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ULTIMATE MATHEMATICS : BY AJAY MITTAL

CHAPTER: D.E

CLASS No: 3

Type : put Bracket = V (reducible to variable separable form)

Ques 1 Solve $\frac{dy}{dx} = \sin(x+y)$

put $x+y=v$

Diff w.r.t x

$$1+\frac{dy}{dx} = \frac{dv}{dx}$$

$$\Rightarrow \frac{dy}{dx} = \frac{dv}{dx} - 1$$

$$\therefore \quad \frac{dv}{dx} - 1 = \sin v$$

$$\Rightarrow \frac{dv}{dx} = 1+\sin v$$

$$\Rightarrow \frac{dv}{1+\sin v} = dx$$

$$\Rightarrow \int \frac{1}{1+\sin v}\,dv = \int dx$$

Rationalize

$$\Rightarrow \int \frac{1-\sin v}{\cos^2 v}\,dv = x$$

$$\Rightarrow \int \sec^2 v - \tan v \sec v \, dv = x$$

Ans

$$\Rightarrow \tan v - \sec v = x + c \Rightarrow \tan(x+y) - \sec(x+y) = x+c$$

Ans:- 2 → Solve

$$(x+y+1)^2 dy = dx \quad ; \quad y(-1) = 0$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{(x+y+1)^2}$$

put $x+y+1 = v$

$$\Rightarrow 1 + \frac{dy}{dx} = \frac{dv}{dx}$$

$$\Rightarrow \frac{dy}{dx} = \frac{dv}{dx} - 1$$

$$\therefore \frac{dv}{dx} - 1 = \frac{1}{v^2}$$

$$\Rightarrow \frac{dv}{dx} = \frac{1}{v^2} + 1$$

$$\Rightarrow \frac{dv}{dx} = \frac{1+v^2}{v^2}$$

$$\Rightarrow \frac{v^2}{v^2+1} dv = dx$$

$$\Rightarrow \int \frac{v^2}{v^2+1} dv = \int dx$$

$$\Rightarrow \int \frac{v^2+1-1}{v^2+1} dv = x$$

$$\Rightarrow \int 1 - \frac{1}{v^2+1} dv = x$$

$$\Rightarrow v - \tan^{-1} v = x + C$$

$$\Rightarrow x+y+1 - \tan^{-1}(x+y+1) = x + C$$

$$\Rightarrow \boxed{y = \tan^{-1}(x+y+1) - 1 + C}$$

put $x = -1$ & $y = 0$

$$0 = 0 - 1 + C$$

$$C = 1$$

$$\therefore \boxed{y = \tan^{-1}(x+y+1)}$$

Ans

Ans 3 → Solve

$$(x-y)(dx+dy) = dx-dy$$

given $y=-1$ & $x=0$

$\Rightarrow \quad x\,dx + x\,dy - y\,dx - y\,dy = dx - dy$

$\Rightarrow \quad dy(x-y+1) = dx(1-x+y)$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{1-x+y}{x-y+1}$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{1-(x-y)}{(x-y)+1}$$

put $x-y=v$

$$1-\frac{dy}{dx} = \frac{dv}{dx}$$

$$\Rightarrow \quad \frac{dy}{dx} = 1-\frac{dv}{dx}$$

$$\therefore \quad 1-\frac{dv}{dx} = \frac{1-v}{v+1}$$

$$\Rightarrow \quad \frac{dv}{dx} = 1-\left(\frac{1-v}{v+1}\right)$$

$$\Rightarrow \quad \frac{dv}{dx} = \frac{v+\cancel{1}-\cancel{1}+v}{v+1}$$

$$\Rightarrow \quad \frac{dv}{dx} = \frac{2v}{v+1}$$

$$\Rightarrow \quad \int \frac{v+1}{v}\,dv = 2\int dx$$

$$\Rightarrow \quad \int 1+\frac{1}{v}\,dv = 2x$$

$\Rightarrow v + \log v = 2x + c$

$\Rightarrow x - y + \log|x-y| = 2x + c$

$\Rightarrow \log|x-y| = y + x + c$

put $x = 0$ & $y = -1$

$\Rightarrow \log 1 = -1 + c$

$\Rightarrow c = 1$

$\therefore \boxed{\log|x-y| = x + y + 1}$ is the Req. solution

— x —

Q.N 14 → Show that the general solution of the D.E $\frac{dy}{dx} + \frac{y^2+y+1}{x^2+x+1} = 0$ is given by $(x + y + 1) = A(1 - x - y - 2xy)$ where A is a parameter

