(1)

SOLUTIONS **WORKSHEET NO: 1** (class - 2)

Chapter: AOI

Ans 1 A(2, 0) B(4, 5) C(6, 3)

Equation of AB

$y-0 = \frac{5}{2}(x-2)$

$y = \frac{5x-10}{2}$

Equation of BC

$y-5 = \frac{-2}{2}(x-4)$

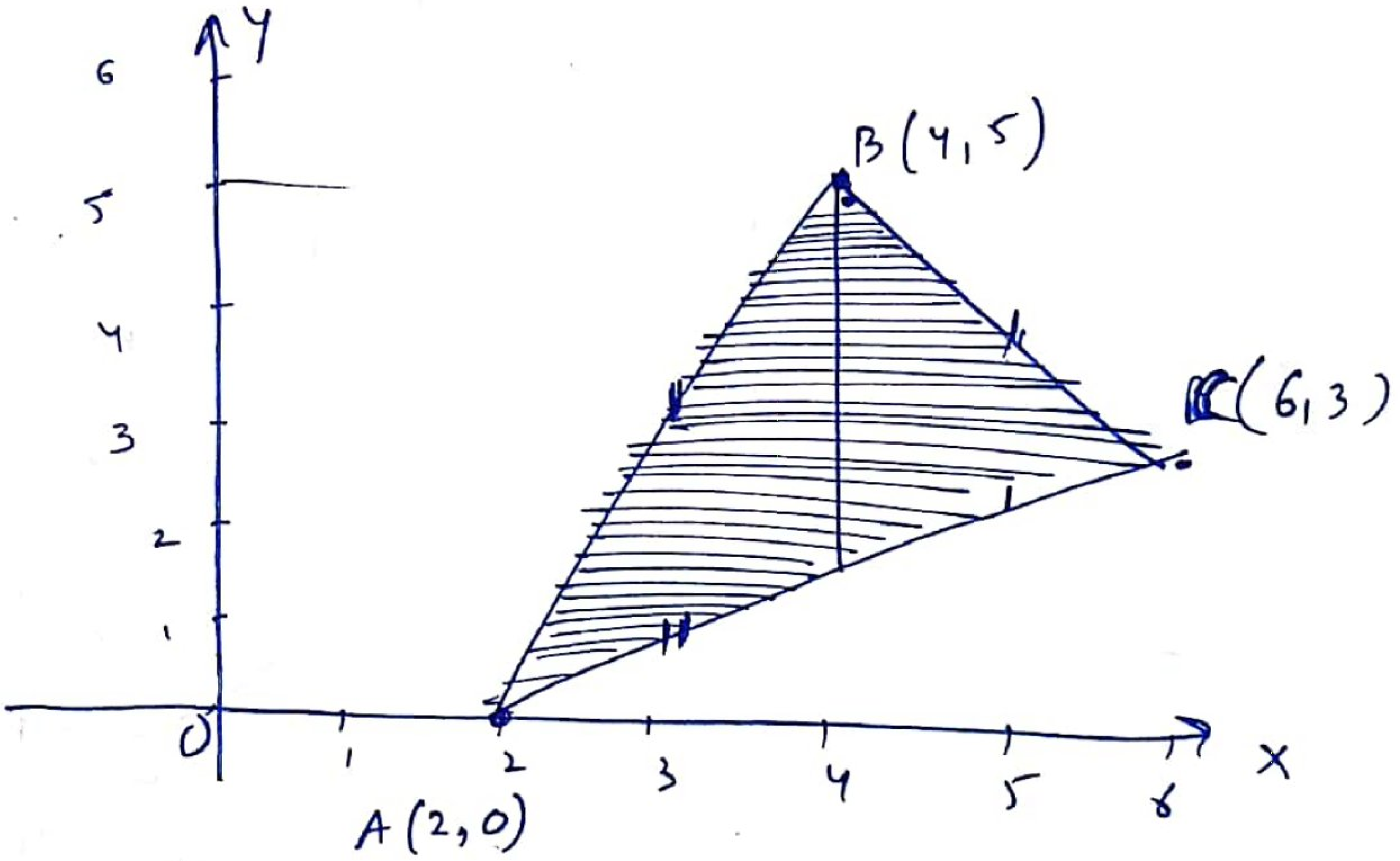
$y-5 = -x+4$

$y = -x+9$

Equation of AC

$y-0 = \frac{3}{4}(x-2)$

$y = \frac{3x-6}{4}$



Required area $= \int_2^4 \left(\frac{5x-10}{2}\right) - \left(\frac{3x-6}{4}\right)dx + \int_4^6 (-x+9) - \left(\frac{3x-6}{4}\right)dx$

$= \frac{1}{4}\int_2^4 (7x-14)\,dx + \frac{1}{4}\int_4^6 (-7x+42)\,dx$

$= \frac{7}{4}\int_2^4 (x-2)\,dx + \frac{7}{4}\int_4^6 (-x+6)\,dx$

$= \frac{7}{4}\left(\frac{x^2}{2} - 2x\right)_2^4 + \frac{7}{4}\left(-\frac{x^2}{2} + 6x\right)_4^6$

$$= \frac{7}{4}\left[\left(\cancel{8}-\cancel{8}\right)-(2-4)\right]+\frac{7}{4}\left[(-18+36)-(-8+24)\right]$$

