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→ ULTIMATE MATHEMATICS: BY AJAY MITTAL →

Chapter: INTEGRATION: CLASS NO:5

Special Integrals

① $I = \int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C$

Proof put $x = a\tan\theta$

$dx = a\sec^2\theta\,d\theta$

$\therefore I = \int \frac{1}{a^2+a^2\tan^2\theta} \cdot a\sec^2\theta\,d\theta$

$= \int \frac{1}{a^2 \cdot \cancel{\sec^2\theta}} \cdot a\,\cancel{\sec^2\theta}\,d\theta$

$= \frac{1}{a}\int d\theta$

$= \frac{1}{a}\theta + C$

$= \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C$ Ans

② $I = \int \frac{1}{\sqrt{a^2-x^2}}\,dx$

put $x = a\sin\theta$

$dx = a\cos\theta\,d\theta$

$I = \int \frac{1}{\sqrt{a^2-a^2\sin^2\theta}} \cdot a\cos\theta\,d\theta$

$= \int \frac{1}{\cancel{a\cos\theta}} \cdot \cancel{a\cos\theta}\,d\theta$

$= \int d\theta = \theta + C = \sin^{-1}\left(\frac{x}{a}\right) + C$ Ans

(3) $I = \int \frac{1}{x^2-a^2}\,dx$

$= \int \frac{1}{(x+a)(x-a)}\,dx$

$= \frac{1}{2a}\int \left(\frac{1}{x-a} - \frac{1}{x+a}\right)dx$

$= \frac{1}{2a}\left[\log|x-a| - \log|x+a|\right] + c$

$= \frac{1}{2a}\log\left|\frac{x-a}{x+a}\right| + C$

(4) $I = \int \frac{1}{\sqrt{x^2+a^2}}\,dx$ $\quad$ Hint put $x = a\tan\theta$

(5) $I = \int \frac{1}{\sqrt{x^2-a^2}}\,dx$ $\quad$ Hint put $x = a\sec\theta$

(6) $I = \int \frac{1}{a^2-x^2}\,dx$ $\quad$ (Hint Same as $\int \frac{1}{x^2-a^2}\,dx$)

Q.N 1 $I = \int \frac{1}{\sqrt{3-9x^2}}\,dx$

$= \frac{1}{3}\int \frac{1}{\sqrt{\left(\frac{1}{\sqrt{3}}\right)^2 - x^2}}\,dx$

$= \frac{1}{3}\sin^{-1}(x\sqrt{3}) + c$

Q.N 2 $I = \int \frac{1}{2-4x^2}\,dx$

$= \frac{1}{4}\int \frac{1}{\left(\frac{1}{\sqrt{2}}\right)^2 - x^2}\,dx$

$I = \frac{1}{4}\left[\frac{1}{2\times\frac{1}{\sqrt{2}}}\log\left|\frac{\frac{1}{\sqrt{2}}+x}{\frac{1}{\sqrt{2}}-x}\right|\right] + c$

$= \frac{1}{4\sqrt{2}}\log\left|\frac{1+\sqrt{2}x}{1-\sqrt{2}x}\right| + c$ Ans

(3) $I = \int \frac{1}{\sqrt{(2-x)^2 - 4}} dx$

$= -\log \left| (2-x) + \sqrt{(2-x)^2 - 4} \right| + C$

<u>Type</u> $\int \frac{1}{\text{Quadratic}} dx$ (perfect square)

$\hookrightarrow ax^2 + bx + c$

(4) $I = \int \frac{1}{2x^2 + 4x + 5} dx$

$= \frac{1}{2} \int \frac{1}{x^2 + 2x + \frac{5}{2}} dx$

$= \frac{1}{2} \int \frac{1}{(x+1)^2 - 1 + \frac{5}{2}} dx$

$= \frac{1}{2} \int \frac{1}{(x+1)^2 + \left(\frac{\sqrt{3}}{\sqrt{2}}\right)^2} dx$

$= \frac{1}{2} \times \frac{\sqrt{2}}{\sqrt{3}} \tan^{-1}\left(\frac{\sqrt{2}(x+1)}{\sqrt{3}}\right) + C$

$= \frac{1}{\sqrt{6}} \tan^{-1}\left(\frac{\sqrt{2}(x+1)}{\sqrt{3}}\right) + C$ Ans

