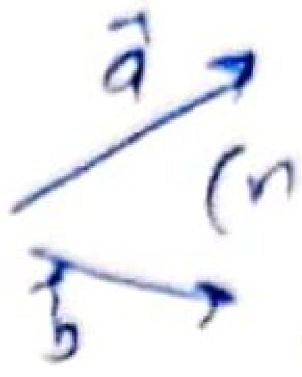
!! जय श्री राधे कृष्णा जय श्री गिरिराज जी महाराज !!

ULTIMATE MATHEMATICS: BY AJAY MITTAL

REVISION: $\boxed{\text{VECTORS \& 3-D}}$ combined. $\boxed{\text{CLASS NO: 1}}$

(1) Collinear vectors $\boxed{\vec{a} = \lambda \vec{b}}$ $\lambda \in R - \{0\}$

Same or parallel support



(not collinear vectors)

$$\boxed{\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}}$$

$$\boxed{\vec{a} \times \vec{b} = \vec{0}}$$

(2) Parallel vectors $\boxed{\vec{a} = \lambda \vec{b}}$ $\lambda$ must be positive

for anti-parallel $\lambda$ is $-ve$

Same $\boxed{\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}}$ and $\boxed{\vec{a} \times \vec{b} = \vec{0}}$

(3) If three "points" are collinear, then vectors made by these points are also collinear vectors

i.e. if A, B, C points are collinear, then $\vec{AB}, \vec{AC}$ are also collinear $\Rightarrow \vec{AB} = \lambda \vec{AC}$ (or) $\vec{AB} \times \vec{AC} = \vec{0}$

(or) $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

(4) Unit vector $\boxed{\hat{a} = \frac{\vec{a}}{|\vec{a}|}}$ ; $\boxed{|\hat{a}| = 1}$

$\hat{i}, \hat{j}, \hat{k}$ are unit vectors along x, y, z axis resp

$\hat{i} \cdot \hat{j} = \hat{j} \cdot \hat{k} = \hat{k} \cdot \hat{i} = 0$

$\hat{i} \times \hat{j} = +\hat{k}$ ; $\hat{j} \times \hat{i} = -\hat{k}$

$\hat{i} \times \hat{i} = \vec{0}$

$\hat{i} \cdot \hat{i} = \hat{j} \cdot \hat{j} = \hat{k} \cdot \hat{k} = 1$

(5) If two vectors are in the same direction / along / parallel

then their unit vector is same

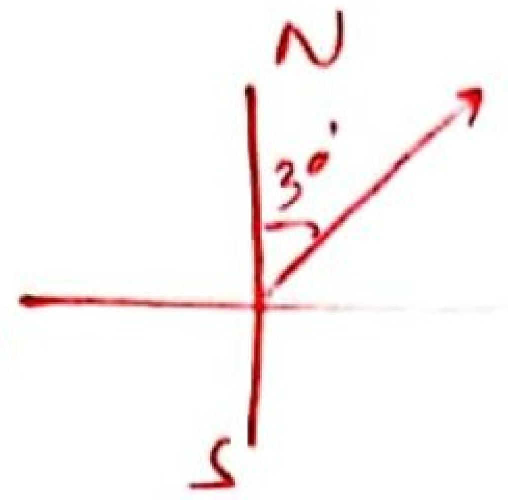


(6) $\boxed{\text{vector} = (\text{Unit vector})(\text{Magnitude})}$

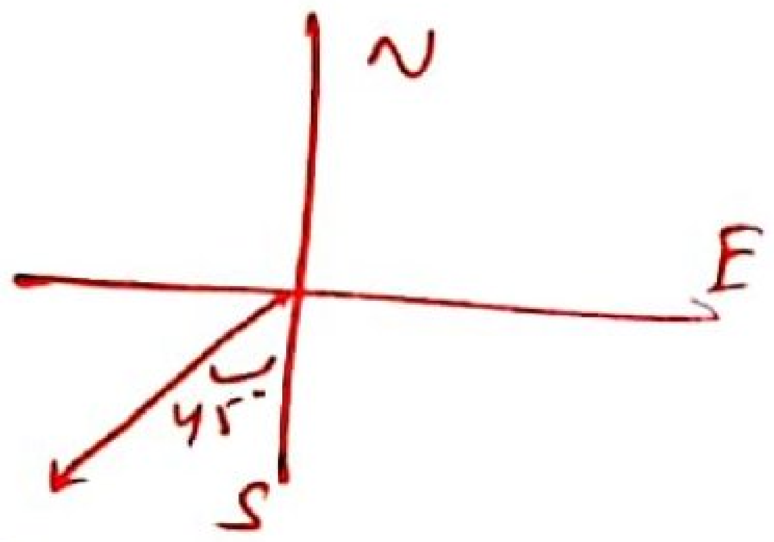
(7) Vector $= a\hat{i} + b\hat{j} + c\hat{k}$ when $a, b, c$ are D.R.s of vector

unit vector $= l\hat{i} + m\hat{j} + n\hat{k}$ when $l, m, n$ are D.C.s of vector

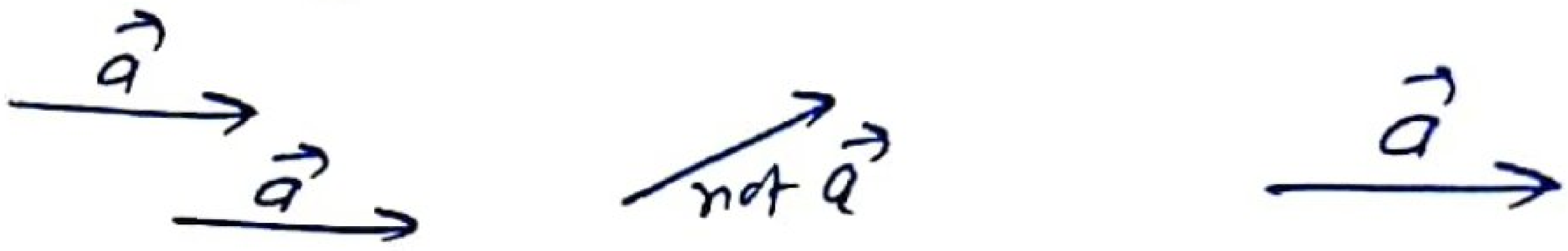
(8) $30^\circ$ east of North




$45^\circ$ west of South.

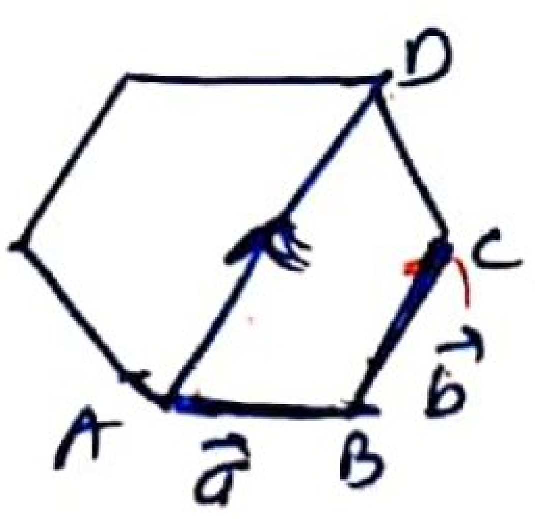



(9) Free vectors:




