ANSWER KEY of EXAM No: 11

Class = 12^{th}

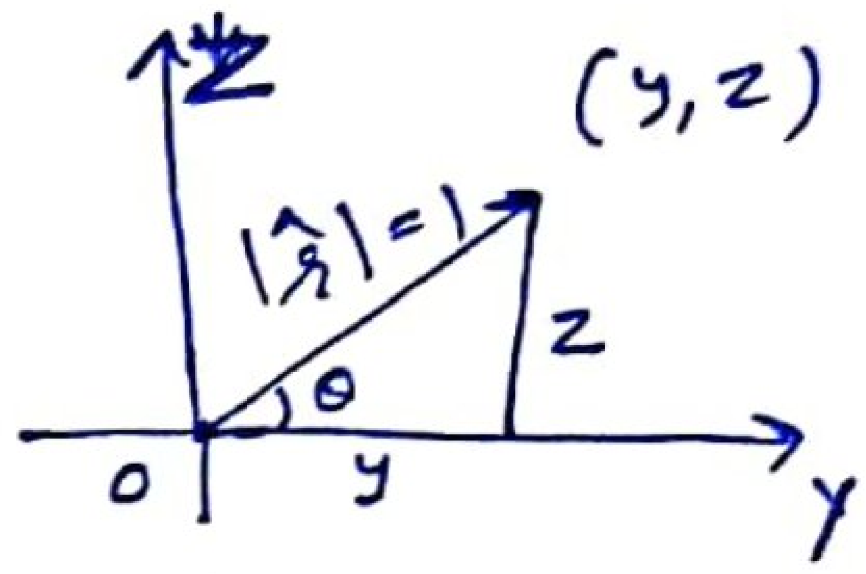
VECTORS & PROBABILITY

PART - A

SECTION : I

Ans 1 Any unit vector in YZ plane is given by

$$\hat{r} = \cos\theta\,\hat{j} + \sin\theta\,\hat{k}$$



$y = \cos\theta$
$z = \sin\theta$

given $\theta = 30°$

$$\therefore \hat{r} = \cos(30°)\hat{j} + \sin(30)\hat{k}$$

$$\boxed{\hat{r} = \frac{\sqrt{3}}{2}\hat{j} + \frac{1}{2}\hat{k}}$$ Ans

Ans 2 → given $x(\hat{i}+\hat{j}+\hat{k})$ is a unit vector

$$\Rightarrow |x\hat{i} + x\hat{j} + x\hat{k}| = 1$$

$$\Rightarrow \sqrt{x^2+x^2+x^2} = 1$$

$$\Rightarrow \sqrt{3x^2} = 1$$

Squaring $3x^2 = 1$

$$\Rightarrow \boxed{x = \pm\frac{1}{\sqrt{3}}}$$ Ans

Ans 3 → For $\vec{a}$ & $\vec{b}$ to be collinear

(•) It is not necessary or always true that $\vec{a}$ & $\vec{b}$ have the same direction & different magnitudes

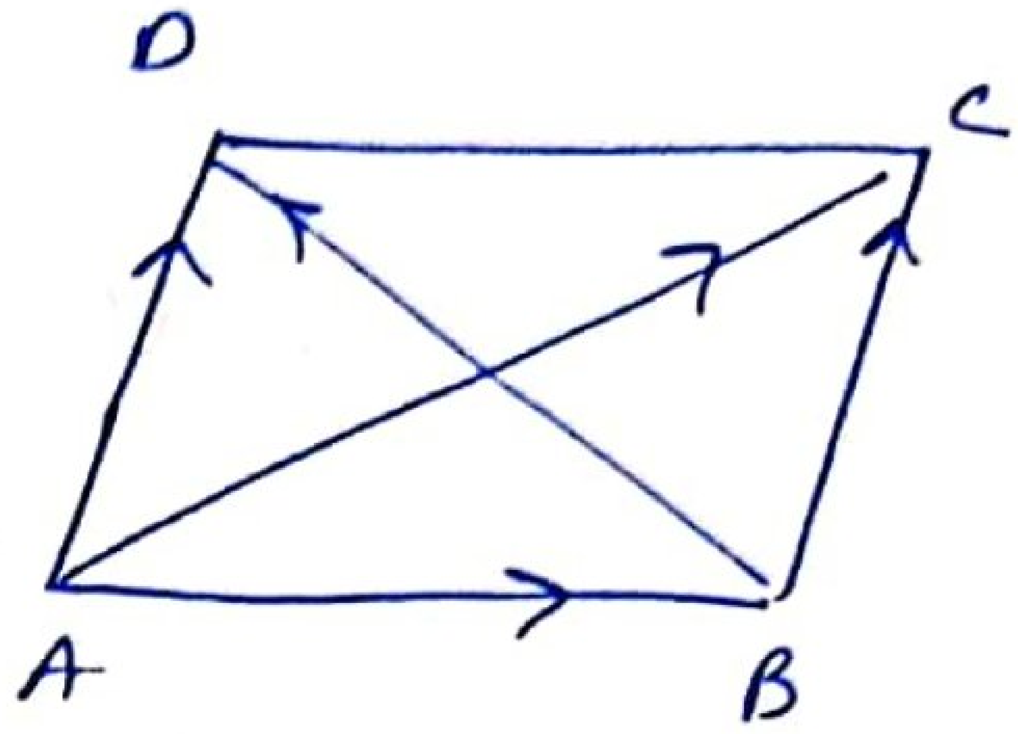
(•) also it is necessary that $\vec{a} = \vec{b}$ or $\vec{a} = -\vec{b}$

(•) For $\vec{a}$ & $\vec{b}$ to be collinear $\vec{a} = \lambda\vec{b}$ or their

Corresponding components must be in equal proportion

$\therefore$ $\boxed{(b) \ \& \ (d) \text{ are Incorrect}}$ Ans

Qn. 4 → In ΔABD (By triangle law)



$\vec{AB} + \vec{BD} = \vec{AD}$

$\Rightarrow \quad \vec{BD} = \vec{AD} - \vec{AB}$

In ΔABC

$\vec{AC} = \vec{AB} + \vec{BC}$

$\vec{AC} = \vec{AB} + \vec{AD}$ --- $\{\because \vec{BC} = \vec{AD}\}$

$\therefore \ \vec{AC} - \vec{BD} = (\vec{AB} + \vec{AD}) - (\vec{AD} - \vec{AB})$

$\boxed{\vec{AC} - \vec{BD} = 2\vec{AB}}$ $\quad \therefore (c)$ Ans

Qn 5 → given $\quad \vec{OA} = \hat{i} + x\hat{j} + 3\hat{k}$, $\vec{OB} = 3\hat{i} + 4\hat{j} + 7\hat{k}$ and

