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← ULTIMATE MATHEMATICS →

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Chapter. Differentiation & Continuity

← CLASS NO = 10 →

Qn 1 If $y = \begin{vmatrix} f(x) & g(x) & h(x) \\ l & m & n \\ a & b & c \end{vmatrix}$, then show that

$$\frac{dy}{dx} = \begin{vmatrix} f'(x) & g'(x) & h'(x) \\ l & m & n \\ a & b & c \end{vmatrix}$$

Soln Diff wrt x

$$\frac{dy}{dx} = \begin{vmatrix} f'(x) & g'(x) & h'(x) \\ l & m & n \\ a & b & c \end{vmatrix} + \begin{vmatrix} f(x) & g(x) & h(x) \\ 0 & 0 & 0 \\ a & b & c \end{vmatrix} + \begin{vmatrix} f(x) & g(x) & h(x) \\ l & m & n \\ 0 & 0 & 0 \end{vmatrix}$$

$$\Rightarrow \frac{dy}{dx} = \begin{vmatrix} f'(x) & g'(x) & h'(x) \\ l & m & n \\ a & b & c \end{vmatrix} \quad \text{Ans}$$

Qn. 2 → Using PMI, show that $\frac{d}{dx}(x^n) = nx^{n-1}$ for all $n \in N$.

Soln Let $P(n): \frac{d}{dx}(x^n) = nx^{n-1}$

$P(1): \frac{d}{dx}(x) = x^0 = 1$

$\Rightarrow 1 = 1$ Clearly $P(1)$ is true

Let $P(k)$ be true

$P(k): \frac{d}{dx}(x^k) = kx^{k-1}$

T.prove $P(k+1)$ is true

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D&C class No-10 (2)

$P(k+1): \frac{d}{dx}\left(x^{k+1}\right) = (k+1)x^k$

LHS $\frac{d}{dx}\left(x^{k+1}\right)$

$= \frac{d}{dx}\left(x^k \cdot x\right)$

$= x^k \frac{d}{dx}(x) + x \cdot \frac{d}{dx}\left(x^k\right)$

$= x^k \cdot (1) + x \cdot k x^{k-1}$ ...(from P(k))

$= x^k + k x^k$

$= (k+1)x^k$

$=$ RHS

$\therefore P(k+1)$ is true

By PMI, $P(n)$ is true for all $n \in N$ Ans

Qn 3 → Using the fact that $\sin(A+B) = \sin A \cos B + \cos A \sin B$ and the differentiation, obtain the sum formula for cosines.

Sol (I) $\sin(A+B) = \sin A \cos B + \cos A \sin B$

Diff wrt A (treating B as constant)

$\Rightarrow \cos(A+B) \cdot (1+0) = \cos A \cdot \cos B + (-\sin A) \cdot \sin B$

$\Rightarrow \cos(A+B) = \cos A \cos B - \sin A \sin B$ Ans

(II) Diff wrt 't'

$\cos(A+B) \cdot \left(\frac{dA}{dt} + \frac{dB}{dt}\right) = \sin A \cdot (-\sin B)\frac{dB}{dt} + \cos B \cdot \cos A \cdot \frac{dA}{dt} + \cos A \cos B \cdot \frac{dB}{dt} + \sin B \cdot (-\sin A)\frac{dA}{dt}$

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$$\Rightarrow \cos(A+B)\left(\frac{dA}{dt}+\frac{dB}{dt}\right) = \cos A\cos B\left(\frac{dA}{dt}+\frac{dB}{dt}\right) - \sin A\sin B\left(\frac{dA}{dt}+\frac{dB}{dt}\right)$$

$$\Rightarrow \cos(A+B) = \cos A\cos B - \sin A\sin B$$ Ans

Qn. 4 → Does there exists a function which is continuous everywhere but not differentiable at exactly two points? Justify your answer

Sol let $f(x) = |x-1| + |x-2|$

Since Modulus function is everywhere continuous & sum of two cont. functions is also continuous.