(10) Like & unlike vectors → same direction & opposite direction respectively

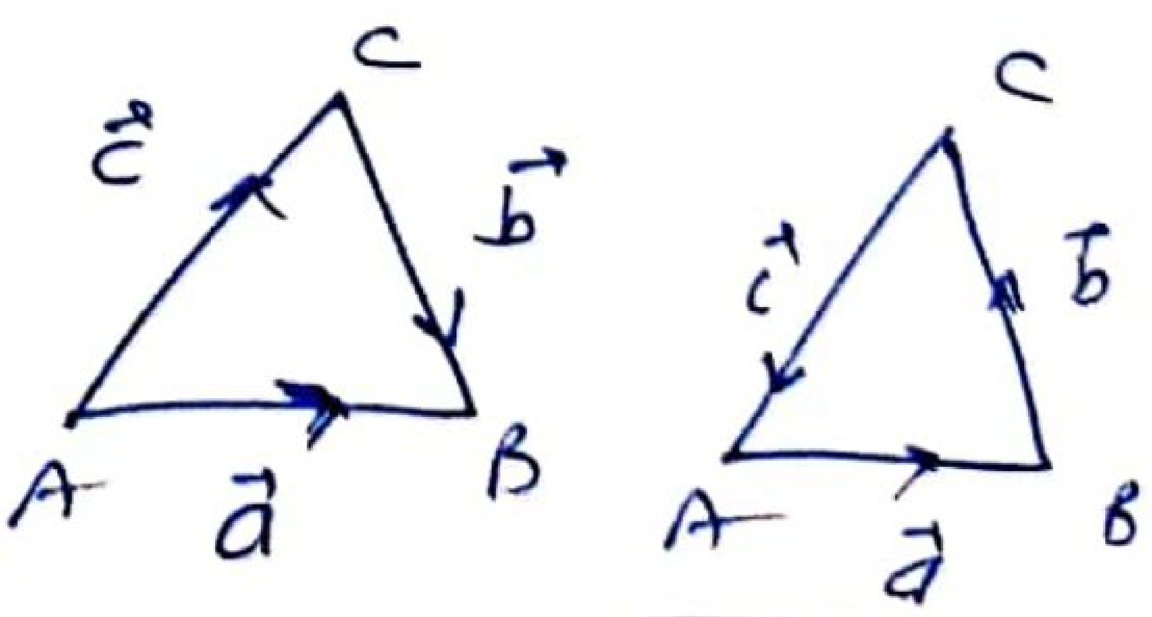
(11) In Regular hexagon.




$\vec{AD} = 2\vec{b}$

(12) Triangle law of addition of vectors

$\boxed{\vec{c} + \vec{b} = \vec{a}}$




$\boxed{\vec{a} + \vec{b} + \vec{c} = \vec{0}}$

(13) position vector of a point let $A(1,2,3)$

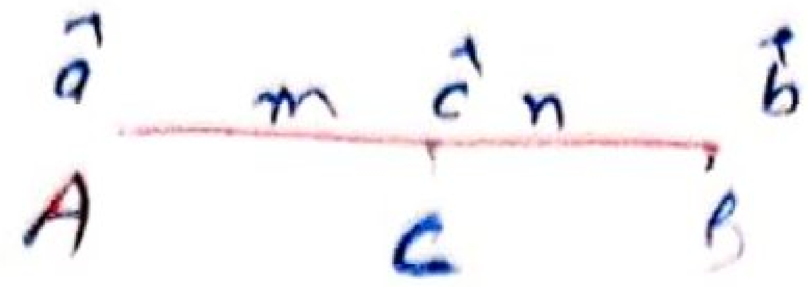
then p.v of point $A = \vec{OA} = \vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$

vector joining two points $A(1,2,3)$ $B(4,-3,4)$

$\vec{AB} = \vec{OB} - \vec{OA} = \vec{b} - \vec{a} = (4-1)\hat{i} + (-3-2)\hat{j} + (4-3)\hat{k}$

$\vec{BA} = \vec{OA} - \vec{OB} = -3\hat{i} + 5\hat{j} - \hat{k}$

(14) Section formula




Internal: $\vec{c} = \dfrac{m\vec{b} + n\vec{a}}{m+n}$

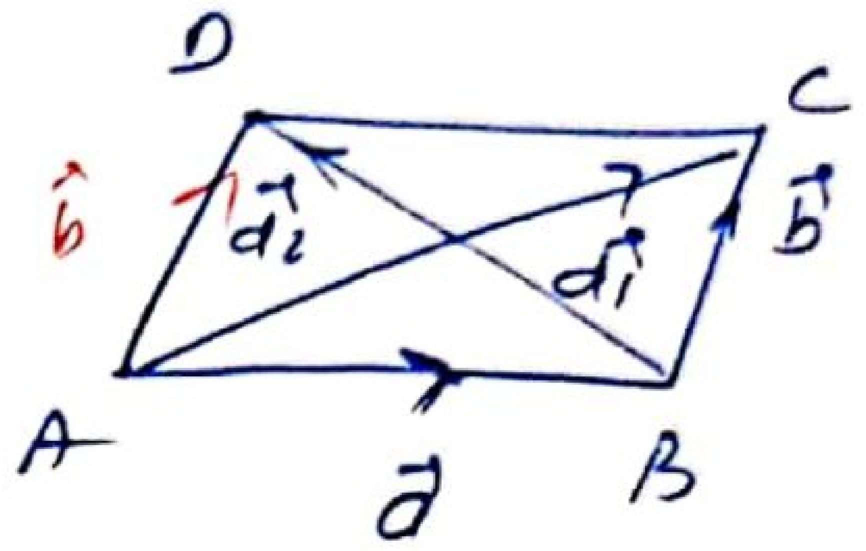
External $\vec{c} = \dfrac{m\vec{b} - n\vec{a}}{m-n}$

Mid point $\vec{c} = \dfrac{\vec{a}+\vec{b}}{2}$

(15) Any unit vector in xy plane $= \cos\theta\,\hat{i} + \sin\theta\,\hat{j}$

(16) Diagonals of a parallelogram

$\vec{d_1} = \vec{b} + \vec{a}$ & $\vec{d_2} = \vec{b} - \vec{a}$




(17) Extra If $\vec{a}$, $\vec{b}$ are any two non-zero, non-collinear vectors and $x$ & $y$ are scalars,

then $x\vec{a} + y\vec{b} = \vec{0} \Rightarrow \boxed{x = y = 0}$

$2\vec{a} + 3\vec{b} \neq \vec{0}$
$\vec{a} \neq -\frac{3}{2}\vec{b}$

(18) $l = \cos\alpha$, $m = \cos\beta$, $n = \cos\gamma$

$l^2 + m^2 + n^2 = 1$ & $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

$\alpha, \beta, \gamma$ are the angles made by vector with X, Y, Z resp

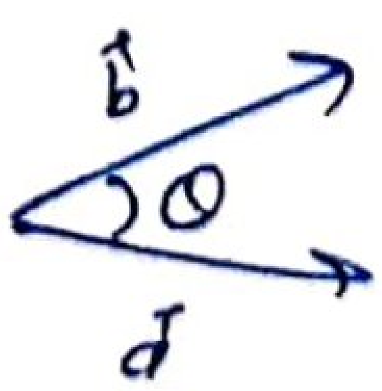
D.R's & D.C's are proportional

there are infinite set of DR's

there are only two set of D.C's

(19) $\vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|\cos\theta$

$\cos\theta = \dfrac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|}$




vectors must be coinitial

(20) for acute angle $\vec{a}\cdot\vec{b} > 0$

for obtuse angle $\vec{a}\cdot\vec{b} < 0$

(21) If $\vec{a}\cdot\vec{b}=0$ then either $\vec{a}=\vec{0}$, $\vec{b}=\vec{0}$ or $\vec{a}\perp\vec{b}$

