॥ जय श्री राधे कृष्णा ॥

Topic : ....

Date. : ....

Page No. : .... (1)

Solution of worksheet No: 3

A·OD

Ans: 1 → given $\frac{dv}{dt} = 9\,cm^3/sec$; $x = 10\,cm$

to find: $\frac{ds}{dt} = ?$

we have

$$V = x^3$$

Diff w.r.t t

$$\frac{dv}{dt} = 3x^2\frac{dx}{dt}$$

$$\Rightarrow 9 = 3(100)\frac{dx}{dt}$$

$$\Rightarrow \frac{dx}{dt} = \frac{3}{100}\,cm/sec$$

Now $S = 6x^2$

Diff w.r.t t

$$\frac{ds}{dt} = 12x\frac{dx}{dt}$$

$$\frac{ds}{dt} = 12\times 10\times\frac{3}{100} = 3.6\,cm^2/sec$$

∴ S.A of cube is increasing at the rate $3.6\,cm^2/sec$ **Ans**

Ans 2 → given: $\frac{dx}{dt} = -3\,cm/min$, $x = 10\,cm$

$\frac{dy}{dt} = 2\,cm/min$; $y = 6\,cm$

(i) $P = 2x + 2y$

Diff w.r.t t

$$\frac{dp}{dt} = 2\frac{dx}{dt} + 2\frac{dy}{dt}$$

$$\frac{dp}{dt} = 2(-3) + 2(2) = -2\,cm/min$$

∴ Perimeter of rectangle is decreasing at the rate $2\,cm/min$ **Ans**

(ii) $A = xy$

Diff w.r.t t

Solution A.O.D. (Worksheet) No: 3 (2)

$$\frac{dA}{dt} = x\frac{dy}{dt} + y\frac{dx}{dt}$$

$$= (10)(2) + (6)(-3)$$

$$\frac{dA}{dt} = 2\ cm^2/min$$

$\therefore$ Area of rectangle is increasing at the rate $2 cm^2/min$ Ans

Ans 3 → given $\frac{dr}{dt} = 5\ cm/sec$; $r = 8cm$

to find: $\frac{dA}{dt} = ?$

we have, $A = \pi r^2$

Diff. w.r.t 't'

$$\frac{dA}{dt} = 2\pi r\frac{dr}{dt}$$

$$\frac{dA}{dt} = 2\pi(8)(5) = 40\pi\ cm^2/sec$$

Ans

$\therefore$ Area of circular wave is increasing at the Rate $40\pi cm^2/sec$

Ans 4 → given: $\frac{dr}{dt} = \frac{1}{2}\ cm/sec$; $r = 1cm$

to find: $\frac{dv}{dt} = ?$

we have $v = \frac{4}{3}\pi r^3$

Diff. w.r.t 't'

$$\frac{dv}{dt} = 4\pi r^2 \cdot \frac{dr}{dt}$$

$$\frac{dv}{dt} = 4\pi(1)^2\left(\frac{1}{2}\right) = 2\pi\ cm^3/sec$$

$\therefore$ volume of air bubble is increasing at the rate $2\pi\ cm^3/sec$ Ans

CLASSTIME

Solution A.O.D (worksheet No:- 3)

Ans 5→ given: diameter = $\frac{3}{2}(2x+1)$

$\therefore$ radius $= \frac{3}{4}(2x+1) = r$

we have, $V = \frac{4}{3}\pi r^3$

$\Rightarrow V = \frac{4}{3}\pi\left[\frac{3}{4}(2x+1)\right]^3$

$\Rightarrow V = \frac{4}{3}\pi \times \frac{27}{64}(2x+1)^3$

$V = \frac{9\pi}{16}(2x+1)^3$

Diff w.r.t 'x'

$\frac{dv}{dx} = \frac{9\pi}{16} \times 3(2x+1)^2 \cdot (2)$

$\frac{dv}{dx} = \frac{27\pi}{8}(2x+1)^2$ Ans.

