DATE
PAGE

Solutions of D-3

BY: AJAY MITTAL

Qns. 1 → $x+3y=5$ and $2x+6y=8$

Here $A=\begin{bmatrix}1&3\\2&6\end{bmatrix}$; $B=\begin{bmatrix}5\\8\end{bmatrix}$ & $X=\begin{bmatrix}x\\y\end{bmatrix}$

$|A| = 6-6=0$ Now we have check $(A\,adj\,A)\,B$

$Adj\,A=\begin{bmatrix}6&-3\\-2&1\end{bmatrix}$

$(A\,adj\,A)\cdot B=\begin{bmatrix}6&-3\\-2&1\end{bmatrix}\begin{bmatrix}5\\8\end{bmatrix}=\begin{bmatrix}30-24\\-10+8\end{bmatrix}=\begin{bmatrix}6\\-2\end{bmatrix}\neq 0$

∴ System is Inconsistent (Misprint in Worksheet).

Qns 2 → Given equations

$3x-y-2z=2$, $0x+2y-z=-1$, $3x-5y=3$

Here $A=\begin{bmatrix}3&-1&-2\\0&2&-1\\3&-5&0\end{bmatrix}$, $B=\begin{bmatrix}2\\-1\\3\end{bmatrix}$

$|A| = 3(0-5)+1(0+3)-2(0-6)$

$= -15+3+12 = 0$

Now we have check $(Adj\,A)\,B$

$Adj\,A=\begin{bmatrix}-5&10&5\\-3&6&3\\-6&12&6\end{bmatrix}$

$(Adj\,A)\,B=\begin{bmatrix}-5&10&5\\-3&6&3\\-6&12&6\end{bmatrix}\begin{bmatrix}2\\-1\\3\end{bmatrix}=\begin{bmatrix}-5\\-3\\-6\end{bmatrix}\neq 0$

Since $|A|=0$ and $(Adj\,A)\,B\neq 0$ ∴ Inconsistent Ans

Q.No 3 → Given equations

$5x-y+4z=5$; $2x+3y+5z=2$; $5x-2y+6z=-1$

then $A=\begin{bmatrix} 5 & -1 & 4 \\ 2 & 3 & 5 \\ 5 & -2 & 6 \end{bmatrix}$; $B=\begin{pmatrix} 5 \\ 2 \\ -1 \end{pmatrix}$

$|A| = 5(18+10) + 1(12-25) + 4(-4-15)$

$= 140 - 13 - 76$

$= 51 \neq 0$

∴ System is Consistent Ans

Q.No 6 → Given equations in (proper form)

$x-y+2z=7$; $3x+4y-5z=-5$; $2x-y+3z=12$

then equations can be written in the form

$$\begin{bmatrix} 1 & -1 & 2 \\ 3 & 4 & -5 \\ 2 & -1 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 7 \\ -5 \\ 12 \end{bmatrix}$$

(or) $AX=B$

where $A=\begin{bmatrix} 1 & -1 & 2 \\ 3 & 4 & -5 \\ 2 & -1 & 3 \end{bmatrix}$; $X=\begin{pmatrix} x \\ y \\ z \end{pmatrix}$; $B=\begin{pmatrix} 7 \\ -5 \\ 12 \end{pmatrix}$

$|A| = 1(12-5) + 1(9+10) + 2(-3-8)$

$= 7 + 19 - 22$

$= 4 \neq 0$

∴ System is Consistent, have unique solution

$Adj A = \begin{bmatrix} 7 & 1 & -3 \\ -19 & -1 & 11 \\ -11 & -1 & 7 \end{bmatrix}$

Solution (D-3)

DATE
PAGE

$$A^{-1} = \frac{1}{|A|} \cdot \text{Adj } A$$

$$A^{-1} = \frac{1}{4}\begin{bmatrix} 7 & 1 & -3 \\ -19 & -1 & 11 \\ -11 & -1 & 7 \end{bmatrix}$$

we have $AX = B$

$$\Rightarrow X = A^{-1}B$$

$$\Rightarrow X = \frac{1}{4}\begin{bmatrix} 7 & 1 & -3 \\ -19 & -1 & 11 \\ -11 & -1 & 7 \end{bmatrix}\begin{bmatrix} 7 \\ -5 \\ 12 \end{bmatrix}$$

$$= \frac{1}{4}\begin{bmatrix} 49 - 5 - 36 \\ -133 + 5 + 132 \\ -77 + 5 + 84 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 8 \\ 4 \\ 12 \end{bmatrix}$$

$\Rightarrow x = 2,\ y = 1,\ z = 3$ is the required solution Ans.

