SOLUTIONS

EXAM NO: 10

CLASS - XII

(Integration & A.O.I)

Ans: 1 → $I = \int_0^{\pi} \frac{x\,dx}{a^2\cos^2 x + b^2\sin^2 x}$ ---- ①

$I = \int_0^{\pi} \frac{(\pi - x)\,dx}{a^2\cos^2(\pi-x) + b^2\sin^2(\pi-x)} dx$ --- (Prop IV) $\int_0^a f(x)dx = \int_0^a f(a-x)\,dx$

$I = \int_0^{\pi} \frac{(\pi - x)\,dx}{a^2\cos^2 x + b^2\sin^2 x} dx$ -- ②

① + ②

$2I = \pi \int_0^{\pi} \frac{1}{a^2\cos^2 x + b^2\sin^2 x} dx$

divide N & D by $\cos^2 x$

$2I = \pi \int_0^{\pi} \frac{\sec^2 x}{a^2 + b^2\tan^2 x} dx$

$\cancel{2}I = \cancel{2}\pi \int_0^{\pi/2} \frac{\sec^2 x\,dx}{a^2 + b^2\tan^2 x}$ --- $\left\{ \begin{array}{l} f(2a-x) = f(x) \\ \int_0^{2a} f(x)dx = 2\int_0^a f(x)dx \end{array} \right\}$

put $\tan x = t$

$\sec^2 x\,dx = dt$

when $x = 0$; $t = 0$

when $x = \pi/2$; $t = \infty$

$\therefore I = \pi \int_0^{\infty} \frac{dt}{a^2 + b^2 t^2}$

$I = \frac{\pi}{b^2} \int_0^{\infty} \frac{1}{(\frac{a}{b})^2 + t^2} dt$

$I = \frac{\pi}{b^2} \times \frac{b}{a} \left[\tan^{-1}\left(\frac{tb}{a}\right)\right]_0^{\infty}$

$$I = \frac{\pi}{ab}\left(\tan^{-1}(\infty) - \tan^{-1}(0)\right)$$

$$I = \frac{\pi}{ab}\left(\frac{\pi}{2} - 0\right)$$

$$\boxed{I = \frac{\pi^2}{2ab}}$$ Ans

Ans 2 * $I = \int \frac{\sqrt{x^2+1}\,[\log(x^2+1) - 2\log x]}{x^4}\,dx$

$$I = \int \frac{\sqrt{x^2+1}\,\log\left(\frac{x^2+1}{x^2}\right)}{x^4}\,dx$$

$$= \int \frac{x\sqrt{1+\frac{1}{x^2}}\cdot\log\left(1+\frac{1}{x^2}\right)}{x^4}\,dx$$

$$= \int \frac{\sqrt{1+\frac{1}{x^2}}\cdot\log\left(1+\frac{1}{x^2}\right)}{x^3}\,dx$$

put $1+\frac{1}{x^2} = t$

$$\frac{-2}{x^3}\,dx = dt$$

$$\Rightarrow \frac{dx}{x^3} = -\frac{dt}{2}$$

$$I = -\frac{1}{2}\int \underset{\text{II}}{\sqrt{t}}\cdot \underset{\text{I}}{\log t}\,dt$$

$$= -\frac{1}{2}\left[\frac{2}{3}\log t\cdot t^{3/2} - \frac{2}{3}\int \frac{1}{t}\cdot t^{3/2}\,dt\right]$$

$$= -\frac{1}{2}\left[\frac{2}{3}\log t\cdot t^{3/2} - \frac{2}{3}\int t^{1/2}\,dt\right]$$

$$= -\frac{1}{2}\left[\frac{2}{3}\cdot\log t\cdot t^{3/2} - \frac{2}{3}\times\frac{2}{3}\,t^{3/2}\right] + C$$

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$$= \frac{-1}{2} \times \frac{2}{3}\left[\log t \cdot t^{3/2} - \frac{2}{3}t^{3/2}\right] + c$$

$$= \frac{-1}{3} \cdot t^{3/2}\left(\log t - \frac{2}{3}\right] + c$$

$$\boxed{I = \frac{-1}{3}\left(1+\frac{1}{x^2}\right)^{3/2}\left[\log\left(1+\frac{1}{x^2}\right) - \frac{2}{3}\right] + c} \text{ Ans}$$

Ans 3 →

$$I = \int_{\pi/6}^{\pi/3} \frac{\sin x + \cos x}{\sqrt{\sin(2x)}}\, dx$$

$$I = \int_{\pi/6}^{\pi/3} \frac{\sin x + \cos x}{\sqrt{1 - 1 + \sin(2x)}}\, dx$$

$$I = \int_{\pi/6}^{\pi/3} \frac{\sin x + \cos x}{\sqrt{1 - (1 - \sin(2x))}}\, dx$$

$$I = \int_{\pi/6}^{\pi/3} \frac{\sin x + \cos x}{\sqrt{1 - (\sin x - \cos x)^2}}\, dx$$

put $\sin x - \cos x = t$

$(\cos x + \sin x)dx = dt$

when $x = \pi/6$

$t = \frac{1}{2} - \frac{\sqrt{3}}{2} = \frac{1-\sqrt{3}}{2}$

when $x = \pi/3$

$t = \frac{\sqrt{3}}{2} - \frac{1}{2} = \frac{\sqrt{3}-1}{2}$

$$I = \int_{\frac{1-\sqrt{3}}{2}}^{\frac{\sqrt{3}-1}{2}} \frac{dt}{\sqrt{1-t^2}}$$

$$I = \left(\sin^{-1} t\right)_{\frac{1-\sqrt{3}}{2}}^{\frac{\sqrt{3}-1}{2}} = \sin^{-1}\left(\frac{\sqrt{3}-1}{2}\right) - \sin^{-1}\left(\frac{1-\sqrt{3}}{2}\right)$$

$$= \sin^{-1}\left(\frac{\sqrt{3}-1}{2}\right) + \sin^{-1}\left(\frac{\sqrt{3}-1}{2}\right)$$

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$$\therefore \boxed{I = 2\sin^{-1}\left(\frac{\sqrt{3}-1}{2}\right)} \quad \underline{\text{ANS}}$$

