SOLUTIONS : WORKSHEET NO: 1 D·E

Ans: 1 $\frac{dy}{dx} - y = \cos x$

here $P = -1$ & $Q = \cos x$

$I.F = e^{\int P dx} = e^{-\int 1 \cdot dx} = e^{-x}$

Solution is given by

$$y \cdot IF = \int Q \times IF \, dx + C$$

$$y \cdot e^{-x} = \int \cos x \cdot e^{-x} dx + C$$

$$y \cdot e^{-x} = I_1 + C \quad \text{-----} (i)$$

where $I_1 = \int \underset{II}{e^{-x}} \cdot \underset{I}{\cos x} \, dx$

$$I_1 = \cos x \cdot \frac{e^{-x}}{-1} - \int -\sin x \cdot \frac{e^{-x}}{-1} \cdot dx$$

$$I_1 = -e^{-x}\cos x - \int \underset{II}{e^{-x}} \cdot \underset{I}{\sin x} \, dx$$

$$I_1 = -e^{-x}\cos x - \left[-\sin x \, e^{-x} + \int \cos x \, e^{-x} dx\right]$$

$$I_1 = -e^{-x}\cos x + e^{-x}\sin x - I_1$$

$$\Rightarrow 2I_1 = e^{-x}(\sin x - \cos x)$$

$$\Rightarrow I_1 = \frac{e^{-x}}{2}(\sin x - \cos x)$$

$\therefore$ equation (i) becomes

$$ye^{-x} = \frac{e^{-x}}{2}(\sin x - \cos x) + C$$

$\Rightarrow \boxed{y = \frac{1}{2}(\sin x - \cos x) + Ce^{x}}$ is the solution Ans

Ans: 2 $y\,dx - (x + 2y^2)\,dy = 0$

$\Rightarrow y\,dx = (x + 2y^2)\,dy$

$\Rightarrow \frac{dx}{dy} = \frac{x + 2y^2}{y}$

$\Rightarrow \frac{dx}{dy} = \frac{x}{y} + 2y$

$\Rightarrow \frac{dx}{dy} - \frac{x}{y} = 2y$

comparing with $\frac{dx}{dy} + Px = Q$

here $P = -\frac{1}{y}$ and $Q = 2y$

$I.F = e^{\int P\,dy} = e^{-\int \frac{1}{y}\,dy} = e^{-\log y} = e^{\log (y)^{-1}} = \frac{1}{y}$

$\therefore \boxed{I.F = \frac{1}{y}}$

Solution is given by

$x \times I.F = \int Q \times I.F\,dy + C$

$\Rightarrow x \times \frac{1}{y} = \int 2\cancel{y} \times \frac{1}{\cancel{y}}\,dy + C$

$\Rightarrow \frac{x}{y} = 2y + C$

$\Rightarrow \boxed{x = 2y^2 + Cy}$ is the solution Ans

Ans: 3 → $\frac{dy}{dx} + y\cot x = 2x + x^2\cot x$; $\boxed{y=0,\ x=\pi/2}$

here $P = \cot x$ & $Q = 2x + x^2 \cot x$

$IF = e^{\int \cot x\, dx} = e^{\log(\sin x)} = \sin x$

Solution

$y \times \sin x = \int \left(2x + x^2 \cot x\right) . \sin x\, dx + C$

$y \sin x = \int 2x \sin x + x^2 . \cos x\, dx + C$

$y \sin x = 2\int x \sin x\, dx + \int \underset{I}{x^2} . \underset{II}{\cos x}\, dx + C$

$\Rightarrow y \sin x = \cancel{2\int x \sin x\, dx} + \left[x^2 \sin x - \int 2x . \cancel{\sin x\, dx}\right] + c$