$$= \frac{7}{4}\left[2\right]+\frac{7}{4}\left[18-16\right]$$

$$= \frac{7}{2}+\frac{7}{2} = 7$$

$\therefore$ $\boxed{\text{Required area} = 7 \text{ square units}}$ Ans

Ans. 2 → (1) $x^2 = y$ (2) $x = 1$ (3) $x = 2$ (4) X-axis

- ✓ parabola
- ✓ vertex $(0,0)$
- ✓ face open +ve y axis

| line $\parallel$ to y-axis | line $\parallel$ to y-axis |

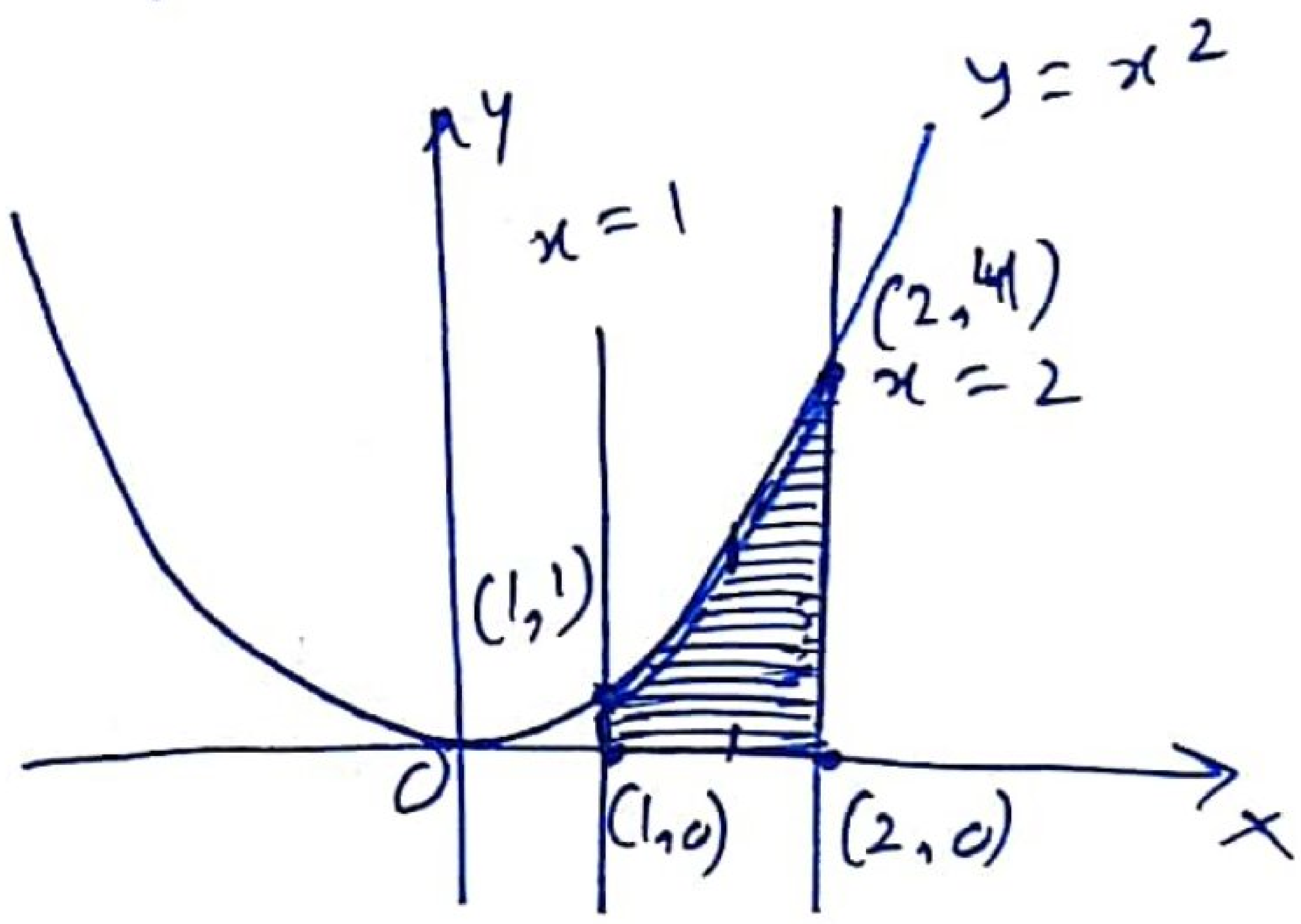
graph

Required Area

$$= \int_1^2 (x^2 - 0)\,dx$$

$$= \left(\frac{x^3}{3}\right)_1^2$$

$$= \frac{8}{3} - \frac{1}{3}$$



$\boxed{\text{Area} = \frac{7}{3} \text{ square units}}$ Ans

Ans 3 → (1) $y^2 = 4ax$ parabola; vertex $(0,0)$; open +ve x axis

(2) $y = mx$: line passing through $(0,0)$

Intersection point

$y = mx$ & $y^2 = 4ax$

