Class No: 4 Differentiation & Continuity (1)

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WORKSHEET No: 3

Ques 1 → $y = \sqrt{\sin x + \sqrt{\sin x + \sqrt{\sin x + \cdots \infty}}}$ show $\frac{dy}{dx} = \frac{\cos x}{2y-1}$

Ques 2 → $y = e^{x + e^{x + e^{x + \cdots \infty}}}$ show $\frac{dy}{dx} = \frac{y}{1-y}$

Ques 3 → $y = (\sqrt{x})^{(\sqrt{x})^{(\sqrt{x})^{\cdots \infty}}}$ show $\frac{dy}{dx} = \frac{y^2}{x(2 - y\log x)}$

Ques 4 → $y = x + \cfrac{1}{x + \cfrac{1}{x + \cfrac{1}{x + \cdots \infty}}}$ show $\frac{dy}{dx} = \frac{y}{2y-x}$

Ques. 5 → $y = (\cos x)^{(\cos x)^{(\cos x)^{\cdots \infty}}}$ show $\frac{dy}{dx} = \frac{y^2 \tan x}{1 - y\log(\cos x)}$

Ques 6 → $y = \tan^{-1}\left(\frac{3x - x^3}{1 - 3x^2}\right)$ find $\frac{dy}{dx}$, if

(i) $-\frac{1}{\sqrt{3}} < x < \frac{1}{\sqrt{3}}$ (ii) $x > \frac{1}{\sqrt{3}}$ (iii) $x < -\frac{1}{\sqrt{3}}$

Ans (i) $\frac{3}{1+x^2}$ (ii) $\frac{3}{1+x^2}$ (iii) $\frac{3}{1+x^2}$

Imp Point:

$\tan\theta = -\tan(-\theta)$	$\cos\theta = \cos(-\theta)$
$\tan\theta = -\tan(\pi - \theta)$	$\cos\theta = -\cos(\pi - \theta)$
$\tan\theta = \tan(\pi + \theta)$	$\cos\theta = -\cos(\pi + \theta)$
$\tan\theta = -\tan(2\pi - \theta)$	$\cos\theta = \cos(2\pi - \theta)$

Class - 4 Worksheet No: 3 (2)

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Qn 7 → $y = \sec^{-1}\left(\frac{1}{2x^2-1}\right)$; $0 < x < \frac{1}{\sqrt{2}}$ find $\frac{dy}{dx}$

Ans $\frac{dy}{dx} = \frac{-2}{\sqrt{1-x^2}}$

HINT put $x = \cos(2\theta)$ and $\sec^{-1}x = \cos^{-1}(1/x)$

Qn 8 → $y = \cos^{-1}\left(2x\sqrt{1-x^2}\right)$; $-\frac{1}{\sqrt{2}} < x < \frac{1}{\sqrt{2}}$ find $\frac{dy}{dx}$

Ans $\frac{dy}{dx} = \frac{-2}{\sqrt{1-x^2}}$

HINT put $x = \sin\theta$

Qn 9 → $y = \tan^{-1}\left(\frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}\right)$; $-1 < x < 1$ find $\frac{dy}{dx}$

Ans $\frac{dy}{dx} = \frac{-x}{\sqrt{1-x^4}}$

Qn 10 → $y = \tan^{-1}\left(\frac{2x}{1-x^2}\right) + \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$; $x \in (0,1)$ find $\frac{dy}{dx}$

Ans $\frac{dy}{dx} = \frac{4}{1+x^2}$

HINT put $x = \tan\theta$

Qn 11 → $y = \tan^{-1}\left(\frac{x}{1+6x^2}\right)$ find $\frac{dy}{dx}$

Ans $\frac{dy}{dx} = \frac{3}{1+9x^2} - \frac{2}{1+4x^2}$

then Diff.

