SOLUTIONS — WORKSHEET NO: 11 (Class No.-14)

INTEGRATION

Ans. 1 → $I = \int_0^{\pi/2} \frac{\sin x - \cos x}{1+\sin x \cdot \cos x}\, dx$

Soln

$$I = \int_0^{\pi/2} \frac{\sin x - \cos x}{1+\sin x \cdot \cos x}\, dx \quad \text{---- (1)}$$

$$I = \int_0^{\pi/2} \frac{\sin(\frac{\pi}{2}-x) - \cos(\frac{\pi}{2}-x)}{1+\sin(\frac{\pi}{2}-x)\cos(\frac{\pi}{2}-x)}\, dx \quad \text{---} \quad (P_{IV})$$

$$I = \int_0^{\pi/2} \frac{\cos x - \sin x}{1+\cos x \sin x}\, dx \quad \text{--- (2)}$$

(1) + (2)

$$2I = \int_0^{\pi/2} \frac{0}{1+\sin x \cos x}\, dx$$

$$2I = 0$$

$$\boxed{I = 0} \quad \text{Ans}$$

Ans. 2 → $I = \int_0^{\pi} \frac{e^{\cos x}}{e^{\cos x} + e^{-\cos x}}\, dx$

Soln

$$I = \int_0^{\pi} \frac{e^{\cos x}}{e^{\cos x} + e^{-\cos x}}\, dx \quad \text{--- (1)}$$

$$I = \int_0^{\pi} \frac{e^{\cos(\pi-x)}}{e^{\cos(\pi-x)} + e^{-\cos(\pi-x)}}\, dx \quad \text{---} \quad (P_{IV})$$

$$I = \int_0^{\pi} \frac{e^{-\cos x}}{e^{-\cos x} + e^{\cos x}}\,dx \quad \text{--- (2)}$$

(1) + (2)

$$2I = \int_0^{\pi} \cancel{\frac{e^{\cos x} + e^{-\cos x}}{e^{\cos x} + e^{-\cos x}}}\,dx$$

$$2I = \int_0^{\pi} 1 \cdot dx$$

$$= (x)_0^{\pi}$$

$$2I = \pi$$

$$\boxed{I = \frac{\pi}{2}} \quad \text{Ans}$$

Ques 3 → $I = \int_0^{\pi/2} \sin(2x) \cdot \log(\cot x)\,dx$

Soln

$$I = \int_0^{\pi/2} \sin(2x) \cdot \log(\cot x)\,dx \quad \text{--- (1)}$$

$$I = \int_0^{\pi/2} \sin\left(2\left(\frac{\pi}{2} - x\right)\right) \cdot \log\left(\cot\left(\frac{\pi}{2} - x\right)\right)dx \quad \text{--- (P IV)}$$

$$I = \int_0^{\pi/2} \sin(\pi - 2x) \cdot \log(\tan x)\,dx$$

$$I = \int_0^{\pi/2} \sin(2x) \cdot \log(\tan x)\,dx \quad \text{--- (2)}$$

(1) + (2)

$$2I = \int_0^{\pi/2} \sin(2x)\left(\log(\cot x) + \log(\tan x)\right)dx$$

$$2I = \int_0^{\pi/2} \sin(2x) \cdot \log(\tan x \cdot \cot x)\, dx$$

$$2I = \int_0^{\pi/2} \sin(2x) \cdot \log(1)\, dx$$

$$2I = 0 \quad \text{-- } \{\because \log 1 = 0\}$$

$\boxed{I = 0}$ Ans

Qn: 4 → $I = \int_0^1 \log\left(\frac{1}{x} - 1\right) dx$

Soln

$$I = \int_0^1 \log\left(\frac{1-x}{x}\right) dx \quad \text{--- (1)}$$

$$I = \int_0^1 \log\left(\frac{1-(1-x)}{1-x}\right) dx \quad \text{-- } (P\ \text{IV})$$

$$I = \int_0^1 \log\left(\frac{x}{1-x}\right) dx \quad \text{-- (2)}$$

(1) + (2)

$$2I = \int_0^1 \log\left(\frac{1-x}{x} \times \frac{x}{1-x}\right) dx$$

$$2I = \int_0^1 \log 1\, dx$$

$$2I = 0$$

$\boxed{I = 0}$ Ans

Qn 5 → $I = \int_0^{\pi/2} \frac{\sin^2 x}{1 + \sin x \cdot \cos x}\, dx$

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Soln $I = \int_0^{\pi/2} \frac{\sin^2 x}{1+\sin x \cdot \cos x} dx$ --- (1)

$$I = \int_0^{\pi/2} \frac{\sin^2(\frac{\pi}{2}-x)}{1+\sin(\frac{\pi}{2}-x)\cos(\frac{\pi}{2}-x)} dx \quad \text{---} \quad (PD)$$

$$I = \int_0^{\pi/2} \frac{\cos^2 x}{1+\cos x \cdot \sin x} dx \quad \text{---} \quad (2)$$

