!! जय श्री गिरिराज जी महाराज जय श्री राधे कृष्णा !!

ULTIMATE MATHEMATICS : BY AJAY MITTAL

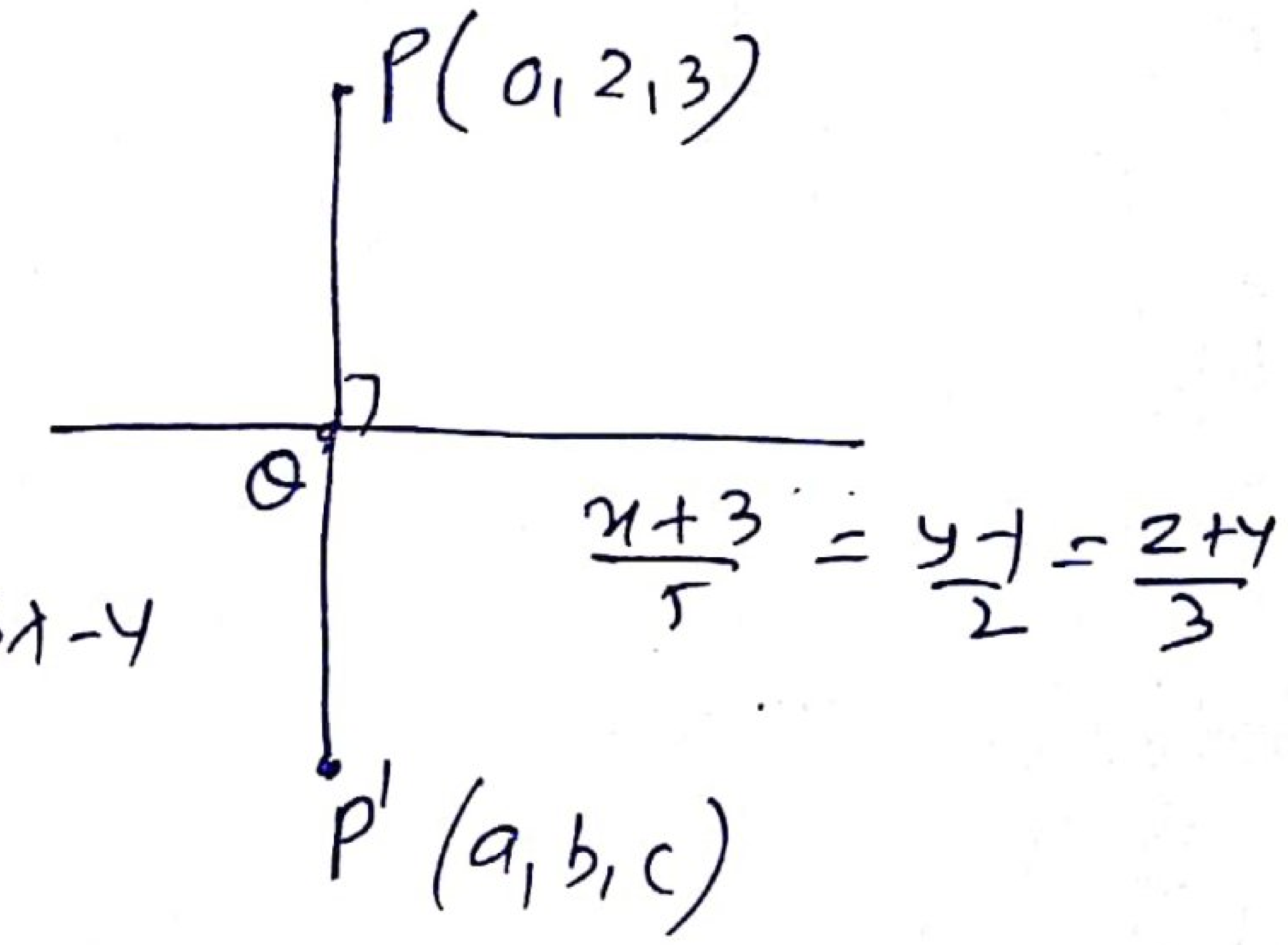
CHAPTER- 3-D $\boxed{\text{CLASS NO- 3}}$

IMP

Ques 1 → Find the foot of the perpendicular from the point $(0,2,3)$ on the line $\frac{x+3}{5} = \frac{y-1}{2} = \frac{z+4}{3}$.
Also find the perpendicular distance / length of ⊥r
Also find the image of the point $(0,2,3)$ in the line.
Also find the equation of the perpendicular

Soln

let $\frac{x+3}{5} = \frac{y-1}{2} = \frac{z+4}{3} = \lambda$



$\Rightarrow x = 5\lambda - 3; \quad y = 2\lambda + 1; \quad z = 3\lambda - 4$

let coordinate of Q is

$Q(5\lambda - 3,\ 2\lambda + 1,\ 3\lambda - 4)$

D.R's of PQ : $5\lambda - 3, \quad 2\lambda - 1, \quad 3\lambda - 7$

DR's of given line: $= 5, 2, 3$

Since PQ ⊥ given line

$\therefore 25\lambda - 15 + 4\lambda - 2 + 9\lambda - 21 = 0$ ---- $\left\{ \begin{array}{l} a_1a_2 + b_1b_2 \\ + c_1c_2 = 0 \end{array} \right.$

$\Rightarrow 38\lambda = 38 \quad \Rightarrow \lambda = 1$

$\therefore$ $\boxed{\text{foot of } \perp^r \text{ is } Q(2,3,-1)}$ Ans

(·) $\perp^r$ distance / length of $\perp^r$ from point P to the line

$$PQ = \sqrt{4+1+16} = \sqrt{21} \text{ units}$$

$$\boxed{PQ = \sqrt{21} \text{ units}} \text{ Ans}$$

(·) IMAGE let $P'(a,b,c)$ be the image of point P in the line

Q is the mid point of PP'