(22) If $\vec{a}\times\vec{b}=\vec{0}$ then either $\vec{a}=\vec{0}$, $\vec{b}=\vec{0}$ or $\vec{a}\parallel\vec{b}$

(23) If $\vec{a}+\vec{b}+\vec{c}=\vec{0}$ then $\vec{a}\times\vec{b}=\vec{b}\times\vec{c}=\vec{c}\times\vec{a}$

(24) If $\vec{a}+\vec{b}+\vec{c}=\vec{0}$ then $|\vec{a}|^2+|\vec{b}|^2+|\vec{c}|^2+2\vec{a}\cdot\vec{b}+2\vec{b}\cdot\vec{c}+2\vec{c}\cdot\vec{a}=0$

To find angle b/w $\vec{a}$ & $\vec{b}$

Use $\vec{a}+\vec{b}=-\vec{c}$

(25) $\vec{a}\times\vec{b}=|\vec{a}||\vec{b}|\sin\theta\,\hat{n}$

$|\vec{a}\times\vec{b}|=|\vec{a}||\vec{b}|\sin\theta$

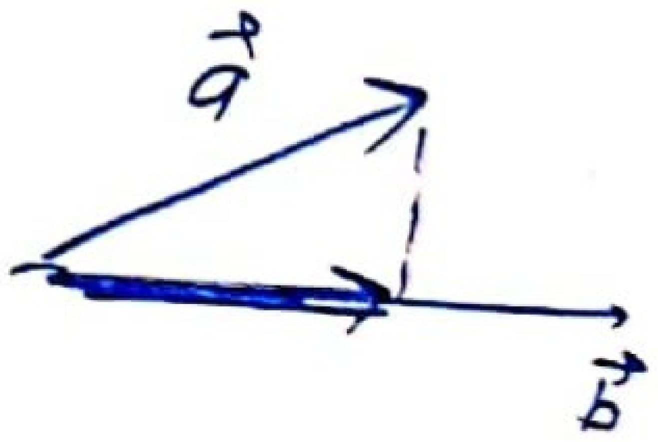
(26) $\vec{a}\cdot\vec{a}=|\vec{a}|^2$ $\quad |\hat{a}+\hat{b}|^2=(\hat{a}+\hat{b})\cdot(\hat{a}+\hat{b})=|\hat{a}|^2+|\hat{b}|^2+2\hat{a}\cdot\hat{b}$

$\vec{a}\times\vec{a}=\vec{0}$

(27) $\vec{a}\cdot(\vec{b}+\vec{c})=\vec{a}\cdot\vec{b}+\vec{a}\cdot\vec{c}=\vec{a}\cdot(\vec{b}+\vec{c})$

$\vec{a}\times(\vec{b}+\vec{c})=\vec{a}\times\vec{b}+\vec{a}\times\vec{c}=\vec{a}\times(\vec{b}+\vec{c})$

(28) Projection of $\vec{a}$ on $\vec{b}$ $=\dfrac{\vec{a}\cdot\vec{b}}{|\vec{b}|}$

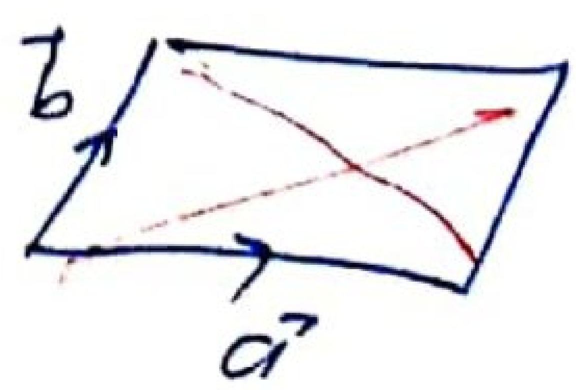



Projection vector $=\left(\dfrac{\vec{a}\cdot\vec{b}}{|\vec{b}|}\right)\hat{b}$

(29) Area of triangle ABC $=\frac{1}{2}|\vec{AB}\times\vec{AC}|$

$=\frac{1}{2}|\vec{AB}\times\vec{CA}|$

(30) Area of parallelogram $=|\vec{a}\times\vec{b}|$

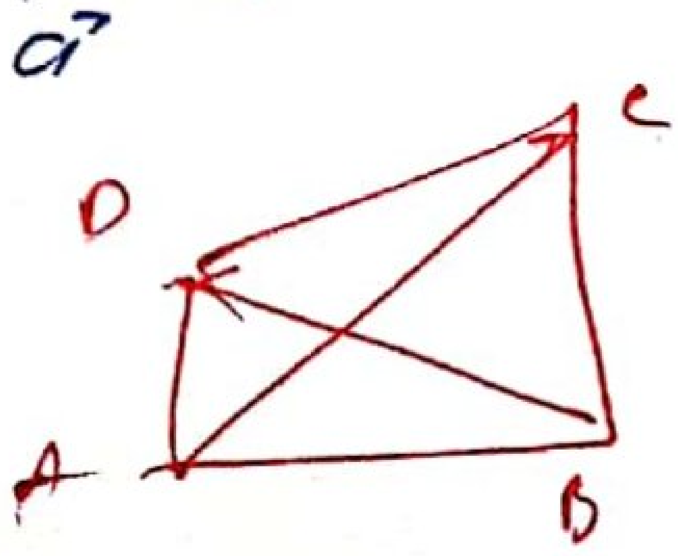



$=\frac{1}{2}|\vec{d_1}\times\vec{d_2}|$

where $\vec{d_1}=\vec{b}+\vec{a}$

$\vec{d_2}=\vec{b}+\vec{a}$

· Area of quadrilateral $=\frac{1}{2}|\vec{AC}\times\vec{BD}|$



(31) $m\vec{a} \cdot n\vec{b} = mn(\vec{a} \cdot \vec{b})$

$m\vec{a} \times n\vec{b} = mn(\vec{a} \times \vec{b})$

$|\lambda \vec{a}| = |\lambda||\vec{a}|$

(32) • any vector $\perp^r$ to $\vec{a} \times \vec{b} = \vec{a} \times \vec{b}$

• but with some given condition take $\lambda(\vec{a} \times \vec{b})$

• $\vec{a} \times \vec{b}$ is a vector normal to the plane containing $\vec{a}$ & $\vec{b}$

(33) there are always two unit vectors normal to the plane i.e ~~upw~~ upward & down ward

$$\therefore \hat{n} = \pm \frac{(\vec{a} \times \vec{b})}{|\vec{a} \times \vec{b}|}$$

Scalar Triple product:

(34) here always $\vec{a}, \vec{b}, \vec{c}$ i.e three vectors are used

Scalar Triple product $= \vec{a} \cdot (\vec{b} \times \vec{c}) = [\vec{a}\ \vec{b}\ \vec{c}] = (\vec{a} \times \vec{b}) \cdot \vec{c}$

$$= \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = \text{Scalar quantity}$$

(35) $[\vec{a}\ \vec{b}\ \vec{c}] = [\vec{b}\ \vec{c}\ \vec{a}] = [\vec{c}\ \vec{a}\ \vec{b}]$

$\vec{a} \cdot (\vec{b} \times \vec{c}) = \vec{b} \cdot (\vec{c} \times \vec{a}) = \vec{c} \cdot (\vec{a} \times \vec{b})$

$(\vec{a} \times \vec{b}) \cdot \vec{c} = (\vec{b} \times \vec{c}) \cdot \vec{a} = (\vec{c} \times \vec{a}) \cdot \vec{b}$

