!! जय श्री राधे कृष्णा !!

(1)

← Solution of worksheet No: 5 (Class-6) →

A.OD. (Maxima - Minima)

Q No 1 → $x+y=60$ ---- (given) --(i)

let $P=xy^3$ --- (to be Max)

$P=(60-y)y^3$ --- {from eq(i)}

$P=60y^3-y^4$

Diff wrt y

$\frac{dP}{dy}=180y^2-4y^3$

for Max/Min put $\frac{dP}{dy}=0$

$\Rightarrow 180y^2-4y^3=0$

$\Rightarrow 4y^2(45-y)=0$

$y=0$, $\boxed{y=45}$

not possible

Diff again wrt y

$\frac{d^2P}{dy^2}=360y-12y^2$

$\left(\frac{d^2P}{dy^2}\right)_{y=45}=360\times45-12(45)^2$

$=16200-24300=-8100<0$

$\therefore$ P is Maximum at $y=45$

put $y=45$ in eq(i)

$x+45=60$

$x=15$

$\therefore$ two numbers are 15 & 45 Ans.

Ans 2 → let $S \rightarrow$ Sum of two numbers x & y

$\therefore S=35$

$\Rightarrow 35=x+y$ ---(i)

let $P=x^2y^5$ --- (to be Max)

$\Rightarrow P = (35-y)^2 y^5$ --- {from eq(i)}

Diff wrt y

$$\frac{dP}{dy} = (35-y)^2 \cdot 5y^4 + y^5 \cdot 2(35-y)(-1)$$

$$= (35-y)y^4\left[(35-y)5 + 2y(-1)\right]$$

$$\frac{dP}{dy} = (35-y)y^4(175-7y)$$

for Max/Min put $\frac{dP}{dy} = 0$

$$\Rightarrow (35-y)(y)^4(175-7y) = 0$$

$$\underbrace{y=35 \; ; \; y=0}_{\text{Not possible}}, \; \boxed{y=25}$$

Diff again wrt y (Product rule in 3 functions)

$$\frac{d^2P}{dy^2} = (-1)y^4(175-7y) + 4y^3(35-y)(175-7y) + (-7)(35-y)y^4$$

$$\left(\frac{d^2P}{dy^2}\right)_{y=25} = 0+0-7(10)(25)^4$$

$$= -70(25)^4 < 0$$

Clearly P is Maximum at $y=25$

put $y=25$ in eq (1)

$$x+25 = 35$$

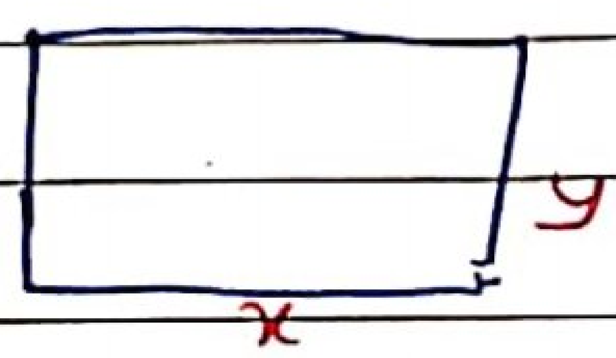
$$x = 10$$

$\therefore$ two numbers are 10 & 25 Ans

QNS 7 $\rightarrow$ let $x \rightarrow$ length of rectangle

$y \rightarrow$ breadth of rectangle

let $P \rightarrow$ perimeter of rectangle



Solution A-OD (Worksheet 5) (3)

$P = 2x + 2y$ ---(1) given

let $A \to$ Area of rectangle

$A = xy$ ... (to be Max)

$A = x\left(\frac{P-2x}{2}\right)$

$A = \frac{1}{2}(Px - 2x^2)$

Diff w.r.t x

$\frac{dA}{dx} = \frac{1}{2}(P - 4x)$

for Max/Min put $\frac{dA}{dx} = 0$

$\frac{1}{2}(P - 4x) = 0$

$\boxed{P = 4x}$ (or) $\boxed{x = P/4}$

Diff w.r.t x

$\frac{d^2A}{dx^2} = \frac{1}{2}(-4) = -2 < 0$

$\therefore$ Area of rectangle is Maximum at $x = P/4$

put $P = 4x$ in eq (1)

$4x = 2x + 2y$

$2x = 2y \Rightarrow x = y$

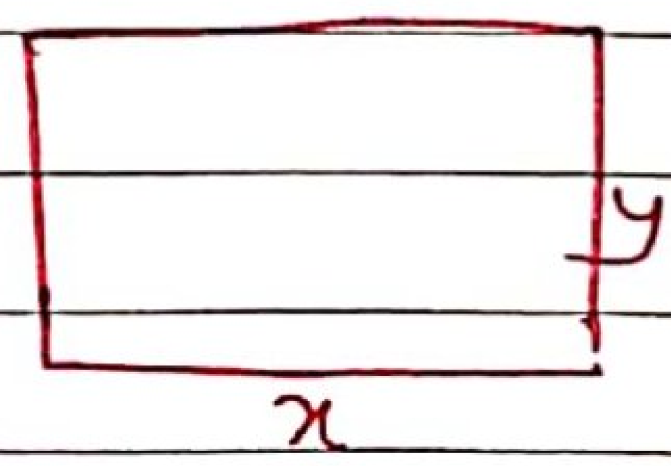
$\Rightarrow l = b$

$\therefore$ Rectangle is a square

Hence square has maximum perimeter **Ans**

Qn 4 → let $x \to$ length of rectangle

$y \to$ breadth of rectangle



$A = xy$ --- given (i)