$2 = \frac{0+a}{2}$ | $3 = \frac{2+b}{2}$ | $-1 = \frac{3+c}{2}$

$\Rightarrow$ $a = 4$ | $b = 4$ | $c = -5$

$\therefore$ $\boxed{\text{Image } P'(4,4,-5)}$ Ans

(·) Equation of PQ (two point form)

$$\boxed{\frac{x-0}{2} = \frac{y-2}{1} = \frac{z-3}{-4}} \text{ Ans}$$

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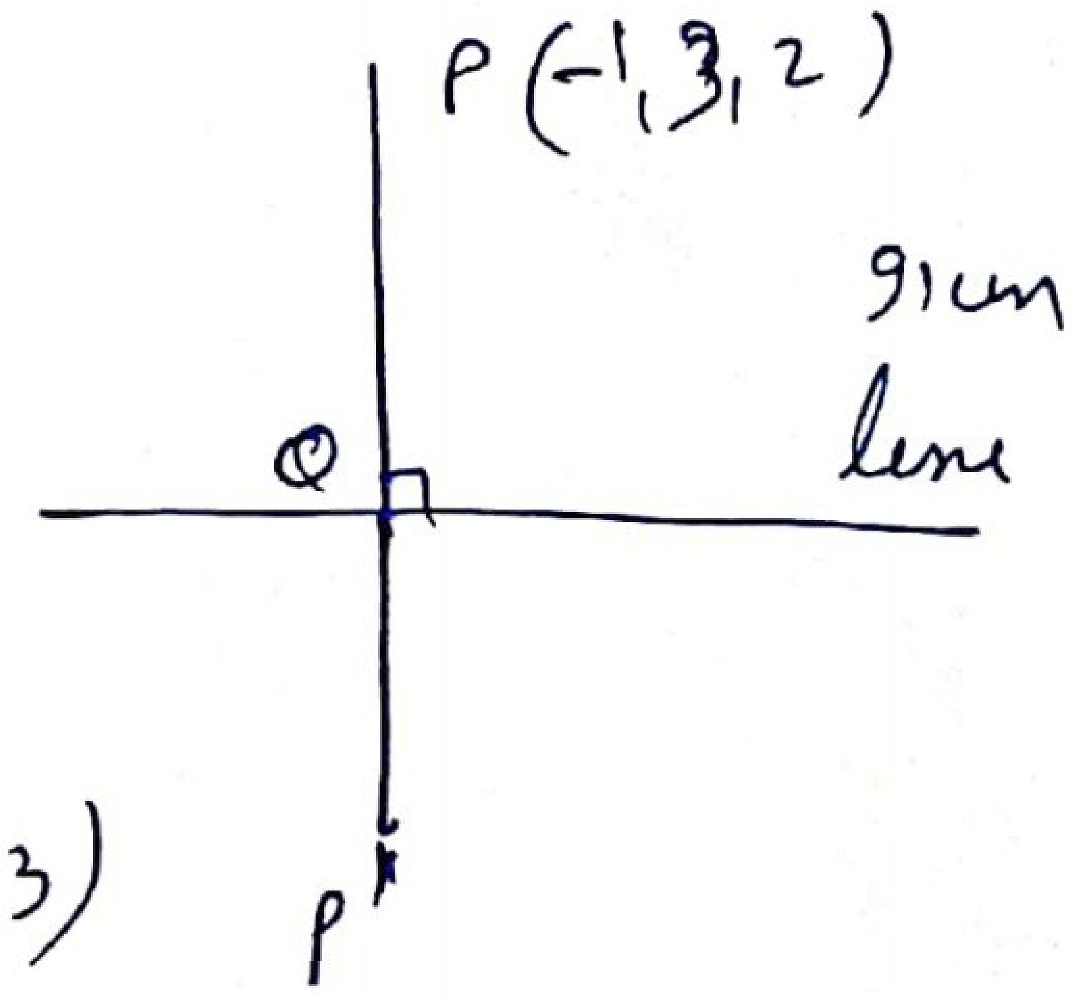
Qns 2 → Find the equation of the perpendicular drawn from the point $P(-1,3,2)$ to the line $\vec{r} = (2\hat{j} + 3\hat{k}) + \lambda(2\hat{i} + \hat{j} + 3\hat{k})$. Also find the perpendicular distance / length of perpendicular from point P to the line. Also find the distance b/w the point P and its image.

Soln

Converting given equation of line in Cartesian form

$$\frac{x-0}{2} = \frac{y-2}{1} = \frac{z-3}{3} = \lambda$$



$\Rightarrow x = 2\lambda \;;\; y = \lambda + 2 \;;\; z = 3\lambda + 3$

let coordinate of Q is $(2\lambda, \lambda+2, 3\lambda+3)$

DR's of PQ: $2\lambda+1, \; \lambda-1, \; 3\lambda+1$

DR's of line: $2, 1, 3$

Since $PQ \perp$ given line

$\Rightarrow \quad 4\lambda + 2 + \lambda - 1 + 9\lambda + 3 = 0$

$\Rightarrow \quad 14\lambda = -4$

$\Rightarrow \quad \lambda = -\frac{2}{7}$

$\therefore$ $\boxed{\text{foot of } \perp^r \text{ is } Q \quad \left(-\frac{4}{7}, \frac{12}{7}, \frac{15}{7}\right)}$

Now equation of $\perp^r$ PQ (Two point form)

$$\frac{x+1}{-\frac{4}{7}+1} = \frac{y-3}{\frac{12}{7}-3} = \frac{z-2}{\frac{15}{7}-2}$$

$$\Rightarrow \frac{x+1}{\frac{3}{7}} = \frac{y-3}{-\frac{9}{7}} = \frac{z-2}{\frac{1}{7}}$$

$$\Rightarrow \frac{x+1}{3} = \frac{y-3}{-9} = \frac{z-2}{1}$$

vector equation of $\perp^r$

$$\boxed{\vec{r} = (-\hat{i}+3\hat{j}+2\hat{k}) + \lambda(3\hat{i}-9\hat{j}+\hat{k})}$$ Ans

(ii) $\perp^r$ distance / length of $\perp^r$

$$PQ = \sqrt{\left(-\frac{4}{7}+1\right)^2 + \left(\frac{12}{7}-3\right)^2 + \left(\frac{15}{7}-2\right)^2}$$

