॥ जय श्री राधे कृष्णा ॥

ULTIMATE MATHEMATICS: BY AJAY MITTAL

INTEGRATION : CLASS NO: 7 (Revision)

① Basic Integration

$$\int (1-2x)^{5/2} dx = \frac{2}{7}\frac{(1-2x)^{7/2}}{(-2)} + c$$

② ~~In~~ Substitution

$\int (\;)^n$ linear ⓧ $= t$

$= \int \sin(\;)$ linear ⓧ $= t$

$= \int e^{(\;)}$ linear ⓧ $= t$

$= \int \sqrt{(\;)}$ linear ⓧ $= t$ (or) t^2

$= \int \frac{\;}{D^r}$ put $= t$

✓ $x\,dx \rightarrow$ put $x^2 = t$

$x^2 dx \rightarrow$ put $x^3 = t$

✓ $\sin x\,dx \rightarrow$ put $\cos x = t$

$\cos x\,dx \rightarrow$ put $\sin x$ ~~dx~~ $= t$

✓ $\sec^2 x\,dx \rightarrow$ put $\tan x = t$

✓ $\frac{1}{x} dx \rightarrow$ put $\log x = t$

✓ $e^x dx \rightarrow e^x = t$

✓ $\int \frac{\;}{e^x + 1}$ दोनों चाहिए

✓ $\sin(2x)dx \rightarrow$ put $\sin^2 x = t$

$\rightarrow$ put $\cos^2 x = t$

✓ $\int \frac{\;}{\text{variable} + \text{constant}}$ $(\;)^2$, linear ⓧ $= t$

① Trigo powers

$$\begin{cases} \sin^2 x \to \dfrac{1-\cos(2x)}{2} \\ \cos^2 x \to \dfrac{1+\cos(2x)}{2} \\ \tan^2 x \to \sec^2 x - 1 \\ \cot^2 x \to \csc^2 x - 1 \end{cases}$$

$$\begin{cases} \int \sin^3 x\,dx = \dfrac{3\sin x - \sin(3x)}{4} \\ \quad \to \int \sin^2 x \cdot \sin x\,dx \\ \int \cos^3 x\,dx = \dfrac{3\cos x + \cos(3x)}{4} \\ \quad \to \int \cos^2 x \cdot \cos x\,dx \end{cases}$$

- $\int \tan^3 x\,dx = \int \tan x \cdot \underbrace{\tan^2 x}_{\sec^2 x - 1}\,dx$
- $\int \cot^3 x\,dx = \int \cot x \cdot \underbrace{\cot^2 x}_{\csc^2 x - 1}\,dx$
- $\int \sin^4 x\,dx \to \int (\underbrace{\sin^2 x}_{\frac{1-\cos(2x)}{2}})^2\,dx$
- $\int \cos^4 x\,dx \to \int (\underbrace{\cos^2 x}_{\frac{1+\cos(2x)}{2}})^2\,dx$
- $\int \tan^4 x\,dx \to \int \tan^2 x \cdot \underbrace{\tan^2 x}_{(\sec^2 x - 1)}\,dx$
- $\int \cot^4 x\,dx \to \int \cot^2 x \cdot ((\csc^2 x - 1)\,dx$
- $\int \sec^4 x\,dx \to \int \sec^2 x \cdot (1+\tan^2 x)\,dx$
- $\int \csc^4 x\,dx \to \int \csc^2 x\,(1+\cot^2 x)\,dx$
- $\begin{cases} \int \sin^8 x\,dx \\ \int \cos^8 x\,dx \end{cases}$ $\quad \int \sin^7 x\,dx \quad \int \cos^7 x\,dx \quad \int \sin^9 x\,dx \quad \int \cos^9 x\,dx \to \int \sin^8 x \cdot \sin x\,dx = \int (1-\cos^2 x)^4 \sin x\,dx$
- $\int \tan^5 x\,dx = \int \tan^3 x \cdot \underbrace{\tan^2 x}_{\sec^2 x - 1}\,dx$
- $\int \cot^5 x\,dx = \int \cot^3 x \cdot ((\csc^2 x - 1)\,dx$

