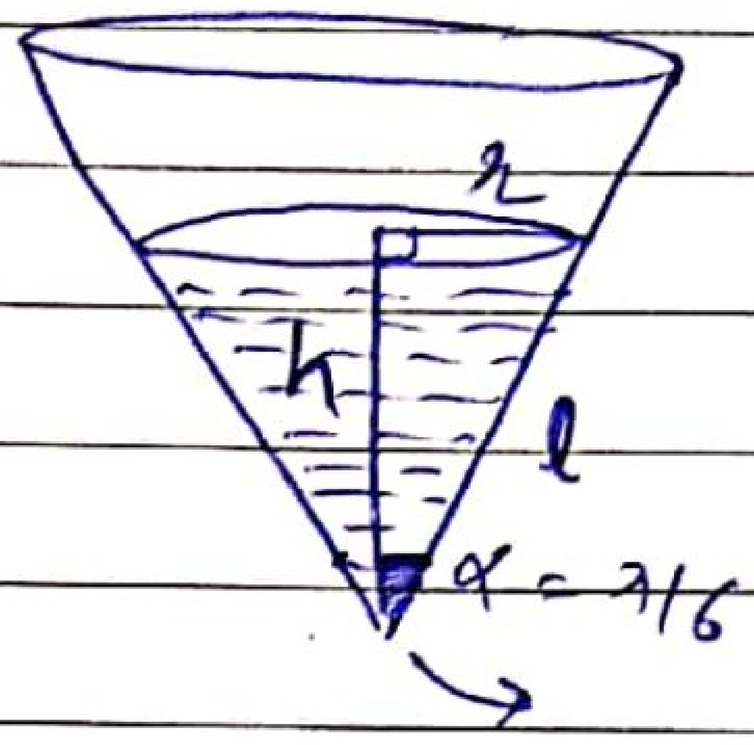
SOLUTION WORKSHEET No: 4 (1)

A·OD

Qn. 1 → given: $\frac{dV}{dt} = -1\ cm^3/sec$

$l = 4\ cm$, $\alpha = \frac{1}{2}\left(\frac{\pi}{3}\right) = \frac{\pi}{6}$

To find: $\frac{dl}{dt} = ?$



$V = \frac{1}{3}\pi r^2 h$

from Δ

$$\sin\left(\frac{\pi}{6}\right) = \frac{r}{l} \quad \& \quad \cos\left(\frac{\pi}{6}\right) = \frac{h}{l}$$

$$\frac{1}{2} = \frac{r}{l} \quad \text{and} \quad \frac{\sqrt{3}}{2} = \frac{h}{l}$$

$$r = \frac{l}{2} \quad \text{and} \quad h = \frac{\sqrt{3}}{2} l$$

$$\therefore V = \frac{1}{3}\pi\left(\frac{l^2}{4}\right)\left(\frac{\sqrt{3}}{2} l\right)$$

$$V = \frac{\sqrt{3}\pi}{24} l^3$$

Diff w.r.t 't'

$$\frac{dV}{dt} = \frac{\sqrt{3}\pi}{24}(3l^2)\frac{dl}{dt}$$

$$\Rightarrow -1 = \frac{\sqrt{3}\pi}{\cancel{24}_{8}}(\cancel{3})(\cancel{16}_{2})\frac{dl}{dt}$$

$$\Rightarrow \frac{dl}{dt} = \frac{-1}{2\sqrt{3}\pi}\ cm/sec$$

$\therefore$ Slant height is decreasing at the rate $\frac{1}{2\sqrt{3}\pi}$ cm/sec **Ans**

Qn 2 →

$V = \frac{4}{3}\pi r^3$	$S = 4\pi r^2$
$\frac{dV}{dr} = 4\pi r^2$	$\frac{dS}{dr} = 8\pi r$

Now $\frac{dV}{dS} = \frac{dV/dr}{dS/dr} = \frac{4\pi r^2}{8\pi r} = \frac{r}{2}$

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Solution A.OD. (W.S 4) (2)

$$\left(\frac{dv}{ds}\right)_{r=2} = \frac{2}{2} = 1 \text{ cm}^3/\text{cm}^2$$ **Ans**

(Note: there is misprint in Units in worksheet)