Ans. 6→ given: $r = 10m$

$\frac{dv}{dt} = 314 \; m^3/hr$

to find $\frac{dh}{dt} = ?$

we have, $V = \pi r^2 h$

$V = \pi(10)^2 h$

Diff w.r.t 't'

$\frac{dV}{dt} = 100\pi \frac{dh}{dt}$

$314 = 100(3.14)\frac{dh}{dt}$

$314 = 314\frac{dh}{dt}$

$\frac{dh}{dt} = 1 \; m/hr$

$\therefore$ depth of wheat is increasing at the rate $1 \; m/hr$

Ans

Solution (Worksheet No-3) A.O.D (4)

Qn 7 → given: $\frac{dy}{dt} = \frac{dx}{dt}$

given. equation of curve

$y = x^2 + 2x$

Diff wrt 't'

$\frac{dy}{dt} = 2x\frac{dx}{dt} + 2\frac{dx}{dt}$

$\Rightarrow \frac{\cancel{dy}}{\cancel{dt}} = \frac{\cancel{dx}}{\cancel{dt}}(2x+2)$

$\Rightarrow 1 = 2x + 2$ ... $\left\{\because \frac{dy}{dt} = \frac{dx}{dt}\right\}$

$\Rightarrow x = -\frac{1}{2}$

put in equation of curve

$y = \frac{1}{4} - 1 = -\frac{3}{4}$

$\therefore$ Required point on the curve is $\left(-\frac{1}{2}, -\frac{3}{4}\right)$ **Ans..**

Qn 8 → given $\frac{d\theta}{dt} = 2 \cdot \frac{d}{dt}(\sin\theta)$

$\Rightarrow \frac{\cancel{d\theta}}{\cancel{dt}} = 2\cos\theta \cdot \frac{\cancel{d\theta}}{\cancel{dt}}$ --- $\left\{\because \frac{d\theta}{dt} \neq 0\right\}$

