D&C

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Differentiation & Continuity

(Class No: 4)

①

Qn 1 → $y = \sqrt{\sin x + \sqrt{\sin x + \sqrt{\sin x + \cdots \infty}}}$

$$y = \sqrt{\sin x + y}$$

Squaring

$$y^2 = \sin x + y$$

Diff w.r.t x

$$2y\frac{dy}{dx} = \cos x + \frac{dy}{dx}$$

$$\Rightarrow \frac{dy}{dx}(2y-1) = \cos x$$

$$\Rightarrow \frac{dy}{dx} = \frac{\cos x}{2y-1} \quad \underline{\text{Ans}}$$

Qn 2 → $y = e^{x+e^{x+\cdots\infty}}$

$$\Rightarrow y = e^{x+y}$$

taking log on both sides

$$\log y = (x+y)\log e$$

$$\Rightarrow \log y = x + y \quad \cdots \{\log e = 1\}$$

Diff w.r.t x

$$\frac{1}{y}\cdot\frac{dy}{dx} = 1 + \frac{dy}{dx}$$

$$\Rightarrow \frac{dy}{dx}\left(\frac{1}{y} - 1\right) = 1$$

$$\Rightarrow \frac{dy}{dx}\left(\frac{1-y}{y}\right) = 1$$

$$\Rightarrow \frac{dy}{dx} = \frac{y}{1-y} \quad \underline{\text{Ans}}$$

DEC Solution ②

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Ans 3 → $y = (\sqrt{x})^{(\sqrt{x})^{(\sqrt{x})^{\cdots\infty}}}$

$\Rightarrow y = (\sqrt{x})^{y}$

$\Rightarrow y = (x)^{y/2}$ --- $\{\because \sqrt{x} = x^{1/2}\}$

taking log on both sides

$\log y = \frac{y}{2} \log x$

Diff wrt x

$\frac{1}{y} \cdot \frac{dy}{dx} = \frac{y}{2} \cdot \frac{1}{x} + \log x \cdot \frac{1}{2} \cdot \frac{dy}{dx}$

$\Rightarrow \frac{dy}{dx}\left(\frac{1}{y} - \frac{\log x}{2}\right) = \frac{y}{2x}$

$\Rightarrow \frac{dy}{dx}\left(\frac{2 - y\log x}{\cancel{2}y}\right) = \frac{y}{\cancel{2}x}$

$\Rightarrow \frac{dy}{dx} = \frac{y^2}{x(2 - y\log x)}$ Ans

Ans 4 → $y = x + \cfrac{1}{x + \cfrac{1}{x + \cfrac{1}{x + \cdots\infty}}}$

$y = x + \frac{1}{y}$

$\Rightarrow y^2 = xy + 1$

Diff wrt x

$2y\frac{dy}{dx} = x\frac{dy}{dx} + y$

$\Rightarrow \frac{dy}{dx}(2y - x) = y \Rightarrow \frac{dy}{dx} = \frac{y}{2y - x}$ Ans

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Qns 5 → $y = (\cos x)^{(\cos x)^{(\cos x) \cdots \infty}}$

$\Rightarrow y = (\cos x)^y$

taking log on both sides

$\log y = \log (\cos x)^y$

$\Rightarrow \log y = y \log (\cos x)$

Diff w.r.t x

$\frac{1}{y} \cdot \frac{dy}{dx} = y \cdot \frac{1}{\cos x} (-\sin x) + \log(\cos x) \cdot \frac{dy}{dx}$

$\Rightarrow \frac{dy}{dx}\left(\frac{1}{y} - \log(\cos x)\right) = -y \tan x$

$\Rightarrow \frac{dy}{dx}\left(\frac{1 - y\log(\cos x)}{y}\right) = -y \tan x$

$\Rightarrow \frac{dy}{dx} = \frac{-y \tan^2 x}{1 - y\log(\cos x)}$ Ans

{worksheet Me -ve sign Bhi ayegay

Misprint

Qn 6 → $y = \tan^{-1}\left(\frac{3x - x^3}{1 - 3x^2}\right)$

put $x = \tan\theta$

$y = \tan^{-1}\left(\frac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta}\right)$