$\vec{OC} = y\hat{i} - 2\hat{j} - 5\hat{k}$

$\vec{AB} = \vec{OB} - \vec{OA} = 2\hat{i} + (4-x)\hat{j} + 4\hat{k}$

$\vec{BC} = \vec{OC} - \vec{OB} = (y-3)\hat{i} - 6\hat{j} - 12\hat{k}$

Since points A, B, C are collinear

$\therefore$ $\vec{AB}$ & $\vec{BC}$ are also collinear

$\therefore$ their corresponding components are in equal ratio

$\Rightarrow \quad \frac{2}{y-3} = \frac{4-x}{-6} = \frac{4}{-12}$

$\Rightarrow \quad \frac{2}{y-3} = \frac{x-4}{6} = \frac{-1}{3}$

$\Rightarrow \frac{2}{y-3} = -\frac{1}{3}$ & $\frac{x-y}{6} = \frac{-1}{3}$

$\Rightarrow 6 = -y+3$ & $3x-12 = -6$

$\Rightarrow y = -3$ & $3x = 6$

$x = 2$

$\therefore \boxed{(x,y) = (2,-3)}$ $\therefore$ (a) Ans

Ans 6 →

Given $\beta = \pi/3$ & $\gamma = \pi/2$ and $\alpha = \theta$

$l = \cos\alpha = \cos\theta$

$m = \cos\beta = \cos\pi/3 = 1/2$

$n = \cos\gamma = \cos\pi/2 = 0$

we have $l^2+m^2+n^2 = 1$

$\Rightarrow \cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

$\Rightarrow \cos^2\theta + \frac{1}{4} + 0 = 1$

$\Rightarrow \cos^2\theta = 3/4$

$\Rightarrow \cos\theta = \pm\frac{\sqrt{3}}{2}$

But θ is obtuse

$\therefore \cos\theta = -\frac{\sqrt{3}}{2}$

$\Rightarrow \theta = \pi - \frac{\pi}{6} \Rightarrow \boxed{\theta = \frac{5\pi}{6}}$ Ans

Ans 7 →

we have $l^2+m^2+n^2 = 1$

$\Rightarrow \cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

$\Rightarrow 1-\sin^2\alpha + 1-\sin^2\beta + 1-\sin^2\gamma = 1$

$\Rightarrow \boxed{\sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2}$ Ans

Ans 8 → given $\vec{p} = 5\hat{i} + \lambda\hat{j} - 3\hat{k}$ & $\vec{q} = \hat{i} + 3\hat{j} - 5\hat{k}$

$\vec{p} + \vec{q} = 6\hat{i} + (\lambda + 3)\hat{j} - 8\hat{k}$

$\vec{p} - \vec{q} = 4\hat{i} + (\lambda - 3)\hat{j} + 2\hat{k}$

given $\vec{p} + \vec{q}$ & $\vec{p} - \vec{q}$ are $\perp^r$

$\Rightarrow (\vec{p} + \vec{q}) \cdot (\vec{p} - \vec{q}) = 0$

$\Rightarrow \left(6\hat{i} + (\lambda + 3) - 8\hat{k}\right) \cdot \left(4\hat{i} + (\lambda - 3) + 2\hat{k}\right) = 0$

$\Rightarrow 24 + \lambda^2 - 9 - 16 = 0$

$\Rightarrow \lambda^2 = 1$

$\Rightarrow \boxed{\lambda = \pm 1}$ $\therefore$ (d) Ans

Ans 9 → given $\vec{a} = 2\hat{i} + 2\hat{j} + 3\hat{k}$, $\vec{b} = -\hat{i} + 2\hat{j} + \hat{k}$

$\vec{c} = 3\hat{i} + \hat{j}$

given $(\vec{a} + \lambda\vec{b}) \perp \vec{c}$

$\Rightarrow (\vec{a} + \lambda\vec{b}) \cdot \vec{c} = 0$

$\Rightarrow \left(2\hat{i} + 2\hat{j} + 3\hat{k} - \lambda\hat{i} + 2\lambda\hat{j} + \lambda\hat{k}\right) \cdot (3\hat{i} + \hat{j}) = 0$

$\Rightarrow \left(\hat{i}(2 - \lambda) + \hat{j}(2 + 2\lambda) + \hat{k}(3 + \lambda)\right) \cdot (3\hat{i} + \hat{j}) = 0$

$\Rightarrow 6 - 3\lambda + 2 + 2\lambda = 0$

$\Rightarrow \boxed{\lambda = 8}$ $\therefore$ (a) Ans

Ans 10 → given $|\vec{a}| = 13$, $|\vec{b}| = 5$ & $\vec{a} \cdot \vec{b} = 60$

we have $\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}| \cos\theta$

$\Rightarrow \quad 60 = (13)(5)\cos\theta$

$\Rightarrow \quad \cos\theta = \frac{12}{13}$

Now $\quad \sin\theta = \sqrt{1-\cos^2\theta} = \sqrt{1-\frac{144}{169}} = \frac{5}{13}$

we have $\quad |\vec{a}\times\vec{b}| = |\vec{a}||\vec{b}|\sin\theta$

$= (\cancel{13})(5)\left(\frac{5}{\cancel{13}}\right)$

$\Rightarrow \boxed{|\vec{a}\times\vec{b}| = 25} \quad \therefore (c)$ Ans

Ques 11 → given $\quad P(A) = 1/2 \quad \& \quad P(B) = 7/12$

$P(\text{not } A \text{ or not } B) = 1/4$

$\Rightarrow \quad P(A' \cup B') = 1/4$

$\Rightarrow \quad P(A\cap B)' = 1/4 \quad$ --- { De-morgan's law }

$\Rightarrow \quad 1 - P(A\cap B) = 1/4$

$\Rightarrow \quad P(A\cap B) = 3/4$

Now $\quad P(A)\cdot P(B) = \frac{1}{2}\times\frac{7}{12} = \frac{7}{24}$

Clearly $\quad P(A\cap B) \neq P(A)\cdot P(B)$

$\therefore$ $\boxed{\text{A \& B are dependent events}}$ $\therefore (b)$ Ans

Ques 12 → given $\quad P(A) = 0.4 \quad , \quad P(B) = p \quad \& \quad P(A\cup B) = 0.6$

we have $\quad P(A\cup B) = P(A) + P(B) - P(A\cap B)$

$\Rightarrow \quad 0.6 = P(A) + P(B) - P(A)\cdot P(B)$

-- { since A & B are independent events

$\Rightarrow 0.6 = 0.4 + p - (0.4)(p)$

$\Rightarrow 0.6 = 0.4 + 0.6p$

$\Rightarrow 0.2 = 0.6p$

$\Rightarrow p = \frac{0.2}{0.6}$

$\Rightarrow \boxed{p = 1/3}$ ∴ (b) Ans

Ans 13 → Given P.D

X :	1	2	3	4
$P(x)$:	c	$2c$	$4c$	$4c$

we have $\Sigma p_i = 1$

$\Rightarrow c + 2c + 4c + 4c = 1$

$\Rightarrow c = \frac{1}{11}$

Now $P(x \le 2) = P(x=1) + P(x=2)$

$= c + 2c$

$= 3c$

$\boxed{P(x \le 2) = \frac{3}{11}}$ ∴ (d) none of these Ans

Ans 14 → total Socks = 9

5 brown & 4 white

A → Event that two Socks drawn are of same colour

total No of ways = $^9C_2$

fav no of ways = $^5C_2 + {}^4C_2$

∴ Req prob $P(A) = \frac{^5C_2 + {}^4C_2}{^9C_2}$

$$= \frac{\frac{5\times 4}{\cancel{2}} + \frac{4\times 3}{\cancel{2}}}{\frac{9\times 8}{\cancel{2}}}$$

$$= \frac{20+12}{72}$$

$$= \frac{32}{72}$$

$$\boxed{P(A) = \frac{4}{9} = \frac{48}{108}} \quad \therefore (d) \quad \text{Ans}$$