(36) $[\vec{a}\ \vec{b}\ \vec{c}] = -[\vec{b}\ \vec{a}\ \vec{c}]$

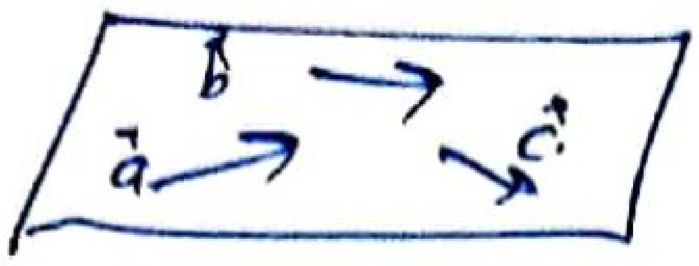
(37) $[\vec{a}\ \vec{a}\ \vec{b}] = 0 = [\vec{a}\ b\ \vec{a}] = [\vec{a}\ \vec{c}\ \vec{c}] = 0$

(38) $[\lambda\vec{a}\ \mu\vec{b}\ \gamma\vec{c}] = \lambda\mu\gamma[\vec{a}\ \vec{b}\ \vec{c}]$

$[\lambda\vec{a}\ \vec{b}\ \vec{c}] = \lambda(\vec{a}\ \vec{b}\ \vec{c})$

(39) vector $\vec{a}, \vec{b}, \vec{c}$ are coplanar if $[\vec{a}\ \vec{b}\ \vec{c}] = 0$

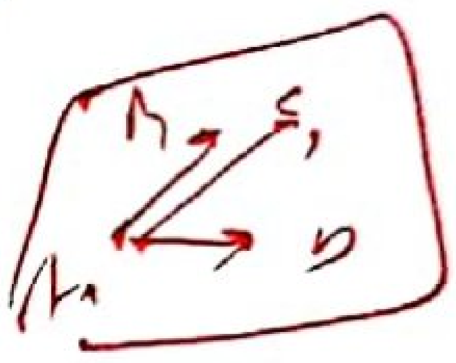
i.e




(40) volume of parallelopiped with three adjacent edges $\vec{a}, \vec{b}, \vec{c} = \left|[\vec{a}\ \vec{b}\ \vec{c}]\right|$ cubic units Ans

(41) four point A, B, C, D are coplanar if

$[\vec{AB}\ \ \vec{AC}\ \ \vec{AD}] = 0$




Remember : (1) Two points are always collinear

(•) To check three points are collinear

(2) Two vectors are always coplanar

To check three vectors are coplanar

(3) Three points are always coplanar

To check four points are coplanar

— x —

(3-D)

(42) equation of line $\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c}$

also $\frac{x-x_1}{l} = \frac{y-y_1}{m} = \frac{z-z_1}{n}$

also $\vec{r} = \vec{a} + \lambda \vec{b}$

where $\vec{b} = a\hat{i} + b\hat{j} + c\hat{k}$

$\vec{a}$ p.v of given point

$(x_1, y_1, z_1)$ is the fixed point

a, b, c are the DR's of line (or) $\vec{b}$

$\vec{a}, \vec{b}, \vec{c}$ are also projections of line on x, y, z axis resp.

for any point on the line always let

$$\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c} = \lambda$$

then $(x, y, z)$ is any point on the line

(43) equation of line in two point form

$$\frac{x-x_1}{x_2-x_1} = \frac{y-y_1}{y_2-y_1} = \frac{z-z_1}{z_2-z_1}$$

(or) $\vec{r} = \vec{a} + \lambda(\vec{b} - \vec{a})$ where $\vec{a}, \vec{b}$ are P.V of given points

(44) when two lines are parallel

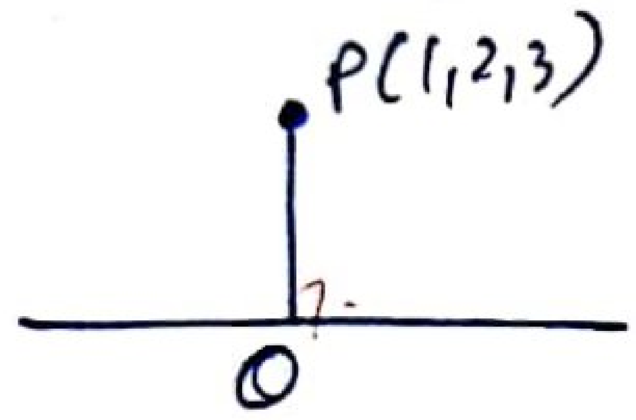
take $\vec{b_1} = \vec{b_2}$

(or) D.R's are same

when two lines are $\perp^r$

always use $a_1a_2 + b_1b_2 + c_1c_2 = 0$

eg



$\frac{x-1}{2} = \frac{y+1}{3} = \frac{z}{4} = \lambda \Rightarrow Q(2\lambda+1,\ 3\lambda-1,\ 4\lambda)$

DR of PQ $= 2\lambda+1-1,\ 3\lambda-1-2,\ 4\lambda-3 = 2\lambda,\ 3\lambda-3,\ 4\lambda-3$

PQ $\perp$ given line $\therefore (2\lambda)(2) + (3\lambda-3)(3) + (4\lambda-3)(4) = 0$

$\lambda =$ $\therefore Q(\ )$

(45) angle b/w two lines

$$\cos\theta = \frac{\vec{b_1}\cdot\vec{b_2}}{|\vec{b_1}|\,|\vec{b_2}|} = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\ \sqrt{a_2^2+b_2^2+c_2^2}}$$

(46) Shortest distance b/w two skew lines / non-parallel / non-intersecting lines

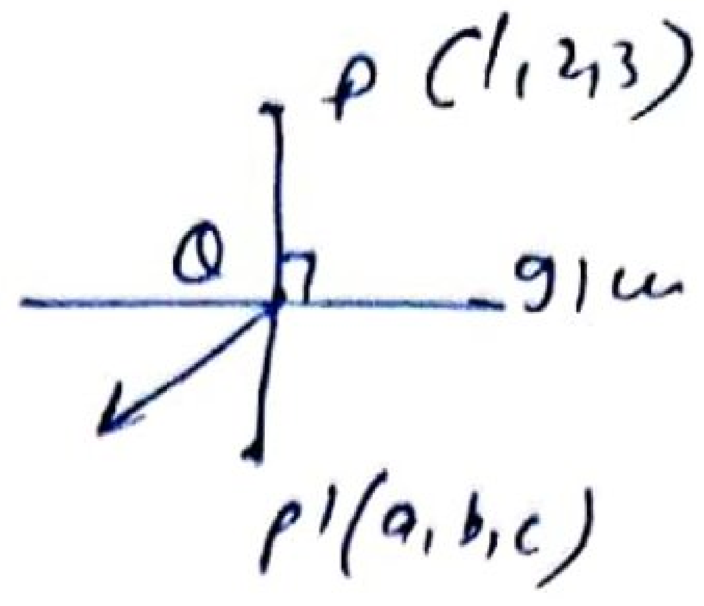
$$= \frac{|(\vec{a_2}-\vec{a_1})\cdot(\vec{b_1}\times\vec{b_2})|}{|\vec{b_1}\times\vec{b_2}|}$$

(47) Distance b/w two parallel lines

$\vec{r} = \vec{a_1} + \lambda\vec{b}$ & $\vec{r} = \vec{a_2} + \mu\vec{b}$

$$\text{distance} = \frac{|(\vec{a_2}-\vec{a_1})\times\vec{b}|}{|\vec{b}|}$$

(48) Image of point in line




(49) Req line is $\perp$ to two given lines
then $\vec{b} = \vec{b_1} \times \vec{b_2}$

required line is $\parallel$ to two given planes
then $\vec{b} = \vec{n_1} \times \vec{n_2}$

