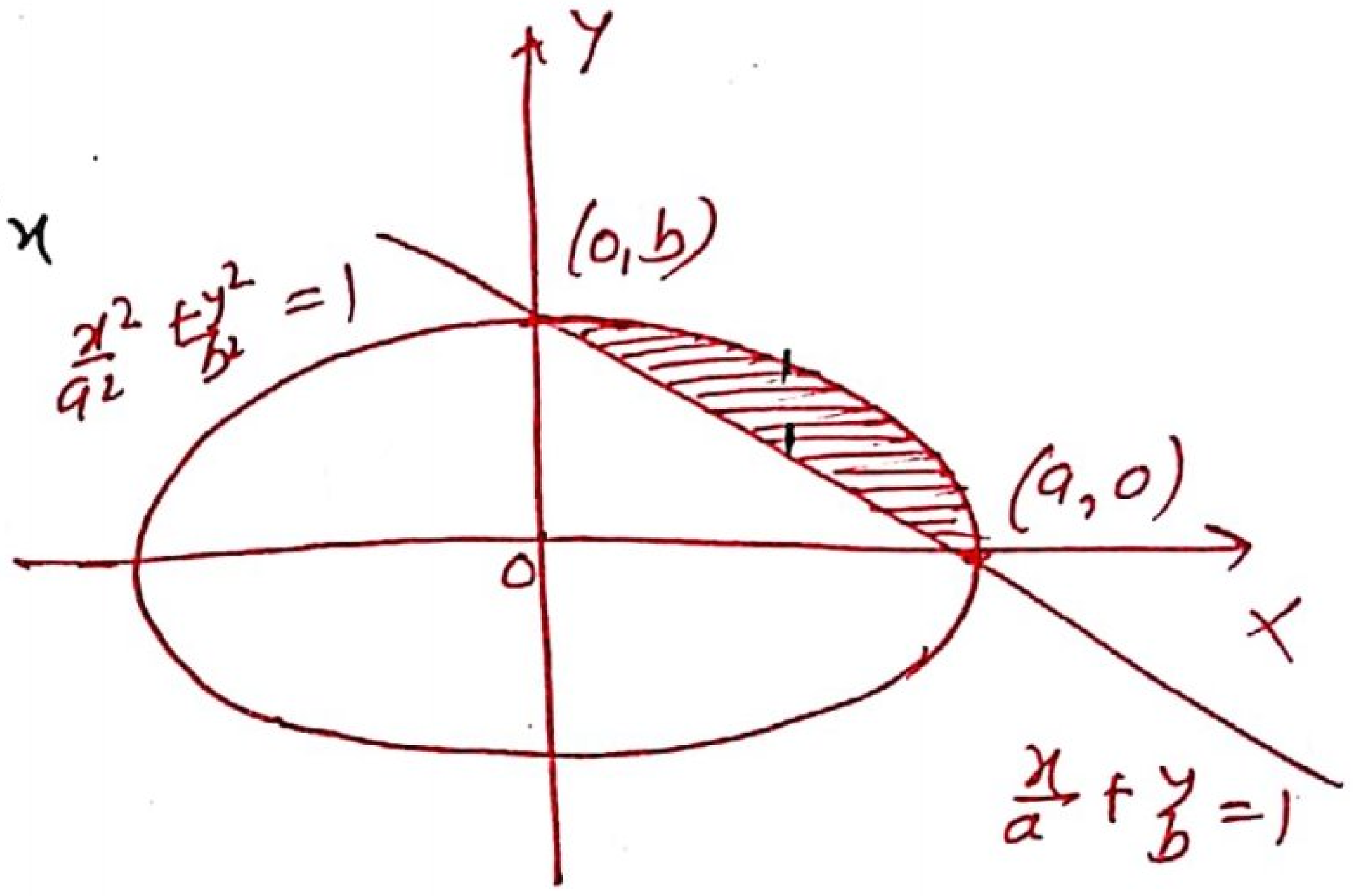
Solutions

WORKSHEET NO: 2 (Class No: 3) ①

CHAPTER A·O·I

Ans: 1 (1) $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ ellipse with centre $(0, 0)$

(2) $\frac{x}{a} + \frac{y}{b} = 1$ line passing through $(a, 0)$ & $(0, b)$



$$\text{Req area} = \int_0^a \frac{b}{a}\sqrt{a^2-x^2} - \frac{b}{a}(a-x)\,dx$$

$$= \frac{b}{a}\int_0^a \sqrt{a^2-x^2} - a + x\,dx$$

$$= \frac{b}{a}\left[\frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right) - ax + \frac{x^2}{2}\right]_0^a$$