HINT $y = \tan^{-1}\left(\frac{3x-2x}{1+6x^2}\right) \Rightarrow y = \tan^{-1}(3x) - \tan^{-1}(2x)$

Qn 12 → $y = \sin^{-1}\left(\frac{2^{x+1}}{1+4^x}\right)$ find $\frac{dy}{dx}$ Ans $\frac{dy}{dx} = \frac{2^{x+1}\cdot\log 2}{1+4^x}$

HINT $2^{x+1} = 2\cdot 2^x = 2^x + 2^x$

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Differentiation & Continuity

CLASS NO: 4

TOPIC: 3 INFINITE SERIES

Ques 1 → $y = x^{x^{x^{\cdots\infty}}}$ find $\frac{dy}{dx}$

Soln

we have $y = x^{\left(x^{x^{\cdots\infty}}\right)}$

Main point
Jahan se Repetation
Start ho wahan
y likh denay

$\Rightarrow y = x^{y}$

Taking log on both sides

$\log y = y \log x$

Diff wrt x

$\frac{1}{y}\cdot\frac{dy}{dx} = y\cdot\frac{1}{x} + \log x\cdot\frac{dy}{dx}$

$\Rightarrow \frac{dy}{dx}\left(\frac{1}{y} - \log x\right) = \frac{y}{x}$

$\Rightarrow \frac{dy}{dx}\left(\frac{1 - y\log x}{y}\right) = \frac{y}{x}$

$\Rightarrow \frac{dy}{dx} = \frac{y^2}{x(1 - y\log x)}$ ANS

Ques 2 → $y = \sqrt{\log x + \sqrt{\log x + \sqrt{\log x + \cdots\infty}}}$

Soln

$y = \sqrt{\log x + \left(\sqrt{\log x + \sqrt{\log x + \cdots\infty}}\right)}$

$y = \sqrt{\log x + y}$

Squaring

$y^2 = \log x + y$

Diff wrt x

$$2y \cdot \frac{dy}{dx} = \frac{1}{x} + \frac{dy}{dx}$$

$$\Rightarrow \frac{dy}{dx}(2y-1) = \frac{1}{x}$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{x(2y-1)} \quad \underline{\text{Ans}}$$

Qns $\underline{3}$ $\rightarrow$ $y = (\tan x)^{(\tan x)^{(\tan x) \cdots \infty}}$ find $\frac{dy}{dx}$ at $x = \frac{\pi}{4}$

Solution $y = (\tan x)^y$

taking log

$$\log y = y \log(\tan x)$$

Diff wrt x

$$\frac{1}{y} \cdot \frac{dy}{dx} = y \cdot \frac{1}{\tan x} \cdot \sec^2 x + \log(\tan x) \cdot \frac{dy}{dx}$$

$$\Rightarrow \frac{dy}{dx}\left(\frac{1}{y} - \log(\tan x)\right) = \frac{y \sec^2 x}{\tan x}$$

$$\Rightarrow \frac{dy}{dx}\left\{\frac{1 - y\log(\tan x)}{y}\right\} = \frac{y \sec^2 x}{\tan x}$$

$$\Rightarrow \frac{dy}{dx} = \frac{y^2 \sec^2 x}{\tan x(1 - y \log \tan x)}$$

put $x = \pi/4$

$$\left(\frac{dy}{dx}\right)_{x=\pi/4} = \frac{y^2 \sec^2(\pi/4)}{\tan(\frac{\pi}{4})\left(1-y\log(\tan\frac{\pi}{4})\right)}$$

$$= \frac{y^2 (\sqrt{2})^2}{(1)(1-y\log 1)}$$

$$= \frac{2y^2}{1-0} \quad \text{---} \{\because \log 1 = 0\}$$

$$= 2y^2$$

$$= 2(1)^2 \quad \text{---} \left\{ \begin{array}{l} \text{since } y = (\tan x)^y \\ \text{when } x = \pi/4 \\ \quad y = (1)^y = 1 \end{array} \right\}$$

$$\left(\frac{dy}{dx}\right)_{x=\pi/4} = 2 \quad \underline{\underline{\text{Ans}}}$$

