Solution of (D-5) BY AJAY MITTAL

Qns. 1 → $\begin{vmatrix} -a^2 & ab & ac \\ ba & -b^2 & bc \\ ca & cb & -c^2 \end{vmatrix}$

taking a, b, c common from R_1, R_2 & R_3 resp

$$= abc \begin{vmatrix} -a & b & c \\ a & -b & c \\ a & b & -c \end{vmatrix}$$

taking a, b, c common from C_1, C_2 & C_3 resp.

$$= a^2b^2c^2 \begin{vmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{vmatrix}$$

$R_2 \to R_2 + R_1$ and $R_3 \to R_3 + R_1$

$$= a^2b^2c^2 \begin{vmatrix} -1 & 1 & 1 \\ 0 & 0 & 2 \\ 0 & 2 & 0 \end{vmatrix}$$

expanding

$$= a^2b^2c^2 \left[-1(0-4) - 1(0) + 1(0) \right]$$

$$= a^2b^2c^2 (4)$$

$$= 4a^2b^2c^2 = \text{Rhs} \quad \text{PROVED.}$$

Qns. 2 → given $\begin{vmatrix} x & x^2 & 1+x^3 \\ y & y^2 & 1+y^3 \\ z & z^2 & 1+z^3 \end{vmatrix} = 0$

Sum property in C_3

$$= \begin{vmatrix} x & x^2 & 1 \\ y & y^2 & 1 \\ z & z^2 & 1 \end{vmatrix} + \begin{vmatrix} x & x^2 & x^3 \\ y & y^2 & y^3 \\ z & z^2 & z^3 \end{vmatrix} = 0$$

taking x, y, z common from R_1, R_2, R_3 resp. from 2nd determinant

(D-5) Solutions → (2)

$$\Rightarrow \begin{vmatrix} x & x^2 & 1 \\ y & y^2 & 1 \\ z & z^2 & 1 \end{vmatrix} + xyz \begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} = 0$$

$C_2 \leftrightarrow C_3$ in (I^{st} det)

$$\Rightarrow - \begin{vmatrix} x & 1 & x^2 \\ y & 1 & y^2 \\ z & 1 & z^2 \end{vmatrix} + xyz \begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} = 0$$

$C_1 \leftrightarrow C_2$ (I^{st} det)

$$\Rightarrow + \begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} + xyz \begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} (1+xyz) = 0$$

$R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$

$$\Rightarrow \begin{vmatrix} 1 & x & x^2 \\ 0 & y-x & y^2-x^2 \\ 0 & z-x & z^2-x^2 \end{vmatrix} (1+xyz) = 0$$

$$\Rightarrow (y-x)(z-x) \begin{vmatrix} 1 & x & x^2 \\ 0 & 1 & y+x \\ 0 & 1 & z+x \end{vmatrix} (1+xyz) = 0$$

$R_3 \to R_3 - R_2$

$$\Rightarrow (y-x)(z-x) \begin{vmatrix} 1 & x & x^2 \\ 0 & 1 & y+x \\ 0 & 0 & z-y \end{vmatrix} (1+xyz) = 0$$

$$\Rightarrow (y-x)(z-x)(z-y)(1+xyz) = 0$$

$\Rightarrow$ But $y-x \neq 0$, $z-x \neq 0$, $z-y \neq 0$ — {since $x \neq y \neq z$ given}

$\therefore 1+xyz = 0$

proved

Ans 3 →
$$\begin{vmatrix} x & x^2 & 1+px^3 \\ y & y^2 & 1+py^3 \\ z & z^2 & 1+pz^3 \end{vmatrix}$$

Sum prop in C_3

$$= \begin{vmatrix} x & x^2 & 1 \\ y & y^2 & 1 \\ z & z^2 & 1 \end{vmatrix} + \begin{vmatrix} x & x^2 & px^3 \\ y & y^2 & py^3 \\ z & z^2 & pz^3 \end{vmatrix}$$

taking x, y, z common from R_1, R_2, R_3 and also p common from C_3 {2nd determ}

$$= \begin{vmatrix} x & x^2 & 1 \\ y & y^2 & 1 \\ z & z^2 & 1 \end{vmatrix} + pxyz \begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix}$$

$C_2 \leftrightarrow C_3$ (Ist det.)

$$= - \begin{vmatrix} x & 1 & x^2 \\ y & 1 & y^2 \\ z & 1 & z^2 \end{vmatrix} + pxyz \begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix}$$

