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ULTIMATE MATHEMATICS: BY AJAY MITTAL

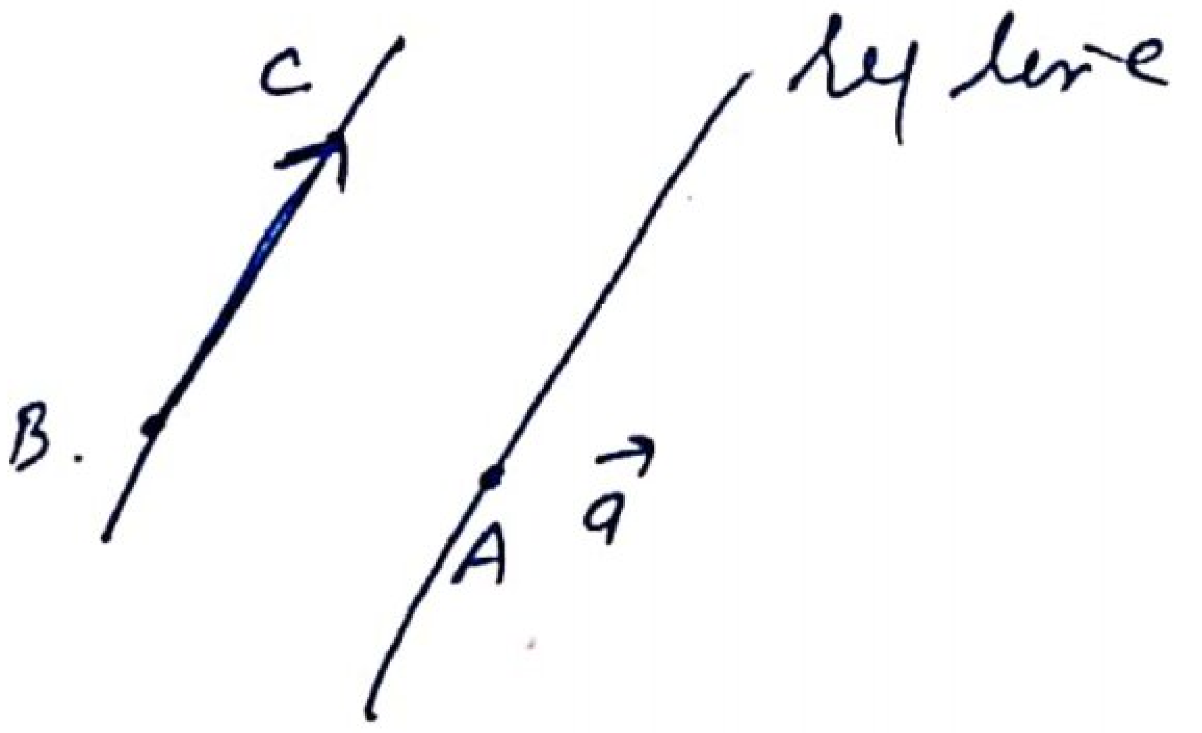
Chapter: 3-D

CLASS No: 2

Qu. 1 Find the vector equation of a line passing through a point with position vector $2\hat{i}-\hat{j}+\hat{k}$ and parallel to the line joining the points $-\hat{i}+4\hat{j}+\hat{k}$ and $\hat{i}+2\hat{j}+2\hat{k}$. Also reduce it to its cartesian form

Sol. Let points are $A(2\hat{i}-\hat{j}+\hat{k})$

$B(-\hat{i}+4\hat{j}+\hat{k})$ & $C(\hat{i}+2\hat{j}+2\hat{k})$



here $\vec{a} = 2\hat{i}-\hat{j}+\hat{k}$

$\vec{b} = \vec{BC} = 2\hat{i}-2\hat{j}+\hat{k}$

equation of line in vector form:

$$\vec{r} = \vec{a}+\lambda\vec{b}$$

$$\Rightarrow \boxed{\vec{r} = (2\hat{i}-\hat{j}+\hat{k})+\lambda(2\hat{i}-2\hat{j}+\hat{k})} \text{ Ans}$$

$$\Rightarrow x\hat{i}+y\hat{j}+z\hat{k} = (2\hat{i}-\hat{j}+\hat{k})+\lambda(2\hat{i}-2\hat{j}+\hat{k})$$

$$\Rightarrow x = 2+2\lambda \;;\; y = -1-2\lambda \;;\; z = 1+\lambda$$

$$\Rightarrow \boxed{\frac{x-2}{2} = \frac{y+1}{-2} = \frac{z-1}{1}} = \lambda$$

cartesian form

Qn 2 → Find the cartesian equation of a line passing through the points $A(2,-1,3)$ and $B(4,2,1)$. Also reduce it in to vector form

