!! जय श्री राधे कृष्णा जय श्री गिरिराज जी महाराज !!

ULTIMATE MATHEMATICS: BY AJAY MITTAL

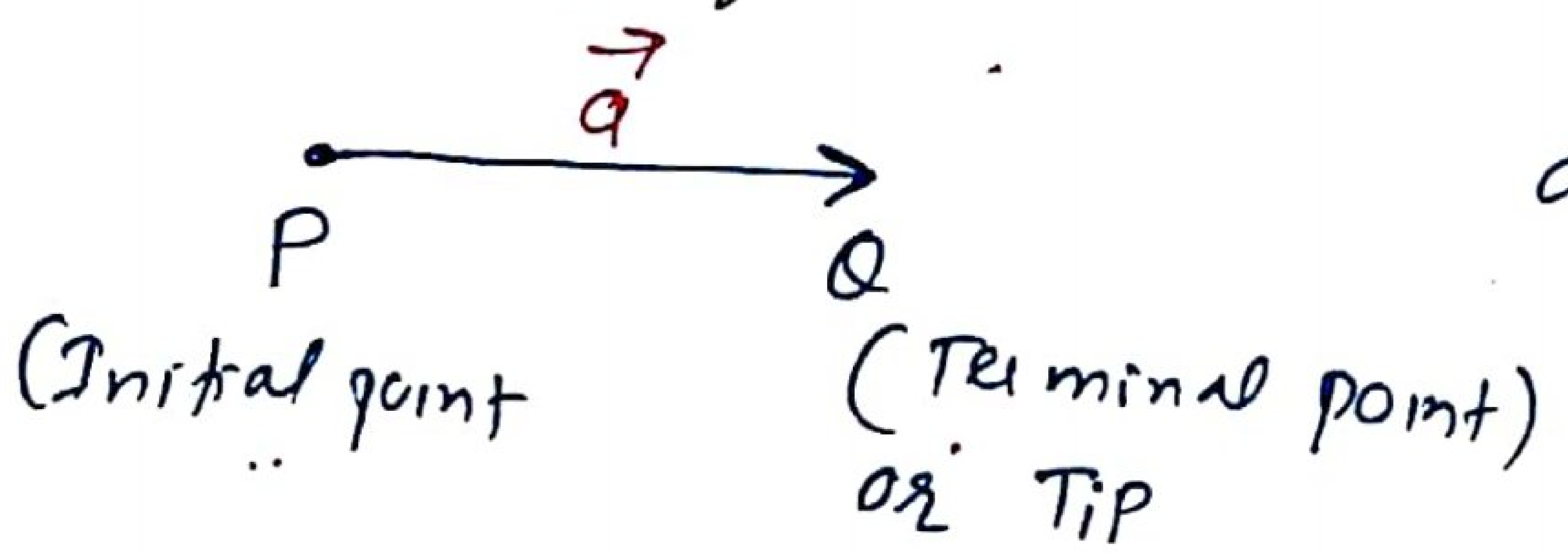
$\boxed{\text{CHAPTER: VECTORS}}$ CLASS No: 1

Two types of quantities :

(1) Scalar quantities : which have only magnitude / length
(Real numbers) (not any fixed direction)
example: distance, mass, volume, density ...

(2) vector quantities: : which have both magnitude and direction
example: Velocity, displacement, acceleration, Force, ----

