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ULTIMATE MATHEMATICS

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RELATION & FUNCTIONS

CLASS NO:4

Qn1: let $A=\{-1,0,1,2\}$, $B=\{-4,-2,0,2\}$

and $f,g : A \to B$ be functions defined by

$f(x)= x^2-x$ and $g(x)= 2\left|x-\frac{1}{2}\right|-1$

Are f and g equal?

Solution

$f(x)= x^2-x$

$f(-1)= 1+1=2$

$f(0)= 0-0=0$

$f(1)= 1-1=0$

$f(2)= 4-2=2$

$\therefore f=\{(-1,2),(0,0)(1,0)(2,2)\}$

$g(x)= 2\left|x-\frac{1}{2}\right|-1$

$g(-1)= 2\left|-\frac{3}{2}\right|-1 = 3-1=2$

$g(0)= 2\left|-\frac{1}{2}\right|-1 = 1-1=0$

$g(1) = 2\left|\frac{1}{2}\right|-1 = 1-1=0$

$g(2) = 2\left|\frac{3}{2}\right|-1 = 3-1=2$

$g=\{(-1,2),(0,0)(1,0)(2,2)\}$

Clearly f and g are equal **Ans**

Qn.2 → let $f: R\to R$ be the Signum function and $g: R\to R$ be the greatest Integer function Then does fog and gof Coincide in $(0,1]$?

XII RCF (class No- 4) (2)

Basics

Concept of composite function

$f(x) = 2x-1 \quad , \quad g(x) = x^2 - x$

$fog = f(g(x)) = f(x^2-x) = 2(x^2-x)-1 = 2x^2-2x-1$

$gof = g(f(x)) = g(2x-1) = (2x-1)^2-(2x-1)$ = Brack open

$fof = f(f(x)) = f(2x-1) = 2(2x-1)-1 = 4x-3$

$gog = g(g(x)) = g(x^2-x) = (x^2-x)^2-(x^2-x)$ = Bracket open

$fog(2) = f(g(2)) = f(2) = 3$

$gof(-1) = g(f(-1)) = g(-3) = 9+3 = 12$

Soln

$f : R \to R$

$f(x) = \begin{cases} -1 & : x<0 \\ 0 & : x=0 \\ 1 & : x>0 \end{cases}$ signum function

$g : R \to R$

$g(x) = [x]$

when $x \in (0,1]$

value of $f = 1$ (Range)

~~Range of f~~ =

when $x \in (0,1]$

value $g = 0, 1.$

when $x \in (0,1]$

$fog = f(g)$

$= f(0) = 0$

$f(1) = 1$

Range of

$\therefore fog = \{0, 1\}$

when $x \in [0,1]$

$gof = g(f)$

$= g(1)$

$= 1$

Range of $gof = \{1\}$

Range are not equal

fog & gof does not coincide in $(0,1]$ Ans

XII REF (class No: 4) (3)

Q.7 → Consider a function $f: [0, \frac{\pi}{2}] \to R$ given by $f(x) = \sin x$ and $g: [0, \frac{\pi}{2}] \to R$ given by $g(x) = \cos x$

Show that f and g are one-one, but $f+g$ is not one-one.

Solution $f: [0, \frac{\pi}{2}] \to R$

$f(x) = \sin x$

one-one: let $x_1, x_2 \in [0, \frac{\pi}{2}]$

and $f(x_1) = f(x_2)$

$\Rightarrow \sin x_1 = \sin x_2$ .... $\{\sin(30^\circ) = \sin(150^\circ)\}$ $\frac{1}{2} = \frac{1}{2}$ but $30^\circ \neq 150^\circ$

