ULTIMATE MATHEMATICS

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CLASS NO: 7

← DIFFERENTIATION & CONTINUITY → CLASS NO: 7

TOPIC CONTINUITY

XI

- (·) $LHL = \lim_{x \to a^-} (f(x))$ put $x = a - h$ & $h \to 0$
- (·) $RHL = \lim_{x \to a^+} (f(x))$ put $x = a + h$ & $h \to 0$
- (·) $x < a$, (·) $x > a$
- (·) If $LHL = RHL$ then $\lim_{x \to a} f(x)$ exists

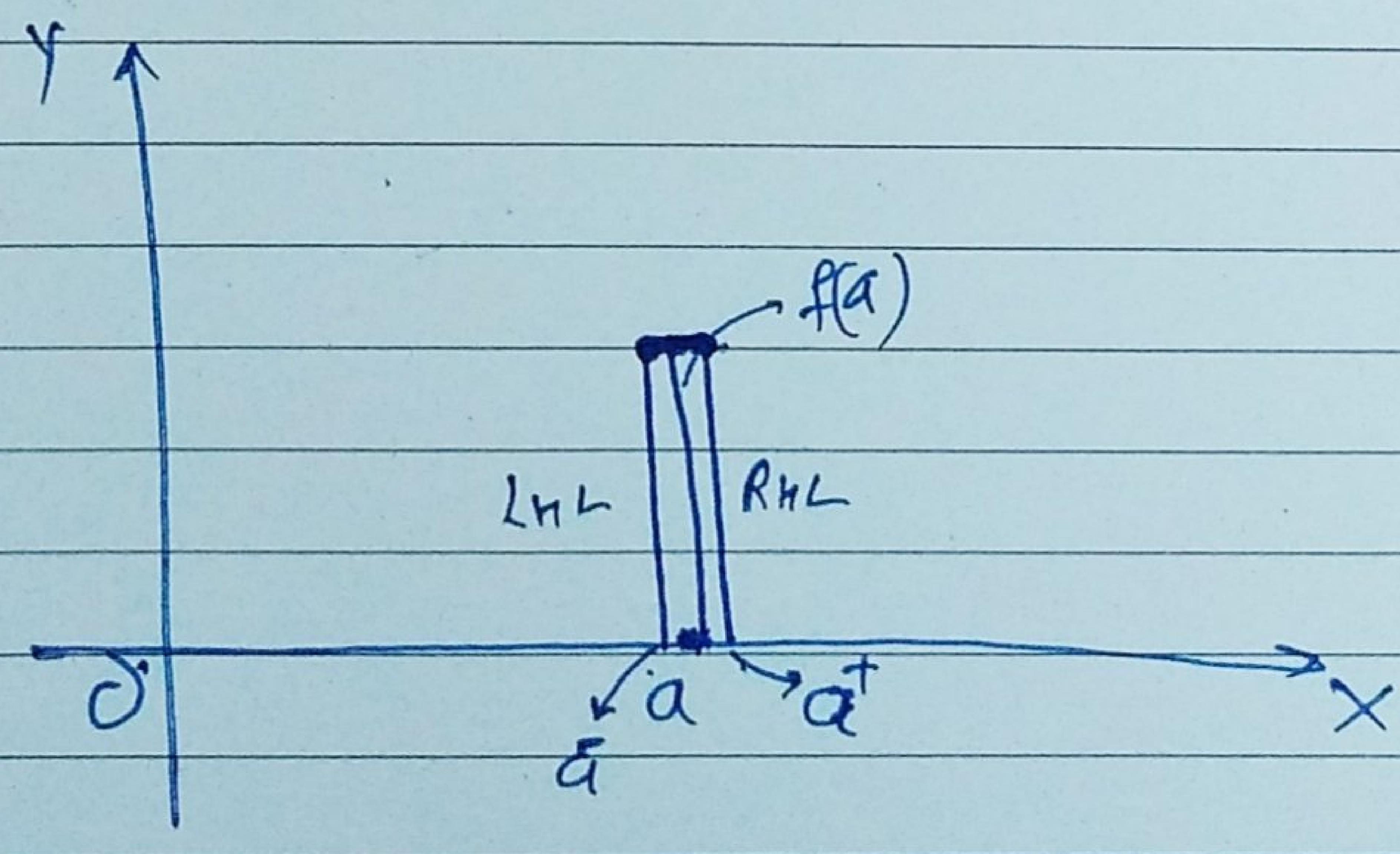
XII

- (·) A function $f(x)$ is said to be continuous at $x = a$

 if $\boxed{LHL = RHL = f(a)}$