(1) + (2)

$$2I = \int_0^{\pi/2} \frac{\sin^2 x + \cos^2 x}{1+\sin x \cdot \cos x} dx$$

$$2I = \int_0^{\pi/2} \frac{1}{1+\sin x \cdot \cos x} dx$$

Divide N & D by $\cos^2 x$

$$2I = \int_0^{\pi/2} \frac{\sec^2 x}{\sec^2 x + \tan x} dx$$

$$2I = \int_0^{\pi/2} \frac{\sec^2 x}{1+\tan^2 x + \tan x} dx$$

put $\tan x = t$ | when $x=0$; $t=0$

$\sec^2 x\, dx = dt$ | when $x = \pi/2$; $t=\infty$

$$2I = \int_0^{\infty} \frac{dt}{t^2+t+1}$$

$$2I = \int_0^{\infty} \frac{1}{(t+\frac{1}{2})^2 - \frac{1}{4}+1} dt$$

$$2I = \int_0^\infty \frac{1}{\left(t+\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}$$

$$2I = \frac{2}{\sqrt{3}}\left[\tan^{-1}\left(\frac{2t+1}{\sqrt{3}}\right)\right]_0^\infty$$

$$2I = \frac{2}{\sqrt{3}}\left(\tan^{-1}(\infty) - \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)\right)$$

$$\cancel{2}I = \frac{\cancel{2}}{\sqrt{3}}\left(\frac{\pi}{2} - \frac{\pi}{6}\right)$$

$$I = \frac{1}{\sqrt{3}}\left(\frac{\pi}{3}\right)$$

$$\boxed{I = \frac{\pi}{3\sqrt{3}}}$$ Ans

Qn. 6 → $I = \int_0^\pi \frac{x}{1+\sin x}\,dx$

Soln

$$I = \int_0^\pi \frac{x}{1+\sin x}\,dx \quad --- (1)$$

$$I = \int_0^\pi \frac{\pi - x}{1+\sin(\pi - x)}\,dx \quad --- \text{(P IV)}$$

$$I = \int_0^\pi \frac{\pi - x}{1+\sin x}\,dx \quad -- (2)$$

(1) + (2)

$$2I = \int_0^\pi \frac{\pi}{1+\sin x}\,dx$$

$$2I = \pi\int_0^\pi \frac{1}{1+\sin x}\,dx$$

Rationalize

$$2I = \pi \int_0^{\pi} \frac{1-\sin x}{\cos^2 x}\, dx$$

$$2I = \pi \int_0^{\pi} \sec^2 x - \tan x \sec x \, dx$$

$$2I = \pi \left[\tan x - \sec x \right]_0^{\pi}$$

$$2I = \pi \left[(\tan \pi - \sec \pi) - (\tan 0 - \sec 0) \right]$$

$$2I = \pi \left[(0 - (-1)) - (0 - 1) \right]$$

$$2I = \pi (1 + 1)$$

$$2I = 2\pi$$

$$\boxed{I = \pi}$$ Ans

Qn. 7 → $$I = \int_0^{\pi/2} \frac{x}{\sin x + \cos x}\, dx$$

Sol

$$I = \int_0^{\pi/2} \frac{x}{\sin x + \cos x}\, dx \quad \text{---} \ (1)$$

$$I = \int_0^{\pi/2} \frac{\pi/2 - x}{\sin(\frac{\pi}{2} - x) + \cos(\frac{\pi}{2} - x)}\, dx \quad \text{---} \quad (P_{IV})$$

$$I = \int_0^{\pi/2} \frac{\frac{\pi}{2} - x}{\cos x + \sin x}\, dx \ - \ (2)$$

(1) + (2)

$$2I = \frac{\pi}{2} \int_0^{\pi/2} \frac{1}{\sin x + \cos x}\, dx$$

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$$2I = \frac{\pi}{2}\int_0^{\pi/2} \frac{1}{\frac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}} + \frac{1-\tan^2(x/2)}{1+\tan^2(x/2)}}\,dx$$

$$2I = \frac{\pi}{2}\int_0^{\pi/2} \frac{\sec^2(x/2)\,dx}{2\tan\frac{x}{2} + 1 - \tan^2(x/2)}$$

