!! जय श्री राधे कृष्णा !!

ULTIMATE MATHEMATICS

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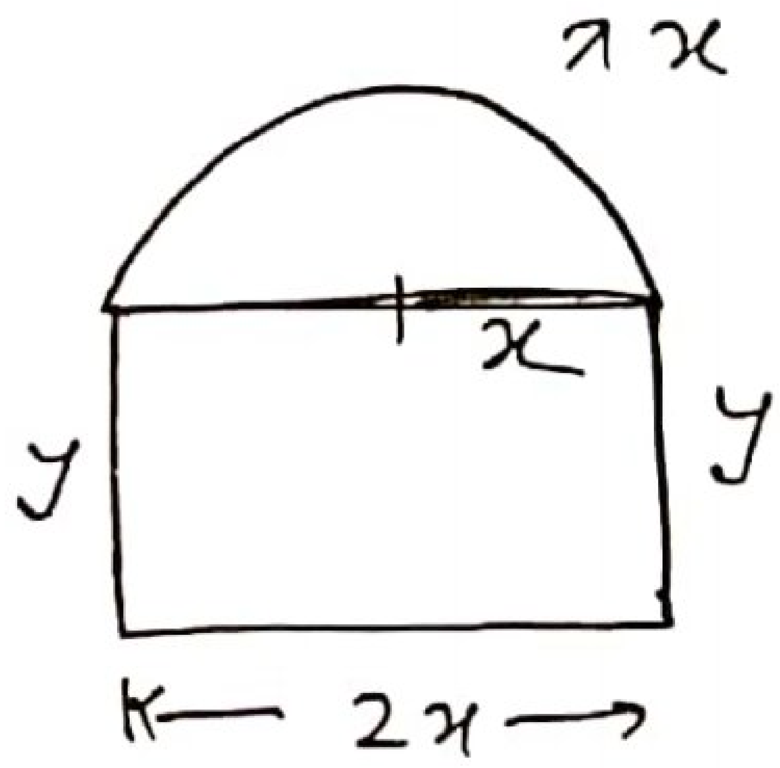
CHAPTER: A.OD $\quad$ CLASS NO: 7

Topic: Maxima Minima (Continued---)

Ans. 1 → A window is in the form of a rectangle surmounted by a semi-circular opening. The total perimeter of the window is 10m. Find the dimensions of the window to admit maximum light through the whole opening

Soln Let: $2x$ → length of rectangle
y → breadth of rectangle



$\because \quad 10 = 2x + y + \pi x + y$

$10 = x(\pi+2) + 2y$ --- (given) -(1)

$\because$ A → area of window

$A = \frac{\pi x^2}{2} + 2xy$ --- (to be Max)

$A = \frac{\pi x^2}{2} + \cancel{2}x\left(\frac{10 - x(\pi+2)}{\cancel{2}}\right)$

$A = \frac{\pi x^2}{2} + 10x - x^2(\pi+2)$

Diff wrt x

$\frac{dA}{dx} = \pi x + 10 - 2x(\pi+2)$

$= \pi x + 10 - 2\pi x - 4x$

$= -\pi x - 4x + 10$

$\frac{dA}{dx} = -x(\pi+4) + 10$

for Max/Min put $\frac{dA}{dx} = 0$

$\Rightarrow -x(\pi+4) + 10 = 0$

$\Rightarrow \boxed{x = \frac{10}{\pi+4}}$

Diff again wrt x

$\frac{d^2A}{dx^2} = -(\pi+4) < 0$

$\therefore$ Area of window is Max at $x = \frac{10}{\pi+4}$

put $x = \frac{10}{\pi+4}$ in eq(i)

$\Rightarrow 10 = \frac{10}{\pi+4}(\pi+2) + 2y$

$\Rightarrow 10 - \frac{10(\pi+2)}{\pi+4} = 2y$

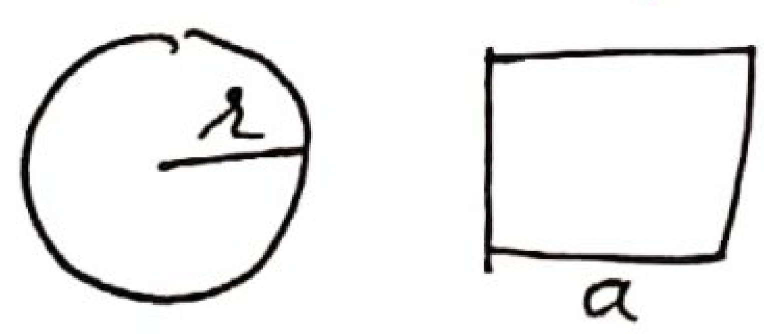
$\Rightarrow \frac{\cancel{10\pi} + 40 - \cancel{10\pi} - 20}{\pi+4} = 2y$

