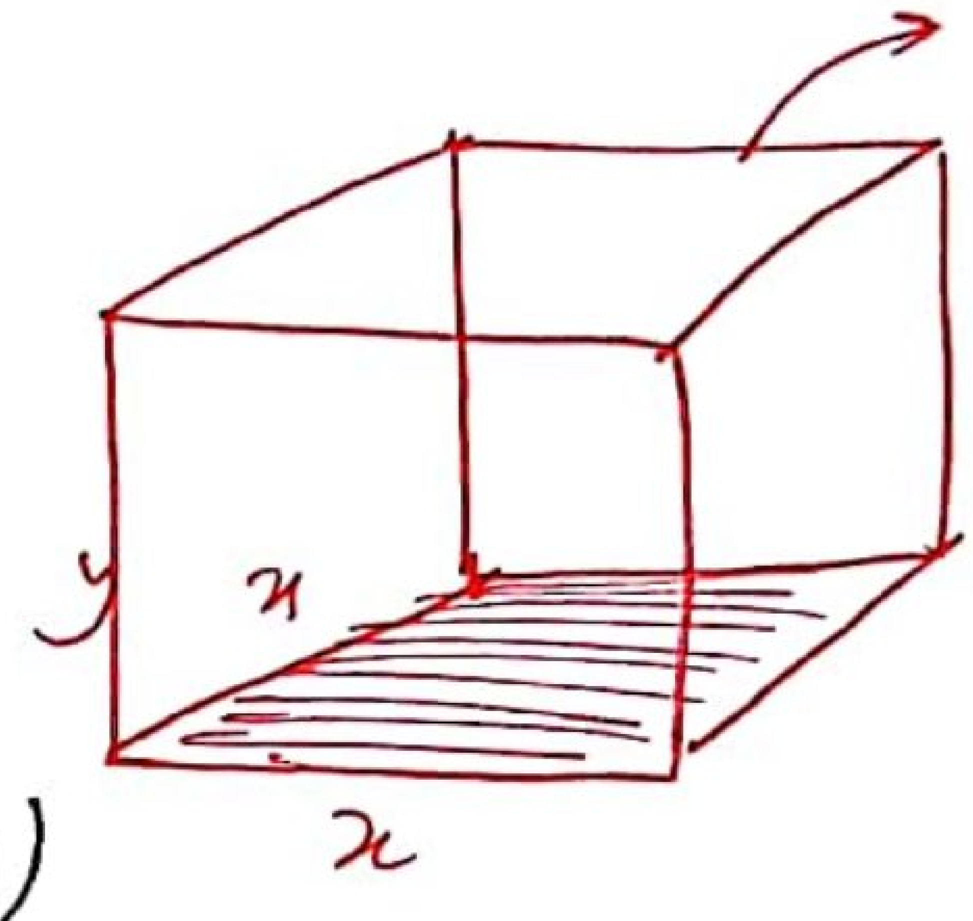
॥ जय श्री साईं कृपा ॥

Solutions : A.OD (WORKSHEET NO: 6)

MAXIMA MINIMA

Ans 1 →



$c^2 = x^2 + 4xy$ -- (given) -- (1)

$V = x^2y$ --- (to be Max)

$V = x^2\left(\frac{c^2 - x^2}{4x}\right)$ -- (from eq(1))

$V = \frac{1}{4}(c^2x - x^3)$

Diff w.r.t x

$\frac{dV}{dx} = \frac{1}{4}(c^2 - 3x^2)$

put $\frac{dV}{dx} = 0$

$\Rightarrow c^2 = 3x^2 \Rightarrow \boxed{c = \sqrt{3}x}$ (or) $\boxed{x = \frac{c}{\sqrt{3}}}$

Diff again

$\frac{d^2V}{dx^2} = \frac{1}{4}(-6x) = -\frac{3x}{2}$

$\left(\frac{d^2V}{dx^2}\right)_{x=\frac{c}{\sqrt{3}}} = -\frac{3c}{2\sqrt{3}} < 0$

$\therefore$ volume of box is Maximum at $x = \frac{c}{\sqrt{3}}$

put $x = \frac{c}{\sqrt{3}}$ in eq (1)

