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ULTIMATE MATHEMATICS

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Chapter: RELATION & FUNCTION

CLASS NO: 2

Qn 1 → Prove that the relation R on the set N×N defined by $(a,b)\,R\,(c,d) \Leftrightarrow a+d=b+c$ is an equivalence relation

Sol (1) Symmetric Relation

let $(a,b)\,R\,(c,d)$

$\Rightarrow a+d=b+c$

$\Rightarrow d+a=c+b$

$\Rightarrow c+b=d+a$

$\Rightarrow (c,d)\,R\,(a,b)$

$\therefore$ R is symmetric Relation

Rough work
$(c,d)\,R\,(a,b)$
$c+b=d+a$
$(a,b) \in R$

(2) Transitive Relation

let $(a,b)\,R\,(c,d)$ and $(c,d)\,R\,(e,f)$

$\Rightarrow a+d=b+c$ & $c+f=d+e$

$\Rightarrow a+d=b+c$ & $d=c+f-e$

$\Rightarrow a+\cancel{c}+f-e=b+\cancel{c}$

$\Rightarrow a+f=b+e$

$\Rightarrow (a,b)\,R\,(e,f)$

$\therefore$ R is transitive relation

Rough
$(a,b)\,R\,(e,f)$
$a+f=b+e$

(3) Reflexive relation

for each $(a,b) \in N\times N$

we always have

$a+b=b+a$

$\Rightarrow (a,b)\,R\,(a,b)$

$\therefore$ R is reflexive relation

Rough.
$(a,b)\,R\,(a,b)$
$a+b=b+a$

$\therefore$ R is an equivalence relation.

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R&F (Class No:- 2) ②

Qn.2 → If R_1 and R_2 are equivalence relations in a set A, Show that $R_1 \cap R_2$ is also an equivalence relation.

Soln XI Sets { If $x \in (A \cap B)$ then $x \in A$ and $x \in B$ } { If $x \in A$ and $x \in B$ then $x \in A \cap B$ }

(1) Symmetric relation

let $(a,b) \in R_1 \cap R_2$

$\Rightarrow (a,b) \in R_1$ and $(a,b) \in R_2$

$\Rightarrow (b,a) \in R_1$ and $(b,a) \in R_2$ --- { $\because R_1$ & R_2 are itself symmetric }

$\Rightarrow (b,a) \in R_1 \cap R_2$

$\therefore R_1 \cap R_2$ is symmetric relation

(2) Transitive relation

let $(a,b) \in R_1 \cap R_2$ and $(b,c) \in R_1 \cap R_2$

$\Rightarrow (a,b) \in R_1$ and $(a,b) \in R_2$ and $(b,c) \in R_1$ and $(b,c) \in R_2$

$\Rightarrow (a,b) \in R_1$ and $(b,c) \in R_1$ and $(a,b) \in R_2$ and $(b,c) \in R_2$

$\Rightarrow (a,c) \in R_1$ and $(a,c) \in R_2$ --- { $\because R_1$ and R_2 are transitive Relation }

$\Rightarrow (a,c) \in R_1 \cap R_2$

$\therefore R_1 \cap R_2$ is a transitive relation

(3) Reflexive relation

for each $a \in A$

$(a,a) \in R_1$ and $(a,a) \in R_2$ --- { $\because R_1$ & R_2 are reflexive relation }

$\Rightarrow (a,a) \in R_1 \cap R_2$

$\therefore R_1 \cap R_2$ is Reflexive relation

$\therefore R_1 \cap R_2$ is an equivalence relation.

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RCF (class No: 2) (3)

Qn. 3 → Let $A = \{1, 2, 3\}$. Then find the number of equivalence relations containing $(1,2)$.

Soln $A = \{1, 2, 3\}$

largest Relation = $A \times A$

$R = \{(1,1)(1,2)(1,3)(2,1)(2,2),(2,3)(3,1)(3,2)(3,3)\}$

✓ $R_1 = \{(1,1)(2,2)(3,3),(1,2),(2,1)\}$

✓ $R_2 = \{(1,1)(2,2)(3,3)(1,2)(2,1),(1,3)(3,1),(2,3)(3,2)\}$

∴ there are two equivalence relation contain $(1,2)$ Ans

Qn. 4 → Let $A = \{1, 2, 3\}$ then show that number of relations containing $(1,2)$ & $(2,3)$ which are reflexive and transitive but not symmetric is three.