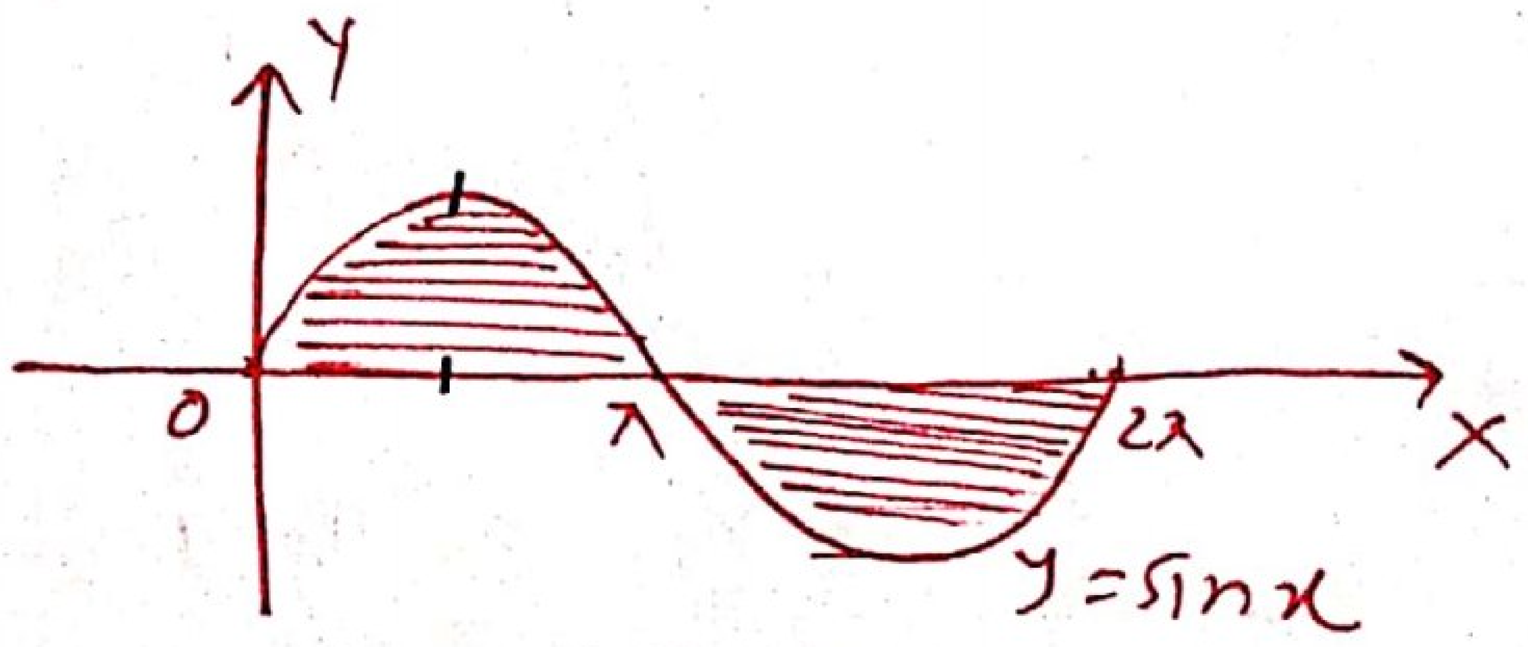
$$= \frac{b}{a}\left[\left(0 + \frac{a^2}{2}\cdot\frac{\pi}{2} - a^2 + \frac{a^2}{2}\right) - (0)\right]$$

