ULTIMATE MATHEMATICS

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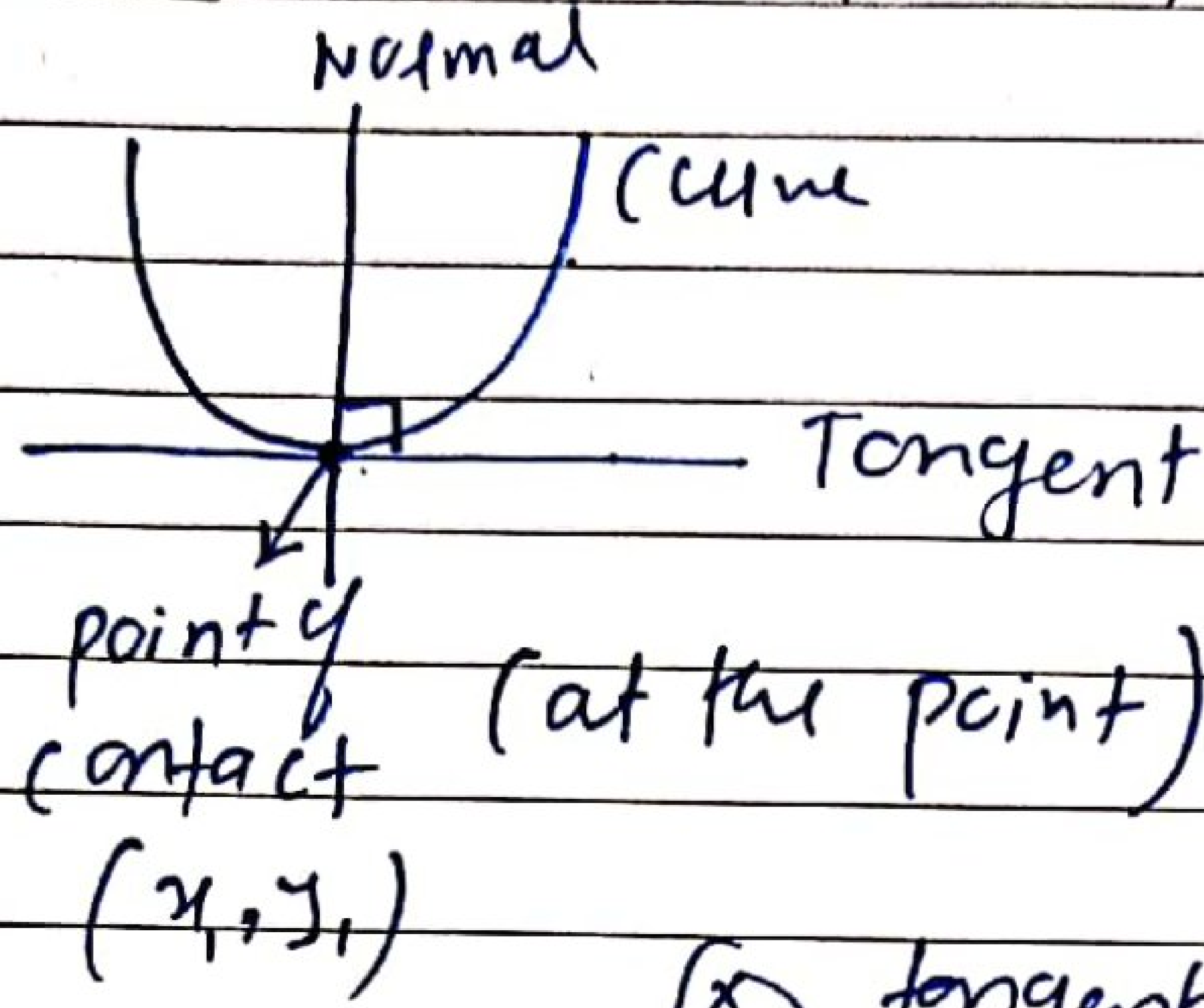
CHAPTER: A·O·D

