|| जय श्री राधे कृष्णा || जय श्री गिरिराज जी महाराज ||

ULTILMATE MATHEMATICS: BY AJAY MITTAL

CHAPTER: AOI

CLASS NO: 2

Qn:1 Find the area bounded by the curves

$y = x^2 + 2$; $y = x$; $x = 0$ and $x = 3$

Soln (1) $y = x^2 + 2$

$x^2 = y - 2$

- ✓ parabola
- ✓ vertex (0, 2)
- ✓ face open +ve y-axis

(2) $y = x$

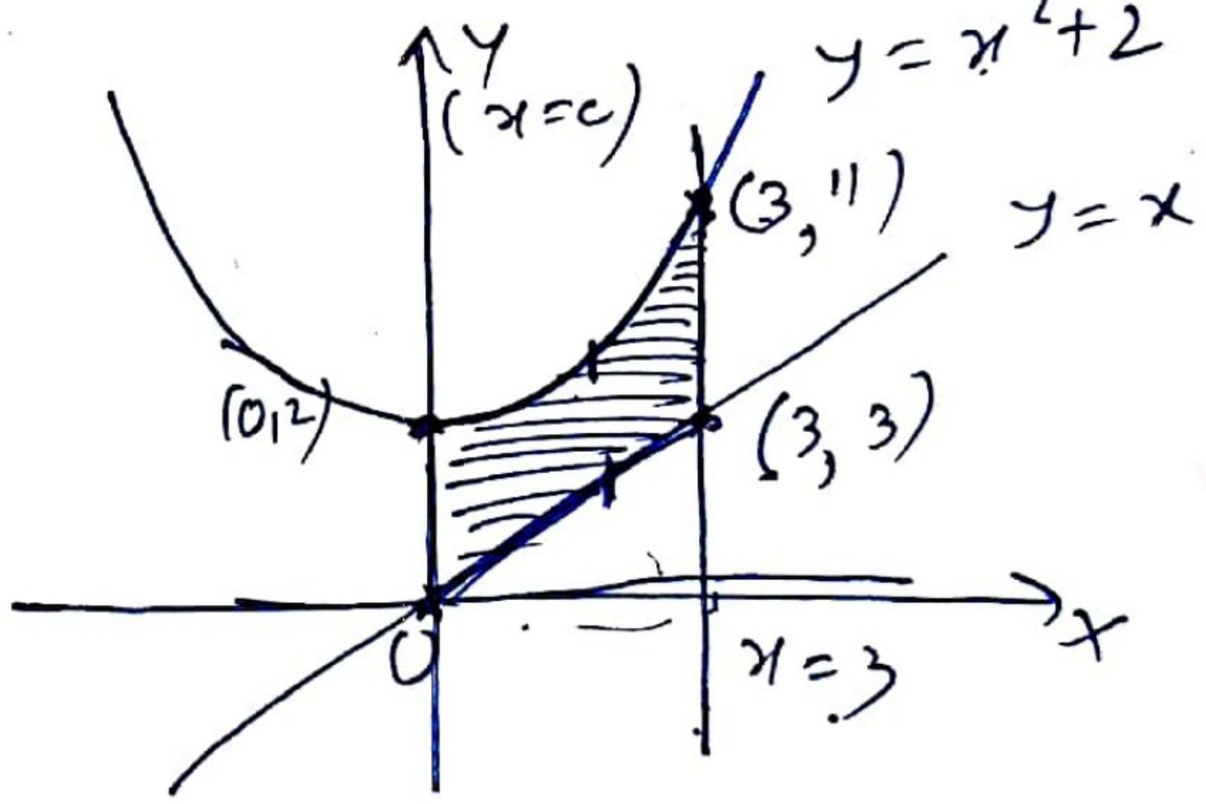
line passing thru (0,0)

(3) $x = 0$

equation of y-axis

(4) $x = 3$

line || to y-axis



Required area

$= \int_0^3 (x^2 + 2) - x \, dx$

$= \left(\frac{x^3}{3} + 2x - \frac{x^2}{2}\right)_0^3$

$= \left(9 + 6 - \frac{9}{2}\right) - (0)$

$= \frac{21}{2}$ Square unit Ans

Qn:2 Find the area bounded by the triangle whose vertices are A(-1, 0) B(1, 3) C(3, 2)

