Solutions **WORKSHEET No: 1** Chapter - 3-D

Ans. 1 let the vertices are $A(3,5,-4)$ $B(-1,1,2)$ $C(-5,-5,-2)$

D.R's of side AB are: $-1-3,\ 1-5,\ 2+4 = -4, -4, 6$

D.c's of side AB are $= \frac{-4}{\sqrt{16+16+36}}, \frac{-4}{\sqrt{16+16+36}}, \frac{6}{\sqrt{16+16+36}}$

$= \frac{-4}{\sqrt{68}}, \frac{-4}{\sqrt{68}}, \frac{6}{\sqrt{68}}$

$= -\frac{2}{\sqrt{17}}, \frac{-2}{\sqrt{17}}, \frac{3}{\sqrt{17}}$

D.R's of side BC are $= -5+1, -5-1, -2-2 = -4, -6, -4$

D.c's of side BC are $= \frac{-4}{\sqrt{68}}, \frac{-6}{\sqrt{68}}, \frac{-4}{\sqrt{68}}$

$= \frac{-2}{\sqrt{17}}, \frac{-3}{\sqrt{17}}, \frac{-2}{\sqrt{17}}$

D.R's of side CA are $= 3+5, 5+5, -4+2 = 8, 10, -2$

D.c's of side CA are $= \frac{8}{\sqrt{64+100+4}}, \frac{10}{\sqrt{64+100+4}}, \frac{-2}{\sqrt{64+100+4}}$

$= \frac{8}{\sqrt{168}}, \frac{10}{\sqrt{168}}, \frac{-2}{\sqrt{168}}$

$= \frac{4}{\sqrt{42}}, \frac{5}{\sqrt{42}}, -\frac{1}{\sqrt{42}}$ Ans

— x —

Ans 2 → Given $\alpha = 90°$, $\beta = 135°$, $\gamma = 45°$

$l = \cos\alpha = \cos(90°) = 0$

$m = \cos\beta = \cos(135°) = \cos(180-45°) = -1/\sqrt{2}$

$n = \cos\gamma = \cos(45°) = 1/\sqrt{2}$

$\therefore$ $\boxed{\text{D.c's are } 0, -1/\sqrt{2}, 1/\sqrt{2}}$ Ans.

Ans 3 → Given $\alpha = \beta = \gamma$

$\Rightarrow \cos\alpha = \cos\beta = \cos\gamma$

$\Rightarrow l = m = n$

we have, $l^2 + m^2 + n^2 = 1$

$\Rightarrow 3l^2 = 1$ ---- $\{\because l = m = n\}$

$\Rightarrow l = \pm \frac{1}{\sqrt{3}}$

$\therefore m = \pm \frac{1}{\sqrt{3}}$ & $n = \pm \frac{1}{\sqrt{3}}$

$\therefore$ $\boxed{\text{D.c's are } \pm \frac{1}{\sqrt{3}}, \pm \frac{1}{\sqrt{3}}, \pm \frac{1}{\sqrt{3}}}$ Ans

— x —

Ans 4 → Given points: $(-1, 0, 2)$ & $(3, 4, 6)$

let Position vector of these points are

$\vec{a} = -\hat{i} + 0\hat{j} + 2\hat{k}$ & $\vec{b} = 3\hat{i} + 4\hat{j} + 6\hat{k}$

Now, vector equation of line is given by

$\vec{r} = \vec{a} + \lambda(\vec{b} - \vec{a})$

$\Rightarrow \vec{r} = -\hat{i} + 2\hat{k} + \lambda\left((3\hat{i} + 4\hat{j} + 6\hat{k}) - (-\hat{i} + 2\hat{k})\right)$

$\Rightarrow \boxed{\vec{r} = -\hat{i} + 2\hat{k} + \lambda(4\hat{i} + 4\hat{j} + 4\hat{k})}$ Ans

— x —

Ans 5 → Given: equation of line

$$\frac{3x-1}{6} = \frac{3-2y}{8} = z - 1$$

(i) Standard form: $\frac{x - 1/3}{2} = \frac{y - 3/2}{-4} = \frac{z-1}{1}$

(2) fixed point on the line : $\left(\frac{1}{3}, \frac{3}{2}, 1\right)$

(3) D.R's of line : 2, −4, 1

(4) D.c's of line = $\frac{2}{\sqrt{4+16+1}}, \frac{-4}{\sqrt{4+16+1}}, \frac{1}{\sqrt{4+16+1}} = \frac{2}{\sqrt{21}}, \frac{-4}{\sqrt{21}}, \frac{1}{\sqrt{21}}$