<u>Qn 5</u> $I = \int \frac{1}{1 - 3x - 3x^2} dx$

<u>Sol</u> $= -\frac{1}{3} \int \frac{1}{x^2 + x - \frac{1}{3}} dx$

$= -\frac{1}{3} \int \frac{1}{\left(x + \frac{1}{2}\right)^2 - \frac{1}{4} - \frac{1}{3}} dx$

$= -\frac{1}{3} \int \frac{1}{\left(x+\frac{1}{2}\right)^2 - \left(\frac{\sqrt{7}}{\sqrt{12}}\right)^2} dx$

$= \frac{1}{3} \int \frac{1}{\left(\frac{\sqrt{7}}{\sqrt{12}}\right)^2 - \left(x + \frac{1}{2}\right)} dx$

$= \frac{1}{3} \times \frac{1}{2\frac{\sqrt{7}}{\sqrt{12}}} \log \left| \frac{\frac{\sqrt{7}}{\sqrt{12}} + x + \frac{1}{2}}{\frac{\sqrt{7}}{\sqrt{12}} - x - \frac{1}{2}} \right| + C$

LTM Ans

(4)

Type $\int \frac{1}{\sqrt{\text{Quadratic}}}\,dx$ ~~Perfect~~ perfect square

Qn: 6 $I = \int \frac{1}{\sqrt{2x^2-3x+1}}\,dx$

$= \frac{1}{\sqrt{2}}\int \frac{1}{\sqrt{x^2-\frac{3}{2}x+\frac{1}{2}}}\,dx$

$= \frac{1}{\sqrt{2}}\int \frac{1}{\sqrt{\left(x-\frac{3}{4}\right)^2-\frac{9}{16}+\frac{1}{2}}}\,dx$ $\qquad \frac{-9+8}{16} = \frac{-1}{16}$

$= \frac{1}{\sqrt{2}}\int \frac{1}{\sqrt{\left(x-\frac{3}{4}\right)^2-\left(\frac{1}{4}\right)^2}}\,dx$

$= \frac{1}{\sqrt{2}}\log\left|\left(x-\frac{3}{4}\right)+\sqrt{x^2-\frac{3}{2}+\frac{1}{2}}\right|+C$ Ans

Q: 7 $I = \int \frac{1}{\sqrt{x(1-2x)}}\,dx$

$= \int \frac{1}{\sqrt{x-2x^2}}\,dx$

$= \int \frac{1}{\sqrt{-2\left(x^2-\frac{x}{2}\right)}}\,dx$

$= \frac{1}{\sqrt{2}}\int \frac{1}{\sqrt{-\left[\left(x-\frac{1}{4}\right)^2-\left(\frac{1}{4}\right)^2\right]}}\,dx$

$= \frac{1}{\sqrt{2}}\int \frac{1}{\sqrt{\left(\frac{1}{4}\right)^2-\left(x-\frac{1}{4}\right)^2}}\,dx$

$= \frac{1}{\sqrt{2}}\sin^{-1}\left(\frac{x-\frac{1}{4}}{\frac{1}{4}}\right)+C = \frac{1}{\sqrt{2}}\sin^{-1}(4x-1)+C$ Ans

Type "After Substitution", $\frac{1}{\text{Quadratic}}$, $\frac{1}{\sqrt{\text{Quadratic}}}$,

