← ULTIMATE MATHEMATICS →

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RELATION & FUNCTION

← CLASS NO: 5 →

Ans 1 → Consider $f: N \to N$, $g: N \to N$ & $h: N \to R$ defined as $f(x) = 2x$; $g(y) = 3y+4$ & $h(z) = \sin z$

Show that $ho(gof) = (hog)of$

Solution LHS

$ho(gof)$
$= ho(gof(x))$
$= ho(g(f(x)))$
$= ho(g(2x))$
$= ho(6x+4)$
$= h(6x+4)$
$= \sin(6x+4)$

RHS

$(hog)of$
$= (hog)of(x)$
$= \cancel{hof}(hog)o(2x)$
$= ho(g(2x))$
$= ho(6x+4)$
$= h(6x+4)$
$= \sin(6x+4)$

Clearly $ho(gof) = (hog)of$

Ans 2 → Give ~~two~~ examples of two functions $f: N \to \cancel{Z}$ and $g: Z \to Z$ such that gof is injective but g is not injective.

Sol Let $g(x) = |x|$

$g(-1) = |-1| = 1$
$g(1) = |1| = 1$

Clearly g is not injective

Let $f(x) = x$

Now

$$\boxed{\begin{aligned} &gof: N \to Z \\ &gof = g(f(x)) \\ &\quad\;\; = g(x) \\ &gof = |x| \end{aligned}}$$

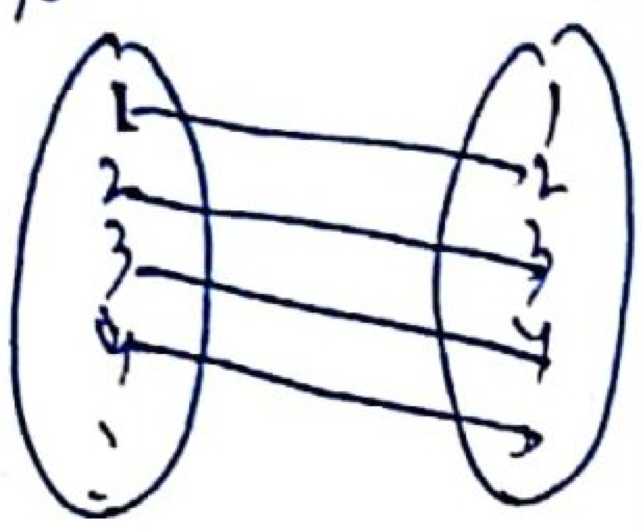
Let $x_1, x_2 \in N$
& $gof(x_1) = gof(x_2)$
~~$g(f(x_1)) = g(f(x_2))$~~
$\cancel{gof}\; |x_1| = |x_2|$
$x_1 = \pm x_2$
$\Rightarrow \boxed{x_1 = x_2}$; $x_1 \neq -x_2$ $\because x_1, x_2 \in N$

Ques 3 → Give examples of two functions $f: N \to N$ and $g: N \to N$ such that gof is onto and f is not on-to

Soln Let $f: N \to N$

$f(x) = x+1$

N N



$1 \in N$ (codomain) but there does not exist any element x in N (domain) such that

$f(x) = x+1 = 1$

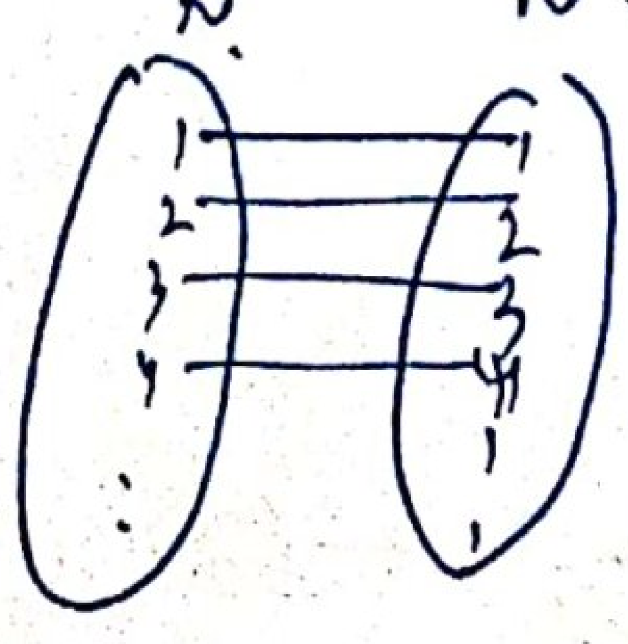
$\therefore f$ is not on-to

Let $g(x) = \begin{cases} x-1 & : x>1 \\ 1 & : x=1 \end{cases}$ $\qquad gof : N \to N$