$C_1 \leftrightarrow C_2$ (Ist det)

$$= + \begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} + pxyz \begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix}$$

$$\Rightarrow \begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} (1+pxyz)$$

$R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$

$$= \begin{vmatrix} 1 & x & x^2 \\ 0 & y-x & y^2-x^2 \\ 0 & z-x & z^2-x^2 \end{vmatrix} (1+pxyz)$$

$$= (y-x)(z-x) \begin{vmatrix} 1 & x & x^2 \\ 0 & 1 & y+x \\ 0 & 1 & z+x \end{vmatrix} (1+pxyz)$$

$R_3 \to R_3 - R_2$

$$= (y-x)(z-x)\begin{vmatrix} 1 & x & x^2 \\ 0 & 1 & y+x \\ 0 & 0 & z-y \end{vmatrix}(1+pxyz)$$

expanding

$$=(\underset{(-)}{y-x})(z-x)(\underset{(-)}{z-y})(1+pxyz)$$

$$= (x-y)(y-z)(z-x)(1+pxyz) \quad \text{Ans.}$$

Ans: 4 → given a, b, c are in AP

$\therefore \; 2b = a+c$

we have
$$\begin{vmatrix} x+1 & x+2 & x+a \\ x+2 & x+3 & x+b \\ x+3 & x+4 & x+c \end{vmatrix}$$

$R_1 \to R_1 + R_3$

$$= \begin{vmatrix} 2x+4 & 2x+6 & 2x+(a+c) \\ x+2 & x+3 & x+b \\ x+3 & x+4 & x+c \end{vmatrix}$$

$$= \begin{vmatrix} 2x+4 & 2x+6 & 2x+2b \\ x+2 & x+3 & x+b \\ x+3 & x+4 & x+c \end{vmatrix}$$

= taking 2 common from R_1

$$= 2\begin{vmatrix} x+2 & x+3 & x+b \\ x+2 & x+3 & x+b \\ x+3 & x+4 & x+c \end{vmatrix}$$

Since R_1 & R_2 are identical

$= 2 \times 0 = 0$ Ans

Ans 5 → given a, b, c are in AP

$\Rightarrow 2b = a+c$

we have
$$\begin{vmatrix} x+2 & x+3 & x+2a \\ x+3 & x+4 & x+2b \\ x+4 & x+5 & x+2c \end{vmatrix}$$

$R_1 \to R_1 + R_3$

$$= \begin{vmatrix} 2x+6 & 2x+8 & 2x+2(a+c) \\ x+3 & x+4 & x+2b \\ x+4 & x+5 & x+2c \end{vmatrix}$$

$$= \begin{vmatrix} 2x+6 & 2x+8 & 2x+4b \\ x+3 & x+4 & x+2b \\ x+4 & x+5 & x+2c \end{vmatrix} \quad \cdots \{2b = a+c\}$$

taking 2 common from R_1

$$= 2\begin{vmatrix} x+3 & x+4 & x+2b \\ x+3 & x+4 & x+2b \\ x+4 & x+5 & x+2c \end{vmatrix}$$

$= 2 \times 0$ --- {Since R_1 & R_2 are identically}

$= 0$ Ans

Qn-6 → α, β, γ are in AP

$\Rightarrow 2\beta = \alpha + \gamma$

we have
$$\begin{vmatrix} x-3 & x-4 & x-\alpha \\ x-2 & x-3 & x-\beta \\ x-1 & x-2 & x-\gamma \end{vmatrix}$$

$R_1 \to R_1 + R_3$

$$= \begin{vmatrix} 2x-4 & 2x-6 & 2x-(\alpha+\gamma) \\ x-2 & x-3 & x-\beta \\ x-1 & x-2 & x-\gamma \end{vmatrix}$$

$$= \begin{vmatrix} 2x-4 & 2x-6 & 2x-2\beta \\ x-2 & x-3 & x-\beta \\ x-1 & x-2 & x-\gamma \end{vmatrix}$$

taking 2 common from R_1

$$= 2\begin{vmatrix} x-2 & x-3 & x-\beta \\ x-2 & x-3 & x-\beta \\ x-1 & x-2 & x-\gamma \end{vmatrix} = 2 \times 0 \quad \{R_1 \,\&\, R_2 \text{ are identically}\}$$

