← ULTIMATE MATHEMATICS →

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INVERSE TRIGO CLASS (I-5)

Q.No 1 Simplify

$\sin^{-1}(2x\sqrt{1-x^2})$; $-\frac{1}{\sqrt{2}} \le x \le \frac{1}{\sqrt{2}}$

put $x = \sin\theta$

$\sin^{-1}(2\sin\theta\sqrt{1-\sin^2\theta})$

$= \sin^{-1}(2\sin\theta\cos\theta)$

$= \sin^{-1}(\sin(2\theta))$

$= 2\theta$

$= 2\sin^{-1}x$ Ans

$-\frac{1}{\sqrt{2}} \le x \le \frac{1}{\sqrt{2}}$

$-\frac{1}{\sqrt{2}} \le \sin\theta \le \frac{1}{\sqrt{2}}$

$-\frac{\pi}{4} \le \theta \le \frac{\pi}{4}$

$-\frac{\pi}{2} \le 2\theta \le \frac{\pi}{2}$

(within Range)

Q.No 2 Simplify $\sin^{-1}(2x\sqrt{1-x^2})$; $\frac{1}{\sqrt{2}} \le x \le 1$

✗ put $x = \sin\theta$

$= \sin^{-1}(2\sin\theta\cos\theta)$

$= \sin^{-1}(\sin(2\theta))$

$\frac{1}{\sqrt{2}} \le x \le 1$

$\frac{1}{\sqrt{2}} \le \sin\theta \le 1$

$\frac{\pi}{4} \le \theta \le \frac{\pi}{2}$ ✗

$\frac{\pi}{2} \le 2\theta \le \pi$

(out of Range)

✓ put $x = \cos\theta$

$= \sin^{-1}(2\cos\theta\sqrt{1-\cos^2\theta})$

$= \sin^{-1}(2\cos\theta\sin\theta)$

$= \sin^{-1}(\sin(2\theta))$

CLASS . I-5

(2)

Page No.

Date :

$\frac{1}{\sqrt{2}} \le x \le 1$

$= 2\theta$

$\frac{1}{\sqrt{2}} \le \cos\theta \le 1$

$= 2\cos^{-1}x$

$0 \le \theta \le \frac{\pi}{4}$

$0 \le 2\theta \le \frac{\pi}{2}$

(Within Range of $\sin^{-1}x$)

Qn 3 Simplify

$$\tan^{-1}\left(\sqrt{\frac{a-x}{a+x}}\right) \; ; \; -a < x < a$$

put $x = a\cos(2\theta)$

$$= \tan^{-1}\left(\sqrt{\frac{a-a\cos(2\theta)}{a+a\cos(2\theta)}}\right)$$

$$= \tan^{-1}\sqrt{\frac{1-\cos(2\theta)}{1+\cos(2\theta)}}$$

$$|x| = \begin{cases} x ; & x \ge 0 \\ -x & x < 0 \end{cases}$$

$$= \tan^{-1}\sqrt{\frac{\cancel{2}\sin^2\theta}{\cancel{2}\cos^2\theta}}$$

$$= \tan^{-1}\sqrt{\tan^2\theta}$$

$$\boxed{\sqrt{x^2} = |x|}$$

$$\cancel{=} \tan^{-1}(\tan\theta) \; \times$$

$$= \tan^{-1}|\tan\theta|$$

$$= \tan^{-1}(\tan\theta)$$

$$= \theta$$

$$= \frac{1}{2}\cos^{-1}(x/a) \quad \underline{\text{Ans}}$$

$-a < x < a$

$-a < a\cos(2\theta) < a$

$-1 < \cos(2\theta) < 1$

$0 < 2\theta < \pi$

$0 < \theta < \frac{\pi}{2}$ — within Range

I st quad

Q4 → Simplify $\tan^{-1}\left(\dfrac{3x-x^3}{1-3x^2}\right)$; $-\infty < x < \dfrac{-1}{\sqrt{3}}$

put $x = \tan\theta$

$\tan^{-1}\left(\dfrac{3\tan\theta - \tan^3\theta}{1-3\tan^2\theta}\right)$

$= \tan^{-1}(\tan(3\theta))$

special

$= \tan^{-1}(\tan(\pi + 3\theta))$

$= \pi + 3\theta$

$= \pi + 3\tan^{-1}x$

$-\infty < x < -\dfrac{1}{\sqrt{3}}$

$-\infty < \tan\theta < -\dfrac{1}{\sqrt{3}}$

$-\dfrac{\pi}{2} < \theta < -\dfrac{\pi}{6}$

$-\dfrac{3\pi}{2} < 3\theta < -\dfrac{\pi}{2}$

(out of range)

adding π

$-\dfrac{\pi}{2} < (\pi + 3\theta) < \dfrac{\pi}{2}$

Within Range

Q5 → Show that $\sec^2(\tan^{-1}2) + \csc^2(\cot^{-1}3) = 15$

$(\sec(\tan^{-1}2))^2 + (\csc(\cot^{-1}3))^2$

$=$ $P=2,\ B=1$ $\quad$ $B=3,\ P=1$

$H=\sqrt{4+1} = \sqrt{5}$ $\quad$ $H=\sqrt{9+1}=\sqrt{10}$

$= [\sec(\sec^{-1}(\sqrt{5}))]^2 + (\csc(\csc^{-1}(\sqrt{10})))^2$

$= (\sqrt{5})^2 + (\sqrt{10})^2$

$= 5 + 10 = 15$

CLASS I-5

4

Page No.

