SOLUTIONS: INTEGRATION : CLASS NO: 5

← WORKSHEET NO: 4 →

Ans. 1 → $I = \int \frac{1}{4x^2 - 4x + 3}\,dx$

$$I = \frac{1}{4}\int \frac{1}{x^2 - x + \frac{3}{4}}\,dx$$

$$= \frac{1}{4}\int \frac{1}{(x-\frac{1}{2})^2 - \frac{1}{4} + \frac{3}{4}}\,dx$$

$$= \frac{1}{4}\int \frac{1}{(x-\frac{1}{2})^2 + (\frac{1}{\sqrt{2}})^2}\,dx$$

$$= \frac{1}{4} \times \frac{1}{\frac{1}{\sqrt{2}}} \tan^{-1}\left(\frac{x - 1/2}{1/\sqrt{2}}\right) + C$$

$$I = \frac{1}{2\sqrt{2}} \tan^{-1}\left(\frac{2x-1}{\sqrt{2}}\right) + C \quad \underline{\text{Ans}}$$

Ans 2 → $I = \int \frac{1}{1 + x - x^2}\,dx$

$$I = -\int \frac{1}{x^2 - x - 1}\,dx$$

$$= -\int \frac{1}{(x-\frac{1}{2})^2 - \frac{1}{4} - 1}\,dx$$

$$= -\int \frac{1}{(x-\frac{1}{2})^2 - (\frac{\sqrt{5}}{2})^2}\,dx$$

$$= \int \frac{1}{(\frac{\sqrt{5}}{2})^2 - (x-\frac{1}{2})^2}\,dx$$

$$= \frac{1}{2 \times \frac{\sqrt{5}}{2}} \log\left|\frac{\frac{\sqrt{5}}{2} + x - \frac{1}{2}}{\sqrt{5}/2 - x + 1/2}\right| + C$$

$$I = \frac{1}{\sqrt{5}} \log\left|\frac{\sqrt{5}+2x-1}{\sqrt{5}-2x+1}\right| + c \quad \text{Ans}$$

Ans No: 3 → $I = \int \frac{x\,dx}{3x^4 - 18x^2 + 11}$

put $x^2 = t$

$2x\,dx = dt \Rightarrow x\,dx = \frac{dt}{2}$

$$\therefore I = \frac{1}{2}\int \frac{dt}{3t^2 - 18t + 11}$$

$$= \frac{1}{2} \times \frac{1}{3}\int \frac{1}{t^2 - 6t + \frac{11}{3}}\,dt$$

$$= \frac{1}{6}\int \frac{1}{(t-3)^2 - 9 + \frac{11}{3}}\,dt$$

$$= \frac{1}{6}\int \frac{1}{(t-3)^2 - \left(\frac{4}{\sqrt{3}}\right)^2}\,dt$$

$$= \frac{1}{6} \times \frac{1}{2 \times \frac{4}{\sqrt{3}}} \log\left|\frac{t-3-\frac{4}{\sqrt{3}}}{t-3+\frac{4}{\sqrt{3}}}\right| + c$$

$$= \frac{\sqrt{3}}{48} \log\left|\frac{\sqrt{3}t - 3\sqrt{3} - 4}{\sqrt{3}t - 3\sqrt{3} + 4}\right| + c$$

$$\therefore I = \frac{\sqrt{3}}{48} \log\left|\frac{\sqrt{3}x^2 - 3\sqrt{3} - 4}{\sqrt{3}x^2 - 3\sqrt{3} + 4}\right| + c \quad \text{Ans}$$

Ans 4 → $I = \int \frac{e^{3x}}{4e^{6x} - 9}\,dx$

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Ans: 4 → $I = \int \frac{e^{3x}}{4e^{6x}-9}\,dx$

put $e^{3x} = t$

$3e^{3x}dx = dt \Rightarrow e^{3x}dx = \frac{dt}{3}$

$I = \frac{1}{3}\int \frac{dt}{4t^2-9}$

$= \frac{1}{3} \times \frac{1}{4}\int \frac{1}{t^2-\left(\frac{3}{2}\right)^2}dt$