$\Rightarrow x_1 = x_2$ .... $\{x_1, x_2 \in [0, \frac{\pi}{2}]\}$

$\therefore f$ is one-one

Similarly $g: [0, \frac{\pi}{2}] \to R$

$g(x) = \cos x$

let $x_1, x_2 \in [0, \pi/2]$

and $g(x_1) = g(x_2)$

$\Rightarrow \cos x_1 = \cos x_2$

$\Rightarrow x_1 = x_2$ .... $\{x_1, x_2$ are in I^{st} quad$\}$

$\therefore g$ is one-one

$\therefore f+g = f(x) + g(x)$ $D_f \cap D_g = [0, \frac{\pi}{2}]$

$f+g = \sin x + \cos x$

$(f+g)(0) = \sin 0^\circ + \cos 0^\circ = 0 + 1 = 1$

$(f+g)\frac{\pi}{2} = \sin(\frac{\pi}{2}) + \cos\frac{\pi}{2} = 1 + 0 = 1$

$0 \to 1$, $\pi/2 \to 1$ Clearly $f+g$ is not one-one

Ans

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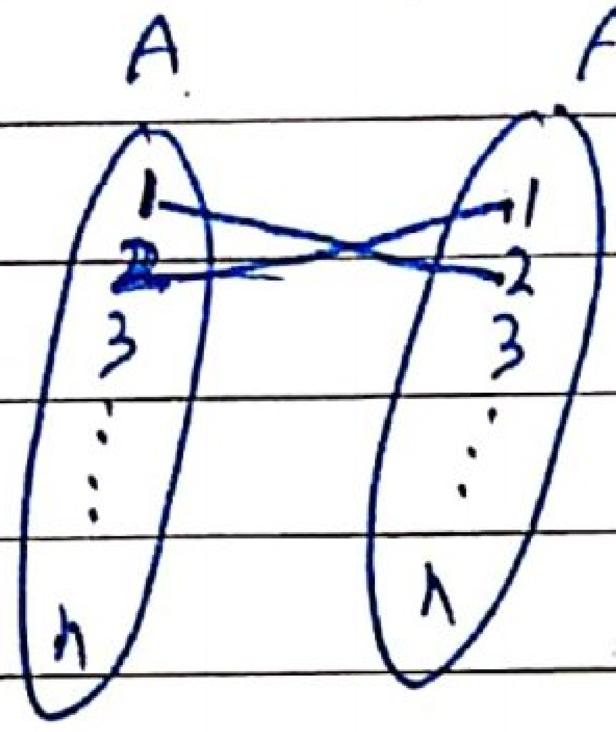
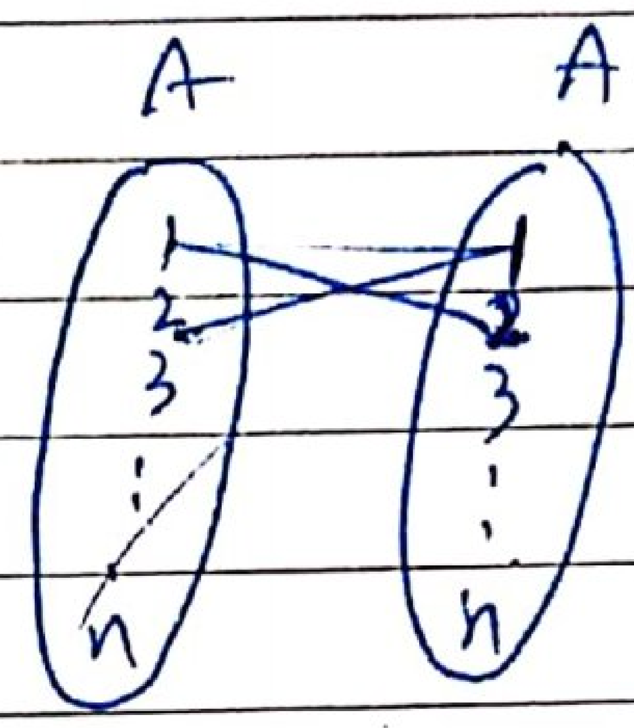
XII R&F class No-4

Q.No 4 → Find the number of one one & number of on-to functions from the set $A=\{1,2,3,\dots n\}$ to itself.