$$= \frac{b}{a}\left[\frac{a^2\pi}{4} - \frac{a^2}{2}\right]$$

$$= \frac{a^2 b}{4a}\left[\pi - 2\right]$$

$$= \frac{ab}{4}(\pi - 2) \text{ square units}$$ **Ans**

Ans: 2 → $y = \sin x$; $x = 0$ & $x = 2\pi$



$$\text{Required area} = 2\int_0^\pi (\sin x - 0)\,dx$$

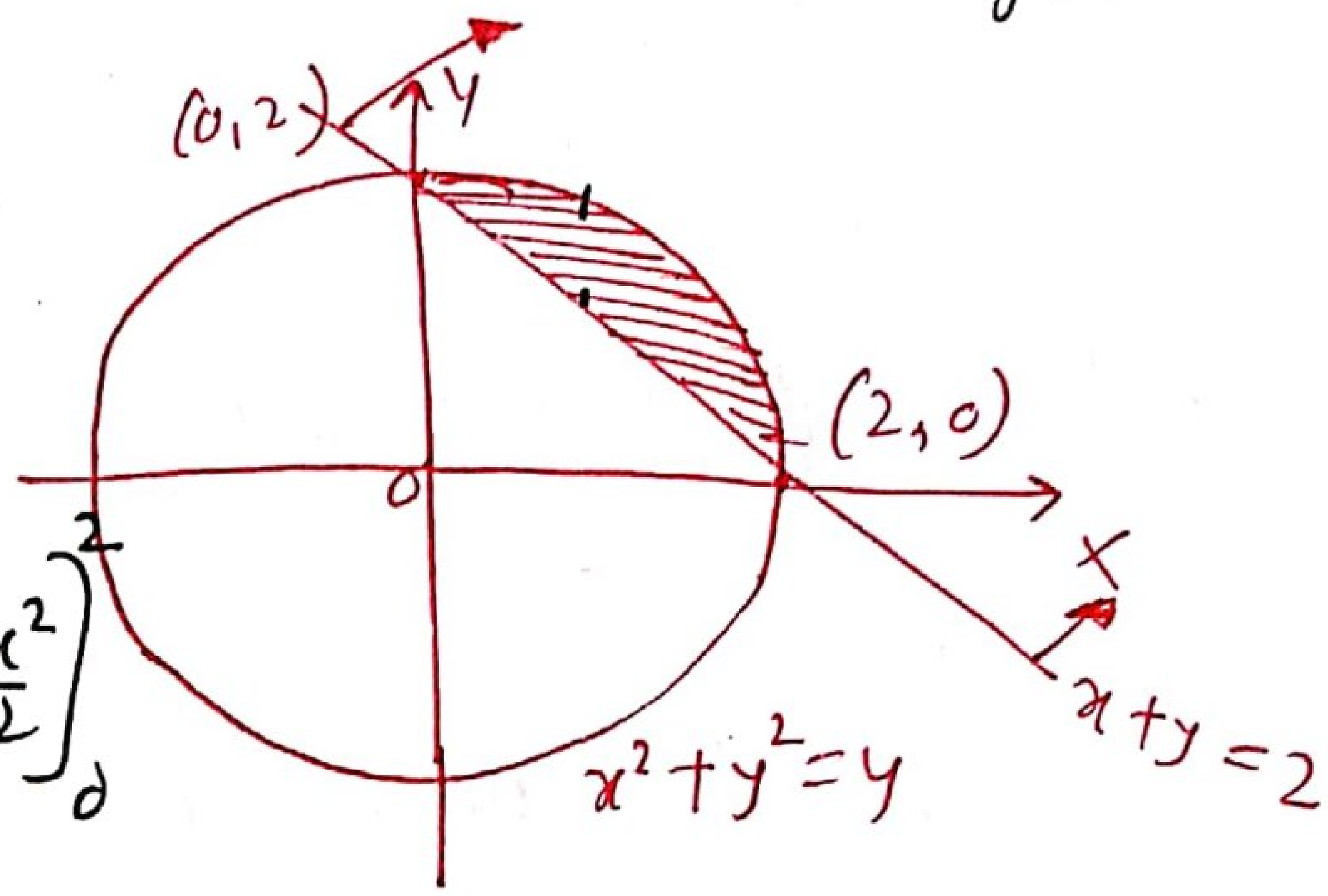
$$= 2(-\cos x)_0^\pi$$

$$= 2[-\cos\pi + \cos 0] = 2[1+1] = 4 \text{ square units}$$ **Ans**

Ans 3 → (1) $x^2+y^2 \le 4$ Circle ; centre $(0,0)$; Radius $= 2$
solution. Inside the circle

(2) $x+y \ge 2$: line passing through $(0,2)$ $(2,0)$
Solution = away from the Origin

