← ULTIMATE MATHEMATICS →

(BY AJAY MITTAL)

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Inverse Trigo CLASS (I-4)

Qns 1 → If $\cos^{-1}x + \cos^{-1}y + \cos^{-1}z = \pi$
show that $x^2+y^2+z^2+2xyz = 1$

Soln

$\cos^{-1}x + \cos^{-1}y = \pi - \cos^{-1}z$

$\cos^{-1}x + \cos^{-1}y = \cos^{-1}(-z)$ --- $\{\cos^{-1}(-x) = \pi - \cos^{-1}x\}$

let $\cos^{-1}x = A$ & $\cos^{-1}y = B$

$\Rightarrow \boxed{\cos A = x}$ & $\boxed{\cos B = y}$

$\Rightarrow \sin A = \sqrt{1-\cos^2 A}$ & $\sin B = \sqrt{1-\cos^2 B}$

$\Rightarrow \boxed{\sin A = \sqrt{1-x^2}}$ & $\boxed{\sin B = \sqrt{1-y^2}}$

~~we have $\cos(A+B)$~~

equation becomes

$A + B = \cos^{-1}(-z)$

$\Rightarrow \cos(A+B) = -z$

$\Rightarrow \cos A \cos B - \sin A \sin B = -z$

$\Rightarrow xy - \sqrt{1-x^2}\sqrt{1-y^2} = -z$

$\Rightarrow xy + z = \sqrt{1-x^2}\sqrt{1-y^2}$

Squaring

$x^2y^2 + z^2 + 2xyz = (1-x^2)(1-y^2)$

$\cancel{x^2y^2} + z^2 + 2xyz = 1 - y^2 - x^2 + \cancel{x^2y^2}$

$x^2 + y^2 + z^2 + 2xyz = 1$ Ans

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Q. 2 → ~~Show~~ Given that

$\cos^{-1}\left(\frac{x}{a}\right) + \cos^{-1}\left(\frac{y}{b}\right) = \alpha$

Show that $\frac{x^2}{a^2} - \frac{2xy}{ab}\cos\alpha + \frac{y^2}{b^2} = \sin^2\alpha$

— Let $\cos^{-1}\left(\frac{x}{a}\right) = A$ & $\cos^{-1}\left(\frac{y}{b}\right) = B$

$\Rightarrow \boxed{\cos A = \frac{x}{a}}$ & $\boxed{\cos B = \frac{y}{b}}$

$\sin A = \sqrt{1-\cos^2 A}$ & $\sin B = \sqrt{1-\cos^2 B}$

$\boxed{\sin A = \sqrt{1-\frac{x^2}{a^2}}}$ & $\boxed{\sin B = \sqrt{1-\frac{y^2}{b^2}}}$

equation becomes

$A + B = \alpha$

$\Rightarrow \cos(A+B) = \cos\alpha$

$\Rightarrow \cos A\cos B - \sin A\sin B = \cos\alpha$

$\Rightarrow \left(\frac{x}{a}\right)\left(\frac{y}{b}\right) - \sqrt{1-\frac{x^2}{a^2}}\sqrt{1-\frac{y^2}{b^2}} = \cos\alpha$

$\Rightarrow \frac{xy}{ab} - \cos\alpha = \sqrt{1-\frac{x^2}{a^2}}\sqrt{1-\frac{y^2}{b^2}}$

Squaring

$\frac{x^2y^2}{a^2b^2} + \cos^2\alpha - \frac{2xy}{ab}\cos\alpha = \left(1-\frac{x^2}{a^2}\right)\left(1-\frac{y^2}{b^2}\right)$

$\Rightarrow \cancel{\frac{x^2y^2}{a^2b^2}} + \cos^2\alpha - \frac{2xy}{ab}\cos\alpha = 1 - \frac{y^2}{b^2} - \frac{x^2}{a^2} + \cancel{\frac{x^2y^2}{a^2b^2}}$

$\Rightarrow \frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{2xy\cos\alpha}{ab} = 1 - \cos^2\alpha = \sin^2\alpha$

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Ques 3 → (SELF) If $\cos^{-1}\left(\frac{x}{2}\right) + \cos^{-1}\left(\frac{y}{3}\right) = \alpha$