Soln here $x_1 = 2,\ y_1 = -1,\ z_1 = 3$

$x_2 = 4,\ y_2 = 2,\ z_2 = 1$

equat. of line in two point form

$$\frac{x-x_1}{x_2-x_1} = \frac{y-y_1}{y_2-y_1} = \frac{z-z_1}{z_2-z_1}$$

$$\Rightarrow \boxed{\frac{x-2}{2} = \frac{y+1}{3} = \frac{z-3}{-2}}$$ Ans

reduction

$$\frac{x-2}{2} = \frac{y+1}{3} = \frac{z-3}{-2} = \lambda \text{ (let)}$$

$$\Rightarrow x = 2\lambda+2\ ;\ y = 3\lambda-1\ ;\ z = -2\lambda+3$$

let $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

$$\vec{r} = (2\lambda+2)\hat{i} + (3\lambda-1)\hat{j} + (-2\lambda+3)\hat{k}$$

$$\boxed{\vec{r} = (2\hat{i} - \hat{j} + 3\hat{k}) + \lambda(2\hat{i} + 3\hat{j} - 2\hat{k})}$$ Ans

Qns 3 → The cartesian equation of a line is $6x-2 = 3y+1 = 2z-2$. Find its direction ratios, fixed point and its vector form

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Solution given equation of line

$$\frac{6x-2}{1} = \frac{3y+1}{1} = \frac{2z-2}{1}$$

$$\Rightarrow \frac{x-1/3}{\frac{1}{6}} = \frac{y+1/3}{\frac{1}{3}} = \frac{z-1}{1/2}$$

$$\Rightarrow \frac{x-1/3}{1} = \frac{y+1/3}{2} = \frac{z-1}{3} \quad \text{(standard form)}$$

DRS 1, 2, 3

fixed point = $(1/3, -1/3, 1)$

here $\vec{a} = \frac{1}{3}\hat{i} - \frac{1}{3}\hat{j} + \hat{k}$

$\vec{b} = \hat{i} + 2\hat{j} + 3\hat{k}$

$\vec{r} = \left(\frac{1}{3}\hat{i} - \frac{1}{3}\hat{j} + \hat{k}\right) + \lambda(\hat{i} + 2\hat{j} + 3\hat{k})$ Ans

Q.No. 4 → If the points $A(-1,3,2)$, $B(-4,2,-2)$ & $C(5,5,\lambda)$ are collinear. Using the concept of equation of line, find value of λ.

Soln equation of line AB

$$\frac{x+1}{-3} = \frac{y-3}{-1} = \frac{z-2}{-4}$$

Since A, B, C are collinear. Point C must satisfy this equation

$$\frac{5+1}{-3} = \frac{5-3}{-1} = \frac{\lambda-2}{-4} \Rightarrow \frac{\lambda-2}{-4} = -2 \Rightarrow \lambda = 10$$

Ans

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Qus 5 → Find the point on the line $\frac{x+2}{3} = \frac{y+1}{2} = \frac{z-3}{2}$ at a distance of $3\sqrt{2}$ from the point $(1,2,3)$.

Soln

Let $\frac{x+2}{3} = \frac{y+1}{2} = \frac{z-3}{2} = \lambda$

$\Rightarrow x = 3\lambda - 2, \; y = 2\lambda - 1; \; z = 2\lambda + 3$

Let $P(3\lambda-2, 2\lambda-1, 2\lambda+3)$ is the required point

Given $Q(1,2,3)$

Given $PQ = 3\sqrt{2}$

$\Rightarrow \sqrt{(3\lambda-3)^2 + (2\lambda-3)^2 + (2\lambda)^2} = 3\sqrt{2}$

$\Rightarrow 9\lambda^2 + 9 - 18\lambda + 4\lambda^2 + 9 - 12\lambda + 4\lambda^2 = 18$

