SOLUTION of DETERMINANTS
CLASS-4 (D-4)
(ULTIMATE MATHEMATICS)

Ans. 1

$$\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$$

$R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$

$$= \begin{vmatrix} 1 & a & a^2 \\ 0 & b-a & b^2-a^2 \\ 0 & c-a & c^2-a^2 \end{vmatrix}$$

taking $(b-a)$ & $(c-a)$ common from R_2 & R_3 Resp.

$$= (b-a)(c-a) \begin{vmatrix} 1 & a & a^2 \\ 0 & 1 & b+a \\ 0 & 1 & c+a \end{vmatrix}$$

$R_3 \to R_3 - R_2$

$$= (b-a)(c-a) \begin{vmatrix} 1 & a & a^2 \\ 0 & 1 & b+a \\ 0 & 0 & c-b \end{vmatrix}$$

expanding

$$= (b-a)(c-a)\left[1(c-b)\right]$$

$$= (b-a)(c-a)(c-b)$$

$$= (a-b)(b-c)(c-a) = RHS$$ **Ans.**

Ans. 2 → LHS

$$\begin{vmatrix} 1 & 1 & 1 \\ x^2 & y^2 & z^2 \\ x & y & z \end{vmatrix}$$

$C_2 \to C_2 - C_1$ and $C_3 \to C_3 - C_1$

$$= \begin{vmatrix} 1 & 0 & 0 \\ x^2 & y^2-x^2 & z^2-x^2 \\ x & y-x & z-x \end{vmatrix}$$

taking $(y-x)$ and $(z-x)$ common from C_2 & C_3

$$= (y-x)(z-x) \begin{vmatrix} 1 & 0 & 0 \\ x^2 & y+x & z+x \\ x & 1 & 1 \end{vmatrix}$$

$C_3 \to C_3 - C_2$

$$= (y-x)(z-x) \begin{vmatrix} 1 & 0 & 0 \\ x^2 & y+x & z-y \\ x & 1 & 0 \end{vmatrix}$$

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expanding

$= (y-x)(z-x)\,[\,0-(z-y)\,]$

$= -(y-x)(z-x)(z-y)$

$= -(x-y)(y-z)(z-x)$ Ans

Misprint in worksheet (-ve bhi ayega)

Ques. 3 →

$$\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix}$$

$C_2 \to C_2 - C_1$ and $C_3 \to C_3 - C_1$

$$= \begin{vmatrix} 1 & 0 & 0 \\ a & b-a & c-b \\ a^3 & (b-a)(b^2+a^2+ab) & (c-b)(c^2+b^2+bc) \end{vmatrix}$$

taking $(b-a)$ and $(c-b)$ common from C_2 & C_3

$$= (b-a)(c-b) \begin{vmatrix} 1 & 0 & 0 \\ a & 1 & 1 \\ a^3 & b^2+a^2+ab & c^2+b^2+bc \end{vmatrix}$$

$C_3 \to C_3 - C_2$

$$= (b-a)(c-b) \begin{vmatrix} 1 & 0 & 0 \\ a & 1 & 0 \\ a^3 & b^2+a^2+ab & c^2-a^2+bc-ab \end{vmatrix}$$

$$= (b-a)(c-b) \begin{vmatrix} 1 & 0 & 0 \\ a & 1 & 0 \\ a^3 & b^2+a^2+ab & (c+a)(c-a)+b(c-a) \end{vmatrix}$$

taking $(c-a)$ common from R_3

$$= (b-a)(c-b)(c-a) \begin{vmatrix} 1 & 0 & 0 \\ a & 1 & 0 \\ a^3 & b^2+a^2+ab & c+a+b \end{vmatrix}$$

expanding

$= (b-a)(c-b)(c-a)\,(1(a+b+c)-0)$

$= (a-b)(b-c)(c-a)(a+b+c)$ = RHS Ans

Q.N. 4 → $\begin{vmatrix} 1 & a^2 & a^3 \\ 1 & b^2 & b^3 \\ 1 & c^2 & c^3 \end{vmatrix}$

