Ans: 1 → Given: equation of line

$$3x+1 = 6y-2 = 1-z$$

$$\Rightarrow \frac{3x+1}{1} = \frac{6y-2}{1} = \frac{1-z}{1}$$

$$\Rightarrow \frac{x+1/3}{1/3} = \frac{y-1/3}{1/6} = \frac{z-1}{-1}$$

$$\Rightarrow \frac{x+1/3}{2} = \frac{y-1/3}{1} = \frac{z-1}{-6} \quad --- \left\{\begin{array}{l}\text{Multiply}\\ \text{Denominator}\\ \text{by } 6\end{array}\right\}$$

this is in the form of $\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c}$

(1) fixed point $(x_1, y_1, z_1) = (-1/3, 1/3, 1)$

(2) Direction Ratios: $a, b, c = 2, 1, -6$

(3) vector form: $\vec{r} = \vec{a} + \lambda \vec{b}$

here $\vec{a} = -\frac{1}{3}\hat{i} + \frac{1}{3}\hat{j} + \hat{k}$ & $\vec{b} = 2\hat{i} + \hat{j} - 6\hat{k}$

$\therefore \vec{r} = \left(-\frac{1}{3}\hat{i} + \frac{1}{3}\hat{j} + \hat{k}\right) + \lambda\left(2\hat{i} + \hat{j} - 6\hat{k}\right)$ Ans

—x—

Ans 2 → Given: position vectors of points are

$-2\hat{i} + 3\hat{j}$; $\hat{i} + \lambda\hat{j} + 3\hat{k}$; $7\hat{i} - \hat{k}$

In coordinates, the given points are

$(-2, 3, 0)$; $(1, \lambda, 3)$; $(7, 0, -1)$

Now equation of line passing through $(-2,3,0)$ & $(7,0,-1)$ is given by

$$\frac{x+2}{9} = \frac{y-3}{-3} = \frac{z-0}{-1} \quad \text{-----} \left\{ \text{By formula } \frac{x-x_1}{x_2-x_1} = \frac{y-y_1}{y_2-y_1} = \frac{z-z_1}{z_2-z_1} \right\}$$

Now point $(1, \lambda, 3)$ also lies on it

--- {Since given points are collinear}

$$\Rightarrow \frac{1+2}{9} = \frac{\lambda-3}{-3} = \frac{3-0}{-1}$$

$$\Rightarrow \frac{1}{3} = \frac{\lambda-3}{-3} = -3$$

Not possible {they must be equal}

$\therefore$ the given points are not collinear

$\boxed{\text{No value of } \lambda}$ Ans. {Note: Misprint in worksheet Answers}

—x—

Ans 3 → given equation of line:

$$\frac{x+2}{3} = \frac{y+1}{2} = \frac{z-3}{2}$$

let $\dfrac{x+2}{3} = \dfrac{y+1}{2} = \dfrac{z-3}{2} = \lambda$

Any point / required point on the line is

$$Q(3\lambda-2,\ 2\lambda-1,\ 2\lambda+3)$$

given point $P(1,3,3)$

given $PQ = 5$

$$\Rightarrow \sqrt{(3\lambda-2-1)^2 + (2\lambda-1-3)^2 + (2\lambda+3-3)^2} = 5$$

$\Rightarrow \sqrt{(3\lambda-3)^2+(2\lambda-4)^2+(2\lambda)^2} = 5$

$\Rightarrow 9\lambda^2+9-18\lambda+4\lambda^2+16-16\lambda+4\lambda^2 = 25$

$\Rightarrow 17\lambda^2 - 34\lambda = 0$

$\Rightarrow 17\lambda^2 - 34\lambda = 0.$

$\Rightarrow 17\lambda(\lambda-2) = 0$

$\Rightarrow \lambda = 0$ or $\lambda = 2$

$\therefore$ Required points on the line are

$\boxed{Q(-2,-1,3) \quad \text{or} \quad Q(4,3,7)}$ Ans

-x-

Ans 4 → given : equation of line

$$\frac{x-3}{1} = \frac{y-1}{2} = \frac{z+1}{-2}$$

D.R's of this line $= 1, 2, -2$

Since, Required line is parallel to the given line

$\therefore$ D.R's of required line are $= \lambda, 2\lambda, -2\lambda$

Point on Required line : $(1,-1,2)$ --- (given)

Now equation of Required line

$$\frac{x-1}{\lambda} = \frac{y+1}{2\lambda} = \frac{z-2}{-2\lambda}$$

$$\Rightarrow \boxed{\frac{x-1}{1} = \frac{y+1}{2} = \frac{z-2}{-2}}$$ Ans

Reduction in vector form.

let $\frac{x-1}{1} = \frac{y+1}{2} = \frac{z-2}{-2} = \lambda$

$\Rightarrow x = \lambda + 1 \;;\; y = 2\lambda - 1 \;;\; z = -2\lambda + 2$

Now $x\hat{i} + y\hat{j} + z\hat{k} = (\lambda+1)\hat{i} + (2\lambda-1)\hat{j} + (-2\lambda+2)\hat{k}$

