!! जय श्री राधे कृष्णा !!

ULTIMATE MATHEMATICS : BY: AJAY MITTAL

CHAPTER: A.O.D CLASS NO: 10

Topic INCREASING - DECREASING FUNCTIONS

(·) given $f(x) =$

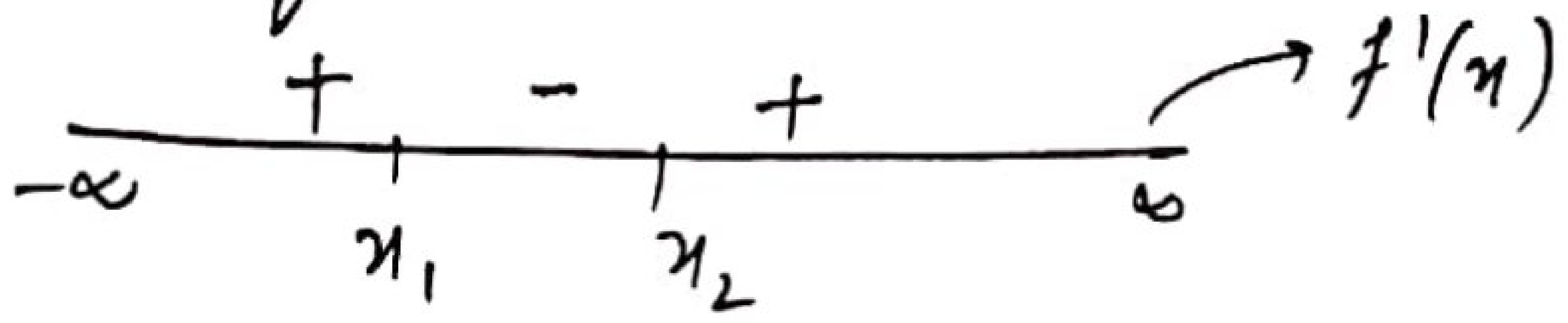
(·) find $f'(x) =$

(·) simplify $f'(x) =$

(·) put $f'(x) = 0$

(·) get values x : critical point / stationary / turning point

(·)



(·) $f(x)$ is strictly increasing, if $f'(x) > 0$; (,)

(·) $f(x)$ is strictly decreasing, if $f'(x) < 0$; (,)

(·) $f(x)$ is increasing if $f'(x) \geq 0$; [,]

(·) $f(x)$ is decreasing if $f'(x) \leq 0$; [,]

Type 1 find the Intervals

Type 2 given Intervals, show that $f(x)$ is ↑ or ↓

Ques: 1 → find the intervals for which given function is strictly increasing or strictly decreasing

$f(x) = -2x^3 - 9x^2 - 12x + 1$

Soln: Diff wrt x

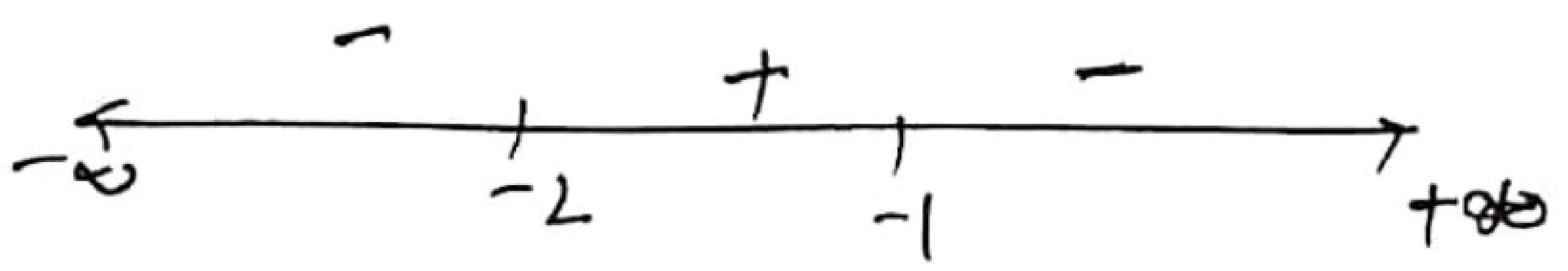
$f'(x) = -6x^2 - 18x - 12$

$f'(x) = -6(x^2 + 3x + 2)$

$= -6(x+1)(x+2)$

put $f'(x) = 0$

$x = -1, \; x = -2$



Intervals	Sign of $f'(x)$	Nature of $f(x)$
$(-\infty, -2)$	$(-)(-)(-)$ $f'(x) < 0$	$f(x)$ is stri $\downarrow$
$(-2, -1)$	$(-)(-)(+)$ $f'(x) > 0$	$f(x)$ is str $\uparrow$
$(-1, \infty)$	$(-)(+)(+)$ $f'(x) > 0$	$f(x)$ is str $\downarrow$