(c) required line is $\parallel$ to plane & $\perp$ to given line
then $\vec{b} = \vec{n} \times \vec{b_1}$

(50) If $2a + b - 3c = 0$
$4a - 3b + 2c = 0$

$$\frac{a}{2-9} = \frac{-b}{4+12} = \frac{c}{-6-4} = \lambda \qquad a = \quad b = \quad c =$$

If line is $\perp$ plane then $\vec{b} = \vec{n}$

(51) equation of plane

general form $ax + by + cz = d \rightarrow$ any constant

a, b, c are DR's of $\vec{n}$

general form = $\vec{r} \cdot \vec{n} = d$

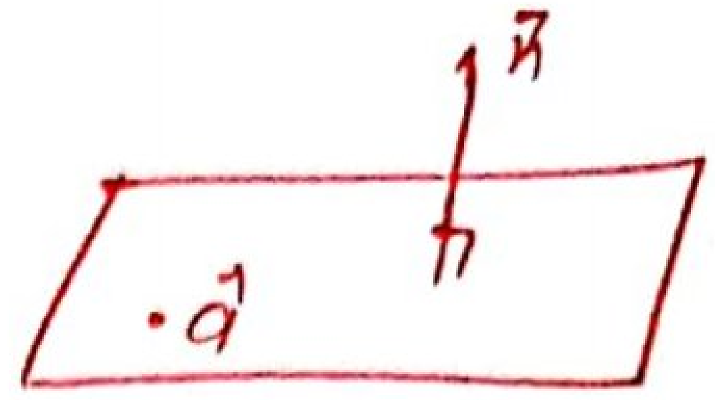
(52) normal form $lx + my + nz = d \rightarrow$ distance of plane from the origin

$\vec{r} \cdot \hat{n} = d$
$\hookrightarrow$ unit vector normal to the plane

$$\hat{n} = \frac{\vec{n}}{|\vec{n}|}$$

53 equation $(\vec{r}-\vec{a})\cdot\vec{n}=0$

(or) $\boxed{\vec{r}\cdot\vec{n}=\vec{a}\cdot\vec{n}}$



54 equation through three non-collinear points

$A(x_1,y_1,z_1)$ $B(x_2,y_2,z_2)$ $C(x_3,y_3,z_3)$

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & y_3-y_1 \end{vmatrix}=0$$

also $(\vec{r}-\vec{a})\cdot\left[(\vec{b}-\vec{a})\times(\vec{c}-\vec{a})\right]=0$

If we get $0=0$

then there are Infinite no of planes passing through these 3 points.

55 Intercept form $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$

this plane meets coordinate Axes at $A(a,a,c)$ $B(a,b,c)$ & $C(0,0,c)$

then DR's of $\vec{n}=\frac{1}{a},\frac{1}{b},\frac{1}{c}$

56 equation of plane passing through the line of Intersection of Intersection of two given planes $\vec{r}\cdot\vec{n_1}=d_1$ & $\vec{r}\cdot\vec{n_2}=d_2$

then $\vec{r}\cdot(\vec{n_1}+\lambda\vec{a_2})=d_1+\lambda d_2$

where $\lambda$ to be found out

Condition 1 plane passes through given point e.g $(1,2,3)$

Condition 2 plane is $\perp^r$ to $3^{rd}$ plane

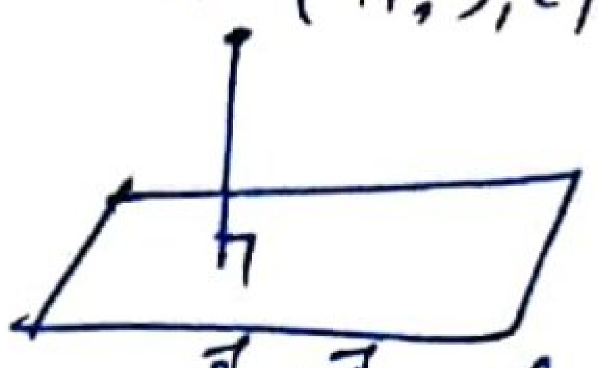
$\therefore \vec{n}\cdot\vec{n_3}=0$

Condition 3 → plane is parallel to given line
i.e. $\vec{b}\cdot\vec{n}=0$

Condition 4 : Distance of plane from a given point is given

(56) Distance of a point from the plane

$\vec{a}\,(x_1, y_1, z_1)$



$\vec{r}\cdot\vec{n}=d$

$ax+by+cz=d$

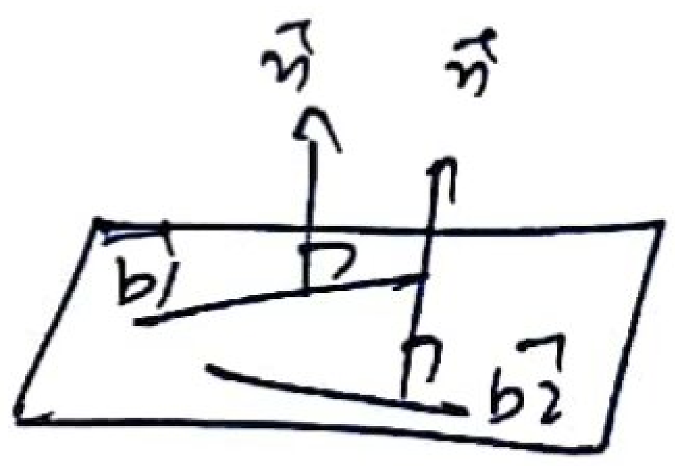
$$\text{distance} = \frac{|ax_1+by_1+cz_1-d|}{\sqrt{a^2+b^2+c^2}}$$

$$= \frac{|\vec{a}\cdot\vec{n}-d|}{|\vec{n}|}$$

(57) equation of plane containing two lines

direct formula
$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}=0$$

(01)



$\vec{n}=\vec{b_1}\times\vec{b_2}$

$\vec{a}=\vec{a_1}$ $\qquad \vec{r}\cdot\vec{n}=\vec{a}\cdot\vec{n}$

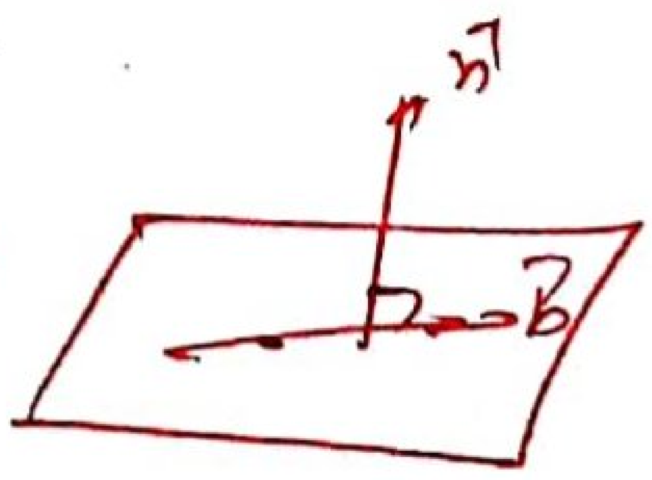
(58) Condition of coplanarity of two lines

given line $\vec{r}=\vec{a_1}+\lambda\vec{b_1}$ & $\vec{r}=\vec{a_2}+\mu\vec{b}$

is $(\vec{a_2}-\vec{a_1})\cdot(\vec{b_1}\times\vec{b_2})=0$ (a)
$$\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & c_2 & c_2 \end{vmatrix}=0$$

