।। जय श्री राधे कृष्णा।। जय श्री गिरिराज जी महाराज ।।

ULTIMATE MATHEMATICS: BY AJAY MITTAL

REVISION : INTEGRATION $\boxed{\text{CLASS NO: 3}}$

Que 1 → If $I_n = \int_0^{\pi/2} x^n \cdot \sin x\, dx$

then evaluate $I_{10} + 90 I_8$

Soln

$$I_{10} = \int_0^{\pi/2} x^{10} \cdot \sin x \, dx$$

$$I_8 = \int_0^{\pi/2} x^8 \cdot \sin x \, dx \quad \ldots (1)$$

we have $I_{10} = \int_0^{\pi/2} \underset{I}{x^{10}} \cdot \underset{II}{\sin x}\, dx$

$$= \left(-x^{10} \cdot \cos x\right)_0^{\pi/2} + 10\int_0^{\pi/2} \underset{I}{x^9} \cdot \underset{II}{\cos x}\, dx$$

$$= [0 - (0)] + 10\left[\left(x^9 \sin x\right)_0^{\pi/2} - 9\int_0^{\pi/2} x^8 \sin x\, dx\right]$$

$$= 10 \cdot \left(x^9 \cdot \sin x\right)_0^{\pi/2} - 90\int_0^{\pi/2} x^8 \cdot \sin x\, dx$$

$$I_{10} = 10\left[\left(\left(\tfrac{\pi}{2}\right)^9\right) - (0)\right] - 90 I_8 \quad \ldots \{\text{from (1)}\}$$

$$\boxed{I_{10} + 90 I_8 = 10 \cdot \left(\tfrac{\pi}{2}\right)^9} \text{ Ans}$$

—x—

Que 2 $I = \int_0^3 [x]\, dx$

Placed

Soln $I = \int_0^1 0\, dx + \int_1^2 1\, dx + \int_2^3 2\, dx$

Ans = 3

Ques 3 → $I = \int_0^{2.2} [x^2]\,dx$ 2

Sol. $I = \int_0^1 0\,dx + \int_1^{\sqrt{2}} 1\,dx + \int_{\sqrt{2}}^{\sqrt{3}} 2\,dx + \int_{\sqrt{3}}^{2} 3\,dx + \int_2^{2.2} 4\,dx$

Ans.

Q. 4 $I = \int_1^2 [3x]\,dx$

Sol. $I = \int_1^{4/3} 3\,dx + \int_{4/3}^{5/3} 4\,dx + \int_{5/3}^{2} 5\,dx$

Key 4
$3x = 4$
$x = 4/3$

Proceed

Ans = 4

Qn 5 → $I = \int_{1/e}^{e} |\log x|\,dx$ (1)

Sol. $I = -\int_{1/e}^{1} \log x\,dx + \int_1^e \log x\,dx$

$= -(x\log x - x)_{1/e}^{1} + (x\log x - x)_1^e$

Proceed Ans $2 - \frac{2}{e}$

Qn. 6 $I = \int_0^{2\pi} |\sin x|\,dx$ (A)

Sol. $I = \int_0^{\pi} \sin x\,dx - \int_{\pi}^{2\pi} \sin x\,dx$

Proceed Ans = 4

7 Qn $I = \int_0^{\pi} |\cos(2x)|\,dx$

Soln
Critical point
$2x = \pi/2 \Rightarrow x = \pi/4$
$2x = 3\pi/2 \Rightarrow x = 3\pi/4$

$I = \int_0^{\pi/4} \cos(2x)\,dx - \int_{\pi/4}^{3\pi/4} \cos(2x)\,dx + \int_{3\pi/4}^{\pi} \cos(2x)\,dx$

proceed - Ans = 2

Qn. 8 ⇒ $I = \int_0^{\pi/2} |\sin x - \cos x|\,dx$

↳ Critical point
$\sin x - \cos x = 0$
$\sin x = \cos x \Rightarrow \tan x = 1$
$x = \pi/4$

Soln $I = \int_0^{\pi/4} |\sin x - \cos x|\,dx + \int_{\pi/4}^{\pi/2} |\sin x - \cos x|\,dx$

