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# ULTIMATE MATHEMATICS - BY AJAY MITTAL

CHAPTER: 3-D

CLASS NO: 4

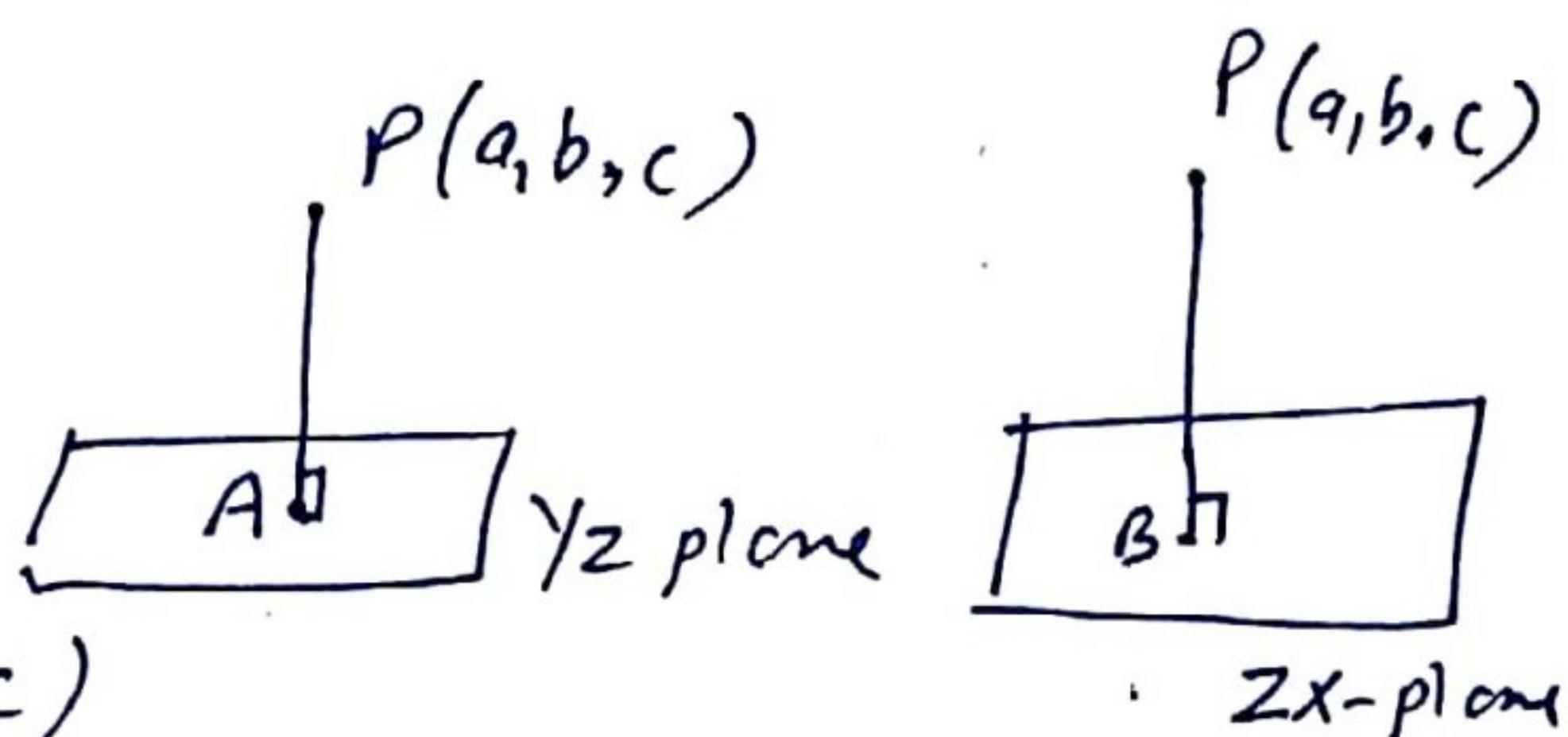
Ques 1 → From a point  $P(a, b, c)$  perpendiculars  $PA$  and  $PB$  are drawn to  $YZ$  and  $Zx$ -planes. Find the equation of the plane  $OAB$

Solution

Coordinate of  $A$  is  $(0, b, c)$

and coordinate of  $B$  is  $(a, 0, c)$

and origin  $O(0, 0, 0)$



equation of plane  $OAB$  is given by

$$\begin{vmatrix} x-0 & y-0 & z-0 \\ 0-0 & b-0 & c-0 \\ a-0 & 0-0 & c-0 \end{vmatrix} = 0 \Rightarrow \begin{vmatrix} x & y & z \\ 0 & b & c \\ a & 0 & c \end{vmatrix} = 0$$

$$\Rightarrow x(bc) - y(-ac) + z(-ab) = 0$$

divide by  $abc$

$$\Rightarrow \boxed{\frac{x}{a} + \frac{y}{b} - \frac{z}{c} = 0} \text{ Ans}$$

Ques 2 → A plane meets the coordinate axes at  $A, B$  and  $C$  respectively such that the centroid of triangle  $ABC$  is  $(1, -2, 3)$ . Find the equation of the plane

(2)