Soln largest Relation $R = \{(1,1)(1,2)(1,3)(2,1)(2,2)(2,3)(3,1)(3,2)(3,3)\}$

$R_1 = \{(1,1)(2,2)(3,3)(1,2)(2,3)(1,3)\}$

$R_2 = \{(1,1)(2,2)(3,3)(1,2)(2,3)(1,3)(2,1)\}$

~~$R_3 = \{(1,1)(2,2)(3,3)(1,2)(2,3)(1,3)(3,1)(\ldots)(3,2)(2,1)$~~

$R_3 = \{(1,1)(2,2)(3,3)(1,2)(2,3)(1,3)(3,2)\}$

∴ Req. no of relation is 3 Ans

CLASSTIME

REF (class No. 2)

Qn. 5 → Given a non empty set X, consider P(X) which is the power set of X.
Define the relation R in P(X) as follows:
A R B if and only if A ⊂ B. Is R is an equivalence relation? Justify your Answer

Soln (NO) Symmetric relation
let A. A R B
⇒ A ⊂ B
then it is not necessary that B ⊂ A
eg A = {1, 2} ; B = {1, 2, 3}
~~Clearly A ⊂ B~~
~~but B ⊄ A~~
B R̸ A
∴ R is not symmetric
∴ R is not an equivalence relation

Qn. 6 → A = ~~{1, 2}~~ {1, 2, 3}
R = {(2, 3)}. Is R is transitive?

Sol It is transitive

Qn. 7 → let R be a relation on N defined as
$R = \{(x, y) : 2x + y = 4\}$. Find the domain and Range of relation R. Also check whether R is reflexive, symmetric and transitive?

$y = 4 - 2x$

Sol $R = \{(1, 2)\}$ Domain = {1}
Range = {2}
$1 \in N$ but $(1,1) \notin R$
$(1,2) \in R$ but $(2,1) \notin R$
✓ It is transitive

REL CLASS NO: 2

Qn. 8 let $A = \{1, 2, 3, 4, 5\}$
Relation on set A given by

$R = \{(a, b) : |a^2 - b^2| < 8\}$ Find domain,
Range, codomain, Roster form
Also check whether R is reflexive, transitive or symmetric

Sol

$R = \{(1,1)\ (2,2)\ (3,3)\ (4,4)\ (5,5)\ (1,2)\ (2,1),$
$(2,3)\ (3,2)\ (3,4)\ (4,3)\}$

Domain $\{1, 2, 3, 4, 5\}$

Range $\{1, 2, 3, 4, 5\}$

Codomain $A = \{1, 2, 3, 4, 5\}$

$(1,2) \in R$ & $(2,3) \in R$ but $(1,3) \notin R$
$\therefore$ not transitive

✓ Symmetric : for each $(a, b) \in R$, $(b, a) \in R$
✓ reflexive for each $a \in A$, $(a, a) \in R$

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RELATION & FUNCTION

(1)

WORKSHEET No: 2

Qn. 1 → let R be a relation on the set A of "ordered pairs" of +ve integers defined by
$(x, y) R (u, v) \Leftrightarrow xv = yu$. Show that R is an equivalence relation.

Qn. 2 → let $f: X \to Y$ be a function. Define a relation R in X given by
$R = \{(a, b) : f(a) = f(b)$. Examine whether R is an equivalence relation or not.

Qn. 3 → Show that the number of equivalence relations in the set $(1, 2, 3)$ containing $(1, 2)$ & $(2, 1)$ is two.

Qn. 4 → let $A = \{1, 2, 3\}$. The find the number of relations containing $(1, 2)$ and $(1, 3)$ which are reflexive and symmetric but not transitive

ANS: 1

Qn. 5 → $A = \{1, 2, 3\}$ Relation on set R
Write the smallest equivalence relation

Qn. 6 → $A = \{1, 2, 3, \ldots 9\}$ and Relation R in $A \times A$ defined by $(a, b) R (c, d) \Leftrightarrow a + d = b + c$
find the "equivalence class" $[(2, 5)]$

Qn. 7 → N set of natural numbers and Relation R on $N \times N$ defined by $(a, b) R (c, d) \Leftrightarrow ad(b + c) = bc(a + d)$
Check whether R is an equivalence or not

Qn. 8 → R is a relation on set Z of integers and given by $R = \{(x, y) : |x - y| \le 1\}$ Check whether R is

CLASSTIME

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reflexive, Symmetric or transitive?

Qn. 9 → Each of the following defines a relation on N.

(i) x is greater than y

(2) $x+y=10$

(3) xy is square of an integer

Determine which of the above relations are reflexive, symmetric and transitive.

Qn 10 → Let $A=\{2,3,4,---17,18\}$

Let R be an equivalence relation on $A \times A$

Given by $(a,b)\,R\,(c,d) \Leftrightarrow ad=bc$

Find the " equivalence class $[(3,2)]$

—x—

CLASSTIME