← ULTIMATE MATHEMATICS →

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Matrices Class No: 6 (M-6)

TOPIC: PMI

Q.No 1 If $A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$ show that

$$A^n = \begin{bmatrix} \cos(n\theta) & \sin(n\theta) \\ -\sin(n\theta) & \cos(n\theta) \end{bmatrix}$$ for all values of $n \in N$

Sol.

let $P(n)$: $A^n = \begin{bmatrix} \cos(n\theta) & \sin(n\theta) \\ -\sin(n\theta) & \cos(n\theta) \end{bmatrix}$

$P(1)$: $A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$

Clearly $P(1)$ is true

let $P(k)$ be true

$$A^k = \begin{bmatrix} \cos(k\theta) & \sin(k\theta) \\ -\sin(k\theta) & \cos(k\theta) \end{bmatrix}$$

to prove $P(k+1)$ is true

$P(k+1)$: $A^{k+1} = \begin{bmatrix} \cos(k+1)\theta & \sin(k+1)\theta \\ -\sin(k+1)\theta & \cos(k+1)\theta \end{bmatrix}$

LHS A^{k+1}

$= A^k . A$

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$$A^{k+1} = A^k A = \begin{bmatrix} \cos(k\theta) & \sin(k\theta) \\ -\sin(k\theta) & \cos(k\theta) \end{bmatrix} \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

$$= \begin{bmatrix} \cos(k\theta)\cos\theta - \sin(k\theta)\sin\theta & \cos(k\theta)\sin\theta + \sin(k\theta)\cos\theta \\ — & — \end{bmatrix}$$

$$= \begin{bmatrix} \cos(k+1)\theta & \sin(k+1)\theta \\ -\sin(k+1)\theta & \cos(k+1)\theta \end{bmatrix}$$

= RHS

∴ $P(k+1)$ is true

∴ By PMI $P(n)$ is true for all $n \in N$.

Given $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$

Show $(aI + bA)^n = a^n I + n a^{n-1} bA$ for all $n \in N$

Let $P(n)$: $(aI + bA)^n = a^n I + n a^{n-1} bA$

$P(1)$: $(aI + bA) = aI + bA$

Clearly $P(1)$ is true

Let $P(k)$ be true

$P(k)$: $(aI + bA)^k = a^k I + k a^{k-1} bA$

$P(k+1)$: $(aI + bA)^{k+1} = a^{k+1} I + (k+1) a^k bA$

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LHS $(aI+bA)^{k+1}$

$= (aI+bA)^k \cdot (aI+bA)$

$= \cancel{a^k I + k a}$

$= (a^k I + k a^{k-1} bA)(aI + bA)$

$= a^{k+1} I + a^k bA + k a^k bA + k a^{k-1} b^2 A^2$

$= a^{k+1} I + a^k bA(k+1) + k a^{k-1} b^2 A^2$

$= a^{k+1} I + a^k bA(k+1) + 0 \ldots \left\{ A^2 = AA = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0 \right\}$

= RHS

∴ P(k+1) is true

By PMI P(n) is true for all $n \in N$

Qu 3 If $AB = BA$ (given)
By PMI show that $AB^n = B^n A$
Further show that $(AB)^n = A^n B^n$ for all $n \in N$

Sol. given $AB = BA$

(i) $P(n)$: $AB^n = B^n A$

$P(1)$: $AB = BA$

Clearly P(1) is true .. (given)

Let P(k) be true

$P(k)$: $AB^k = B^k A$

TP P(k+1) is true: $P(k+1)$: $AB^{k+1} = B^{k+1} A$

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LHS AB^{k+1}

$= AB^k \cdot B$

$= B^k AB$ --- (From P(k))

$= B^k BA$ -- (given)

$= B^{k+1} A$

= RHS

∴ P(k+1) is true

∴ By PMI P(n) is true for all value $n \in N$

(ii) $P(n): (AB)^n = A^n B^n$

$P(1): AB = AB$

clearly P(1) is true

let P(k) be true

$P(k): (AB)^k = A^k B^k$

TP P(k+1) is true

$P(k+1): (AB)^{k+1} = A^{k+1} B^{k+1}$

LHS $(AB)^{k+1}$

$= (AB)^k (AB)$

$= A^k B^k AB$ --- (From P(k))

$= A^k B^k BA$ --- (given)

$= A^k B^{k+1} A$

$= A^k AB^{k+1}$ --- {From Result of part (i)}

$= A^{k+1} B^{k+1}$ = RHS

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Q.N.4 given $A = \begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix}$ $f(x) = x^2 - 4x + 7$