Representation of a vector

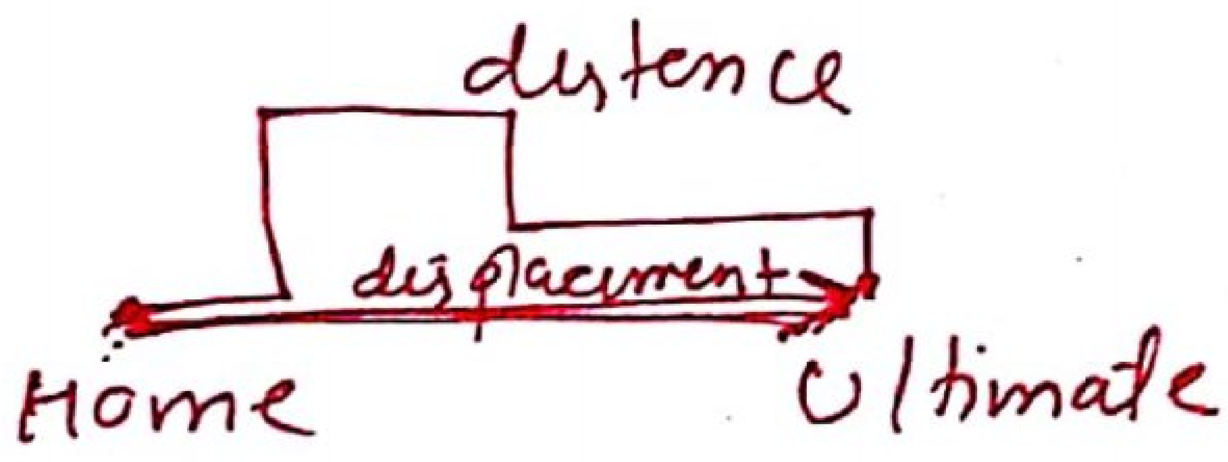


denoted by $\overrightarrow{PQ}$

$\overrightarrow{PQ}$

generally denoted by $\vec{a}, \vec{b}, \vec{c}, ----$

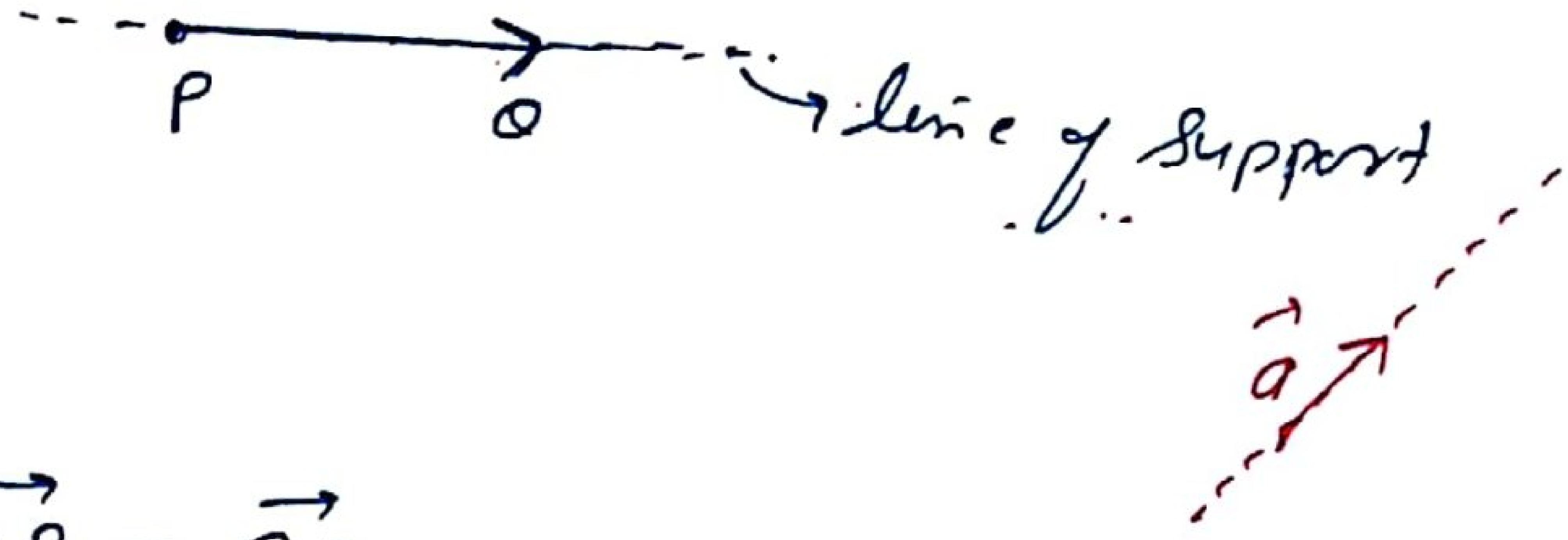
$\overleftarrow{QP}$ ✗	$\overrightarrow{PQ}$ ✓
$\vec{P}$ ✗	$\vec{a}$ ✓
$\vec{ab}$ ✗	



Magnitude (length) : $|\overrightarrow{PQ}|$ (or) $|\vec{a}|$

or simply PQ or a

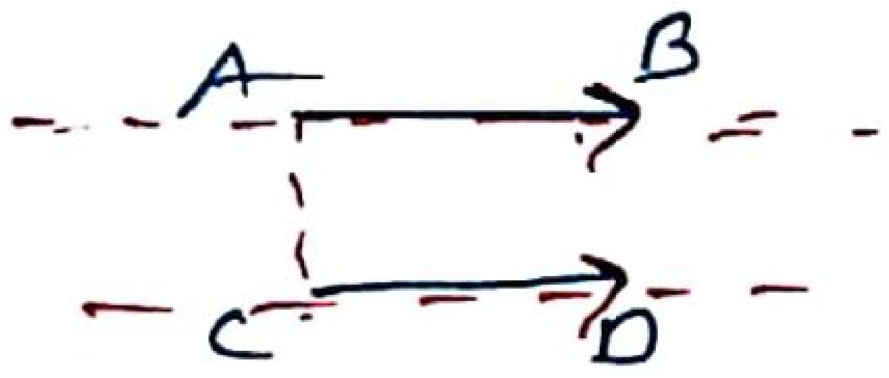
(·) line of support



Types of vectors

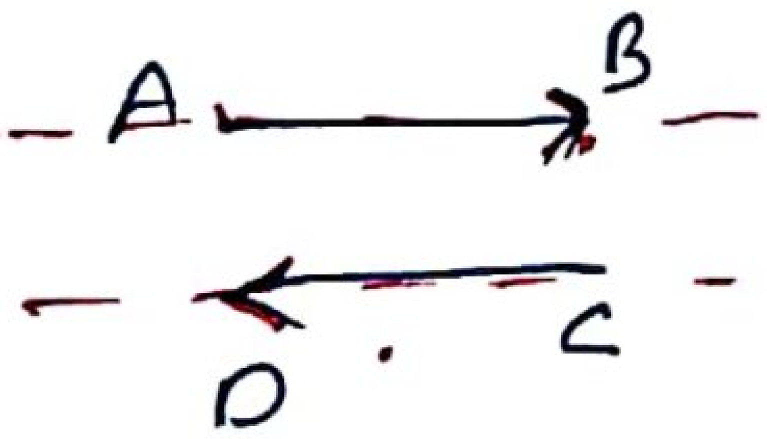
(1) **Equal vectors** $\vec{AB} = \vec{CD}$

if (1) Same magnitude $|\vec{AB}| = |\vec{CD}|$
(2) Same direction / sense
(3) Same or parallel line of support



(OR)

(2) **Opposite vectors** : (1) Same magnitude
(2) but opposite direction
(3) Same or parallel line of support



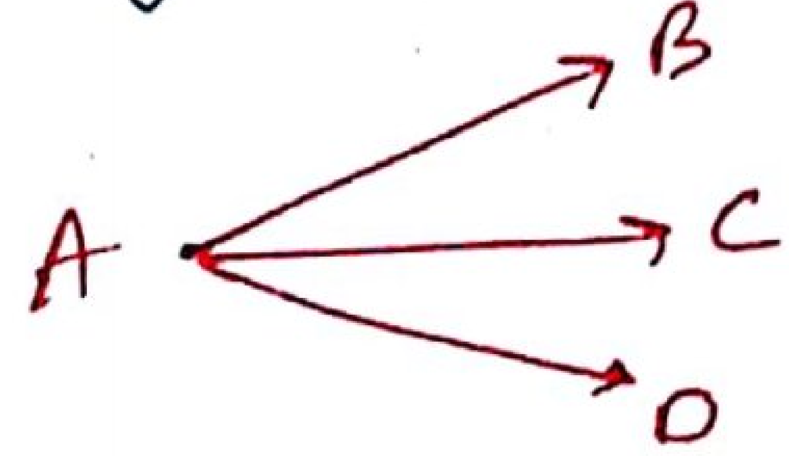
$\vec{AB} = -\vec{CD}$ but $|\vec{AB}| = |\vec{CD}|$