- (·) If $\boxed{\lim_{x \to a} f(x) = f(a)}$

graph



$a = 2$

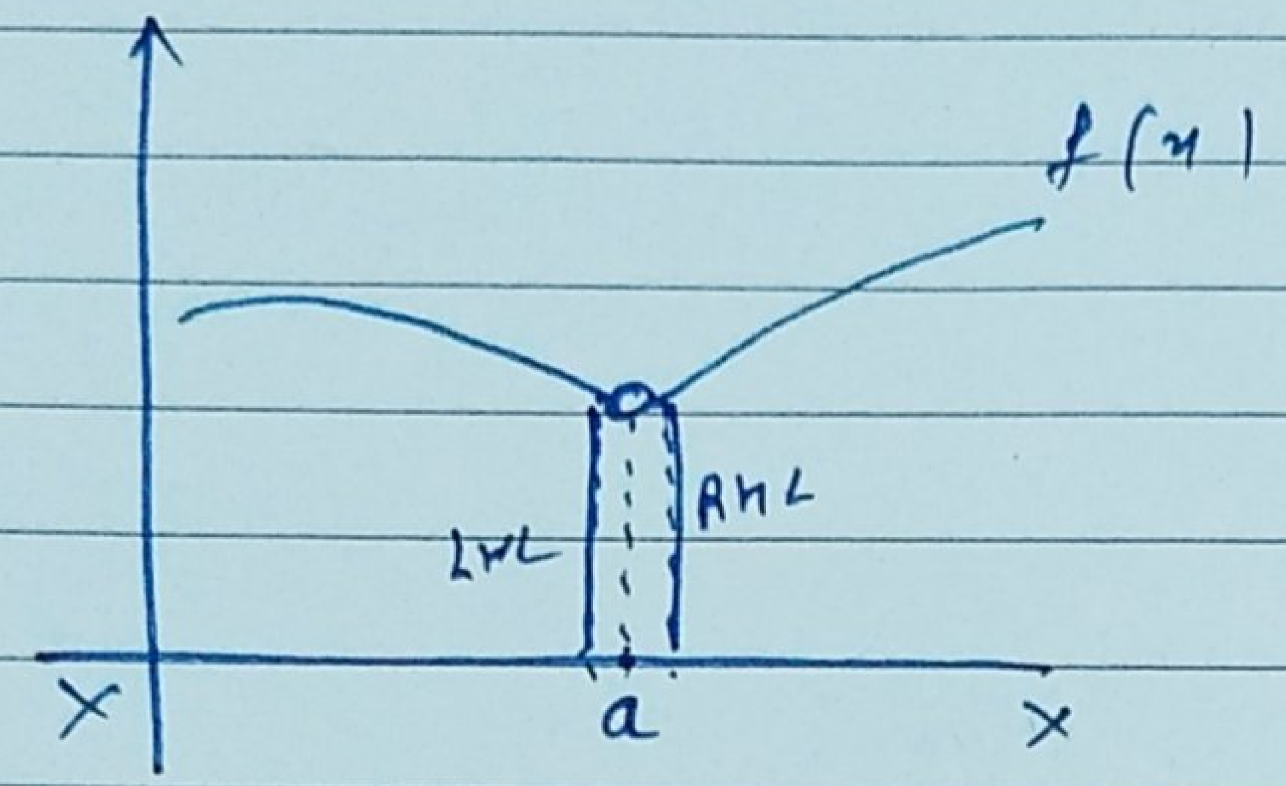
2^-

1.99999

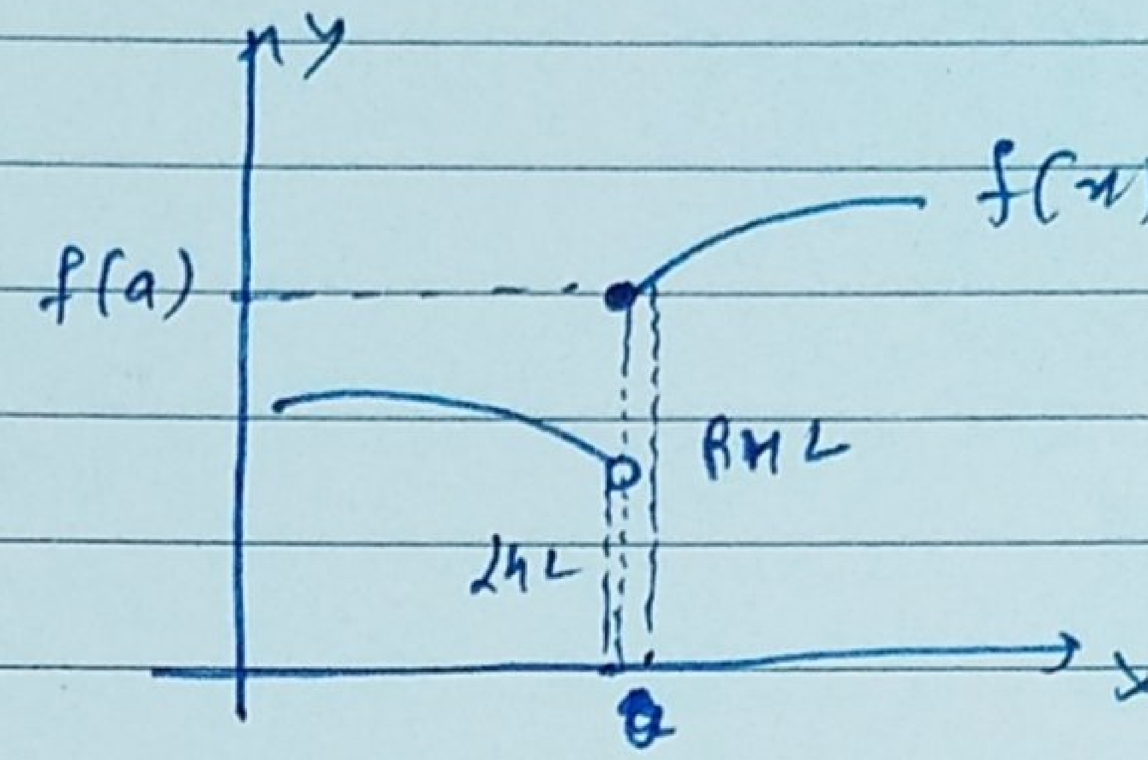
2^+

2.0001

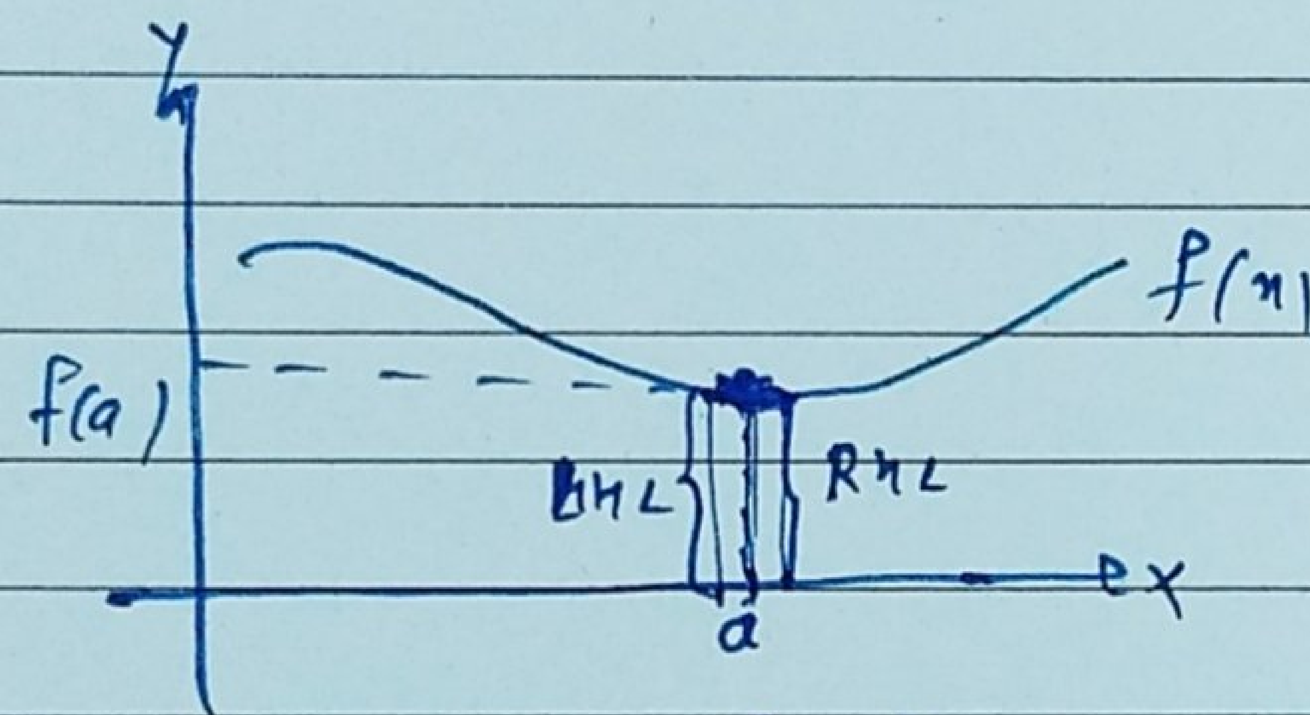
D&C Class No: 7



LHL = RHL But f(a) not defined



LHL ≠ RHL



LHL = RHL = f(a)

Theorem If f and g are continuous at $x = a$, then

(·) $f+g$ is continuous at $x = a$

(·) $f-g$ " " " $x = a$

(·) $f \cdot g$ " " " $x = a$

(·) kf " " " $x = a$

