SOLUTIONS $\boxed{\text{EXAM NO: 9}}$ (CLASS - 12th)

$\boxed{\text{SECTION: A}}$

Ans 1 $R = \{(a, b) : a \le b^3\}$

Symmetric $(1, 2) \in R$ as $1 \le 2^3$

but $(2, 1) \notin R$ as $2 \nleq 1^3$

$\therefore$ R is not symmetric

Reflexive $\frac{1}{2} \in R$ (Real number set)

but $(\frac{1}{2}, \frac{1}{2}) \notin R$ as $\frac{1}{2} \nleq (\frac{1}{2})^3$

Transitive

$(9, 4) \in R$ and $(4, 2) \in R$

as $9 \le 4^3$ and $4 \le 2^3$

but $9 \nleq 2^3$

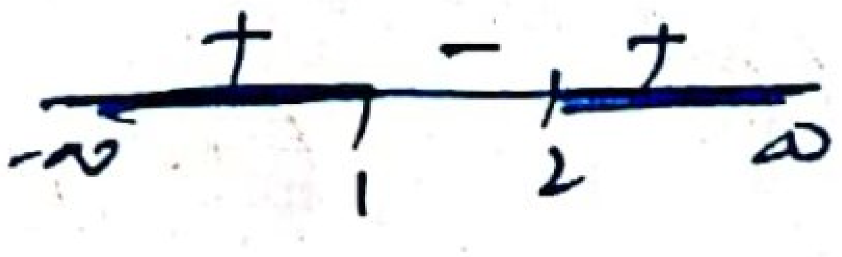
$\therefore (9, 2) \notin R$ R is not transitive.

Ans 2 → (i) $f(x) = \sqrt{x^2 - 3x + 2}$

$f(x)$ exists for all $x \in$ such that

$x^2 - 3x + 2 \ge 0$

$\Rightarrow (x-1)(x-2) \ge 0$



$\therefore$

Domain: $x \in (-\infty, 1] \cup [2, \infty)$ Ans

(2) $f(x) = \dfrac{1}{\sqrt{x+2}}$

$f(x)$ exists for all $x \in$ such that

$x + 2 > 0$

$\Rightarrow x > -2$ $\therefore$ Domain $x \in (-2, \infty)$ Ans

Ans 3 →

$$f(x) = x^2-4x+5$$

let $y = f(x)$

$$\Rightarrow \quad y = x^2-4x+5$$

$$\Rightarrow \quad x^2-4x+(5-y)=0$$

By Quadratic formula

$$x = \frac{4 \pm \sqrt{16-4(5-y)}}{2}$$

$$x = \frac{4 \pm \sqrt{4y-4}}{2}$$

$$x = \frac{4 \pm 2\sqrt{y-1}}{2}$$

$$x = 2 \pm \sqrt{y-1}$$

x exists such that

$$y-1 \geq 0$$

$$\Rightarrow \quad y \geq 1$$

$\therefore$ Range $= [1, \infty)$ Ans

Alternate method

$$f(x) = x^2-4x+5$$

$$f(x) = (x-2)^2-4+5$$

$$f(x) = (x-2)^2+1$$

Min value of $f(x)$ is when $(x-2)=0$

$$\therefore \quad f_{min} = 1$$

Maximum value of $f(x) = \infty$

$\therefore$ Range $= [1, \infty)$ Ans

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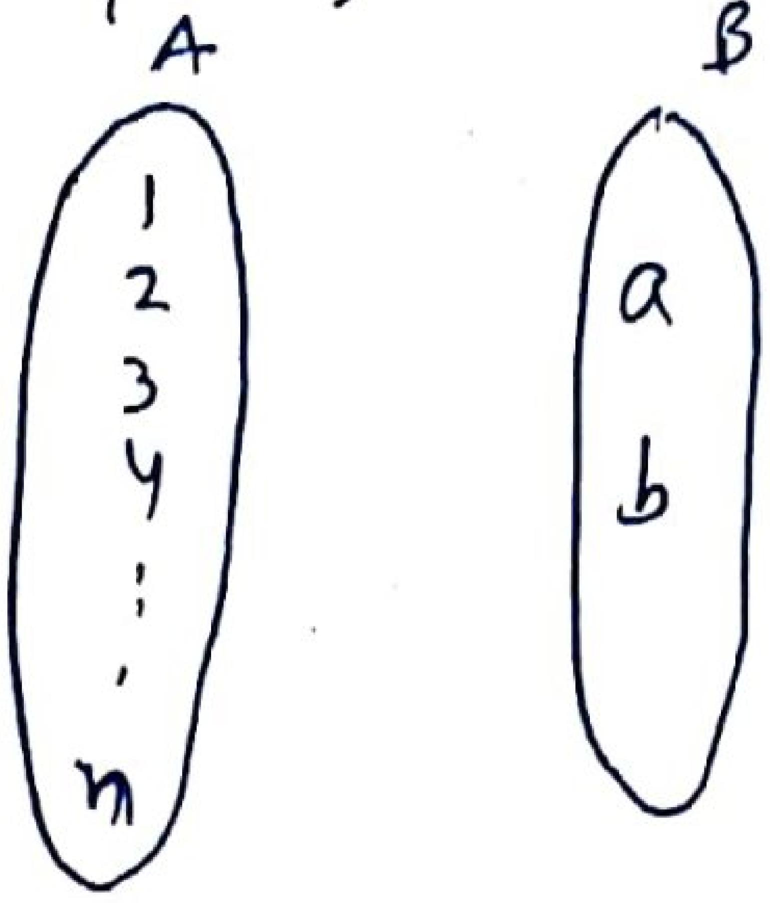
Ans 4 → $A = \{1, 2, 3, 4, \ldots n\}$ & $B = \{a, b\}$