Qn. 3 → given $\frac{dv}{dt} \propto S$

$$\Rightarrow \frac{dv}{dt} = kS$$ {where k is −ve (since volume is decreasing)}

$$\Rightarrow \frac{d}{dt}\left(\frac{4}{3}\pi r^3\right) = k(4\pi r^2)$$

$$\Rightarrow \frac{4\pi}{3}(3r^2)\frac{dr}{dt} = k(4\pi r^2)$$

$$\frac{dr}{dt} = k$$ {where k is −ve}

Clearly radius is decreasing at the constant rate **Ans**

Qn. 4 → given $\frac{dA}{dt} = k$ (k is constant)

$$\frac{d}{dt}(\pi r^2) = k$$

$$\Rightarrow \pi(2r)\frac{dr}{dt} = k$$

$$\Rightarrow \frac{dr}{dt} = \frac{k}{2\pi r}$$

Now $P = 2\pi r$ (circumference)

$$\frac{dP}{dt} = 2\pi\frac{dr}{dt}$$

$$= 2\pi\left(\frac{k}{2\pi r}\right)$$

$$\frac{dP}{dt} = \frac{k}{r}$$

$$\Rightarrow \frac{dP}{dt} \propto \frac{1}{r}$$ **PROVED.**

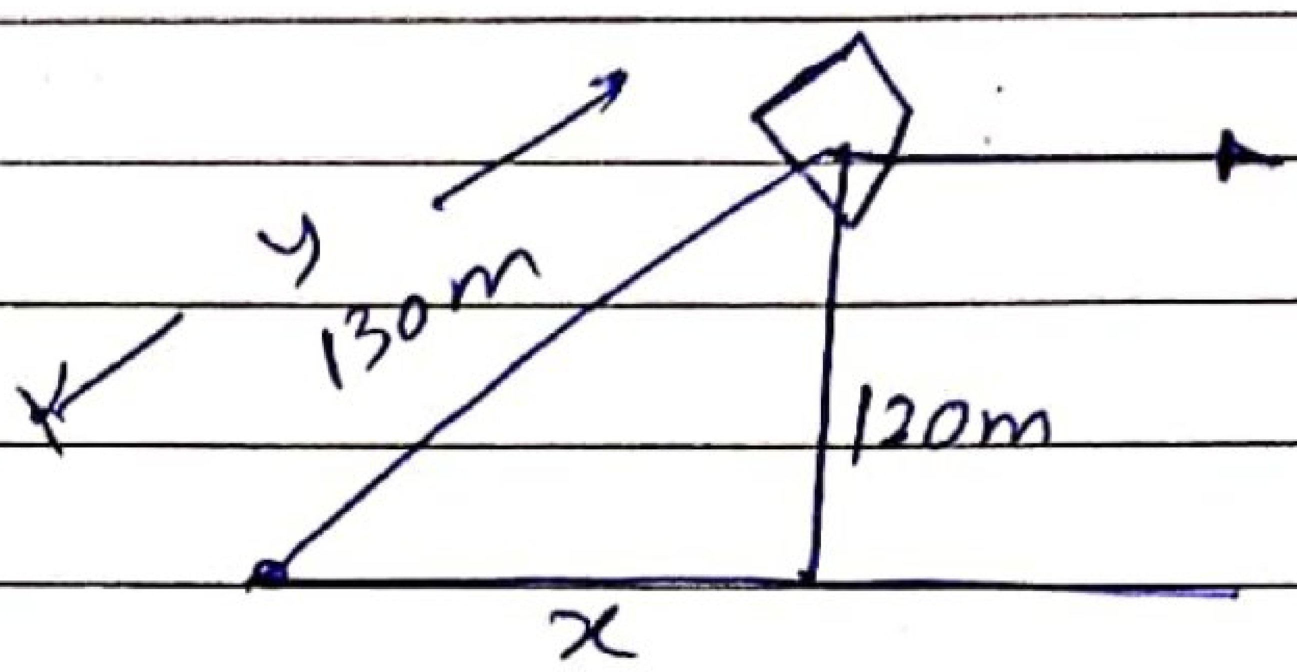
CLASSTIME

Solution A.OD. (W.S. 4) (3)

