(1)

— ULTIMATE MATHEMATICS —
(BY: AJAY MITTAL: 9891067390)

INVERSE TRIGO Class I-3

Qn 1 Show that $\tan^{-1}\left(\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right) = \frac{\pi}{4} - \frac{1}{2}\cos^{-1}$

Divide by $\sqrt{1+x}$

$$= \tan^{-1}\left(\frac{1-\sqrt{\frac{1-x}{1+x}}}{1+\sqrt{\frac{1-x}{1+x}}}\right)$$

$$= \tan^{-1}(1) - \tan^{-1}\left(\sqrt{\frac{1-x}{1+x}}\right)$$

put $\boxed{x = \cos(2\theta)}$

$$= \frac{\pi}{4} - \tan^{-1}\sqrt{\frac{1-\cos(2\theta)}{1+\cos(2\theta)}}$$

$$= \frac{\pi}{4} - \tan^{-1}\sqrt{\frac{\cancel{2}\sin^2\theta}{\cancel{2}\cos^2\theta}}$$

$$= \frac{\pi}{4} - \tan^{-1}(\tan\theta)$$

$$= \frac{\pi}{4} - \theta \quad \text{(replace } \theta\text{)}$$

$$= \frac{\pi}{4} - \frac{1}{2}\cos^{-1}x \quad \text{Ans}$$

Qn 2 Simplify $\cot^{-1}\left(\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right)$

$$= \tan^{-1}\left(\frac{\sqrt{1+\sin x} - \sqrt{1-\sin x}}{\sqrt{1+\sin x} + \sqrt{1-\sin x}}\right)$$

Divide by $\sqrt{1+\sin x}$

$$= \tan^{-1}\left(\frac{1 - \sqrt{\frac{1-\sin x}{1+\sin x}}}{1 + \sqrt{\frac{1-\sin x}{1+\sin x}}}\right)$$

$$= \tan^{-1}(1) - \tan^{-1}\sqrt{\frac{1-\sin x}{1+\sin x}}$$

$$= \frac{\pi}{4} - \tan^{-1}\sqrt{\frac{1-\cos\left(\frac{\pi}{2}-x\right)}{1+\cos\left(\frac{\pi}{2}-x\right)}}$$

$$= \frac{\pi}{4} - \tan^{-1}\sqrt{\frac{\cancel{2}\sin^2\left(\frac{\pi}{4}-\frac{x}{2}\right)}{\cancel{2}\cos^2\left(\frac{\pi}{4}-\frac{x}{2}\right)}}$$

$$= \frac{\pi}{4} - \tan^{-1}\sqrt{\tan^2\left(\frac{\pi}{4}-\frac{x}{2}\right)}$$

$$= \frac{\pi}{4} - \tan^{-1}\left(\tan\left(\frac{\pi}{4}-\frac{x}{2}\right)\right)$$

$$= \cancel{\frac{\pi}{4}} - \cancel{\frac{\pi}{4}} + \frac{x}{2}$$

$$= \frac{x}{2}$$ Ans

Page (3)

Page No.

Date :

Class I - 3

SOLVE

3 Solve $\tan^{-1}(2x) + \tan^{-1}(3x) = \frac{\pi}{4}$

$\Rightarrow \tan^{-1}\left(\frac{2x+3x}{1-(2x)(3x)}\right) = \frac{\pi}{4}$

condition $\boxed{xy < 1}$

$\Rightarrow \tan^{-1}\left(\frac{5x}{1-6x^2}\right) = \frac{\pi}{4}$

$\Rightarrow \frac{5x}{1-6x^2} = \tan\left(\frac{\pi}{4}\right)$

$\Rightarrow \frac{5x}{1-6x^2} = 1$

$\Rightarrow 5x = 1-6x^2$

$\Rightarrow 6x^2 + 5x - 1 = 0$

$\Rightarrow 6x^2 + 6x - x - 1 = 0$

$\Rightarrow 6x(x+1) - 1(x+1) = 0$

$\Rightarrow (x+1)(6x-1) = 0$

$\Rightarrow x = -1$ (or) $x = 1/6$

Check $(2x)(3x) = 6x^2$

when $x = -1$ $\Rightarrow 6(-1)^2 = 6 > 1$ (x)

when $x = 1/6$ $\Rightarrow 6\left(\frac{1}{6}\right)^2 = \frac{1}{6} < 1$ ✓

$\boxed{\text{Solution } x = 1/6}$ Ans.

Class I-3

Page No.