$= 0$ Ans

Ans. 7 → $\begin{vmatrix} x & x^2 & yz \\ y & y^2 & zx \\ z & z^2 & xy \end{vmatrix}$

$R_1 \to xR_1$, $R_2 \to yR_2$ and $R_3 \to zR_3$

$= \frac{1}{xyz}\begin{vmatrix} x^2 & x^3 & xyz \\ y^2 & y^3 & xyz \\ z^2 & z^3 & xyz \end{vmatrix}$

taking xyz common from C_3

$= \frac{xy\cancel{z}}{x\cancel{y}z}\begin{vmatrix} x^2 & x^3 & 1 \\ y^2 & y^3 & 1 \\ z^2 & z^3 & 1 \end{vmatrix}$

$R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$

$= \begin{vmatrix} x^2 & x^3 & 1 \\ y^2-x^2 & y^3-x^3 & 0 \\ z^2-x^2 & z^3-x^3 & 0 \end{vmatrix}$

taking $(y-x)$ & $(z-x)$ common from R_2 & R_3 resp

$= (y-x)(z-x)\begin{vmatrix} x^2 & x^3 & 1 \\ y+x & y^2+x^2+xy & 0 \\ z+x & z^2+x^2+zx & 0 \end{vmatrix}$

$R_3 \to R_3 - R_2$

$= (y-x)(z-x)\begin{vmatrix} x^2 & x^3 & 1 \\ y+x & y^2+x^2+xy & 0 \\ z-y & z^2-y^2+x(z-y) & 0 \end{vmatrix}$

taking $(z-y)$ common from R_3

$= (y-x)(z-x)(z-y)\begin{vmatrix} x^2 & x^3 & 1 \\ y+x & y^2+x^2+xy & 0 \\ 1 & z+y+x & 0 \end{vmatrix}$

expanding

$$= (y-x)(z-x)(z-y)\left[x^2(0) - x^3(0) + 1(y+x)(z+y+x) - (x^2+y^2+xy)\right]$$

$$= \underset{(-)}{(y-x)}(z-x)\underset{(-)}{(z-y)}\left[zy + \cancel{y^2} + xy + zx + \cancel{yx} + \cancel{x^2} - \cancel{x^2} - \cancel{y^2} - \cancel{xy}\right]$$

$$= (x-y)(y-z)(z-x)(xy+yz+zx) = RHS \quad \underline{\text{Ans}}$$

Ans. 8 →
$$\begin{vmatrix} a^2+1 & ab & ac \\ ab & b^2+1 & bc \\ ca & cb & c^2+1 \end{vmatrix}$$

$R_1 \to aR_1$, $R_2 \to bR_2$, $R_3 \to cR_3$

$$= \frac{1}{abc}\begin{vmatrix} a^3+a & a^2b & a^2c \\ ab^2 & b^3+b & b^2c \\ c^2a & c^2b & c^3+c \end{vmatrix}$$

taking a, b, c common from C_1, C_2 & C_3 resp

$$= \frac{\cancel{abc}}{\cancel{abc}}\begin{vmatrix} a^2+1 & a^2 & a^2 \\ b^2 & b^2+1 & b^2 \\ c^2 & c^2 & c^2+1 \end{vmatrix}$$

$R_1 \to R_1 + R_2 + R_3$

$$= \begin{vmatrix} 1+a^2+b^2+c^2 & 1+a^2+b^2+c^2 & 1+a^2+b^2+c^2 \\ b^2 & b^2+1 & b^2 \\ c^2 & c^2 & c^2+1 \end{vmatrix}$$

taking $1+a^2+b^2+c^2$ common from R_1

$$= (1+a^2+b^2+c^2)\begin{vmatrix} 1 & 1 & 1 \\ b^2 & b^2+1 & b^2 \\ c^2 & c^2 & c^2+1 \end{vmatrix}$$

$C_2 \to C_2 - C_1$ and $C_3 \to C_3 - C_1$

$$= (1+a^2+b^2+c^2)\begin{vmatrix} 1 & 0 & 0 \\ b^2 & 1 & b^2 \\ c^2 & 0 & 1 \end{vmatrix}$$

expanding

$= (1+a^2+b^2+c^2)(1)$

$= 1+a^2+b^2+c^2 = RHS$ **PROVED**

Q.No. 9 →
$$\begin{vmatrix} -bc & b^2+bc & c^2+bc \\ a^2+ac & -ac & c^2+ac \\ a^2+ab & b^2+ab & -ab \end{vmatrix}$$