$c^2 = \frac{c^2}{3} + 4\frac{c}{\sqrt{3}}y$

$\Rightarrow \frac{2c^2}{3} = \frac{4c}{\sqrt{3}}y$

$\Rightarrow y = \frac{\sqrt{3}c}{6}$

$V_{max} = x^2y$

$= \frac{c^2}{3} \times \frac{\sqrt{3}c}{6}$

$= \frac{c^3}{6\sqrt{3}}$ cubic units

Proved

Ans 2. Let the point on the curve is $P(x, y)$

given: equation of curve

$$y^2 = 4x \quad \text{----(given)} \quad \text{--(1)}$$

let $Q(2,1)$ be the given point

let $S \to$ distance b/w P & Q

$$S = \sqrt{(x-2)^2 + (y-1)^2} \quad \text{--- (to be Min)}$$

$$S = \sqrt{\left(\frac{y^2}{4} - 2\right)^2 + (y-1)^2} \quad \text{--- (from (i))}$$

$$S = \sqrt{\frac{y^4}{16} + 4 - \cancel{y^2} + \cancel{y^2} + 1 - 2y}$$

$$S = \sqrt{y^4 - 32y + 80}$$

$$S^2 = y^4 - 32y + 80$$

let $S^2 = Z$

then S is Max/Min as according to Z is Max/Min

$$Z = y^4 - 32y + 80$$

Diff w.r.t y

$$\frac{dz}{dy} = 4y^3 - \cancel{32y}\ 32$$

put $\frac{dz}{dy} = 0$

$$4y^3 = 32$$

$$y^3 = 8 \Rightarrow y = 2$$

put $y=2$ in eq (1)

$\Rightarrow 4 = 4x$

$\Rightarrow x = 1$

Diff again w.r.t. y

$$\frac{d^2z}{dy^2} = 12y^2$$

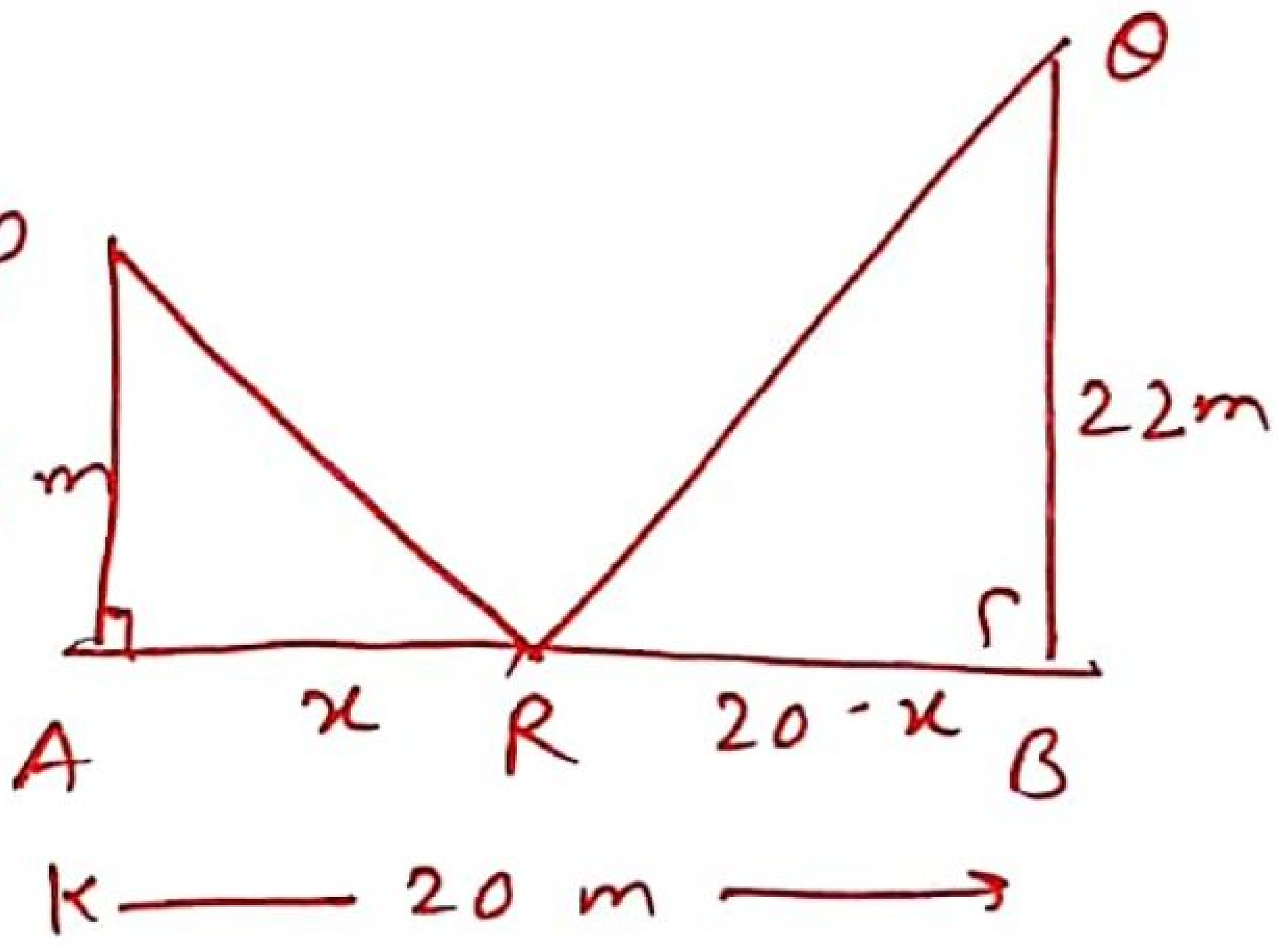
$$\left(\frac{d^2z}{dy^2}\right)_{y=2} = 12(4) = 48 > 0$$

