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WORKSHEET I-2

Inverse - TRIGO

Qn 1 → Simplify $\tan^{-1}\sqrt{\dfrac{1-\cos x}{1+\cos x}}$ Ans $\dfrac{x}{2}$

Qn 2 → Simplify $\tan^{-1}\left(\dfrac{\cos x}{1+\sin x}\right)$ Ans $\dfrac{\pi}{4}-\dfrac{x}{2}$

Qn 3 → Simplify $\tan^{-1}\left(\dfrac{1-\sin x}{\cos x}\right)$ Ans $\dfrac{\pi}{4}-\dfrac{x}{2}$

Qn 4 → Simplify $\tan^{-1}\left(\dfrac{x}{\sqrt{a^2-x^2}}\right)$ Ans $\tan^{-1}\left(\dfrac{x}{a}\right)$

Qn 5 → Simplify $\tan^{-1}\sqrt{\dfrac{a-x}{a+x}}$ Ans $\dfrac{1}{2}\cos^{-1}\left(\dfrac{x}{a}\right)$

Qn 6 → Simplify $\cos^{-1}\left(\dfrac{x}{\sqrt{x^2+a^2}}\right)$ Ans $\dfrac{\pi}{2}-\tan^{-1}(x/a)$ OR $\cot^{-1}\left(\dfrac{x}{a}\right)$

Qn 7 → Show $\tan^{-1}\left(\dfrac{1}{2}\right)+\tan^{-1}\left(\dfrac{1}{5}\right)+\tan^{-1}\left(\dfrac{1}{8}\right)=\pi/4$

Qn 8 → Show $\tan^{-1}\left(\dfrac{1}{5}\right)+\tan^{-1}\left(\dfrac{1}{7}\right)+\tan^{-1}\left(\dfrac{1}{3}\right)+\tan^{-1}\left(\dfrac{1}{8}\right)=\dfrac{\pi}{4}$

Qn 9 → Show $2\tan^{-1}\left(\dfrac{1}{5}\right)+\tan^{-1}\left(\dfrac{1}{7}\right)+2\tan^{-1}\left(\dfrac{1}{8}\right)=\dfrac{\pi}{4}$

Qn 10 → Show $\tan\left[2\tan^{-1}\left(\dfrac{1}{5}\right)-\dfrac{\pi}{4}\right]=-\dfrac{7}{17}$

Hint write $\dfrac{\pi}{4}=\tan^{-1}(1)$

Qn 11 → ~~Show~~ Simplify $\tan^{-1}\left(\sqrt{1+x^2}-x\right)$ Ans $\dfrac{1}{2}\cot^{-1}x$

Q 12 → Simplify $\tan^{-1}\left(\dfrac{\sqrt{1+x^2}+1}{x}\right)$ Ans $\dfrac{\pi}{2}-\dfrac{1}{2}\tan^{-1}x$

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[from Qn 13 to Qn 16
HINT Convert in to $\tan^{-1}(\ )$ by
using Conversion P, B, H Method]

Qn 13 → Show $\cos^{-1}\left(\frac{4}{5}\right) + \cos^{-1}\left(\frac{12}{13}\right) = \cos^{-1}\left(\frac{33}{65}\right)$

Qn 14 → Show $\sin^{-1}\left(\frac{3}{5}\right) + \sin^{-1}\left(\frac{8}{17}\right) = \cos^{-1}\left(\frac{36}{85}\right)$

Qn 15 → Show that $\sin^{-1}\left(\frac{4}{5}\right) + \sin^{-1}\left(\frac{5}{13}\right) + \sin^{-1}\left(\frac{16}{65}\right) = \frac{\pi}{2}$

Qn 16 → Show $\sin^{-1}\left(\frac{4}{5}\right) + 2\tan^{-1}\left(\frac{1}{3}\right) = \frac{\pi}{2}$

—x—

Inverse TRIGO (I-2)

Conversion $\tan^{-1}\left(\frac{x}{1}\right)$ $P = x$, $B = 1$, $H = \sqrt{x^2+1}$

$$= \sin^{-1}\left(\frac{x}{\sqrt{x^2+1}}\right) = \cos^{-1}\left(\frac{1}{\sqrt{x^2+1}}\right) = \sec^{-1}\left(\sqrt{x^2+1}\right)$$