$-ve \rightarrow$ shows opposite direction

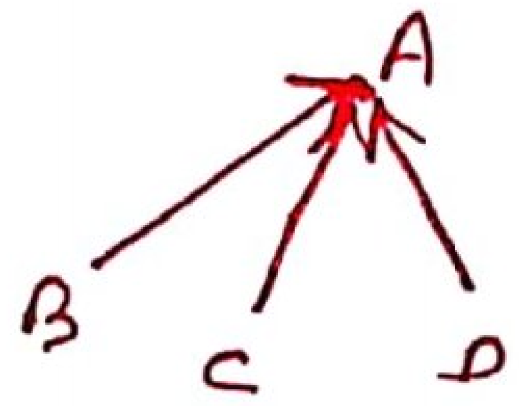
$\vec{AB}$ is always equals to $-\vec{BA}$

(3) **Like vectors & unlike vectors** : like vectors when they have same sense of direction and unlike when they have opposite directions
(Magnitude can be different / unequal / equal)

(4) Coinitial vectors : vectors having same initial point

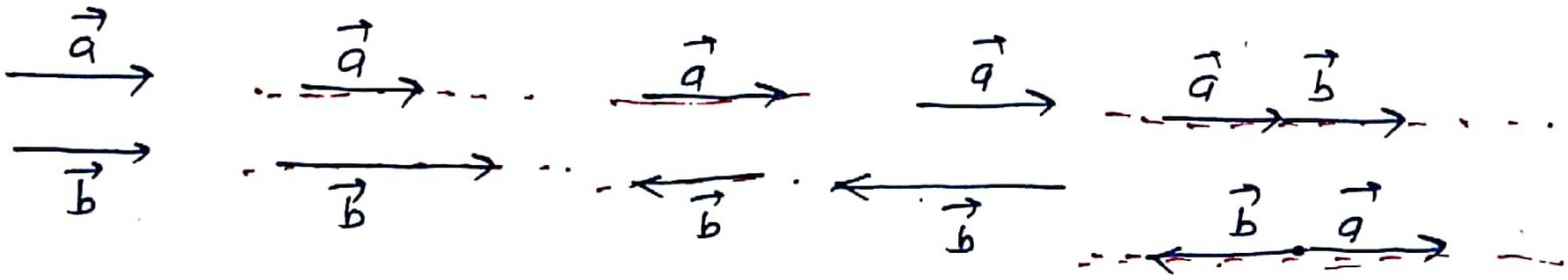


(5) Co-terminous vectors : vectors having same terminal point

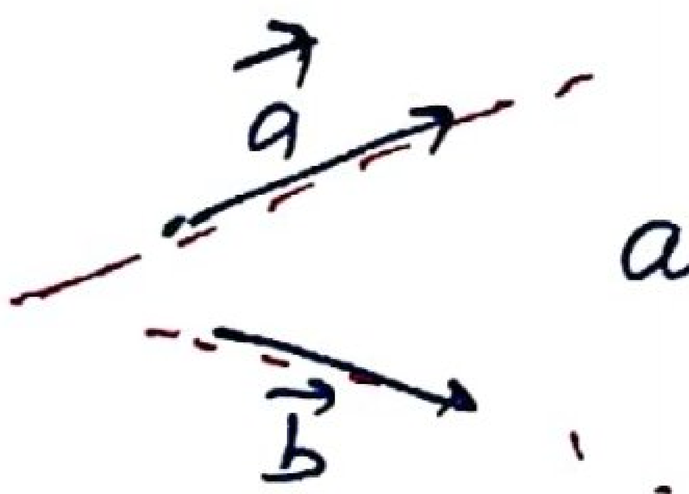


(6) **COLLINEAR / PARALLEL VECTORS**

(i) vectors having same or parallel support are called collinear vectors



In all these: $\vec{a}$ & $\vec{b}$ are collinear vectors



are not collinear vectors

Important If $\boxed{\vec{a} = \lambda \vec{b}}$ or $\boxed{\vec{b} = \lambda \vec{a}}$ then

$\vec{a}$ & $\vec{b}$ are collinear or parallel

where $\lambda \to$ is a scalar

e.g. $\vec{a} = 3\vec{b}$ represents that $\vec{a}$ & $\vec{b}$ are in same direction and magnitude of $\vec{a}$ = 3 times the mag. of $\vec{b}$