Soln

$$\frac{dy}{dx} = -\frac{(y^2+y+1)}{x^2+x+1}$$

$$\Rightarrow \int \frac{dy}{y^2+y+1} = -\int \frac{dx}{x^2+x+1}$$

$$\Rightarrow \int \frac{1}{(y+\frac{1}{2})^2 - \frac{1}{4} + 1} dy = -\int \frac{1}{(x+\frac{1}{2})^2 - \frac{1}{4} + 1} dx$$

$$= \int \frac{1}{(y+\frac{1}{2})^2 + (\frac{\sqrt{3}}{2})^2} dy = -\int \frac{1}{(x+\frac{1}{2})^2 + (\frac{\sqrt{3}}{2})^2} dx$$

$$\Rightarrow \frac{2}{\sqrt{3}} \tan^{-1}\left(\frac{2y+1}{\sqrt{3}}\right) = -\frac{2}{\sqrt{3}} \tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right) + C$$

$$\Rightarrow \frac{2}{\sqrt{3}}\left(\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)+\tan^{-1}\left(\frac{2y+1}{\sqrt{3}}\right)\right)=C$$

$$\Rightarrow \tan^{-1}\left[\frac{\frac{2x+1}{\sqrt{3}}+\frac{2y+1}{\sqrt{3}}}{1-\left(\frac{2x+1}{\sqrt{3}}\right)\left(\frac{2y+1}{\sqrt{3}}\right)}\right]=\frac{\sqrt{3}}{2}C$$

$$\Rightarrow \frac{\frac{2x+2y+2}{\cancel{\sqrt{3}}}}{\frac{3-4xy-2x-2y-1}{\cancel{3}\sqrt{3}}}=\tan\left(\frac{\sqrt{3}}{2}c\right)$$

$$\Rightarrow \frac{\cancel{2}\sqrt{3}(x+y+1)}{2-\cancel{4}xy-2x-\cancel{2y}}=\tan\left(\frac{\sqrt{3}}{2}c\right)$$

$$\Rightarrow \frac{\sqrt{3}(x+y+1)}{1-x-y-2xy}=\tan\left(\frac{\sqrt{3}}{2}c\right)$$

$$\Rightarrow \frac{x+y+1}{1-x-y-2xy}=\frac{1}{\sqrt{3}}\tan\left(\frac{\sqrt{3}}{2}c\right) \rightarrow \text{constant} = A$$

$$\Rightarrow \boxed{x+y+1=A(1-x-y-2xy)}$$ where A is a parameter

is the Required general solution

Que 5 → At any point (x, y) of a curve, the slope of the tangent is twice the slope of the line segment joining the point of contact to the point $(-4, -3)$. Find the equation of the curve given that it passes through $(-2, 1)$

Soln Given $\frac{dy}{dx} = 2\left(\frac{y+3}{x+4}\right)$

$\Rightarrow \frac{dy}{y+3} = 2\frac{dx}{x+4}$

$\Rightarrow \int \frac{1}{y+3}\,dy = 2\int \frac{1}{x+4}\,dx$

$\Rightarrow \log|y+3| = 2\log|x+4| + \log c$

$\Rightarrow \log|y+3| - \log|x+4|^2 = \log c$

$\Rightarrow \log\left|\frac{y+3}{(x+4)^2}\right| = \log c$

$\Rightarrow \left|\frac{y+3}{(x+4)^2}\right| = c$

put $x = -2$ & $y = 1$

$\Rightarrow \frac{4}{4} = c$

$\Rightarrow c = 1$

$\therefore \left|\frac{y+3}{(x+4)^2}\right| = 1$

$\Rightarrow \frac{y+3}{(x+4)^2} = \pm 1$

equation of curve

$\boxed{y+3 = (x+4)^2}$

or

$y+3 = -(x+4)^2$

rejected

(General solution)

⇒ No of arbitrary constant / parameters = order of the D.E

Particular solution: No. of arbitrary constant / parameter always equal to zero

Qn 6 Verify that the given function is a solution of the given D.E

$xy = \log y + c$ D.E: $y' = \dfrac{y^2}{1-xy}$

Diff w.r.t x

$\Rightarrow x\dfrac{dy}{dx} + y = \dfrac{1}{y}\cdot\dfrac{dy}{dx}$

$\Rightarrow \dfrac{dy}{dx}\left(x - \dfrac{1}{y}\right) = -y$

$\Rightarrow \dfrac{dy}{dx}\left(\dfrac{xy-1}{y}\right) = -y$

$\Rightarrow \dfrac{dy}{dx} = \dfrac{-y^2}{xy-1} = \dfrac{y^2}{1-xy}$ hence verified

Qn 7 → Verify that function $y = c_1 e^{ax}\cos(bx) + c_2 e^{ax}\sin(bx)$ where c_1 & c_2 are parameters / arbitrary constants is a solution of the D.E

$$\frac{d^2y}{dx^2} - 2a\frac{dy}{dx} + (a^2+b^2)y = 0$$

Soln Given $y = c_1 e^{ax}\cos(bx) + c_2 e^{ax}\sin(bx)$

$$y = e^{ax}\left(c_1\cos(bx) + c_2\sin(bx)\right) \quad \text{---(i)}$$

Diff

$$\frac{dy}{dx} = e^{ax}\left(-c_1 b\sin(bx) + bc_2\cos(bx)\right) + \underbrace{\left(c_1\cos(bx) + c_2\sin(bx)\right)e^{ax}}\cdot a$$

$$\frac{dy}{dx} = e^{ax}\left(-c_1 b\sin(bx) + bc_2\cos(bx)\right) + ay \quad \text{--- \{from (i)\}} \quad \text{--- (2)}$$

