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XII SOLUTIONS of WORKSHEET NO. 2

→ RELATION FUNCTION →

Ans 1 → $(x,y) R (u,v) \Leftrightarrow xv = yu$

Symmetric

let $(x,y) R (u,v)$

$\Rightarrow xv = yu$

$\Rightarrow vx = uy$

$\Rightarrow uy = vx$

$\Rightarrow (u,v) R (x,y)$

$\therefore$ R is symmetric

Rough work
$(u,v) R (x,y)$
$uy = vx$

Transitive let $(x,y) R (u,v)$ & $(u,v) R (a,b)$

$\Rightarrow xv = yu$ and $ub = va$

$\Rightarrow xv = yu$ and $v = \frac{ub}{a}$

$\Rightarrow x\left(\frac{\cancel{u}b}{a}\right) = y\cancel{u}$

$\Rightarrow xb = ya$

$\Rightarrow (x,y) R (a,b)$

$\therefore$ R is transitive

Rough work
$(x,y) R (a,b)$
$xb = ya$

Reflexive for each $(x,y) \in A$

we always have $xy = yx$

$\Rightarrow (x,y) R (x,y)$

$\therefore$ R is Reflexive

$\therefore$ R is an equivalence relation Ans

Rough work
$(x,y) R (x,y)$
$xy = yx$

Ans 2 → $R = \{(a,b) : f(a) = f(b)\}$

Symmetric let $(a,b) \in R$

$\Rightarrow f(a) = f(b)$

$\Rightarrow f(b) = f(a)$

$\Rightarrow (b,a) \in R$

$\therefore$ R is Symmetric.

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XII Solutions (Worksheet No: 2) R&F (2)

(2) Transitive: let $(a,b) \in R$ & $(b,c) \in R$
$\Rightarrow f(a) = f(b)$ and $f(b) = f(c)$
$\Rightarrow f(a) = f(c)$
$\Rightarrow (a,c) \in R$
$\therefore$ R is transitive

(3) Reflexive: for each $a \in X$ (given set)
$f(a) = f(a)$
$\Rightarrow (a,a) \in R$
$\therefore$ R is reflexive
$\therefore$ R is an equivalence relation. Ans

Ans 3 → given set $A = \{1,2,3\}$
R (largest Relation) $= \{(1,1)(2,2)(3,3)(1,2)(1,3)(2,1)(2,3)(3,1)(3,2)\}$

$R_1 = \{(1,1)(2,2)(3,3)(1,2)(2,1)\}$ 1st equivalence relation

$R_2 = \{(1,1)(2,2)(3,3)(1,2)(2,1)(1,3)(3,1)(2,3)(3,2)\}$ 2nd equivalence relation

$\therefore$ Ans = 2

Ans 4 → given set $A = \{1,2,3\}$

$R_1 = \{(1,1)(2,2)(3,3)(1,2)(1,3)(2,1)(3,1)\}$
this relation is reflexive, symmetric
but not transitive
Since $(2,1) \in R$ & $(1,3) \in R$ but $(2,3) \notin R$

Ans. 5 → $A = \{1,2,3\}$
Smallest equivalence relation
$R = \{(1,1)(2,2),(3,3)\}$ Ans

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RTF Solutions Worksheet no- 9 (3)

Ans 6 → $A = \{1,2,3,4 \dots 9\}$

$(a,b) \, R \, (c,d) \Leftrightarrow a+d = b+c$

equivalence class $[(2,5)]$

here $a = 2$ & $b = 5$

we have to find (c,d) pairs such that

$2+d = 5+c$

$\Rightarrow d-c = 3$

pick values of c & d from A set such that $d-c=3$

$\therefore$ Equivalence class $[(2,5)] = \{(1,4)\ (2,5)\ (3,6)\ (4,7)\ (5,8)\ (6,9)\}$ Ans

Ans. 7 → $(a,b) \, R \, (c,d) \Leftrightarrow ad(b+c) = bc(a+d)$

Symmetric

let $(a,b) \, R \, (c,d)$

$\Rightarrow ad(b+c) = bc(a+d)$

$\Rightarrow da(c+b) = cb(d+a)$

$\Rightarrow cb(d+a) = da(c+b)$

$\therefore (c,d) \, R \, (a,b)$

$\therefore R$ is symmetric

Rough work: $(c,d) \, R \, (a,b)$

$cb(d+a) = da(c+b)$

Transitive let $(a,b) \, R \, (c,d)$ & $(c,d) \, R \, (e,f)$

$\Rightarrow ad(b+c) = bc(a+d)$ & $cf(d+e) = de(c+f)$

$\Rightarrow \frac{b+c}{bc} = \frac{a+d}{ad}$ & $\frac{d+e}{de} = \frac{c+f}{cf}$

