Topic : .... Date. : .... Page No. : ....

Solutions

← WORKSHEET NO. 2 →

A.O.D (Tangent & Normal)

Qn 1 → given curves $C_1: 4x = y^2$ & $C_2: 4xy = k$

Solving these equations, we get

$$y^3 = k$$

$$\Rightarrow y = k^{1/3} \text{ put in (1)}$$

$$\Rightarrow 4x = k^{2/3}$$

$$\Rightarrow x = \frac{k^{2/3}}{4}$$

∴ point of Intersection / point of Contact $= \left(\frac{k^{2/3}}{4}, k^{1/3}\right)$

Diff C_1

$$4 = 2y\frac{dy}{dx}$$

$$\Rightarrow \frac{dy}{dx} = \frac{2}{y}$$

Slope of $T_1 = \frac{2}{k^{1/3}}$

Diff C_2

$$4\left(x\frac{dy}{dx} + y\right) = 0$$

$$\Rightarrow \frac{dy}{dx} = \frac{-y}{x}$$

slope of $T_2 = \frac{-k^{1/3}}{\frac{k^{2/3}}{4}}$

$$= \frac{-4k^{1/3}}{k^{2/3}}$$

Since curve cut at right angles

$$\therefore m_1 m_2 = -1$$

$$\left(\frac{2}{k^{1/3}}\right)\left(\frac{-4k^{1/3}}{k^{2/3}}\right) = -1$$

$$\cancel{-}8 = \cancel{-}k^{2/3}$$

cubing both sides

$$\boxed{512 = k^2}$$ proved

Soln. A.O.D (W.S 2) (2)

Qn. 2 → Given $C_1 : \; y^2 = 8x$ & $C_2 : \; 2x^2 = y^2$

Given point of Int. $= (1, 2\sqrt{2})$

Diff C_1 $\quad 2y\frac{dy}{dx} = 8$ | Diff C_2 $\quad 4x = 2y\frac{dy}{dx}$

$\Rightarrow \frac{dy}{dx} = \frac{4}{y}$ | $\frac{dy}{dx} = \frac{2x}{y}$

Slope of $T_1 = \frac{4}{2\sqrt{2}} = \frac{2}{\sqrt{2}} = \sqrt{2}$ | Slope of $T_2 = \frac{2}{2\sqrt{2}} = \frac{1}{\sqrt{2}}$

Now $\quad m_1 m_2 = (\cancel{\sqrt{2}})\left(\frac{1}{\cancel{\sqrt{2}}}\right) = 1$ (which is not equal to -1)

$\therefore$ Curves "do not" cut ~~also~~ orthogonally)

Note: (there is a misprint in the worksheet)

Qn. 3 → Let the point of Contact be (x_1, y_1)

Given equation of curve

$$y = \frac{1}{x^2 - 2x + 3}$$

Diff. wrt x

$$\frac{dy}{dx} = \frac{-1}{(x^2 - 2x + 3)^2} \cdot (2x - 2)$$

Slope of tangent at $(x_1, y_1) = \frac{-(2x_1 - 2)}{(x_1^2 - 2x_1 + 3)^2}$

also slope of tangent $= 0$ ... (given)

$$\Rightarrow \frac{-(2x_1 - 2)}{(x_1^2 - 2x_1 + 3)^2} = 0$$

Topic : .... Date. : .... Page No. : ....

Soln. W.S 2 (A.OD) (3)

$\Rightarrow 2x_1 = 2$

$\Rightarrow x_1 = 1$

also we have

$y_1 = \dfrac{1}{x_1^2 - 2x_1 + 3}$ --- {∵ (x_1, y_1) lies on the curve}

put $x_1 = 1$ $\quad y_1 = \dfrac{1}{1-2+3} = \dfrac{1}{2}$

∴ point of contact $= (1, \frac{1}{2})$

Now equation of tangent at $(1, \frac{1}{2})$ with slope $= 0$ is

$y - \frac{1}{2} = 0(x-1)$

$\Rightarrow \boxed{2y - 1 = 0}$ Ans

Qn. 4 → Let the point of contact be (x_1, y_1)

equation of curve $\quad y = \dfrac{1}{x-3}$

Diff. w.r.t x $\quad \dfrac{dy}{dx} = \dfrac{-1}{(x-3)^2}$

Slope of tangent at $(x_1, y_1) = \dfrac{-1}{(x_1-3)^2}$

also slope of tangent $= 2$ --- (given)

