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ULTIMATE MATHEMATICS → 1

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CHAPTER $\boxed{\text{A.O.D}}$ CLASS NO: 3

TOPIC: DERIVATIVE AS A RATE MEASURE

(·) $\frac{dy}{dt}$ → rate of change of y wrt 't' (time)

→ rate measure

(·) Mensuration

$\frac{dv}{dt}$: cm^3/sec

$\frac{dr}{dt}$: cm/sec

$\frac{ds}{dt}$: cm^2/sec

(·) $\frac{dv}{dt} = +ve$ (Increase)

$\frac{dv}{dt} = -ve$ (decrease)

Que 1 → An edge of a variable cube is increasing at the rate of 3 cm/sec. How fast is the volume of the cube increasing when the edge is 10 cm ~~long~~ long?

Soln let x → side (edge) of the cube

given: $\frac{dx}{dt} = 3\, cm/sec$

$x = 10$ cm (Initial value)

To find: $\frac{dv}{dt} = ?$

$V = x^3$

Diff wrt 't' $\frac{dv}{dt} = 3x^2 \cdot \frac{dx}{dt}$

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A.O.D. (Class No 3) (2)

$$\Rightarrow \frac{dv}{dt} = 3(100)(3)$$
$$= 900\ cm^3/sec$$

$\therefore$ volume of cube is increasing at the rate $900\ cm^3/sec$ Ans

Ques 2 → The volume of a cube is increasing at the rate of $8\ cm^3/sec$. How fast is the surface area increasing when the length of an edge is 12 cm?

Soln Let $x \to$ side of cube

Given. $\frac{dv}{dt} = 8\ cm^3/sec$

$x = 12$ cm (Initial value)

To find: $\frac{ds}{dt} = ?$

we have $V = x^3$

Diff wrt t $\frac{dv}{dt} = 3x^2 \cdot \frac{dx}{dt}$

$$8 = 3(144)\frac{dx}{dt}$$

$$\Rightarrow \frac{dx}{dt} = \frac{8}{3 \times 144}\ cm/sec$$

we have $S = 6x^2$

Diff wrt 't' $\frac{ds}{dt} = 12x \cdot \frac{dx}{dt}$

$$\frac{ds}{dt} = \cancel{12 \times 12} \times \frac{8}{\cancel{3 \times 144}} = \frac{8}{3}\ cm^2/sec$$

$\therefore$ S.A of cube is increasing at the rate $8/3\ cm^2/sec$ Ans

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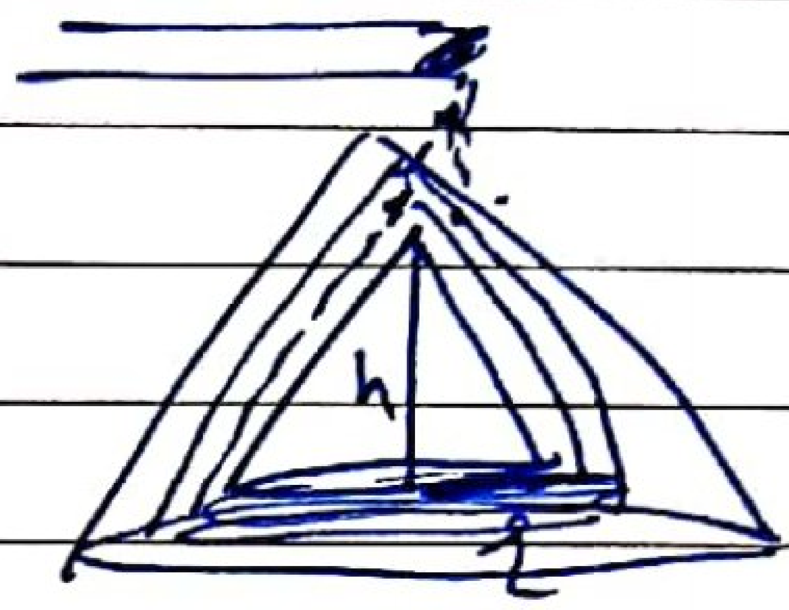
A.OD (Class No: 3) (3)

Ques 3 → Sand is pouring from a pipe at the rate of $12 cm^3/sec$. The falling sand forms a cone on the ground in such a way that the height of the cone is always one-sixth of the radius of the base. How fast is the height of the sand cone increasing when the height is 4cm?