Required Area $= \int_0^2 \sqrt{4-x^2} - (2-x)\, dx$



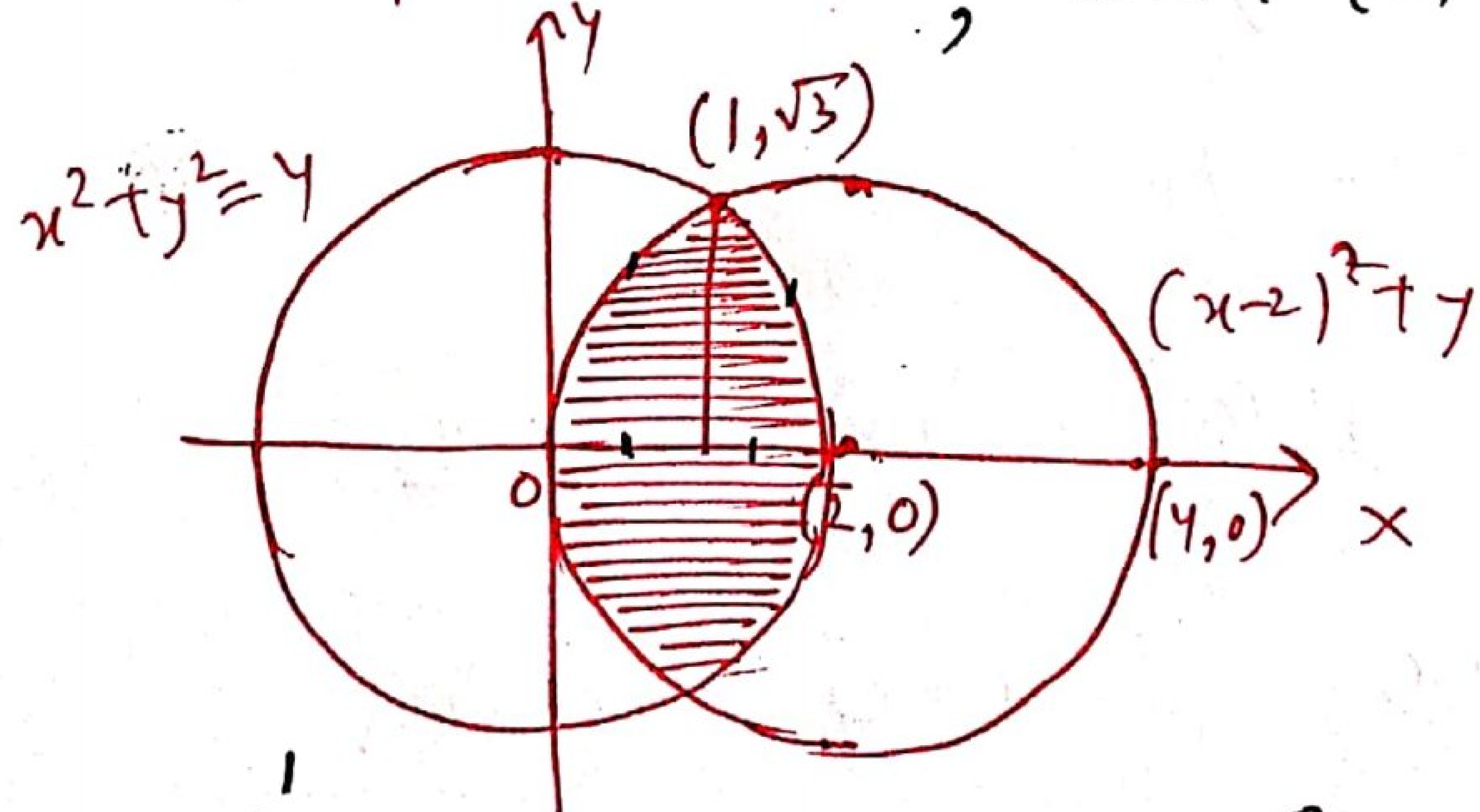
$= \left[\frac{x}{2}\sqrt{4-x^2} + 2\sin^{-1}\left(\frac{x}{2}\right) - 2x + \frac{x^2}{2} \right]_0^2$

