!! जय श्री राधे राधे जय श्री गिरिराज जी महाराज !!

ULTIMATE MATHEMATICS: BY AJAY MITTAL

CHAPTER: INTEGRATION   CLASS: NO: 11

partial fraction (continued)....

Type: 4 even power of x

Qn: 1 $I = \int \frac{x^2+3}{(x^2+1)(x^2+2)} dx$

let $x^2 = y$ (temp.)

$\therefore \frac{x^2+3}{(x^2+1)(x^2+2)} = \frac{y+3}{(y+1)(y+2)}$

let $\frac{y+3}{(y+1)(y+2)} = \frac{A}{y+1} + \frac{B}{y+2}$

$\Rightarrow y+3 = A(y+2) + B(y+1)$

$1 = A + B$

$3 = 2A + B$

$-2 = -A$ $A = 2$ $B = -1$

$\therefore I = \int \frac{2}{x^2+1} - \frac{1}{x^2+2} dx$

$= 2\int \frac{1}{x^2+1} dx - \int \frac{1}{x^2+2} dx$

$I = 2\tan^{-1}(x) - \frac{1}{\sqrt{2}} \tan^{-1}\left(\frac{x}{\sqrt{2}}\right) + C$ Ans

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Qn. 2 $I = \int \frac{1}{x^4 - 1}\, dx$

$$I = \int \frac{1}{(x^2+1)(x^2-1)}\, dx$$

let $x^2 = y$ (temp.)

$$\frac{1}{(x^2+1)(x^2-1)} = \frac{1}{(y+1)(y-1)}$$

let $\frac{1}{(y+1)(y-1)} = \frac{A}{y+1} + \frac{B}{y-1}$

$$\Rightarrow 1 = A(y-1) + B(y+1)$$

$$0 = \cancel{A} + B$$

$$1 = -\cancel{A} + B$$

$$1 = 2B$$

$B = 1/2$ $A = -1/2$

$$\therefore I = \int \frac{-1}{2(x^2+1)} + \frac{1}{2(x^2-1)}\, dx$$

$$= -\frac{1}{2}\int \frac{1}{x^2+1}\, dx + \frac{1}{2}\int \frac{1}{x^2-1}\, dx$$

$$= -\frac{1}{2}\tan^{-1}(x) + \frac{1}{2} \times \frac{1}{2}\log\left|\frac{x-1}{x+1}\right| + C$$ **Ans**

Qn. 3 $I = \int \frac{(x^2+1)(x^2+2)}{(x^2+3)(x^2+4)}\, dx$

$$\begin{array}{r|l} & 1 \\ x^4+7x^2+12 & x^4+3x^2+2 \\ & -(x^4+7x^2+12) \\ \hline & -4x^2-10 \end{array}$$

$$\therefore I = \int 1 - \frac{4x^2+10}{(x^2+3)(x^2+4)}\, dx$$

$$\Rightarrow \quad x - \int \frac{4x^2+10}{(x^2+3)(x^2+4)}\, dy$$

let $x^2 = y$ (temp)

