Solution

WORKSHEET NO: 3

RELATION & FUNCTION

Ans 1 $f : R \to R$
$f(x) = x^4$

one-one $f(-1) = (-1)^4 = 1$
also $f(1) = (1)^4 = 1$
clearly f is not one-one

ONTO $-1 \in R$ (codomain), but there does not exists any element x in R (domain) such that
$f(x) = x^4 = -1$
$\therefore f$ is not on-to **Ans**

Qn 2 $f : R - \{3\} \to R - \{1\}$
$$f(x) = \frac{x-2}{x-3}$$

one-one let $x_1, x_2 \in R - \{3\}$ (domain)
& $f(x_1) = f(x_2)$
$$\Rightarrow \frac{x_1 - 2}{x_1 - 3} = \frac{x_2 - 2}{x_2 - 3}$$
$$\Rightarrow \cancel{x_1 x_2} - 3x_1 - 2x_2 + \cancel{6} = \cancel{x_1 x_2} - 2x_1 - 3x_2 + \cancel{6}$$
$$\Rightarrow -x_1 = -x_2$$
$$\Rightarrow \boxed{x_1 = x_2} \quad \therefore f \text{ is one-one}$$

ON-TO let $y = f(x)$
$$\Rightarrow y = \frac{x-2}{x-3}$$
$$\Rightarrow xy - 3y = x - 2$$
$$\Rightarrow x(y-1) = 3y - 2$$

Soln REF (w.s 3)

$\Rightarrow \quad x = \frac{3y-2}{y-1}$

for each $y \in R-\{1\}$ (codomain), there exists an element x in Domain such that

$f(x) = f\left(\frac{3y-2}{y-1}\right)$

$$= \frac{\frac{3y-2}{y-1} - 2}{\frac{3y-2}{y-1} - 3} = \frac{3y - \cancel{2} - 2y + \cancel{2}}{\cancel{3y} - 2 - \cancel{3y} + 3} = \frac{y}{1} = y$$

$\therefore f$ is onto

Alternate Method

let $y = f(x)$

$\Rightarrow y = \frac{x-2}{x-3}$

$\Rightarrow xy - 3y = x - 2$

$\Rightarrow x(y-1) = 3y-2$

$\Rightarrow x = \frac{3y-2}{y-1}$

for Range $\quad y-1 \neq 0$

$\qquad y \neq 1$

$\therefore$ Range $= R-\{1\}$ which is equal to codomain

$\therefore$ Range = Codomain

$\therefore f$ is ON-TO Ans

Ques 3 → $f: R \to R$

$f(x) = 3-4x$

(easy hai do yourself)

Ans 4 → $f: R \to R$

(1) Signum function

$$f(x) = \begin{cases} 1 ; & x > 0 \\ -1 ; & x < 0 \\ 0 ; & x = 0 \end{cases}$$

one-one

$f(1) = 1$

also $f(2) = 1$ {∵ when $x > 0$, $f(x) = 1$}

Clearly f is not one-one

ON-TO

Range = $\{1, -1, 0\}$

but Codomain = R

⇒ Range ≠ Codomain

∴ f is not on-to

OR

$2 \in R$ (Codomain) but this is not the image of any element in domain

∴ f is not on-to

(2) greatest Integer function

$f(x) = [x]$

one-one $f(1.1) = [1.1] = 1$

$f(1.2) = [1.2] = 1$

Clearly f is not one-one

ONTO $1.4 \in R$ (codomain) but there does not exists any element x in R (domain) Such that $f(x) = [x] = 1.4$

∴ f is not on-to

(3) Modulus function

$$f(x) = |x|$$

one-one $f(-1) = |-1| = 1$

$f(1) = |1| = 1$

Clearly f is not one-one

ON-TO $-2 \in R$ (codomain) but there does not exists any element x in R (domain) such that $f(x) = |x| = -2$

