ULTIMATE MATHEMATICS

(By: AJAY MITTAL)

Solutions of Worksheet No: M 2 (Matrices)

Ans: 1 → $A = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$

given $A^2 = kA - 2I$

$$\Rightarrow \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}\begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix} = k\begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix} - 2\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix} = \begin{bmatrix} 3k & -2k \\ 4k & -2k \end{bmatrix} - \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix} = \begin{bmatrix} 3k-2 & -2k \\ 4k & -2k-2 \end{bmatrix}$$

$\Rightarrow 1 = 3k-2$ | $-2 = -2k$ | $4 = 4k$ | $-4 = -2k-2$

$\Rightarrow 3k = k$ | $k = 1$ | $k = 1$ | $-2k = -2$

$k = 1$ | | | $k = 1$

$\therefore \boxed{k=1}$ Ans

Ans 2 → $A = \begin{bmatrix} 1 & 0 \\ -1 & 7 \end{bmatrix}$

given $A^2 - 8A + kI = 0$

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ -1 & 7 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ -1 & 7 \end{bmatrix} - 8\begin{bmatrix} 1 & 0 \\ -1 & 7 \end{bmatrix} + k\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = 0$$

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ -8 & 49 \end{bmatrix} - \begin{bmatrix} 8 & 0 \\ -8 & 56 \end{bmatrix} + \begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix} = 0$$

$$\Rightarrow \begin{bmatrix} -7 & 0 \\ 0 & -7 \end{bmatrix} + \begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix} = 0$$

$$\Rightarrow \begin{bmatrix} -7+k & 0 \\ 0 & -7+k \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$\Rightarrow -7+k=0$ | $-7\cancel{+4}\ k=0$

$\Rightarrow \boxed{k=7}$ Ans

Qns 3 → $A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$

given $A^2 = \lambda A + \mu I$

$$\Rightarrow \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} = \lambda\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} + \mu\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 7 & 12 \\ 4 & 7 \end{bmatrix} = \begin{bmatrix} 2\lambda & 3\lambda \\ \lambda & 2\lambda \end{bmatrix} + \begin{bmatrix} \mu & 0 \\ 0 & \mu \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 7 & 12 \\ 4 & 7 \end{bmatrix} = \begin{bmatrix} 2\lambda+\mu & 3\lambda \\ \lambda & 2\lambda+\mu \end{bmatrix}$$

$\Rightarrow 7 = 2\lambda+\mu$ | $12 = 3\lambda$ | $4 = \lambda$ | $7 = 2\lambda+\mu$

$7 = 8+\mu$ | $\boxed{\lambda=4}$ | $\boxed{\lambda=4}$ | $7 = 8+\mu$

$\boxed{\mu=-1}$ | | | $\boxed{\mu=-1}$

$\boxed{\lambda=4 \ \& \ \mu=-1}$ Ans

Qns 4 → $A = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$

$f(x) = x^2 - 5x + 6$

$f(A) = A^2 - 5A + 6I$

$$\begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}\begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix} - 5\begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix} + 6\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & -1 & 2 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{bmatrix} - \begin{bmatrix} 10 & 0 & 5 \\ 10 & 5 & 15 \\ 5 & -5 & 0 \end{bmatrix} + \begin{bmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} -5 & -1 & -3 \\ -1 & -7 & -10 \\ -5 & 4 & -2 \end{bmatrix} + \begin{bmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{bmatrix}$$

$$f(A) = \begin{bmatrix} 1 & -1 & -3 \\ -1 & -1 & -10 \\ -5 & 4 & 4 \end{bmatrix}$$ Ans

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Qn. 5 we have $\begin{bmatrix} 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 0 \\ 2 & 0 & 1 \\ 1 & 0 & 2 \end{bmatrix} \begin{bmatrix} 0 \\ 2 \\ x \end{bmatrix} = 0$

$\Rightarrow \begin{bmatrix} 6 & 2 & 4 \end{bmatrix} \begin{bmatrix} 0 \\ 2 \\ x \end{bmatrix} = 0$

$\Rightarrow \begin{bmatrix} 0 + 4 + 4x \end{bmatrix} = 0$

$\Rightarrow \begin{bmatrix} 4 + 4x \end{bmatrix} = \begin{bmatrix} 0 \end{bmatrix}$

