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DIFFERENTIATION & CONTINUITY

CLASS NO: 8

Qn. 1 Find the value of 'a' so that the function

$$f(x) = \begin{cases} a\sin\left(\frac{\pi}{2}x + \frac{\pi}{2}\right) & : x \geq 0 \\ \frac{\tan x - \sin x}{x^3} & : x < 0 \end{cases}$$ is continuous at $x = 0$

Soln

$$LHL = \lim_{x \to 0^-}\left(\frac{\tan x - \sin x}{x^3}\right)$$

put $x = 0 - h = -h$ & $h \to 0$

$$LHL = \lim_{h \to 0}\left(\frac{\tan(-h) - \sin(-h)}{(-h)^3}\right)$$

$$= \lim_{h \to 0}\left(\frac{-\tan h + \sin h}{-h^3}\right)$$

$$= \lim_{h \to 0}\left(\frac{\tan h - \sin h}{h^3}\right)$$

$$= \lim_{h \to 0}\left(\frac{\frac{\sin h}{\cos h} - \sin h}{h^3}\right)$$

$$= \lim_{h \to 0}\left(\frac{\sin h - \sin h \cos h}{h^3 \cdot \cos h}\right)$$

$$= \lim_{h \to 0}\left(\frac{\sin h(1 - \cos h)}{h^3 \cdot \cos h}\right)$$

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$$= \lim_{h \to 0} \left(\frac{\sin h \cdot 2\sin^2(h/2)}{h^3 \cdot \cos h} \right)$$

$$= \lim_{h \to 0} \left[\frac{\frac{\sin h}{h} \cdot \cancel{h} \times 2 \frac{\sin^2\left(\frac{h}{2}\right)}{\frac{h^2}{4}} \times \frac{\cancel{h^2}}{4}}{\cancel{h^3} \cdot \cos h} \right]$$

$$= \frac{2 \times \frac{1}{4}}{1} = \frac{1}{2} \quad \therefore \text{LHL} = 1/2$$

$$\text{RHL} = \lim_{x \to 0^+} \left(a \sin\left(\frac{\pi}{2}x + \frac{\pi}{2}\right)\right)$$

put $x = 0 + h = h$ & $h \to 0$

$$= \lim_{h \to 0} \left(a \sin\left(\frac{\pi}{2}h + \frac{\pi}{2}\right)\right)$$

$$= a\sin\left(\frac{\pi}{2}\right) = a \times 1 = a$$

RHL = a

$$f(0) = a\sin\left(\frac{\pi}{2}\right) = a \times 1 = a$$

Since $f(x)$ is cont. at $x = 0$

LHL = RHL = $f(0)$

$$\frac{1}{2} = a = a$$

$\Rightarrow$ $a = 1/2$ Ans

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Ques 2 → For what value of λ is the function

special

$$f(x) = \begin{cases} \lambda(x^2 - 2x) & ; x \le 0 \\ 4x + 1 & ; x > 0 \end{cases}$$

at $x = 0$? at $x = 1$? and at $x = -1$?

Sol: Cont. at $x = 0$

$LHL = \lim_{x \to 0^-} \left(\lambda(x^2 - 2x) \right)$

put $x = 0 - h = -h$ & $h \to 0$

$LHL = \lim_{h \to 0} \left(\lambda(h^2 + 2h) \right)$

$\boxed{LHL = 0}$

$RHL = \lim_{x \to 0^+} (4x + 1)$ put $x = 0 + h$ & $h \to 0$

$RHL = \lim_{h \to 0} (4h + 1)$

$\boxed{RHL = 1}$ $f(0) = 0$

Since we are given $f(x)$ is cont at $x = 0$

$0 = 1 = 0$ (false)

there is NO value of λ Ans

Cont at $x = 1$

$LHL = \lim_{x \to 1^-} (4x + 1)$ put $x = 1 - h$ & $h \to 0$

$LHL = \lim_{h \to 0} \left(4(1 - h) + 1 \right) = 5$

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$RHL = \lim_{x \to 1^+} (4x+1)$ put $x = 1+h$ & $h \to 0$

$RHL = \lim_{h \to 0} (4(1+h)+1)) = 4+1 = 5$

$f(1) = (4(1)+1) = 4+1 = 5$

$f(x)$ is cont at $x = 1$

$5 = 5 = 5$ (True)