(2) $\sin x$, $\cos x$ multiply हो same power

$\int \sin^2 x \cdot \cos^2 x \, dx = \int (\sin x \cos x)^2 dx = \int \left(\frac{\sin(2x)}{2}\right)^2 = \frac{1}{4}\int \sin^2(2x)\, dx$

$\int \sin^3 x \cos^3 x \, dx$

$= \int \sin^4 x \cos^4 x \, dx$

$= \int \sin^5 x \cdot \cos^5 x \, dx$

(3) $\sin x$, $\cos x$ multiply हो with Diff. power.

(·) $\int \sin^3 x \cdot \cos^4 x \, dx = \int \sin^2 x \cdot \cos^4 x \sin x \, dx = \int (1-t^2) \cdot t^4 \, dt$

(·) $\int \sin^3 x \cdot \cos^5 x \, dx \rightarrow \int \sin^2 x \cdot \cos^5 x \sin x \, dx = \int (1-t^2) \cdot t^5 \, dt$

(·) $\int \sin^2 x \cdot \cos^4 x \, dx = \int \sin^2 x \cdot \cos^2 x \cdot \cos^2 x \, dx$

(·) $\int \sin^6 x \, dx = \int (\sin^3 x)^2 dx = \int \left(\frac{3\sin x - \sin(3x)}{4}\right)^2$

$\int \left(\frac{\sin(2x)}{2}\right)^2 \cdot \left(\frac{1+\cos(2x)}{2}\right) dx$

Bekar

$\int \left(\frac{1+\cos(2x)}{2}\right)\left(\frac{1+\cos(2x)}{2}\right)^2$

$\frac{1}{8}\int (1-\cos(2x))(1+\cos^2(2x)+2\cos(2x))\, dx$

open the Bracket

(4) $\tan x$ & $\sec x$, $\cot x$ & $\operatorname{cosec} x$ multiply.

(·) $\int \sec^n x \cdot \tan x \, dx = \int \sec^{n-1} x \cdot \sec x \tan x \, dx$

$= \int t^{n-1} \, dt$

(5) Rahalu

$\int \frac{1}{1\pm\sin x}$, $\int \frac{1}{1\pm\cos x}$ $\int \frac{\sin x}{1\pm\sin x}$. $\int \frac{\cos x}{1\pm\cos x}$,

(6) $\sin(A\pm B)$, $\cos(A\pm B)$ formula

"जैसा angle नीचे है, वैसा ही ऊपर बनाओ।

$\int \frac{1}{\sin(x-a)\cdot\sin(x-b)}$ — M & D by $\sin(a-b)$

$\int \frac{1}{\cos(x+a)\cos(x+b)}$ — $\sin(a-b)$

$\int \frac{1}{\sin(x-a)\cdot\cos(x+b)}$ — $\cos(a+b)$

(7) Separate (after adjustment)

(8) If $D^r \to \sin x$ & $\cos x$ Multiply में — Sum of power even

Divide N & D by $\cos^{\text{even}} x$ → $\tan x = t$ $\sec^2 x\,dx = dt$

(9) $\int \frac{\ln x}{\sqrt{\ln x}}$ same ऊपर बनाओ। $=t^2$

$\int \ln x \sqrt{\ln x}$ $=t^2$

$\int \frac{\text{Quad}}{\sqrt{\ln x}}$ $=t^2$

$\int \text{Quad}\sqrt{\ln x}$ $=t^2$

$\int \frac{\ln x}{(\ln x)^n}$ $=t$

$\int \ln x\,(\ln x)^n$ $=t$

(10) Divide when degree of $N^r \geq$ degree of D^r then divide

$\int \frac{N}{D}dx = \int Q + \frac{R}{D}\,dx$

(11) take common & then put 't'

$$\int \frac{(x^4 - x)^{3/4}}{x^7}\,dx = \int \frac{\cancel{x^3}\left(1-\frac{1}{x^3}\right)^{3/4}}{\cancel{x^7}\,x^4}\,dx$$

put $1-\frac{1}{x^3} = t \Rightarrow \frac{3}{x^4}\,dx = dt$

$$= \frac{1}{3}\int t^{3/4}\,dt$$

(12) Special Integral (6) $\to$ after Substitution $\int \frac{}{\text{variable + const.}}$ $\to$ $(\ )^2 = t$