Solution

Let  $A(a, 0, 0)$ ,  $B(0, b, 0)$   
 $\& C(0, 0, c)$

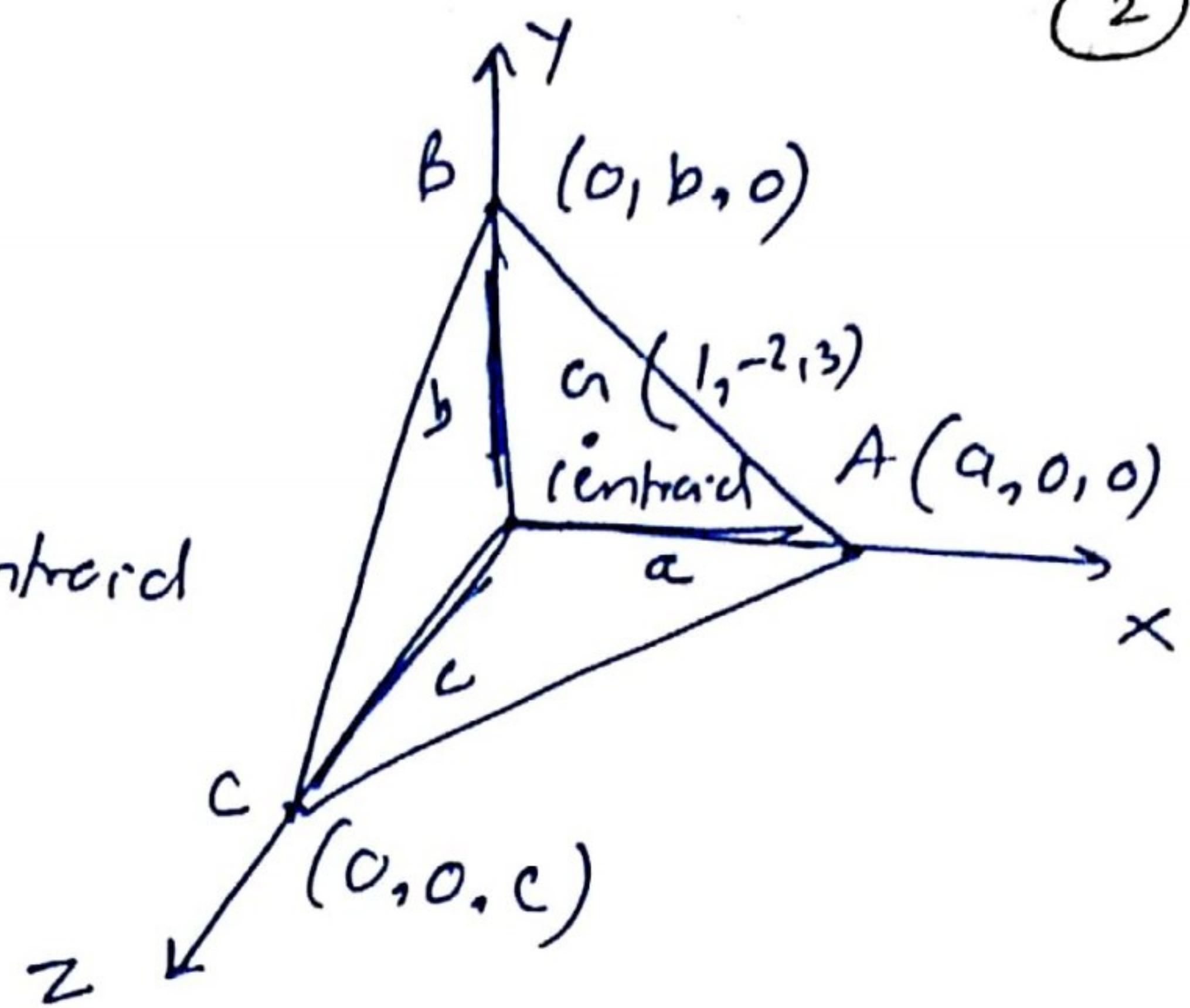
Now  $G(1, -2, 3)$  is the centroid

Then

$$l = \frac{a+0+0}{3} \Rightarrow a=3$$

$$-2 = \frac{0+b+0}{3} \Rightarrow b=-6$$

$$3 = \frac{0+0+c}{3} \Rightarrow c=9$$



Now equation of plane in Intercept form

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$$

$$\Rightarrow \frac{x}{3} - \frac{y}{6} + \frac{z}{9} = 1$$

$$\Rightarrow \boxed{6x - 3y + 2z = 18}$$
 Ans

Ques 3 → Let  $\vec{n}$  be a vector of magnitude  $2\sqrt{3}$  such that it makes equal acute angles with the coordinate axes.

Find the vector and cartesian equation of the plane passing through the point  $(1, -1, 2)$  and normal to  $\vec{n}$ .

Solutn

Given  $\alpha = \beta = \gamma$

$$\Rightarrow \cos \alpha = \cos \beta = \cos \gamma$$

$$\Rightarrow l = m = n$$

we have

$$l^2 + m^2 + n^2 = 1$$

(3)

$$\Rightarrow 3l^2 = 1$$

$$\Rightarrow l = \frac{1}{\sqrt{3}} \quad \left\{ l \neq -\frac{1}{\sqrt{3}} ; \text{ since acute angles}\right.$$

$$\therefore l = m = n = \frac{1}{\sqrt{3}}$$

Name  $\vec{n} = l\hat{i} + m\hat{j} + n\hat{k}$

$$\Rightarrow \vec{n} = \frac{1}{\sqrt{3}}\hat{i} + \frac{1}{\sqrt{3}}\hat{j} + \frac{1}{\sqrt{3}}\hat{k}$$

Given  $|\vec{n}| = 2\sqrt{3}$

we know that  $\vec{n}' = |\vec{n}|\vec{n}$

$$\vec{n}' = 2\sqrt{3} \left( \frac{1}{\sqrt{3}}\hat{i} + \frac{1}{\sqrt{3}}\hat{j} + \frac{1}{\sqrt{3}}\hat{k} \right)$$

$$\Rightarrow \boxed{\vec{n}' = 2\hat{i} + 2\hat{j} + 2\hat{k}}$$

Let position vector of given point  $(1, -1, 2)$  is

$$\vec{a} = \hat{i} - \hat{j} + 2\hat{k}$$

Vector equation of plane :

$$(\vec{r} - \vec{a}) \cdot \vec{n}' = 0$$

$$\Rightarrow \vec{r} \cdot \vec{n}' = \vec{a} \cdot \vec{n}'$$

$$\Rightarrow \vec{r} \cdot (2\hat{i} + 2\hat{j} + 2\hat{k}) = (\hat{i} - \hat{j} + 2\hat{k}) \cdot (2\hat{i} + 2\hat{j} + 2\hat{k})$$

$$\Rightarrow \vec{r} \cdot (2\hat{i} + 2\hat{j} + 2\hat{k}) = 2 - 2 + 4$$

$$\Rightarrow \boxed{\vec{r} \cdot (\hat{i} + \hat{j} + \hat{k}) = 2}$$

$$(x\hat{i} + y\hat{j} + z\hat{k}) \cdot (\hat{i} + \hat{j} + \hat{k}) = 2$$

$$\Rightarrow \boxed{x + y + z = 2} \text{ Cartesian equation of plane} \quad \underline{\text{Ans.}}$$

Ques 4 → find the equation of the plane that bisects the line segment joining the points  $(1, 2, 3)$  and  $(3, 4, 5)$  and is at right angle to it.

Soln let the given points are

$$A(1, 2, 3) \text{ & } B(3, 4, 5)$$

let  $C$  be the Mid point of  $AB$

∴ Coordinates of  $C$  is  $(2, 3, 4)$

Given  $AB \perp$  plane

∴  $\vec{BA}$  is normal to the plane

$$\Rightarrow \vec{BA} = \vec{n}$$

{ You can take also  
 $\vec{AB} = \vec{n}$  }

$$\Rightarrow \vec{n} = -2\hat{i} - 2\hat{j} - 2\hat{k}$$

Point  $C$  is on the plane

Let its position vector is  $\vec{a} = 2\hat{i} + 3\hat{j} + 4\hat{k}$

Now equation of plane is

$$(\vec{r} - \vec{a}) \cdot \vec{n} = 0$$

$$\Rightarrow \vec{r} \cdot \vec{n} = \vec{a} \cdot \vec{n}$$

$$\Rightarrow \vec{r} \cdot (-2\hat{i} - 2\hat{j} - 2\hat{k}) = (2\hat{i} + 3\hat{j} + 4\hat{k}) \cdot (-2\hat{i} - 2\hat{j} - 2\hat{k})$$

