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ULTIMATE MATHEMATICS: BY AJAY MITTAL

CHAPTER: INTEGRATION: CLASS NO: 10

Topic Partial fraction.

$$\int \frac{dx}{(\quad)(\quad)(\quad)} = \int \frac{}{(\quad)} + \frac{}{(\quad)} + \frac{}{(\quad)}\,dx$$

linear $(x+2)$ — quadratic x^2+4 — linear repeating $(x+1)^3$

Type: 1 all are linear factors

Q.N: 1 $I = \int \frac{2x+1}{(x+1)(x+2)(x+3)}\,dx$

let $\frac{2x+1}{(x+1)(x+2)(x+3)} = \frac{A}{x+1} + \frac{B}{x+2} + \frac{C}{x+3}$

$\Rightarrow 2x+1 = A(x+2)(x+3) + B(x+1)(x+3) + C(x+1)(x+2)$

$\Rightarrow 2x+1 = A(x^2+5x+6) + B(x^2+4x+3) + C(x^2+3x+2)$

equating the coefficients of x^2, x, constant

$0 = A + B + C \longrightarrow C = -A - B$

$2 = 5A + 4B + 3C$

$1 = 6A + 3B + 2C$

$\therefore 2 = 2A + 0B$

$1 = 4A + B$

$\Rightarrow$ $2A = 3A + 3B$

$2A = 4A + 2B$

$0 = -A + B$

$A \neq 0$

$1 = -2A$	$C = \frac{1}{2} - 3$
$A = -\frac{1}{2}$	$C = -\frac{5}{2}$
$B = 3$	

$$\therefore I = \int -\frac{1}{2(x+1)} + \frac{3}{(x+2)} + \frac{5}{2}\left(\frac{1}{x+3}\right) dx$$

$$I = -\frac{1}{2}\log|x+1| + 3\log|x+2| - \frac{5}{2}\log|x+3| + C \quad \underline{\text{Ans}}$$

Method II

$$2x+1 = A(x+2)(x+3) + B(x+1)(x+3) + C(x+1)(x+2)$$

put $x=-1$	put $x=-2$	$x=-3$
$-1 = A(+1)(2)$	$-3 = B(-1)(1)$	$-5 = C((-2)(-1)$
$A = -1/2$	$B = 3$	$C = -5/2$

Qn 2 $I = \int \frac{x^3}{(x-1)(x-2)} dx$

$$\therefore I = \int (x+3) + \frac{7x-6}{(x-1)(x-2)} dx$$

$$I = \frac{x^2}{2} + 3x + \int \frac{7x-6}{(x-1)(x-2)} dx$$

Let $\frac{7x-6}{(x-1)(x-2)} = \frac{A}{x-1} + \frac{B}{x-2}$

$$7x-6 = A(x-2) + B(x-1)$$

$7 = A + B$

$-6 = -2A - B$

$1 = -A$

$A = -1$ $\quad B = 8$

$$x^2-3x+2 \;\big)\; x^3 \;\big(\; x+3$$
$$-(x^3-3x^2+2x)$$
$$3x^2-2x$$
$$-(3x^2-9x+6)$$
$$7x-6$$

$$\therefore I = \frac{x^2}{2} + 3x + \int \frac{-1}{x-1} + \frac{8}{x-2} dx$$

$$= \frac{x^2}{2} + 3x - \log|x-1| + 8\log|x-2| + C \quad \underline{\text{Ans}}$$

Q.N: 3 $I = \int \frac{3x}{(x^2+2)(x^2+5)} dx$

put $x^2 = t$

$x\,dx = \frac{dt}{2}$

$\therefore I = \frac{3}{2} \int \frac{dt}{(t+2)(t+5)}$ Proved

Q.N: 4 Tricky

$I = \int \frac{1}{\sin x - \sin(2x)} dx$

$= \int \frac{1}{\sin x - 2\sin x \cos x}$

$= \int \frac{1}{\sin x (1 - 2\cos x)}$

$= \int \frac{\sin x}{\sin^2 x (1 - 2\cos x)}$

$= \int \frac{\sin x}{(1 - \cos^2 x)(1 - 2\cos x)}$

put $\cos x = t$

$\sin x\,dx = -dt$

$I = -\int \frac{dt}{(1-t^2)(1-2t)}$

$I = -\int \frac{1}{(1+t)(1-t)(1-2t)} dt$ (Proved)

let $\frac{1}{(1+t)(1-t)(1-2t)} = \frac{A}{1+t} + \frac{B}{1-t} + \frac{C}{1-2t}$

