!! जय श्री राधे कृष्णा !!

XII ULTIMATE MATHEMATICS: BY: AJAY MITTAL

A.O.D CLASS NO: 9

Topic Maxima - Minima (without word problems)

① I^{st} derivative test (Local Maxima & local Minima)

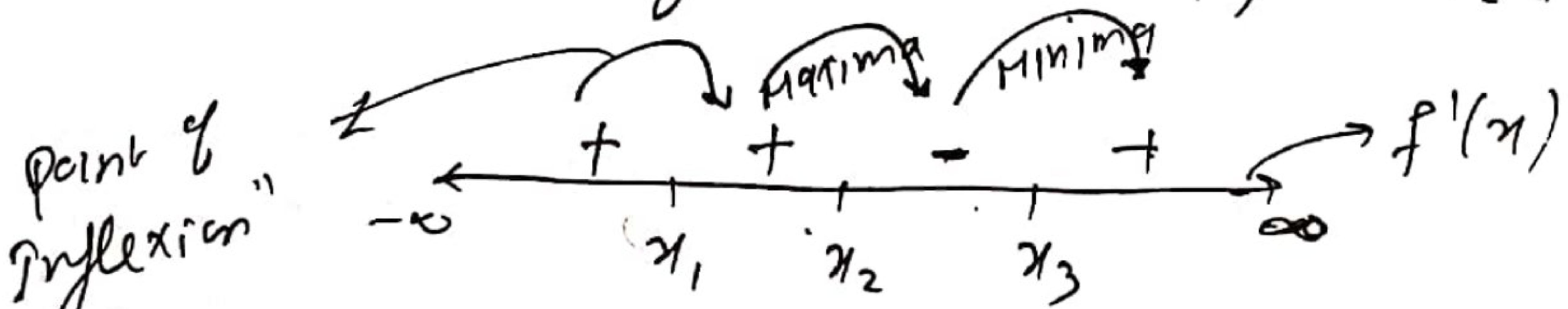
(·) given $f(x) =$

(·) find $f'(x) =$

(·) Simplify $f'(x) =$

(·) put $f'(x) = 0$

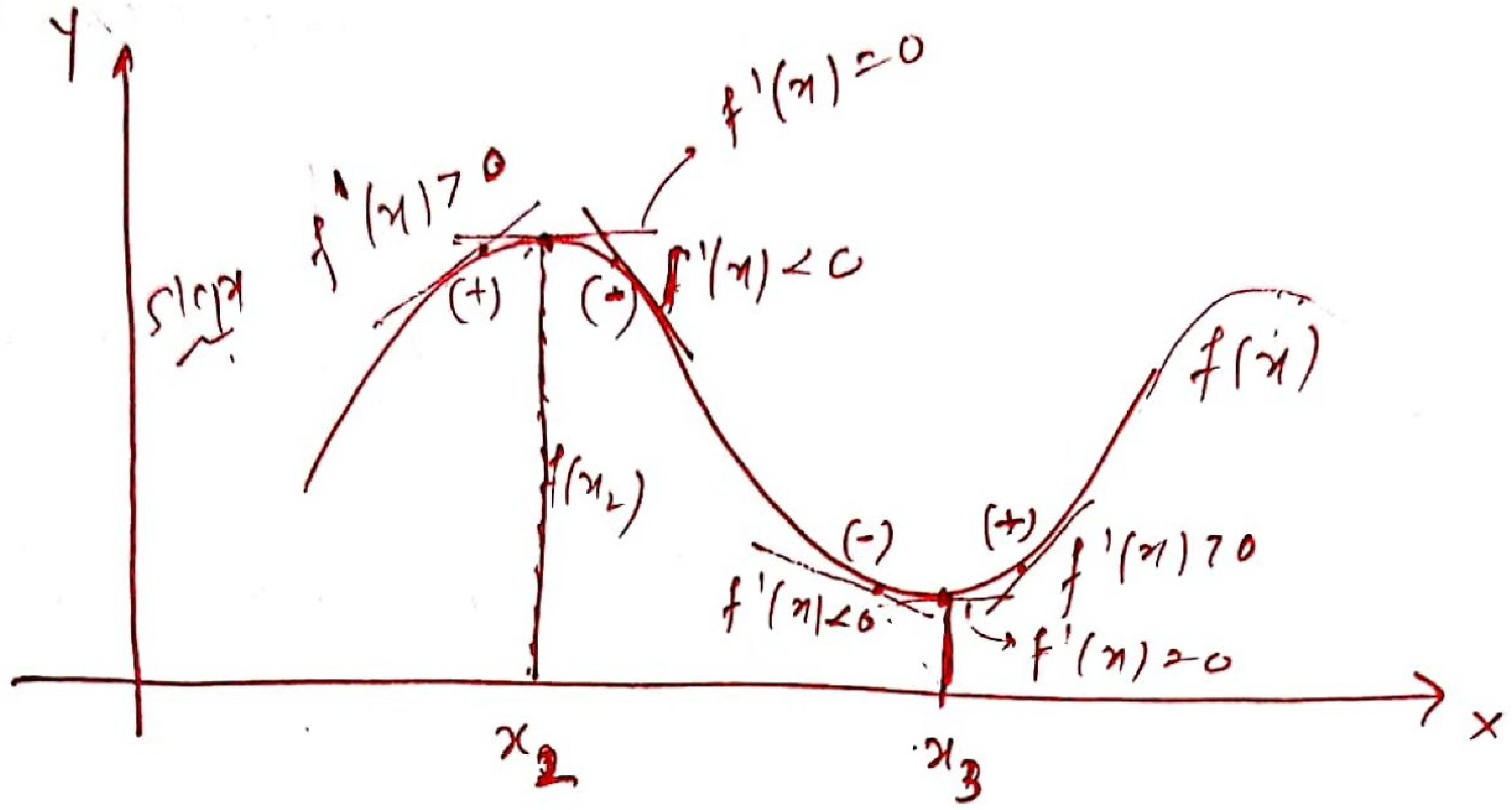
get value of x ; $x = x_1$, $x = x_2$, $x = x_3$