$P = 2x + 2y$ --- (to be Minimize)

$P = 2x + \frac{2A}{x}$ --- (from eq (i))

CLASSTIME

Solution A.O.D (worksheet -5)

Diff w.r.t x

$$\frac{dP}{dx} = 2 - \frac{2A}{x^2}$$

for Max | Min put $dP/dx = 0$

$$2 - \frac{2A}{x^2} = 0$$

$\Rightarrow$ $\boxed{A = x^2}$ or $\boxed{x = \sqrt{A}}$

Diff again w.r.t x

$$\frac{d^2P}{dx^2} = +\frac{4A}{x^3}$$

$$\left(\frac{d^2P}{dx^2}\right)_{x=\sqrt{A}} = \frac{4A}{(A)\sqrt{A}} = \frac{4}{\sqrt{A}} > 0$$

$\therefore$ Perimeter of rectangle is minimum at $x = \sqrt{A}$

put $A = x^2$ in eq (1)

$\therefore x^2 = xy$

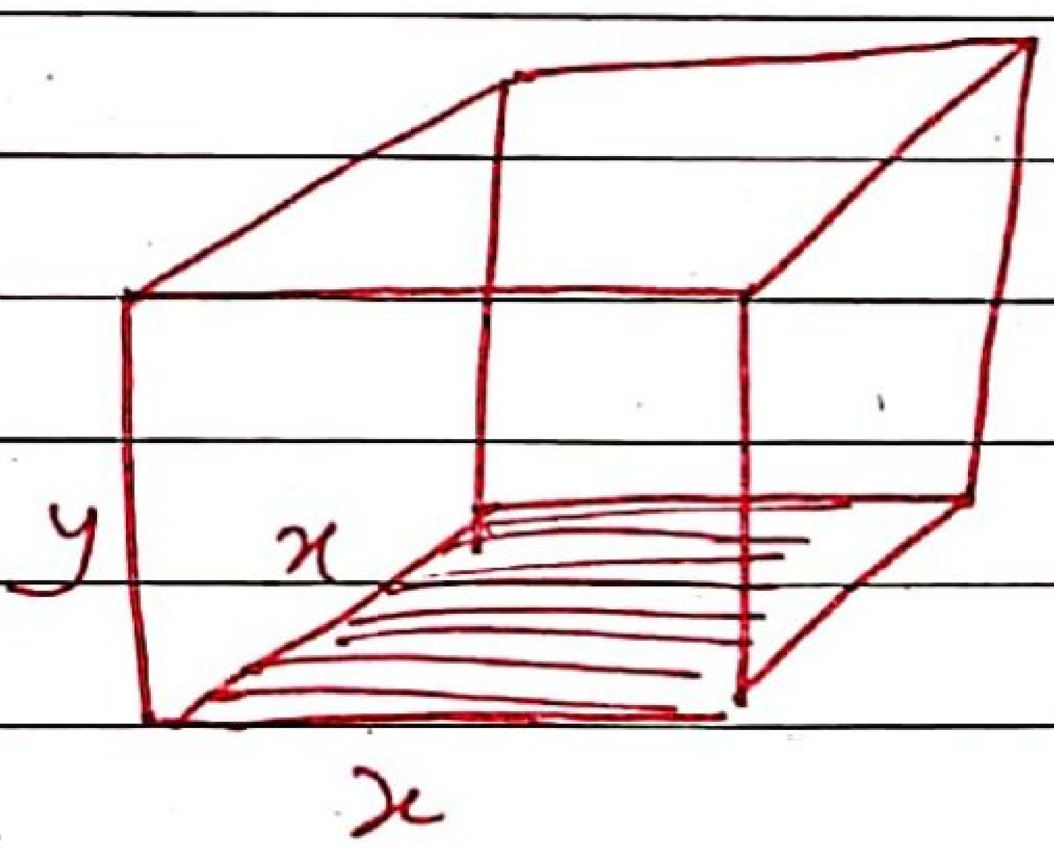
$\Rightarrow x = y$

$\therefore l = b$ $\Rightarrow$ rectangle is a square

$\therefore$ Square has minimum perimeter Ans

Qn 5 → let $x \to$ length & breadth

$y \to$ height of box



let $V \to$ volume of box (cuboid)