$= -\int_0^{\pi/4} (\sin x - \cos x)\,dx + \int_{\pi/4}^{\pi/2} (\sin x - \cos x)\,dx$

proceed Ans = $2\sqrt{2} - 2$

Qn. 9 ⇒ $I = \int_0^{1} \frac{\log(1+x)}{1+x^2}\,dx$

Soln
put $x = \tan\theta$
$dx = \sec^2\theta\,d\theta$

when $x = 0$; $\theta = 0$
when $x = 1$; $\theta = \pi/4$

$$I = \int_0^{\pi/4} \frac{\log(1+\tan\theta)}{\cancel{1+\tan^2\theta}} \cdot \cancel{\sec^2\theta}\, d\theta$$

$$I = \int_0^{\pi/4} \log(1+\tan\theta) \quad \text{-- (1)}$$

$$I = \int_0^{\pi/4} \log\left(1+\tan\left(\tfrac{\pi}{4}-\theta\right)\right) d\theta \quad \text{-- (P IV)}$$

$$I = \int_0^{\pi/4} \log\left(1+\frac{1-\tan\theta}{1+\tan\theta}\right) d\theta$$

$$I = \int_0^{\pi/4} \log\left(\frac{2}{1+\tan\theta}\right) d\theta \quad \text{- (2)}$$

(1) + (2)

$$2I = \int_0^{\pi/4} \log(2)\, d\theta$$

$$= (\theta \log 2)_0^{\pi/4}$$

$$2I = \frac{\pi}{4}\log 2$$

$$I = \frac{\pi}{8}\log 2 \quad \text{Ans}$$

Qn. 10 → $I = \int_{-2}^{2} \frac{x^2}{1+5^x} dx$ - (1)

Sol. $I = \int_{-2}^{2} \frac{(-x)^2}{1+5^{-x}} dx$ -- $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$

$$I = \int_{-2}^{2} \frac{x^2}{1+5^{-x}} dx$$

$$I = \int_{-2}^{2} \frac{5^x \cdot x^2}{5^x + 1} dx \quad -(2)$$

(1)+(2)

$$2I = \int_{-2}^{2} \frac{x^2 + 5^x \cdot x^2}{1+5^x} dx$$

$$2I = \int_{-2}^{2} \frac{x^2\cancel{(1+5^x)}}{\cancel{1+5^x}} dx$$

$$2I = \left(\frac{x^3}{3}\right)_{-2}^{2}$$

proceed (Ans = 8/3)

Q.No-11 $\quad I = \int_{-\pi/2}^{\pi/2} \frac{\cos x}{1+e^{-x}} dx$ $\quad$ Ans = 1

Q.No-12 * $\quad I = \int_{0}^{\pi/2} \log|\tan x + \cot x| \, dx$

Sol $\quad I = \int_{0}^{\pi/2} \log(\tan x + \cot x) dx \quad \left\{ \begin{array}{l} 0 < x < \pi/2 \\ \tan x + \cot x > 0 \end{array} \right\}$

$$I = \int_{0}^{\pi/2} \log\left(\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}\right) dx$$

$$I = \int_{0}^{\pi/2} \log\left(\frac{1}{\sin x \cos x}\right) dx$$

$$I = -\int_{0}^{\pi/2} \log(\sin x \cdot \cos x) dx$$

(6)

$$I = -\int_0^{\pi/2} \log(\sin x)\,dx - \int_0^{\pi/2} \log(\cos x)\,dx$$

let $I_1 = \int_0^{\pi/2} \log(\sin x)\,dx \quad \cdots (1)$

$I_1 = \int_0^{\pi/2} \log(\cos x)\,dx \quad \cdots (2) \quad \cdots$ P(IV)

(1) + (2)

$$2I_1 = \int_0^{\pi/2} \log(\sin x \cdot \cos x)\,dx$$

$$2I_1 = \int_0^{\pi/2} \log\left(\frac{\sin(2x)}{2}\right)dx$$

$$2I_1 = \int_0^{\pi/2} \log(\sin(2x))\,dx - \int_0^{\pi/2} \log 2\,dx$$

$$2I_1 = \int_0^{\pi/2} \log(\sin(2x))\,dx - (x \log 2)_0^{\pi/2}$$

$$2I_1 = \int_0^{\pi/2} \log(\sin(2x))\,dx - \frac{\pi}{2}\log 2$$

$$2I_1 = I_2 - \frac{\pi}{2}\log 2$$

where $I_2 = \int_0^{\pi/2} \log(\sin(2x))\,dx$

put $2x = t$, $dx = \frac{dt}{2}$ | when $x = 0$, $t = 0$; $x = \pi/2$, $t = \pi$

