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← ULTIMATE MATHEMATICS →

(BY: AJAY MITTAL : 9891067390)

(DETERMINANTS : Class 5) (D5)

Ques: 1 If $x \neq y \neq z$ and $\begin{vmatrix} x & x^2 & 1+x^3 \\ y & y^2 & 1+y^3 \\ z & z^2 & 1+z^3 \end{vmatrix} = 0$ then

show that $xyz = -1$

Soln

$$\begin{vmatrix} x & x^2 & 1 \\ y & y^2 & 1 \\ z & z^2 & 1 \end{vmatrix} + \begin{vmatrix} x & x^2 & x^3 \\ y & y^2 & y^3 \\ z & z^2 & z^3 \end{vmatrix} = 0$$

taking x, y, z common from R_1, R_2, R_3 resp

(from 2^{nd} determinant)

$$\Rightarrow \begin{vmatrix} x & x^2 & 1 \\ y & y^2 & 1 \\ z & z^2 & 1 \end{vmatrix} + xyz \begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} = 0$$

$C_2 \leftrightarrow C_3$

$$\Rightarrow -\begin{vmatrix} x & 1 & x^2 \\ y & 1 & y^2 \\ z & 1 & z^2 \end{vmatrix} + xyz \begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} = 0$$

$C_1 \leftrightarrow C_2$

$$\Rightarrow \begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} + xyz \begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} (1 + xyz) = 0$$

get

$$(x-y)(y-z)(z-x)(1+xyz) = 0$$

But $x-y \neq 0$, $y-z \neq 0$, $z-x \neq 0$ ... {since $x \neq y \neq z$}

$\Rightarrow 1+xyz = 0 \Rightarrow \boxed{xyz = -1}$ Ans

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Qn 2 If a, b, c are in AP show that

$$\begin{vmatrix} x+2 & x+3 & x+2a \\ x+3 & x+4 & x+2b \\ x+4 & x+5 & x+2c \end{vmatrix} = 0$$

Soln $\boxed{2b = a+c}$

$R_1 \to R_1 + R_3$

$$= \begin{vmatrix} 2x+6 & 2x+8 & 2x+2(a+c) \\ x+3 & x+4 & x+2b \\ x+4 & x+5 & x+2c \end{vmatrix}$$

$$= \begin{vmatrix} 2x+6 & 2x+8 & 2x+4b \\ x+3 & x+4 & x+2b \\ x+4 & x+5 & x+2c \end{vmatrix}$$

taking 2 common from R_1

$$= 2\begin{vmatrix} x+3 & x+4 & x+2b \\ x+3 & x+4 & x+2b \\ x+4 & x+5 & x+2c \end{vmatrix}$$

$= 2 \times 0$ -- {since R_1 & R_2 are identically

$= 0$ Ans

Qn 3 If a, b, c are +ve show that the value of the determinant $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$ is −ve

Soln $C_1 \to C_1 + C_2 + C_3$

$$= \begin{vmatrix} a+b+c & b & c \\ a+b+c & c & a \\ a+b+c & a & b \end{vmatrix}$$

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$$= (a+b+c)\begin{vmatrix} 1 & b & c \\ 1 & c & a \\ 1 & a & b \end{vmatrix}$$

$R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$

$$= (a+b+c)\begin{vmatrix} 1 & b & c \\ 0 & c-b & a-c \\ 0 & a-b & b-c \end{vmatrix}$$

expanding along R_1

$$= (a+b+c)\Big((c-b)(b-c) - (a-b)(a-c)\Big)$$

$$= (a+b+c)\left(\cancel{bc} - c^2 - b^2 + bc - a^2 + ac + ab - \cancel{bc}\right)$$