Solution Given vertices

$A(-1,0)$ $B(1,3)$ $C(3,2)$

Equation of AB

$$y-0 = \frac{3}{2}(x+1)$$

$$y = \frac{3x+3}{2}$$

Eq of BC

$$y-3 = \frac{-1}{2}(x-1)$$

$$2y-6 = -x+1$$

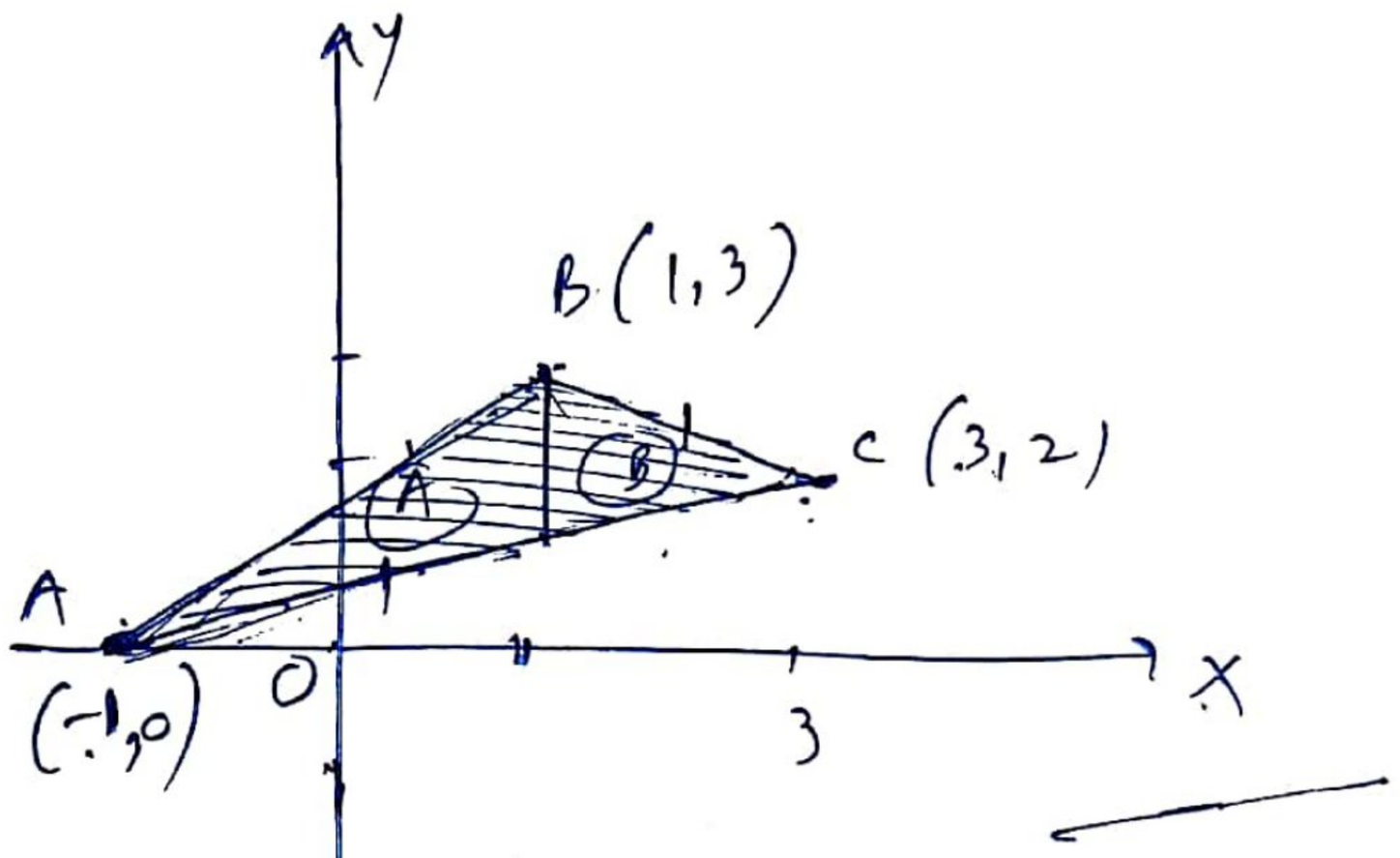
$$y = \frac{-x+7}{2}$$

Eq of AC

$$y-0 = \frac{2}{4}(x+1)$$

$$4y = 2x+2$$

$$y = \frac{x+1}{2}$$



Required area

$$= \int_{-1}^{1} \left(\frac{3x+3}{2}\right) - \left(\frac{x+1}{2}\right) dx + \int_{1}^{3} \left(\frac{-x+7}{2}\right) - \left(\frac{x+1}{2}\right) dx$$

$$= \frac{1}{2}\int_{-1}^{1} (2x+2)\,dx + \frac{1}{2}\int_{1}^{3} -2x+6\,dx$$

$$= \int_{-1}^{1} x+1\,dx + \int_{1}^{3} -x+3\,dx$$

$$= \left(\frac{x^2}{2}+x\right)_{-1}^{1} + \left(-\frac{x^2}{2}+3x\right)_{1}^{3}$$

$\square$ Ans.

Qn.3 Find the area bounded by the lines

$2x+y=4$; $3x-2y=6$ & $x-3y=-5$

Soln

$2x+y=4$ ---(1)

$3x-2y=6$ --(2)

$x-3y=-5$ --(3)

Solv (1) & (2)

$4x+2y=8$

$3x-2y=6$

$7x=14$

$x=2, y=0$

$A(2,0)$

Solv (2) & (3),

$3x-2y=6$

$3x-9y=-15$

$7y=21$

$y=3, x=4$

$B(4,3)$

Solv (1) & (3)

$6x+3y=12$

$x-3y=-5$

$7x=7$

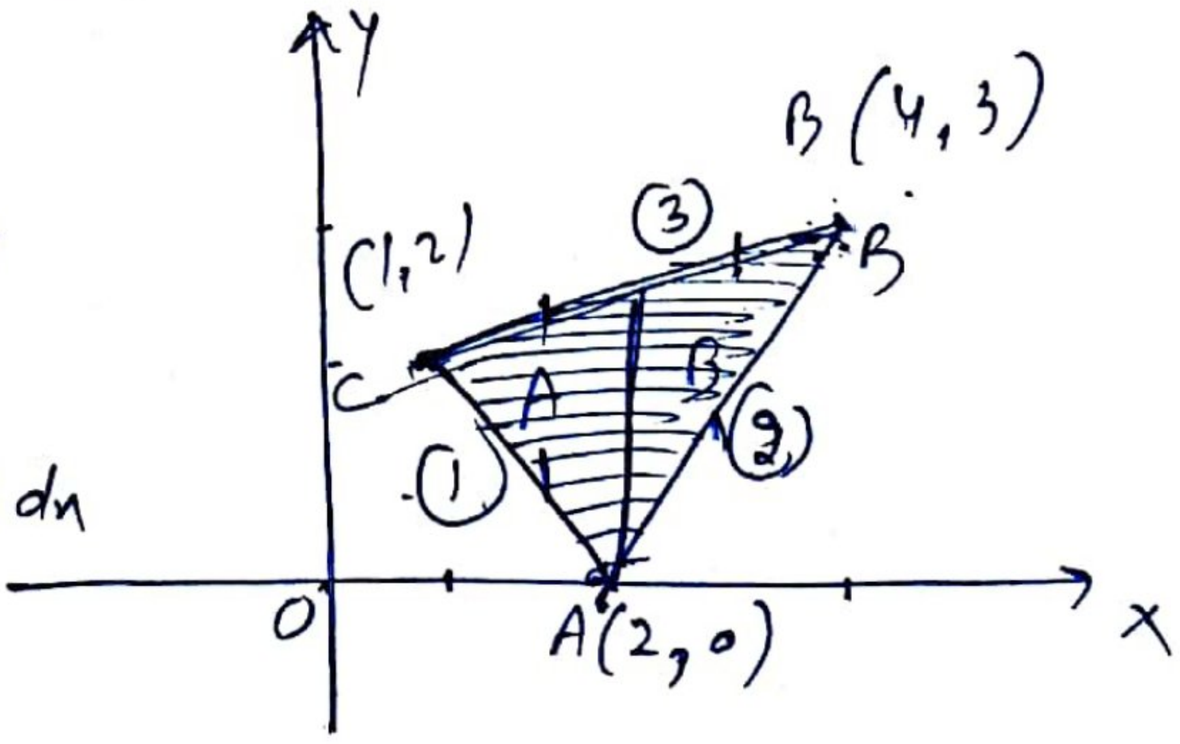
$x=1 \quad y=2$

$C(1,2)$

$$\text{Area} = \int_1^2 \left(\frac{x+5}{3}\right) - (4-2x)\,dx + \int_2^4 \left(\frac{x+5}{3}\right) - \left(\frac{3x-6}{2}\right) dx$$

