

## EXAM NO: 11

## VECTORS & PROBABILITY

MAX. MARKS: 80

Max. TIME: 150 Min

Reading Time: 10 Min

### PART A

1) ~~Set~~ consists

### SECTION - A

Ques 1 + Write down a unit vector in Y-Z plane, making an angle of  $30^\circ$  with the direction of X-axis

Ques 2 + Find the value of  $x$  for which  $x(i + j + k)$  is a unit vector

Ques 3 + If  $\vec{a}$  &  $\vec{b}$  are two collinear vectors, then which of the following are incorrect?

(a)  $\vec{b} = \lambda \vec{a}$  for some scalar  $\lambda$

(b)  $\vec{a} = \pm \vec{b}$  (c) the respective components of  $\vec{a}$  &  $\vec{b}$  are proportional (d) both the vectors  $\vec{a}$  &  $\vec{b}$  have the same direction but different magnitudes

Ques 4 + ABCD is a parallelogram with AC and BD as diagonals, then  $\vec{AC} - \vec{BD} =$

(a)  $4\vec{AB}$  (b)  $3\vec{AB}$  (c)  $2\vec{AB}$  (d)  $\vec{AB}$

Ques 5 + If three points A, B & C have position vectors

$i + x\hat{j} + 3\hat{k}$ ;  $3\hat{i} + 4\hat{j} + 7\hat{k}$  and  $5\hat{i} - 2\hat{j} - 5\hat{k}$  respectively

are collinear, then  $(x, y) =$

(a)  $(2, -3)$  (b)  $(-2, 3)$  (c)  $(-2, -3)$  (d)  $(2, 3)$

Ques 6  $\rightarrow$  A unit vector  $\vec{r}$  makes  $\frac{\pi}{3}$  &  $\frac{\pi}{2}$  with  $\hat{j}$  &  $\hat{k}$  respectively and an obtuse angle  $\theta$  with  $\hat{i}$ . Find  $\theta$  (2)

Ques 7  $\rightarrow$  If a vector makes angles  $\alpha, \beta, \gamma$  with  $ox, oy$  and  $oz$  respectively, then write the value of  $\sin^2\alpha + \sin^2\beta + \sin^2\gamma$

Ques 8  $\rightarrow$  If  $\vec{p} = 5\hat{i} + \lambda\hat{j} - 3\hat{k}$  and  $\vec{q} = \hat{i} + 3\hat{j} - 5\hat{k}$ , then value of  $\lambda$ , for which  $(\vec{p} + \vec{q})$  &  $(\vec{p} - \vec{q})$  are perpendicular is  
 (a)  $\frac{3}{20}$  (b)  $-\frac{20}{3}$  (c)  $\frac{1}{3}$  (d) none of these

Ques 9  $\rightarrow$  If  $\vec{a} = 2\hat{i} + 2\hat{j} + 3\hat{k}$ ,  $\vec{b} = -\hat{i} + 2\hat{j} + \hat{k}$  and  $\vec{c} = 3\hat{i} + \hat{j}$  such that  $\vec{a} + \lambda\vec{b}$  is perpendicular to  $\vec{c}$ , then value of  $\lambda$  is  
 (a) 8 (b) 10 (c)  $\pm 8$  (d) 12

Ques 10  $\rightarrow$  If  $|\vec{a}| = 13$ ,  $|\vec{b}| = 5$  and  $\vec{a} \cdot \vec{b} = 60$ , then  $|\vec{a} \times \vec{b}| =$

(a) 15 (b)  $\pm 25$  (c) 25 (d) none of these

Ques 11  $\rightarrow$  events A and B are such that  $P(A) = \frac{1}{2}$ ,  $P(B) = \frac{7}{12}$  and  $P(\text{not } A \text{ or not } B) = \frac{11}{4}$ . Which of the following is correct

- (a) Events A & B are independent (b) A & B are dependent
- (c) A & B are mutually exclusive (d) A & B are exhaustive

(3)

Ques 12 \* If  $P(A) = 0.4$ ,  $P(B) = p$ ,  $P(A \cup B) = 0.6$

and A and B are independent events, then value of  $p$  is

- (a)  $\frac{2}{3}$  (b)  $\frac{1}{3}$  (c)  $\frac{3}{4}$  (d) 0.6

Ques 13 \* Given probability distribution table

X :	1	2	3	4
$P(X)$ :	c	$2c$	$4c$	$4c$

then value of  $P(X \leq 2)$  is

- (a) 0.2 (b) 0.9 (c) 0.3 (d) none of these

Ques 14 \* A bag contains 5 brown and 4 white socks.