$V = x^2 y$ --- (given) --(i)

let $S \to$ S.A of cuboid

$$S = 2x^2 + 2xy + 2xy$$

$$S = 2x^2 + 4xy \quad \text{--- (to be Min)}$$

$$S = 2x^2 + 4x\left(\frac{V}{x^2}\right) \quad \text{--- (from eq (i))}$$

$$S = 2x^2 + \frac{4V}{x}$$

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Solution A.OD (W.S - 5) 3

D/f wrt x

$$\frac{ds}{dx} = 4x - \frac{4V}{x^2}$$

for Max / Min put $\frac{ds}{dx} = 0$

$$\cancel{4}x = \frac{\cancel{4}V}{x^2}$$

$$\boxed{x^3 = V} \quad \text{or} \quad \boxed{x = \sqrt[3]{V}}$$

D/f again

$$\frac{d^2s}{dx^2} = 4 + \frac{8V}{x^3}$$

$$\left(\frac{d^2s}{dx^2}\right)_{x=\sqrt[3]{V}} = 4 + \frac{8V}{V} = 12 > 0$$

$\therefore$ S.A of cuboid / box is Minimum at $x = \sqrt[3]{V}$

put $V = x^3$ in eq (1)

$$x^3 = x^2 y$$

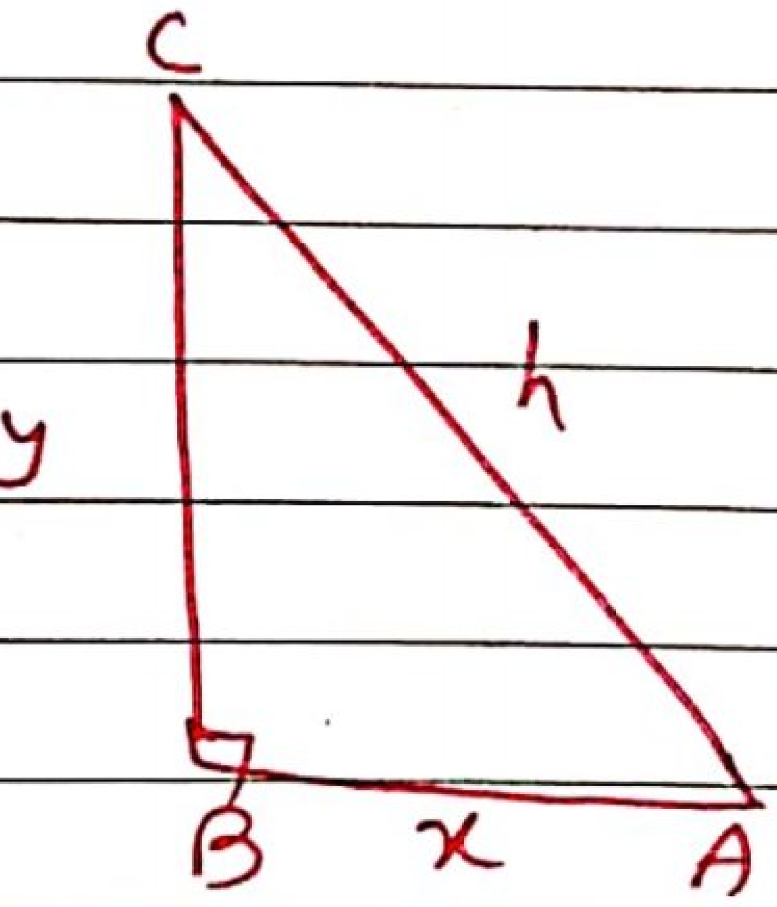
$\Rightarrow x = y$

$\Rightarrow$ length = breadth = height

$\therefore$ cuboid is a cube. Ans

Qn 6 → let $x \rightarrow$ base

$y \rightarrow$ altitude



$h \rightarrow$ hypotenuse (given)

$h^2 = x^2 + y^2$ ---- ~~(given)~~ (given) --(i)

A → Area of triangle

$$A = \frac{1}{2}xy \quad \text{--- (to be Max)}$$

$$A = \frac{1}{2}x\sqrt{h^2 - x^2} \quad \text{--- (from eq (i))}$$

Squaring

CLASSTIME

Scanned with CamScanner

Solution A.O.D (W.S - 5) (6)

$$A^2 = \frac{1}{4} x^2 (h^2 - x^2)$$

$$A^2 = \frac{1}{4} (h^2 x^2 - x^4)$$

Let $A^2 = z$ then A is Max/Min as according to z is Max/Min

$$z = \frac{1}{4} (h^2 x^2 - x^4)$$

Diff. w.r.t x

$$\frac{dz}{dx} = \frac{1}{4} (2x h^2 - 4x^3)$$

for Max/Min put $\frac{dz}{dx} = 0$

$$2x h^2 = 4x^3$$

$$h^2 = 2x^2 \Rightarrow \boxed{h = \sqrt{2} x} \text{ or } \boxed{x = \frac{h}{\sqrt{2}}}$$

Diff again w.r.t. x

$$\frac{d^2 z}{dx^2} = \frac{1}{4} (2h^2 - 12x^2)$$

$$\left(\frac{d^2 z}{dx^2}\right)_{x = \frac{h}{\sqrt{2}}} = \frac{1}{4} (2h^2 - 6h^2) = \frac{-4h^2}{4} = -h^2 < 0$$