Soln

$\frac{dv}{dt} = 12 cm^3/sec$



$h = \frac{1}{6}r$

$h = 4cm$ (Instantaneous value)

to find: $\frac{dh}{dt}$

we have $V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi(6h)^2 h$ ... $\{\because h = \frac{1}{6}r\}$

$V = 12\pi h^3$

Diff w.r.t 't'

$\frac{dv}{dt} = 12\pi\left(3h^2 . \frac{dh}{dt}\right)$

$\cancel{12} = \cancel{12}\pi(3\times16)\frac{dh}{dt}$

$\frac{dh}{dt} = \frac{1}{48\pi} cm/sec$

$\therefore$ height of the cone is $\uparrow$ at the rate $\frac{1}{48\pi} cm/sec$ $\frac{dh}{dt}$

Qn. 4 → A particle moves along the curve $6y = x^3 + 2$. Find the points on the curve at which the y-coordinate is changing 8 times as fast as the x-coordinate.

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A.O.D (Class No: 3) (4)

Sol: Given equation of curve

$$6y = x^3 + 2$$

$$\frac{dy}{dt} = 8\frac{dx}{dt}$$

to find point on the curve (x, y)

Diff equation of curve w.r.t 't'

$$6\frac{dy}{dt} = 3x^2 \cdot \frac{dx}{dt}$$

$$\Rightarrow 6\left(8\frac{dx}{dt}\right) = 3x^2\frac{dx}{dt} \quad \cdots \left\{\because \text{given } \frac{dy}{dt} = 8\frac{dx}{dt}\right\}$$

$$\Rightarrow 48 = 3x^2 \quad \cdots \left\{\because \frac{dx}{dt} \neq 0\right\}$$

$$\Rightarrow x^2 = 16$$

$$\Rightarrow x = \pm 4$$

put in eq of curve

put $x = 4$: $6y = 64 + 2 = 66 \Rightarrow y = 11$

$x = -4$: $6y = -64 + 2 = -62 \Rightarrow y = \frac{-62}{6} = \frac{-31}{3}$

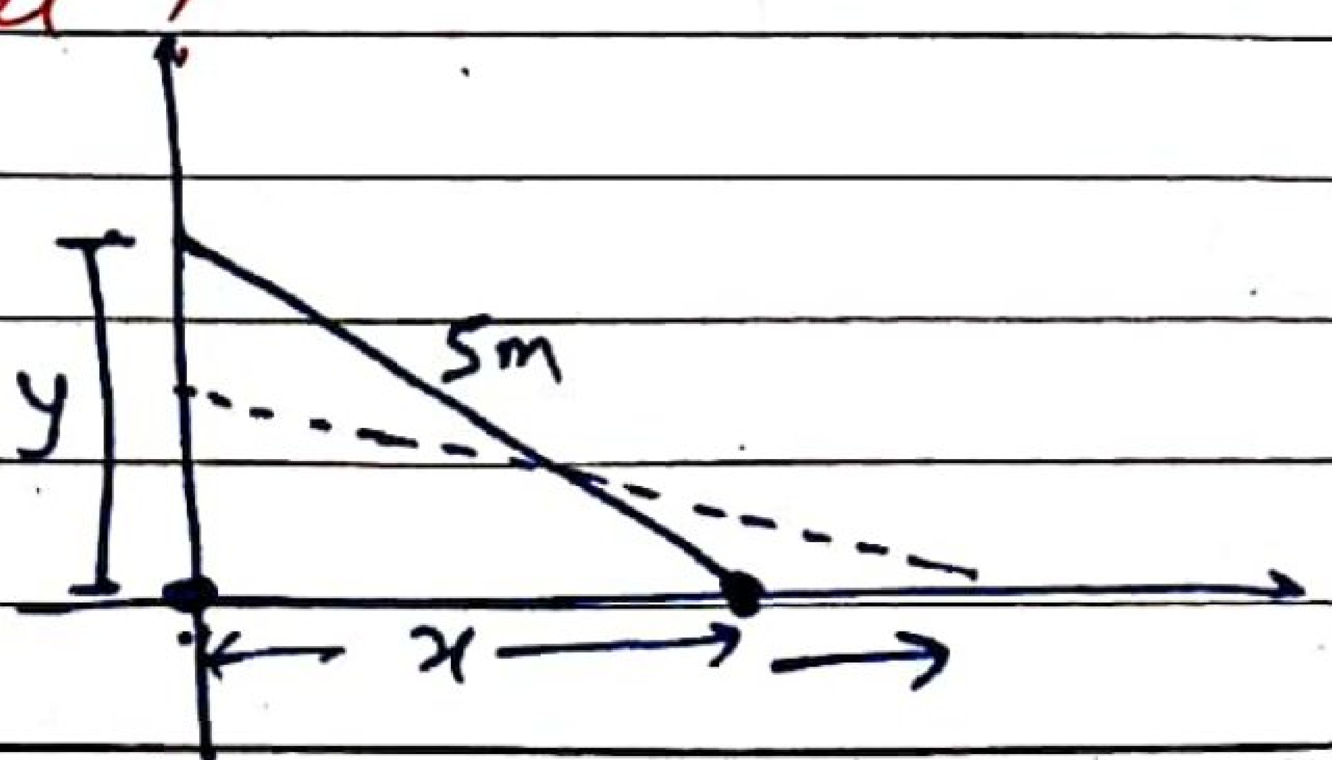
$\therefore$ Required points are $(4, 11)$ & $\left(-4, \frac{-31}{3}\right)$ Ans