Qn 15 → let $\vec{a} = 0\hat{i} - \frac{3}{4}\hat{j} + 0\hat{k}$

$$|\vec{a}| = \sqrt{\frac{9}{16}} = \frac{3}{4}$$

$$\hat{a} = \frac{-\frac{3}{4}\hat{j}}{\frac{3}{4}}$$

$$\hat{a} = -\hat{j}$$

$\boxed{\text{Unit vector in the direction opposite to } \hat{a} \text{ is } \hat{j}}$ Ans

Qn. 16 → let $\vec{AB} = 2\hat{i}$ & $\vec{AC} = -3\hat{j}$

$$\text{Area of } \Delta ABC = \frac{1}{2}|\vec{AB} \times \vec{AC}|$$

$$= \frac{1}{2}|2\hat{i} \times -3\hat{j}|$$

$$= \frac{1}{2}|-6\hat{k}| \quad \cdots \{\because \hat{i}\times\hat{j} = \hat{k}\}$$

$$= \frac{1}{2}\sqrt{0+0+36}$$

$$= \frac{1}{2}(6)$$

$\boxed{\text{Area of } \Delta ABC = 3 \text{ square units}}$ Ans

SECTION- II

ON 17 → let $E_1 \rightarrow$ Vinay processed the form

$E_2 \rightarrow$ Sonia " " "

$E_3 \rightarrow$ Iqbal " " "

$A \rightarrow$ An error is committed

Given

$P(E_1) = \frac{50}{100}$ | $P(A/E_1) = 0.06 = \frac{6}{100}$

$P(E_2) = \frac{20}{100}$ | $P(A/E_2) = 0.04 = \frac{4}{100}$

$P(E_3) = \frac{30}{100}$ | $P(A/E_3) = 0.03 = \frac{3}{100}$

(i) Required conditional prob $= P(A/E_2)$

$\boxed{\text{Req prob} = \frac{4}{100} = 0.04 \therefore (b)}$ Ans

(ii) Required probability $= P(E_2 \cap A)$

we know

$$P(A/E_2) = \frac{P(A \cap E_2)}{P(E_2)}$$

$$\Rightarrow P(A \cap E_2) = P(E_2) \times P(A/E_2)$$

$$= \frac{20}{100} \times \frac{4}{100} = \frac{80}{10000}$$

$\boxed{\text{Req prob} = 0.08 \therefore (c)}$ Ans

(iii) Required prob $= P(A)$

By total law

$$P(A) = P(E_1)P(A/E_1) + P(E_2)P(A/E_2) + P(E_3)P(A/E_3)$$

$$= \frac{50}{100} \times \frac{6}{100} + \frac{20}{100} \times \frac{4}{100} + \frac{30}{100} \times \frac{3}{100}$$

$$= \frac{470}{10000}$$

$\therefore$ $\boxed{\text{Req prob} = 0.047}$ $\therefore$ (b) Ans

(ix) Required prob $= 1 - P(E_1/A)$

By Bayes theorem

$$P(E_1/A) = \frac{P(E_1)\,P(A/E_1)}{P(E_1)P(A/E_1) + P(E_2)\,P(A/E_2) + P(E_3)\,P(A/E_3)}$$

$$= \frac{\frac{50}{100} \times \frac{6}{100}}{\frac{50}{100} \times \frac{6}{100} + \frac{20}{100} \times \frac{4}{100} + \frac{30}{100} \times \frac{3}{100}}$$

$$= \frac{300}{470}$$

$$= \frac{30}{47}$$

$\therefore$ Req prob $= 1 - \frac{30}{47}$

$\boxed{\text{Req prob} = \frac{17}{47}}$ $\therefore$ (d) Ans

(x) Req prob $\sum_{i=1}^{3} P(E_i/A) = P(E_1/A) + P(E_2/A) + P(E_3/A)$

~~P(...)~~ In Bayes theorem, it is always true that

(i) $P(E_1) + P(E_2) + P(E_3) = 1$

also (ii) $P(E_1/A) + P(E_2/A) + P(E_3/A) = 1$

$\therefore$ $\boxed{\sum_{i=1}^{3} P(E_i/A) = 1}$ $\therefore$ (d) Ans

Ans 18 → Let $E_1 \to$ very hard working student is selected

$E_2 \to$ Regular but no so hardworking student is selected

$E_3 \to$ careless and irregular student is selected

let $A \to$ Student selected do not get good marks in the final examination

then $P(E_1) = \frac{10}{60}$ | $P(A/E_1) = 0.002 = \frac{2}{1000}$

$P(E_2) = \frac{30}{60}$ | $P(A/E_2) = 0.02 = \frac{2}{100}$

$P(E_3) = \frac{20}{60}$ | $P(A/E_3) = 0.20 = \frac{2}{10}$

(i) Req prob = $P(E_3/A)$

By Bayes theorem

$$P(E_3/A) = \frac{P(E_3)\,P(A/E_3)}{P(E_1)\,P(A/E_1) + P(E_2)\,P(A/E_2) + P(E_3)\,P(A/E_3)}$$

$$= \frac{\frac{20}{\cancel{60}} \times \frac{2}{10}}{\frac{10}{\cancel{60}} \times \frac{2}{1000} + \frac{30}{\cancel{60}} \times \frac{2}{100} + \frac{20}{60} \times \frac{2}{10}}$$

$$= \frac{4}{\frac{2}{100} + \frac{6}{10} + 4}$$

$$= \frac{400}{2 + 60 + 400}$$

$$= \frac{400}{462}$$

$\boxed{\text{Req Prob} = \frac{200}{231}}$ (b) Ans

(ii) Req prob $P(E_1/A) + P(E_2/A)$

we know that

$P(E_1/A) + P(E_2/A) + P(E_3/A) = 1$

$\rightarrow P(E_1/A) + P(E_2/A) + \frac{200}{231} = 1$ --- { from (i) part

$\rightarrow P(E_1/A) + P(E_2/A) = \frac{31}{231}$

$\therefore \boxed{\text{Req prob} = \frac{31}{231}}$ $\therefore$ (a) Ans

(iii) Req prob $= P(A)$

By total law of prob

$P(A) = P(E_1)P(A/E_1) + P(E_2)P(A/E_2) + P(E_3)P(A/E_3)$

$= \frac{10}{60} \times \frac{2}{1000} + \frac{30}{60} \times \frac{2}{100} + \frac{20}{60} \times \frac{2}{10}$

$= \frac{2\cancel{0}}{6000\cancel{0}} + \frac{6\cancel{0}}{60\cancel{0}\cancel{0}} + \frac{4\cancel{0}}{60\cancel{0}}$