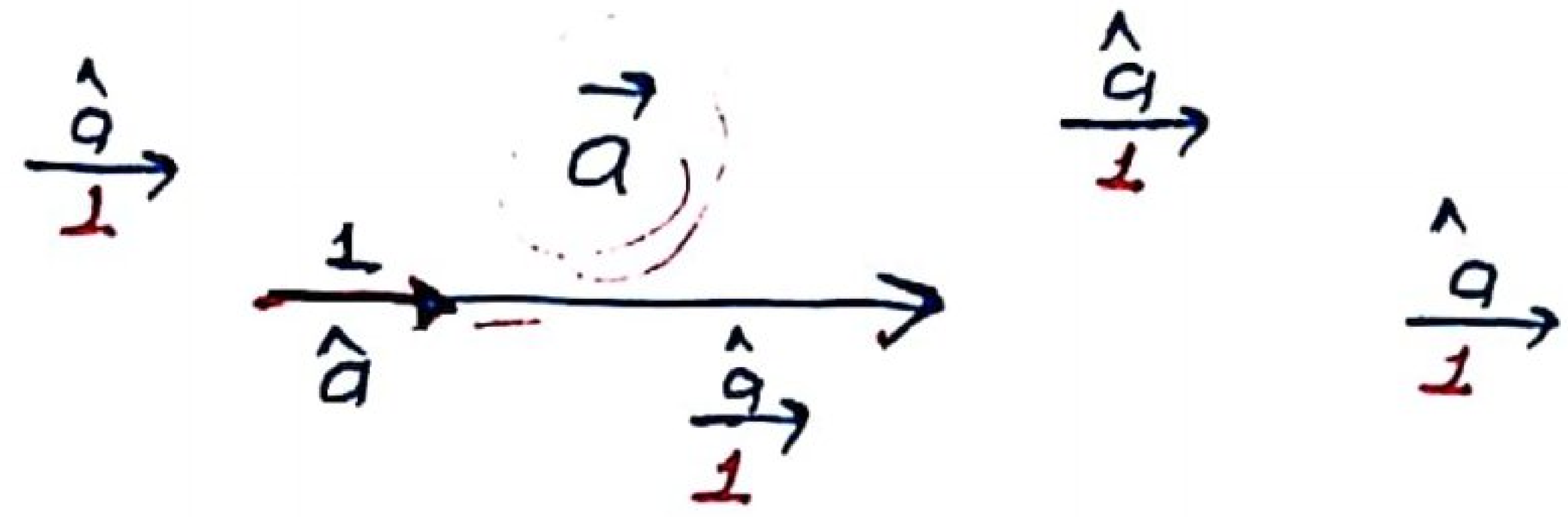
$|\vec{a}| = 3|\vec{b}|$

(7) UNIT VECTOR

(·) A vector whose magnitude is 1 (unity)

(·) The unit vector in the direction of $\vec{a}$ is denoted by $\hat{a}$ (cap)

$|\hat{a}| = 1$



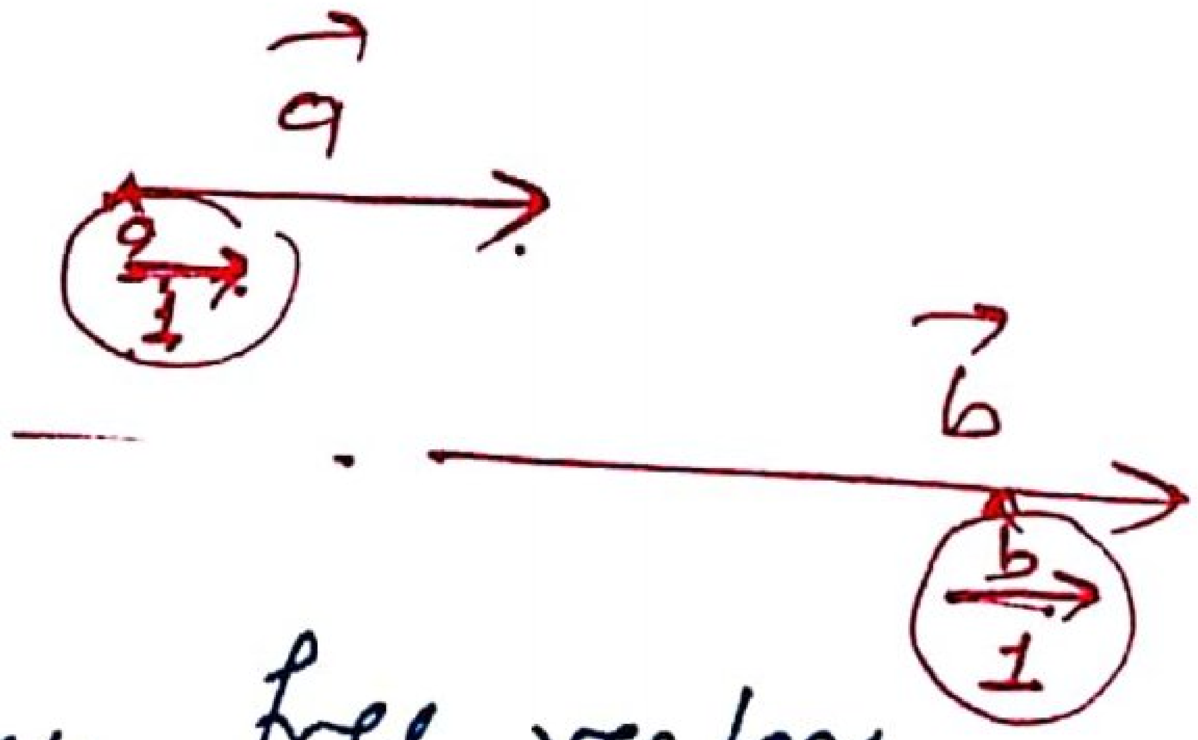
<u>Imp</u> Any vector in the direction of $\vec{a}$ with length or magnitude 1 is called unit vector of $\vec{a}$ i.e $\hat{a}$

<u>IMP</u> $\boxed{\text{Unit vector} = \dfrac{\text{Vector}}{\text{Magnitude}}}$ (or) $\hat{a} = \dfrac{\vec{a}}{|\vec{a}|}$

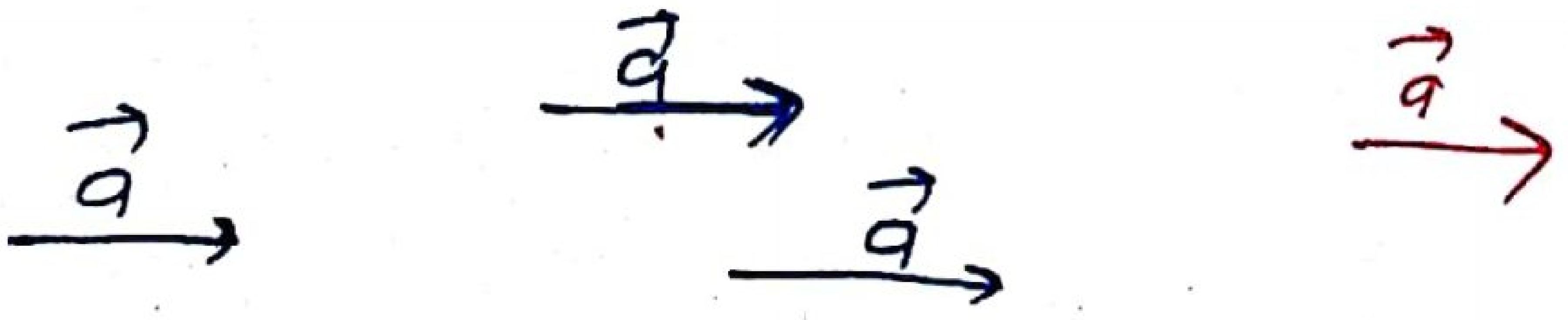
$\Rightarrow$ $\boxed{\text{Vector} = (\text{Magnitude})\,(\text{unit vector})}$

