ANSWER KEY / CORRECT SOLUTIONS



EXAM NO: 7

**Ans. 1**

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

For upper triangular Matrix elements above the principal diagonal must be zero

i.e $a_{12} = 0$ ; $a_{13} = 0$; $a_{23} = 0$ and so on.

$\therefore$ $\boxed{i < j}$ (B) ANS

**Ans. 2** →

$$A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$$

$$f(x) = (1+x)(1-x) = 1 - x^2$$

$$f(A) = I - A^2$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 5 & 4 \\ 4 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} -4 & -4 \\ -4 & -4 \end{bmatrix} = \boxed{-4\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}}$$

$\therefore$ (A) Ans

**Ans 3** →

$$\begin{bmatrix} 1 & x & 1 \end{bmatrix}\begin{bmatrix} 1 & 3 & 2 \\ 2 & 5 & 1 \\ 15 & 3 & 2 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \\ x \end{bmatrix} = 0$$

$$\Rightarrow \begin{bmatrix} 2x+16 & 6+5x & 4+x \end{bmatrix}\begin{bmatrix} 1 \\ 2 \\ x \end{bmatrix} = 0$$

$$\Rightarrow \left[2x+16+12+10x+4x+x^2\right] = 0$$

$$\Rightarrow \left[x^2+16x+28\right] = [0]$$

$$\Rightarrow x^2 + 16x + 28 = 0$$

$$\Rightarrow (x+2)(x+14) = 0$$

$\boxed{x = -2 \quad x = -14}$

$\boxed{x = -2}$ (C) Ans

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Ans-4 → $a_{ij} = i^2 - j^2$

$a_{ji} = j^2 - i^2$

$a_{ji} = -(i^2 - j^2)$

$\boxed{a_{ji} = -a_{ij}}$

also $a_{11} = 0$ ; $a_{22} = 0$ and so-on

∴ A must be Skew Symmetric Matrix ∴ (A) Ans

Ans-5 → $A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

$A^2 = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$

$A^4 = A^2A^2 = \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}\begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} 8 & 8 \\ 8 & 8 \end{bmatrix}$

$A' = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

Given $A^4 = kA'$

$\begin{bmatrix} 8 & 8 \\ 8 & 8 \end{bmatrix} = k\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

$\Rightarrow 8\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} = k\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

$\Rightarrow \boxed{k = 8}$ (C) Ans

Ans-6 → $A = \begin{bmatrix} b & 2 & a \\ 2 & 1 & -2 \\ -2 & 2 & -1 \end{bmatrix}$ ; $A' = \begin{bmatrix} b & 2 & -2 \\ 2 & 1 & 2 \\ a & -2 & -1 \end{bmatrix}$

Given. $AA' = 9I$

$$\begin{bmatrix} b & 2 & a \\ 2 & 1 & -2 \\ -2 & 2 & -1 \end{bmatrix}\begin{bmatrix} b & 2 & -2 \\ 2 & 1 & 2 \\ a & -2 & -1 \end{bmatrix} = \begin{bmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{bmatrix}$$

$$\begin{bmatrix} b^2+4+a^2 & 2b+2-2a & -2b+4-a \\ 2b+2-2a & 9 & 0 \\ -2b+4-a & 0 & 9 \end{bmatrix} = \begin{bmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{bmatrix}$$

$\rightarrow$ $2b+2-2a=0$

$2a-2b=2$

$\boxed{a-b=1}$

$-2b+4-a=0$

$\boxed{a+2b=4}$

$a^2+b^2+4=9$

$4+1+4=9$

$\therefore$ Satisfied

Solving $3b=3$

$\boxed{b=1 \quad \& \quad a=2}$

$\therefore$ (B) Ans

**Ans 7**

$$\Delta = \begin{vmatrix} \cos(\alpha+\beta) & -\sin(\alpha+\beta) & \cos(2\beta) \\ \sin\alpha & \cos\alpha & \sin\beta \\ -\cos\alpha & \sin\alpha & \cos\beta \end{vmatrix}$$

expanding

$$\Delta = \cos(\alpha+\beta)\left(\cos\alpha\cos\beta - \sin\alpha\sin\beta\right) + \sin(\alpha+\beta)\left(\sin\alpha\cos\beta+\cos\alpha\sin\beta\right)$$
$$+\cos(2\beta)\left(\sin^2\alpha+\cos^2\alpha\right)$$