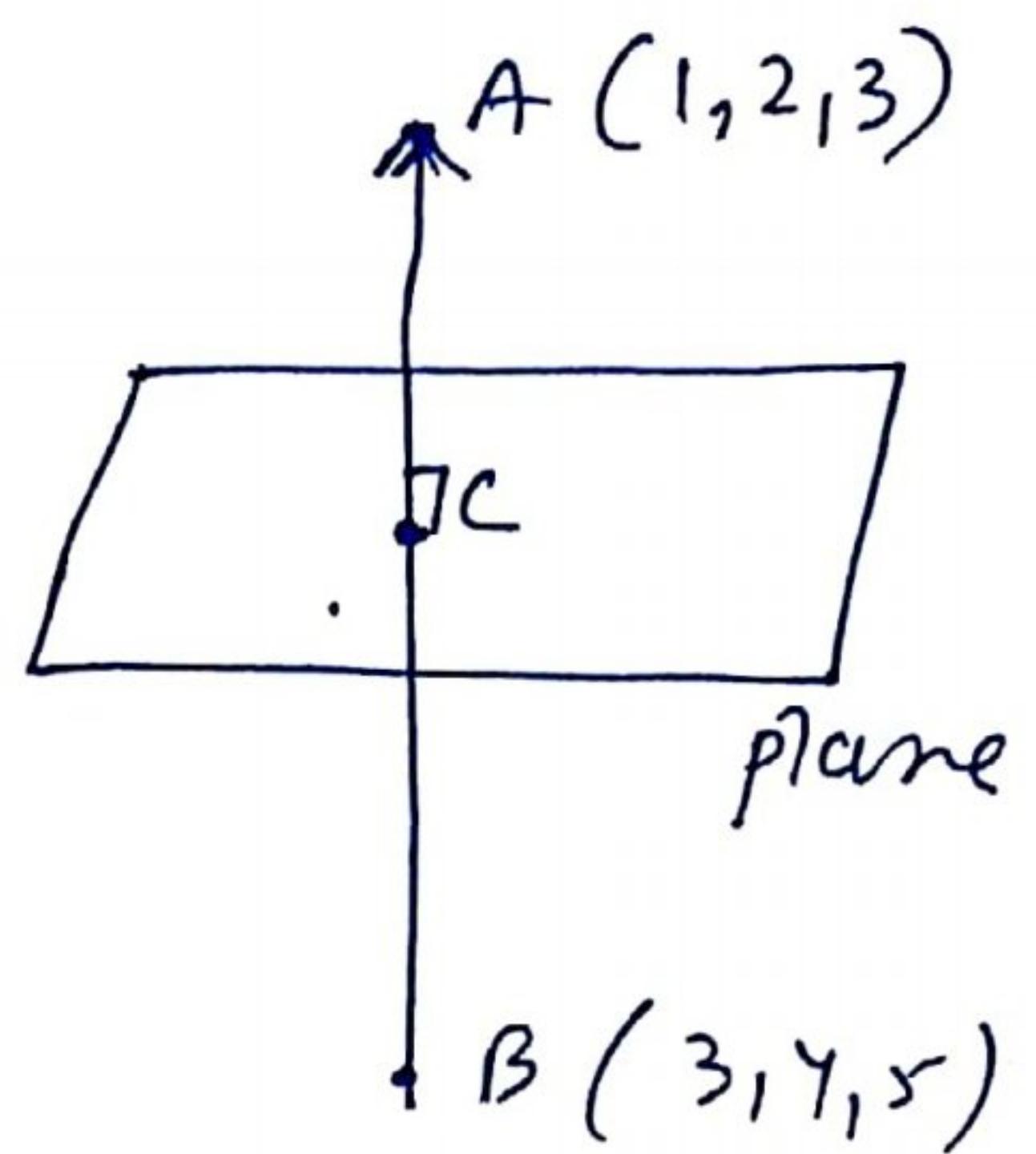
$$\Rightarrow \vec{r} \cdot (-2\hat{i} - 2\hat{j} - 2\hat{k}) = -4 - 6 - 8$$

$$\Rightarrow \vec{r} \cdot (-2\hat{i} - 2\hat{j} - 2\hat{k}) = -18$$

$$\Rightarrow \boxed{\vec{r} \cdot (\hat{i} + \hat{j} + \hat{k}) = 9} \text{ vector form}$$

$$(x\hat{i} + y\hat{j} + z\hat{k}) \cdot (\hat{i} + \hat{j} + \hat{k}) = 9 \Rightarrow \boxed{x + y + z = 9} \text{ cartesian form}$$

Ans



(1)

Ques 5 → Reduce the equation  $\vec{r} \cdot (-3\hat{i} + 4\hat{j} - 12\hat{k}) + 5 = 0$  to normal form and hence find the length of perpendicular from the origin to the plane

Solution Given equation:

$$\vec{r} \cdot (-3\hat{i} + 4\hat{j} - 12\hat{k}) + 5 = 0$$

$$\Rightarrow \vec{r} \cdot (-3\hat{i} + 4\hat{j} - 12\hat{k}) = -5$$

$$\Rightarrow \vec{r} \cdot (3\hat{i} - 4\hat{j} + 12\hat{k}) = 5$$

Divide by  $\sqrt{9 + 16 + 144} = 13$

$$\Rightarrow \left[ \vec{r} \cdot \left( \frac{3}{13}\hat{i} - \frac{4}{13}\hat{j} + \frac{12}{13}\hat{k} \right) = \frac{5}{13} \right] \text{normal form}$$

Comparing with  $\vec{r} \cdot \vec{n} = d$

$$\text{then } d = \frac{5}{13}$$

$$\therefore \left[ \text{Distance of the plane from the origin} = \frac{5}{13} \right] \text{Ans}$$

Ques 6 → Find the equation of a plane which is at a distance of  $3\sqrt{3}$  from the origin and the normal to which is equally inclined with the coordinate axes.

Solution Given  $d = 3\sqrt{3}$

$$\text{Given } \alpha = \beta = \gamma$$

$$\Rightarrow \cos \alpha = \cos \beta = \cos \gamma$$

$$\Rightarrow l = m = n$$

$$\text{Now } l^2 + m^2 + n^2 = 1$$

$$\Rightarrow 3l^2 = 1$$

(6)

$$\Rightarrow l = \pm \frac{1}{\sqrt{3}}$$

$$\therefore l = m = n = \pm \frac{1}{\sqrt{3}}$$

now  $\vec{n} = l\hat{i} + m\hat{j} + n\hat{k}$

$$\Rightarrow \vec{n} = \pm \frac{1}{\sqrt{3}}(1\hat{i} + 1\hat{j} + 1\hat{k})$$

Equation of plane (normal form)

$$\vec{x} \cdot \vec{n} = d$$

$$\Rightarrow \vec{x} \cdot \left( \pm \frac{1}{\sqrt{3}}(1\hat{i} + 1\hat{j} + 1\hat{k}) \right) = 3\sqrt{3}$$

$$\Rightarrow \boxed{\vec{x} \cdot (1\hat{i} + 1\hat{j} + 1\hat{k}) = \pm 9} \text{ vector form}$$

$$(or) (x\hat{i} + y\hat{j} + z\hat{k}) \cdot (1\hat{i} + 1\hat{j} + 1\hat{k}) = \pm 9$$

$$\Leftrightarrow \boxed{x + y + z = \pm 9} \text{ cartesian form } \underline{\underline{Ans}}$$