$\therefore$ $f(x)$ is everywhere continuous.

$$f(x) = \begin{cases} -2x+3 & ; \quad x<1 \\ 1 & ; \quad 1 \le x < 2 \\ 2x-3 & ; \quad x \ge 2 \end{cases}$$

Diff. at $x=1$

LHD $\neq$ RHD

Diff at $x=2$

LHD $\neq$ RHD

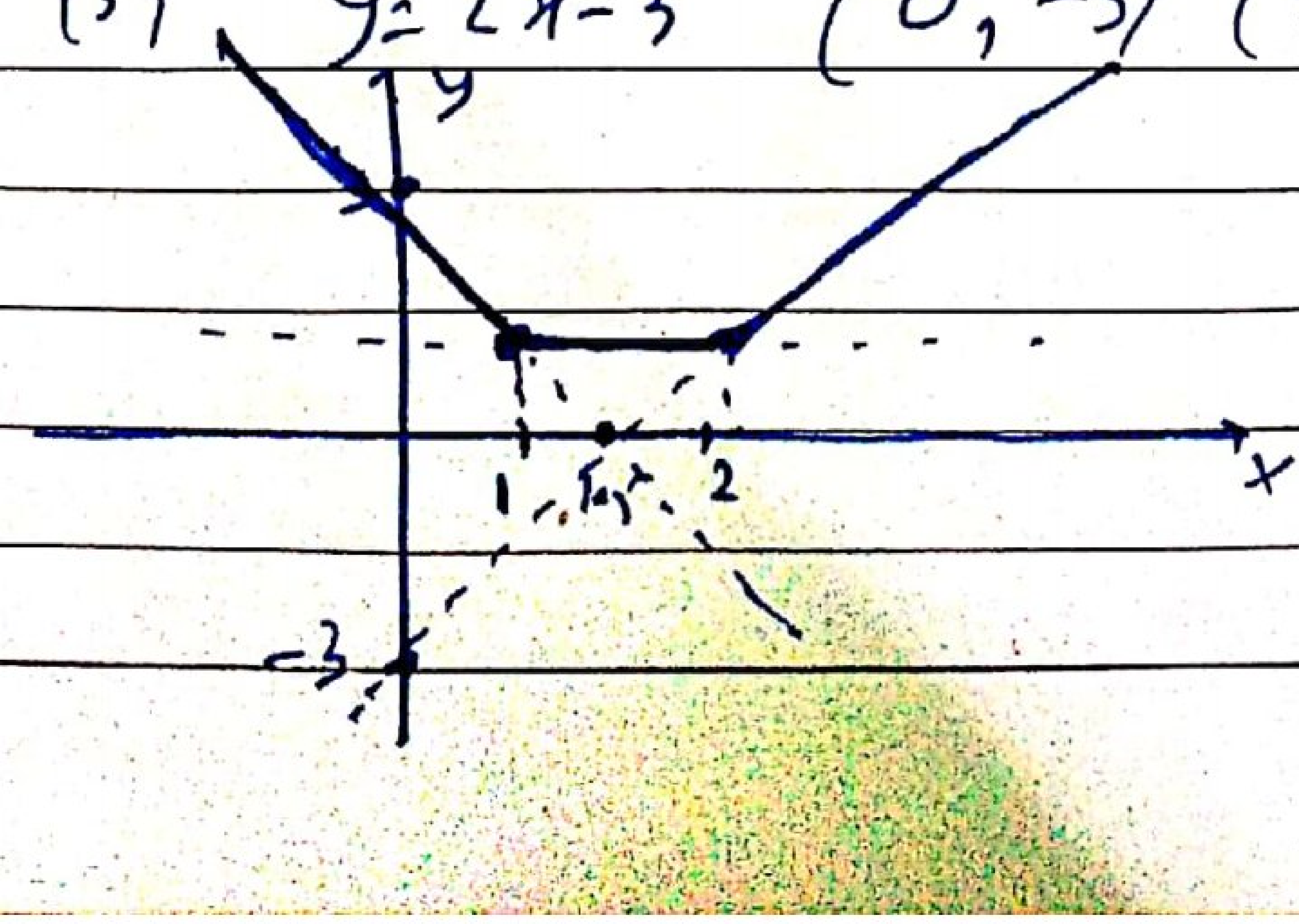
$\therefore$ $f(x)$ is not Diff at $x=1$ & $x=2$ Ans

(OR) Graphically

(1) $y = -2x+3$ $(0,3)$ $(3/2, 0)$

(2) $y = 1$

(3) $y = 2x-3$ $(0,-3)$ $(3/2, 0)$



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Qu. 5 → $f(x) = |x|^3$ show that $f''(x)$ exists for all real x and find it.