ANS: 4 → $I = \int \log(\log x) + \frac{1}{(\log x)^2}\, dx$

put $\log x = t$

$x = e^t$

$dx = e^t\, dt$

$$I = \int e^t \left(\log t + \frac{1}{t^2}\right) dt$$

$$= \int e^t \left(\log t + \frac{1}{t} - \frac{1}{t} + \frac{1}{t^2}\right) dt$$

$$= \int e^t \left(\log t + \frac{1}{t}\right) dt - \int e^t \left(\frac{1}{t} - \frac{1}{t^2}\right) dt$$

$$= \int \underset{\text{II}}{e^t} \cdot \underset{\text{I}}{\log t}\, dt + \int e^t \cdot \frac{1}{t}\, dt - \left[\int \underset{\text{II}}{e^t} \cdot \underset{\text{I}}{\frac{1}{t}}\, dt - \int e^t \cdot \frac{1}{t^2}\, dt\right]$$

$$= \log t \cdot e^t - \cancel{\int \frac{1}{t} \cdot e^t\, dt} + \cancel{\int e^t \cdot \frac{1}{t}\, dt} - \left[\frac{1}{t} \cdot e^t + \cancel{\int \frac{1}{t^2} \cdot e^t\, dt} - \cancel{\int e^t \cdot \frac{1}{t^2}\, dt}\right]$$

$$= e^t \cdot \log t - \frac{1}{t} e^t$$

$$= e^t \left(\log t - \frac{1}{t}\right) + C$$

$$\boxed{I = x\left(\log(\log x) - \frac{1}{\log x}\right) + C} \quad \underline{\text{ANS}}$$

ANS 5 → $I = \int_0^{\pi} \frac{x \tan x}{\sec x + \tan x}\, dx$

$$I = \int_0^{\pi} \frac{x \sin x}{1 + \sin x}\, dx \quad \text{-----}(i)$$

(5)

$$I = \int_0^{\pi} \frac{(\pi-x)\sin(\pi-x)}{1+\sin(\pi-x)}\,dx \quad \cdots \left\{ \int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx \right\}$$

$$I = \int_0^{\pi} \frac{(\pi-x)\sin x}{1+\sin x}\,dx \quad \text{--(2)}$$

(1) + (2)

$$2I = \pi \int_0^{\pi} \frac{\sin x}{1+\sin x}\,dx$$

$$2I = \pi \int_0^{\pi} \sin x \cdot \frac{(1-\sin x)}{1-\sin^2 x}\,dx$$

$$2I = \pi \int_0^{\pi} \frac{\sin x - \sin^2 x}{\cos^2 x}\,dx$$

$$2I = \pi \int_0^{\pi} \tan x \sec x - \tan^2 x\,dx$$

$$2I = \pi \int_0^{\pi} \sec x \tan x - (\sec^2 x - 1)\,dx$$

$$2I = \pi \left[\sec x - \tan x + x \right]_0^{\pi}$$

$$2I = \pi \left[(\sec\pi - \tan\pi + \pi) - (\sec 0 - \tan 0 + 0) \right]$$

$$2I = \pi \left[(-1 - 0 + \pi) - (1) \right]$$

$$2I = \pi(\pi - 2)$$

$$\boxed{I = \frac{\pi}{2}(\pi-2)} \quad \text{Ans}$$

Ans 6 →

$$I = \int \frac{\sin^{-1}\sqrt{x} - \cos^{-1}\sqrt{x}}{\sin^{-1}\sqrt{x} + \cos^{-1}\sqrt{x}}\,dx$$

By property $\sin^{-1}\sqrt{x} + \cos^{-1}\sqrt{x} = \pi/2$

$$I = \int \frac{\sin^{-1}\sqrt{x} - \left(\frac{\pi}{2} - \sin^{-1}\sqrt{x}\right)}{\frac{\pi}{2}} dx$$

$$I = \frac{2}{\pi} \int 2\sin^{-1}\sqrt{x} - \frac{\pi}{2} \, dx$$

$$I = \frac{4}{\pi} \int \sin^{-1}\sqrt{x} \, dx - \int dx$$

$$I = \frac{4}{\pi} I_1 - x + c$$

when $I_1 = \int \sin^{-1}\sqrt{x} \, dx$

put $x = t^2$

$dx = 2t \, dt$

$$I_1 = 2\int \underset{\mathrm{I}}{\sin^{-1} t} \cdot \underset{\mathrm{II}}{t} \, dt$$

$$I_1 = 2\left[\sin^{-1} t \cdot \frac{t^2}{2} - \frac{1}{2}\int \frac{1}{\sqrt{1-t^2}} \cdot t^2 dt \right]$$

$$I_1 = 2\left[\frac{t^2}{2} \cdot \sin^{-1} t + \frac{1}{2}\int \frac{1-t^2-1}{\sqrt{1-t^2}} dt \right]$$

$$I_1 = 2\left[\frac{t^2}{2} \cdot \sin^{-1} t + \frac{1}{2}\int \sqrt{1-t^2} - \frac{1}{\sqrt{1-t^2}} dt \right]$$

$$I_1 = t^2 \cdot \sin^{-1} t + \left(\frac{t}{2}\sqrt{1-t^2} + \frac{1}{2}\sin^{-1} t - \sin^{-1} t \right)$$

$$I_1 = t^2 \cdot \sin^{-1} t + \frac{t}{2}\sqrt{1-t^2} - \frac{1}{2}\sin^{-1} t$$

$$I_1 = \left(t^2 - \frac{1}{2}\right)\sin^{-1} t + \frac{t}{2}\sqrt{1-t^2}$$

$$I_1 = \left(x - \frac{1}{2}\right)\sin^{-1}\sqrt{x} + \frac{1}{2}\sqrt{x}\sqrt{1-x}$$

$$\therefore I = \frac{4}{\pi}\left[\frac{(2x-1)\sin^{-1}\sqrt{x}}{2} + \frac{\sqrt{x-x^2}}{2}\right] - x + C$$

$$\boxed{I = \frac{2}{\pi}\left[(2x-1)\sin^{-1}\sqrt{x} + \sqrt{x-x^2}\right] - x + C}$$ Ans

