

XII

EXAM NO: 7

(MIXED

OBJECTIVE + SUBJECTIVE)

MATRICES & DETERMINANTS

MARKS: 70 | TIME: 2 hr

SECTION: A: OBJECTIVE (TWO MARKS EACH) : 30

Q.1.1 A square matrix $A = [a_{ij}]_{n \times n}$ is called a lower triangular matrix if $a_{ij} = 0$ for

(A) $i = j$ (B) $i < j$ (C) $i > j$ (D) none of these

Q.1.2 If $A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$ and $f(x) = (1+x)(1-x)$, then $f(A)$ is

(A) $-4 \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ (B) $-8 \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ (C) $4 \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ (D) none of these

Q.1.3 If $[1 \ x \ 1] \begin{bmatrix} 1 & 3 & 2 \\ 2 & 5 & 1 \\ 15 & 3 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ x \end{bmatrix} = 0$; then $x =$

(A) -7 (B) -11 (C) -2 (D) 14

Q.1.4 If a square matrix $A = [a_{ij}]$; $a_{ij} = i^2 - j^2$ is of even order, then

(A) A is a skew-symmetric matrix (B) Null matrix

(C) A is a symmetric matrix (D) neither symmetric nor skew-symmetric

Q.1.5 If $A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ satisfies $A^4 = kA'$, then value of k is

(A) 1 (B) 4 (C) 8 (D) 10

Q.1.6 If $A = \begin{bmatrix} b & 2 & a \\ 2 & 1 & -2 \\ -2 & 2 & -1 \end{bmatrix}$ is a matrix satisfying $AA' = 9I$ then value of a & b are respectively

(A) $1, 2$ (B) $2, 1$ (C) $-1, 2$ (D) $-2, 1$

Q_{11.7} → The value of $\Delta = \begin{vmatrix} \cos(\alpha+\beta) & -\sin(\alpha+\beta) & \cos(2\beta) \\ \sin\alpha & \cos\alpha & \sin\beta \\ -\cos\alpha & \sin\alpha & \cos\beta \end{vmatrix}$ is independent of

- (A) α (B) β (C) α and β (D) none of these

~~Q_{11.8} → If the points (a_1, b_1) , (a_2, b_2) & (a_1+a_2, b_1+b_2) are collinear then~~

Q_{11.8} → If the points (a_1, b_1) , (a_2, b_2) & (a_1+a_2, b_1+b_2) are collinear then

- (A) $a_1 b_2 = a_2 b_1$ (B) $a_1 + a_2 = b_1 + b_2$ (C) $a_2 b_2 = a_1 b_1$
(D) $a_1 b_2 + a_2 b_1 = 0$

Q_{11.9} → If the system of equations ~~$x+y+z=2$~~ $x+y+z=2$, $2x+y-z=3$ & $3x+2y+kz=4$ has a unique solution, if k is not equal to

- (A) 4 (B) -4 (C) 0 (D) 1

Q_{11.10} → If for the non-singular matrix A , $A^3 = A^2$, then A^{-1} is equal to

- (A) A (B) A^2 (C) I (D) none of these

Q_{11.11} → order 3×3 ; $|3AB| = 405$; $|A| = 3$, then $|B| =$

- (A) 81 (B) -5 (C) 5 (D) 27

(3)

Qns 12 \rightarrow order 4×4 ; $|\text{Adj } A| = -27$, then $|-A| =$

(A) 3 (B) -3 (C) $\frac{1}{3}$ (D) 81

Qns 13 \rightarrow If the area of triangle ABC with vertices $A(1,3)$ $B(0,0)$ $C(k,0)$ is 3 square units, then value of k is

(A) 2 (B) 3 (C) 4 (D) 5

Qns 14 \rightarrow let $f(t) = \begin{vmatrix} \cos t & t & 1 \\ 2 \sin t & t & 2t \\ \sin t & t & t \end{vmatrix}$ then $\lim_{t \rightarrow 0} \left(\frac{f(t)}{t^2} \right)$ is equal to

