← ULTIMATE MATHEMATICS →

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DIFFERENTIATION & CONTINUITY

← CLASS NO: 2 →

logarithms

(·) Basic properties

① $\log(AB) = \log A + \log B$

$\log(ABC) = \log A + \log B + \log C$

② $\log(A/B) = \log A - \log B$

③ $\log(m)^n = n\log m$

(x) $\log A \cdot \log B$

(x) $\frac{\log A}{\log B}$

(x) $(\log m)^n$

$\log 1 = 0$

$\log e = 1$

Base of $\log x$

$y = \log_e x$ = Natural log

base

Diff $\frac{dy}{dx} = \frac{1}{x}$

$\boxed{y = \log_2 x \quad \frac{dy}{dx} \neq \frac{1}{x}}$

Property: Change of base

$\log_a b = \frac{\log b}{\log a}$

$y = \log_2 x$

$y = \frac{\log x}{\log 2}$

$\frac{dy}{dx} = \frac{1}{x\log 2}$

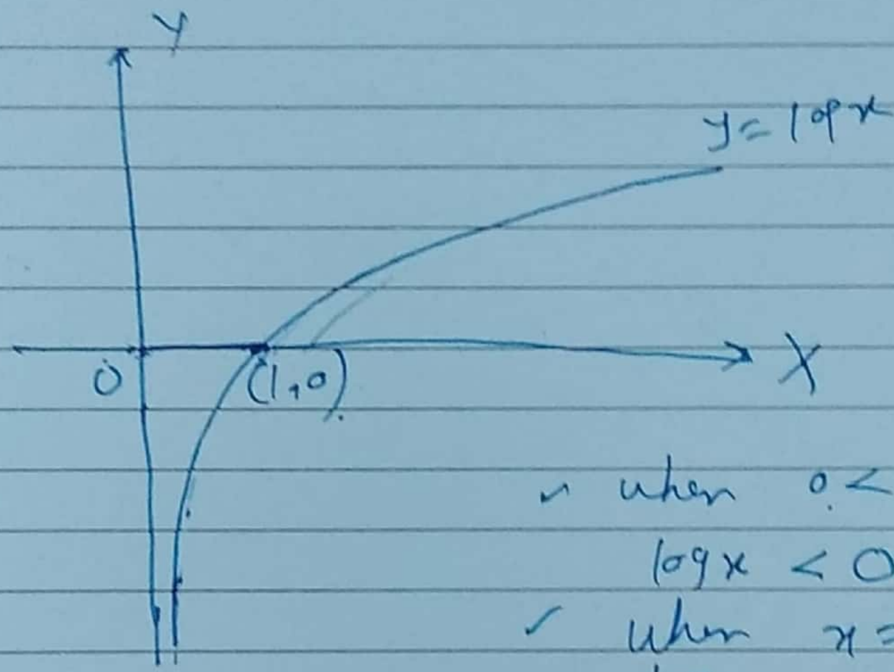
(•) $y = \log x$ Domain

$\log x$ exists only when $\boxed{x > 0}$

✓ $\log 0$ ✗

$\log(-ve)$ ✗

(•) Graph



✓ when $0 < x < 1$
$\log x < 0$

✓ when $x = 1$
$\log x = 0$

✓ when $x > 1$
$\log x > 0$

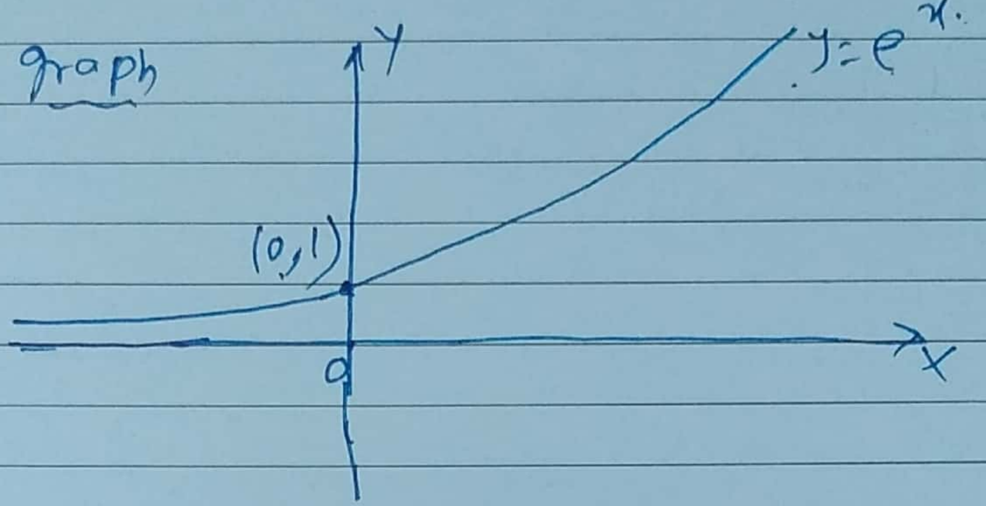
✓ Exponential function

$(\cancel{\text{Base}}) = (\text{Constant})^{\text{function}}$

	$y = e^x$	$y = \log x$	
Domain (•)	$x \in R$	(•) $x > 0$	(Domain)
Range (•)	$y > 0$ $\quad \boxed{e^x > 0}$	(•) $y \in R$	(Range)

(•) $y = \log_e x$ $\Rightarrow$ If $\log_e x = y$ then $x = e^y$

(·) graph



(·) $e \approx 2.73$ (Approx)

(·) expiring $e^x = 1 + \frac{x^1}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \dots \infty$

← TOPIC:- 1 LOGARITHMIC DIFFERENTIATION →

(·) $y = (f(x))^{g(x)}$

taking log on both sides

$\log y = \log (f(x))^{g(x)}$

$\log y = g(x) \cdot \log(f(x))$

Diff wrt x