Show that $9x^2 - 12xy\cos\alpha + 4y^2 = 36\sin^2\alpha$

Ques 4 → NCERT Show that $\sin^{-1}\left(\frac{12}{13}\right) + \cos^{-1}\left(\frac{4}{5}\right) + \tan^{-1}\left(\frac{63}{16}\right) = \pi$

Sol. Let $\sin^{-1}\left(\frac{12}{13}\right) = A$; $\cos^{-1}\left(\frac{4}{5}\right) = B$; $\tan^{-1}\left(\frac{63}{16}\right) = C$

$\Rightarrow \boxed{\sin A = \frac{12}{13}}$; $\boxed{\cos B = \frac{4}{5}}$; $\tan C = \frac{63}{16}$

$\cos A = \sqrt{1 - \frac{144}{169}}$; $\sin B = \sqrt{1 - \frac{16}{25}}$; $\tan C = \frac{63}{16}$

$\boxed{\cos A = \frac{5}{13}}$; $\boxed{\sin B = \frac{3}{5}}$

$\tan(A) = \frac{12}{5}$; $\tan B = \frac{3}{4}$; $\tan C = \frac{63}{16}$

Now $\tan(A+B)$

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$\tan(A+B) = \frac{\frac{12}{5} + \frac{3}{4}}{1 - \frac{12}{5} \times \frac{3}{4}}$$

$$\Rightarrow \tan(A+B) = \frac{48 + 15}{20 - 36}$$

$$\rightarrow \tan(A+B) = \frac{63}{-16} = -\frac{63}{16}$$

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$\tan(-\theta) = -\tan\theta$

$\tan(\pi-\theta) = -\tan\theta$

$\tan(2\pi-\theta) = -\tan\theta$

$\Rightarrow \tan(A+B) = -\tan C$

$\Rightarrow \tan(A+B) = \tan(\pi - C)$

$\Rightarrow A+B = \pi - C$

$\Rightarrow A+B+C = \pi$

Replace A, B, C

$\Rightarrow \sin^{-1}\left(\frac{12}{13}\right) + \cos^{-1}\left(\frac{4}{5}\right) + \tan^{-1}\left(\frac{63}{16}\right) = \pi$ Ans

Que 5 → Solve $\tan^{-1}\left(\frac{1-x}{1+x}\right) = \frac{1}{2}\tan^{-1}x \quad ; \quad x > 0$

NCERT

$\Rightarrow \tan^{-1}(1) - \tan^{-1}x = \frac{1}{2}\tan^{-1}x$

$\Rightarrow \frac{\pi}{4} = \frac{1}{2}\tan^{-1}x + \tan^{-1}x$

$\Rightarrow \frac{\pi}{4} = \frac{3}{2}\tan^{-1}x$

$\Rightarrow \frac{2\pi}{12} = \tan^{-1}x$

$\Rightarrow \tan^{-1}x = \frac{\pi}{6}$

$\Rightarrow x = \tan\left(\frac{\pi}{6}\right)$

$\Rightarrow \boxed{x = 1/\sqrt{3}}$ Ans

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Qn 6 → NCERT Show that

$$\frac{9\pi}{8} - \frac{9}{4}\sin^{-1}\left(\frac{1}{3}\right) = \frac{9}{4}\sin^{-1}\left(\frac{2\sqrt{2}}{3}\right)$$

LHS $\frac{9\pi}{8} - \frac{9}{4}\sin^{-1}\left(\frac{1}{3}\right)$

$= \frac{9}{4}\left(\frac{\pi}{2} - \sin^{-1}\frac{1}{3}\right)$

$= \frac{9}{4}\cos^{-1}\left(\frac{1}{3}\right)$ --- $\left\{\sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}\right\}$

= conversion

here $B = 1$, $H = 3$

$P = \sqrt{9-1} = \sqrt{8} = 2\sqrt{2}$

$= \frac{9}{4}\sin^{-1}\left(\frac{2\sqrt{2}}{3}\right)$

Qn 7 NCERT 1M Show that $\tan^{-1}\sqrt{x} = \frac{1}{2}\cos^{-1}\left(\frac{1-x}{1+x}\right)$