Sol $f(x) = \begin{cases} (x)^3 & : x \geq 0 \\ (-x)^3 & ; x < 0 \end{cases}$

$f(x) = \begin{cases} x^3 & : x \geq 0 \\ -x^3 & : x < 0 \end{cases}$

✓ when $x < 0$, $f(x) = -x^3$ which is a polynomial function and it is everywhere differentiable

✓ when $x > 0$; $f(x) = x^3$ " " " "
" " " " " " " " "

✓ Diff at $x = 0$

$$LHD = \lim_{x \to 0^-} \left(\frac{-x^3 - f(0)}{x - 0} \right)$$

$$= \lim_{x \to 0^-} \left(\frac{-x^3}{x} \right)$$

$$= \lim_{x \to 0^-} (-x^2)$$ put $x = 0 - h = -h$, $h \to 0$

$$LHD = \lim_{h \to 0} (-h^2) = 0$$

$$RHD = \lim_{x \to 0^+} \left(\frac{x^3 - 0}{x - 0} \right)$$

$$= \lim_{x \to 0^+} (x^2)$$

$RHD = 0$

Since $LHD = RHD$

$\therefore$ $f(x)$ is differentiable for all $x \in R$

$\therefore$ $f'(x)$ exists

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$$f'(x) = \begin{cases} 3x^2 ; & x \geq 0 \\ -3x^2 ; & x < 0 \end{cases}$$

✓ when $x > 0$, $f'(x) = 3x^2$ which is a polynomial function and it is everywhere differentiable

✓ when $x < 0$, $f'(x) = -3x^2$ - - - - - - -

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✓ Diff at $x = 0$

$$LHD = \lim_{x \to 0^-} \left(\frac{-3x^2 - f'(0)}{x - 0} \right)$$

$$= \lim_{x \to 0^-} \left(\frac{-3x^2 - 0}{x - 0} \right)$$

$$= \lim_{x \to 0^-} (-3x) \quad \text{put } x = 0 - h \ \& \ h \to 0$$

$$LHD = \lim_{h \to 0} (3h) = 0$$

Similay $RHD = 0$

Since $LHD = RHD$

$\therefore f'(x)$ is differentiable for all $x \in R$

$\therefore f''(x)$ exists.

and $f''(x) = \begin{cases} 6x ; & x \geq 0 \\ -6x ; & x < 0 \end{cases}$ Ans

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D&C class No: 10 (6)

Topic : Differentiation of a function w.r.t another function

Qn 6 Diff. $\sin x$ w.r.t $\log x$

let $u = \sin x$; $v = \log x$

Diff w.r.t x.

$$\frac{du}{dx} = \cos x \quad \Big| \quad \frac{dv}{dx} = \frac{1}{x}$$

$$\frac{du}{dv} = \frac{du/dx}{dv/dx}$$

$$= \frac{\cos x}{\frac{1}{x}} = x\cos x \quad \text{Ans}$$

Qn 7 Diff $\sin^2 x$ w.r.t $e^{\cos x}$

Sol let $u = \sin^2 x$ & $v = e^{\cos x}$

$$\frac{du}{dx} = 2\sin x.\cos x \quad \Big| \quad \frac{dv}{dx} = e^{\cos x}\cdot(-\sin x)$$

$$\frac{du}{dv} = \frac{du/dx}{dv/dx} = \frac{2\cancel{\sin x}\cos x}{e^{\cos x}\cdot(-\cancel{\sin x})}$$

$$\frac{du}{dv} = \frac{-2\cos x}{e^{\cos x}} \quad \text{Ans}$$

Qn 8 $y = \sin^{-1}\left(\frac{2^{x+1}}{1+4^x}\right)$ find $\frac{dy}{dx}$

$$y = \sin^{-1}\left(\frac{2\cdot 2^x}{1+(2^x)^2}\right) \quad \text{put } 2^x = \tan\theta$$

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$y = \sin^{-1}\left(\frac{2\tan\theta}{1+\tan^2\theta}\right)$

$y = \sin^{-1}\left(\sin(2\theta)\right)$

$y = 2\theta$ --- {put $2^x = \tan\theta$}

replace θ

$y = 2\tan^{-1}(2^x)$

Diff w.r.t x

$$\frac{dy}{dx} = 2\cdot\frac{1}{1+4^x}\cdot\frac{d}{dx}(2^x)$$

$$= \frac{2}{1+4^x}\cdot 2^x\cdot\log 2 = \frac{2^{x+1}\cdot\log 2}{1+4^x} \quad \underline{\text{Ans}}$$