$= \frac{1}{12} \times \frac{1}{2\times\frac{3}{2}} \log\left|\frac{t-3/2}{t+3/2}\right| + C$

$= \frac{1}{36}\log\left|\frac{2t-3}{2t+3}\right| + C$

$\therefore I = \frac{1}{36}\log\left|\frac{2e^{3x}-3}{2e^{3x}+3}\right| + C$ Ans

Ans: 5 → $I = \int \frac{1}{\sqrt{2x^2+3x-2}}\,dx$

$I = \frac{1}{\sqrt{2}}\int \frac{1}{\sqrt{x^2+\frac{3}{2}x-1}}\,dx$

$= \frac{1}{\sqrt{2}}\int \frac{1}{\sqrt{\left(x+\frac{3}{4}\right)^2-\frac{9}{16}-1}}\,dx$

$= \frac{1}{\sqrt{2}}\int \frac{1}{\sqrt{\left(x+\frac{3}{4}\right)^2-\left(\frac{5}{4}\right)^2}}\,dx$

as it is

$I = \frac{1}{\sqrt{2}}\log\left|\left(x+\frac{3}{4}\right)+\sqrt{x^2+\frac{3}{2}x-1}\right| + C$ Ans

Qn: 6 → $I = \int \frac{1}{\sqrt{x(1-2x)}}\,dx$

$$I = \int \frac{1}{\sqrt{x-2x^2}}\,dx$$

$$I = \int \frac{1}{\sqrt{-2\left(x^2-\frac{x}{2}\right)}}\,dx$$

$$= \frac{1}{\sqrt{2}}\int \frac{1}{\sqrt{-\left[\left(x-\frac{1}{4}\right)^2-\left(\frac{1}{4}\right)^2\right]}}\,dx$$

$$= \frac{1}{\sqrt{2}}\int \frac{1}{\sqrt{\left(\frac{1}{4}\right)^2-\left(x-\frac{1}{4}\right)^2}}\,dx$$

$$= \frac{1}{\sqrt{2}}\sin^{-1}\left(\frac{x-\frac{1}{4}}{\frac{1}{4}}\right)+C$$

$$I = \frac{1}{\sqrt{2}}\sin^{-1}(4x-1)+C \quad \text{Ans}$$

Qn. 7 → $I = \int \frac{1}{\sqrt{7-3x-2x^2}}\,dx$

$$I = \int \frac{1}{\sqrt{-2\left(x^2+\frac{3}{2}x-\frac{7}{2}\right)}}\,dx$$

$$= \frac{1}{\sqrt{2}}\int \frac{1}{\sqrt{-\left[\left(x+\frac{3}{4}\right)^2-\frac{9}{16}-\frac{7}{2}\right]}}\,dx$$

$$= \frac{1}{\sqrt{2}}\int \frac{1}{\sqrt{-\left[\left(x+\frac{3}{4}\right)^2-\left(\frac{\sqrt{65}}{4}\right)^2\right]}}\,dx$$

$$= \frac{1}{\sqrt{2}}\int \frac{1}{\sqrt{\left(\frac{\sqrt{65}}{4}\right)^2-\left(x+\frac{3}{4}\right)^2}}\,dx$$

$$I = \frac{1}{\sqrt{2}} \sin^{-1}\left(\frac{x+\frac{3}{4}}{\frac{\sqrt{65}}{4}}\right) + C$$

$$I = \frac{1}{\sqrt{2}} \sin^{-1}\left(\frac{4x+3}{\sqrt{65}}\right) + C \quad \underline{\text{ANS}}$$

$\underline{\text{QN. 8}} \rightarrow \quad I = \int \sqrt{\operatorname{cosec} x - 1}\, dx$

$$I = \int \sqrt{\frac{1}{\sin x} - 1}\, dx$$

$$= \int \sqrt{\frac{1-\sin x}{\sin x}}\, dx$$

$$= \int \sqrt{\frac{1-\sin x}{\sin x} \times \frac{1+\sin x}{1+\sin x}}\, dx$$

$$= \int \frac{\cos x}{\sqrt{\sin^2 x + \sin x}}\, dx$$

put $\sin x = t$
$\cos x\, dx = dt$

$$\therefore I = \int \frac{dt}{\sqrt{t^2+t}}$$

$$= \int \frac{1}{\sqrt{(t+\frac{1}{2})^2 - (\frac{1}{2})^2}}\, dt$$

as it is

$$= \log\left|\left(t+\frac{1}{2}\right) + \sqrt{t^2+t}\right| + C$$

$$I = \log\left|\left(\sin x + \frac{1}{2}\right) + \sqrt{\sin^2 x + \sin x}\right| + C \quad \underline{\text{ANS}}$$

