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CHAPTER: LINEAR PROGRAMMING PROBLEM (LPP).

CLASS NO: 1

(·) LPP

Maximize / Minimize

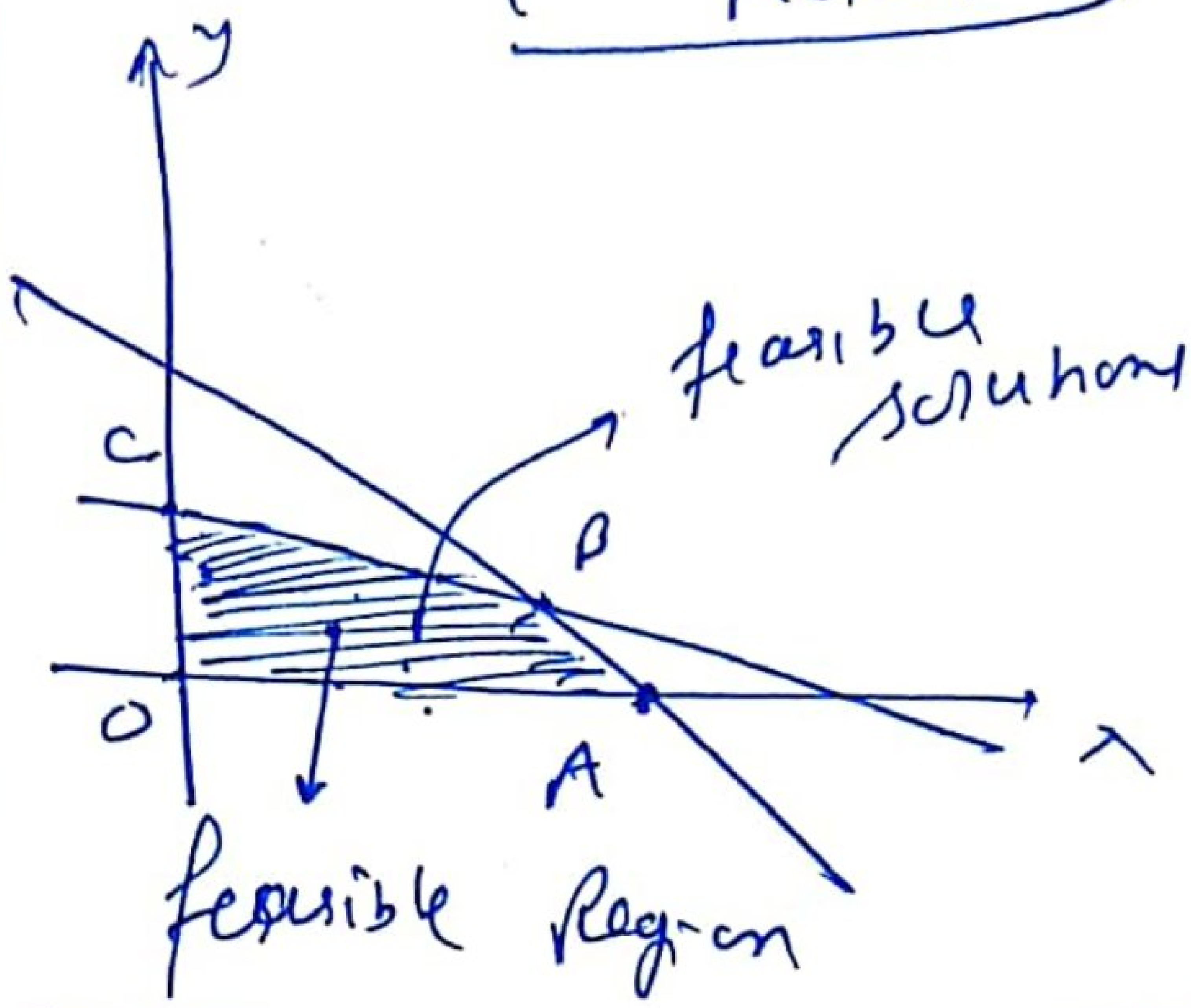
$$Z = 2x + 3y$$

Subject to constraints

Min two
$$\begin{cases} x + y \le 4 \\ 2x - y \ge 5 \\ x - y = 6 \end{cases}$$

and $x \ge 0, y \ge 0$

Corner point Method



optimal solution: value of x & y for Z_{max} / Z_{min}

(·) Z → objective function

(·) Maximum or Minimum value of Z → optimal value of Z