&

Any



Qn.4 → Find the area bounded by the curves

$y=x|x|$; x-axis, ordinates $x=-1$ & $x=1$

Soln (1) $y=x|x|$

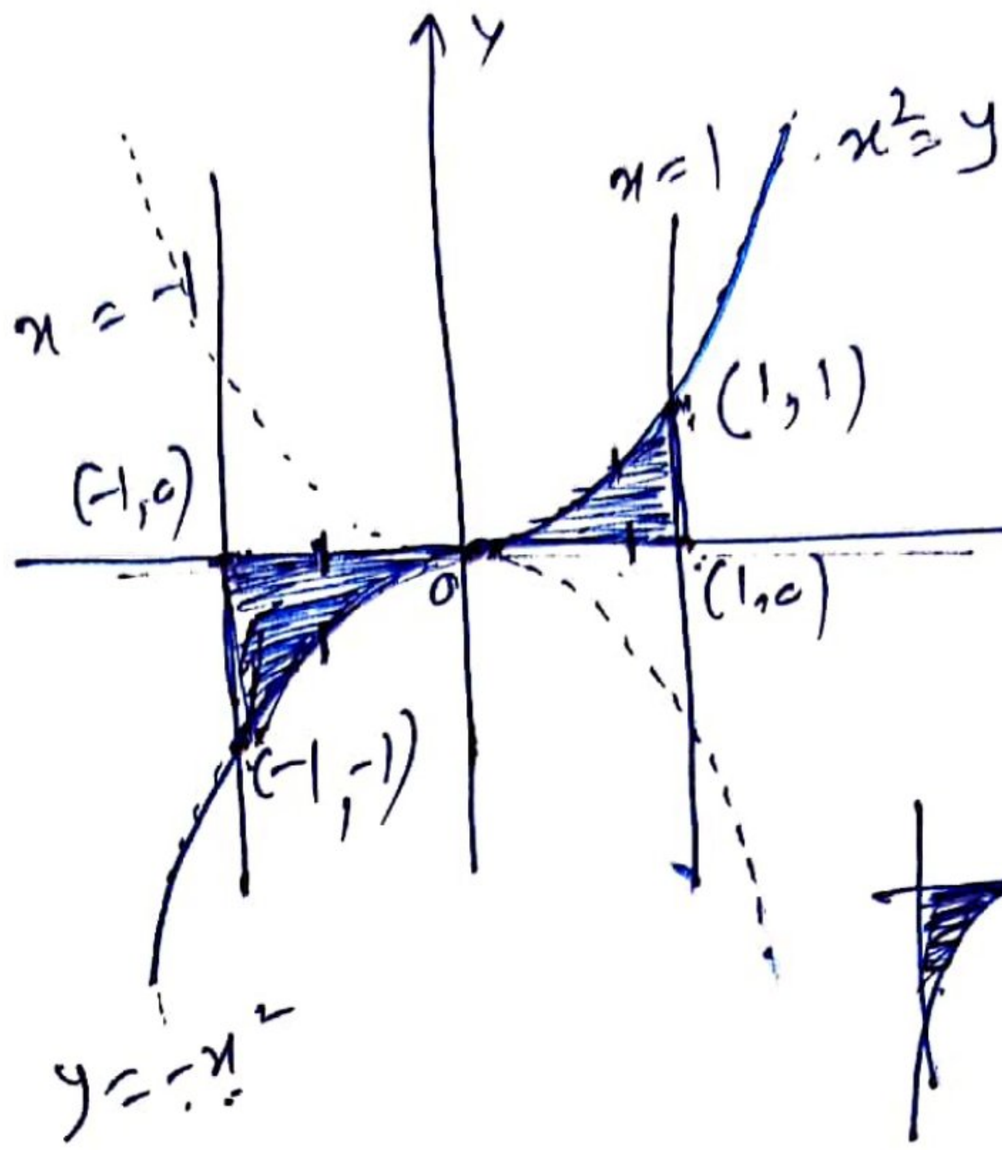
$$y=\begin{cases} x^2 ; & x\geq 0 \\ -x^2 ; & x<0 \end{cases} \Rightarrow \boxed{x^2=y} : x\geq 0 \qquad \boxed{x^2=-y} ; \; x<0$$

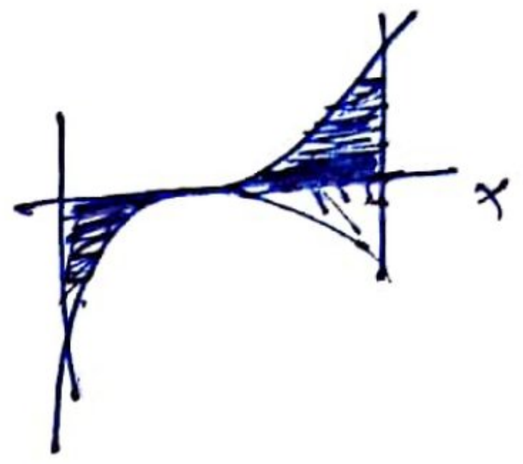
Ry area

$$= \int_{-1}^{0} 0-(-x^2)\,dx + \int_{0}^{1} x^2\,dx$$

$$= \left(\frac{x^3}{3}\right)_{-1}^{0} + \left(\frac{x^3}{3}\right)_{0}^{1}$$

$$= \square \text{ sq}$$





Qn. 5 Find the area the Region

$\{(x,y): 0 \le y \le x^2+1;\ 0 \le y \le x+1;\ 0 \le x \le 2\}$

(1) $y \ge 0$;

(2) $x^2+1 \ge y$

$x^2 \ge y-1$

parabola

vertex $(0,1)$

Soln. outside

(3) $y \le x+1$

line

points $(0,1)$ $(-1,0)$

Soln $0 \le 1$

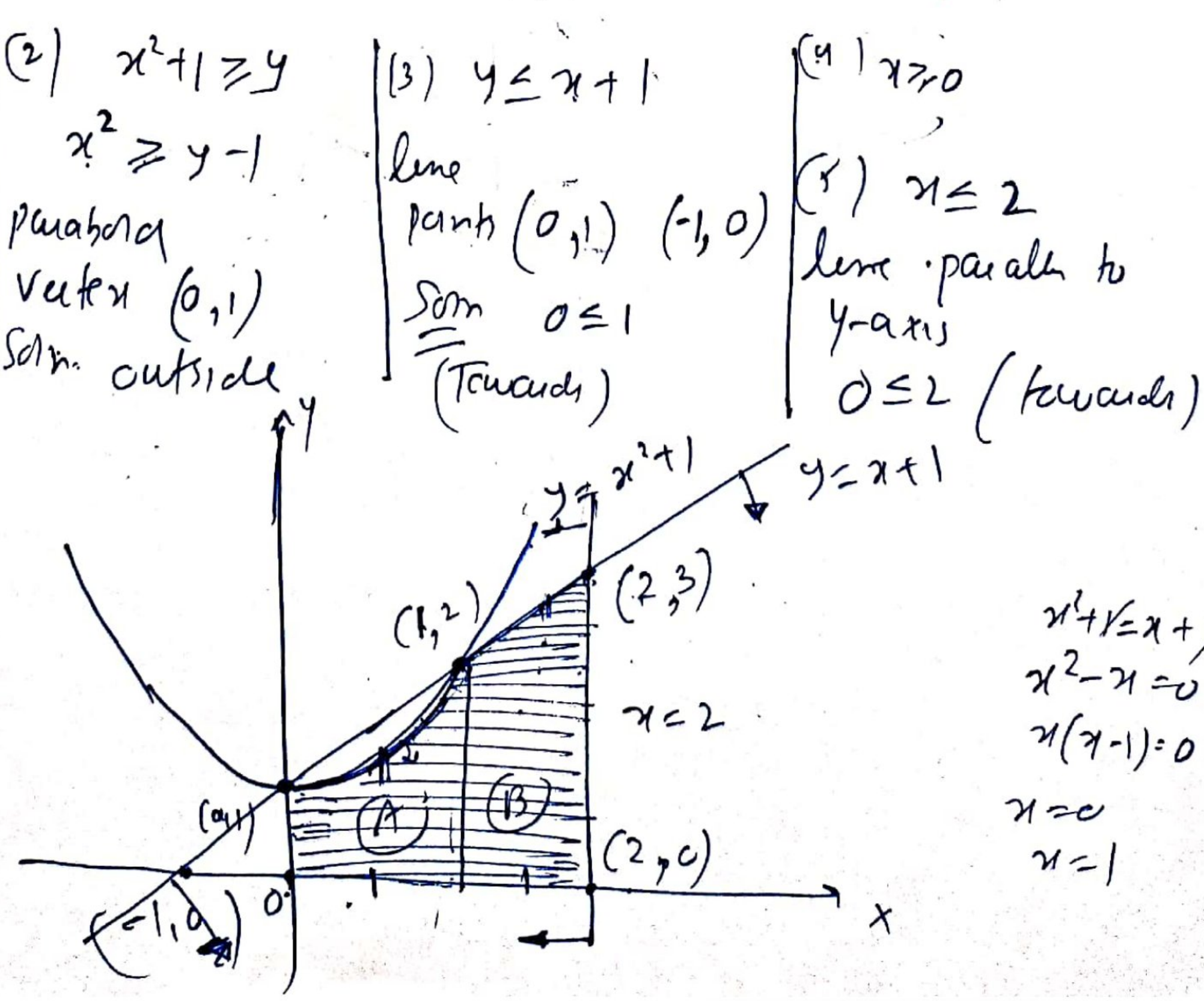
(Towards)