Number of Injections / one-one function $= 0$

Since when 1 connects to a and 2 can connects only to b remaining all elements of domain not covered

$\therefore$ No of one-one function $= 0$

Number of surjections / on-to functions $= 2^n - 2$

Since every element of domain can be connected to any element of codomain in 2 ways

$\therefore$ no of way of connecting n elements $= 2 \times 2 \times 2 \ldots n$ times $= 2^n$

out of these 2^n ways, there are two ways in which all the elements of domain connect to either 'a' or 'b'

$\therefore$ No of surjections $= 2^n - 2$ Ans

Ans 5 → $A = \{1, 2, 3\}$

$R_1 = \{(1,1)\ (2,2)\ (3,3)\}$

$R_2 = \{(1,1)\ (2,2)\ (3,3)\ (1,2)\ (2,1)\}$

$R_3 = \{(1,1)\ (2,2)\ (3,3)\ (2,3)\ (3,2)\}$

$R_4 = \{(1,1)\ (2,2)\ (3,3)\ (1,3)\ (3,1)\}$

$R_5 = \{(1,1)\ (2,2)\ (3,3)\ (1,2)\ (2,1)\ (1,3)\ (3,1)\ (2,3)\ (3,2)\}$

$\therefore \boxed{\text{Ans} = 5}$

Ans 6 → $f(x) = \frac{1}{2-\cos x}$

Domain: $f(x)$ exists for all x such that

$2-\cos x \neq 0$

$\cos x \neq 2$ {which is already true}

∴ for all x, $f(x)$ exists

$\boxed{\text{Domain} = R}$

Range: we have $-1 \leq \cos x \leq 1$

$\Rightarrow \quad 1 \geq -\cos x \geq -1$

$\Rightarrow \quad 3 \geq 2-\cos x \geq 1$

(or) $\quad 1 \leq 2-\cos x \leq 3$

Reciprocal (sign change)

$\Rightarrow \quad 1 \geq \frac{1}{2-\cos x} \geq \frac{1}{3}$

$\Rightarrow \quad 1 \geq f(x) \geq \frac{1}{3}$

∴ $f(x) \in \left[\frac{1}{3}, 1\right]$

∴ $\boxed{\text{Range} = \left[\frac{1}{3}, 1\right]}$ Ans

Ans 7 → $x = e^t \cos t$ & $y = e^t \sin t$ at $t = \pi/4$

Diff wrt t

$\frac{dx}{dt} = -e^t \sin t + \cos t \cdot e^t = e^t(\cos t - \sin t)$

$\frac{dy}{dt} = e^t \cos t + e^t \sin t = e^t(\cos t + \sin t)$

$$\frac{dy}{dx} = \frac{e^t(\cos t + \sin t)}{e^t(\cos t - \sin t)}$$

$$m = \left(\frac{dy}{dx}\right)_{t=\pi/4} = \frac{\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}}{\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}} = \frac{\sqrt{2}}{0} = \infty$$

$\Rightarrow$ $m = \infty$

$\Rightarrow$ tangent is parallel to Y-axis

$\therefore$ $\boxed{\text{tangent makes angle } \pi/2 \text{ with X-axis}}$ Ans

Ans 8 → $y = \sin x$ at $(0,0)$

Diff wrt x

$$\frac{dy}{dx} = \cos x$$

Slope of ~~~~ tangent at $(0,0)$ = $\cos 0 = 1$

Slope of Normal at $(0,0)$ $= -1$ (-ve reciprocal)