$gof = g(f(x)) = g(x+1) = \begin{cases} x+1-1 & ; x>1 \\ 1 & ; x=1 \end{cases}$

$gof = \begin{cases} x & : x>1 \\ 1 & : x=1 \end{cases}$

N N



clearly Range = codomain = N

$\therefore gof$ is onto

Qn 4 Show that if $f: A \to B$ and $g: B \to C$ are one-one then $gof: A \to C$ is also one-one.

Soln

let $x_1, x_2 \in A$ (domain of gof)

and $(gof)(x_1) = (gof)(x_2)$

$\Rightarrow g(f(x_1)) = g(f(x_2))$

$\Rightarrow f(x_1) = f(x_2) \ldots$ {If $g(m) = g(n)$ then $m = n$ if g is one-one}

$\Rightarrow x_1 = x_2 \ldots$ {$\because$ f is also one-one}

$\therefore$ gof is also one-one function

If f is one-one and $f(m) = f(n)$ then $m = n$

Qn 5 → Show that $f: N \to N$ given by

$$f(x) = \begin{cases} x+1 : \text{ if } x \text{ is odd} \\ x-1 : \text{ if } x \text{ is even} \end{cases}$$

is both one-one & on-to

Soln

One-one

Case I: let x_1, x_2 are both odd

& $f(x_1) = f(x_2)$

$\Rightarrow x_1 + 1 = x_2 + 1$

$\Rightarrow x_1 = x_2$

$\therefore$ f is one-one

Case II: when x_1, x_2 are both even

& $f(x_1) = f(x_2)$

$\Rightarrow x_1 - \cancel{1} = x_2 - \cancel{1}$

$\Rightarrow x_1 = x_2$

$\therefore$ f is one-one

Case III: let $x_1 \to$ odd, $x_2 \to$ even

& $f(x_1) = f(x_2)$

$x_1 + 1 = x_2 - 1$

$x_2 - x_1 = 2$

not possible

Case IV

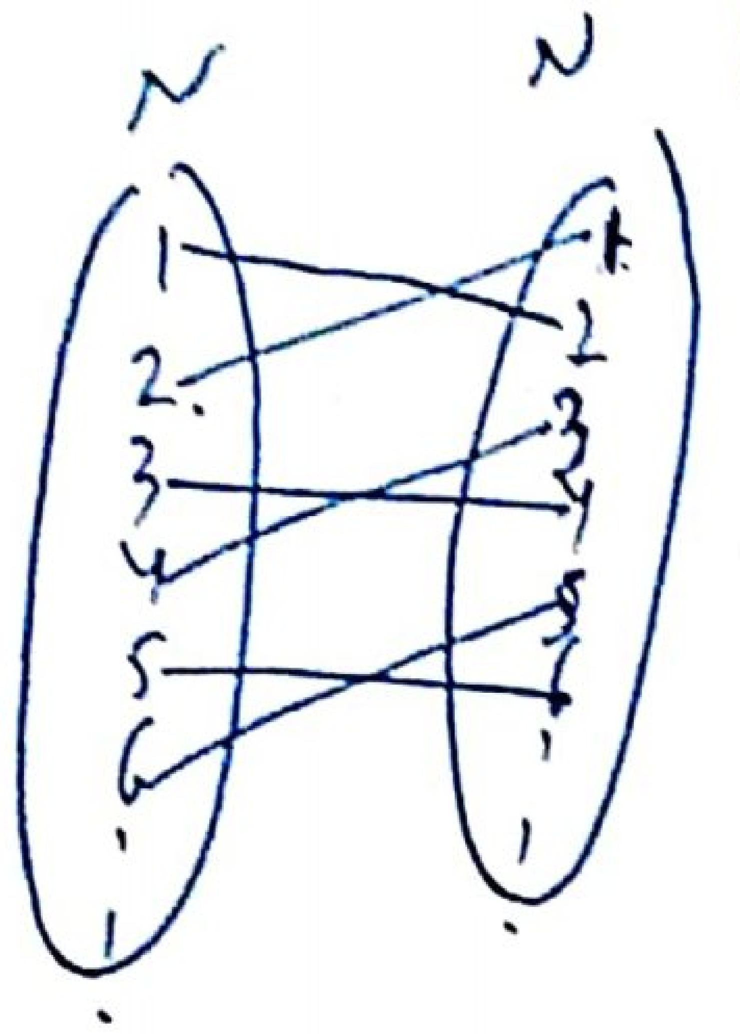
Same

not possible

$\therefore$ f is one-one

XII REF class No: 5 (4)