(·) decision variables: → x & y

(·) optimisation problem →

(·) linear constraints → inequality / equation / Restrictions

(·) $x \ge 0, y \ge 0$ → Non-negative Restrictions

Ques: 1 Minimize

$$Z = 200x + 500y$$

Subject to constraints

$$x + 2y \geq 10$$
$$3x + 4y \leq 24$$
$$\& \; x \geq 0, \; y \geq 0$$

$$3x + 8y = 30$$
$$3x + 4y = 24$$
$$2y = 6$$

Soln

$x + 2y \geq 10$ points $(0,5)$ & $(10,0)$; (away)

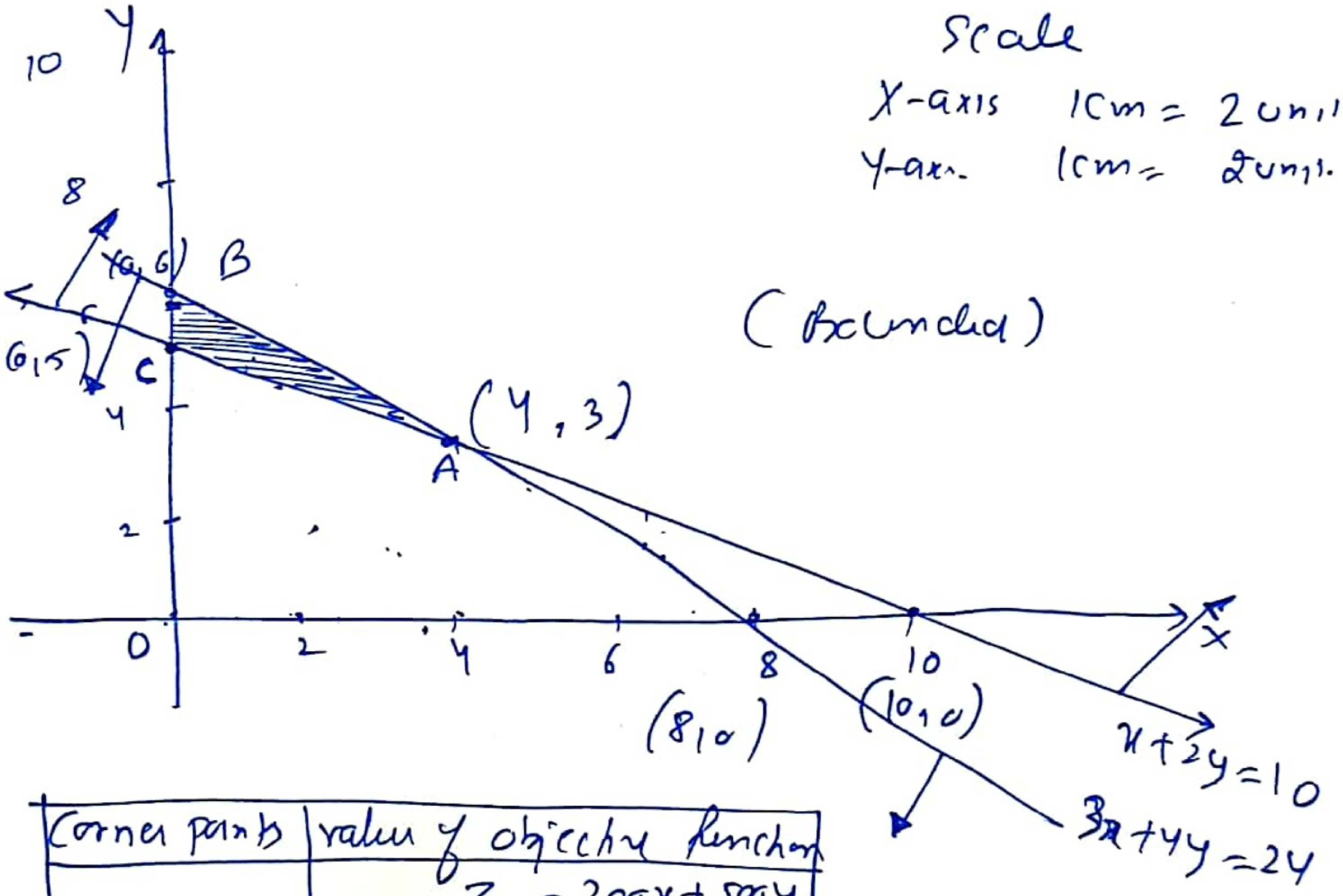
$3x + 4y \leq 24$ points $(0,6)$ $(8,0)$ soln (towards)