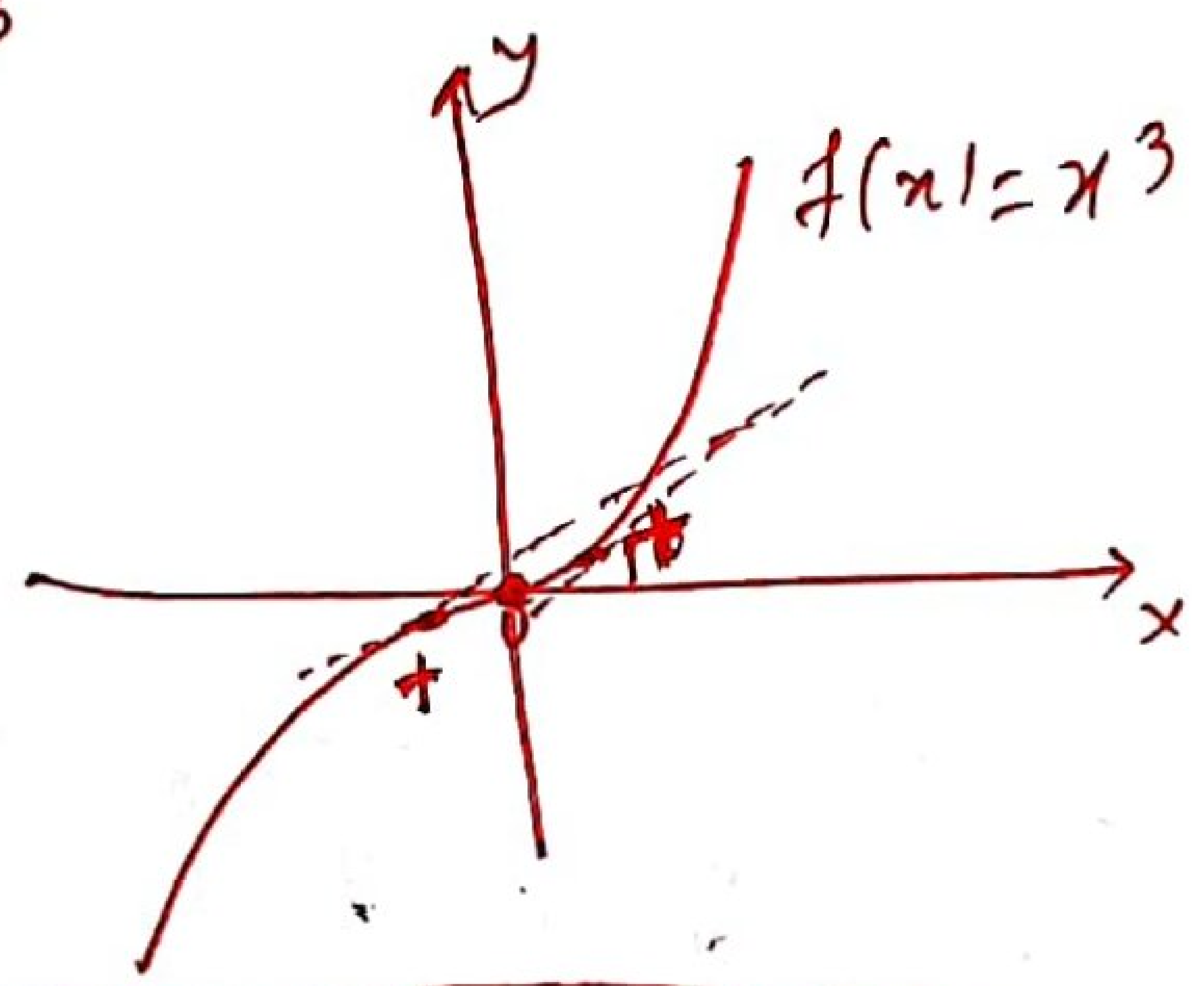
{ local Maximum value = $f(x_2)$
here $x = x_2$ is the point of local Maxima

{ $x = x_3$ → is the point of local Minima
$f(x_3)$ → is the local Maximum value

$x = x_1$ → is the point of Inflexion



e.g. $f(x) = x^3$



Imp — points at which $f'(x)=0$ (or) $f'(x)$ is not defined
"Critical points / Stationary points"

2nd derivative Test (Local Maxima & Local Minima)

(·) given $f(x)$
(·) find $f'(x) =$
(·) Simplify $f'(x) =$
(·) put $f'(x) = 0$

$x = x_1$, $x = x_2$, $x = x_3$

(·) $f''(x)$ find.

- $f''(x_3) > 0$ (point of Minima)
  $f(x_3) \to$ local Min. value
- $f''(x_2) < 0$ (point of local Max.)
  $f(x_2) \to$ local Max. value
- $f''(x_1) = 0$ (Test fails)
  go to Ist derivative test

