← ULTIMATE MATHEMATICS →

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CHAPTER: INVERSE TRIGO (I-1)

Trigonometry

✓ $\sin x$ = value

↓

angle

eg $\sin(\pi/3) = \frac{\sqrt{3}}{2}$

$\tan x = \frac{\sin x}{\cos x}$

$\operatorname{Cosec} x = \frac{1}{\sin x}$

Inverse Trigonometry

$\sin^{-1} x \neq \frac{1}{\sin x}$

$\sin^{-1} x$ = angle

↓

value

eg $\sin^{-1}(\frac{1}{2}) = \pi/6$

(x) $\tan^{-1} x \neq \frac{\sin^{-1} x}{\cos^{-1} x}$

(x) $\operatorname{cosec}^{-1} x \neq \frac{1}{\sin^{-1} x}$

(X) $\sin^{-1}(\pi/3)$

(P-I) (principal value Branch)

$\sin^{-1}(\sin x) = x$; $x \in [-\frac{\pi}{2}, \frac{\pi}{2}]$ ← Range

$\tan^{-1}(\tan x) = x$; $x \in (-\frac{\pi}{2}, \frac{\pi}{2})$

$\cos^{-1}(\cos x) = x$; $x \in [0, \pi]$ -- $\{\cos\pi = -1\}$

$\operatorname{Cosec}^{-1}(\operatorname{Cosec}(x)) = x$; $x \in [\frac{-\pi}{2}, \frac{\pi}{2}] - \{0\}$

$\sec^{-1}(\sec(x)) = x$; $x \in [0, \pi] - \{\frac{\pi}{2}\}$

$\cot^{-1}(\cot x) = x$, $x \in (0, \pi)$

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P - II

$\sin^{-1}x + \cos^{-1}x = \pi/2$

$\tan^{-1}x + \cot^{-1}x = \pi/2$

$\operatorname{Sec}^{-1}x + \operatorname{Cosec}^{-1}x = \pi/2$

Proof (i Let $\sin^{-1}x = \theta$

$\Rightarrow x = \sin\theta$

$\Rightarrow x = \cos(\frac{\pi}{2} - \theta)$

$\Rightarrow \cos^{-1}x = \frac{\pi}{2} - \theta$

$\Rightarrow \theta + \cos^{-1}x = \pi/2$

$\Rightarrow \sin^{-1}x + \cos^{-1}x = \pi/2$ proved

P - III

$\operatorname{Cosec}^{-1}x = \sin^{-1}\left(\frac{1}{x}\right)$; $x > 0$

$\operatorname{Sec}^{-1}x = \cos^{-1}\left(\frac{1}{x}\right)$; $x > 0$

$\cot^{-1}x = \tan^{-1}\left(\frac{1}{x}\right)$; $x > 0$

P - IV

Proof Let $\operatorname{Cosec}^{-1}x = \theta$

$\Rightarrow x = \operatorname{Cosec}\theta$

$\Rightarrow x = \frac{1}{\sin\theta}$

$\Rightarrow \sin\theta = \frac{1}{x}$

$\Rightarrow \theta = \sin^{-1}\left(\frac{1}{x}\right)$

$\Rightarrow \operatorname{Cosec}^{-1}x = \sin^{-1}(1/x)$ Proved

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P-IV

$\sin^{-1}(-x) = -\sin^{-1}x$

$\tan^{-1}(-x) = -\tan^{-1}x$

$\operatorname{cosec}^{-1}(-x) = -\operatorname{cosec}^{-1}x$

$\cos^{-1}(-x) = \pi - \cos^{-1}x$

$\sec^{-1}(-x) = \pi - \sec^{-1}x$

$\cot^{-1}(-x) = \pi - \cot^{-1}x$

Proof

Let $\sin^{-1}(-x) = \theta$

$-x = \sin\theta$

$x = -\sin\theta$

$x = \sin(-\theta)$

$\sin^{-1}x = -\theta$

$-\sin^{-1}x = \theta$

$-\sin^{-1}x = \sin^{-1}(-x)$

Let $\cos^{-1}(-x) = \theta$

$-x = \cos\theta$

$x = -\cos\theta$

$x = \cos(\pi - \theta)$

$\cos^{-1}x = \pi - \theta$

$\cos^{-1}x = \pi - \cos^{-1}(-x)$

$\cos^{-1}(-x) = \pi - \cos^{-1}x$

Questions

(1) find the value of $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) + \cos^{-1}\left(-\frac{1}{2}\right) + \cot^{-1}(\sqrt{3})$

$= -\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) + \pi - \cos^{-1}\left(\frac{1}{2}\right) + \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$

$= -\frac{\pi}{3} + \pi - \frac{\pi}{3} + \frac{\pi}{6}$

$= \pi - \frac{2\pi}{3} + \frac{\pi}{6} = \frac{6\pi - 4\pi + \pi}{6} = \frac{3\pi}{6} = \frac{\pi}{2}$

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Qn 2 Find the value of $\sin^{-1}\left(\sin\left(\frac{2\pi}{3}\right)\right) + \cos^{-1}\left(\cos\left(\frac{4\pi}{3}\right)\right)$

$= \sin^{-1}\left(\sin\left(\pi - \frac{\pi}{3}\right)\right) + \cos^{-1}\left(\cos\left(\pi + \frac{\pi}{3}\right)\right)$ (II, III)