special Integrals

Q.8 $I = \int \frac{x}{x^4+x^2+1}\,dx$

put $x^2 = t$

$2x\,dx = dt \Rightarrow x\,dx = \frac{dt}{2}$

$\Rightarrow \frac{1}{2}\int \frac{1}{t^2+t+1}\,dt$

perfect square

Q.9 $I = \int \frac{\sin(2x)\,dx}{\sin^4 x + \cos^2 x + 5}$

$= \int \frac{\sin(2x)\,dx}{\sin^4 x + 1 - \sin^2 x + 5}$

put $\sin^2 x = t$

$\sin(2x)\,dx = dt$

$I = \int \frac{dt}{t^2 - t + 6}$

proceed

Q.10 → $I = \int \frac{1}{\sqrt{1-e^{2x}}}\,dx$

$= \int \frac{1}{\sqrt{1-\frac{1}{e^{-2x}}}}\,dx$

$= \int \frac{e^{-x}}{\sqrt{e^{-2x}-1}}\,dx$

put $e^{-x} = t$

$e^{-x}\,dx = -dt$

$I = -\int \frac{dt}{\sqrt{t^2-1}}$

$= -\log|t+\sqrt{t^2-1}| + C$

$= -\log|e^{-x}+\sqrt{e^{-2x}-1}| + C$ Ans

Q. 11

$$I = \int \frac{\sin(2x)\cdot\cos(2x)}{\sqrt{9-\cos^4(2x)}}\,dx$$

$\cos^4(2x) \downarrow (\cos^2(2x))^2 \to$ linear (x)

put $\cos^2(2x) = t$

$$-2\cos(2x)\,\sin(2x)\cdot 2\,dx = dt$$

$$\sin(2x)\cos(2x)\,dx = -\frac{dt}{4}$$

$$I = -\frac{1}{4}\int \frac{dt}{\sqrt{3^2-t^2}}$$

$$= -\frac{1}{4}\sin^{-1}\left(\frac{\cos^2(2x)}{3}\right) + C$$

Trick

$$\int \frac{}{\text{const} \pm \text{variable}}$$

variable $\downarrow$ $(\;)^2$ $\to$ linear (x) $\to$ put $(\;) = t$

Q. 12

$$I = \int \frac{\sqrt{x}\cdot dx}{\sqrt{a^3-x^3}}$$

$x^3 \downarrow (x^{3/2})^2$

put $x^{3/2} = t$

$$\frac{3}{2}x^{1/2}\,dx = dt$$

$$\sqrt{x}\,dx = \frac{2}{3}\,dt$$

$$I = \frac{2}{3}\int \frac{dt}{\sqrt{(a^{3/2})^2-t^2}}$$

$$= \frac{2}{3}\sin^{-1}\left(\frac{x^{3/2}}{a^{3/2}}\right) + C \quad \text{Ans}$$

Q.N 13 $I = \int \sqrt{\sec x - 1}\, dx$

$I = \int \sqrt{\frac{1}{\cos x} - 1}\, dx$

$= \int \sqrt{\frac{1-\cos x}{\cos x}}\, dx$

$= \int \sqrt{\frac{1-\cos x}{\cos x} \times \frac{1+\cos x}{1+\cos x}}\, dx.$

$= \int \frac{\sin x}{\sqrt{\cos^2 x + \cos x}}\, dx$

put $\cos x = t$

$\sin x\, dx = -dt$

$I = -\int \frac{dt}{\sqrt{t^2+t}}$

Process

Q.N 14 $I = \int \frac{3x^5}{1+x^{12}}\, dx$

put $x^6 = t$

$6x^5\, dx = dt$

$x^5\, dx = \frac{dt}{6}$

$I = \frac{1}{2}\int \frac{dt}{1+t^2}$

$= \frac{1}{2}\tan^{-1}(t) + c = \frac{1}{2}\tan^{-1}(x^6) + c$

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Qn 15 → $I = \int \sqrt{\frac{\sin(x-\alpha)}{\sin(x+\alpha)}}\, dx$

Tricky

$$I = \int \sqrt{\frac{\sin(x-\alpha)}{\sin(x+\alpha)} \times \frac{\sin(x-\alpha)}{\sin(x-\alpha)}}\, dx$$

$$= \int \frac{\sin(x-\alpha)}{\sqrt{\sin(x+\alpha).\sin(x-\alpha)}}\, dx$$

$$= \int \frac{\sin(x-\alpha)}{\sqrt{\sin^2 x - \sin^2\alpha}}\, dx$$

Formula XI

$\sin(A+B).\sin(A-B) = \sin^2 A - \sin^2 B$

$$= \int \frac{\sin x \cos\alpha - \cos x \sin\alpha}{\sqrt{\sin^2 x - \sin^2\alpha}}\, dx$$

$$= \cos\alpha \int \frac{\sin x\, dx}{\sqrt{\sin^2 x - \sin^2\alpha}} - \sin\alpha \int \frac{\cos x}{\sqrt{\sin^2 x - \sin^2\alpha}}\, dx$$