$\overset{+\quad +}{\overline{\quad\ x_1 \quad}}$

(3) Absolute Maximum value & Absolute minimum value :-

✓ given closed Interval (always) e.g $x \in [2,5]$

(·) given $f(x) =$

(·) find $f'(x) =$

(·) simplify $f'(x) =$

(·) put $f'(x) = 0$

get value of x ; $x = 1$, $x = 3$, $x = 4$

(·) go to $f(x)$

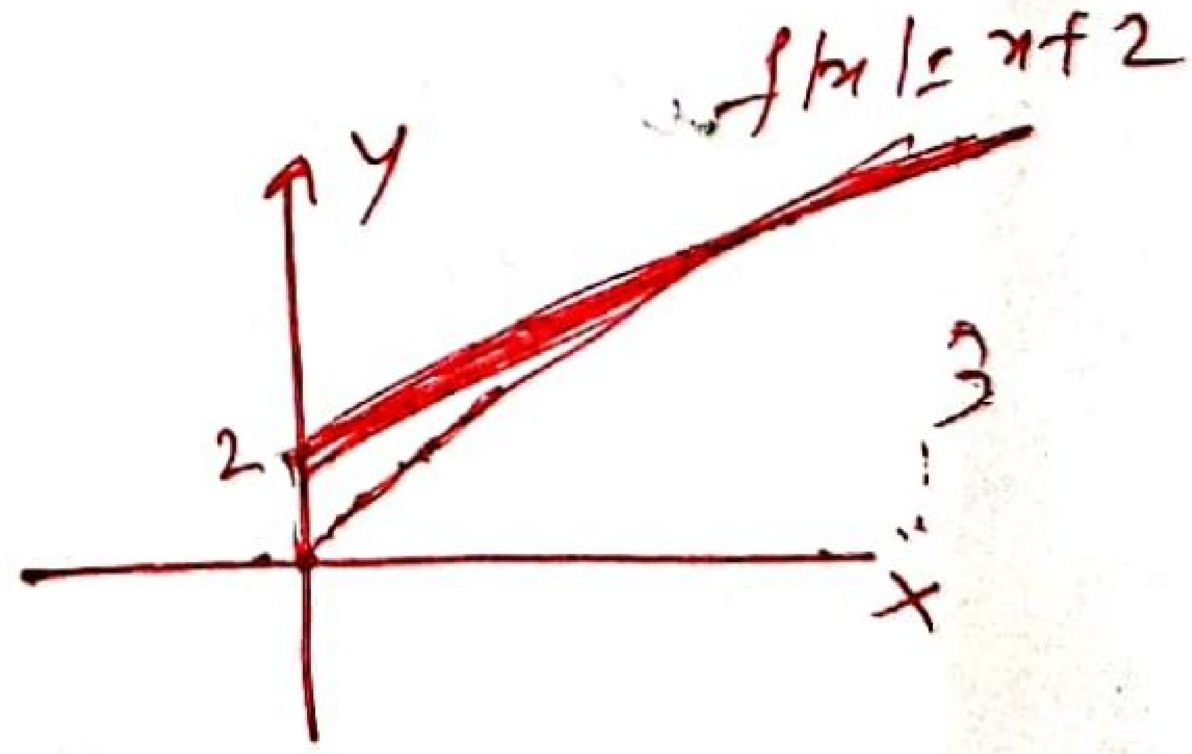
$f(3) = 10$ ← Absolute (global) ~~Max~~ Min value

$f(4) = 20$

$f(2) = 30$

$f(5) = 40$ ← Absolute Max value (global)

eg $f(x) = x+2$; $x \in (0,1)$



$f'(x) = 1$

given $[0,1]$ ✓

$f(0) = 2$ ← Min

$f(1) = 3$ ← Max

QUESTIONS:

Qns 1 find the points of local Maxima, points of local minima, local Max value & local Min value