①

TOPIC: 1 TANGENTS & NORMALS

XII (·)

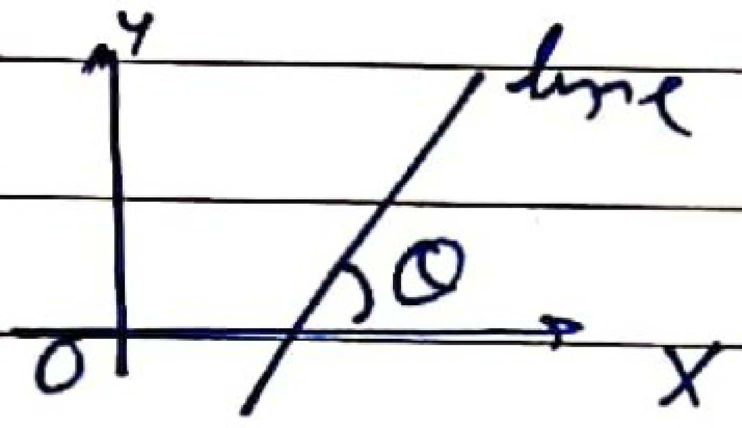


(at the point)

ⓧ tangent from/through the point $(2,3)$

XI (·) $\overline{XI}$ class (Straight lines)

- ✓ $ax + by + c = 0$
- ✓ $m = \tan\theta$



- ✓ $m = \dfrac{y_2 - y_1}{x_2 - x_1}$
- ✓ $m = \dfrac{-\text{coff of } x}{\text{coff of } y}$
- ✓ $m_1 = m_2$
- ✓ $m_1 m_2 = -1$ (-ve reciprocal)
- ✓ Slope of X-axis $= 0$
- ✓ Slope of Y-axis $= \frac{1}{0} = \infty$
- ✓ $\tan\theta = \left|\dfrac{m_1 - m_2}{1 + m_1 m_2}\right|$
- ✓ point slope form
  $y - y_1 = m(x - x_1)$
- ✓ two point $y - y_1 = \left(\dfrac{y_2 - y_1}{x_2 - x_1}\right)(x - x_1)$

A.O.D. class No-1

XII - always given equation of curve

✓ Diff. equation of curve and get

$$\frac{dy}{dx} = \frac{2x}{3y}$$

✓ Slope of tangent / gradient at (x_1, y_1)

$$\left(\frac{dy}{dx}\right)_{(x_1,y_1)} = \frac{2x_1}{3y_1}$$

✓ Slope of Normal at $(x_1, y_1) = \frac{-1}{\left(\frac{dy}{dx}\right)_{(x_1,y_1)}} = \frac{-3y_1}{2x_1}$

✓ equation of tangent at (x_1, y_1)

$$y - y_1 = m(x - x_1)$$

✓ point of contact
✓ slope

✓ equation of Normal at (x_1, y_1)

$$y - y_1 = -\frac{1}{m}(x - x_1)$$

Q.N-1 → Find the points on the curve $y = x^3 - 11x + 5$ at which the tangent ~~has the equation~~ is parallel to the line $y = x - 11$. Also find the equation of tangents at these points.

Soln (·) let the point of contact / point on the curve is (x_1, y_1)

(·) equation of curve

$$y = x^3 - 11x + 5 \quad \text{-----(i)}$$

(·) Diff wrt x

$$\frac{dy}{dx} = 3x^2 - 11$$

(·) Slope of tangent at $(x_1, y_1) = 3x_1^2 - 11$

(·) Slope of given line $y = x - 11 = 1$

$$(x - y - 11 = 0) = -\frac{1}{-1} = 1$$

(∴) Since they are parallel

$$3x_1^2 - 11 = 1$$

$$\Rightarrow 3x_1^2 = 12$$

$$\Rightarrow x_1^2 = 4$$

$$\Rightarrow \boxed{x_1 = \pm 2}$$

(∴) also we have

$$y_1 = x_1^3 - 11x_1 + 5 \quad \ldots \left\{ \because (x_1, y_1) \text{ lies on the curve} \right\}$$

