SOLUTIONS : A·O·D WORKSHEET NO: 7

(Class No: 9)

Ans. 1 → $f(x) = \cos^2 x + \sin x \quad ; \quad x \in [0, \pi]$

Diff w.r.t x

$$f'(x) = -2\cos x \sin x + \cos x$$

$$f'(x) = \cos x(-2\sin x + 1)$$

put $f'(x) = 0$

$$\Rightarrow \cos x(1 - 2\sin x) = 0$$

$$\Rightarrow \cos x = 0 \qquad \Big| \qquad 1 - 2\sin x = 0$$

$$x = \pi/2 \qquad \Big| \qquad \sin x = 1/2$$

$$x = \frac{\pi}{6} \; ; \; x = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$$

$$f(0) = 1 + 0 = 1$$

$$f\left(\frac{\pi}{6}\right) = \cos^2\left(\frac{\pi}{6}\right) + \sin\left(\frac{\pi}{6}\right) = \frac{3}{4} + \frac{1}{2} = \frac{5}{4}$$

$$f\left(\frac{5\pi}{6}\right) = \cos^2\left(\frac{5\pi}{6}\right) + \sin\left(\frac{5\pi}{6}\right) = \frac{3}{4} + \frac{1}{2} = \frac{5}{4}$$

$$f(\pi/2) = 0 + 1 = 1$$

$$f(\pi) = 1 + 0 = 1$$

∴ Absolute Min value = 1 and

Absolute Max. value = 5/4 **Ans**

Ans. 2 → $f(x) = (x-2)^4 \cdot (x+1)^5$

Diff w.r.t x

$$f'(x) = (x-2)^4 \cdot 5(x+1)^4 + (x+1)^5 \cdot 4(x-2)^3$$

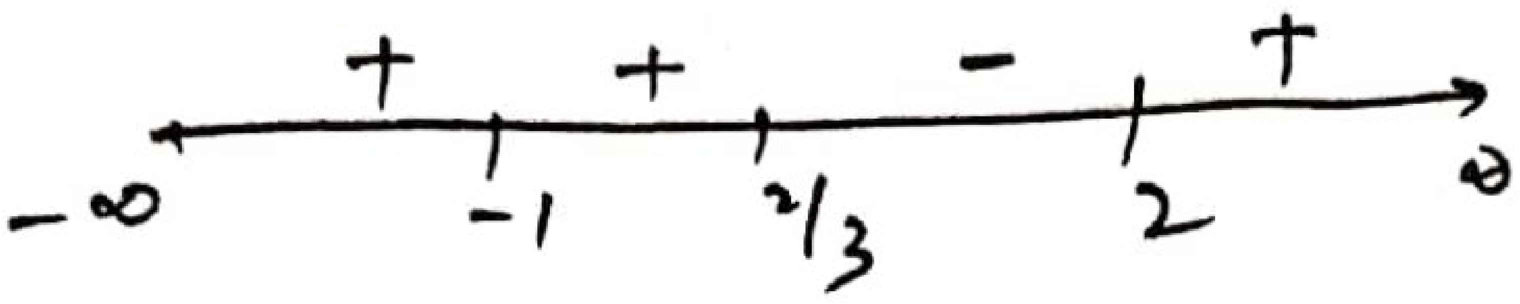
$$= (x-2)^3 \cdot (x+1)^4 \left[5x - 10 + 4x + 4\right]$$

$$f'(x) = (x-2)^3(x+1)^4\,(9x-6)$$

put $f'(x) = 0$

$x = 2$, $x = -1$, $x = 2/3$ (Critical points)

Ist derivative test



at $x = 2$ $f'(x)$ changes its sign from $-ve$ to $+ve$

$\therefore$ $x = 2$ is point of local Minima

at $x = 2/3$: $f'(x)$ changes its sign from $+ve$ to $-ve$

$\therefore$ $x = 2/3$ is point of local Maxima

at $x = -1$; $f'(x)$ does not change its sign

$\therefore$ $x = -1$ is the point of Inflexion Ans

(Note: Misprint in worksheet Answer)