Equation of Normal at $(0,0)$

$y - 0 = -1(x - 0)$

$\Rightarrow$ $y = -x$

$\Rightarrow$ $\boxed{x + y = 0}$ Ans

Ans 9 → $f(x) = x^x$

taking log on both sides

$\log(f(x)) = x \log x$

Diff

$$\frac{1}{f(x)} \cdot f'(x) = 1 + \log x$$

$\Rightarrow f'(x) = x^x(1+\log x)$

for stationary point, put $f'(x) = 0$

$x^x(1+\log x) = 0$

$\Rightarrow 1+\log x = 0 \quad \cdots \{\because x^x \neq 0\}$

$\Rightarrow \log x = -1$

$\Rightarrow x = e^{-1}$

$\therefore \boxed{x = \frac{1}{e}}$ Ans

Ans 10 → $f(x) = x^2 - 8x + 17$

$f(x) = (x-4)^2 - 16 + 17$

$f(x) = (x-4)^2 + 1$

$f(x)$ is minimum when $x - 4 = 0$

$\therefore$ $\boxed{\text{Minimum value of } f(x) = 1}$ Ans

alternate method

$f(x) = x^2 - 8x + 17$

$f'(x) = 2x - 8$

put $f'(x) = 0$

$x = 4$

$f''(x) = 2 > 0 \quad$ so $f(x)$ is minimum

$f_{min} = 16 - 32 + 17 = 1$

$\therefore$ $\boxed{\text{Min value of } f(x) = 1}$ Ans

SECTION : B

Ans 11

$f : A \times B \to B \times A$

$f(a, b) : (b, a)$

one-one

let (a, b) & $(c, d) \in A \times B$ (domain)

and $f(a, b) = f(c, d)$

$\Rightarrow (b, a) = (d, c)$

$\Rightarrow b = d$ and $a = c$

$\Rightarrow (a, b) = (c, d)$

$\therefore f$ is one-one

ON-TO

(·) $n(A \times B) = n(B \times A)$ -- {always}

(·) also f is one-one -- {just proved}

$\Rightarrow$ ~~codo~~ codomain = Range

$\therefore f$ must be on-to

$\therefore f$ is a bijective function **Ans**

Ans 12-A $f : R_+ \to [-5, \infty)$

$f(x) = 9x^2 + 6x - 5$

one-one let $x_1, x_2 \in R_+$ (domain)

and $f(x_1) = f(x_2)$

$\Rightarrow 9x_1^2 + 6x_1 - \cancel{5} = 9x_2^2 + 6x_2 - \cancel{5}$

$\Rightarrow 9(x_1^2 - x_2^2) + 6(x_1 - x_2) = 0$

$\Rightarrow (x_1-x_2)\,(9x_1+9x_2+6)=0$

$\Rightarrow x_1-x_2=0$ (or) $9x_1+9x_2+6=0$

(Rejected) Since $x_1, x_2 \in R_+$

$\therefore \boxed{x_1=x_2}$

f is one-one function

<u>ON-TO</u> let $y=f(x)$

$\Rightarrow$ $y=9x^2+6x-5$

$\Rightarrow$ $9x^2+6x-(5+y)=0$

$$x=\frac{-6\pm\sqrt{36+36(5+y)}}{18}$$

$$x=\frac{-6\pm 6\sqrt{1+5+y}}{18}$$

$$x=\frac{-1\pm\sqrt{6+y}}{3}$$ But $x \neq \frac{-1-\sqrt{6+y}}{3}$ $\{x\in R_+ y$

for each $y\in[-5,\infty)$, there exists an element x in domain (R_+) such that

$$f(x)=9\left(\frac{-1+\sqrt{6+y}}{3}\right)^2+6\left(\frac{-1+\sqrt{6+y}}{3}\right)-5$$

$$=\cancel{9}\left(\frac{1+6+y-2\sqrt{6+y}}{\cancel{9}}\right)-2+2\sqrt{6+y}-5$$

$$=\cancel{7}-\cancel{2\sqrt{6+y}}+\cancel{2\sqrt{6+y}}-\cancel{7}+y$$

$f(x)=y$

$\therefore$ f is on-to

Hence f is a bijective function Ans

Ans: 13 → Symmetric

let $(a,b) \in R_1 \cap R_2$

$\Rightarrow (a,b) \in R_1$ and $(a,b) \in R_2$

$\Rightarrow (b,a) \in R_1$ and $(b,a) \in R_2$ --- {$\because$ R_1 & R_2 are symmetric Relations}