Qn-1 Simplify $\tan^{-1}\left(\frac{1+\sin x}{\cos x}\right)$

$$= \tan^{-1}\left(\frac{1+\cos\left(\frac{\pi}{2}-x\right)}{\sin\left(\frac{\pi}{2}-x\right)}\right)$$

$$= \tan^{-1}\left(\frac{\cancel{2}\cos^{\cancel{2}}\left(\frac{\pi}{4}-\frac{x}{2}\right)}{\cancel{2}\sin\left(\frac{\pi}{4}-\frac{x}{2}\right)\cancel{\cos\left(\frac{\pi}{4}-\frac{x}{2}\right)}}\right)$$

$$= \tan^{-1}\left(\cot\left(\frac{\pi}{4}-\frac{x}{2}\right)\right)$$

$$= \tan^{-1}\left(\tan\left(\frac{\pi}{2}-\left(\frac{\pi}{4}-\frac{x}{2}\right)\right)\right)$$

$$= \frac{\pi}{2}-\frac{\pi}{4}+\frac{x}{2} = \frac{\pi}{4}+\frac{x}{2}$$

Qn 2 Simplify $\tan^{-1}\left(\sqrt{1+x^2}+x\right)$

put $x = \tan\theta$

$$= \tan^{-1}\left(\sqrt{1+\tan^2\theta}+\tan\theta\right)$$

$$= \tan^{-1}\left(\sec\theta+\tan\theta\right)$$

$$= \tan^{-1}\left(\frac{1}{\cos\theta}+\frac{\sin\theta}{\cos\theta}\right) = \tan^{-1}\left(\frac{1+\sin\theta}{\cos\theta}\right)$$

$= \tan^{-1}\left(\frac{1+\sin\theta}{\cos\theta}\right)$

$= \tan^{-1}\left(\frac{1+\cos(\frac{\pi}{2}-\theta)}{\sin(\frac{\pi}{2}-\theta)}\right)$

$= \tan^{-1}\left(\frac{\cancel{2}\cos^2(\frac{\pi}{4}-\frac{\theta}{2})}{\cancel{2}\sin(\frac{\pi}{4}-\frac{\theta}{2})\cancel{\cos(\frac{\pi}{4}-\frac{\theta}{2})}}\right)$

$= \tan^{-1}\left(\cot(\frac{\pi}{4}-\frac{\theta}{2})\right)$

$= \tan^{-1}\left(\tan\left(\frac{\pi}{2}-(\frac{\pi}{4}-\frac{\theta}{2})\right)\right)$

$= \frac{\pi}{2} - \frac{\pi}{4} + \frac{\theta}{2}$

$= \frac{\pi}{4} + \frac{\theta}{2}$

replace θ

$\boxed{\text{put } x = \tan\theta \quad \tan^{-1}x = \theta}$

$= \frac{\pi}{4} + \frac{\tan^{-1}x}{2}$ Ans

Q.No.3 ~~Simplify~~ ~~Show~~ Show $\tan^{-1}(\frac{1}{5}) + \tan^{-1}(\frac{1}{7}) + \tan^{-1}(\frac{1}{3}) + \tan^{-1}(\frac{1}{8}) = \frac{\pi}{4}$

$\tan^{-1}\left(\frac{x+y}{1-xy}\right) + \tan^{-1}\left(\frac{x+y}{1-xy}\right)$

Qn 4 Show that $2\tan^{-1}\left(\frac{1}{2}\right) + \tan^{-1}\left(\frac{1}{7}\right) = \tan^{-1}\left(\frac{31}{17}\right)$

Ans

$$\tan^{-1}\left(\frac{2\times\frac{1}{2}}{1-\frac{1}{4}}\right) + \tan^{-1}\left(\frac{1}{7}\right)$$

$$\tan^{-1}\left(\frac{1}{\frac{3}{4}}\right) + \tan^{-1}(1/7)$$

$$= \tan^{-1}\left(\frac{4}{3}\right) + \tan^{-1}(1/7)$$

$$= \tan^{-1}\left(\frac{x+y}{1-xy}\right) = \tan^{-1}\left(\frac{31}{17}\right)$$