$\lambda \in R$ for every value of λ $f(x)$ is cont. at $x = 1$

(c) Cont. at $x = -1$

$LHL = \lim_{x \to -1^-} (\lambda(x^2-2x))$

put $x = -1-h$ & $h \to 0$

$RHL = \lim_{h \to 0} (\lambda[(-1-h)^2 - 2(-1-h))]$

$= \lambda[(1) + 2] = 3\lambda$

$= \cancel{\lambda+2}\ 3\lambda$

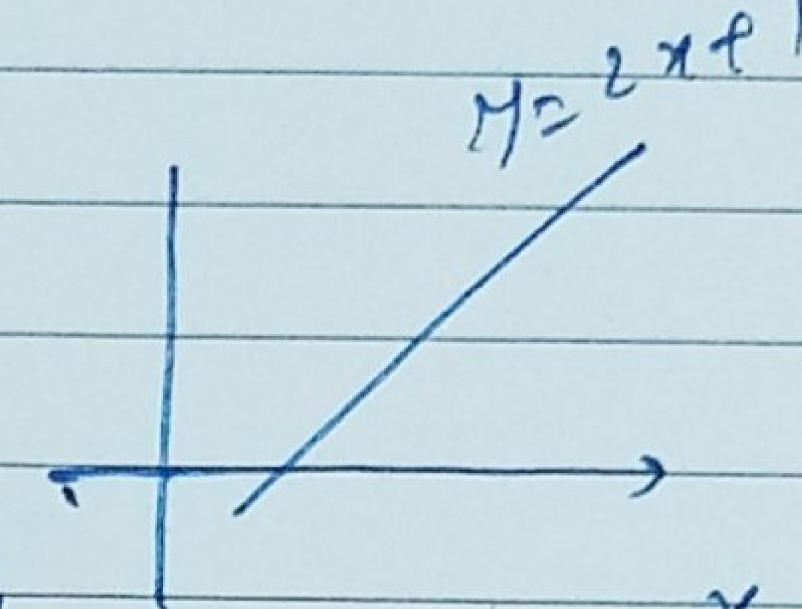
$RHL = \cancel{\lambda+2}\ 3\lambda$ $f(-1) = \lambda(1+2) = 3\lambda$