(1) $\int \frac{1}{x^{2/3}\sqrt{x^{2/3}-4}}\,dx$

$$= \int \frac{1}{x^{2/3}\sqrt{(x^{1/3})^2-4}}\,dx$$

put $x^{1/3} = t \to \frac{1}{3}(x)^{-2/3}dx = dt \to \frac{1}{3}\frac{1}{x^{2/3}}dx = dt$

$$= 3\int \frac{dt}{\sqrt{t^2-2^2}}$$

(13) $\int \frac{1}{\text{Quadratic}}\,dx$

(14) $\int \frac{1}{\sqrt{\text{Quadra.}}}\,dx$

(·) $\int \frac{1}{\sqrt{4x^2-8}}\,dx$

$$= \int \frac{1}{\sqrt{(2x)^2 - (2\sqrt{2})^2}}$$

$$= \int \frac{1}{\sqrt{1-x-2x^2}}\,dx$$

$$= \int \frac{1}{\sqrt{-2\left(x^2+\frac{x}{2}-\frac{1}{2}\right)}}\,dx$$

$$\to \frac{1}{\sqrt{2}}\int \frac{1}{\sqrt{-\left[\left(x+\frac{1}{4}\right)^2-\frac{1}{16}-\frac{1}{2}\right]}}\,dx$$

$$= \frac{1}{\sqrt{2}}\int \frac{1}{\sqrt{-\left[\left(x+\frac{1}{4}\right)^2-\left(\frac{3}{4}\right)^2\right]}}\,dx$$

$-\frac{1}{2}+\frac{2}{4}$ ... $+\left(\frac{1}{4}\right)^2$

(14) Alter Subn. then $\int \frac{1}{\text{Quad}}$ $\int \frac{1}{\sqrt{\text{Quad}}}$

(15) $\int \frac{\text{linear}}{\text{Quadratic}} dx$ Der. अपने बनाओ

(a) $N^r = A \cdot \frac{d}{dx}(D^r) + B$

find A & B by equating coefficient of x & const term.

$$\int \frac{3x-1}{1-x-2x^2} dx = 3\int \frac{x-1/3}{1-x-2x^2} = -\frac{3}{4}\int \frac{-4x+\frac{4}{3}-1+1}{1-x-2x^2}$$

$$= -\frac{3}{4}\int \frac{(-4x-1)+7/3}{\text{''}} dx$$

$$= -\frac{3}{4}\int \frac{-4x-1}{\underbrace{1-x-2x^2}_{=t}} dx - \frac{7}{4}\int \frac{1}{\underbrace{1-x-2x^2}_{P.S}} dx$$

(16) $I = \int \frac{\text{lin}}{\sqrt{\text{Quadratic}}} dx$

(17) $\int \frac{\text{linear}}{\text{Special Integral}}$ → Separate

(i) $\int \frac{3x-1}{\sqrt{25-5x^2}} dx = 3\int \frac{x}{\sqrt{25-5x^2}} dx - \int \frac{1}{\sqrt{25-5x^2}}$

put $25-5x^2 = t$

(18) Single $\sin x, \cos x$

$\int \frac{1}{\sin x, \cos x, \text{const}}$

$\sin x = \frac{2\tan(x/2)}{1+\tan^2(x/2)}$ $\cos x = \frac{1-\tan^2(x/2)}{1+\tan^2(x/2)}$

put $\tan(x/2) = t$