ULTIMATE MATHEMATICS= BY AJAY MITTAL

PROBABILITY : $\boxed{\text{CLASS No: 3}}$ (12th class)

Ans 1 A die is loaded in such a way that an even number is twice likely to occur as an odd number if the die is tossed twice, find the P·D of random variable X representing the pufect square in the two tosses.

Soln

let P(each even number) = 2 P(each odd number)

let P(each odd number) = p

∴ P(each even number) = $2p$

we have

$$P(1) + P(2) + P(3) + P(4) + P(5) + P(6) = 1$$
$$p + 2p + p + 2p + p + 2p = 1$$
$$9p = 1$$
$$\boxed{p = \frac{1}{9}}$$

∴ P(odd) = $1/9$ & P(even) = $\frac{2}{9}$

P(getting a pufect square) = P(1,4) = $\frac{1}{9} + \frac{2}{9} = \frac{1}{3}$

P(not getting a pufect square) = $\frac{2}{3}$

X → denotes the number of pufect square

X → 0, 1, 2

P(X=0) = P(getting No pufect square) = $\frac{2}{3} \times \frac{2}{3} = \frac{4}{9}$

$P(x=1) = P(\text{getting } 1 \text{ perfect square ou}) = \left(\frac{1}{3} \times \frac{2}{3}\right) \times 2 = \frac{4}{9}$

$P(x=2) = P(\text{getting } 2 \text{ perfect square ou}) = \frac{1}{3} \times \frac{1}{3} = \frac{1}{9}$

x	0	1	2
$P(x)$	$\frac{4}{9}$	$\frac{4}{9}$	$\frac{1}{9}$

$\underline{\underline{A_1}}$

Qno 2 → Two numbers are selected without replacement from the first six +ve Integers. let x denotes the larger of the two numbers. Find P.D of x
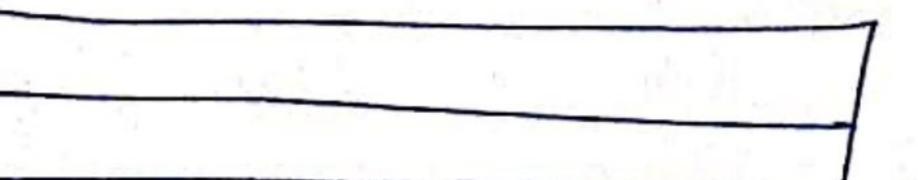
Soln

1,2,3,4,5,6

$x \to$ denotes the larger number

$x \to$ can take values $2, 3, 4, 5, 6$

$P(x=2) = P(\text{larger no } 2) = P((1,2),(2,1)) = \left(\frac{1}{6} \times \frac{1}{5}\right) \times 2 = \frac{2}{30}$

$P(x=3) = P(\text{larger no. } 3) = P\left[(1,3),(3,1),(2,3),(3,2)\right] = \left(\frac{1}{6} \times \frac{1}{5}\right) \times 4 = \frac{4}{30}$

$P(x=4)$

$P(x=5)$

$P(x=6) = \left(\frac{1}{6} \times \frac{1}{5}\right) \times 10 = \frac{10}{30}$

$\underline{A_1}$

Qus 3 → A fair coin is tossed until a head or five tails occur - If x denotes the number tosses of the coin . Find ~~Mean~~ P.D of x

Son) let $x \to$ denotes the number tossen

$x \to$ can take values $1, 2, 3, 4, 5$

$$S = \{H, TH, TTH, TTTH, TTTTH, TTTTT\}$$

$P(x=1) = \frac{1}{2}$

$P(x=2) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

$P(x=3) = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$

$P(x=4) = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{16}$

$P(x=5) = \left(\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \right) \times 2 = \frac{2}{32}$

x	
$P(x)$	

Qus 4 → A random variable X can take all non-negative Integral values and the probability that X takes the value r is proportional to α^r $(0 < \alpha < 1)$ find $P(x=0)$

