!! जय श्री राधे कृष्णा !!

← ULTIMATE MATHEMATICS: BY AJAY MITTAL,

CHAPTER:- INTEGRATION $\boxed{\text{CLASS NO: 4}}$

Type $\boxed{\text{Take common and then put} = t}$

Q=1

$$I = \int \frac{(x^4-1)^{1/4}}{x^6}\,dx$$

$$= \int \frac{x\left(1-\frac{1}{x^4}\right)^{1/4}}{x^6}\,dx$$

$$= \int \frac{\left(1-\frac{1}{x^4}\right)^{1/4}}{x^5}\,dx$$

put $1-\frac{1}{x^4} = t$

$$\Rightarrow \frac{4}{x^5}\,dx = dt \Rightarrow \frac{dx}{x^5} = \frac{dt}{4}$$

$$\therefore I = \frac{1}{4}\int t^{1/4}\,dt$$

$$= \frac{1}{4}\times\frac{4}{5}(t)^{5/4} + C$$

$$= \frac{1}{5}\left(1-\frac{1}{x^4}\right)^{5/4} + C \quad \text{Ans}$$

Q.2

$$I = \int \frac{1}{x(x^4+x)^{3/4}}\,dx$$

$$= \int \frac{1}{x^4\left(1+\frac{1}{x^3}\right)^{3/4}}\,dx$$

put $1+\frac{1}{x^3}=t$

$$-\frac{3}{x^4}dx = dt \Rightarrow \frac{dx}{x^4} = \frac{-dt}{3}$$

$$\therefore I = \frac{-1}{3}\int \frac{dt}{t^{3/4}}$$

$$= \frac{-1}{3}\int t^{-3/4}\,dt$$

$$= -\frac{1}{3}\times 4\,t^{1/4} + C$$

$$= \frac{-4}{3}\left(1+\frac{1}{x^3}\right)^{1/4} + c \quad \underline{\underline{\text{Ans}}}$$

Qu. 3

$$I = \int \frac{1}{x(x^n-1)}\,dx$$

$$= \int \frac{1}{x^{n+1}\left(1-\frac{1}{x^n}\right)}\,dx$$

put $1-\frac{1}{x^n}=t$

$$\frac{n}{x^{n+1}}\,dx = dt \Rightarrow \frac{dx}{x^{n+1}} = \frac{dt}{n}$$

$$\therefore I = \frac{1}{n}\int \frac{1}{t}\,dt$$

$$= \frac{1}{n}\log\left|1-\frac{1}{x^n}\right| + c \quad \underline{\underline{\text{Ans}}}$$

Type. Miscellaneous Ques

Que. 4 If $f'(x) = 4x^3 - \frac{3}{x^4}$ such that $f(2) = 0$ then find $f(x)$

Sol $\boxed{f(x) = \int f'(x)\,dx}$

$$f(x) = \int 4x^3 - 3x^{-4}\,dx$$

$$= \frac{4x^4}{4} - \frac{3x^{-3}}{-3} + C$$

$$f(x) = x^4 + \frac{1}{x^3} + c$$

given $f(2) = 0$

$$\Rightarrow 0 = 16 + \frac{1}{8} + C$$

$$0 = \frac{129}{8} + C \Rightarrow C = -\frac{129}{8}$$

$$\therefore f(x) = x^4 + \frac{1}{x^3} - \frac{129}{8}$$ Ans

Que. 5 → $I = \int \frac{1}{1+\tan x}\,dx$

$$I = \int \frac{1}{1+\frac{\sin x}{\cos x}}\,dx$$

$$= \int \frac{\cos x}{\cos x + \sin x}\,dx$$

$$= \frac{1}{2}\int \frac{2\cos x}{\cos x + \sin x}\,dx$$

$$= \frac{1}{2}\int \frac{\cos x + \cos x + \sin x - \sin x}{\cos x + \sin x}\,dx$$

$$I = \frac{1}{2}\int \frac{(\cos x + \sin x) + (\cos x - \sin x)}{\cos x + \sin x}\,dx$$

$$= \frac{1}{2}\int \frac{\cancel{\cos x + \sin x}}{\cancel{\cos x + \sin x}}\,dx + \frac{1}{2}\int \frac{\cos x - \sin x}{\cos x + \sin x}\,dx$$