$$\frac{4x^2+10}{(x^2+3)(x^2+4)} = \frac{4y+10}{(y+3)(y+4)}$$

let $\dfrac{4y+10}{(y+3)(y+4)} = \dfrac{A}{y+3} + \dfrac{B}{y+4}$

process

Ans 4 special

$$I = \int \frac{\tan\theta + \tan^3\theta}{1+\tan^3\theta}\, d\theta$$

$$= \int \frac{\tan\theta(1+\tan^2\theta)}{1+\tan^3\theta}\, d\theta$$

$$= \int \frac{\tan\theta \cdot \sec^2\theta\, d\theta}{1+\tan^3\theta}$$

put $\tan\theta = t \Rightarrow \sec^2\theta\, d\theta = dt$

$$\therefore I = \int \frac{t\, dt}{t^3+1}$$

$$= \int \frac{t\, dt}{(t+1)(t^2-t+1)}$$

L Q

let $\dfrac{t}{(t+1)(t^2-t+1)} = \dfrac{A}{t+1} + \dfrac{Bt+C}{t^2-t+1}$

$\Rightarrow \quad t = A(t^2-t+1) + (Bt+C)(t+1)$

$t = A(t^2-t+1) + (Bt^2 + Bt + Ct + C)$

$0 = A + B \quad \Rightarrow \quad B = -A$

$1 = -A + B + C \quad \Rightarrow \quad 1 = -2A + \cancel{C}$

$0 = A + C \qquad\qquad\qquad 0 = A + \cancel{C}$

$\underline{1 = -3A}$ $\quad A = -1/3$

$B = 1/3 \qquad C = 1/3$

$$\therefore \; I = \int \frac{-1}{3(t+1)} + \frac{\frac{1}{3}t + 1/3}{t^2-t+1}\,dt$$

$$= -\frac{1}{3}\int \frac{1}{t+1}\,dt + \frac{1}{3}\int \frac{t+1}{t^2-t+1}\,dt$$

$$I = -\frac{1}{3}\log|t+1| + \frac{1}{3} I_1 + C$$

$$I_1 = \int \frac{t+1}{t^2-t+1}\,dt$$

$$= \frac{1}{2}\int \frac{2t+2-1+1}{t^2-t+1}\,dt$$

$$= \frac{1}{2}\int \frac{(2t-1)+3}{t^2-t+1}\,dt$$

$$= \frac{1}{2}\int \frac{2t-1}{t^2-t+1}\,dt + \frac{3}{2}\int \frac{1}{t^2-t+1}\,dt$$

put $t^2-t+1 = z$

$(2t-1)\,dt = dz$

$$I_1 = \frac{1}{2}\int \frac{dz}{z} + \frac{3}{2}\int \frac{1}{(t-\frac{1}{2})^2 - \frac{1}{4} + 1}\,dt$$

$$= \frac{1}{2}\log|z| + \frac{3}{2}\int \frac{1}{(t-\frac{1}{2})^2 + (\frac{\sqrt{3}}{2})^2}\,dt$$

$$= \frac{1}{2}\log|t^2-t+1| + \frac{3}{2}\times\frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{2t-1}{\sqrt{3}}\right)$$

$$I_1 = \frac{1}{2}\log|t^2-t+1| + \sqrt{3}\tan^{-1}\left(\frac{2t-1}{\sqrt{3}}\right)$$

$$\therefore I = -\frac{1}{3}\log|t+1| + \frac{1}{3}\left[\frac{1}{2}\log|t^2-t+1| + \sqrt{3}\tan^{-1}\left(\frac{2t-1}{\sqrt{3}}\right)\right] + C$$

$$I = -\frac{1}{3}\log|1+\tan\theta| + \frac{1}{6}\log|\tan^2\theta - \tan\theta + 1| + \frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{2\tan\theta - 1}{\sqrt{3}}\right) + C$$

Ans

Ans. 5 (Special)

$$I = \int \frac{\sin x}{\sin(4x)}\,dx$$

$$I = \int \frac{\sin x}{2\sin(2x)\cos(2x)}\,dx$$

$$= \int \frac{\cancel{\sin x}}{4\cancel{\sin x}\cos x\cdot\cos(2x)}\,dx$$

$$= \frac{1}{4}\int \frac{1}{\cos x\cdot\cos(2x)}\,dx$$

$$= \frac{1}{4}\int \frac{1}{\cos x\cdot(1-2\sin^2 x)}\,dx$$

$$I = \frac{1}{4}\int \frac{\cos x}{(1-\sin^2 x)(1-2\sin^2 x)}\,dx$$

put $\sin x = t \Rightarrow \cos x\,dx = dt$

$$I = \frac{1}{4}\int \frac{dt}{(1-t^2)(1-2t^2)}$$

let $t^2 = y$ (temp)