$\Rightarrow \boxed{y = \frac{10}{\pi+4}}$

Dimension : $\left.\begin{array}{l} \text{length} = 2x = \frac{20}{\pi+4}\ m \\ \text{Breadth} = y = \frac{10}{\pi+4}\ m \end{array}\right\}$ Ans

Que 2 → The sum of the perimeter of a circle and square is 'k', where k is a constant. Prove that the sum of their areas is least when the side of square is double the Radius of the circle

Sol: (.) $a \to$ side of the square
$r \to$ Radius of circle



(i) $K = 2\pi r + 4a$ --- (given) ..(1)

(ii) $A \to$ sum of their areas

$$A = \pi r^2 + a^2 \quad \text{--- (to be Min)}$$

$$A = \pi r^2 + \left(\frac{K - 2\pi r}{4}\right)^2 \quad \text{--- \{from eq (i)\}}$$

$$A = \pi r^2 + \frac{K^2 + 4\pi^2 r^2 - 4K\pi r}{16}$$

Diff w.r.t r

$$\frac{dA}{dr} = 2\pi r + \frac{(2r)4\pi^2 - 4K\pi}{16}$$

$$= 2\pi r + \frac{2\pi^2 r - K\pi}{4}$$

$$= \frac{8\pi r + 2\pi^2 r - K\pi}{4}$$

$$\frac{dA}{dr} \Rightarrow \frac{r(8\pi + 2\pi^2) - K\pi}{4}$$

for Max/Min put $\frac{dA}{dr} = 0$

$$\frac{r(8\pi + 2\pi^2) - K\pi}{4} = 0$$

$$\Rightarrow r(8\pi + 2\pi^2) = K\pi$$

$$\Rightarrow \boxed{r(8 + 2\pi) = K}$$

$$\Rightarrow r = \frac{K}{2\pi + 8}$$

Diff again w.r.t r

$$\frac{d^2A}{dr^2} = \frac{8\pi + 2\pi^2}{4} > 0$$

$\therefore$ sum of their area is least (Min)

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put $k = \cancel{8}(2\pi r + \cancel{8})$ in eq (i)

$\Rightarrow \quad 2\cancel{\pi r} + \cancel{8}1 = 2\cancel{\pi r} + 4a$

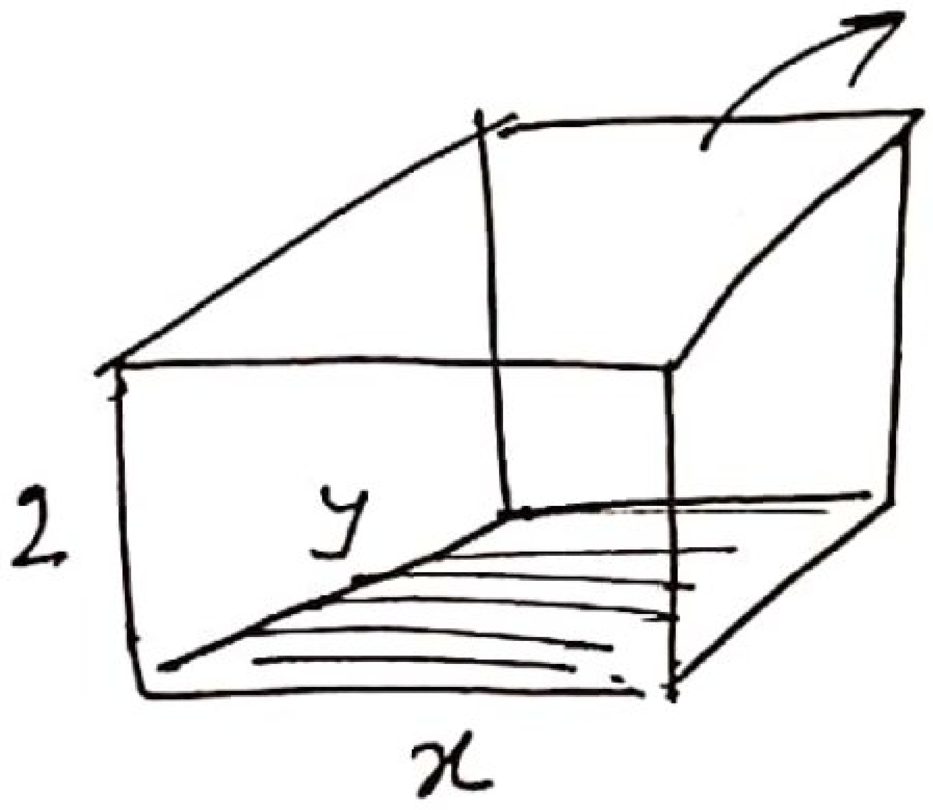
$\cancel{8}1 = 4a$

$21 = a$

$\therefore$ Side of square = double the Radius of circle

Ans

Ques 3 → A tank with rectangular base and rectangular sides, open at the top is to be constructed so that its depth is 2m and volume is $8 m^3$. If building of tank costs Rs 70 per m^2 for the base and Rs 45 per m^2 for sides. What is the cost of least expensive tank?