Ques 5 → $y = 130\,m$

given $\dfrac{dx}{dt} = 52\ m/sec$

to find $\dfrac{dy}{dt} = ?$



By pythagoras

$$16900 = 14400 + x^2$$
$$\Rightarrow x^2 = 2500$$
$$\Rightarrow x = 50$$

again

$$y^2 = 14400 + x^2$$

Diff w.r.t 't'

$$\cancel{2}y\frac{dy}{dt} = \cancel{2}x\frac{dx}{dt}$$

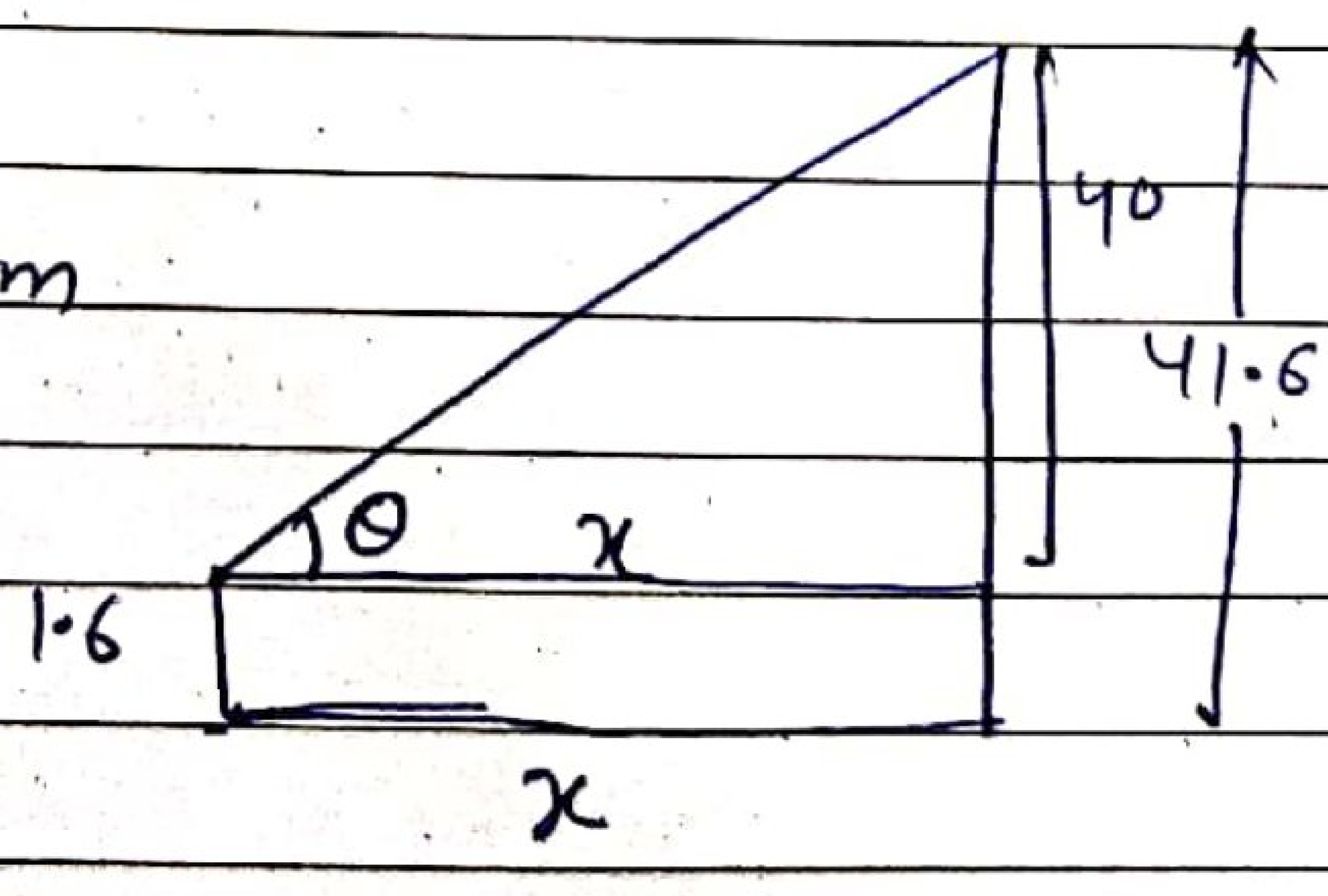
$$130\frac{dy}{dt} = 50(52)$$

$$\frac{dy}{dt} = \frac{5\cancel{0} \times 52}{13\cancel{0}} = \frac{\cancel{52}^{4}}{\cancel{13}} \times 5 = 20$$