$\Rightarrow y \sin x = x^2 \sin x + c$

put $y=0$ & $x = \pi/2$

$\Rightarrow 0 = \frac{\pi^2}{4} . \sin\left(\frac{\pi}{2}\right) + c$

$\Rightarrow c = -\frac{\pi^2}{4}$

$\therefore y \sin x = x^2 \sin x - \frac{\pi^2}{4}$

$\Rightarrow \boxed{y = x^2 - \frac{\pi^2}{4 \sin x}}$ Ans

Ans 4 → Slope of tangent at $(x, y) = \frac{dy}{dx}$

Now given $\frac{dy}{dx} = x + xy$

$\Rightarrow \frac{dy}{dx} - xy = x$

Comparing with $\frac{dy}{dx} + Py = Q$

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Here $P = -x$ & $Q = x$

$I.F = e^{\int P.dx} = e^{-\int x\, dx} = e^{-\frac{x^2}{2}}$

<u>Solution</u>

$y \times I.F = \int Q \times IF\, dx + c$

$\Rightarrow y.e^{-x^2/2} = \int x.e^{-\frac{x^2}{2}}\, dx + C$

put $-\frac{x^2}{2} = t$

$\Rightarrow -x\,dx = dt$

$\Rightarrow x\,dx = -dt$

$\therefore y.e^{-x^2/2} = -\int e^t dt + C$

$\Rightarrow y.e^{-x^2/2} = -e^t + C$

$\Rightarrow ye^{-x^2/2} = -e^{-x^2/2} + C$

$\Rightarrow y = -1 + C.e^{x^2/2}$

this curve passing through the point $(0,1)$

put $x = 0$ & $y = 1$

$\Rightarrow 1 = -1 + Ce^0$

$\Rightarrow 1 = -1 + C$

$\Rightarrow C = 2$

$\therefore \boxed{y = -1 + 2e^{x^2/2}}$ is the required equation of curve <u>Ans</u>

Ans 5 → $\frac{dy}{dx} - 3y\cot x = \sin(2x)$; $y(\frac{\pi}{2}) = 2$

Comparing with $\frac{dy}{dx} + Py = Q$

here $P = -3\cot x$ & $Q = \sin(2x)$

$$I.F = e^{\int P dx} = e^{-3\int \cot x\, dx} = e^{-3\log(\sin x)} = e^{\log(\sin x)^{-3}}$$

$$\Rightarrow \boxed{I.F = \frac{1}{\sin^3 x}}$$

solution

$$y \times IF = \int Q \times IF\, dx + C$$

$$\Rightarrow y \cdot \frac{1}{\sin^3 x} = \int \sin(2x) \cdot \frac{1}{\sin^3 x}\, dx + C$$

$$\Rightarrow y \cdot \frac{1}{\sin^3 x} = 2\int \sin x \cdot \cos x \cdot \frac{1}{\sin^3 x}\, dx + C$$

$$= 2\int \frac{\cos x}{\sin^2 x}\, dx + C$$

$$= 2\int \cot x \cdot \operatorname{cosec} x\, dx + C$$

$$\Rightarrow \frac{y}{\sin^3 x} = -2\operatorname{cosec} x + C$$

$$\Rightarrow y = -2\sin^2 x + C \cdot \sin^3 x$$

put $x = \pi/2$ & $y = 2$

$$\Rightarrow 2 = -2 + C$$

$$\Rightarrow C = 4$$

$\therefore$ $\boxed{y = -2\sin^2 x + 4\sin^3 x}$ is the Required solution

Ans 6 → $x\frac{dy}{dx} + y - x + xy\cot x = 0$

$$\Rightarrow \quad x\frac{dy}{dx} + y(1 + x\cot x) = x$$

divide by x

$$\Rightarrow \quad \frac{dy}{dx} + y\left(\frac{1}{x} + \cot x\right) = 1$$

here $P = \frac{1}{x} + \cot x$ & $Q = 1$

$$IF = e^{\int P\,dx} = e^{\int \frac{1}{x} + \cot x\, dx} = e^{\log x + \log(\sin x)}$$

$$= e^{\log(x\sin x)}$$

$$\boxed{I.F = x\sin x}$$

Solution

$$y \times IF = \int (Q \times IF)\,dx + C$$

$$\Rightarrow y(x\sin x) = \int \underset{I}{x}\,\underset{II}{\sin x}\,dx + C$$

$$\Rightarrow xy\sin x = -x\cos x + \int \cos x\,dx + C$$

$$\Rightarrow xy\sin x = -x\cos x + \sin x + C$$

$$\Rightarrow y = -\frac{x\cos x}{x\sin x} + \frac{\sin x}{x\sin x} + \frac{C}{x\sin x}$$

$$\Rightarrow \boxed{y = -\cot x + \frac{1}{x} + \frac{C}{x\sin x}}$$ Ans

Ans 7 → $(1+x^2)\frac{dy}{dx} + 2xy = \frac{1}{1+x^2}$; $y = 0, x = 1$

divide by $(1+x^2)$

$$\Rightarrow \frac{dy}{dx} + \frac{2x}{1+x^2}\cdot y = \frac{1}{(1+x^2)^2}$$

here $P = \frac{2x}{1+x^2}$ & $Q = \frac{1}{(1+x^2)^2}$

$I.F = e^{\int \frac{2x}{1+x^2} dx}$ put $1+x^2 = t$

$2x\,dx = dt$

$I.F = e^{\int \frac{dt}{t}} = e^{\log t} = t = 1+x^2$

$\therefore \boxed{IF = 1+x^2}$

solution $y \times IF = \int (Q \times IF)\, dx + C$

$\Rightarrow y(1+x^2) = \int \frac{1}{(1+x^2)^{\cancel{2}}} \cdot \cancel{(1+x^2)}\, dx + C$