Ans. 3 → (1) $f(x) = \frac{x}{2} + \frac{2}{x}$; $x > 0$

$$f'(x) = \frac{1}{2} - \frac{2}{x^2}$$

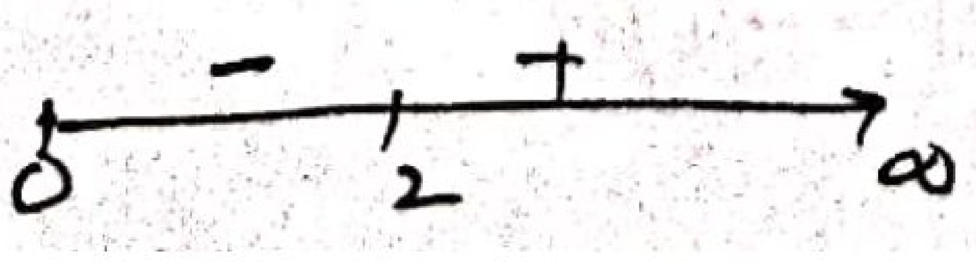
$$= \frac{x^2-4}{2x^2}$$

$$f'(x) = \frac{(x+2)(x-2)}{2x^2}$$

Critical points : $x = -2$; $x = 2$, $x = 0$ → Rejected {$\because x > 0$}

$x = -2$ → Rejected {$\because x > 0$

$\therefore$ $\boxed{x = 2}$ only



{$\because x > 0$}

at $x=2$: $f'(x)$ changes its sign from $-ve$ to $+ve$

$\therefore$ $x=2$ is point of local Minima

local Minimum value $= f(2)$

$$f(2) = \frac{2}{2} + \frac{2}{2} = 1+1 = 2$$

No point of local Maxima

$\therefore$ No local Maximum value

ANS

{Note: Misprint in Worksheet Answer}

(2) $f(x) = x\sqrt{1-x}$; $x > 0$

$$f'(x) = x \cdot \frac{1}{2\sqrt{1-x}}(-1) + \sqrt{1-x}\cdot(1)$$

$$= \frac{-x + 2(1-x)}{2\sqrt{1-x}}$$

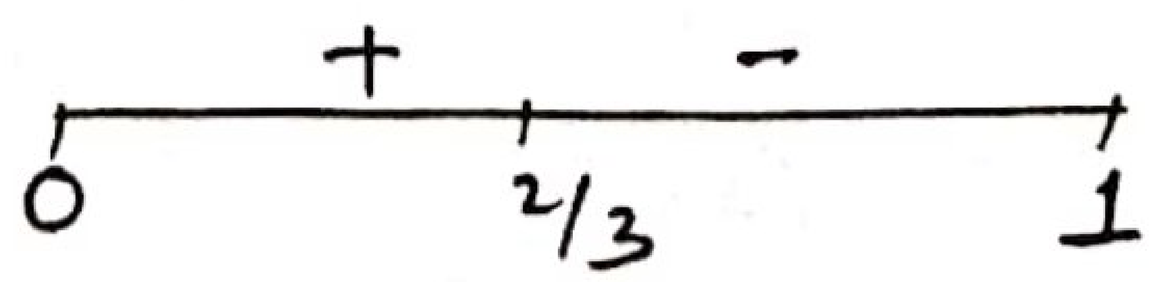
$$f'(x) = \frac{2-3x}{2\sqrt{1-x}}$$

Critical points $x = 2/3$ & $x = 1$

{ Remember: $f(x)$ is not defined / does not exists when $x > 1$ $\because$ $\sqrt{-ve}$ aajayega }

and $x > 0$ (given)

