Solutions

WORKsheet No: 2 (class -3)

Integration

Qn: 1 $I = \int \frac{1}{\sin^2 x \cdot \cos^2 x} dx$

divide by $\cos^4 x$

$I = \int \frac{\sec^4 x}{\tan^2 x} dx$

$= \int \frac{(1+\tan^2 x)\cdot \sec^2 x}{\tan^2 x} dx$

put $\tan x = t \Rightarrow \sec^2 x\, dx = dt$

$I = \int \frac{(1+t^2)\, dt}{t^2}$

$= \int \left(\frac{1}{t^2} + 1\right) dt$

$= -\frac{1}{t} + t + C$

$= -\frac{1}{\tan x} + \tan x + C$ **Ans**

Qn: 2 $I = \int \frac{\sec x}{\sec x + \tan x} dx$ (change in to $\sin x$, $\cos x$)

$= \int \frac{1}{1+\sin x} dx$

Rationalize

$= \int \frac{1-\sin x}{\cos^2 x} dx$

$= \int \sec^2 x - \tan x \sec x \, dx$

$= \tan x - \sec x + C$

Qn 3 $I = \int \frac{\cot x}{\operatorname{cosec} x - \cot x} dx$

$= \int \frac{\cos x}{1 - \cos x} dx$ (Rationalize)

$$I = \int \frac{\cos x(1+\cos x)}{\sin^2 x} dx$$

$$= \int \frac{\cos x + \cos^2 x}{\sin^2 x} dx$$

$$= \int \cot x \cdot \operatorname{cosec} x + \cot^2 x \, dx$$

$$= \int \operatorname{cosec} x \cdot \cot x + \operatorname{cosec}^2 x - 1 \, dx$$

$$= -\operatorname{cosec} x - \cot x - x + C$$ Ans

Ans 4 $\quad I = \int \frac{\operatorname{cosec} x}{\operatorname{cosec} x - \cot x} dx$

$$I = \int \frac{1}{1-\cos x} dx \quad \text{(Rationalize)}$$

$$= \int \frac{1+\cos x}{\sin^2 x}$$

$$= \int \operatorname{cosec}^2 x + \operatorname{cosec} x \cdot \cot x \, dx$$

$$= -\cot x - \operatorname{cosec} x + C$$ Ans

Ans 5 → $\quad I = \int \frac{x+2}{(x+1)^2} dx$

put $x+1 = t$

$dx = dt$

$$I = \int \frac{(t-1)+2}{t^2} \cdot dt$$

$$= \int \frac{t+1}{t^2} dt$$

$$= \int \frac{1}{t} + \frac{1}{t^2} dt$$

$$= \log|t| - \frac{1}{t} + C \Rightarrow \log|x+1| - \frac{1}{x+1} + C$$ Ans

Ans 6 $I = \int (7x-2)\sqrt{3x+2}\, dx$

put $3x+2 = t^2$

$3dx = 2t\,dt$

$dx = \frac{2}{3} t\, dt$

$I = \frac{2}{3}\int \left(7\left(\frac{t^2-2}{3}\right) - 2\right) \cdot t \cdot t\, dt$

$= \frac{2}{9}\int (7t^2 - 20) t^2\, dt$

$= \frac{2}{9}\int (7t^4 - 20t^2)\, dt$

$= \frac{2}{9}\left(\frac{7t^5}{5} - \frac{20t^3}{3} \right] + C$

$= \frac{2}{9}\left[\frac{7(3x+2)^{5/2}}{5} - 20\frac{(3x+2)^{3/2}}{3} \right] + C$ ANS