$$= \cos(\alpha+\beta)\cdot\cos(\alpha+\beta) + \sin(\alpha+\beta)\sin(\alpha+\beta) + \cos(2\beta)$$

$$= \cos^2(\alpha+\beta) + \sin^2(\alpha+\beta) + \cos(2\beta)$$

$$= 1+\cos(2\beta)$$

$$= 2\cos^2(\beta)$$

$\boxed{\text{It is Independent of } \alpha}$ $\therefore$ (A) Ans

**Ans 8** points $(a_1, b_1)$, $(a_2, b_2)$, $(a_1+a_2, b_1+b_2)$ are collinear

$$\Rightarrow \begin{vmatrix} a_1 & b_1 & 1 \\ a_2 & b_2 & 1 \\ a_1+a_2 & b_1+b_2 & 1 \end{vmatrix} = 0$$

(4)

$\Rightarrow a_1\left(\cancel{b_2}-b_1-\cancel{b_2}\right)-b_1\left(\cancel{a_2}+a_1-\cancel{a_2}\right)+1\left(a_2b_1+\cancel{a_2b_2}-a_1b_2-\cancel{a_2b_2}\right)=0$

$\Rightarrow -\cancel{a_1b_1}+\cancel{a_1b_1}+a_2b_1-a_1b_2=0$

$\Rightarrow \boxed{a_2b_1=a_1b_2}$ $\therefore$ (A) Ans

**Qn: 9 →** $x+y+z=2$ ; $2x+y-z=3$ & $3x+2y+kz=4$

for unique solution $|A|\neq 0$

$$\Rightarrow \begin{vmatrix} 1 & 1 & 1 \\ 2 & 1 & -1 \\ 3 & 2 & k \end{vmatrix} \neq 0$$

$\Rightarrow 1(k+2)-1(2k+3)+1(4-3)\neq 0$

$\Rightarrow k+2-2k-3+1\neq 0$

$\Rightarrow -k\neq 0$

$\Rightarrow \boxed{k\neq 0}$ $\therefore$ (C) Ans

**Qn: 10 →** given $A^3=A^2$

Pre multiply by $A^{-1}$

$A^{-1}A^3=A^{-1}A^2$

$A^{-1}A\cdot A^2=A^{-1}A\,A$

$IA^2=IA$

$A^2=A$

again pre multiply by $A^{-1}$

$A^{-1}A^2=A^{-1}A$

$A^{-1}A\,A=I$

$IA=I$

$A=I$

Again pre-multiply by $A^{-1}$

$A^{-1}A=A^{-1}I$

$I=A^{-1}$

$\therefore \boxed{A^{-1}=I}$

(C) Ans

Ans 11 → $n=3$ ; $|3AB| = 405$ ; $|A'| = 3$

we know $|A'| = |A|$

$\therefore |A| = 3$

$|3AB| = 405$

$\Rightarrow 3^3 |A| |B| = 405$

$\Rightarrow 27 \times 3 |B| = 405$

$\Rightarrow \boxed{|B| = 5}$ $\therefore$ (C) Ans

Ans. 12 $n=4$ ; $|Adj A| = -27$

$|Adj A| = |A|^{n-1}$

$\Rightarrow -27 = |A|^3$

$\Rightarrow |A| = -3$

$|-A| = (-1)^4 |A| = |A| = -3$

$\therefore \boxed{|-A| = -3}$ $\therefore$ (B) Ans

Ans. 13 → $A(1,3)$ $B(0,0)$ $C(k,0)$

$$3 = \frac{1}{2} \begin{vmatrix} 1 & 3 & 1 \\ 0 & 0 & 1 \\ k & 0 & 1 \end{vmatrix}$$