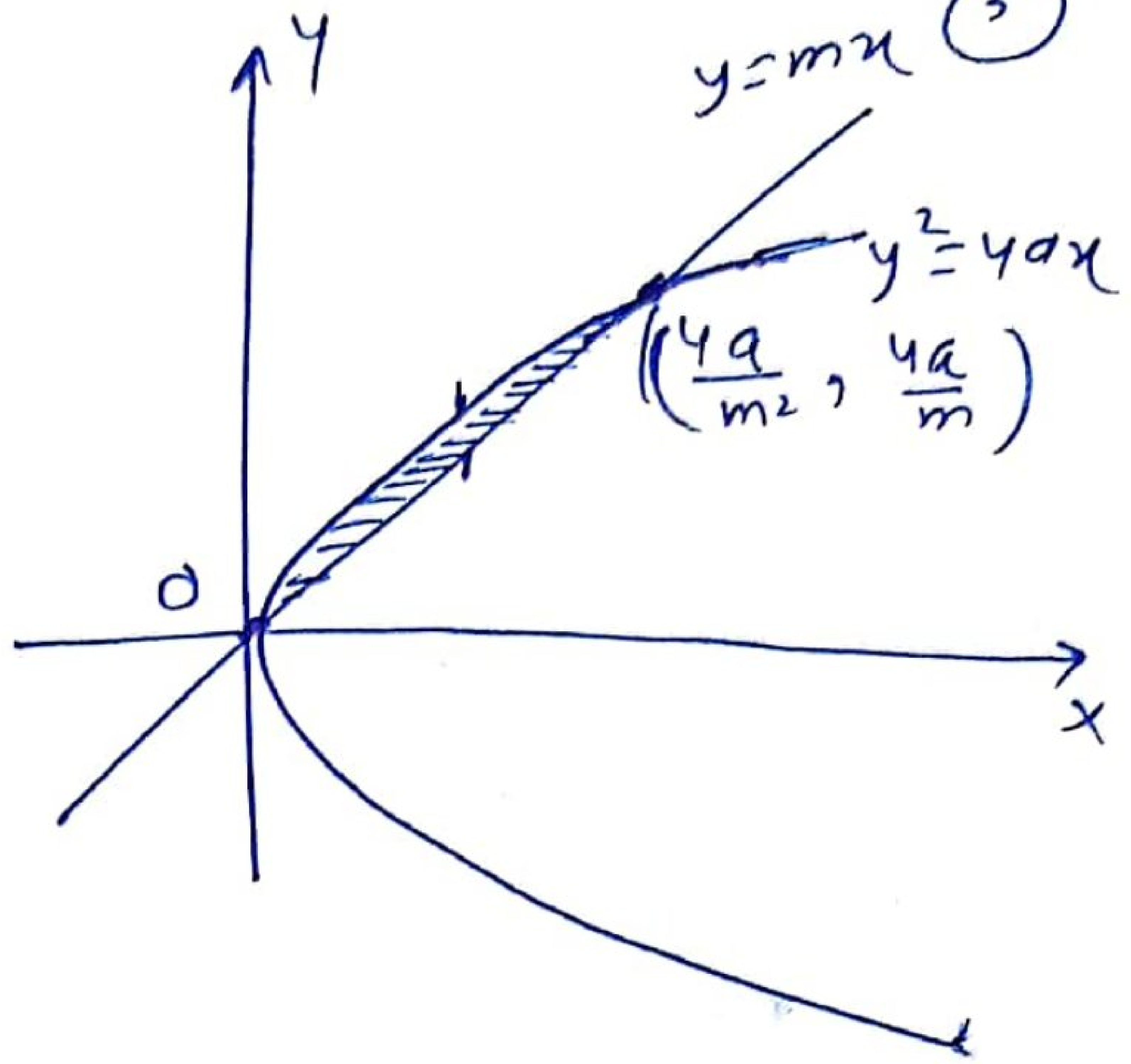
$\Rightarrow m^2x^2 = 4ax$

$\Rightarrow m^2x = 4a$

$\Rightarrow x = \frac{4a}{m^2}$

$\therefore y = \frac{4a}{m}$

$\therefore \left(\frac{4a}{m^2}, \frac{4a}{m}\right)$



Required area $= \int_0^{\frac{4a}{m^2}} (2\sqrt{a}\sqrt{x} - mx)\,dx$

$= \left[\frac{2}{3}\times 2\sqrt{a}\,x^{3/2} - m\frac{x^2}{2}\right]_0^{\frac{4a}{m^2}}$

$= \left[\frac{4\sqrt{a}}{3}\left(\frac{4a}{m^2}\right)^{3/2} - \frac{m}{2}\left(\frac{4a}{m^2}\right)^2\right] - [0]$

$= \frac{4\sqrt{a}}{3}\cdot\frac{4a}{m^2}\cdot\frac{2\sqrt{a}}{m} - \frac{m}{2}\left(\frac{16a^2}{m^4}\right) \cdots \left\{ x^{3/2} = x\sqrt{x} \right\}$

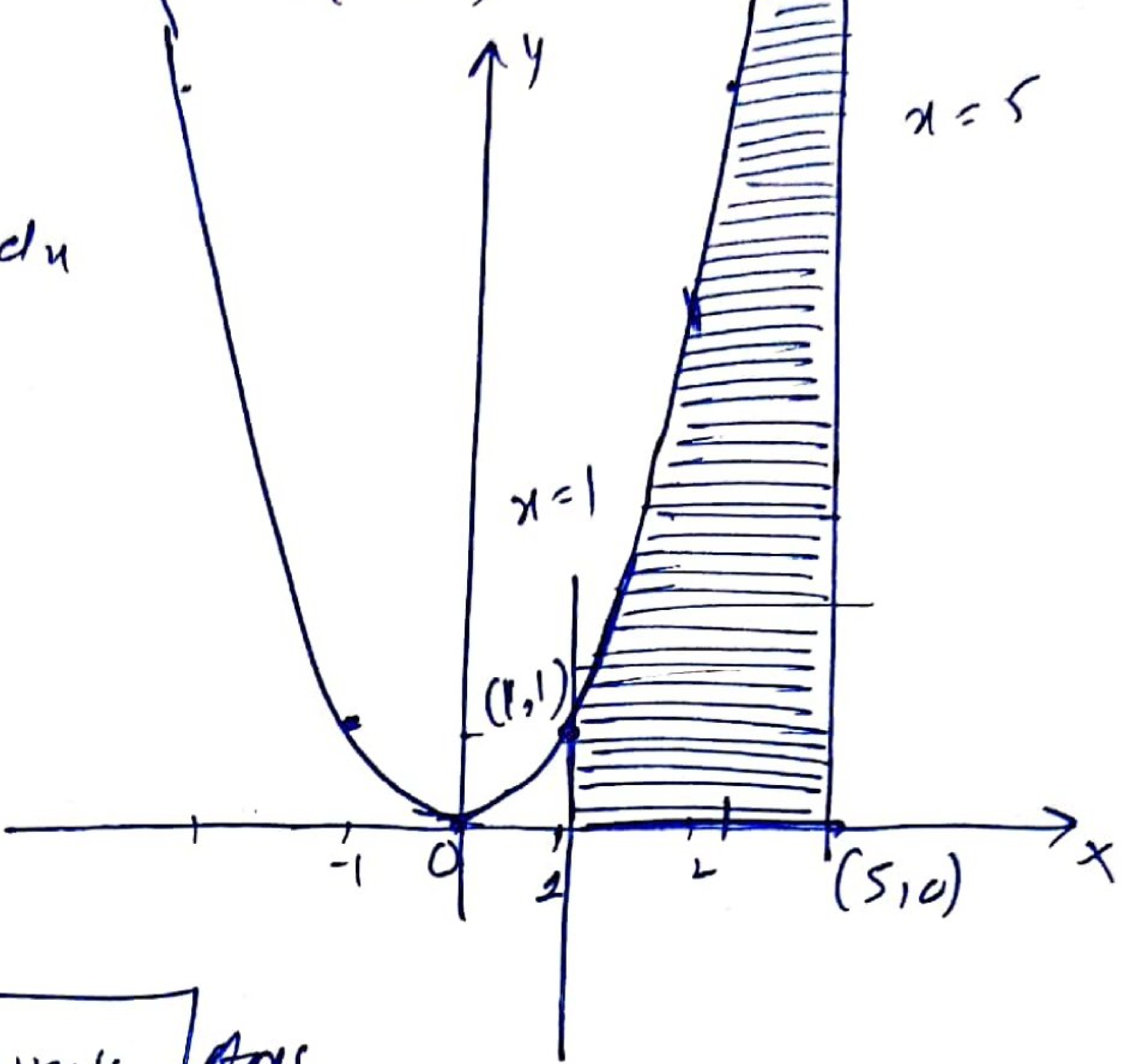
$= \frac{32a^2}{3m^3} - \frac{8a^2}{m^3}$