$= \frac{2 + 60 + 400}{6000}$

$= \frac{462}{6000}$

$\boxed{\text{Req prob} = \frac{231}{3000}}$ (b) Ans

(iv) Req prob $= P(E_1/A)$

By Bays theorem $P(E_1/A) = \dfrac{\frac{10}{60} \times \frac{2}{1000}}{\frac{10}{60} \times \frac{2}{1000} + \frac{30}{60} \times \frac{2}{100} + \frac{20}{100} \times \frac{2}{10}}$

$$= \frac{\frac{2\cancel{00}}{600\cancel{00}}}{\frac{462}{600\cancel{0}}}$$

$$= \frac{2}{462}$$

$\therefore$ $\boxed{\text{Req prob} = \frac{1}{231}}$ $\therefore$ (a) Ans

(V) Req prob $= 1 - P(E_1/A)$

$= 1 - \frac{1}{231}$ --- { from (IV) part}

$\boxed{\text{Req prob} = \frac{230}{231}}$ $\therefore$ (b) Ans

PART-B

SECTION: III

QN. 19 $S = \{MFS, MSF, FSM, FMS, SMF, SFM\}$

Sample space

$A \rightarrow$ Son on one end

$B \rightarrow$ Father in the Middle

$A = \{SMF, SFM, MFS, FMS\}$

$B = \{MFS, SFM\}$

$A \cap B = \{MFS, SFM\}$

$P(A \cap B) = \frac{2}{6}$

$P(B) = \frac{2}{6}$

By conditional prob : $P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{2/6}{2/6} = 1$ Ans

Ans 20 → total balls = 25

10 white & 15 black

Req prob = P(getting one is white and other is black) = $\underbrace{\text{Two cases}}$

$= P(\text{getting } 1^{st} \text{ white \& } 2^{nd} \text{ black}) + P(\text{getting } 1^{st} \text{ black \& } 2^{nd} \text{ white})$

$= \left(\frac{10}{25} \times \frac{15}{24}\right) + \left(\frac{15}{25} \times \frac{10}{24}\right)$

$= \frac{150}{600} + \frac{150}{600}$

$= \frac{300}{600} = 1/2$

$\therefore \boxed{\text{Req prob} = 1/2}$ Ans

Ans 21 → let X → denotes the no of milk chocholates

X → can take values 0, 1, 2

$P(X=0) = P(\text{No Milk Chocholate}) = \frac{4}{6} \times \frac{3}{5} = \frac{12}{30} = \frac{6}{15}$

$P(X=1) = P(\text{1 Milk chocholate}) = \left(\frac{2}{6} \times \frac{4}{5}\right) \times {}^2C_1$

$= \frac{8 \times 2}{30} = \frac{16}{30}$

$P(X=2) = P(\text{2 Milk Chocholate}) = \frac{2}{6} \times \frac{1}{5} = \frac{2}{30}$

P.D

X	0	1	2
$P(X)$	$\frac{12}{30}$	$\frac{16}{30}$	$\frac{2}{30}$

Clearly $P(X=1)$ is the highest

$\therefore$ getting 1 Milk Chocholate is the most likely outcome Ans

Ans 22 → Given $P(A) = 3/8$; $P(B) = 1/2$; $P(A \cap B) = 1/4$

Required prob $= P(B'/A') = \dfrac{P(B' \cap A')}{P(A')}$

(By conditional prob)

$$= \frac{P(A \cup B)'}{1 - P(A)}$$

$$= \frac{1 - P(A \cup B)}{1 - P(A)}$$

$$= \frac{1 - [P(A) + P(B) - P(A \cap B)]}{1 - P(A)}$$

$$= \frac{1 - \left[\frac{3}{8} + \frac{1}{2} - \frac{1}{4}\right]}{1 - \frac{3}{8}}$$

$$= \frac{1 - \left(\frac{3+4-2}{8}\right)}{\frac{5}{8}}$$

$$= \frac{3/8}{5/8}$$

$\therefore$ $\boxed{\text{Req prob} = 3/5}$ Ans

Ans. 23 → Given $A(2,3,5)$ $B(3,5,8)$ $C(2,7,8)$

$\vec{AB} = \hat{i} + 2\hat{j} + 3\hat{k}$

$\vec{AC} = 0\hat{i} + 4\hat{j} + 3\hat{k}$

Area of $\Delta ABC = \frac{1}{2} |\vec{AB} \times \vec{AC}|$

Now $\vec{AB} \times \vec{AC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ 0 & 4 & 3 \end{vmatrix} = -6\hat{i} - 3\hat{j} + 4\hat{k}$

$|\vec{AB} \times \vec{AC}| = \sqrt{36+9+16} = \sqrt{61}$

$\therefore$ $\boxed{\text{Area of } \Delta ABC = \frac{1}{2}\sqrt{61} \text{ square units}}$ Ans

Qn. 24 → given $\vec{a}\cdot\vec{b} = \vec{a}\cdot\vec{c}$ & $\vec{a}\times\vec{b} = \vec{a}\times\vec{c}$

$\Rightarrow \vec{a}\cdot\vec{b} - \vec{a}\cdot\vec{c} = 0$ and $\vec{a}\times\vec{b} - \vec{a}\times\vec{c} = \vec{0}$

$\Rightarrow \vec{a}\cdot(\vec{b}-\vec{c}) = 0$ and $\vec{a}\times(\vec{b}-\vec{c}) = \vec{0}$

$\Rightarrow \vec{a} = \vec{0}$ or $\vec{b}-\vec{c} = \vec{0}$ and $\vec{a} = \vec{0}$ or $\vec{b}-\vec{c} = \vec{0}$

or $\vec{a} \perp (\vec{b}-\vec{c})$ or $\vec{a} \parallel (\vec{b}-\vec{c})$

But given $\vec{a} \neq \vec{0}$ and $\vec{a}$ cannot $\perp$ & $\parallel$ simultaneously to $\vec{b}-\vec{c}$

$\therefore$ $\vec{b}-\vec{c} = \vec{0}$

$\Rightarrow$ $\boxed{\vec{b} = \vec{c}}$ Ans

Ques 25 → Given $\vec{a}\times\vec{b} = \vec{c}\times\vec{d}$ & $\vec{a}\times\vec{c} = \vec{b}\times\vec{d}$

To prove $\vec{a}-\vec{d}$ is parallel to $\vec{b}-\vec{c}$

we have to prove $(\vec{a}-\vec{d})\times(\vec{b}-\vec{c}) = \vec{0}$

LHS $(\vec{a}-\vec{d})\times(\vec{b}-\vec{c})$

$= \vec{a}\times\vec{b} - \vec{a}\times\vec{c} - \vec{d}\times\vec{b} + \vec{d}\times\vec{c}$

$= \vec{a}\times\vec{b} - \vec{a}\times\vec{c} + \vec{b}\times\vec{d} - \vec{c}\times\vec{d}$ ... $\{\because \vec{a}\times\vec{b} = -\vec{b}\times\vec{a}\}$

$= \vec{a} \times \vec{b} - \vec{a} \times \vec{c} + \vec{a} \times \vec{c} - \vec{a} \times \vec{b}$

$= \vec{0} + \vec{0}$ ... {from given}

$= \vec{0}$

$\boxed{\text{Hence } \vec{a} - \vec{d} \text{ is parallel to } \vec{b} - \vec{c}}$ Ans