$R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$

$$= \begin{vmatrix} 1 & a^2 & a^3 \\ 0 & (b+a)(b-a) & (b-a)(b^2+a^2+ab) \\ 0 & (c+a)(c-a) & (c-a)(c^2+a^2+ac) \end{vmatrix}$$

taking $(b-a)$ & $(c-a)$ common from R_2 & R_3 resp

$$= (b-a)(c-a) \begin{vmatrix} 1 & a^2 & a^3 \\ 0 & b-a & b^2+a^2+ab \\ 0 & c-a & c^2+a^2+ac \end{vmatrix}$$

$R_3 \to R_3 - R_2$

$$= (b-a)(c-a) \begin{vmatrix} 1 & a^2 & a^3 \\ 0 & b-a & b^2+a^2+ab \\ 0 & c-b & c^2-b^2+ac-ab \end{vmatrix}$$

$$= (b-a)(c-a) \begin{vmatrix} 1 & a^2 & a^3 \\ 0 & b-a & b^2+a^2+ab \\ 0 & c-b & (c+b)(c-b)+a(c-b) \end{vmatrix}$$

taking $(c-b)$ common from R_3

$$= (b-a)(c-a)(c-b) \begin{vmatrix} 1 & a^2 & a^3 \\ 0 & b+a & b^2+a^2+ab \\ 0 & 1 & c+b+a \end{vmatrix}$$

expanding

$$= (b-a)(c-a)(c-b)\left((b+a)(a+b+c) - 1(b^2+a^2+ab)\right)$$

$$= (b-a)(c-a)(c-b)\left[ab + \cancel{b^2} + bc + \cancel{a^2} + \cancel{ab} + ac - \cancel{b^2} - \cancel{a^2} - \cancel{ab}\right]$$

$$= (b-a)(c-a)(c-b)(ab+bc+ac)$$

$$= (a-b)(b-c)(c-a)(ab+bc+ac) = \text{RHS}$$ Ans.

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Ans 5 → $\begin{vmatrix} x & y & z \\ x^2 & y^2 & z^2 \\ x^3 & y^3 & z^3 \end{vmatrix}$

taking x, y, z common from C_1, C_2, C_3 resp.

$= xyz \begin{vmatrix} 1 & 1 & 1 \\ x & y & z \\ x^2 & y^2 & z^2 \end{vmatrix}$

$C_2 \to C_2 - C_1$ and $C_3 \to C_3 - C_1$

$= xyz \begin{vmatrix} 1 & 0 & 0 \\ x & y-x & z-x \\ x^2 & (y+x)(y-x) & (z+x)(z-x) \end{vmatrix}$

$= xyz\,(y-x)(z-x) \begin{vmatrix} 1 & 0 & 0 \\ x & 1 & 1 \\ x^2 & y+x & z+x \end{vmatrix}$

$C_3 \to C_3 - C_2$

$= (xyz)(y-x)(z-x) \begin{vmatrix} 1 & 0 & 0 \\ x & 1 & 0 \\ x^2 & y+x & z-y \end{vmatrix}$

expanding

$= xyz\,(y-x)(z-x)(z-y)$

$= xyz\,(x-y)(y-z)(z-x) =$ RHS Ans.

Ans 6 → $\begin{vmatrix} x+4 & 2x & 2x \\ 2x & x+4 & 2x \\ 2x & 2x & x+4 \end{vmatrix}$

$C_1 \to C_1 + C_2 + C_3$

$= \begin{vmatrix} 5x+4 & 2x & 2x \\ 5x+4 & x+4 & 2x \\ 5x+4 & 2x & x+4 \end{vmatrix}$

take $(5x+y)$ common from C_1

$$= (5x+y)\begin{vmatrix} 1 & 2x & 2x \\ 1 & x+y & 2x \\ 1 & 2x & x+y \end{vmatrix}$$

