dx}(\sin(x^3)) + \sin^4(x^3) \cdot 3\cos^2(x^2) \cdot \frac{d}{dx}(\cos(x^2))$$

$$= \cos^3(x^2) \cdot 4\sin^3(x^3) \cdot \cos(x^3) \cdot 3x^2 + \sin^4(x^3) \cdot 3\cos^2(x^2) \cdot (-\sin(x^2)) \cdot 2x$$

Ans

(18) $y = \dfrac{x+\sin x}{x-\sin x}$

Quotient Rule

$$\frac{dy}{dx} = \frac{(x-\sin x)\cdot\frac{d}{dx}(x+\sin x) - (x+\sin x)\cdot\frac{d}{dx}(x-\sin x)}{(x-\sin x)^2}$$

$$= \frac{(x-\sin x)\cdot(1+\cos x) - (x+\sin x)\cdot(1-\cos x)}{(x-\sin x)^2}$$

open the brackets

$= \cancel{(x - \sin x}$

$\sin x\cos x$

$$\frac{dy}{dx} = \frac{\cancel{x} + x\cos x - \sin x - \cancel{\sin x\cos x} - \cancel{x} + x\cos x - \sin x +}{(x-\sin x)^2}$$

$$\frac{dy}{dx} = \frac{2x\cos x - 2\sin x}{(x-\sin x)^2}$$ Ans

(19) $y = \dfrac{x\sin x}{\log x}$

Diff Quotient Rule $\to$ (Product rule)

$$\frac{dy}{dx} = \frac{\log x\cdot\frac{d}{dx}(x\sin x) - x\sin x\cdot\frac{d}{dx}(\log x)}{(\log x)^2}$$

$$\frac{dy}{dx} = \frac{\log x\cdot(x\cos x + \sin x) - x\sin x\cdot\left(\frac{1}{x}\right)}{(\log x)^2}$$

Ans

— x —