Scale

X-axis 1cm = 2 units

Y-axis 1cm = 2 units

(Bounded)



Corner points	value of objective function $Z = 200x + 500y$
A (4,3)	$Z = 800 + 1500 = 2300$
B (0,6)	$Z = 0 + 3000 = 3000$
C (0,5)	$Z = 0 + 2500 = 2500$

∴ Z is Minimum at (4,3) (optimal solution)

∴ Min value of Z is 2300 (optimal value)

Qu. 2 → Minimize $Z = x + 2y$

Subject to
$x + 2y \geq 100$
$2x - y \leq 0$
$2x + y \leq 200$
& $x, y \geq 0$

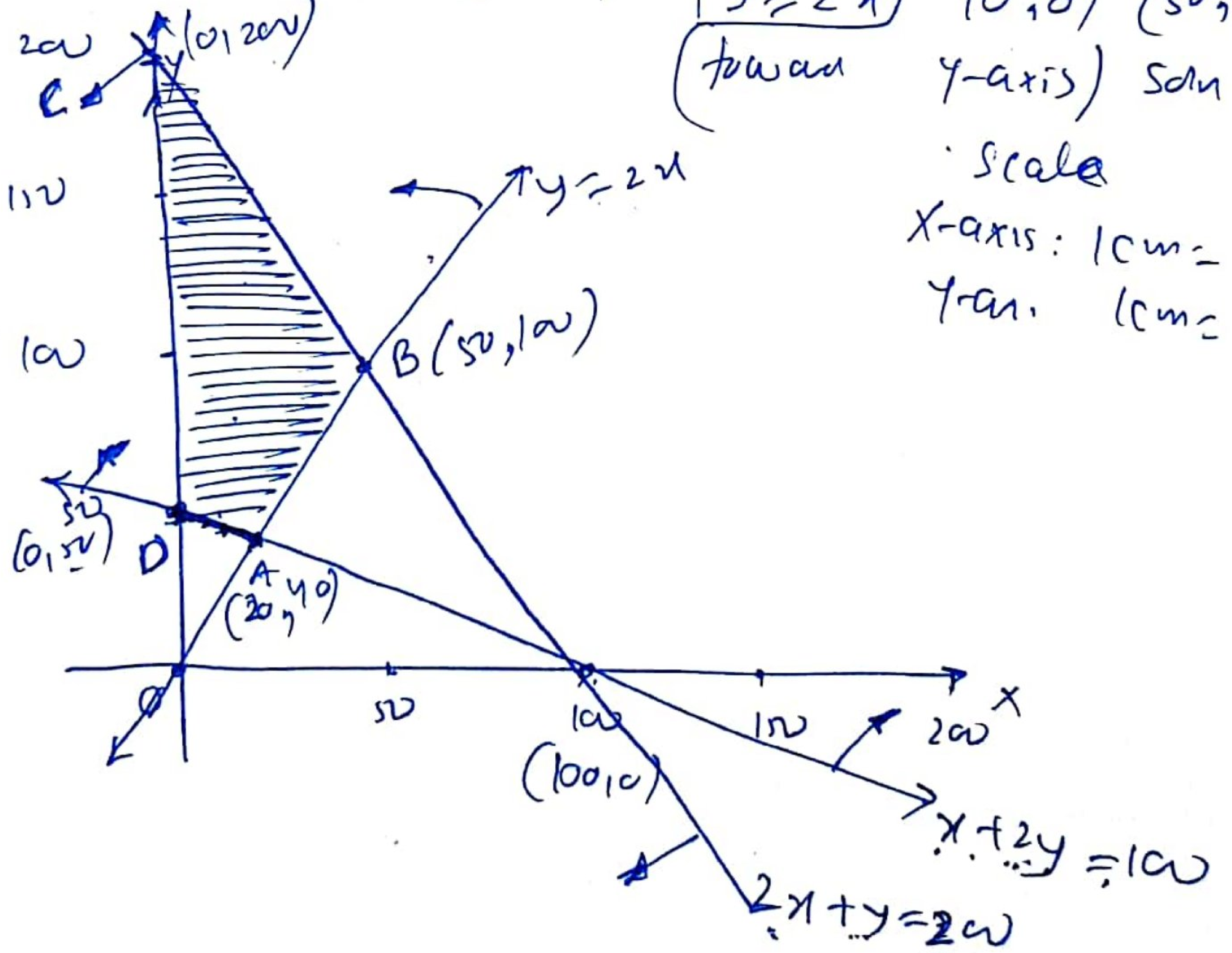
Soln

(i) $x + 2y \geq 100$; points $(0, 50)$, $(100, 0)$
soln $0 \geq 100$ (away)

(ii) $2x + y \leq 200$; pts. $(0, 200)$ & $(100, 0)$
soln $0 \leq 200$ (Towards)

(iii) $2x - y \leq 0 \Rightarrow$ $\boxed{y \geq 2x}$ $(0, 0)$ $(50, 100)$
(toward y-axis) soln

Scale
X-axis : 1cm = 50 units
Y-axis : 1cm = 50 units



Corner points	value of objective function $Z = x + 2y$
A (20, 40)	$Z = 20 + 80 = 100$
B (50, 100)	$Z = 50 + 200 = 250$
C (0, 200)	$Z = 0 + 400 = 400$
D (0, 50)	$Z = 0 + 100 = 100$

→ Minimum

∴ (Multiple Solution)

∴ Z is Minimum at all the points on the line joining A(20,40) & D(0,50)

& Min value of $Z = 100$ (optimal value)

(Infinite ~~optimal~~ optimal solution)