Qn 5 Simplify $\tan^{-1}\left(\frac{a\cos x - b\sin x}{b\cos x + a\sin x}\right)$

Divide by $b\cos x$

$$= \tan^{-1}\left(\frac{\frac{a}{b} - \tan x}{1 + \frac{a}{b}\tan x}\right)$$

$$= \tan^{-1}\left(\frac{a}{b}\right) - \tan^{-1}(\tan x)$$

$$= \tan^{-1}\left(\frac{a}{b}\right) - x \quad \text{Ans}$$

INVERSE TRIGO CLASS-2 (I-2)

P-V

$$\tan^{-1}x + \tan^{-1}y = \tan^{-1}\left(\frac{x+y}{1-xy}\right) \; : \; xy < 1$$

$$\tan^{-1}x - \tan^{-1}y = \tan^{-1}\left(\frac{x-y}{1+xy}\right) \; ; \; xy > -1$$

P-VI

$$\tan^{-1}\left(\frac{x+y}{1-xy}\right) = \tan^{-1}x + \tan^{-1}y$$

$$\tan^{-1}\left(\frac{x-y}{1+xy}\right) = \tan^{-1}x - \tan^{-1}y$$

P-VII

$$2\tan^{-1}x = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$$

$$2\tan^{-1}x = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$$

$$2\tan^{-1}x = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$$

P-VIII

✓ $1-\cos x = 2\sin^2(x/2)$

✓ $1+\cos x = 2\cos^2(x/2)$

✓ $\sin x = 2\sin(x/2)\cos(x/2)$

(·)✓ $1-\sin x = 1-\cos\left(\frac{\pi}{2}-x\right) = 2\sin^2\left(\frac{\pi}{4}-\frac{x}{2}\right)$

(·)✓ $1+\sin x = 1+\cos\left(\frac{\pi}{2}-x\right) = 2\cos^2\left(\frac{\pi}{4}-\frac{x}{2}\right)$

(·)✓ $\cos x = \sin\left(\frac{\pi}{2}-x\right) = 2\sin\left(\frac{\pi}{4}-\frac{x}{2}\right)\cos\left(\frac{\pi}{4}-\frac{x}{2}\right)$

$$1 - \cos(2\theta) = 2\sin^2\theta$$
$$1 + \cos(2\theta) = 2\cos^2\theta$$

P-IX Substitutions

$\sqrt{1-x^2}$: put $x = \sin\theta / \cos\theta$

$\sqrt{1+x^2}$ put $x = \tan\theta / \cot\theta$

$\sqrt{x^2-1}$ put $x = \sec\theta / \operatorname{cosec}\theta$

$\sqrt{a^2-x^2}$: put $x = a\sin\theta / a\cos\theta$

$\sqrt{x^2+a^2}$: put $x = a\tan\theta / a\cot\theta$

$\sqrt{x^2-a^2}$. put $x = a\sec\theta / a\operatorname{cosec}\theta$

$\sqrt{\frac{a^2-x^2}{a^2+x^2}}$ put $x^2 = a\cos(2\theta)$

$\sqrt{\frac{1-x^2}{1+x^2}}$ put $x^2 = \cos(2\theta)$

$\sqrt{\frac{a-x}{a+x}}$ put $x = a\cos(2\theta)$

— x —

① Simplify

$\tan^{-1}(\cot x)$
$= \tan^{-1}(\tan(\frac{\pi}{2}-x))$
$= \frac{\pi}{2} - x$

(·) $\cos^{-1}(\sin x)$
$= \cos^{-1}(\cos(\frac{\pi}{2}-x))$
$= \frac{\pi}{2} - x$

(·) Conversions

$\sin^{-1}(\frac{3}{5})$ $\frac{3 \to P}{5 \to H}$

$B = \sqrt{H^2-P^2}$
$= \sqrt{25-9} = 4$

$= \cos^{-1}\left(\frac{4}{5}\right) = \tan^{-1}\left(\frac{3}{4}\right) = \sec^{-1}\left(\frac{5}{4}\right)$