Solution: $\boxed{\text{WORKSHEET No: 3}}$ (D.E)

Ans 1

$$e^x dy + (ye^x + 2x)dx = 0$$

$$\Rightarrow \frac{dy}{dx} = -\frac{(ye^x+2x)}{e^x}$$

$$\Rightarrow \frac{dy}{dx} = -y - \frac{2x}{e^x}$$

$$\Rightarrow \frac{dy}{dx} + y = -\frac{2x}{e^x}$$

Comp with $\frac{dy}{dx} + Py = Q$

here $P = 1$; $Q = -\frac{2x}{e^x}$

$$I \cdot F = e^{\int P dx} = e^{\int 1 \cdot dx} = e^x$$

Solution

$$y \times IF = \int (Q \times IF)\, dx + c$$

$$\Rightarrow ye^x = -2\int \frac{x}{\cancel{e^x}} \times \cancel{e^x}\, dx + c$$

$$\Rightarrow ye^x = -2 \cdot \left(\frac{x^2}{2}\right) + c$$

$$\Rightarrow \boxed{ye^x + x^2 = c}$$ Ans

Ans 2 ▸

$$\frac{ydx - xdy}{y} = 0$$

$$\Rightarrow ydx = xdy$$

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$$\frac{dy}{dx} = \frac{y}{x}$$

$$\Rightarrow \frac{dy}{y} = \frac{dx}{x}$$

$$\Rightarrow \int \frac{dy}{y} = \int \frac{dx}{x}$$

$$\Rightarrow \log|y| = \log|x| + \log c$$

$$\Rightarrow \log|y| = \log|cx|$$

$$\Rightarrow \boxed{|y| = |cx|}$$ Ans

OR $$y = \pm cx$$

$$\boxed{y = c_1 x}$$ Ans when $(c_1 = \pm c)$

Ans 3 →

(a) $$y - \cos y = x \quad \text{---(i)}$$

Diff. wrt x

$$\frac{dy}{dx} + \sin y \frac{dy}{dx} = 1$$

$$\frac{dy}{dx}(1 + \sin y) = 1$$

$$\frac{dy}{dx} = \frac{1}{1 + \sin y}$$

T.P $$(y \sin y + \cos y + x)\frac{dy}{dx} = y$$

L.H.S $$(y \sin y + \cos y + x) \cdot \frac{1}{1 + \sin y}$$

$$= (y \sin y + y) \cdot \frac{1}{1 + \sin y} \quad \text{---} \left\{ \begin{array}{l} \text{from eq (i)} \\ \cos y + x = y \end{array} \right.$$

$$= y(\cancel{\sin y + 1}) \cdot \frac{1}{\cancel{1 + \sin y}}$$

$= y$ PROVED Ans

(K) $xy = ae^{x} + be^{-x} + x^2$ ----(1)

Diff wrt x

$$x\frac{dy}{dx} + y = ae^{x} - be^{-x} + 2x$$

Diff again wrt x

$$x\frac{d^2y}{dx^2} + \frac{dy}{dx} + \frac{dy}{dx} = ae^{x} + be^{x} + 2$$

$$\Rightarrow x\frac{d^2y}{dx^2} + 2\frac{dy}{dx} = (xy - x^2) + 2 \quad \text{from eq (1)}$$

$$\Rightarrow x\frac{d^2y}{dx^2} + 2\frac{dy}{dx} - xy + x^2 - 2 = 0$$

PROVED

(4) (b) $y = x\sin x$

Diff wrt x

$$\frac{dy}{dx} = x\cos x + \sin x$$

T.P $\quad x\frac{dy}{dx} = y + x\sqrt{x^2 - y^2}$

L.H.S $\quad x\frac{dy}{dx}$

$$= x(x\cos x + \sin x)$$

$$= x^2\cos x + x\sin x$$

R.H.S $\quad y + x\sqrt{x^2 - y^2}$

$$= x\sin x + x\sqrt{x^2 - x^2\sin^2 x}$$

$$= x\sin x + x^2\sqrt{1 - \sin^2 x}$$

$$= x\sin x + x^2\cos x$$

Clearly LHS = RHS PROVED

(d) $y = e^{x}(a\cos x + b\sin x)$ ---(1)

Diff wrt x

$$\frac{dy}{dx} = e^{x}(-a\sin x + b\cos x) + (a\cos x + b\sin x)e^{x}$$

$$\frac{dy}{dx} = e^{x}(-a\sin x + b\cos x) + y \quad \text{--- \{from eq(1)\}} \quad \text{--- (2)}$$

Diff again

$$\frac{d^2y}{dx^2} = e^{x}(-a\cos x - b\sin x) + (-a\sin x + b\cos x)e^{x} + \frac{dy}{dx}$$

$$\frac{d^2y}{dx^2} = -y + \frac{dy}{dx} - y + \frac{dy}{dx} \quad \text{--- \{from (1) \& (2)\}}$$

$$\Rightarrow \frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = 0 \quad \text{PROVED}$$

