SOLUTIONS: INTEGRATION

$\boxed{\text{WORKSHEET NO: 8}}$ (class - 10)

Qns: 1 → $I = \int \frac{2x-1}{(x-1)(x+2)(x-3)} dx$

let $\frac{2x-1}{(x-1)(x+2)(x-3)} = \frac{A}{x-1} + \frac{B}{x+2} + \frac{C}{x-3}$

$\Rightarrow \quad 2x-1 = A(x+2)(x-3) + B(x-1)(x-3) + C(x-1)(x+2)$

$\Rightarrow 2x-1 = A(x^2-x-6) + B(x^2-4x+3) + C(x^2+x-2)$

equating coefficients of x^2, x & constant term

$0 = A + B + C \Rightarrow C = -A - B$

$2 = -A - 4B + C$ $\Rightarrow$ $2 = -2A - 5B$ $\times 2$

$-1 = -6A + 3B - 2C$ $\quad -1 = -4A + 5B$

$4 = -4A - 10B$

$-5 = 15B$

$\Rightarrow \boxed{B = -1/3}$ $\boxed{A = -\frac{1}{6}}$ $\boxed{C = \frac{1}{2}}$

$\therefore I = \int \frac{-1}{6(x-1)} - \frac{1}{3(x+2)} + \frac{1}{2(x-3)} dx$

$I = -\frac{1}{6}\log|x-1| - \frac{1}{3}\log|x+2| + \frac{1}{2}\log|x-3| + C$ Ans

Qns: 2 → $I = \int \frac{\cos\theta}{(2+\sin\theta)(3+4\sin\theta)} d\theta$

put $\sin\theta = t$

$\cos\theta d\theta = dt$

$I = \int \frac{dt}{(2+t)(3+4t)}$

let $\dfrac{1}{(2+t)(3+4t)} = \dfrac{A}{2+t} + \dfrac{B}{3+4t}$

$1 = A(3+4t) + B(2+t)$

equating cofficient of t and constant term

$0 = 4A + B \quad \times 2 \Rightarrow 0 = 8A + \cancel{2B}$

$1 = 3A + 2B \qquad\qquad 1 = 3A + \cancel{2B}$

$-1 = 5A$

$\boxed{A = -1/5}$ $\boxed{B = \frac{4}{5}}$

$\therefore I = \int \dfrac{-1}{5(2+t)} + \dfrac{4}{5(3+4t)} \, dt$

$I = -\frac{1}{5}\log|2+t| + \frac{\cancel{4}}{5} \times \frac{1}{\cancel{4}} \log|3+4t| + C$

$I = -\frac{1}{5}\log|2+\sin\theta| + \frac{1}{5}\log|3+4\sin\theta| + C$ Ans

Ques 3 → $I = \int \dfrac{(x-1)(x-2)(x-3)}{(x-4)(x-5)(x-6)} \, dx$

degree is ~~equal~~ equal, we have to divide

(Ek Shortcut hai : divide karen to pakka quotient 1 ayega

$$x^3 \ldots \overline{)\, x^3 \ldots \,}^{\,1}$$
$$\underline{x^3 \ldots}$$
Remainder

Shortcut

let $\dfrac{(x-1)(x-2)(x-3)}{(x-4)(x-5)(x-6)} = 1 + \dfrac{A}{x-4} + \dfrac{B}{(x-5)} + \dfrac{C}{(x-6)}$

$\Rightarrow (x-1)(x-2)(x-3) = (x-4)(x-5)(x-6) + A(x-5)(x-6)$
$+ B(x-4)(x-6) + C(x-4)(x-5)$

put $x = 4$

$(3)(2)(1) = A(-1)(-2)$

$6 = 2A$

$\boxed{A = 3}$

put $x = 5$

$(4)(3)(2) = B(1)(-1)$

$24 = -B$

$\boxed{B = -24}$

put $x = 6$

$(5)(4)(3) = C(2)(1)$

$60 = 2C$

$\boxed{C = 30}$

$$\therefore I = \int 1 + \frac{3}{x-4} - \frac{24}{x-5} + \frac{30}{x-6}\,dx$$

$$I = x + 3\log|x-4| - 24\log|x-5| + 30\log|x-6| + c \quad \text{Ans}$$

Qn 4 → $I = \int \frac{x^2+1}{(x-1)^2(x+3)}\,dx$

let $\frac{x^2+1}{(x-1)^2(x+3)} = \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{x+3}$