Qn 26 →

Lhs $|\vec{a} \times \vec{b}|^2$

$= (|\vec{a}||\vec{b}| \sin\theta)^2$

$= |\vec{a}|^2 |\vec{b}|^2 \sin^2\theta$

$= |\vec{a}|^2 |\vec{b}|^2 (1 - \cos^2\theta)$

$= |\vec{a}|^2 |\vec{b}|^2 - |\vec{a}|^2 |\vec{b}|^2 \cos^2\theta$

$= (\vec{a} \cdot \vec{a})(\vec{b} \cdot \vec{b}) - (\vec{a} \cdot \vec{b})^2$

$= \begin{vmatrix} \vec{a} \cdot \vec{a} & \vec{a} \cdot \vec{b} \\ \vec{a} \cdot \vec{b} & \vec{b} \cdot \vec{b} \end{vmatrix} = \text{Rhs}$ Ans

Qn. 27 → Given $\vec{a} = 2x^2\hat{i} + 4x\hat{j} + \hat{k}$ & $\vec{b} = 7\hat{i} - 2\hat{j} + x\hat{k}$

Given that angle b/w them is obtuse {$\because \cos\theta$ is -ve}

$\Rightarrow \vec{a} \cdot \vec{b} < 0$

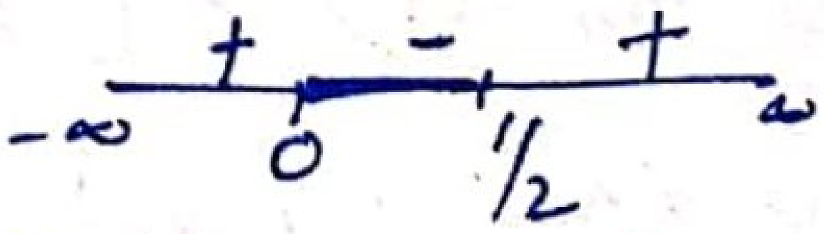
$\Rightarrow (2x^2\hat{i} + 4x\hat{j} + \hat{k}) \cdot (7\hat{i} - 2\hat{j} + x\hat{k}) < 0$

$\Rightarrow 14x^2 - 8x + x < 0$

$\Rightarrow 14x^2 - 7x < 0$

$\Rightarrow 7x(2x - 1) < 0$

$\Rightarrow x(2x - 1) < 0$



$\therefore x \in (0, \tfrac{1}{2})$ Ans

Scanned with CamScanner

Ans 28 → given $\vec{a}+\vec{b}+\vec{c}=\vec{0}$

$\Rightarrow \vec{a}+\vec{b}=-\vec{c}$

$\Rightarrow |\vec{a}+\vec{b}|=|-\vec{c}|$

$\Rightarrow |\vec{a}+\vec{b}|=|\vec{c}|$

Squaring $\Rightarrow |\vec{a}+\vec{b}|^2=|\vec{c}|^2$

$\Rightarrow |\vec{a}|^2+|\vec{b}|^2+2\vec{a}\cdot\vec{b}=|\vec{c}|^2$

$\Rightarrow |\vec{a}|^2+|\vec{b}|^2+2|\vec{a}||\vec{b}|\cos\theta=|\vec{c}|^2$

$\Rightarrow 9+25+2(3)(5)\cos\theta=49$

$\Rightarrow 30\cos\theta=49-34$

$\Rightarrow 30\cos\theta=15$

$\Rightarrow \cos\theta=1/2$

$\Rightarrow \boxed{\theta=\pi/3}$ Ans

SECTION – IV

Ans 29 → let the two unit vectors are $\hat{a}$ & $\hat{b}$ given their sum is also unit vector