$= \left[0 + 2 \times \frac{\pi}{2} - 4 + 2\right] - [0]$

$= (\pi - 2)$ square units **Ans**

Ans 4 → (1) $(x-2)^2+y^2=4$ Circle, Centre $(2,0)$, Radius $= 2$

(2) $x^2+y^2=4$ Circle, Centre $(0,0)$, Radius $= 2$



Required area $= 2\int_0^1 \sqrt{4-(x-2)^2}\, dx + 2\int_1^2 \sqrt{4-x^2}\, dx$

$= 2\left[\frac{(x-2)}{2}\sqrt{4-(x-2)^2} + 2\sin^{-1}\left(\frac{x-2}{2}\right) \right]_0^1 + 2\left[\frac{x}{2}\sqrt{4-x^2} + 2\sin^{-1}\left(\frac{x}{2}\right) \right]_1^2$

$$=2\left[\left(-\frac{1}{2}\sqrt{3}+2\sin^{-1}\left(-\frac{1}{2}\right)\right)-\left(0+2\sin^{-1}(-1)\right)\right]+2\left[\left(0+2\sin^{-1}(1)\right)-\left(\frac{1}{2}\sqrt{3}+2\sin^{-1}\frac{1}{2}\right)\right]$$

$$=2\left[-\frac{\sqrt{3}}{2}-\frac{\pi}{3}+\pi\right]+2\left[\pi-\frac{\sqrt{3}}{2}-\frac{\pi}{3}\right]$$