(·) $\frac{f}{g}$ " " " $x = a$ {provided $g(a) \neq 0$}

(·) $f \circ g$ is also continuous at $x = a$

(·) $\frac{0}{0}$, $\frac{\infty}{\infty}$, $\infty - \infty$ (Indeterminate form)

(·)

D&C — class No: 7 — (3)

$$|x| = \begin{cases} x & : x \geq 0 \\ -x & : x < 0 \end{cases}$$

<u>Type-1</u> Check the continuity of the function at $x = a$

Q.-1 $$f(x) = \begin{cases} \frac{|x|}{x} & ; x \neq 0 \\ 1 & ; x = 0 \end{cases}$$

Check the cont. of $f(x)$ at $x = 0$

Soln Redefine the function

$$f(x) = \begin{cases} \frac{x}{x} & ; x > 0 \\ -\frac{x}{x} & ; x < 0 \\ 1 & ; x = 0 \end{cases}$$

$$f(x) = \begin{cases} 1 & ; x > 0 \\ -1 & ; x < 0 \\ 1 & ; x = 0 \end{cases}$$

$$LHL = \lim_{x \to 0^-} (-1) = -1$$

$$RHL = \lim_{x \to 0^+} (1) = 1$$

$f(0) = 1$

$LHL \neq RHL$ $\therefore f(x)$ is discontinuous at $x = 0$ Ans

CLASS NO = 7

Q.-4 → examine the continuity of $f(x) = \begin{cases} \dfrac{\cos x}{\frac{\pi}{2} - x} & ; x \neq \frac{\pi}{2} \\ 1 & ; x = \frac{\pi}{2} \end{cases}$ at $x = \pi/2$