$f(x) = 3x^4 + 4x^3 - 12x^2 + 12$

Soln Diff wrt x

$f'(x) = 12x^3 + 12x^2 - 24x$

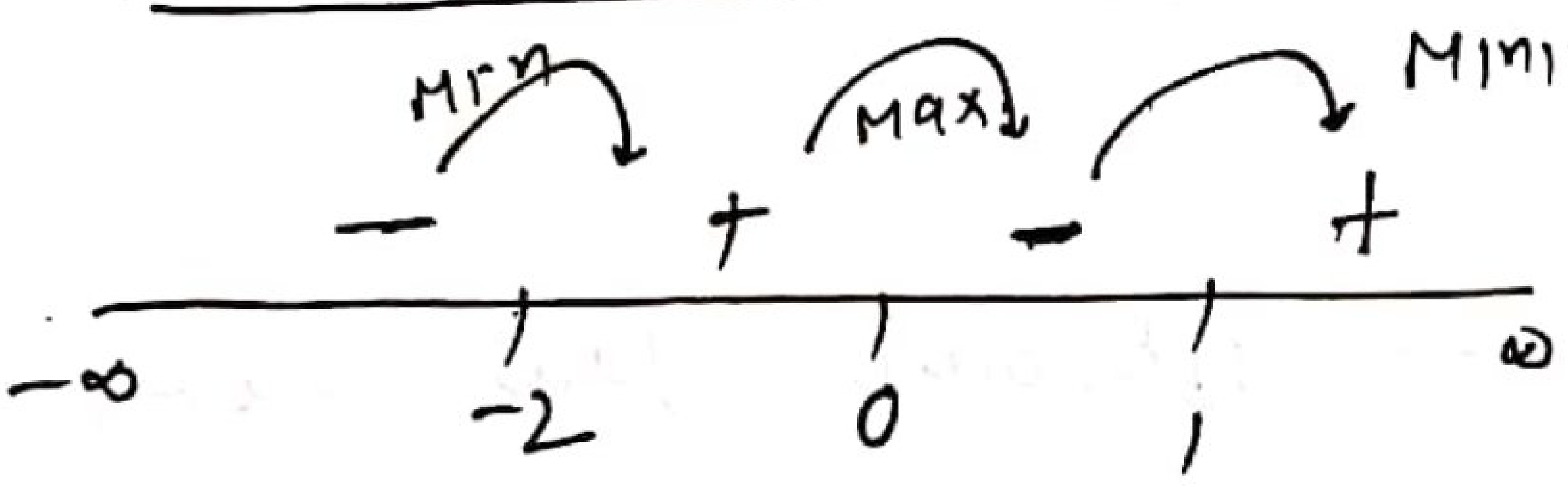
$= 12x(x^2 + x - 2)$

$f'(x) = 12x(x+2)(x-1)$

$(-)(+)(-)$
$(-)(-)(-)$

put $f'(x) = 0$

$\boxed{x = 0, \quad x = -2, \quad x = 1}$



∴ $x = -2$ & $x = 1$ are the points of local Minima

local Minimum values

$f(-2) =$

$f(1) =$

∴ $x = 0$ is the point of local Maxima

$f(0) =$ local Maximum value $= 12$ Ans

OR

$f''(x) = 36x^2 + 24x - 24$

$f''(1) = 36 + \cancel{24} - \cancel{24} = 36 > 0$ (point of local Min)

$f(1) \rightarrow$ local Min value (Procedure)

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Ques 2 → Find all (Previous) question)

$f(x) = \sin x - \cos x \quad ; \quad 0 < x < 2\pi$

Sol:

Diff w.r.t x

$f'(x) = \cos x + \sin x$

put $f'(x) = 0$

$\Rightarrow \cos x + \sin x = 0$

$\Rightarrow \sin x = -\cos x$

$\Rightarrow \tan x = -1$

$x = \pi - \pi/4 = 3\pi/4$ also $x = 2\pi - \pi/4 = 7\pi/4$

$\boxed{x = 3\pi/4, \quad x = 7\pi/4}$

$\cos(330) + \sin(330)$

$\cos(360-30) + \sin(360-30)$

$\frac{\sqrt{3}}{2} \quad -\frac{1}{2}$

Max Min

$+ \quad - \quad +$

$0 \quad 3\pi/4 \quad 7\pi/4 \quad 2\pi$

(i) $x = 3\pi/4$ → point of local Maxima

local Max value $= f(3\pi/4) = \sin(3\pi/4) - \cos(3\pi/4)$

$= \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} = \frac{2}{\sqrt{2}} = \sqrt{2}$