$\Rightarrow \frac{1}{c} + \frac{1}{b} = \frac{1}{d} + \frac{1}{a}$ & $\frac{1}{e} + \frac{1}{d} = \frac{1}{f} + \frac{1}{c}$

$\Rightarrow \frac{1}{e} + \cancel{\frac{1}{d}} - \frac{1}{f} + \frac{1}{b} = \cancel{\frac{1}{d}} + \frac{1}{a}$

$\Rightarrow \frac{1}{e} + \frac{1}{b} = \frac{1}{f} + \frac{1}{a}$

$\Rightarrow$ ~~(a,b)~~

Rough work: $(a,b) \, R \, (e,f)$

$af(b+e) = be(a+f)$

$\frac{b+e}{be} = \frac{a+f}{af}$

$\frac{1}{e} + \frac{1}{b} = \frac{1}{f} + \frac{1}{a}$

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$\frac{b+e}{eb} = \frac{a+f}{af}$

$\Rightarrow af(b+e) = be(a+f)$

$\Rightarrow (a,b) \; R \; (e,f)$

$\therefore R$ is transitive.

Reflexive

for each $(a,b) \in N \times N$

we always have $ab = ba$

and $a+b = b+a$

$\Rightarrow ab(b+a) = ba(a+b)$

$\Rightarrow (a,b) \; R \; (a,b)$

$\therefore R$ is reflexive

$\therefore R$ is an equivalence relation Ans

Rough work

$(a,b) \; R \; (a,b)$

$ab(b+a) = ba(a+b)$

Qn. 8 → Relation on Z

$R = \{(x, y) : |x-y| \leq 1\}$

Symmetric: let $(x, y) \in R$

$\Rightarrow |x-y| \leq 1$

$\Rightarrow |y-x| \leq 1$

$\Rightarrow (y, x) \in R$ $\therefore$ R is Symmetric

Reflexive: for each $a \in Z$

$|a-a| = 0 \leq 1$ $\therefore (a, a) \in R$

$\therefore$ R is transitive

Transitive (not): Since $(1,2) \in R$ & $(2,3) \in R$

as $|1-2| \leq 1$ & $|2-3| \leq 1$

but $(1,3) \notin R$

Since $|1-3| = 2 \nleq 1$

$\therefore$ R is not transitive Ans

Qn. 9 → Relation on N

(i) $xRy \Leftrightarrow x > y$

not symmetric

$(2,1) \in R$

as $2 > 1$

but $(1,2) \notin R$

Since $1 \ngtr 2$

not reflexive $1 \in N$

but $(1,1) \notin R$

Since $1 \ngtr 1$

Transitive let $(x, y) \in R$ & $(y, z) \in R$

$\Rightarrow x > y$ and $y > z$

$\Rightarrow x > z$

$\Rightarrow (x, z) \in R$

$\therefore$ R is transitive.

(ii) $x R y \Leftrightarrow x+y=10$

Symmetric let $(x,y) \in R$

$\Rightarrow x+y=10$

$\Rightarrow y+x=10$

$\Rightarrow (y,x) \in R$

$\therefore R$ is symmetric

Reflexive

$1 \in N$

but $(1,1) \notin R$

Since $1+1 \neq 0$

$\therefore R$ is not transitive

Transitive:

$(4,6) \in R$ & $(6,4) \in R$

Since $4+6=0$ & $6+4=0$

but $(4,4) \notin R$

Since $4+4 \neq 10$

$\therefore R$ is transitive

(iii) $x R y \Leftrightarrow xy$ is square of an integer

Symmetric let $(x,y) \in R$

$\Rightarrow xy = \lambda^2 \quad \ldots \{\lambda \in z\}$

$\Rightarrow yx = \lambda^2 \quad \ldots \{\because xy = yx\}$

$\therefore (y,x) \in R$

Reflexive for each $x \in N$

$(x)(x) = x^2$ which is square of an Integer

$\Rightarrow (x,x) \in R$

$\therefore R$ is reflexive

Transitive: (~~...~~ R & ~~...~~ R

Qn 10 → Equivalence class $[(3,2)]$

here $a = 3$, $b = 2$

we have to find value of (c, d)

such that

$$3d = 2c$$

$$\Rightarrow \frac{d}{c} = \frac{2}{3}$$

we have to pick (c, d) from given

set $A = \{2, 3, 4, 5, \ldots 17, 18\}$

$\therefore$ Equivalence class $[(3,2)] = \{(3,2), (6,4), (9,6)\ (12,8)\ (15,10)\ (18,12)\}$

Ans