$= \sin^{-1}\left(\sin\frac{\pi}{3}\right) + \cos^{-1}\left(-\cos\frac{\pi}{3}\right)$

$= \frac{\pi}{3} + \pi - \cos^{-1}\left(\cos\frac{\pi}{3}\right)$

$= \frac{\pi}{3} + \pi - \frac{\pi}{3} = \pi$ Ans

Qn 3 Find $\tan^{-1}\left(\tan\frac{3\pi}{4}\right) + \cos^{-1}\left(\cos\frac{5\pi}{6}\right) + \sin^{-1}\left(\sin\left(13\frac{\pi}{6}\right)\right)$

$= \tan^{-1}\left(\tan\left(\pi - \frac{\pi}{4}\right)\right) + \frac{5\pi}{6} + \sin^{-1}\left(\sin\left(2\pi + \frac{\pi}{6}\right)\right)$ (II, I)

$= \tan^{-1}\left(-\tan\frac{\pi}{4}\right) + \frac{5\pi}{6} + \sin^{-1}\left(\sin\frac{\pi}{6}\right)$

$= -\tan^{-1}\left(\tan\frac{\pi}{4}\right) + \frac{5\pi}{6} + \frac{\pi}{6}$

$= -\frac{\pi}{4} + \frac{5\pi}{6} + \frac{\pi}{6} = -\frac{\pi}{4} + \pi = \frac{3\pi}{4}$ Ans

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Qn 4 If $\sin\left[\sin^{-1}x + \cos^{-1}(1/3)\right] = 1$ find value of x

$\Rightarrow \sin^{-1}x + \cos^{-1}(1/3) = \sin^{-1}(1)$

$\Rightarrow \sin^{-1}x + \cos^{-1}(1/3) = \pi/2$

($\sin^{-1}x$ and $\cos^{-1}(1/3)$ are same)

$\Rightarrow \boxed{x = 1/3}$ Ans --- { $\because \sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}$ }

(I - 1)

ON. 5 → find value of $\sin\left(\frac{\pi}{3} - \sin^{-1}\left(-\frac{1}{2}\right)\right)$

$= \sin\left(\frac{\pi}{3} + \sin^{-1}\left(\frac{1}{2}\right)\right)$

$= \sin\left(\frac{\pi}{3} + \frac{\pi}{6}\right)$

$= \sin\left(\frac{\pi}{2}\right) = 1$ Ans

WORKSHEET 1 —x—

ON. 6 → Find the value of $\sin^{-1}\left(\sin\frac{3\pi}{4}\right) + \cos^{-1}\left(\cos\frac{3\pi}{4}\right) + \tan^{-1}\left(\tan\frac{3\pi}{4}\right)$

ON. 7 → Find the value of

$\tan^{-1}\left(\tan\frac{5\pi}{3}\right) + \cos^{-1}\left(\cos\left(\frac{5\pi}{3}\right)\right) + \sin^{-1}\left(\sin\left(\frac{5\pi}{3}\right)\right)$

ON. 8 → Find the value of

$\sin^{-1}\left(-\frac{1}{\sqrt{2}}\right) + \cos^{-1}\left(-\frac{1}{\sqrt{2}}\right) + \tan^{-1}(-1)$

ON. 9 → Find the value of

$\operatorname{cosec}^{-1}(-2) - \sec^{-1}(-2) + \cot^{-1}(-1)$

ON. 10 → If $\cot\left(\sec^{-1}(2) + \operatorname{cosec}^{-1}(x)\right) = 0$ find x

ON. 11 → If $\cos\left(\tan^{-1}(3x) + \cot^{-1}(5)\right) = 0$ find x

ON. 12 → Find value of $\tan^{-1}\left(2\cos\left(2\sin^{-1}\frac{1}{2}\right)\right)$

Q.No. 13 → Find the value of $\cos^{-1}(\cos(1540^\circ))$

Q.No. 14 → Find the value of $\sin^{-1}(\sin(-600^\circ))$

Q.No. 15 → If $4\sin^{-1}x + \cos^{-1}x = \pi$ Find value of x

Q.No. 16 → If $\sin^{-1}x - \cos^{-1}x = \frac{\pi}{6}$, Find value of x

Q.No. 17 → Find value of $\cos\left[\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right) + \frac{\pi}{4}\right]$

Q.No. 18 → Find value of $\tan(\sec^{-1}a + \operatorname{cosec}^{-1}a)$

Q.No. 19 → If $\sin^{-1}x + \sin^{-1}y + \sin^{-1}z = \frac{3\pi}{2}$

Logical QN

Find value of $x+y+z$

Q.No. 20 → If $\cos^{-1}\alpha + \cos^{-1}\beta + \cos^{-1}\gamma = 3\pi$

Find $\alpha^2+\beta^2+\gamma^2$

← ANSWERS →

6) $3\pi/4$

7) $-\pi/3$

8) $\pi/4$

9) $-\frac{\pi}{12}$

10) $x=2$

11) $x=5/3$

12) $\pi/4$

(13) 100°

(14) 60°

(15) $x=1/2$

(16) $x=\frac{\sqrt{3}}{2}$

(17) $-\left(\frac{\sqrt{3}+1}{2\sqrt{2}}\right)$ Hint: $\cos(15^\circ)=\cos(45^\circ-30^\circ)$

(18) ∞

(19) 3

(20) 3