$\Rightarrow y = \tan^{-1}(\tan(3\theta))$

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(i) $\frac{-1}{\sqrt{3}} < x < \frac{1}{\sqrt{3}}$

$\Rightarrow \frac{-1}{\sqrt{3}} < \tan\theta < \frac{1}{\sqrt{3}}$

$\Rightarrow \frac{-\pi}{6} < \theta < \frac{\pi}{6}$

$\Rightarrow \frac{-\pi}{2} < 3\theta < \frac{\pi}{2}$ (within range of $\tan^{-1}x$)

$\Rightarrow \tan^{-1}(\tan(3\theta))$

$= 3\theta$

$y = 3\tan^{-1}x$ --- {replace θ by $\tan^{-1}x$}

Diff w.r.t x

$\frac{dy}{dx} = \frac{3}{1+x^2}$ Ans

(ii) $x > \frac{1}{\sqrt{3}}$

$\Rightarrow \tan\theta > \frac{1}{\sqrt{3}}$

$\Rightarrow \theta > \frac{\pi}{6}$

$\Rightarrow 3\theta > \frac{\pi}{2}$ (outside range of $\tan^{-1}x$)

$\Rightarrow -3\theta < -\frac{\pi}{2}$

$\Rightarrow \pi - 3\theta < \pi - \frac{\pi}{2}$

$\Rightarrow (\pi - 3\theta) < \pi/2$ (within range)

$\Rightarrow \tan^{-1}(\tan(3\theta))$ (Main point)

$= \tan^{-1}(-\tan(\pi - 3\theta))$

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$= -\tan^{-1}(\tan(\pi - 3\theta))$ $\longrightarrow$ within range

$= -(\pi - 3\theta)$

$y = -\pi + 3\theta$

$y = -\pi + 3\tan^{-1}x$

Diff w.r.t x

$\frac{dy}{dx} = 0 + \frac{3}{1+x^2} = \frac{3}{1+x^2}$ Ans

(iii) $x < -\frac{1}{\sqrt{3}}$

$\Rightarrow \tan\theta < -\frac{1}{\sqrt{3}}$

$\Rightarrow \theta < -\frac{\pi}{6}$

$\Rightarrow 3\theta < -\frac{\pi}{2}$ (outside range of $\tan^{-1}x$)

$\Rightarrow \pi + 3\theta < \pi - \frac{\pi}{2}$

$\Rightarrow \pi + 3\theta < \frac{\pi}{2}$ (within range)

$\Rightarrow \tan^{-1}(\tan(3\theta))$

$= \tan^{-1}(\tan(\pi + 3\theta))$ $\longrightarrow$ within Range

$y = \pi + 3\theta$

$y = \pi + 3\tan^{-1}x$

Diff w.r.t x

$\frac{dy}{dx} = 0 + \frac{3}{1+x^2} = \frac{3}{1+x^2}$ Ans

$1+x^2$ $\quad 1+x^2$

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(7) QN

$y = \sec^{-1}\left(\frac{1}{2x^2-1}\right) \quad ; \quad 0 < x < \frac{1}{\sqrt{2}}$

put $\quad x = \cos\theta$

$y = \sec^{-1}\left(\frac{1}{2\cos^2\theta - 1}\right)$

$= \sec^{-1}\left(\frac{1}{\cos(2\theta)}\right)$

$= \cos^{-1}(\cos(2\theta)) \quad --- \left\{\text{since } \sec^{-1}x = \cos^{-1}\left(\frac{1}{x}\right)\right\}$

given $\quad 0 < x < 1/\sqrt{2}$

$\Rightarrow 0 < \cos\theta < \frac{1}{\sqrt{2}}$

$\Rightarrow \frac{\pi}{4} < \theta < \frac{\pi}{2}$

$\Rightarrow \frac{\pi}{2} < 2\theta < \pi \quad$ (within Range of $\cos^{-1}x$)