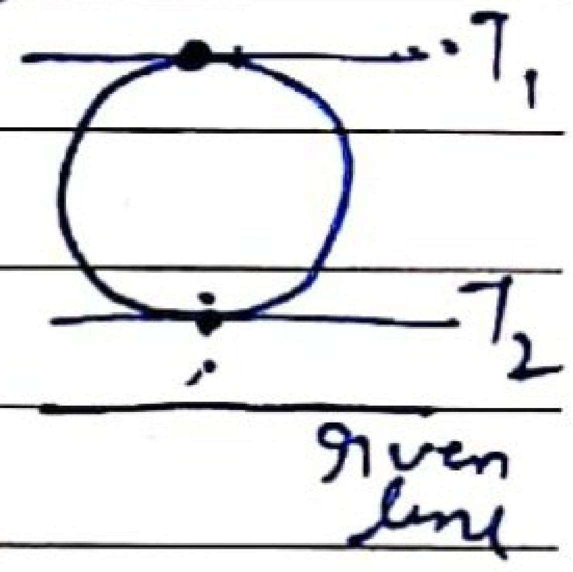
for $x_1 = 2$ $\quad y_1 = 8 - 22 + 5$

$y_1 = -9$ $\quad \therefore$ point $(2, -9)$

for $x_1 = -2$

$y_1 = -8 + 22 + 5$ $\quad y_1 = 19$ point $(-2, 19)$

∴ points on the curve are $(2,-9)$ & $(-2, 19)$



Equation of Tangent at $(2,-9)$

$$y + 9 = 1(x - 2)$$

$$y + 9 = x - 2$$

$$\Rightarrow x - y - 11 = 0 \quad \text{Ans}$$

Equation of Tangent at $(-2, 19)$

$$y - 19 = 1(x + 2)$$

$$\Rightarrow y - 19 = x + 2$$

$$\Rightarrow x - y + 21 = 0 \quad \text{Ans}$$

Qn.2 → Find the equation of the normals to the curve $y = x^3 + 2x + 6$ which are parallel to the line $x + 14y + 4 = 0$

Sol: (∴) Let the point of contact be (x_1, y_1)

(∴) given equation of curve

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A.OD Class No: 1

$y = x^3 + 2x + 6$

(·) Diff wrt x

$\frac{dy}{dx} = 3x^2 + 2$

(·) slope of tangent at $(x_1, y_1) = 3x_1^2 + 2$

(·) slope of ~~Normal~~ Normal at $(x_1, y_1) = \frac{-1}{3x_1^2+2}$

(·) slope of given line $(x + 14y + 4 = 0) = -\frac{1}{14}$

(·) Since normal is parallel to the given line

$\therefore \frac{-1}{3x_1^2+2} = \frac{-1}{14}$

$\Rightarrow 3x_1^2 + 2 = 14$

$\Rightarrow 3x_1^2 = 12$

$\Rightarrow x_1^2 = 4$

$\boxed{x_1 = \pm 2}$

(·) also we have,

$y_1 = x_1^3 + 2x_1 + 6$ ... { $\because (x_1, y_1)$ lies on the curve }

(·) for $x_1 = 2$ $\quad y_1 = 8 + 4 + 6 = 18$ $\therefore$ point $(2, 18)$

(·) for $x_1 = -2$ $\quad y_1 = -8 - 4 + 6 = -6$ $\therefore$ point $(-2, -6)$

(·) equation of Normal at $(2, 18)$

$y - 18 = -\frac{1}{14}(x - 2)$

$\Rightarrow 14y - 252 = -x + 2$

$\Rightarrow x + 14y - 254 = 0$ Ans

(·) Similarly 2nd equation of Normal (self)

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A.O) Clan No. 1

Ques 3 → Find the equation of the Normal to the curve $x^2 = 4y$ "passing through" the point $(1,2)$

Soln (·) Let the point of contact is (x_1, y_1)