A man pulls out two socks. The probability that these are of the same ~~different~~ colour is

- (a)  $\frac{5}{108}$  (b)  $\frac{18}{108}$  (c)  $\frac{30}{108}$  (d)  $\frac{48}{108}$

Ques 15 \* Find a unit vector in the direction opposite to  $-\frac{3}{4}\hat{j}$

Ques 16 \* Find the area of the triangle whose two sides are represented by the vectors  $2\hat{i}$  and  $-3\hat{j}$

SECTION - B (case study question)

Ques 17 \* In an office, three employees Vinay, Sonia and Iqbal process incoming copies of a certain form. Vinay processes 50% of the forms. Sonia processes 20%

and Iqbal the remaining 30% of the farms. (4)

Vinay has an error rate of 0.06. Sonia has an error rate of 0.04 and Iqbal has an error rate of 0.03

Answer the following (any four)

(i) The conditional probability that an error is committed in processing given that Sonia processed the form is

- (a) 0.0210 (b) 0.04 (c) 0.47 (d) 0.06

(ii) The probability that Sonia processed the form and committed an error is:

- (a) 0.005 (b) 0.006 (c) 0.008 (d) 0.68

(iii) the total probability of committing an error in processing the form is:

- (a) 0 (b) 0.047 (c) 0.234 (d) 1

(iv) The manager of the company wants to do a quality check. During inspection he selects a form at random from the day's output of processed forms. If the form selected at random has an error, the probability that the form is NOT processed by Vinay is.

- (a) 1 (b)  $30/47$  (c)  $20/47$  (d)  $17/47$

(v) Let A be the event of committing an error in processing the form and let  $E_1$ ,  $E_2$  &  $E_3$  be the events that Vinay, Sonia and Iqbal

(3)

Proceeded for farm - The value of  $\sum_{i=1}^3 P(E_i/A)$  is

- (a) 0 (b) 0.03 (c) 0.06 (d) 1

Q24. 18 → There are three categories of students in a class of 60 students:

A = very hard working students

B = regular but not so hardworking

C = Careless and irregular

It is known that 10 students are in category A, 30 in category B and rest in category C.

It is also found that probability of students of category A, unable to get good marks in the final year examination is 0.002, of category B it is 0.02, and of category C it is 0.20

Answer the following (any four)

(i) If a student selected at random was found to be the one who could not get good marks in the examination, then the probability that this student is of category C is

- (a)  $\frac{20}{231}$  (b)  $\frac{200}{231}$  (c)  $\frac{31}{231}$  (d)  $\frac{21}{231}$

(ii) Assume that a student selected at random was found to be the one who could not get good marks in the examination. The true probability

(8)

that this student is either of category A or  
of category B is

- (a)  $\frac{31}{231}$  (b)  $\frac{200}{231}$  (c)  $\frac{201}{231}$  (d)  $\frac{21}{231}$

(ii) The probability that the student is unable  
to get good marks in the examination is

- (a)  $\frac{231}{300}$  (b)  $\frac{231}{3000}$  (c)  $\frac{770}{1000}$  (d) 0.007

(iii) A student selected at random was found  
to be one who could not get good marks in the  
examination. The probability that this student is  
of category A is

- (a)  $\frac{1}{231}$  (b)  $\frac{200}{231}$  (c)  $\frac{230}{231}$  (d) none of them

(iv) A student selected at random was found  
to be the one who could not get good marks in  
the examination. The probability that this student  
is NOT of category A is

- (a) 0 (b)  $\frac{230}{231}$  (c)  $\frac{21}{231}$  (d) 1

PART - B

SECTION = III

Qn. 19 → Mother, father and son line up at random for a family picture. Find  $P(A|B)$ , if A and B are defined as follows:

A: Son on one end      B: Father is in the middle

Qn. 20 → A bag contains 10 white and 115 black balls. Two balls are drawn without replacement. What is the probability that one is white and ~~other~~ other is black.

Qn. 21 → A refrigerator box ~~contains~~ contains 2 milk chocolates and 4 dark chocolates. Two chocolates are drawn at random without replacement. Find the probability distribution of the number of milk chocolates. ~~What~~ what is the most likely outcome?