$\Rightarrow |\hat{a}|=1$, $|\hat{b}|=1$ and $|\hat{a}+\hat{b}|=1$

$\Rightarrow |\hat{a}+\hat{b}|^2=1$

$\Rightarrow |\hat{a}|^2+|\hat{b}|^2+2\hat{a}\cdot\hat{b}=1$

$\Rightarrow 1+1+2\hat{a}\cdot\hat{b}=1$

$\Rightarrow \hat{a}\cdot\hat{b}=-1/2$

Now $|\hat{a}-\hat{b}|^2 = |\hat{a}|^2 + |\hat{b}|^2 - 2\hat{a}\cdot\hat{b}$

$= 1 + 1 - 2(-1/2)$

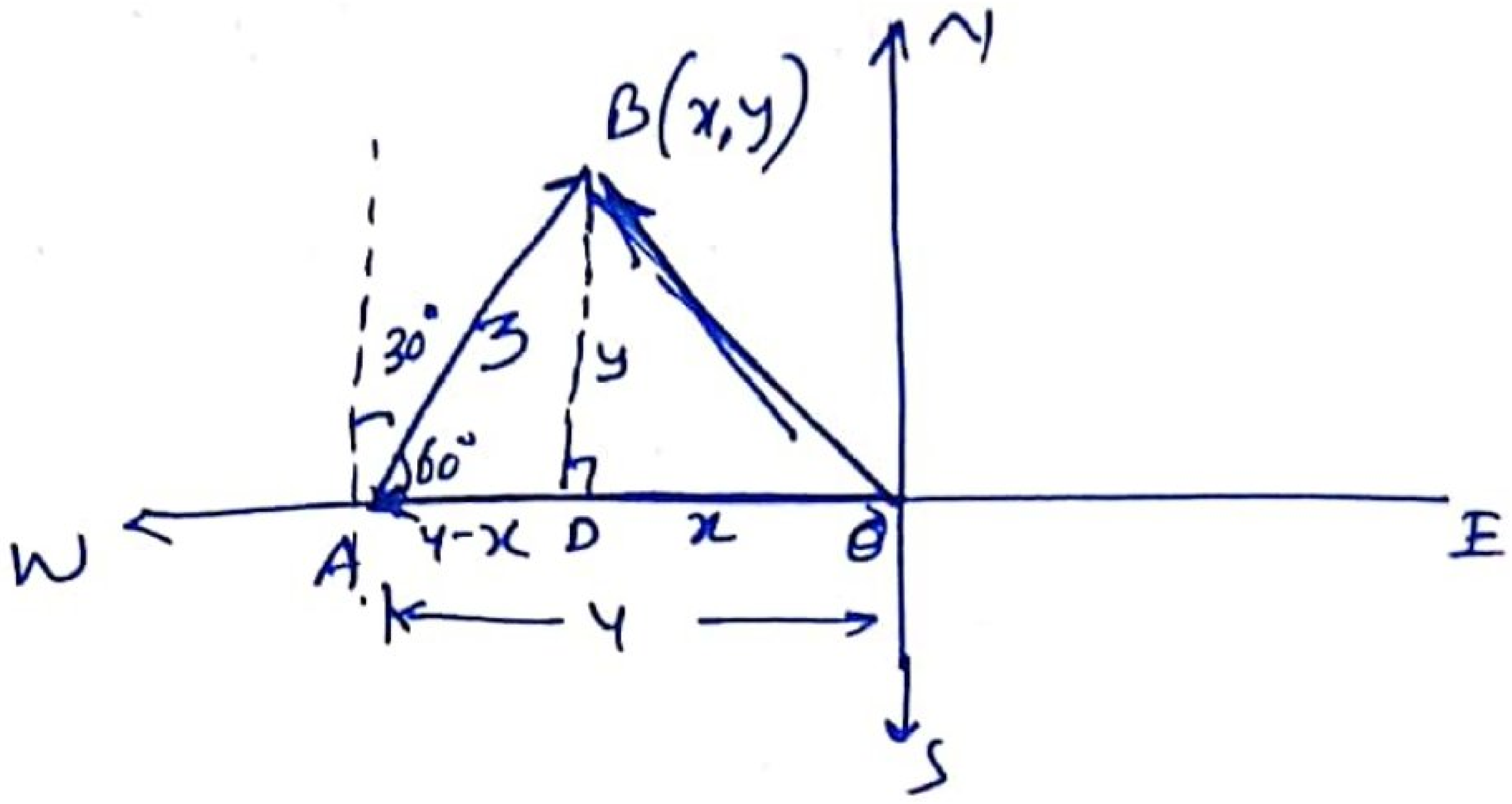
$= 1+1+1$

$|\hat{a}-\hat{b}|^2 = 3$

$\Rightarrow \boxed{|\hat{a}-\hat{b}| = \sqrt{3}}$ Ans

Ans 30 A

Required $= \vec{OB}$



In ΔABD

$\sin(60°) = \frac{y}{3}$

$\Rightarrow \frac{\sqrt{3}}{2} = \frac{y}{3}$

$y = \frac{3\sqrt{3}}{2}$

$\cos(60°) = \frac{4-x}{3}$

$\Rightarrow \frac{1}{2} = \frac{4-x}{3}$

$\Rightarrow 3 = 8-2x$

$\Rightarrow x = 5/2$

$\therefore$ Coordinate of B is $\left(-\frac{5}{2}, \frac{3\sqrt{3}}{2}\right)$

Now $\vec{OB} = \left(-\frac{5}{2}-0\right)\hat{i} + \left(\frac{3\sqrt{3}}{2}-0\right)\hat{j} = -\frac{5}{2}\hat{i} + \frac{3\sqrt{3}}{2}\hat{j}$

$|\vec{OB}| = \sqrt{\frac{25}{4}+\frac{27}{4}} = \sqrt{\frac{52}{4}} = \sqrt{13}$ km

Ans

Ans 31 → given $\vec{a} = \hat{i} + 4\hat{j} + 2\hat{k}$

$\vec{b} = 3\hat{i} - 2\hat{j} + 7\hat{k}$

$\vec{c} = 2\hat{i} - \hat{j} + 4\hat{k}$

given $\vec{d} \perp \vec{a}$ & $\vec{d} \perp \vec{b}$ & $\vec{c} \cdot \vec{d} = 15$

$\Rightarrow \vec{d} = \lambda (\vec{a} \times \vec{b})$

$\Rightarrow \vec{d} = \lambda \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 4 & 2 \\ 3 & -2 & 7 \end{vmatrix} = \lambda (32\hat{i} - \hat{j} - 14\hat{k})$

given $\vec{c} \cdot \vec{d} = 15$

$\Rightarrow (2\hat{i} - \hat{j} + 4\hat{k}) \cdot \lambda (32\hat{i} - \hat{j} - 14\hat{k}) = 15$

$\Rightarrow \lambda (64 + 1 - 56) = 15$

$\Rightarrow \lambda (9) = 15$

$\Rightarrow \boxed{\lambda = \frac{5}{3}}$

$\therefore \vec{d} = \frac{5}{3}(32\hat{i} - \hat{j} - 14\hat{k})$

$\boxed{\vec{d} = \frac{160}{3}\hat{i} - \frac{5}{3}\hat{j} - \frac{70}{3}\hat{k}}$ Ans

Ans 32 → let $\vec{\beta} = 6\hat{i} - 3\hat{j} - 6\hat{k}$

& $\vec{\alpha} = \hat{i} + \hat{j} + \hat{k}$

let $\vec{\beta} = \vec{\beta_1} + \vec{\beta_2}$

Such that $\vec{\beta_1} \parallel \vec{\alpha}$ & $\vec{\beta_2} \perp \vec{\alpha}$

$\Rightarrow \vec{\beta_1} = \lambda \vec{\alpha}$

$\Rightarrow \vec{\beta_1} = \lambda\hat{i} + \lambda\hat{j} + \lambda\hat{k}$

Now $\vec{\beta} = \vec{\beta_1} + \vec{\beta_2}$

$\Rightarrow \vec{\beta_2} = \vec{\beta} - \vec{\beta_1}$

$\Rightarrow \vec{\beta_2} = 6\hat{i} - 3\hat{j} - 6\hat{k} - \lambda\hat{i} - \lambda\hat{j} - \lambda\hat{k}$

$\Rightarrow \vec{\beta_2} = \hat{i}(6-\lambda) + \hat{j}(-3-\lambda) + \hat{k}(-6-\lambda)$

Given $\vec{\beta_2} \perp \vec{\alpha}$

$\Rightarrow \vec{\beta_2} \cdot \vec{\alpha} = 0$

$\Rightarrow \left(\hat{i}(6-\lambda) + \hat{j}(-3-\lambda) + \hat{k}(-6-\lambda)\right) \cdot (\hat{i} + \hat{j} + \hat{k}) = 0$

$\Rightarrow 6 - \lambda - 3 - \lambda - 6 - \lambda = 0$

$\Rightarrow -3 - 3\lambda = 0$

$\Rightarrow \lambda = -1$

$\therefore \vec{\beta_1} = -\hat{i} - \hat{j} - \hat{k}$ & $\vec{\beta_2} = 7\hat{i} - 2\hat{j} - 5\hat{k}$

Now $\vec{\beta_1} + \vec{\beta_2} = -\hat{i} - \hat{j} - \hat{k} + 7\hat{i} - 2\hat{j} - 5\hat{k}$

$= 6\hat{i} - 3\hat{j} - 6\hat{k} = \vec{\beta}$ (verified)

Ans

Ans 33 $\rightarrow$ Given $\vec{a} \times \vec{b} = \vec{c}$ and $\vec{b} \times \vec{c} = \vec{a}$

$\Rightarrow \vec{c} \perp \vec{a}$ & $\vec{c} \perp \vec{b}$ and $\vec{a} \perp \vec{b}$ & $\vec{a} \perp \vec{c}$

$\Rightarrow \vec{a} \perp \vec{b} \perp \vec{c}$

$\therefore \vec{a}$ & $\vec{b}$ & $\vec{c}$ are mutually perpendicular vectors

We have $\vec{a} \times \vec{b} = \vec{c}$ and $\vec{b} \times \vec{c} = \vec{a}$

$\Rightarrow |\vec{a} \times \vec{b}| = |\vec{c}|$ and $|\vec{b} \times \vec{c}| = |\vec{a}|$

$\Rightarrow |\vec{a}||\vec{b}| \sin\left(\frac{\pi}{2}\right) = |\vec{c}|$ and $|\vec{b}||\vec{c}| \sin\left(\frac{\pi}{2}\right) = |\vec{a}|$

$\Rightarrow |\vec{a}||\vec{b}| = |\vec{c}| =$ and $|\vec{b}||\vec{c}| = |\vec{a}|$

$\Rightarrow |\vec{b}|\, \cancel{|\vec{a}|}\, |\vec{b}| = \cancel{|\vec{a}|}$

$\Rightarrow |\vec{b}|^2 = 1$

$\Rightarrow |\vec{b}| = 1$

$\therefore |\vec{c}| = |\vec{a}|$

Ans.