Ans 7 → $I = \int_{-1}^{3/2} |x\sin(\pi x)|\, dx$

(·) $-1 < x < -\frac{1}{2}$; $-\pi < \pi x < -\frac{\pi}{2}$

$|x\sin(\pi x)| = x\sin(\pi x)$

(·) $-\frac{1}{2} < x < 0$; $-\frac{\pi}{2} < \pi x < 0$

$|x\sin(\pi x)| = x\sin(\pi x)$

(·) $0 < x < \frac{1}{2}$; $0 < \pi x < \frac{\pi}{2}$

$|x\sin(\pi x)| = x\sin(\pi x)$

(·) $\frac{1}{2} < x < 1$; $\frac{\pi}{2} < \pi x < \pi$

$|x\sin(\pi x)| = x\sin(\pi x)$

(·) $1 < x < \frac{3}{2}$; $\pi < \pi x < \frac{3\pi}{2}$

$|x\sin(\pi x)| = -x\sin(\pi x)$

$$\therefore I = \int_{-1}^{1} x\sin(\pi x)\, dx - \int_{1}^{3/2} x\sin(\pi x)\, dx$$

Let $I_1 = \int \underset{I}{x} \cdot \underset{II}{\sin(\pi x)}\, dx$

$$I_1 = -\frac{x\cos(\pi x)}{\pi} + \frac{1}{\pi}\int \cos(\pi x)\,dx$$

$$I_1 = -\frac{x\cos(\pi x)}{\pi} + \frac{\sin(\pi x)}{\pi^2}$$

$$\therefore I = \left[-\frac{x\cos(\pi x)}{\pi} + \frac{\sin(\pi x)}{\pi^2}\right]_{-1}^{1} - \left[-\frac{x\cos(\pi x)}{\pi} + \frac{\sin(\pi x)}{\pi^2}\right]_{1}^{3/2}$$

$$= \left(\frac{1}{\pi}+0\right) - \left(-\frac{1}{\pi}+0\right) - \left[\left(0-\frac{1}{\pi^2}\right) - \left(\frac{1}{\pi}+0\right)\right]$$

$$= \frac{1}{\pi} + \frac{1}{\pi} + \frac{1}{\pi^2} + \frac{1}{\pi}$$

$$\therefore \boxed{I = \frac{3}{\pi} + \frac{1}{\pi^2}} \text{ Ans}$$

Ans 8 →

$$I = \int_0^{\pi} \log(1+\cos x)\,dx \quad \text{---(1)}$$

$$I = \int_0^{\pi} \log(1+\cos(\pi-x))\,dx \quad \ldots \left\{\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx \quad P\text{-}IV\right\}$$

$$I = \int_0^{\pi} \log(1-\cos x)\,dx \quad \text{--(2)}$$

(1) + (2)

$$2I = \int_0^{\pi} \log\big((1+\cos x)(1-\cos x)\big)\,dx$$

$$2I = \int_0^{\pi} \log(1-\cos^2 x)\,dx$$

$$2I = \int_0^{\pi} \log(\sin^2 x)\,dx$$

$$\cancel{2}I = \cancel{2}\int_0^{\pi} \log(\sin x)\,dx$$

$$I = \int_0^{\pi} \log(\sin x)\,dx$$

$$I = 2\int_0^{\pi/2} \log(\sin x)\,dx \quad \cdots \left\{ \int_0^{2a} f(x)\,dx = 2\int_0^{a} f(x)\,dx \right.$$
$$\left. \text{if } f(2a-x) = f(x) \right\}$$

$$\frac{I}{2} = \int_0^{\pi/2} \log(\sin x)\,dx \quad -- (3)$$

$$\frac{I}{2} = \int_0^{\pi/2} \log(\cos x)\,dx \quad -- (4)$$

(3) + (4)

$$I = \int_0^{\pi/2} \log(\sin x.\cos x)\,dx$$

$$I = \int_0^{\pi/2} \log\left(\frac{\sin(2x)}{2}\right)dx$$

$$I = \int_0^{\pi/2} \log(\sin(2x))\,dx - \int_0^{\pi/2} \log 2\,dx$$

$$I = \int_0^{\pi/2} \log(\sin(2x))\,dx - (x\log 2)_0^{\pi/2}$$

$$I = \int_0^{\pi/2} \log(\sin(2x))\,dx - \frac{\pi}{2}\log 2$$

$$I = I_1 - \frac{\pi}{2}\log 2$$

where $I_1 = \int_0^{\pi/2} \log(\sin(2x))\,dx$

put $2x = t$ | when $x=0$; $t=0$

$dx = \frac{dt}{2}$ | when $x = \frac{\pi}{2}$; $t = \pi$

$$I_1 = \frac{1}{2}\int_0^{\pi} \log(\sin t)\,dt$$

$$I_1 = \frac{1}{2} \times 2 \int_0^{\pi/2} \log(\sin t)\, dt \quad \text{--- \{ P VI \}}$$

$$I_1 = \int_0^{\pi/2} \log(\sin x)\, dx \quad \text{-- } \left\{ \int_a^b f(x)\,dx = \int_a^b f(t)\,dt \right\}$$

$$\boxed{I_1 = \frac{I}{2}}$$

$$\therefore I = \frac{I}{2} - \frac{\pi}{2} \lg 2$$

$$I - \frac{I}{2} = -\frac{\pi}{2} \lg 2$$

$$\Rightarrow \frac{I}{\cancel{2}} = -\frac{\pi}{\cancel{2}} \lg 2$$

$$\Rightarrow \boxed{I = -\pi \lg 2} \quad \text{Ans}$$

Qn. 14 → $I = \int \frac{(x^2+1)(x^2+2)}{(x^2+3)(x^2+4)}\, dx$

Divide

$$x^4+7x^2+12 \overline{\big) \, x^4+3x^2+2 \,} \; 1$$
$$-(x^4+7x^2+12)$$
$$\overline{-4x^2-10}$$

$$I = \int 1 - \frac{4x^2+10}{(x^2+3)(x^2+4)}\, dx$$

$$I = x - \int \frac{4x^2+10}{(x^2+3)(x^2+4)}\, dx$$

let $x^2 = y$

$$\therefore \frac{4x^2+10}{(x^2+3)(x^2+4)} = \frac{4y+10}{(y+3)(y+4)}$$

let $\frac{4y+10}{(y+3)(y+4)} = \frac{A}{y+3} + \frac{B}{y+4}$

$$\Rightarrow 4y+10 = A(y+4) + B(y+3)$$

comp coefficients of y and constant term

$4 = A + B$

$10 = 4A + 3B$

Solving we get

$$\begin{aligned} \cancel{16} &= \cancel{4A} + 4B \\ \cancel{10} &= \cancel{4A} + 3B \\ \hline 6 &= B \end{aligned}$$