LHS $\tan^{-1}\sqrt{x}$

$= \frac{1}{2}\left(2\tan^{-1}\sqrt{x}\right)$

$= \frac{1}{2}\cos^{-1}\left(\frac{1-(\sqrt{x})^2}{1+(\sqrt{x})^2}\right)$

$= \frac{1}{2}\cos^{-1}\left(\frac{1-x}{1+x}\right)$

= RHS

Q8 → Simplify

(iii) $\tan\left\{\frac{1}{2}\left[\sin^{-1}\left(\frac{2x}{1+x^2}\right)+\cos^{-1}\left(\frac{1-y^2}{1+y^2}\right)\right]\right\}$

$=\tan\left\{\frac{1}{2}\left[2\tan^{-1}x+2\tan^{-1}y\right]\right\}$

$=\tan\left(\tan^{-1}x+\tan^{-1}y\right)$

$=\tan\left(\tan^{-1}\left(\frac{x+y}{1-xy}\right)\right)$

$=\frac{x+y}{1-xy}$ Ans

Q → If $\boxed{a>b>c}$

show that $\cot^{-1}\left(\frac{ab+1}{a-b}\right)+\cot^{-1}\left(\frac{bc+1}{b-c}\right)+\cot^{-1}\left(\frac{ca+1}{c-a}\right)=\pi$

prop. $\boxed{\cot^{-1}x=\tan^{-1}\left(\frac{1}{x}\right)}$; $x>0$

Ly $\tan^{-1}\left(\frac{a-b}{1+ab}\right)+\tan^{-1}\left(\frac{b-c}{1+bc}\right)+\cot^{-1}\left(\frac{ca+1}{-(a-c)}\right)$

$=\tan^{-1}a-\cancel{\tan^{-1}b}+\cancel{\tan^{-1}b}-\tan^{-1}c+\pi-\cot^{-1}\left(\frac{ca+1}{a-c}\right)$

--- $\{\cot^{-1}(-x)=\pi-\cot^{-1}x\}$

$=\tan^{-1}a-\tan^{-1}c+\pi-\tan^{-1}\left(\frac{a-c}{1+ca}\right)$

$=\cancel{\tan^{-1}a}-\cancel{\tan^{-1}c}+\pi-\{\cancel{\tan^{-1}a}-\cancel{\tan^{-1}c}\}$

$=\pi$ Ans

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Qn 10 → If $a_1, a_2, a_3, \ldots a_n$ are in A.P with common difference 'd', then find

$$\tan\left[\tan^{-1}\left(\frac{d}{1+a_1a_2}\right)+\tan^{-1}\left(\frac{d}{1+a_2a_3}\right)+\tan^{-1}\left(\frac{d}{1+a_3a_4}\right)+\cdots\cdots\tan^{-1}\left(\frac{d}{1+a_{n-1}a_n}\right)\right]$$

Soln $$\tan\left[\tan^{-1}\left(\frac{a_2-a_1}{1+a_1a_2}\right)+\tan^{-1}\left(\frac{a_3-a_2}{1+a_2a_3}\right)+\tan^{-1}\left(\frac{a_4-a_3}{1+a_3a_4}\right)+\cdots\tan^{-1}\left(\frac{a_n-a_{n-1}}{1+a_{n-1}a_n}\right)\right]$$

$$=\tan\left[\cancel{\tan^{-1}(a_2)}-\tan^{-1}(a_1)+\cancel{\tan^{-1}(a_3)}-\cancel{\tan^{-1}(a_2)}+\tan^{-1}(a_4)-\cancel{\tan^{-1}(a_3)}+\cdots\cdots\tan^{-1}(a_n)-\tan^{-1}(a_{n-1})\right]$$

$$=\tan\left[-\tan^{-1}(a_1)+\tan^{-1}(a_n)\right]$$

$$=\tan\left(\tan^{-1}(a_n)-\tan^{-1}(a_1)\right)$$

$$=\tan\left[\tan^{-1}\left(\frac{a_n-a_1}{1+a_na_1}\right)\right]$$

$$=\frac{a_n-a_1}{1+a_1a_n}$$ Ans