$\therefore f$ is not on-to

Qn. 5 → $f: N \to N$

$f(x) = x-1 \quad ; x \geq 2$

$f(1) = f(2) = 1$

one-one given $f(1) = 1$

$f(2) = 1$

Clearly f is not one-one

on-to let $y = f(x)$

$y = x - 1$

$x = y + 1$

for $x \geq 2$, we have $y \geq 2$

{ e.g. $x = 3$; $3 = y + 1 \Rightarrow y = 2$

$x = 4$; $4 = y + 1 \Rightarrow y = 3$

also $y = 1$ when $x = 1$ & $x = 2$

$\therefore$ Range $=$ 1, 2, 3, -- ∞ $= N =$ codomain

Since Range = Codomain

$\therefore$ f is on-to **Ans**

Qn 6 → $f(n) = \begin{cases} \frac{n+1}{2} & : n \text{ is odd} \\ \frac{n}{2} & : n \text{ is even} \end{cases}$

$f(1): \frac{1+1}{2} = \frac{2}{2} = 1$

$f(2) = \frac{2}{2} = 1$

Clearly f is not one-one

(no need to check on-to)

$\therefore$ f is not bijective **Ans**

Qn 7 → $f: N \to [6, \infty)$

$f(x) = 4x^2 + 12x + 15$

one-one let $x_1, x_2 \in N$ (domain)

and $f(x_1) = f(x_2)$

$\Rightarrow 4x_1^2 + 12x_1 + \cancel{15} = 4x_2^2 + 12x_2 + \cancel{15}$

$\Rightarrow 4(x_1^2 - x_2^2) + 12(x_1 - x_2) = 0$

$\Rightarrow (x_1 - x_2)[4x_1 + 4x_2 + 12] = 0$

$\Rightarrow x_1 - x_2 = 0$ | but $4x_1 + 4x_2 + 12 \neq 0$ Since $x_1, x_2 \in N$

$\therefore x_1 = x_2$

$\therefore$ f is one-one

On-to Let $y = f(x)$

$\Rightarrow y = 4x^2 + 12x + 15$

$\Rightarrow 4x^2 + 12x + (15 - y) = 0$

Quadratic formula

$$x = \frac{-12 \pm \sqrt{144 - 16(15-y)}}{8}$$

$$x = \frac{-12 \pm \sqrt{16y - 96}}{8}$$

$$x = \frac{-12 \pm 4\sqrt{y-6}}{8}$$

$$x = \frac{-3 \pm \sqrt{y-6}}{2}$$

But $x \neq \frac{-3 - \sqrt{y-6}}{2}$; as $x \in N$

$\therefore x = \frac{-3 + \sqrt{y-6}}{2}$

for each $y \in [6, \infty)$ there exist an element x in domain (N) such that $f(x) = f\left(\frac{-3 + \sqrt{y-6}}{2}\right)$

$$= 4\left(\frac{-3 + \sqrt{y-6}}{2}\right)^2 + 12\left(\frac{-3 + \sqrt{y-6}}{2}\right) + 15$$

$\vdots$

$= y$

$\therefore f$ is onto

OR Range: $y - 6 \geq 0$

$y \geq 6$

$y \in [6, \infty)$ = Codomain $\therefore f$ is onto

$\because f$ is bijective

$\therefore f$ is invertible

and $f^{-1}(y) = \frac{-3+\sqrt{y-6}}{2}$

and $f^{-1}(x) = \frac{-3+\sqrt{x-6}}{2}$ } Ans

Qn. 8 → $\boxed{\text{easy hai}}$ (do yourself)

$f: R \to R$ and $f(x) = 4x+3$

Qn. 9 → $f: R_+ \to [4, \infty)$

$f(x) = x^2+4$

one-one let $x_1, x_2 \in R_+$ (domain)

