!! जय श्री राधे कृष्णा। जय श्री गिरिराज जी महाराज !!

ULTIMATE MATHEMATICS : BY AJAY MITTAL

CHAPTER: PROBABILITY

CLASS NO: 1

(·) Trial : Tossing a coin

(·) Event : getting head / tail

(·) Events : A, B, C ---

(·) P(A) → Prob of event A (Occurrence of event A)

(·) P(A') → Prob of not A

(·) $P(A) + P(A') = 1$

(·) Mutually exclusive events :

$A \cap B = \phi$; $P(A \cap B) = 0$

(·) Exhaustive events

$A \cup B = S$; $P(A \cup B) = 1$

(·) Mutually exclusive & Exhaustive events

$P(A) + P(B) = 1$

(एक ही 2 एक पक्का)

A, B, C
1/2, 1/3

Formula

(1) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A \cup B)$ → either-or, or, atleast one

$P(A \cap B)$ → and, both, common, as well as, simultaneously occur

(2) $P(A' \cap B') = 1 - P(A \cup B)$

neither A nor B

(3) $P(A \cap B') = P(A) - P(A \cap B)$

only A, A but not B, A alone

(4) $P(B \cap A') = P(B) - P(A \cap B)$

(5) $P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) + P(A \cap B \cap C)$

(6) $\boxed{P(\text{atleast one}) = 1 - P(\text{none})}$

- x -

(7) $P(A' \cup B') = P(A \cap B)' = 1 - P(A \cap B)$

Sample Space = S = {all possible outcomes}

3 coin

(4) (2) (1)

S = { HHH, HHT, HTH, HTT, THH, THT, TTH, TTT }

4 coin

(8)

S = { 8H 4H 2H H, 8T 4T 2T T }

Cards total = 52

(2) Colour Black = 26

Red = 26

(4) Suits diamond (R) = 13

Heart (R) = 13

club (B) = 13

Space (B) = 13

face cards (12) K Q J

(4) (4) (4)

Ace (A) = 4

Shortcuts

$^nC_0 = 1$

$^nC_1 = n$

$^nC_2 = \frac{n(n-1)}{2}$

$^nC_3 = \frac{n(n-1)(n-2)}{6}$

$^nC_n = 1$

$^nC_1 = {}^nC_{n-1}$

$^{10}C_8 = {}^{10}C_2$

Topic 1 Conditional probability

(·) $P(A|B)$ = Prob of occurance of event A given that event B has already occured.

(·) $P(A)$ = Prob of occure of event A

(·) $P(B|A)$ = vice versa

(·) $P(A|B) = \frac{P(A \cap B)}{P(B)}$ ✓ (Required: A; Given: B)

$P(B|A) = \frac{P(A \cap B)}{P(A)}$

Formal

A → Required event

B → given event

$A = \{ \dots \}$

$B = \{ \dots \}$

$A \cap B = \{ \dots \}$

$P(A \cap B) =$

$P(B) =$

$P(A|B) = \frac{P(A \cap B)}{P(B)}$

Qn 1 A family has two children. What is the probability that both the children are boys given that atleast one of them is a boy?

Soln $S = \{BB, BG, GB, GG\}$

$A \rightarrow$ both the children are boys

$B \rightarrow$ atleast one is a boy

$A = \{BB\}$; $B = \{BG, GB, BB\}$

$A \cap B = \{BB\}$

$P(A \cap B) = 1/4$; $P(B) = 3/4$

By Bayes $P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{1/4}{3/4} = 1/3$ Ans

Qn 2 → A die is thrown three times. Events A & B are defined as below

A: 4 on the third throw

B: 6 on the first and 5 on the second throw

Find A/B

Soln $A = \{(1,1,4), (1,2,4) \text{ ---- } (6,6,4)\}$

$B = \{(6,5,1), (6,5,2) \text{ ---- } (6,5,6)\}$

$A \cap B = \{(6,5,4)\}$

$P(A \cap B) = \frac{1}{216}$; $P(B) = \frac{6}{216}$

$P(A|B) = \frac{1/216}{6/216} = 1/6$ Ans

Qns 3 → Mother, father & son line up at random for a family picture

E: Son on one end F: Father in Middle

Find $P(E|F)$

Son $S = \{MFS, MSF, FSM, FMS, SMF, SFM\}$

$E = \{SFM, SMF, MFS, FMS\}$

$F = \{MFS, SFM\}$

$E \cap F = \{MFS, SFM\}$

$P(E \cap F) = \frac{2}{6}$.

$P(F) = 2/6$

$P(E|F) = \frac{2/6}{2/6} = 1$ Ans

Qn 4 → Consider the experiment of tossing a coin. If the coin shows head, toss it again; but if it shows tail, then throw a dice. Find the Conditional probability of the event that the dice shows a number greater than 4 given that there is atleast one tail.