(5) vector form of line : $\vec{r} = \left(\frac{1}{3}\hat{i} + \frac{3}{2}\hat{j} + \hat{k}\right) + \lambda\left(2\hat{i} - 4\hat{j} + \hat{k}\right)$

(6) Any point on the line

let $\frac{x-1/3}{2} = \frac{y-3/2}{-4} = \frac{z-1}{1} = \lambda$

$\Rightarrow \quad x = 2\lambda + 1/3 \;;\quad y = -4\lambda + 3/2 \;;\quad z = \lambda + 1$

∴ Any point on the line is $\left(2\lambda + \frac{1}{3}, -4\lambda + \frac{3}{2}, \lambda + 1\right)$

—x—

Ans 6 → given lines: $\frac{x+3}{3} = \frac{y-1}{5} = \frac{z+3}{4}$ and

$\frac{x+1}{1} = \frac{y-4}{1} = \frac{z-5}{2}$

here $a_1 = 3, \quad b_1 = 5, \quad c_1 = 4$

$a_2 = 1, \quad b_2 = 1, \quad c_2 = 2$

angle b/w two lines is given by

$$\cos\theta = \frac{|a_1a_2 + b_1b_2 + c_1c_2|}{\sqrt{a_1^2 + b_1^2 + c_1^2}\,\sqrt{a_2^2 + b_2^2 + c_2^2}}$$

$$\cos\theta = \frac{|3 + 5 + 8|}{\sqrt{9+25+16}\,\sqrt{1+1+4}}$$

$$\Rightarrow \cos\theta = \frac{16}{\sqrt{50}\,\sqrt{6}}$$

$$= \frac{16}{5\sqrt{2} \times \sqrt{2}\,\sqrt{3}}$$

$$= \frac{16}{10\sqrt{3}}$$

$$= \frac{8}{5\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

$$\cos\theta = \frac{8\sqrt{3}}{15}$$

$$\Rightarrow \boxed{\theta = \cos^{-1}\left(\frac{8\sqrt{3}}{15}\right)}$$ Ans

-x-

Ans 7 → given lines: $\vec{r} = (\hat{i}+\hat{j}) + \lambda(2\hat{i}-\hat{j}+\hat{k})$

and $\vec{r} = 2\hat{i}+\hat{j}-\hat{k} + \mu(3\hat{i}-5\hat{j}+2\hat{k})$

here, $\vec{a_1} = \hat{i}+\hat{j}$; $\vec{b_1} = 2\hat{i}-\hat{j}+\hat{k}$

$\vec{a_2} = 2\hat{i}+\hat{j}-\hat{k}$; $\vec{b_2} = 3\hat{i}-5\hat{j}+2\hat{k}$

$\vec{a_2}-\vec{a_1} = \hat{i}+0\hat{j}-\hat{k}$

$$\vec{b_1}\times\vec{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & 1 \\ 3 & -5 & 2 \end{vmatrix} = 3\hat{i}-\hat{j}-7\hat{k}$$

$|\vec{b_1}\times\vec{b_2}| = \sqrt{9+1+49} = \sqrt{59}$

Shortest distance b/w two lines is given by

$$\text{Distance} = \frac{|(\vec{a_2}-\vec{a_1})\cdot(\vec{b_1}\times\vec{b_2})|}{|\vec{b_1}\times\vec{b_2}|}$$

$$= \frac{|(\hat{i}+0\hat{j}-\hat{k})\cdot(3\hat{i}-\hat{j}-7\hat{k})|}{\sqrt{59}}$$

$$= \frac{|3+7|}{\sqrt{59}}$$

$\therefore$ $\boxed{\text{Distance} = \frac{10}{\sqrt{59}} \text{ units}}$ Ans

-x-

Ans 8 → Given lines:

$$\vec{r} = (1-\lambda)\hat{i} + (\lambda-2)\hat{j} + (3-2\lambda)\hat{k} \quad \text{and}$$

$$\vec{r} = (\mu+1)\hat{i} + (2\mu-1)\hat{j} - (2\mu+1)\hat{k}$$

rearranging these equations in standard form

$$\vec{r} = (\hat{i} - 2\hat{j} + 3\hat{k}) + \lambda(-\hat{i} + \hat{j} - 2\hat{k})$$

and $\vec{r} = (\hat{i} - \hat{j} - \hat{k}) + \mu(\hat{i} + 2\hat{j} - 2\hat{k})$

here $\vec{a_1} = \hat{i} - 2\hat{j} + 3\hat{k}$; $\vec{b_1} = -\hat{i} + \hat{j} - 2\hat{k}$