$\therefore$ here



at $x = 2/3$ $f'(x)$ changes its sign from $+ve$ to $-ve$

$\therefore$ $x=2/3$ is the point of local Maxima

local Maximum value $= f(2/3)$

$$f(2/3) = \frac{2}{3}\sqrt{1-\frac{2}{3}}$$

$$= \frac{2}{3} \times \frac{1}{\sqrt{3}} = \frac{2}{3\sqrt{3}} \text{ or } \frac{2\sqrt{3}}{9}$$ ✓

No Minimum value ANS

(3) $f(x) = \sin x + \frac{1}{2}\cos(2x) \quad ; \; 0 \le x \le \pi/2$

$f'(x) = \cos x + \frac{1}{\cancel{2}}(-\sin(2x)) \cdot \cancel{2}$

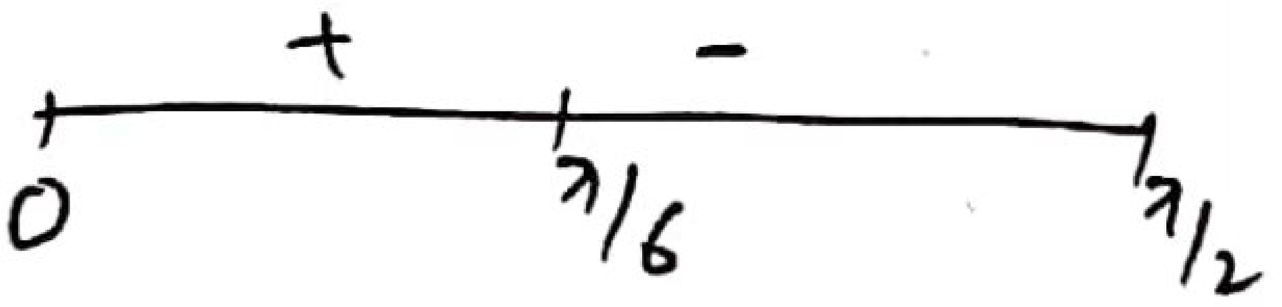
$f'(x) = \cos x - \sin(2x)$

$= \cos x - 2\sin x \cos x$

$f'(x) = \cos x(1 - 2\sin x)$

put $f'(x) = 0$

$\cos x = 0$	$\sin x = 1/2$
$x = \pi/2$	$x = \pi/6$



Imp
Note : here Critical point is also $x = \pi/2$

Imp but by Ist derivative test, we cannot Check at $x = \pi/2$ because it is an end point

So please go to 2nd derivative Test

$f''(x) = -\sin x - 2\cos(2x)$

$f''(\pi/6) = -\sin(\pi/6) - 2\cos(\pi/3) = -\frac{1}{2} - 2\left(\frac{1}{2}\right) = -3/2 < 0$

$\therefore f(x)$ is Maximum at $x = \pi/6$

local Maximum value $= f(\pi/6) = \sin(\pi/6) + \frac{1}{2}\cos(\pi/3)$

local Maximum value $= \frac{1}{2} + \frac{1}{4} = = 3/4$

Now $f''(\pi/2) = -\sin(\pi/2) - 2\cos(\pi) = -1 - 2(-1) = 1 > 0$

$\therefore x = \pi/2$ is the point of local Minima

local Minimum value $= f(\pi/2)$

$f(\pi/2) = \sin(\pi/2) + \frac{1}{2}\cos(\pi) = 1 + \frac{1}{2}(-1) = 1/2$ Ans

(4) $f(x) = 2\sin x - x$; $-\frac{\pi}{2} \le x \le \pi/2$

$f'(x) = 2\cos x - 1$

put $f'(x) = 0$

$2\cos x - 1 = 0$

$\Rightarrow \cos x = 1/2$

$x = -\frac{\pi}{3}$ also $x = \pi/3$

$-$ $+$ $-$

$-\pi/2$ $-\pi/3$ $\pi/3$ $\pi/2$

$x = -\pi/3$ is the point of local Minima

and local Minimum value $= f(-\pi/3)$

$= 2\sin(-\pi/3) - \frac{\pi}{3} = -2\left(-\frac{\sqrt{3}}{2}\right) - \frac{\pi}{3} = -\sqrt{3} - \pi/3$