<u>IMP</u>

<u>VIMP</u> If $\vec{a}$ & $\vec{b}$ are in same direction, parallel, along then $\boxed{\hat{a} = \hat{b}}$ <u>IMP</u>

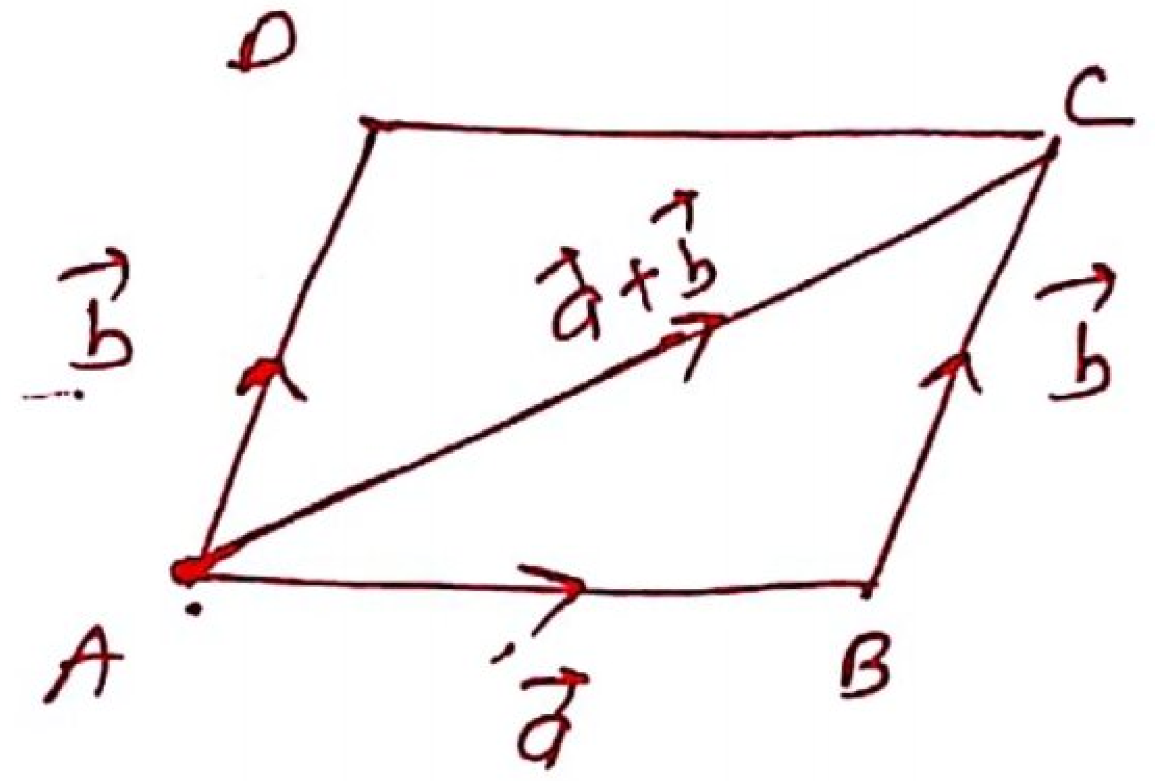


(·) Free vectors : vectors are always free vectors.



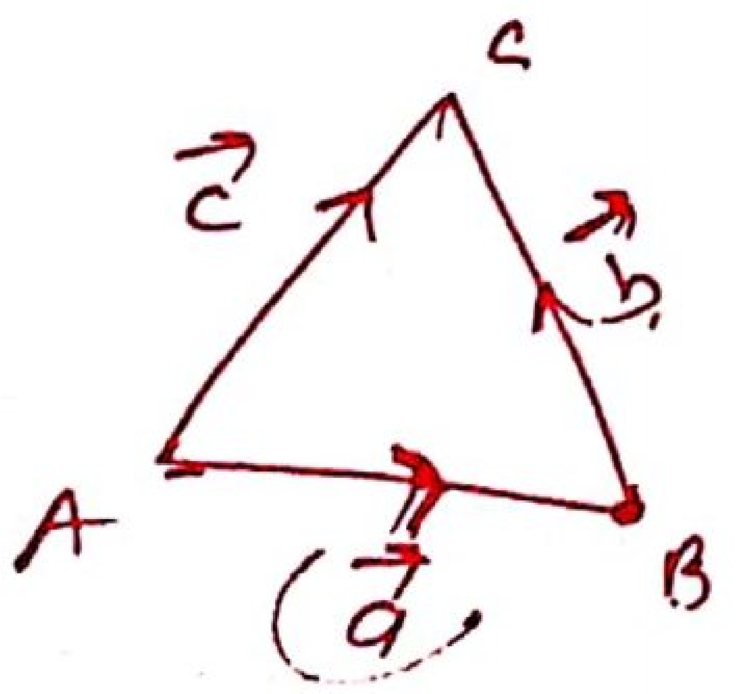
Parallelogram law of addition of vectors:

Let two vectors $\vec{a}$ & $\vec{b}$ represents the two adjacent sides of a parallelogram, then their sum $(\vec{a}+\vec{b})$ is represented by the diagonal of the parallelogram which is coinitial with the given vectors