and $f(x_1) = f(x_2)$

$\Rightarrow x_1^2 + \cancel{4} = x_2^2 + \cancel{4}$

$\Rightarrow x_1^2 = x_2^2$

$\Rightarrow x_1 = \pm x_2$ but $x_1 \neq -x_2$

$\{\because x_1, x_2 \in R_+\}$

$\therefore \boxed{x_1 = x_2}$

$\therefore f$ is one-one

on-to let $y = f(x)$

$\Rightarrow y = x^2+4$

$\Rightarrow x^2 = y-4$

$\Rightarrow x = \pm\sqrt{y-4}$

But $x \neq -\sqrt{y-4}$ ... $\{\because x \in R_+\}$

$\Rightarrow x = \sqrt{y-4}$

for range $\quad y-4 \geq 0$

$y \geq 4$

$y \in [4, \infty)$ = codomain also

$\therefore$ Range = codomain

$\therefore f$ is onto

$\therefore f$ is bijective

$\therefore f$ is invertible

and $\left.\begin{array}{l} f^{-1}(y) = \sqrt{y-4} \\ f^{-1}(x) = \sqrt{x-4} \end{array}\right\}$ Ans

Qn 10 → $f(x) = (3-x^3)^{1/3}$

$fof(x) = f(f(x))$

$= f(3-x^3)^{1/3}$

$= \left[3 - \{(3-x^3)^{1/3}\}^3\right]^{1/3}$

$= \left[3 - (3-x^3)\right]^{1/3}$

$= [x^3]^{1/3}$

$= x$ Ans

Qn. 11 → $f: R - \{-\frac{4}{3}\} \to R - \{\frac{4}{3}\}$ → (Note there is misprint in worksheet)

$f(x) = \frac{4x}{3x+4}$

one-one: let $x_1, x_2 \in R - \{-\frac{4}{3}\}$ (domain)

& $f(x_1) = f(x_2)$

$\Rightarrow \dfrac{4x_1}{3x_1+4} = \dfrac{4x_2}{3x_2+4}$

$\Rightarrow \cancel{12x_1x_2} + 16x_1 = \cancel{12x_1x_2} + 16x_2$

$\Rightarrow 16x_1 = 16x_2$

$\Rightarrow \boxed{x_1 = x_2}$ $\therefore$ f is one-one

on-to let $y = f(x)$

$y = \dfrac{4x}{3x+4}$

$3xy + 4y = 4x$

$x(3y-4) = -4y$

$x = \dfrac{-4y}{3y-4}$

for range $3y-4 \neq 0$

$y \neq 4/3$

$\therefore y \in R - \{\frac{4}{3}\}$ = codomain

$\Rightarrow$ Range = codomain

$\therefore$ f is onto

$\therefore$ f is Invertible

and $f^{-1}(y) = \dfrac{-4y}{3y-4}$

$f^{-1}(x) = \dfrac{-4x}{3x-4}$ Ans

Qn 12 → $f: R \to R$

$f(x) = \dfrac{x}{x^2+1}$

One-one let $x_1, x_2 \in R$

$$\Rightarrow \frac{x_1}{x_1^2+1} = \frac{x_2}{x_2^2+1}$$

$$\Rightarrow x_1x_2^2 + x_1 = x_1^2x_2 + x_2$$

$$\Rightarrow x_1x_2^2 - x_1^2x_2 + x_1 - x_2$$

$$\Rightarrow x_1x_2(x_2 - x_1) - 1(x_2 - x_1) = 0$$

$$(x_2 - x_1)(x_1x_2 - 1) = 0$$

$x_1 = x_2$; also $x_1x_2 = 1$

$\therefore f$ is not one-one

(OR) $f(2) = \frac{2}{4+1} = \frac{2}{5}$

$$f(\tfrac{1}{2}) = \frac{\frac{1}{2}}{\frac{1}{4}+1} = \frac{\frac{1}{2}}{\frac{5}{4}} = \frac{4}{10} = \frac{2}{5}$$

Clearly two different elements $2, \frac{1}{2}$ has same image $= \frac{2}{5}$ in codomain

