|| जय श्री राधे कृष्णा ||

← ULTIMATE MATHEMATICS: BY AJAY MITTAL →

CHAPTER : INTEGRATION

CLASS NO: 3

Type. Sinx & cosx in multiplication with different powers

① $I = \int \sin^3 x \cdot \cos^4 x \, dx$

$= \int \sin^2 x \cdot \cos^4 x \cdot \sin x \, dx$

$= \int (1-\cos^2 x) \cdot \cos^4 x \cdot \sin x \, dx$

put $\cos x = t$

$\sin x \, dx = -dt$

$\therefore I = -\int (1-t^2) \cdot t^4 \, dt$

$= -\int t^4 - t^6 \, dt$

$= -\left[\frac{t^5}{5} - \frac{t^7}{7}\right] + C$

$I = -\left[\frac{\cos^5 x}{5} - \frac{\cos^7 x}{7}\right] + C$ Ans

② $I = \int \sin^3 x \cdot \cos^5 x \, dx$

$= \int \sin^2 x \cdot \cos^5 x \cdot \sin x \, dx$

$= \int (1-\cos^2 x) \cdot \cos^5 x \cdot \sin x \, dx$

put $\cos x = t$

$\sin x = -dt$

Proceed

(3) $I = \int \frac{\sin^9 x}{\cos^4 x}\,dx$

$= \int \frac{\sin^8 x \cdot \sin x\,dx}{\cos^4 x}$

$= \int \frac{(1-\cos^2 x)^4 \cdot \sin x\,dx}{\cos^4 x}$

put $\cos x = t \Rightarrow \sin x\,dx = -dt$

$I = -\int \frac{(1-t^2)^4\,dt}{t^4}$

$= \int \frac{(1+t^4-2t^2)^2}{t^4}\,dt$

$= -\int \frac{1+t^8+4t^4+2t^4-4t^6-4t^2}{t^4}\,dt$

$= -\int t^{-4}+t^4+4+2-4t^2-\frac{4}{t^2}\,dt$

$= -\left[\frac{t^{-3}}{-3}+\frac{t^5}{5}+6t-\frac{4t^3}{3}+\frac{4}{t}\right]+C$

$= -\left[\frac{-1}{3\cos^3 x}+\frac{\cos^5 x}{5}+6\cos x-\frac{4\cos^3 x}{3}+\frac{4}{\cos x}\right]+C$ Ans

Type: $\sin x$ & $\cos x$ in multiplication in denominator (with sum of powers = even) = m

→ Divide N & D by $\cos^m x$

→ put $\tan x = t$ $\quad \sec^2 x\,dx = dt$

(4) $I = \int \frac{1}{\sin^3 x \cdot \cos^5 x} dx$

Divide Nr & Dr by $\cos^8 x$

$= \int \frac{\sec^8 x}{\tan^3 x} dx$

$= \int \frac{\sec^6 x \cdot \sec^2 x dx}{\tan^3 x}$

$= \int \frac{(1+\tan^2 x)^3 \cdot \sec^2 x dx}{\tan^3 x}$

put $\tan x = t \Rightarrow \sec^2 x dx = dt$

$= \int \frac{(1+t^2)^3 \cdot dt}{t^3}$

$= \int \frac{1+t^6+3t^2+3t^4}{t^3} dt$

t^{-1}

$\frac{t^0}{0}$

$= \int t^{-3} + t^3 + \frac{3}{t} + 3t \, dt$

$= \frac{t^{-2}}{-2} + \frac{t^4}{4} + 3\log|t| + \frac{3t^2}{2} + c$

$= -\frac{1}{2\tan^2 x} + \frac{\tan^4 x}{4} + 3\log|\tan x| + \frac{3\tan^2 x}{2} + c$ Ans

(5) $I = \int \sqrt{\sec^3 x \cdot \operatorname{cosec}^5 x} \, dx$

$= \int \frac{1}{\cos^{3/2} x \cdot \sin^{5/2} x} dx$

$$I = \int \frac{1}{\cos^{3/2}x \cdot \sin^{5/2}x}\,dx$$

divide by $\cos^4 x$

$$I = \int \frac{\sec^4 x\,dx}{\tan^{5/2}x}$$

$$= \int \frac{(1+\tan^2 x)\,\sec^2 x}{\tan^{5/2}x}\,dx$$

put $\tan x = t \Rightarrow \sec^2 x\,dx = dt$

$$I = \int \frac{(1+t^2)\,dt}{t^{5/2}}$$

$2 - \frac{5}{2}$

$$= \int t^{-5/2} + t^{-1/2}\,dt$$

$$= \frac{-2}{3}t^{-3/2} + 2t^{1/2} + C$$

$$= \frac{-2}{3}\cdot\frac{1}{\tan^{3/2}x} + 2\sqrt{\tan x} + C \quad \text{Ans}$$