$\Rightarrow (b,a) \in R_1 \cap R_2$

$\therefore R_1 \cap R_2$ is Symmetric Relation

Reflexive

for all $a \in A$

$(a,a) \in R_1$ and $(a,a) \in R_2$ --- {$\because$ R_1 & R_2 are reflexively}

$\Rightarrow (a,a) \in R_1 \cap R_2$

$\therefore R_1 \cap R_2$ is reflexive relation

Transitive

let $(a,b) \in R_1 \cap R_2$ and $(b,c) \in R_1 \cap R_2$

$\Rightarrow (a,b) \in R_1$ and $(a,b) \in R_2$ and $(b,c) \in R_1$ and $(b,c) \in R_2$

$\Rightarrow (a,b) \in R_1$ & $(b,c) \in R_1$ and $(a,b) \in R_2$ & $(b,c) \in R_2$

$\Rightarrow (a,c) \in R_1$ and $(a,c) \in R_2$ -- {Since R_1 & R_2 are transitive}

$\Rightarrow (a,c) \in R_1 \cap R_2$

$\therefore R_1 \cap R_2$ is transitive relation

$\therefore R_1 \cap R_2$ is an Equivalence relation Ans

Ans 14 → equation of curves

$C_1: \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ & $C_2: xy = c^2$

let the point of Intersection / point of Contact is (x_1, y_1)

Differentiate

$$\frac{2x}{a^2} - \frac{2y\frac{dy}{dx}}{b^2} = 0 \quad \& \quad x\frac{dy}{dx} + y = 0$$

$$\frac{y\frac{dy}{dx}}{b^2} = \frac{x}{a^2} \quad \& \quad \frac{dy}{dx} = \frac{-y}{x}$$

$$\Rightarrow \frac{dy}{dx} = \frac{b^2 x}{a^2 y}.$$

Slope $m_1 = \frac{b^2 x_1}{a^2 y_1}$ & Slope $m_2 = \frac{-y_1}{x_1}$

Since two curves cut orthogonally / $\perp^r$

$\therefore m_1 m_2 = -1$

$$\Rightarrow \frac{b^2 \cancel{x_1}}{a^2 \cancel{y_1}} \times \frac{\cancel{-}\cancel{y_1}}{\cancel{x_1}} = \cancel{-}1$$

$\Rightarrow \boxed{b^2 = a^2}$ is the required condition Ans

Ans 15 → $f(x) = x + \frac{1}{x}$

Diff w.r.t x $f'(x) = 1 - \frac{1}{x^2}$

for Max / Min put $f'(x) = 0$

$\Rightarrow 1 - \frac{1}{x^2} = 0$

$\Rightarrow \frac{1}{x^2} = 1$

$\Rightarrow x^2 = 1$

$\Rightarrow \boxed{x = \pm 1}$

Diff again $f''(x) = \frac{2}{x^3}$

$f''(1) = \frac{2}{1} = 2 > 0 \therefore f(x)$ is Minimum at $x = 1$

$f''(-1) = \frac{2}{-1} = -2 < 0 \therefore f(x)$ is Maximum at $x = -1$

Min value of $f(x) = f(1) = 1 + \frac{1}{1} = 2$

Max. value of $f(x) = f(-1) = -1 + \frac{1}{-1} = -2$

Clearly $\boxed{\text{Maximum value} < \text{Min value}}$ ANS

Ans 16 → $f(x) = \sin(2x) - x \quad ; x \in [-\frac{\pi}{2}, \frac{\pi}{2}]$

$f'(x) = 2\cos(2x) - 1$

for Max/Min, put $f'(x) = 0$

$\Rightarrow 2\cos(2x) - 1 = 0$

$\Rightarrow \cos(2x) = \frac{1}{2} = \cos(\frac{\pi}{3})$ or $\cos(\frac{-\pi}{3})$

$\Rightarrow 2x = \frac{\pi}{3}$ & $2x = -\frac{\pi}{3}$

$\Rightarrow \boxed{x = \frac{\pi}{6}}, \boxed{x = -\frac{\pi}{6}}$

Now

$$f\left(-\frac{\pi}{2}\right) = \sin(-\pi) + \frac{\pi}{2} = 0 + \frac{\pi}{2} = \frac{\pi}{2}$$

$$f\left(-\frac{\pi}{6}\right) = \sin\left(-\frac{\pi}{3}\right) + \frac{\pi}{6} = -\frac{\sqrt{3}}{2} + \frac{\pi}{6}$$

$$f\left(\frac{\pi}{6}\right) = \sin\left(\frac{\pi}{3}\right) - \frac{\pi}{6} = \frac{\sqrt{3}}{2} - \frac{\pi}{6}$$

$$f\left(\frac{\pi}{2}\right) = \sin(\pi) - \frac{\pi}{2} = 0 - \frac{\pi}{2} = -\frac{\pi}{2}$$