On-to $f(x) = \begin{cases} x+1 : & x \text{ is odd} \\ x-1 ; & x \text{ is even} \end{cases}$



for each odd number $(2n-1) \in N$ Codomain

there exists an even number $(2n) \in N$ (domain)

& for each even number $(2n) \in N$ (codomain)

there exist an odd number $(2n-1) \in N$ (domain)

$\therefore$ Range = Codomain

$\therefore$ f is onto

Another method to check Invertible function

process: A function $f: X \to Y$ is defined to be Invertible,

theorem: if there exists a function $g: Y \to X$ such that

$gof = I_X$ and $fog = I_Y$. The function

g is called inverse of f and denoted by f^{-1}

Q.6 → Let $f: N \to Y$ be a function defined as

$f(x) = 4x+3$. Show that f is invertible. Find

Inverse, when $Y = \{ y = 4x+3 ;$ for some $x \in N\}$

Sol: Consider an arbitrary element y of Set Y

let $y = f(x)$

$y = 4x+3$

$\Rightarrow x = \frac{y-3}{4}$

let g be a function defined as

$g: Y \to N$

$g(y) = \frac{y-3}{4}$

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Now $gof = g(f(x)) = g(4x+3) = \frac{4x+\cancel{3}-\cancel{3}}{4} = x$

$\therefore gof = I_N$

$fog = f(g(y)) = f\left(\frac{y-3}{4}\right) = \cancel{4}\left(\frac{y-3}{\cancel{4}}\right) + 3 = y$

$fog = I_y$

Since $gof = I_N$ & $fog = I_y$

$\therefore f$ is invertible

and g is the inverse of f given by

$f^{-1} = \frac{y-3}{4}$ Ans

Qn 7 → Let $f: N \to R$ be a function defined as $f(x) = 4x^2 + 12x + 15$. Show that $f: N \to S$ where S is the range of f, is invertible. Find the inverse of f.

Sol: Let y be an arbitrary element of set S

let $y = f(x)$

$y = 4x^2 + 12x + 15$

$\Rightarrow 4x^2 + 12x + (15-y) = 0$

Quadratic formula

$x = \frac{-12 \pm \sqrt{144 - 16(15-y)}}{8}$

$x = \frac{-12 \pm \sqrt{144 - 240 + 16y}}{8}$

$x = \frac{-12 \pm \sqrt{16y - 96}}{8}$

$$x = \frac{-12 \pm 4\sqrt{y-6}}{8}$$

$$x = \frac{-3 \pm \sqrt{y-6}}{2}$$

$$\boxed{x = \frac{-3+\sqrt{y-6}}{2}} \quad \& \quad x = \frac{-3-\sqrt{y-6}}{2}$$

ⓧ rejected
$\because x \in N$

let g be a function defined as

$g : S \to N$

$$g(y) = \frac{-3+\sqrt{y-6}}{2}$$

$$gof = g(f(x)) = g(4x^2+12x+15)$$

$$= \frac{-3+\sqrt{4x^2+12x+15-6}}{2}$$

$$= \frac{-3+\sqrt{4x^2+12x+9}}{2}$$

$$= \frac{-3+\sqrt{(2x+3)^2}}{2}$$

$$= \frac{\cancel{-3} + 2x \cancel{+3}}{2}$$

$$gof = \frac{2x}{2} = x = I_N$$

Now $fog = f(g(y)) = f\left(\frac{-3+\sqrt{y-6}}{2}\right)$

$$= 4\left(\frac{-3+\sqrt{y-6}}{2}\right)^2 + 12\left(\frac{-3+\sqrt{y-6}}{2}\right) + 15$$

$$= \cancel{4}\left(\frac{9+y-6-6\sqrt{y-6}}{\cancel{4}}\right) + \left(-18 + 6\sqrt{y-6}\right) + 15$$

$$= y = I_S \quad \therefore f \text{ is invertible}$$