$= \frac{32a^2 - 24a^2}{3m^3}$

$\boxed{\text{Req area} = \frac{8a^2}{3m^3} \text{ square units}}$ Ans

Qn. 4 → (1) $y = x^4$ (2) $x = 1$ (3) $x = 5$ (4) X-axis

Point $(0,0)$ $(1,1)$ $(2,16)$ $(-1,1)$ $(2,16)$



Required area $= \int_{1}^{5} (x^4 - 0)\, dx$

$= \left(\frac{x^5}{5}\right)_1^5$

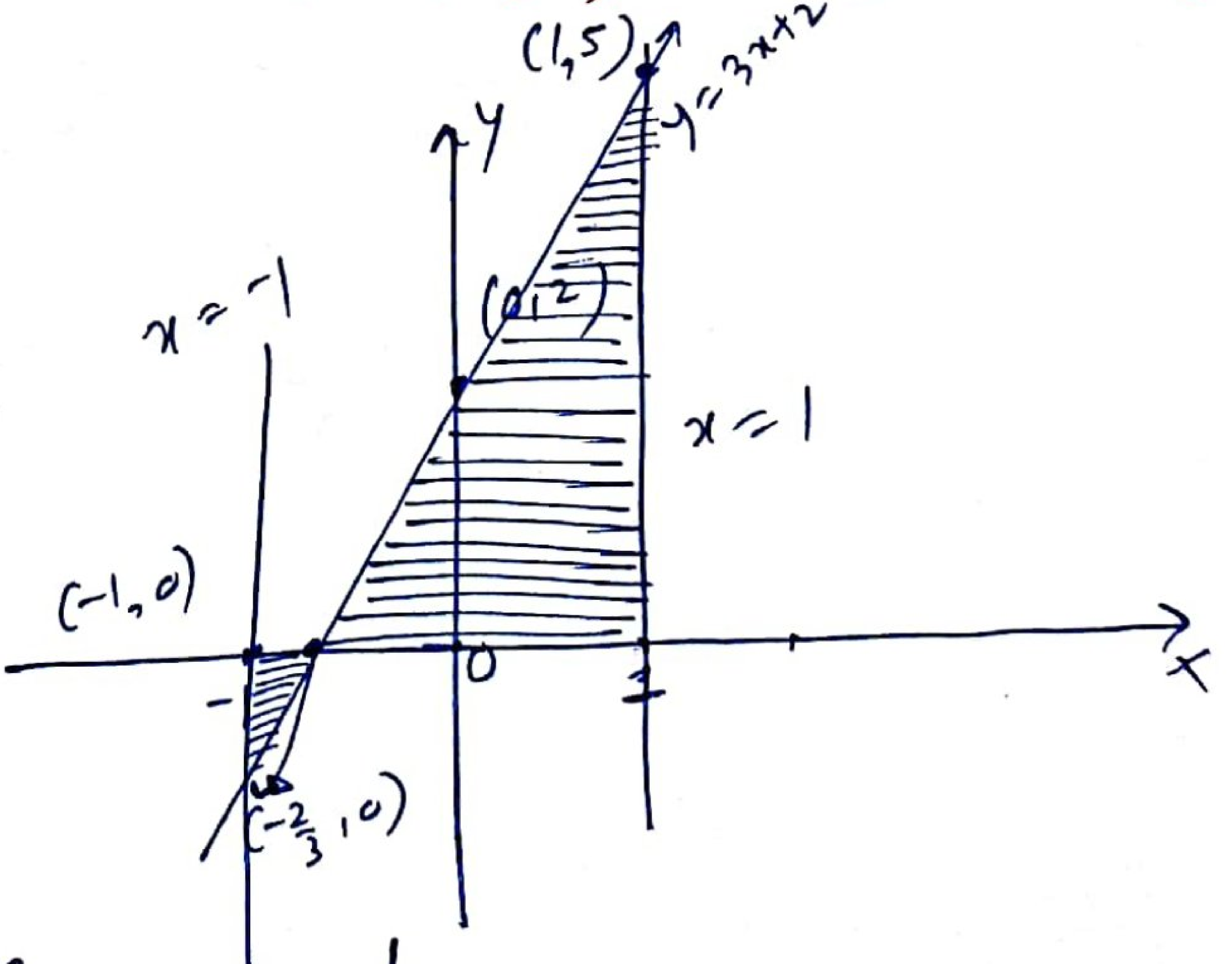
$= \left(\frac{3125}{5} - \frac{1}{5}\right)$

$= (625 - 0.2)$

$\boxed{\text{Area} = 624.8 \text{ square units}}$ Ans

Qn. 5 → (1) $y = 3x + 2$ (2) $x = -1$ (3) $x = 1$ (4) X-axis

point $(0,2)$ $\left(-\frac{2}{3}, 0\right)$



Required area

$= \int_{-1}^{-2/3} 0 - (3x+2)\, dx$

$+ \int_{-2/3}^{1} (3x+2) - 0\, dx$

$= -\left(\frac{3x^2}{2} + 2x\right)_{-1}^{-2/3} + \left(\frac{3x^2}{2} + 2x\right)_{-2/3}^{1}$

$= -\left[\left(\frac{2}{3} - \frac{4}{3}\right) - \left(\frac{3}{2} - 2\right)\right] + \left[\left(\frac{3}{2} + 2\right) - \left(\frac{2}{3} - \frac{4}{3}\right)\right]$

$$= -\left[-\frac{2}{3}+\frac{1}{2}\right]+\left[\frac{7}{2}+\frac{2}{3}\right]$$