Multiply R_1, R_2, R_3 by a, b, c resp.

$$= \frac{1}{abc}\begin{vmatrix} -abc & ab^2+abc & ac^2+abc \\ a^2b+abc & -abc & bc^2+abc \\ a^2c+abc & b^2c+abc & -abc \end{vmatrix}$$

taking a, b, c common from C_1, C_2 & C_3

$$= \frac{abc}{abc}\begin{vmatrix} -bc & ab+ac & ac+ab \\ ab+bc & -ac & bc+ab \\ ac+bc & bc+ac & -ab \end{vmatrix}$$

$R_1 \to R_1 + R_2 + R_3$

$$= \begin{vmatrix} ab+bc+ca & ab+bc+ca & ab+bc+ca \\ ab+bc & -ac & bc+ab \\ ac+bc & bc+ac & -ab \end{vmatrix}$$

taking $(ab+bc+ca)$ common from R_1

$$= (ab+bc+ca)\begin{vmatrix} 1 & 1 & 1 \\ ab+bc & -ac & bc+ab \\ ac+bc & bc+ac & -ab \end{vmatrix}$$

$C_2 \to C_2 - C_1$ and $C_3 \to C_3 - C_1$

$$= (ab+bc+ca)\begin{vmatrix} 1 & 0 & 0 \\ ab+bc & -(ab+bc+ac) & 0 \\ ac+bc & 0 & -(ab+bc+ac) \end{vmatrix}$$

expanding

$= (ab+bc+ac)\left[1(ab+bc+ca)^2\right]$

$= (ab+bc+ca)^3 =$ RHS ANS

Qn 10 → $\begin{vmatrix} x+y & x & x \\ 5x+4y & 4x & 2x \\ 10x+8y & 8x & 3x \end{vmatrix}$

Sum property in C_1

$= \begin{vmatrix} x & x & x \\ 5x & 4x & 2x \\ 10x & 8x & 3x \end{vmatrix} + \begin{vmatrix} y & x & x \\ 4y & 4x & 2x \\ 8y & 8x & 3x \end{vmatrix}$

x common from C_1, C_2, C_3 y, x common from C_1, C_2 & C_3

$= x^3 \begin{vmatrix} 1 & 1 & 1 \\ 5 & 4 & 2 \\ 10 & 8 & 3 \end{vmatrix} + yx^2 \begin{vmatrix} 1 & 1 & 1 \\ 4 & 4 & 2 \\ 8 & 8 & 3 \end{vmatrix}$

$C_2 \to C_2 - C_1$ & $C_3 \to C_3 - C_1$ C_1 & C_2 are identical

$= x^3 \begin{vmatrix} 1 & 0 & 0 \\ 5 & -1 & -3 \\ 10 & -2 & -7 \end{vmatrix} + yx^2(0)$

expanding $x^3\left(1(7-6) - 0 + 0\right) + 0$

$= x^3(1)$

$= x^3 =$ RHS ANS

Qn 11 → $\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix}$

Taking a, b, c common from R_1, R_2 & R_3 resp

$$= abc \begin{vmatrix} \frac{1}{a}+1 & \frac{1}{a} & \frac{1}{a} \\ \frac{1}{b} & \frac{1}{b}+1 & \frac{1}{b} \\ \frac{1}{c} & \frac{1}{c} & \frac{1}{c}+1 \end{vmatrix}$$

$R_1 \to R_1 + R_2 + R_3$

$$= abc \begin{vmatrix} 1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c} & 1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c} & 1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c} \\ \frac{1}{b} & \frac{1}{b}+1 & \frac{1}{b} \\ \frac{1}{c} & \frac{1}{c} & \frac{1}{c}+1 \end{vmatrix}$$

$$= abc\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right) \begin{vmatrix} 1 & 1 & 1 \\ 1/b & 1/b+1 & 1/b \\ 1/c & 1/c & 1/c+1 \end{vmatrix}$$

$C_2 \to C_2 - C_1$ & $C_3 \to C_3 - C_1$

$$= abc\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right) \begin{vmatrix} 1 & 0 & 0 \\ 1/b & 1 & 1 \\ 1/c & 0 & 1 \end{vmatrix}$$

