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← ULTIMATE MATHEMATICS: BY AJAY MITTAL →

CHAPTER : INTEGRATION

CLASS NO: 2

(1) Evaluate $I = \int \sin^2 x \, dx$

$$I = \int \frac{1-\cos(2x)}{2} dx$$

$$= \frac{1}{2}\int 1-\cos(2x) \, dx$$

$$= \frac{1}{2}\left[x - \frac{\sin(2x)}{2}\right] + c$$

(2) $I = \int \tan^2 x \, dx$

$$I = \int (\sec^2 x - 1) \, dx$$

$$= \tan x - x + c$$

(3) $I = \int \sin^3 x \, dx$

$$= \frac{1}{4}\int 3\sin x - \sin(3x) + c$$

$$= \frac{1}{4}\left[-3\cos x + \frac{\cos(3x)}{3}\right] + c$$

(4) $I = \int \sin^4 x \, dx$

$$= \int (\sin^2 x)^2 \, dx$$

$$= \int \left(\frac{1-\cos(2x)}{2}\right)^2 dx$$

$$I = \frac{1}{4}\int 1 + \cos^2(2x) - 2\cos(2x)\,dx$$

$$= \frac{1}{4}\int 1 + \frac{1+\cos(4x)}{2} - 2\cos(2x)\,dx$$

$$= \frac{1}{8}\int 3 + \cos(4x) - 4\cos(2x)\,dx$$

$$I = \frac{1}{8}\left[3x + \frac{\sin(4x)}{4} - 4\frac{\sin(2x)}{2}\right] + C$$

(5) $I = \int \tan^3 x\,dx$

$$= \int \tan^2 x \cdot \tan x\,dx$$

$$= \int (\sec^2 x - 1)\cdot \tan x\,dx$$

$$= \int (\tan x \cdot \sec^2 x - \tan x)\,dx$$

$$= \int \tan x \cdot \sec^2 x\,dx - \int \tan x\,dx$$

put $\tan x = t$ in I^{st} Integral

$\sec^2 x\,dx = dt$

$$\therefore I = \int t\,.dt - \log|\sec x|$$

$$= \frac{t^2}{2} - \log|\sec x| + C$$

$$I = \frac{\tan^2 x}{2} - \log|\sec x| + C \quad \text{Ans}$$

(6) $I = \int \cot^4(3x)\,dx$

$= \int \cot^2(3x) \cdot \cot^2(3x)\,dx$

$= \int \cot^2(3x) \cdot (\operatorname{cosec}^2(3x) - 1)\,dx$

$= \int \cot^2(3x) \cdot \operatorname{cosec}^2(3x) - \cot^2(3x)\,dx$

$= \int \cot^2(3x) \cdot \operatorname{cosec}^2(3x)\,dx - \int \cot^2(3x)\,dx$

put $\cot(3x) = t$

$-\operatorname{cosec}^2(3x) \cdot 3\,dx = dt$

$\operatorname{cosec}^2(3x)\,dx = -\frac{dt}{3}$

$\therefore I = -\frac{1}{3}\int t^2\,dt - \int \operatorname{cosec}^2(3x) - 1\,dx$

$= -\frac{1}{3}\left(\frac{t^3}{3}\right) - \left(-\frac{\cot(3x)}{3} - x\right) + c$

$I = \frac{\cot^3(3x)}{9} + \frac{\cot(3x)}{3} + x + c$ Ans

(7) $I = \int \sec^4 x\,dx$

$= \int \sec^2 x \cdot \sec^2 x\,dx$

$= \int (1 + \tan^2 x) \cdot \sec^2 x\,dx$

$= \int \sec^2 x\,dx + \int \tan^2 x \sec^2 x\,dx$

put $\tan x = t$ in 2nd Integral

$I = \tan x + \int t^2\,dt$

$I = \tan x + \frac{\tan^3 x}{3} + c$

Ans

8. $I = \int \sin^5 x \, dx$

$= \int \sin^4 x \cdot \sin x \, dx$

$= \int (\sin^2 x)^2 \cdot \sin x \, dx$

$= \int (1-\cos^2 x)^2 \cdot \sin x \, dx$

put $\cos x = t$

$-\sin x \, dx = dt \Rightarrow \sin x \, dx = -dt$

$I = -\int (1-t^2)^2 \, dt$

$= -\int 1 + t^4 - 2t^2 \, dt$

$= -\left[t + \frac{t^5}{5} - \frac{2t^3}{3}\right] + C$

$I = -\left[\cos x + \frac{\cos^5 x}{5} - \frac{2\cos^3 x}{3}\right] + C$

9. $I = \int \tan^5 x \, dx$

$= \int \tan^3 x \cdot \tan^2 x \, dx$

$= \int \cancel{\sec} \tan^3 x \cdot (\sec^2 x - 1) \, dx$

$= \int \tan^3 x \cdot \sec^2 x \, dx - \int \tan^3 x \, dx$

put $\tan x = t$ in I^{st} Int.