$\therefore$ Z is Minimum

$\therefore$ Distance is Minimum at $y=2$

$\therefore$ point on the curve is $(1,2)$ Ans

Ans 3 →



ΔAPR $\quad PR^2 = 16^2 + x^2$

ΔRBQ $\quad QR^2 = 22^2 + (20-x)^2$

let

$$S = PR^2 + QR^2 \quad \text{--- (to be Min)}$$

$$S = (16)^2 + x^2 + (22)^2 + (20-x)^2$$

Diff w.r.t. x

$$\frac{dS}{dx} = 2x + 2(20-x)(-1)$$

$$= 2x - 40 + 2x$$

$$\frac{dS}{dx} = 4x - 40$$

put $\frac{dS}{dx} = 0$

$$\Rightarrow \boxed{x = 10}$$

Scanned with CamScanner

Diff again wrt x

$$\frac{d^2S}{dx^2} = 4 > 0$$

$\therefore$ S is Minimum at $x = 10$

$\therefore$ Required distance = 10 m **Ans**

Ans 4 → Price of one item = $5 - \frac{x}{100}$

Revenue = price × quantity

$$R = \left(5 - \frac{x}{100}\right)x = 5x - \frac{x^2}{100}$$

Total cost $C = \frac{x}{5} + 500$

$$P = R - C$$

$$P = 5x - \frac{x^2}{100} - \frac{x}{5} - 500$$

$$\frac{dP}{dx} = 5 - \frac{2x}{100_{50}} - \frac{1}{5}$$

$$= \frac{250 - x - 10}{50} = \frac{240 - x}{50}$$

put $\frac{dP}{dx} = 0$

$$\frac{240 - x}{50} = 0 \Rightarrow x = 240$$

(note: Misprint in worksheet)

Diff again

$$\frac{d^2P}{dx^2} = \frac{-1}{50} < 0$$

$\therefore$ Profit is Max at $x = 240$

$\therefore$ Required No of items = 240 **Ans**

Scanned with CamScanner

Q.N. 5 → let point is $P(x,y)$

given point $Q(0,5)$

equation of curve $x^2 = 2y$ --- (given) --(1)

let $s \to$ distance b/w P & Q

$s = PQ = \sqrt{(x-0)^2 + (y-5)^2}$

$s = \sqrt{2y + y^2 + 25 - 10y}$ --- {from (1) $x^2 = 2y$}

$s = \sqrt{y^2 - 8y + 25}$

Squaring $s^2 = y^2 - 8y + 25$

let $s^2 = z$

$z = y^2 - 8y + 25$

$\frac{dz}{dy} = 2y - 8$ put $\frac{dz}{dy} = 0$

$\boxed{y = 4}$

Diff again $\frac{d^2z}{dy^2} = 2 > 0$

$\therefore$ z is Minimum

$\therefore$ Distance is Minimum at $y = 4$

put $y = 4$ in eq (1)