put $\cos x + \sin x = t$

$\Rightarrow (-\sin x + \cos x)\,dx = dt$

$$= \frac{1}{2}x + \frac{1}{2}\int \frac{dt}{t}$$

$$I = \frac{1}{2}x + \frac{1}{2}\log|\cos x + \sin x| + C \quad \text{Ans}$$

Q.n 6 $I = \int \frac{1}{1+e^{x}}\,dx$

$$= \int \frac{1}{1+\frac{1}{e^{-x}}}\,dx$$

$$= \int \frac{e^{-x}}{e^{-x}+1}\,dx$$

put $e^{-x} + 1 = t$

$-e^{-x}\,dx = dt \Rightarrow e^{-x}\,dx = -dt$

$$\therefore I = -\int \frac{dt}{t}$$

$$= -\log|e^{-x}+1| + C \quad \text{Ans}$$

Q.n 7 $I = \int \frac{1}{\sqrt{1-3e^{-2x}}}\,dx$

$\sqrt{e^{2x}} = e^{x}$

$$= \int \frac{1}{\sqrt{1-\frac{3}{e^{2x}}}}\,dx$$

$$\Rightarrow \int \frac{e^x}{\sqrt{e^{2x}-3}}\,dx \qquad e^{2x}=(e^x)^2$$

put $e^x = t$

$e^x dx = dt$

$$I = \int \frac{dt}{\sqrt{t^2-3}} = \int \frac{dt}{\sqrt{t^2-(\sqrt{3})^2}}$$

$$= \log\left|t+\sqrt{t^2-3}\right|+C$$

$$I = \log\left|e^x+\sqrt{e^{2x}-3}\right|+C \quad \text{Ans}$$

Qn. 8 → $I = \int \frac{\sqrt{\tan x}}{\sin x \cdot \cos x}\,dx$

~~Put~~ Divide N & D by $\cos^2 x$

$$I = \int \frac{\sqrt{\tan x}\,.\sec^2 x}{\tan x}\,dx$$

put $\tan x = t \Rightarrow \sec^2 x\,dx = dt$

$$I = \int \frac{\sqrt{t}}{t}\,dt$$

$$= \int \frac{1}{\sqrt{t}}\,dt$$

$$= 2\sqrt{t}+C$$

$$= 2\sqrt{\tan x}+C \quad \text{Ans}$$

Qn. 9 → $I = \int \frac{\cos(2x)}{(\cos x+\sin x)^2}\,dx$

$$= \int \frac{\cos^2 x-\sin^2 x}{(\cos x+\sin x)^2}\,dx = \int \frac{\cos x-\sin x}{\cos x+\sin x}\,dx \quad \text{Proceed}$$

Ans 10

$$I = \int \frac{\cos(2x) - \cos(2\alpha)}{\cos x - \cos\alpha}\, dx$$

$$= \int \frac{2\cos^2 x \cancel{-1} - 2\cos^2\alpha \cancel{+1}}{\cos x - \cos\alpha}\, dx$$

$$= 2\int \frac{\cos^2 x - \cos^2\alpha}{\cos x - \cos\alpha}\, dx$$

$$= 2\int \cos x + \cos\alpha \, dx$$

$$= 2\left[\sin x + x\cos\alpha\right] + C \quad \text{Ans}$$

Ans 11 →

$$I = \int e^{3\log x}\,(x^4+1)^{-1}\, dx$$

$$\boxed{e^{\log x} = x,\quad e^{3\log x} = e^{\log x^3} = x^3}$$

$$= \int \frac{x^3}{x^4+1}\, dx$$

put $x^4 + 1 = t$

$4x^3 dx = dt \Rightarrow x^3 dx = \frac{dt}{4}$

$$I = \frac{1}{4}\int \frac{dt}{t} \quad \text{(follow)}$$

Ans 12

$$I = \int \frac{1}{\sqrt{\sin^3 x \cdot \sin(x+\alpha)}}\, dx$$

$$= \int \frac{1}{\sqrt{\sin^3 x \cdot (\sin x\cos\alpha + \cos x\sin\alpha)}}\, dx$$

$$= \int \frac{1}{\sqrt{\sin^4 x \cdot (\cos\alpha + \cot x \cdot \sin\alpha)}}\, dx$$

$$I = \int \frac{\operatorname{cosec}^2 x}{\sqrt{\cos\alpha + \cot x \cdot \sin\alpha}}\, dx$$

put $\cos\alpha + \cot x \cdot \sin\alpha = t$

$$-\operatorname{cosec}^2 x \cdot \sin\alpha \, dx = dt$$

$$\operatorname{cosec}^2 x \, dx = -\frac{dt}{\sin\alpha}$$

$$\therefore I = \frac{-1}{\sin\alpha} \int \frac{dt}{\sqrt{t}}$$

$$= \frac{-1}{\sin\alpha} \cdot 2\sqrt{\cos\alpha + \cot x \sin\alpha} + c \quad \underline{\text{Ans}}$$

<u>Qu. 13</u> → $I = \int \frac{\sin^8 x - \cos^8 x}{1 - 2\sin^2 x \cdot \cos^2 x}\, dx$

$$= \int \frac{(\sin^4 x + \cos^4 x)(\sin^4 x - \cos^4 x)}{1 - 2\sin^2 x \cos^2 x}\, dx$$

$$= \int \frac{(\sin^4 x + \cos^4 x)(\sin^2 x + \cos^2 x)(\sin^2 x - \cos^2 x)}{1 - 2\sin^2 x \cos^2 x}\, dx$$

$$= \int \frac{(\sin^4 x + \cos^4 x) \cdot (\sin^2 x - \cos^2 x)}{(\sin^2 x + \cos^2 x)^2 - 2\sin^2 x \cos^2 x}\, dx$$

$$= \int \frac{(\sin^4 x + \cos^4 x)(\sin^2 x - \cos^2 x)}{\sin^4 x + \cos^4 x + \cancel{2\sin^2 x \cos^2 x} - \cancel{2\sin^2 x \cos^2 x}}\, dx$$