$$= -\left[\frac{-4+3}{6}\right]+\left[\frac{21+4}{6}\right]$$

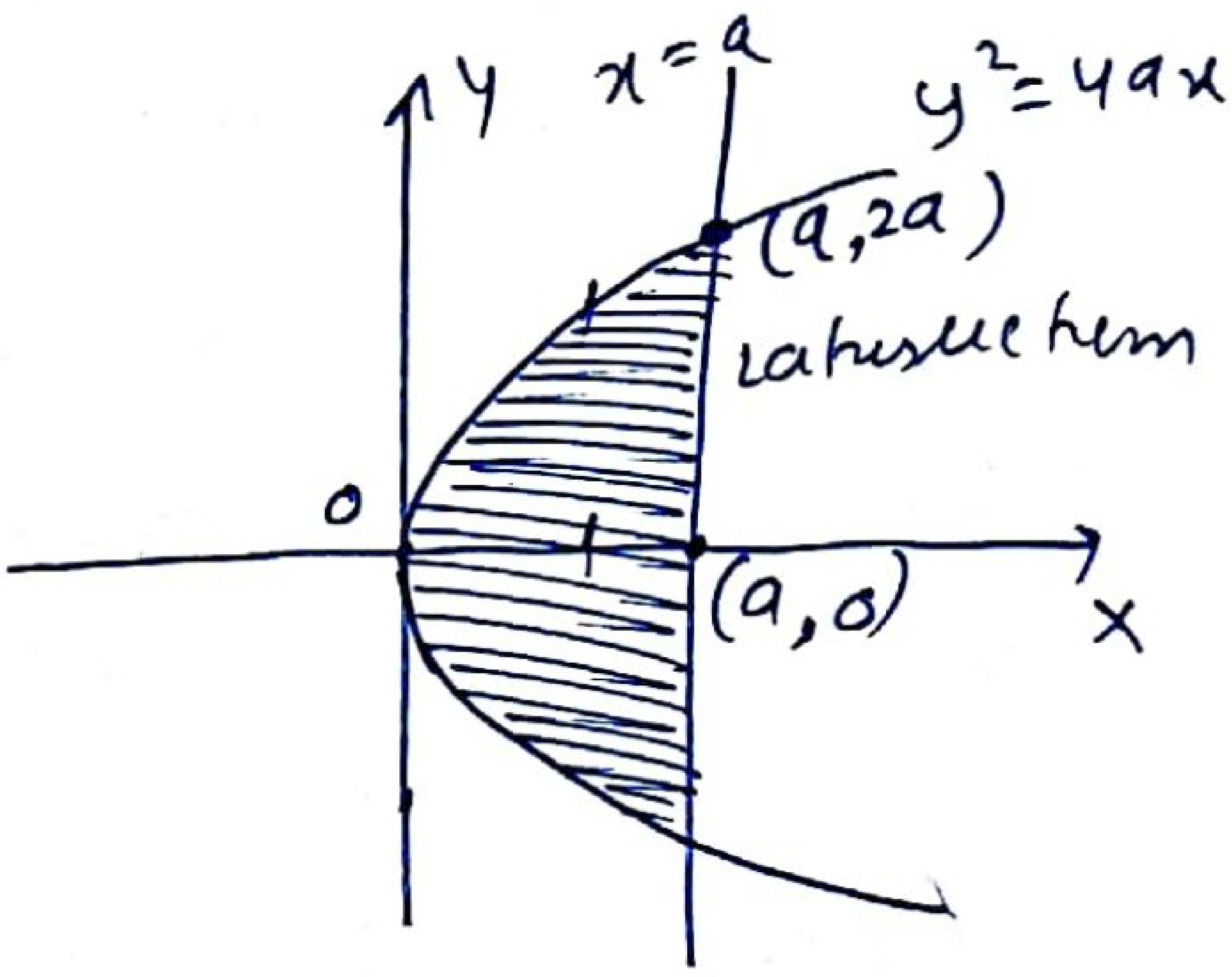
$$= \frac{1}{6}+\frac{25}{6}$$

$$= \frac{26}{6} = \frac{13}{3}$$

$\therefore$ $\boxed{\text{Required area} = \frac{13}{3}}$ square units **Ans**

Qu. 6 → $y^2 = 4ax$ & its Latusrectum

equation of latusrectum for parabola $y^2 = 4ax$ is $x = a$



Required area (Due to Symmetry)

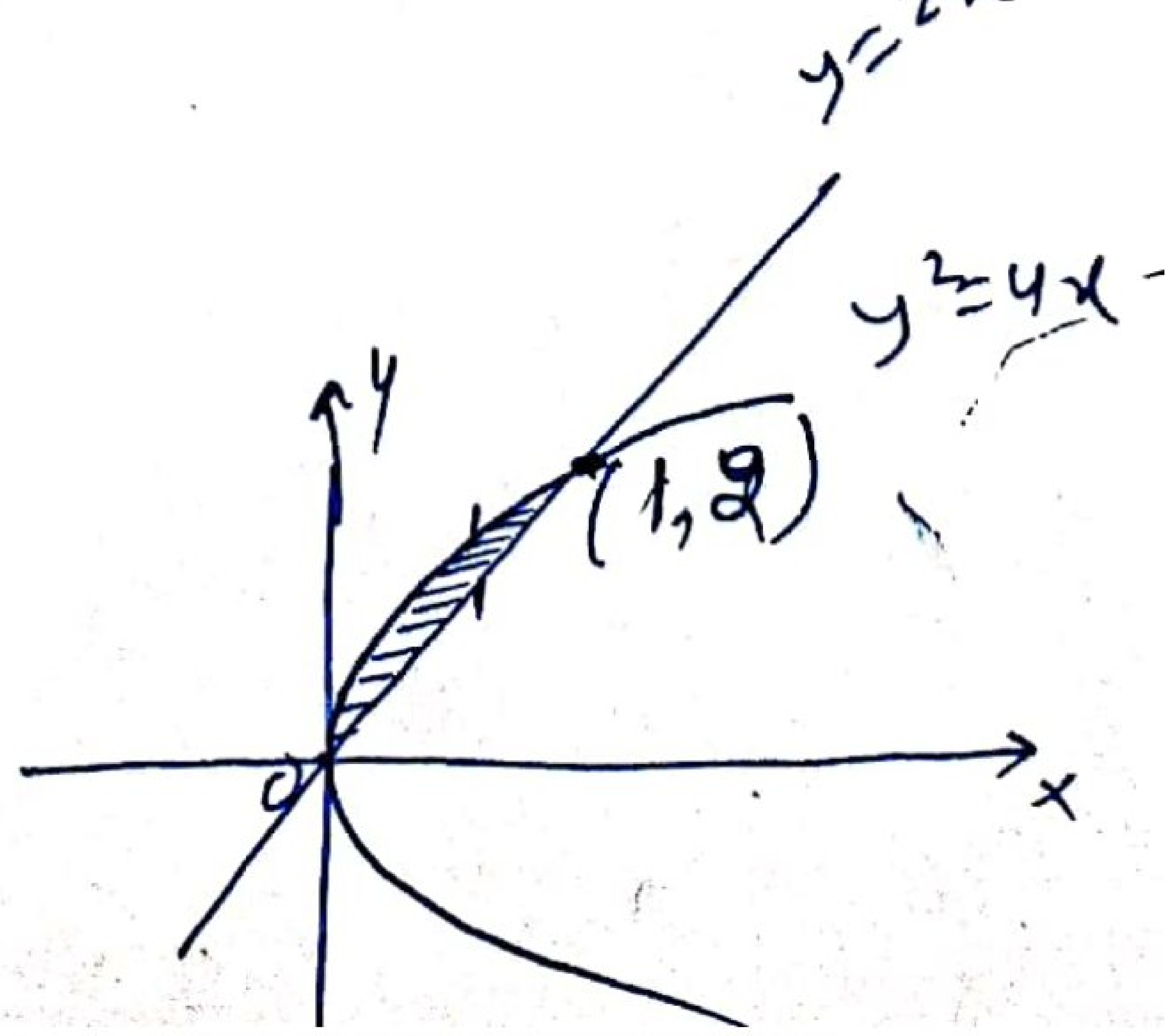
$$= 2\int_0^a (2\sqrt{a}\sqrt{x} - 0)\,dx$$

$$= \frac{2}{3}\times 4\sqrt{a}\left[(x)^{3/2}\right]_0^a$$

$$= \frac{8}{3}\sqrt{a}\left(a^{3/2} - 0\right)$$

$$= \frac{8}{3}\sqrt{a}\left(a\sqrt{a}\right)$$

$\boxed{\text{Area} = \frac{8a^2}{3} \text{ square units}}$ **Ans**



Qu. 7 → $y^2 = 4x$ and $2x = y$

Int point $4x = (2x)^2$

$\Rightarrow 4x = 4x^2$

$4x^2 - 4x = 0 \Rightarrow x = 0, \; x = 1$

Required area $= \int_0^1 2\sqrt{x} - 2x \, dx$

$= \left[2 \times \frac{2}{3} (x)^{3/2} - x^2 \right]_0^1$

$= \left[\frac{4}{3} - 1 \right] - 0$

$\boxed{\text{Required area} = \frac{1}{3} \text{ sq. units}}$

Q.N. 8 → (1) $y = 2x + 1$

(2) $y = 3x + 1$

(3) $x = 4$

Solving (1) & (2)