$$\therefore I_2 = \frac{1}{2}\int_0^{\pi} \log(\sin t)\,dt$$

$$I_2 = \frac{1}{2} \times 2 \int_0^{\pi/2} \log(\sin t)\,dt \quad \cdots \left\{ \int_0^{2a} f(x)\,dx = 2\int_0^a f(x)\,dx \right\}$$

$$I_2 = \int_0^{\pi/2} \log(\sin x)\,dx \quad \cdots \quad (P\ I)$$

$$I_2 = I_1$$

$$2I_1 = I_1 - \cancel{\pi\log}\ \frac{\pi}{2}\log 2$$

$$I_1 = -\frac{\pi}{2}\log 2$$

$$\therefore I = -\left(-\frac{\pi}{2}\log 2\right) - \left(-\frac{\pi}{2}\log 2\right)$$

$$I = \pi\log 2 \quad \text{Ans}$$

Qn. 13 $I = \int_0^{\pi} \frac{x}{1-\cos\alpha.\sin x}\,dx \quad \cdots (1)$

Sol: $I = \int_0^{\pi} \frac{(\pi - x)}{1-\cos\alpha\sin x}\,dx \quad \cdots (2) \quad \cdots (P\ IV)$

(1) + (2)

$$2I = \pi\int_0^{\pi} \frac{1}{1-\cos\alpha\sin x}\,dx$$

$$2I = \pi\int_0^{\pi} \frac{1}{1-\cos\alpha . \frac{2\tan(x/2)}{1+\tan^2(x/2)}}\,dx$$

$$2I = \pi\int_0^{\pi} \frac{\sec^2(x/2)}{1+\tan^2(x/2) - 2\cos\alpha . \tan(x/2)}\,dx$$

put $\tan(x/2) = t$ | $x = 0 \quad t = 0$

$\sec^2(x/2)\, dx = 2dt$ | $x = \pi \quad t = \infty$

$$2/I = 2\pi \int_0^\infty \frac{dt}{t^2 - 2\cos\alpha \cdot t + 1}$$

$$I = \pi \int_0^\infty \frac{1}{(t - \cos\alpha)^2 - \cos^2\alpha + 1} dt$$

$$I = \pi \int_0^\infty \frac{1}{(t - \cos\alpha)^2 + \sin^2\alpha} dt$$

$$= \frac{\pi}{\sin\alpha} \left[\tan^{-1}\left(\frac{t - \cos\alpha}{\sin\alpha} \right) \right]_0^\infty$$

$$= \frac{\pi}{\sin\alpha} \left(\tan^{-1}(\infty) - \tan^{-1}(-\cot\alpha) \right)$$

$$= \frac{\pi}{\sin\alpha} \left(\frac{\pi}{2} + \tan^{-1}\left(\tan\left(\frac{\pi}{2} - \alpha\right)\right) \right)$$

$$= \frac{\pi}{\sin\alpha} \left(\frac{\pi}{2} + \frac{\pi}{2} - \alpha \right)$$

$$\boxed{I = \frac{\pi(\pi - \alpha)}{\sin\alpha}}$$ Ans

Scanned with CamScanner

Qn 14 $I = \int_{-\pi}^{\pi} (\cos(ax) - \sin(bx))^2 \, dx$

Sol:

$$I = \int_{-\pi}^{\pi} \underbrace{\cos^2(ax)}_{\text{even}} + \underbrace{\sin^2(bx)}_{\text{even}} - 2 \underbrace{\sin(bx)\cos(ax)}_{\text{odd}} \, dx$$

$$= \int_{-\pi}^{\pi} \frac{1+\cos(2ax)}{2} + \frac{1-\cos(2bx)}{2} - \{\sin(b+a)x + \sin(b-a)x\} \, dx$$

$$= \frac{1}{2}\int_{-\pi}^{\pi} 2 + \cos(2ax) - \cos(2bx) - 2\sin(b+a)x - 2\sin(b-a)x \, dx$$

don't do this method (lengthy)

$$= \frac{1}{2}\left[2x + \frac{\sin(2ax)}{2a} - \frac{\sin(2bx)}{2b} + \frac{2\cos(b+a)x}{b+a} + \frac{2\cos(b-a)x}{b-a} \right]_{-\pi}^{\pi}$$

$$= \frac{1}{2}\left[2\pi + \frac{\sin(2a\pi)}{2a} - \frac{\sin(2b\pi)}{2b} \cdots \right.$$