$$\therefore \frac{1}{(1-t^2)(1-2t^2)} = \frac{1}{(1-y)(1-2y)}$$

let $\frac{1}{(1-y)(1-2y)} = \frac{A}{1-y} + \frac{B}{1-2y}$

$$1 = A(1-2y) + B(1-y)$$

$$0 = -2A - \cancel{B}$$

$$1 = A + \cancel{B}$$

$$1 = -A$$

$A = -1$ $\quad$ $B = 2$

$$\therefore I = \frac{1}{4}\int \frac{-1}{1-t^2} + \frac{2}{1-2t^2}\,dt$$

$$= -\frac{1}{4}\int \frac{1}{1-t^2}\,dt + \frac{1}{2}\int \frac{1}{1-2t^2}\,dt$$

$$= -\frac{1}{4}\times\frac{1}{2}\log\left|\frac{1+t}{1-t}\right| + \frac{1}{4}\int \frac{1}{\left(\frac{1}{\sqrt{2}}\right)^2 - t^2}\,dt$$

$$I = -\frac{1}{8}\log\left|\frac{1+t}{1-t}\right| + \frac{1}{4}\times\frac{1}{2\times\frac{1}{\sqrt{2}}}\log\left|\frac{1+\sqrt{2}t}{1-\sqrt{2}t}\right| + C \quad \text{Ans}$$

Type x^4 वाली type

(·) $\int \frac{x^2+1}{x^4+1}\,dx$ (·) $\int \frac{x^2-1}{x^4+1}\,dx$

- ✓ $D^r \to x^4$
- ✓ No odd power of x
- ✓ Constant of D^r must be +ve
- ✓ D^r Constant $=$ (N^r constant$)^2$

(·) Divide N & D by x^2

(·) N^r we get $\left(1+\frac{1}{x^2}\right)dx$ or $\left(1-\frac{1}{x^2}\right)dx$

(·) In D^r put $x-\frac{1}{x}=t$ or $x+\frac{1}{x}=t$

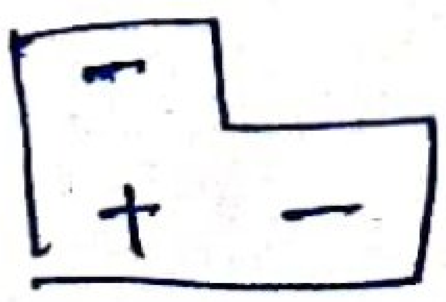
(·) $a^2+b^2=(a+b)^2-2ab = (a-b)^2+2ab$

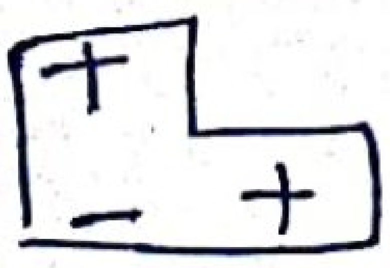
Qn 6 $I=\int \frac{x^2-1}{x^4+1}\,dx$

Divide by x^2

$$I=\int \frac{1-\frac{1}{x^2}}{x^2+\frac{1}{x^2}}\,dx$$

$$=\int \frac{1-\frac{1}{x^2}\,dx}{\left(x+\frac{1}{x}\right)^2-2}$$





put $x+\frac{1}{x}=t$

$\left(1-\frac{1}{x^2}\right)dx=dt$

$$I=\int \frac{dt}{t^2-2}$$

$$=\frac{1}{2\sqrt{2}}\log\left|\frac{t-\sqrt{2}}{t+\sqrt{2}}\right|+c$$

$$=\frac{1}{2\sqrt{2}}\log\left|\frac{x+\frac{1}{x}-\sqrt{2}}{x+\frac{1}{x}+\sqrt{2}}\right|+c$$

Ans

Qn. 7 $I = \int \frac{x^2+1}{x^4+x^2+1} dx$ (Single set)

पहेलगा

Divide by x^2

$$I = \int \frac{1+\frac{1}{x^2}}{x^2+\frac{1}{x^2}+1} dx$$

$$= \int \frac{1+\frac{1}{x^2}\, dx}{(\underbrace{x-\frac{1}{x}}_{t})^2+2+1}$$

$$= \int \frac{dt}{t^2+3}$$

$$= \frac{1}{\sqrt{3}} \tan^{-1}\left(\frac{t}{\sqrt{3}}\right) + C$$

$$I = \frac{1}{\sqrt{3}} \tan^{-1}\left(\frac{x-\frac{1}{x}}{\sqrt{3}}\right) + C \quad \text{Ans}$$