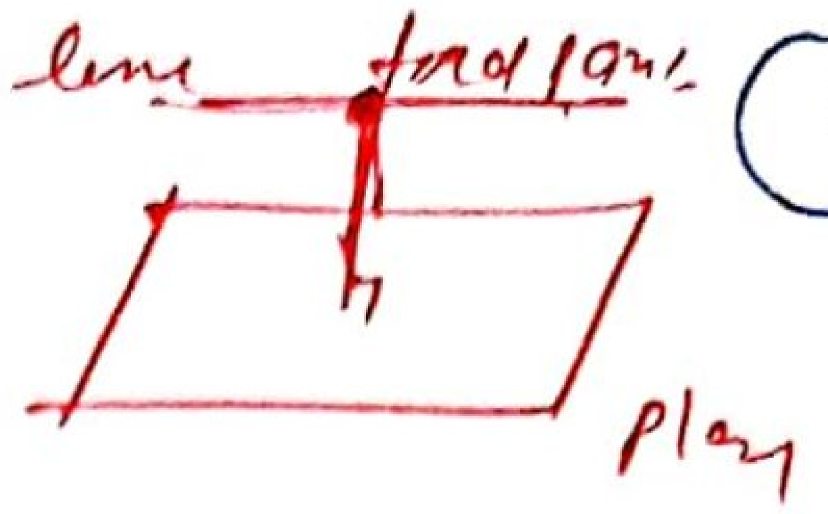
(59) condition plane contains the line

fixed point of line must satisfy equation of plane



& $\vec{b}\cdot\vec{n}=0$

(60) distance b/w line & plane




to find this distance
line must be parallel to the plane
i.e $\vec{b}\cdot\vec{n}=0$

then req. distance = distance b/w fixed point of the line & plane

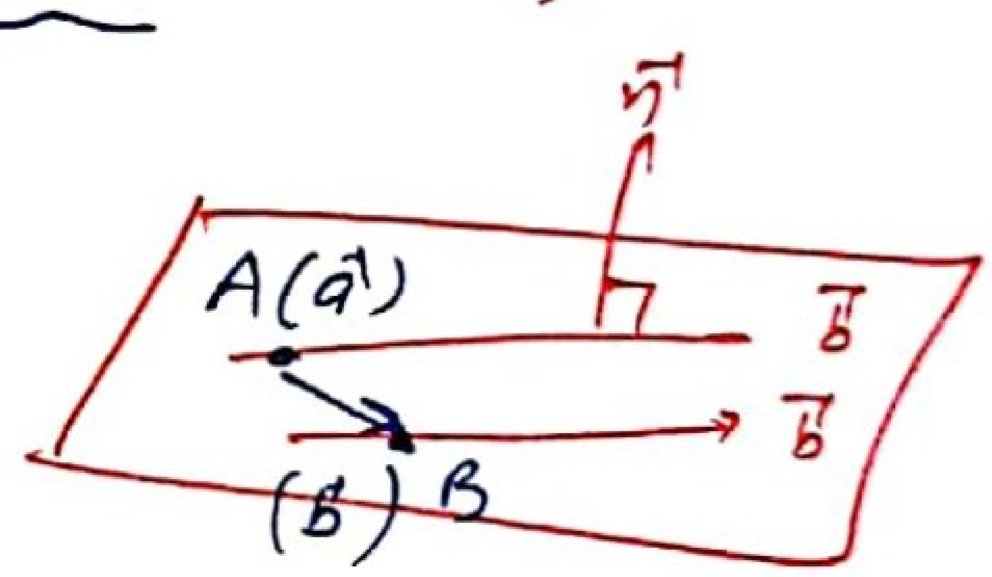
$$= \frac{|ax_1+by_1+cz_1-d|}{|\vec{n}|}$$

If $\vec{b}\cdot\vec{n}\neq 0$
then line must intersect the plane
$\therefore$ distance $=0$

(61) equation of plane containing two parallel lines

prepare $\vec{AB}$ now $\vec{n}=\vec{b}\times\vec{AB}$

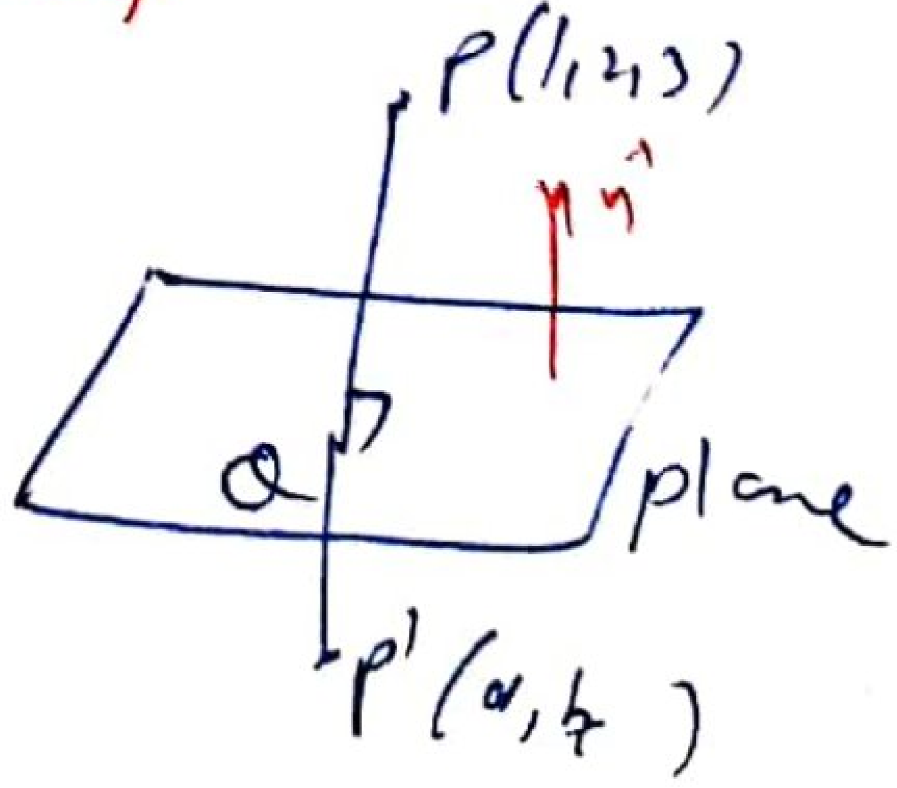
$\vec{r}\cdot\vec{n}=\vec{a}\cdot\vec{n}$