$$= \int (\sin^2 x - \cos^2 x)\, dx$$

$$= -\int \cos(2x)\, dx = -\frac{\sin(2x)}{2} + c \quad \underline{\text{Ans}}$$

Qn 14 $I = \int \frac{\cos x - \sin x}{1 + \sin(2x)}\, dx$

$= \int \frac{\cos x - \sin x}{\sin^2 x + \cos^2 x + 2\sin x \cos x}\, dx$

$= \int \frac{\cos x - \sin x}{(\sin x + \cos x)^2}\, dx$

put $\sin x + \cos x = t$

$(\cos x - \sin x)\, dx = dt$

$I = \int \frac{dt}{t^2}$

$= -\frac{1}{\sin x + \cos x} + c$ Ans

Qn. 15 $I = \int \frac{1}{x - \sqrt{x}}\, dx$

$= \int \frac{1}{\sqrt{x}(\sqrt{x} - 1)}\, dx$

put $\sqrt{x} - 1 = t$

$\frac{1}{2\sqrt{x}}\, dx = dt \Rightarrow \frac{dx}{\sqrt{x}} = 2dt$

$\therefore I = 2\int \frac{dt}{t}$

$= 2\log|\sqrt{x} - 1| + c$ Ans

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Qn 16 →

$$I = \int \cos^{-1}(\sin x)\,dx$$

$$= \int \cos^{-1}(\cos(\tfrac{\pi}{2}-x))$$

$$= \int (\tfrac{\pi}{2}-x)\,dx$$

$$= \frac{x\pi}{2} - \frac{x^2}{2} + C \quad \text{Ans}$$

Qn 17 →

$$I = \int 2^{2^{2^x}} \cdot 2^{2^x} \cdot 2^x\,dx$$

put $2^{2^{2^x}} = t$

$$\Rightarrow 2^{2^{2^x}} \cdot \log 2 \cdot 2^{2^x} \cdot \log 2 \cdot 2^x \cdot \log 2\,dx = dt$$

$$\Rightarrow 2^{2^{2^x}} \cdot 2^{2^x} \cdot 2^x (\log 2)^3 dx = dt$$

$$\Rightarrow 2^{2^{2^x}} \cdot 2^{2^x} \cdot 2^x\,dx = \frac{dt}{(\log 2)^3}$$

$$\therefore I = \frac{1}{(\log 2)^3}\int dt$$

$$= \frac{1}{(\log 2)^3} \cdot t + C$$

$$= \frac{1}{(\log 2)^3} \cdot 2^{2^{2^x}} + C \quad \text{Ans}$$

Qn-18 → $I = \int \frac{(e^{2x}-1)}{e^{2x}+1}\,dx$

$= \int \frac{\cancel{e^{x}}(e^{x}-e^{-x})}{\cancel{e^{x}}(e^{x}+e^{-x})}\,dx$

put $e^{x}+e^{-x} = t$

$(e^{x}-e^{-x})\,dx = dt$

$I = \int \frac{dt}{t} = \log|t| + c$

$= \log|e^{x}+e^{-x}| + c$ Ans

Qn-19 → $I = \int \frac{\sin(2x)}{a^2\sin^2 x + b^2\cos^2 x}\,dx$

put $a^2\sin^2 x + b^2\cos^2 x = t$

$a^2\sin(2x) - b^2\sin(2x)\,dx = dt$

$\sin(2x)\,dx = \frac{dt}{a^2-b^2}$

$I = \frac{1}{a^2-b^2}\int \frac{dt}{t}$

$= \frac{1}{a^2-b^2}\log|a^2\sin^2 x + b^2\cos^2 x| + c$ Ans

$\sin(2x)\,dx$

$\sin^2 x = t$

$\rightarrow 2\sin x\cos x\,dx = dt$

$\sin(2x)\,dx = dt$

$\cos^2 x = t$

$\rightarrow -2\cos x\sin x\,dx = dt$

$\sin(2x)\,dx = -dt$

Qn-20 → $\int \tan(x-\alpha)\tan(x+\alpha)\tan(2x)\,dx$

Sol $2x = (x+\alpha) + (x-\alpha)$

$\Rightarrow \tan(2x) = \tan\big(\overset{A}{(x+\alpha)} + \overset{B}{(x-\alpha)}\big)$

$$\Rightarrow \tan(2x) = \frac{\tan(x+\theta) + \tan(x-\theta)}{1 - \tan(x+\theta)\tan(x-\theta)}$$

$$\Rightarrow \tan(2x) - \tan(2x)\tan(x+\theta)\tan(x-\theta) = \tan(x+\theta) + \tan(x-\theta)$$

$$\Rightarrow \tan(2x) - \tan(x+\theta) - \tan(x-\theta) = \tan(2x)\tan(x+\theta)\cdot\tan(x-\theta)$$

$$\therefore I = \int \tan(2x) - \tan(x+\theta) - \tan(x-\theta)\,dx$$

$$= \frac{1}{2}\log|\sec(2x)| - \log|\sec(x+\theta)| - \log|\sec(x-\theta)| + C$$

Ans