$\sec^2 x \, dx = dt$

$I = \int t^3 \, dt - \int \tan x \cdot \tan^2 x \, dx$

$I = \frac{t^4}{4} - \int \tan x \cdot (\sec^2 x - 1)\, dx$

$I = \frac{t^4}{4} - \int \tan x \cdot \sec^2 x\, dx + \int \tan x \cdot dx$

put $\tan x = t$

$\sec^2 x\, dx = dt$

$I = \frac{t^4}{4} - \int t \cdot dt + \log|\sec x|$

$I = \frac{t^4}{4} - \frac{t^2}{2} + \log|\sec x| + C$

$I = \frac{\tan^4 x}{4} - \frac{\tan^2 x}{2} + \log|\sec x| + C$ Ans

Q. 10 $I = \int \cos^7 x\, dx$

$= \int \cos^6 x \cdot \cos x\, d$

$= \int (\cos^2 x)^3 \cdot \cos x\, dx$

$= \int (1 - \sin^2 x)^3 \cdot \cos x\, dx$

put $\sin x = t \Rightarrow \cos x\, dx = dt$

$I = \int (1 - t^2)^3 \cdot dt$

$= \int 1 - t^6 - 3t^2 + 3t^4\, dt$

$= \left[t - \frac{t^7}{7} - \frac{\cancel{3}t^3}{\cancel{3}} + \frac{3t^5}{5}\right] + C$

$I = \sin x - \frac{\sin^7 x}{7} - \sin^3 x + \frac{3\sin^5 x}{5} + C$ Ans

Q = 11 $\quad I = \int \sin^6 x \, dx$

$$= \int (\sin^3 x)^2 \, dx$$

$$= \int \left(\frac{3\sin x - \sin(3x)}{4} \right)^2 dx$$

$$= \frac{1}{16} \int 9\sin^2 x + \sin^2(3x) - 6\sin x \cdot \sin(3x) \, dx$$

$$= \frac{1}{16} \int 9\left(\frac{1-\cos(2x)}{2}\right) + \left(\frac{1-\cos(6x)}{2}\right) - 3\left(\cos(2x) - \cos(4x)\right) dx$$

$$= \frac{1}{32} \int 9 - 9\cos(2x) + 1 - \cos(6x) - 6\cos(2x) + 6\cos(4x) \, dx$$

$$= \frac{1}{32} \int 10 - 15\cos(2x) - \cos(6x) + 6\cos(4x) \, dx$$

$$= \frac{1}{32} \left[10x - \frac{15\sin(2x)}{2} - \frac{\sin(6x)}{6} + \frac{6\sin(4x)}{4} \right] + C$$

Ans

Type: $\sin x$ & $\cos x$ in multiplication with difference angles

- ✓ $2\sin A \cos B = \sin(A+B) + \sin(A-B)$
- ✓ $2\cos A \sin B =$
- ✓ $2\cos A \cos B =$
- ✓ $2\sin A \sin B =$

Q = 12 $\quad I = \int \cos(3x) \cdot \sin(2x) \, dx$

$$I = \frac{1}{2} \int 2\cos(3x) \sin(2x) \, dx$$

$$= \frac{1}{2} \int \sin(5x) + \sin(x) \, dx$$

$$I = \frac{1}{2} \left[-\frac{\cos(5x)}{5} + \cos x \right] + C$$

Ans

Qn-13 Evaluate $I = \int \sin x \cdot \sin(2x) \cdot \sin(3x)\, dx$

$$= \frac{1}{2}\int \left(2\sin x \cdot \sin(2x)\right) \cdot \sin(3x)\, dx$$

$$= \frac{1}{2}\int \left(\cos(x) - \cos(3x)\right) \cdot \sin(3x)\, dx$$

$$= \frac{1}{2}\int \left(\sin(3x) \cdot \cos x - \sin(3x)\cos(3x)\right) dx$$

$$= \frac{1}{4}\int 2\sin(3x)\cos x - 2\sin(3x)\cos(3x)\, dx$$

$$= \frac{1}{4}\int \sin(4x) + \sin(2x) - \sin(6x)\, dx$$

$$= \frac{1}{4}\left[\frac{-\cos(4x)}{4} - \frac{\cos(2x)}{2} + \frac{\cos(6x)}{6}\right] + C$$

Qn-14 $I = \int \frac{\sin(4x)}{\sin x}\, dx$

$$= \int \frac{2\sin(2x)\cos(2x)}{\sin x}\, dx$$

$$= \int \frac{4\cancel{\sin x} \cdot \cos x \cdot \cos(2x)}{\cancel{\sin x}}\, dx$$

$$= 2\int 2\cos x \cdot \cos(2x)\, dx$$

Proceed

Type sinx & cosx in multiplication with ~~different~~ same power

Q.15 $I = \int \sin^2 x \cdot \cos^2 x \, dx$

$= \int (\sin x \cos x)^2 \, dx$

$= \int \left(\frac{\sin(2x)}{2}\right)^2 dx$

$= \frac{1}{4}\int \sin^2(2x) \, dx$

$= \frac{1}{4}\int \frac{1-\cos(4x)}{2} \, dx$

$= \frac{1}{8}\left[x - \frac{\sin(4x)}{4}\right] + C$

Q: 15 $I = \int \sin^4 x \cdot \cos^4 x \, dx$

$= \int (\sin x \cos x)^4 \, dx$

$= \int \left(\frac{\sin(2x)}{2}\right)^4 dx$

$= \frac{1}{16}\int \sin^4(2x) \, dx$

Procedure