Ques 7 \* Find the equation of the plane passing through

the point  $(1, 1, -1)$  and perpendicular to the planes

$$x + 2y + 3z = 7 \text{ and } 2x - 3y + 4z = 0$$

Solution Given planes  $x + 2y + 3z = 7$  (or)  $\vec{x} \cdot (1 + 2\hat{j} + 3\hat{k}) = 7$   
 and  $2x - 3y + 4z = 0$  (or)  $\vec{x} \cdot (2\hat{i} - 3\hat{j} + 4\hat{k}) = 0$

then  $\vec{n}_1 = \hat{i} + 2\hat{j} + 3\hat{k}$

$\vec{n}_2 = 2\hat{i} - 3\hat{j} + 4\hat{k}$

let  $\vec{n}$  be the normal vector to the required plane

Since, Required plane is  $\perp^{\circ}$  to the given planes

(7)

$$\therefore \vec{n} \perp \vec{n}_1 \text{ & } \vec{n} \perp \vec{n}_2$$

$$\Rightarrow \vec{n} = \lambda (\vec{n}_1 \times \vec{n}_2)$$

$$\Rightarrow \vec{n} = \lambda \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ 2 & -3 & 4 \end{vmatrix}$$

$$\Rightarrow \vec{n} = \lambda (17\hat{i} + 2\hat{j} - 7\hat{k})$$

Let position vector of given point  $(1, 1, -1)$  is  
 $\vec{a} = \hat{i} + \hat{j} - \hat{k}$

Now, equation of plane is

$$(\vec{r} - \vec{a}) \cdot \vec{n} = 0$$

$$\Rightarrow \vec{r} \cdot \vec{n} = \vec{a} \cdot \vec{n}$$

$$\Rightarrow \vec{r} \cdot \lambda(17\hat{i} + 2\hat{j} - 7\hat{k}) = (\hat{i} + \hat{j} - \hat{k}) \cdot \lambda(17\hat{i} + 2\hat{j} - 7\hat{k})$$

$$\Rightarrow \vec{r} \cdot (17\hat{i} + 2\hat{j} - 7\hat{k}) = 17 + 2 - 7$$

$$\Rightarrow \vec{r} \cdot (17\hat{i} + 2\hat{j} - 7\hat{k}) = 26$$

$$\Rightarrow (x\hat{i} + y\hat{j} + z\hat{k}) \cdot (17\hat{i} + 2\hat{j} - 7\hat{k}) = 26$$

$$\Rightarrow \boxed{17x + 2y - 7z = 26} \quad \underline{\text{Ans}}$$

Alternate Method (Cayley Method)

$$\text{Given planes } x + 2y + 3z = 7$$

$$\text{and } 2x - 3y + yz = 0$$

D.R's of normal vector to 1st plane = 1, 2, 3

D.R's of normal vector to 2nd plane = 2, -3, 1

Let D.R's of the normal vector to required plane =  $a, b, c$

(8)

Since required plane is perpendicular to the given planes

$$\therefore a + 2b + 3c = 0$$

$$2a - 3b + 4c = 0$$

$$\Rightarrow \frac{a}{8+9} = \frac{-b}{4-6} = \frac{c}{-3-4} = \lambda$$

$$\Rightarrow a = 17\lambda ; b = 2\lambda ; c = -7\lambda$$

Point on plane is  $(1, 1, -1)$

here  $x_1 = 1, y_1 = 1, z_1 = -1$

Cartesian equation of plane is

$$(x-x_1)a + (y-y_1)b + (z-z_1)c = 0$$

$$\Rightarrow (x-1)(17\lambda) + (y-1)(2\lambda) + (z+1)(-7\lambda) = 0$$

$$\Rightarrow 17x - 17 + 2y - 2 - 7z - 7 = 0$$

$$\Rightarrow [17x + 2y - 7z = 26] \quad \underline{\text{Any}}$$

Ques 8 → Find the equation of the plane passing through the points  $(2, 1, -1)$  &  $(-1, 3, 4)$  and perpendicular to the plane  $x-2y+4z=10$

Solution

given plane

$$x - 2y + 4z = 10$$

$$(08) \quad \vec{n} \cdot (i - 2j + 4k) = 10$$

$$\text{here } \vec{n}_1 = i - 2j + 4k$$

let  $\vec{n}$  = normal vector to the required plane

since planes are  $\perp^r$

$$\therefore \vec{n} \perp \vec{n}_1$$

$$\text{Now } \vec{AB} = (-1-2)i + (3-1)j + (4+1)k \\ \vec{AB} = -3i + 2j + 5k$$

$\vec{n}$  also  $\perp^r$  to  $\vec{AB}$

$$\Rightarrow \vec{n} = \lambda (\vec{n}_1 \times \vec{AB})$$

$$\Rightarrow \vec{n} = \lambda \begin{vmatrix} i & j & k \\ 1 & -2 & 4 \\ -3 & 2 & 5 \end{vmatrix}$$