$\Rightarrow \cos^{-1}(\cos(2\theta))$

$= 2\theta$

$y = 2\cos^{-1}x \quad --- \{\text{replace } \theta \text{ by } \cos^{-1}x\}$

$\frac{dy}{dx} = \frac{-2}{\sqrt{1-x^2}}$ Ans

QN 8 → $\quad y = \cos^{-1}\left(2x\sqrt{1-x^2}\right) \; , \; \frac{-1}{\sqrt{2}} < x < \frac{1}{\sqrt{2}}$

put $\quad x = \sin\theta$

$y = \cos^{-1}($ ...

$y = \cos^{-1}((2\sin\theta\cos\theta))$

$y = \cos^{-1}(\sin(2\theta))$

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$y = \cos^{-1}\left(\cos\left(\frac{\pi}{2} - 2\theta\right)\right)$

Now $\quad -\frac{1}{\sqrt{2}} < x < \frac{1}{\sqrt{2}}$

$\Rightarrow -\frac{1}{\sqrt{2}} < \sin\theta < \frac{1}{\sqrt{2}}$

$\Rightarrow -\frac{\pi}{4} < \theta < \frac{\pi}{4}$

$\Rightarrow -\frac{\pi}{2} < 2\theta < \frac{\pi}{2}$

$\Rightarrow \frac{\pi}{2} > -2\theta > -\frac{\pi}{2}$

$\Rightarrow \frac{\pi}{2} + \frac{\pi}{2} > \left(\frac{\pi}{2} - 2\theta\right) > \frac{\pi}{2} - \frac{\pi}{2}$

$\Rightarrow \pi > \left(\frac{\pi}{2} - 2\theta\right) > 0$

or $\quad 0 < \left(\frac{\pi}{2} - 2\theta\right) < \pi \quad$ (within Range of $\cos^{-1}$)

$\Rightarrow y = \cos^{-1}\left(\cos\left(\frac{\pi}{2} - 2\theta\right)\right)$

$\Rightarrow y = \frac{\pi}{2} - 2\theta$

replace θ

$y = \frac{\pi}{2} - 2\sin^{-1}x$

Diff w.r.t. x

$\frac{dy}{dx} = 0 - \frac{2}{\sqrt{1-x^2}} = \frac{-2}{\sqrt{1-x^2}}$ Ans

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Qn 9 → $y = \tan^{-1}\left(\frac{\sqrt{1+x^2} + \sqrt{1-x^2}}{\sqrt{1+x^2} - \sqrt{1-x^2}}\right)$; $-1 < x < 1$

Divide by $\sqrt{1+x^2}$

$\Rightarrow y = \tan^{-1}\left(1 + \sqrt{\frac{1-x^2}{\ldots}}\right.$

$$\frac{\quad 1+x^2}{1-\sqrt{\frac{1-x^2}{1+x^2}}}\Bigg)$$

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$$y = \tan^{-1}(1) + \tan^{-1}\left(\sqrt{\frac{1-x^2}{1+x^2}}\right)$$

put $x^2 = \cos(2\theta)$

$$y = \frac{\pi}{4} + \tan^{-1}\sqrt{\frac{1-\cos(2\theta)}{1+\cos(2\theta)}}$$

$$y = \frac{\pi}{4} + \tan^{-1}\sqrt{\frac{\cancel{2}\sin^2\theta}{\cancel{2}\cos^2\theta}}$$

$$y = \frac{\pi}{4} + \tan^{-1}\sqrt{\tan^2\theta}$$

$$y = \frac{\pi}{4} + \tan^{-1}|\tan\theta|$$

we have

$-1 < x < 1$

$0 < x^2 < 1$

$0 < \cos(2\theta) < 1$

$0 < 2\theta < \frac{\pi}{2}$

$0 < \theta < \frac{\pi}{4}$

$\longrightarrow$ I^{st} quadrant

$$y = \frac{\pi}{4} + \tan^{-1}(\tan\theta)$$

$\longrightarrow$ within Range of $\tan^{-1}x$)

$$y = \frac{\pi}{4} + \theta$$

$$y = \frac{\pi}{4} + \frac{1}{2}\cos^{-1}x \quad \cdots \Big\{ \text{replace } \theta$$