Que 3 Minimize

$$Z = -50x + 20y$$

Subject to Constraints

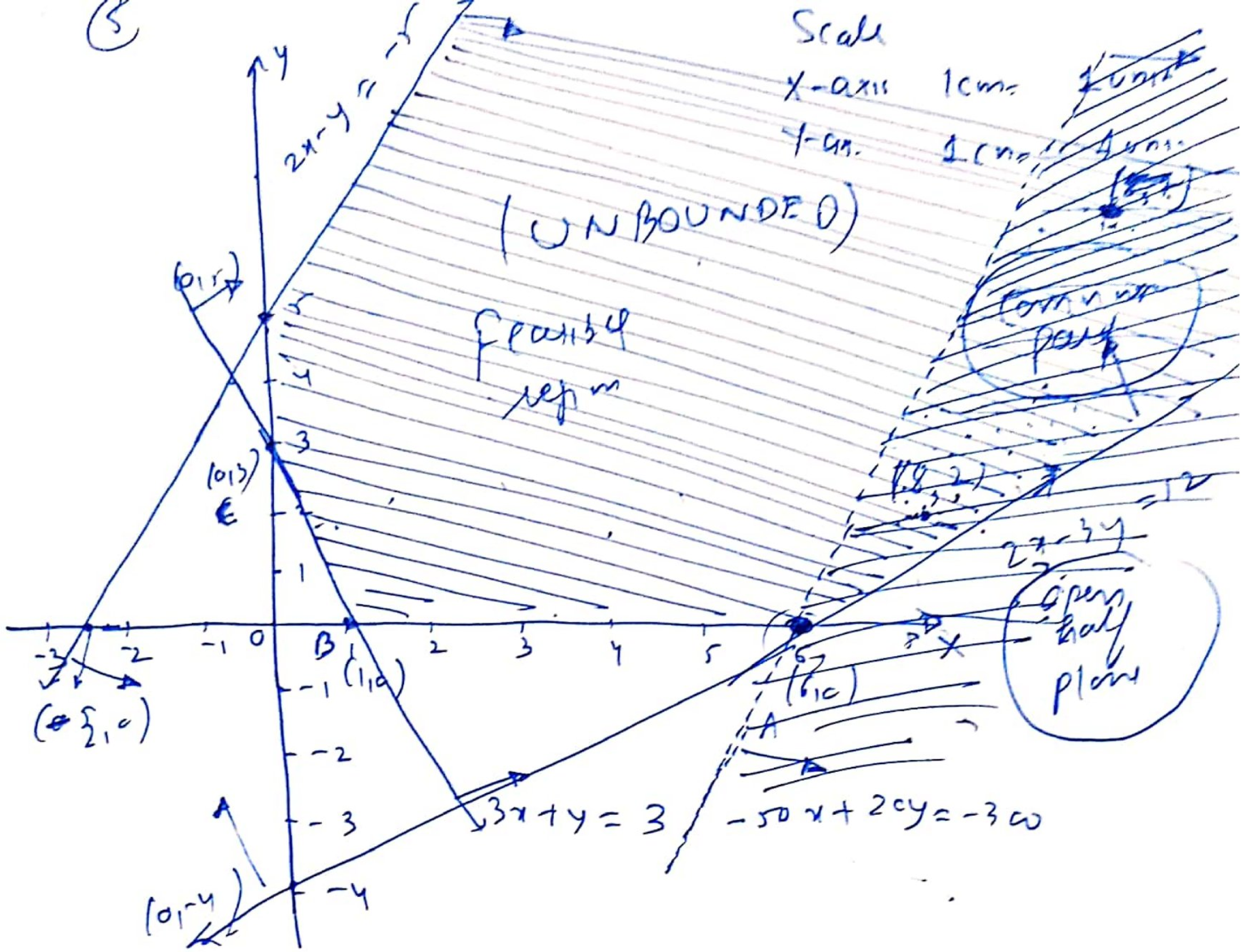
$2x - y \geq -5$; $3x + y \geq 3$ & $2x - 3y \leq 12$ & $x, y \geq 0$

Solⁿ: (1) $2x - y \geq -5$ points $(0, 5)$ $(-\frac{5}{2}, 0)$ solⁿ $0 \geq -5$ (towards)

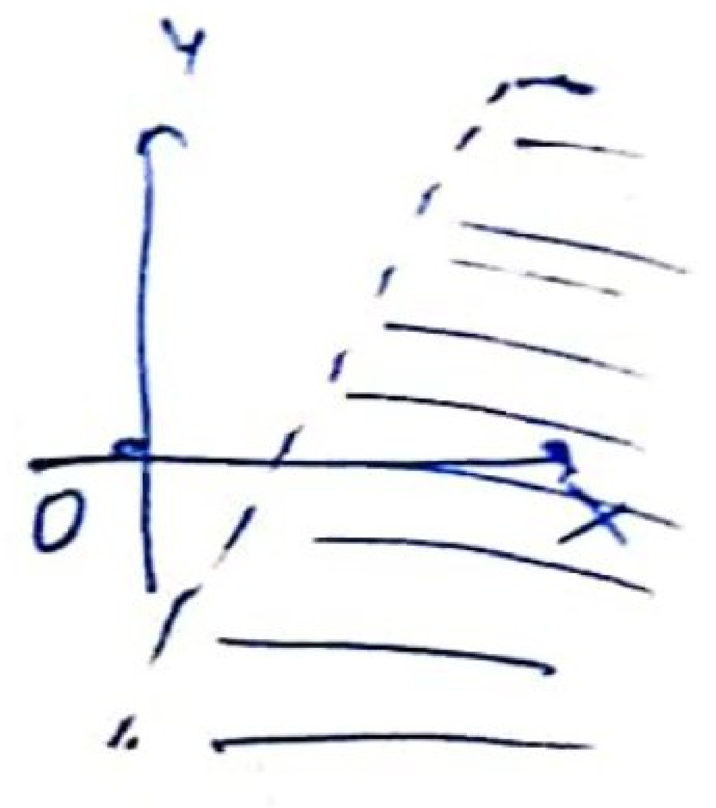
(2) $3x + y \geq 3$ points $(0, 3)$ $(1, 0)$ solⁿ $0 \geq 3$ (away)

(3) $2x - 3y \leq 12$ points $(0, -4)$ $(6, 0)$ solⁿ $0 \leq 12$ (Towards)

5



Corner point	value of objective function $Z = -50x + 20y$
A (6,0).	$Z = -300$
B (1,0)	$Z = -50$
C (0,3)	$Z = 60$
D (0,5)	$Z = 100$



Special.