Type:-2 Linear & quadratic factor

Q15 $I = \int \frac{2x+1}{(x+1)(x^2+4)}\,dx$

Let $\frac{2x+1}{(x+1)(x^2+4)} = \frac{A}{x+1} + \frac{Bx+C}{x^2+4}$

$2x+1 = A(x^2+4) + (Bx+C)(x+1)$

$2x+1 = A(x^2+4) + (Bx^2 + Bx + Cx + C)$

Equate coefficient of x^2, x, constant

$0 = A + B \rightarrow B = -A$

$2 = B + C \Rightarrow 2 = -A + C$

$1 = 4A + C \quad 1 = 4A + C$

$1 = -5A$

$A = -1/5$

$B = 1/5$

$C = 2 - 1/5$

$C = -9/5$

OR

put $x=-1$ | put $x=0$ | put $x=1$

$-1 = 5A$ | $1 = -4/5 + C$ | $3 = -1 + (B + 9/5)2$

$A = -1/5$ | $C = 9/5$ | $4 = 2B + 18/5$

$4 - 18/5 = 2B$

$B = 1/5$

$\therefore I = \int \frac{-1}{5(x+1)} + \frac{\frac{1}{5}x + \frac{9}{5}}{x^2+4}\,dx$

$I = -\frac{1}{5}\int \frac{1}{x+1}dx + \frac{1}{5}\int \frac{x}{x^2+4}dx + \frac{9}{5}\int \frac{1}{x^2+4}dx$

put $x^2+4=t$ in 2nd Integral

$$x\,dx = \frac{dt}{2}$$

$$\therefore I = -\frac{1}{5}\log|x+1| + \frac{1}{10}\int\frac{dt}{t} + \frac{9}{5}\times\frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right)+c$$

$$I = -\frac{1}{5}\log|x+1| + \frac{1}{10}\log|x^2+4| + \frac{9}{10}\tan^{-1}\left(\frac{x}{2}\right)+C$$

Ans

Qu 6 $I = \int \frac{x^4}{(x-1)(x^2+1)}\,dx$

$x^3-x^2+x-1 \;\overline{)\; x^4}\;\; x+1$

$x^4 - x^3 + x^2 - x$

$x^3 - x^2 + x$

$x^3 - x^2 + x - 1$

1

$$\therefore I = \int (x+1) + \frac{1}{(x-1)(x^2+1)}\,dx$$

$$I = \frac{x^2}{2} + x + \int \frac{1}{(x-1)(x^2+1)}\,dx$$

Let $\frac{1}{(x-1)(x^2+1)} = \frac{A}{x-1} + \frac{Bx+C}{x^2+1}$

(Process)

Q=7 $I = \int \frac{1}{x^3+x^2+x+1}\,dx$

$$I = \int \frac{1}{x^2(x+1)+1(x+1)}\,dx$$

$$= \int \frac{1}{(x+1)(x^2+1)}\,dx$$

(Process)

Type 3 Linear repeating factor

Ques $I = \int \frac{x+3}{(x+1)(x+2)^2}\,dx$

Let $\frac{x+3}{(x+1)(x+2)} = \frac{A}{x+1} + \frac{B}{(x+2)} + \frac{C}{(x+2)^2}$