Diff again

$$\frac{d^2y}{dx^2} = e^{ax}\left(-c_1 b^2\cos(bx) - b^2c_2\sin(bx)\right) + \underbrace{\left(-c_1 b\sin(bx) + bc_2\cos(bx)\right)\cdot e^{ax}}\cdot a + a\frac{dy}{dx}$$

(from (2))

$$\frac{d^2y}{dx^2} = -b^2\cdot \underbrace{e^{ax}\left(c_1\cos(bx) + c_2\sin(bx)\right)} + a\left(\frac{dy}{dx} - ay\right) + a\frac{dy}{dx}$$

$$\frac{d^2y}{dx^2} = -b^2y + a\frac{dy}{dx} - a^2y + a\frac{dy}{dx}$$

$$\boxed{\frac{d^2y}{dx^2} - 2a\frac{dy}{dx} + (a^2+b^2)y = 0}$$ hence verified

Qn. 8 Prove that $x^2-y^2 = c(x^2+y^2)^2$ is the general (or) verify solution of the D.E $(x^3-3xy^2)dx = (y^3-3x^2y)dy$ where c is a parameter.

Soln Given $x^2-y^2 = c(x^2+y^2)^2$ ---(i)

Diff wrt x

$$2x-2y\frac{dy}{dx} = 2c(x^2+y^2)\cdot\left(2x+2y\frac{dy}{dx}\right)$$

$$\Rightarrow x-y\frac{dy}{dx} = 2c(x^2+y^2)\left(x+y\frac{dy}{dx}\right)$$

$$\Rightarrow x-y\frac{dy}{dx} = 2\frac{(x^2-y^2)\cdot\cancel{(x^2+y^2)}}{(x^2+y^2)^{\cancel{2}}}\left(x+y\frac{dy}{dx}\right) \quad \text{--- \{from eq (i)\}}$$

$$\Rightarrow x-y\frac{dy}{dx} = \frac{2(x^2-y^2)}{x^2+y^2}\cdot\left(x+y\frac{dy}{dx}\right)$$

$$\Rightarrow x^3-x^2y\frac{dy}{dx}+y^2x-y^3\frac{dy}{dx} = 2x^3+2x^2y\frac{dy}{dx}-2y^2x-2y^3\frac{dy}{dx}$$

$$\Rightarrow \frac{dy}{dx}\left(-x^2y-y^3-2x^2y+2y^3\right) = 2x^3-2y^2x-x^3-y^2x$$

$$\Rightarrow \frac{dy}{dx}\left(y^3-3x^2y\right) = x^3-3y^2x$$

$$\Rightarrow dy\left(y^3-3x^2y\right) = \left(x^3-3y^2x\right)dx$$

Ans verified

"DON'T" from Differential Equation (D·E)

example: 4, 5, 6, 7, 8, 25, 14

Exercise 9.3 (Complete)

Miscellaneous Qns 3, Qn. 5, 15

exercise 9.4: Qns 20, 21, 22, 19

"DON'T" FROM INTEGRATION

example: 23, 24, 25, 26

exercise 7.7 (complete)

exercise 7.8 (complete)

Miscelleaneous : Qns No 40

— x —

WORKSHEET NO: 3 — D.E. Class No: 3

Ques 1 → Find the general solution of D.E

$$e^x dy + (ye^x + 2x)dx = 0$$

Ans $ye^x + x^2 = C$

Ques 2 → Find the general solution of D.E $\dfrac{ydx - xdy}{y} = 0$

Ans $cx = y$

Ques 3 → Verify that the given functions is a solution of the corresponding differential equation

(a) $y - \cos y = x$; $(y\sin y + \cos y + x)\dfrac{dy}{dx} = y$

(b) $y = x\sin x$; $x\dfrac{dy}{dx} = y + x\sqrt{x^2 - y^2}$

(c) $xy = ae^x + be^{-x} + x^2$; $x\dfrac{d^2y}{dx^2} + 2\dfrac{dy}{dx} - xy + x^2 - 2 = 0$

(d) $y = e^x(a\cos x + b\sin x)$; $\dfrac{d^2y}{dx^2} - 2\dfrac{dy}{dx} + 2y = 0$

Ques 4 → Solve $\cos\left(\dfrac{dy}{dx}\right) = a$; $y = 1,\ x = 0$

Ans $\cos\left(\dfrac{y-2}{x}\right) = a$

Ques 5 → ~~Sol~~ Solve $(x^3 + x^2 + x + 1)\dfrac{dy}{dx} = 2x^2 + x$; $y(0) = 1$

Ans $y = \frac{1}{4}\left[\log (x+1)^2 (x^2+1)^3\right] - \frac{1}{2}\tan^{-1}x + 1$

Ques 6 → Solve $e^x \tan y\, dx + (1 - e^x)\sec^2 y\, dy = 0$

Ans $\tan y = C(1 - e^x)$