Ans 7 → $I = \int \frac{8x+13}{\sqrt{4x+7}}\, dx$

put $4x+7 = t^2$

$4dx = 2t\,dt$

$dx = \frac{t\,dt}{2}$

$I = \frac{1}{2}\int \frac{8\left(\frac{t^2-7}{4}\right)+13}{t} \cdot t\,dt$

$= \frac{1}{8}\int 8t^2 - 4\, dt$

$= \frac{1}{8}\left[\frac{8t^3}{3} - 4t \right] + C$

$= \frac{1}{8}\left[\frac{8(4x+7)^{3/2}}{3} - 4(4x+7)^{1/2} \right] + C$ Ans

Qn. 8 → $I=\int \frac{2x-1}{(x-1)^4}dx$

put $x-1=t$
$dx=dt$

$$I=\int \frac{2(t+1)-1}{t^4}dt$$

$$=\int \frac{2t+1}{t^4}dt$$

$$=\int 2t^{-3}+t^{-4}\,dt$$

$$=\frac{2t^{-2}}{-2}+\frac{t^{-3}}{-3}+c$$

$$=-\frac{1}{t^2}-\frac{1}{3t^3}+c$$

$$=-\frac{1}{(x-1)^2}-\frac{1}{3(x-1)^3}+c \quad \text{Ans}$$

Qn. 9 → $I=\int (5x+3)\sqrt{2x-1}\,dx$

put $2x-1=t^2$
$\cancel{2}dx=\cancel{2}t\,dt$

$$I=\int\left(5\left(\frac{t^2+1}{2}\right)+3\right)\cdot t\cdot t\,dt$$

$$=\frac{1}{2}\int (5t^2+11)t^2\,dt$$

$$=\frac{1}{2}\int\left(5t^4+11t^2\right)dt$$

$$=\frac{1}{2}\left[\frac{\cancel{5}t^5}{\cancel{5}}+\frac{11t^3}{3}\right]+c$$

$$=\frac{1}{2}\left[(2x-1)^{5/2}+\frac{11}{3}(2x-1)^{3/2}\right]+c \quad \text{Ans}$$

Ans 10 $I = \int (x^2+2)\sqrt{1-2x}\, dx$

put $1-2x = t^2$

$-2dx = 2t\,dt$

$dx = -t\,dt$

$$= -\int \left(\left(\frac{1-t^2}{2}\right)^2 + 2\right) \cdot t \cdot t\, dt$$

$$= -\int \left(\frac{1+t^4-2t^2}{4} + 2\right) \cdot t^2\, dt$$

$$= -\frac{1}{4}\int (9t^2 + t^6 - 2t^4)\, dt$$

$$= -\frac{1}{4}\left[\frac{9t^3}{3} + \frac{t^7}{7} - \frac{2t^5}{5}\right] + C$$

$$= -\frac{1}{4}\left[3(1-2x)^{3/2} + \frac{(1-2x)^{7/2}}{7} - \frac{2}{5}(1-2x)^{5/2}\right] + C$$

Ans

Ans 11 $I = \int \frac{1}{\cos(x+a)\cos(x+b)}\, dx$

$$= \frac{1}{\sin(a-b)}\int \frac{\sin(a-b)}{\cos(x+a)\cdot\cos(x+b)}\, dx$$

$$= \frac{1}{\sin(a-b)}\int \frac{\sin(a-b+x-x)}{\cos(x+a)\cos(x+b)}\, dx$$

$$= \frac{1}{\sin(a-b)}\int \frac{\sin((x+a)-(x+b))}{\cos(x+a)\cdot\cos(x+b)}\, dx$$

$$= \frac{1}{\sin(a-b)}\int \frac{\sin(x+a)\cdot\cancel{\cos(x+b)}}{\cos(x+a)\,\cancel{\cos(x+b)}} - \frac{\cancel{\cos(x+a)}\cdot\sin(x+b)}{\cancel{\cos(x+a)}\cos(x+b)}\, dx$$

$$= \frac{1}{\sin(a-b)} \int \tan(x+a) - \tan(x+b)\, dx$$

$$= \frac{1}{\sin(a-b)} \left[\log|\sec(x+a)| - \log|\sec(x+b)| \right] + c \quad \underline{\text{Ans}}$$

Qn 12

$$I = \int \frac{\sin(x+a)}{\sin(x+b)}\, dx$$

$$= \int \frac{\sin(x+a+b-b)}{\sin(x+b)}\, dx$$

$$= \int \frac{\sin(\underset{A}{(x+b)} + \underset{B}{(a-b)})}{\sin(x+b)}\, dx$$

$$= \int \frac{\sin(x+b)\cdot\cos(a-b)}{\sin(x+b)} + \frac{\cos(x+b)\sin(a-b)}{\sin(x+b)}\, dx$$

$$= \int \cos(a-b) + \cot(x+b)\sin(a-b)\, dx$$

$$= x\cos(a-b) + \log|\sin(x+b)| \cdot \sin(a-b) + c \quad \underline{\text{Ans}}$$