$\Rightarrow \dfrac{-1}{(x_1-3)^2} = 2$

$\Rightarrow -\frac{1}{2} = (x-3)^2$

which is not possible (imaginary roots)

∴ ~~...~~

CLASSTIME

Topic : .... Date. : .... Page No. : ....

Soln A-OD (W-12) (4)

$\therefore$ there is no tangent to the curve having slope 2 Ans

Qn. 5 → given equation of curve

$$\left(\frac{x}{a}\right)^n + \left(\frac{y}{b}\right)^n = 2$$

Diff. w.r.t. x

$$n\left(\frac{x}{a}\right)^{n-1}\cdot\left(\frac{1}{a}\right) + n\left(\frac{y}{b}\right)^{n-1}\cdot\frac{1}{b}\cdot\frac{dy}{dx} = 0$$

given point of contact is (a,b)

$$\therefore n\left(\frac{a}{a}\right)^{n-1}\cdot\left(\frac{1}{a}\right) + n\left(\frac{b}{b}\right)^{n-1}\cdot\frac{1}{b}\left(\frac{dy}{dx}\right)_{(a,b)} = 0$$

$$\Rightarrow \frac{n}{a} + \frac{n}{b}\left(\frac{dy}{dx}\right)_{(a,b)} = 0$$

$$\Rightarrow \frac{\cancel{n}}{b}\frac{dy}{dx} = \frac{-\cancel{n}}{a}$$

$$\Rightarrow \left(\frac{dy}{dx}\right)_{(a,b)} = \frac{-b}{a} \quad (\text{slope of tangent})$$

Now equation of tangent at (a,b) with slope $= \frac{-b}{a}$ is

$$y - b = \frac{-b}{a}(x-a)$$

$$\Rightarrow ay - ab = -bx + ab$$

$$\Rightarrow bx + ay = 2ab$$

divide by ab

$$\frac{x}{a} + \frac{y}{b} = 2$$ $\therefore$ this line touches the curve Ans

CLASSTIME

A.O.D (W.S 2)

Qn 6 → given equation of curve

$$y^2 = ax^3 + b \quad ----(i)$$

Diff. w.r.t x

$$2y\frac{dy}{dx} = 3ax^2$$

$$\frac{dy}{dx} = \frac{3ax^2}{2y}$$

given point of contact (2,3)

$\therefore$ slope of tangent at $(2,3) = \frac{3a(4)}{2(3)} = \frac{12a}{6} = 2a$

given equation of Tangent: $y = 4x - 5$

slope of this tangent $= 4$

Since both are slopes of tangents

$$\Rightarrow 2a = 4$$

$$\Rightarrow a = 2$$

We know, point of contact also lies on the curve

from eq (1)

$$9 = 8a + b$$

put $a = 2$

$$9 = 16 + b \Rightarrow b = -7$$

$\therefore \boxed{a = 2,\ b = -7}$ Ans

Qn 7 → given equation of the tangent

$x = \sin(3t)$; $y = \cos(2t)$ & $t = \pi/4$

Diff wrt t

$$\frac{dx}{dt} = 3\cos(3t) \quad \& \quad \frac{dy}{dt} = -2\sin(2t)$$

$$\frac{dy}{dx} = \frac{-2\sin(2t)}{3\cos(3t)}$$

Slope of tangent at $(t = \pi/4)$ $= \dfrac{-2\sin(\pi/2)}{3\cos\left(\frac{3\pi}{4}\right)}$

$$= \frac{-2\times 1}{\left(-\frac{1}{\sqrt{2}}\right)3} = \frac{2\sqrt{2}}{3}$$

Now point of contact $= \left(\sin\left(\frac{3\pi}{4}\right), \cos\left(\frac{\pi}{2}\right)\right)$