$\Rightarrow \boxed{\vec{r} = (\hat{i} - \hat{j} + 2\hat{k}) + \lambda(\hat{i} + 2\hat{j} - 2\hat{k})}$ Ans

—x—

Qn. 5 → given lines

$\frac{x-5}{5\lambda+2} = \frac{2-y}{5} = \frac{1-z}{-1}$ & $\frac{x}{1} = \frac{2y+1}{4\lambda} = \frac{1-z}{-3}$

Converting in to standard form

$\frac{x-5}{5\lambda+2} = \frac{y-2}{-5} = \frac{z-1}{1}$ & $\frac{x}{1} = \frac{y+1/2}{2\lambda} = \frac{z-1}{3}$

here $a_1 = 5\lambda+2 \;;\; b_1 = -5 \;,\; c_1 = 1$

$a_2 = 1 \;,\; b_2 = 2\lambda \;,\; c_2 = 3$

5

given that given lines are $\perp^r$

$\therefore a_1a_2 + b_1b_2 + c_1c_2 = 0$

$\Rightarrow 5\lambda + 2 - 10\lambda + 3 = 0$

$\Rightarrow 5\lambda = 5$

$\Rightarrow \boxed{\lambda = 1}$ Ans

—x—

Qn. 6 → given lines (In standard form)

$$\frac{x-8}{8} = \frac{y+9}{-16} = \frac{z-10}{7} \quad \text{and}$$

$$\frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}$$

D.R's of Ist line = $8, -16, 7$

D.R's of 2nd line = $3, 8, -5$

let D.R's of Required line = a, b, c

Since, required line is $\perp^r$ to the given lines

$\therefore \quad 8a - 16b + 7c = 0$

$3a + 8b - 5c = 0$

$$\Rightarrow \frac{a}{80-56} = \frac{-b}{-40-21} = \frac{c}{64+48} = \lambda$$

$\Rightarrow a = 24\lambda \; ; \; b = 61\lambda \; ; \; c = 112\lambda$

$\therefore$ D.R's of required line = $24\lambda, 61\lambda, 112\lambda$

point on Required line = $(1, 2, -4)$

Now equation of Required line

$$\frac{x-1}{24\cancel{\lambda}} = \frac{y-2}{61\cancel{\lambda}} = \frac{z+4}{112\cancel{\lambda}} \quad \text{----}\left\{ \text{By formula } \frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c} \right\}$$

$$\Rightarrow \boxed{\frac{x-1}{24} = \frac{y-2}{61} = \frac{z+4}{112}}$$ Ans

—x—

Ans 7 → given lines

$$\vec{r} = \hat{i} + \lambda(2\hat{i} + \hat{j} - 3\hat{k})$$

and $\vec{r} = (2\hat{i} + \hat{j} - \hat{k}) + \mu(\hat{i} + \hat{j} + \hat{k})$

here $\vec{b_1} = 2\hat{i} + \hat{j} - 3\hat{k}$

$\vec{b_2} = \hat{i} + \hat{j} + \hat{k}$

let $\vec{b}$ is a vector parallel to required line

given Required line is $\perp^r$ to the given lines

$\therefore \quad \vec{b} \perp \vec{b_1}$ and $\vec{b} \perp \vec{b_2}$

$\Rightarrow \quad \vec{b} = \lambda(\vec{b_1} \times \vec{b_2})$

$$\Rightarrow \vec{b} = \lambda \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{vmatrix} = \lambda(4\hat{i} - 5\hat{j} + \hat{k})$$

position vector on required line

$$\vec{a} = \hat{i} + \hat{j} - 3\hat{k} \quad \text{--- (given)}$$

Now equation of required line

$$\vec{r} = \vec{a} + t\vec{b}$$

$$\Rightarrow \vec{r} = (\hat{i} + \hat{j} - 3\hat{k}) + \cancel{t}\left(\lambda(4\hat{i} - 5\hat{j} + \hat{k})\right)$$

$$\Rightarrow \boxed{\vec{r} = (\hat{i} + \hat{j} - 3\hat{k}) + \mu(4\hat{i} - 5\hat{j} + \hat{k})} \quad \text{--- } \{\text{where } \mu = t\lambda\}$$

ANS

—x—

ANS 8 → given D.R's of 1st line

$a_1 = a$, $b_1 = b$, $c_1 = c$

D.R's of 2nd line

$a_2 = b - c$, $b_2 = c - a$, $C_2 = a - b$

Now angle b/w two lines

$$\cos\theta = \frac{|a_1a_2 + b_1b_2 + c_1c_2|}{\sqrt{a_1^2 + b_1^2 + c_1^2}\ \sqrt{a_2^2 + b_2^2 + c_2^2}}$$

$$\cos\theta = \frac{|\cancel{ab} - \cancel{ac} + \cancel{bc} - \cancel{ab} + \cancel{ac} - \cancel{bc}|}{\sqrt{a^2 + b^2 + c^2}\ \sqrt{(b-c)^2 + (c-a)^2 + (a-b)^2}}$$