Qn 13 →

$$I = \int \frac{1}{\sin(x-a)\cos(x-b)}\, dx$$

$$= \frac{1}{\cos(a-b)} \int \frac{\cos(a-b)}{\sin(x-a)\cos(x-b)}\, dx$$

$$= \frac{1}{\cos(a-b)} \int \frac{\cos(a-b+x-x)}{\sin(x-a)\cdot\cos(x-b)}\, dx$$

$$= \frac{1}{\cos(a-b)} \int \frac{\cos((x-b)-(x-a))}{\sin(x-a)\cdot\cos(x-b)}\, dx$$

$$= \frac{1}{\cos(a-b)} \int \frac{\cos(x-b)\cos(x-a) + \sin(x-b)\sin(x-a)}{\sin(x-a)\cdot\cos(x-b)}$$

$$= \frac{1}{\cos(a-b)} \int \cot(x-a) + \cot(x-b)\, dx$$

$$= \frac{1}{\cos(a-b)} \left[\log|\sin(x-a)| + \log|\sin(x-b)|\right] + C \quad \underline{\text{Ans}}$$

Qu. 14 → $I = \int \frac{\sin(2x)}{\sin(x-\pi/3)\cdot\sin(x+\pi/3)}\, dx$

$$I = \int \frac{\sin(x+x+\pi/3-\pi/3)\, dx}{\sin(x-\pi/3)\cdot\sin(x+\pi/3)}$$

$$= \int \frac{\sin\left((\overset{A}{x+\pi/3}) + (\overset{B}{x-\pi/3})\right)}{\sin(x-\pi/3)\cdot\sin(x+\pi/3)}\, dx$$

$$= \int \frac{\cancel{\sin(x+\pi/3)}\cos(x-\pi/3)}{\sin(x-\pi/3)\cdot\cancel{\sin(x+\pi/3)}} + \frac{\cos(x+\pi/3)\cdot\cancel{\sin(x-\pi/3)}}{\cancel{\sin(x-\pi/3)}\cdot\sin(x+\pi/3)}\, dx$$

$$= \int \cot(x-\pi/3) + \cot(x+\pi/3)\, dx$$

$$= \log|\sin(x-\pi/3)| + \log|\sin(x+\pi/3)| + C$$

Qu. 15 $I = \int \tan^3 x\cdot\sec^3 x\, dx$

$$= \int \tan^2 x\cdot\sec^2 x\cdot\sec x\tan x\, dx$$

$$= \int (\sec^2 x - 1)\cdot\sec^2 x\cdot\sec x\tan x\, dx$$

put $\sec x = t$

$\sec x \tan x \, dx = dt$

$$I = \int (t^2-1)t^2 \, dt$$

$$= \int (t^4 - t^2) \, dt$$

$$= \frac{t^5}{5} - \frac{t^3}{3} + C$$

$$I = \frac{\sec^5 x}{5} - \frac{\sec^3 x}{3} + C$$ Ans

Qn 16 $I = \int \tan^5 x \cdot \sec^4 x \, dx$

$$= \int \tan^5 x \cdot \sec^2 x \cdot \sec^2 x \, dx$$

$$= \int \tan^5 x \cdot (1+\tan^2 x) \sec^2 x \, dx$$

put $\tan x = t$

$\sec^2 x \, dx = dt$

$$= \int t^5 (1+t^2) \, dt$$

$$= \int (t^5 + t^7) \, dt$$

$$= \frac{t^6}{6} + \frac{t^8}{8} + C$$

$$= \frac{\tan^6 x}{6} + \frac{\tan^8 x}{8} + C$$ Ans

Qn. 17 → $I = \int \cot^5 x \cdot \operatorname{cosec}^4 x \, dx$

$$= \int \cot^5 x \cdot \operatorname{cosec}^2 x \cdot \operatorname{cosec}^2 x \, dx$$