Que 5: A ladder 5m long is leaning against a wall. The bottom of the ladder is pulled along the ground, away from the wall, at the rate of 2cm/sec. How fast is its height on the wall decreasing when the foot of the ladder is 4m away from the wall?

Sol: $\frac{dx}{dt} = 2\,cm/sec = .02\,m/sec$

to find $\frac{dy}{dt} = ?$

$x = 4\,m$ (Initial)



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A.CD (class No 3)

we have, $x^2+y^2=25$

for $x=4$

$16+y^2=25$

$\boxed{y=3}$

again, $x^2+y^2=25$

Diff wrt 't'

$2x\frac{dx}{dt}+2y\frac{dy}{dt}=0$

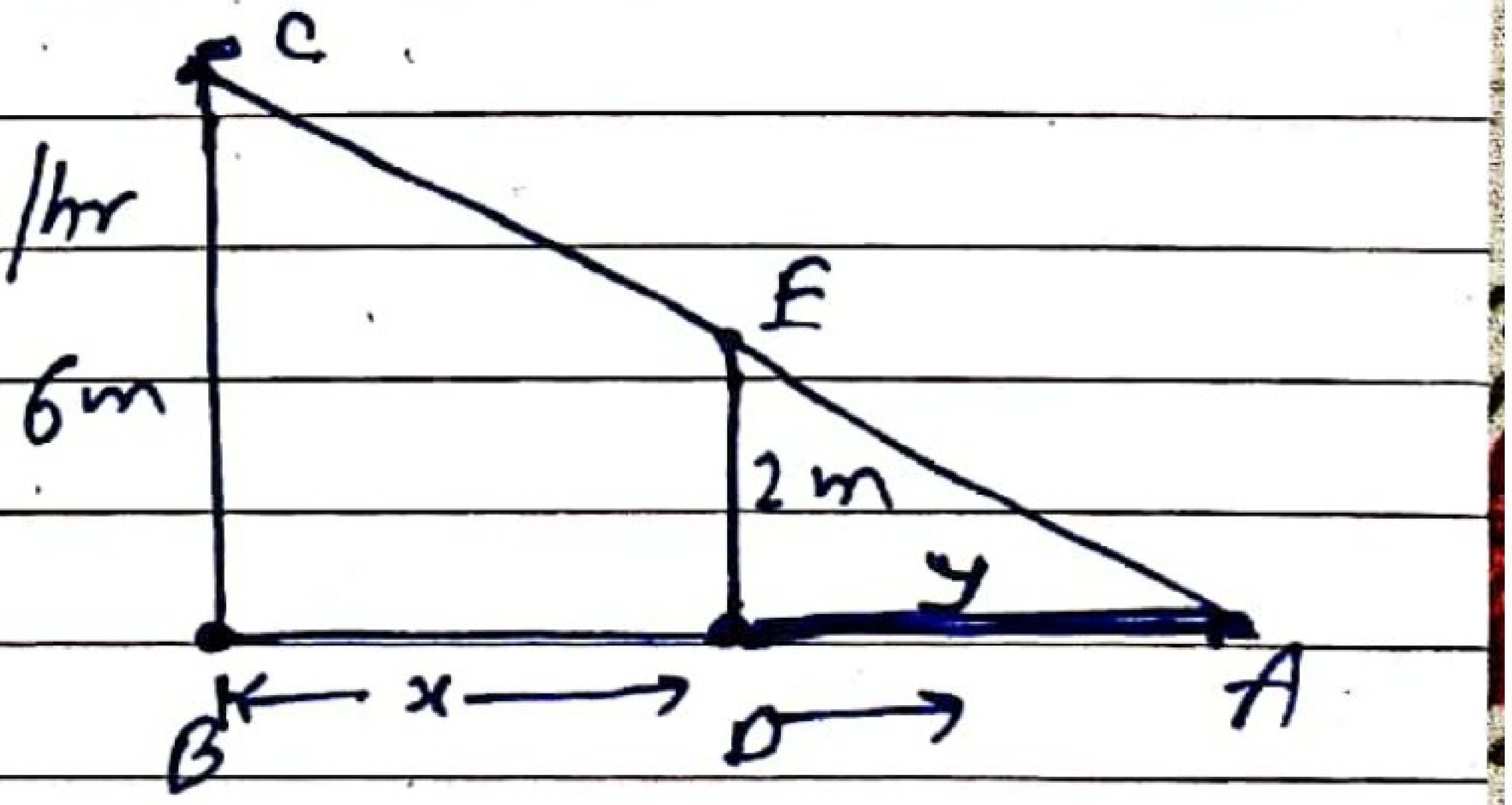
$\Rightarrow x\frac{dx}{dt}+y\frac{dy}{dt}=0$