Greater value of $f(x) = \pi/2$

least value of $f(x) = -\pi/2$

$$\boxed{\text{Difference} = \frac{\pi}{2} - \left(-\frac{\pi}{2}\right) = \pi}$$ Ans

Ans 17 → $9y^2 = x^3$ --- { Misprint in Question paper }

let point of contact be (x_1, y_1)

Diff. wrt x

$$18y\frac{dy}{dx} = 3x^2$$

$$\frac{dy}{dx} = \frac{x^2}{6y}$$

Slope of Tangent at $(x_1, y_1) = \frac{x_1^2}{6y_1}$

Slope of Normal at $(x_1, y_1) = \frac{-6y_1}{x_1^2}$

also slope of Normal $= \pm 1$ { Since Normal makes equal Intercepts with Axes $\therefore \theta = 45^\circ$, $\theta = 135^\circ$ }

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$$\Rightarrow \frac{-6y_1}{x_1^2} = \pm 1$$

$$\Rightarrow 6y_1 = \pm x_1^2$$

$$\Rightarrow y_1 = \frac{\pm x_1^2}{6}$$

also we have

$$9y_1^2 = x_1^3 \quad \text{---} \left\{\because (x_1, y_1) \text{ lies on the curve}\right\}$$

$$\Rightarrow 9\left(\frac{x_1^4}{36}\right) = x_1^3$$

$$\Rightarrow x_1^4 = 4x_1^3$$

$$\Rightarrow x_1^4 - 4x_1^3 = 0$$

$$\Rightarrow x_1^3(x_1 - 4) = 0$$

$$\Rightarrow x_1 = 0 \; ; \quad x_1 = 4$$

$$y_1 = 0 \qquad 9y_1^2 = 64$$

$$y_1 = \pm \frac{8}{3}$$

But $(0,0)$ rejected $\{\because$ line which makes equal Intercepts with Axes can't pass through the origin$\}$

$$\therefore \boxed{\text{Required points are } \left(4, \pm \frac{8}{3}\right)} \quad \text{Ans}$$

Ans 18 → $f(x) = \dfrac{4\sin x - 2x - x\cos x}{2 + \cos x}$

By default Range is $[0, 2\pi]$

Diff wrt x

$$f'(x) = \frac{(2+\cos x)\left[4\cos x - 2 - \{-x\sin x + \cos x\}\right] - (4\sin x - 2x - x\cos x)(-\sin x)}{(2+\cos x)^2}$$

$$= \frac{(2+\cos x)(3\cos x + x\sin x - 2) + (4\sin^2 x - 2x\sin x - x\sin x\cos x)}{(2+\cos x)^2}$$

$$= \frac{6\cos x \cancel{+2x\sin x} - 4 + 3\cos^2 x \cancel{+ x\sin x\cos x} - 2\cos x + 4\sin^2 x \cancel{- 2x\sin x} \cancel{- x\sin x\cos x}}{(2+\cos x)^2}$$

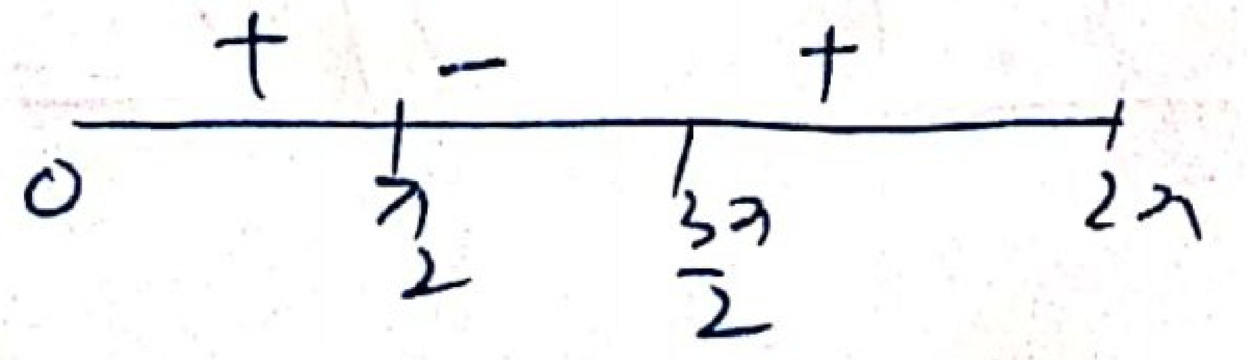
$$= \frac{4\cos x \cancel{-4} + 3\cos^2 x \cancel{+4} - 4\cos^2 x}{(2+\cos x)^2}$$