$\vec{a_2} = \hat{i} - \hat{j} - \hat{k}$; $\vec{b_2} = \hat{i} + 2\hat{j} - 2\hat{k}$

$$\vec{a_2} - \vec{a_1} = 0\hat{i} + \hat{j} - 4\hat{k}$$

$$\vec{b_1} \times \vec{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 1 & -2 \\ 1 & 2 & -2 \end{vmatrix} = 2\hat{i} - 4\hat{j} - 3\hat{k}$$

$$|\vec{b_1} \times \vec{b_2}| = \sqrt{4+16+9} = \sqrt{29}$$

$$\text{Distance} = \frac{|(\vec{a_2} - \vec{a_1}) \cdot (\vec{b_1} \times \vec{b_2})|}{|\vec{b_1} \times \vec{b_2}|}$$

$$= \frac{|(\hat{j} - 4\hat{k}) \cdot (2\hat{i} - 4\hat{j} - 3\hat{k})|}{\sqrt{29}}$$

$$= \frac{|0-4+12|}{\sqrt{29}}$$

$\therefore$ $\boxed{\text{Distance} = \frac{8}{\sqrt{29}} \text{ units}}$ Ans

-x-

Q. 10 → given equations.

$$\frac{1-x}{3} = \frac{7y-14}{2p} = \frac{z-3}{2} \text{ and } \frac{7-7x}{3p} = \frac{y-5}{1} = \frac{6-z}{5}$$

converting in to standard form

$$\frac{x-1}{-3} = \frac{y-2}{\frac{2p}{7}} = \frac{z-3}{2} \text{ and } \frac{x-1}{\frac{-3p}{7}} = \frac{y-5}{1} = \frac{z-6}{-5}$$

here

$a_1 = -3, \quad b_1 = \frac{2p}{7}, \quad c_1 = 2$

$a_2 = \frac{-3p}{7}, \quad b_2 = 1, \quad c_2 = -5$

Since lines are perpendicular

$\therefore a_1 a_2 + b_1 b_2 + c_1 c_2 = 0$

$\Rightarrow \frac{9p}{7} + \frac{2p}{7} - 10 = 0$

$\Rightarrow 11p - 70 = 0$

$\Rightarrow \boxed{p = \frac{70}{11}}$ Ans

-x-

Ans 11 → Points on Ist line: $(1,-1,2)$ & $(3,4,-2)$

D.R's of this line: $a_1 = 3-1,\ \underbrace{4+1}_{b_1},\ \underbrace{-2-2}_{c_1}$

$\Rightarrow a_1 = 2;\ b_1 = 5,\ c_1 = -4$

Points on 2nd line: $(0,3,2)$ & $(3,5,6)$

D.R's of this line: $a_2 = 3-0;\ b_2 = 5-3,\ c_2 = 6-2$

$a_2 = 3,\ b_2 = 2,\ c_2 = 4$

Now $a_1 a_2 + b_1 b_2 + c_1 c_2 = 6 + 10 - 16 = 0$

$\boxed{\text{Hence, given lines are perpendicular}}$ Ans

—x—

Ans 12 → D.R's of Ist line

$a_1 = 2-4,\quad b_1 = 3-k,\quad c_1 = 4-8$

$a_1 = -2,\quad b_1 = 3-k,\quad c_1 = -4$

D.R's of 2nd line:

$a_2 = 1+1,\quad b_2 = 2+2,\quad c_2 = 5-1$

$a_2 = 2,\quad b_2 = 4,\quad c_2 = 4$

given that lines are parallel

$\therefore \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} \Rightarrow \frac{-2}{2} = \frac{3-k}{4} = \frac{-4}{4}$

$\Rightarrow \frac{3-k}{4} = -1 \Rightarrow \boxed{k = 7}$ Ans