$\Rightarrow 6 = |1(0) - 3(-k) + 1(0)|$

$\Rightarrow 6 = |3k|$

$\Rightarrow 3k = \pm 6$

$\Rightarrow k = \pm 2$

$\boxed{k=2}$ (A) Ans

**Qn. 14 →** $f(t) = \begin{vmatrix} \cos t & t & 1 \\ 2\sin t & t & 2t \\ \sin t & t & t \end{vmatrix}$

$$f(t) = \cos t(-t^2) - t(\cancel{2t\sin t} - \cancel{2t\sin t}) + 1(2t\sin t - t\sin t)$$

$$f(t) = -t^2\cos t + t\sin t$$

Now $\lim_{t\to 0}\left(\frac{f(t)}{t^2}\right) = \lim_{t\to 0}\left(\frac{-t^2\cos t + t\sin t}{t^2}\right)$

$$= \lim_{t\to 0}\left(-\cos t + \frac{\sin t}{t}\right)$$

$$= -1 + 1 = \boxed{0}$$ ∴ (A) Ans

**Qn. 15 →** $A = \frac{1}{\pi}\begin{bmatrix} \sin^{-1}(\pi x) & \tan^{-1}\left(\frac{x}{\pi}\right) \\ \sin^{-1}\left(\frac{x}{\pi}\right) & \cot^{-1}(\pi x) \end{bmatrix}$

$$B = \frac{1}{\pi}\begin{bmatrix} -\cos^{-1}(\pi x) & \tan^{-1}\left(\frac{x}{\pi}\right) \\ \sin^{-1}\left(\frac{x}{\pi}\right) & -\tan^{-1}(\pi x) \end{bmatrix}$$

$$A - B = \frac{1}{\pi}\begin{bmatrix} \sin^{-1}(\pi x) + \cos^{-1}(\pi x) & 0 \\ 0 & \cot^{-1}(\pi x) + \tan^{-1}(\pi x) \end{bmatrix}$$

$$= \frac{1}{\pi}\begin{bmatrix} \frac{\pi}{2} & 0 \\ 0 & \frac{\pi}{2} \end{bmatrix}$$

$$= \frac{1}{\pi} \times \frac{\pi}{2}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\boxed{A - B = \frac{1}{2} I}$$ ∴ (D) Ans

Qn. 16 → let A → denotes the quantity sold in two Markets

$$A = \begin{array}{c} \\ \text{I} \\ \text{II} \end{array} \begin{array}{c} \begin{array}{ccc} x & y & z \end{array} \\ \begin{bmatrix} 10000 & 2000 & 18000 \\ 6000 & 20000 & 8000 \end{bmatrix}_{2\times 3} \end{array}$$

B → denotes the S.P per unit of each product

$$B = \begin{array}{c} x \\ y \\ z \end{array} \begin{bmatrix} 2.5 \\ 1.5 \\ 1 \end{bmatrix}$$

C → denotes the C.P per unit of each product

$$C = \begin{array}{c} x \\ y \\ z \end{array} \begin{bmatrix} 2 \\ 1 \\ .50 \end{bmatrix}$$

P → profit per unit of each product

$$P = B - C = \begin{bmatrix} 0.5 \\ 0.5 \\ 0.5 \end{bmatrix}$$

Profit Matrix $= AP$

$$= \begin{array}{c} \text{I} \\ \text{II} \end{array} \begin{bmatrix} 10000 & 2000 & 18000 \\ 6000 & 20000 & 8000 \end{bmatrix} \begin{bmatrix} 0.5 \\ 0.5 \\ 0.5 \end{bmatrix}$$

$$= \begin{array}{c} \text{I} \\ \text{II} \end{array} \begin{bmatrix} 15000 \\ 17000 \end{bmatrix}$$

∴ ~~the~~ profit from Market I = Rs 15000

& profit from Market II = Rs 17000

& gross profit = Rs 15000 + Rs 17000

$= \boxed{\text{Rs } 32000}$ Ans

Qn 17 → given $A' = A$ and $B' = B$

Case I let $AB = BA$

T.P $AB \to$ Symmetric

let $P = AB$

$\Rightarrow P' = (AB)'$

$\Rightarrow P' = B'A'$

$\Rightarrow P' = BA$ ... {Given $B' = B$ & $A' = A$}

$\Rightarrow P' = AB$ ... {Given $AB = BA$}

$\Rightarrow P' = P$

$\therefore$ P is a symmetric Matrix

Conversely let $AB \to$ symmetric Matrix

T.P $AB = BA$

we have $(AB)' = AB$

$\Rightarrow B'A' = AB$

$\Rightarrow BA = AB$ ... {Given $B' = B$ & $A' = A$}

$\therefore$ A & B commute

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Qn: 18 → given $A = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$