$x = \cos(2\theta)$

$\cos^{-1}x = 2\theta$

$\theta = \frac{1}{2}\cos^{-1}x\Big\}$

Diff wrt x

$$\frac{dy}{dx} = 0 - \frac{1}{2\sqrt{1-x^2}} = \frac{-1}{2\sqrt{1-x^2}}$$

$2\sqrt{1-x^2}$

Ans

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Qn 10 $y = \tan^{-1}\left(\frac{2x}{1-x^2}\right) + \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$; $0 < x < 1$

put $x = \tan\theta$

$y = \tan^{-1}\left(\frac{2\tan\theta}{1-\tan^2\theta}\right) + \cos^{-1}\left(\frac{1-\tan^2\theta}{1+\tan^2\theta}\right)$

$y = \tan^{-1}(\tan(2\theta)) + \cos^{-1}(\cos(2\theta))$

we have $0 < x < 1$

$\Rightarrow 0 < \tan\theta < 1$

$\Rightarrow 0 < \theta < \frac{\pi}{4}$

$\Rightarrow \underbrace{0 < 2\theta < \frac{\pi}{2}}$ → within range of $\tan^{-1}x$ and also $\cos^{-1}x$

$y = \tan^{-1}(\tan(2\theta)) + \cos^{-1}(\cos(2\theta))$

$y = 2\theta + 2\theta$

$y = 4\theta$

$\Rightarrow y = 4\tan^{-1}x$

Diff wrt x

$\frac{dy}{dx} = \frac{4}{1+x^2}$ Ans

Qn. 11 → $y = \tan^{-1}\left(\frac{x}{1+6x^2}\right)$

$y = \tan^{-1}\left(\frac{3x-2x}{1+6x^2}\right)$

$\Rightarrow y = \tan^{-1}(3x) - \tan^{-1}(2x)$ ---{property $\tan^{-1}\left(\frac{x-y}{1+xy}\right) = \tan^{-1}x - \tan^{-1}y$}

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Diff wrt x

$\frac{dy}{dx} = \frac{1}{1+(3x)^2} \cdot 3 - \frac{1}{1+(2x)^2} \cdot 2$

$\Rightarrow \frac{dy}{dx} = \frac{3}{1+9x^2} - \frac{2}{1+4x^2}$ Ans

Qn 12 → $y = \sin^{-1}\left(\frac{2^{x+1}}{1+4^x}\right)$

$y = \sin^{-1}\left(\frac{2 \cdot 2^x}{1+(2^x)^2}\right)$ --- {$4^x = (2^x)^2$}

$\Rightarrow$ ~~$y = \sin^{-1}\left(\frac{2^x \cdot 2}{1+(2^x)^2}\right)$~~ Put $2^x = \tan\theta$

$\Rightarrow y = \sin^{-1}\left(\frac{2\tan\theta}{1+\tan^2\theta}\right)$

$\Rightarrow y = \sin^{-1}(\sin(2\theta))$

$y = 2\theta$

$y = 2\tan^{-1}(2^x)$ ---{replace θ by $\tan^{-1}(2^x)$}

Diff wrt x

$\frac{dy}{dx} = 2 \cdot \frac{1}{1+(2^x)^2} \cdot 2^x \cdot \log 2$

$= 2 \cdot \quad 2^x \log 2$

$$1+4^x$$

$$\frac{dy}{dx} = \frac{2^{x+1}\cdot \log 2}{1+4^x} \quad \underline{\underline{Ans}}$$