$2x + 1 = 3x + 1$

$x = 0$

$y = 1$

$\therefore A(0,1)$

Solving (2) & (3)

$x = 4$

$y = 12 + 1 = 13$

$B(4,13)$

Solving (1) & (3)

$x = 4$

$y = 8 + 1 = 9$

$\therefore C(4, 9)$

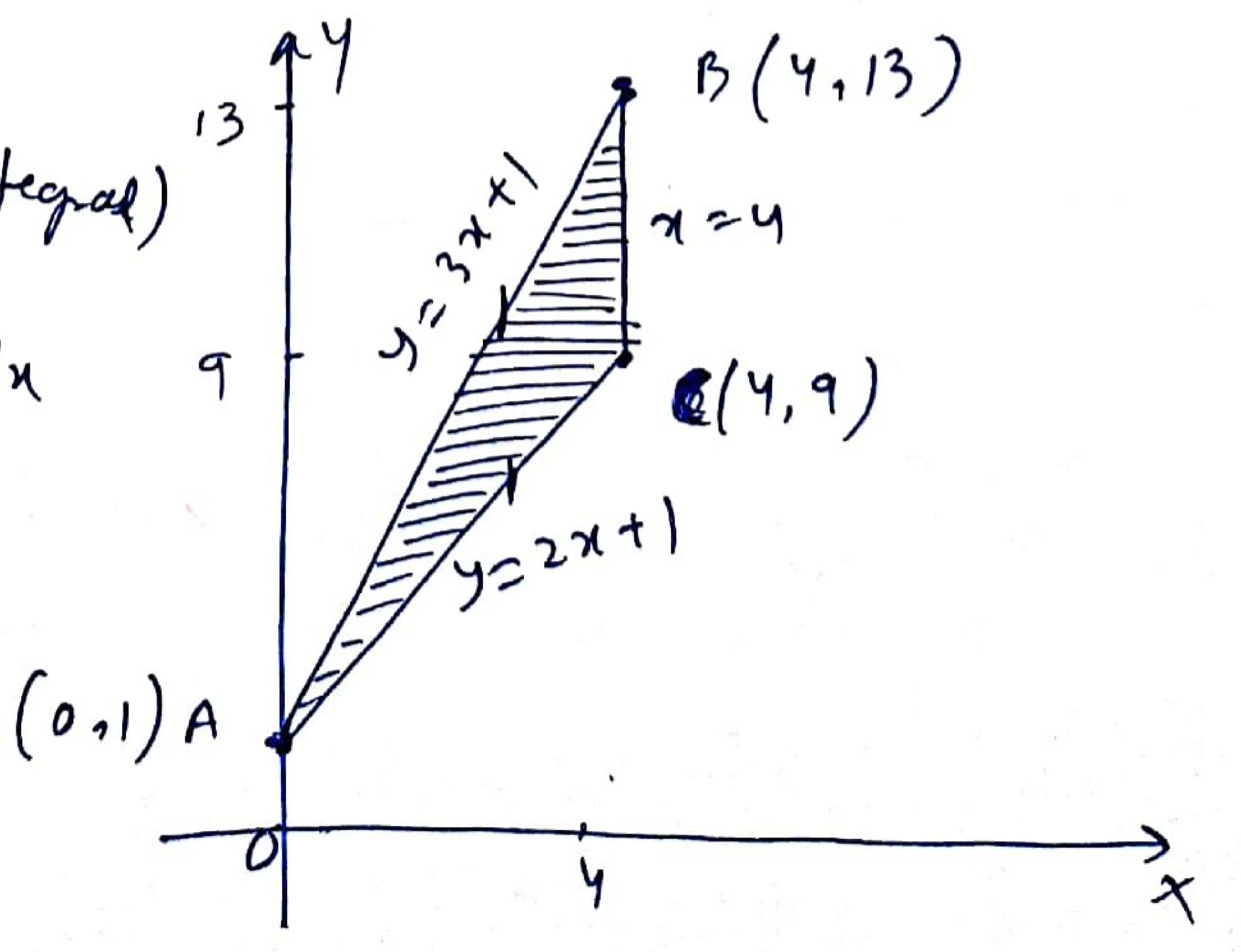
Required area (only 1 Integral)