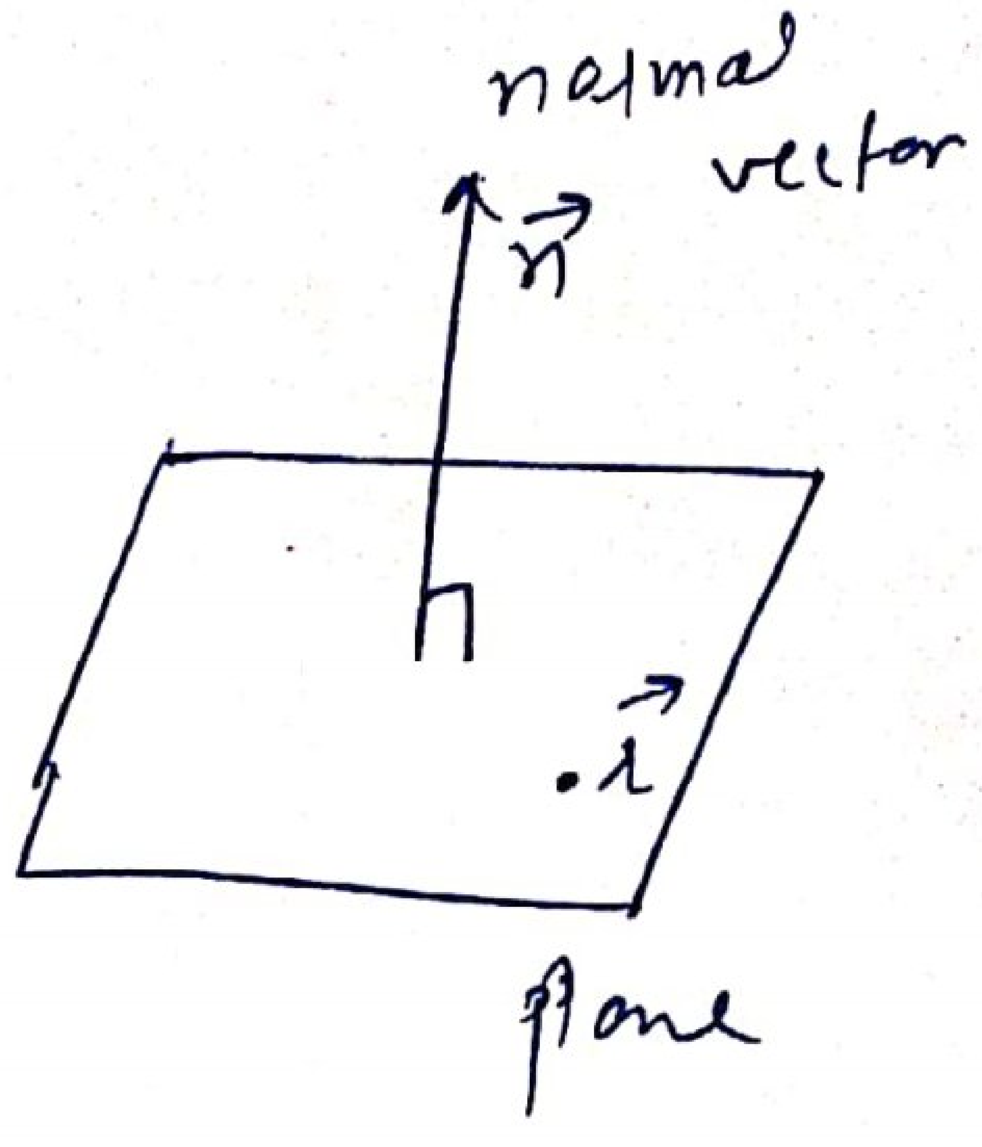
$$= \sqrt{\frac{9}{49} + \frac{81}{49} + \frac{1}{49}}$$

$$= \sqrt{\frac{91}{49}}$$

$$\boxed{PQ = \frac{\sqrt{91}}{7} \text{ units}}$$

(iii) $PP' = 2PQ = \frac{2\sqrt{91}}{7}$ units Ans

PLANES



(·) $\vec{n} = a\hat{i} + b\hat{j} + c\hat{k}$

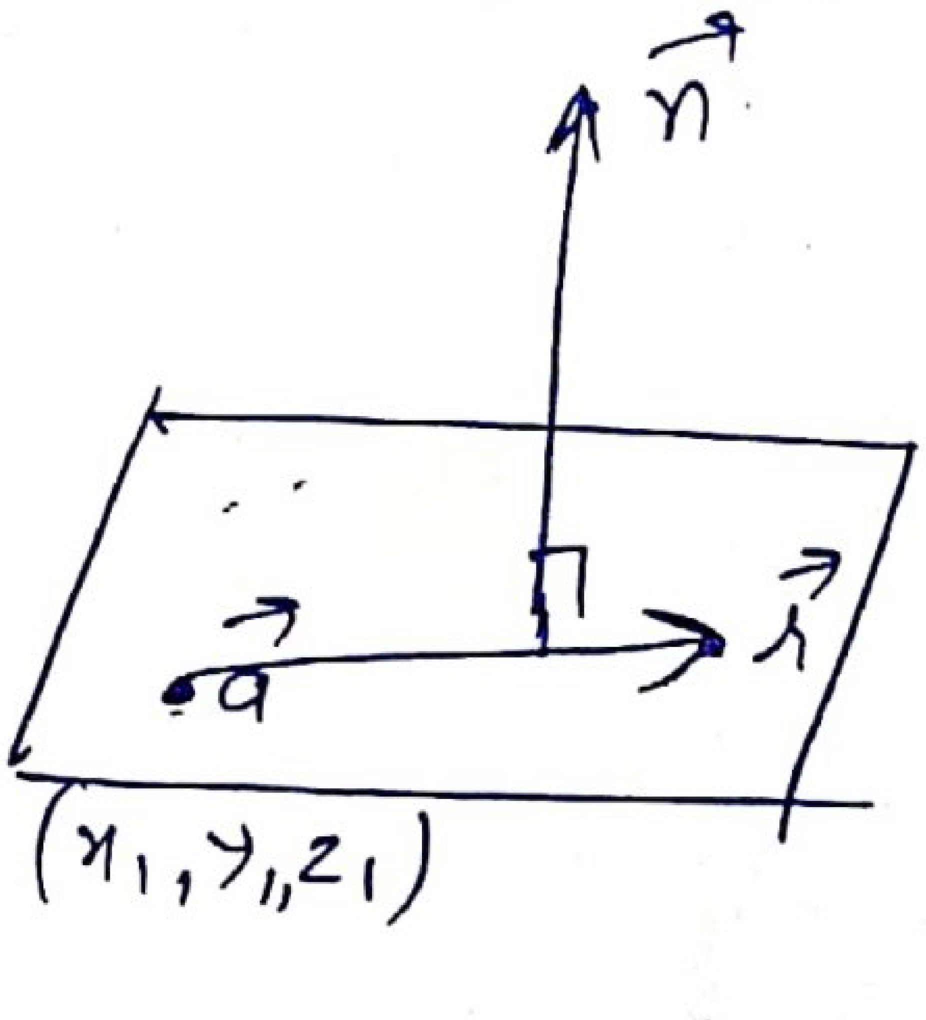
a, b, c are the DR's of $\vec{n}$

(·) general equation of plane

$$\boxed{\begin{array}{l} ax + by + cz = d \\ \vec{r}\cdot\vec{n} = d \end{array}}$$