$$= \frac{4\cos x - 4\cos^2 x}{(2+\cos x)^2}$$

$$f'(x) = \frac{\cos x(4 - \cos x)}{(2+\cos x)^2} = 0$$

$\cos x = 0$; $4 - \cos x \neq 0$ $\because \cos x \neq 4$

$\Rightarrow x = \frac{\pi}{2}$; $x = \frac{3\pi}{2}$



for $x \in [0, \frac{\pi}{2}) \cup (\frac{3\pi}{2}, 2\pi]$ $f(x)$ is strictly increasing

for $x \in (\frac{\pi}{2}, \frac{3\pi}{2})$ $f(x)$ is strictly decreasing Ans

Ans 19 → $y(1+x^2) = 2-x$

Since curve cuts X-axis ∴ $y=0$

$\Rightarrow 0(1+x^2) = 2-x$

$\Rightarrow \boxed{x=2}$

∴ point of contact / intersection is $(2,0)$

Differentiate equation of curve

$y(2x) + (1+x^2)\frac{dy}{dx} = -1$

$\Rightarrow \frac{dy}{dx} = \frac{-1-2xy}{1+x^2}$

Slope of Tangent at $(2,0)$ $= \frac{-1}{1+4} = \frac{-1}{5}$

Now equation of Tangent at $(2,0)$

$y-0 = \frac{-1}{5}(x-2)$

$\Rightarrow 5y = -x+2$

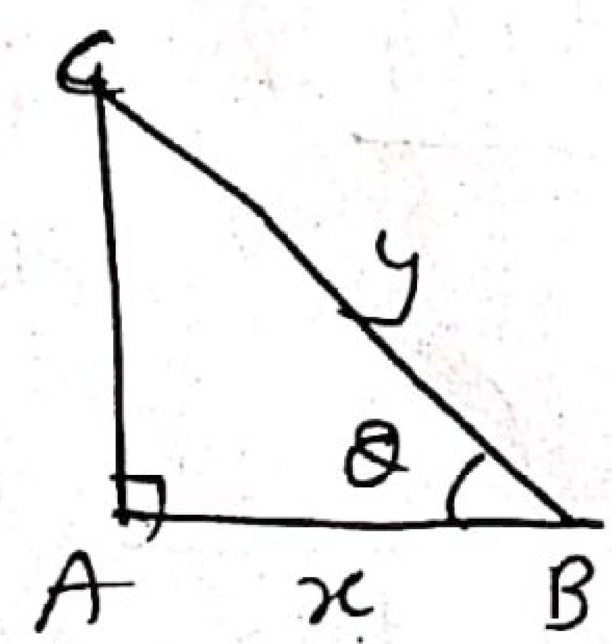
$\Rightarrow \boxed{x+5y-2=0}$ Ans

SECTION : C

Ans 20 → let $AB = x$
& $BC = y$

let $S \rightarrow$ be their sum

$S = x+y$ --- (Given) --- (1)



Let $A \rightarrow$ area of $\triangle ABC$

$$A = \frac{1}{2} x \sqrt{y^2 - x^2} \quad \text{--- (to be Max)}$$

$$A = \frac{1}{2} x \sqrt{(S-x)^2 - x^2} \quad \text{--- \{from eq (1)\}}$$

$$A = \frac{1}{2} x \sqrt{S^2 - 2Sx}$$

Squaring

$$A^2 = \frac{1}{4} x^2 (S^2 - 2Sx)$$

$$A^2 = \frac{1}{4} (S^2 x^2 - 2Sx^3)$$

Let $A^2 = z$

then A is Max/Min as according to z is Max/Min.

$$z = \frac{1}{4} (S^2 x^2 - 2Sx^3)$$

$$\frac{dz}{dx} = \frac{1}{4}\left(\cancel{\ }\right) = \frac{1}{4} (2xS^2 - 6Sx^2)$$

for Max/Min put $\frac{dz}{dx} = 0$

$$2S^2 x = 6Sx^2$$

$$\Rightarrow \cancel{x = \frac{2S}{2}} \Rightarrow 2S^2 x = 6Sx^2$$

$$\Rightarrow \boxed{x = \frac{S}{3}}$$

Diff again

$$\frac{d^2 z}{dx^2} = \frac{1}{4} (2S^2 - 12Sx)$$

$$\left(\frac{d^2 z}{dx^2}\right)_{x = S/3} = \frac{1}{4} (2S^2 - 4S^2) = \frac{-2S^2}{4} < 0$$

$\therefore$ Z is Max.