(4) $x \ge 0$,

(5) $x \le 2$

line parallel to y-axis

$0 \le 2$ (towards)



$x^2+1 = x+1$

$x^2 - x = 0$

$x(x-1) = 0$

$x = 0$

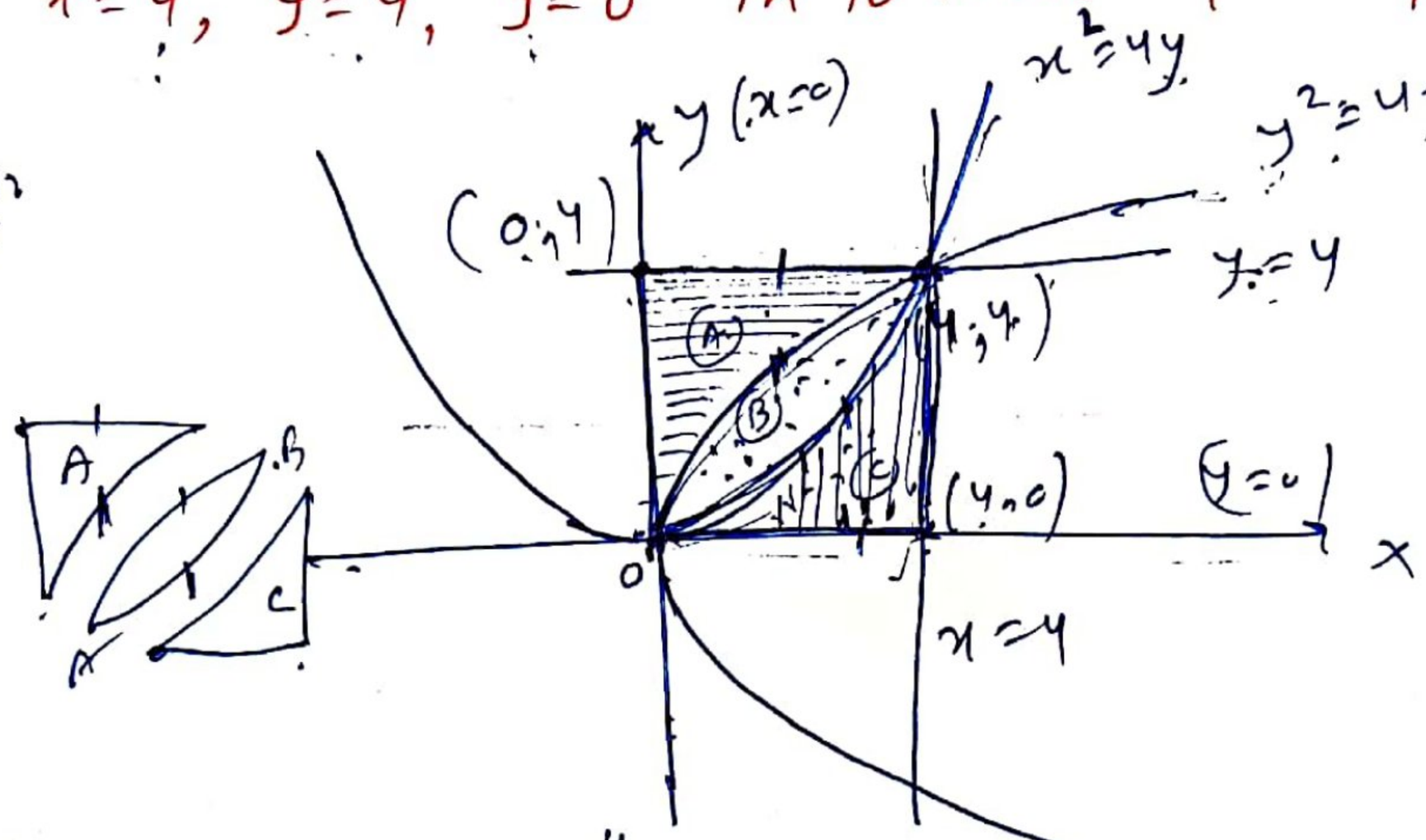
$x = 1$

By area $= \int_0^1 (x^2+1) - (0)\, dx + \int_1^2 (x-1)\, dy$

$= \square$ Sq. units

Q.N.6 → Show that the curves $y^2 = 4x$ and $4y = x^2$ divide the area of the square bounded by $x=0$, $x=4$, $y=4$, $y=0$ in to three equal parts.

Soln:



$\left(\frac{x^2}{4}\right)^2 = 4x$

$x^4 = 64x$

$x^4 - 64x = 0$

$x(x^3 - 64) = 0$

$x = 0, \; x = 4$

Area of Region $= A = \int_0^4 (4 - 2\sqrt{x})\, dx = \frac{16}{3}$ Sq. units.

Ar. of Region $B = = \int_0^4 \left(2\sqrt{x} - \frac{x^2}{4}\right) dx = \frac{16}{3}$ Sq. units.

Ar. of Region $C = \int_0^4 \left(\frac{x^2}{4} - 0\right) dx = \frac{16}{3}$ Sq. units.