$$\cos\theta = 0$$

$$\Rightarrow \boxed{\theta = \pi/2}$$

ANS

—x—

Ans. 9 →

D'R's of X-axis = 1, 0, 0

Required line is parallel to X-axis

∴ D'R's of required line = $\lambda, 0, 0$

point on Required line = origin = $(0, 0, 0)$

Equation of Required line

$$\frac{x-0}{\lambda} = \frac{y-0}{0} = \frac{z-0}{0}$$

$$\Rightarrow \boxed{\frac{x}{1} = \frac{y}{0} = \frac{z}{0}}$$ ANS

Note : Remember

Equation of X-axis : $\frac{x}{1} = \frac{y}{0} = \frac{z}{0}$

Equation of Y-axis = $\frac{x}{0} = \frac{y}{1} = \frac{z}{0}$

Equation of Z-axis = $\frac{x}{0} = \frac{y}{0} = \frac{z}{1}$

Ans 10 → let $\frac{x+1}{3} = \frac{y+3}{5} = \frac{z+5}{7} = \lambda$

any point on this line is $(3\lambda-1, 5\lambda-3, 7\lambda-5)$

let $\frac{x-2}{1} = \frac{y-4}{3} = \frac{z-6}{5} = \mu$

any point on this line is $(\mu+2, 3\mu+4, 5\mu+6)$

If two lines Intersect, then for some value of

λ & μ we must have

$3\lambda - 1 = \mu + 2$ | $5\lambda - 3 = 3\mu + 4$ | $7\lambda - 5 = 5\mu + 6$

$3\lambda - \mu = 3$ | $5\lambda - 3\mu = 7$ | $7\lambda - 5\mu = 11$

Solving these equations

we get $\lambda = 1/2$ & $\mu = -3/2$

these value of λ & μ satisfies the 3rd equation

$\therefore$ two lines Intersect

Now $\underline{\lambda = 1/2}$ point is $\left(\frac{3}{2} - 1, \frac{5}{2} - 3, \frac{7}{2} - 5\right)$

$= \left(1/2, -1/2, -3/2\right)$

for $\underline{\mu = \frac{-3}{2}}$ point is $\left(-\frac{3}{2} + 2, -\frac{9}{2} + 4, -\frac{15}{2} + 6\right)$

$= \left(1/2, -1/2, -3/2\right)$

$\therefore$ $\boxed{\text{point of Intersection is } \left(1/2, -1/2, -3/2\right)}$ Ans

(Note: Misprint in worksheet Answer)

Q.N 11 → Given equations (vector form)

$\vec{r} = 3\hat{i} + 2\hat{j} - 4\hat{k} + \lambda(\hat{i} + 2\hat{j} + 2\hat{k})$

$\vec{r} = 5\hat{i} - 2\hat{j} + \mu(3\hat{i} + 2\hat{j} + 6\hat{k})$

Converting these equations in to Cartesian form

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$$\frac{x-3}{1} = \frac{y-2}{2} = \frac{z+4}{2} = \lambda$$

and $\frac{x-5}{3} = \frac{z+2}{2} = \frac{z-0}{6} = \mu$

Any point on I^{st} line $= (\lambda+3, 2\lambda+2, 2\lambda-4)$

Any point on 2^{nd} line $= (3\mu+5, 2\mu-2, 6\mu)$

If the lines Intersect, then

$\lambda+3 = 3\mu+5$	$2\lambda+2 = 2\mu-2$	$2\lambda-4 = 6\mu$
$\lambda-3\mu = 2$	$2\lambda-2\mu = -4$	$2\lambda-6\mu = 4$

solving these

$4\mu = -8$

$\mu = -2$ & $\lambda = -4$

clearly value of λ & μ satisfy the remaining equation

Hence, given lines Intersect

put $\lambda = -4$: point is $(-4+3, -8+2, -8-4)$
$= (-1, -6, -12)$

for $\mu = -2$ point is $(-1, -6, -12)$

$\boxed{\text{Hence point of Intersection is } (-1, -6, -12)}$

Ans

Qn. 12 →

Ist line passes through the points $(2, 3, -1)$ & $(1, -2, 0)$

D.R's of this line are $= 1-2, -2-3, 0+1$

$= -1, -5, 1$

2nd line passes through the points $(3, -4, 1)$ & $(2, 1, 3)$

D.R's of this line are $= 2-3, 1+4, 3-1$

$= -1, 5, 2$

Let D.R's of required line are $= a, b, c$

Since required line is $\perp^r$ to the given lines

$\therefore \quad -a - 5b + c = 0$

$-a + 5b + 2c = 0$

$$\frac{a}{-10-5} = \frac{-b}{-2+1} = \frac{c}{-5-5} = \lambda$$

$\Rightarrow a = -15\lambda \quad ; \quad b = \lambda \quad ; \quad c = -10\lambda$

Point on required line is $(2, -1, 3)$ --- (given)

Equation of required line is given by

$$\frac{x-2}{-15\lambda} = \frac{y+1}{\lambda} = \frac{z-3}{-10\lambda} \quad \text{--- } \left\{ \text{By formula } \frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c} \right\}$$

$$\Rightarrow \boxed{\frac{x-2}{-15} = \frac{y+1}{1} = \frac{z-3}{-10}} \quad \text{Ans}$$

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