Ans 4 →

$$\cos\left(\frac{dy}{dx}\right) = a \quad ; \quad y = 1 \text{ \& } x = 0$$

$$\Rightarrow \frac{dy}{dx} = \cos^{-1} a$$

$$\Rightarrow dy = \cos^{-1} a \, dx$$

$$\Rightarrow \int dy = \int \cos^{-1} a \, dx$$

$$\Rightarrow y = x\cos^{-1} a + c$$

put $x = 0$ & $y = 1$

$$\Rightarrow 1 = 0 + c \quad \Rightarrow c = 1$$

$$\therefore y = x\cos^{-1} a + 1$$

$$\Rightarrow \frac{y-1}{x} = \cos^{-1} a$$

$$\Rightarrow \boxed{\cos\left(\frac{y-1}{x}\right) = a} \text{ Ans}$$

{Note: Misprint in worksheet Answer}

Ans 5 →

$$(x^3+x^2+x+1)\frac{dy}{dx} = 2x^2+x \quad ; \; y(0)=1$$

$$\Rightarrow \frac{dy}{dx} = \frac{2x^2+x}{x^3+x^2+x+1}$$

$$\Rightarrow \frac{dy}{dx} = \frac{2x^2+x}{(x^2+1)(x+1)}$$

$$\Rightarrow \int dy = \int \frac{2x^2+x}{(x+1)(x^2+1)}\,dx$$

let $\frac{2x^2+x}{(x+1)(x^2+1)} = \frac{A}{x+1} + \frac{Bx+C}{x^2+1}$

$$\Rightarrow 2x^2+x = A(x^2+1) + (Bx+C)(x+1)$$

$$\Rightarrow 2x^2+x = A(x^2+1) + (Bx^2+Bx+Cx+C)$$

$2 = A+B$

$1 = B+C$

$0 = A+C$ $\quad C=-A$ $\quad \therefore$ $1 = B-\cancel{A}$

$2 = \cancel{A}+B$

$3 = 2B$

$B = 3/2$

$A = 2-\frac{3}{2}$

$A = 1/2$ $\quad C = -1/2$

$$\therefore \int dy = \int \frac{1}{2(x+1)} + \frac{\frac{3}{2}x - \frac{1}{2}}{x^2+1}\,dx$$

$$y = \frac{1}{2}\int \frac{1}{x+1}\,dx + \frac{3}{2}\int \frac{x}{x^2+1}\,dx - \frac{1}{2}\int \frac{dx}{x^2+1}\,dx$$

put $x^2+1=t$

$x\,dx = \frac{dt}{2}$

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$$y = \frac{1}{2}\log|x+1| + \frac{3}{4}\int \frac{dt}{t} - \frac{1}{2}\tan^{-1}x + c$$

$$y = \frac{1}{2}\log|x+1| + \frac{3}{4}\log|x^2+1| - \frac{1}{2}\tan^{-1}x + c$$

put $x = 0$ & $y = 1$

$$1 = 0 + 0 - 0 + c \Rightarrow c = 1$$

$$\therefore \boxed{y = \frac{1}{2}\log|x+1| + \frac{3}{4}\log|x^2+1| - \frac{1}{2}\tan^{-1}x + 1}$$

NO need to simplify

$$y = \frac{1}{4}\left[\log(x+1)^2 + \log(x^2+1)^3\right] - \frac{1}{2}\tan^{-1}x + 1$$

$$\boxed{y = \frac{1}{4}\log(x+1)^2(x^2+1)^3 - \frac{1}{2}\tan^{-1}x + 1}$$ Ans

Qns 6 →

$$e^x \tan y\, dx + (1-e^x)\sec^2 y\, dy = 0$$

$$\frac{dy}{dx} = \frac{-e^x \tan y}{(1-e^x)\sec^2 y}$$

$$\Rightarrow \frac{\sec^2 y}{\tan y}\, dy = -\frac{e^x}{1-e^x}\, dx$$

$$\Rightarrow \int \underbrace{\frac{\sec^2 y\, dy}{\tan y}}_{t} = \int \frac{e^x}{\underbrace{e^x - 1}_{z}}\, dx$$

$$\Rightarrow \int \frac{dt}{t} = \int \frac{dz}{z}$$

$$\Rightarrow \log|t| = \log|z| + \log c$$

$$\Rightarrow \log|t| = \log|cz|$$

$$\Rightarrow |t| = |cz|$$

$$t = \pm cz$$

$$t = c_1 z \quad \{\pm c = c_1\}$$

$$\boxed{\tan y = c_1(e^x - 1)}$$

Ans

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