$B = 6$ & $A = -2$

$\therefore \; I = x - \int \frac{-2}{x^2+3} + \frac{6}{x^2+4}\, dx$

$= x + 2\int \frac{1}{x^2+3}\, dx - 6\int \frac{1}{x^2+4}\, dx$

$$\boxed{I = x + \frac{2}{\sqrt{3}} \tan^{-1}\left(\frac{x}{\sqrt{3}}\right) - 3\tan^{-1}\left(\frac{x}{2}\right) + c}$$ **Ans**

Ans 10 → $I = \int \frac{5x+3}{\sqrt{(x+3)(2-x)}}\, dx$

$I = \int \frac{5x+3}{\sqrt{-x^2-x+6}}\, dx$

$= 5\int \frac{x+\frac{3}{5}}{\sqrt{-x^2-x+6}}\, dx$

$= -\frac{5}{2}\int \frac{-2x-\frac{6}{5}-1+1}{\sqrt{-x^2-x+6}}\, dx$

$= -\frac{5}{2}\int \frac{(-2x-1)-\frac{1}{5}}{\sqrt{-x^2-x+6}}\, dx$

$I = -\frac{5}{2}\int \frac{-2x-1}{\sqrt{-x^2-x+6}}\, dx + \frac{1}{2}\int \frac{1}{\sqrt{-x^2-x+6}}\, dx$

put $-x^2-x+6=t$ in I^{st} Integral

$(-2x-1)\,dx = dt$

$$I = -\frac{5}{2}\int \frac{dt}{\sqrt{t}} + \frac{1}{2}\int \frac{1}{\sqrt{-(x^2+x-6)}}\,dx$$

$$= -\frac{5}{\cancel{2}} \times \cancel{2}\sqrt{t} + \frac{1}{2}\int \frac{1}{\sqrt{-\left[\left(x+\frac{1}{2}\right)^2-\frac{1}{4}-6\right]}}\,dx$$

$$= -5\sqrt{-x^2-x+6} + \frac{1}{2}\int \frac{1}{\sqrt{-\left\{\left(x+\frac{1}{2}\right)^2-\left(\frac{5}{2}\right)^2\right\}}}\,dx$$

$$= -5\sqrt{-x^2-x+6} + \frac{1}{2}\int \frac{1}{\sqrt{\left(\frac{5}{2}\right)^2-\left(x+\frac{1}{2}\right)^2}}\,dx$$

$$\boxed{I = -5\sqrt{(x+3)(2-x)} + \frac{1}{2}\sin^{-1}\left(\frac{2x+1}{5}\right) + C}$$ Ans

Qn 11 →
$$I = \int_0^{\pi/2} \frac{\cos^2 x}{\cos^2 x + 4\sin^2 x}\,dx$$

divide N and D by $\cos^4 x$

$$I = \int_0^{\pi/2} \frac{\sec^2 x\,dx}{\sec^2 x + 4\tan^2 x\sec^2 x}$$

$$= \int_0^{\pi/2} \frac{\sec^2 x\,dx}{\sec^2 x\,(1+4\tan^2 x)}$$

$$I = \int_0^{\pi/2} \frac{\sec^2 x\,dx}{(1+\tan^2 x)(1+4\tan^2 x)}$$

put $\tan x = t$ | when $x=0$; $t=0$

$\sec^2 x\,dx = dt$ | when $x=\pi/2$; $t=\infty$

$$I = \int_0^\infty \frac{dt}{(1+t^2)(1+4t^2)}$$

let $t^2 = y$

$$\therefore \frac{1}{(1+t^2)(1+4t^2)} = \frac{1}{(1+y)(1+4y)}$$

let $\frac{1}{(1+y)(1+4y)} = \frac{A}{1+y} + \frac{B}{1+4y}$

$$\Rightarrow 1 = A(1+4y) + B(1+y)$$

Comp coff of y and constant term

$$0 = 4A + \cancel{B}$$
$$1 = A + \cancel{B}$$
$$-1 = 3A$$

$\boxed{A = -\frac{1}{3}}$ & $\boxed{B = \frac{4}{3}}$

$$\therefore I = \int_0^\infty \frac{-1}{3(1+t^2)} + \frac{4}{3} \cdot \frac{1}{(1+4t^2)} \, dt$$

$$= -\frac{1}{3}\int_0^\infty \frac{1}{1+t^2}\,dt + \frac{\cancel{4}}{3} \cdot \frac{1}{\cancel{4}}\int_0^\infty \frac{1}{\left(\frac{1}{2}\right)^2 + t^2}\,dt$$

$$= -\frac{1}{3}\left(\tan^{-1} t\right)_0^\infty + \frac{1}{3} \times 2 \left[\tan^{-1}(2t)\right]_0^\infty$$

$$= -\frac{1}{3}\left(\frac{\pi}{2} - 0\right) + \frac{2}{3}\left(\frac{\pi}{2} - 0\right)$$

$$= -\frac{\pi}{6} + \frac{\pi}{3}$$

$\boxed{I = \pi/6}$ ANS

Ans. 12 $\quad 2x+y=4 \quad \ldots (1)$

$3x-2y=6 \quad \ldots (2)$

$x-3y=-5 \quad \ldots (3)$

Solving (1) & (2)