Sol:

$$LHL = \lim_{x \to \frac{\pi}{2}^-} \left(\frac{\cos x}{\frac{\pi}{2} - x} \right)$$

put $x = \frac{\pi}{2} - h$ & $h \to 0$

$$LHL = \lim_{h \to 0} \left(\frac{\cos\left(\frac{\pi}{2} - h\right)}{\frac{\pi}{2} - \left(\frac{\pi}{2} - h\right)} \right)$$

$$= \lim_{h \to 0} \left(\frac{\sin h}{h} \right) = 1$$

$$RHL = \lim_{x \to \frac{\pi}{2}^+} \left(\frac{\cos x}{\frac{\pi}{2} - x} \right)$$

put $x = \frac{\pi}{2} + h$ & $h \to 0$

$$RHL = \lim_{h \to 0} \left(\frac{\cos\left(\frac{\pi}{2} + h\right)}{\frac{\pi}{2} - \left(\frac{\pi}{2} + h\right)} \right)$$

$$= \lim_{h \to 0} \left(\frac{\cancel{-}\sin h}{\cancel{-}h} \right) = 1$$

$f\left(\frac{\pi}{2}\right) = 1$

Since $LHL = RHL = f(\pi/2)$

$\therefore f(x)$ is cont. at $x = \frac{\pi}{2}$ Ans

CLASSTIME

Topic : D&C   Date. :   Page No. : 5

CLASS No.- 7

Qn 3 → Discuss the Continuity of $f(x) = |x-2| + |x-1|$ at $x=1$ & $x=2$

Soln

$f(x) = |x-2| + |x-1|$
(2)   (1)

$|x-3|$
$-$ (3) $+$

$f(x) = |x-1| + |x-2|$

(1)   (2)

$$f(x) = \begin{cases} -(x-1) - (x-2) & ; \quad x < 1 \\ (x-1) - (x-2) & ; \quad 1 \le x < 2 \\ (x-1) + (x-2) & ; \quad x \ge 2 \end{cases}$$

$$f(x) = \begin{cases} -2x + 3 & : \quad x < 1 \\ 1 & \quad 1 \le x < 2 \\ 2x - 3 & ; \quad x \ge 2 \end{cases}$$

Cont. at $x = 2$

$LHL = \lim_{x \to 2^-} (1) = 1$

$RHL = \lim_{x \to 2^+} (2x - 3)$ put $x = 2+h$ & $h \to 0$

$= \lim_{h \to 0} (2(2+h) - 3) = 1$

$f(2) = 2(2) - 3 = 1$

$LHL = RHL = f(2)$ $f(x)$ is cont at $x = 2$

Ans

Topic : DEC   Date. : CLASS NO 7   Page No. : 6

Qn. 4 → $f(x) = \begin{cases} x\sin\left(\frac{1}{x}\right) & ; x \neq 0 \\ 0 & ; x = 0 \end{cases}$

Show $f(x)$ is continuous at $x = 0$

Soln

$LHL = \lim_{x \to 0^-} \left(x \sin\left(\frac{1}{x}\right)\right)$

put $x = 0 - h = -h$ & $h \to 0$

$LHL = \lim_{h \to 0} \left(-h \cdot \sin\left(\frac{-1}{h}\right)\right)$

$= \lim_{h \to 0} \left(h \sin\left(\frac{1}{h}\right)\right)$

$= 0 \times \sin\left(\frac{1}{0}\right)$

$= 0 \times$ (an oscillating number b/w -1 & 1)

$= 0$

Similarly $RHL = 0$

$f(0) = 0$

$f(x)$ is continuous at $x = 0$ Ans

$\cancel{\lim_{h \to 0} \left(h \frac{\sin\left(\frac{1}{h}\right)}{\frac{1}{h}} \times \frac{1}{h} \right)}$ ✗

$\lim_{x \to 0} \left(\frac{\sin x}{x} \right) = 1$

CLASSTIME

Topic : .... Date. : .... Page No. : 7

DEC Class No: 7

Type = II

Q 5 → $f(x) = \begin{cases} \frac{1-\cos(kx)}{x \sin x} & ; x \neq 0 \\ \frac{1}{2} & ; x = 0 \end{cases}$ is continous at $x=0$ find value of 'k'

