Solutions

DEC

Solutions of worksheet : 1

Diff and Continuity

1→ $y = (\log x)^x + x^{\log x}$

$y = u + v$

Diff wrt x

$$\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx} \quad \text{----(1)}$$

$u = (\log x)^x$

taking log

$\log u = x \log(\log x)$

Diff wrt x

$$\frac{1}{u} \cdot \frac{du}{dx} = x \cdot \frac{1}{\log x} \cdot \frac{1}{x} + \log(\log x)$$

$$\frac{du}{dx} = (\log x)^x \left[\frac{1}{\log x} + \log(\log x) \right]$$

$$\frac{du}{dx} = \frac{(\log x)^x}{\log x} \left[1 + \log x \cdot \log(\log x) \right]$$

$v = x^{\log x}$

taking log

$\log v = \log x \cdot \log x$

Diff wrt x

$$\frac{1}{v} \cdot \frac{dv}{dx} = \log x \cdot \frac{1}{x} + \log x \cdot \frac{1}{x}$$

$$\frac{dv}{dx} = x^{\log x} \cdot \frac{2 \log x}{x}$$

$$\frac{dv}{dx} = 2x^{\log x - 1} \cdot \log x$$

$$\frac{dy}{dx} = \frac{(\log x)^x}{\log x} \left[1 + \log x \cdot \log(\log x) \right] + 2x^{\log x - 1} \cdot \log x$$

Ans..

D & C Solution

Worksheet 1

Ans 2 → Please do yourself (easy hai)

Ans 3 → $y = (x\cos x)^x + (x\sin x)^{1/x}$

$y = u + v$

Diff wrt x

$$\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx} \quad \text{--- (1)}$$

$u = (x\cos x)^x$

taking log

$$\log u = x \log(x\cos x)$$

Diff wrt x

$$\frac{1}{u}\cdot\frac{du}{dx} = \cancel{x}\cdot\frac{1}{\cancel{x}\cos x}\cdot(-x\sin x + \cos x) + \log(x\cos x)\cdot$$

$$\frac{du}{dx} = (x\cos x)^x\left[(-x\tan x + 1) + \log(x\cos x)\right]$$

$v = (x\sin x)^{1/x}$

taking log

$$\log v = \frac{1}{x}\log(x\sin x)$$

Diff wrt x

$$\frac{1}{v}\cdot\frac{dv}{dx} = \frac{1}{x}\left[\frac{1}{x\sin x}(x\cos x + \sin x)\right] + \log(x\sin x)\left(\frac{-1}{x^2}\right)$$

$$= \frac{x\cot x + 1}{x^2} + \log(x\sin x)\cdot\left(\frac{-1}{x^2}\right)$$

$$\frac{dv}{dx} = (x\sin x)^{1/x}\left[\frac{x\cot x + 1 - \log(x\sin x)}{x^2}\right]$$

from equation (1)

$$\therefore \frac{dy}{dx} = (x\cos x)^x\left[1 - x\tan x + \log(x\cos x)\right] + (x\sin x)^{\frac{1}{x}}\left[\frac{x\cot x + 1 - \log(x\sin x)}{x^2}\right]$$

Ans

Ans 4 → $y = \left(x+\frac{1}{x}\right)^x + x^{1+\frac{1}{x}}$

Solution let $u = \left(x+\frac{1}{x}\right)^x$ & $v = x^{1+\frac{1}{x}}$

$\Rightarrow y = u + v$

Diff both sides w.r.t x

$$\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx} \quad \text{---- (i)}$$

Consider $u = \left(x+\frac{1}{x}\right)^x$

taking log on both sides

$$\Rightarrow \log u = x\log\left(x+\frac{1}{x}\right)$$

Diff w.r.t x

$$\frac{1}{u}\cdot\frac{du}{dx} = x\cdot\frac{1}{x+\frac{1}{x}}\cdot\left(1-\frac{1}{x^2}\right) + \log\left(x+\frac{1}{x}\right)\cdot 1$$

$$\frac{du}{dx} = \left(x+\frac{1}{x}\right)^x\left[\frac{\cancel{x^2}}{x^2+1}\cdot\left(\frac{x^2-1}{\cancel{x^2}}\right) + \log\left(x+\frac{1}{x}\right)\right]$$

$$\frac{du}{dx} = \left(x+\frac{1}{x}\right)^x\left[\frac{x^2-1}{x^2+1} + \log\left(x+\frac{1}{x}\right)\right]$$

