ULTIMATE MATHEMATICS (D-6)

← (WORKSHEET No: D-6)

← other properties of determinants →

(1) $|kA| = k^n|A|$ where $n \to$ is the order of the A matrix

e.g if 3x3 matrix, then $n=3$
4x4 matrix, then $n=4$

(2) $|Adj\ A| = |A|^{n-1}$

(3) $|A'| = |A|$

(4) $|AB| = |A||B|$

(5) $Adj(AB) = (Adj\ B)(Adj\ A)$

(6) $(AB)^{-1} = B^{-1}A^{-1}$

(7) $A(Adj\ A) = (Adj\ A)A = |A|I$

(8) $(A^{-1})^{-1} = A$

(9) $|A^{-1}| = \frac{1}{|A|}$

(10) $(A')^{-1} = (A^{-1})'$

(11) Determinant of a skew-symm matrix of odd order $= 0$

(12) $|-A| = \begin{cases} |A| ; & \text{if } n \to \text{even} \\ -|A| ; & \text{if } n \to \text{odd} \end{cases}$

$|(-1)A|$

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(13) If $\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{3} \end{vmatrix}$

then

$$a_{11} C_{11} + a_{12} C_{12} + a_{13} C_{13} = \Delta$$
$$a_{11} C_{11} + a_{21} C_{21} + a_{31} C_{31} = \Delta$$

then $a_{11} C_{21} + a_{12} C_{22} + a_{13} C_{23} = 0$

$$a_{11} C_{12} + a_{21} C_{22} + a_{31} C_{32} = 0$$

(14) Matrix A is invertible / non-singular if $|A| \neq 0$. $A^{-1} = \frac{1}{|A|} \cdot \text{Adj} A$

Matrix A is non-invertible / Singular if $|A| = 0$

(15) Matrix B is the Inverse of A / A is the Inverse of B if $\boxed{AB = I = BA}$

(16) ~~Proxy~~ ~~AdjA~~ $\text{Adj}(A') = (\text{Adj} A)'$

(17) $\text{Adj}(\text{Adj} A) = |A|^{n-2} \cdot A$

(18) $|\text{Adj}(\text{Adj} A)| = |A|^{(n-1)^2}$

(19) $|A^n| = |A|^n$

ULTIMATE MATHEMATICS

D-6 (BY: AJAY MITTAL: 9891067390)

Q1 Show that determinant of a skew symmetric matrix of odd order is zero

Sol. let A be any skew-symm matrix

$\Rightarrow A' = -A$

$\Rightarrow |A'| = |-A|$

$\Rightarrow |A| = -|A|$ ... { since $n \to$ odd $\therefore |-A| = -|A|$ }

$\Rightarrow |A| + |A| = 0$

$\Rightarrow 2|A| = 0$

$\Rightarrow |A| = 0$. proof

Qn 2 Show that $|A^{-1}| = \frac{1}{|A|}$

order $n \times n$

Sol. $|A^{-1}| = \left|\underbrace{\frac{1}{|A|}}_{k} \text{Adj}A\right| = \left(\frac{1}{|A|}\right)^n |\text{Adj}A|$

$= \frac{1}{|A|^n} . |A|^{n-1}$

$= \frac{1}{|A|}$

Qn 3 Show that Inverse of a matrix is unique

Sol let A be a matrix

$\rightarrow$ let B and C be its inverse

$\Rightarrow \quad AB = BA = I$

also $\quad AC = CA = I$

Now consider. $\quad AB = I$

$$C(AB) = CI$$

$$\Rightarrow (CA)B = C$$

$$\Rightarrow IB = C$$

$$\Rightarrow B = C$$

$\therefore$ Matrix A has unique Inverse proved

Qns-4 → order 3×3 ; $|A| = 3$; find $|2A| = ?$

Soln $|2A| = 2^3|A| = 8\times(3) = 24$ Ans

Qns 5 → order 2×2 ; $|3A| = 243$; find $|A| = ?$

Soln here $n=2$

$|3A| = 243$

$3^2|A| = 243 \quad \ldots \{|kA| = k^n|A|\}$

$\Rightarrow 9|A| = 243$

$\Rightarrow |A| = 27$ Ans

Qns 6 → order 3×3 ; $|A| = 5$; find $|Adj\,A|$

Solution here $n=3$

$|Adj\,A| = |A|^{n-1}$

$|Adj\,A| = (5)^{3-1} = 5^2 = 25$ Ans

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Ques 7 → order 3×3 ; $|Adj A| = 64$; Find $|A| = ?$