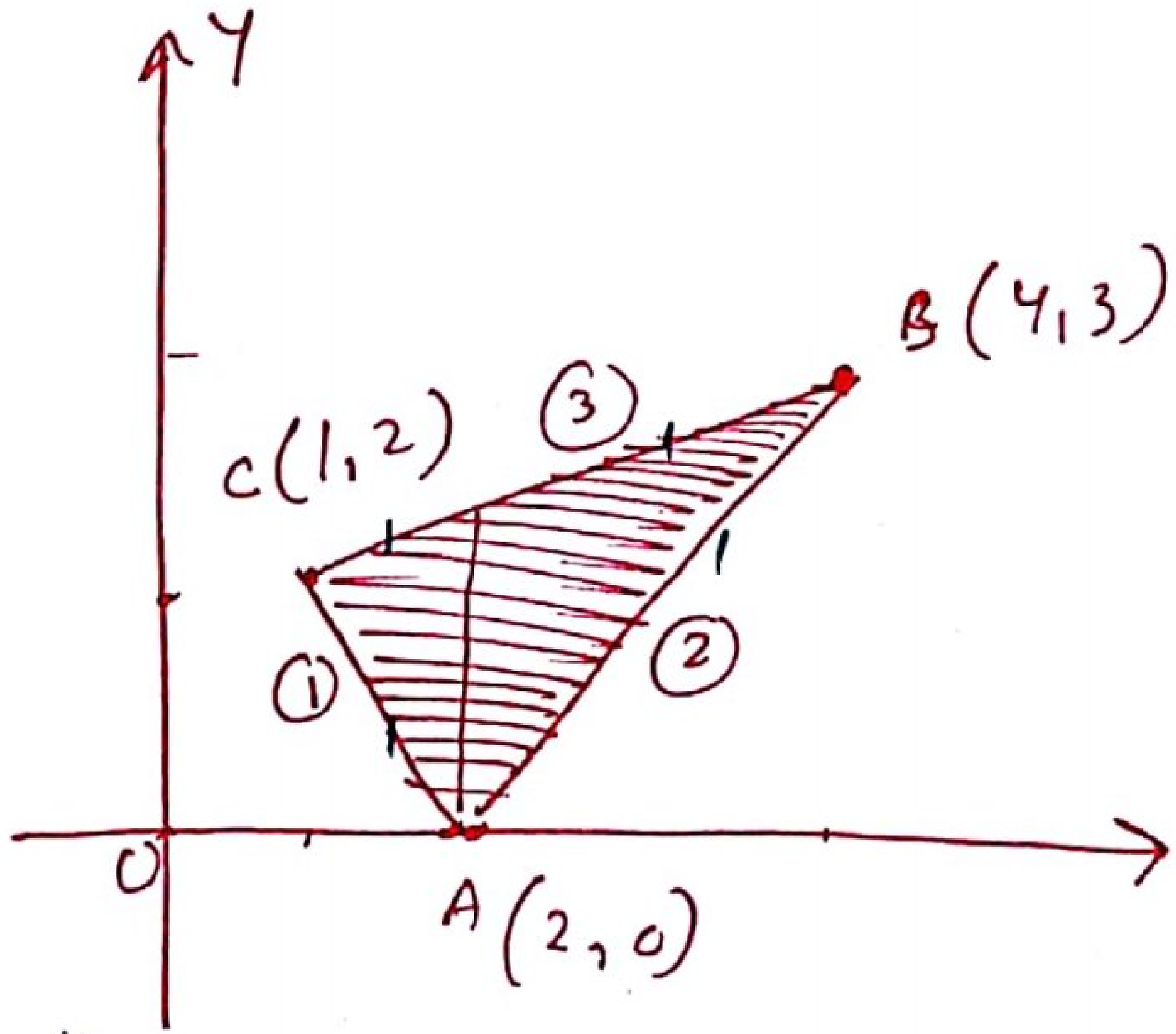
$2x+y=4$
$3x-2y=6$
$4x+2y=8$
$7x=14$
$x=2 \;;\; y=0$
$A(2,0)$

Solving (2) & (3)

$3x-2y=6$
$3x-9y=-15$
$7y=21$
$y=3 \;,\; x=4$
$\therefore B(4,3)$

Solving (1) & (3)

$2x+y=4$
$2x-6y=-10$
$7y=14$
$y=2 \;;\; x=1$
$\therefore C(1,2)$



Required area

$$= \int_1^2 \left(\frac{x+5}{3}\right)-(4-2x)\,dx + \int_2^4 \left(\frac{x+5}{3}\right)-\left(\frac{3x-6}{2}\right)dx$$

$$= \int_1^2 \frac{x+5-12+6x}{3}\,dx + \int_2^4 \frac{2x+10-9x+18}{6}\,dx$$

$$= \frac{1}{3}\int_1^2 (7x-7)\,dx + \frac{1}{6}\int_2^4 (28-7x)\,dx$$

$$= \frac{7}{3}\int_1^2 (x-1)\,dx + \frac{7}{6}\int_2^4 (4-x)\,dx$$

$$= \frac{7}{3}\left(\frac{x^2}{2}-x\right)_1^2 + \frac{7}{6}\left(4x-\frac{x^2}{2}\right)_2^4$$

$$= \frac{7}{3}\left[(2-2)-\left(\frac{1}{2}-1\right)\right] + \frac{7}{6}\left[(16-8)-(8-2)\right]$$

$$= \frac{7}{3}\left[\frac{1}{2}\right] + \frac{7}{6}\left[8-6\right]$$

$= \frac{7}{6} + \frac{14}{6} = \frac{21}{6} = \frac{7}{2}$

$\therefore$ $\boxed{\text{Required area} = \frac{7}{2} \text{ square units}}$ Ans