Qn. 22 → If  $P(A) = \frac{3}{8}$ ,  $P(B) = \frac{1}{2}$  &  $P(A \cap B) = \frac{1}{4}$

Find  $P(B'|A')$

Qn. 23 → Using vectors find the area of the triangle with vertices  $A(2, 3, 5)$ ,  $B(3, 5, 8)$ ,  $C(2, 7, 8)$

Qn. 24 → If  $\vec{a}, \vec{b}, \vec{c}$  are vectors such that  $\vec{a} \cdot \vec{b} = \vec{a} \cdot \vec{c}$ ;  $\vec{a} \times \vec{b} = \vec{a} \times \vec{c}$ ;  $\vec{a} \neq \vec{0}$ , then

(8)

Show that  $\vec{b} = \vec{c}$

Ques. 25  $\rightarrow$  If  $\vec{a} \times \vec{b} = \vec{c} \times \vec{d}$  and  $\vec{a} \times \vec{c} = \vec{b} \times \vec{d}$

Show that  $\vec{d} - \vec{a}$  is parallel to  $\vec{b} - \vec{c}$

Ques. 26  $\rightarrow$  Show that  $|\vec{a} \times \vec{b}|^2 = \begin{vmatrix} \vec{a} \cdot \vec{a} & \vec{a} \cdot \vec{b} \\ \vec{a} \cdot \vec{b} & \vec{b} \cdot \vec{b} \end{vmatrix}$

Ques. 27  $\rightarrow$  ~~Find~~ Find the values of  $x$  for which the angle between the vectors  $\vec{a} = 2x^2\hat{i} + 4x\hat{j} + \hat{k}$  and  $\vec{b} = \hat{i} - 2\hat{j} + x\hat{k}$  is obtuse

Ques. 28  $\rightarrow$  If  $\vec{a} + \vec{b} + \vec{c} = \vec{0}$ ;  $|\vec{a}| = 3$ ,  $|\vec{b}| = 5$ ,  $|\vec{c}| = 7$   
Find the angle between  $\vec{a}$  &  $\vec{b}$

### SECTION-IV

Ques. 29  $\rightarrow$  If the sum of two unit vectors is a unit vector. Find the magnitude of their difference

Ques. 30  $\rightarrow$  A girl walks 4km towards west, then she walks 3km in a direction  $30^\circ$  east of north and stops. Determine the girl's displacement from her initial point of departure.

Ques. 31  $\rightarrow$  If  $\vec{a} = \hat{i} + 4\hat{j} + 2\hat{k}$ ;  $\vec{b} = 3\hat{i} - 2\hat{j} + 7\hat{k}$   
and  $\vec{c} = 2\hat{i} - \hat{j} + 4\hat{k}$ . Find a vector  $\vec{d}$  which

is perpendicular to both  $\vec{a}$  &  $\vec{b}$  and  $\vec{c} \cdot \vec{d} = 15$  (9)

Qn. 32 → Decompose the vector  $6\hat{i} - 3\hat{j} - 6\hat{k}$  in to two vectors ; one is parallel and other is perpendicular to  $\hat{i} + \hat{j} + \hat{k}$

Qn. 33 → If  $\vec{a}, \vec{b}, \vec{c}$  are three non-zero vectors such that  $\vec{a} \times \vec{b} = \vec{c}$  and  $\vec{b} \times \vec{c} = \vec{a}$ .  
~~Please~~ Please find that  $\vec{a}, \vec{b}, \vec{c}$  are mutually  $\perp$  vectors such that  $|\vec{b}|=1$  and  $|\vec{c}|=|\vec{a}|$

Qn. 34 → A card from a pack of 52 cards is lost. From the remaining cards of the pack, two cards are drawn and found to be both diamonds. Find the probability that the lost card being a diamond.

Qn. 35 → Consider the experiment of tossing a coin. If the coin shows head, toss it again but if it shows tail, then throw a dice. Find the conditional probability of the event that the die shows a number greater than 4, given that there is atleast one tail

SECTION = V

Qn 36  $\rightarrow$  prove that in a  $\triangle ABC$ ,  $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$   
 and  $\cos A = \frac{b^2 + c^2 - a^2}{2bc}$ ; where  $a, b, c$  represent  
 the magnitudes of the sides opposite to the  
 angles  $A, B, C$  respectively

Qn. 37  $\rightarrow$  If  $\vec{a}, \vec{b}, \vec{c}$  are three mutually  $\perp^{\sim}$  vectors  
 of equal magnitudes. Prove that  $\vec{a} + \vec{b} + \vec{c}$   
 is equally inclined with vectors  $\vec{a}, \vec{b}$  &  $\vec{c}$ .  
 Also find the angle

Qn. 38  $\rightarrow$  Bag I contains 3 red & 4 black balls and  
 Bag II contains 4 red and 5 black balls.  
 One ball is transferred from Bag I to bag II  
 and then a ball is drawn from bag II.

- (1) Find the probability that ball so drawn is white
- (2) If the ball so draw is white, then what is the probability that the transferred ball is black

-x-