Stop

OR

DO this

$$I = \int_{-\pi}^{\pi} \underbrace{\cos^2(ax)\,dx}_{\text{even}} + \int_{-\pi}^{\pi} \underbrace{\sin^2(ax)\,dx}_{\text{even}} - 2\int_{-\pi}^{\pi} \underbrace{\sin(bx)\cos(ax)\,dx}_{\text{odd}}$$

$$I = 2\int_{0}^{\pi} \cos^2(ax)\,dx + 2\int_{0}^{\pi} \sin^2(ax)\,dx - 0$$

Proceed Ans $\boxed{2\pi + \frac{\sin(2a\pi)}{2a} - \frac{\sin(2b\pi)}{2b}}$

Q.No. 15 → $I = \int_0^a \sin^{-1}\sqrt{\frac{x}{a+x}}\, dx$

Sol → put $x = a\tan^2\theta$ | $x=0,\ \theta=0$

$dx = 2a\tan\theta\cdot\sec^2\theta\, d\theta$ | $x=a,\ \theta=\pi/4$.

$I = 2a\int_0^{\pi/4} \sin^{-1}\sqrt{\frac{a\tan^2\theta}{a+a\tan^2\theta}}\cdot \tan\theta\cdot\sec^2\theta\, d\theta$

$= 2a\int_0^{\pi/4} \sin^{-1}\left(\sqrt{\sin^2\theta}\right)\cdot \tan\theta\ \sec^2\theta\, d\theta$

$= 2a\int_0^{\pi/4} \theta\cdot\tan\theta\ \cancel{\sec}\ \sec^2\theta\, d\theta$

$I = 2a\int_0^{\pi/4} \underbrace{\theta}_{I}\cdot\ \underbrace{\tan\theta\ \sec^2\theta}_{II}\, d\theta$

$I = 2a\left[\left(\theta\,\frac{\tan^2\theta}{2}\right)_0^{\pi/4} - \frac{1}{2}\int_0^{\pi/4} 1.\tan^2\theta\, d\theta\right]$

$---\left\{\int \tan\theta\sec^2\theta\, d\theta\right.$ put $\tan\theta = t$ $\sec^2\theta\, d\theta = t$

$\left.\Rightarrow \int t\, dt = \frac{t^2}{2} = \frac{\tan^2\theta}{2}\right\}$

$I = 2a\left[\left(\frac{\pi}{4}\times\frac{1}{2} - 0\right) - \frac{1}{2}\int_0^{\pi/4} \sec^2\theta - 1\, d\theta\right]$

$= 2a\left(\frac{\pi}{8} - \frac{1}{2}\left(\tan\theta - \theta\right)_0^{\pi/4}\right)$

Ans

$= 2a\left(\frac{\pi}{8} - \frac{1}{2}\left(1 - \frac{\pi}{4}\right)\right) = a\left[\frac{\pi}{4} - 1 + \frac{\pi}{4}\right] = a\left(\frac{\pi}{2} - 1\right)$

Q.16 → $I = \int_{\pi/3}^{\pi/2} \frac{\sqrt{1+\cos x}}{(1-\cos x)^{5/2}}\, dx$

Sol. $I = \int_{\pi/3}^{\pi/2} \frac{\sqrt{2\cos^2(x/2)}}{\left(2\sin^2(x/2)\right)^{5/2}}\, dx$

$= \int_{\pi/3}^{\pi/2} \frac{\sqrt{2}\cos(x/2)}{4\sqrt{2}\sin^5(x/2)}\, dx$

$= \frac{1}{4}\int_{\pi/3}^{\pi/2} \frac{\cos(x/2)\, dx}{\sin^5(x/2)}$

put $\sin(x/2) = t$ | $x = \pi/3$; $t = 1/2$

$\cos(x/2)dx = 2dt$ | $x = \pi/2$; $t = 1/\sqrt{2}$

$I = \frac{1}{4} \times 2 \int_{1/2}^{1/\sqrt{2}} \frac{dt}{t^5}$ Proceed

Ans $= 3/2$

Q.17 → $I = \int_0^{1/2} \frac{dx}{(1+x^2)\sqrt{1-x^2}}$

Sol. put $x = \sin\theta$ | $x = 0$, $\theta = 0$

$dx = \cos\theta\, d\theta$ | $x = 1/2$; $\theta = \pi/6$

$I = \int_0^{\pi/6} \frac{\cos\theta\, d\theta}{(1+\sin^2\theta) \cdot \cos\theta}$