$= \int_0^4 (3x+1) - (2x+1) \, dx$

$= \int_0^4 x \, dx$

$= \left(\frac{x^2}{2} \right)_0^4$

$= 8 - 0$

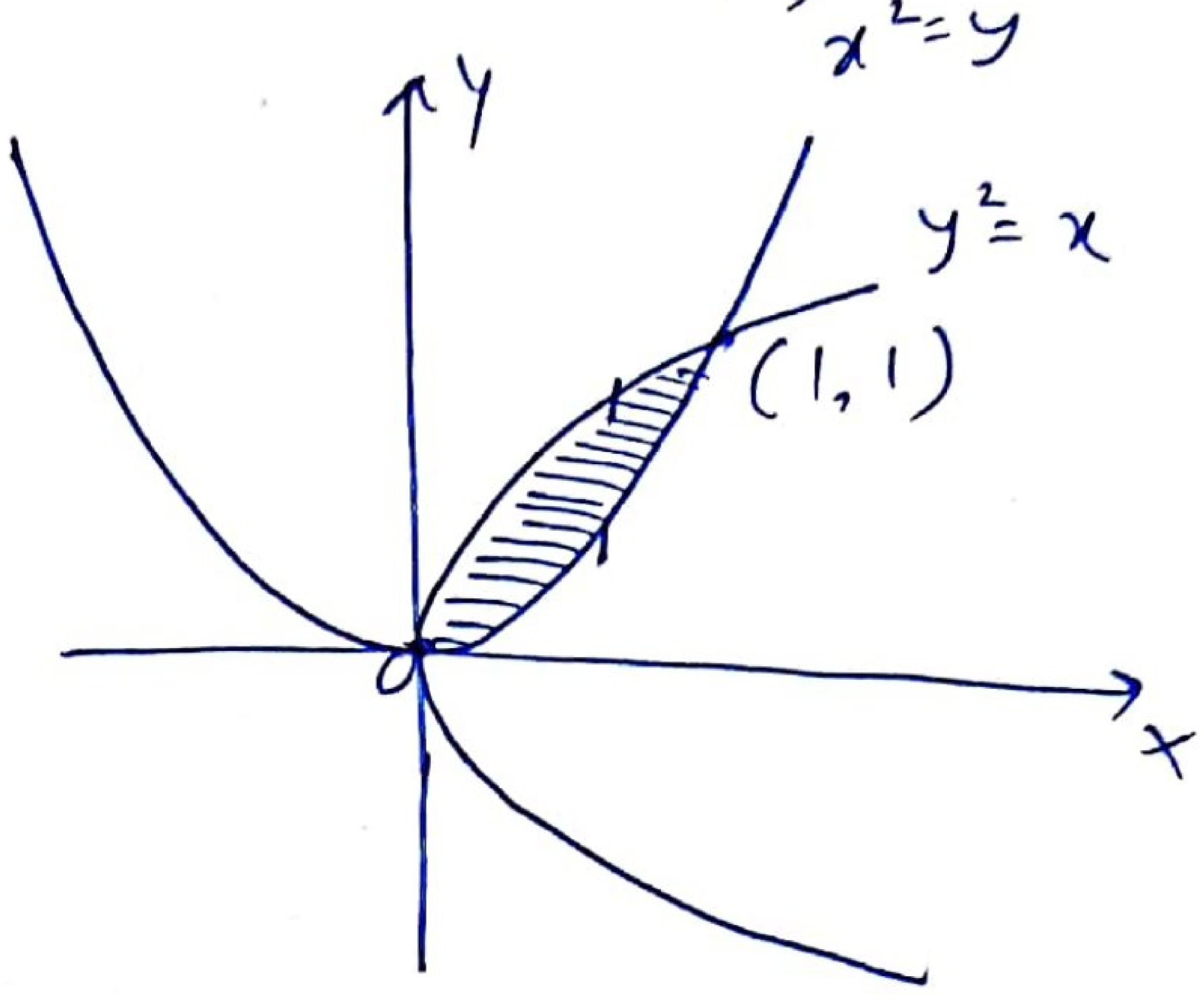


$\therefore \boxed{\text{Req. Area} = 8 \text{ sq. units}}$ Ans

Qn. 9 → (1) $y = x^2$ (2) $y^2 = x$

Both are parabolas with vertices $(0,0)$

Int. point



$y^2 = x$ & $x^2 = y$

$\Rightarrow x^4 = x$

$\Rightarrow x^4 - x = 0$

$x(x^3 - 1) = 0$

$\Rightarrow x = 0 \quad x = 1$

$y = 1$

$\therefore (1,1)$

Required area $= \int_0^1 \sqrt{x} - x^2 \, dx$

$= \left(\frac{2}{3} x^{3/2} - \frac{x^3}{3} \right)_0^1$

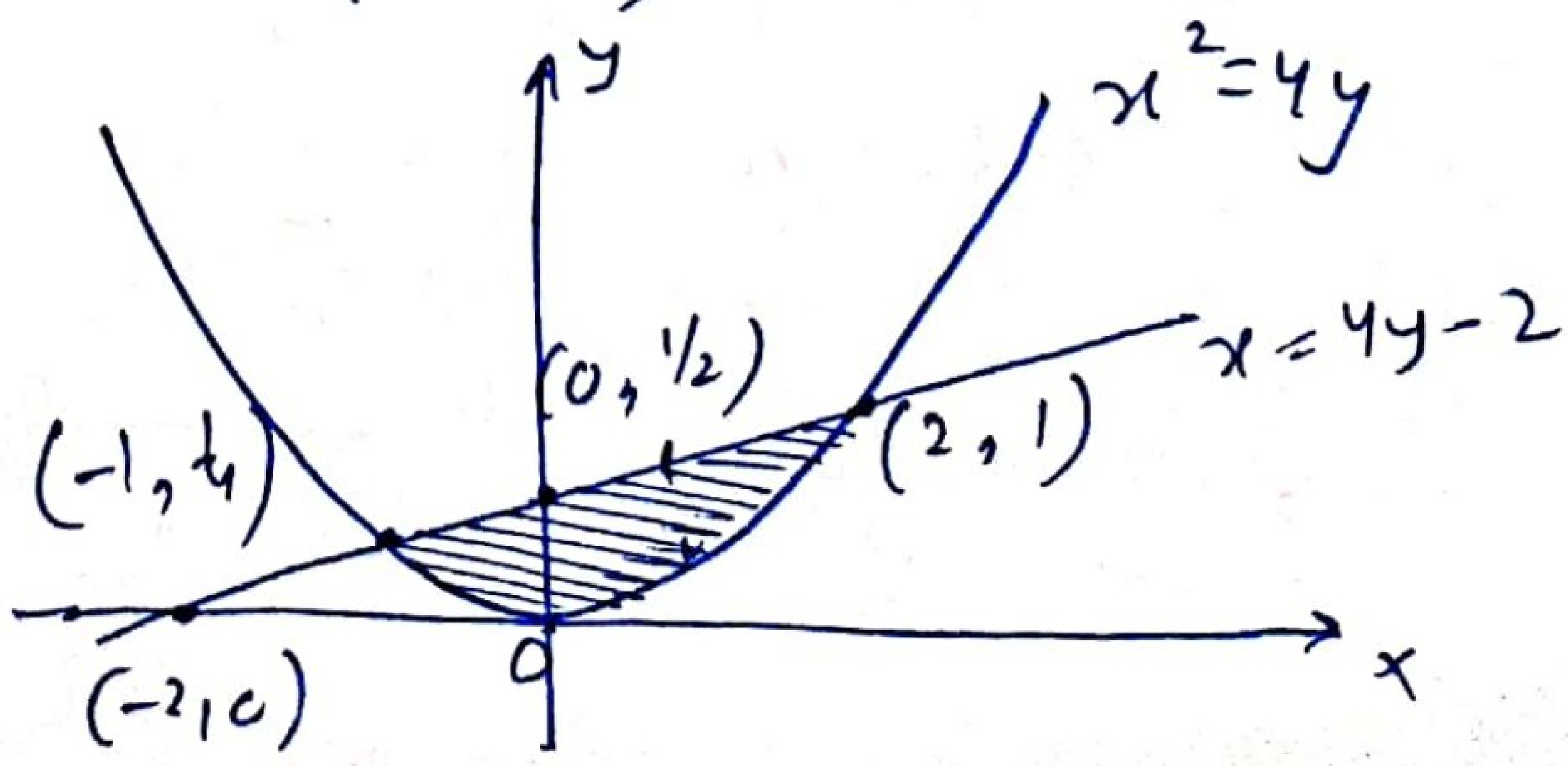
$= \left(\frac{2}{3} - \frac{1}{3} \right) - 0$