Ques 13 → $\{(x,y): 0 \le y \le x^2+1;\ 0 \le y \le x+1;\ 0 \le x \le 2\}$

(·) $x^2+1 \geq y$

$x^2 \geq y-1$

parabola; vertex $(0,1)$ solution outside the parabola

(·) $y \le x+1$

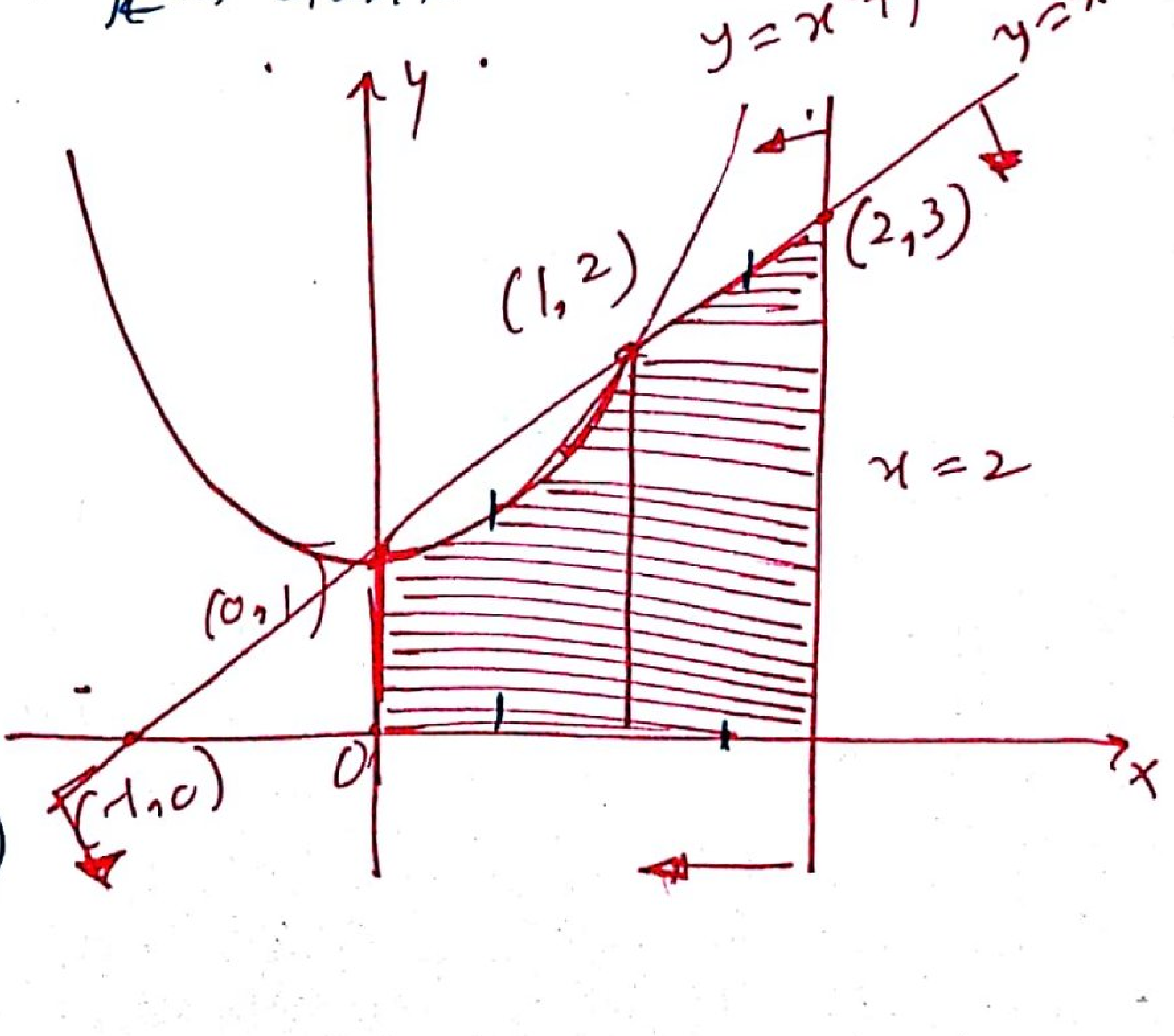
line; points $(0,1)$, $(-1,0)$; solution towards origin

(·) $x \le 2$

line; parallel to y-axis; solution towards origin

(·) $x \geq 0$; $y \geq 0$;

solution in I^{st} Quadrant



Required area

$= \int_0^1 (x^2+1)\,dx + \int_1^2 (x+1)\,dx$

$= \left(\frac{x^3}{3}+x\right)_0^1 + \left(\frac{x^2}{2}+x\right)_1^2$

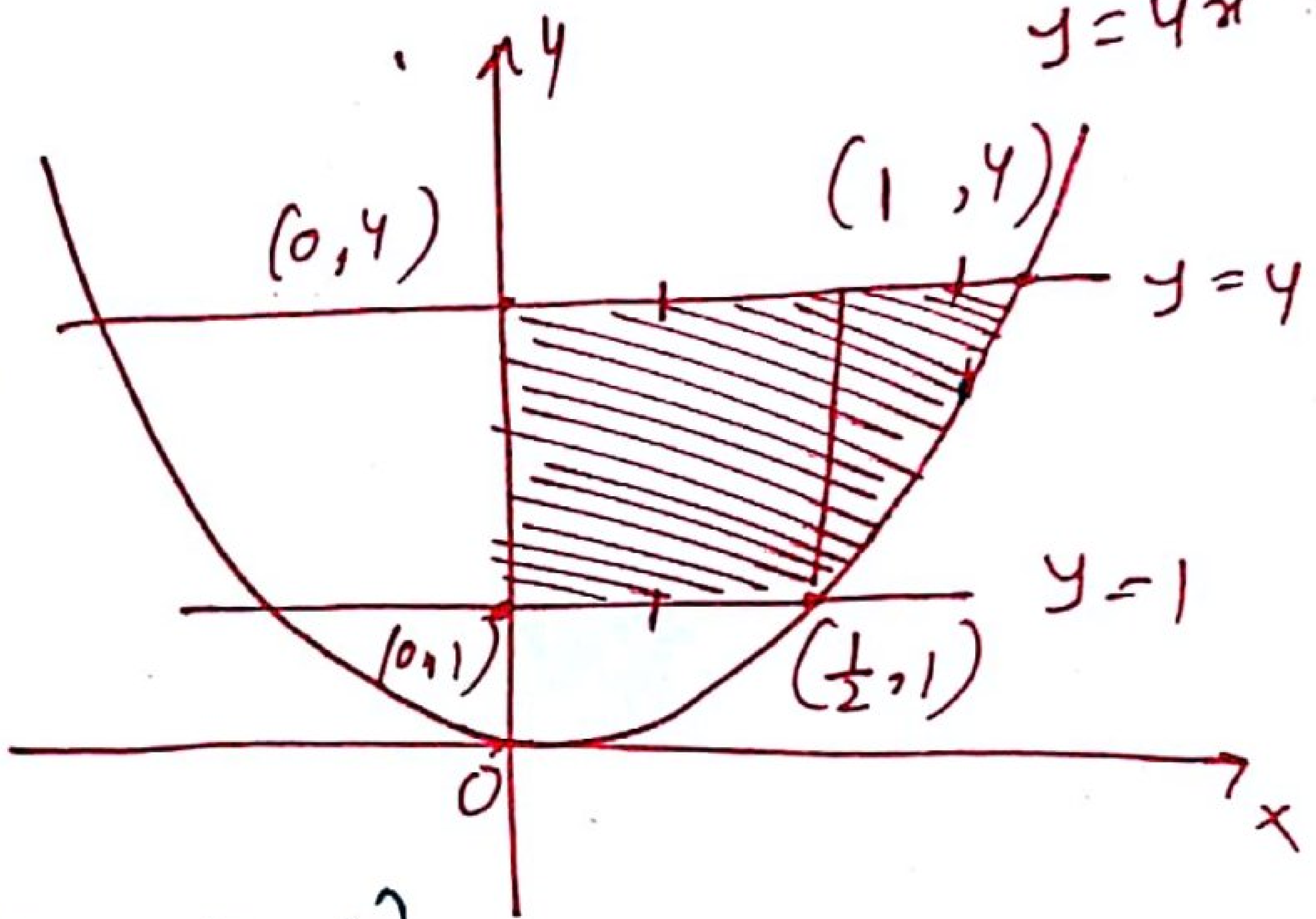
$= \left(\frac{1}{3}+1\right) - 0 + (2+2) - \left(\frac{1}{2}+1\right)$