Now consider $v = (x)^{1+\frac{1}{x}}$

taking log on both sides

$$\log v = \left(1+\frac{1}{x}\right) \cdot \log x$$

Diff w.r.t x

$$\frac{1}{v} \cdot \frac{dv}{dx} = \left(1+\frac{1}{x}\right) \cdot \frac{1}{x} + \log x \cdot \left(\frac{-1}{x^2}\right)$$

$$\frac{dv}{dx} = (x)^{1+\frac{1}{x}} \left[\frac{x+1}{x^2} - \frac{\log x}{x^2} \right]$$

$$\frac{dv}{dx} = (x)^{1+\frac{1}{x}} \left[\frac{x+1-\log x}{x^2} \right]$$

put in eq (i)

$$\frac{dy}{dx} = \left(x+\frac{1}{x}\right)^x \left[\frac{x^2-1}{x^2+1} + \log\left(x+\frac{1}{x}\right)\right] + x^{1+\frac{1}{x}} \left[\frac{x+1-\log x}{x^2}\right]$$

Ans

Ques: 5 → $y = x^{x\cos x} + \frac{x^2+1}{x^2-1}$. Find $\frac{dy}{dx}$

Solution let $u = x^{x\cos x}$ & $v = \frac{x^2+1}{x^2-1}$

$\therefore y = u + v$

Diff w.r.t x

$$\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx} \quad \text{---(i)}$$

Consider $u = x^{x\cos x}$

taking log on both sides

$$\log u = (x\cos x) \cdot \log x$$

Diff w.r.t x

$$\frac{1}{y}\cdot\frac{dy}{dx} = (\cancel{x}\cos x)\cdot\frac{1}{\cancel{x}} + \log x\cdot(-x\sin x + \cos x)$$

$$\frac{dy}{dx} = x^{x\cos x}\left[\cos x - x\sin x\cdot\log x + \log x\cdot\cos x\right)$$

common

$$\frac{dy}{dx} = x^{x\cos x}\left(\cos x\cdot(1+\log x) - x\sin x\cdot\log x\right)$$

Consider $V = \frac{x^2+1}{x^2-1}$

(No need to take log)
Quotient Rule lagao

Diff wrt x

$$\frac{dV}{dx} = \frac{(x^2-1)\cdot(2x) - (x^2+1)\cdot(2x)}{(x^2-1)^2}$$

$$= \frac{2x(\cancel{x^2}-1-\cancel{x^2}-1)}{(x^2-1)^2}$$

$$\frac{dV}{dx} = \frac{-4x}{(x^2-1)^2}$$

put in equation (1)

$$\therefore \frac{dy}{dx} = x^{x\cos x}\left[\cos x\cdot(1+\log x) - x\sin x\cdot\log x\right] - \frac{4x}{(x^2-1)^2}$$

Ans

Q. 6 → $y = (x+3)^2\cdot(x+4)^3\cdot(x+5)^4$

Sol^n taking log on both sides

DEC Solution Worksheet 1

$\log y = \log(x+3)^2 + \log(x+4)^3 + \log(x+5)^4$

$\log y = 2\log(x+3) + 3\log(x+4) + 4\log(x+5)$

Diff wrt x

$\frac{1}{y} \cdot \frac{dy}{dx} = 2 \cdot \frac{1}{x+3} + \frac{3}{x+4} + \frac{4}{x+5}$

$\frac{dy}{dx} = y\left(\frac{2}{x+3} + \frac{3}{x+4} + \frac{4}{x+5}\right)$

$\frac{dy}{dx} = (x+3)^2(x+4)^3 \cdot (x+5)^4 \cdot \left[\frac{2}{x+3} + \frac{3}{x+4} + \frac{4}{x+5}\right]$

L.C.M

$= (x+3)^2(x+4)^3(x+5)^4\left[\frac{2(x+4)(x+5) + 3(x+3)(x+5) + 4(x+3)(x+4)}{(x+3)(x+4)(x+5)}\right]$

$= (x+3)(x+4)^2(x+5)^3$ (open the Bracket)

$\frac{dy}{dx} = (x+3)(x+4)^2(x+5)^3 \cdot (9x^2 + 70x + 133)$ Ans

---

Ques 7 → $y = \cos x \cdot \cos(2x) \cdot \cos(3x)$

taking log on both sides

$\log y = \log(\cos x) + \log(\cos(2x)) + \log(\cos(3x))$

Diff wrt x

$$\frac{1}{y}\cdot\frac{dy}{dx} = \frac{1}{\cos x}(-\sin x) + \frac{1}{\cos(2x)}(-2\sin(2x)) + \frac{1}{\cos(3x)}(-3\sin(3x))\Big]$$

$$\frac{dy}{dx} = y\left[-\tan x - 2\tan(2x) - 3\tan(3x)\right)$$

$$\frac{dy}{dx} = -\cos x\cdot\cos(2x)\cdot\cos(3x)\left(\tan x + 2\tan(2x) + 3\tan(3x)\right)$$

Ans.