Type $\int \frac{1}{1 \pm \sin x}\,dx$; $\int \frac{1}{1 \pm \cos x}\,dx$; $\int \frac{\sin x}{1 \pm \sin x}\,dx$; $\int \frac{\cos x}{1 \pm \cos x}\,dx$

Rationalize

Ex. 6 $I = \int \frac{1}{1-\sin x}\,dx$

Rationalize $I = \int \frac{1+\sin x}{1-\sin^2 x}\,dx$

$$= \int \frac{1+\sin x}{\cos^2 x}\,dx$$

$$I = \int \sec^2 x + \sec x \tan x \, dx$$

$$I = \tan x + \sec x + C \quad \text{Ans}$$

Q.7 $$I = \int \frac{\tan x}{\sec x + \tan x} \, dx$$

$$= \int \frac{\frac{\sin x}{\cos x}}{\frac{1}{\cos x} + \frac{\sin x}{\cos x}} \, dx$$

$$= \int \frac{\sin x}{1 + \sin x} \, dx$$

Rationally
$$= \int \frac{\sin x \cdot (1 - \sin x)}{1 - \sin^2 x} \, dx$$

$$= \int \frac{\sin x + \sin^2 x}{\cos^2 x} \, dx$$

$$= \int \tan x \cdot \sec x + \tan^2 x \, dx$$

$$= \int \sec x \tan x + \sec^2 x - 1 \, dx$$

$$= \sec x + \tan x - x + C \quad \text{Ans}$$

Type $\sin(A \pm B)$; $\cos(A \pm B)$ formula based

Q.8 $$I = \int \frac{\sin(x + a)}{\sin x} \, dx$$

$$= \int \frac{\sin x \cdot \cos a + \cos x \cdot \sin a}{\sin x} \, dx$$

$$= \int \cos a + \cot x \cdot \sin a \, dx$$

$$= x \cos a + \log|\sin x| \cdot \sin a + C$$

Ans

$\frac{a}{b+c}$ $\frac{a+b}{c}$

Q. 9 $I = \int \frac{\sin x}{\sin(x+\alpha)} dx$

Adjustment

$= \int \frac{\sin(x+\alpha-\alpha)}{\sin(x+\alpha)} dx$

$= \int \frac{\sin(x+\alpha)\cos\alpha - \cos(x+\alpha).\sin\alpha}{\sin(x+\alpha)} dx$

$= \int \cos\alpha - \cot(x+\alpha)\sin\alpha \, dx$

$= x\cos\alpha - \log|\sin(x+\alpha)| \cdot \sin\alpha + C$

Q. 10 $I = \int \frac{\sin(2x)}{\sin(5x)\sin(3x)} dx$ $(5x-3x)$ → formula → separate

Q. 11 $I = \int \frac{1}{\sin(x+a)\sin(x+b)} dx$

$I = \frac{1}{\sin(a-b)} \int \frac{\sin(a-b)}{\sin(x+a).\sin(x+b)} dx$

$= \frac{1}{\sin(a-b)} \int \frac{\sin(a-b+x-x)}{\sin(x+a)\sin(x+b)} dx$

$= \frac{1}{\sin(a-b)} \int \frac{\sin((x+a)-(x+b))}{\sin(x+a)\sin(x+b)} dx$

$= \frac{1}{\sin(a-b)} \int \frac{\sin(x+a)\cdot\cos(x+b) - \cos(x+a)\cdot\sin(x+b)}{\sin(x+a)\cdot\sin(x+b)} dx$