$\Rightarrow 4x + 4 = 0$

$\Rightarrow \boxed{x = -1}$ Ans

Qn. 6 we have $\begin{bmatrix} x & -5 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix} \begin{bmatrix} x \\ 4 \\ 1 \end{bmatrix} = 0$

$\Rightarrow \begin{bmatrix} x-2 & -10 & 2x-8 \end{bmatrix} \begin{bmatrix} x \\ 4 \\ 1 \end{bmatrix} = 0$

$\Rightarrow \begin{bmatrix} x^2 - \cancel{2x} - 40 + \cancel{2x} - 8 \end{bmatrix} = 0$

$\Rightarrow \begin{bmatrix} x^2 - 48 \end{bmatrix} = \begin{bmatrix} 0 \end{bmatrix}$

$\Rightarrow x^2 - 48 = 0 \quad \Rightarrow \boxed{x = \pm 4\sqrt{3}}$ Ans

Ans 7 $\quad A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$

$f(x) = x^2 - 5x + 7$

to prove A is a root of the polynomial we have to show $f(A) = 0$

$f(A) = A^2 - 5A + 7I$

$\Rightarrow \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} - 5\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} + 7\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} - \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$

$= \begin{bmatrix} -7 & 0 \\ 0 & -7 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$

$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0$ proved

Ans 8 $\quad A = \begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix}, B = \begin{bmatrix} a & 1 \\ b & -1 \end{bmatrix}$

given $(A+B)^2 = A^2 + B^2$

$\Rightarrow (A+B)(A+B) = A^2 + B^2$

$\Rightarrow A^2 + AB + BA + B^2 = A^2 + B^2$

$\Rightarrow AB + BA = A^2 + B^2 - A^2 - B^2$

$\Rightarrow AB + BA = 0$

$\Rightarrow \begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix}\begin{bmatrix} a & 1 \\ b & -1 \end{bmatrix} + \begin{bmatrix} a & 1 \\ b & -1 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix} = 0$

$\Rightarrow \begin{bmatrix} a-b & 2 \\ 2a-b & 3 \end{bmatrix} + \begin{bmatrix} a+2 & -a-1 \\ b-2 & -b+1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$$\Rightarrow \begin{bmatrix} 2a-b+2 & -a+1 \\ 2a+\cancel{b}-\cancel{b}-2 & 2-b \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$\Rightarrow 2a \cancel{-b} + 2 = 0$ | $-a+1=0$ | $2a-2=0$ | $2-b=0$

$\Rightarrow 2a=2$ | $a=1$ | $a=1$ | $b=2$

$a=1$

$\therefore$ $\boxed{a=1 \ \& \ b=2}$ Ans

9 already solved in Notes (given)

10 (we will do in 3^{rd} class)

11) $F(x) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$\therefore F(y) = \begin{bmatrix} \cos y & -\sin y & 0 \\ \sin y & \cos y & 0 \\ 0 & 0 & 1 \end{bmatrix}$ .. {replace x by y}

$F(x+y) = \begin{bmatrix} \cos(x+y) & -\sin(x+y) & 0 \\ \sin(x+y) & \cos(x+y) & 0 \\ 0 & 0 & 1 \end{bmatrix}$ --- {replace x by $x+y$}

taking LHS $F(x) \cdot F(y)$

$$= \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos y & -\sin y & 0 \\ \sin y & \cos y & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} \cos x\cos y - \sin x\sin y & -\cos x\sin y - \sin x\cos y & 0 \\ \sin x\cos y + \cos x\sin y & \cos x\cos y - \sin x\sin y & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

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$$= \begin{bmatrix} \cos(x+y) & -\sin(x+y) & 0 \\ \sin(x+y) & \cos(x+y) & 0 \\ 0 & 0 & 1 \end{bmatrix} = F(x+y)$$

Proved

Qn 12 given $X\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} = \begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{bmatrix}$

$$\begin{bmatrix} & \\ & \end{bmatrix}_{2\times 2}\begin{bmatrix} & & \\ & & \end{bmatrix}_{2\times 3} = \begin{bmatrix} & & \\ & & \end{bmatrix}_{2\times 3}$$

same

Ans

let $X = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$\therefore$ given equation becomes

