(D-3)

॥ जय श्री गिरिराज जी महाराज ॥

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ULTIMATE MATHEMATICS

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(D-3) Determinants: Class No: 3

IMP Topic: Solving system of linear equations

4M/6M

(·) Sum: $a_1x + b_1y + c_1z = d_1$

$a_2x + b_2y + c_2z = d_2$

$a_3x + b_3y + c_3z = d_3$

(·) these equations can be written in the matrix form

$$\begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & c_3 & c_3 \end{bmatrix}_{3\times 3} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}$$

A X B

$\Rightarrow \boxed{AX = B}$

$\Rightarrow A^{-1}AX = A^{-1}B$

$\Rightarrow IX = A^{-1}B$

$\Rightarrow \boxed{X = A^{-1}B}$ (Matrix Method)

(·) $A^{-1} = \frac{1}{|A|}\,\text{Adj}\,A$

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(·) Conditions of Consistency:

(1) If $|A| \neq 0$

∴ System is Consistent ✓

∴ Unique Solution

(2) If $|A| = 0$ and $(\text{Adj}A)\,B = O$

∴ System: Consistent

∴ Infinite Many ≠ Solution:

(3) If $|A| = 0$ but $(\text{Adj}A)\,B \neq O$

∴ System: Inconsistent

∴ NO Solution:

Qn 1→ Solve the system of linear equations (By Matrix Method)

$2x+y+z=1$; $x-2y-z=3/2$; $3y-5z=9$

Sol: these equations can be written in the matrix form

$$\begin{bmatrix} 2 & 1 & 1 \\ 1 & -2 & -1 \\ 0 & 3 & -5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 3/2 \\ 9 \end{bmatrix}$$

A X B

$\Rightarrow AX = B$

$\Rightarrow A^{-1}AX = A^{-1}B$

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$\Rightarrow \quad IX = A^{-1}B$

$\Rightarrow \quad \boxed{X = A^{-1}B}$ ✓

$$|A| = \begin{vmatrix} (+) & (-) & (+) \\ 2 & 1 & 1 \\ 1 & -2 & -1 \\ 0 & 3 & -5 \end{vmatrix}$$

$|A| = 2(13) - 1(-5) + 1(3)$

$|A| = 26 + 5 + 3 = 34 \neq 0$ (Unique solution)

System (matrix).

$C_{11} = 13 \qquad C_{12} = +5 \qquad C_{13} = 3$

$C_{21} = +8 \qquad C_{22} = -10 \qquad C_{23} = -6$

$C_{31} = 1 \qquad C_{32} = +3 \qquad C_{33} = -5$

$$\text{Adj } A = \begin{bmatrix} 13 & 8 & 1 \\ 5 & -10 & 3 \\ 3 & -6 & -5 \end{bmatrix}$$

$$\text{Now } A^{-1} = \frac{1}{34}\begin{bmatrix} 13 & 8 & 1 \\ 5 & -10 & 3 \\ 3 & -6 & -5 \end{bmatrix}$$

$$X = A^{-1}B = \frac{1}{34}\begin{bmatrix} 13 & 8 & 1 \\ 5 & -10 & 3 \\ 3 & -6 & -5 \end{bmatrix}\begin{bmatrix} 1 \\ 3/2 \\ 9 \end{bmatrix}$$

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$$X = \frac{1}{34}\begin{bmatrix} 34 \\ 17 \\ -51 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 1/2 \\ -3/2 \end{bmatrix}$$

$\Rightarrow$ **$x = 1,\ y = 1/2,\ z = -3/2$** is the required solution.

Check

$= 2(1) + 1/2 - \frac{3}{2}$

$= 2 + \frac{1}{2} - 3/2$

$= 5/2 - 3/2$

$= 1$ — x —

Special Ques II

$$A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}$$

By Eqn $A^{-1} = \frac{1}{|A|}\,\text{Adj}\,A = \frac{1}{\square}\begin{bmatrix} & & \\ & & \\ & & \end{bmatrix}$

Given equations

$x + 2y + z = 4$; $-x + y + z = 0$, $x - 3y + z = 2$

these equations can be written in the form

$A'X = B$.

$$\boxed{X = (A')^{-1}B}$$

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Imp $\boxed{(A')^{-1} = (A^{-1})'}$ ✓

$X = (A^{-1})' B$

$X = \frac{1}{\square}[\quad][\quad]$

$X = \begin{bmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{bmatrix}$

$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{bmatrix}$

$\boxed{x = \quad , \quad y = \quad , \quad z = }$ Solution.