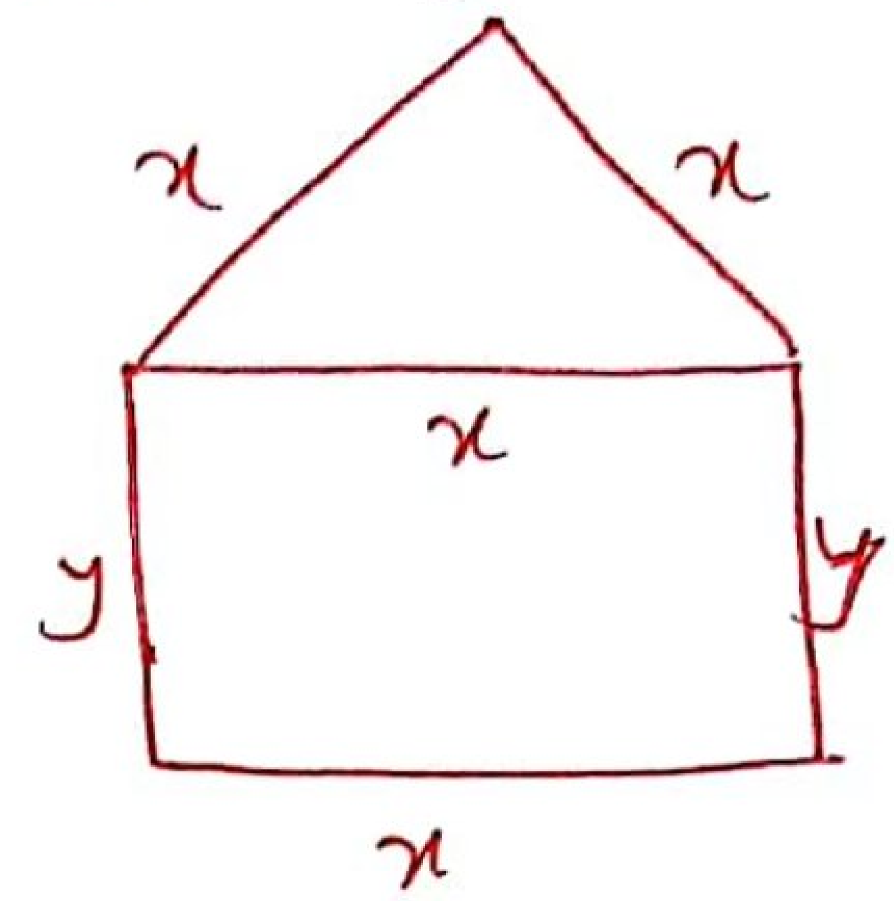
$x^2 = 8y$

$x = \pm 2\sqrt{2}$

$\therefore$ Required points are $(\pm 2\sqrt{2}, 4)$ Ans.

Qn 6 :- $x \rightarrow$ length

$y \rightarrow$ breadth



$12 = 3x + 2y$ -- (given) --(1)

$A = xy + \frac{\sqrt{3}}{4}x^2$ -- (to be Max)

$A = x\left(\frac{12-3x}{2}\right) + \frac{\sqrt{3}}{4}x^2$

$A = \frac{1}{2}(12x - 3x^2) + \frac{\sqrt{3}}{4}x^2$

$\frac{dA}{dx} = \frac{1}{2}(12 - 6x) + \frac{\sqrt{3}x}{2}$

$= \frac{12 - 6x + \sqrt{3}x}{2}$

$\frac{dA}{dx} = \frac{12 - x(6-\sqrt{3})}{2} = 0$

$\Rightarrow x = \frac{12}{6-\sqrt{3}}$

Diff again

$\frac{d^2A}{dx^2} = -\frac{(6-\sqrt{3})}{2} < 0$

$\therefore$ Area of window is Maximum at $x = \frac{12}{6-\sqrt{3}}$

Put $x = \frac{12}{6-\sqrt{3}}$ in eq(1)