$$\vec{n} = \lambda (-18i - 17j - 4k)$$

let position vector of point A (2, 1, -1) is

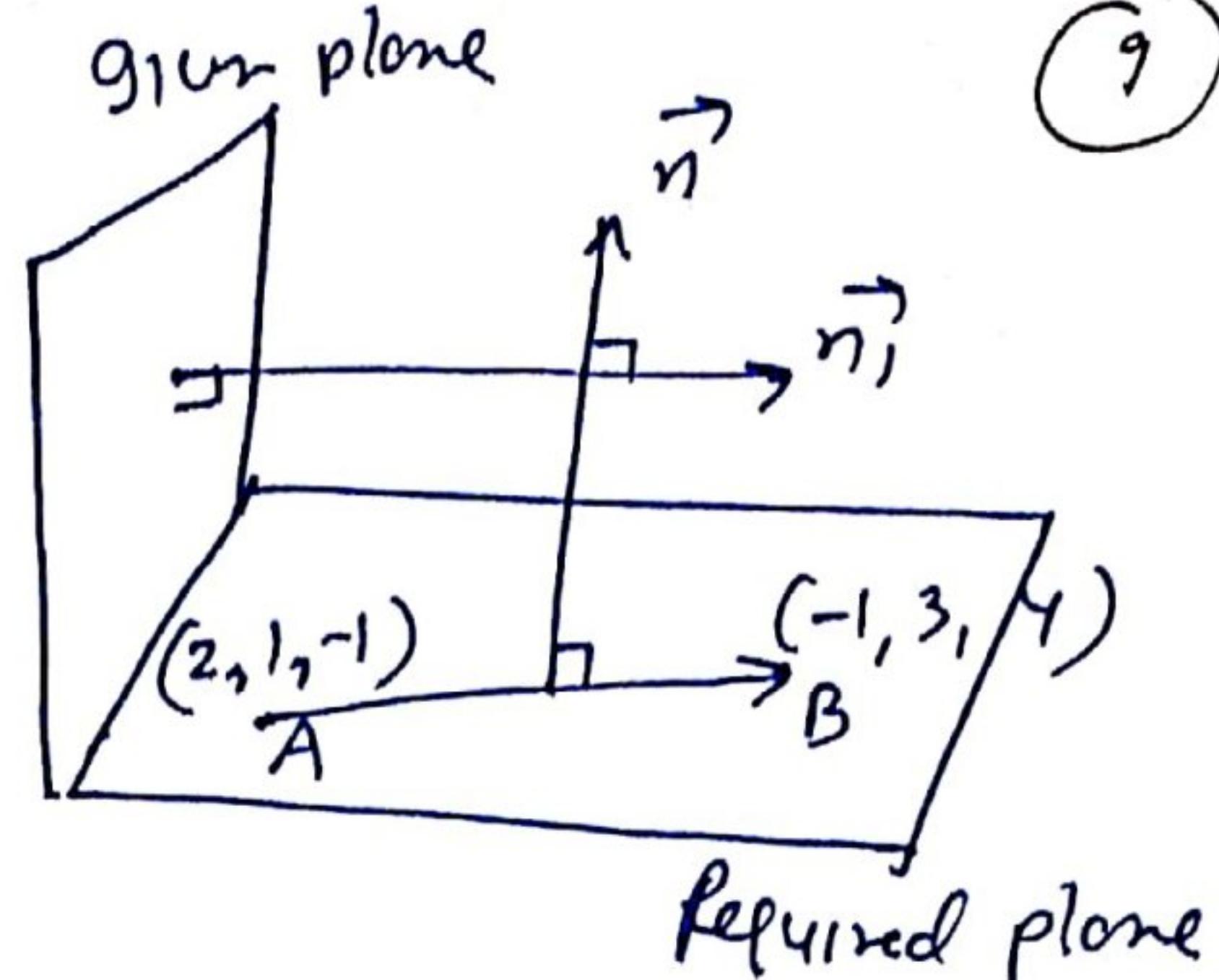
$$\vec{a} = 2i + j - k$$

equation of plane is

$$(\vec{r} - \vec{a}) \cdot \vec{n} = 0$$

$$\Rightarrow \vec{r} \cdot \vec{n} = \vec{a} \cdot \vec{n}$$

$$\Rightarrow \vec{r} \cdot \lambda(-18i - 17j - 4k) = (2i + j - k) \cdot (\lambda(-18i - 17j - 4k))$$



(9)

$$\Rightarrow \vec{r} \cdot (-18\vec{i} - 17\vec{j} - 4\vec{k}) = -36 - 17 + 4$$

$$\Rightarrow \vec{r} \cdot (-18\vec{i} - 17\vec{j} - 4\vec{k}) = -49$$

$$\Rightarrow \boxed{\vec{r} \cdot (18\vec{i} + 17\vec{j} + 4\vec{k}) = 49}$$

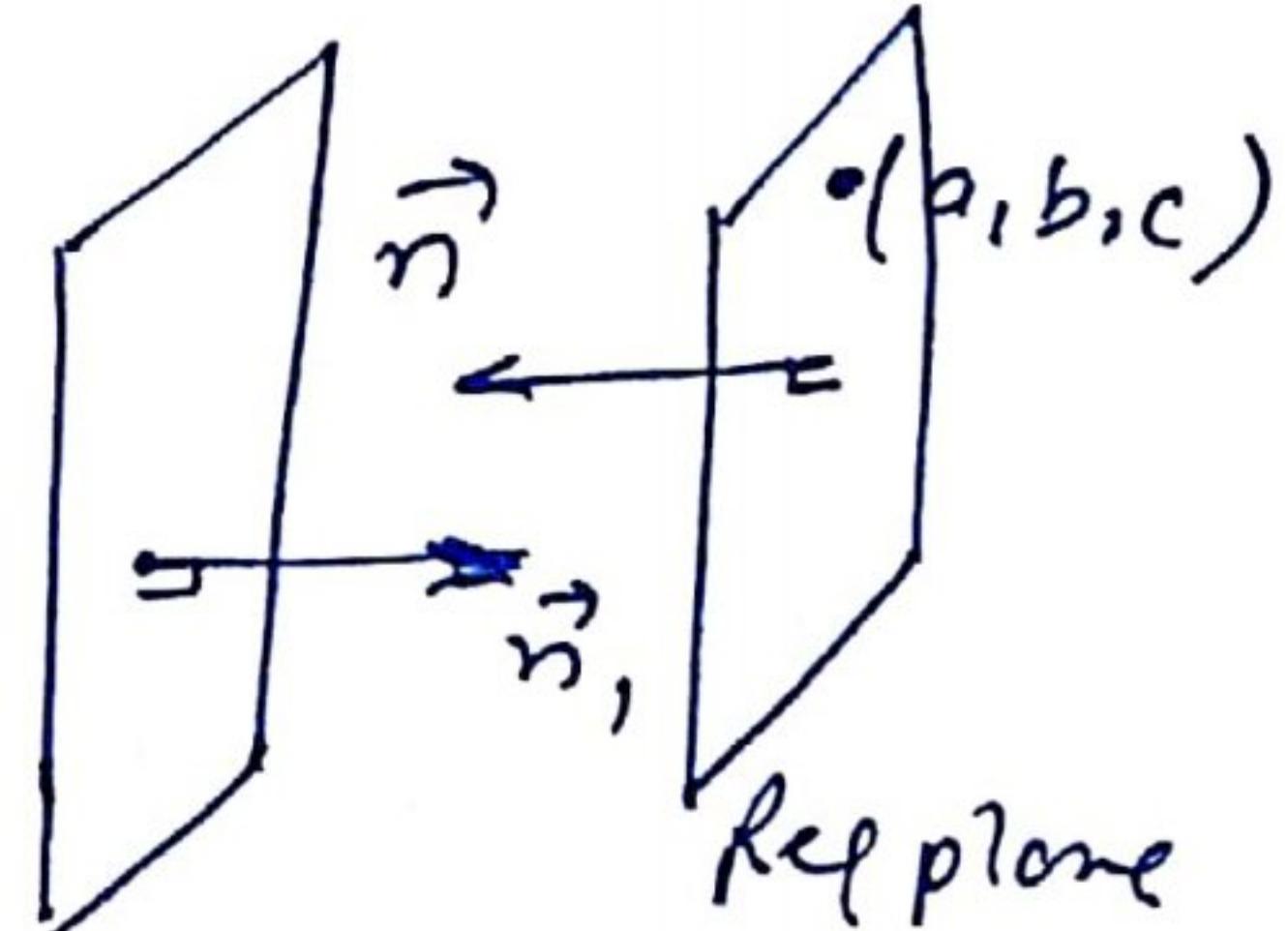
$$\Rightarrow (x\vec{i} + y\vec{j} + z\vec{k}) \cdot (18\vec{i} + 17\vec{j} + 4\vec{k}) = 49$$

$$\Rightarrow \boxed{18x + 17y + 4z = 49} \quad \underline{\text{Ans}}$$

Ques 9 → Find the equation of the plane passing through the point  $(a, b, c)$  and parallel to the plane  $\vec{r} \cdot (\vec{i} + \vec{j} + \vec{k}) = 2$