$$=-\sqrt{3}-\frac{2\pi}{3}+2\pi+2\pi-\sqrt{3}-\frac{2\pi}{3}$$

$$=4\pi-\frac{4\pi}{3}-2\sqrt{3}$$

$$=\left(\frac{8\pi}{3}-2\sqrt{3}\right)$$ square units **Ans**

Ans: 5 →

(1) $x^2+y^2=8x$

$x^2-8x+y^2=0$

$(x-4)^2-16+y^2=0$

$\Rightarrow (x-4)^2+y^2=16$

Circle centre $(4,0)$ Radius $=4$

(2) $y^2=4x$ parabola ; vertex $(0,0)$; open +ve x-axis

Int. point

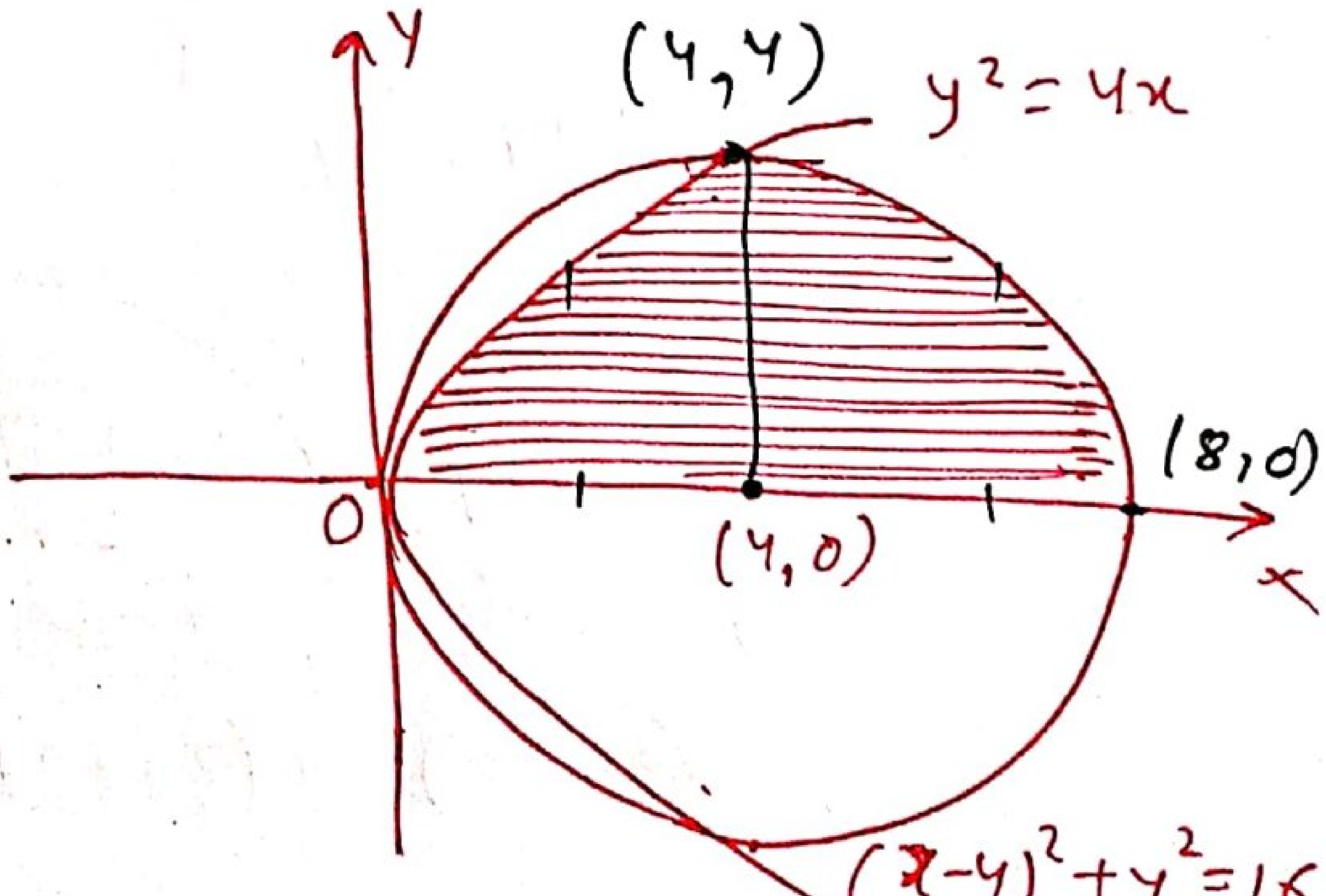
$x^2+y^2=8x$ & $y^2=4x$

$\Rightarrow x^2+4x=8x$

$\Rightarrow x^2-4x=0$

$\Rightarrow x(x-4)=0$

$\Rightarrow x=0;\ x=4$



Required area $=\int_0^4 2\sqrt{x}\,dx+\int_4^8\sqrt{16-(x-4)^2}\,dx$

$$=2\times\frac{2}{3}\left(x^{3/2}\right)_0^4+\left[\frac{(x-4)}{2}\sqrt{16-(x-4)^2}+8\sin^{-1}\left(\frac{x-4}{4}\right)\right]_4^8$$