Qns 4 → $y = a^{x^{a^{x^{\cdots\infty}}}}$ find $\frac{dy}{dx}$

Soln $y = a^{x^{\left(a^{x^{\cdots\infty}}\right)}}$

$y = a^{x^y}$ ---- {remember here $y \neq (a^x)^y$}

taking log

$\log y = x^y . \log a$

taking again log

$\log(\log y) = \log(x^y . \log a)$

$\Rightarrow \log(\log y) = y\log x + \log(\log a)$

Diff. w.r.t x

$$\frac{1}{\log y}\cdot\frac{1}{y}\cdot\frac{dy}{dx} = y\cdot\frac{1}{x} + \log x\,\frac{dy}{dx} + 0$$

$$\Rightarrow \frac{dy}{dx}\left(\frac{1}{y\log y} - \log x\right) = \frac{y}{x}$$

$$\Rightarrow \frac{dy}{dx}\left(\frac{1 - y\log x\cdot\log y}{y\log y}\right) = \frac{y}{x}$$

$$\Rightarrow \frac{dy}{dx} = \frac{y^2\log y}{x(1 - y\log x\log y)}$$ Ans

Ques 5 → $y = \cfrac{\sin x}{1+\cfrac{\cos x}{1+\cfrac{\sin x}{1+\cfrac{\cos x}{1+\cdots\infty}}}}$ find $\frac{dy}{dx}$

Solution

$$y = \cfrac{\sin x}{1+\cfrac{\cos x}{1+y}}$$

$$\Rightarrow y = \frac{(1+y)\sin x}{1+y+\cos x}$$

$$\Rightarrow y + y^2 + y\cos x = \sin x + y\sin x$$

Diff w.r.t x

Proceed yourself $\quad \frac{dy}{dx} = \frac{(1+y)\cos x + y\sin x}{1+2y+\cos x - \sin x}$

Topic: 4 Differentiation of Inverse Trigo function

Qns 1 → Differentiate $y = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$ when

(i) $x \in (-1, 1)$ (ii) $x \in (1, \infty)$ (iii) $x \in (-\infty, -1)$

Solution We have $y = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$

put $x = \tan\theta$

$\Rightarrow y = \sin^{-1}\left(\frac{2\tan\theta}{1+\tan^2\theta}\right)$

$\Rightarrow y = \sin^{-1}(\sin(2\theta))$

(i) $x \in (-1, 1)$ (or) $-1 < x < 1$

$\Rightarrow -1 < \tan\theta < 1$

$\Rightarrow -\frac{\pi}{4} < \theta < \frac{\pi}{4}$

$\Rightarrow -\frac{\pi}{2} < 2\theta < \frac{\pi}{2}$ (within Range of $\sin^{-1}x$)

$\Rightarrow y = \sin^{-1}(\sin(2\theta))$

$\Rightarrow y = 2\theta$

replace θ

$y = 2\tan^{-1}x$

Diff w.r.t x

$y = \frac{2}{1+x^2}$ Ans

(ii) x ~~$\in (-\infty$~~, $x \in (1, \infty)$ (or) $1 < x < \infty$

$y = \sin^{-1}(\sin(2\theta))$

$\Rightarrow 1 < \tan\theta < \infty$

$\Rightarrow \frac{\pi}{4} < \theta < \frac{\pi}{2}$

$\Rightarrow \frac{\pi}{2} < 2\theta < \pi$ (out of Range of $\sin^{-1}x$)

Imp points
$$\begin{cases} \sin(2\theta) = -\sin(-2\theta) \\ \sin(2\theta) = \sin(\pi-2\theta) \\ \sin(2\theta) = -\sin(\pi+2\theta) \\ \sin(2\theta) = -\sin(2\pi-2\theta) \end{cases}$$

