SOLUTIONS:

WORKSHEET No: 9 (Class No: 11)

INTEGRATION

Qns 1 $I = \int \frac{x^2}{(x^2+2)(2x^2+1)} dx$

(Type even power of x)

let $x^2 = y$ (temp)

$\therefore \frac{x^2}{(x^2+2)(2x^2+1)} = \frac{y}{(y+2)(2y+1)}$

let $\frac{y}{(y+2)(2y+1)} = \frac{A}{y+2} + \frac{B}{2y+1}$

$y = A(2y+1) + B(y+2)$

$1 = 2A + B$

$0 = A + 2B$ $\boxed{B = -1/3}$ $\boxed{A = \frac{2}{3}}$

$\Rightarrow 0 = 2A + 4B$

$1 = -3B$

$\therefore I = \int \frac{2}{3(x^2+2)} - \frac{1}{3(2x^2+1)} dy$

$I = \frac{2}{3}\int \frac{1}{x^2+2} dx - \frac{1}{3}\int \frac{1}{2x^2+1} dx$

$= \frac{2}{3} \times \frac{1}{\sqrt{2}} \tan^{-1}\left(\frac{x}{\sqrt{2}}\right) - \frac{1}{6}\int \frac{dx}{x^2 + \left(\frac{1}{\sqrt{2}}\right)^2}$

$I = \frac{\sqrt{2}}{3} \tan^{-1}\left(\frac{x}{\sqrt{2}}\right) - \frac{1}{6} \times \sqrt{2} \tan^{-1}(\sqrt{2}x) + C$ Ans

Ans:- 2 $I = \int \frac{x^2}{(x^2+1)(3x^2+4)} dx$

let $x^2 = y$ (temp)

$\therefore \frac{x^2}{(x^2+1)(3x^2+4)} = \frac{y}{(y+1)(3y+4)}$

let $\frac{y}{(y+1)(3y+4)} = \frac{A}{y+1} + \frac{B}{3y+4}$

$y = A(3y+4) + B(y+1)$

$1 = 3A + B$

$0 = 4A + B$

$1 = -A$

$\boxed{A = -1}$ $\boxed{B = 4}$

$\therefore I = \int \frac{-1}{x^2+1} + \frac{4}{3x^2+4} dx$

$= -\int \frac{1}{x^2+1} dx + \frac{4}{3}\int \frac{1}{x^2+\left(\frac{2}{\sqrt{3}}\right)^2} dx$

$I = -\tan^{-1}x + \frac{4}{3} \times \frac{\sqrt{3}}{2} \tan^{-1}\left(\frac{x\sqrt{3}}{2}\right) + C$

$I = -\tan^{-1}x + \frac{2\sqrt{3}}{3} \tan^{-1}\left(\frac{\sqrt{3}x}{2}\right) + C$ Ans

Ans: 3 $\rightarrow$ $I = \int \frac{5x^2+20x+6}{x^3+2x^2+x} dx$

$I = \int \frac{5x^2+20x+6}{x(x^2+2x+1)} dx$

$I = \int \frac{5x^2+20x+6}{x(x+1)^2} dx$

let $\dfrac{5x^2+20x+6}{x(x+1)^2} = \dfrac{A}{x} + \dfrac{B}{x+1} + \dfrac{C}{(x+1)^2}$

$\Rightarrow 5x^2+20x+6 = A(x+1)^2 + B(x+1) + Cx$

$\Rightarrow 5x^2+20x+6 = A(x^2+2x+1) + B(x+1) + Cx$

$5 = A$

$20 = 2A+B+C$ $\quad \boxed{A=5} \quad \boxed{B=1}$

$6 = A+B$ $\quad 20 = 10+1+C \quad C=9$

$\therefore I = \int \dfrac{5}{x} + \dfrac{1}{x+1} + \dfrac{9}{(x+1)^2}\, dx$

$I = 5\log|x| + \log|x+1| - \dfrac{9}{x+1} + C$ ANS

Ques. 4 $I = \int \dfrac{1}{x^3-1}\, dx$

$I = \int \dfrac{1}{(x-1)(x^2+x+1)}\, dx$

let $\dfrac{1}{(x-1)(x^2+x+1)} = \dfrac{A}{x-1} + \dfrac{Bx+C}{x^2+x+1}$

$1 = A(x^2+x+1) + (Bx+C)(x-1)$

$1 = A(x^2+x+1) + (Bx^2 - Bx + Cx - C)$

$0 = A+B \Rightarrow B = -A$

$0 = A-B+C \Rightarrow 0 = 2A+C$

$1 = A - C \Rightarrow 1 = A - C$

$1 = 3A$

$A = 1/3$ $\quad B = -1/3$; $\quad C = -2/3$

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$$\therefore I = \int \frac{1}{3}\frac{1}{(x-1)} + \frac{-\frac{1}{3}x - \frac{2}{3}}{x^2+x+1}\,dx$$