Son) $X = 0, 1, 2, 3, \ldots$

$P(x=r) \propto \alpha^r$

$P(x=r) = \lambda \alpha^r$

we have

$$P(x=0) + P(x=1) + P(x=2) + P(x=3) + \cdots = 1$$

$$\lambda \alpha^0 + \lambda \alpha^1 + \lambda \alpha^2 + \lambda \alpha^3 + \cdots = 1$$

$$\lambda \left(1 + \alpha + \alpha^2 + \alpha^3 + \cdots \alpha \right) = 1$$

$\leftarrow$ G.p $a=1$, $r=\alpha \longrightarrow \cdots \{0 < \alpha < 1\}$

$$\Rightarrow \quad \lambda \left(\frac{1}{1-\alpha} \right) = 1$$

$$\Rightarrow \boxed{\lambda = 1 - \alpha}$$

$$\therefore P(x=r) = (1-\alpha) \cdot \alpha^r$$

$$P(x=0) = (1-\alpha) \alpha^0$$

$$= 1 - \alpha \qquad \text{An}$$

$$- \times -$$

BAYE'S THEOREM :

$A \rightarrow$ given eunt

$F_1 \rightarrow$
$F_2 \rightarrow$ } Required eunt
$F_3 \rightarrow$ (Minimum two)
$\vdots$

$$\boxed{P(A \mid E) = \text{given}}$$

Meaning
WIH

$$P(E_1) + P(E_1) + P(E_2) + \cdots = 1$$

$$P(E_1 \mid A) = \frac{P(E_1) \cdot P(A \mid E_1)}{P(E_1) P(A \mid E_1) + P(E_2) P(A \mid E_2) + P(E_3) P(A \mid E_3)}$$

$$+ \cdots$$

Que 1 There are three bags : BAG - I contain 3R, 7G bulls
Bag II contain 4R, 6G balls & Bag III contain
5R & 5G bulls.
One of the bag is selected, and a ball is drawn, and it is found to be red. Find the probability that the good ball came from bag II ?

Soln

Let $A \to$ the ball drawn is Red

$F_1 \to$ Bag I is to selected
$E_2 \to$ Bag II. " "
$E_3 \to$ Bag III " "

$P(E_1) = 1/3 \qquad P(E_2) = 1/3 \qquad P(E_3) = 1/3$

$P(A|E_1) = \overset{\nearrow \text{Red}}{3/10}_{\searrow \text{Bag I}} \qquad P(A|E_2) = \frac{4}{10} \qquad ; P(A|E_3) = \frac{5}{10}$

By prob $P(E_1|A) = \dfrac{\frac{1}{3} \times \frac{4}{10}}{\left(\frac{1}{3} \times \frac{3}{10}\right) + \left(\frac{1}{3} \times \frac{4}{10}\right) + \left(\frac{1}{3} \times \frac{5}{10}\right)}$

$= \dfrac{4}{3+4+5} = \dfrac{4}{12} = \dfrac{1}{3} \quad \underline{\text{Ans}}$

Que 2 In a bolt factory, machines A, B & manufactures respectively 25%, 35% & 40% of the total bolts. Of their outputs 5, 4 & 2% are respectively defective bolts. A bolt is drawn at random from the product. If the bolt drawn is found to be defective.

what is the probability that if is manufactured by machine c?

Solⁿ let A → defective bolt is drawn

$E_1 →$ Bolt is manufactured by Machine A

$E_2 →$ " " " " B

$E_2 →$ " " " " C

$P(E_1) = \frac{25}{100}$; $P(E_2) = \frac{35}{700}$; $P(E_3) = \frac{40}{700}$

$P(A|E_1) = \frac{5}{100}$; $P(A|E_2) = \frac{4}{700}$ & $P(A|E_3) = \frac{2}{100}$

→ defect bolt

↳ Mach A

By prob $P(E_3/A) = \dfrac{\frac{40}{700} \times \frac{2}{100}}{\left(\frac{25}{100} \times \frac{5}{100}\right) + \left(\frac{35}{700} \times \frac{4}{100}\right) + \left(\frac{40}{700} \times \frac{2}{100}\right)}$

$= \dfrac{80}{125 + 140 + 80} = \dfrac{80}{345}$

Qⁿ 3 → There are three coins. One is two headed coin, another is biased coin that comes up heads 75% of the time and the third is an unbiased coin. one of the three coins is chosen at random and tossed, it shows head, what is the probability that it was the two headed coin?