$12 = \frac{36}{6-\sqrt{3}} + 2y$

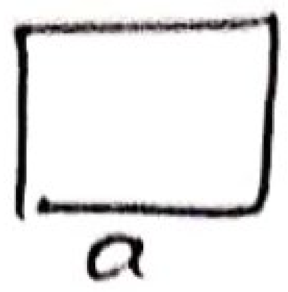
$\Rightarrow \frac{72 - 12\sqrt{3} - 36}{6-\sqrt{3}} = 2y$

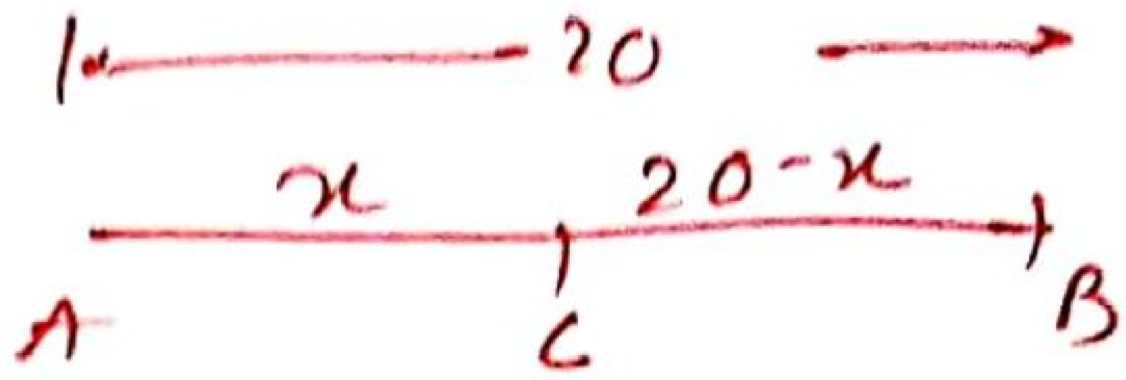
$\frac{36 - 12\sqrt{3}}{6-\sqrt{3}} = 2y$

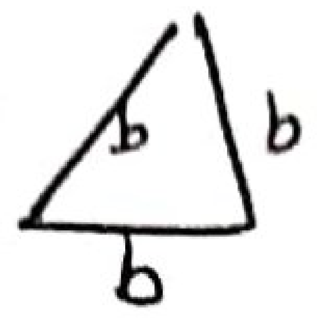
$y = \frac{18 - 6\sqrt{3}}{6-\sqrt{3}}$ m

Ans

Q. 7

$\underset{A \qquad\qquad C}{\overline{\quad x \quad}} \equiv$  $\Rightarrow 4a = x$



$\underset{C \qquad\qquad B}{\overline{\quad 20-x \quad}} \equiv$  $\Rightarrow 3b = 20-x$