(A) 0 (B) -1 (C) 2 (D) 3

Qns 15 \rightarrow If $A = \frac{1}{\lambda} \begin{bmatrix} \sin^{-1}(\lambda x) & \tan^{-1}(\frac{x}{\lambda}) \\ \sin^{-1}(\frac{x}{\lambda}) & \cot^{-1}(\lambda x) \end{bmatrix}$ &

$B = \frac{1}{\lambda} \begin{bmatrix} -\cos^{-1}(\lambda x) & \tan^{-1}(\frac{x}{\lambda}) \\ \sin^{-1}(\frac{x}{\lambda}) & -\tan^{-1}(\lambda x) \end{bmatrix}$ then $A-B$ is

equal to

(A) I (B) O (C) $2I$ (D) $\frac{1}{2}I$

SECTION: B : SUBJECTIVE (FOUR MARKS EACH) = 40

Qns 16 \rightarrow A manufacturer produces three products X, Y, Z which he sells in two markets. Annual sales are indicated below:

Market	Products		
I	10000	2000	18000
II	6000	20000	8000

(4)

If unit sale prices of x, y and z are Rs 2.50, Rs 1.50 & Rs 1.00 respectively and unit cost price of these products are Rs 2.00, Rs 1.00 & 50 paise respectively. Find the gross profit

Ques-17 \rightarrow If A and B are symmetric matrices of the same order, then show that AB is symmetric if and only if A and B commute, i.e. $AB = BA$

Ques-18 \rightarrow Let $A = \begin{bmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix}$ such that

$A^{32} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$, then value of α is

- (A) 0 (B) $\frac{\pi}{16}$ (C) $\frac{\pi}{64}$ (D) $\frac{\pi}{32}$

(Note: Give complete explanation:
0 marks only for answer)

Ques-19 \rightarrow Let $A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}$ & $IOB = \begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & \alpha \\ 1 & -2 & 3 \end{bmatrix}$
if B is the Inverse of A , then find α

Ques-20 \rightarrow Let $A = \begin{bmatrix} \cos^2 \theta & \sin \theta \cos \theta \\ \cos \theta \sin \theta & \sin^2 \theta \end{bmatrix}$ and $B = \begin{bmatrix} \cos^2 \phi & \sin \phi \cos \phi \\ \cos \phi \sin \phi & \sin^2 \phi \end{bmatrix}$
then $AB = O$, if

- (A) $O = n\pi$, $n = 0, 1, 2, \dots$
(B) $O + \phi = (2n+1)\frac{\pi}{2}$; $n \in \mathbb{Z}$
(C) $\theta = \phi + (2n+1)\frac{\pi}{2}$; $n \in \mathbb{Z}$
(D) none of them

(Note: Give complete explanation)

(5)

Qns. 21 → given $A = \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$ & $B = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$

find BA and use this to solve the system of equations $y + 2z = 7$, $x - y = 3$; $2x + 3y + 4z = 17$

Qn. 22 → for the matrix $A = \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix}$ find the numbers a & b such that $A^2 + aA + bI = 0$.
Hence find A^{-1}

Qn. 23 → find matrix X such that

$$\begin{bmatrix} 3 & 2 \\ 7 & 5 \end{bmatrix} X \begin{bmatrix} -1 & 1 \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} 2 & -1 \\ 0 & 4 \end{bmatrix}$$

Qn. 24 → If $\begin{bmatrix} 1 & -\tan\theta \\ \tan\theta & 1 \end{bmatrix} \begin{bmatrix} 1 & \tan\theta \\ -\tan\theta & 1 \end{bmatrix}^{-1} = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$
 then find a & b

Qn. 25 → If $f(x) = ax^2 + bx + c$ is a quadratic polynomial such that $f(1) = 8$; $f(2) = 11$ and $f(-3) = 6$ find $f(x)$ using matrix method

— x —