Ans 8 → $(\cos x)^y = (\cos y)^x$

taking log on both sides

$y\log(\cos x) = x\log(\cos y)$

Diff wrt x

$$\Rightarrow y\cdot\frac{1}{\cos x}(-\sin x) + \log(\cos x)\cdot\frac{dy}{dx} = x\cdot\frac{1}{\cos y}(-\sin y)\cdot\frac{dy}{dx} + \log(\cos y)\cdot 1$$

$$\Rightarrow -y\tan x + \log(\cos x)\frac{dy}{dx} = -x\tan y\frac{dy}{dx} + \log(\cos y)$$

$$\Rightarrow \frac{dy}{dx}\left(\log(\cos x) + x\tan y\right) = \log(\cos y) + y\tan x$$

$$\Rightarrow \frac{dy}{dx} = \frac{\log(\cos y) + y\tan x}{\log(\cos x) + x\tan y}$$ Ans

Solution

DEC worksheet 1

Qn 9 → $x^y = y^x$ (do yourself easy hai)

self

Qn 10 → $y = x^{x^x}$

(already done in Notes / lecture)

Qn 11 → $y^x + x^y = 1$

Solution let $y^x = u$ and $x^y = v$

$\therefore u + v = 1$

Diff wrt x

$$\frac{du}{dx} + \frac{dv}{dx} = 0 \quad ---(1)$$

Consider $u = y^x$

taking log on both sides

$$\log u = x \log y$$

Diff wrt x

$$\frac{1}{u} \cdot \frac{du}{dx} = x \cdot \frac{1}{y} \frac{dy}{dx} + \log y \cdot 1$$

$$\frac{du}{dx} = y^x \left[\frac{x}{y} \frac{dy}{dx} + \log y \right]$$

Consider $v = x^y$

taking log on both sides

$$\log v = y \log x$$

Diff wrt x

Page No.
Date :

$$\frac{1}{v}\cdot\frac{dv}{dx} = y\cdot\frac{1}{x} + \log x\cdot\frac{dy}{dx}$$

$$\Rightarrow \frac{dv}{dx} = x^y\left[\frac{y}{x} + \log x\cdot\frac{dy}{dx}\right]$$

Put in equation (i)

$$\Rightarrow y^x\left[\frac{x}{y}\frac{dy}{dx} + \log y\right] + x^y\left(\frac{y}{x} + \log x\cdot\frac{dy}{dx}\right) = 0$$

$$\Rightarrow \frac{dy}{dx}\left(y^x\cdot\frac{x}{y} + x^y\cdot\log x\right) = -y^x\cdot\log y - x^y\cdot\frac{y}{x}$$

$$\Rightarrow \frac{dy}{dx} = -\frac{\left(y^x\cdot\log y + x^{y-1}\cdot y\right)}{y^{x-1}\cdot x + x^y\log y}$$ Ans

---

QN. 12 → $x^y + y^x + x^x = a^b$

Solⁿ let $x^y = u$; $y^x = v$ and $x^x = w$

$\Rightarrow u + v + w = a^b$ → constant ka derivative $= 0$

Diff w.r.t x

$$\frac{du}{dx} + \frac{dv}{dx} + \frac{dw}{dx} = 0 \quad ---(i)$$

$u = x^y$

$\log u = y\log x$

Diff

$$\frac{1}{u}\cdot\frac{du}{dx} = y\cdot\frac{1}{x} + \log x\cdot\frac{dy}{dx}$$

$$\frac{du}{dx} = x^y\left[\frac{y}{x} + \log x \cdot \frac{dy}{dx}\right]$$

Consider $v = y^x$

$\log v = x \log y$

Diff wrt x

$$\frac{1}{v} \cdot \frac{dv}{dx} = x \cdot \frac{1}{y} \cdot \frac{dy}{dx} + \log y \cdot 1$$

$$\frac{dv}{dx} = y^x\left(\frac{x}{y}\frac{dy}{dx} + \log y\right)$$

Consider $w = x^x$

$\log w = x \log x$

Diff wrt x

$$\frac{1}{w}\frac{dw}{dx} = x\frac{1}{x} + \log x \cdot 1$$

$$\frac{dw}{dx} = x^x(1 + \log x)$$

put all in equation (i)