$\therefore 3\lambda = 3\lambda = 3\lambda \Rightarrow \lambda \in R$

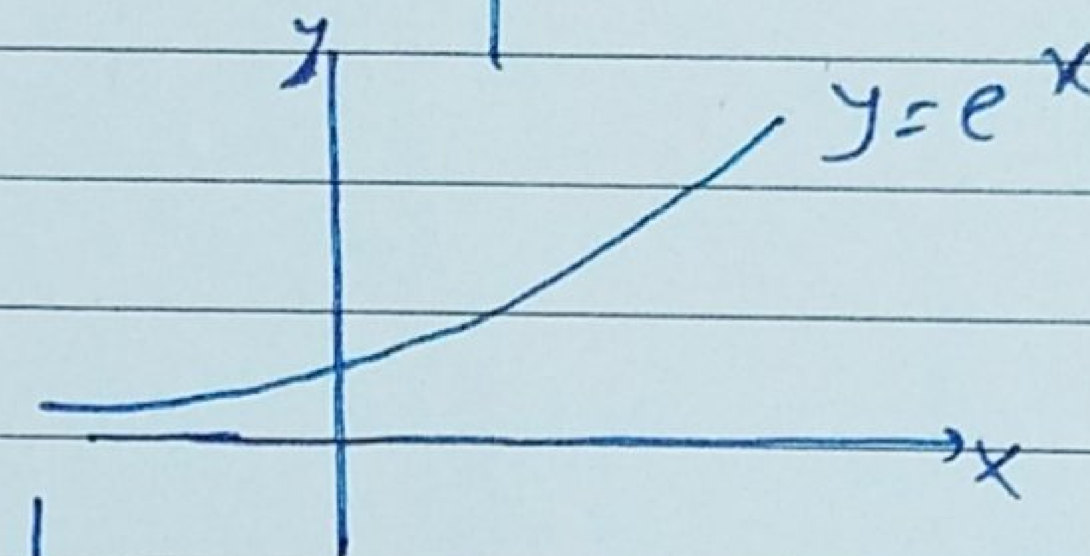
CLASSTIME

everywhere continuous functions

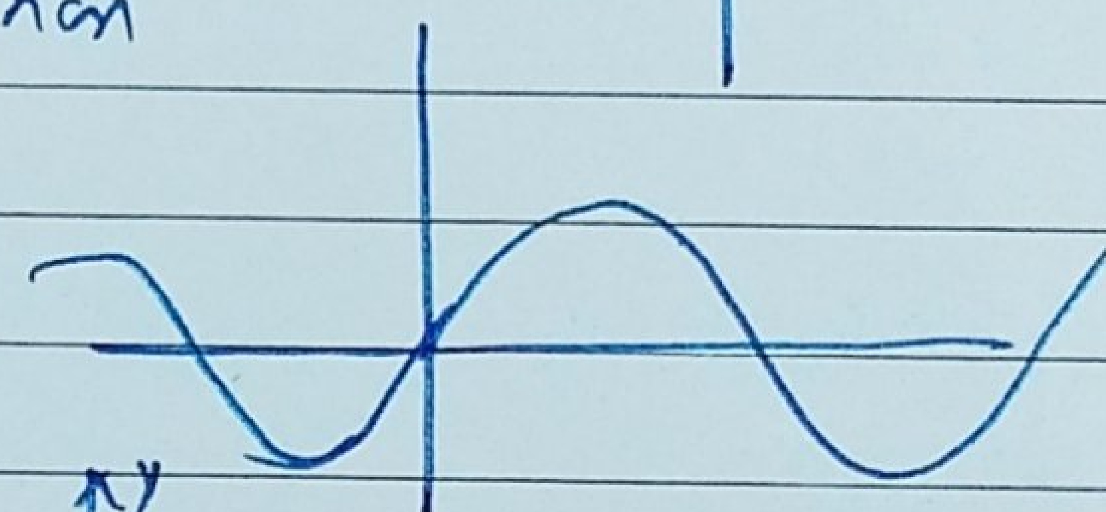
(1) polynomial function



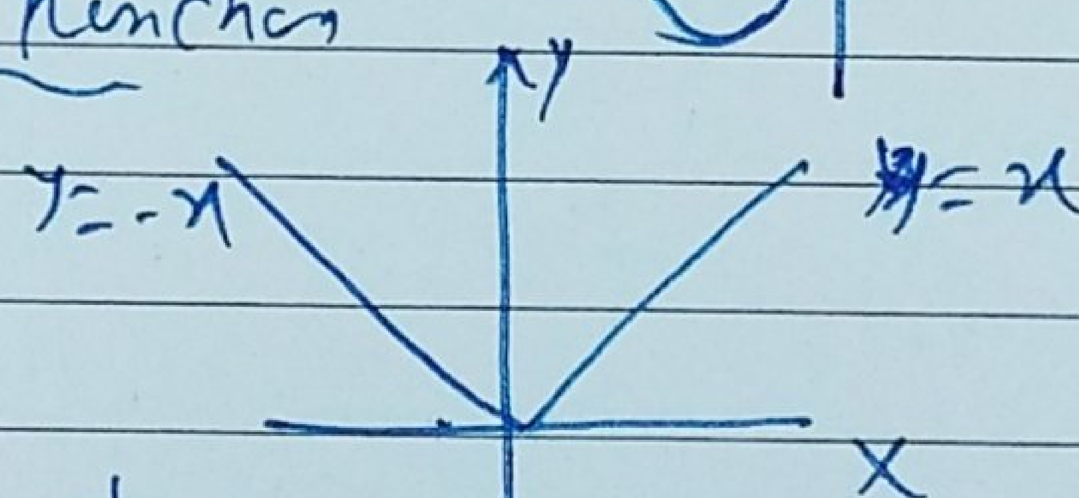
(2) exponential function



(3) sine, cosine function

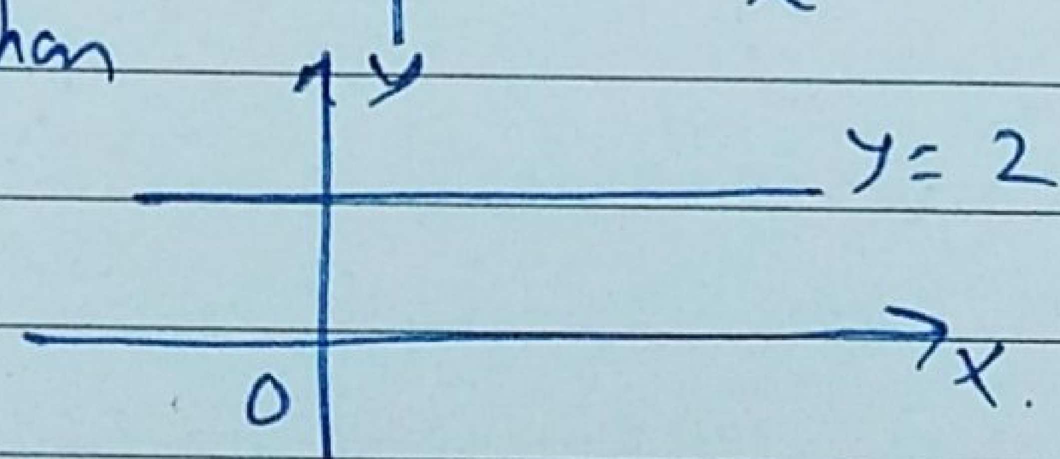


(4) Modulus function

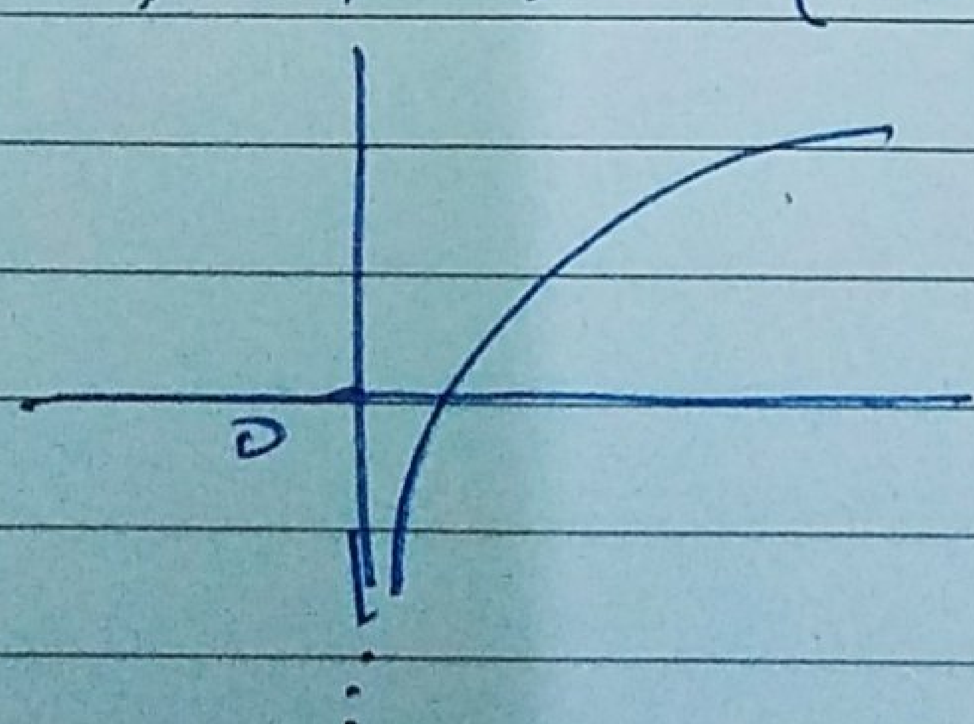


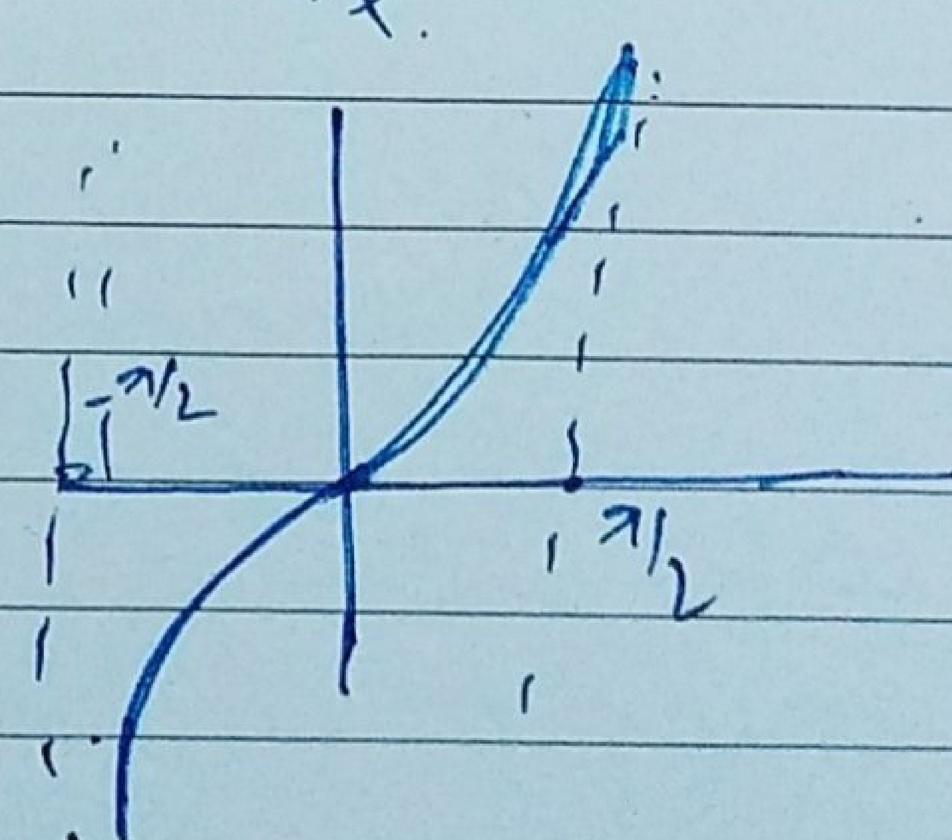
(5) constant function

$f(x) = k$



(6) $\tan x$, $\log x$ (domain)





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QN (or) Discuss the continuity of the function. Find all the points of discontinuity

