XII

!! जय श्री श्याम कृपा !!

DURATION: 3 hrs
MARKS: 80

ULTIMATE MATHEMATICS : BY AJAY MITTAL

SUBJECTIVE TEST: EXAM NO : 5 (overall)

SECTION: A (ONE MARKS EACH)

Ans. 1 → For the set $A = \{1, 2, 3\}$, define a relation R in the set A as follows: $R = \{(1,1), (2,2), (3,3), (1,3)\}$ write the ordered pair to be added to R to make it the smallest equivalence relation ______

Ans. 2 → Set A has 3 elements and the set B has 4 elements then the number of injective mappings that can be defined from A to B is

(A) 144 (B) 12 (C) 24 (D) 64

Ans 3 → Let R be the relation defined in N by aRb if $2a+3b=30$ then $R =$ ______

Ans 4 → The value of $\sin\left[2\cot^{-1}\left(-\frac{5}{12}\right)\right]$ is

(A) $\frac{110}{169}$ (B) $\frac{120}{169}$ (C) $\frac{-120}{169}$ (D) $\frac{169}{120}$

Ans 5 → Write one branch of $\sec^{-1}x$ other than Principle value Branch ______

Ans : 6 → The Principal value of the expression $\cos^{-1}[\cos(-680^\circ)]$ is

(A) $\frac{2\pi}{9}$ (B) $\frac{-2\pi}{9}$ (C) $\frac{34\pi}{9}$ (D) $\frac{\pi}{9}$

Ans. 7 → If $\begin{bmatrix} 2x & 3 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ -3 & 0 \end{bmatrix} \begin{bmatrix} x \\ 8 \end{bmatrix} = 0$ then x equals to

(A) $\frac{23}{2}$ (B) $0, \frac{23}{2}$ (C) 0 (D) $\frac{-23}{2}, 0$

Ans. 8 → If A and B are two skew-symmetric matrices of same order, then AB is symmetric matrix if

(A) $A = B$ (B) $AB = BA$ (C) $AB = -BA$ (D) none of these

Ans. 9 → If matrix $A = [a_{ij}]_{2\times2}$ where $a_{ij} = 1 \; ; i \neq j$
$= 0 \; ; i = j$
then A^2 is equal to

(A) A (B) O (C) I (D) none of these

Ans. 10 → The maximum value of $\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+\sin\theta & 1 \\ 1+\cos\theta & 1 & 1 \end{vmatrix}$ is

(A) $\frac{\sqrt{3}}{2}$ (B) 1 (C) $\sqrt{2}$ (D) $\frac{1}{2}$

Ans. 11 → If the value of a third order determinant is 12, then the value of the determinant formed by replacing each element by its cofactor will be

(A) 12 (B) 144 (C) $(12)^3$ (D) $(12)^4$

Ans. 12 → If $f(x) = |\cos x|$, find $f'(3\pi/4)$ = ________

Ans. 13 → The function $f(x) = [x]$ where $[x]$ is a greatest integer function, is continuous at

(A) 3 (B) 2 (C) 1 (D) 1.5

Qns 14 → Differential coefficient of $\sec(\tan^{-1}x)$ is

(A) $\frac{x}{1+x^2}$ (B) $\frac{1}{\sqrt{1+x^2}}$ (C) $x\sqrt{1+x^2}$ (D) $\frac{x}{\sqrt{1+x^2}}$

Qns 15 → The number of points at which the function $f(x) = \frac{1}{\log|x|}$ is discontinuous is

(A) 2 (B) 4 (C) 3 (D) 1

Qn 16 → Derivative of x^2 w.r.t x^3 is __________

Qn 17 → The slope of the normal to the curve $x = 1 - a\sin\theta$ and $y = b\cos^2\theta$ at $\theta = \pi/2$ is