A → Sum of their areas

$$A = a^2 + \frac{\sqrt{3}}{4} b^2 \text{ -- (to be Min)}$$

$$A = \frac{x^2}{16} + \frac{\sqrt{3}}{4}\left(\frac{20-x}{3}\right)^2$$

$$A = \frac{x^2}{16} + \frac{\sqrt{3}}{36}(20-x)^2$$

$$\frac{dA}{dx} = \frac{2x}{16} + \frac{\sqrt{3}}{36} \cdot 2(20-x)(-1)$$

$$= \frac{x}{8} - \frac{\sqrt{3}}{18}(20-x)$$

$$= \frac{9x - 80\sqrt{3} + 4\sqrt{3}x}{72}$$

$$\frac{dA}{dx} = \frac{x(9+4\sqrt{3}) - 80\sqrt{3}}{72} = 0$$

$$\boxed{x = \frac{80\sqrt{3}}{9+4\sqrt{3}}}$$

Diff again

$$\frac{d^2A}{dx^2} = \frac{9+4\sqrt{3}}{72} > 0$$

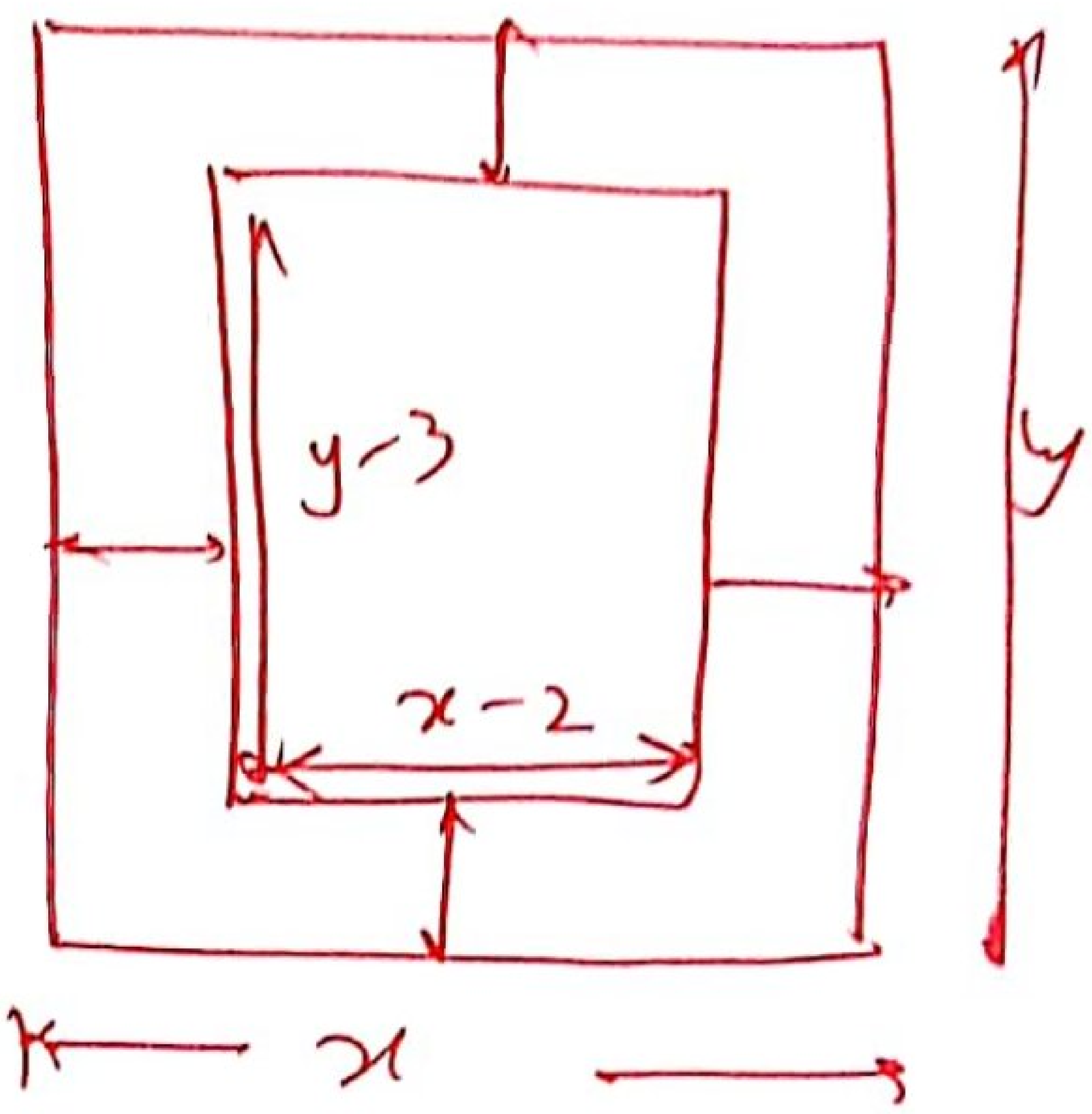
∴ Sum of the area is least

~~pls~~ length of Ist piece $= \dfrac{80\sqrt{3}}{9+4\sqrt{3}}$ m

length of 2nd piece $= 20 - x = 20 - \dfrac{80\sqrt{3}}{9+4\sqrt{3}}$

$= \dfrac{180}{9+4\sqrt{3}}$ m Ans

Q.8 → $x \to$ length of page
$y \to$ breadth of page



Margin in sides = 2 cm
Margin top & bottom = 3 cm

∴ length of printed page = $(x-2)$ cm
breadth " " = $(y-3)$ cm

$150 = xy$ --- (Given) --(1)

let $A \to$ area of printed matter

$A = (x-2)(y-3)$ --- (to be Max)