Solⁿ let A → Coin shows head

$E_1 →$ two headed coin is selected

$E_2 →$ biased " " '

$E_3 →$ unbiased " " '

$P(E_1) = 1/3$; $P(E_2) = 1/3$ $P(E_3) = 1/3$

$P(\overline{A|E_1}) = 1$ (hard) $P(A|E_2) = 3/4$ $P(A|E_3) = 1/2$

Req prob $P(E_1|A) = \dfrac{\frac{1}{3} \times 1}{(\frac{1}{3} \times 1) + (\frac{1}{3} \times \frac{3}{4}) + (\frac{1}{3} \times \frac{1}{2})}$

$= \boxed{}$ _An_

Qu. 4 → A letter is known to have come either come from TATANAGAR or CALCUTTA. On the envelope just two consecutive letters TA are visible. what is the probability that letter has come from CALCUTTA?

Soln) A → (TA) is visible on envelope

E_1 → letter came from TATANAGAR

E_2 → " " " CALCUTTA

$P(E_1) = 1/2$ $P(E_2) = 1/2$

$P(A|E_1) = \dfrac{2}{8}$ (TA) └ tatanagar

$P(A|E_2) = \dfrac{1}{7}$

Req prob $P(E_2|A) =$ value put _An_

Qn1 given

X	0	1	2
P(X)	$3c^3$	$4c-10c^2$	$5c-1$

find

(i) c   (ii) $P(x < 2)$ (iii) $P(1 < x \le 2)$

Qn 2 → let X be a random variable which assumes values x_1, x_2, x_3, x_4 such that

$$2P(x = x_1) = 3P(x = x_2) = P(x = x_3) = 5P(x = x_4)$$

Find the probability distribution of X.

Qn 3 → Two cards are drawn from 52 cards. Find the probability distribution of Number of face-cards.

Qn 4 → Five defective bolts are accidentally mixed with twenty good ones. If four bolts are drawn at random from this lot. Find the probability distribution of the number of defective bolts.

Qn 5 → A pair of dice is thrown. Let X be the random variable which denotes the minimum of the two numbers which appear. Find P.D of X

Qn. 6 → Given three identical boxes I, II, III, each containing two coins. In box I, both coins are good coins, In box II both are silver coins

and in box Ⅲ there is one gold and one silver coin. A person chooses a box at random and takes out a coin. If the coin is gold, what is the probability that the other coin in the box is also of gold?

Qu. 7 → Two groups are competing for the position on the Board of Directors of a Corporation. The probabilities that the first and second group will win are 0.6 & 0.4 respectively. Further, if the first group wins, the probability of introducing a new product is 0.7 and the corresponding probability is 0.3 if the second group wins. Find the probability that the new product introduced was by the second group?

Qu. 8 → of the students in a college, it is known that 60% reside in hostel and 40% are day scholars. Previous year results report that 30% of all students who reside in hostel attain grade A and 20% of day scholars attain grade A. At the end of the year, one student is chosen at random from the college and he has an A grade. what is the probability that the chosen student is a hostlier?

Ques 9 → Suppose a girl throws a die. If she gets a 5 or 6, she tosses a coin three times and notes the number of heads. If she gets 1, 2, 3 or 4, she tosses a coin once and notes whether a head or tails is obtained. If she obtained exactly one head, what is the probability that she threw 1, 2, 3 or 4 with the dice?

Qu 10 → Suppose 5% of men and 0.25% of women have grey hair. A grey haired person is chosen. What is the probability of this person being male? Assume that there are equal number of males and females

Qu 11 → Bag I contains 3 Red and 4 black balls and Bag II contains 4 Red and 5 black balls. One ball is transferred from Bag I to Bag II and then a ball is drawn from Bag II. The ball so drawn is found to be Red in colour. Find the probability that the transferred ball is black.

Qu 12 → A card from a pack of 52 cards is lost. From the remaining cards of the pack two cards are drawn and are found to be both diamonds. Find the probability the lost card being a diamond   — × —