$\therefore f$ is not one-one

ONTO. let $y = f(x)$

$$y = \frac{x}{x^2+1}$$

$$x^2y + y = x$$

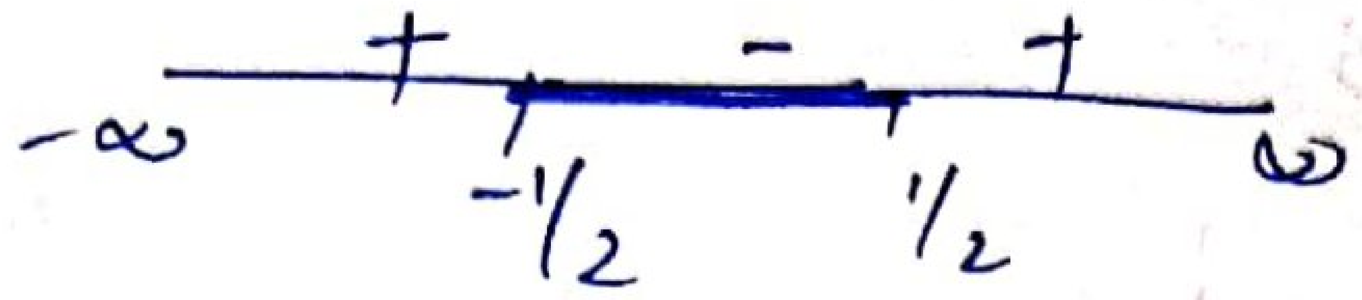
$$\Rightarrow x^2y - x + y = 0$$

Quadratic formula $x = \frac{1 \pm \sqrt{1-4y^2}}{2y}$

for range $1-4y^2 \geq 0$ & $2y \neq 0$

$\Rightarrow 4y^2 - 1 \leq 0$ and $y \neq 0$

$(2y-1)(2y+1) \leq 0$



$\therefore$ Range $[-\frac{1}{2}, \frac{1}{2}] - \{0\}$

but Codomain $= R$ (given)

$\Rightarrow$ Range $\neq$ Codomain

$\therefore f$ is not on-to

OR

$1 \in R$ (Codomain)

when we put $f(x) = 1$

then $1 = \frac{x}{x^2+1}$

$\Rightarrow x^2 + 1 = x$

$\Rightarrow x^2 + 1 - x = 0$

(No Real roots)

$\therefore$ there is no value of x in domain such that $f(x) = \frac{x}{x^2+1} = 1$

$\therefore f(x)$ is not on-to

Ans 13 $f: \{(5,2)\ (6,3)\}$

$g = \{(2,5), (3,6)\}$

here $f(5) = 2$ | $g(2) = 5$

$f(6) = 3$ | $g(3) = 6$

Now

fog

$= f(g(2)) = f(5) = 2$

$f(g(3)) = f(6) = 3$

$\therefore fog = \{(2,2), (3,3)\}$ Ans

Qn 14 → $f = \{(1,2)\ (3,5), (4,1)\}$

$g = \{(2,3), (5,1), (1,3)\}$

$f(1) = 2$	$g(2) = 3$
$f(3) = 5$	$g(5) = 1$
$f(4) = 1$	$g(1) = 3$

$fog = f(g(2)) = f(3) = 5$

$f(g(5)) = f(1) = 2$

$f(g(1)) = f(3) = 5$

$\therefore fog = \{(2,5)\ (5,2)\ (1,5)\}$ Ans

$gof = g(f(1)) = g(2) = 3$

$g(f(3)) = g(5) = 1$

$g(f(4)) = g(1) = 3$

$\therefore gof = \{(1,3)\ (3,1)\ (4,3)\}$ Ans

Qn. 15 $f(x) = \dfrac{x}{\sqrt{1+x^2}}$

$fof(x) = f(f(x))$

$= f\left(\dfrac{x}{\sqrt{1+x^2}}\right)$

$= \dfrac{\frac{x}{\sqrt{1+x^2}}}{\sqrt{1+\frac{x^2}{1+x^2}}}$

$= \dfrac{\frac{x}{\sqrt{1+x^2}}}{\frac{\sqrt{1+2x^2}}{\sqrt{1+x^2}}}$

$fof = \dfrac{x}{\sqrt{1+2x^2}}$

$fofof(x) = f\left(\dfrac{x}{\sqrt{1+2x^2}}\right)$

$= \dfrac{\frac{x}{\sqrt{1+2x^2}}}{\sqrt{1+\frac{x^2}{1+2x^2}}}$

$= \dfrac{x}{\sqrt{1+3x^2}}$ Ans