$A = (x-2)\left(\dfrac{150}{x} - 3\right)$

$A = 150 - 3x - \dfrac{300}{x} + 6$

$\dfrac{dA}{dx} = -3 + \dfrac{300}{x^2} = 0$

$x^2 = 100$

$x = 10$

$\dfrac{d^2A}{dx^2} = -\dfrac{600}{x^3}$

$\left(\dfrac{d^2A}{dx^2}\right)_{x=10} = \dfrac{-600}{1000} < 0$

$\therefore$ Area of Printed Matter is Maximum

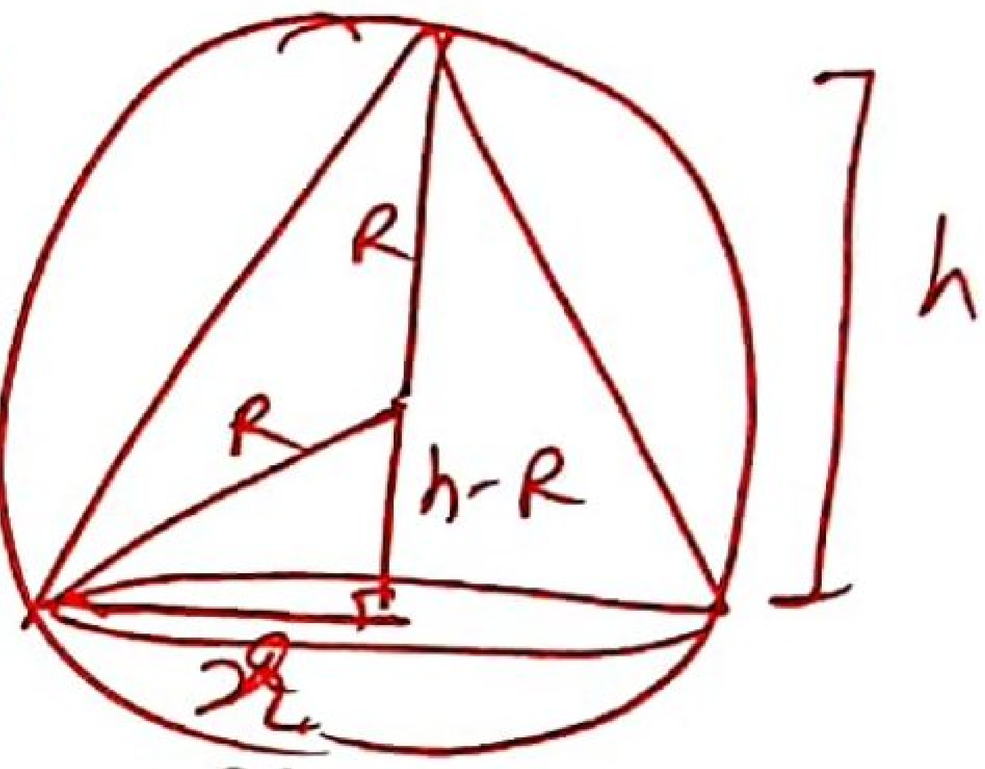
put $y = 10$ in eq (1)

$150 = 10x$

$\Rightarrow x = 15$

$\therefore l = 15$ cm $\quad b = 10$ cm $\quad$ Ans

Ans 9 $\quad R^2 = (h-R)^2 + r^2$ -- (given)



$V = \frac{1}{3}\pi r^2 h$ -- (Max)

$V = \frac{1}{3}\pi\left(R^2 - (h-R)^2\right)h$ -- (from (1))

$V = \frac{1}{3}\pi\left(\cancel{R^2} - \cancel{R^2} - h^2 + 2hR\right)h$

$V = \frac{1}{3}\pi\left(-h^3 + 2h^2R\right)$

$\frac{dV}{dh} = \frac{1}{3}\pi\left(-3h^2 + 4hR\right) = 0$

$\boxed{h = \frac{4R}{3}}$

$\frac{d^2V}{dh^2} = \frac{1}{3}\pi(-6h + 4R)$

$\left(\frac{d^2V}{dh^2}\right)_{h=\frac{4R}{3}} = \frac{1}{3}\pi(-8R + 4R) = -\frac{4\pi R}{3} < 0$

$\therefore V \to$ Max

$\therefore h = \frac{4R}{3} = \frac{2}{3}(2R)$

$= \frac{2}{3} \times$ diameter of sphere $\quad$ Proved