

→ **ULTIMATE MATHEMATICS** → (1)

XII TEST No: 4

CHAPTER: DIFFERENTIATION & CONTINUITY

Q. No. 1 → value of a for which $f(x) = \begin{cases} \frac{1-\cos(4x)}{x^2} & ; x < 0 \\ a & ; x = 0 \\ \frac{\sqrt{x}}{\sqrt{16+\sqrt{x}}-4} & ; x > 0 \end{cases}$ is continuous at $x=0$ is

(A) 3 (B) 4 (C) 6 (D) none of these

Q. No. 2 → value of a & b for which $f(x) = \begin{cases} \frac{1-\sin^3 x}{3\cos^2 x} & ; x < \frac{\pi}{2} \\ \text{ } & ; x = \frac{\pi}{2} \\ \frac{b(1-\sin x)}{(x-2x)^2} & ; x > \frac{\pi}{2} \end{cases}$ is continuous at $x = \frac{\pi}{2}$ are

(A) $a=2, b=\frac{1}{4}$ (B) $a=2, b=4$
(C) $a=\frac{1}{2}, b=4$ (D) none of these

Q. No. 3 → value of a & b so that $f(x) = \begin{cases} x + a\sqrt{2}\sin x & ; 0 \leq x < \pi/4 \\ 2x\cot x + b & ; \pi/4 \leq x < \pi/2 \\ a\cos(2x) - b\sin x & ; \pi/2 \leq x \leq \pi \end{cases}$ becomes continuous on $[0, \pi]$

(A) $a=-\frac{\pi}{6}, b=-\frac{\pi}{12}$ (B) $a=\frac{\pi}{6}, b=\frac{\pi}{12}$ (C) $a=-\frac{\pi}{6}, b=\frac{\pi}{12}$
(D) $a=\frac{\pi}{6}, b=-\frac{\pi}{12}$

Q. No. 4 → value of a & b so that $f(x) = \begin{cases} x^2 & ; x \leq c \\ ax+b & ; x > c \end{cases}$ is differentiable at $x=c$ are

(A) $a=2c, b=c^2$ (B) $a=-2c, b=-c^2$
(C) $a=2c, b=-c^2$ (D) $a=-2c, b=c^2$

Q. No. 5 → At what points on the curve $y=12(x+1)(x-2)$; $x \in [-1, 2]$, the tangent is parallel to x -axis (Using Rolle's theorem)

(A) $(2, 27)$ (B) $(\frac{1}{2}, -27)$ (C) $(-\frac{1}{2}, 27)$ (D) $(-\frac{1}{2}, -\frac{1}{27})$

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Qn 6 → At what point on the parabola $y = (x-4)^2$, the tangent is parallel to the chord joining the points $(4,0)$ & $(5,1)$

- (A) $(\frac{9}{2}, 4)$ (B) $(9, \frac{1}{4})$ (C) $(\frac{9}{4}, 2)$ (D) none of these

Qn 7 → Derivative of $\log\left[\tan\left(\frac{\pi}{4} + \frac{x}{2}\right)\right]$ is

- (A) $\cos x$ (B) $\sec x$ (C) $\operatorname{cosec} x$ (D) $\tan x$

Qn 8 → Derivative of $y = [x + \sqrt{x^2 + a^2}]^n$ is

- (A) $\frac{dy}{dx} = \frac{y}{\sqrt{x^2 + a^2}}$ (B) $\frac{dy}{dx} = \frac{n^2 y}{\sqrt{x^2 + a^2}}$ (C) $\frac{dy}{dx} = \frac{ny}{x^2 + a^2}$

(D) $\frac{dy}{dx} = \frac{ny}{\sqrt{x^2 + a^2}}$

Qn 9 → derivative of $\log \sqrt{\frac{1+\tan x}{1-\tan x}}$ is

- (A) $\sec x$ (B) $\csc(2x)$ (C) $\sec(2x)$ (D) $\operatorname{cosec}(2x)$

Qn 10 → If $y = \tan^{-1}\left(\frac{\sqrt{1+x^2} + \sqrt{1-x^2}}{\sqrt{1+x^2} - \sqrt{1-x^2}}\right)$; $-1 < x < 1$, then $\frac{dy}{dx} =$