$\therefore$ Area of triangle ΔABC is Maximum at $x = \frac{s}{3}$

put $x = \frac{s}{3}$ in eq (1)

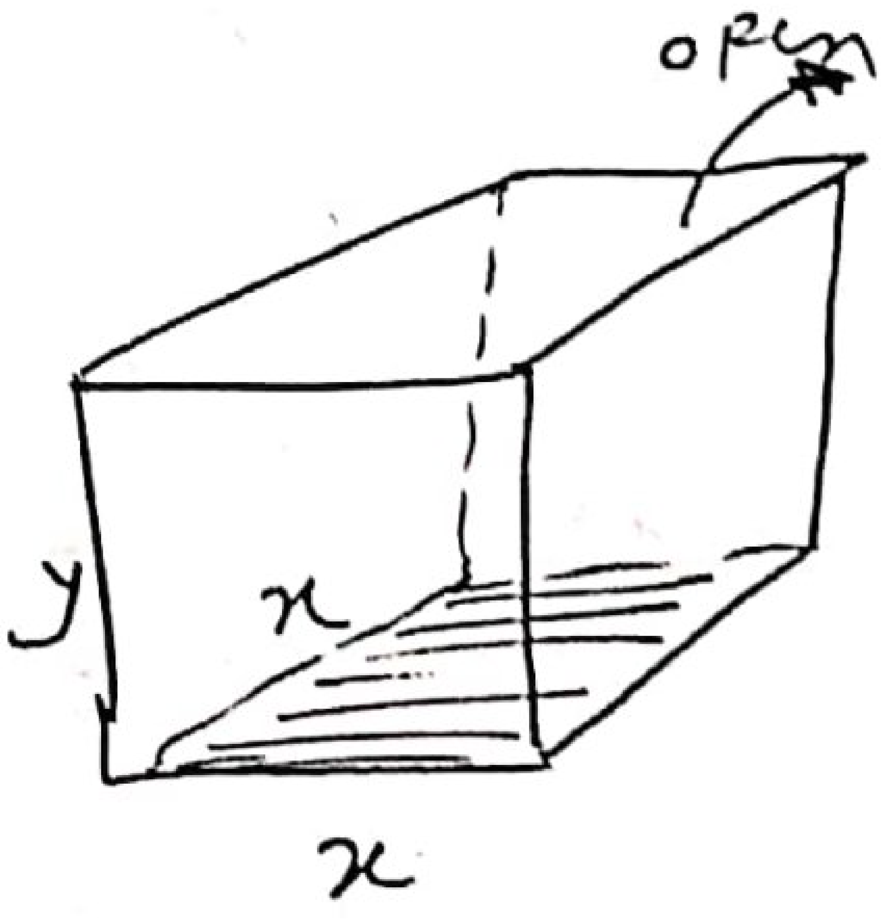
$$s = \frac{s}{3} + y$$

$$\Rightarrow \boxed{y = \frac{2s}{3}}$$

Now In ΔABC let θ be the angle b/w x & y

$$\therefore \cos\theta = \frac{x}{y} = \frac{\frac{s}{3}}{\frac{2s}{3}} = \frac{1}{2}$$

$$\rightarrow \boxed{\theta = \frac{\pi}{3}}$$ Ans



ONS: 21 →

let x be l & b of box

y ~~be~~ height of box

$$c^2 = x^2 + 2xy + 2xy$$

$$\rightarrow c^2 = x^2 + 4xy \quad \text{--- (given)} \quad \text{--- (1)}$$

let $V \rightarrow$ volume of box

$$V = x^2 y \quad \text{--- (to be Max)}$$

$$V = x^2 \left(\frac{c^2 - x^2}{4x} \right)$$

$$V = \frac{1}{4}(c^2 x - x^3)$$

Diff wrt x

$$\frac{dv}{dx} = \frac{1}{4}\left(c^2 - 3x^2\right)$$

for Max/Min put $\frac{dv}{dx} = 0$

$\Rightarrow c^2 = 3x^2$

$\Rightarrow \boxed{x = \frac{c}{\sqrt{3}}}$

Diff again

$$\frac{d^2v}{dx^2} = \frac{1}{4}(-6x)$$

$$\left(\frac{d^2v}{dx^2}\right)_{x=\frac{c}{\sqrt{3}}} = \frac{1}{4}\left(-\frac{6c}{\sqrt{3}}\right) < 0$$