$\therefore$ z is Max

$\therefore$ Area of ~~tri~~ triangle is Maximum at $x = h/\sqrt{2}$

put $h = \sqrt{2} x$ in eq (i)

$$2x^2 = x^2 + y^2$$

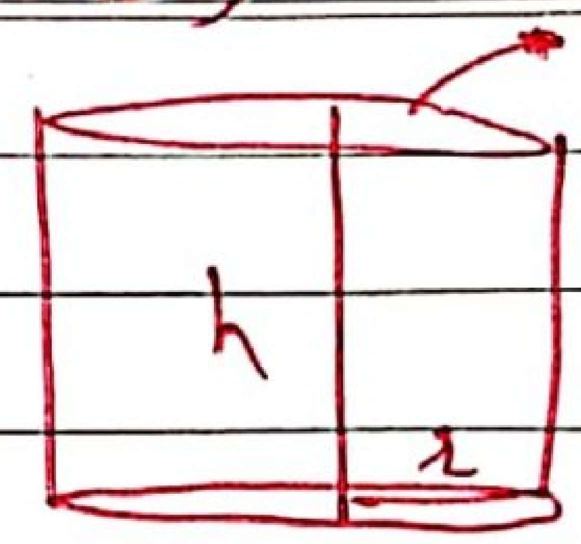
$$\Rightarrow x^2 = y^2$$

$$\Rightarrow x = y$$

$\Rightarrow$ triangle is isosceles Ans

Solution A.O.D (W.S 5)

Qn. 7 → let $h \to$ height of cone
$r \to$ Radius of cone



$V = \pi r^2 h$ --- (given) --(1)

$S = \pi r^2 + 2\pi r h$ ---(to be Minimized)

$S = \pi r^2 + 2\cancel{\pi} r\left(\frac{V}{\cancel{\pi} r^2}\right)$ --- (from eq (1))

$S = \pi r^2 + \frac{2V}{r}$

Diff w.r.t r

$\frac{dS}{dr} = 2\pi r - \frac{2V}{r^2}$

for Max/Min put $\frac{dS}{dr} = 0$

$\cancel{2}\pi r = \frac{\cancel{2}V}{r^2}$

$\Rightarrow \boxed{V = \pi r^3}$ $r^3 = \frac{V}{\pi}$

Diff again w.r.t r

$\frac{d^2S}{dr^2} = 2\pi + \frac{4V}{r^3}$

$\left(\frac{d^2S}{dr^2}\right)_{r^3 = \frac{V}{\pi}} = 2\pi + 4\pi = 6\pi > 0$

$\therefore$ T.S.A of Cylinder is Minimum at $r^3 = V/\pi$

put $V = \pi r^3$ in eq (1)

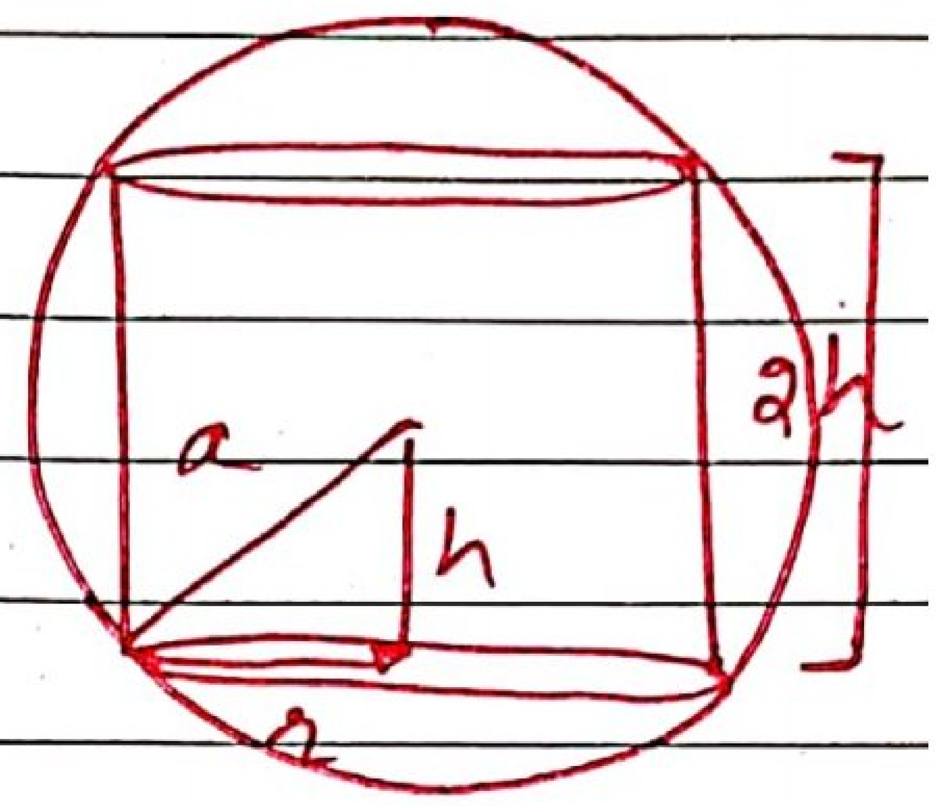
$\cancel{\pi} r^3 = \cancel{\pi} r^2 h$

$r = h$

$\therefore$ Radius of base = height of the cylinder Ans

Qn. 8 → let $2h \to$ height of cylinder
$r \to$ Radius of cylinder

Solution A·OD (WS 5)