where d → any constant

① equation of a plane passing through a given point and whose normal vector is known



$$\boxed{(\vec{r} - \vec{a})\cdot\vec{n} = 0}$$

where $\vec{a} = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$

Reason: $(\vec{r} - \vec{a}) \perp \vec{n}$

$\Rightarrow (\vec{r} - \vec{a})\cdot\vec{n} = 0$

Proof: $\vec{r}\cdot\vec{n} - \vec{a}\cdot\vec{n} = 0$

$$\boxed{\vec{r}\cdot\vec{n} = \vec{a}\cdot\vec{n}}$$

$$\boxed{(x - x_1)a + (y - y_1)b + (z - z_1)c = 0}$$

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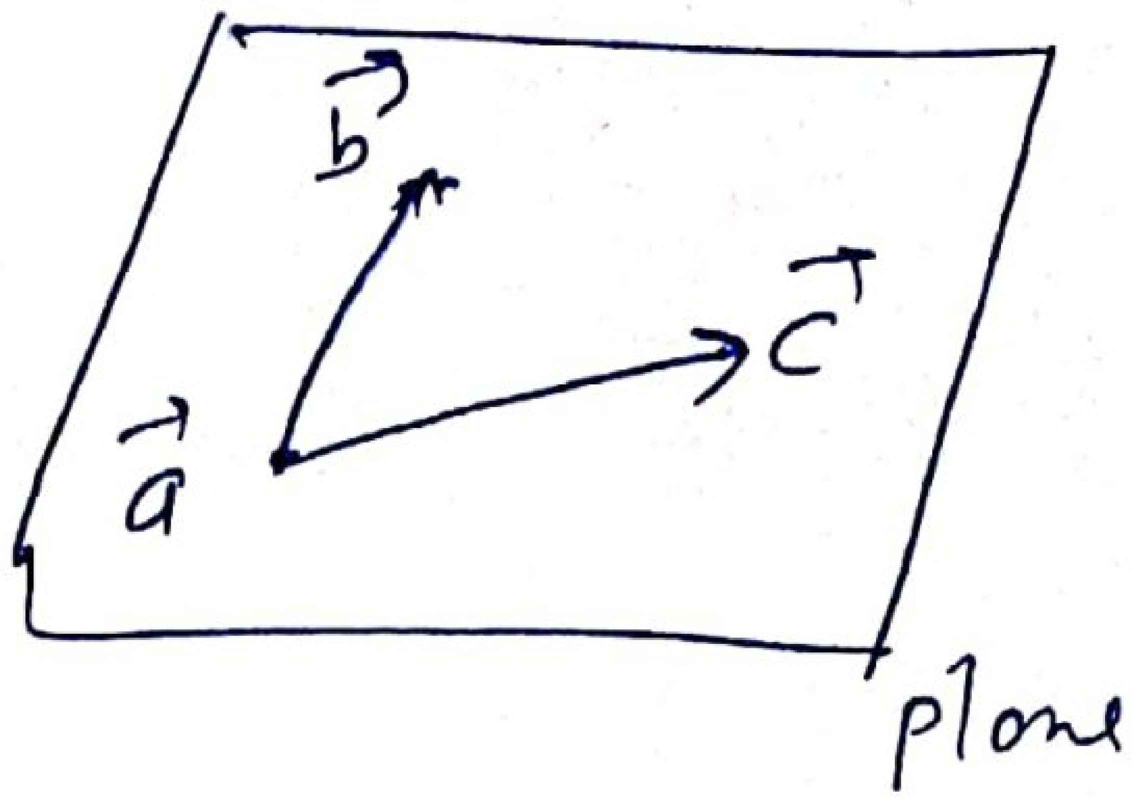
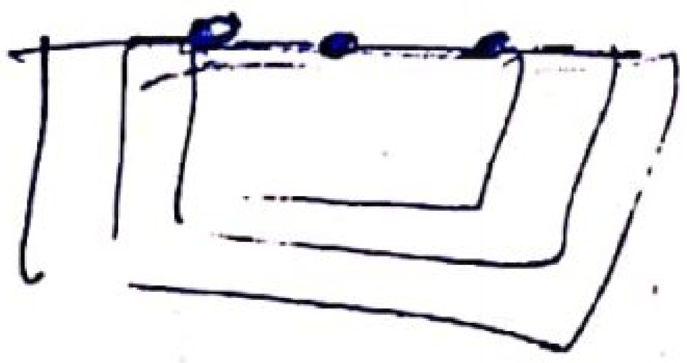
(6)

(2) Equation of a plane passing through three non-collinear points

$$\boxed{(\vec{r}-\vec{a})\cdot\left((\vec{b}-\vec{a})\times(\vec{c}-\vec{a})\right)=0}$$