$\Rightarrow 17\lambda^2 - 30\lambda = 0$

$\Rightarrow \lambda(17\lambda - 30) = 0$

$\lambda = 0$ or $\lambda = \frac{30}{17}$

∴ required points are $(-2,-1,3)$ or

$\left(\frac{54}{17}, \frac{43}{17}, \frac{111}{17}\right)$ Ans

Qu. 6 → Prove that the lines $x = ay + b$; $z = cy + d$ and $x = a'y + b'$; $z = c'y + d'$ are perpendicular if $aa' + cc' + 1 = 0$

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Soln
given lines $x = ay+b$; $z = cy+d$

$$\Rightarrow \quad \frac{x-b}{a} = y \quad \& \quad \frac{z-d}{c} = y$$

$$\Rightarrow \quad \frac{x-b}{a} = y = \frac{z-d}{c}$$

$$\Rightarrow \quad \frac{x-b}{a} = \frac{y-0}{1} = \frac{z-d}{c}$$

DR's of this line $= a, 1, c$

2nd line (Similar) $\quad \frac{x-b'}{a'} = \frac{y-0}{1} = \frac{z-d'}{c'}$

DR's of this line : $a', 1, c'$

Since lines are $\perp^r$

$\therefore \quad \boxed{aa'+1+cc'=0}$ proved

Ques 7 → Find the equation of a line parallel to the line $\frac{x-2}{3} = \frac{2y+1}{2} = \frac{5-z}{1}$ and passing through the point $(1,-1,0)$

Soln : standard equation of given line

$$\frac{x-2}{3} = \frac{y+1/2}{1} = \frac{z-5}{-1}$$

D.R's of this line $= 3, 1, -1$

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Since Required line is parallel to given line

∴ D.R's of required line are $= 3\lambda, \lambda, -\lambda$

given point on Required line: $(1,-1,0)$

Equation of Req. line

$$\frac{x-1}{3\lambda} = \frac{y+1}{\lambda} = \frac{z-0}{-\lambda}$$

$$\Rightarrow \boxed{\frac{x-1}{3} = \frac{y+1}{1} = \frac{z-0}{-1}} \quad \text{Ans}$$

Qn. 8 → Find the Cartesian equation of the line passing through the point $(-1,3,-2)$ and perpendicular to the lines

$$\frac{x}{1} = \frac{y}{2} = \frac{z}{3} \quad \text{and} \quad \frac{x+2}{-3} = \frac{y-1}{2} = \frac{z+1}{5}$$

Soln Let D.R's of required line are $= a, b, c$

D.R's of I^st line $= 1, 2, 3$

D.R's of 2^nd line $= -3, 2, 5$

Since Required line is $\perp$ to the given lines

$$a+2b+3c=0$$
$$-3a+2b+5c=0$$

Method

$$\frac{a}{10-6} = \frac{-b}{5+9} = \frac{c}{2+6} = \lambda$$

$\Rightarrow a = 4\lambda, \; b = -14\lambda \; ; \; c = 8\lambda$

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∴ equation of line is

$$\frac{x+1}{4\lambda} = \frac{y-3}{-14\lambda} = \frac{z+2}{8\lambda}$$

$$\Rightarrow \boxed{\frac{x+1}{2} = \frac{y-3}{-7} = \frac{z+2}{4}}$$ Ans

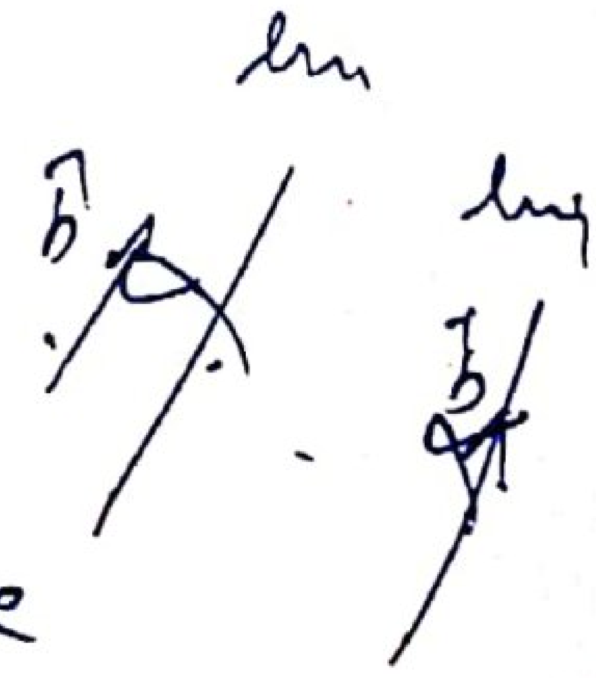
Q.No. 9 → A line passes through $(2, -1, 3)$ and is perpendicular to the lines $\vec{r} = (\hat{i}+\hat{j}-\hat{k}) + \lambda(2\hat{i}-2\hat{j}+\hat{k})$ and $\vec{r} = (2\hat{i}-\hat{j}+3\hat{k}) + \mu(\hat{i}+2\hat{j}+2\hat{k})$. Find vector equation and cartesian equation