Special Case: Let $A = \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$

$C = \begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}$

Now $CA = \begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix} \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$

$CA = \begin{bmatrix} 8 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & 8 \end{bmatrix}$

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$$\Rightarrow CA = 8\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$$

$$\Rightarrow \boxed{CA = 8I}$$ ✓

$$CAA^{-1} = 8IA^{-1}$$

$$CI = 8A^{-1}$$

$$C = 8A^{-1}$$

$$\boxed{A^{-1} = \frac{1}{8}C}$$ ✓

$$A^{-1} = \frac{1}{8}\begin{bmatrix}-4 & 4 & 4\\ -7 & 1 & 3\\ 5 & -3 & -1\end{bmatrix}$$ ✓

Given equations

$x - y + z = 4$; $x - 2y - 2z = 9$, $2x + y + 3z = 1$

These equations can be written in the form

$$\underbrace{\begin{bmatrix}1 & -1 & 1\\ 1 & -2 & -2\\ 2 & 1 & 3\end{bmatrix}}_{A}\underbrace{\begin{bmatrix}x\\ y\\ z\end{bmatrix}}_{X} = \underbrace{\begin{bmatrix}4\\ 9\\ 1\end{bmatrix}}_{B}$$

$AX = B$.

$\Rightarrow X = A^{-1}B$

$\Rightarrow X = \frac{1}{8}\begin{bmatrix} & & \\ & & \\ & & \end{bmatrix}\begin{bmatrix}4\\ 9\\ 1\end{bmatrix} = \begin{bmatrix}=\\ =\\ =\end{bmatrix}$

$x =$, $y =$, $z =$ is the required solution.

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Q.No 1 → Examine the consistency of system of equations:
$x+3y=5$ and $2x+6y=8$

Q.No 2 → Examine the consistency of system of equations:
$3x-y-2z=2$; $2y-z=-1$ and $3x-5y=3$

Q.No 3 → Examine the consistency of system of equations:
$5x-y+4z=5$; $2x+3y+5z=2$ and $5x-2y+6z=-1$

Q.No 4 → Solve the system of linear equations using Matrix method
$4x-3y=3$ and $3x-5y=7$

Q.No 5 → Solve the system of linear equations:
$2x+y+z=1$; $x-2y-z=\frac{3}{2}$ and $3y-5z=9$

Q.No 6 → Solve the system of linear equations
$x-y+2z=7$; $4y+3x=5z-5$ and $2x+3z=12+y$

Q.No 7 → Solve the system of linear equations
$\frac{2}{x}+\frac{3}{y}+\frac{10}{z}=4$; $\frac{4}{x}-\frac{6}{y}+\frac{5}{z}=1$ & $\frac{6}{x}+\frac{9}{y}-\frac{20}{z}=2$

Q.No 8 → Solve the system of linear equations
$2yz-3xz+3xy=10xyz$; $yz+xz+xy=10xyz$
and $3yz-xz+2xy=13xyz$

Q.No 9 → If $A=\begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$, find A^{-1}. Using A^{-1} solve the system of equations $2x-3y+5z=11$;
$3x+2y-4z=-5$ and $x+y-2z=-3$

Q.No 10 → The sum of three numbers is 6. If we multiply third number by 3 and add

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second number to it, we get 11. By adding first and third numbers, we get double of the second number. Represent it algebraically and find the numbers using matrix method.

Q.N. 11 → If $A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}$ find A^{-1} and hence solve the system of equations

$x + 2y + z = 4$; $-x + y + z = 0$, $x - 3y + z = 2$

Q.N. 12 → The cost of 4 kg onion, 3 kg wheat and 2 kg rice is Rs 60. The cost of 2 kg onion, 4 kg wheat and 6 kg rice is Rs 90. The cost of 6 kg onion, 2 kg wheat and 3 kg rice is Rs 70. Find cost of each item per kg by matrix Method

Q.N. 13 → Use product $\begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix} \begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix}$ to solve the system of equations $x - y + 2z = 1$, $2y - 3z = 1$ and $3x - 2y + 4z = 2$

Q.N. 14 → Determine the product $\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix} \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$ and hence solve the equations

$x - y + z = 4$; $x - 2y - 2z = 9$ & $2x + y + 3z = 1$

Q.N. 15 → If $A = \begin{bmatrix} 7 & 2 & -6 \\ -2 & 1 & -3 \\ -4 & 2 & 5 \end{bmatrix}$ & $B = \begin{bmatrix} 1 & -2 & 0 \\ 2 & 1 & 3 \\ 0 & -2 & 1 \end{bmatrix}$ Find BA and hence solve the equations $x - 2y = 10$; $2x + y + 3z = 8$ and $-2y + z = 7$.

ANSWERS

1) Consistent	4). $x = \frac{-6}{11}$, $y = \frac{-19}{11}$	7) $x=2, y=3, z=5$	10). 1, 2, 3	13) 0, 5, 3
2) Inconsistent	5). $x=1, y=1/2, z=-3/2$	8). $x=1/2, y=1/3, z=1/5$	11) 9/5, 2/5, 7/5	14) 3, −2, −1
3). Consistent	6). $x=2, y=1, z=3$	9). $x=1, y=2, z=3$	12) Rs 5, Rs 8, Rs 8	15) 4, −3, 1