(·) given equation of curve
$$x^2 = 4y$$

(·) Diff w.r.t x
$$2x = 4\frac{dy}{dx}$$
$$\Rightarrow \frac{dy}{dx} = \frac{x}{2}$$

(·) Slope of Tangent at $(x_1, y_1) = \frac{x_1}{2}$

(·) Slope of Normal at $(x_1, y_1) = \frac{-2}{x_1}$

* equation of Normal
$$y - y_1 = \frac{-2}{x_1}(x - x_1)$$

(·) $(1,2)$ lies on it
$$\Rightarrow 2 - y_1 = \frac{-2}{x_1}(1 - x_1)$$
$$\Rightarrow \cancel{2x_1} - x_1 y_1 = -2 + \cancel{2x_1}$$
$$\Rightarrow \boxed{x_1 y_1 = 2} \quad ..$$

(·) also we have,
$$\boxed{x_1^2 = 4y_1} \quad \ldots \{\because (x_1, y_1) \text{ lies on the curve}\}$$
$$\Rightarrow x_1^2 = 4\left(\frac{2}{x_1}\right)$$
$$\Rightarrow x_1^3 = 8$$
$$\Rightarrow \boxed{x_1 = 2} \Rightarrow y_1 = 1 \quad \therefore \text{ point of contact } (2,1)$$

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A.O.D class No: 1

(·) Equation of normal at $(2,1)$

$$y-1=\frac{-2}{2}(x-2)$$

$$\Rightarrow y-1=-x+2$$

$$\Rightarrow x+y-3=0 \quad \underline{\text{Ans}}$$

Qn. 4 → Find the equation of the tangent to the curve $x=1-\cos\theta,\ y=\theta-\sin\theta$ at $\theta=\pi/4$

Sol

(·) Given equation of curve

$$x=1-\cos\theta, \quad y=\theta-\sin\theta$$

(·) point of contact $\left(1-\frac{1}{\sqrt2}, \frac{\pi}{4}-\frac{1}{\sqrt2}\right)$

(·) Diff w.r.t θ

$$\frac{dx}{d\theta}=\sin\theta \quad ; \quad \frac{dy}{d\theta}=1-\cos\theta$$

$$\frac{dy}{dx}=\frac{1-\cos\theta}{\sin\theta}$$

(·) Slope of tangent at $\theta=\pi/4$ $=\dfrac{1-\frac{1}{\sqrt2}}{\frac{1}{\sqrt2}}=\sqrt2-1$

(·) equation of tangent at $\left(1-\frac{1}{\sqrt2}, \frac{\pi}{4}-\frac{1}{\sqrt2}\right)$

$$y-\left(\frac{\pi}{4}-\frac{1}{\sqrt2}\right)=(\sqrt2-1)\left(x-\left(1-\frac{1}{\sqrt2}\right)\right)$$

$$\Rightarrow y-\left(\frac{\pi}{4}-\frac{1}{\sqrt2}\right)=(\sqrt2-1)x-\frac{(\sqrt2-1)^2}{\sqrt2}$$

$$\Rightarrow y-\frac{\pi}{4}+\frac{1}{\sqrt2}=(\sqrt2-1)x-\frac{(3-2\sqrt2)}{\sqrt2}$$

$$\Rightarrow (\sqrt2-1)x-y-\left(\frac{3-2\sqrt2}{\sqrt2}\right)-\frac{1}{\sqrt2}+\frac{\pi}{4}=0$$

Ans

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A.01) Class No: 1 (7)

Qn. 5 → Find the equation of Tangent and Normal to the curve $y = \frac{x-7}{(x-2)(x-3)}$ at the point where it cuts the x-axis.