$$x^y\left[\frac{y}{x} + \log x \cdot \frac{dy}{dx}\right] + y^x\left[\frac{x}{y} \cdot \frac{dy}{dx} + \log y\right] + x^x(1 + \log x) = 0$$

$$\Rightarrow \left(x^{y-1} \cdot y + x^y \log x \frac{dy}{dx}\right) + \left(y^{x-1} \cdot x \frac{dy}{dx} + y^x \log y\right) + x^x(1 + \log x) = 0$$

take $\frac{dy}{dx}$ common

D & C Worksheet 1 (Solution)

$$\frac{dy}{dx}\left(x^y \log x + x \cdot y^{x-1}\right) = -y^x \log y - y x^{y-1} - x^x(1+\log x)$$

$$\Rightarrow \frac{dy}{dx} = \frac{-\left[y^x \log y + y x^{y-1} + x^x(1+\log x)\right]}{x^y \log x + x y^{x-1}}$$ Ans

---

Ques 13 → $xy = e^{x-y}$

Soln taking log on both sides

$$\log(xy) = \log(e^{x-y})$$

$$\Rightarrow \log x + \log y = (x-y) \quad \cdots \{\log e = 1\}$$

Diff wrt x

$$\frac{1}{x} + \frac{1}{y}\cdot\frac{dy}{dx} = 1 - \frac{dy}{dx}$$

$$\Rightarrow \frac{dy}{dx}\left(\frac{1}{y}+1\right) = 1 - \frac{1}{x}$$

$$\Rightarrow \frac{dy}{dx}\left(\frac{1+y}{y}\right) = \frac{x-1}{x}$$

$$\Rightarrow \frac{dy}{dx} = \frac{y(x-1)}{x(y+1)}$$ Ans

Ques 14 → $y = (\sin x)^x + \sin^{-1}\sqrt{x}$

Soln let $u = (\sin x)^x$ & $v = \sin^{-1}\sqrt{x}$

DEC

Worksheet 1
(SOLUTION)

$\therefore y = u + v$

Diff wrt x

$$\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx} \quad \text{---(i)}$$

Consider $u = (\sin x)^x$

$$\log u = x \log(\sin x)$$

Diff

$$\frac{1}{u} \cdot \frac{du}{dx} = x \cdot \frac{1}{\sin x}(\cos x) + \log(\sin x) \cdot 1$$

$$\Rightarrow \frac{du}{dx} = (\sin x)^x \left(x \cot x + \log(\sin x) \right)$$

Consider $v = \sin^{-1}\sqrt{x}$

(Direct differentiate)

$$\frac{dv}{dx} = \frac{1}{\sqrt{1-(\sqrt{x})^2}} \cdot \frac{1}{2\sqrt{x}}$$

$$= \frac{1}{\sqrt{1-x}} \frac{1}{2\sqrt{x}}$$

$$\frac{dv}{dx} = \frac{1}{2\sqrt{x-x^2}}$$

$$\therefore \frac{dy}{dx} = (\sin x)^x \left[x \cot x + \log(\sin x) \right] + \frac{1}{2\sqrt{x-x^2}}$$

Ans

Qns 15 → $f(x) = (1+x)(1+x^2)(1+x^4)(1+x^8)$

Soln Taking log on both sides

DEC

Worksheet 1

(SOLUTION)

$$\log(f(x)) = \log(1+x) + \log(1+x^2) + \log(1+x^4) + \log(1+x^8)$$

diff wrt x

$$\frac{1}{f(x)} \cdot f'(x) = \frac{1}{1+x} + \frac{1}{1+x^2}(2x) + \frac{1}{1+x^4}(4x^3) + \frac{1}{1+x^8}\cdot(8x^7)$$

$$f'(x) = f(x)\left[\frac{1}{1+x} + \frac{2x}{1+x^2} + \frac{4x^3}{1+x^4} + \frac{8x^7}{1+x^8}\right]$$

$$= (1+x)(1+x^2)(1+x^4)(1+x^8)\left[\frac{1}{1+x} + \frac{2x}{1+x^2} + \frac{4x^3}{1+x^4} + \frac{8x^7}{1+x^8}\right]$$

for $f'(1)$ put $x = 1$

$$f'(1) = (2)(2)(2)(2)\left(\frac{1}{2} + \frac{2}{2} + \frac{4}{2} + \frac{8}{2}\right)$$

$$= 16\left(\frac{1}{2} + 1 + 2 + 4\right)$$

$$= 16\left(\frac{1+2+4+8}{2}\right)$$

$$= 8 \times 15$$

$$f'(1) = 120$$ Ans