$a \rightarrow$ Radius of Sphere

$a^2 = h^2 + r^2$ --- (given) ---(i)

$V \rightarrow$ volume of cylinder

$V = \pi r^2 (2h)$ --- (to be Max)

$V = \pi (a^2 - h^2)(2h)$ from eq (i)

$V = 2\pi (a^2 h - h^3)$

$\frac{dV}{dh} = 2\pi (a^2 - 3h^2)$

for Max/Min; put $\frac{dV}{dh} = 0$

$a^2 = 3h^2$

$\boxed{a = \sqrt{3}h}$ $\boxed{h = a/\sqrt{3}}$

Diff again wrt h

$\frac{d^2V}{dh^2} = 2\pi(-6h)$

$= -12\pi h$

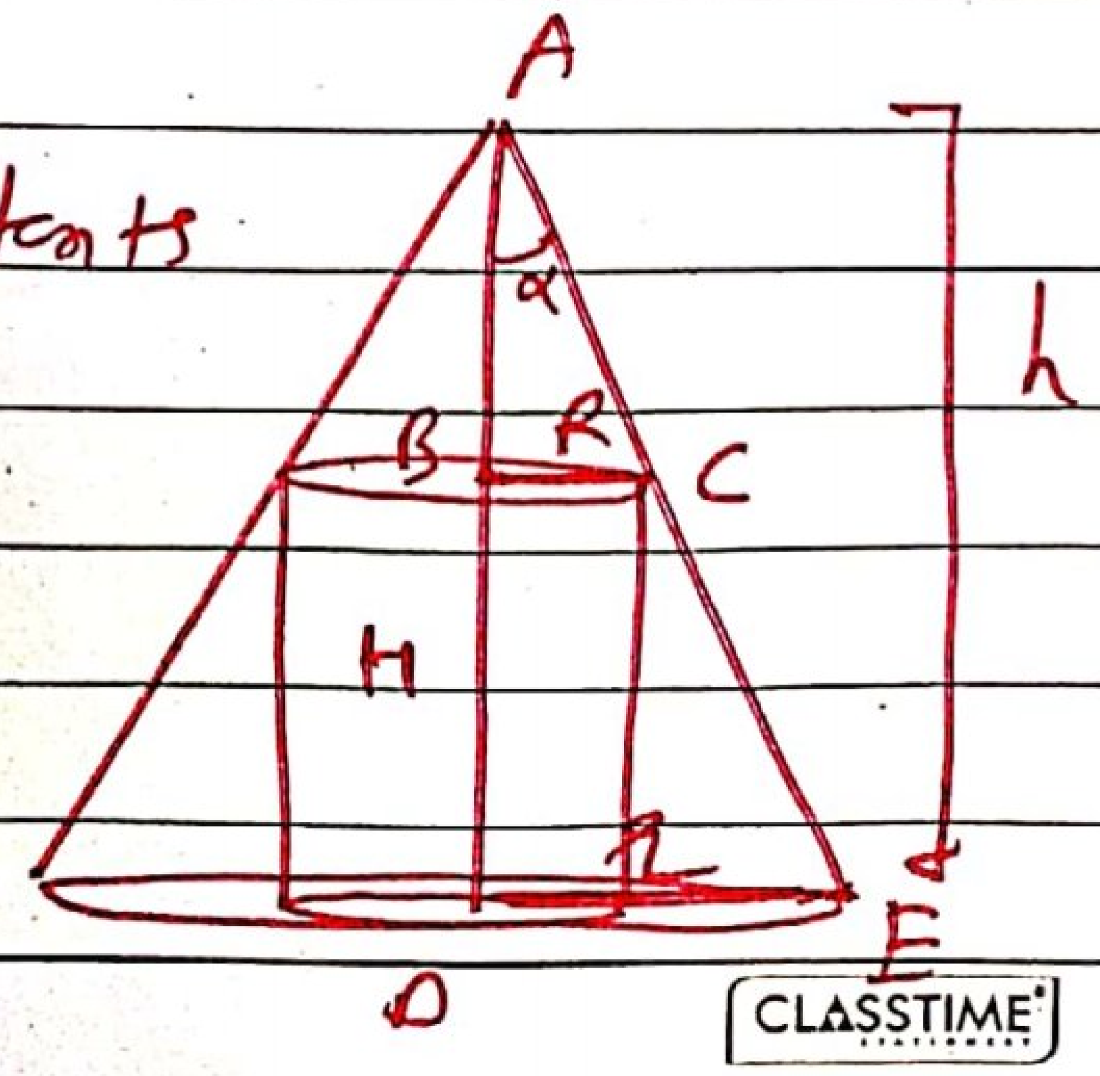
$\left(\frac{d^2V}{dh^2}\right)_{h = \frac{a}{\sqrt{3}}} = -12\pi\left(\frac{a}{\sqrt{3}}\right) < 0$

$\therefore$ volume of the cylinder is Max at $h = a/\sqrt{3}$

Since height $= 2h$

$= \frac{2a}{\sqrt{3}}$ Proved

Qn. 9 $\rightarrow$ $h, \alpha, r \rightarrow$ dimensions of cone $\therefore$ they are constants



let $R \rightarrow$ Radius of cylinder

$H \rightarrow$ height of cylinder

In ΔABC

$\tan\alpha = \frac{R}{AB}$

Solution A·O·D (W·S - 5)