Soln: given lines: $\vec{r} = (\hat{i}+\hat{j}-\hat{k}) + \lambda(2\hat{i}-2\hat{j}+\hat{k})$

$\vec{r} = (2\hat{i}-\hat{j}-3\hat{k}) + \mu(\hat{i}+2\hat{j}+2\hat{k})$

here $\vec{b_1} = 2\hat{i}-2\hat{j}+\hat{k}$

$\vec{b_2} = \hat{i}+2\hat{j}+2\hat{k}$



let $\vec{b}$ is vector parallel to required line

Since req. line is $\perp^r$ to the given lines

$\Rightarrow \vec{b} \perp \vec{b_1}$ & $\vec{b} \perp \vec{b_2}$

$\Rightarrow \boxed{\vec{b} = t(\vec{b_1} \times \vec{b_2})}$

$$\Rightarrow \vec{b} = t\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -2 & 1 \\ 1 & 2 & 2 \end{vmatrix} = t(-6\hat{i}-3\hat{j}+6\hat{k})$$

P.V of given point is $\vec{a} = 2\hat{i}-\hat{j}+3\hat{k}$

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Required equation of line

$\vec{r} = \vec{a} + t\vec{b}$

$\Rightarrow \boxed{\vec{r} = (2\hat{i} - \hat{j} + 3\hat{k}) + t(-6\hat{i} - 3\hat{j} + 6\hat{k})}$ Ans

Cartesian form

$$\boxed{\frac{x-2}{-6} = \frac{y+1}{-3} = \frac{z-3}{6}}$$ Ans

Qn. 10 ⇒ Find the equation of the line passing through the point $(1,-1,1)$ and perpendicular to the lines joining the points $(4,3,2)$, $(1,-1,0)$ and $(1,2,-1)$ & $(2,1,1)$

Soln Ist line passes through $(4,3,2)$ & $(1,-1,0)$

D.R's of this line $= -3, -4, -2$

2nd line passes through $(1,2,-1)$ & $(2,1,1)$

DR's of this line $= 1, -1, 2$

let DR's of req. line $= a, b, c$

Since Required line $\perp^r$ to the given lines

$$-3a - 4b - 2c = 0$$
$$a - b + 2c = 0$$

get $a =$ $b =$ $c =$

Q.No. 11 → Show that the lines

$\vec{r} = (\hat{i}+\hat{j}-\hat{k}) + \lambda(3\hat{i}-\hat{j})$ & $\vec{r} = (4\hat{i}-\hat{k}) + \mu(2\hat{i}+3\hat{k})$

Intersect. Also find point of Intersection.

Sol Converting given vector equations in to Cartesian form

$$\frac{x-1}{3} = \frac{y-1}{-1} = \frac{z+1}{0} = \lambda$$

and

$$\frac{x-4}{2} = \frac{y-0}{0} = \frac{z+1}{3} = \mu$$

Any point on I^{st} line $= (3\lambda+1, -\lambda+1, -1)$

Any point on 2^{nd} line $= (2\mu+4, 0, 3\mu-1)$

If two lines Intersect then for same value of λ & μ

$3\lambda+1 = 2\mu+4$ | $-\lambda+1=0$ | $-1 = 3\mu-1$

$\lambda = 1$ | $\mu = 0$

$3+1 = 0+4$

$4=4$ ∴ λ & μ satisfy the 3^{rd} equation

∴ lines Intersect

put value of λ : point is $(4, 0, -1)$

put value of μ : point is $(4, 0, -1)$

∴ Clearly lines Intersect at $(4, 0, -1)$

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WORKSHEET NO: 2 $\boxed{3\text{-}D}$