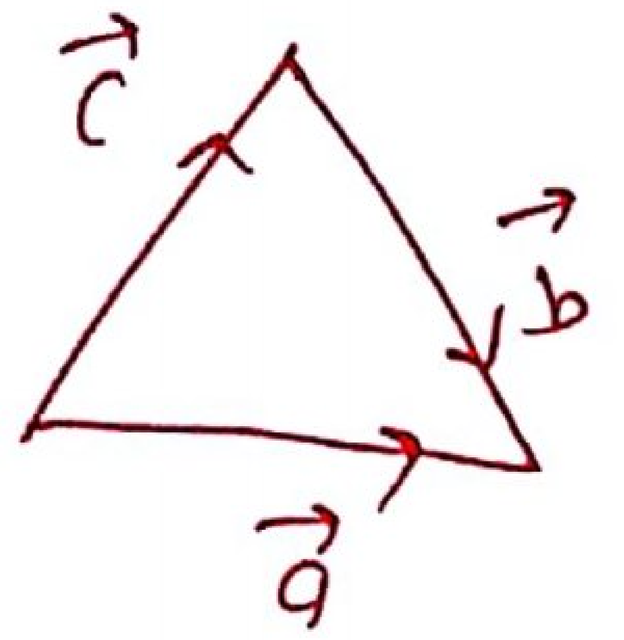
Triangle law of addition of vectors

If two vectors are represented by the two sides of a triangle in the same order, then their sum is represented by the third side taken in "reverse" order

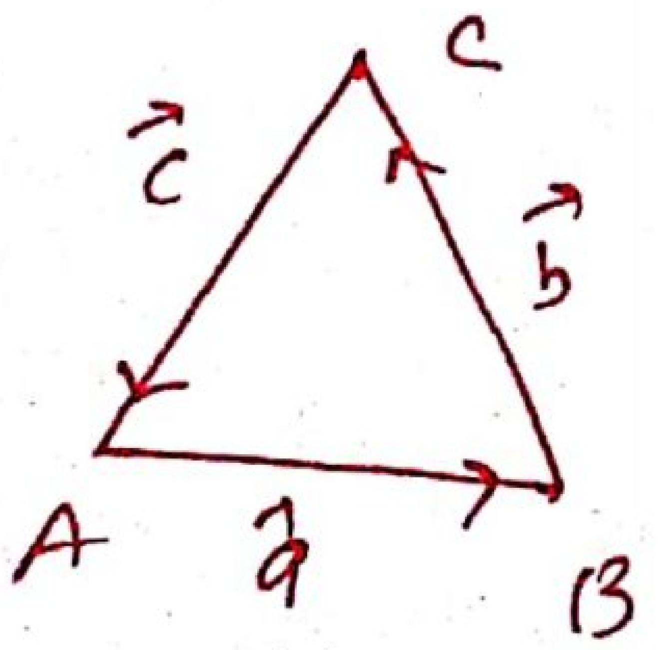


$\vec{AB} + \vec{BC} = \vec{AC}$

$\vec{a} + \vec{b} = \vec{c}$



$\vec{c} + \vec{b} = \vec{a}$



$\vec{AB} + \vec{BC} = -\vec{AC}$

$\vec{AB} + \vec{BC} + \vec{AC} = \vec{0}$ → zero vector

$\vec{a}+\vec{b}+\vec{c} = \vec{0}$

$\boxed{\vec{a}+\vec{b} = -\vec{c}}$

(·) Null / zero vector: A vector whose initial & terminal points are coincident is null vector denoted by $\vec{0}$

$\vec{AA} = \vec{0}$

(·) $\vec{AB} = \vec{CD}$

then $\vec{AB} - \vec{CD} = \vec{0}$ ↓ vector

(·) If $AB = CD$

then $AB - CD = 0$ ↓ scalar

(·) Proper vectors: vectors other than null vector are ~~coll~~ called proper vectors.

Some properties

(·) $\vec{a} + \vec{b} = \vec{b} + \vec{a}$ (Commutative)

(·) $(\vec{a} + \vec{b}) + \vec{c} = \vec{a} + (\vec{b} + \vec{c})$ (Associative)

(·) $\vec{a} + \vec{0} = \vec{0} + \vec{a} = \vec{a}$ (existence of additive Identity)

(·) $\vec{a} + (-\vec{a}) = \vec{0}$ (existence of additive Inverse)

(·) $n(m\vec{a}) = m(n)\vec{a} = mn\,\vec{a}$

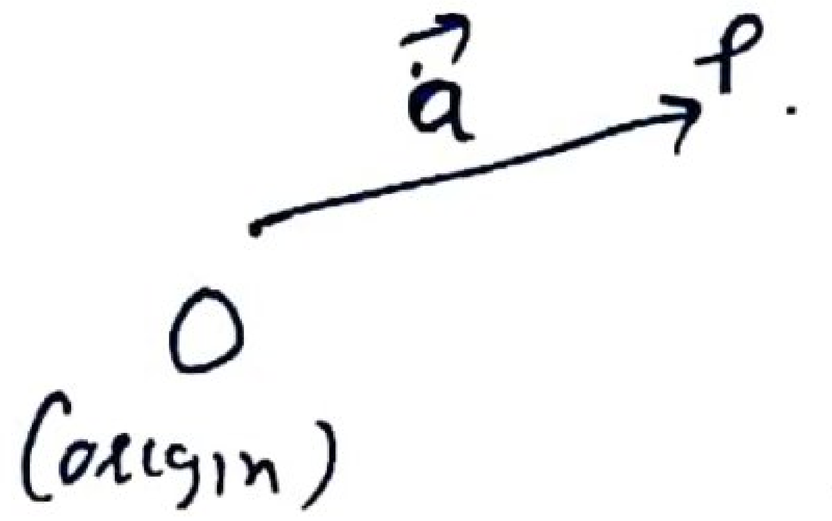
(·) $m(\vec{a} + \vec{b}) = m\vec{a} + m\vec{b}$