$R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$

$$= (5x+y)\begin{vmatrix} 1 & 2x & 2x \\ 0 & y-x & 0 \\ 0 & 0 & y-x \end{vmatrix}$$

taking $(y-x)$ common from R_2 & R_3

$$= (5x+y)(y-x)^2\begin{vmatrix} 1 & 2x & 2x \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

expanding

$$= (5x+y)(y-x)^2(1)$$

$$= (5x+y)(y-x)^2 = \text{RHS}$$ ANS

Ans 7 →

$$= \begin{vmatrix} y+k & y & y \\ y & y+k & y \\ y & y & y+k \end{vmatrix}$$

$C_1 \to C_1 + C_2 + C_3$

$$= (3y+k)\begin{vmatrix} 1 & y & y \\ 1 & y+k & y \\ 1 & y & y+k \end{vmatrix}$$

$R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$

$$= (3y+k)\begin{vmatrix} 1 & y & y \\ 0 & k & 0 \\ 0 & 0 & k \end{vmatrix}$$

expanding

$$= (3y+k)(k^2) = \text{RHS}$$ ANS

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Ons 8 →
$$\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$$

$R_1 \to R_1 + R_2 + R_3$

$$= \begin{vmatrix} a+b+c & a+b+c & a+b+c \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$$

$$= (a+b+c) \begin{vmatrix} 1 & 1 & 1 \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$$

$C_2 \to C_2 - C_1$ and $C_3 \to C_3 - C_1$

$$= (a+b+c) \begin{vmatrix} 1 & 0 & 0 \\ 2b & -(a+b+c) & 0 \\ 2c & 0 & -(a+b+c) \end{vmatrix}$$

expanding

$$= (a+b+c) \left[(a+b+c)^2 \right]$$

$$= (a+b+c)^3 = \text{RHS}$$ Ans

Ons 9 →
$$= \begin{vmatrix} x+y+2z & x & y \\ z & y+z+2x & y \\ z & x & z+x+2y \end{vmatrix}$$

$C_1 \to C_1 + C_2 + C_3$

$$= \begin{vmatrix} 2(x+y+z) & x & y \\ 2(x+y+z) & y+z+2x & y \\ 2(x+y+z) & x & z+x+2y \end{vmatrix}$$

$$= 2(x+y+z) \begin{vmatrix} 1 & x & y \\ 1 & 2x+y+z & y \\ 1 & x & x+2y+z \end{vmatrix}$$

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$R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$

$$= 2(x+y+z)\begin{vmatrix} 1 & x & y \\ 0 & x+y+z & 0 \\ 0 & 0 & x+y+z \end{vmatrix}$$

expanding

$$= 2(x+y+z)\left[(x+y+z)^2 - 0\right]$$

$$= 2(x+y+z)^3 = \text{RHS} \quad \text{ANS}$$

Qn 16 →

$$\begin{vmatrix} 3a & -a+b & -a+c \\ -b+a & 3b & -b+c \\ -c+a & -c+b & 3c \end{vmatrix}$$

$C_1 \to C_1 + C_2 + C_3$

$$= \begin{vmatrix} a+b+c & -a+b & -a+c \\ a+b+c & 3b & -b+c \\ a+b+c & -c+b & 3c \end{vmatrix}$$

$$= (a+b+c)\begin{vmatrix} 1 & -a+b & -a+c \\ 1 & 3b & -b+c \\ 1 & -c+b & 3c \end{vmatrix}$$

$R_2 \to R_2 - R_1$ & $R_3 \to R_3 - R_1$

$$= (a+b+c)\begin{vmatrix} 1 & -a+b & -a+c \\ 0 & 2b+a & -b+a \\ 0 & -c+a & 2c+a \end{vmatrix}$$

expanding

$$= (a+b+c)\left[(2b+a)(2c+a) - (-c+a)(-b+a)\right]$$

$$= (a+b+c)\left[4bc + 2ab + 2ac + \cancel{a^2} - bc + ac + ab - \cancel{a^2}\right]$$

$$= (a+b+c)(3ab + 3bc + 3ac)$$

$$= 3(a+b+c)(ab+bc+ac) = \text{Ans}$$

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Qns 11)

$$\begin{vmatrix} a & a+b & a+2b \\ a+2b & a & a+b \\ a+b & a+2b & a \end{vmatrix}$$