$S = \{(H,H), (H,T), (T,1), (T,2), (T,3), (T,4), (T,5), (T,6)\}$

A → getting a no. more than 4

B → getting atleast one tail

$A = \{(T,5), (T,6)\}$

$B = \{(H,T), (T,1), (T,2) \cdots (T,6)\}$

$A \cap B = \{(T,5), (T,6)\}$

(X) $P(A \cap B) = \frac{2}{8}$ (X)

Equally likely outcomes

$P(A \cap B) = \left(\frac{1}{2} \times \frac{1}{8}\right) \times 2 = \frac{1}{6}$

$P(B) = \left(\frac{1}{2} \times \frac{1}{2}\right) + \left(\frac{1}{2} \times \frac{1}{6}\right) \times 6 = \frac{1}{4} + \frac{1}{2} = 3/4$

$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{1/6}{3/4} = \frac{4}{18} = \frac{2}{9}$ Ans

—x—

Worksheet No: 1 (PROBABILITY) 12th Class

Qns:1 Ten cards numbered 1 to 10 are placed in a box, mixed up thoroughly and then one card is drawn randomly. If it is known that the number on the card is more than 3, what is the probability that it is an even number? Ans= 4/7

Qns 2 A dice is thrown twice and the sum of the numbers appearing is observed to be 6. What is the probability that the number 4 has appeared atleast once?

Ans = 2/5

Qns 3 → A black and a red dice are rolled.

(a) Find the conditional probability of obtaining a sum greater than 9, given that the black die resulted in 5 Ans 1/3

(b) Find the conditional probability of obtaining the sum 8, given that the red die resulted in a number less than 4 Ans 1/9

Qns 4 → If a family has two children, what is the conditional probability that both are girls given that

(i) the youngest is a girl Ans 1/2

(2) atleast one is a girl Ans 1/3

Qns 5 → In a school, there are 1000 students, out of which 430 are girls. It is known that out of 430, 10% of the girls study in class XII. What is the

probability that a student chosen randomly studies in class XII given that the chosen student is a girl? Ans $\frac{1}{10}$

Qns 6 → An instructor has a question bank consisting of 300 easy True/false questions, 200 difficult True/false Questions, 500 easy multiple choice questions and 400 difficult multiple choice questions. If a question is selected at random from the question bank, what is the probability that it will be an easy question given that it is a multiple choice question? Ans $\frac{5}{9}$

Qns 7 → Consider the experiment of throwing a die, if a multiple of 3 comes up, throw the die again and if any other number comes, toss a coin. Find the conditional probability of the event "the coin shows a tail" given that 'at least one die shows a 3' Ans = 0

Qns 8 → If $P(A) = \frac{6}{11}$, $P(B) = \frac{5}{11}$, $P(A \cup B) = \frac{7}{11}$

Find $P(A|B)$ & $P(B/A)$ Ans 4/5 & 2/3

Qns 9 → If $2P(A) = P(B) = \frac{5}{13}$ and $P(A/B) = \frac{2}{5}$

Find $P(A \cup B)$ Ans $\frac{11}{26}$

Qus 10 → In a hostel, 60% of the students read Hindi newspaper, 40% read English newspaper and 20% read both Hindi and English newspaper. A student is selected at random

(a) Find the prob. that She reads neither Hindi nor English newspapers

(b) If She reads Hindi newspaper, find the probability that She reads English newspaper.

(c) If She reads English newspaper, find the probability that She reads Hindi newspaper

Ans (a) $\frac{1}{5}$ (b) $1/3$ (c) $1/2$

Qus 11 → A and B are two events such that $P(A) \neq 0$. Find $P(B/A)$; if

(i) A is a subset of B (ii) $A \cap B = \phi$ Ans (i) 1 (ii) 0

Qus 12 → A couple has two children. Find the probability that both children are females, if it is known that the elder child if a female Ans $1/2$

Qus 13 → Given ~~P(A' ∪ B')~~

$P(A' \cup B') = 2/3$ & $P(A \cup B) = 5/9$

find $P(A') + P(B')$ Ans = $\frac{10}{9}$

Qns 14 → An electronic assembly consists of two subsystems say A & B. From previous testing procedures, the following probabilities are assumed to be known:

P(A fails) = 0·2

P(B fails alone) = 0·15

P(A and B fail) = 0·15

Find

(i) P(A fails / B has failed) Ans 0·5

(ii) P(A fails alone) Ans 0·05

Qns 15 → If $P(B) = \frac{3}{5}$, $P(A/B) = 1/2$

$P(A \cup B) = \frac{4}{5}$, then find $P(A \cup B)' + P(A' \cup B)$

Ans = 1

Qn 16 → Given $P(A) = \frac{7}{13}$, $P(B) = \frac{9}{13}$, $P(A \cup B) = \frac{12}{13}$

find $P(A'/B)$ Ans = $\frac{5}{9}$