$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \end{vmatrix}=0$$

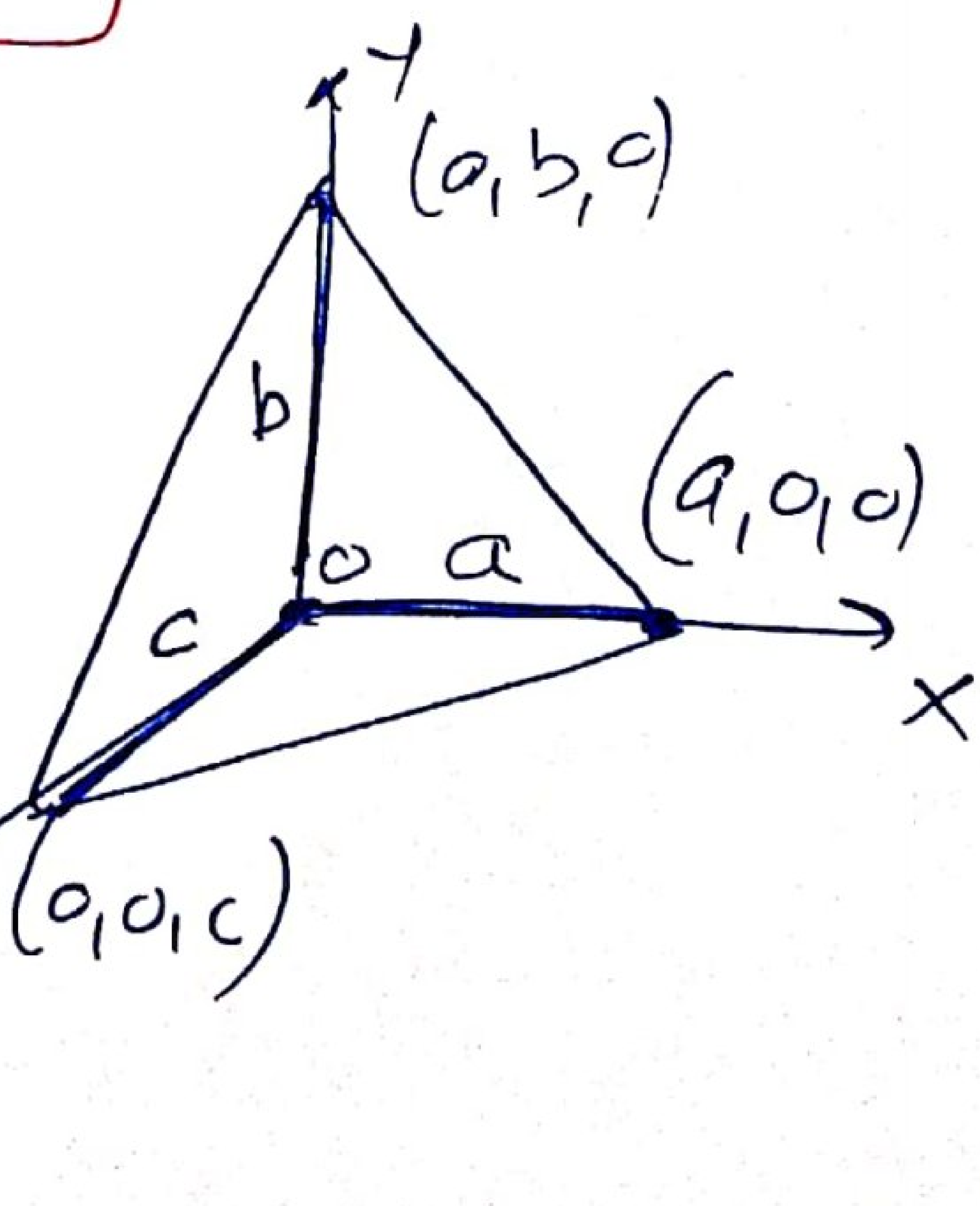


{Note: If three points are collinear then there will be infinite no. of planes.}

(3) ⇒ Intercept form of a plane

here a, b, c are the x-int, y-int, & z-int resp

$$\boxed{\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1}$$



Normal form of a plane

vector $\boxed{\vec{r} \cdot \hat{n} = d}$

where $d \rightarrow \perp^r$ distance of the plane from the origin

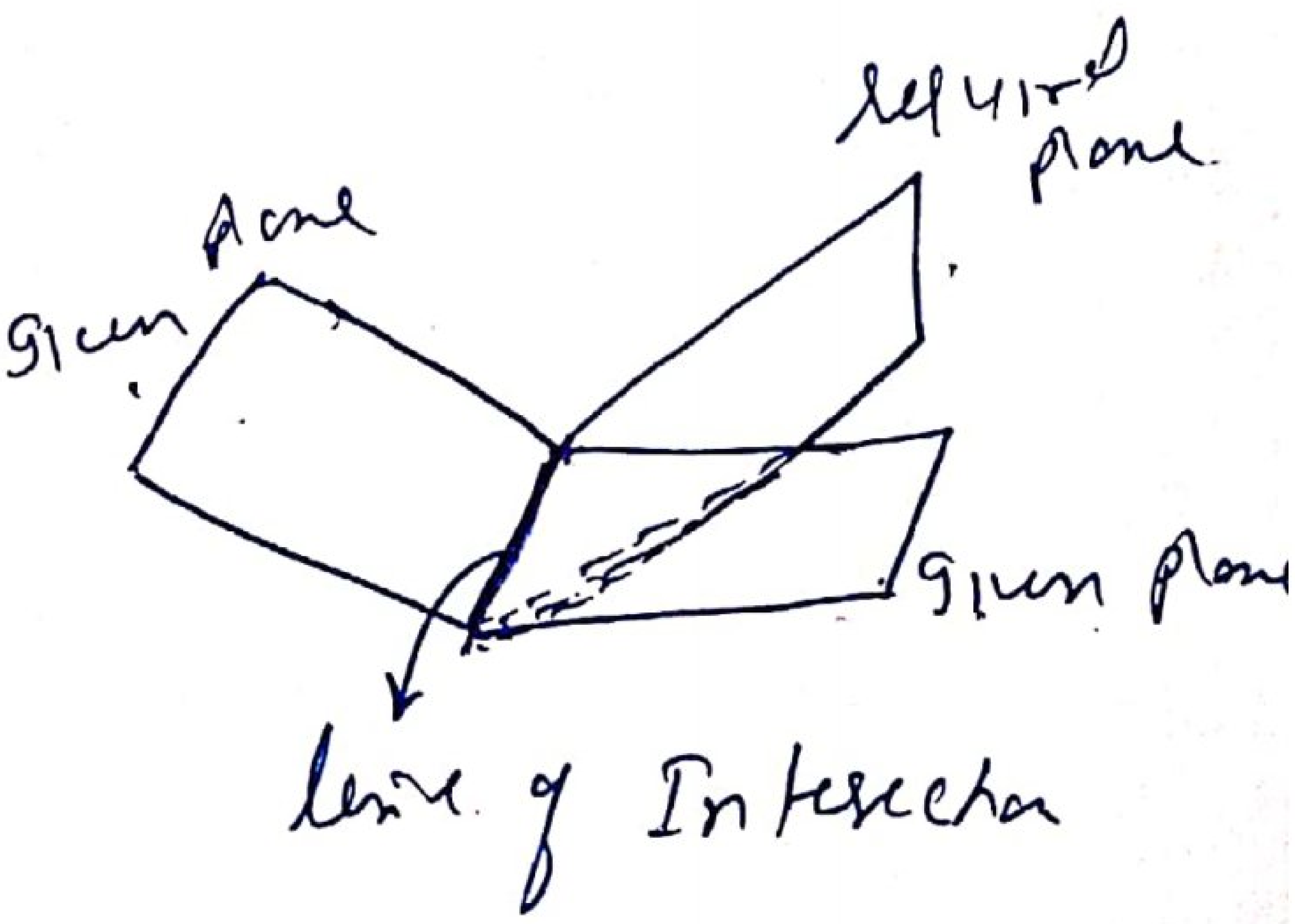
Cartesian $(x\hat{i} + y\hat{j} + z\hat{k}) \cdot (l\hat{i} + m\hat{j} + n\hat{k}) = d$