$$= \frac{1}{3}\log|x-1| - \frac{1}{3}\int \frac{x+2}{x^2+x+1}\,dx$$

$$I = \frac{1}{3}\log|x-1| - \frac{1}{3}I_1 + C$$

where $I_1 = \displaystyle\int \frac{x+2}{x^2+x+1}\,dx$

$$= \frac{1}{2}\int \frac{2x+4+1-1}{x^2+x+1}\,dx$$

$$= \frac{1}{2}\int \frac{(2x+1)+3}{x^2+x+1}\,dx$$

$$= \frac{1}{2}\int \frac{2x+1}{\underbrace{x^2+x+1}_{t}}\,dx + \frac{3}{2}\int \frac{1}{x^2+x+1}\,dx$$

$$= \frac{1}{2}\int \frac{dt}{t} + \frac{3}{2}\int \frac{1}{\left(x+\frac{1}{2}\right)^2 - \frac{1}{4} + 1}\,dx$$

$$= \frac{1}{2}\log|x^2+x+1| + \frac{3}{2}\int \frac{1}{\left(x+\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}\,dx$$

$$= \frac{1}{2}\log|x^2+x+1| + \frac{3}{\cancel{2}}\times\frac{\cancel{2}}{\sqrt{3}}\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)$$

$$\therefore I = \frac{1}{2}\log|x-1| - \frac{1}{3}\left\{\frac{1}{2}\log|x^2+x+1| + \sqrt{3}\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)\right\} + c$$

$$I = \frac{1}{2}\log|x-1| - \frac{1}{6}\log|x^2+x+1| - \frac{\sqrt{3}}{3}\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right) + c$$

Ans

Qn. 5 $I = \int \frac{\sin(2x)}{(1+\sin x)(3\sin x - 2)} dx$

Sol.

$I = 2\int \frac{\sin x \cos x}{(1+\sin x)(3\sin x - 2)} dx$

put $\sin x = t \rightarrow \cos x \, dx = dt$

$I_1 = 2\int \frac{t \, dt}{(1+t)(3t-2)}$

let $\frac{t}{(1+t)(3t-2)} = \frac{A}{1+t} + \frac{B}{3t-2}$

$\Rightarrow t = A(3t-2) + B(1+t)$

$\Rightarrow 1 = 3A + B$

$0 = -2A + B$

$1 = 5A$

$A = 1/5$

$B = 2/5$

$\therefore I = 2\int \frac{1}{5(t+1)} + \frac{2}{5(3t-2)} dt$

$= \frac{2}{5}\int \frac{1}{t+1} dt + \frac{4}{5}\int \frac{1}{3t-2} dt$

$= \frac{2}{5} \log|t+1| + \frac{4}{5} \frac{\log|3t-2|}{3} + C$

$= \frac{2}{5} \log|\sin x + 1| + \frac{4}{15} \log|3\sin x - 2| + C$ Ans

Q. 6 → $I = \int \frac{x^3 - 1}{x^3 + x}\, dx$

$$\begin{array}{r|l} & 1 \\ x^3+x & x^3 - 1 \\ & -(\cancel{x^3} + x) \\ \hline & -x - 1 \end{array}$$

Divide

$$I = \int 1 - \frac{x+1}{x^3+x}\, dx$$

$$I = x - \int \frac{x+1}{x(x^2+1)}\, dx$$

let $\frac{x+1}{x(x^2+1)} = \frac{A}{x} + \frac{Bx+C}{x^2+1}$

$$x+1 = A(x^2+1) + (Bx+C)x$$

$$x+1 = A(x^2+1) + (Bx^2+Cx)$$

$0 = A + B$

$1 = C$

$1 = A$

$\boxed{A=1}$ $\boxed{C=1}$ $\boxed{B=-1}$

$$\therefore I = x - \int \frac{1}{x} + \frac{-x+1}{x^2+1}\, dx$$

$$I = x - \log|x| + \int \frac{x}{x^2+1}\, dx - \int \frac{1}{x^2+1}\, dx$$

put $x^2 + 1 = t$

$x\,dx = \frac{dt}{2}$

$$I = x - \log|x| + \frac{1}{2}\int \frac{dt}{t} - \tan^{-1}x$$

$$I = x - \log|x| + \frac{1}{2}\log|x^2+1| - \tan^{-1}x + C$$

Ans

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Qn 7 → $I = \int \frac{x^2-3}{x^4+2x^2+9}\,dx$