Ans 34 →

let $E_1 \rightarrow$ the card lost is a diamond

$E_2 \rightarrow$ the card lost is not a diamond

$A \rightarrow$ the two card drawn are both diamonds

$P(E_1) = \frac{13}{52} = \frac{1}{4}$

$P(E_2) = \frac{39}{52} = \frac{3}{4}$

$P(A/E_1) = \frac{{}^{12}C_2}{{}^{51}C_2}$

$P(A/E_2) = \frac{{}^{13}C_2}{{}^{51}C_2}$

By Bayes theorem

Req prob $= P(E_1/A) = \dfrac{P(E_1)P(A/E_1)}{P(E_1)P(A/E_1) + P(E_2)P(A/E_2)}$

Scanned with CamScanner

$$= \frac{\cancel{\frac{1}{4}} \times \frac{{}^{12}C_2}{\cancel{{}^{51}C_2}}}{\cancel{\frac{1}{4}} \times \frac{{}^{12}C_2}{\cancel{{}^{51}C_2}} + \cancel{\frac{3}{4}} \times \frac{{}^{13}C_2}{\cancel{{}^{51}C_2}}}$$

$$= \frac{\frac{\cancel{12} \times 11}{\cancel{2}}}{\frac{\cancel{12} \times 11}{\cancel{2}} + 3 \times \frac{13 \times \cancel{12}}{\cancel{2}}}$$

$$= \frac{11}{11 + 39}$$

$$\boxed{\text{Req prob} = \frac{11}{50}}$$ Ans

Ans 35 → Sample space

$S = \{HH, HT, T1, T2, T3, T4, T5, T6\}$

Sample space outcomes are "NOT EQUALLY LIKELY"

$A \rightarrow$ die shows number more than 4

$B \rightarrow$ there is atleast one tail

$A = \{T5, T6\}$

$B = \{HT, T1, T2, T3, T4, T5, T6\}$

$A \cap B = \{T5, T6\}$

$P(A \cap B) = \left(\frac{1}{2} \times \frac{1}{6}\right) + \left(\frac{1}{2} \times \frac{1}{6}\right) = \frac{2}{12} = \frac{1}{6}$

$P(B) = \left(\frac{1}{2} \times \frac{1}{2}\right) + \left(\frac{1}{2} \times \frac{1}{6}\right) \times 6 = \frac{1}{4} + \frac{1}{2} = 3/4$

Req prob (By conditional prob)

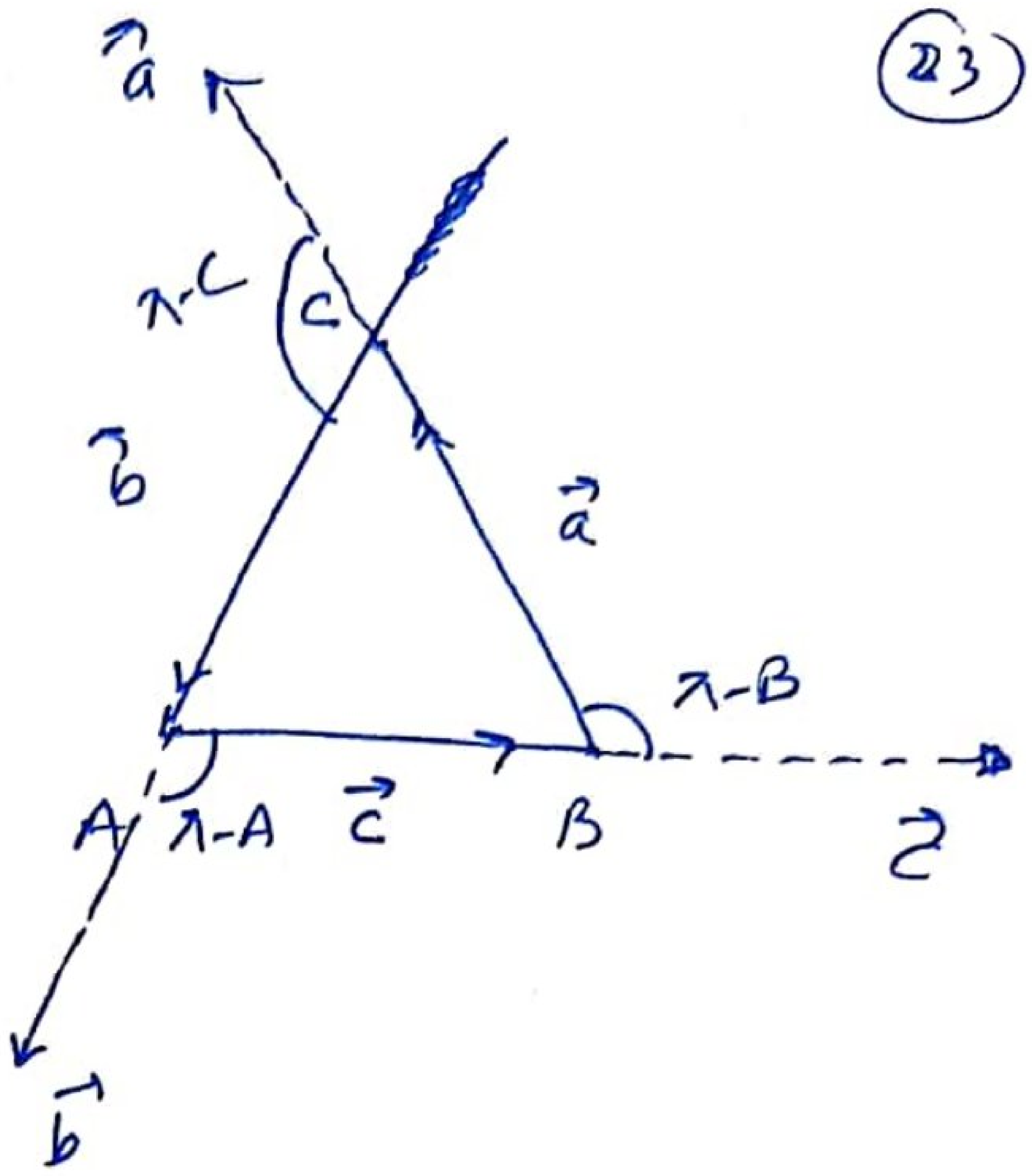
$P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{1/6}{3/4} = \frac{4}{18}$

$\therefore \boxed{\text{Req prob} = \frac{2}{9}}$ Ans

Scanned with CamScanner

SECTION - V

Ans 36 →



$|\vec{a}| = a$

$|\vec{b}| = b$

$|\vec{c}| = c$

In ΔABC (By triangle law)

$\vec{a} + \vec{b} + \vec{c} = \vec{0}$

$\Rightarrow \quad \vec{a} \times (\vec{a} + \vec{b} + \vec{c}) = \vec{a} \times \vec{0}$

$\Rightarrow \quad \vec{0} + \vec{a} \times \vec{b} + \vec{a} \times \vec{c} = \vec{0}$