(62) angle b/w line & plane

$$\sin\theta=\frac{\vec{b}\cdot\vec{n}}{|\vec{b}||\vec{n}|}$$

(63) foot of $\perp$ / Image of point to the plane




DR of PQ = DR of $\vec{n}$

equation of PQ : $\frac{x-x_1}{a}=\frac{y-y_1}{b}=\frac{z-z_1}{c}$

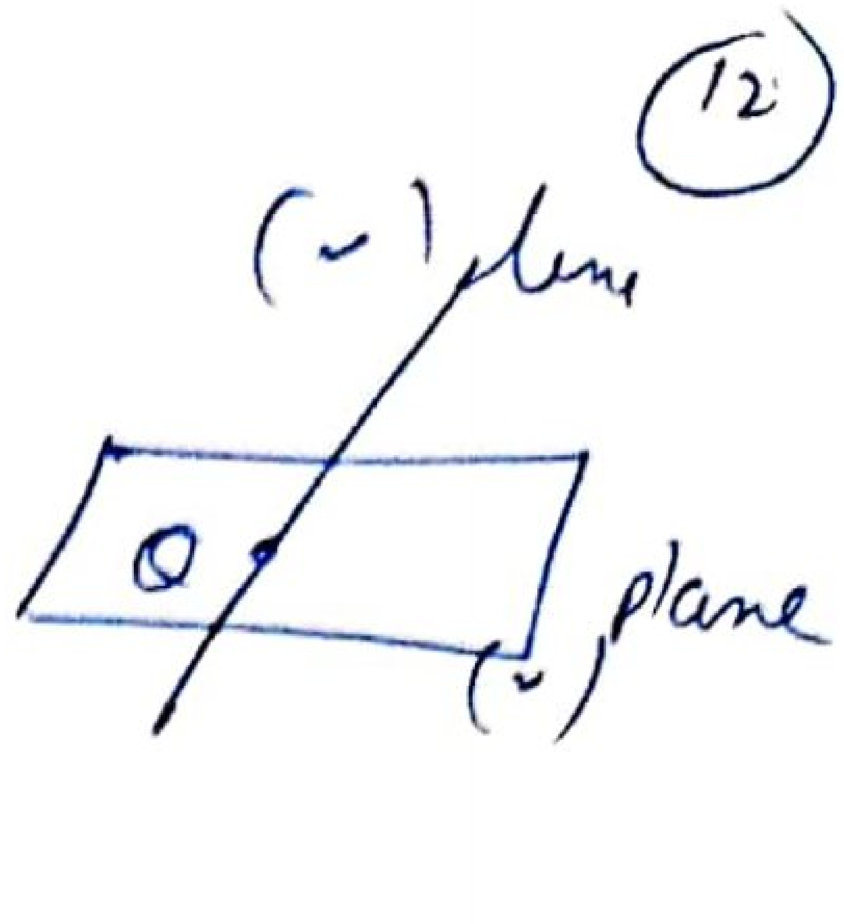
get any point Q (in terms of λ)

Q also lies on plane

get (λ=) $\therefore$ Q( ✓ ) $\therefore$ Image P'( ✓ )

## (64) Intersection point of line & plane

equation of line = ✓
plane = ✓
} cartesian form




let line = $\lambda$

get Q ($\lambda$ terms) put in equation of plane

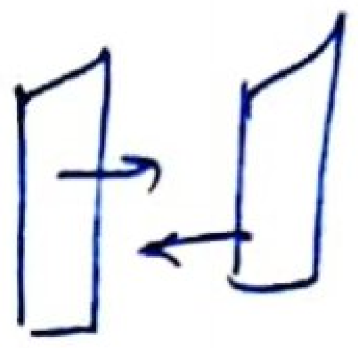
get $\lambda$ =

$\therefore Q(\ )$ Ans

## (65)

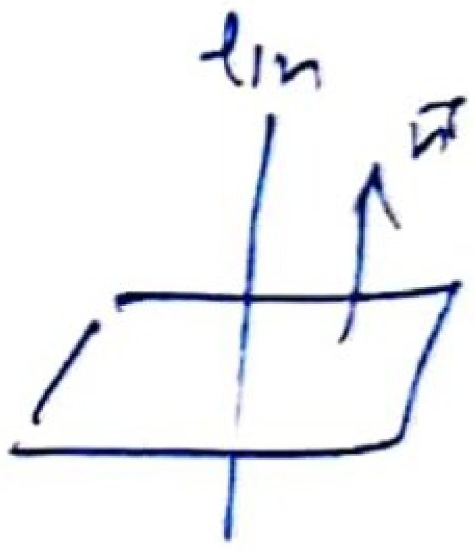
✓ plane $\parallel^{r}$ to given plane

then $\vec{n} = \vec{n_1}$



✓ plane $\perp^{r}$ to given line

then $\vec{n} = \vec{b}$




✓ plane $\perp^{r}$ to two given planes

$\vec{n} = \vec{n_1} \times \vec{n_2}$

✓ plane $\parallel^{r}$ to given lines (two)

$\vec{n} = \vec{b_1} \times \vec{b_2}$

✓ plane passing through two points A( ) & B( ) & $\perp^{r}$ to given plane

$\vec{n} = \vec{n_1} \times \vec{AB}$

✓ plane passing through two points A( ) & B( ) & $\parallel^{r}$ to given line

$\vec{n} = \vec{b} \times \vec{AB}$

## (66) distance b/w two parallel planes

$ax + by + cz = d_1$
$ax + by + cz = d_2$

$$D_{\parallel} = \frac{|d_1 - d_2|}{\sqrt{a^2+b^2+c^2}}$$

WORKSHEET NO:1 REVISION $\boxed{3\text{-}D}$ & $\boxed{\text{VECTORS}}$ Combined

**Qu-1** Find the Position vector of a point R which divides the line segment joining P and Q whose position vectors are $2\vec{a}+\vec{b}$ and $\vec{a}-3\vec{b}$ externally in the ratio 1:2. Also, Show that P is the mid point of the line segment RQ

Ans $\vec{OR} = 3\vec{a}+5\vec{b}$

**Qu-2** → The position vectors of the point P, Q, R are $\hat{i}+2\hat{j}+3\hat{k}$, $-2\hat{i}+3\hat{j}+5\hat{k}$ and $7\hat{i}-\hat{k}$ respectively. Prove that P, Q and R are collinear

**Qu-3** → Show that the points A, B and C with position vectors $\vec{a} = 3\hat{i}-4\hat{j}-4\hat{k}$, $\vec{b} = 2\hat{i}-\hat{j}+\hat{k}$ & $\vec{c} = \hat{i}-3\hat{j}-5\hat{k}$ respectively form the vertices of a right angled triangle.

**Qu-4** → Find a vector of magnitude of 5 units parallel to the resultant of the vectors $\vec{a} = 2\hat{i}+3\hat{j}-\hat{k}$ and $\vec{b} = \hat{i}-2\hat{j}+\hat{k}$

Ans $\frac{\sqrt{5}}{\sqrt{2}}(3\hat{i}+\hat{j})$

**Qu-5** → If $\vec{a} = \hat{i}+\hat{j}+\hat{k}$, $\vec{b} = 4\hat{i}-2\hat{j}+3\hat{k}$ and $\vec{c} = \hat{i}-2\hat{j}+\hat{k}$ find a vector of magnitude 6 units which is parallel to the vector $2\vec{a}-\vec{b}+3\vec{c}$

Ans $2\hat{i}-4\hat{j}+4\hat{k}$

**Qu-6** Find the value of p for which the vectors $\vec{a} = 3\hat{i}+2\hat{j}+9\hat{k}$ and $\vec{b} = \hat{i}+p\hat{j}+3\hat{k}$ are
(1) $\perp^r$ (2) parallel

Ans (1) −15 (2) 2/3

**Qu-7** → Let $\vec{a} = 4\hat{i}+5\hat{j}-\hat{k}$, $\vec{b} = \hat{i}-4\hat{j}+5\hat{k}$ and $\vec{c} = 3\hat{i}+\hat{j}-\hat{k}$. Find a vector $\vec{d}$ which is $\perp^r$ to both $\vec{a}$ & $\vec{b}$ satisfying $\vec{c}\cdot\vec{d} = 21$

Ans $7\hat{i}-7\hat{j}-7\hat{k}$

Q.No 8 → If $\vec{a}+\vec{b}+\vec{c}=\vec{0}$, $|\vec{a}|=3$, $|\vec{b}|=5$; $|\vec{c}|=7$