$= \frac{4}{3}(8-0) + [(0 + 8\sin^{-1}(1)) - (0)]$

$= \frac{32}{3} + 8 \times \frac{\pi}{2}$

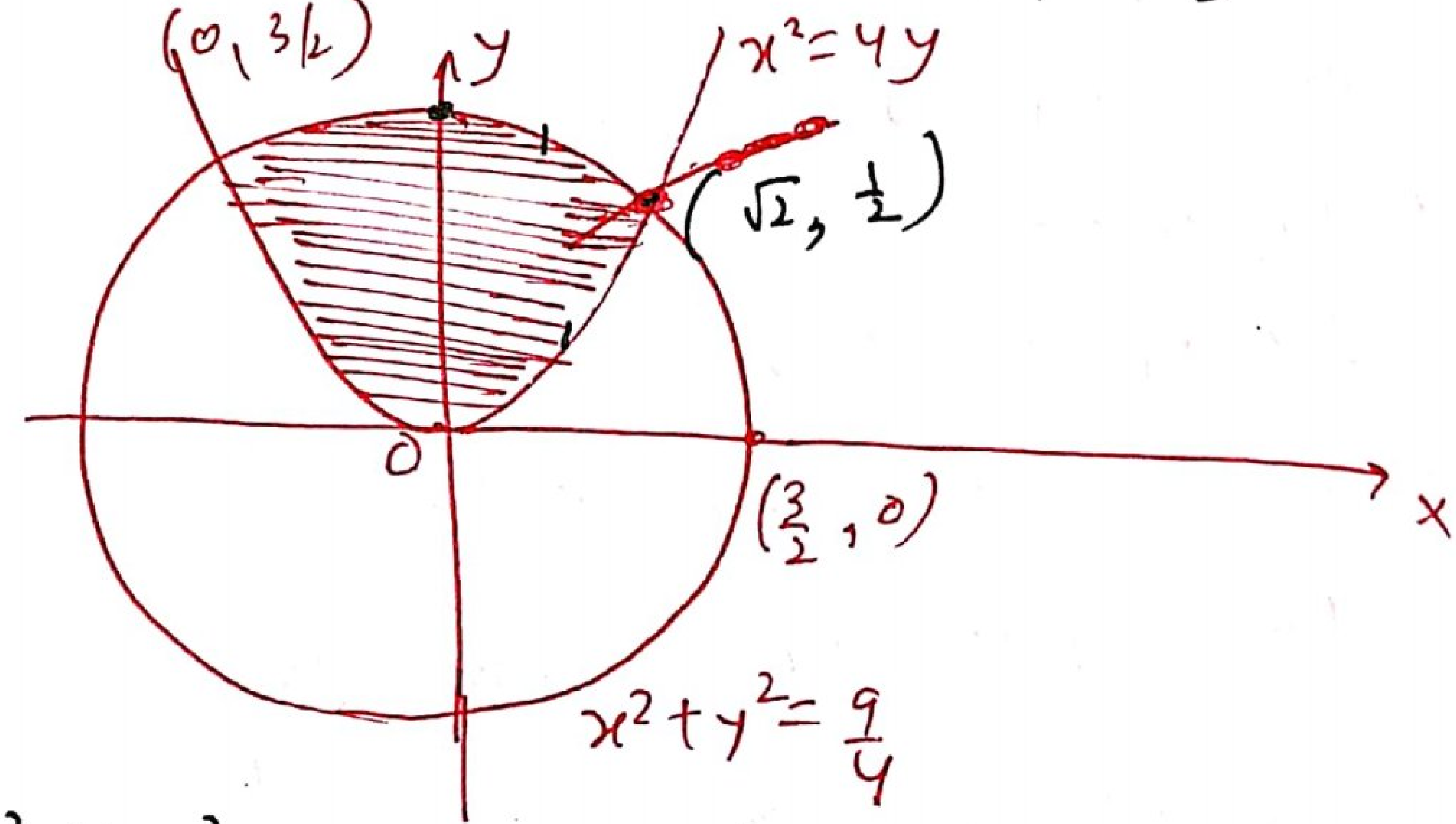
$= 4\pi + \frac{32}{3}$

$= \frac{4}{3}(8 + 3\pi)$ square units Ans

Ans 6 →

(1) $4x^2 + 4y^2 = 9$; circle, centre $(0,0)$
(or) $x^2 + y^2 = \frac{9}{4}$; Radius $= 3/2$

(2) $x^2 = 4y$; parabola, vertex $(0,0)$
open the y-axis



Int. point $4x^2 + 4y^2 = 9$ & $x^2 = 4y$

$\Rightarrow 16y + 4y^2 = 9$

$\Rightarrow 4y^2 + 16y - 9 = 0$

$\Rightarrow 4y^2 + 18y - 2y - 9 = 0$

$\Rightarrow 2y(2y+9) - 1(2y+9) = 0$

$\Rightarrow y = \frac{1}{2}$; $y = -\frac{9}{2}$ (Rejected)

$x = \sqrt{2}$

Required area $= 2\int_0^{\sqrt{2}} \sqrt{\frac{9}{4} - x^2} - \frac{x^2}{4}\, dx$

$$= 2\left[\frac{x}{2}\sqrt{\frac{9}{4}-x^2} + \frac{9}{8}\sin^{-1}\left(\frac{2x}{3}\right) - \frac{x^3}{12}\right]_0^{\sqrt{2}}$$

$$= 2\left[\left(\frac{\sqrt{2}}{2}\times\frac{1}{2} + \frac{9}{8}\sin^{-1}\left(\frac{2\sqrt{2}}{3}\right) - \frac{2\sqrt{2}}{12}\right) - 0\right]$$

$$= \frac{\sqrt{2}}{2} + \frac{9}{4}\sin^{-1}\left(\frac{2\sqrt{2}}{3}\right) - \frac{\sqrt{2}}{3}$$

$$= \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{3} + \frac{9}{4}\sin^{-1}\left(\frac{2\sqrt{2}}{3}\right)$$