$-50x + 20y < -300$

point (0, −15) (6, 0)

soln $0 < -300$ (away)

Since there is common region b/w feasible region & open half plane

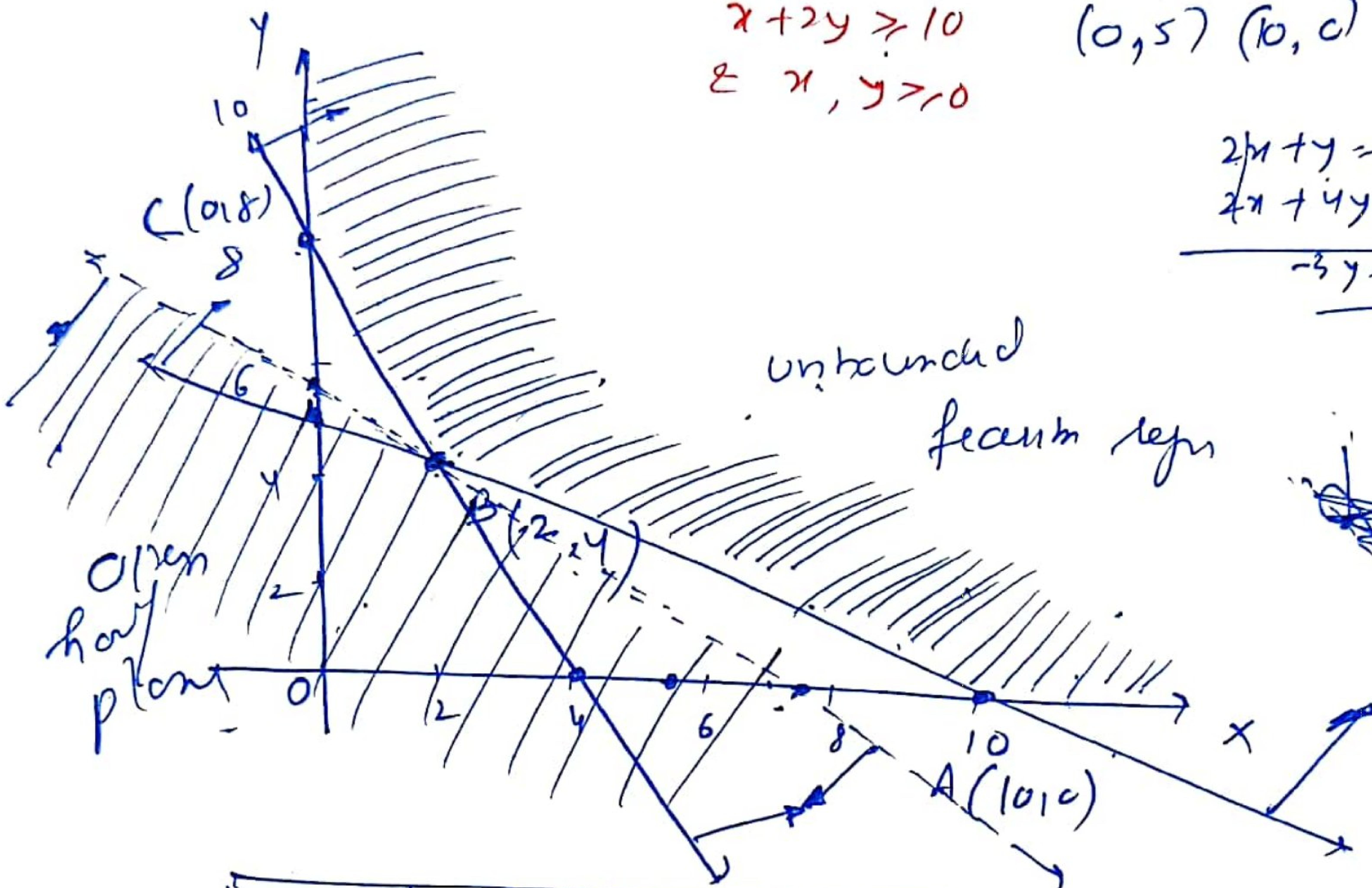
∴ Z cannot be minimized subject to given constraints