$\Rightarrow x^2+1 = A(x-1)(x+3) + B(x+3) + C(x-1)^2$

$\Rightarrow x^2+1 = A(x^2+2x-3) + B(x+3) + C(x^2-2x+1)$

Equating coefficients

$1 = A + C$

$0 = 2A + B - 2C$

$1 = -3A + 3B + C$

Solving these equations, we get

$\boxed{A = 3/8}$ $\boxed{B = \frac{1}{2}}$ $\boxed{C = \frac{5}{8}}$

$$\therefore I = \int \frac{1}{8(x-1)} + \frac{1}{2(x-1)^2} + \frac{5}{8(x+3)}\,dx$$

$$I = \frac{1}{8}\log|x-1| - \frac{1}{2(x-1)} + \frac{5}{8}\log|x+3| + c \quad \text{Ans}$$

Qn 5 → $I = \int \frac{x^2+x+1}{(x-1)^3}\,dx$

Two Methods ① partial fraction

let $\frac{x^2+x+1}{(x-1)^3} = \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{(x-1)^3}$

Method II

put $x-1=t$

$dx = dt$

$I = \int \frac{(t+1)^2 + (t+1) + 1}{t^3}\,dt$

$= \int \frac{t^2 + 3t + 3}{t^3}\,dt$

$= \int \frac{1}{t} + \frac{3}{t^2} + 3t^{-3}\,dt$

$= \log|t| - \frac{3}{t} - \frac{3}{2}t^{-2} + c$

$I = \log|x-1| - \frac{3}{x-1} - \frac{3}{2(x-1)^2} + c$ Ans

Ans: 6 → $I = \int \frac{3x+5}{x^3-x^2-x+1}\,dx$

$I = \int \frac{3x+5}{x^2(x-1) -1(x-1)}\,dx$

$I = \int \frac{3x+5}{(x^2-1)(x-1)}\,dx$

$I = \int \frac{3x+5}{(x+1)(x-1)^2}\,dx$

(Solution w.s = 8)

let $\frac{3x+5}{(x+1)(x-1)^2} = \frac{A}{x+1} + \frac{B}{x-1} + \frac{C}{(x-1)^2}$

$\Rightarrow 3x+5 = A(x-1)^2 + B(x+1)(x-1) + C(x+1)$

$\Rightarrow 3x+5 = A(x^2-2x+1) + B(x^2-1) + C(x+1)$

equating coefficients

$0 = A + B$

$3 = -2A + C$

$5 = A - B + C$

Solving these equations we get

$\boxed{A = 1/2}$ $\boxed{B = -1/2}$ $\boxed{C = 4}$

$\therefore I = \int \frac{1}{2(x+1)} - \frac{1}{2(x-1)} + \frac{4}{(x-1)^2} \, dx$

$I = \frac{1}{2}\log|x+1| - \frac{1}{2}\log|x-1| - \frac{4}{x-1} + c$ ANS

Ans. 7 → $I = \int \frac{2}{(1-x)(1+x^2)} dx$

let $\frac{2}{(1-x)(1+x^2)} = \frac{A}{1-x} + \frac{Bx+C}{1+x^2}$

$\Rightarrow 2 = A(1+x^2) + (Bx+C)(1-x)$

$\Rightarrow 2 = A(1+x^2) + (Bx - Bx^2 + C - Cx)$

equating coefficients of x^2, x & constant

$0 = A - B \rightarrow A = B$

$0 = B - C \Rightarrow 0 = A - C$

$2 = A + C \rightarrow 2 = A + C$

$\Rightarrow 2 = 2A$

$A = 1$ $B = 1$ $C = 1$

Scanned with CamScanner

$$\therefore I = \int \frac{1}{1-x} + \frac{x+1}{1+x^2}\,dx$$

$$= \frac{\log|1-x|}{(-1)} + \int \frac{x}{1+x^2}\,dx + \int \frac{1}{1+x^2}\,dx$$

put $1+x^2 = t$

$x\,dx = \frac{dt}{2}$

$$= -\log|1-x| + \frac{1}{2}\int \frac{dt}{t} + \tan^{-1}x$$

$$I = -\log|x-1| + \frac{1}{2}\log|1+x^2| + \tan^{-1}x + C \quad \underline{\text{ANS}}$$