$$= \int \cot^5 x \cdot (1+\cot^2 x) \cdot \csc^2 x \, dx$$

put $\cot x = t$

$\csc^2 x \, dx = -dt$

$$I = -\int t^5 (1+t^2) \, dt$$

$$= -\int (t^5 + t^7) dt$$

$$= -\left(\frac{t^6}{6} + \frac{t^8}{8}\right) + c$$

$$= -\left(\frac{\cot^6 x}{6} + \frac{\cot^8 x}{8}\right) + c$$

Qn. 18 → $I = \int \sec^n x \cdot \tan x \, dx$

$$= \int \sec^{n-1} x \cdot \sec x \tan x \, dx$$

put $\sec x = t$

$\sec x \tan x \, dx = dt$

$$I = \int t^{n-1} \cdot dt$$

$$= \frac{t^n}{n} + c$$

$$I = \frac{\sec^n x}{n} + c \quad \text{Ans}$$

Qn 19 → $I = \int \frac{1}{\sqrt{\sin^3 x \cdot \cos^5 x}} dx$

$$I = \int \frac{1}{\sin^{3/2} x \cdot \cos^{5/2} x} dx \qquad \frac{3}{2} + \frac{5}{2} = 4 \to \text{even}$$

Divide by $\cos^4 x$

$$= \int \frac{\sec^4 x}{\tan^{3/2} x} dx$$

$$= \int \frac{(1+\tan^2 x) \cdot \sec^2 x}{\tan^{3/2} x} dx$$

put $\tan x = t$
$\sec^2 x \, dx = dt$

$$= \int \frac{(1+t^2) \cdot dt}{t^{3/2}}$$

$$= \int t^{-3/2} + t^{1/2} dt$$

$$= -2t^{-1/2} + \frac{2}{3} t^{3/2} + c$$

$$= \frac{-2}{\sqrt{\tan x}} + \frac{2}{3} (\tan x)^{3/2} + c \quad \text{Ans}$$

Q.20 $I = \int \sec^{4/3} x \cdot \operatorname{cosec}^{8/3} x \, dx$

$$= \int \frac{1}{\cos^{4/3} x \cdot \sin^{8/3} x} dx \qquad \frac{4}{3} + \frac{8}{3} = \frac{12}{3} = 4$$

Divide by $\cos^4 x$

$$= \int \frac{\sec^4 x}{\tan^{8/3} x} dx$$

$$= \int \frac{(1+\tan^2 x) \cdot \sec^2 x \, dx}{\tan^{8/3} x}$$

put $\tan x = t \Rightarrow \sec^2 x \, dx = dt$

$$I = \int \frac{(1+t^2) \cdot dt}{t^{8/3}}$$

$$= \int t^{-8/3} + t^{-2/3}\, dt$$

$$= -\frac{3}{5} t^{-5/3} + 3\, t^{1/3} + c$$

$$= \frac{-3}{5(\tan x)^{5/3}} + 3(\tan x)^{1/3} + c \quad \underline{\text{Ans}}$$

Qn 21 $I = \int \frac{1}{\sin x \cdot \cos^5 x} dx$

Divide by $\cos^6 x$

$$= \int \frac{\sec^6 x\, dx}{\tan x}$$

$$= \int \frac{\sec^4 x \cdot \sec^2 x\, dx}{\tan x}$$

$$= \int \frac{(1+\tan^2 x)^2 \cdot \sec^2 x\, dx}{\tan x}$$

put $\tan x = t \Rightarrow \sec^2 x\, dx = dt$

$$= \int \frac{(1+t^2)^2 \cdot dt}{t}$$

$$= \int \frac{1 + t^4 + 2t^2}{t} dt$$

$$= \int \frac{1}{t} + t^3 + 2t\, dt$$

$$= \log|t| + \frac{t^4}{4} + \frac{2t^2}{2} + c$$

$$= \log|\tan x| + \frac{\tan^4 x}{4} + \tan^2 x + c \quad \underline{\text{Ans}}$$

Qn 22 Same Divide by $\cos^4 x$ (self)