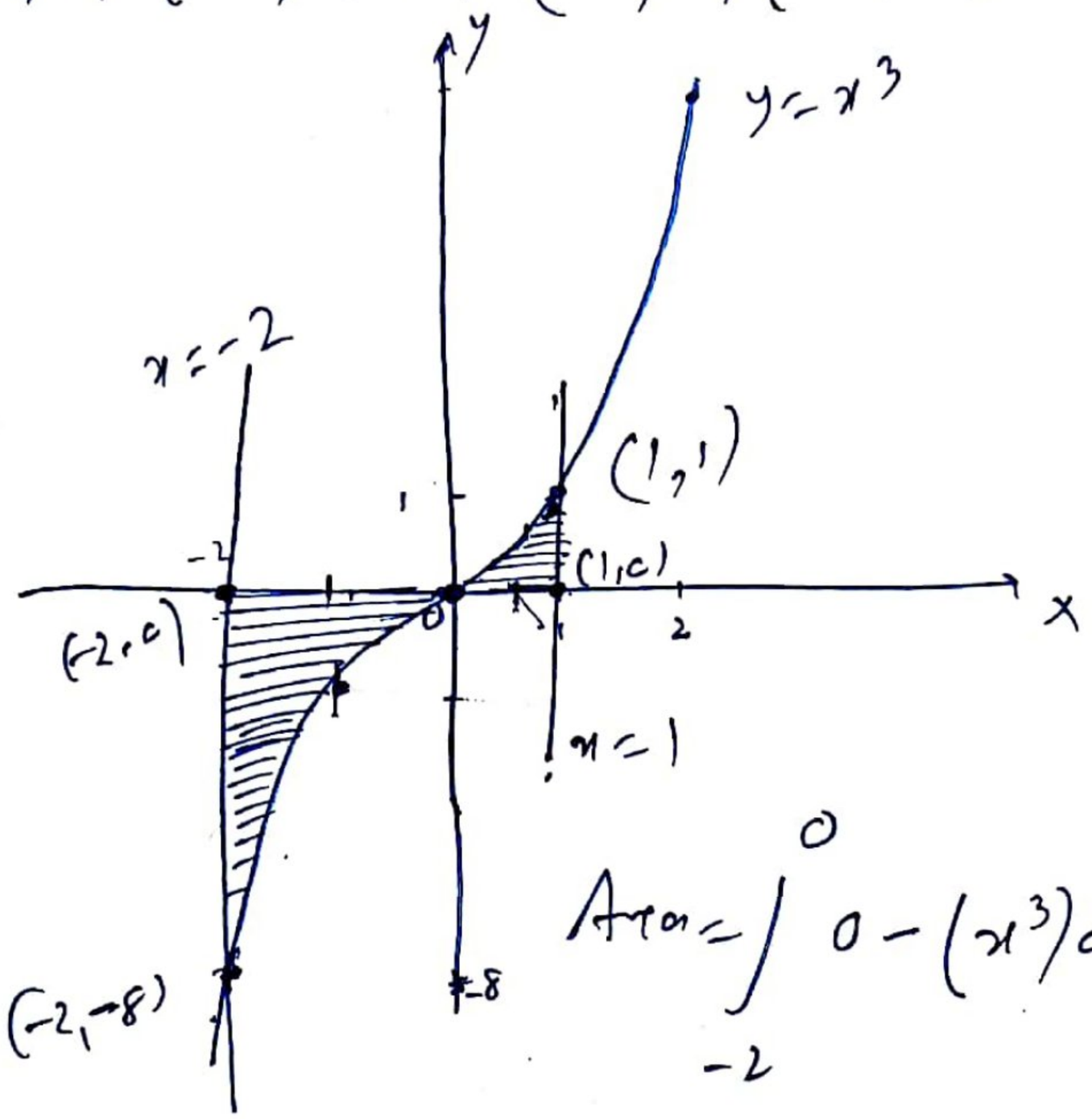
Hence

Qn:-7 Find the area bounded by the curves

$y = x^3$; x-axis ; $x = -2$ and $x = 1$

Sol:- (1) $y = x^3$

Point (0,0) (1,1) (2,8) (-1,-1) (-2,-8)