Date :

Que 4 → Solve $2\tan^{-1}(\cos x) = \tan^{-1}(2\operatorname{cosec} x)$

$\Rightarrow \tan^{-1}\left(\frac{2\cos x}{1-\cos^2 x}\right) = \tan^{-1}(2\operatorname{cosec} x)$

$\Rightarrow \tan^{-1}\left(\frac{2\cos x}{\sin^2 x}\right) = \tan^{-1}(2\operatorname{cosec} x)$

$\Rightarrow \frac{2\cos x}{\sin^2 x} = 2\operatorname{cosec} x$

$\Rightarrow \cos x = \operatorname{cosec} x \times \sin^2 x$

$\Rightarrow \cos x = \sin x$

$\Rightarrow \frac{\sin x}{\cos x} = 1$

$\tan x = 1$

$\boxed{x = \frac{\pi}{4}}$ Ans

Que 5 → Solve $\sin^{-1}(1-x) - 2\sin^{-1}x = \frac{\pi}{2}$
Special

$\Rightarrow \sin^{-1}(1-x) = \frac{\pi}{2} + 2\sin^{-1}x$

$\Rightarrow 1-x = \sin\left(\frac{\pi}{2} + 2\sin^{-1}x\right)$

$\Rightarrow 1-x = \cos(2\sin^{-1}x)$

$\Rightarrow 1-x = 1 - 2\sin^2(\sin^{-1}x)$ ... $\{\cos(2\theta) = 1-2\sin^2\theta\}$

$\Rightarrow 1-x = 1 - 2(\sin(\sin^{-1}x))^2$ ... $\{\sin^2\theta = (\sin\theta)^2\}$

$1-x = 1-2x^2$

$\Rightarrow 2x^2 - x = 0$

class XII - 3

$\Rightarrow x(2x-1)=0$

$\rightarrow x=0$ or $x=1/2$

Check
~~On Lt~~ put $x=0$

$\sin^{-1}(1-0) - 2\sin^{-1}(0)$

$= \frac{\pi}{2} - 0$

$= \pi/2 = \text{R.H.S}$ ✓

When $x=1/2$ $\quad \sin^{-1}\left(1-\frac{1}{2}\right) - 2\sin^{-1}\left(\frac{1}{2}\right)$

$= \sin^{-1}(1/2) - 2\times\frac{\pi}{6}$

$= \frac{\pi}{6} - \frac{\pi}{3} = -\frac{\pi}{6} \neq \frac{\pi}{2}$ ✗

$\therefore \boxed{x=0}$ solution

Qn6 → Solve $\tan^{-1}(x-1) + \tan^{-1}x + \tan^{-1}(x+1) = \tan^{-1}(3x)$

hint

Qn7 → If $y = \cot^{-1}(\sqrt{\cos x}) - \tan^{-1}(\sqrt{\cos x})$
Show that $\sin y = \tan^2\left(\frac{x}{2}\right)$

Sol $\quad y = \frac{\pi}{2} - \tan^{-1}(\sqrt{\cos x}) - \tan^{-1}\sqrt{\cos x}$

$y = \frac{\pi}{2} - 2\tan^{-1}\sqrt{\cos x}$

$y = \frac{\pi}{2} - \cos^{-1}\left(\frac{1-\cos x}{1+\cos x}\right) \rightarrow \left\{2\tan^{-1}x = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)\right.$

Class 12-3

$$y = \frac{\pi}{2} - \cancel{\tan}\cos^{-1}\left(\frac{2\sin^2(x/2)}{2\cos^2(x/2)}\right)$$

$$y = \boxed{\frac{\pi}{2} - \cos^{-1}\left(\tan^2(x/2)\right)}$$

$$y = \sin^{-1}\left(\tan^2(x/2)\right) \qquad \left\{\sin^{-1}x + \cos^{-1}y = \pi/2\right\}$$

$$\sin y = \tan^2(x/2) \quad \text{Ans}$$

Qn 8 → Show that

$$\tan\left(\frac{\pi}{4} + \frac{1}{2}\cos^{-1}\left(\frac{a}{b}\right)\right) + \tan\left(\frac{\pi}{4} - \frac{1}{2}\cos^{-1}\left(\frac{a}{b}\right)\right) = \frac{2b}{a}$$