$\therefore$ local Minimum value $= -\sqrt{3} - \pi/3$ at $x = -\pi/3$

$x = \pi/3$ is the point of local Maxima

and local Maximum value $= f(\pi/3)$

$f(\pi/3) = 2\sin(\pi/3) - \pi/3 = \sqrt{3} - \pi/3$ ANS

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(5) $f(x) = \sin^4 x + \cos^4 x \quad ; \quad 0 < x < \pi/2$

$f'(x) = 4\sin^3 x \cdot \cos x + 4\cos^3 x \cdot (-\sin x)$

$= 4\sin x \cos x \left(\sin^2 x - \cos^2 x \right)$

$= -2\sin(2x) \cdot \left(\cos^2 x - \sin^2 x \right)$

$= -2\sin(2x) \cdot \cos(2x)$

$f'(x) = -\sin(4x)$

put $f'(x) = 0$

$-\sin(4x) = 0$

$\Rightarrow \sin(4x) = 0$

$\Rightarrow 4x = 0$	$4x = \pi$	$4x = 2\pi$
$x = 0$	$x = \pi/4$	$x = \pi/2$
(Rejected) $\because x > 0$		rejected $\because x < \pi/2$

$-$ $+$

0 $\pi/4$ $\pi/2$

$x = \pi/4$ is the point of local Minima and local Minimum value $= f(\pi/4)$

$f(\pi/4) = \sin^4(\pi/4) + \cos^4(\pi/4) = \left(\frac{1}{\sqrt{2}}\right)^4 + \left(\frac{1}{\sqrt{2}}\right)^4$

$= \frac{1}{4} + \frac{1}{4} = 1/2$

$\therefore$ local Minimum value $= 1/2$ at $x = \pi/4$ Ans

QNS: 7 (1) $f(x) = 3x^4 - 8x^3 + 12x^2 - 48x + 1 \; ; \; x \in [1,4]$

$f'(x) = 12x^3 - 24x^2 + 24x - 48$

$= 12(x^3 - 2x^2 + 2x - 4)$

$= 12(x^2(x-2) + 2(x-2))$

$f'(x) = 12(x^2+2)(x-2)$

put $f'(x) = 0$

$\boxed{x = 2} \qquad \because x^2 + 2 \neq 0$

$f(1) = 3 - 8 + 12 - 48 + 1 = -40$

$f(2) = 48 - 64 + 48 - 96 + 1 = -63$

$f(4) = 768 - 512 + 192 - 192 + 1 = 257$

$\therefore$ Absolute Max. value $= 257$ Ans

and Absolute Min value $= -63$
(Misprint in worksheet Ans)

(2) $f(x) = (x-2)\sqrt{x-1} \; ; \; x \in [1,9]$

$f'(x) = (x-2) \cdot \frac{1}{2\sqrt{x-1}} + \sqrt{x-1} \cdot (1)$

$= \frac{x - 2 + 2x - 2}{2\sqrt{x-1}}$

$f'(x) = \frac{3x-4}{2\sqrt{x-1}}$

critical points: $x = 4/3$ & $x = 1$

$f(1) = (-1)\sqrt{0} = 0$

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$f(4/3) = \left(\frac{4}{3}-2\right)\sqrt{\frac{4}{3}-1} = -\frac{2}{3}\sqrt{\frac{1}{3}} = -\frac{2}{(3)^{3/2}}$

$f(9) = (7)\sqrt{8} = 14\sqrt{2}$

$\therefore$ Absolute Min value $= \frac{-2}{(3)^{3/2}}$ Ans

and Absolute Max. value $= 14\sqrt{2}$

{Note:- Misprint in worksheet Ans}

— x —