$AB = \frac{R}{\tan\alpha}$

Now $H = h - AB$

$\boxed{H = h - \frac{R}{\tan\alpha}}$

let $S \to$ C·S·A of Cylinder

$S = 2\pi RH$ ... (to be Maximized)

$S = 2\pi R\left(h - \frac{R}{\tan\alpha}\right)$

$S = 2\pi\left(Rh - \frac{R^2}{\tan\alpha}\right)$

Diff w.r.t R

$\frac{dS}{dR} = 2\pi\left(h - \frac{2R}{\tan\alpha}\right)$

for Max/Min put $\frac{dS}{dR} = 0$

$2\pi\left(h - \frac{2R}{\tan\alpha}\right) = 0$

$\boxed{h = \frac{2R}{\tan\alpha}}$ or $\boxed{R = \frac{h\tan\alpha}{2}}$

Diff again w.r.t R

$\frac{d^2S}{dR^2} = 2\pi\left(-\frac{2}{\tan\alpha}\right) < 0$

$\therefore$ C·S·A of Cylinder is Maximum

Now In $\underbrace{\Delta ADE}$

$\tan\alpha = \frac{r}{h}$

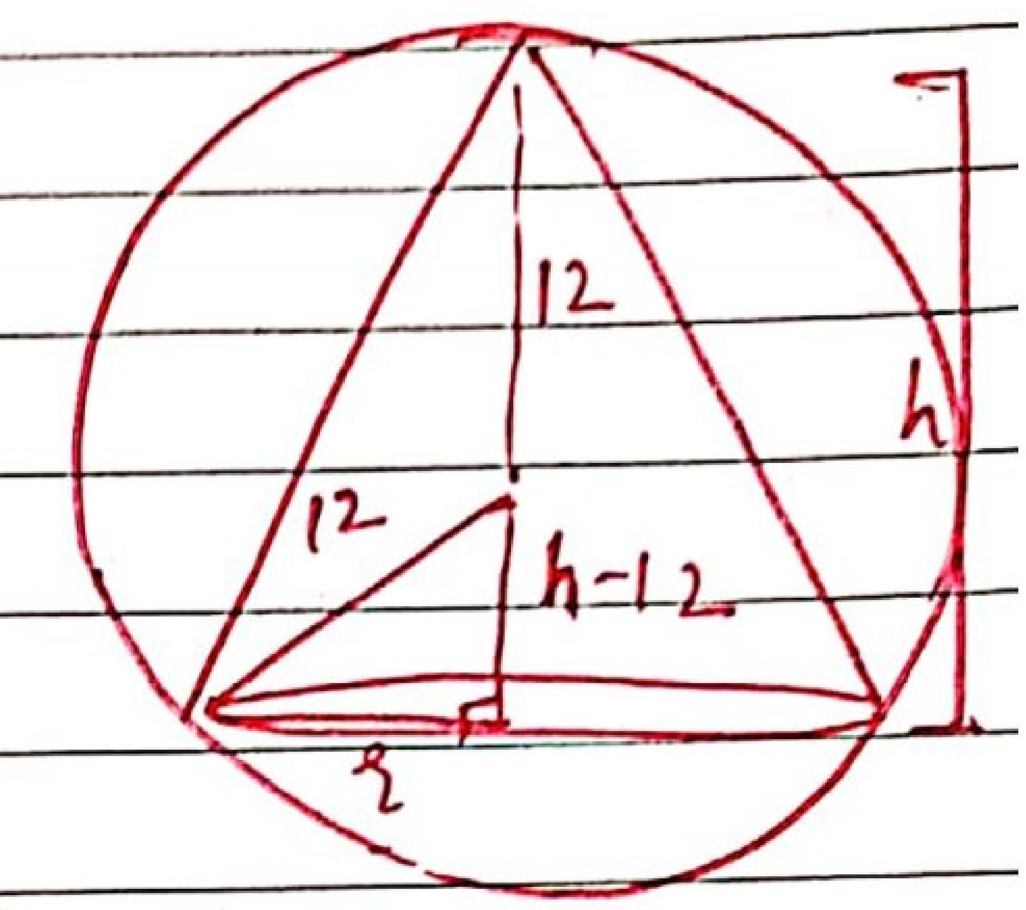
$\Rightarrow \frac{2R}{h} = \frac{r}{h}$ ... $\left\{\because R = \frac{h\tan\alpha}{2}\right\}$

$\Rightarrow R = \frac{1}{2}r$

$\therefore$ Radius of Cylinder = $\frac{1}{2}$ Radius of Cone Ans.