Date :

Qn 6 → Simplify

$$\sin^{-1}\left(\frac{\sqrt{1+x}+\sqrt{1-x}}{2}\right)$$

put $x = \cos(2\theta)$

$$= \sin^{-1}\left(\frac{\sqrt{1+\cos(2\theta)}+\sqrt{1-\cos(2\theta)}}{2}\right)$$

$$= \sin^{-1}\left(\frac{\sqrt{2\cos^2\theta}+\sqrt{2\sin^2\theta}}{2}\right)$$

$$= \sin^{-1}\left(\frac{\sqrt{2}\cos\theta+\sqrt{2}\sin\theta}{2}\right)$$

$$= \sin^{-1}\left(\frac{1}{\sqrt{2}}\cos\theta+\frac{1}{\sqrt{2}}\sin\theta\right)$$

$$= \sin^{-1}\left(\sin\frac{\pi}{4}\cos\theta+\cos\frac{\pi}{4}\sin\theta\right)$$

$$= \sin^{-1}\left(\sin\left(\frac{\pi}{4}+\theta\right)\right)$$

$$= \frac{\pi}{4}+\theta$$

$$= \frac{\pi}{4}+\frac{1}{2}\cos^{-1}x \quad \text{Ans}$$

~~Qn 7~~ ~~$\cos^{-1}(3\sqrt{1-x^2}+4\sqrt{\ }$~~

Q. 7 Simplify $\cos^{-1}\left(\frac{3\sqrt{1-x^2}+4x}{5}\right)$

put $x = \sin\theta$

CLASS I-5

$$= \cos^{-1}\left(\frac{3\sqrt{1-\sin^2\theta} + 4\sin\theta}{5}\right)$$

$$= \cos^{-1}\left(\frac{3\cos\theta + 4\sin\theta}{5}\right)$$

$$= \cos^{-1}\left(\frac{3}{5}\cos\theta + \frac{4}{5}\sin\theta\right)$$

let $\cos\alpha = 3/5$

$\sin\alpha = \sqrt{1-\cos^2\alpha} = \sqrt{1-\frac{9}{25}} = \frac{4}{5}$

$$= \cos^{-1}\left(\cos\alpha\cos\theta + \sin\alpha\sin\theta\right)$$

$$= \cos^{-1}\left(\cos(\theta-\alpha)\right)$$

$$= \theta - \alpha$$

$$= \sin^{-1}x - \cos^{-1}(3/5) \quad \text{Ans}$$

8 → evaluate $\sin\left(3\sin^{-1}(0.4)\right)$

let $\sin^{-1}(0.4) = \theta$

$\Rightarrow \sin\theta = 0.4 \longrightarrow$ use

it ~~con~~ becomes

$= \sin(3\theta) \longrightarrow$ to find

$= 3\sin\theta - 4\sin^3\theta$

$= 3(0.4) - 4(0.4)^3$

$= 1.2 - 0.256 = 0.944$

Q.8 → Find the domain of

$$y = \cos^{-1}(x^2-4)$$

Sol:

$$y = \cos^{-1}(x^2-4)$$

$$\cos y = x^2-4$$

We know that

$$-1 \le \cos y \le 1$$

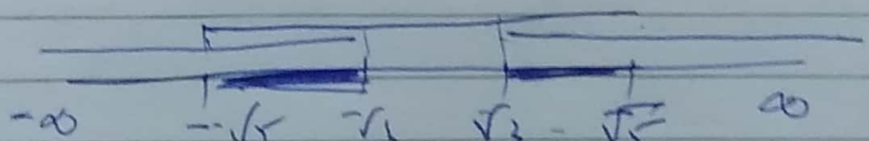
$$-1 \le x^2-4 \le 1$$

$$3 \le x^2 \le 5 \qquad \text{(adding 4)}$$

$$\sqrt{3} \le |x| \le \sqrt{5}$$

Case	$\lvert x\rvert \le a$ $-a \le x \le a$	$\lvert x\rvert \ge a$ $x \le -a$ (or) $x \ge a$
	Consider $\lvert x\rvert \le \sqrt{5}$ $-\sqrt{5} \le x \le \sqrt{5}$	Consider $\lvert x\rvert \ge \sqrt{3}$ $x \le -\sqrt{3}$ (or) $x \ge \sqrt{3}$

Intersection



$\therefore x \in [-\sqrt{5}, -\sqrt{3}] \cup [\sqrt{3}, \sqrt{5})$ Ans