or $\boxed{lx + my + nz = d}$

Qu. (5) Equation of a plane passing the the Intersection or line of Intersection of two given planes

given planes

$$\vec{r} \cdot \vec{n_1} = d_1$$
$$\& \quad \vec{r} \cdot \vec{n_2} = d_2$$



Required plane

$$\boxed{\vec{r} \cdot (\vec{n_1} + \lambda \vec{n_2}) = d_1 + \lambda d_2}$$

where λ to be found out

(6) ~~ca~~ angle b/w two planes

$$\cos\theta = \frac{|\vec{n_1} \cdot \vec{n_2}|}{|\vec{n_1}||\vec{n_2}|}$$

$$\cos\theta = \frac{|a_1a_2 + b_1b_2 + c_1c_2|}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}$$

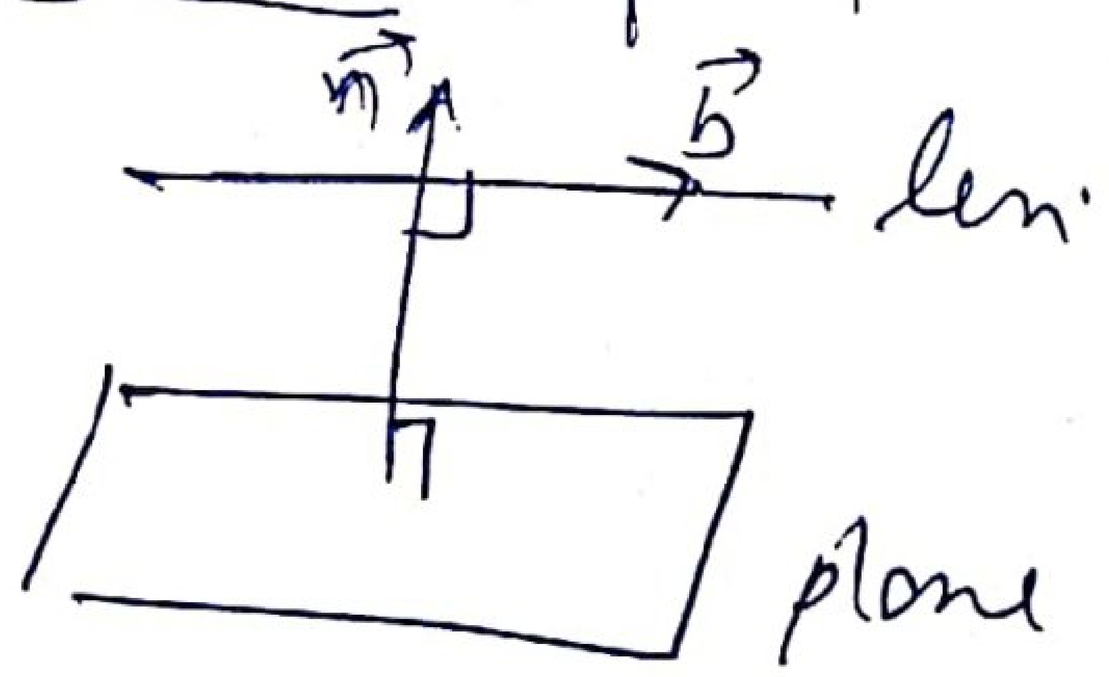
If two planes are $\perp^r$ then $\vec{n_1} \cdot \vec{n_2} = 0$

$a_1a_2 + b_1b_2 + c_1c_2 = 0$

If two planes are $\parallel^l$ $\vec{n_1} \parallel \vec{n_2} \Rightarrow \vec{n_1} = \lambda \vec{n_2}$