Solution A-OD (W-S-5)

Qn 10 → let $h \to$ height of cone
$r \to$ Radius of cone



$(12)^2 = (h-12)^2 + r^2$ ---(given) ---(i)

$V \to$ volume of cone

$V = \frac{1}{3}\pi r^2 h$ -- (to be Max)

$V = \frac{1}{3}\pi\left(144 - (h-12)^2\right)h$ --- (from eq (1))

$V = \frac{1}{3}\pi(-h^2 + 24h)h$

$V = \frac{1}{3}\pi(-h^3 + 24h^2)$

$\frac{dV}{dh} = \frac{1}{3}\pi(-3h^2 + 48h)$

for Max/Min put $\frac{dV}{dh} = 0$

$3h^2 = 48h$

$\Rightarrow \boxed{h = 16}$

Diff again w.r.t h

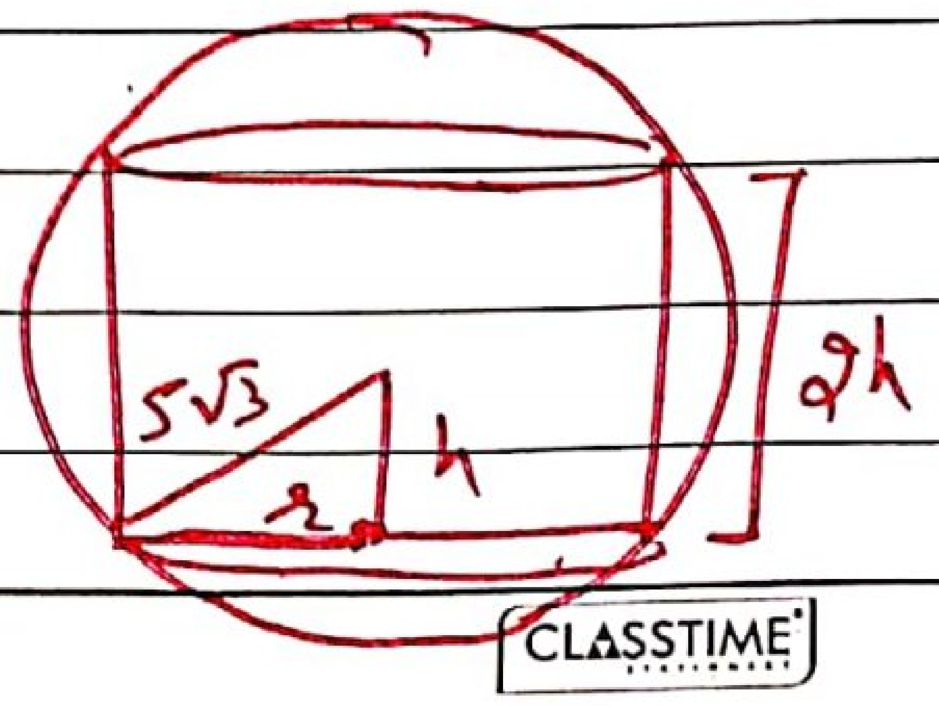
$\frac{d^2V}{dh^2} = \frac{1}{3}\pi(-6h + 48)$

$\left(\frac{d^2V}{dh^2}\right)_{h=16} = \frac{1}{3}\pi(-96 + 48) = \frac{-48\pi}{3} < 0$

∴ volume of cone is Max at $h = 16$

∴ height of cone = 16 cm Ans

Qn. 11 → let $2h \to$ height of cylinder
$r \to$ Radius of cylinder



$(5\sqrt{3})^2 = h^2 + r^2$

$75 = h^2 + r^2$ --- (given) --(1)

Solution A·OD (W-S-5) 11

V → volume of cylinder

$V = \pi r^2 (2h)$ --- (to be Max)

$V = \pi (75 - h^2)(2h)$ --- (from eq(1))

$V = 2\pi (75h - h^3)$

$\frac{dV}{dh} = 2\pi (75 - 3h^2)$

for Max/Min put $\frac{dV}{dh} = 0$

$75 = 3h^2$

$\boxed{h = 5}$

diff. again w.r.t h

$\frac{d^2V}{dh^2} = 2\pi(-6h)$

$\left(\frac{d^2V}{dh^2}\right)_{h=5} = 2\pi(-30) = -60\pi < 0$

∴ volume of ~~co~~ cone is Max at $h = 5$

put $h = 5$ in eq (1)

$75 = 25 + r^2$

$\boxed{r^2 = 50}$

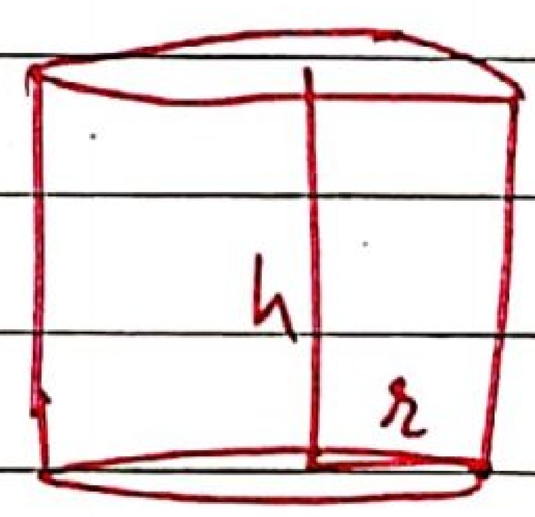
volume of cylinder $= \pi r^2 (2h)$

$= \pi (50)(10)$

$= 500\pi \text{ cm}^3$ Ans

Qn. 12 → r → radius of cylinder

h → height of cylinder



$2156 = \pi r^2 h$ -- (given) --(1)

let S → T.S.A of cylinder

$S = 2\pi rh + 2\pi r^2$ --- ~~(given)~~ (to be Minimized)