$$f(x) = |x+1| + |x-1|$$

Sol

$$f(x) = |x+1| + |x-1|$$

$$\leftarrow (-1) \leftrightarrow (1) \rightarrow$$

$$f(x) = \begin{cases} -(x+1)-(x-1) \;; & x < -1 \\ (x+1)-(x-1) & -1 \le x < 1 \\ (x+1)+(x-1) & x \ge 1 \end{cases}$$

$$f(x) = \begin{cases} -2x & : \; x < -1 \\ 2 & : \; -1 \le x < 1 \\ 2x & : \; x \ge 1 \end{cases}$$

(i) when $x < -1$

$f(x) = -2x$ which is a polynomial function and it is everywhere continuous

$\therefore f(x)$ is continuous for all $x < -1$

(i) when $-1 \le x < 1$

$f(x) = 2$ which is a constant function and it is everywhere continuous

(i) when $x > 1$

$f(x) = 2x$ ....

(i) cont at $x = -1$ ✓ | (i) cont at $x = 1$ ✓

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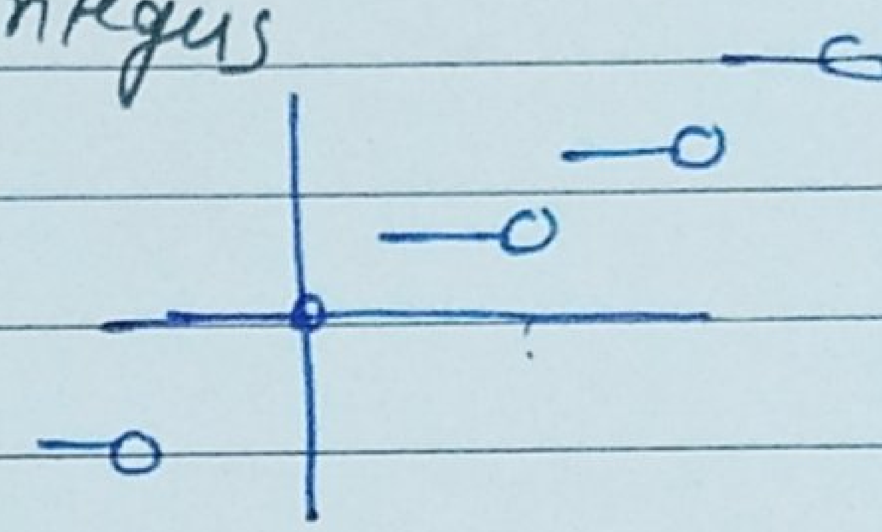
Qns Prove that the greatest Integer function $[x]$ is not continuous at all the Integers



Sol

$[1 \text{ to } 2] = 1$

$[7 \text{ to } 8] = 7$

$[7 \text{ to } 9] = ?$ X

let $k \in Z$

$$f(x) = \begin{cases} k-1 & ; \ k-1 \le x < k \\ k & ; \ k \le x < k+1 \end{cases}$$

Cont at $x = k$

$$LHL = \lim_{x \to k^-} (k-1) = k-1$$

$$RHL = \lim_{x \to k^+} (k) = k$$

$$f(k) = k$$

But $LHL \neq RHL$

$\therefore f(x)$ is not Cont at $x = k$

Since $k \in Z$ Ans

$\therefore f(x)$ is not cont at all the Integers.