$C_1 \to C_1 + C_2 + C_3$

$$= \begin{vmatrix} 3(a+b) & a+b & a+2b \\ 3(a+b) & a & a+b \\ 3(a+b) & a+2b & a \end{vmatrix}$$

$$= 3(a+b) \begin{vmatrix} 1 & a+b & a+2b \\ 1 & a & a+b \\ 1 & a+2b & a \end{vmatrix}$$

$R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$

$$= 3(a+b) \begin{vmatrix} 1 & a+b & a+2b \\ 0 & -b & -b \\ 0 & b & -2b \end{vmatrix}$$

$R_3 \to R_3 + R_2$

$$= 3(a+b) \begin{vmatrix} 1 & a+b & a+2b \\ 0 & -b & -b \\ 0 & 0 & -3b \end{vmatrix}$$

expanding

$$= 3(a+b) \left(3b^2\right)$$

$$= 9(a+b)b^2 = \text{RHS} \quad \text{Ans}$$

Qns 12 →

$$\begin{vmatrix} 1 & x & x^2 \\ x^2 & 1 & x \\ x & x^2 & 1 \end{vmatrix}$$

Two Methods   Method 1 (easy)

$R_2 \to R_2 - x^2 R_1$ and $R_3 \to R_3 - x R_1$

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$$= \begin{vmatrix} 1 & x & x^2 \\ 0 & 1-x^3 & x-x^4 \\ 0 & 0 & 1-x^3 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & x & x^2 \\ 0 & 1-x^3 & x(1-x^3) \\ 0 & 0 & 1-x^3 \end{vmatrix}$$

taking $(1-x^3)$ common from R_2 & R_3

$$= (1-x^3)^2 \begin{vmatrix} 1 & x & x^2 \\ 0 & 1 & x \\ 0 & 0 & 1 \end{vmatrix}$$

expanding

$= (1-x^3)^2 (1)$

$= (1-x^3)^2$ **Ans**

Ans 13 → $\begin{vmatrix} \alpha & \alpha^2 & \beta+\gamma \\ \beta & \beta^2 & \gamma+\alpha \\ \gamma & \gamma^2 & \alpha+\beta \end{vmatrix}$

$C_3 \to C_3 + C_1$

$$= \begin{vmatrix} \alpha & \alpha^2 & \alpha+\beta+\gamma \\ \beta & \beta^2 & \alpha+\beta+\gamma \\ \gamma & \gamma^2 & \alpha+\beta+\gamma \end{vmatrix}$$

$$= (\alpha+\beta+\gamma) \begin{vmatrix} \alpha & \alpha^2 & 1 \\ \beta & \beta^2 & 1 \\ \gamma & \gamma^2 & 1 \end{vmatrix}$$

$R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$

$$= (\alpha+\beta+\gamma) \begin{vmatrix} \alpha & \alpha^2 & 1 \\ \beta-\alpha & \beta^2-\alpha^2 & 0 \\ \gamma-\alpha & \gamma^2-\alpha^2 & 0 \end{vmatrix}$$

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Taking $(\beta-\alpha)$ & $(\gamma-\alpha)$ common from R_2 & R_3

$$= (\alpha+\beta+\gamma)(\beta-\alpha)(\gamma-\alpha)\begin{vmatrix} \alpha & \alpha^2 & 1 \\ 1 & \beta+\alpha & 0 \\ 1 & \gamma+\alpha & 0 \end{vmatrix}$$

$R_3 \to R_3 - R_2$

$$= (\alpha+\beta+\gamma)(\beta-\gamma)(\gamma-\alpha)\begin{vmatrix} \alpha & \alpha^2 & 1 \\ 1 & \beta+\gamma & 0 \\ 0 & \gamma-\beta & 0 \end{vmatrix}$$

expanding

$$= (\alpha+\beta+\gamma)(\beta-\gamma)(\gamma-\alpha)\left[1(\gamma-\beta)\right]$$

$$= (\alpha+\beta+\gamma)(\alpha-\beta)(\beta-\gamma)(\gamma-\alpha) = RHS \quad \text{ANS}$$

Ons 14 →

$$\begin{vmatrix} a & b-c & c-b \\ a-c & b & c-a \\ a-b & b-a & c \end{vmatrix}$$