$$= \left(\frac{1}{\sqrt{2}}, 0\right)$$

equation of Tangent at $\left(\frac{1}{\sqrt{2}}, 0\right)$ is given by

$$y - 0 = \frac{2\sqrt{2}}{3}\left(x - \frac{1}{\sqrt{2}}\right)$$

$$y - 0 = \frac{2\sqrt{2}}{3}\left(x - \frac{1}{\sqrt{2}}\right)$$

$$y = \frac{2\sqrt{2}x}{3} - \frac{2}{3}$$

$$\Rightarrow 3y = 2\sqrt{2}x - 2$$

$$\Rightarrow 2\sqrt{2}x - 3y - 2 = 0 \quad \text{Ans}$$

Qn 8 → Let the point of contact is (x_1, y_1)
equation of curve

$$y = x^2 + 3x + 4$$

Diff wrt x

$$\frac{dy}{dx} = 2x + 3$$

Slope of tangent at $(x_1, y_1) = 2x_1 + 3$

equation of tangent at (x_1, y_1) is

$y - y_1 = (2x_1 + 3)(x - x_1)$

this tangent passes through the point $(0,0)$

$\Rightarrow -y_1 = (2x_1+3)(-x_1)$

$\Rightarrow \quad y_1 = 2x_1^2 + 3x_1 \quad --- (1)$

also we have

$y_1 = x_1^2 + 3x_1 + 4 \quad -- (2)$

from (1) & (2)

$2x_1^2 + \cancel{3x_1} = x_1^2 + \cancel{3x_1} + 4$

$\Rightarrow x_1^2 = 4$

$x_1 = \pm 2$ put in eq (1)

$x_1 = 2 \quad y_1 = 8 + 6 = 14$

$x_1 = -2 \quad y_1 = 8 - 6 = 2$

$\therefore$ Required points are $(2, 14)$ & $(-2, 2)$ Ans

Ans 9 → equation of curve $y = 2x^2 + 3\sin x$

it crosses the Y-axis $\therefore x = 0$

$\Rightarrow y = 0 + 3\sin(0) = 0$

$\therefore$ point of contact is $(0,0)$

Diff w.r.t x

$\frac{dy}{dx} = 4x + 3\cos x$

Soln A.00 (W.S 1) (8)

Slope of Tangent at (0, 0) $= 0 + 3\cos(0) = 3$

Slope of Normal $= \frac{-1}{3}$ (-ve reciprocal)

Equation of Normal at (0, 0)

$$y - 0 = \frac{-1}{3}(x - 0)$$

$$3y = -x \Rightarrow \boxed{x + 3y = 0} \text{ Ans}$$

Q.No. 10 $y = 3x^2 - 9x + 8$

Diff wrt x $\frac{dy}{dx} = 6x - 9$

Slope of Tangent at $(x_1, y_1) = 6x_1 - 9$

also slope of Tangent $= \pm 1$ {∵ tangent makes equal angles with axes; $\theta = 45$, $\theta = 135$}

$\Rightarrow 6x_1 - 9 = \pm 1$

$\Rightarrow 6x_1 - 9 = 1$	$6x_1 - 9 = -1$
$\Rightarrow x_1 = \frac{5}{3}$	$x_1 = \frac{4}{3}$

also we have

$y_1 = 3x_1^2 - 9x_1 + 8$ --- {∵ (x_1, y_1) lies on the curve}

for $x_1 = \frac{5}{3}$ $y_1 = 3\left(\frac{25}{9}\right) - 9\left(\frac{5}{3}\right) + 8 = \frac{4}{3}$

for $x_1 = \frac{4}{3}$ $y_1 = 3\left(\frac{16}{9}\right) - 9\left(\frac{4}{3}\right) + 8 = \frac{4}{3}$

∴ Required points are $\left(\frac{5}{3}, \frac{4}{3}\right)$ & $\left(\frac{4}{3}, \frac{4}{3}\right)$ Ans