$\frac{1}{y} \cdot \frac{dy}{dx} =$ product rule

$\frac{dy}{dx} = y$ [product rule]

$\frac{dy}{dx} = (f(x))^{g(x)} \cdot$ (product rule)

Qn 1 Differentiate w.r.t x

$$y = (\sin x)^{\sqrt{x}}$$

Soln taking log on both sides

$$\log y = \log(\sin x)^{\sqrt{x}}$$

$$\Rightarrow \log y = \sqrt{x} \cdot \log(\sin x)$$

Diff w.r.t x

$$\frac{1}{y} \cdot \frac{dy}{dx} = \sqrt{x} \cdot \frac{1}{\sin x} \cdot \cos x + \log(\sin x) \cdot \frac{1}{2\sqrt{x}}$$

$$\frac{dy}{dx} = (\sin x)^{\sqrt{x}} \left[\sqrt{x} \cot x + \frac{\log(\sin x)}{2\sqrt{x}} \right]$$ Ans

Qn 2 $$y = (\log x)^{\log x}$$

Soln taking log on both sides

$$\log y = \log(\log x)^{\log x}$$

$$\Rightarrow \log y = \log x \cdot \log(\log x)$$

Diff w.r.t x

$$\frac{1}{y} \cdot \frac{dy}{dx} = \cancel{\log x} \cdot \frac{1}{\cancel{\log x}} \cdot \frac{1}{x} + \log(\log x) \cdot \frac{1}{x}$$

$$\frac{dy}{dx} = (\log x)^{\log x} \left[\frac{1}{x} + \frac{\log(\log x)}{x} \right]$$

$$\frac{dy}{dx} = \frac{(\log x)^{\log x}}{x} \left[1 + \log(\log x) \right]$$ Ans

Ans 3 → $y = (\sin x)^{\cos x} + (\tan^{-1}x)^{\log x}$

$$* \log(A+B) \neq \log A + \log B$$

→ $y = u + v$

Diff wrt x

$$\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx} \quad \text{----}(i)$$

Consider $u = (\sin x)^{\cos x}$

taking log

$$\log u = \log(\sin x)^{\cos x}$$

$$\Rightarrow \log u = \cos x \cdot \log(\sin x)$$

Diff wrt x

$$\frac{1}{u}\cdot\frac{du}{dx} = \cos x \cdot \frac{1}{\sin x}\cdot \cos x + \log(\sin x)\cdot(-\sin x)$$

$$\frac{dy}{dx} = (\sin x)^{\cos x}\left[\cos x \cdot \cot x - \sin x \cdot \log(\sin x)\right]$$