← **DIFFERENTIABILITY** →

A function $f(x)$ is said to be differentiable at $x = a$ if

~~LHD~~ $\boxed{L.H.D = R.H.D}$

$x<a$

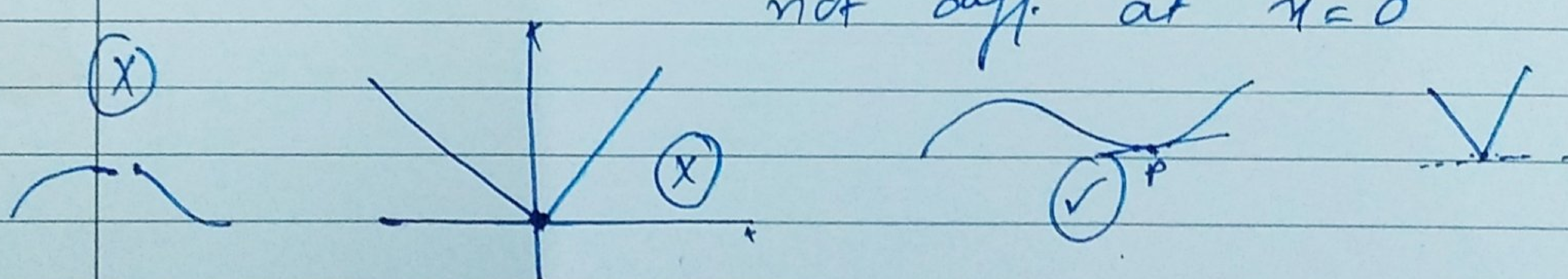
$$LHD = \lim_{x\to a^-}\left(\frac{f(x)-f(a)}{x-a}\right) \quad x=a$$

$x>a$

$$RHD = \lim_{x\to a^+}\left(\frac{f(x)-f(a)}{x-a}\right) \quad x=a$$

(·) Every Differentiable function is continuous. but every continuous function need not be differentiable

eg $f(x) = |x|$ ~~but~~ is cont not
not diff. at $x=0$



Que Show that $f(x) = |x-2|$ is not differentiable at $x=2$

Sol
$$f(x) = \begin{cases} x-2 & : x-2 \geq 0 \ ; \ x \geq 2 \\ -x+2 & , x-2 < 0 \ ; \ x < 2 \end{cases}$$

$$LHD = \lim_{x\to 2^-}\left(\frac{-x+2-f(2)}{x-2}\right)$$

$$= \lim_{x\to 2^-}\left(\frac{-x+2-0}{x-2}\right)$$

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$$LHD = \lim_{x \to 2^-} \left(\frac{-(x-2)}{x-2} \right)$$

$$LHD = \lim_{x \to 2^-} (-1) = -1$$

$$RHD = \lim_{x \to 2^+} \left(\frac{f(x) - f(2)}{x-2} \right)$$

$$= \lim_{x \to 2^+} \left(\frac{x-2-0}{x-2} \right)$$

$$= \lim_{x \to 2^+} (1) = 1$$

$RHD = 1$

Since $LHD \neq RHD$

$\therefore f(x)$ is not Differentiable at $x = 2$ Ans

Qn 2 $f(x) = [x]$ Check the Differentiability of $f(x)$ at $x = 1$ & $x = 2$; $0 < x < 3$

Ans: $f(x) = \begin{cases} 0 & ; 0 < x < 1 \\ 1 & ; 1 \leq x < 2 \\ 2 & ; 2 \leq x < 3 \end{cases}$

Diff at $x = 1$

$$LHD = \lim_{x \to 1^-} \left(\frac{f(x) - f(1)}{x-1} \right)$$

$= \lim_{x \to 1^-} \left(0 - 1 \right)$ put $x = 1-h$ & $h \to 0$

$$\lim_{x \to 1^+} \left(\frac{}{x-1} \right)$$

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$$LHD = \lim_{h \to 0} \left(\frac{-1}{\cancel{1}-h-\cancel{1}} \right) = \lim_{h \to 0} \left(\frac{1}{h} \right) = \infty$$

$$RHD = \lim_{x \to 1^+} \left(\frac{f(x) - f(1)}{x-1} \right)$$

$$= \lim_{x \to 1^+} \left(\frac{1-1}{x-1} \right) = \lim_{x \to 0} \left(\frac{0}{x-1} \right) = 0$$

$LHD \neq RHD$

$\therefore f(x)$ is not differentiable at $x = 1$

Similarly $f(x)$ is not Diff at $x = 2$ Ans

—x—