$\Rightarrow 4(.02)+3.\frac{dy}{dt}=0$

$\Rightarrow \frac{dy}{dt}=\frac{-.08}{3}$ m/sec $=\frac{-8}{3}$ cm/sec

$\therefore$ hight of ladder on the wall is decreasing at the rate $\frac{8}{3}$ cm/sec Ans

Qn. 6 → A man of height 2m walks at a uniform speed of 5km/hr away from a lamp post which is 6m high. Find the rate at which the length of his shadow increases.



Soln :

given $\frac{dx}{dt}=5$ km/hr $=5000$ m/hr

to find $\frac{dy}{dt}=?$

$\Delta ABC \sim \Delta ADE$

$\frac{^3\cancel{6}}{\cancel{2}}=\frac{x+y}{y}$

$\rightarrow 3y=x+y$

$\rightarrow 2y=x$

Diff wrt 't'

$\rightarrow 2\frac{dy}{dt}=\frac{dx}{dt}$

$2\frac{dy}{dt}=5000$

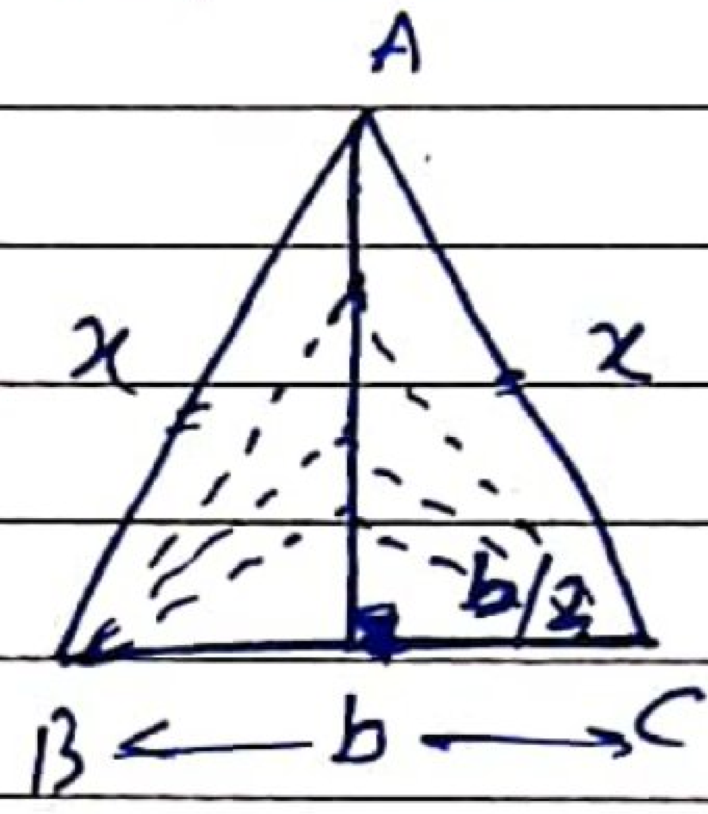
$\frac{dy}{dt}=2500$ m/hr $=2.5$ km/hr Ans