Sol $f: A \to A$

$A=\{1,2,3 \dots n\}$





no of one-one function $= n \times (n-1)(n-2) \dots 1$

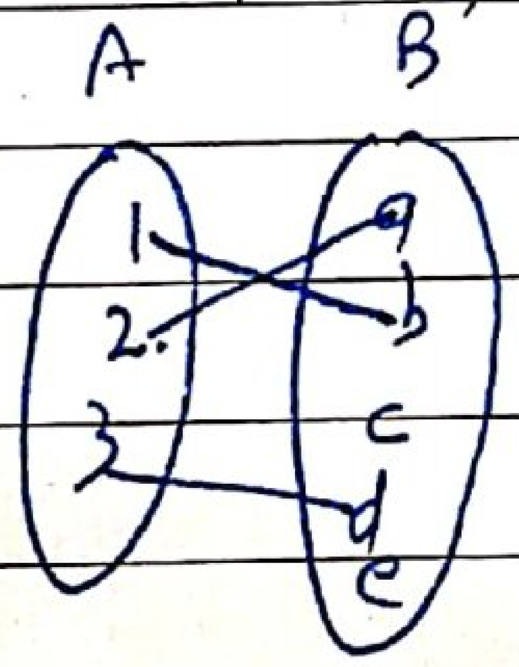
$= n!$

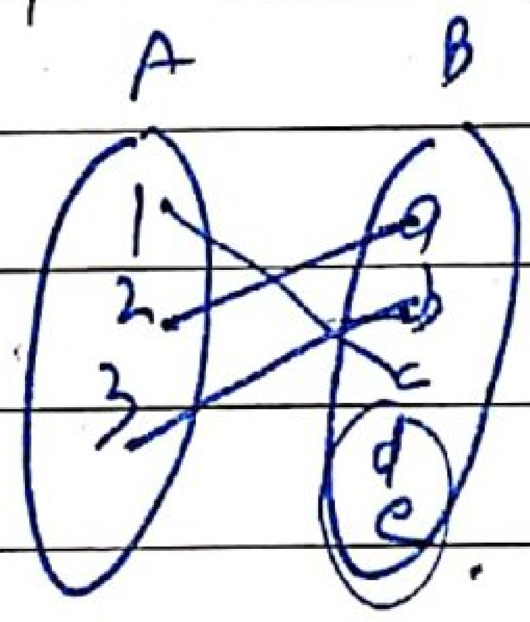
no of on-to functions

$= n \times (n-1)(n-2) \dots 1$

$= n!$

Q5 $A=\{1,2,3\}$ $B=\{a,b,c,d,e\}$





one-one

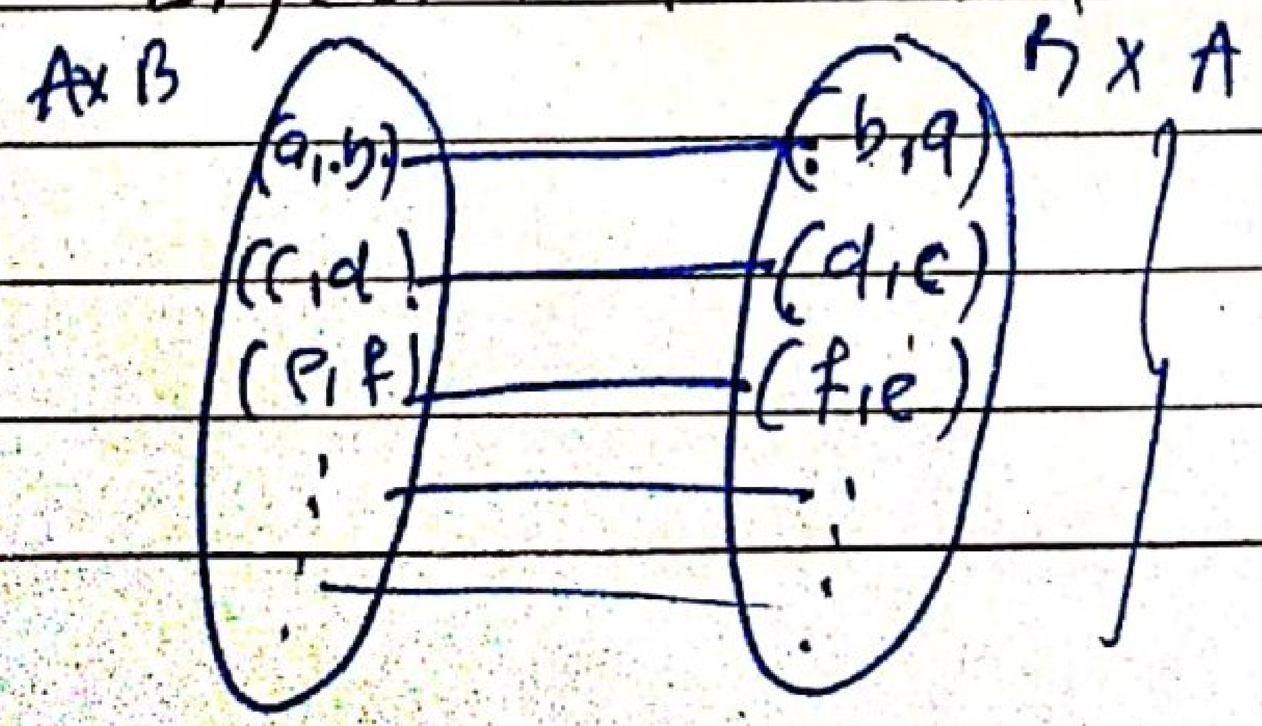
$5 \times 4 \times 3$

$= 60$

$3 \times 2 \times 1 \dots ?$

No of on-to function = 0

Q.No 6 → Let A and B are two sets. Show that (Confluent) $f: A \times B \to B \times A$ such that $f(a,b) = (b,a)$ is bijective function



let 10 pairs

10 pairs

CLASSTIME

Sol **one-one function**

let (a,b) & $(c,d) \in A\times B$

and $f(a,b) = f(c,d)$

$\Rightarrow (b,a) = (d,c)$

$\Rightarrow b=d$ & $a=c$

Now $(a,b) = (c,d)$

$\therefore f$ is one-one function

on-to function

(∵) $n(A\times B) = n(B\times A)$

(∵) also f is one-one function (just proved)

(∵) Codomain = Range

$\therefore f$ must be on to

$\therefore f$ is Bijective function Ans