- (A) $\frac{-x}{\sqrt{1-x^4}}$ (B) $\frac{x}{\sqrt{1-x^4}}$ (C) $\frac{x^2}{\sqrt{1-x^4}}$ (D) $\frac{-2x}{\sqrt{1-x^4}}$

Qn 11 → Derivative of $\cot^{-1}\left(\frac{\sqrt{1+\sin x} - \sqrt{1-\sin x}}{\sqrt{1+\sin x} + \sqrt{1-\sin x}}\right)$; $0 < x < \frac{\pi}{2}$ is

- (A) 2 (B) $\frac{1}{2}$ (C) $-\frac{1}{2}$ (D) none of them

Qn 12 → If $\sqrt{1-x^6} + \sqrt{1-y^6} = a(x^3 - y^3)$ then $\frac{dy}{dx} =$

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(A) $\frac{x^2}{y^2} \sqrt{\frac{1-x^6}{1-y^6}}$ (B) $\frac{x^2}{y^2} \sqrt{\frac{1-y^6}{1-x^6}}$ (C) $\frac{y^2}{x^2} \sqrt{\frac{1-x^6}{1-y^6}}$ (D) none of these

Qn 13 \rightarrow If $x^m y^n = (x+y)^{m+n}$ then $\frac{dy}{dx}$ at (4,2) is

(A) 2 (B) $\frac{1}{2}$ (C) 4 (D) 0

Qn 14 \rightarrow If $y = (f(x))^{(f(x))^{(f(x)) \dots \infty}}$, then $\frac{dy}{dx}$ at $x = \frac{1}{4}$ is

(A) $\frac{1}{2}$ (B) 1 (C) ∞ (D) 2

Qn 15 \rightarrow If $x = \sqrt{a} \sin t$, $y = \sqrt{a} \cos t$ then $\frac{dy}{dx} =$

(A) $\frac{y}{x}$ (B) $-\frac{x}{y}$ (C) $-\frac{y}{x}$ (D) $\frac{y}{x}$

Qn 16 \rightarrow Derivative of $\cos^{-1}(4x^3-3x)$ w.r.t $\tan^{-1}\left(\frac{\sqrt{1-x^2}}{x}\right)$ is

(A) 2 (B) -3 (C) $3x$ (D) 3

Qn 17 \rightarrow $y = e^{a \cos^{-1} x}$ satisfies the equation

(A) $\frac{d^2 y}{dx^2} - x \frac{dy}{dx} - a^2 y = 0$ (B) $(1-x^2) \frac{d^2 y}{dx^2} - \frac{dy}{dx} - a^2 y = 0$

(C) $(1-x^2) \frac{d^2 y}{dx^2} - x \frac{dy}{dx} - a^2 y = 0$ (D) $(1-x^2) \frac{d^2 y}{dx^2} + x \frac{dy}{dx} - a^2 y = 0$

Qn 18 \rightarrow $x = 2 \cos t - \cos(2t)$, $y = 2 \sin t - \sin(2t)$ then $\frac{d^2 y}{dx^2}$ at $t = \frac{\pi}{2}$ is

(A) $-\frac{3}{2}$ (B) $\frac{3}{2}$ (C) -2 (D) 0

Qn 19 \rightarrow If $x = a(\cos \theta + \theta \sin \theta)$, $y = a(\sin \theta - \theta \cos \theta)$ then $\frac{d^2 y}{dx^2}$ at $\theta = \frac{\pi}{3}$

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(A) $\frac{24}{\lambda}$ (B) $\frac{24}{a}$ (C) $\frac{24}{a\lambda}$ (D) $\frac{3\sqrt{3}}{8\lambda}$

Q. No 20 \rightarrow If $f(x) = (1+x)(1+x^2)(1+x^4)(1+x^8)$,
then $f'(1)$ is equal to

(A) 124 (B) 130 (C) 154 (D) none of these