--- $\{\because \log|a-b| = \log|b-a|\}$

Ans: 8 → $I = \int \frac{5x}{(x+1)(x^2-4)}\,dx$

$I = \int \frac{5x}{(x+1)(x+2)(x-2)}\,dx$

(please do yourself : easy hai)
(Same as Ans No- 1)

Ans. 9 → $I = \int \frac{x^4}{(x-1)(x^2+1)}\,dx$

Divide degree of N^r > degree of D^r

$$\begin{array}{r|l} & x+1 \\ x^3-x^2+x-1 & x^4 \\ & -(x^4-x^3+x^2-x) \\ \hline & x^3-x^2+x \\ & -(x^3-x^2+x-1) \\ \hline & 1 \end{array}$$

$$I = \int x+1 + \frac{1}{(x-1)(x^2+1)}\,dx$$

$$I = \frac{x^2}{2} + x + \int \frac{1}{(x-1)(x^2+1)}\,dx$$

$$I = \frac{x^2}{2} + x + I_1 + c$$

when $I_1 = \int \frac{1}{(x-1)(x^2+1)} dx$

let $\frac{1}{(x-1)(x^2+1)} = \frac{A}{x-1} + \frac{Bx+C}{x^2+1}$

$$1 = A(x^2+1) + (Bx+C)(x-1)$$

$$\Rightarrow 1 = A(x^2+1) + (Bx^2 - Bx + Cx - C)$$

equating coefficients

$$0 = A + B \Rightarrow B = -A$$
$$0 = -B + C \Rightarrow 0 = A + C$$
$$1 = A - C \Rightarrow 1 = A - C$$
$$1 = 2A$$

$A = 1/2$ $B = -1/2$ $C = -1/2$

$$\therefore I_1 = \int \frac{1}{2(x-1)} + \frac{-\frac{1}{2}x - \frac{1}{2}}{(x^2+1)} dx$$

$$I_1 = \frac{1}{2} \log|x-1| - \frac{1}{2} \int \frac{x}{x^2+1} dx - \frac{1}{2} \int \frac{1}{x^2+1} dx$$

put $x^2 + 1 = t$

$x\,dx = \frac{dt}{2}$

$$I_1 = \frac{1}{2} \log|x-1| - \frac{1}{4} \int \frac{dt}{t} - \frac{1}{2} \tan^{-1} x$$

$$I_1 = \frac{1}{2} \log|x-1| - \frac{1}{4} \log|x^2+1| - \frac{1}{2} \tan^{-1} x$$

$$\therefore I = \frac{x^2}{2} + x + \frac{1}{2} \log|x-1| - \frac{1}{4} \log|x^2+1| - \frac{1}{2} \tan^{-1} x + c \quad \text{Ans}$$

(Solution W.S = 8)

Qn. 10 → $I = \int \frac{1}{x - x^3} dx$

$I = \int \frac{1}{x(1-x^2)} dx$

$I = \int \frac{1}{x(1+x)(1-x)} dx$

Let $\frac{1}{x(1+x)(1-x)} = \frac{A}{x} + \frac{B}{1+x} + \frac{C}{1-x}$

$\Rightarrow 1 = A(1+x)(1-x) + B(x)(1-x) + C(x)(1+x)$

$\Rightarrow 1 = A(1-x^2) + (Bx - Bx^2) + (Cx + Cx^2)$

equating coefficients

$0 = -A - B + C$

$0 = B + C$

$\boxed{1 = A}$ $C = 1/2$ $B = -1/2$

$\therefore I = \int \frac{1}{x} - \frac{1}{2(1+x)} + \frac{1}{2(1-x)} dx$

$I = \log|x| - \frac{1}{2}\log|1+x| + \frac{1}{2}\frac{\log|1-x|}{(-1)} + c$

$I = \log|x| - \frac{1}{2}\log|1+x| - \frac{1}{2}\log|1-x| + C$

$I = 2\log|x| - (\log|1+x| + \log|1-x|) + \underbrace{2C}_{\text{New Constant}}$

$I = \log|x^2| - \log|1-x^2| + C_1$

$I = \log\left|\frac{x^2}{1-x^2}\right| + C_1$ Ans

—x—