Divide N & D by x^2

$$I = \int \frac{1-\frac{3}{x^2}}{x^2+\frac{9}{x^2}+2}\,dx$$

$$I = \int \frac{1-\frac{3}{x^2}}{\left(x+\frac{3}{x}\right)^2-6+2}\,dx$$

$$I = \int \frac{1-\frac{3}{x^2}}{\left(x+\frac{3}{x}\right)^2-4}\,dx$$

put $x+\frac{3}{x}=t \Rightarrow \left(1-\frac{3}{x^2}\right)dx = dt$

$$I = \int \frac{dt}{t^2-4}$$

$$= \frac{1}{2\times 2}\log\left|\frac{t-2}{t+2}\right|+C$$

$$I = \frac{1}{4}\log\left|\frac{x+\frac{3}{x}-2}{x+\frac{3}{x}+2}\right|+C$$

$$I = \frac{1}{4}\log\left|\frac{x^2-2x+3}{x^2+2x+3}\right|+C$$ Ans

Qn.8 → $I = \int \frac{1}{x^4+x^2+1}\,dx$

Divide by x^2

$$I = \int \frac{\frac{1}{x^2}}{x^2+\frac{1}{x^2}+1}\,dx$$

$$= \frac{1}{2}\int \frac{\frac{2}{x^2}}{x^2+\frac{1}{x^2}+1}\,dx$$

$$I = \frac{1}{2}\int \frac{\frac{1}{x^2} + \frac{1}{x^2} + 1 - 1}{x^2 + \frac{1}{x^2} + 1}\, dx$$

$$I = \frac{1}{2}\int \frac{1+\frac{1}{x^2}}{x^2+\frac{1}{x^2}+1}\, dx \quad - \frac{1}{2}\int \frac{1-\frac{1}{x^2}}{x^2+\frac{1}{x^2}+1}\, dx$$

$$= \frac{1}{2}\int \frac{1+\frac{1}{x^2}}{\left(x-\frac{1}{x}\right)^2+2+1}\, dx \quad - \frac{1}{2}\int \frac{1-\frac{1}{x^2}}{\left(x+\frac{1}{x}\right)^2-2+1}\, dx$$

put $x - \frac{1}{x} = t$ $\qquad$ put $x+\frac{1}{x} = z$

$\left(1+\frac{1}{x^2}\right)dx = dt$ $\qquad$ $\left(1-\frac{1}{x^2}\right)dx = dz$

$$I = \frac{1}{2}\int \frac{dt}{t^2+3} \quad - \frac{1}{2}\int \frac{dz}{z^2-1}$$

$$= \frac{1}{2} \times \frac{1}{\sqrt{3}} \tan^{-1}\left(\frac{t}{\sqrt{3}}\right) - \frac{1}{2} \times \frac{1}{2\times 1} \log\left|\frac{z-1}{z+1}\right| + c$$

$$= \frac{1}{2\sqrt{3}} \tan^{-1}\left(\frac{x-\frac{1}{x}}{\sqrt{3}}\right) - \frac{1}{4} \log\left|\frac{x+\frac{1}{x}-1}{x+\frac{1}{x}+1}\right| + c$$

$$I = \frac{1}{2\sqrt{3}} \tan^{-1}\left(\frac{x^2-1}{\sqrt{3}\, x}\right) - \frac{1}{4} \log\left|\frac{x^2-x+1}{x^2+x+1}\right| + c \quad \underline{\text{Ans}}$$