Scanned with CamScanner

$$I = \int_0^{\pi/6} \frac{1}{1+\sin^2\theta}\, d\theta$$

divide by $\cos^2\theta$

Proceed Ans $\frac{1}{\sqrt{2}} \tan^{-1}\left(\sqrt{\frac{2}{3}}\right)$

Q.No. 18 → $I = \int_0^{\pi/2} \frac{\sin^2 x}{\sin x + \cos x}\, dx$ — (1)

$$I = \int_0^{\pi/2} \frac{\cos^2 x}{\cos x + \sin x}\, dx \quad \text{--(2)} \quad \text{-- (P IV)}$$

$$2I = \int_0^{\pi/2} \frac{1}{\sin x + \cos x}\, dx$$

Proceed Ans $-\frac{1}{\sqrt{2}} \log(\sqrt{2}-1)$ Ans

Q.No. 19 → $I = \int_0^1 \underset{\mathrm{II}}{x} \underset{\mathrm{I}}{(\tan^{-1}x)^2}\, dx$

$$I = \left((\tan^{-1}x)^2 \cdot \frac{x^2}{2}\right)_0^1 - \frac{2}{2}\int_0^1 \frac{\tan^{-1}x}{1+x^2} \cdot x^2\, dx$$

put $\tan^{-1}x = t$ | $x=0$; $t=0$

$\frac{1}{1+x^2}dx = dt$ | $x=1$; $t=\pi/4$

$$I = \left(\frac{\pi^2}{16}\cdot\frac{1}{2} - 0\right) - \int_0^{\pi/4} t \tan^2 t\, dt$$

$$I = \frac{\pi^2}{32} - \int_0^{\pi/4} t\cdot(\sec^2 t - 1)\, dt$$

Proceed Ans $\frac{\pi^2}{16} - \frac{\pi}{4} + \frac{1}{2}\log 2$

Scanned with CamScanner

Qn. 20 → $I = \int_{-1}^{1} \frac{x^3 + |x| + 1}{x^2 + 2|x| + 1}\, dx$

Sol.

$I = \cancel{\int_{-1}^{0} \frac{x^3 - x + 1}{x^2 - 2x + 1}} \quad \cancel{\int_{0}^{1}}$

Sol

$I = \int_{-1}^{1} \frac{x^3}{x^2 + 2|x| + 1}\, dx + \int_{-1}^{1} \frac{|x| + 1}{x^2 + 2|x| + 1}$

$f(x) = \frac{x^3}{x^2 + 2|x| + 1}$

$f(-x) = \frac{-x^3}{x^2 + 2|x| + 1} = -f(x)$

$\therefore$ odd

even

$g(x) = \frac{|x| + 1}{x^2 + 2|x| + 1}$

$g(-x) = g(x)$

even

$\therefore \ I = 0 + 2\int_{0}^{1} \frac{|x| + 1}{x^2 + 2|x| + 1}\, dx$

$I = 2\int_{0}^{1} \frac{x + 1}{x^2 + 2x + 1}\, dx$

$= 2\int_{0}^{1} \frac{x + 1}{(x + 1)^2}\, dx$

$= \cancel{2(\ldots)} \quad 2\left(\log|x + 1|\right)_0^1$

$= 2(\log 2)$

$I = 2\log 2$ A

Q.No. 21 → $I = \int_{-2}^{2} |x\cos(\pi x)|\,dx$

$f(x) = |x\cos(\pi x)|$ $\qquad |-2| = 2$

$f(-x) = |-x\cos(-\pi x)|$ $\qquad |-x| = |x|$

$= |-x\cos(\pi x)|$

$= |x\cos(\pi x)|$

$= f(x)$ → even

$I = 2\int_{0}^{2} |x\cos(\pi x)|\,dx$

$0 < x < 1/2 \Rightarrow 0 < \pi x < \pi/2 \Rightarrow |\overset{(+)}{x}\,\overset{(+)}{\cos(\pi x)}| = x\cos(\pi x)$

$1/2 < x < 1 \Rightarrow \pi/2 < \pi x < \pi \Rightarrow |\overset{(+)}{x}\,\overset{(-)}{\cos(\pi x)}| = -x\cos(\pi x)$