(·) $(m+n)\vec{a} = m\vec{a} + n\vec{a}$

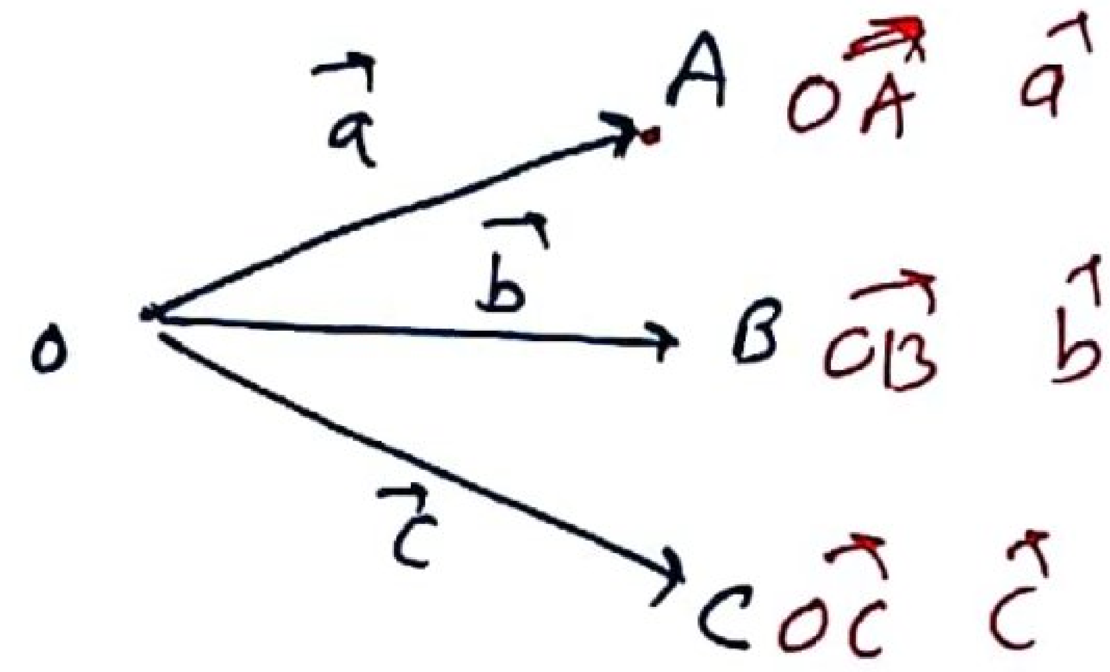
{ m & n are Scalars }

Position vector of a point :

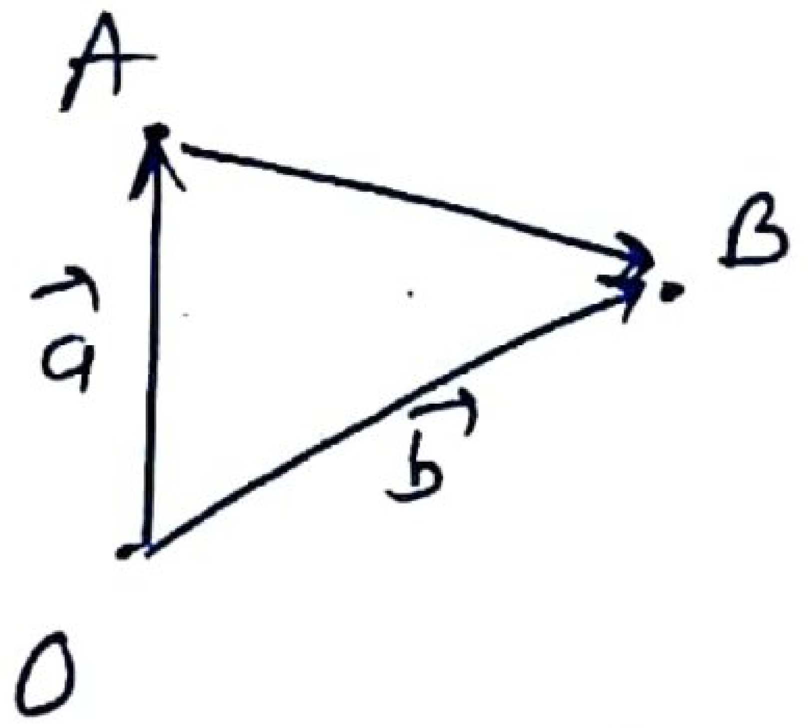
If a point O is fixed as the origin in space and P is any point, then $\vec{OP}$ is called the position vector of point P w.r.t O



$\vec{OP} = \vec{a}$.



Imp



By triangle law

$\vec{OA} + \vec{AB} = \vec{OB}$

$\vec{AB} = \vec{OB} - \vec{OA}$

$\Rightarrow$ $\vec{AB}$ = P.V of point B − P.V of point A

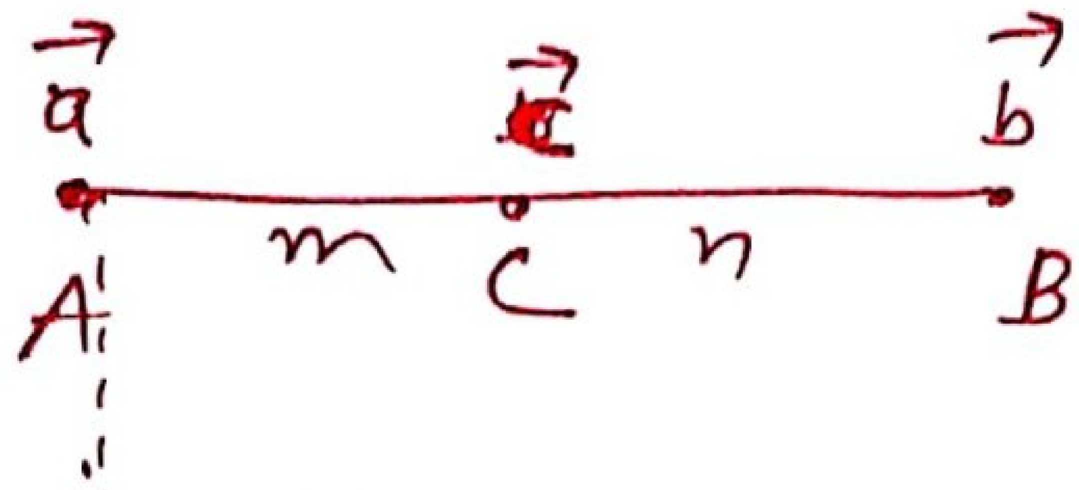
$\Rightarrow$ $\boxed{\vec{AB} = \vec{b} - \vec{a}}$ (or) $\boxed{\vec{AB} = \vec{OB} - \vec{OA}}$