Soln (·) $x \to$ length of tank

$y \to$ breadth ,,

(·) $8 = 2xy$

$\Rightarrow \quad xy = 4$ -- (given) --(1)

(·) $C \to$ total cost of the tank

$C = 70xy + 4y(45) + 4x(45)$

$C = 70xy + 180y + 180x$ --- (to be Min)

$C = 70(4) + 180\left(\frac{4}{x}\right) + 180x$

Diff wrt x

$\frac{dC}{dx} = -\frac{720}{x^2} + 180$

for Max/Min put $\frac{dC}{dx} = 0$

$\Rightarrow \quad -\frac{720}{x^2} + 180 = 0$

$\Rightarrow \quad \frac{\cancel{720}^{4}}{x^2} = 1\cancel{80}$

$\Rightarrow \quad x^2 = 4$

$\Rightarrow \quad \boxed{x = 2}$

Diff again w.r.t x

$$\frac{d^2C}{dx^2} = \frac{1440}{x^3}$$

$$\left(\frac{d^2C}{dx^2}\right)_{x=2} = \frac{1440}{8} > 0$$

$\therefore$ Cost of the tank is least

put $x = 2$ in eq (i)

$\Rightarrow \quad 2y = 4$

$\Rightarrow \quad y = 2$

Total Min. Cost $\quad C_{min} = 70(2)(2) + 180(2) + 180(2)$

$= 280 + 720$

$= 1000$

$\therefore$ Min Cost = Rs 1000 Ans

QN. 4 → Show that the right circular cone of least curved surface and given volume has an altitude equal to $\sqrt{2}$ times the radius of the base.

(3 Mar.)

Soln. (i) $h \to$ height ...
$r \to$ Radius of cone
$l \to$ slant height ...

(i) $V = \frac{1}{3}\pi r^2 h$ --- (given) ---(1)

(i) $S \to$ C.S.A of Cone

$S = \pi r l$ --- (to be Min)

$S = \pi r\sqrt{h^2 + r^2}$

$S = \pi r\sqrt{\left(\frac{3V}{\pi r^2}\right)^2 + r^2}$

$S = \pi r\sqrt{\frac{9V^2}{\pi^2 r^4} + r^2}$

$S = \pi r\sqrt{\frac{9V^2 + \pi^2 r^6}{\pi^2 r^4}}$

$S = \frac{\pi r}{\pi r^2}\sqrt{9V^2 + \pi^2 r^6}$

$S = \frac{1}{r}\sqrt{9V^2 + \pi^2 r^6}$

Squaring $S^2 = \frac{1}{r^2}(9V^2 + \pi^2 r^6)$

$S^2 = \frac{9V^2}{r^2} + \pi^2 r^4$

let $S^2 = Z$

$\therefore$ S is Max/Min as according to Z is Max/Min

$$z = \frac{9v^2}{r^2} + \pi^2 r^4$$

Diff w.r.t. r

$$\frac{dz}{dr} = \frac{-18v^2}{r^3} + 4\pi^2 r^3$$

for Max/Mini put $\frac{dz}{dr} = 0$

$$\frac{18v^2}{r^3} = 4\pi^2 r^3$$

$$\Rightarrow 18v^2 = 4\pi^2 r^6$$

$$\Rightarrow 9v^2 = 2\pi^2 r^6$$

$$\Rightarrow v^2 = \frac{2\pi^2 r^6}{9}$$

$$\Rightarrow \boxed{v = \frac{\sqrt{2}\pi r^3}{3}}$$

Diff again w.r.t. r

$$\frac{d^2z}{dr^2} = \frac{54v^2}{r^4} + 12\pi^2 r^2 > 0$$

~~Clea~~ Clearly z is Minimum

$\therefore$ C.S.A of Cone is Minimum/Least

put value of V in eq (1)

$$\frac{\sqrt{2}\cancel{\pi} r^3}{\cancel{3}} = \frac{1}{\cancel{3}}\cancel{\pi} r^2 h$$

$$\sqrt{2}\, r = h$$

$\therefore$ altitude is $\sqrt{2}$ times the Radius of base

Qn. 5 → A wire of length 28 m is to be cut in to two pieces. One of the pieces is to be made in to a square and other in to a circle. What should be the length of the two pieces so that the combined area of the square and circle is minimum?