$\boxed{\text{Req area} = \frac{1}{3} \text{ sq. units}}$ Ans

Qn 10 → $x^2 = 4y$ & $x = 4y - 2$

vertex $(0,0)$ points $(0, \frac{1}{2})$ & $(-2, 0)$

Int. point



$x^2 = 4y$

$\Rightarrow x^2 = x + 2$

$\Rightarrow x^2 - x - 2 = 0$

$\Rightarrow (x-2)(x+1) = 0$

$x=2$; $x=-1$ $\therefore (2,1)$ & $(-1, 1/4)$
$y=1$; $y=1/4$

Required area $= \int_{-1}^{2} \left(\frac{x+2}{4}\right) - \frac{x^2}{4}\, dx$

$= \frac{1}{4}\int_{-1}^{2} x+2-x^2\, dx$

$= \frac{1}{4}\left[\frac{x^2}{2}+2x-\frac{x^3}{3}\right]_{-1}^{2}$

$= \frac{1}{4}\left[\left(2+4-\frac{8}{3}\right)-\left(\frac{1}{2}-2+\frac{1}{3}\right)\right]$

$= \frac{1}{4}\left[\left(\frac{10}{3}\right)-\left(\frac{3-12+2}{6}\right)\right]$

$= \frac{1}{4}\left[\frac{10}{3}+\frac{7}{6}\right]$

$= \frac{1}{4}\left[\frac{20+7}{6}\right] = \frac{27}{24} = \frac{9}{8}$

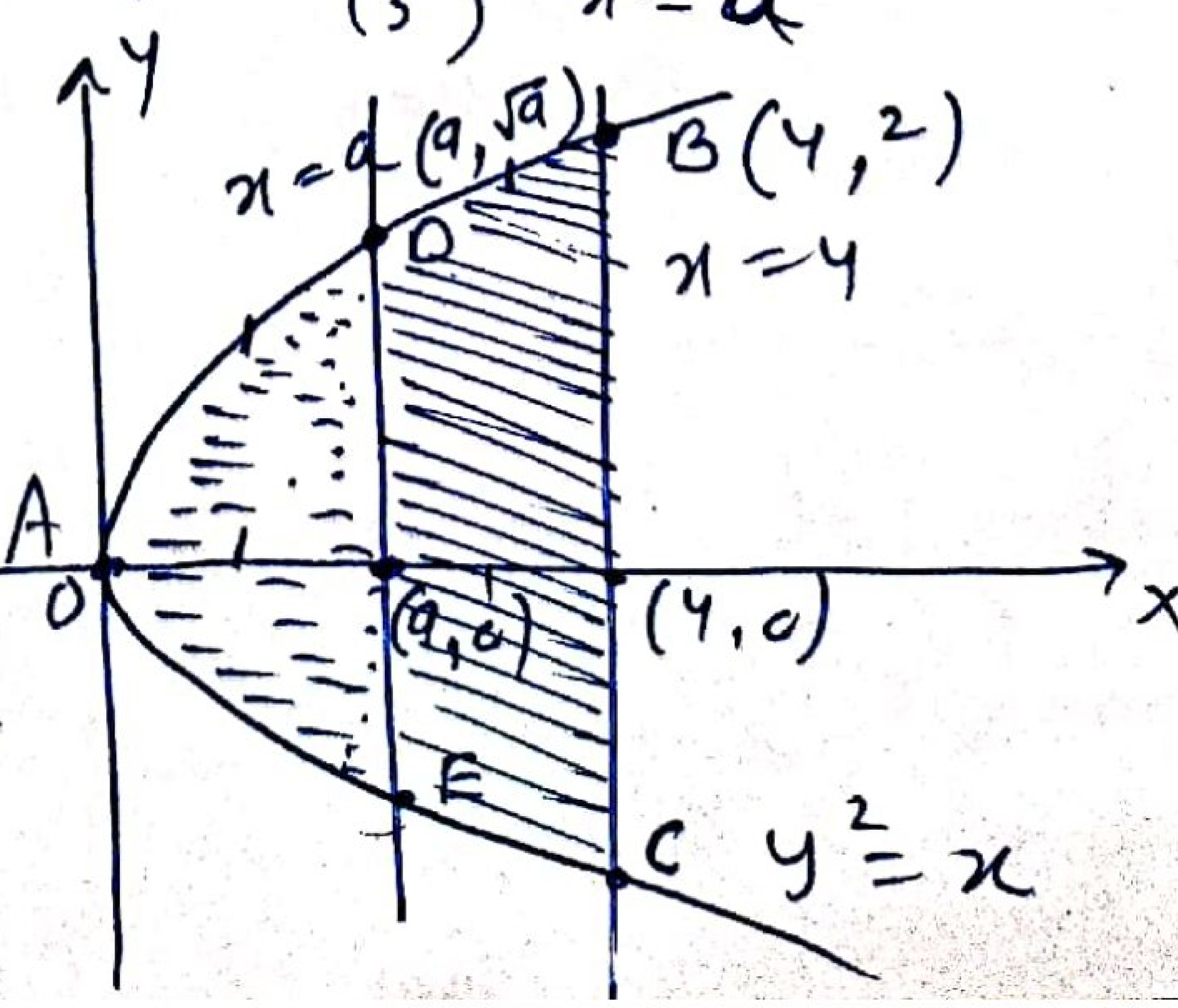
$\therefore$ $\boxed{\text{Required area} = \frac{9}{8} \text{ square units}}$ Ans

Que 11 → (1) $y^2=x$ (2) $x=4$ (3) $x=a$

given area

Area of ADEA = Area of DBCED

$\Rightarrow 2\int_0^a (\sqrt{x}-0)\,dx = 2\int_a^4 (\sqrt{x}-0)\,dx$



$$\Rightarrow \cancel{2}\left[\frac{2}{3}x^{3/2}\right]_0^a = \cancel{2}\left[\frac{2}{3}x^{3/2}\right]_a^4$$

$$\Rightarrow \frac{2}{3}(a)^{3/2} - 0 = \frac{2}{3}(4)^{3/2} - \frac{2}{3}a^{2/3}$$

$$\Rightarrow \frac{4}{3}(a)^{3/2} = \frac{2}{3}(4)^{3/2}$$

$$\Rightarrow \frac{4}{\cancel{3}}(a)^{3/2} = \frac{2}{\cancel{3}}(8)$$

$$\Rightarrow a^{3/2} = 4$$

$$\Rightarrow \boxed{a = 4^{2/3}}$$ Ans

-x-