Qn 9 → $y = \log_7(\log x)$ find $\frac{dy}{dx}$

Sol

$y = \frac{\log(\log x)}{\log 7}$ $\log_a b = \frac{\log b}{\log a}$

Diff $\frac{dy}{dx} = \frac{1}{\log 7}\cdot\frac{1}{\log x}\cdot\frac{1}{x}$ Ans

Qn 10 → examine the applicability of Rolle's theorem in

(1) $f(x) = [x]$; $x \in [-2, 2]$

(2) $f(x) = x^2 - 1$; $x \in [1, 2]$

Sol (1) $f(x) = [x]$

$f(-2) = [-2] = -2$

$f(2) = [2] = 2$

Clearly $f(-2) \neq f(2)$ ∴ R.T is not applicable

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D & C Class No:- 10

Q. 11 → If u, v, w are functions of x then show that $\frac{d}{dx}(u \cdot v \cdot w) = \frac{du}{dx} \cdot v \cdot w + \frac{dv}{dx} \cdot w \cdot u + u \cdot v \frac{dw}{dx}$ in two ways

(1) by repeated application of product rule
(2) By logarithmic differentiation

Sol ① let $y = u \cdot v \cdot w$

$$y = \underset{\mathrm{I}}{(uv)}\,\underset{\mathrm{II}}{w}$$

Diff w.r.t x

$$\frac{dy}{dx} = (uv) \cdot \frac{dw}{dx} + w\left(u \cdot \frac{dv}{dx} + v \cdot \frac{du}{dx}\right)$$

$$= u \cdot v \cdot \frac{dw}{dx} + w \cdot u \cdot \frac{dv}{dx} + w \cdot v \cdot \frac{du}{dx}$$

② $y = u \cdot v \cdot w$

$$\log y = \lg(u \cdot v \cdot w)$$

$$\lg y = \lg u + \lg v + \lg w$$

Diff w.r.t x

$$\frac{1}{y} \cdot \frac{dy}{dx} = \frac{1}{u} \cdot \frac{du}{dx} + \frac{1}{v} \cdot \frac{dv}{dx} + \frac{1}{w} \cdot \frac{dw}{dx}$$

$$\frac{dy}{dx} = uvw\left(\frac{1}{u} \cdot \frac{du}{dx} + \frac{1}{v} \cdot \frac{dv}{dx} + \frac{1}{w} \cdot \frac{dw}{dx}\right)$$

$$\frac{dy}{dx} = v \cdot w \frac{du}{dx} + u \cdot w \frac{dv}{dx} + u \cdot v \cdot \frac{dw}{dx}$$

Ans

Q. 12 → $y = 2\sqrt{\cot(x^2)}$ find $\frac{dy}{dx}$

Soln

$$\frac{dy}{dx} = \cancel{2} \cdot \frac{1}{\cancel{2}\sqrt{\cot(x^2)}} \cdot \left(-\operatorname{cosec}^2(x^2)\right) \cdot 2x$$

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$$\frac{dy}{dx} = \frac{-2x \cdot \operatorname{cosec}^2(x^2)}{\sqrt{\cot(x^2)}}$$

$$\frac{dy}{dx} = -2x \cdot \frac{\frac{1}{\sin^2(x^2)}}{\frac{\sqrt{\cos(x^2)}}{\sqrt{\sin(x^2)}}}$$

$$= \frac{-2x \cdot \sqrt{\sin(x^2)}}{\sin(x^2) \cdot \sin(x^2) \cdot \sqrt{\cos x^2}}$$

$$\frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}$$

$$= \frac{-2x}{\sqrt{\sin(x^2)} \cdot \sin(x^2) \cdot \sqrt{\cos(x^2)}}$$

$$= \frac{-2x}{\sin(x^2)\sqrt{\sin(x^2) \cdot \cos(x^2)}}$$

$$= \frac{-2x}{\sin(x^2) \cdot \sqrt{\frac{2\sin(x^2) \cdot \cos(x^2)}{2}}}$$

$$\frac{dy}{dx} = \frac{-2\sqrt{2}x}{\sin(x^2) \cdot \sqrt{\sin(2x^2)}}$$ Ans