$\therefore$ shadow of Man is ↑ at the rate 2.5 km/hr

CLASSTIME

A.O.D (class No: 3) (6)

Q.No 7 → Two equal sides of an isosceles triangle with fixed base 'b' are decreasing at the rate of 3 cm/sec. How fast is the area decreasing when the two equal sides are equal to the base?



Sol

Let the equal sides $= x$

$$\frac{dx}{dt} = -3\text{ cm/sec}$$

$x = b$ (Initial value)

to find : $\frac{dA}{dt} = ?$

$$A = \frac{1}{2}(\text{base})(\text{altitude})$$

$$A = \frac{1}{2}(b)\sqrt{x^2 - \frac{b^2}{4}}$$

$$A = \frac{b}{4}\sqrt{4x^2 - b^2}$$

Diff. w.r.t 't'

$$\frac{dA}{dt} = \frac{b}{4} \cdot \frac{1}{2\sqrt{4x^2-b^2}} \cdot 8x\frac{dx}{dt}$$

$$\frac{dA}{dt} = \frac{b(b)(-3)}{\sqrt{4b^2-b^2}}$$

$$= \frac{-3b^2}{\sqrt{3}\,b}$$

$$\frac{dA}{dt} = -\sqrt{3}\,b \text{ cm}^2/\text{sec}$$

∴ Area of Δ is ↓ at the rate $\sqrt{3}\,b$ cm²/sec

Ans

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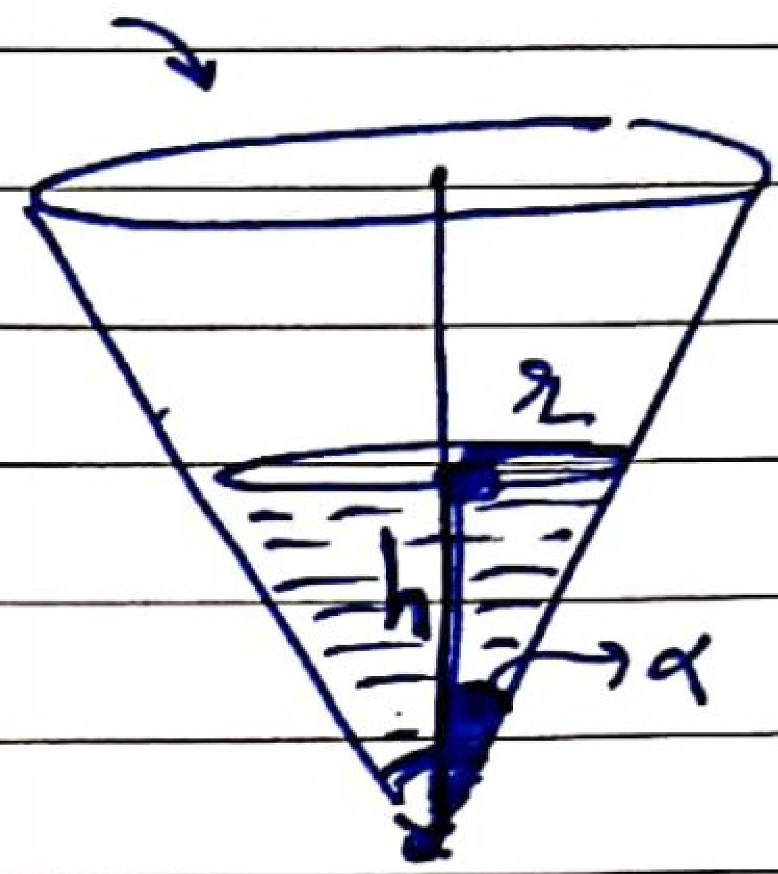
A.CD Class No:3

Q1.8 A water tank has the shape of an inverted right circular cone with its axis vertical and vertex lowermost. Its "semi-vertical" angle is $\tan^{-1}(0.5)$. Water is poured in to it at a constant rate of 5 cubic meter per hour. Find the rate at which the level of water is rising when the depth of water in the tank is 4m.