Qns 4 and Qn 5 → same as Qn no 6 (Do yourself)

Qn 7 → Given equations

$$\frac{2}{x} + \frac{3}{y} + \frac{10}{z} = 4 \ ; \ \frac{4}{x} - \frac{6}{y} + \frac{5}{z} = 1 \ \& \ \frac{6}{x} + \frac{9}{y} - \frac{20}{z} = 2$$

these equations can be written in the form

$$\begin{bmatrix} 2 & 3 & 10 \\ 4 & -6 & 5 \\ 6 & 9 & -20 \end{bmatrix}\begin{bmatrix} 1/x \\ 1/y \\ 1/z \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix}$$

(or) $AX = B$

Solution (Q-3)

$$A = \begin{bmatrix} 2 & 3 & 10 \\ 4 & -6 & 5 \\ 6 & 9 & -20 \end{bmatrix} \quad ; \quad B = \begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix} \quad ; \quad X = \begin{bmatrix} 1/x \\ 1/y \\ 1/z \end{bmatrix}$$

$$|A| = 2(120 - 45) - 3(-80 - 30) + 10(36 + 36)$$

$$= 150 + 330 + 720 = 1200 \neq 0$$

$\therefore$ System is consistent and have unique Solution

$$Adj\,A = \begin{bmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \cdot Adj\,A = \frac{1}{1200} \begin{bmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{bmatrix}$$

$$AX = B$$

$$\Rightarrow X = A^{-1}B$$

$$\Rightarrow \dot{X} = \frac{1}{1200} \begin{bmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{bmatrix} \begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix}$$

$$= \frac{1}{1200} \begin{bmatrix} 300 + 150 + 150 \\ 440 - 100 + 60 \\ 288 + 0 - 48 \end{bmatrix}$$

$$= \frac{1}{1200} \begin{bmatrix} 600 \\ 400 \\ 240 \end{bmatrix}$$

$$\begin{bmatrix} 1/x \\ 1/y \\ 1/z \end{bmatrix} = \begin{bmatrix} 1/2 \\ 1/3 \\ 1/5 \end{bmatrix}$$

$\Rightarrow 1/x = 1/2 \Rightarrow x = 2$

$1/y = 1/3 \Rightarrow y = 3$ Ans

$1/z = 1/5 \Rightarrow z = 5$

(Q3) Solutions

(5)

Qns 8 → Given equations

$2yz - 3xz + 3xy = 10xyz$

$yz + xz + xy = 10xyz$

$3yz - xz + 2xy = 13xyz$

Divide all these equations by xyz

$\frac{2}{x} - \frac{3}{y} + \frac{3}{z} = 10$

$\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 10$

$\frac{3}{x} - \frac{1}{y} + \frac{2}{z} = 13$

Proceed Same as Qns no: 7 (Do yourself)

Qns 9 → Given $A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$

$|A| = 2(-4+4) + 3(-6+4) + 5(3-2)$

$|A| = 0 - 6 + 5 = -1$

$Adj A = \begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix}$

$A^{-1} = -\frac{1}{1}\begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix} = \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix}$

Given equations

$2x - 3y + 5z = 11$; $3x + 2y - 4z = -5$

and $x + y - 2z = -3$

these equations can be written in the form

$$AX = B$$

where $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$; $B = \begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix}$

$\Rightarrow X = A^{-1}B$ → already found

$$\Rightarrow X = \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix} \begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix}$$

$$\Rightarrow X = \begin{bmatrix} 0 - 5 + 6 \\ -22 - 45 + 69 \\ -11 - 25 + 39 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

Ans

$\therefore x=1, \quad y=2, \quad z=3$ is the required solution

---

Qns 10 → let the three numbers are x, y, z

According to question

$$x+y+z = 6 \quad \text{---(i)}$$

$$y+3z = 11 \quad \text{-(2)}$$

$$x+z = 2y \Rightarrow x-2y+z = 0 \quad \text{--(3)}$$

Now Do yourself

Qns 11) given $A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}$

$$|A| = 1(1+3) + 1(2+3) + 1(2-1)$$

(D-3) Solutions

$|A| = 4 + 5 + 1 = 10$

$$\text{Adj } A = \begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix}$$

$$A^{-1} = \frac{1}{10}\begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix}$$

Given equations $x + 2y + z = 4$; $-x + y + z = 0$ and $x - 3y + z = 2$

These equations can be written in the form

$$AX = B \quad \text{--- (Important point: equations ke coefficients se jo matrix banti hai wo transpose of hai A matrix ki)}$$

$$\Rightarrow X = (A')^{-1} B$$

$$\Rightarrow X = (A^{-1})' B$$

$\longrightarrow$ Property $(A')^{-1} = (A^{-1})'$

$$\Rightarrow X = \frac{1}{10}\begin{bmatrix} 4 & -5 & 1 \\ 2 & 0 & -2 \\ 2 & 5 & 3 \end{bmatrix}\begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{10}\begin{bmatrix} 16 + 0 + 2 \\ 8 + 0 - 4 \\ 8 + 0 + 6 \end{bmatrix} = \frac{1}{10}\begin{bmatrix} 18 \\ 4 \\ 14 \end{bmatrix}$$