∴ string is out at the rate of $20\ m/sec$ **Ans**

Ques. 6 →

$\dfrac{dx}{dt} = 2\ m/sec$; $x = 30\ m$



to find $\dfrac{d\theta}{dt} = ?$

$$\tan\theta = \frac{40}{x}$$

Diff w.r.t t

$$\sec^2\theta\,\frac{d\theta}{dt} = \frac{-40}{x^2}\,\frac{dx}{dt} \quad \text{------ (i)}$$

Now By Pythagoras $H^2 = x^2 + 1600$

$$H^2 = (30)^2 + 1600 = 2500$$
$$H = 50$$

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Solution A.O.D. (W.S.-4) (4)

$\cos\theta = \frac{Base}{Hyp} = \frac{30}{50} = \frac{3}{5}$

$\Rightarrow \sec\theta = 5/3$

$\therefore$ equation (i) becomes

$\frac{25}{9}\frac{d\theta}{dt} = \frac{-40}{900}(2)$

$\frac{d\theta}{dt} = -\frac{8\cancel{0}\times 9}{25\times 9\cancel{00}} = \frac{-4}{125}$ Rad/sec

$\therefore$ angle of elevation is decreasing at the rate $\frac{4}{125}$ Rad/sec ANS

Qn. 7 → $C_1: y^2 = 4x$

$C_2: x^2+y^2-6x+1=0$

Solving these equations

$x^2+4x-6x+1=0$

$x^2-2x+1=0$

$(x-1)^2=0$

$\Rightarrow x=1$ put in C_1

$y^2=4$

$\Rightarrow y=\pm 2$

$\therefore$ point of Intersection are $(1,2)$ & $(1,-2)$

Diff C_1 w.r.t. x

$2y\frac{dy}{dx}=4$

$\frac{dy}{dx}=\frac{2}{y}$

Diff C_2 w.r.t. x

$2x+2y\frac{dy}{dx}-6=0$

$x+y\frac{dy}{dx}-3=0$

CLASSTIME

Soln. (W.S.4) A.O.D (5)

$$\frac{dy}{dx} = \frac{3-x}{y}$$

Consider point $(1,2)$

Slope of (T_1) at $(1,2) = \frac{2}{2} = 1$

Slope of (T_2) at $(1,2) = \frac{3-1}{2} = 1$

Since $m_1 = m_2$ $\therefore$ curves touch at $(1,2)$

Consider point $(1,-2)$

Slope of (T_1) at $(1,-2) = \frac{2}{-2} = -1$

Slope of (T_2) at $(1,-2) = \frac{3-1}{-2} = -1$

Since $m_1 = m_2$ $\therefore$ curves touch each other at $(1,-2)$

Hence two given curves touch each other

Ans

Ans 8 → let the point of contact is (x_1, y_1)

given equation of curve

$$\sqrt{x} + \sqrt{y} = 4$$

Diff w.r.t x

$$\frac{1}{2\sqrt{x}} + \frac{1}{2\sqrt{y}} \cdot \frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = -\frac{\sqrt{y}}{\sqrt{x}}$$

Slope of Tangent at $(x_1, y_1) = -\frac{\sqrt{y_1}}{\sqrt{x_1}}$

also Slope of Tangent $= \pm 1$ $\left\{ \because \text{tangent is equally inclined to the axes } \therefore \theta = 45° \; \& \; \theta = 135° \right.$

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Solution A·OD (W·S4)

$\therefore \frac{-\sqrt{y_1}}{\sqrt{x}} = \pm 1$

Squaring $\frac{y_1}{x_1} = 1$

$y_1 = x_1$

also have

$\sqrt{x_1} + \sqrt{y_1} = 4$ --- [$\because (x_1, y_1)$ lies on the curve]

$\Rightarrow \sqrt{x_1} + \sqrt{x_1} = 4$

$\Rightarrow 2\sqrt{x_1} = 4$

$\Rightarrow \sqrt{x_1} = 2$

Squaring $x_1 = 4 \Rightarrow y_1 = 4$

$\therefore$ Point on the curve is $(4, 4)$ **Ans**

On Q7

$C_1 = xy = 4$ --- (1)

$C_2 = x^2 + y^2 = 8$ --- (2)

Solving these equation

$x^2 + \frac{16}{x^2} = 8$

$x^4 + 16 = 8x^2$

$x^4 - 8x^2 + 16 = 0$

$\Rightarrow (x^2 - 4)^2 = 0$

$\Rightarrow x^2 - 4 = 0$

$\Rightarrow x = \pm 2$ put in eq (i)