Ans 9 → $I = \int \frac{\cos x \, dx}{\sqrt{\sin^2 x - 2\sin x - 3}}$

put $\sin x = t$
$\cos x \, dx = dt$

$I = \int \frac{dt}{\sqrt{t^2 - 2t - 3}}$

$= \int \frac{1}{\sqrt{(t-1)^2 - 1 - 3}} dt$

$= \int \frac{1}{\sqrt{(t-1)^2 - (2)^2}} dt$

as it is

$= \log \left| (t-1) + \sqrt{t^2 - 2t - 3} \right| + C$

$I = \log \left| (\sin x - 1) + \sqrt{\sin^2 x - 2\sin x - 3} \right| + C$ Ans

Ans-10 → $I = \int \frac{1}{x^{2/3} \sqrt{x^{2/3} - 4}} dx$

$= \int \frac{1}{x^{2/3} \sqrt{(x^{1/3})^2 - (2)^2}} dx$ --- { Concept $\int \frac{N^r}{\text{variable} \pm \text{constant}} dx$

$(x^{1/3})^2$ → Not linear

∴ put $x^{1/3} = t$

$\frac{1}{3} x^{-2/3} dx = dt$

$\frac{1}{x^{2/3}} dx = 3 dt$

$I = 3 \int \frac{dt}{\sqrt{t^2 - (2)^2}}$

$= 3 \log \left| t + \sqrt{t^2 - 4} \right| + C$

$$I = 3\log\left| x^{1/3} + \sqrt{x^{2/3} - 4} \right| + C \quad \text{Ans}$$

Qn 11 → $I = \int \frac{\sin(8x)}{\sqrt{9 + \sin^4(4x)}} dx$

$$I = \int \frac{\sin(8x)}{\sqrt{(3)^2 + \underbrace{(\sin^2(4x))^2}_{\text{Not linear}}}} dx$$

∴ put $\sin^2(4x) = t$

$2\sin(4x)\cdot\cos(4x)\cdot 4\, dx = dt$

$\sin(8x)\, dx = \frac{dt}{4}$

$$\therefore I = \frac{1}{4}\int \frac{dt}{\sqrt{(3)^2 + t^2}}$$

$$= \frac{1}{4}\log\left| t + \sqrt{t^2 + 9} \right| + C$$

$$I = \frac{1}{4}\log\left| \sin^2(4x) + \sqrt{\sin^4(4x) + 9} \right| + C \quad \text{Ans}$$

Qn 12 → $I = \int \frac{1}{e^x + e^{-x}} dx$

$$I = \int \frac{1}{e^x + \frac{1}{e^x}} dx$$

$$= \int \frac{e^x}{e^{2x} + 1} dx$$

put $e^x = t$

$e^x dx = dt$

$$\therefore I = \int \frac{dt}{t^2 + 1}$$

$I = \tan^{-1}(t) + C$

$I = \tan^{-1}(e^x) + C$ ANS

Qn. 13 → $I = \int \frac{1}{\sqrt{(1-x^2)\left(9+(\sin^{-1}x)^2\right)}} dx$

$I = \int \frac{1}{\sqrt{1-x^2}\sqrt{9+(\sin^{-1}x)^2}} dx$

put $\sin^{-1}x = t$

$\frac{1}{\sqrt{1-x^2}} dx = dt$

$I = \int \frac{dt}{\sqrt{9+t^2}}$

$= \log\left|t + \sqrt{9+t^2}\right| + C$

$I = \log\left|\sin^{-1}x + \sqrt{9+(\sin^{-1}x)^2}\right| + C$ Ans

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