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} = \begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} a+4b & 2a+5b & 3a+6b \\ c+4d & 2c+5d & 3c+5d \end{bmatrix} = \begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{bmatrix}$$

$a+4b=-7$
$2a+5b=-8$
Solve $a=1$, $b=-2$
it also satisfies the equation $3a+6b=-9$

$c+4d=2$
$2c+5d=4$
Solve $c=2$, $d=0$
it also satisfies the equation $3c+5d=6$

$$\therefore X = \begin{bmatrix} 1 & -2 \\ 2 & 0 \end{bmatrix}$$ Ans

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Qn 13 given $\begin{bmatrix} 2 & -1 \\ 1 & 0 \\ -3 & 4 \end{bmatrix} A = \begin{bmatrix} -1 & -8 & -10 \\ 1 & -2 & -5 \\ 9 & 22 & 15 \end{bmatrix}$

$$\begin{bmatrix} & \end{bmatrix}_{3\times 2}\begin{bmatrix} & \end{bmatrix}_{2\,3} = \begin{bmatrix} & \end{bmatrix}_{3\times 3}$$

same

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let $A = \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix}$

$\therefore$ Given equation becomes

$$\begin{bmatrix} 2 & -1 \\ 1 & 0 \\ -3 & 4 \end{bmatrix}\begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix} = \begin{bmatrix} -1 & -8 & -10 \\ 1 & -2 & -5 \\ 9 & 22 & 15 \end{bmatrix}$$

(proceed yourself)

Ans 14 $\begin{bmatrix} 1 & -4 \\ 3 & -2 \end{bmatrix} X = \begin{bmatrix} -16 & -6 \\ 7 & 2 \end{bmatrix}$

let $X = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$$\Rightarrow \begin{bmatrix} 1 & -4 \\ 3 & -2 \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} -16 & -6 \\ 7 & 2 \end{bmatrix}$$

(proceed yourself)

Ans 15 Given $CD - AB = 0$

$\Rightarrow CD = AB$

$$\begin{bmatrix} & \end{bmatrix}_{2\times 2}\begin{bmatrix} & \end{bmatrix}_{-\,-} = \begin{bmatrix} & \end{bmatrix}_{2\times 2}\begin{bmatrix} & \end{bmatrix}_{2\times 2}$$

let $D = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

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$$\begin{bmatrix} 2 & 5 \\ 3 & 8 \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 5 & 2 \\ 7 & 4 \end{bmatrix}$$

$$\rightarrow \begin{bmatrix} 2a+5c & 2b+5d \\ 3a+8c & 3b+8d \end{bmatrix} = \begin{bmatrix} 3 & 0 \\ 43 & 22 \end{bmatrix}$$

$2a+5c = 3$
$3a+8c = 43$
Solve $a = -191$
$c = 77$

$2b+5d = 0$
$3b+8d = 22$
Solve
$a = -110$, $d = 44$

$$\therefore D = \begin{bmatrix} -191 & -110 \\ 77 & 44 \end{bmatrix} \text{ Ans}$$

Qn 16 given $2A - 3B + 5C = 0$
to find matrix A

$$2A = 3B - 5C$$

$$2A = 3\begin{bmatrix} -2 & 2 & 0 \\ 3 & 1 & 4 \end{bmatrix} - 5\begin{bmatrix} 2 & 0 & -2 \\ 7 & 1 & 6 \end{bmatrix}$$

$$2A = \begin{bmatrix} -16 & 6 & 10 \\ -26 & -2 & -18 \end{bmatrix}$$

$$\therefore A = \frac{1}{2}\begin{bmatrix} -16 & 6 & 10 \\ -26 & -2 & -18 \end{bmatrix} = \begin{bmatrix} -8 & 3 & 5 \\ -13 & -1 & -9 \end{bmatrix} \text{ Ans}$$

Qn 17 given $A^2 = B$

$$\begin{bmatrix} \alpha & 0 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} \alpha & 0 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 5 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} \alpha^2 & 0 \\ \alpha+1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 5 & 1 \end{bmatrix}$$

$\alpha^2 = 1$ | $\alpha+1 = 5$

$\alpha = \pm 1$ | $\alpha = 4$ $\rightarrow$ Since no common value of α