$1 < x < 3/2 \Rightarrow \pi < \pi x < 3\pi/2 \rightarrow |\overset{(+)}{x}\,\overset{(-)}{\cos(\pi x)}|$

$= -x\cos(\pi x)$

$3/2 < x < 2 \Rightarrow 3\pi/2 < \pi x < 2\pi$

$\Rightarrow |x\cos(\pi x)| = x\cos(\pi x)$

$$\Rightarrow I = 2\left[\int_0^{1/2} x\cos(\pi x)\,dx - \int_{1/2}^{3/2} x\cos(\pi x)\,dx + \int_{3/2}^{2} x\cos(\pi x)\,dx\right]$$

$$I_1 = \int \underset{I}{x}\,\underset{II}{\cos(\pi x)}\,dx$$

Proceed

Ans $= \frac{8}{\pi}$

Qn. 22 → If $x = \int_0^y \frac{dt}{\sqrt{1+9t^2}}$ and $\frac{d^2y}{dx^2} = ay$

find value of a

Soln

$$x = \frac{1}{3}\int_0^y \frac{1}{\sqrt{\left(\frac{1}{3}\right)^2 + t^2}}\,dt$$

$$x = \frac{1}{3}\left(\log\left|t + \sqrt{\tfrac{1}{9}+t^2}\right|\right)_0^y$$

$$x = \frac{1}{3}\left[\log\left|y + \sqrt{\tfrac{1}{9}+y^2}\right| - \log\left|\tfrac{1}{3}\right|\right]$$

$$x = \frac{1}{3}\left(\log\left|y + \sqrt{\tfrac{1}{9}+y^2}\right| - \frac{1}{3}\log\left(\tfrac{1}{3}\right)\right)$$

Diff w.r.t y

$$\frac{dx}{dy} = \frac{1}{3}\cdot\frac{1}{y+\sqrt{\frac{1}{9}+y^2}}\cdot\left(1 + \frac{1}{\cancel{2}\sqrt{\frac{1}{9}+y^2}}\,\cancel{2}y\right) - 0$$

$$\frac{dx}{dy} = \frac{1}{3}\cdot\frac{1}{\cancel{y+\sqrt{\frac{1}{9}+y^2}}}\left(\frac{\cancel{\sqrt{\frac{1}{9}+y^2}+y}}{\sqrt{\frac{1}{9}+y^2}}\right)$$

$$\frac{dx}{dy} = \frac{1}{3}\cdot\frac{1}{\sqrt{\frac{1}{9}+y^2}}$$

$$\Rightarrow \frac{dy}{dx} = 3\sqrt{\frac{1}{9}+y^2}$$

Diff w.r.t x

$$\frac{d^2y}{dx^2} = 3\cdot\frac{1}{\cancel{2}\sqrt{\frac{1}{9}+y^2}}\cdot \cancel{2}y\cdot\frac{dy}{dx}$$

$$= 3\cdot\frac{1}{\cancel{\sqrt{\frac{1}{9}+y^2}}}\cdot 3y\cancel{\sqrt{\frac{1}{9}+y^2}}$$

$$\frac{d^2y}{dx^2} = 9y = ay \;\ldots\text{(given)}$$

$$\Rightarrow a=9 \quad \text{Ans}$$

Qn. 23 → $I = \int_0^{\pi/2} \frac{\tan x}{1+m^2\tan^2 x}\,dx$

Sol $I = \int_0^{\pi/2} \frac{\frac{\sin x}{\cos x}}{1+m^2\cdot\frac{\sin^2 x}{\cos^2 x}}\,dx$