e.g. $\vec{CD} = \vec{OD} - \vec{OC}$

= final − Initial

SECTION FORMULA

Internal division



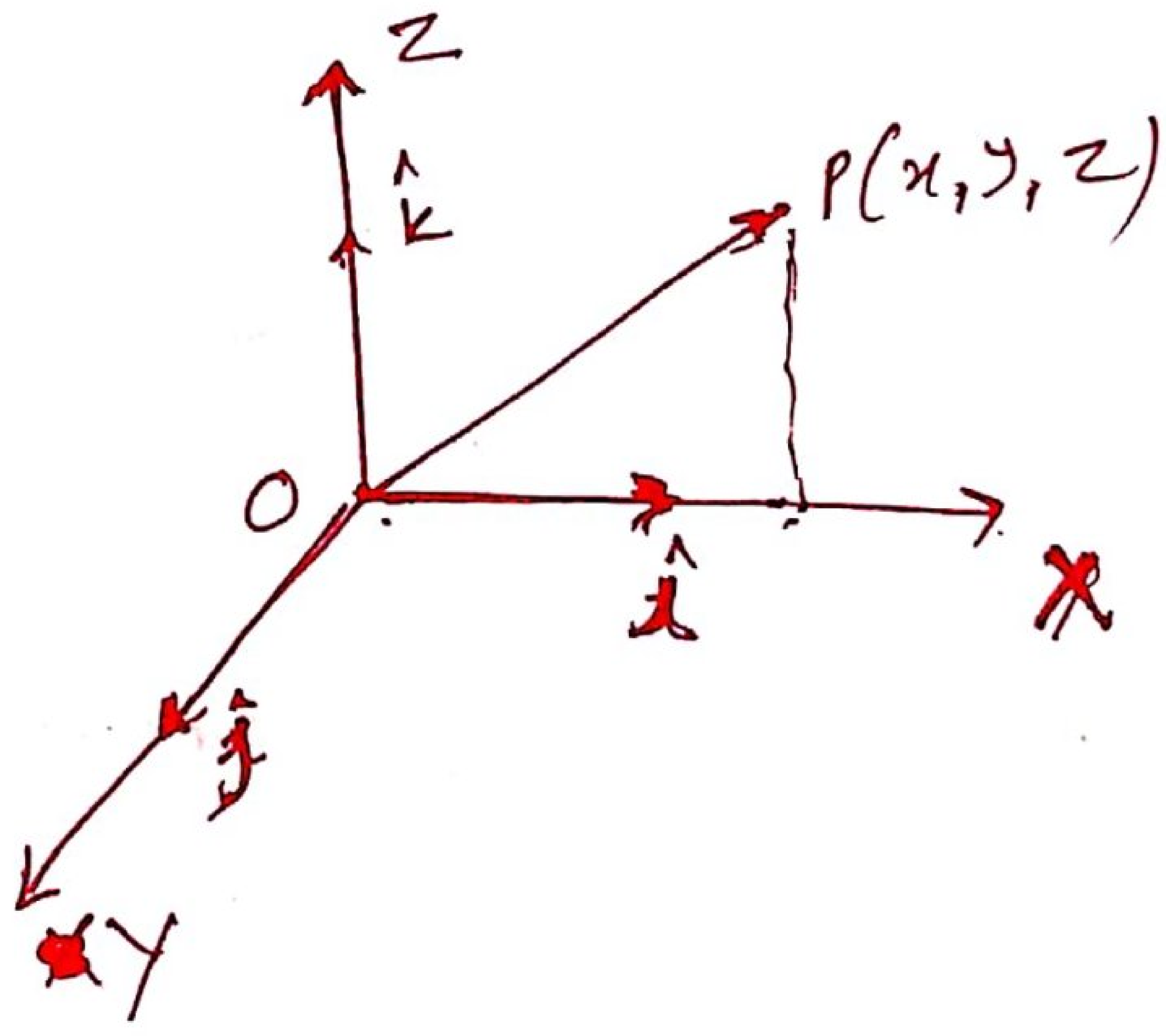
$$\vec{c} = \frac{m\vec{b} + n\vec{a}}{m+n}$$

(or) O $$\vec{OC} = \frac{m(\vec{OB}) + n(\vec{OA})}{m+n}$$

External division

$$\vec{c} = \frac{m\vec{b} - n\vec{a}}{m-n}$$

COMPONENTS OF A VECTOR



Let $P(x, y, z)$ be any point in the space

$\hat{i}, \hat{j}, \hat{k}$ are the unit vectors along X-axis, Y-axis, Z-axis respectively

$$\boxed{\vec{OP} = x\hat{i} + y\hat{j} + z\hat{k}}$$

$$|\vec{OP}| = \sqrt{x^2 + y^2 + z^2}$$

$x\hat{i}, y\hat{j}, z\hat{k}$ are the components of $\vec{OP}$

(•) vector joining two points

$$\boxed{\vec{AB} = (x_2-x_1)\hat{i} + (y_2-y_1)\hat{j} + (z_2-z_1)\hat{k}}$$

$O\,(0,0,0)$

$$\underset{(x_1,y_1,z_1)}{A} \longrightarrow \underset{(x_2,y_2,z_2)}{B}$$

$\vec{OA}$ = p.v. of point A $= (x_1-0)\hat{i} + (y_1-0)\hat{j} + (z_1-0)\hat{k}$

$= x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$

$\vec{OB}$ = p.v of point B $= x_2\hat{i} + y_2\hat{j} + z_2\hat{k}$

$\vec{AB} = \vec{OB} - \vec{OA}$

$= (x_2-x_1)\hat{i} + (y_2-y_1)\hat{j} + (z_2-z_1)\hat{k}$

IMP If a point P has coordinate $(2,-3,4)$
then p.v of point P is $\vec{OP} = 2\hat{i} - 3\hat{j} + 4\hat{k}$