$$I = \int \cot(x+b) - \cot(x+a)\, dx$$

$$= \log|\sin(x+b)| - \log|\sin(x+a)| + C$$

$$I = \log\left|\frac{\sin(x+b)}{\sin(x+a)}\right| + C$$

Qn 12 $I = \int \frac{1}{\sin(x+a)\cdot\cos(x-b)}\, dx$

D & M by $\cos(a+b)$

$$= \frac{1}{\cos(a+b)}\int \frac{\cos(a+b)}{\sin(x+a)\cos(x-b)}\, dx$$

$$= \frac{1}{\cos(a-b)}\int \frac{\cos(a+b+x-x)}{\sin(x+a)\cdot\cos(x-b)}\, dx$$

$$= \frac{1}{\cos(a-b)}\int \frac{\cos((x+a)-(x-b))}{\sin(x+a)\cdot\cos(x-b)}\, dx$$

$$= \frac{1}{\cos(a-b)}\int \frac{\cos(x+a)\cos(x-b) + \sin(x+a)\cdot\sin(x-b)}{\sin(x+a)\cdot\cos(x-b)}\, dx$$

$$= \frac{1}{\cos(a-b)}\int \cot(x+a) + \tan(x-b)\, dx$$

$$= \frac{1}{\cos(a-b)}\left[\log|\sin(x+a)| + \log|\sec(x-b)|\right] + C$$

Type (·) $\int \frac{\text{linear}}{\sqrt{\text{linear}}}\,dx$ (·) $\int \text{linear}\sqrt{\text{linear}}$

$\text{linear} \to t^2$

(·) $\int \frac{\text{linear}}{(\text{linear})^n}$ (·) $\int \text{linear}\,(\text{linear})^n$

$\text{linear} = t$

(·) $\int \frac{\text{Quad}}{\sqrt{\text{linear}}}$ (·) $\int \text{Quad}\sqrt{\text{linear}}$

$\text{linear} = t^2$

(·) $\int$

Q = 13 $I = \int \frac{2x+3}{\sqrt{3x+4}}\,dx$

Put $3x+4 = t^2$

$3dx = 2t\,dt$

$dx = \frac{2}{3}t\,dt$

$\therefore I = \frac{2}{3}\int \frac{2\left(\frac{t^2-4}{3}\right)+3}{\cancel{t}}\cdot\cancel{t}\,dt$

$= \frac{2}{9}\int (2t^2+1)\,dt$

$= \frac{2}{9}\left[2\frac{t^3}{3}+t\right]+C$

$= \frac{2}{9}\left[\frac{2}{3}(3x+4)^{3/2}+(3x+4)^{1/2}\right]+C$ Ans

OR $I = \int \frac{2x+3}{\sqrt{3x+4}}\,dx$

$= 2\int \frac{x+3/2}{\sqrt{3x+4}}\,dx$

$$= \frac{2}{3}\int \frac{3x + 9/2}{\sqrt{3x+4}}\, dx$$

$$= \frac{2}{3}\int \frac{3x + \frac{9}{2} + 4 - 4}{\sqrt{3x+4}}\, dx$$

$$= \frac{2}{3}\int \frac{(3x+4) + 1/2}{\sqrt{3x+4}}\, dx$$

$$= \frac{2}{3}\int \sqrt{3x+4} + \frac{1}{2\sqrt{3x+4}}\, dx$$

$$= \frac{2}{3}\left[\frac{2}{3}\frac{(3x+4)^{3/2}}{3} + \frac{1}{\cancel{2}} \times \frac{\cancel{2}\sqrt{3x+4}}{3}\right] + C$$

$$= \frac{2}{3}\left[\frac{2}{9}(3x+4)^{3/2} + \frac{1}{3}(3x+4)^{1/2}\right] + C$$

Q. 14 $I = \int (x^2+1)\sqrt{3-5x}\, dx$

put $3-5x = t^2$

$-5dx = 2t\,dt \Rightarrow dx = -\frac{2}{5}t\,dt$

$$I = -\frac{2}{5}\int \left(\left(\frac{3-t^2}{5}\right)^2 + 1\right) . t \cdot t\,dt$$

$$= -\frac{2}{5}\int \left(\frac{9+t^4-6t^2}{25} + 1\right) t^2\, dt$$

$$= \frac{-2}{125}\int \cancel{9t^2} + \cancel{t^6}\,(34t^2 + t^6 - 6t^4)\, dt$$

proceed

WORKSHEET NO: 2 (class - 3)

INTEGRATION

Qn 1 $I = \int \frac{1}{\sin^2 x \cdot \cos^2 x} dx$ — (1) Ans $-\frac{1}{\tan x} + \tan x + c$