Qn-4 Minimize $Z = 50x + 70y$

Subject to constraints
$2x + y \geq 8$ (0,8) (4,0)
$x + 2y \geq 10$ (0,5) (10,0)
& $x, y \geq 0$

$2x + y = 8$
$2x + 4y = 20$

$-3y = -12$



	$Z = 50x + 70y$
A(10,0)	Z = 500
B(2,4)	Z = 100 + 280 = 380
C(0,8)	Z = 560

$50x + 70y < 380$
$(0, \frac{38}{7})$ & $(\frac{38}{5}, 0)$ (towards)
5.4 7.6

there is NO common points b/w open half plane & feasible region

$\therefore$ Z is Minimum at (2,4)

Min value of Z is 380 Ans

Qn. 5 Minimize

$$Z = 60x + 80y$$

Subject to constraints

$3x + 4y \geq 8$ $(0, 2)$ $(\frac{8}{3}, 0)$

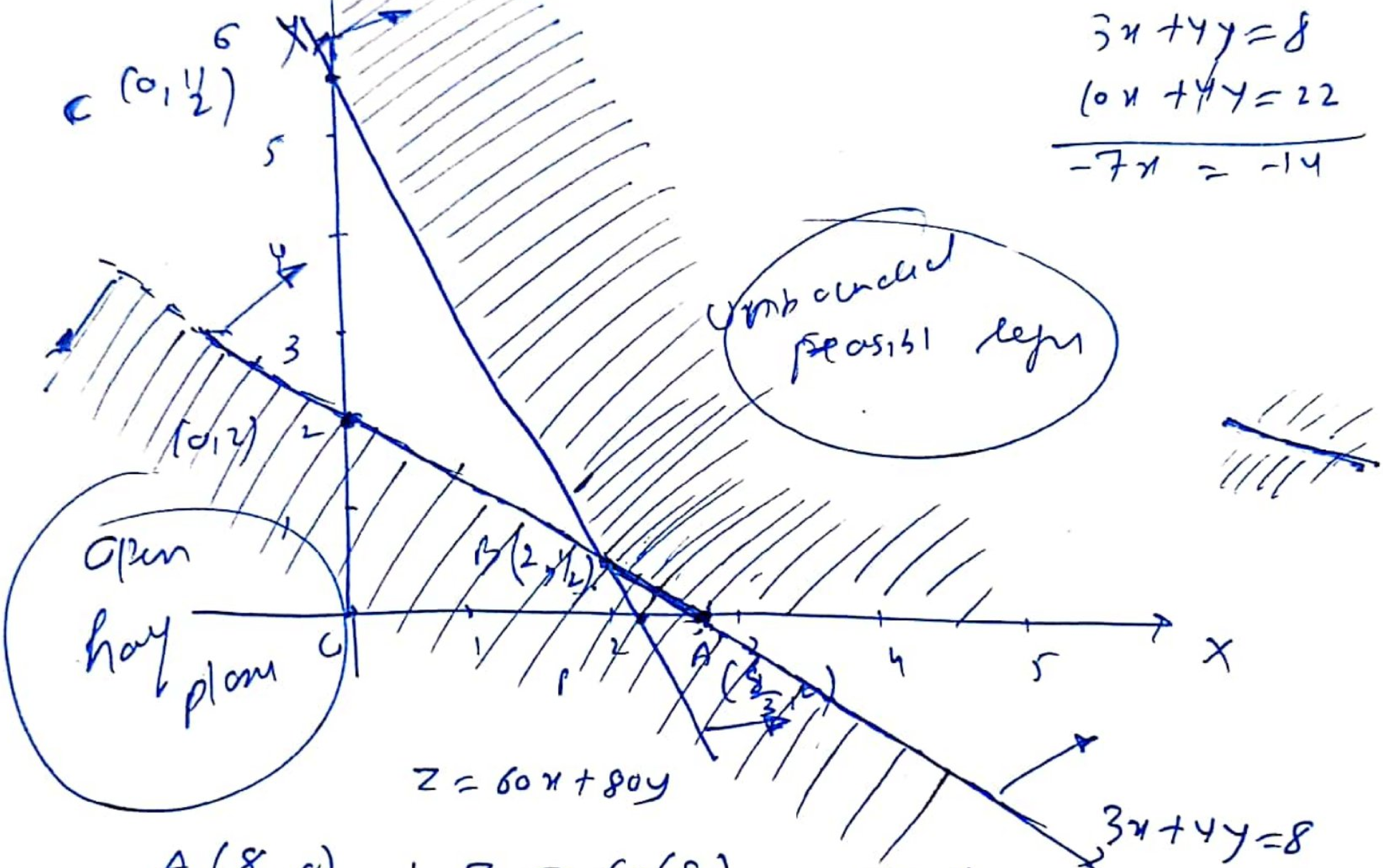
$5x + 2y \geq 11$ $(0, \frac{11}{2})$ $(\frac{11}{5}, 0)$