$\therefore$ volume of box is Maximum at $x = \frac{c}{\sqrt{3}}$

put $x = \frac{c}{\sqrt{3}}$ in eq(1)

$$c^2 = \frac{c^2}{3} + \frac{4c}{\sqrt{3}}y$$

$$\Rightarrow \frac{2c^2}{3} = \frac{4c}{\sqrt{3}}y$$

$$\Rightarrow \boxed{y = \frac{\sqrt{3}c}{6}}$$

Max volume $V_{max} = x^2 y$

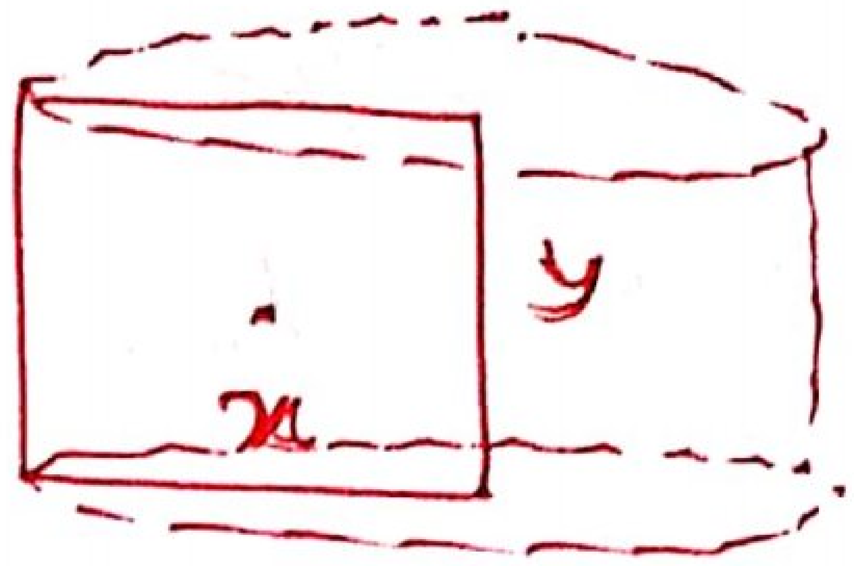
$$= \frac{c^2}{3} \times \frac{\sqrt{3}c}{6}$$

$$\boxed{V_{max} = \frac{c^3}{6\sqrt{3}} \text{ cubic units}}$$

Ans

Ans 22 →

let $x \rightarrow$ length of rectangle
$y \rightarrow$ breadth " "



$36 = 2x + 2y$

$\Rightarrow x + y = 18$ --- (given) --(1)

Radius of virtual cylinder = x
height of " " = y

let $V \rightarrow$ volume of virtual cylinder

$V = \pi x^2 y$ --- (to be Max)

$V = \pi x^2 (18 - x)$ --- {from eq (1)}

$V = \pi (18x^2 - x^3)$

$\frac{dV}{dx} = \pi (36x - 3x^2)$

for Max / Min put $\frac{dV}{dx} = 0$

$\Rightarrow 36x = 3x^2$

$\Rightarrow \boxed{x = 12}$

Diff again

$\frac{d^2V}{dx^2} = \pi (36 - 6x)$

$\left(\frac{d^2V}{dx^2}\right)_{x=12} = \pi (36 - 72) = -36\pi < 0$

$\therefore$ volume of cylinder is Max at $x = 12$

put $x=12$ in eq (1) — length = 12 cm
we get $y=6$ — Breadth = 6 cm

(Dimension)

Now $V_{max} = \pi x^2 y$
$= \pi (144)(6)$

$$\boxed{V_{max} = 864\pi \text{ cm}^3}$$ Ans

Ans 28 ⇒

$(\alpha,\beta) R (\gamma,\delta) \Rightarrow \alpha+\delta = \beta+\gamma$

Symmetric let $(\alpha,\beta) R (\gamma,\delta)$
$\Rightarrow \alpha+\delta = \beta+\gamma$
$\Rightarrow \delta+\alpha = \gamma+\beta$
$\Rightarrow \gamma+\beta = \delta+\alpha$
$\Rightarrow (\gamma,\delta) R (\alpha,\beta)$ ∴ R is Symmetric.

Reflexive for each $(\alpha,\beta) \in A\times A$
we always have $\alpha+\beta = \beta+\alpha$
$\Rightarrow (\alpha,\beta) R (\beta,\alpha)$ ∴ R is reflexive

Transitive
let $(\alpha,\beta) R (\gamma,\delta)$ and $(\gamma,\delta) R (a,b)$
$\Rightarrow \alpha+\delta = \beta+\gamma$ and $\gamma+b = \delta+a$
$\Rightarrow \alpha+\delta = \beta+\gamma$ and $\delta = \gamma+b-a$
$\Rightarrow \alpha+\cancel{\gamma}+b-a = \beta+\cancel{\gamma}$
$\Rightarrow \alpha+b = \beta+a$
$\Rightarrow (\alpha,\beta) R (a,b)$ ∴ R is transitive

∴ R is an equivalence Relation Ans

Equivalence class $[(3,6)] = \{(1,4)(2,5)(3,6)(4,7)(5,8)(6,9)\}$ Ans