Show $f(A) = 0$. Hence find A^5

(i) $f(A) = A^2 - 4A + 7I$

$\vdots$

$= 0$

(ii) we have $A^2 - 4A + 7I = 0$

$\boxed{A^2 = 4A - 7I}$

multiply by A

$A^3 = 4A^2 - 7A$

$\rightarrow A^3 = 4(4A - 7I) - 7A$

$A^3 = 9A - 28I$

multiply by A

$A^4 = 9A^2 - 28A$

$A^4 = 9(4A - 7I) - 28A$

$A^4 = 8A - 63I$

multiply by A

$A^5 = 8A^2 - 63A$

$A^5 = 8(4A - 7I) - 63A$

$A^5 = -31A - 56I$

$A^5 = -31\begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix} - 56\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$A^5 = \begin{bmatrix} -62 & -93 \\ 31 & -62 \end{bmatrix} - \begin{bmatrix} 56 & 0 \\ 0 & 56 \end{bmatrix} = \begin{bmatrix} -118 & -93 \\ 31 & -118 \end{bmatrix}$ Ans

॥ जय श्री गिरिराज जी महाराज ॥

Qns: 1 → If $A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$, then show that

$A^n = \begin{bmatrix} \cos(n\theta) & \sin(n\theta) \\ -\sin(n\theta) & \cos(n\theta) \end{bmatrix}$ for all $n \in N$.

Qns: 2 → If $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$, then show that

$(aI + bA)^n = a^n I + n a^{n-1} bA$ for all $n \in N$

Qns. 3 → If $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ show that $A^n = \begin{bmatrix} 3^{n-1} & 3^{n-1} & 3^{n-1} \\ 3^{n-1} & 3^{n-1} & 3^{n-1} \\ 3^{n-1} & 3^{n-1} & 3^{n-1} \end{bmatrix}$

for all $n \in N$

Qn. 4 → If A and B are square matrices of the same order such that $AB = BA$, then prove that by induction $AB^n = B^n A$. Further prove that $(AB)^n = A^n B^n$ for all $n \in N$

Qn. 5 → If $A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$, then prove that $A^n = \begin{bmatrix} 1+2n & -4n \\ n & 1-2n \end{bmatrix}$

for all $n \in N$

Qn. 6 → If $A = \begin{bmatrix} a & 1 \\ 0 & a \end{bmatrix}$, then show $A^n = \begin{bmatrix} a^n & na^{n-1} \\ 0 & a^n \end{bmatrix}$ for all $n \in N$

Qn. 7 → If $A = \begin{bmatrix} a & b \\ 0 & 1 \end{bmatrix}$ then show that $A^n = \begin{bmatrix} a^n & \dfrac{b(a^n-1)}{a-1} \\ 0 & 1 \end{bmatrix}$

for all $n \in N$

Qn. 8 → If $A = \begin{bmatrix} \cos\theta & i\sin\theta \\ i\sin\theta & \cos\theta \end{bmatrix}$ then show $A^n = \begin{bmatrix} \cos(n\theta) & i\sin(n\theta) \\ i\sin(n\theta) & \cos(n\theta) \end{bmatrix}$

for all $n \in N$

Qns 9 → If $A = \begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix}$ and $f(x) = x^2 - 4x + 7$. Show that $f(A) = 0$. Using this result find A^5

Qns 10 → If $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$ Show that $A^2 - 5A + 7I = 0$ Hence find A^4

Qns. 11 → If $A = diag(a, b, c)$ Show that $A^n = diag(a^n, b^n, c^n)$ for all $n \in N$

Qns. 12 → Give an example of two matrices A and B such that
(i) $A \neq 0$, $B \neq 0$, $AB = 0$, $BA \neq 0$
(ii) $A \neq 0$, $B \neq 0$, $AB = BA = 0$

Qns. 13 → If $A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ and $A^4 = \lambda A$. Find value of λ

Qns. 14 → If $A = \begin{bmatrix} i & 0 \\ 0 & i \end{bmatrix}$, then find A^{4n}

Qns. 15 → If A is skew symm matrix and n is an even natural no. then show that A^n is a symm matrix and when n is a odd natural no. then show that A^n is a skew symm matrix

← ANSWERS →

(9) $A^5 = \begin{bmatrix} -118 & -93 \\ 31 & -118 \end{bmatrix}$

(10) $A^4 = \begin{bmatrix} 39 & 55 \\ -55 & -16 \end{bmatrix}$

(12) (i) $A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$ & $B = \begin{bmatrix} 0 & 0 \\ 3 & 0 \end{bmatrix}$

(ii) $A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ & $B = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$

(13) $\lambda = 8$

(14) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$