$\therefore$ $f(x)$ is strictly $\uparrow$ in $(-2, -1)$

$f(x)$ is stri $\downarrow$ in $(-\infty, -2) \cup (-1, \infty)$ Ans

Qn. 2 → Find the Intervals in which $f(x) = \sin x + \cos x$; $0 \le x \le 2\pi$ is strictly $\uparrow$ and strictly $\downarrow$

Sol: $f(x) = \sin x + \cos x$

$f'(x) = \cos x - \sin x$

put $f'(x) = 0$

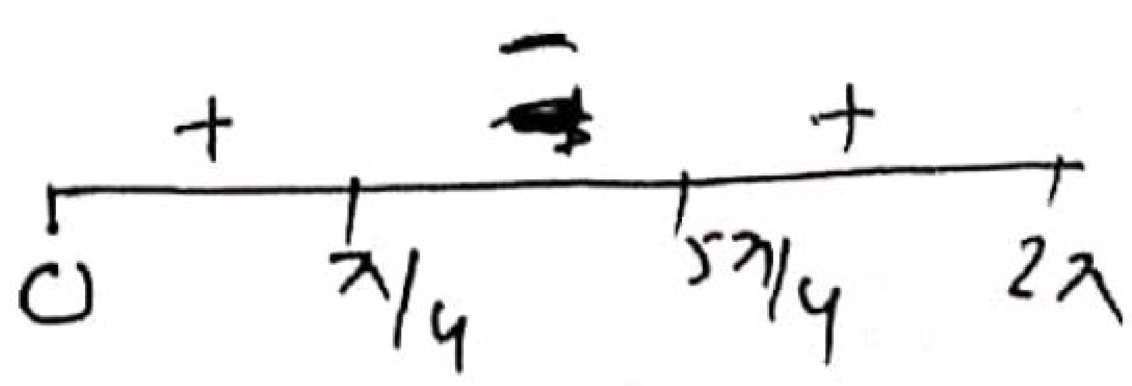
$\cos x = \sin x$

$\tan x = 1$

$x = \pi/4$

$x = \pi + \pi/4$

$x = 5\pi/4$



Intervals	Sign of $f'(x)$	Nature of $f(x)$
$[0, \pi/4)$	$f'(x) > 0$	$f(x)$ is str ↑
$(\pi/4, 5\pi/4)$	$f'(x) < 0$	$f(x)$ is str ↓
$(5\pi/4, 2\pi]$	$f'(x) > 0$	$f(x)$ is str ↑

$\therefore f(x)$ is strly ↑ in $[0, \pi/4) \cup (5\pi/4, 2\pi]$

$f(x)$ is str ↓ in $(\pi/4, 5\pi/4)$ Ans

Ques 3 → Separate the Interval $[0, \pi/2]$ into to sub-Intervals in which $f(x) = \sin^4 x + \cos^4 x$ is ~~Gt~~ Increasing or decreasing.

Sol: $f(x) = \sin^4 x + \cos^4 x$

Diff $f'(x) = 4\sin^3 x \cdot \cos x - 4\cos^3 x \sin x$

$f'(x) = 4\sin x \cos x(\sin^2 x - \cos^2 x)$

$f'(x) = -2\sin(2x)\cdot\cos(2x)$

$f'(x) = -\sin(4x)$

put $f'(x) = 0$

$\sin(4x) = 0$

$4x = 0$ $x = 0$	$4x = \pi$ $x = \pi/4$	$4x = 2\pi$ $x = \pi/2$	$4x = 3\pi$ $x = \frac{3\pi}{4}$ (x)