put $\tan(x/2) = t$

$\frac{1}{2}\sec^2(x/2)dx = dt$ | when $x = 0$, $t = 0$

$\sec^2(\frac{x}{2})dx = 2dt$ | when $x = \pi/2$; $t = 1$

$$\therefore 2I = \frac{\pi}{\cancel{2}} \times \cancel{2}\int_0^1 \frac{dt}{-t^2+2t+1}$$

$$2I = -\pi\int_0^1 \frac{1}{t^2-2t-1}\,dt$$

$$2I = -\pi\int_0^1 \frac{1}{(t-1)^2-(\sqrt{2})^2}\,dt$$

$$2I = \pi\int_0^1 \frac{1}{(\sqrt{2})^2-(t-1)^2}\,dt$$

$$2I = \pi \times \frac{1}{2\sqrt{2}}\left(\log\left|\frac{\sqrt{2}+t-1}{\sqrt{2}-t+1}\right|\right)_0^1$$

$$2I = \frac{\pi}{2\sqrt{2}}\left[\log\left|\frac{\sqrt{2}}{\sqrt{2}}\right| - \log\left|\frac{\sqrt{2}-1}{\sqrt{2}+1}\right|\right]$$

$$2I = \frac{\pi}{2\sqrt{2}}\left[-\log\left|\frac{\sqrt{2}-1}{\sqrt{2}+1} \times \frac{\sqrt{2}-1}{\sqrt{2}-1}\right|\right]$$

$$2I = \frac{-\pi}{2\sqrt{2}} \log\left[(\sqrt{2}-1)^2\right]$$

$$\cancel{2}I = -\frac{\pi}{2\sqrt{2}} \times \cancel{2} \log(\sqrt{2}-1)$$

$$\boxed{I = -\frac{\pi}{2\sqrt{2}} \log(\sqrt{2}-1)}$$ Ans

Qn 8 + $I = \int_0^{\pi} \frac{x \tan x}{\sec x \cdot \operatorname{cosec} x}\, dx$

Soln $I = \int_0^{\pi} \frac{x \frac{\sin x}{\cancel{\cos x}}}{\frac{1}{\cancel{\cos x}} \cdot \frac{1}{\sin x}}\, dx$

$$I = \int_0^{\pi} x \sin^2 x \, dx \quad \text{---- (1)}$$

$$I = \int_0^{\pi} (\pi - x) \sin^2(\pi - x)\, dx \quad \text{---- (P IV)}$$

$$I = \int_0^{\pi} (\pi - x) \sin^2 x \, dx \quad \text{-- (2)}$$

(1) + (2)

$$2I = \pi \int_0^{\pi} \sin^2 x \, dx$$

$$2I = \frac{\pi}{2} \int_0^{\pi} 1 - \cos(2x)\, dx$$

$$2I = \frac{\pi}{2}\left(x - \frac{\sin(2x)}{2}\right)_0^{\pi}$$

$$2I = \frac{\pi}{2}\left[(\pi - 0) - (0)\right] \Rightarrow \boxed{I = \frac{\pi^2}{4}}$$ Ans

(9)

Ans. 9 →

$$I = \int_0^{\pi/2} 2\log(\cos x) - \log(\sin(2x))\,dx$$

$$I = \int_0^{\pi/2} \log(\cos^2 x) - \log(\sin(2x))\,dx$$

$$I = \int_0^{\pi/2} \log\left(\frac{\cos^2 x}{\sin(2x)}\right) dx$$

$$I = \int_0^{\pi/2} \log\left(\frac{\cos^{\cancel{2}} x}{2\sin x\,\cancel{\cos x}}\right) dx$$

$$I = \int_0^{\pi/2} \log\left(\frac{\cot x}{2}\right) dx \quad \text{--- (1)}$$

$$I = \int_0^{\pi/2} \log\left(\frac{\cot(\pi/2 - x)}{2}\right) dx \quad \text{--- (P IV)}$$

$$I = \int_0^{\pi/2} \log\left(\frac{\tan x}{2}\right) dx \quad \text{--- (2)}$$

(1) + (2)

$$2I = \int_0^{\pi/2} \log\left(\frac{\cot x}{2} \cdot \frac{\tan x}{2}\right) dx$$

$$2I = \int_0^{\pi/2} \log\left(\frac{1}{4}\right) dx$$

$$2I = \int_0^{\pi/2} \log 1 - \log 4\,dx$$

$$2I = -\left(x \log 4\right)_0^{\pi/2}$$

$$2I = -\frac{\pi}{2}\log 4$$

$$\boxed{I = -\frac{\pi}{4}\log 4} \quad \text{or} \quad I = -\frac{\pi}{4}\log(2)^2 \Rightarrow \boxed{I = -\frac{\pi}{2}\log 2}$$ Ans