Find the angle b/w $\vec{a}$ & $\vec{b}$ Ans $= \pi/3$

Q.No 9 → If a unit vector $\vec{a}$ makes $\pi/4$ with $\hat{i}$, $\pi/3$ with $\hat{j}$ and acute angle $\theta$ with $\hat{k}$. Find the components of $\vec{a}$

Ans $\frac{1}{\sqrt{2}}\hat{i}, \frac{1}{2}\hat{j}, \frac{1}{2}\hat{k}$

Q.No 10 → Show that the vectors $2\hat{i}-\hat{j}+\hat{k}$, $\hat{i}-3\hat{j}-5\hat{k}$, $3\hat{i}-4\hat{j}-4\hat{k}$ form the sides of a right angled triangle

Q.No 11 → Find the values of 'a' for which the vector $\vec{r}=(a^2-4)\hat{i}+2\hat{j}-(a^2-9)\hat{k}$ makes acute angles with the coordinate axes Ans $(-3,-2)\cup(2,3)$

Hint $\vec{r}\cdot\hat{i}>0$ ; $\vec{r}\cdot\hat{j}>0$ ; $\vec{r}\cdot\hat{k}>0$

Q.No 12 → Find the projection of $\vec{b}+\vec{c}$ on $\vec{a}$ where $\vec{a}=2\hat{i}-2\hat{j}+\hat{k}$ & $\vec{b}=\hat{i}+2\hat{j}-2\hat{k}$ and $\vec{c}=2\hat{i}-\hat{j}+4\hat{k}$

Ans $=2$

Q.No 13 → Let $\vec{a},\vec{b},\vec{c}$ be unit vectors such that $\vec{a}\cdot\vec{b}=\vec{a}\cdot\vec{c}=0$ and the angle b/w $\vec{b}$ & $\vec{c}$ is $\pi/6$. Prove that $\vec{a}=\pm 2(\vec{b}\times\vec{c})$

Q.No 14 → Find a unit vector $\perp^r$ to each of $\vec{a}+\vec{b}$ & $\vec{a}-\vec{b}$ where $\vec{a}=3\hat{i}+2\hat{j}+2\hat{k}$ & $\vec{b}=\hat{i}+2\hat{j}-2\hat{k}$ Ans $\pm\frac{1}{3}(2\hat{i}-2\hat{j}-\hat{k})$

Q.No 15 → value of $(\hat{i}\times\hat{j})\cdot\hat{k}+(\hat{j}\times\hat{k})\cdot\hat{j}=?$ Ans $=2$

Q.No 16 → value of area of ||gm determined by the vectors $2\hat{i}$ and $3\hat{j}=?$ Ans 6 sq. units

Qn 17 Find $\lambda$ if $(2\hat{i}+6\hat{j}+14\hat{k})\times(\hat{i}-\lambda\hat{j}+7\hat{k})=\vec{0}$ Ans = $-3$

Qn. 18 If $\vec{a}$ is a unit vector such that $\vec{a}\times\hat{i}=\hat{j}$ find $\vec{a}\cdot\hat{i}$. Ans 0

Qn 19 If $|\vec{a}\times\vec{b}|=4$ & $|\vec{a}\cdot\vec{b}|=2$ then $|\vec{a}|^2|\vec{b}|^2=$
(a) 6 (b) 2 (c) 20 (d) 8 Ans. C

Qn. 20 If $\vec{a}$ is any vector, then $(\vec{a}\times\hat{i})^2+(\vec{a}\times\hat{j})^2+(\vec{a}\times\hat{k})^2=$
(a) $|\vec{a}|^2$ (b) $2|\vec{a}|^2$ (c) $3|\vec{a}|^2$ (d) $4|\vec{a}|^2$ Ans = b

Qn. 21 → The vectors $\vec{a}$ & $\vec{b}$ satisfy the equation $2\vec{a}+\vec{b}=\vec{p}$ and $\vec{a}+2\vec{b}=\vec{q}$ where $\vec{p}=\hat{i}+\hat{j}$ and $\vec{q}=\hat{i}-\hat{j}$. If $\theta$ is the angle b/w $\vec{a}$ & $\vec{b}$, then
(a) $\cos\theta=\frac{4}{5}$ (b) $\sin\theta=\frac{1}{\sqrt{2}}$ (c) $\cos\theta=-\frac{4}{5}$ (d) $\cos\theta=-\frac{3}{5}$ Ans (c)

Qn 22 The vector $(\cos\alpha\cos\beta)\hat{i}+(\cos\alpha\sin\beta)\hat{j}+(\sin\alpha)\hat{k}$ is a (A) null vector (B) unit vector (C) constant vector (D) none of these

Qn 23 → If $\vec{a}\cdot\hat{i}=\vec{a}\cdot(\hat{i}+\hat{j})=\vec{a}\cdot(\hat{i}+\hat{j}+\hat{k})=1$, then $\vec{a}=$
(A) $\vec{0}$ (B) $\hat{i}$ (C) $\hat{j}$ (D) $\hat{i}+\hat{j}+\hat{k}$ Ans (b)

Qn 24 → If the vectors $\hat{i}-2x\hat{j}+3y\hat{k}$ & $\hat{i}+2x\hat{j}-3y\hat{k}$ are $\perp^r$, then the locus of $(x,y)$ is
(A) circle (B) ellipse (C) hyperbola (D) none of these Ans (b)

Qn 25 The values of $x$ for which the angle between $\vec{a}=2x^2\hat{i}+4x\hat{j}+\hat{k}$, $\vec{b}=7\hat{i}-2\hat{j}+x\hat{k}$

is obtuse and the angle b/w $\vec{b}$ & the Z-axis is acute and less than $\pi/6$ are

(A) $x > 1/2$ or $x < 0$ (B) $0 < x < 1/2$ (C) $\frac{1}{2} < x < 15$ (D) $\phi$

Ans = B

Q.n 26 → Find the value of $\lambda$ so that the vectors $\vec{a} = \lambda\hat{i} - 2\hat{j} + 4\hat{k}$, $\vec{b} = -2\hat{i} + 4\hat{j} - 2\hat{k}$ & $\vec{c} = 4\hat{i} - 2\hat{j} - 2\hat{k}$ are coplanar Ans $\lambda = -2$

Q.n 27 → Show that the four points whose position vectors are $6\hat{i} - 7\hat{j}$, $16\hat{i} - 29\hat{j} - 4\hat{k}$, $3\hat{j} - 6\hat{k}$ and $2\hat{i} + 5\hat{j} + 10\hat{k}$ are coplanar

Q.n 28 → Find the value of '$\lambda$' for which the four points with position vector $3\hat{i} - 2\hat{j} - \hat{k}$, $2\hat{i} + 3\hat{j} - 4\hat{k}$, $-\hat{i} + \hat{j} + 2\hat{k}$ and $4\hat{i} + 5\hat{j} + \lambda\hat{k}$ are coplanar

Hint take $[\vec{AB}\ \vec{AC}\ \vec{AD}] = 0$ Ans $\lambda = \frac{-146}{17}$

Q.n 29 → If the vectors $\vec{\alpha} = a\hat{i} + a\hat{j} + c\hat{k}$, $\vec{\beta} = \hat{i} + \hat{k}$ and $\vec{\gamma} = c\hat{i} + c\hat{j} + b\hat{k}$ are coplanar, then Prove that $c$ is the geometric mean of $a$ & $b$

Q.n 30 → Find the volume of the parallelopiped whose Co-terminous edges are $2\hat{i} + 3\hat{j} + 4\hat{k}$, $\hat{i} + 2\hat{j} - \hat{k}$ & $3\hat{i} - \hat{j} + 2\hat{k}$ Ans 37 cubic units

—x—