$$= (a+b+c)\left(-a^2 - b^2 - c^2 + ab + bc + ac\right)$$

$$= -(a+b+c)\left(a^2 + b^2 + c^2 - ab - bc - ac\right)$$

$$= -\frac{1}{2}(a+b+c)\left(2a^2 + 2b^2 + 2c^2 - 2ab - 2bc - 2ac\right)$$

$$= -\frac{1}{2}(a+b+c)\left((a-b)^2 + (b-c)^2 + (c-a)^2\right)$$

Clearly value of Δ is $-ve$ **Ans**

Q. 4 Show that $\begin{vmatrix} a^2+1 & ab & ac \\ ab & b^2+1 & bc \\ ca & cb & c^2+1 \end{vmatrix} = (1+a^2+b^2+c^2)$

$R_1 \to aR_1$, $R_2 \to bR_2$, $R_3 \to cR_3$

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$$= \frac{1}{abc}\begin{vmatrix} a^3+a & a^2b & a^2c \\ ab^2 & b^3+b & b^2c \\ c^2a & c^2b & c^3+c \end{vmatrix}$$

taking a, b, c common from C_1, C_2, C_3 resp

$$= \frac{abc}{abc}\begin{vmatrix} a^2+1 & a^2 & a^2 \\ b^2 & b^2+1 & b^2 \\ c^2 & c^2 & c^2+1 \end{vmatrix}$$

$R_1 \to R_1+R_2+R_3$

(proceed)

Q.no 5 Show that $\begin{vmatrix} -bc & b^2+bc & c^2+bc \\ a^2+ac & -ac & c^2+ac \\ a^2+ab & b^2+ab & -ab \end{vmatrix} = (ab+bc+ca$

Soln $R_1 \to aR_1$, $R_2 \to bR_2$ & $R_3 \to cR_3$

$$= \frac{1}{abc}\begin{vmatrix} -abc & ab^2+abc & ac^2+abc \\ a^2b+abc & -abc & bc^2+abc \\ ca^2+abc & b^2c+abc & -abc \end{vmatrix}$$

taking a, b, c common from C_1, C_2, C_3 resp

$$= \frac{abc}{abc}\begin{vmatrix} -bc & ab+ac & ac+ab \\ ab+bc & -ac & bc+ab \\ ac+bc & bc+ac & -ab \end{vmatrix}$$

$R_1 \to R_1+R_2+R_3$

proceed

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<u>Q No. 6</u> Show that $\begin{vmatrix} b+c & q+r & y+z \\ c+a & r+p & z+x \\ a+b & p+q & x+y \end{vmatrix} = 2\begin{vmatrix} a & p & x \\ b & q & y \\ c & r & z \end{vmatrix}$

$R_1 \to R_1 + R_2 + R_3$

$$\Rightarrow \begin{vmatrix} 2(a+b+c) & 2(p+q+r) & 2(x+y+z) \\ c+a & r+p & z+y \\ a+b & p+q & x+y \end{vmatrix}$$

$$= 2\begin{vmatrix} a+b+c & p+q+r & x+y+z \\ c+a & r+p & z+x \\ a+b & p+q & x+y \end{vmatrix}$$

$R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$

$$= 2\begin{vmatrix} a+b+c & p+q+r & x+y+z \\ -b & -q & -y \\ -c & -r & -z \end{vmatrix}$$

$R_1 \to R_1 + R_2 + R_3$

$$= 2\begin{vmatrix} a & p & x \\ -b & -q & -y \\ -c & -r & -z \end{vmatrix}$$

$$= 2(-1)^2\begin{vmatrix} a & p & x \\ b & q & y \\ c & r & z \end{vmatrix}$$

$$= 2\begin{vmatrix} a & p & x \\ b & q & y \\ c & r & z \end{vmatrix}$$ <u>Ans</u>

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Qn 7 Show that $\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} = abc\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)$

$= abc + bc + ca + ab$

take a, b, c common from R_1, R_2 & R_3

$$= abc \begin{vmatrix} \frac{1}{a}+1 & \frac{1}{a} & \frac{1}{a} \\ \frac{1}{b} & \frac{1}{b}+1 & \frac{1}{b} \\ \frac{1}{c} & \frac{1}{c} & \frac{1}{c}+1 \end{vmatrix}$$