Solution

(i) $\overset{x}{\overline{A \quad\quad C}} \equiv \square_a \Rightarrow 4a = x$

$A \xleftarrow{\quad 28\,m \quad} B$, $AC = x$, $CB = 28 - x$

$\overset{28-x}{\overline{C \quad\quad A}} \equiv \bigcirc_r \Rightarrow 2\pi r = 28 - x$

(i) let $A \to$ combined area of $\square$ & $\odot$

$$A = a^2 + \pi r^2 \quad \text{... (to be Min)}$$

$$A = \frac{x^2}{16} + \pi\left(\frac{28-x}{2\pi}\right)^2$$

$$A = \frac{x^2}{16} + \cancel{\pi}\left(\frac{28^2 + x^2 - 56x}{4\pi^{\cancel{2}}}\right)$$

Diff wrt x

$$\frac{dA}{dx} = \frac{x}{8} + \frac{2x - 56}{4\pi}$$

$$\frac{dA}{dx} = \frac{\pi x + 4x - 112}{8\pi}$$

$$\frac{dA}{dx} = \frac{x(\pi+4) - 112}{8\pi}$$

for Max/Min put $\frac{dA}{dx} = 0$

$$\boxed{x = \frac{112}{\pi+4}}$$

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Diff. again w.r.t. x

$$\frac{d^2A}{dx^2} = \frac{\pi+4}{8\pi} > 0$$

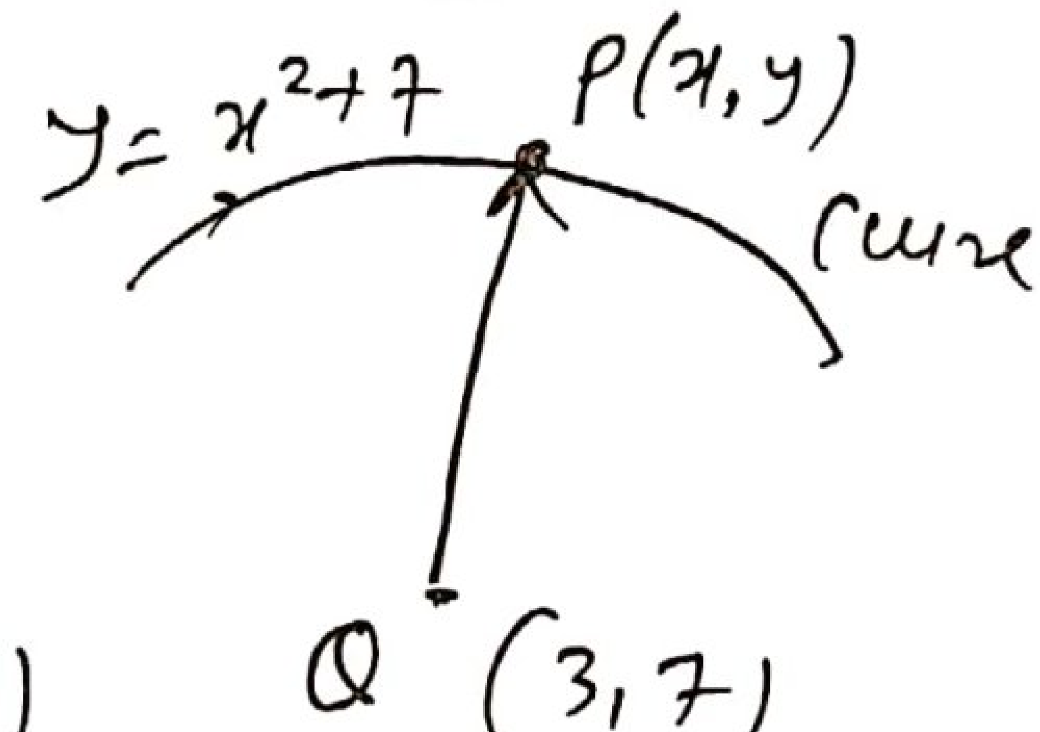
$\therefore$ Combined area of $\square$ & $\odot$ is Minimum

$\therefore$ length of 1st piece $= x\,m = \frac{112}{\pi+4}\,m$ (Ans)

length of 2nd piece $= (28-x)\,m = 28 - \frac{112}{\pi+4} = \frac{28\pi}{\pi+4}\,m$

Qn. 6 → An apache helicopter of enemy is flying along the curve given by $y = x^2+7$. A soldier placed at $(3,7)$ wants to shoot the helicopter when it is nearest to him. Find the nearest distance.