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

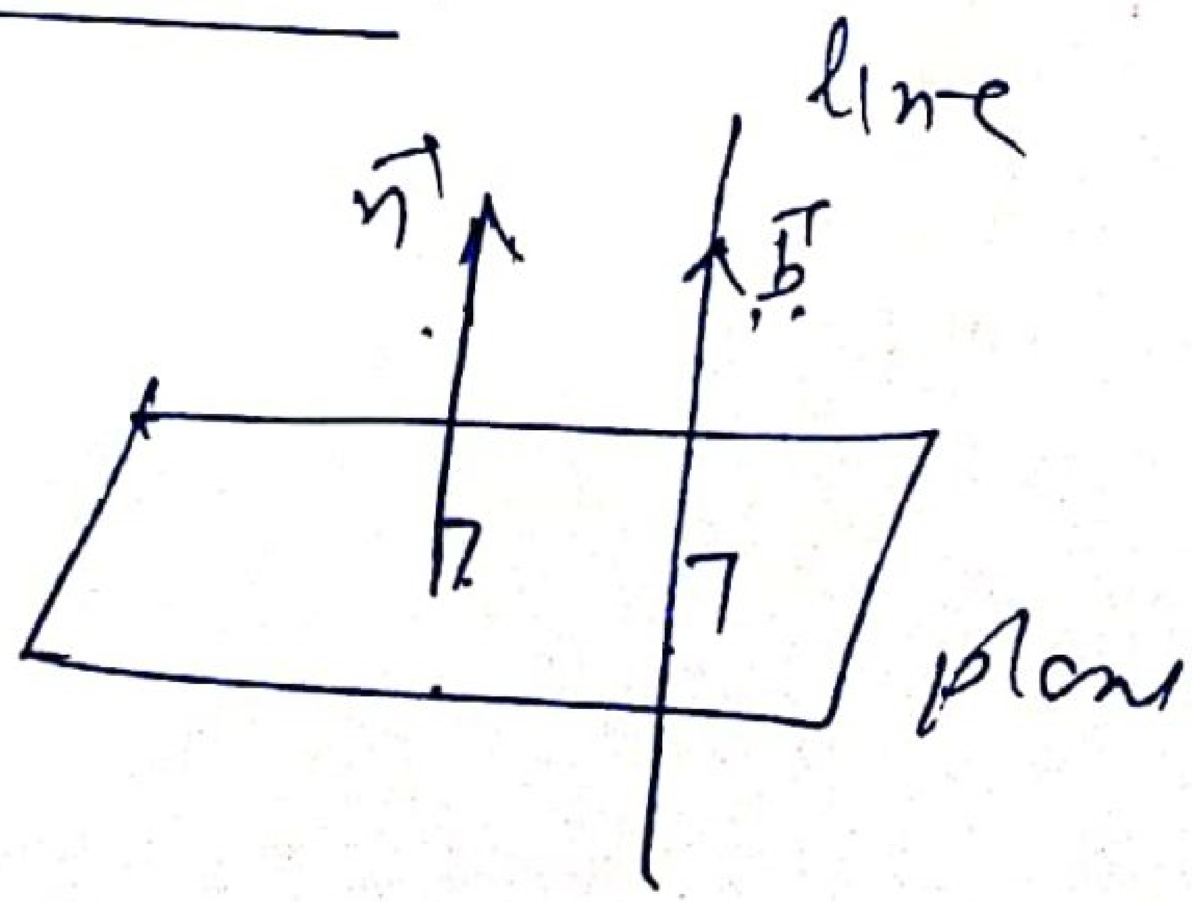
If line is parallel to the plane



$\vec{n} \perp \vec{b}$

$\Rightarrow \vec{n} \cdot \vec{b} = 0$

If line is $\perp^r$ to the plane



$\vec{n} \parallel^l$ line

$\therefore \vec{n} \parallel \vec{b}$

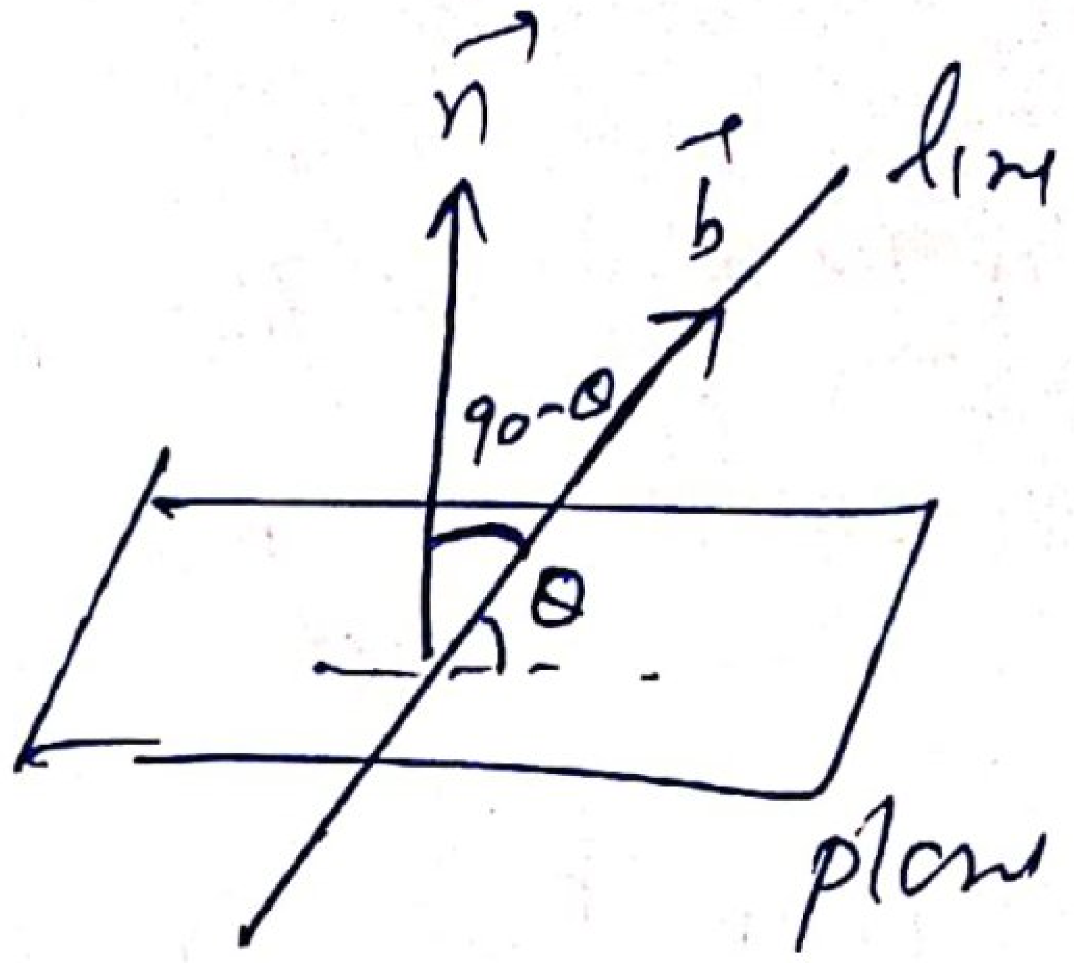
$\Rightarrow \vec{n} = \lambda \vec{b}$

7) Angle b/w line and plane

line $\vec{r} = \vec{a} + \lambda\vec{b}$

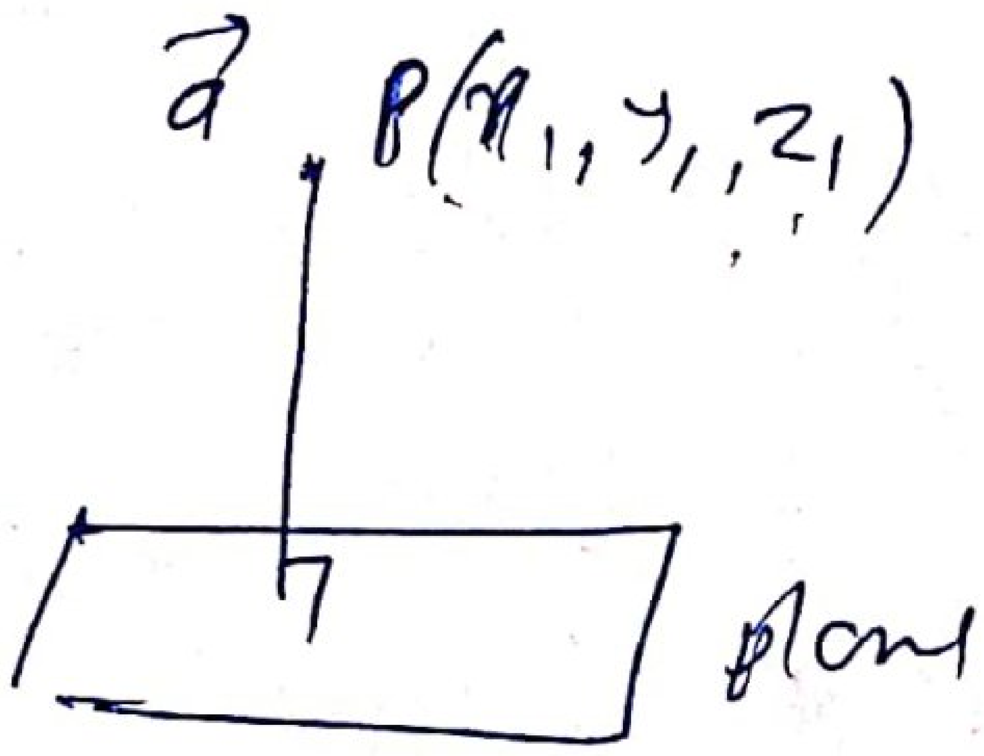
plane $\vec{r} \cdot \vec{n} = d$

$$\sin\theta = \frac{|\vec{b} \cdot \vec{n}|}{|\vec{b}||\vec{n}|}$$



8) ⊥r distance / distance of a point from a plane

$$\text{Distance} = \frac{|\vec{a} \cdot \vec{n} - d|}{|\vec{n}|}$$



$\vec{r} \cdot \vec{n} = d$

$ax + by + cz = d$

$$\text{Distance} = \frac{|ax_1 + by_1 + cz_1 - d|}{\sqrt{a^2 + b^2 + c^2}}$$

— x —

WORKSHEET NO: 3 | Chapter - 3 D

Ques. 1 → Find the foot of the perpendicular from $(0,2,7)$ on the line $\frac{x+2}{-1} = \frac{y-1}{3} = \frac{z-3}{-2}$ **Ans** $\left(-\frac{5}{2}, \frac{5}{2}, 2\right)$

Ques 2 → Find the length of the perpendicular draw from the point $\hat{i}+6\hat{j}+3\hat{k}$ to the line
$\vec{r} = (\hat{j}+2\hat{k}) + \lambda(\hat{i}+2\hat{j}+3\hat{k})$ **Ans** $\sqrt{13}$ units

Ques 3 → Find the image of the point $(1,6,3)$ in the line $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$ **Ans** $(1,0,7)$

Ques. 4 → $A(1,0,4)$ $B(0,-11,3)$ $C(2,-3,1)$ are three points and D is the foot of $\perp^r$ from A on BC. Find the distance b/w point A and its image on line BC

Hint: First find equation of BC ; $D\left(\frac{22}{9}, \frac{-11}{9}, \frac{5}{9}\right)$

Hint: $AA' = 2AD$

Ques. 5 → Vertices B and C of ΔABC lie along the line $\frac{x+2}{2} = \frac{y-1}{1} = \frac{z-0}{4}$. Find the area of the triangle given that A has coordinates $(1,-1,2)$ and line segment BC has length 5 units **Area** $= \frac{5}{2}\sqrt{\frac{71}{7}}$

Ques. 6 → The line $\frac{x-3}{3} = \frac{y-2}{1} = \frac{z-1}{0}$ is

(a) parallel to x-axis
(b) parallel to y-axis
(c) parallel to z-axis
(d) perpendicular to z-axis

Ans (d)