Qn.9 → $I = \int \frac{x^2}{x^4+5x^2+1}\, dx$

Sol. Divide by x^2

$$I = \int \frac{1}{x^2+5+\frac{1}{x^2}}\, dx$$

$$I = \frac{1}{2}\int \frac{2}{x^2+\frac{1}{x^2}+5}\,dx$$

$$= \frac{1}{2}\int \frac{1+1+\frac{1}{x^2}-\frac{1}{x^2}}{x^2+\frac{1}{x^2}+5}\,dx$$

$$= \frac{1}{2}\int \frac{\left(1+\frac{1}{x^2}\right)}{x^2+\frac{1}{x^2}+5}\,dx + \frac{1}{2}\int \frac{1-\frac{1}{x^2}}{x^2+\frac{1}{x^2}+5}\,dx$$

$$= \frac{1}{2}\int \frac{1+\frac{1}{x^2}}{\underbrace{\left(x-\frac{1}{x}\right)}_{t}^2+2+5}\,dx + \frac{1}{2}\int \frac{1-\frac{1}{x^2}}{\underbrace{\left(x+\frac{1}{x}\right)}_{z}^2-2+5}$$

$$I = \frac{1}{2}\int \frac{dt}{t^2+7} + \frac{1}{2}\int \frac{dz}{z^2+3}$$

$$= \frac{1}{2}\times\frac{1}{\sqrt{7}}\tan^{-1}\left(\frac{t}{\sqrt{7}}\right) + \frac{1}{2}\times\frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{z}{\sqrt{3}}\right)+C$$

$$I = \frac{1}{2\sqrt{7}}\tan^{-1}\left(\frac{x-\frac{1}{x}}{\sqrt{7}}\right) + \frac{1}{2\sqrt{3}}\tan^{-1}\left(\frac{x+\frac{1}{x}}{\sqrt{3}}\right)+C$$

$$I = \frac{1}{2\sqrt{7}}\tan^{-1}\left(\frac{x^2-1}{\sqrt{7}\,x}\right) + \frac{1}{2\sqrt{3}}\tan^{-1}\left(\frac{x^2+1}{\sqrt{3}\,x}\right)+C$$

<u>Ans</u>

<u>Qn 10</u> → $I = \int \sqrt{\cot x}\,dx$

<u>Soln</u> put $\cot x = t^2$

$$-\operatorname{cosec}^2 x\,dx = \cancel{dt}\ 2t\,dt$$

$$dx = -\frac{2t\,dt}{\operatorname{cosec}^2 x}$$

$$dx = \frac{-2t\,dt}{1+\cot^2 x}$$

$$dx = \frac{-2t\,dt}{1+t^4}$$

$$\therefore I = -2\int t \cdot \frac{t\,dt}{t^4+1}$$

$$= -2\int \frac{t^2}{t^4+1}\,dt$$

Divide by t^2

$$= -2\int \frac{1}{t^2+\frac{1}{t^2}}\,dt$$

$$= -\int \frac{1+1+\frac{1}{t^2}-\frac{1}{t^2}}{t^2+\frac{1}{t^2}}\,dt$$

$$I = -\int \frac{1+\frac{1}{t^2}}{t^2+\frac{1}{t^2}}\,dt - \int \frac{1-\frac{1}{t^2}}{t^2+\frac{1}{t^2}}\,dt$$

$$= -\int \frac{1+\frac{1}{t^2}}{(t-\frac{1}{t})^2+2}\,dt - \int \frac{1-\frac{1}{t^2}}{(t+\frac{1}{t})^2-2}\,dt$$

let u $\qquad$ let v

$$= -\int \frac{du}{u^2+2} - \int \frac{dv}{v^2-2}$$

$$= -\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{u}{\sqrt{2}}\right) - \frac{1}{2\times\sqrt{2}}\log\left|\frac{v-\sqrt{2}}{v+\sqrt{2}}\right| + C$$

$$= \frac{-1}{\sqrt{2}}\tan^{-1}\left(\frac{2t^2-1}{\sqrt{2}\,t}\right) - \frac{1}{2\sqrt{2}}\log\left|\frac{t+\frac{1}{t}-\sqrt{2}}{t+\frac{1}{t}+\sqrt{2}}\right| + C$$

$$I = \frac{-1}{\sqrt{2}}\tan^{-1}\left(\frac{2\cot x - 1}{\sqrt{2}\sqrt{\cot x}}\right) - \frac{1}{2\sqrt{2}}\log\left|\frac{\cot x + 1 - \sqrt{2}\sqrt{\cot x}}{\cot x + 1 + \sqrt{2}\sqrt{\cot x}}\right| + C$$

Ans