Sol Since $f(x)$ is continous at $x=0$

$\therefore \boxed{\lim_{x \to 0} f(x) = f(0)}$

$\Rightarrow \lim_{x \to 0} \left(\frac{1-\cos(kx)}{x \sin x} \right) = \frac{1}{2}$

$\Rightarrow \lim_{x \to 0} \left(\frac{2\sin^2\left(\frac{kx}{2}\right)}{x \sin x} \right) = \frac{1}{2}$

$\boxed{\lim_{x \to 0} \left(\frac{\sin^2 x}{x^2} \right) = 1}$

$\Rightarrow \lim_{x \to 0} \left(\frac{2 \frac{\sin^2\left(\frac{kx}{2}\right)}{\frac{k^2x^2}{4}} \times \frac{k^2 \cancel{x^2}}{4}}{\cancel{x} \frac{\sin x}{x} \times \cancel{x}} \right) = \frac{1}{2}$

$\Rightarrow 2 \times \frac{k^2}{4} = \frac{1}{2}$

$\Rightarrow \frac{k^2}{2} = \frac{1}{2}$

$\Rightarrow k^2 = 1$

$\Rightarrow$ $(k = \pm 1)$ Ans

Topic : .... Date. : .... Page No. : ....

DEC CLASS No: 7 (8)

Q.n.6 $\to$ $f(x) = \begin{cases} \dfrac{1-\cos(4x)}{x^2} & ; x < 0 \\ a & ; x = 0 \\ \dfrac{\sqrt{x}}{\sqrt{16+\sqrt{x}}-4} & ; x > 0 \end{cases}$ is continuous at $x = 0$ Find value of 'a'.

Sol. $LHL = \lim_{x \to 0^-} \left(\dfrac{1-\cos(4x)}{x^2} \right)$

$= \lim_{x \to 0^-} \left(\dfrac{2\sin^2(2x)}{x^2} \right)$

= put $x = 0 - h = -h$ & $h \to 0$

$= \lim_{h \to 0} \left(\dfrac{2\sin^2(-2h)}{(-h)^2} \right)$

$= \lim_{h \to 0} \left(\dfrac{2\sin^2(2h)}{4h^2} \times 4 \right) = 8 \times 1 = 8$

RHL $\lim_{x \to 0^+} \left(\dfrac{\sqrt{x}}{\sqrt{16+\sqrt{x}}-4} \right)$ put $x = 0 + h = h$ & $h \to 0$

$= \lim_{h \to 0} \left(\dfrac{\sqrt{h}}{\sqrt{16+\sqrt{h}}-4} \right)$

$= \lim_{h \to 0} \left(\dfrac{\sqrt{h}\,(\sqrt{16+\sqrt{h}}+4)}{(\sqrt{16+\sqrt{h}}-4)(\sqrt{16+\sqrt{h}}+4)} \right)$

$= \lim_{h \to 0} \left(\frac{\sqrt{h}\,(\sqrt{16+\sqrt{h}}+4)}{16+\sqrt{h}-16} \right) = \sqrt{16+0}+4$

$= 4+4 = 8$

$f(0) = a$ since $f(x)$ is cont.

$8 = 8 = a \Rightarrow (a=8)$ Ans

CLASSTIME

Topic : .... Date. : .... Page No. : ....

ULTIMATE MATHEMATICS

DIFFERENTIATION & CONTINUITY

WORKSHEET NO: 6

Ques 1 Show that $f(x) = \begin{cases} \frac{\sin x}{x} + \cos x & ; x \neq 0 \\ 2 & ; x = 0 \end{cases}$ is continuous at $x=0$

Ques 2 Find the value of 'λ' so that $f(x)$ is continuous at $x = -1$

$$f(x) = \begin{cases} \frac{x^2 - 2x - 3}{x+1} & ; x \neq -1 \\ \lambda & ; x = -1 \end{cases}$$