Sol Given $\alpha = \tan^{-1}(0.5)$

$\frac{dv}{dt} = 5\,m^3/hr$

To find: $\frac{dh}{dt} = ?$

$h = 4m$ (at instant value)

We have $V = \frac{1}{3}\pi r^2 h$

Given $\alpha = \tan^{-1}(0.5) \Rightarrow \tan\alpha = \frac{1}{2}$

$\frac{r}{h} = \frac{1}{2} \Rightarrow r = \frac{h}{2}$

$V = \frac{1}{3}\pi\left(\frac{h^2}{4}\right)h$

$V = \frac{\pi}{12}h^3$

Diff w.r.t 't'

$\frac{dv}{dt} = \frac{\pi}{12}(3h^2)\frac{dh}{dt}$

$5 = \frac{\pi}{12}(3 \times 16)\frac{dh}{dt}$

$\frac{dh}{dt} = \frac{5}{4\pi}\,m/hr$ Ans

∴ level of water is rising at the rate $\frac{5}{4\pi}\,m/hr$ Ans

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A·O·D

WORKSHEET No: 3

(Rate Measure)

Ans 1 → The volume of a cube is increasing at a rate of 9 cubic centimeters per second. How fast is the surface area increasing when the length of an edge is 10 cm? **Ans** $\frac{ds}{dt} = 3.6 \text{ cm}^2/\text{sec}$

Ans 2 → The length x of a rectangle is decreasing at the rate of 3 cm/minute and the width y is increasing at the rate of 2 cm/minute. When $x = 10$ cm and $y = 6$ cm. Find the rate of change of

(a) the ~~perim~~ perimeter **Ans** $\frac{dP}{dt} = -2 \text{ cm/min}$

(b) the area of the rectangle **Ans** $\frac{dA}{dt} = 2 \text{ cm}^2/\text{min}$

Ans 3 → A stone is dropped in to a quite lake and waves moves in circles at the speed of 5 cm/sec. At the instant when the radius of the circular wave is 8 cm, how fast is the enclosed area increasing? **Ans** $80\pi \text{ cm}^2/\text{sec}$

Ans 4 → The radius of an air bubble is increasing at the rate of $\frac{1}{2}$ cm/sec. At what rate is the volume of the bubble increasing when the radius is 1 cm? **Ans** $2\pi \text{ cm}^3/\text{sec}$

Ans 5 → A balloon, which always remains spherical, has a variable diameter $\frac{3}{2}(2x+1)$. Find the rate of ~~ch~~ change of its volume w.r.t x. **Ans** $\frac{dV}{dx} = \frac{27}{8}\pi(2x+1)^2$

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Q.No. 6 → A cylindrical tank of radius 10m is being filled with wheat at the rate of 314 cubic meter per hour. Find the rate at which the depth of wheat is increasing. Ans $1\ m/hr$

Q.No. 7 → A particle moves along the curve $y = x^2 + 2x$. At what points on the curve y coordinate is changing as fast as x coordinate (same rate)
Ans $\left(\frac{-1}{2}, \frac{-3}{4}\right)$

Q.No. 8 → Find an angle θ, which increases twice as fast as its sine. Ans $\theta = \pi/3$

Q.No. 9 → A man 180cm tall walks at a rate of $2\ m/sec$ away from a source of light that is 9m above the ground. How fast is the length of his shadow increasing when he is 3m away from the source of light? Ans $0.1\ m/sec$

Q.No. 10 → An inverted cone has a depth of 10cm and a base of 5cm. Water is poured in to it at the rate of $\frac{3}{2}$ c.c per minute. Find the rate at which the level of water in the cone is rising when the depth is 4m. Ans $\frac{3}{8\pi}$ cm/min.

Q.No. 11 → The volume of a cube is increasing at a constant rate. Prove that the increase in surface area varies inversely as the length of the edge of the cube.

Hint to prove $\frac{ds}{dt} \propto \frac{1}{x}$

CLASSTIME