$$A^2 = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix} \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$$

$$A^2 = \begin{bmatrix} \cos^2\alpha - \sin^2\alpha & -\sin\alpha\cos\alpha - \sin\alpha\cos\alpha \\ \sin\alpha\cos\alpha + \sin\alpha\cos\alpha & -\sin^2\alpha + \cos^2\alpha \end{bmatrix}$$

$$A^2 = \begin{bmatrix} \cos(2\alpha) & -\sin(2\alpha) \\ \sin(2\alpha) & \cos(2\alpha) \end{bmatrix}$$

$$A^3 = A^2 A$$

$$= \begin{bmatrix} \cos(2\alpha) & -\sin(2\alpha) \\ \sin(2\alpha) & \cos(2\alpha) \end{bmatrix} \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$$

$$= \begin{bmatrix} \cos(2\alpha)\cos\alpha - \sin(2\alpha)\sin\alpha & -\sin\alpha\cos(2\alpha) - \sin(2\alpha)\cos\alpha \\ \sin(2\alpha)\cos\alpha + \cos(2\alpha)\sin\alpha & -\sin\alpha\sin(2\alpha) + \cos(2\alpha)\cos\alpha \end{bmatrix}$$

$$A^3 = \begin{bmatrix} \cos(3\alpha) & -\sin(3\alpha) \\ \sin(3\alpha) & \cos(3\alpha) \end{bmatrix}$$

due to symmetry

$$A^{32} = \begin{bmatrix} \cos(32\alpha) & -\sin(32\alpha) \\ \sin(32\alpha) & \cos(32\alpha) \end{bmatrix}$$

given $A^{32} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

$$\Rightarrow \begin{bmatrix} \cos(2\alpha) & -\sin(32\alpha) \\ \sin(32\alpha) & \cos(32\alpha) \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

$\Rightarrow \cos(32\alpha) = 0$ | $\cancel{-}\sin(32\alpha) = \cancel{-}1$

$32\alpha = \pi/2$ | $32\alpha = \pi/2$

$$\Rightarrow \boxed{\alpha = \frac{\pi}{64}}$$ (c) Ans

Qn 19 given B is the Inverse of A

$\Rightarrow AB = I$ (or) $BA = I$

Given $A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}$ & $B = \frac{1}{10}\begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & \alpha \\ 1 & -2 & 3 \end{bmatrix}$

$AB = I$

$$\Rightarrow \frac{1}{10}\begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & \alpha \\ 1 & -2 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \frac{1}{10}\begin{bmatrix} 10 & 0 & 5-\alpha \\ 0 & 10 & \alpha-5 \\ 0 & 0 & \alpha+5 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$\Rightarrow \frac{5-\alpha}{10} = 0$ | $\frac{\alpha-5}{10} = 0$ | $\frac{\alpha+5}{10} = 1$

$\Rightarrow \alpha = 5$ | $\alpha = 5$ | $\alpha = 5$

$\therefore \boxed{\alpha = 5}$ Ans

**Q~ 20 →** Given $AB = 0$

$$\Rightarrow \begin{bmatrix} \cos^2\theta & \cos\theta\sin\theta \\ \cos\theta\sin\theta & \sin^2\theta \end{bmatrix}\begin{bmatrix} \cos^2\phi & \cos\phi\sin\phi \\ \cos\phi\sin\phi & \sin^2\phi \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} \cos^2\theta\cos^2\phi + \cos\theta\sin\theta\cos\phi\sin\phi & \cos^2\theta\cos\phi\sin\phi + \cos\theta\sin\theta\sin^2\phi \\ \cos\theta\sin\theta\cos^2\phi + \sin^2\theta\cos\phi\sin\phi & \cos\theta\sin\theta\cos\phi\sin\phi + \sin^2\theta\sin^2\phi \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} \cos\theta\cos\phi.\cos(\theta-\phi) & \cos\theta\sin\phi.\cos(\theta-\phi) \\ \cos\phi\sin\theta.\cos(\theta-\phi) & \sin\theta\sin\phi.\cos(\theta-\phi) \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$\Rightarrow \cos(\theta-\phi) = 0$