Expanding

$$= abc\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right) \left[1(1) - 0 + 0\right]$$

$$= abc\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right) = \underline{RHS}$$

open the bracket

$= abc + bc + bc + ab = \underline{RHS}$ Proved

Qn 12 → $\Delta = \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$

$C_1 \to C_1 + C_2 + C_3$

$$= \begin{vmatrix} a+b+c & b & c \\ a+b+c & c & a \\ a+b+c & a & b \end{vmatrix}$$

taking $(a+b+c)$ common from C_1

(x')   (Q 5) Solution (x . a (''))

$$= (a+b+c)\begin{vmatrix} 1 & b & c \\ 1 & c & a \\ 1 & a & b \end{vmatrix}$$

$R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$

$$= (a+b+c)\begin{vmatrix} 1 & b & c \\ 0 & c-b & a-c \\ 0 & a-b & b-c \end{vmatrix}$$

expanding

$$= (a+b+c)\left[(c-b)(b-c) - (a-b)(a-c)\right]$$

$$= (a+b+c)\left(\cancel{bc} - c^2 - b^2 + bc - a^2 + ac + ab - \cancel{bc}\right)$$

$$= (a+b+c)\left(-a^2 - b^2 - c^2 + ab + bc + ca\right)$$

$$= -(a+b+c)\left(a^2 + b^2 + c^2 - ab - bc - ca\right)$$

multiply and divide by 2

$$= -\frac{1}{2}(a+b+c)\left(2a^2 + 2b^2 + 2c^2 - 2ab - 2bc - 2ca\right)$$

$$= -\frac{1}{2}(a+b+c)\left[(a^2+b^2-2ab) + (b^2+c^2-2bc) + (c^2+a^2-2ca)\right]$$

$$= -\frac{1}{2}(a+b+c)\left((a-b)^2 + (b-c)^2 + (c-a)^2\right)$$

Clearly the value of determinant is -ve Ans

{ Since $(a+b+c)$ is +ve and sum of square can never be -ve }

Qn 13 Given $\begin{vmatrix} b+c & c+a & a+b \\ c+a & a+b & b+c \\ a+b & b+c & c+a \end{vmatrix} = 0$

(D-5) solution

$C_1 \to C_1 + C_2 + C_3$

$$\Rightarrow \begin{vmatrix} 2(a+b+c) & c+a & a+b \\ 2(a+b+c) & a+b & b+c \\ 2(a+b+c) & b+c & c+a \end{vmatrix} = 0$$

$$\Rightarrow 2(a+b+c) \begin{vmatrix} 1 & c+a & a+b \\ 1 & a+b & b+c \\ 1 & b+c & c+a \end{vmatrix} = 0$$

$R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$

$$\Rightarrow 2(a+b+c) \begin{vmatrix} 1 & c+a & a+b \\ 0 & b-c & c-a \\ 0 & b-a & c-b \end{vmatrix} = 0$$

expanding

$$\Rightarrow 2(a+b+c)\left[(b-c)(c-b) - (b-a)(c-a)\right] = 0$$

$$\Rightarrow 2(a+b+c)\left(\cancel{bc} - b^2 - c^2 + bc - \cancel{bc} + ab + ac - a^2\right) = 0$$

$$\Rightarrow 2(a+b+c)\left(a^2 + b^2 + c^2 - ab - bc - ca\right) = 0$$

$$\Rightarrow (a+b+c)\left(2a^2 + 2b^2 + 2c^2 - 2ab - 2bc - 2ca\right) = 0$$

$$\Rightarrow (a+b+c)\left((a-b)^2 + (b-c)^2 + (c-a)^2\right) = 0$$

$\Rightarrow$ either $a+b+c=0$ (or) $(a-b)^2 + (b-c)^2 + (c-a)^2 = 0$

this is possible only when

$a-b=0$; $b-c=0$ and $c-a=0$

$\Rightarrow a=b$; $b=c$; $c=a$

$\Rightarrow a=b=c$

$\therefore$ either $a+b+c=0$ (or) $a=b=c$ Proved

Ans: 14 → LHS
$$\begin{vmatrix} a+bx & c+dx & p+qx \\ ax+b & cx+d & px+q \\ u & v & w \end{vmatrix}$$