$R_1 \to R_1 + R_2 + R_3$

DETERMINANTS (D-5)

Topic WORKSHEET No:5 Date

Qn 1 → Show that $\begin{vmatrix} -a^2 & ab & ac \\ ba & -b^2 & bc \\ ca & cb & -c^2 \end{vmatrix} = 4a^2b^2c^2$

Qn 2 → If x, y, z are different and $\begin{vmatrix} x & x^2 & 1+x^3 \\ y & y^2 & 1+y^3 \\ z & z^2 & 1+z^3 \end{vmatrix} = 0$

then show that $1+xyz=0$

Qn 3 → Show that $\begin{vmatrix} x & x^2 & 1+px^3 \\ y & y^2 & 1+py^3 \\ z & z^2 & 1+pz^3 \end{vmatrix} = (1+pxyz)(x-y)(y-z)(z-x)$

Qn 4 → If a, b, c are in A.P then show that

$$\begin{vmatrix} x+1 & x+2 & x+a \\ x+2 & x+3 & x+b \\ x+3 & x+4 & x+c \end{vmatrix} = 0$$

Qn 5 → If a, b, c are in A.P, then show that

$$\begin{vmatrix} x+2 & x+3 & x+2a \\ x+3 & x+4 & x+2b \\ x+4 & x+5 & x+2c \end{vmatrix} = 0$$

Qn 6 → If α, β, γ are in AP then show that

$$\begin{vmatrix} x-3 & x-4 & x-\alpha \\ x-2 & x-3 & x-\beta \\ x-1 & x-2 & x-\gamma \end{vmatrix} = 0$$

Qn 7 → Show that $\begin{vmatrix} x & x^2 & yz \\ y & y^2 & zx \\ z & z^2 & xy \end{vmatrix} = (x-y)(y-z)(z-x)(xy+yz+zx)$

Qn 8 → Show that $\begin{vmatrix} a^2+1 & ab & ac \\ ab & b^2+1 & bc \\ ca & cb & c^2+1 \end{vmatrix} = (1+a^2+b^2+c^2)$

Qn 9 → Show that $\begin{vmatrix} -bc & b^2+bc & c^2+bc \\ a^2+ac & -ac & c^2+ac \\ a^2+ab & b^2+ab & -ab \end{vmatrix} = (ab+bc+ca)^3$

Topic ____________________ Date ________

Qns 10 → Show that $\begin{vmatrix} x+y & x & x \\ 5x+4y & 4x & 2x \\ 10x+8y & 8x & 3x \end{vmatrix} = x^3$

Qns 11 → Show that $\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} = abc\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)$

$= abc + bc + ca + ab$

Qn 12 → If a, b, c are positive and unequal, show that the value of the determinant $\Delta = \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$ is positive

Qns 13 → If $\Delta = \begin{vmatrix} b+c & c+a & a+b \\ c+a & a+b & b+c \\ a+b & b+c & c+a \end{vmatrix} = 0$ show that either $a+b+c=0$ or $a=b=c$

Qn 14 → Show that $\begin{vmatrix} a+bx & c+dx & p+qx \\ ax+b & cx+d & px+q \\ u & v & w \end{vmatrix} = (1-x^2)\begin{vmatrix} a & c & p \\ b & d & q \\ u & v & w \end{vmatrix}$

Qns 15 → Show that $\begin{vmatrix} b+c & a & a \\ b & c+a & b \\ c & c & a+b \end{vmatrix} = 4abc$

Qn 16 → Show that $\begin{vmatrix} a^2 & bc & ac+c^2 \\ a^2+ab & b^2 & ac \\ ab & b^2+bc & c^2 \end{vmatrix} = 4a^2b^2c^2$

Qn 17 Show that $\begin{vmatrix} b+c & q+r & y+z \\ c+a & r+p & z+x \\ a+b & p+q & x+y \end{vmatrix} = 2\begin{vmatrix} a & p & x \\ b & q & y \\ c & r & z \end{vmatrix}$