$\Rightarrow \theta-\phi =$ odd multiple of $\pi/2$

$\Rightarrow \theta-\phi = (2n+1)\pi/2$

$\Rightarrow \boxed{\theta = \phi + (2n+1)\pi/2;\ n \in Z}$ $\therefore$ (c) Ans

Ans 21) $A = \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$ $B = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$

$$BA = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$$

$$BA = \begin{bmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{bmatrix} = 6I$$

$$BA = 6I$$

$$\Rightarrow B^{-1}BA = 6B^{-1}I$$

$$\Rightarrow IA = 6B^{-1}$$

$$\Rightarrow A = 6B^{-1}$$

$$\Rightarrow \boxed{B^{-1} = \frac{1}{6}A}$$

given equation

$x - y = 3$ ; $2x + 3y + 4z = 17$ & $y + 2z = 7$

these equations can be written in Matrix form

$$\begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 17 \\ 7 \end{bmatrix}$$

(or) $BX = C$

when $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ & $C = \begin{bmatrix} 3 \\ 17 \\ 7 \end{bmatrix}$

Now $BX = C$

$\Rightarrow X = B^{-1}C$ and $B^{-1} = \frac{1}{6}A$

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$\Rightarrow X = \frac{1}{8} A C$

$$= \frac{1}{8}\begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}\begin{bmatrix} 3 \\ 17 \\ 7 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{8}\begin{bmatrix} 12 \\ -6 \\ 24 \end{bmatrix}$$

$\Rightarrow \boxed{x = 2, \quad y = -1, \quad z = 4}$ is the required solution.

Q.N. 22 $\rightarrow$ $A = \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix}$

given $A^2 + aA + bI = 0$

$$\Rightarrow \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix} + a\begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix} + b\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = 0$$

$$\Rightarrow \begin{bmatrix} 11 & 8 \\ 4 & 3 \end{bmatrix} + \begin{bmatrix} 3a & 2a \\ a & a \end{bmatrix} + \begin{bmatrix} b & 0 \\ 0 & b \end{bmatrix} = 0$$

$$\Rightarrow \begin{bmatrix} 11 + 3a + b & 8 + 2a \\ 4 + a & 3 + a + b \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$\Rightarrow 11 + 3a + b = 0$ | $2a + 8 = 0$ | $4 + a = 0$ | $3 + a + b = 0$

$11 - 12 + b = 0$ | $\boxed{a = -4}$ | $\boxed{a = -4}$ | $3 - 4 + b = 0$

$\boxed{b = 1}$ | | | $b = 1$

$\therefore \boxed{a = -4 \ \& \ b = 1}$ put in given equation

$A^2 - 4A + I = 0$

Pre-multiply by $A^{-1}$

$$A^{-1}A^2 - 4A^{-1}A + A^{-1}I = 0$$

$$A^{-1}AA - 4I + A^{-1} = 0$$

$$IA - 4I + A^{-1} = 0$$

$$A^{-1} = 4I - A$$

$$A^{-1} = 4\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}$$ Ans

Qn. 24 →

given $\begin{bmatrix} 1 & -\tan\theta \\ \tan\theta & 1 \end{bmatrix}\begin{bmatrix} 1 & \tan\theta \\ -\tan\theta & 1 \end{bmatrix}^{-1} = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$

let $A = \begin{bmatrix} 1 & -\tan\theta \\ \tan\theta & 1 \end{bmatrix}$ & $B = \begin{bmatrix} 1 & \tan\theta \\ -\tan\theta & 1 \end{bmatrix}$

we have to find $B^{-1}$

$$|B| = 1+\tan^2\theta$$

$$Adj\,B = \begin{bmatrix} 1 & -\tan\theta \\ \tan\theta & 1 \end{bmatrix}$$

$$B^{-1} = \frac{1}{1+\tan^2\theta}\begin{bmatrix} 1 & -\tan\theta \\ \tan\theta & 1 \end{bmatrix}$$