Qn 7 → The D.R's of a line ⊥ʳ to the lines $\frac{x-7}{2} = \frac{y+17}{-3} = \frac{z-6}{1}$ and $\frac{x+5}{1} = \frac{y+3}{2} = \frac{z-4}{-2}$ are proportional to

Ans (a)

(a) 4, 5, 7 (b) 4, −5, 7 (c) 4, −5, −7 (d) −4, 5, 7

Qn. 8 → Find the equation of the plane passing through the points (1, 1, 0) ; (1, 2, 1) ; (−2, 2, −1) Ans $2x+3y-3z=5$

Qn. 9 → Show that the four points (0, −1, −1), (−4, 4, 4) (4, 5, 1) (3, 9, 4) are coplanar

Hint Find equation of plane using any three points and then Satisfy fourth point in the equation formed.

Qn. 10 → A plane meets the coordinate axes in A, B, C such that the centroid of triangle ABC is the point (p, q, r). Show that the equation of the plane is $\frac{x}{p} + \frac{y}{q} + \frac{z}{r} = 3$

Qn 11 → Reduce equation of plane $2x-y+z-5=0$ in to Intercept form & hence find Intercepts

Ans $\frac{5}{2}, -5, 5$

Qn. 12 → Find the equation of plane in vector and cartesian form Passing through the point (3, −3, 1) and normal to the line joining the points (3, 4, −1) and (2, −1, 5) Ans $\vec{r}\cdot(\hat{i}+5\hat{j}-6\hat{k}) = -18$; $x+5y-6z=-18$

Qn 13 → A vector $\vec{n}$ of magnitude 8 units is inclined to x-axis at 45°, y-axis at 60° and an acute angle with z-axis. If a plane passes through a point $(\sqrt{2}, -1, 1)$ and is normal to $\vec{n}$. Find equation of plane in vector form

ANS $\vec{r}\cdot(\sqrt{2}\hat{i}+\hat{j}+\hat{k}) = 2$

Q.No. 14 → Find the angle between the line $\frac{x+1}{3} = \frac{y-1}{2} = \frac{z-2}{4}$ and the plane $2x+y-3z+4=0$

ANS $\theta = \sin^{-1}\left(\frac{-4}{\sqrt{406}}\right)$

Q.No. 15 → Find the equation of the plane determined by the points $A(3,-1,2)$ $B(5,2,4)$ $C(-1,-1,6)$ and hence find the distance between the plane and the point $P(6,5,9)$

ANS $3x-4y+3z=19$

$\frac{4}{\sqrt{34}}$ units

Q.No. 16 → If the points $(1,1,\lambda)$ & $(-3,0,1)$ are equidistant from the plane $\vec{r}\cdot(3\hat{i}+4\hat{j}-12\hat{k})+13=0$. Find value of λ

ANS $\lambda=1$ or $\lambda = 7/3$

Q.No. 17 → Find the value of λ such that line $\frac{x-2}{6} = \frac{y-1}{\lambda} = \frac{z+5}{-4}$ is perpendicular to the plane $3x-y-2z=7$

ANS $\lambda=26$

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