Solution Given plane  $\vec{r} \cdot (\vec{i} + \vec{j} + \vec{k}) = 2$

$$\text{then } \vec{n}_1 = \vec{i} + \vec{j} + \vec{k}$$



Let  $\vec{n}$  = normal vector to the plane

Since planes are parallel

$$\therefore \vec{n} \parallel \vec{n}_1$$

$$\Rightarrow \vec{n} = \lambda \vec{n}_1$$

$$\Rightarrow \vec{n} = \lambda (\vec{i} + \vec{j} + \vec{k})$$

Let position vector of given point  $(a, b, c)$  is

$$\vec{a} = a\vec{i} + b\vec{j} + c\vec{k}$$

Vector equation of plane is

$$(\vec{r} - \vec{a}) \cdot \vec{n} = 0$$

(11)

$$\Rightarrow \vec{r} \cdot \vec{n} = \vec{a} \cdot \vec{n}$$

$$\Rightarrow \cancel{\vec{r} \cdot \lambda(\vec{i} + \vec{j} + \vec{k})} =$$

$$\Rightarrow \vec{r} \cdot (\lambda(\vec{i} + \vec{j} + \vec{k})) = (a\vec{i} + b\vec{j} + c\vec{k}) - \cancel{\lambda(\vec{i} + \vec{j} + \vec{k})}$$

$$\Rightarrow \boxed{\vec{r} \cdot (\vec{i} + \vec{j} + \vec{k}) = a + b + c}$$

$$(\text{or}) \quad \boxed{x + y + z = a + b + c} \quad \underline{\underline{=}}$$

Ques 10 → Show that the plane  $18x + 17y + 4z = 19$   
 Contains the line  $\vec{r} = -\vec{i} + 3\vec{j} + 4\vec{k} + \lambda(3\vec{i} - 2\vec{j} - 5\vec{k})$

Soln: To Show: plane contains the line  
 we have to prove two conditions:

- (1) Point (fixed point) on the line must satisfy the equation of plane
- (2)  $\vec{b}$  of line must be  $\perp^r$  to  $\vec{n}$  of plane

Plane:  $18x + 17y + 4z = 19$

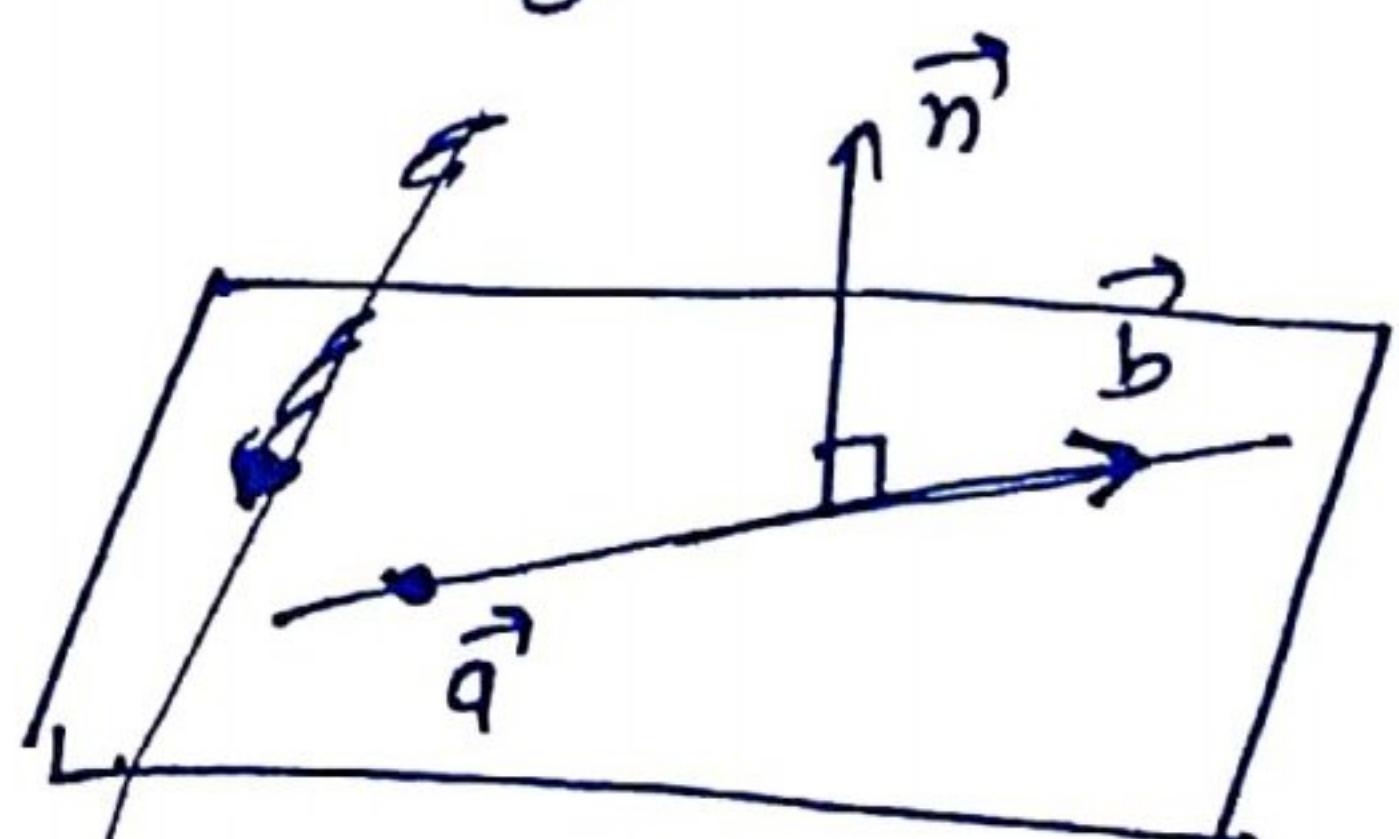
here  $\vec{n} = 18\vec{i} + 17\vec{j} + 4\vec{k}$

line  $\vec{r} = -\vec{i} + 3\vec{j} + 4\vec{k} + \lambda(3\vec{i} - 2\vec{j} - 5\vec{k})$

here fixed point is  $(-1, 3, 4)$  &  $\vec{b} = 3\vec{i} - 2\vec{j} - 5\vec{k}$

$$-18 + 51 + 16$$

$$= \frac{19}{\text{RHS}} \quad \therefore \text{fixed point satisfy equation of plane}$$



$$\text{Ans} \quad \vec{b} \cdot \vec{n}$$

(12)

$$= (3\vec{i} - 2\vec{j} - 5\vec{k}) \cdot (18\vec{i} + 17\vec{j} + 4\vec{k})$$

$$= 54 - 34 - 20$$

$$= 0$$

$$\Rightarrow \vec{b} \perp \vec{n}$$

$\therefore$  Plane contains the line. Ans

Ques  $\underline{\underline{11}} \rightarrow$  Find the equation of the plane with Intercept 3 on the Y-axis and parallel to ZOX plane