∴ given equation becomes

$$\Rightarrow \begin{bmatrix} 1 & -\tan\theta \\ \tan\theta & 1 \end{bmatrix}\frac{1}{1+\tan^2\theta}\begin{bmatrix} 1 & -\tan\theta \\ \tan\theta & 1 \end{bmatrix} = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$$

$$\Rightarrow \frac{1}{1+\tan^2\theta}\begin{bmatrix} 1-\tan^2\theta & -2\tan\theta \\ 2\tan\theta & 1-\tan^2\theta \end{bmatrix} = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} \frac{1-\tan^2\theta}{1+\tan^2\theta} & -\frac{2\tan\theta}{1+\tan^2\theta} \\ \frac{2\tan\theta}{1+\tan^2\theta} & \frac{1-\tan^2\theta}{1+\tan^2\theta} \end{bmatrix} = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} \cos(2\theta) & -\sin(2\theta) \\ \sin(2\theta) & \cos(2\theta) \end{bmatrix} = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$$

$$\therefore \boxed{a = \cos(2\theta) \ \& \ b = \sin(2\theta)} \text{ Ans}$$

Ans- 23 → given $\begin{bmatrix} 3 & 2 \\ 7 & 5 \end{bmatrix} \times \begin{bmatrix} -1 & 1 \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} 2 & -1 \\ 0 & 4 \end{bmatrix}$

let $A = \begin{bmatrix} 3 & 2 \\ 7 & 5 \end{bmatrix}$ & $B = \begin{bmatrix} -1 & 1 \\ -2 & 1 \end{bmatrix}$ & $C = \begin{bmatrix} 2 & -1 \\ 0 & 4 \end{bmatrix}$

$\therefore \ A \times B = C$

$\Rightarrow A^{-1}A \times BB^{-1} = A^{-1}CB^{-1}$

$\Rightarrow I \times I = A^{-1}CB^{-1}$

$\Rightarrow X = A^{-1}CB^{-1}$

$|A| = 15 - 14 = 1$

$Adj\,A = \begin{bmatrix} 5 & -2 \\ -7 & 3 \end{bmatrix}$

$A^{-1} = \frac{1}{1}\begin{bmatrix} 5 & -2 \\ -7 & 3 \end{bmatrix}$

$|B| = -1 + 2 = 1$

$Adj\,B = \begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix}$

$B^{-1} = \frac{1}{1} = \begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix}$

$\therefore \ X = \begin{bmatrix} 5 & -2 \\ -7 & 3 \end{bmatrix}\begin{bmatrix} 2 & -1 \\ 0 & 4 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix}$

$X = \begin{bmatrix} 10 & -13 \\ -14 & 19 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix}$

$$X = \begin{bmatrix} -16 & 3 \\ 24 & -5 \end{bmatrix}$$ Ans

Q.N. 25 → $f(x) = ax^2 + bx + c$

$f(1) = 8$ ; $f(2) = 11$ ; $f(-3) = 6$

$\Rightarrow$ $8 = a + b + c$

$11 = 4a + 2b + c$

$6 = 9 - 3b + c$

then equations can be written in the form

$AX = B$

$$\begin{bmatrix} 1 & 1 & 1 \\ 4 & 2 & 1 \\ 9 & -3 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 8 \\ 11 \\ 6 \end{bmatrix}$$

or $X = A^{-1}B$

$|A| = 1(2+3) - 1(4-9) + 1(-12-18)$

$|A| = 5 + 5 - 30 = -20$

$$\text{Adj } A = \begin{bmatrix} 5 & -4 & -1 \\ 5 & -8 & +3 \\ -30 & 12 & -2 \end{bmatrix}$$

$$X = -\frac{1}{20} \begin{bmatrix} 5 & -4 & -1 \\ 5 & -8 & 3 \\ -30 & 12 & -2 \end{bmatrix} \begin{bmatrix} 8 \\ 11 \\ 6 \end{bmatrix}$$

$$X = -\frac{1}{20} \begin{bmatrix} 40 - 44 - 6 \\ 40 - 88 + 18 \\ -240 + 132 - 12 \end{bmatrix}$$

$$\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 1/2 \\ 3/2 \\ 6 \end{bmatrix}$$

$\therefore a = 1/2$, $b = 3/2$, $c = 6$

$\therefore$ $\boxed{f(x) = \frac{1}{2}x^2 + \frac{3}{2}x + 6}$ Ans

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