Qn 2 $I = \int \frac{\sec x}{\sec x + \tan x} dx$ — (2) Ans $\tan x - \sec x + c$

Qn 3 $I = \int \frac{\cot x}{\operatorname{cosec} x - \cot x} dx$ — (3) Ans $-\operatorname{cosec} x - \cot x - x + c$

Qn 4 $I = \int \frac{\operatorname{cosec} x}{\operatorname{cosec} x - \cot x} dx$ — (4) Ans $-\cot x - \operatorname{cosec} x + c$

Qn 5 $I = \int \frac{x+2}{(x+1)^2} dx$ — (5) Ans $\log|x+1| - \frac{1}{x+1} + c$

Qn 6 $I = \int (7x-2)\sqrt{3x+2}\, dx$ — (6) Ans $\frac{2}{9}\left[\frac{7(3x+2)^{5/2}}{5} - \frac{20(3x+2)^{3/2}}{3}\right] + c$

Qn 7 $I = \int \frac{8x+13}{\sqrt{4x+7}} dx$ — (7) Ans $\frac{1}{8}\left[\frac{8(4x+7)^{3/2}}{3} - 4(4x+7)^{1/2}\right] + c$

Qn 8 $I = \int \frac{2x-1}{(x-1)^4} dx$ — (8) Ans $-\frac{1}{(x-1)^2} - \frac{1}{3(x-1)^3} + c$

Qn 9 $I = \int (5x+3)\sqrt{2x-1}\, dx$ — (9) Ans $\frac{1}{2}\left[(2x-1)^{5/2} + \frac{11}{3}(2x-1)^{3/2}\right] + c$

Qn 10 $I = \int (x^2+2)\sqrt{1-2x}\, dx$ — (10) Ans $-\frac{1}{4}\left[3(1-2x)^{3/2} + \frac{(1-2x)^{7/2}}{7} - \frac{2}{5}(1-2x)^{5/2}\right]$

Qn 11 $I = \int \frac{1}{\cos(x+a)\cos(x+b)} dx$ — (11) $\frac{1}{\sin(a-b)}\left[\log|\sec(x+a)| - \log|\sec(x+b)|\right] + c$

Qn 12 $I = \int \frac{\sin(x+a)}{\sin(x+b)} dx$ — (12) Ans $x\cos(a-b) + \log|\sin(x+b)| \cdot \sin(a-b) + c$

Qn 13 $I = \int \frac{1}{\sin(x+a)\cos(x-b)} dx$ — (13) $\frac{1}{\cos(a-b)}\left[\log|\sin(x-a)| + \log|\sin(x-b)|\right] + c$

Qn 14 $I = \int \frac{\sin(2x)}{\sin(x-\pi/3)\sin(x+\pi/3)} dx$ — (14) $\log|\sin(x-\pi/3)| + \log|\sin(x+\pi/3)| + c$

Q.15 $I = \int \tan^3 x \cdot \sec^3 x \, dx$ (15) Ans $\frac{\sec^5 x}{5} - \frac{\sec^3 x}{3} + c$

Q.16 $I = \int \tan^5 x \cdot \sec^4 x \, dx$ (16) $\frac{\tan^6 x}{6} + \frac{\tan^8 x}{8} + c$

Q.17 $I = \int \cot^5 x \cdot \operatorname{cosec}^4 x \, dx$ (17) $-\left(\frac{\cot^6 x}{6} + \frac{\cot^8 x}{8}\right) + c$

Q.18 $I = \int \sec^n x \cdot \tan x \, dx$ (18) Ans $\frac{\sec^n x}{n} + c$

Q.19 $I = \int \frac{1}{\sqrt{\sin^3 x \cdot \cos^5 x}} dx$ (19) $\frac{-2}{\sqrt{\tan x}} + \frac{2}{3}(\tan x)^{3/2} + c$

Q.20 $I = \int \sec^{4/3} x \cdot \operatorname{cosec}^{8/3} x \, dx$ (20) $\frac{-3}{5(\tan x)^{5/3}} + 3(\tan x)^{1/3} + c$

Q.21 $I = \int \frac{1}{\sin x \cdot \cos^5 x} dx$ (21) $\log|\tan x| + \frac{\tan^4 x}{4} + \tan^2 x + c$

Q.22 $I = \int \frac{1}{\sin^3 x \cdot \cos x} dx$ (22) say.

—x—

Scanned with CamScanner