Consider $v = (\tan^{-1}x)^{\log x}$

taking log

$$\log v = \log(\tan^{-1}x)^{\log x}$$

$$\Rightarrow \log v = \log x \cdot \log(\tan^{-1}x)$$

Diff

$$\frac{1}{v}\cdot\frac{dv}{dx} = \log x \cdot \frac{1}{\tan^{-1}x}\cdot\frac{1}{1+x^2} + \log(\tan^{-1}x)\cdot\frac{1}{x}$$

$$\frac{dv}{dx} = (\tan^{-1}x)^{\log x}\left[\frac{\log x}{\tan^{-1}x\cdot(1+x^2)} + \frac{\log(\tan^{-1}x)}{x}\right]$$

∴ equation (i) becomes

$$\frac{dy}{dx} = \boxed{} + \boxed{}$$ Ans

Q.No. 4 → $y = (x\sin x)^x + (\cos x)^{x\sin x}$

$y = u + v$

Diff

$$\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx} \quad \text{---(1)}$$

Consider $u = (x\sin x)^x$

taking log

$\log u = \log(x\sin x)^x$

$\Rightarrow \log u = x.\log(x\sin x)$

Diff wrt x

$$\frac{1}{u}\cdot\frac{du}{dx} = \cancel{x}\cdot\frac{1}{\cancel{x}\sin x}\cdot(x\cos x + \sin x) + \log(x\sin x)\cdot 1$$

$$\frac{du}{dx} = (x\sin x)^x\left[x\cot x + 1 + \log(x\sin x)\right]$$

Consider $v = (\cos x)^{x\sin x}$

taking log

$\log v = \log(\cos x)^{x\sin x}$

$\Rightarrow \log v = x\sin x . \log(\cos x)$

proceed

CLASS NO: 2

Topic: D&C

Date.:

Qn 5

$$y = (x^x)^x$$

$$(2^3)^2 = 2^6$$

$$y = x^{x^2}$$

taking log on both sides

$$\log y = \log(x^{x^2})$$

$$\log y = x^2 \cdot \log x$$

Diff

$$\frac{1}{y} \cdot \frac{dy}{dx} = x^2 \cdot \frac{1}{x} + \log x \cdot 2x$$

$$\frac{dy}{dx} = (x^x)^x \cdot [x + 2x\log x]$$ Ans

Qn 6

$$y = x^{x^x}$$

$$2^{3^2} = 2^9$$

taking log

$$\log y = \log(x^{x^x})$$

$$\Rightarrow \log y = x^x \cdot \log x$$

take log again

$$\log(\log y) = \log(\underset{A}{x^x} \cdot \underset{B}{\log x})$$

~~Diff w.r.t.~~

~~$\frac{1}{\log y} \cdot \frac{1}{y} \cdot \frac{dy}{dx}$~~

$$\log(\log y) = \log(x^x) + \log(\log x)$$

$$\log(\log y) = x\log x + \log(\log x)$$

Diff

$\log(\log y) = x\log x + \log(\log x)$

Diff wrt x

$$\frac{1}{\log y}\cdot\frac{1}{y}\cdot\frac{dy}{dx} = x\cdot\frac{1}{\cancel{x}} + \log x\cdot 1 + \frac{1}{\log x}\cdot\frac{1}{x}$$

$$\frac{dy}{dx} = y\cdot\log y\left[\log x + \frac{1}{x\log x}\right]$$

$$\frac{dy}{dx} = x^{x^{x}}\cdot x^{x}\cdot\log x\left[\log x + \frac{1}{x\log x}\right) \underline{\underline{\text{Ans}}}$$

← DIFFERENTIATION & CONTINUITY →

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(WORKSHEET No: 1)

Qn 1 → $y = (\log x)^x + x^{\log x}$ find $\frac{dy}{dx}$

Qn. 2 → $y = (\sin x)^{\cos x} + x^{\sin x}$ find $\frac{dy}{dx}$

Qn 3 → $y = (x\cos x)^x + (x\sin x)^{\frac{1}{x}}$. Find $\frac{dy}{dx}$

Qn 4 → $y = \left(x + \frac{1}{x}\right)^x + (x)^{1+\frac{1}{x}}$ find $\frac{dy}{dx}$

Qn 5 → $y = x^{x\cos x} + \frac{x^2+1}{x^2-1}$

Qn. 6 → $y = (x+3)^2 \cdot (x+4)^3 \cdot (x+5)^4$. find $\frac{dy}{dx}$

HINT: $\log(ABC) = \log A + \log B + \log C$
(take directly log on both sides)

Qn 7 → $y = \cos x \cdot \cos(2x) \cdot \cos(3x)$ find $\frac{dy}{dx}$

HINT:- take directly log on both sides

Qn 8 → $(\cos x)^y = (\cos y)^x$ find $\frac{dy}{dx}$

HINT take log on both sides and last me $\frac{dy}{dx}$ common

Qn 9 → If $x^y = y^x$ find $\frac{dy}{dx}$

Qn 10 → $y = x^{x^x}$. Find $\frac{dy}{dx}$

D&C

WORKSHEET No 1

Topic : .... Date. : ....