$x = 2$ $2y = 4 \Rightarrow y = 2$

$x = -2$ $-2y = 4 \Rightarrow y = -2$

$\therefore$ Point of Intersection are $(2, 2)$ & $(-2, -2)$

Diff. C_1 w.r.t x

$x\frac{dy}{dx} + y = 0$

$\frac{dy}{dx} = \frac{-y}{x}$

Diff. C_2 w.r.t x

$2x + 2y\frac{dy}{dx} = 0$

$\frac{dy}{dx} = \frac{-x}{y}$

Slope of (T_1) at $(2,2)$ $= \frac{-2}{2} = -1$

Slope of (T_2) at $(2,2)$ $= \frac{-2}{2} = -1$

Since $m_1 = m_2$ at $(2,2)$

$\therefore$ curve touch each other at point $(2,2)$

Now Slope of (T_1) at $(-2,-2) = \frac{-(-2)}{-2} = -1$

Slope of (T_2) at $(-2,-2) = \frac{-(-2)}{-2} = -1$

Since $m_1 = m_2$ at $(-2,-2)$

$\therefore$ Curves touch also at point $(-2,-2)$

Hence curves touch each other. Ans

Qn 10 → equation of curve

$x = t^2 + 3t - 8$ and $y = 2t^2 - 2t - 5$

point of contact $(2,-1)$

Since point of contact lies on the curve

$\therefore$ put $x = 2$ & $y = -1$

$\Rightarrow 2 = t^2 + 3t - 8$ and $-1 = 2t^2 - 2t - 5$

$\Rightarrow t^2 + 3t - 10 = 0$ and $2t^2 - 2t - 4 = 0$

Solution (W.S 4) A.O.D

$(t+5)(t-2)=0$ and $t^2-t-2=0$

$t=-5$ (or) $t=2$ and $(t-2)(t+1)=0$

$t=2$ (or) $t=-1$

take the common value of t $\therefore t=2$

Diff x w.r.t t

$\frac{dx}{dt}=2t+3$

Diff y w.r.t t

$\frac{dy}{dt}=4t-2$

Now $\frac{dy}{dx}=\frac{dy/dt}{dx/dt}=\frac{4t-2}{2t+3}$

Slope of Tangent at $t=2$ $=\frac{8-2}{4+3}=\frac{6}{7}$ Ans

Now equation of Tangent at $(2,-1)$ is given by

$y+1=\frac{6}{5}(x-2)$

$\Rightarrow 5y+5=6x-12$

$\Rightarrow 6x-5y-17=0$

Qu.11 → Curve C_1: $x^3-3xy^2+2=0$

Curve C_2: $3x^2y-y^3-2=0$

(it is not easy to solve these equations)

$\therefore$ let point of Intersection is (x_1,y_1)

Diff C_1 w.r.t x

$3x^2-3\left(x\cdot 2y\frac{dy}{dx}+y^2\right)=0$

$x^2-2xy\frac{dy}{dx}-y^2=0$

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Solution A.O.D. (W.S.-4)

$$\frac{dy}{dx} = \frac{x^2-y^2}{2xy}$$

(m_1) Slope of (T_1) at (x_1,y_1) $= \dfrac{x_1^2-y_1^2}{2x_1y_1}$

Diff. C_2 w.r.t x

$$3\left(x^2\frac{dy}{dx} + y\cdot 2x\right) - 3y^2\frac{dy}{dx} = 0$$

$$x^2\frac{dy}{dx} + 2xy - y^2\frac{dy}{dx} = 0$$

$$\frac{dy}{dx}(x^2-y^2) = -2xy$$

$$\frac{dy}{dx} = \frac{-2xy}{x^2-y^2}$$

(m_2) Slope of (T_2) at (x_1,y_1) $= \dfrac{-2x_1y_1}{x_1^2-y_1^2}$

Now m_1m_2

$$= \left(\frac{\cancel{x_1^2-y_1^2}}{\cancel{2x_1y_1}}\right)\left(\frac{-\cancel{2x_1y_1}}{\cancel{x_1^2-y_1^2}}\right)$$

$$= -1$$

Since $m_1m_2 = -1$

$\therefore$ curves cut ~~other~~ orthogonally / perpendicular

Ans