Soln :- Equation of curve $y = \frac{x-7}{(x-2)(x-3)}$

Since it cuts the X-axis ∴ $y = 0$

$\Rightarrow 0 = \frac{x-7}{(x-2)(x-3)}$

$\Rightarrow 0 = x-7$

$\Rightarrow x = 7$

∴ point of contact $(7, 0)$

∴ $y = \frac{x-7}{x^2-5x+6}$

Diff wrt x

$\frac{dy}{dx} = \frac{(x^2-5x+6)(1) - (x-7)(2x-5)}{(x^2-5x+6)^2}$

∴ Slope of Tangent at $(7,0)$

$\left(\frac{dy}{dx}\right)_{(7,0)} = \frac{(49-35+6)}{(49-35+6)^2} = \frac{20}{(20)^2} = $

Slope of tangent $= \frac{1}{20}$

∴ Slope of Normal at $(7,0) = -20$

∴ Eq. of Tangent at $(7,0) =$

$y - 0 = \frac{1}{20}(x-7)$

∴ Eq. of Normal at $(7,0)$ $y - 0 = -20(x-7)$

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(A.OD Class No: 1)

Qu. 6 → The equation of the Tangent at (2,3) on the curve $y^2 = ax^3 + b$ is $y = 4x - 5$. Find the values of a and b.

Sol: (·) equat of curve. $y^2 = ax^3 + b$

(·) (2,3) lies on the curve

$$9 = 8a + b \quad \text{----} (1)$$

(·) Diff equat of curve

$$2y\frac{dy}{dx} = 3ax^2$$

$$\frac{dy}{dx} = \frac{3ax^2}{2y}$$

(·) Slope of Tangent at $(2,3) = \frac{12a}{6} = 2a$

(·) also slope of Tangent $(y = 4x - 5) = 4$

$$\Rightarrow 2a = 4$$

$$\Rightarrow a = 2 \quad \text{put in } (1)$$

$$9 = 16 + b$$

$$b = -7$$

$\therefore a = 2, b = -7$ Ans

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A.O.D

WORKSHEET No. 1
(Tangents & Normals)

Ques 1 Find a point on the curve $y=(x-2)^2$ at which the tangent is parallel to the chord joining the points $(2,0)$ & $(4,0)$ **Ans** $(3,1)$

Ques 2 Find points at which the tangent to the curve $y=x^3-3x^2-9x+7$ is parallel to x-axis **Ans** $(3,-20)$ & $(-1,12)$

Ques 3 Find the slope of Tangent and Normal to the curve $x=1-a\sin\theta$, $y=b\cos^2\theta$ at $\theta=\pi/2$ **Ans** (T): $\frac{2b}{a}$; (N): $\frac{-a}{2b}$

Ques 4 Find the equation of tangent and normal to the given curve $y=x^4-6x^3+13x^2-10x+5$ at $(0,5)$ **Ans** (T): $10x+y=5$; (N): $x-10y+50=0$

Ques 5 Find the equation of Tangent and Normal to the parabola $y^2=4ax$ at the point $(at^2, 2at)$ **Ans** $x-ty+at^2=0$
$tx+y=2at+at^3$

Ques 6 Find the equation of the tangent line to the curve $y=x^2-2x+7$ which is
(i) parallel to the line $2x-y+9=0$
(2) perpendicular to the line $5y-15x=13$

Ans (1) $y-2x-3=0$ (2) $36y+12x-227=0$

Ques 7 Find the equation of the tangent to the curve

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$y = \sqrt{3x-2}$ which is parallel to the line
$4x - 2y + 5 = 0$

Ans $48x - 24y = 23$

Ans 8 → Find points on the curve $\frac{x^2}{9} + \frac{y^2}{16} = 1$
at which the tangents are
(i) parallel to x-axis
(2) parallel to y-axis

Ans (1) $(0, \mp 4)$ (2) $(\mp 3, 0)$

Qn 9 → Find the points on the curve $y = x^3$ at which the slope of the tangent is equal to the y-coordinate of the point

Ans $(0, 0)$ $(3, 27)$

Qn 10 → For the curve $x^2 + y^2 - 2x - 3 = 0$, find all the points at which tangent "passes through" the origin

Ans $(0, 0)$ $(1, 2)$ $(-1, -2)$

— x —