Qn. 11 → If $x^y + y^x = 1$ find $\frac{dy}{dx}$

HINT $u+v=1 \Rightarrow \frac{du}{dx} + \frac{dv}{dx} = 0$

and last me $\frac{dy}{dx}$ common lena

Qn 12 → If $x^y + y^x + x^x = a^b$ find $\frac{dy}{dx}$

HINT $u+v+w = a^b \Rightarrow \frac{du}{dx} + \frac{dv}{dx} + \frac{dw}{dx} = 0$

and last me $\frac{dy}{dx}$ common lena.

Qn. 13 → If $xy = e^{x-y}$ find $\frac{dy}{dx}$

Qn 14 → If $y = (\sin x)^x + \sin^{-1}\sqrt{x}$ find $\frac{dy}{dx}$

Qn 15 → If $f(x) = (1+x)\cdot(1+x^2)\cdot(1+x^4)\cdot(1+x^8)$

find $f'(1)$

HINT Take log on both sides

$\log(ABCD) = \log A + \log B + \log C + \log D$

and last me put $x=1$ to find $f'(1)$

→ ANSWERS →

(1) $\frac{dy}{dx} = \frac{(\log x)^x}{\log x}\left[1 + \log x \cdot \log(\log x)\right] + 2x^{\log x - 1} \cdot \log x$

(2) $\frac{dy}{dx} = x^{\sin x}\left[\frac{\sin x}{x} + \cos x \cdot \log x\right] + (\sin x)^{\cos x}\left[\cos x \cdot \cot x - \sin x \cdot \log(\sin x)\right]$

(3) $\frac{dy}{dx} = (x\cos x)^x\left[1 - x\tan x + \log(x\cos x)\right] +$

D&C WORKSHEET No. 1

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$$+ (x\sin x)^{\frac{1}{x}} \left[\frac{x\cot x + 1 - \log(x\sin x)}{x^2} \right]$$

(4) $\frac{dy}{dx} = \left(x+\frac{1}{x}\right)^x \left[\frac{x^2-1}{x^2+1} + \log\left(x+\frac{1}{x}\right)\right] + x^{1+\frac{1}{x}} \left[\frac{x+1-\log x}{x^2} \right]$

(5) $\frac{dy}{dx} = x^{x\cos x} \left[\cos x \cdot (1+\log x) - x\sin x \log x \right] - \frac{4x}{(x^2-1)^2}$

(6) $\frac{dy}{dx} = (x+3)(x+4)^2(x+5)^3 \cdot \left[9x^2 + 70x + 133 \right]$

(7) $\frac{dy}{dx} = -\cos x \cdot \cos(2x) \cdot \cos(3x) \left[\tan x + 2\tan(2x) + 3\tan(3x) \right]$

(8) $\frac{dy}{dx} = \frac{y\tan x + \log(\cos y)}{x\tan y + \log(\cos x)}$

(9) $\frac{dy}{dx} = \frac{y}{x} \cdot \left(\frac{x\log y - y}{y\log x - x} \right)$

(10) already done in notes (class)

$\frac{dy}{dx} = x^{x^x} \cdot x^x \cdot \log x \left[\log x + \frac{1}{x\log x} \right]$

(11) $\frac{dy}{dx} = -\left[\frac{yx^{y-1} + y^x \log y}{x^y \log x + xy^{x-1}} \right]$

(12) $\frac{dy}{dx} = -\left[\frac{y^x \log y + y \cdot x^{y-1} + x^x(1+\log x)}{x \cdot y^{x-1} + x^y \log x} \right]$

DEC WORKSHEET NO = 1

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(13) $\frac{dy}{dx} = \frac{y(x-1)}{x(y+1)}$

(14) $\frac{dy}{dx} = (\sin x)^x \left[x\cot x + \log(\sin x)\right] + \frac{1}{2\sqrt{x-x^2}}$

(15) $f'(1) = 120$

— x —