& $x, y \geq 0$

$3x + 4y = 8$

$10x + 4y = 22$

$-7x = -14$



$Z = 60x + 80y$

$A(\frac{8}{3}, 0)$; $Z = 60(\frac{8}{3}) = 160$

$B(2, \frac{1}{2})$ $Z = 120 + 40 = 160$ Min

$Z = (0, \frac{11}{2})$ $Z = 0 + 80(\frac{11}{2}) = 440$

$60x + 80y < 160$

$(0, 2)$ $(\frac{8}{3}, 0)$ Soln $0 < 160$ (towards)

No common region b/w open half plane & feasible region

$\therefore$ Z is Min at all the points on the line joining $A(\frac{8}{3}, 0)$ & $B(2, \frac{1}{2})$ & Min $Z = 160$ Ans

Qn. 6 given $Z = px + qy$

$2x + y \le 10$; $x + 3y \le 15$; $x, y \ge 0$

→ given corner points $(0,0)$, $(5,0)$, $(3,4)$ and $(0,5)$

Write the condition on p & q so that Maximum of Z occurs at both $(3,4)$ & $(0,5)$

Soln

$$Z = px + qy$$

at $(3,4)$ & $(0,5)$ value of Z is same

$Z = 3p + 4q$ and $Z = 5q$

$\rightarrow 3p + 4q = 5q$

$\Rightarrow \boxed{3p = q}$ Ans

Do only Exercise 12.1 with examples

WORKSHEET (LPP)

Q.No. 1 Maximise $Z = 4x + y$

such that $x + y \le 50$; $3x + y \le 90$ & $x, y \ge 0$

Ans $Z_{max} = 120$ at $(30, 0)$

Q.No. 2 Maximize & Minimize $Z = 3x + 9y$

such that $x + 3y \le 60$; $x + y \ge 10$; $x \le y$ & $x, y \ge 0$

Ans $Z_{max} = 180$ at all the points on the line joining $(15, 15)$ & $(0, 20)$ & $Z_{min} = 60$ at $(15, 5)$

Q.No. 3 → Minimise & Maximise ~~Z = 5x + 10y~~ $Z = 5x + 10y$

such that $x + 2y \le 120$; $x + y \ge 60$; $x - 2y \ge 0$ & $x, y \ge 0$

Ans Min $Z = 300$ at $(60, 0)$

Max $Z = 600$ at all the points on the line joining $(120, 0)$ & $(60, 30)$

Q.No. 4 Maximize $Z = -x + 2y$

such that $x \ge 3$; $x + y \ge 5$; $x + 2y \ge 6$, $y \ge 0$

Ans Z has no maximum value

Q.No. 5 Maximize $Z = x + y$

such that $x - y \le -1$; $-x + y \le 0$ & $x, y \ge 0$

Ans No feasible Region & hence No Max value —x—