$-$ $+$

0 $\pi/4$ $\pi/2$

$\therefore f(x)$ is ↑ in $[0, \pi/4]$ & ↓ in $[\pi/4, \pi/2]$ Ans

Q No: 4 → Prove that $y = \frac{4\sin x}{2+\cos x} - x$ is an Increasing function of x in $[0, \pi/2]$

Type: 2

Soln

$$f(x) = \frac{4\sin x}{2+\cos x} - x$$

Diff w.r.t x

$$f'(x) = \frac{(2+\cos x)(4\cos x) - (4\sin x)(-\sin x)}{(2+\cos x)^2} - 1$$

$$f'(x) = \frac{8\cos x + 4\cos^2 x + 4\sin^2 x - 4 - \cos^2 x - 4\cos x}{(2+\cos x)^2}$$

$$= \frac{4\cos x - \cos^2 x}{(2+\cos x)^2}$$

$$f'(x) = \frac{\cos x(4-\cos x)}{(2+\cos x)^2}$$

for $x \in [0, \pi/2]$

$\cos x \geq 0$ --- { x is in Ist quad }

$4 - \cos x > 0$ --- { $-1 \leq \cos\theta \leq 1$ }

$(2+\cos x)^2 > 0$ ..

$$\therefore \frac{\cos x(4-\cos x)}{(2+\cos x)^2} \geq 0$$

$\Rightarrow f'(x) \geq 0$

$\therefore f(x)$ is $\uparrow$ in $[0, \pi/2]$ Ans

Ques 5 → Show that $f(x) = \log(1+x) - \frac{2x}{2+x}$; $x > -1$,
Type 2 is an increasing function of x throughout its domain.

Soln

$f(x) = \log(1+x) - \frac{2x}{2+x}$ domain $(-1, \infty)$

$$f'(x) = \frac{1}{1+x} - \left(\frac{(2+x)(2) - (2x)(1)}{(2+x)^2}\right)$$

$$= \frac{1}{1+x} - \frac{(4 + \cancel{2x} - \cancel{2x})}{(2+x)^2}$$

$$= \frac{\cancel{4} + x^2 + \cancel{4x} - \cancel{4} - \cancel{4x}}{(1+x)(2+x)^2}$$

$$f'(x) = \frac{x^2}{(1+x)(2+x)^2}$$

extra

If domain not given

$y = \log(1+x) - \frac{2x}{2+x}$

$1+x > 0$ & $2+x \neq 0$

$x > -1$ & $x \neq -2$

$x \in (-1, \infty)$ domain

for $x \in (-1, \infty)$

$x^2 \geq 0$

$1+x > 0$

$(2+x)^2 > 0$

$$\Rightarrow \frac{x^2}{(1+x)(2+x)^2} \geq 0$$

$\Rightarrow f'(x) \geq 0$

$\therefore f(x)$ is ↑ in its domain Ans

Ques No: 6 Find the Intervals in which the function $f(x) = \dfrac{4\sin x - 2x - x\cos x}{2+\cos x}$ is strictly ↑ or strict ↓

Sol: Diff wrt x

$$f'(x) = \frac{(2+\cos x)\left(4\cos x - 2 - \{-x\sin x + \cos x\}\right) - \left(4\sin x - 2x - x\cos x\right)(-\sin x)}{(2+\cos x)^2}$$

$$= \frac{(2+\cos x)(3\cos x - 2 + x\sin x) + (4\sin^2 x - 2x\sin x - x\sin x\cos x)}{(2+\cos x)^2}$$

$$= \frac{6\cos x - 4 + \cancel{2x\sin x} + 3\cos^2 x - 2\cos x + \cancel{x\sin x\cos x} + 4\sin^2 x - \cancel{2x\sin x} - \cancel{x\sin x\cos x}}{(2+\cos x)^2}$$