$$I = \int_0^{\pi/2} \frac{\sin x\cdot\cos x}{\cos^2 x+m^2\sin^2 x}\,dx$$

$$I = \frac{1}{2}\int_0^{\pi/2} \frac{\sin(2x)\,dx}{\cos^2 x+m^2\sin^2 x}$$

put $\cos^2 x + m^2\sin^2 x = t$

$-\sin(2x) + m^2\sin(2x)\,dx = dt$

when $x=0$; $t=1$

when $x=\pi/2$ $t=m^2$

$$\sin(2x)(m^2-1)\,dx = dt$$

$$\sin(2x)\,dx = \frac{dt}{m^2-1}$$

$$\therefore \quad I = \frac{1}{2}\frac{1}{m^2-1}\int_1^{m^2}\frac{dt}{t}$$

$$= \frac{1}{2}\frac{1}{m^2-1}\left(\log|t|\right)_1^{m^2}$$

$$= \frac{1}{2}\frac{1}{m^2-1}\cdot\left(\log(m^2)-\log 1\right)$$

$$I = \frac{1}{\cancel{2}}\left(\frac{\cancel{2}\log m}{m^2-1}\right)$$ Ans

$$I = \frac{\log m}{m^2-1}$$

WORKSHEET NO: 3 REVISION: INTEGRATION

Qns. 1 → $\int_1^2 [3x]\,dx$ ANS $= 4$

Qn 2 → $\int_{1/e}^{e} |\log x|\,dx$ Ans $2 - \frac{2}{e}$

Qn 3 → If $I_n = \int_0^{\pi/2} x^n \sin x\,dx$, then find $I_{10} + 90 I_8$ Ans $10\left(\frac{\pi}{2}\right)^9$

Qn 4 → $\int_0^1 \frac{\log(1+x)}{1+x^2}\,dx$ Ans $\frac{\pi}{8}\log 2$

Qn 5 → $\int_{-2}^{2} \frac{x^2}{1+5^x}\,dx$ Ans $8/3$

Qn 6 → $\int_{-1}^{1} \frac{x^3+|x|+1}{x^2+2|x|+1}\,dx$ Ans $2\log 2$

Qn 7 → $\int_{-\pi/2}^{\pi/2} \frac{\cos x}{1+e^{-x}}\,dx$ Ans $= 1$

Qn. 8 → $\int_0^{\pi/2} |\sin x - \cos x|\,dx$ Ans $2\sqrt{2} - 2$

Qn 9 → $\int_0^{\pi} |\cos(2x)|\,dx$ Ans $= 2$

Qn 10 → $\int_0^a \sin^{-1}\sqrt{\frac{x}{a+x}}\,dx$ Ans $a\left(\frac{\pi}{2} - 1\right)$

Qn 11 → $\int_{-\pi}^{\pi} (\cos(ax) - \sin(bx))^2\,dx$

Ans $2\pi + \frac{\sin(2a\pi)}{2a} - \frac{\sin(2b\pi)}{2b}$

Qn. 12 → $I = \int_0^{\pi} \frac{x}{1-\cos\alpha \cdot \sin x}\,dx$ Ans $\frac{\pi(\pi-\alpha)}{\sin\alpha}$

Qn. 13 → $\int_0^{\pi/2} \log|\tan x + \cot x|\,dx$ Ans $\pi \log 2$

Qn. 14 → $\int_{\pi/3}^{\pi/2} \frac{\sqrt{1+\cos x}}{(1-\cos x)^{5/2}}\,dx$ Ans $3/2$

Qn. 15 → $\int_{-2}^{2} |x\cos(\pi x)|\,dx$ Ans $= \frac{8}{\pi}$

Qn. 16 → If $x = \int_0^{y} \frac{dt}{\sqrt{1+9t^2}}$ and $\frac{d^2y}{dx^2} = ay$
find value of a Ans $a = 9$

Qn. 17 → $\int_0^1 x(\tan^{-1}x)^2\,dx$ Ans $\frac{\pi^2}{16} - \frac{\pi}{4} + \frac{1}{2}\log 2$

Qn. 18 → $\int_0^{\pi/2} \frac{\tan x}{1+m^2\tan^2 x}\,dx$ Ans $\frac{\log m}{m^2-1}$

Qn. 19 → $\int_0^{1/2} \frac{dx}{(1+x^2)\sqrt{1-x^2}}$ Ans $\frac{1}{\sqrt{2}}\tan^{-1}\left(\sqrt{\frac{2}{3}}\right)$

Qn. 20 → $\int_2^8 \frac{\sqrt{10-x}}{\sqrt{x}+\sqrt{10-x}}\,dx$ Ans $I = 3$

Qn. 21 → $\int_0^{\pi/2} \frac{\sin^2 x}{\sin x + \cos x}\,dx$ Ans $\frac{-1}{\sqrt{2}}\log(\sqrt{2}-1)$

Qn. 22 → $I = \int_{-1}^{2} |x+1| + |x| + |x-1|\,dx$ Ans $\frac{19}{2}$

Qn. 23 → $I = \int_0^1 \frac{1}{e^x+e^{-x}}\,dx$ Ans $\tan^{-1}(e) - \frac{\pi}{4}$