Soln here $n = 3$

$|Adj A| = 64$

$\Rightarrow |A|^{3-1} = 64$ --- $\{|Adj A| = |A|^{n-1}\}$

$\Rightarrow |A|^2 = 64$

$\Rightarrow |A| = \pm 8$ Ans

Ques 8 → order 3×3 ; $|2 Adj A| = 128$; find $|A'| = ?$

Soln here $n=3$; $|2 Adj A| = 128$

$= 2^3 |Adj A| = 128$ --- $\{|kA| = k^n |A|\}$

$\Rightarrow 8 |A|^{3-1} = 128$ --- $\{|Adj A| = |A|^{n-1}\}$

$\Rightarrow |A|^2 = 16$

$\Rightarrow |A| = \pm 4$

Since $|A| = |A'|$ $\therefore$ $|A'| = \pm 4$ Ans

Ques 9 → order 3×3 ; $|2AB| = 120$; $|A| = 5$, find $|-B| = ?$

Soln $n = 3$, $|A| = 5$

$|2AB| = 2^3 |AB| = 120$ --- $\{|kA| = k^n |A|\}$

$\Rightarrow 8|A||B| = 120$ --- $\{|AB| = |A||B|\}$

$\Rightarrow 8(5)|B| = 120$

$\Rightarrow |B| = 3$

Since n is odd ; $\therefore |-B| = -|B|$

$\therefore |-B| = -3$ Ans

Qn 10 → ~~with~~ $A = \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}$ find $A(\text{Adj } A)$ without actually finding Adj A

Soln we know that $A(\text{Adj} A) = |A| I$

$|A| = \begin{vmatrix} 3 & 2 \\ 1 & 4 \end{vmatrix} = 12 - 2 = 10$

$\therefore A(\text{Adj} A) = 10 I = \begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix}$ Ans

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Qn 11 → given $A = \begin{bmatrix} 1 & 2 \\ 3 & 9 \end{bmatrix}$ and $\text{Adj } B = \begin{bmatrix} -1 & 2 \\ 3 & 6 \end{bmatrix}$

find $\text{Adj}(BA)$

Solyn. $\text{Adj } A = \begin{bmatrix} 9 & -2 \\ -3 & 1 \end{bmatrix}$

we know that $\text{Adj}(BA) = (\text{Adj} A)(\text{Adj} B)$

$= \begin{bmatrix} 9 & -2 \\ -3 & 1 \end{bmatrix} \begin{bmatrix} -1 & 2 \\ 3 & 6 \end{bmatrix}$

$= \begin{bmatrix} -15 & 6 \\ 6 & 0 \end{bmatrix}$ Ans

Qn 12 → order 4×4 find $|A^{-1}|$

Soln $n = 4$ $|A^{-1}| = \left| \frac{1}{|A|} \cdot \text{Adj} A \right|$

$= \left(\frac{1}{|A|}\right)^4 \cdot |\text{Adj} A|$

$= \frac{1}{|A|^4} \cdot |A|^3$

$= \frac{1}{|A|}$ Ans

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Qn 13 → order 2×2

$A = \begin{bmatrix} 1 & 3 \\ 1 & 6 \end{bmatrix}$ find $Adj(AdjA) = ?$

Soln We know that

$Adj(AdjA) = |A|^{n-2} A$

$|A| = \begin{vmatrix} 1 & 3 \\ 1 & 6 \end{vmatrix} = 6-3 = 3$

$\therefore Adj(AdjA) = (3)^{3-2} A$

$= 3\begin{bmatrix} 1 & 3 \\ 1 & 6 \end{bmatrix}$

$Adj(AdjA) = \begin{bmatrix} 3 & 9 \\ 3 & 18 \end{bmatrix}$ Ans

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Qn 14 order 2×2 $A = \begin{bmatrix} 3 & 1 \\ 2 & -1 \end{bmatrix}$

find $|Adj(AdjA)| = ?$

Soln $|A| = \begin{vmatrix} 3 & 1 \\ 2 & -1 \end{vmatrix} = -3-2 = -5$

$|Adj(AdjA)| = |A|^{(n-1)^2}$

$= (-5)^{(2-1)^2}$

$= (-5)^1$

$= -5$ Ans

Qn 15 $A = \begin{bmatrix} 3 & -2 \\ -5 & 4 \end{bmatrix}$

find $|A^6| = ?$

Soln $|A| = \begin{vmatrix} 3 & -2 \\ -5 & 4 \end{vmatrix} = 12 - 10 = 2$

$|A^6| = |A|^6$

$= (2)^6$

$|A^6| = 64$ **Ans**

Qn 16 If $A = \begin{bmatrix} 3 & -1 & 2 \\ x & 1 & 4 \\ 1 & 2 & 3 \end{bmatrix}$ find value of x

so that matrix A is Invertible

Soln for Invertible matrix

$|A| \neq 0$

$3(3-8) + 1(3x-4) + 2(2x-1) \neq 0$

$\Rightarrow -15 + 3x - 4 + 4x - 2 \neq 0$

$\Rightarrow 7x \neq 21$

$\Rightarrow x \neq 3$ **Ans**.