Qn 10 → $I = \int_0^{\pi/2} \frac{1}{1+\sqrt{\cot x}}\,dx$

$$I = \int_0^{\pi/2} \frac{1}{1+\sqrt{\cot x}}\,dx \quad \text{--- (1)}$$

$$I = \int_0^{\pi/2} \frac{1}{1+\sqrt{\cot(\frac{\pi}{2}-x)}}\,dx \quad \text{--- (P IV)}$$

$$I = \int_0^{\pi/2} \frac{1}{1+\sqrt{\tan x}}\,dx$$

$$I = \int_0^{\pi/2} \frac{1}{1+\frac{1}{\sqrt{\cot x}}}\,dx$$

$$I = \int_0^{\pi/2} \frac{\sqrt{\cot x}}{\sqrt{\cot x}+1}\,dx \quad \text{--- (2)}$$

(1) + (2)

$$2I = \int_0^{\pi/2} \cancel{\frac{1+\sqrt{\cot x}}{\sqrt{\cot x}+1}}\,dx$$

$$2I = [x]_0^{\pi/2}$$

$$2I = \pi/2$$

$$\boxed{I = \pi/4} \quad \text{Ans}$$

Qn 11 → $I = \int_0^{\pi/2} \frac{\sin^n x}{\sin^n x + \cos^n x}\,dx$

$$I = \int_0^{\pi/2} \frac{\sin^n x}{\sin^n x + \cos^n x}\,dx \quad \text{--- (1)}$$

$$I = \int_0^{\pi/2} \frac{\sin^n\left(\frac{\pi}{2}-x\right)}{\sin^n\left(\frac{\pi}{2}-x\right) + \cos^n\left(\frac{\pi}{2}-x\right)}\,dx \;--\; (P\ \text{IV})$$

$$I = \int_0^{\pi/2} \frac{\cos^n(x)}{\cos^n(x)+\sin^n(x)}\,dx \;--\; (2)$$

(1) + (2)

$$2I = \int_0^{\pi/2} \cancel{\frac{\sin^n x + \cos^n x}{\cos^n x + \sin^n x}}\,dx$$

$$2I = (x)_0^{\pi/2}$$

$$2I = \pi/2$$

$$\boxed{I = \pi/4} \quad \underline{\text{Ans}}$$

Qn. 12 → $I = \int_0^{\pi/2} \frac{x \sin x \cos x}{\sin^4 x + \cos^4 x}\,dx$

$$I = \int_0^{\pi/2} \frac{x \sin x \cos x}{\sin^4 x + \cos^4 x}\,dx \;---\; (1)$$

$$I = \int_0^{\pi/2} \frac{\left(\frac{\pi}{2}-x\right)\sin\left(\frac{\pi}{2}-x\right)\cos\left(\frac{\pi}{2}-x\right)}{\sin^4\left(\frac{\pi}{2}-x\right) + \cos^4\left(\frac{\pi}{2}-x\right)}\,dx \;---\; (P\ \text{IV})$$

$$I = \int_0^{\pi/2} \frac{\left(\frac{\pi}{2}-x\right)\cos x \cdot \sin x}{\cos^4 x + \sin^4 x}\,dx \;--\; (2)$$

(1) + (2)

$$2I = \frac{\pi}{2}\int_0^{\pi/2} \frac{\sin x \cdot \cos x}{\sin^4 x + \cos^4 x}\,dx$$

Divide N & D by $\cos^4 x$

$$2I = \frac{\pi}{2}\int_0^{\pi/2} \frac{\tan x \cdot \sec^2 x}{\tan^4 x + 1}\,dx$$

put $\tan^2 x = t$

$2\tan x \sec^2 x\, dx = dt$

$\tan x \sec^2 x\, dx = \frac{dt}{2}$

when $x = 0;\ t = 0$

when $x = \frac{\pi}{2}$; $t = \infty$

$$2I = \frac{\pi}{4}\int_0^{\infty} \frac{dt}{t^2+1}$$

$$2I = \frac{\pi}{4}\left(\tan^{-1} t\right)_0^{\infty}$$

$$2I = \frac{\pi}{4}\left(\tan^{-1}\infty - \tan^{-1} 0\right)$$

$$2I = \frac{\pi}{4}\left(\frac{\pi}{2}\right)$$

$\boxed{I = \frac{\pi^2}{16}}$ Ans

— x —

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