$$\text{Area} = \int_{-2}^{0} 0 - (x^3)\,dx + \int_{0}^{1} x^3\,dx$$

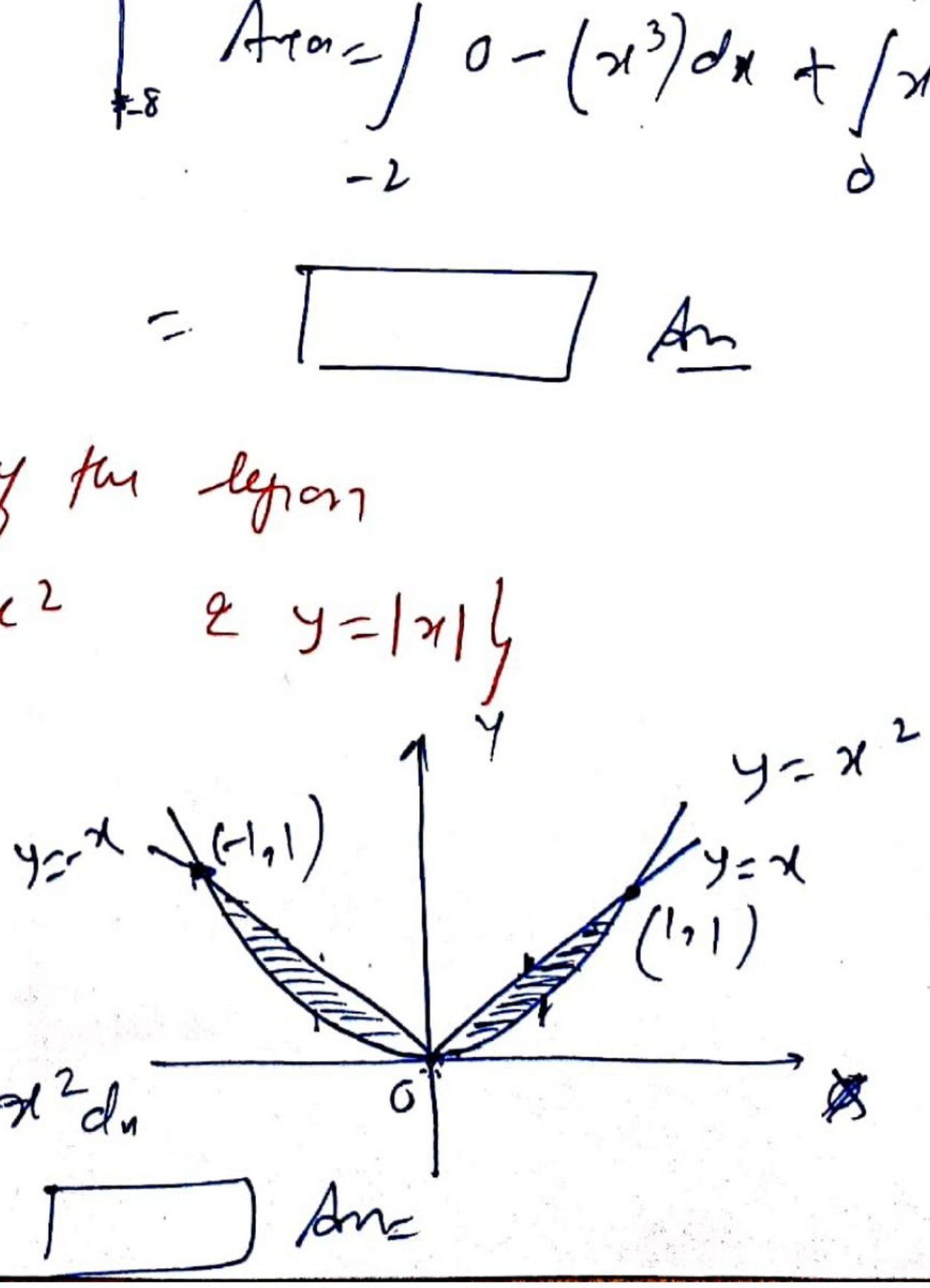
$$= \square \quad \text{Ans}$$

Qn:-8 Find the area of the region

$\{(x,y) : y \geq x^2 \text{ & } y = |x|\}$

Sol (1) $x^2 \leq y$

(2) $y = |x|$



$$\text{Area} = 2\int_{0}^{1} x - x^2\,dx$$

$\square$ Ans

Qn 9 → Find the area bounded by the curve

$|x| + |y| = 1$

Soln (1) $x + y = 1$; $x \geq 0$, $y \geq 0$

$(0, 1)$ $(1, 0)$

(2) $x - y = 1$; $x \geq 0$, $y < 0$

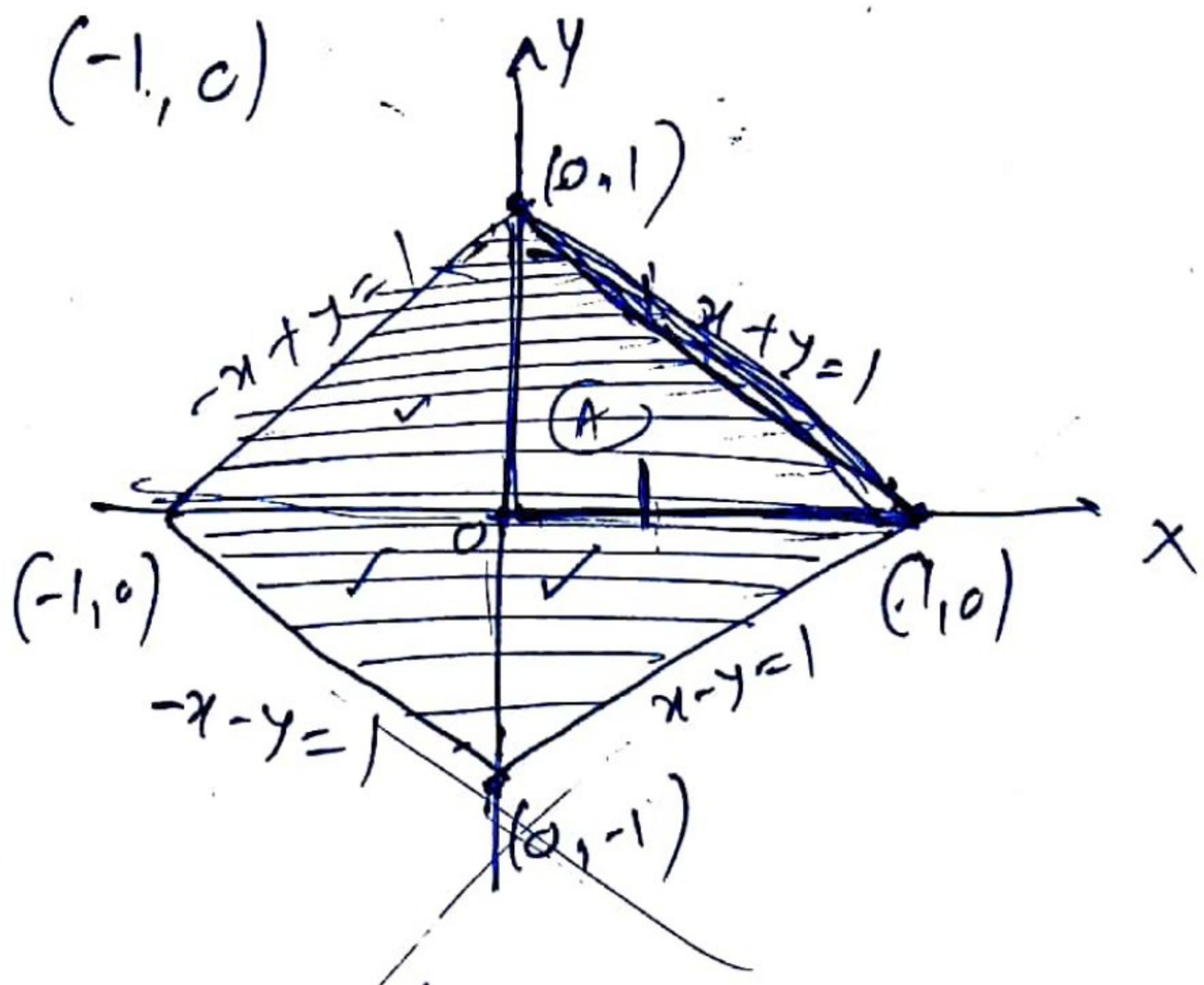
$(0, -1)$ $(1, 0)$

(3) $-x - y = 1$; $x < 0$, $y < 0$

$(0, -1)$ $(-1, 0)$

(4) $-x + y = 1$; $x < 0$ & $y \geq 0$

$(0, 1)$ $(-1, 0)$



By area = $4 \int_0^1 (1 - x)\, dx$

$= \square$ sq

Q~10 Sketch the graph $y = |x+3|$ and evaluate $\int_{-6}^{0} |x+3|\,dx$. what does this value represent on the graph