$$= \cos\alpha \int \frac{\sin x\, dx}{\sqrt{\cancel{1} - \cos^2 x - \cancel{1} + \cos^2\alpha}} - \sin\alpha \int \frac{\cos x\, dx}{\sqrt{\sin^2 x - \sin^2\alpha}}$$

put $\cos x = t$ | put $\sin x = z$

$\sin x\, dx = -dt$ | $\cos x\, dx = dz$

$$= -\cos\alpha \int \frac{dt}{\sqrt{\cos^2\alpha - t^2}} - \sin\alpha \int \frac{dz}{\sqrt{z^2 - \sin^2\alpha}}$$

$$= -\cos\alpha \sin^{-1}\left(\frac{t}{\cos\alpha}\right) - \sin\alpha \log\left| z + \sqrt{z^2 - \sin^2\alpha}\right| + C$$

$$= -\cos\alpha \sin^{-1}\left(\frac{\cos x}{\cos\alpha}\right) - \sin\alpha \log\left| \sin x + \sqrt{\sin^2 x - \sin^2\alpha}\right| + C \quad \text{Ans}$$

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INTEGRATION: WORKSHEET NO: 4

(CLASS NO: 5)

Qns 1 $\int \frac{1}{4x^2-4x+3}\,dx$ — Ans $\frac{1}{2\sqrt{2}}\tan^{-1}\left(\frac{2x-1}{\sqrt{2}}\right)+C$

Qn 2 → $\int \frac{1}{1+x-x^2}\,dx$ — Ans $\frac{1}{\sqrt{5}}\log\left|\frac{\sqrt{5}-1+2x}{\sqrt{5}+1-2x}\right|+C$

Qn 3 → $\int \frac{x}{3x^4-18x^2+11}\,dx$ — Ans $\frac{\sqrt{3}}{48}\log\left|\frac{\sqrt{3}x^2-3\sqrt{3}-4}{\sqrt{3}x^2-3\sqrt{3}+4}\right|+C$

Qn 4 → $\int \frac{e^{3x}}{4e^{6x}-9}\,dx$ — Ans $\frac{1}{36}\log\left|\frac{2e^{3x}-3}{2e^{3x}+3}\right|+C$

Qn 5 → $\int \frac{1}{\sqrt{2x^2+3x-2}}\,dx$ — An $\frac{1}{\sqrt{2}}\log\left|\left(x+\frac{3}{4}\right)+\sqrt{x^2+\frac{3}{2}x-1}\right|+C$

Qn 6 → $\int \frac{1}{\sqrt{x(1-2x)}}\,dx$ — An $\frac{1}{\sqrt{2}}\sin^{-1}(4x-1)+C$

Qn 7 → $\int \frac{1}{\sqrt{7-3x-2x^2}}\,dx$ — Ans $\frac{1}{\sqrt{2}}\sin^{-1}\left(\frac{4x+3}{\sqrt{65}}\right)+C$

Qn 8 → $\int \sqrt{\operatorname{cosec}x-1}\,dx$ — Ans $\log\left|\left(\sin x+\frac{1}{2}\right)+\sqrt{\sin^2x+\sin x}\right|+C$

Qn 9 → $\int \frac{\cos x\,dx}{\sqrt{\sin^2x-2\sin x-3}}$ — Ans $\log\left|(\sin x-1)+\sqrt{\sin^2x-2\sin x-3}\right|+C$

Qn 10 → $\int \frac{1}{x^{2/3}\sqrt{x^{2/3}-4}}\,dx$ — Ans $3\log\left|x^{1/3}+\sqrt{x^{2/3}-4}\right|+C$

Qn 11 → $\int \frac{\sin(8x)}{\sqrt{9+\sin^4(4x)}}\,dx$ — An $\frac{1}{4}\log\left|\sin^2(4x)+\sqrt{9+\sin^4(4x)}\right|+C$

Q.M 12 → $I = \int \frac{1}{e^{x}+e^{-x}} dx$ Ans $= \tan^{-1}(e^{x}) + C$

Q.M 13 → $\int \frac{1}{\sqrt{(1-x^2)(9+(\sin^{-1}x)^2)}} dx$ Ans $= \log\left|\sin^{-1}x + \sqrt{9+(\sin^{-1}x)^2}\right| + C$

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