Qn. 24 → $\int_0^{\pi/2} \sqrt{1-\sin(2x)}\, dx$ Ans $2(\sqrt{2}-1)$

Qn. 25 → $\int_0^{\pi} x \log(\sin x)\, dx$ Ans $-\frac{\pi^2}{2}\log 2$

Qn. 26 → $\int_0^{1} x \log(1+2x)\, dx$ Ans $\frac{3\log 3}{8}$

Qn. 27 → $\int_0^{2} [x^2]\, dx$ Ans $5-\sqrt{2}-\sqrt{3}$

Qn 28 → $\int_0^{2} |x^2-3x+2|\, dx$ Ans 1

Qn. 29 → $\int_0^{2\pi} \frac{1}{1+e^{\sin x}}\, dx$ Ans $= \pi$

Qn. 30 → $\int_0^{\infty} \frac{\log x}{1+x^2}\, dx$ Ans $= 0$

Qn 31 → $\cancel{\int_{-\pi/2}^{\pi/2} \frac{\cos x}{1+e^{x}}\, dx}$ $\int_0^{\pi/2} \frac{x \sin x \cos x}{\sin^4 x + \cos^4 x}\, dx$ Ans $= \frac{\pi^2}{16}$

Qn 32 → $\int_{-a}^{a} \sqrt{\frac{a-x}{a+x}}\, dx$ Ans $a\pi$

Qn 33 → $\int_0^{\pi} \frac{x\, dx}{25\sin^2 x + 16\cos^2 x}$ Ans $\frac{\pi^2}{20}$

Qn 34 → $\int_0^{\pi} \frac{x \tan x}{\sec x + \cos x}\, dx$ Ans $\frac{\pi^2}{4}$

Qn. 35 → $\int_0^{3/2} |x \cos(\pi x)|\, dx$ Ans $\frac{5}{2\pi} - \frac{1}{\pi^2}$

Qn. 36 → $\int_0^{\pi/2} \frac{\sin x + \cos x}{3+\sin(2x)}\, dx$ Ans $\frac{1}{4}\log 3$

Qn 37 → $\int_{\pi/4}^{\pi/2} e^{2x}\left(\frac{1-\sin(2x)}{1-\cos(2x)}\right) dx$ Ans $\frac{1}{2}e^{\pi/2}$

Qn 38 → $\int_0^{\pi/4} \frac{\sec x}{1+2\sin^2 x}\,dx$ Ans $\frac{1}{3}\log(\sqrt{2}+1)+\frac{\sqrt{2}\pi}{12}$

HINT $\sec x = \frac{1}{\cos x}$ → then partial fraction → type 4

Qn 39 → $\int_{-1/2}^{1/2} \cos x \log\left(\frac{1+x}{1-x}\right)dx$ Ans = 0

HINT odd function

Qn 40 → $\int_0^{a} \frac{1}{\cancel{x}+\sqrt{a^2-x^2}}\,dx$ Ans $\pi/4$

HINT put $x = a\sin\theta$

Qn 41 → $\int_0^1 \frac{1-x^2}{x^4+x^2+1}$ Ans $\frac{1}{2}(\log 3)$

Qn 42 → $\int_0^1 \frac{24x^3}{(1+x^2)^4}\,dx$ Ans = 1

Qn 43 → $\int_0^a \frac{x^4}{\sqrt{a^2-x^2}}\,dx$ Ans $\frac{3\pi a^4}{16}$

HINT put $x = a\sin\theta$

Qn 44 → $\int_0^{\pi/2} \frac{\cos x\,dx}{(\cos\frac{x}{2}+\sin\frac{x}{2})^3}$ Ans $2-\sqrt{2}$

HINT $\cos x = \cos^2(\frac{x}{2}) - \sin^2(\frac{x}{2})$ → then a^2-b^2 formula

Qn 45 → $\int_0^1 \frac{x\tan^{-1}x}{(1+x^2)^{3/2}}\,dx$ Ans $\frac{4-\pi}{2\sqrt{2}}$

Qn 46 → $\int_0^1 (\cos^{-1}x)^2\,dx$ Ans $\pi-2$

Qn 47 → $\int_0^{\pi/2} \frac{x+\sin x}{1+\cos x}\,dx$ Ans $\pi/2$

Qn 48 → $\int_1^3 \frac{\log x}{(x+1)^2}\,dx$ Ans $\frac{3}{4}\log 3 - \log 2$ HINT By parts

—— x ——