Sols When plane is parallel to ZOX plane

then Y-axis is  $\perp^{\circ}$  to the plane

$$\text{Hence, here } \vec{n} = \vec{j}$$

Point on the plane =  $(0, 3, 0)$  -- { Given Y-int = 3 }

$$\text{Let P.v of this point } \vec{a} = \vec{0} + 3\vec{j} + 0\vec{k} = 3\vec{j}$$

$$\text{equation of plane } (\vec{r} - \vec{a}) \cdot \vec{n} = 0$$

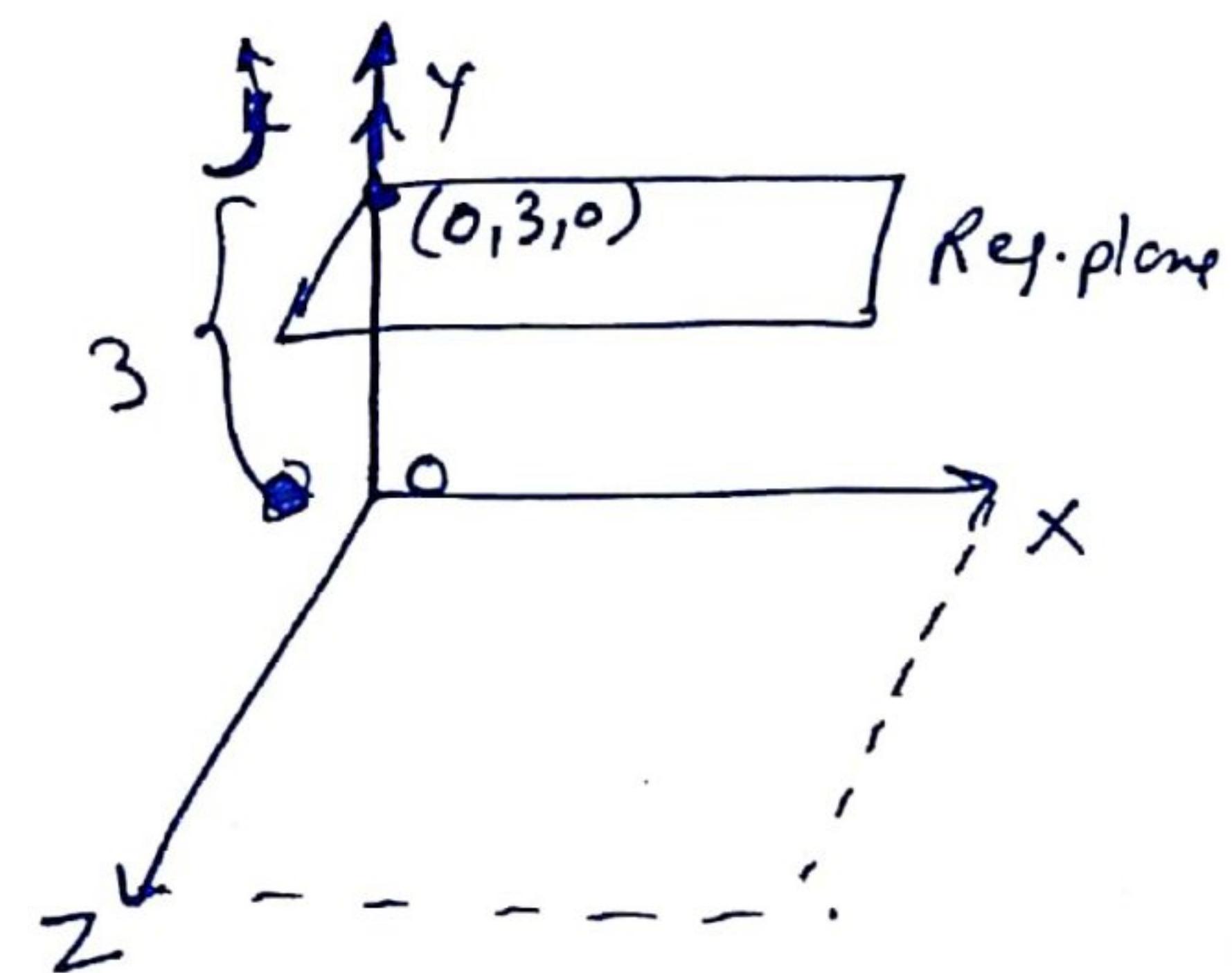
$$\Rightarrow \vec{r} \cdot \vec{n} = \vec{a} \cdot \vec{n}$$

$$\Rightarrow \vec{r} \cdot \vec{j} = (3\vec{j}) \cdot \vec{j}$$

$$\Rightarrow \boxed{\vec{r} \cdot \vec{j} = 3}$$

$$(x\vec{i} + y\vec{j} + z\vec{k}) \cdot \vec{j} = 3$$

$$\Rightarrow \boxed{y = 3} \quad \underline{\text{Ans}}$$



$\left\{ \begin{array}{l} \text{we can write} \\ \text{directly } \underline{\text{Ans}} \\ \boxed{y=3} \quad \text{for} \\ \text{one marking} \end{array} \right.$

Qn 12 → find the equation of a plane passing through  
 the line of intersection of the planes  
 $\vec{r} \cdot (i + 3j - k) = 5$  and  $\vec{r} \cdot (2i - j + k) = 3$  and  
 passing through the point  $(2, 1, -2)$

Soln Given planes  $\vec{r} \cdot (i + 3j - k) = 5$  and  
 $\vec{r} \cdot (2i - j + k) = 3$

here  $\vec{n}_1 = i + 3j - k ; d_1 = 5$   
 $\vec{n}_2 = 2i - j + k ; d_2 = 3$

equation of required plane.

$$\vec{r} \cdot (\vec{n}_1 + \lambda \vec{n}_2) = d_1 + \lambda d_2$$

$$\Rightarrow \vec{r} \cdot (i + 3j - k + 2\lambda i - \lambda j + \lambda k) = 5 + 3\lambda$$

$$\Rightarrow \vec{r} \cdot (i(1+2\lambda) + j(3-\lambda) + k(-1+\lambda)) = 5 + 3\lambda \dots (1)$$

$$\Rightarrow (x i + y j + z k) \cdot (i(1+2\lambda) + j(3-\lambda) + k(-1+\lambda)) = 5 + 3\lambda$$

$$\Rightarrow x(1+2\lambda) + y(3-\lambda) + z(-1+\lambda) = 5 + 3\lambda$$

Now given  $(2, 1, -2)$  lies on it

$$2 + 4\lambda + 3 - \lambda + 2 - 2\lambda = 5 + 3\lambda$$

$$-2\lambda = -2 \Rightarrow \lambda = 1$$

(13)