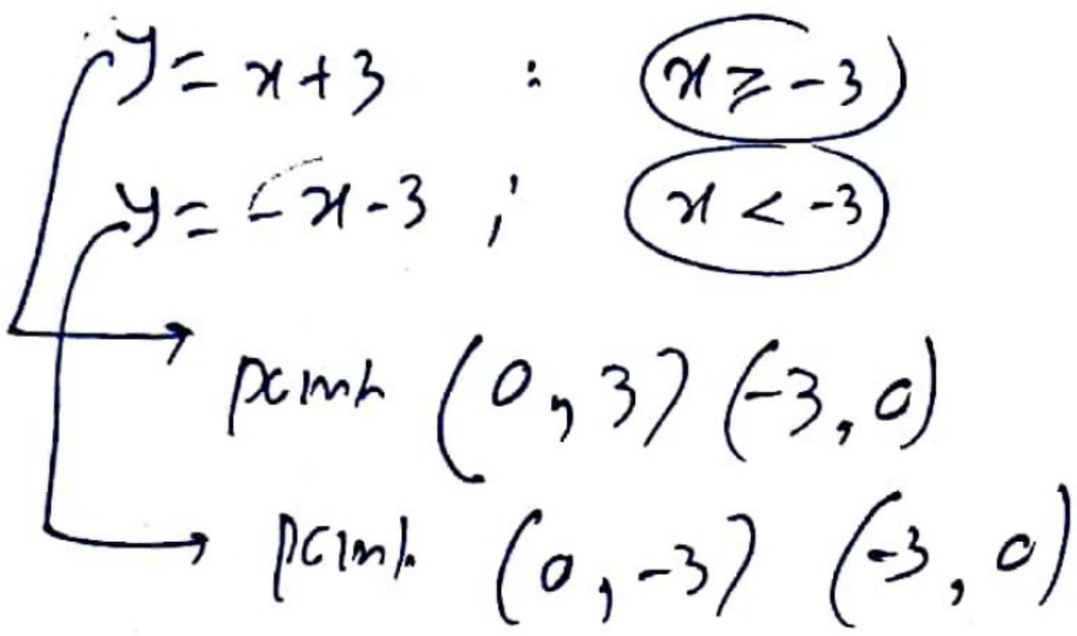
Sol

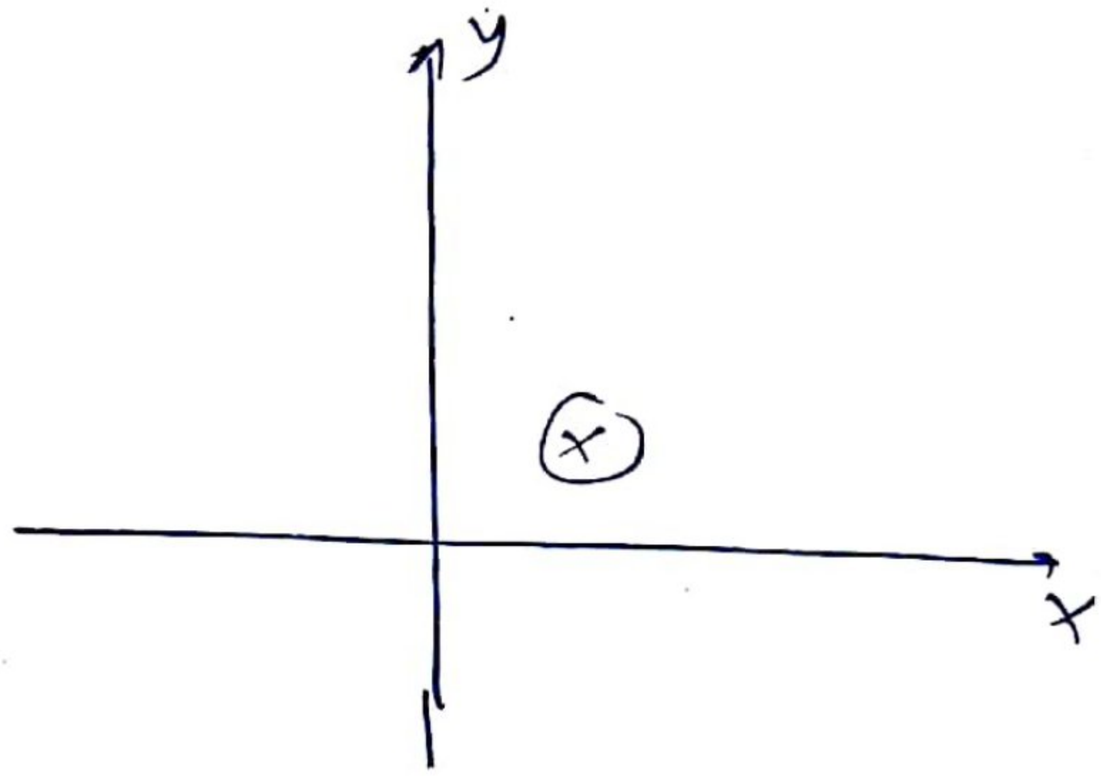
$$y = |x+3|$$

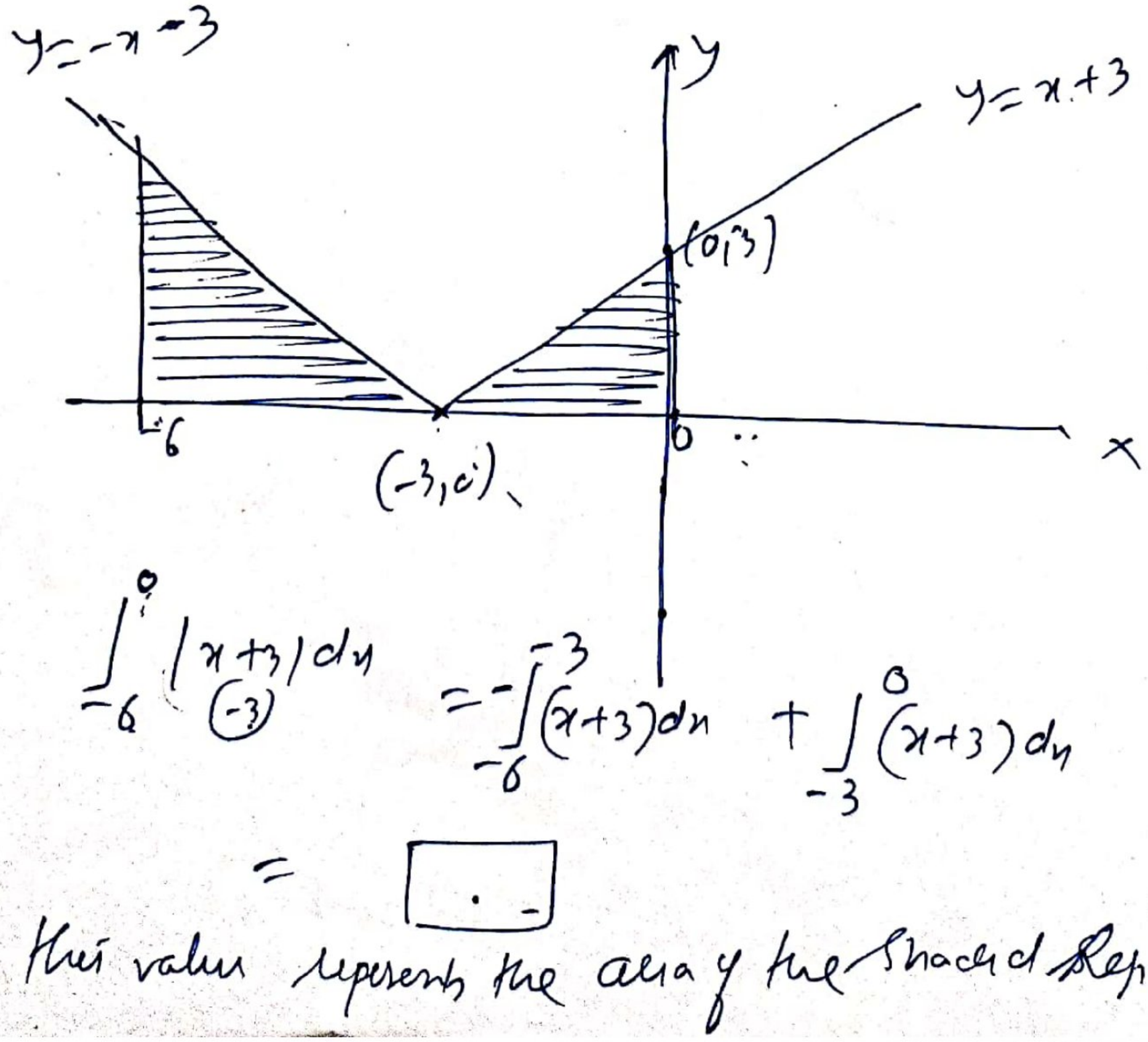
$$y = \begin{cases} x+3 : & x+3 \geq 0 \\ -x-3 : & x+3 < 0 \end{cases}$$