Qn. 8 $I = \int \frac{x^2}{x^4+1} dx$ (double set)

Divide by x^2

$$I = \int \frac{1}{x^2+\frac{1}{x^2}} dx$$

$$= \frac{1}{2}\int \frac{2}{x^2+\frac{1}{x^2}} dx$$

$$= \frac{1}{2}\int \frac{1+1+\frac{1}{x^2}-\frac{1}{x^2}}{x^2+1/x^2} dx$$

$$= \frac{1}{2}\int \frac{1+\frac{1}{x^2}\, dx}{x^2+\frac{1}{x^2}} + \frac{1}{2}\int \frac{1-\frac{1}{x^2}\, dx}{x^2+\frac{1}{x^2}}$$

$$= \frac{1}{2}\int \frac{1+\frac{1}{x^2}\, dx}{(x-\frac{1}{x})^2+2} + \frac{1}{2}\int \frac{1-\frac{1}{x^2}}{(x+\frac{1}{x})^2-2} dx$$

t ← $x-\frac{1}{x}$, $x+\frac{1}{x}$ → z

Process

Q.no 9 $I = \int \frac{1}{x^4 - 3x^2 + 1} dx$ (doubt)

Divide by x^2

$$I = \int \frac{\frac{1}{x^2}}{x^2 + \frac{1}{x^2} - 3} dx$$

$$= \frac{1}{2}\int \frac{\frac{2}{x^2}}{x^2 + \frac{1}{x^2} - 3} dx$$

$$= \frac{1}{2}\int \frac{\frac{1}{x^2} + \frac{1}{x^2} + 1 - 1}{x^2 + \frac{1}{x^2} - 3} dx$$

$$= \frac{1}{2}\int \frac{1 + \frac{1}{x^2}}{x^2 + \frac{1}{x^2} - 3} dx - \frac{1}{2}\int \frac{1 - \frac{1}{x^2}}{x^2 + \frac{1}{x^2} - 3} dx$$

$$= \frac{1}{2}\int \frac{1 + \frac{1}{x^2}}{(x - \frac{1}{x})^2 + 2 - 3} - \frac{1}{2}\int \frac{1 - \frac{1}{x^2}}{(x + \frac{1}{x})^2 - 2 - 3} dx$$

$x - \frac{1}{x} = t$ put $x + \frac{1}{x} = z$

$(1 + \frac{1}{x^2}) dx = dt$ $(1 - \frac{1}{x^2}) dx = dz$

$$I = \frac{1}{2}\int \frac{dt}{t^2 - 1} - \frac{1}{2}\int \frac{dz}{z^2 - 5}$$

$$= \frac{1}{2} \times \frac{1}{2} \log\left|\frac{t - 1}{t + 1}\right| - \frac{1}{2} \times \frac{1}{2\sqrt{5}} \log\left|\frac{z - \sqrt{5}}{z + \sqrt{5}}\right| + C$$

Ans

Q. 10

Special

$$I = \int \sqrt{\tan\theta}\, d\theta$$

put $\tan\theta = t^2$

$$\sec^2\theta\, d\theta = 2t\, dt$$

$$d\theta = \frac{2t\, dt}{\sec^2\theta}$$

$$d\theta = \frac{2t\, dt}{1+\tan^2\theta}$$

$$d\theta = \frac{2t\, dt}{1+t^4}$$

$$\therefore I = \int t \cdot \frac{2t\, dt}{1+t^4}$$

$$I = 2\int \frac{t^2}{t^4+1}\, dt \quad \text{(dour set)}$$

Divide by t^2

$$I = 2\int \frac{1}{t^2+\frac{1}{t^2}}\, dt$$

$$I = \int \frac{1+1+\frac{1}{t^2}-\frac{1}{t^2}}{t^2+\frac{1}{t^2}}\, dt$$

$$= \int \frac{1+\frac{1}{t^2}}{t^2+\frac{1}{t^2}}\, dt + \int \frac{1-\frac{1}{t^2}}{t^2+\frac{1}{t^2}}\, dt$$