$= \frac{4}{3} + 4 - \frac{3}{2}$

$= \frac{8+24-9}{6} = \frac{23}{6}$

$\therefore$ $\boxed{\text{Required area} = \frac{23}{6} \text{ square units}}$ Ans

Qn 14 →
(·) $y = 4x^2$
$\Rightarrow 4x^2 = y$ parabola ; vertex (0,0)
(·) $x = 0$; equation of Y-axis
(·) $y = 1$; line parallel to X-axis
(·) $y = 4$; line parallel to X-axis



Required area

$= \int_0^{1/2} (4-1)dx + \int_{1/2}^{1} (4-4x^2)dx$

$= (3x)_0^{1/2} + \left(4x - \frac{4x^3}{3}\right)_{1/2}^{1}$

$= \left(\frac{3}{2} - 0\right) + \left[\left(4 - \frac{4}{3}\right) - \left(\frac{4}{2} - \frac{4}{3\times 8}\right)\right]$

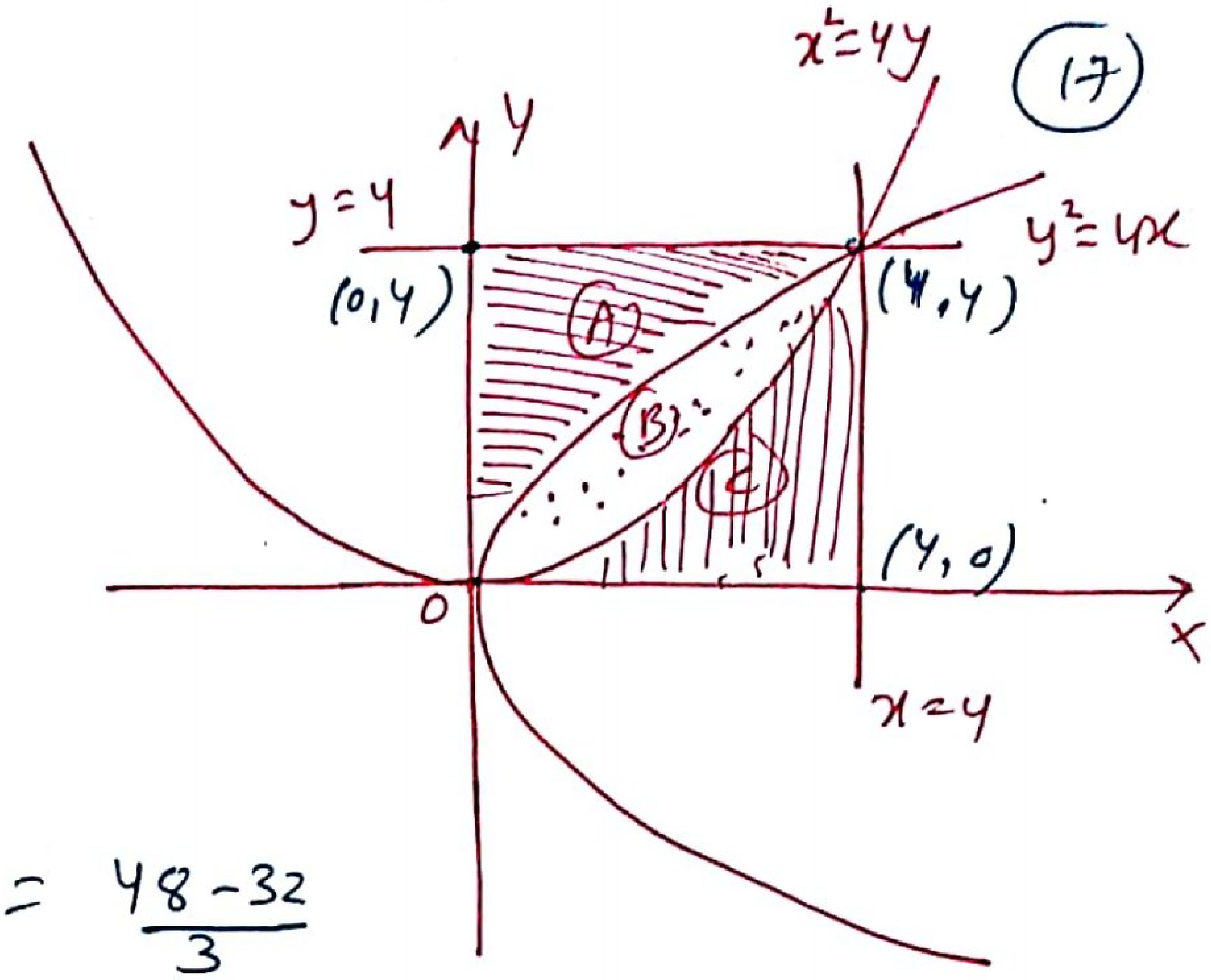
$= \frac{3}{2} + \frac{8}{3} - 2 + \frac{1}{6}$