$x+3 = A(x+2)^2 + B(x+1)(x+2) + C(x+1)$

$x+3 = A(x^2+4x+4) + B(x^2+3x+2) + C(x+1)$

$0 = A + B \longrightarrow B = -A$

$1 = 4A + 3B + C \Rightarrow 1 = 2A + C$

$3 = 4A + 2B + C \Rightarrow 3 = 2A + C$

$-2 = 2A$

$A = 2$
$B = -2$
$C = -1$

~~$A = -2$~~ ~~$B = 2$~~ ~~$C = 7$~~

OR

put $x = -1$ | put $x = -2$ | put $x = 0$

$2 = A$ | $1 = -C$ | $3 = 8 + 2B - 1$

$C = -1$ | | $4 = 8 + 2B$

$-4 = 2B$

$B = -2$

$\therefore I = \int \frac{2}{x+1} - \frac{2}{x+2} - \frac{1}{(x+2)^2}\,dx$

$I = 2\log|x+1| - 2\log|x+2| + \frac{1}{x+2} + C$ Ans

Qn:9 $I = \int \frac{3x+5}{x^3-x^2-x+1}\,dx$

$= \int \frac{3x+5}{x^2(x-1)-1(x-1)}\,dx$

$= \int \frac{3x+5}{(x-1)(x^2-1)}\,dx$

$= \int \frac{3x+5}{(x+1)(x-1)^2}\,dx$

Now $\frac{3x+5}{(x+1)(x-1)^2} = \frac{A}{x+1} + \frac{B}{x-1} + \frac{C}{(x-1)^2}$

(Process)

Qn-10 $I = \int \frac{3x+5}{(x-1)^3}\,dx$

partial fraction — put $x-1=t$ — substitution $\int \frac{\text{lower}}{(\text{lower})^n}$

Q=11 $I = \int \frac{x^2+2}{(x-1)^3}\,dx$

partial fraction — put $x-1=t$.

$dx=dt$

$\int \frac{(t+1)^2+2}{t^3}\,dt$

← INTEGRATION: WORKSHEET NO: 8 →

(Class No: 10)

Q.No:1 $I = \int \frac{2x-1}{(x-1)(x+2)(x-3)}\,dx$ Ans $-\frac{1}{6}\log|x-1| - \frac{1}{3}\log|x+2| + \frac{1}{2}\log|x-3| + C$

Q.No:2 → $I = \int \frac{\cos\theta}{(2+\sin\theta)(3+4\sin\theta)}\,d\theta$ Ans $-\frac{1}{5}\log|2+\sin\theta| + \frac{1}{5}\log|3+4\sin\theta| + C$

Q.No.3 → $I = \int \frac{(x-1)(x-2)(x-3)}{(x-4)(x-5)(x-6)}\,dx$ Ans $x + 3\log|x-4| - 24\log|x-5| + 30\log|x-6| + C$

(Hint: First divide)

Q.No.4 → $\int \frac{x^2+1}{(x-1)^2(x+3)}\,dx$ Ans $\frac{3}{8}\log|x-1| - \frac{1}{2(x-1)} + \frac{5}{8}\log|x+3| + C$

Q.No.5 → $\int \frac{x^2+x+1}{(x-1)^3}\,dx$ Ans $\log|x-1| - \frac{3}{x-1} - \frac{3}{2(x-1)^2} + C$

{Hint Two methods: Partial fraction (or) put $x-1=t$}

Q.No.6 → $\int \frac{3x+5}{x^3-x^2-x+1}\,dx$ Ans $\frac{1}{2}\log|x+1| - \frac{1}{2}\log|x-1| - \frac{4}{x-1} + C$

Q.No.7 → $\int \frac{2}{(1-x)(1+x^2)}\,dx$ Ans $-\log|x-1| + \frac{1}{2}\log|1+x^2| + \tan^{-1}x + C$

Q.No.8 → $\int \frac{5x}{(x+1)(x^2-4)}\,dx$ Ans $\frac{5}{3}\log|x+1| - \frac{5}{2}\log|x+2| + \frac{5}{6}\log|x-2| + C$

Q.No.9 → $\int \frac{x^4}{(x-1)(x^2+1)}\,dx$ Ans $\frac{x^2}{2} + x + \frac{1}{2}\log|x-1| - \frac{1}{4}\log(x^2+1) - \frac{1}{2}\tan^{-1}x + C$

Q.No.10 → $\int \frac{1}{x-x^3}\,dx$ Ans $\frac{1}{2}\log\left|\frac{x^2}{1-x^2}\right| + C$

— x —