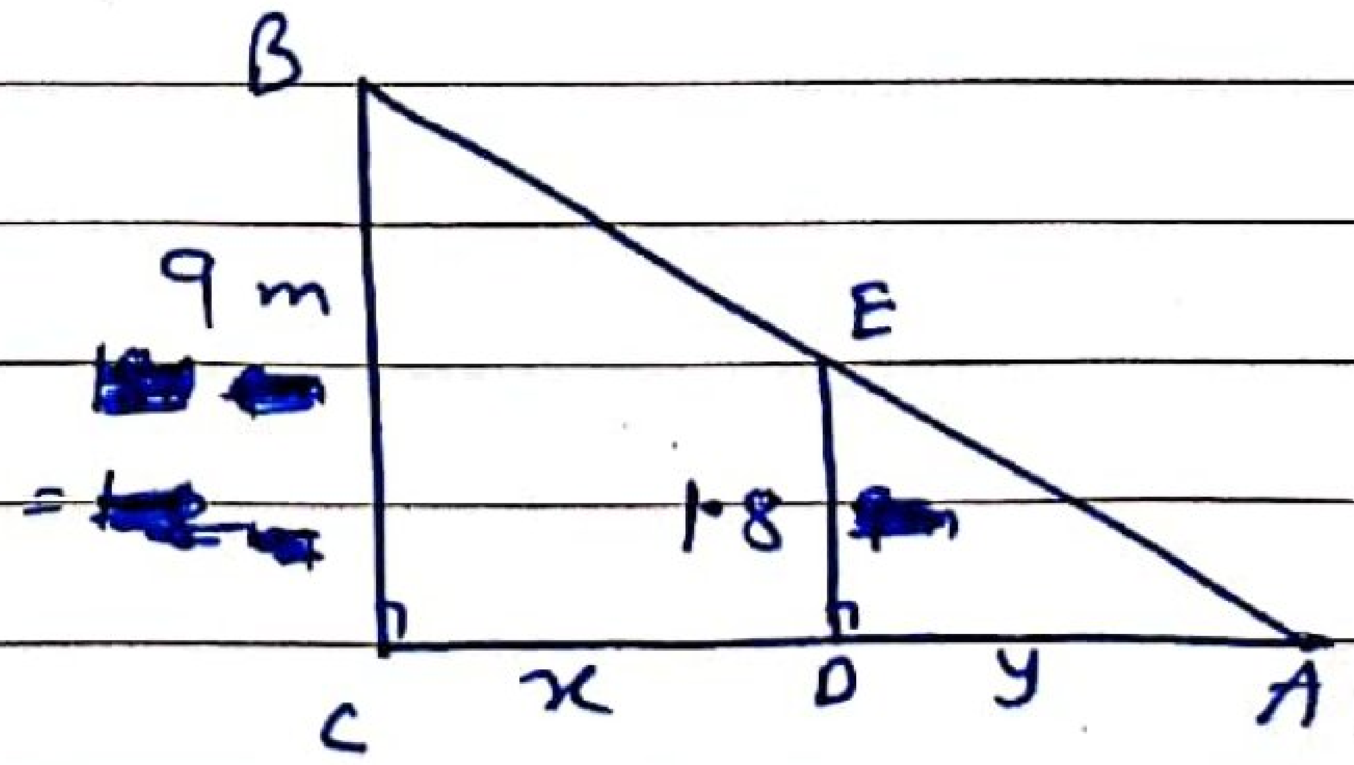
$\Rightarrow 1 = 2\cos\theta$

$\Rightarrow \cos\theta = \frac{1}{2}$ $\therefore \theta = \pi/3$ **Ans.**

Qn 9 → given $\frac{dx}{dt} = 2$ m/sec

$x = 3m$

to find: $\frac{dy}{dt}$



$\Delta ABC \sim \Delta ADE$

$\frac{9}{1.8} = \frac{x+y}{y}$

$\Rightarrow 5y = x + y$

$\Rightarrow 4y = x$

Solution A.O.D (Worksheet No: 3) (5)

Diff w.r.t 't'

$$4\frac{dy}{dt} = \frac{dx}{dt}$$

$$4\frac{dy}{dt} = 2$$

$$\Rightarrow \frac{dy}{dt} = 0.5 \text{ m/sec}$$

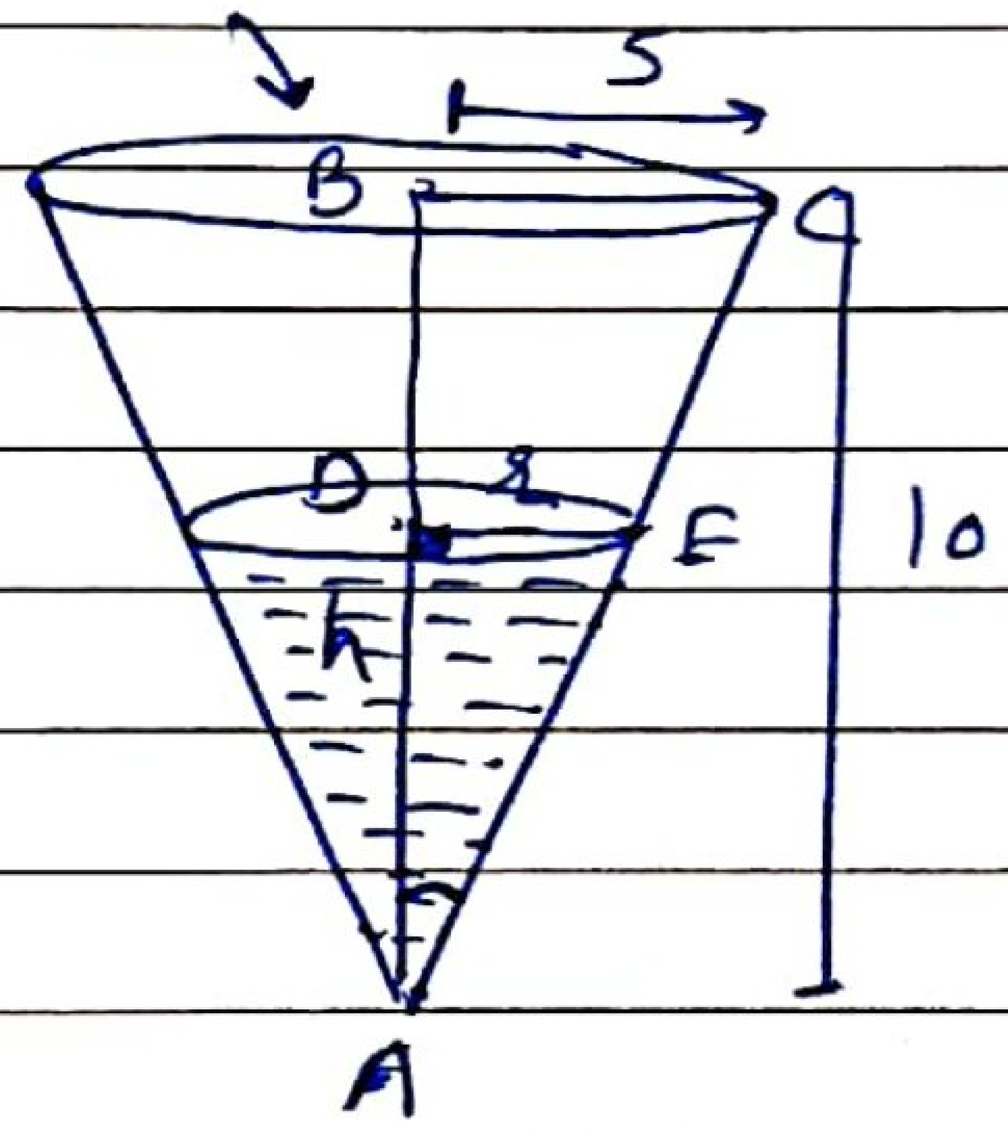
$\therefore$ Shadow is increasing at the rate $= 0.5$ m/sec Ans