(ii) $x = 7\pi/4$ → point of local Minima

local Min. value $= f(7\pi/4)$

$= \sin(7\pi/4) - \cos(7\pi/4)$

$= \sin(2\pi - \pi/4) - \cos(2\pi - \pi/4)$

$= -\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} = -\frac{2}{\sqrt{2}} = -\sqrt{2}$ Ans

Ques 3 → Find the absolute Max. value & Absolute Minimum value of $f(x) = 4x - \frac{1}{2}x^2$

$x \in [-2, \frac{9}{2}]$

Sol

Diff. w.r.t x

$f'(x) = 4 - x$

put $f'(x) = 0$

$x = 4$

$f(4) = 16 - 8 = 8$ ← Absolute Max value

$f(-2) = -8 - 1 = -9$ ← Absolute Min value

$f(\frac{9}{2}) = 18 - \frac{81}{4} = -\frac{9}{4}$

Q. 4 Find the Maximum value & Minimum value

(1) $f(x) = |x+2| - 1$. (2) $f(x) = -|x+1| + 3$

(3) $f(x) = |\sin(4x) + 3|$ (4) $f(x) = x + 1$; $x \in (-1, 1)$

Sol

(1)

$|x+2| \geq 0$

$|x+2| - 1 \geq -1$

$f(x) \geq -1$

Min value $= -1$

No Max value

(2)

$|x+1| \geq 0$

$-|x+1| \leq 0$

$-|x+1| + 3 \leq 3$

$f(x) \leq 3$

Max value $= 3$

No Min value

(3)

$-1 \leq \sin(4x) \leq 1$

$2 \leq \sin(4x) + 3 \leq 4$

$0 \leq |\sin(4x) + 3| \leq 4$

$0 \leq f(x) \leq 4$

Min value $= 0$

Max. value $= 4$

(4)

$-1 < x < 1$

$0 < x + 1 < 2$

$0 < f(x) < 2$

No Max

No Min value

Qn 5 → Prove that the following functions do not have maxima or minima

(i) $f(x) = e^x$ (2) $f(x) = \log x$ (3) $f(x) = x^3 + x^2 + x + 1$

Soln (1) $f(x) = e^x$

$f'(x) = e^x$

put $f'(x) = 0 \Rightarrow e^x = 0$

but $e^x \neq 0$ $\{e^{-\infty} = 0\}$

(2) $f(x) = \log x$

$f'(x) = \frac{1}{x}$

$\frac{1}{x} \neq 0$

(3) $f(x) = x^3 + x^2 + x + 1$

$f'(x) = 3x^2 + 2x + 1 = 0$

↳ No real roots ; $(D < 0)$

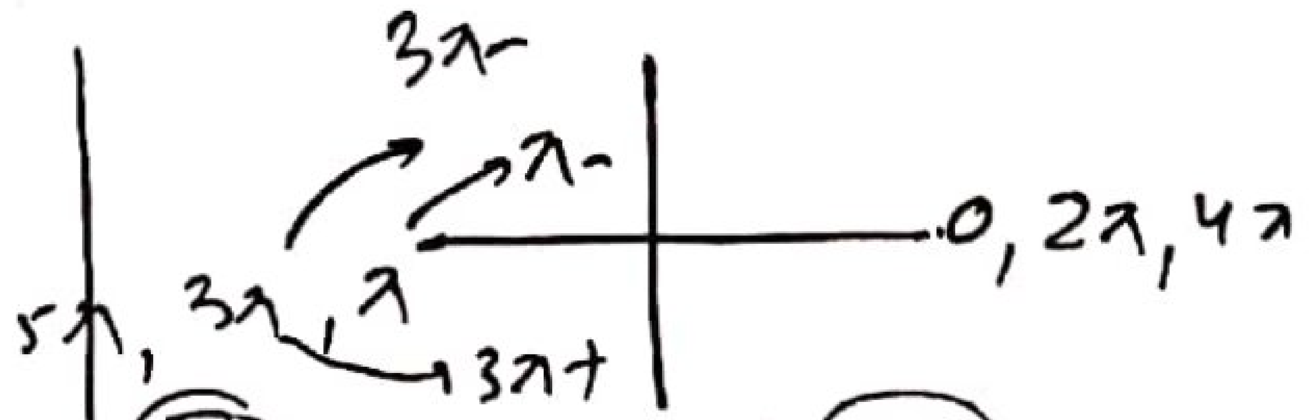
Qn-6 $f(x) = x + \sin(2x)$; $x \in [0, 2\pi]$