$= \frac{9+16-12+1}{6}$

$= \frac{14}{6} = 7/3$

∴ $\boxed{\text{Required area} = 7/3 \text{ square units}}$ Ans

Qn 15 → $y^2 = 4x$: parabola , vertex (0,0) open +ve X-axis
$x^2 = 4y$: parabola vertex (0,0) open +ve Y-axis
$x = 0$: equation of Y-axis
$x = 4$: line parallel to Y-axis
$y = 0$: equation of X-axis
$y = 4$: line parallel to X-axis



Area of Region A

$= \int_0^4 4-2\sqrt{x}\,dx$

$= \left(4x - \frac{4}{3}x^{3/2}\right)_0^4$

$= \left(16 - \frac{4}{3}\times 8\right) - (0) = \frac{48-32}{3}$

$= \frac{16}{3}$

$\therefore$ Area of Region A $= \frac{16}{3}$ square units

Area of Region B $= \int_0^4 2\sqrt{x} - \frac{x^2}{4}\,dx$

$= \left[\frac{4}{3}(x)^{3/2} - \frac{x^3}{12}\right]_0^4$

$= \frac{4}{3}(8) - \frac{64}{12} = \frac{32}{3} - \frac{16}{3} = \frac{16}{3}$

$\therefore$ Area of Region $= \frac{16}{3}$ square units

Area of Region C

$= \int_0^4 \frac{x^2}{4}\,dx$

$= \left(\frac{x^3}{12}\right)_0^4$

$= \frac{64}{12} = \frac{16}{3}$ square units

$\therefore$ two curves divide the area of square in to three equal parts Ans

Qn. 16 → $I = \int_0^{\pi/2} \sin(2x)\,\tan^{-1}(\sin x)\,dx$

$$I = 2\int_0^{\pi/2} \sin x . \cos x \cdot \tan^{-1}(\sin x)\,dx$$

put $\sin x = t$ | when $x = 0$; $t = 0$

$(\cos x)\,dx = dt$ | when $x = \frac{\pi}{2}$; $t = 1$

$$\therefore I = 2\int_0^1 \underset{\text{II}}{t} \cdot \underset{\text{I}}{\tan^{-1}t}\,dt$$

$$= 2\left[\left(\tan^{-1}t \cdot \frac{t^2}{2}\right)_0^1 - \frac{1}{2}\int_0^1 \frac{1}{1+t^2} \cdot t^2\,dt\right]$$

$$= 2\left[\left(\frac{\pi}{4} \cdot \frac{1}{2} - 0\right) - \frac{1}{2}\int_0^1 \frac{t^2+1-1}{1+t^2}\,dt\right]$$

$$= 2\left[\frac{\pi}{8} - \frac{1}{2}\int_0^1 1 - \frac{1}{1+t^2}\,dt\right]$$

$$= 2\left[\frac{\pi}{8} - \frac{1}{2}\left[\left(t - \tan^{-1}t\right)\right]_0^1\right]$$

$$= \frac{\pi}{4} - \left(1 - \frac{\pi}{4}\right)$$

$$= \frac{\pi}{4} - 1 + \frac{\pi}{4}$$

$\boxed{I = \frac{\pi}{2} - 1}$ Ans

Ans 17 $I = \int \frac{1}{x^{1/2} + x^{1/3}}\, dx$

put $x = t^6$

$dx = 6t^5\, dt$

$$I = 6\int \frac{t^5\, dt}{t^3 + t^2}$$

$$= 6\int \frac{t^3}{t+1}\, dt$$

$$= 6\int (t^2 - t + 1) - \frac{1}{t+1}\, dt$$

$$= 6\left[\frac{t^3}{3} - \frac{t^2}{2} + t - \log|t+1|\right] + c$$

$$= 6\left[\frac{x^{1/2}}{3} - \frac{x^{1/3}}{2} + x^{1/6} - \log|x^{1/6}+1|\right] + c \quad \text{Ans}$$

$$\begin{array}{r|l} & t^2 - t + 1 \\ t+1 & t^3 \\ & -(t^3 + t^2) \\ & -t^2 \\ & -(-t^2 - t) \\ & t \\ & -(t+1) \\ & -1 \end{array}$$

Qn. 18 $I = \int_1^5 |x-2| + |x-4| + |x-3|\, dx$

$$I = \int_1^5 |x-2| + |x-3| + |x-4|\, dx$$

$1 \leftarrow 2 \longleftrightarrow 3 \longleftrightarrow 4 \rightarrow$

$$I = \int_1^2 -(x-2) - (x-3) - (x-4)\, dx + \int_2^3 (x-2) - (x-3) - (x-4)\, dx$$

$$+ \int_3^4 (x-2) + (x-3) - (x-4)\, dx + \int_4^5 (x-2) + (x-3) + (x-4)\, dx$$

$$I = \int_1^2 (-3x + 9)\, dx + \int_2^3 (-x + 5)\, dx + \int_3^4 (x-1)\, dx + \int_4^5 (3x - 9)\, dx$$

$=$

$$I = \left(-\frac{3x^2}{2} + 9x\right)\Big|_1^2 + \left(-\frac{x^2}{2} + 5x\right)\Big|_2^3 + \left(\frac{x^2}{2} - x\right)\Big|_3^4 + \left(\frac{3x^2}{2} - 9x\right)\Big|_4^5$$

$$= (-6+18) - \left(-\frac{3}{2} + 9\right) + \left(-\frac{9}{2} + 15\right) - (-2+10) + (8-4) - \left(\frac{9}{2} - 3\right) + \left(\frac{75}{2} - 45\right) - (24-36)$$

$$= 12 - \frac{15}{2} + \frac{21}{2} - 8 + 4 - \frac{3}{2} - \frac{15}{2} + 12$$

$$= 20 - \frac{12}{2}$$

$$= \frac{28}{2} = 14$$

$\therefore \boxed{I = 14}$ Ans