Qn: 1 → The cartesian equation of a line is $3x+1 = 6y-2 = 1-z$. Find the fixed point, direction ratios and also its vector equation

Ans $\left(-\frac{1}{3}, \frac{1}{3}, 1\right)$; $2, 1, -6$; $\vec{r} = -\frac{1}{3}\hat{i} + \frac{1}{3}\hat{j} + \hat{k} + \lambda(2\hat{i} + \hat{j} - 6\hat{k})$

Qn 2 → If the points with position vectors $-2\hat{i} + 3\hat{j}$, $\hat{i} + \lambda\hat{j} + 3\hat{k}$ and $7\hat{i} - \hat{k}$ are collinear. Using the concept of equation of line, find the value of λ Ans $\lambda = 2$

Qn 3 → Find the points on the line $\frac{x+2}{3} = \frac{y+1}{2} = \frac{z-3}{2}$ at a distance of 5 units from the point $P(1,3,3)$

Ans $(4,3,7)$ & $(-2,-1,3)$

Qn 4 → Find the cartesian equation of a line passing through $(1,-1,2)$ and parallel to the line whose equation is $\frac{x-3}{1} = \frac{y-1}{2} = \frac{z+1}{-2}$. Also reduce in vector form

Ans $\frac{x-1}{1} = \frac{y+1}{2} = \frac{z-2}{-2}$; $\vec{r} = (\hat{i} - \hat{j} + 2\hat{k}) + \lambda(\hat{i} + 2\hat{j} - 2\hat{k})$

Qn. 5 → Find the value of λ so that the lines are perpendicular to each other

$$\frac{x-5}{5\lambda+2} = \frac{2-y}{5} = \frac{1-z}{-1} \text{ and } \frac{x}{1} = \frac{2y+1}{4\lambda} = \frac{1-z}{-3}$$

Ans $\lambda = 1$

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Qn. 6 → Determine the equation of the line passing through the point $(1, 2, -4)$ and perpendicular to the lines $\frac{x-8}{8} = \frac{y+9}{-16} = \frac{10-z}{-7}$ and $\frac{15-x}{-3} = \frac{2y-58}{16} = \frac{5-z}{5}$

ANS $\frac{x-1}{24} = \frac{y-2}{61} = \frac{z+4}{112}$

Qn. 7 → Find the equation of the line passing through the point $(\hat{i}+\hat{j}-3\hat{k})$ and perpendicular to the lines $\vec{r} = \hat{i} + \lambda(2\hat{i}+\hat{j}-3\hat{k})$ and $\vec{r} = (2\hat{i}+\hat{j}-\hat{k}) + \mu(\hat{i}+\hat{j}+\hat{k})$

ANS $\vec{r} = (\hat{i}+\hat{j}-3\hat{k}) + \lambda(4\hat{i}-5\hat{j}+\hat{k})$

Qn. 8 → Find the angle b/w two lines whose direction ratios are proportional to a, b, c and $b-c, c-a, a-b$ ANS $\pi/2$

Qn. 9 → Find the equation of a line parallel to x-axis and passing through the origin ANS $\frac{x}{1} = \frac{y}{0} = \frac{z}{0}$

Qn. 10 → Show that the lines $\frac{x+1}{3} = \frac{y+3}{5} = \frac{z+5}{7}$ and $\frac{x-2}{1} = \frac{y-4}{3} = \frac{z-6}{5}$ intersect. Also find their point of Intersection ANS $(10, 14, 4)$

Qn. 11 → Show that the lines $\vec{r} = 3\hat{i}+2\hat{j}-4\hat{k}+\lambda(\hat{i}+2\hat{j}+2\hat{k})$ and $\vec{r} = 5\hat{i}-2\hat{j}+\mu(3\hat{i}+2\hat{j}+6\hat{k})$ are Intersecting. Hence, find their point of Intersection ANS $(-1, -6, -12)$

Qn. 12 → Find the equation of a line passing through $(2, -1, 3)$ and $\perp^r$ to the lines joining the points $(2, 3, -1), (1, -2, 0)$ and $(3, -4, 1), (2, 1, 3)$

ANS = self

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