$\Rightarrow x = 9/5$

$y = 2/5$

$z = 7/5$

Ans

(D-3) Solutions

Qns 12 → let cost of onion is Rs x/kg
cost of wheat is Rs y/kg
cost of Rice is Rs z/kg

According to Question

$$4x + 3y + 2z = 60$$
$$2x + 4y + 6z = 90$$
$$6x + 2y + 3z = 70$$

Now proceed yourself

Qns 13 → let $A = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}$ & let $C = \begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix}$

to find AC

$$AC = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix} \begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix}$$

$$AC = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\boxed{AC = I}$$

↳ from this equation we can find A^{-1}

$A^{-1}AC = A^{-1}I$ (Pre-multiply by A^{-1})

$IC = A^{-1}$

$C = A^{-1}$

$\therefore A^{-1} = \begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix}$

(Q.3) Solutions (9)

Given equations

$x-y+2z=1$; $0x+2y-3z=1$ & $3x-2y+4z=2$

These equations can be written in the form

$AX=B$

$\Rightarrow X=A^{-1}B$ → already found

$$\Rightarrow \begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} -2+0+2 \\ 9+2-6 \\ 6+1-4 \end{bmatrix}$$

$$\begin{bmatrix} \\ \\ \end{bmatrix} = \begin{bmatrix} 0 \\ 5 \\ 3 \end{bmatrix} \Rightarrow x=0, \quad y=5, z=3 \quad \underline{\text{Ans}}$$

---

14. Do yourself (same as Q.No. No: 13)

---

5. Given $A=\begin{bmatrix} 7 & 2 & -6 \\ -2 & 1 & -3 \\ -4 & 2 & 5 \end{bmatrix}$ & $B=\begin{bmatrix} 1 & -2 & 0 \\ 2 & 1 & 3 \\ 0 & -2 & 1 \end{bmatrix}$

To find BA

$$BA=\begin{bmatrix} 1 & -2 & 0 \\ 2 & 1 & 3 \\ 0 & -2 & 1 \end{bmatrix} \begin{bmatrix} 7 & 2 & -6 \\ -2 & 1 & -3 \\ -4 & 2 & 5 \end{bmatrix}$$

$$=\begin{bmatrix} 11 & 0 & 0 \\ 0 & 11 & 0 \\ 0 & 0 & 11 \end{bmatrix}$$

$$=11\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$\boxed{BA=11I}$

(Q-3) Solution (10)

Given equations

$x - 2y = 10$; $2x + y + 3z = 8$ and

$0x - 2y + z = 7$

These equations can be written in the form

$$\begin{bmatrix} 1 & -2 & 0 \\ 2 & 1 & 3 \\ 0 & -2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{pmatrix} 10 \\ 8 \\ 7 \end{pmatrix}$$

$$BX = C$$

where $B = \begin{bmatrix} 1 & -2 & 0 \\ 2 & 1 & 3 \\ 0 & -2 & 1 \end{bmatrix}$; $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$; $C = \begin{bmatrix} 10 \\ 8 \\ 7 \end{bmatrix}$

We already found

$$BA = 11\,I$$

then we have to find B^{-1} (not A^{-1}!)

↳ Remember this

Imp

$$B^{-1}BA = 11\,B^{-1}I \quad \text{--- (premultiply by } B^{-1})$$

$$IA = 11\,B^{-1}$$

$$A = 11\,B^{-1}$$

$$B^{-1} = \frac{1}{11}A$$

$$B^{-1} = \frac{1}{11}\begin{bmatrix} 7 & 2 & -6 \\ -2 & 1 & -3 \\ -4 & 2 & 5 \end{bmatrix}$$

We have $BX = C$

$$X = B^{-1}C$$

$$X = \frac{1}{11}\begin{bmatrix} 7 & 2 & -6 \\ -2 & 1 & -3 \\ -4 & 2 & 5 \end{bmatrix}\begin{bmatrix} 10 \\ 8 \\ 7 \end{bmatrix} = \frac{1}{11}\begin{bmatrix} 70 + 16 - 42 \\ -20 + 8 - 21 \\ -40 + 16 + 35 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ -3 \\ 1 \end{bmatrix} \Rightarrow \begin{bmatrix} x = 4 \\ y = -3 \\ z = 1 \end{bmatrix}$$ Ans