$R_1 \to R_1 - xR_2$

$$= \begin{vmatrix} a-ax^2 & c-cx^2 & p-px^2 \\ ax+b & cx+d & px+q \\ u & v & w \end{vmatrix}$$

$$= \begin{vmatrix} a(1-x^2) & c(1-x^2) & p(1-x^2) \\ ax+b & cx+d & px+q \\ u & v & w \end{vmatrix}$$

taking $(1-x^2)$ common from R_1

$$= (1-x^2) \begin{vmatrix} a & c & p \\ ax+b & cx+d & px+q \\ u & v & w \end{vmatrix}$$

$R_2 \to R_2 - xR_1$

$$= (1-x^2) \begin{vmatrix} a & c & p \\ b & d & q \\ u & v & w \end{vmatrix} = RHS$$

PROVED

Ans 15 →
$$\begin{vmatrix} b+c & a & a \\ b & c+a & b \\ c & c & a+b \end{vmatrix}$$

$R_1 \to R_1 + R_2 + R_3$

$$= \begin{vmatrix} 2(b+c) & 2(c+a) & 2(a+b) \\ b & c+a & b \\ c & c & a+b \end{vmatrix}$$

taking 2 common from R_1

$$= 2 \begin{vmatrix} b+c & c+a & a+b \\ b & c+a & b \\ c & c & a+b \end{vmatrix}$$

$R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$

$$= 2\begin{vmatrix} b+c & c+a & a+b \\ -c & 0 & -a \\ -b & -a & 0 \end{vmatrix}$$

$R_1 \to R_1 + R_2 + R_3$

$$= 2\begin{vmatrix} 0 & c & b \\ -c & 0 & -a \\ -b & -a & 0 \end{vmatrix}$$

expanding

$$= 2\left[\,0 - c(0 - ab) + b(ac)\right]$$

$$= 2(abc + abc) = 2(2abc) = 4abc \text{ - proved}$$

Qn 16 →
$$\begin{vmatrix} a^2 & bc & ac + c^2 \\ a^2+ab & b^2 & ac \\ ab & b^2+bc & c^2 \end{vmatrix}$$

taking a, b, c common from C_1, C_2, C_3 resp

$$= abc\begin{vmatrix} a & c & a+c \\ a+b & b & a \\ b & b+c & c \end{vmatrix}$$

$R_1 \to R_1 + R_2 + R_3$

$$= abc\begin{vmatrix} 2(a+b) & 2(b+c) & 2(a+c) \\ a+b & b & a \\ b & b+c & c \end{vmatrix}$$

$$= 2abc\begin{vmatrix} a+b & b+c & a+c \\ a+b & b & a \\ b & b+c & c \end{vmatrix}$$

$R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$

$$= 2abc \begin{vmatrix} a+b & b+c & a+c \\ 0 & -c & -c \\ -a & 0 & -a \end{vmatrix}$$

$R_1 \to R_1 + R_2 + R_3$

$$= 2abc \begin{vmatrix} b & b & 0 \\ 0 & -c & -c \\ -a & 0 & -a \end{vmatrix}$$

expanding

$$= 2abc\left(b(ca-0) - b(0-ac) + 0 \right)$$

$$= 2abc\left(abc + abc \right)$$

$$= 2abc\left(2abc \right)$$

$$= 4a^2b^2c^2$$

Qu 17 →
$$\begin{vmatrix} b+c & q+r & y+z \\ c+a & r+p & z+x \\ a+b & p+q & x+y \end{vmatrix}$$

$R_1 \to R_1 + R_2 + R_3$

$$= \begin{vmatrix} 2(a+b+c) & 2(p+q+r) & 2(x+y+z) \\ c+a & r+p & z+x \\ a+b & p+q & x+y \end{vmatrix}$$

$$= 2\begin{vmatrix} a+b+c & p+q+r & x+y+z \\ c+a & r+p & z+x \\ a+b & p+q & x+y \end{vmatrix}$$

$R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$

$$= 2\begin{vmatrix} a+b+c & p+q+r & x+y+z \\ -b & -q & -y \\ -c & -r & -z \end{vmatrix}$$

$R_1 \to R_1 + R_2 + R_3$

(D-5) Solutions

$$= 2\begin{vmatrix} a & p & x \\ -b & -q & -y \\ -c & -r & -z \end{vmatrix}$$

taking (-ve) Sign common from R_2 & R_3

$$= 2(-1)^2\begin{vmatrix} a & p & x \\ b & q & y \\ c & r & z \end{vmatrix}$$

$$= 2\begin{vmatrix} a & p & x \\ b & q & y \\ c & r & z \end{vmatrix} = \underline{Rhs}$$

<u>Ans</u>