$S = 2\pi r\left(\frac{2156}{\pi r^2}\right) + 2\pi r^2$ --- (from eq(1))

$S = \frac{4312}{r} + 2\pi r^2$

Diff wrt r

$\frac{dS}{dr} = \frac{-4312}{r^2} + 4\pi r$

for Max/Min put $\frac{dS}{dr} = 0$

$\frac{4312}{r^2} = 4\pi r$

$\Rightarrow 1078 = \pi r^3$

$\Rightarrow 1078 = \frac{22}{7} \times r^3$

$\Rightarrow \frac{\overset{49}{\cancel{1078}} \times 7}{\cancel{22}} = r^3$

$\Rightarrow r^3 = 7 \times 7 \times 7$

$\Rightarrow \boxed{r = 7}$

Diff again

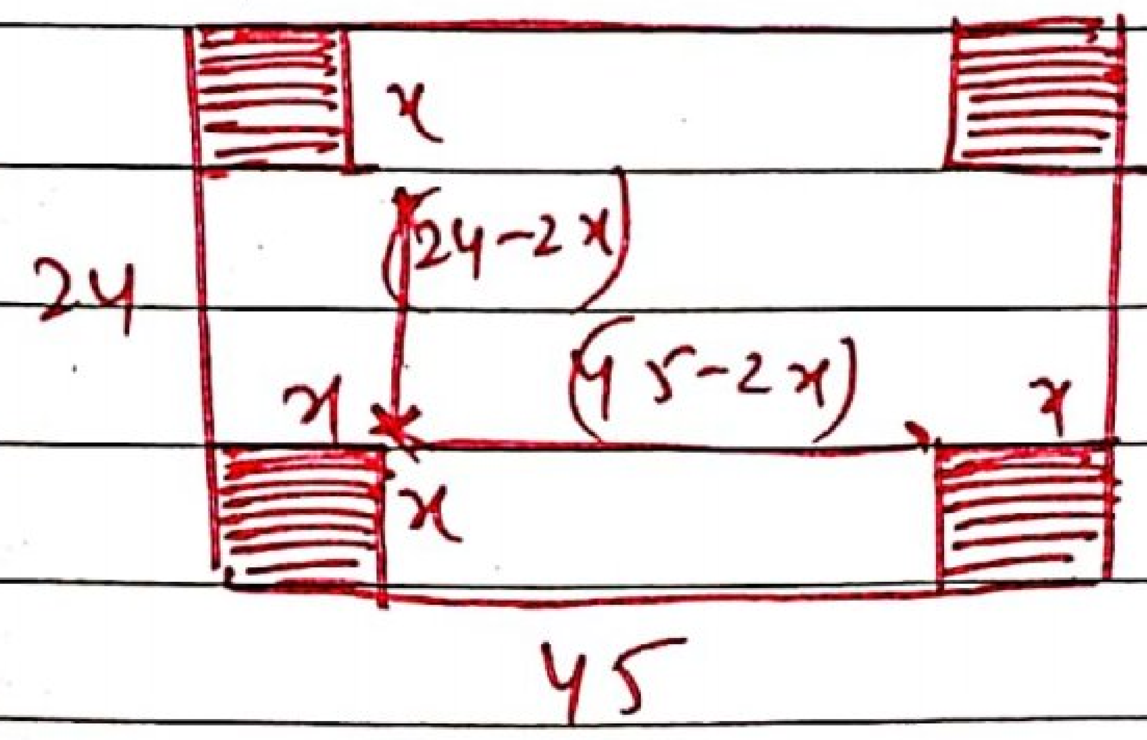
$\frac{d^2S}{dr^2} = \frac{8624}{r^3} + 4\pi$

$\left(\frac{d^2S}{dr^2}\right)_{r=7} = \frac{8624}{(7)^3} + 4\pi > 0$

$\therefore$ T.S.A of cylinder is Max at $r = 7$ cm

Hence Radius of cylinder = 7 cm Ans

Ans 13 → let the side of the square to be cut off = 3cm



length = 45 - 2x

breadth = 24 - 2x

height = x

V → volume of box/cuboid

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Solution A·OD (W·S - 5) (13)

$V = (45-2x)(24-2x)(x)$ ... (for Max)

$V = 1080x - 138x^2 + 4x^3$

Diff wrt x

$\frac{dV}{dx} = 12x^2 - 276x + 1080$

for Max/Min put $\frac{dV}{dx} = 0$

$12x^2 - 276x + 1080 = 0$

$x^2 - 23x + 90 = 0$

$(x-18)(x-5) = 0$

$x = 18$; $\boxed{x = 5}$

(not possible
$B = 24 - 2x$
$B = 24 - 36 = -12$ negative)

$\therefore$ Volume of box $= (45-10)(24-10)(5)$

$= (35)(14)(5)$

$= 2450 \text{ cm}^3$

Note (there is a Misprint in worksheet)

—x—