(14)

$$\boxed{\vec{r} \cdot (3\hat{i} + 2\hat{j} + 0\hat{k}) = 8} \text{ Ans}$$

Ques 13 Find the equation of plane which contains the line of intersection of the planes  $x+2y+3z-4=0$  and  $2x+y-z+5=0$  and perpendicular to the plane  $5x+3y+6z+8=0$

Sols Convert given equation of planes in to vector form

$$\vec{r} \cdot (\hat{i} + 2\hat{j} + 3\hat{k}) = 4 \quad \dots (1)$$

$$\vec{r} \cdot (2\hat{i} + \hat{j} - \hat{k}) = -5 \quad \dots (2)$$

$$\vec{r} \cdot (5\hat{i} + 3\hat{j} + 6\hat{k}) = -8 \quad \dots (3)$$

$$\text{then } \vec{n}_1 = \hat{i} + 2\hat{j} + 3\hat{k} \quad ; \quad d_1 = 4$$

$$\vec{n}_2 = 2\hat{i} + \hat{j} - \hat{k} \quad ; \quad d_2 = -5$$

$$\vec{n}_3 = 5\hat{i} + 3\hat{j} + 6\hat{k}$$

equation of refined plane

$$\vec{r} \cdot (\vec{n}_1 + \lambda \vec{n}_2) = d_1 + \lambda d_2$$

$$\Rightarrow \vec{r} \cdot (\hat{i} + 2\hat{j} + 3\hat{k} + 2\lambda\hat{i} + \hat{j} - \lambda\hat{k}) = 4 - 5\lambda$$

$$\Rightarrow \vec{r} \cdot (\hat{i}(1+2\lambda) + \hat{j}(2+\lambda) + \hat{k}(3-\lambda)) = 4 - 5\lambda \quad \dots (i)$$

$$\text{then } \vec{n} = \hat{i}(1+2\lambda) + \hat{j}(2+\lambda) + \hat{k}(3-\lambda)$$

(15)

Since required plane is  $\perp$  to the 3<sup>rd</sup> plane

$$\therefore \vec{n} \perp \vec{n}_3.$$

$$\Rightarrow \vec{n} \cdot \vec{n}_3 = 0$$

$$\Rightarrow (i(1+2\lambda) + j(2+\lambda) + k(3-\lambda)) \cdot (5i + 3j + 6k) = 0$$

$$\Rightarrow 5 + 10\lambda + 6 + 3\lambda + 18 - 6\lambda = 0$$

$$7\lambda = -29$$

$$\Rightarrow \boxed{\lambda = \frac{-29}{7}} \quad \text{put in eq(i)}$$

$$\Rightarrow \vec{n} \cdot \left( i\left(1 - \frac{58}{7}\right) + j\left(2 - \frac{29}{7}\right) + k\left(3 + \frac{29}{7}\right) \right) = 4 + \frac{145}{7}$$

$$\Rightarrow \vec{n} \cdot \left( i(-51) + j(-15) + k(50) \right) = 173$$

$$\Rightarrow (x_i + y_j + z_k) \cdot (-51i - 15j + 50k) = 173$$

$$\Rightarrow \boxed{-51x - 15y + 50z = 173} \quad \underline{\underline{\Delta}}$$

Ques 1 Find the equation of the plane containing the line of intersection of the plane  $x+y+z=6$  and

$2x+3y+yz+5=0$  and passing through the Point  $(1,1,1)$

$$\underline{\text{Ans}} \quad 20x + 23y + 26z - 69 = 0$$

Ques 2 Find the equation of the plane through the line of intersection of  $\vec{r} \cdot (2\hat{i} - 3\hat{j} + 4\hat{k}) = 1$  and

$\vec{r} \cdot (\hat{i} - \hat{j}) + 4 = 0$  and perpendicular to  $\vec{r} \cdot (2\hat{i} - \hat{j} + \hat{k}) + 8 = 0$

$$\underline{\text{Ans}} \quad \vec{r} \cdot (-5\hat{i} + 2\hat{j} + 12\hat{k}) = 47$$

Ques 3 Find the equation of the plane passing through the point

$2\hat{i} + \hat{j} - \hat{k}$  and passing through the line of intersection of the planes  $\vec{r} \cdot (\hat{i} + 3\hat{j} - \hat{k}) = 0$  and  $\vec{r} \cdot (\hat{j} + 2\hat{k}) = 0$

$$\underline{\text{Ans}} \quad \vec{r} \cdot (\hat{i} + 9\hat{j} + 11\hat{k}) = 0$$

Ques 4 Find the equation of the plane which is perpendicular

to the plane  $5x + 3y - 6z + 8 = 0$  and passing through the line of intersection of the planes  $x + 2y + 3z - 4 = 0$

and  $2x + y - z + 5 = 0$   $\underline{\text{Ans}} \quad 33x + 45y + 50z - 41 = 0$

Ques 5 Find the vector equation of the plane passing through the points  $(2, 1, -1)$  and  $(-1, 3, 4)$  and perpendicular to the plane  $x - 2y + 4z = 10$

$$\underline{\text{Ans}} \quad 18x + 17y + 4z - 49 = 0$$

Ques 6 → Find the equation of the plane passing through the point  $(-1, 3, 2)$  and perpendicular to each of the planes  $x+2y+3z=5$  and  $3x+3y+z=0$

Ans  $7x-8y+3z+25=0$

Ques 7 → Find the equation of the plane passing through the points  $i-j+2k$  and  $2i-2j+2k$  and which is perpendicular to the plane  $\vec{r} \cdot (6i-2j+2k) = 9$

Ans  $\vec{r} \cdot (i+j-2k) + 4 = 0$

Ques 8 → Find the value of  $\lambda$  so that the planes

$$3x-6y-2z=7 \text{ and } 2x+y-\lambda z=5 \text{ are } \perp^{\pi} \text{ to each other}$$

Ans  $\lambda=0$

Ques 9 → Find the vector equation of a plane which is at a distance of 6 units from the origin and has  $2, -1, 2$  as the direction ratios of a normal to it

Ans  $\vec{r} \cdot \left( \frac{2}{3}i - \frac{1}{3}j + \frac{2}{3}k \right) = 6$

Ques 10 → Find the equation of the plane which bisects the line segment joining the points  $(-1, 2, 3)$  and  $(3, -5, 6)$  at right angles Ans  $4x-7y+3z-28=0$

Ques 11 → Find the Intercepts made on the coordinate axis by the plane  $2x+y-2z=3$  and also find the direction cosines of the normal to the plane

Ans  $\frac{3}{2}, 3, -\frac{3}{2}; \frac{2}{3}, \frac{1}{3}, -\frac{2}{3}$

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