Q.7 → Show that the function $f: R \to \{x \in R : -1 < x < 1\}$ defined by $f(x) = \frac{x}{1+|x|}$ is one-one and on-to function.

~~lengthier~~

Sol

Case I when $x \geq 0$; $|x| = x$

$\therefore f(x) = \frac{x}{1+x}$

one-one let $x_1, x_2 \in [0,\infty)$

and $f(x_1) = f(x_2)$

$\Rightarrow \frac{x_1}{1+x_1} = \frac{x_2}{1+x_2}$

$\Rightarrow x_1 + \cancel{x_1x_2} = x_2 + \cancel{x_1x_2}$

$\Rightarrow x_1 = x_2$ $\therefore f$ is one-one when $x \geq 0$

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XII REF class No: 4

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on-to function $f(x) = \frac{x}{1+x}$; $x \geq 0$

then $f(x) \in [0,1)$

let $y = \frac{x}{1+x}$

$\Rightarrow y + xy = x$

$\Rightarrow x(y-1) = -y$

$\Rightarrow x = \frac{-y}{y-1}$

~~Clearly~~ for each value of $y \in [0,1)$, there exits an element x in $[0,\infty)$

such that $f(x) = f\left(\frac{-y}{y-1}\right) = \frac{\frac{-y}{y-1}}{1-\frac{y}{y-1}}$