$$= \int \frac{1+\frac{1}{t^2}}{(t-\frac{1}{t})^2+2}\, dt + \int \frac{1-\frac{1}{t^2}}{(t+\frac{1}{t})^2-2}\, dt$$

let u

let = v

$t-\frac{1}{t}=u$ & $t+\frac{1}{t}=v$

$\left(1+\frac{1}{t^2}\right)dt=du$ $\left(1-\frac{1}{t^2}\right)dt=dv$

$\therefore I=\int\frac{du}{u^2+2}+\int\frac{dv}{v^2-2}$

$I=\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{u}{\sqrt{2}}\right)+\frac{1}{2\sqrt{2}}\log\left|\frac{v-\sqrt{2}}{v+\sqrt{2}}\right|+C$

$=\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{t-\frac{1}{t}}{\sqrt{2}}\right)+\frac{1}{2\sqrt{2}}\log\left|\frac{t+\frac{1}{t}-\sqrt{2}}{t+\frac{1}{t}+\sqrt{2}}\right|+C$

$=\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{t^2-1}{\sqrt{2}\,t}\right)+\frac{1}{2\sqrt{2}}\log\left|\frac{t^2-\sqrt{2}t+1}{t^2+\sqrt{2}t+1}\right|+C$

$I=\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{\tan\theta-1}{\sqrt{2\tan\theta}}\right)+\frac{1}{2\sqrt{2}}\log\left|\frac{\tan\theta-\sqrt{2\tan\theta}+1}{\tan\theta+\sqrt{2\tan\theta}+1}\right|+C$

Ans

Qn 11 $I=\int\frac{1}{\sin^4x+\cos^4x}dx$

Divide by $\cos^4x$

$I=\int\frac{\sec^4x}{\tan^4x+1}dx$

$=\int\frac{\sec^2x\cdot\sec^2x\,dx}{\tan^4x+1}$

$=\int\frac{(\tan^2+1)\cdot\sec^2x\,dx}{\tan^4x+1}$

put $\tan x=t$

$\sec^2x\,dx=dt$

$I=\int\frac{t^2+1}{t^4+1}dt$ (Single sum)

$=\int\frac{1+\frac{1}{t^2}}{t^2+\frac{1}{t^2}}dt$

$=\int\frac{1+\frac{1}{t^2}}{\left(t-\frac{1}{t}\right)^2+2}dt$

$\underbrace{t-\frac{1}{t}}_{=z\,(\text{let})}$

$I=\int\frac{dz}{z^2+2}$ (Proceed)

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INTEGRATION

WORKSHEET No: 9

(Class No = 11)

Qn: 1 $I = \int \frac{x^2+1}{(x^2+2)(2x^2+1)}\,dx$ Ans $\frac{1}{3\sqrt{2}}\left[\tan^{-1}\left(\frac{x}{\sqrt{2}}\right) + \tan^{-1}\left(\sqrt{2}x\right)\right] + C$

Qn.2 → $I = \int \frac{x^2}{(x^2+1)(3x^2+4)}\,dx$

(Answers I will give you in solutions of this worksheet)

Qn.3 → $I = \int \frac{5x^2+20x+6}{x^3+2x^2+x}\,dx$

Qn. 4 → $I = \int \frac{1}{x^3-1}\,dx$

Qn. 5 → $I = \int \frac{\sin(2x)}{(1+\sin x)(3\sin x-2)}\,dx$

(It's a test type Assignment)

Qn 6 → $I = \int \frac{x^3-1}{x^3+x}\,dx$

Qn 7 → $I = \int \frac{x^2-3}{x^4+2x^2+9}\,dx$

Qn 8 $I = \int \frac{1}{x^4+x^2+1}\,dx$

Qn. 9 $I = \int \frac{x^2}{x^4+5x^2+1}\,dx$

Qn 10 $I = \int \sqrt{\cot x}\,dx$