Soln: Let position of helicopter is $P(x,y)$



(·) Given soldier position $Q(3,7)$

(·) $y = x^2+7$ --- (given) ---(1)

(·) let $D \rightarrow$ distance b/w P & Q

$$D = \sqrt{(x-3)^2+(y-7)^2} \text{ -- (to be Min)}$$

$$D = \sqrt{(x-3)^2+(x^2+\cancel{7}-\cancel{7})^2} \text{ --- \{ from eq (1) \}}$$

$$D = \sqrt{x^2+9-6x+x^4}$$

Squaring

$$D^2 = x^4+x^2-6x+9$$

let $D^2 = Z$
then Distance is Max/Min ; as accordingly to ~~athore~~
Z is Max/Min

$$Z = x^4 + x^2 - 6x + 9$$

Diff wrt x

$$\frac{dZ}{dx} = 4x^3 + 2x - 6$$

put $\frac{dZ}{dx} = 0$

$$2x^3 + x - 3 = 0$$

$$\Rightarrow (x-1)(2x^2+2x+3) = 0$$

$$\Rightarrow x = 1 \qquad 2x^2+2x+3 = 0$$

(Rejected No real roots)

$$\begin{array}{r|l} & 2x^2+2x+3 \\ x-1 & 2x^3+x-3 \\ & -(2x^3-2x^2) \\ \hline & 2x^2+x-3 \\ & -(2x^2-2x) \\ \hline & 3x-3 \\ & 3x-3 \\ \hline & \times \end{array}$$

Diff again

$$\frac{d^2Z}{dx^2} = 12x^2 + 2$$

$$\left(\frac{d^2Z}{dx^2}\right)_{x=1} = 12 + 2 = 14 > 0$$

$\therefore$ Z is Minimum at $x = 1$

$\therefore$ Distance b/w soldier & helicopter is Minimum

put $x = 1$ in eq (1)

$$y = 1 + 7 \Rightarrow y = 8$$

$\therefore$ position of helicopter $(1, 8)$

Minimum Distance $PO = \sqrt{4+1} = \sqrt{5}$ units Ans

WORKSHEET No: 6 (Class No: 7)

A·OD (Maxima Minima)

Ans 1 An open box with a square base is to be made out of a given quantity of card board of area c^2 square units. Show that the maximum volume of the box is $\frac{c^3}{6\sqrt{3}}$ cubic units

Ans 2 → Find the point on the curve $y^2 = 4x$ which is nearest ~~nearest~~ to the point $(2,1)$

Ans $(1,2)$

Ans 3 → Let AP and BQ be two vertical poles at points A and B respectively. If AP = ~~16m~~ 16m, BQ = 22m, AB = 20 m, then find the distance of a point R on AB from the point A such that $RP^2 + RQ^2$ is minimum

Ans 10 m

Ans 4 → Manufacturer can sell x items at a price of Rs $\left(5 - \frac{x}{100}\right)$ each. The cost price of x items is Rs $\left(\frac{x}{5} + 500\right)$. Find the number of items he should sell to earn maximum profit Ans $x = 20$

HINT Profit = Revenue − Cost

~~Reven~~. Revenue = price × quantity $= \left(5 - \frac{x}{100}\right)x$

Ans 5 → Find the point on the parabola $x^2 = 2y$ which is closest to the point $(0,5)$

Ans $(\pm 2\sqrt{2}, 4)$

A.O.D (Worksheet No = 6)

Q.No 6 → A large window has the shape of a rectangle surmounted by an equilateral triangle. If the perimeter of the window is 12 m. Find the dimensions of the rectangle that will produce the largest area of the window

Q.No 7 → A wire of length 20 m is to be cut in to two pieces. One of the pieces will be bent in to shape of a square and the other in to shape of an equilateral triangle. Where the wire should be cut so that the sum of the areas of the square and triangle is minimum?

Ans $\frac{80\sqrt{3}}{9+4\sqrt{3}}$ m, $\frac{180}{9+4\sqrt{3}}$ m

Q.No 8 → The total area of a page is 150 cm^2. The combined width of the margin at the top and bottom is 3 cm and the side 2 cm. What must be the dimensions of the page in order that the area of the printed matter may be maximum?

Ans $l = 15$ cm ; $b = 10$ cm

Q.No 9 → Show that the cone of greatest volume which can be inscribed in a given sphere has an altitude equal to 2/3 of the diameter of the sphere.

— x —