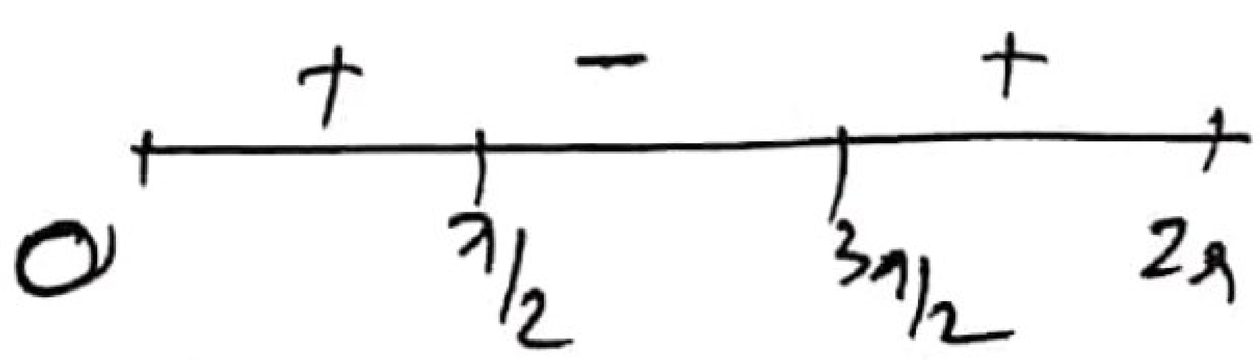
$$= \frac{4\cos x - 4 + 3\cos^2 x + 4(1-\cos^2 x)}{(2+\cos x)^2}$$

$$= \frac{4\cos x - \cos^2 x}{(2+\cos x)^2}$$

$$f'(x) = \frac{\cos x(4-\cos x)}{(2+\cos x)^2}$$

Put $f'(x) = 0$

$\cos x = 0$	$4 - \cos x = 0$
$x = \pi/2$	$\cos x \neq 4$
$x = 3\pi/2$	



$\therefore f(x)$ is Str ↑ in $(0, \pi/2) \cup (3\pi/2, 2\pi)$

& Str ↓ in $(\pi/2, 3\pi/2)$

Q.N: 7 → Show that $f(x) = \tan^{-1}(\sin x + \cos x)$; $x > 0$
Type-2
is always strictly increasing in $(0, \pi/4)$

Sol: $f(x) = \tan^{-1}(\sin x + \cos x)$

Diff. w.r.t x

$$f'(x) = \frac{1}{1+(\sin x + \cos x)^2} \cdot (\cos x - \sin x)$$

$$f'(x) = \frac{\cos x(1 - \tan x)}{1+(\sin x + \cos x)^2}$$

for $x \in (0, \pi/4)$

$\cos x > 0$

$1 - \tan x > 0$ --- $\left\{ \begin{array}{c} 0 < x < \pi/4 \\ 0 < \tan x < 1 \end{array} \right\}$

$1 + (\sin x + \cos x)^2 > 0$

$$\therefore \frac{\cos x\,(1 - \tan x)}{1 + (\sin x + \cos x)^2} > 0$$

$\Rightarrow$ $f'(x) > 0$

$\therefore$ $f(x)$ is strict. ↑ in $(0, \pi/4)$ Ans

Q.N: 8 → Find the intervals in which the function $f(x) = x^2 e^{-x}$ is ↑ or ↓

Sol: $f(x) = x^2 e^{-x}$

Diff $f'(x) = -x^2 \cdot e^{-x} + e^{-x} \cdot 2x$

$f'(x) = x \cdot e^{-x}(-x+2)$

put $f'(x) = 0$

$x(e^{-x})(2-x) = 0$ $\boxed{e^{\pm x} > 0}$

$x = 0$ | $e^{-x} \neq 0$ | $x = 2$

$-$ $+$ $-$

$-\infty$ 0 2 ∞

$\therefore f(x)$ is $\uparrow$ for $x \in [0,2]$ &

$\downarrow$ in $(-\infty, 0] \cup [2, \infty)$ Ans

Q.No-9 → Show that the function
Type-2 $f(x) = x^3 - 3x^2 + 4x$ is Strictly Increasing on R

Soln $f(x) = x^3 - 3x^2 + 4x$

Imp $f'(x) = 3x^2 - 6x + 4$

$f'(x) = 3x^2 - 6x + 3 + 1$

$= 3(x^2 - 2x + 1) + 1$

$f'(x) = 3(x-1)^2 + 1$

for $x \in (-\infty, \infty)$

$(x-1)^2 \geq 0$

$\Rightarrow 3(x-1)^2 \geq 0$

$\Rightarrow 3(x-1)^2 + 1 > 0$

$\Rightarrow f'(x) > 0$

$\therefore f(x)$ is strictly $\uparrow$ in R

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