Sol

$$\text{let } \frac{1}{2}\cos^{-1}\left(\frac{a}{b}\right) = x$$

$$= \tan\left(\frac{\pi}{4} + x\right) + \tan\left(\frac{\pi}{4} - x\right)$$

$$= \frac{1+\tan x}{1-\tan x} + \frac{1-\tan x}{1+\tan x}$$

LCM

$$= \frac{2(1+\tan^2 x)}{1-\tan^2 x} \quad \cdots \left\{\cos(2\theta) = \frac{1-\tan^2\theta}{1+\tan^2\theta}\right\}$$

$$= \frac{2}{\cos(2x)}$$

$$= \frac{2}{\cos\left(2\times\frac{1}{2}\cos^{-1}\frac{a}{b}\right)} = \frac{2}{\cos\left(\cos^{-1}\frac{a}{b}\right)} = \frac{2}{\frac{a}{b}} = \frac{2b}{a}$$

Page No.

Date :

WORKSHEET

(I-3)

Qn 1 → Simplify $\tan^{-1}\left(\dfrac{\sqrt{1+\cos x}+\sqrt{1-\cos x}}{\sqrt{1+\cos x}-\sqrt{1-\cos x}}\right)$

Qn 2 → Simplify $\cot^{-1}\left(\dfrac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right)$

Qn 3 → ~~Simplify~~

Qn 3 Simplify $\tan^{-1}\left(\dfrac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right)$

Qn 4 → Simplify $\tan^{-1}\left(\dfrac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}\right)$

Qn 5 → If $\tan^{-1}\left(\dfrac{\sqrt{1+x^2}-\sqrt{1-x^2}}{\sqrt{1+x^2}+\sqrt{1-x^2}}\right)=\alpha$

then prove that $x^2=\sin(2\alpha)$

Qn 6 → Solve $\tan^{-1}(2x)+\tan^{-1}(3x)=\dfrac{\pi}{4}$

Qn 7 Solve $\tan^{-1}\left(\dfrac{x-1}{x-2}\right)+\tan^{-1}\left(\dfrac{x+1}{x+2}\right)=\dfrac{\pi}{4}$

Qn 8 Solve $\tan^{-1}\left(\dfrac{x-1}{x+1}\right)+\tan^{-1}\left(\dfrac{2x-1}{2x+1}\right)=\tan^{-1}\left(\dfrac{23}{36}\right)$

Qn 9 → Solve $2\tan^{-1}(\sin x)=\tan^{-1}(2\sec x)$

Page No.

Date :

Qn 10 → Solve $\sin^{-1}(1-x) - 2\sin^{-1}x = \frac{\pi}{2}$

Qn 11 → Solve $\tan^{-1}(x-1) + \tan^{-1}x + \tan^{-1}(x+1) = \tan^{-1}(3x)$

Qn 12 → Solve $\tan^{-1}(x+1) + \tan^{-1}(x-1) = \tan^{-1}\left(\frac{8}{31}\right)$

Qn 13 → find value of $\tan^{-1}\left(\frac{x}{y}\right) - \tan^{-1}\left(\frac{x-y}{x+y}\right)$

Qn 14 → Show that

$$\tan\left(\frac{\pi}{4} + \frac{1}{2}\cos^{-1}\left(\frac{a}{b}\right)\right) + \tan\left(\frac{\pi}{4} - \frac{1}{2}\cos^{-1}\left(\frac{a}{b}\right)\right) = \frac{2b}{a}$$

Qn 15 → If $y = \cot^{-1}(\sqrt{\cos x}) - \tan^{-1}(\sqrt{\cos x})$

Show that $\sin y = \tan^2(x/2)$

Qn 16 → Show that $\sin\left[\cot^{-1}\left(\cos\left(\tan^{-1}x\right)\right)\right] = \sqrt{\frac{x^2+1}{x^2+2}}$

(HINT) use Conversion

Qn 17 → Show that

$$\cos\left[\tan^{-1}\left(\sin\left(\cot^{-1}x\right)\right)\right] = \sqrt{\frac{x^2+1}{x^2+2}}$$

HINT

use conversion

Page No.
Date :

ANSWERS (I-3)

$\frac{\pi}{4} + \frac{x}{2}$

$\frac{x}{2}$

$\frac{\pi}{4} - \frac{1}{2}\cos^{-1}x$

$\frac{\pi}{4} + \frac{1}{2}\cos^{-1}(x^2)$

$x = 1/6$

$x = \pm\frac{1}{\sqrt{2}}$

$x = -3/8$

$x = \pi/4$

$x = 0$

$x = 0, \ 1/2, \ -1/2$

$x = 1/4$

$\pi/4$