here $\frac{\pi}{2} < 2\theta < \pi$

$$-\frac{\pi}{2} > -2\theta > -\pi$$

$$\pi - \frac{\pi}{2} > (\pi-2\theta) > \pi-\pi$$

$$\frac{\pi}{2} > (\pi-2\theta) > 0$$

(or) $0 < (\pi-2\theta) < \frac{\pi}{2}$ within Range

$\Rightarrow y = \sin^{-1}(\sin(2\theta))$

$\Rightarrow y = \sin^{-1}(\sin(\pi-2\theta))$

$\Rightarrow y = \pi - 2\theta$

$\Rightarrow y = 2 - \tan^{-1} x$ {replace θ}

Diff

$$\frac{dy}{dx} = -\frac{1}{1+x^2} \quad \underline{\underline{\text{Ans}}}$$

(iii) $x \in (-\infty, -1)$ (or) $-\infty < x < -1$

$y = \sin^{-1}(\sin(2\theta))$

$\Rightarrow -\infty < \tan\theta < -1$

$\Rightarrow -\frac{\pi}{2} < \theta < -\frac{\pi}{4} \rightarrow$ out of range

$\Rightarrow -\pi < 2\theta < -\frac{\pi}{2}$

$$\pi - \pi < (\pi + 2\theta) < \pi - \frac{\pi}{2}$$

$$0 < (\pi + 2\theta) < \frac{\pi}{2} \longrightarrow \text{(within Range)}$$

$$\Rightarrow \quad y = \sin^{-1}(\sin(2\theta))$$

$$\Rightarrow y = \sin^{-1}\left(-\sin(\pi + 2\theta)\right)$$

$$\Rightarrow y = -\sin^{-1}(\sin(\pi + 2\theta))$$

$$\Rightarrow \quad y = -(\pi + 2\theta)$$

$$y = -(\pi + 2\tan^{-1}x) \quad \text{--- \{replace } \theta\text{\}}$$

$$\Rightarrow y = -\pi - 2\tan^{-1}x$$

Diff wrt x

$$\frac{dy}{dx} = 0 - \frac{2}{1+x^2} = \frac{-2}{1+x^2} \quad \underline{\text{Ans}}$$

QN: 2 → $y = \tan^{-1}\left(\sqrt{\frac{1+\sin x}{1-\sin x}}\right)$; $-\frac{\pi}{2} < x < \frac{\pi}{2}$

Solution

$$y = \tan^{-1}\sqrt{\frac{1+\sin x}{1-\sin x}}$$

$$y = \tan^{-1}\sqrt{\frac{1+\cos(\frac{\pi}{2}-x)}{1-\cos(\frac{\pi}{2}-x)}}$$

$$y = \tan^{-1}\sqrt{\frac{\cancel{2}\cos^2(\frac{\pi}{4}-\frac{x}{2})}{\cancel{2}\sin^2(\frac{\pi}{4}-\frac{x}{2})}} \Rightarrow y = \tan^{-1}\sqrt{\cot^2(\frac{\pi}{4}-\frac{x}{2})}$$

DEC

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$$y = \tan^{-1}\left|\cot\left(\frac{\pi}{4}-\frac{x}{2}\right)\right|$$

$$-\frac{\pi}{2} < x < \frac{\pi}{2}$$

$$-\frac{\pi}{4} < \frac{x}{2} < \frac{\pi}{4}$$

$$\frac{\pi}{4} > -\frac{x}{2} > -\frac{\pi}{4}$$

$$\left(\frac{\pi}{4}+\frac{\pi}{4}\right) > \frac{\pi}{4}-\frac{x}{2} > 0$$

$$\frac{\pi}{2} > \left(\frac{\pi}{4}-\frac{x}{2}\right) > 0$$

or $0 < \left(\frac{\pi}{4}-\frac{x}{2}\right) < \frac{\pi}{2}$ → Ist quad.

$$y = \tan^{-1}\left(\cot\left(\frac{\pi}{4}-\frac{x}{2}\right)\right)$$

$$y = \tan^{-1}\left(\tan\left(\frac{\pi}{2}-\left(\frac{\pi}{4}-\frac{x}{2}\right)\right)\right)$$

$$y = \tan^{-1}\left(\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)\right)$$

again $-\frac{\pi}{2} < x < \frac{\pi}{2}$

$$-\frac{\pi}{4} < \frac{x}{2} < \frac{\pi}{4}$$

$$\frac{\pi}{4}-\frac{\pi}{4} < \left(\frac{\pi}{4}+\frac{x}{2}\right) < \frac{\pi}{4}+\frac{\pi}{4}$$

$$0 < \left(\frac{\pi}{4}+\frac{x}{2}\right) < \frac{\pi}{2}$$ → within Range of $\tan^{-1}x$

$$y = \tan^{-1}\left(\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)\right)$$

$$y = \frac{\pi}{4}+\frac{x}{2}$$

Diff w.r.t x

$$\frac{dy}{dx} = 0 + \frac{1}{2} = \frac{1}{2} \quad \text{Ans}$$