$\Rightarrow y(1+x^2) = \tan^{-1}x + C$

put $x = 1$ & $y = 0$

$\Rightarrow 0 = \tan^{-1}(1) + C$

$\Rightarrow \boxed{C = -\frac{\pi}{4}}$

$\therefore \boxed{y(1+x^2) = \tan^{-1}x - \frac{\pi}{4}}$ Ans

Ans 8 →

$$\left[\frac{e^{-2\sqrt{x}}}{\sqrt{x}} - \frac{y}{\sqrt{x}} \right] \frac{dx}{dy} = 1$$

$\Rightarrow \frac{dy}{dx} = \frac{e^{-2\sqrt{x}}}{\sqrt{x}} - \frac{y}{\sqrt{x}}$

$\Rightarrow \frac{dy}{dx} + \frac{y}{\sqrt{x}} = \frac{e^{-2\sqrt{x}}}{\sqrt{x}}$

here $P = \frac{1}{\sqrt{x}}$ & $Q = \frac{e^{-2\sqrt{x}}}{\sqrt{x}}$

$I.F = e^{\int P dx} = e^{\int \frac{1}{\sqrt{x}} dx} = e^{2\sqrt{x}}$ $\therefore \boxed{I.F = e^{2\sqrt{x}}}$

Solution

$$y \times IF = \int (Q \times IF)\,dx + C$$

$$\Rightarrow y e^{2\sqrt{x}} = \int \frac{\cancel{e^{-2\sqrt{x}}}}{\sqrt{x}} \cdot \cancel{e^{2\sqrt{x}}}\,dx + C$$

$$\Rightarrow y e^{2\sqrt{x}} = \int \frac{1}{\sqrt{x}}\,dx + C$$

$$\Rightarrow \boxed{y e^{2\sqrt{x}} = 2\sqrt{x} + C}$$ Ans

Ans 9 → $(x+y)\frac{dy}{dx} = 1$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{x+y}$$

$$\Rightarrow \frac{dx}{dy} = x+y$$

$$\Rightarrow \frac{dx}{dy} - x = y$$

Comp with $\frac{dx}{dy} + Px = Q$

here $P = -1$ & $Q = y$

$$I.F = e^{\int P dy} = e^{\int -1 \cdot dy} = e^{-y}$$

Solution

$$x \times IF = \int (Q \times IF)\,dy + C$$

$$\Rightarrow x e^{-y} = \int \underset{I}{y} \cdot \underset{II}{e^{-y}}\,dy + c$$

$$\Rightarrow x e^{-y} = \frac{y e^{-y}}{-1} + \int e^{-y} \cdot dy + c$$

$$\Rightarrow x e^{-y} = -y e^{-y} - e^{-y} + C$$

$$\Rightarrow x = -y - 1 + c e^{y} \Rightarrow \boxed{x + y + 1 = c e^{y}}$$ Ans

Qu 10 →

$$(1-y^2)\frac{dx}{dy} + yx = ay$$

$$\Rightarrow \quad \frac{dx}{dy} + \frac{yx}{1-y^2} = \frac{ay}{1-y^2}$$

Comp with $\frac{dx}{dy} + Px = Q$

here $P = \frac{y}{1-y^2}$; $Q = \frac{ay}{1-y^2}$

$$I.F = e^{\int P dy}$$

$$IF = e^{\int \frac{y}{1-y^2} dy}$$

put $1-y^2 = t$

$$y\,dy = -\frac{dt}{2}$$

$$\therefore IF = e^{-\frac{1}{2}\int \frac{dt}{t}} = e^{-\frac{1}{2}\int \frac{dt}{t}}$$

$$= e^{-\frac{1}{2}\log t}$$

$$= e^{\log(t)^{-1/2}}$$

$$= t^{-1/2} \quad \text{-----} \{\because e^{\log x} = x\}$$

$$= \frac{1}{\sqrt{t}}$$

$$\boxed{IF = \frac{1}{\sqrt{1-y^2}}}$$

Ans