(A) $\frac{a}{2b}$ (B) $-\frac{2b}{a}$ (C) $\frac{2b}{a}$ (D) none of them

Qns 18 → Maximum value of $f(x) = -|x+1| + 3$ is

(A) 2 (B) 3 (C) 1 (D) 4

Qns:- 19 → Derivative of $\tan^{-1}\left(\frac{1+\sin x}{\cos x}\right) =$ __________

Qn- 20 → $B'AB$ is symmetric matrix if A is __________

SECTION : B (TWO MARKS EACH)

Qns 21 → Find the slope of the tangent to the curve $x = t^2 + 3t - 8$; $y = 2t^2 - 2t - 5$ at the point $(2, -1)$

Qn 22 → find the smallest value of the polynomial $x^3 - 18x^2 + 96x$ in $[0,9]$

Qn. 23 → If $x^y = e^{x-y}$ then show that

$$\frac{dy}{dx} = \frac{\log x}{(1+\log x)^2}$$

Qn. 24 → $f(x) = x|x|$ show that $f(x)$ is differentiable at $x=0$ i.e. show LHD = RHD

Qn. 25 → find matrix A such that

$$\begin{bmatrix} 2 & -1 \\ 1 & 0 \\ -3 & 4 \end{bmatrix} A = \begin{bmatrix} -1 & -8 & -10 \\ 1 & -2 & -5 \\ 9 & 22 & 15 \end{bmatrix}$$

Qn. 26 → $f: R_+ \to [4, \infty)$ and $f(x) = x^2 + 4$. Show that f is bijective

SECTION: C (FOUR MARKS EACH)

Qn. 27 → find the equation of the Normal to the curve $x^2 = 4y$ which "passes" through the point (1,2)

Qn. 28 → If $x = \sin t$ and $y = \sin(pt)$ then show that

$$(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + p^2 y = 0$$

Qn. 29 → Find the values of a and b so that the

$$f(x) = \begin{cases} \dfrac{1-\sin^3 x}{3\cos^2 x} & ; \quad x < \pi/2 \\ a & ; \quad x = \pi/2 \\ \dfrac{b(1-\sin x)}{(\pi-2x)^2} & ; \quad x > \pi/2 \end{cases}$$ is continuous at $x = \pi/2$

Ans 30 → If $A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$ & $B = \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$ are two square matrices, find AB and "hence" solve the system of linear equations

$x - y = 3$; $2x + 3y + 4z = 17$ and $y + 2z = 7$

Ans- 31 → Using matrix multiplication, to divide Rs 30,000 in two different types of bonds. The first bond pays 5% interest per year and the second bond pays 7% Interest per year. How to divide Rs 30,000 among the two types of bonds. If the first fund must obtain an annual total interest of Rs 2000

Ans 32 → Prove that

$$\tan\left(\frac{\pi}{4} + \frac{1}{2}\cos^{-1}\frac{a}{b}\right) + \tan\left(\frac{\pi}{4} - \frac{1}{2}\cos^{-1}\frac{a}{b}\right) = \frac{2b}{a}$$

SECTION - D (SIX MARKS EACH)

Ans 33 → (a) Let R be a relation on set A of ordered pairs of the Integers (natural numbers) defined by

$(x,y)\ R\ (u,v)$ if and only if $xv = yu$. Show that R is an equivalence relation.

(b) Let $A = \{1,2,3\}$, then find the number of equivalence relations containing $(1,2)$

Que 34 → If $A = \begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix}$ show that $A^2 - 4A + 7I = 0$ and "hence" find A^{-1} and also A^4

Que 35 → Show that the volume of the largest cone that can be inscribed in a sphere of radius R is $\frac{8}{27}$ of the volume of the sphere

Que 36 → (a) If $y\sqrt{1-x^2} + x\sqrt{1-y^2} = a$ show that $\frac{dy}{dx} = -\sqrt{\frac{1-y^2}{1-x^2}}$

(b) Show derivative of $\cot^{-1}\left(\frac{\sqrt{1+\sin x} + \sqrt{1-\sin x}}{\sqrt{1+\sin x} - \sqrt{1-\sin x}}\right)$ is independent of x

— x —

End of TEST