$C_1 \to C_1 + C_3$ and $C_2 \to C_2 + C_3$

$$= \begin{vmatrix} a+c-b & 0 & c-b \\ 0 & b+c-a & c-a \\ a+c-b & b+c-a & c \end{vmatrix}$$

taking common $(a+c-b)$ & $(b+c-a)$ from C_1 & C_2 Resp

$$= (a+c-b)(b+c-a)\begin{vmatrix} 1 & 0 & c-b \\ 0 & 1 & c-a \\ 1 & 1 & c \end{vmatrix}$$

expanding

$$= (a+c-b)(b+c-a)\left[1(\cancel{c}-\cancel{c}+a) + (c-b)(-1)\right]$$

$$= (a+c-b)(b+c-a)(a+c+b) = RHS$$

$$= (a+c-b)(b+c-a)(a+b-c) \quad \text{Ans}$$

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QN 15 → given $\begin{vmatrix} x+a & x & x \\ x & x+a & x \\ x & x & x+a \end{vmatrix} = 0$

$C_1 \to C_1 + C_2 + C_3$

$\Rightarrow \begin{vmatrix} 3x+a & x & x \\ 3x+a & x+a & x \\ 3x+a & x & x+a \end{vmatrix} = 0$

$\Rightarrow (3x+a) \begin{vmatrix} 1 & x & x \\ 1 & x+a & x \\ 1 & x & x+a \end{vmatrix} = 0$

$R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$

$\Rightarrow (3x+a) \begin{vmatrix} 1 & x & x \\ 0 & a & 0 \\ 0 & 0 & a \end{vmatrix} = 0$

expanding

$\Rightarrow (3x+a)(a^2) = 0$

$\Rightarrow 3x + a = 0 \Rightarrow \boxed{x = \frac{-a}{3}}$ Ans

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QN 16 → given $\begin{vmatrix} 3x-8 & 3 & 3 \\ 3 & 3x-8 & 3 \\ 3 & 3 & 3x-8 \end{vmatrix} = 0$

$C_1 \to C_1 + C_2 + C_3$

$\Rightarrow \begin{vmatrix} 3x-2 & 3 & 3 \\ 3x-2 & 3x-8 & 3 \\ 3x-2 & 3 & 3x-8 \end{vmatrix} = 0$

⇒ taking $(3x-2)$ common from C_1

$\Rightarrow (3x-2) \begin{vmatrix} 1 & 3 & 3 \\ 1 & 3x-8 & 3 \\ 1 & 3 & 3x-8 \end{vmatrix} = 0$

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$R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$

$$\Rightarrow (3x-2)\begin{vmatrix} 1 & 3 & 3 \\ 0 & 3x-11 & 0 \\ 0 & 0 & 3x-11 \end{vmatrix} = 0$$

expanding

$$\Rightarrow (3x-2)(3x-11)^2 = 0$$

$$\Rightarrow \boxed{x = 2/3} \quad \& \quad \boxed{x = \frac{11}{3}} \longrightarrow$$ (Misprint in worksheet)

Ans

Qn 17 →
$$\begin{vmatrix} 1 & 1+p & 1+p+q \\ 2 & 3+2p & 1+3p+2q \\ 3 & 6+3p & 1+6p+3q \end{vmatrix}$$

$R_2 \to R_2 - 2R_1$ and $R_3 \to R_3 - 3R_1$

$$= \begin{vmatrix} 1 & 1+p & 1+p+q \\ 0 & 1 & p-1 \\ 0 & 3 & -2+3p \end{vmatrix}$$

expanding

$$= 1\left(-2+\cancel{3p}-\cancel{3p}+3\right)$$

$$= 1 \text{ Ans}$$

Qn 18 →
$$\begin{vmatrix} a & a+b & a+b+c \\ 2a & 3a+2b & 4a+3b+2c \\ 3a & 6a+3b & 10a+6b+3c \end{vmatrix}$$

taking a common from C_1

$$= a\begin{vmatrix} 1 & a+b & a+b+c \\ 2 & 3a+2b & 4a+3b+2c \\ 3 & 6a+3b & 10a+6b+3c \end{vmatrix}$$