Ans $\lambda = -4$

Ques 3 Find the value of 'k' so that

$f(x) = \begin{cases} \frac{1-\cos(2x)}{2x^2} & ; x \neq 0 \\ k & ; x = 0 \end{cases}$ is continuous at $x = 0$

Ans $k = 1$

Ques 4 If $f(x) = \begin{cases} 3ax + b & ; x > 1 \\ 11 & ; x = 1 \\ 5ax - 2b & ; x < 1 \end{cases}$ is continuous

at $x=1$, find the values of a & b

Ans $a=3$, $b=2$

Ques 5 Show that the function $f(x) = 2x - |x|$ is continuous at $x = 0$

Hint. $|x| = \begin{cases} x & ; x \geq 0 \\ -x & ; x < 0 \end{cases}$

Qn 6 → Discuss the continuity of the function $f(x) = |x-3| + |x+1|$ at $x = -1$ and $x = 3$

CLASSTIME

Topic : .... Date. : .... Page No. : .... ②

D&C Worksheet No. 6

Qn 7 → If $f(x) = \begin{cases} \frac{1-\cos(2x)}{2x^2} & ; x<0 \\ k & ; x=0 \\ \frac{x}{|x|} & ; x>0 \end{cases}$ is continuous at $x=0$. Find the value of k

Ans $\boxed{k=1}$

Qn. 8 → $f(x) = \begin{cases} \frac{1-\sin^3 x}{3\cos^2 x} & ; x < \pi/2 \\ a & ; x = \pi/2 \\ \frac{b(1-\sin x)}{(\pi-2x)^2} & ; x > \frac{\pi}{2} \end{cases}$ is continuous at $x = \pi/2$. Find the value of a & b

Ans $a = 1/2$, $b = 4$

Qn 9 → Find the value of 'a' & 'b' if the function

$f(x) = \begin{cases} 1 & ; x \le 3 \\ ax+b & ; 3<x<5 \\ 7 & ; x \ge 5 \end{cases}$ is continuous at $x = 3$ and $x = 5$

Ans $a = 3$, $b = -8$

Qn. 10 → Find the value of 'k' if the function

$f(x) = \begin{cases} \frac{k\cos x}{\pi - 2x} & ; x \ne \frac{\pi}{2} \\ 3 & ; x = \pi/2 \end{cases}$ is continuous at $x = \pi/2$

Ans $k = 6$

Qn 11 → $f(x) = \begin{cases} \frac{x-4}{|x-4|} + a & ; x < 4 \\ \dots \end{cases}$ is continuous

$\begin{cases} a+b & ; \quad x=4 \\ \dfrac{x-4}{|x-4|}+b & ; \quad x>4 \end{cases}$ at $x=4$

Find value of a & b

Ans $a=1,\ b=-1$

CLASSTIME

Topic : .... Date. : .... Page No. : ....

DEC Worksheet No: 6

(3)

Ans 12 → Find the relationship b/w a & b so that

$f(x)=\begin{cases} ax+1 & : \quad x\le 3 \\ bx+3 & ; \quad x>3 \end{cases}$ is continuous at $x=3$

Ans $3a-3b=2$

Ans 13 → Discuss the continuity of the function $f(x)=|x|+|x-1|$ at $x=0,\ x=1$

Ans 14 → Show that the function $f(x)=\begin{cases} \dfrac{\sin(3x)}{\tan(2x)} & ; \quad x<0 \\ 3/2 & ; \quad x=0 \\ \dfrac{\log(1+3x)}{e^{2x}-1} & ; \quad x>0 \end{cases}$

is continuous at $x=0$

Ans 15 → If $f(x)=\begin{cases} x+a\sqrt{2}\sin x & ; \quad 0\le x<\pi/4 \\ 2x\cot x+b & . \quad \pi/4\le x<\pi/2 \\ a\cos(2x)-b\sin x & , \quad \frac{\pi}{2}\le x\le \pi \end{cases}$

is continuous at $x=\frac{\pi}{4}$ and at $x=\pi/2$

Find value of a & b Ans $a=\pi/6,\ b=-\pi/12$

—x—