$\Rightarrow \quad \vec{a} \times \vec{b} = \vec{c} \times \vec{a} \quad \text{---} (1)$

Again $\quad \vec{a} + \vec{b} + \vec{c} = \vec{0}$

$\Rightarrow \quad \vec{b} \times (\vec{a} + \vec{b} + \vec{c}) = \vec{b} \times \vec{0}$

$\Rightarrow \quad \vec{b} \times \vec{a} + \vec{0} + \vec{b} \times \vec{c} = \vec{0}$

$\Rightarrow \quad \vec{b} \times \vec{c} = \vec{a} \times \vec{b} \quad \text{--} (2)$

from (1) & (2)

$\vec{a} \times \vec{b} = \vec{b} \times \vec{c} = \vec{c} \times \vec{a}$

$\Rightarrow \quad |\vec{a} \times \vec{b}| = |\vec{b} \times \vec{c}| = |\vec{c} \times \vec{a}|$

$\Rightarrow \quad |\vec{a}|\,|\vec{b}| \sin(\pi - C) = |\vec{b}|\,|\vec{c}| \sin(\pi - A) = |\vec{c}|\,|\vec{a}| \sin(\pi - B)$

$\Rightarrow \quad ab \sin C = bc \sin A = ca \sin B$

divide by abc

$\Rightarrow \quad \dfrac{\sin C}{c} = \dfrac{\sin A}{a} = \dfrac{\sin B}{b}$

$\Rightarrow \quad \dfrac{\sin A}{a} = \dfrac{\sin B}{b} = \dfrac{\sin C}{c} \quad$ Proved again we have $\vec{a}+\vec{b}+\vec{c}=\vec{0}$

$$\Rightarrow \quad \vec{b}+\vec{c}=-\vec{a}$$

$$\Rightarrow \quad |\vec{b}+\vec{c}| = |-\vec{a}|$$

$$\Rightarrow \quad |\vec{b}+\vec{c}|^2 = |\vec{a}|^2$$

$$\Rightarrow \quad |\vec{b}|^2+|\vec{c}|^2 + 2\,\vec{b}\cdot\vec{c} = |\vec{a}|^2$$

$$\Rightarrow \quad |\vec{b}|^2+|\vec{c}|^2 + 2|\vec{b}||\vec{c}|\cos(\pi - A) = |\vec{a}|^2$$

$$\Rightarrow \quad b^2+c^2+2bc(-\cos A) = a^2$$

$$\Rightarrow \quad b^2+c^2-a^2 = 2bc\cos A$$

$$\Rightarrow \quad \boxed{\cos A = \frac{b^2+c^2-a^2}{2bc}}$$ Ans

Ans 37 → given $\vec{a}\perp\vec{b}\perp\vec{c}$

$$\Rightarrow \vec{a}\cdot\vec{b}=0, \quad \vec{b}\cdot\vec{c}=0, \quad \vec{c}\cdot\vec{a}=0$$

given $|\vec{a}| = |\vec{b}| = |\vec{c}|$

let $|\vec{a}| = |\vec{b}| = |\vec{c}| = \lambda$

we have $|\vec{a}+\vec{b}+\vec{c}|^2 = |\vec{a}|^2+|\vec{b}|^2+|\vec{c}|^2 + 2\vec{a}\cdot\vec{b} + 2\vec{b}\cdot\vec{c} + 2\vec{c}\cdot\vec{a}$

$$\Rightarrow |\vec{a}+\vec{b}+\vec{c}|^2 = \lambda^2+\lambda^2+\lambda^2+0+0+0$$

$$\Rightarrow |\vec{a}+\vec{b}+\vec{c}|^2 = 3\lambda^2$$

$$\Rightarrow |\vec{a}+\vec{b}+\vec{c}| = \sqrt{3}\,\lambda$$

let α, β, γ are the angles made by $\vec{a}+\vec{b}+\vec{c}$ with $\vec{a}, \vec{b}$ & $\vec{c}$ respectively

Now $\cos\alpha = \dfrac{(\vec{a}+\vec{b}+\vec{c})\cdot\vec{a}}{|\vec{a}+\vec{b}+\vec{c}|\ |\vec{a}|}$

$= \dfrac{|\vec{a}|^2 + 0 + 0}{|\vec{a}+\vec{b}+\vec{c}|\ |\vec{a}|}$

$= \dfrac{\lambda^2}{(\sqrt{3}\lambda)(\lambda)}$

$= \dfrac{1}{\sqrt{3}}$

$\Rightarrow \alpha = \cos^{-1}(1/\sqrt{3})$

$\cos\beta = \dfrac{(\vec{a}+\vec{b}+\vec{c})\cdot\vec{b}}{|\vec{a}+\vec{b}+\vec{c}|\ |\vec{b}|}$

$= \dfrac{\vec{a}\cdot\vec{b} + |\vec{b}|^2 + \vec{c}\cdot\vec{b}}{|\vec{a}+\vec{b}+\vec{c}|\ |\vec{b}|}$

$= \dfrac{0 + \lambda^2 + 0}{(\sqrt{3}\lambda)(\lambda)}$

$= 1/\sqrt{3}$

$\Rightarrow \beta = \cos^{-1}(1/\sqrt{3})$

Similarly $\gamma = \cos^{-1}(1/\sqrt{3})$

$\therefore$ $\vec{a}+\vec{b}+\vec{c}$ is equally inclined to $\vec{a}$, $\vec{b}$ & $\vec{c}$ and the angle is $\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$ Ans

Qx38 A

$$\underset{\text{I}}{\boxed{\begin{matrix}3W\\4B\end{matrix}}} \longrightarrow \underset{\text{II}}{\boxed{\begin{matrix}4W\\5B\end{matrix}}} \longrightarrow \text{white ball}$$

let $E_1 \to$ white ball is transferred from bag I to bag II

$E_2 \to$ black ball " " " I to II

$A \to$ ball draw from 2^{nd} bag is white

$P(E_1) = \frac{3}{7}$ & $P(E_2) = \frac{4}{7}$

$P(A|E_1) = \frac{5}{10}$ & $P(A|E_2) = \frac{4}{10}$

(i) Total law of prob

$$P(A) = P(E_1)\,P(A|E_1) + P(E_2)\,P(A|E_2)$$

$$= \frac{3}{7} \times \frac{5}{10} + \frac{4}{7} \times \frac{4}{10}$$

$$= \frac{15}{70} + \frac{16}{70}$$

$\therefore$ $\boxed{\text{Req prob} = \frac{31}{70}}$ Ans

(ii) Baye's theorem

Required prob $= P(E_2/A)$

$$P(E_2/A) = \frac{P(E_2)\,P(A/E_2)}{P(E_1)\,P(A/E_1) + P(E_2)\,P(A/E_2)}$$

$$= \frac{\cancel{\frac{4}{7}} \times \frac{4}{10}}{\cancel{\frac{3}{7}} \times \frac{5}{10} + \cancel{\frac{4}{7}} \times \frac{4}{10}}$$

$$= \frac{16}{15+16}$$

$\therefore$ $\boxed{\text{Req prob} = \frac{16}{31}}$ Ans

—x—