$y = x+3 \quad : \quad (x \geq -3)$

$y = -x-3 \quad ; \quad (x < -3)$







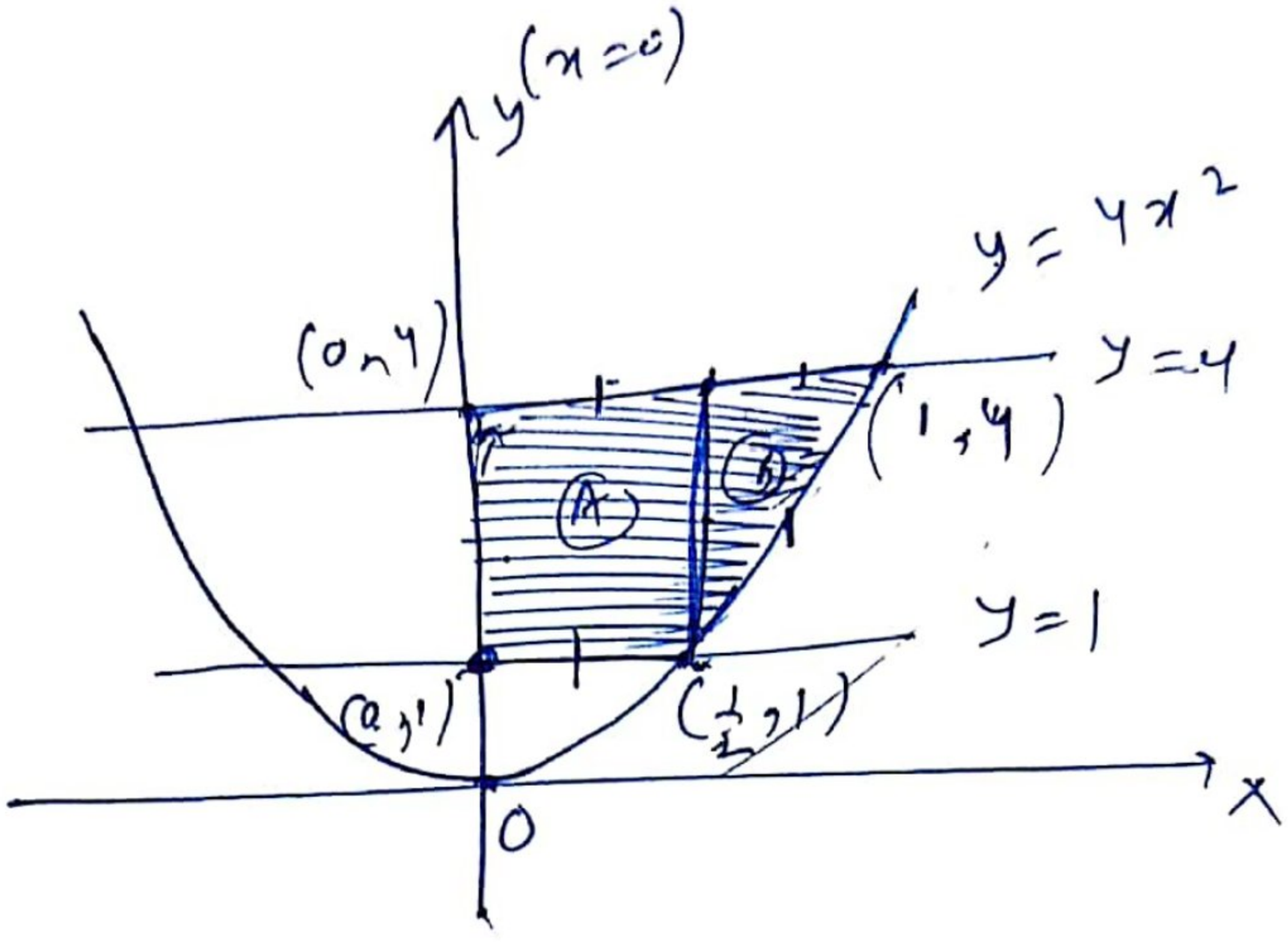
(ii) $\int_{-6}^{0} |x+3|\,dx = -\int_{-6}^{-3} (x+3)\,dx + \int_{-3}^{0} (x+3)\,dx$

$= \square$

(iii) that value represents the area of the shaded Region on the graph.

Q.No: 11 → Find the area in the 1st quadrant and bounded by $y = 4x^2$; $x = 0$; $y = 1$, $y = 4$

Sol:
(1) $4x^2 = y$
(2) $x = 0$
(3) $y = 1$
(4) $y = 4$



Required Area

$$= \int_0^{1/2} (4 - 1)\,dx + \int_{1/2}^{1} (4 - 4x^2)\,dx$$

□ Ans

—x—

~~Def:~~ **Deleted Questions from NCERT**

(All Circles & ELLIPSE QUESTIONS)

Examples: 1, 2, 4, 5, 7, 8, 10

Exercise: 8.1 : Qns 4, 5, 6, 7, 12

Exercise 8.2 : Qns 1, 2, 6

Miscellaneous: Qns 8, 9, 15, 18

—x—

WORKSHEET NO: 1 | Chapter. AOI

(Class No: 2)

Q1: Find the area bounded by the triangle whose vertices are A(2,0) B(4,5) C(6,3) | Ans 7

Q1. 2 → Find the area bounded by the curves $y = x^2$; $x = 1$, $x = 2$ and X-axis | Ans = 7/3

Q No 3 → Find the area enclosed between the parabola $y^2 = 4ax$ and the line $y = mx$ | Ans = $\frac{8a^2}{3m^3}$

Q No. 4 → Find the area under the curve $y = x^4$; $x = 1$, $x = 5$ and X-axis | Ans 624.8

(HINT: Find Points) (parabola type shape)

Q No 5 → Find the area of the region bounded by the line $y = 3x + 2$; X-axis and the ordinates $x = -1$ and $x = 1$ | Ans = $\frac{13}{3}$

Q. 6 → Find the area of the parabola $y^2 = 4ax$ and its latus rectum (Ans $\frac{8}{3}a^2$)

(HINT: equation of Latus rectum: $x = a$)

Q No 7 → Find the area under the curve $y^2 = 4x$ and $2x = y$ | Ans = 1/3

Q No 8 → Find the area of the triangles whose sides have the equations $y = 2x + 1$; $y = 3x + 1$; $x = 4$ | Ans = 8

Q. 9 → Find the area bounded by the parabolas $y = x^2$ and $y^2 = x$ | Ans = 1/3

Qn-10 → Find the area bounded by the parabola $x^2 = 4y$ and the line $x = 4y - 2$

Ans = $\frac{9}{8}$

Qn 11 → The area between $x = y^2$ and $x = 4$ is divided ~~en~~ to ~~two~~ two equal parts by the line $x = a$ . . Find the value of a

Ans = $a = 4^{2/3}$

— x —