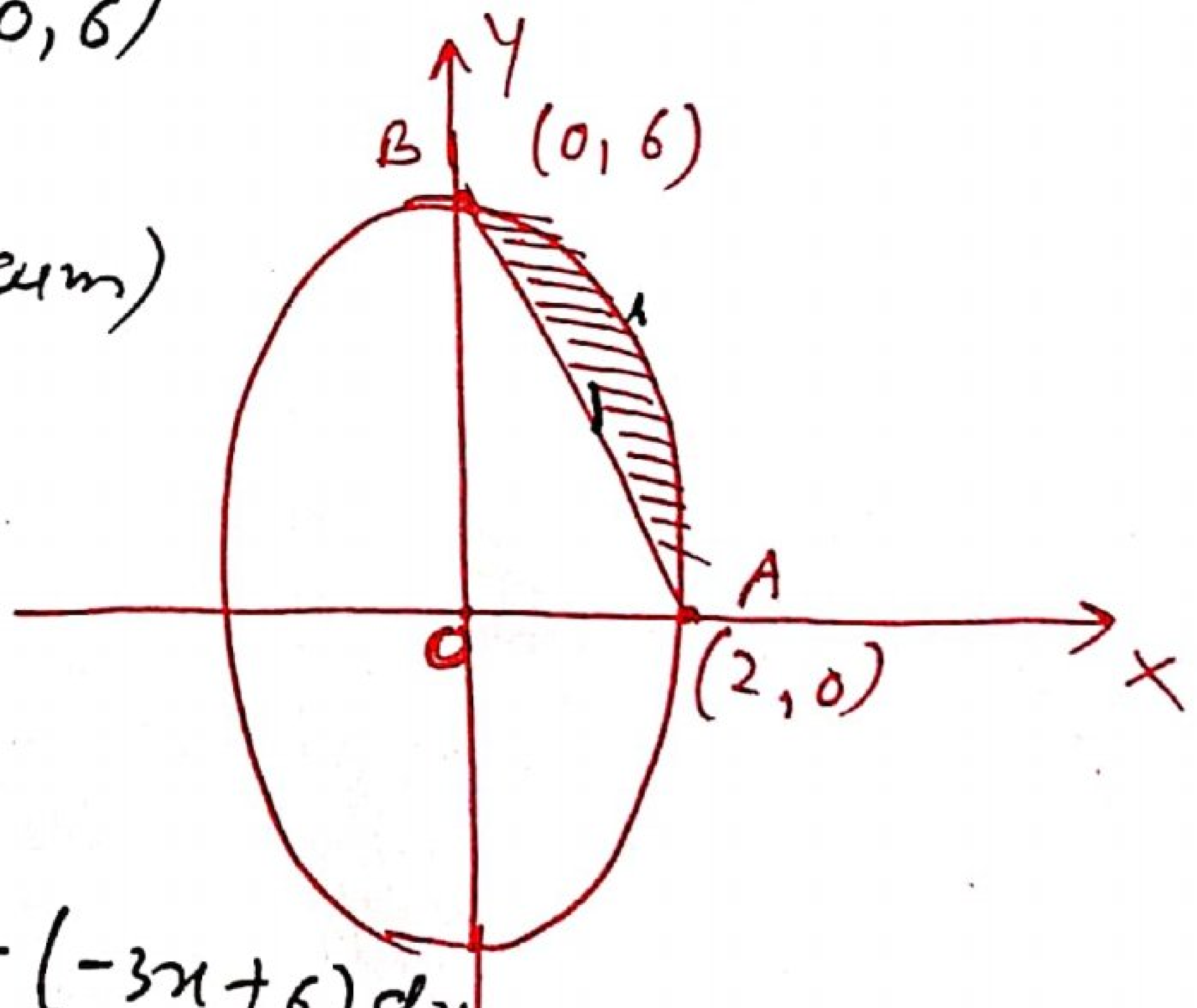
$$= \frac{\sqrt{2}}{6} + \frac{9}{4}\sin^{-1}\left(\frac{2\sqrt{2}}{3}\right) \text{ square units} \quad \underline{\text{Ans}}$$

Ans 7 → (1) $9x^2 + y^2 = 36$

$$\frac{x^2}{4} + \frac{y^2}{36} = 1 \Rightarrow a = 2 \text{ \& } b = 6 \quad (b > a)$$

ellipse ; Centre $(0, 0)$ — vertical ellipse

vertices $(2, 0)$ & $(0, 6)$



equation of chord AB (two point form)

$$y - 0 = \frac{6}{-2}(x - 2)$$

$$\boxed{y = -3x + 6}$$

$$\text{Required area} = \int_0^2 3\sqrt{4-x^2} - (-3x+6)\,dx$$

$$= 3\int_0^2 \sqrt{4-x^2} + x - 2\,dx$$

$$= 3\left[\frac{x}{2}\sqrt{4-x^2} + 2\sin^{-1}\left(\frac{x}{2}\right) + \frac{x^2}{2} - 2x\right]_0^2$$

$$= 3\left[\left(2\times\frac{\pi}{2} + 2 - 4\right) - (0)\right]$$

$$= 3(\pi - 2) = 3\pi - 6 \text{ square units} \quad \underline{\text{Ans}}$$

— x —