$= \frac{-y}{y-1-y}$

$= y$

$\therefore f$ is on to when $x \geq 0$

Case II $|x| = -x$; when $x < 0$

$\therefore f(x) = \frac{x}{1-x}$

one-one ~~st~~ let $x_1, x_2 \in (-\infty, 0)$

$f(x_1) = f(x_2)$

$\vdots$

$x_1 = x_2$ $\therefore f$ is one-one when $x<0$

on-to when $x<0$ then $f(x) \in (-1,0)$

proceed

CLASSTIME

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XII REF (class = 4)

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Que 8 → let $f : \{1, 3, 4\} \to \{1, 2, 5\}$
$g : \{1, 2, 5\} \to \{1, 3\}$ given by
$f = \{(1, 2), (3, 5), (4, 1)\}$
$g = \{(1, 3), (2, 3), (5, 1)\}$ write down $g \circ f$

Soln

$g \circ f : \{1, 3, 4\} \to \{1, 3\}$

output Input

$f(1) = 2$	$g(1) = 3$
$f(3) = 5$	$g(2) = 3$
$f(4) = 1$	$g(5) = 1$

$g \circ f$ $g(f)$
$g(2) = 3$
$g(5) = 1$
$g(1) = 3$

$\therefore$ $g \circ f = \{(2, 3), (5, 1), (1, 3)\}$ Ans

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XII

RELATION & FUNCTION

WORKSHEET No: 3 (class - 4)

Qn. 1 → Check whether the function $f: R \to R$ given by $f(x) = x^4$ is one-one and onto?

Qn. 2 → Let $A = R - \{3\}$ and $B = R - \{1\}$. Consider the function $f(x) = \frac{x-2}{x-3}$. Is f one-one and on-to? Justify your answer.

Qn. 3 → Is the function $f: R \to R$ defined by $f(x) = 3 - 4x$ bijective? Justify your answer.

Qn. 4 → Show that signum function, greatest Integer function, Modulus function defined on $f: R \to R$ are neither one-one nor on-to function.

Qn. 5 → Show that the function $f: N \to N$ given by $f(1) = f(2) = 1$ and $f(x) = x - 1$ for every $x > 2$ is on-to but not one-one

Qn. 6 → Let $f: N \to N$ defined by $f(n) = \begin{cases} \frac{n+1}{2}; & \text{if } n \text{ is odd} \\ \frac{n}{2}; & \text{if } n \text{ is even} \end{cases}$

State whether the function f is bijective. Justify your answer

Qn. 7 → Let $f: N \to [6, \infty)$ be a function defined as $f(x) = 4x^2 + 12x + 15$. Show that f is invertible and find inverse

CLASSTIME

XII Worksheet No:- 3 (R&F) class = 4 (2)

Qn. 8 → Consider $f: R \to R$ given by $f(x) = 4x+3$. Show that f is invertible. Also find the inverse.

Qn. 9 → Consider $f: R_+ \to [4, \infty)$ given by $f(x) = x^2+4$. Show that f is invertible and find the inverse.

Qn. 10 → Let $f: R \to R$ given by $f(x) = (3-x^3)^{1/3}$. Write $fof(x)$

Qn. 11 Let $f: R - \{-\frac{4}{3}\} \to R$ be a function defined as $f(x) = \frac{4x}{3x+4}$. Show that f is invertible and find the inverse.

Qn. 12 → Show that the function $f: R \to R$ defined by $f(x) = \frac{x}{x^2+1}$ is neither one-one nor onto

Qn. 13 → If $f = \{(5,2), (6,3)\}$ and $g = \{(2,5), (3,6)\}$ write fog

Qn. 14 → Let $f = \{(1,2)\ (3,5)\ (4,1)\}$ & $g = \{(2,3), (5,1)\ (1,3)\}$ Write fog and gof

Qn. 15 → Let $f: R \to R$ defined by $f(x) = \frac{x}{\sqrt{1+x^2}}$. Find $(fofof)(x)$

—^—