(Note: "there is Mistake/Misprint in Worksheet Ans")

Ans 10 → given $\frac{dv}{dt} = \frac{3}{2}$ cm³/min

$h = 4$m

to find: $\frac{dh}{dt} = ?$



Now $\Delta ABC \sim \Delta ADE$

$$\frac{10}{h} = \frac{5}{r}$$

$$\Rightarrow \boxed{r = \frac{h}{2}}$$

Now we have $V = \frac{1}{3}\pi r^2 h$ (volume of water cone)

$$V = \frac{1}{3}\pi\left(\frac{h^2}{4}\right)h \quad \text{---} \left\{\because r = \frac{h}{2}\right\}$$

$$V = \frac{1}{3}\frac{\pi}{4}h^3$$

Diff w.r.t 't'

$$\frac{dv}{dt} = \frac{1}{3}\cdot\frac{\pi}{4}(3h^2)\frac{dh}{dt}$$

$$\frac{3}{2} = \frac{1}{3}\cdot\frac{\pi}{4}(3)(16)\frac{dh}{dt}$$

$$\Rightarrow \frac{dh}{dt} = \frac{3}{8\pi} \text{ cm/min}$$ Ans

$\therefore$ level of water is rising at the rate $3/8\pi$ cm/min

Solution A.O.D. (worksheet No: 3) (6)

Qn: 11 → given: $\frac{dV}{dt} = k$ (constant)

To prove $\frac{ds}{dt} \propto \frac{1}{x}$

we have $V = x^3$

Diff. w.r.t 't'

$$\frac{dV}{dt} = 3x^2 \cdot \frac{dx}{dt}$$

$$k = 3x^2 \frac{dx}{dt}$$

$$\Rightarrow \frac{dx}{dt} = \frac{k}{3x^2}$$

Now $S = 6x^2$

Diff. w.r.t 't'

$$\frac{ds}{dt} = 12x \frac{dx}{dt}$$

$$= 12x \left(\frac{k}{3x^2}\right)$$

$$\frac{ds}{dt} = \frac{4k}{x}$$

$\frac{ds}{dt} \propto \frac{1}{x}$ --- {∵ 4k is constant}

Proved

—x—