find absolute Max value & absolute Minimum value

Soln $f'(x) = 1 + 2\cos(2x)$

put $f'(x) = 0$

$\cos(2x) = -\frac{1}{2}$



II nd

$2x = \pi - \frac{\pi}{3} = 2\pi/3$

$\boxed{x = \pi/3}$

III

$2x = \pi + \pi/3 = 4\pi/3$

$\boxed{x = 2\pi/3}$

VI

$2x = 3\pi - \pi/3$

$2x = 8\pi/3$

$x = 4\pi/3$

VII

$2x = 3\pi + \pi/3$

$2x = 10\pi/3$

$x = 5\pi/3$

$f(x) = x + \sin(2x)$

$f(0) = 0 + \sin(0) = 0$

$f(2\pi) = 2\pi + \sin(4\pi) = 2\pi + 0 = 2\pi$

$f(\pi/3) = \frac{\pi}{3} + \sin\left(\frac{2\pi}{3}\right) = \frac{\pi}{3} + \frac{\sqrt{3}}{2}$

$f(2\pi/3) = \frac{2\pi}{3} + \sin\left(\frac{4\pi}{3}\right) = \frac{2\pi}{3} - \frac{\sqrt{3}}{2}$

$f(4\pi/3) = \frac{4\pi}{3} + \sin\left(\frac{8\pi}{3}\right) = \frac{4\pi}{3} + \sin\left(3\pi - \frac{\pi}{3}\right) = \frac{4\pi}{3} + \frac{\sqrt{3}}{2}$

$f(5\pi/3) = \frac{5\pi}{3} + \sin\left(\frac{10\pi}{3}\right) = \frac{5\pi}{3} + \sin\left(3\pi + \frac{\pi}{3}\right) = \frac{4\pi}{3} - \frac{\sqrt{3}}{2}$

$\therefore$ 0 is the Absolute Min value and it occurs at $x = 0$

✓ 2π is the absolute Max. value and it occurs at $x = 2\pi$ Ans

Qns 7 Find absolute Maximum & absolute Minimum value of $f(x) = 12x^{4/3} - 6x^{1/3}$; $x \in [-1, 1]$

Sol

$f(x) = 12x^{4/3} - 6x^{1/3}$

$f'(x) = 16x^{1/3} - 2x^{-2/3}$

$= 16x^{1/3} - \frac{2}{x^{2/3}}$

$f'(x) = \frac{16x - 2}{x^{2/3}} = \frac{2(8x - 1)}{x^{2/3}}$

$$f'(x) = \frac{2(8x-1)}{x^{2/3}}$$

$f'(x)=0 \Rightarrow x = 1/8$

Note $f'(x)$ not defined at $x=0$

$x = 0$ & $1/8$ are the critical points

given $f(x) = 12x^{4/3} - 6x^{1/3}$

$f(0) = 0$.

$f(1) = 12 - 6 = 6$

$f(-1) = 12(-1)^{4/3} - 6(-1)^{1/3} = 12 + 6 = 18$

$f(1/8) = 12\left(\frac{1}{8}\right)^{4/3} - 6\left(\frac{1}{8}\right)^{1/3} = \frac{12}{16} - \frac{6}{2} = \frac{3}{4} - 3 = \frac{-9}{4}$

$\therefore$ Absolute Min value $= -9/4$
Absolute Max value $= 18$ Ans

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