XII | EXAM NO: 8 | (SUBJECTIVE)

INVERSE TRIGO + DIFFERENTIATION CONTINUITY

DO ANY 13 QUESTIONS (4 MARKS EACH)

MARKS: 52
TIME: 2 HRS

Qu. 1 → If $y = e^{a\cos^{-1}x}$ Show that

$$(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - a^2y = 0$$

Qu. 2 → If $(x-a)^2 + (y-b)^2 = c^2$ Show that

$$\frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}}$$ is a constant and independent of a and b

Qu. 3 → If $x = a(\cos t + t\sin t)$ and $y = a(\sin t - t\cos t)$

find $\frac{d^2y}{dx^2}$

Qu. 4 → If $x = \frac{\sin^3 t}{\sqrt{\cos(2t)}}$ and $y = \frac{\cos^3 t}{\sqrt{\cos(2t)}}$. Find $\frac{dy}{dx}$

Qu. 5 → $y = (x\cos x)^x + (x\sin x)^{1/x} + (x)^{x\sin x}$. Find $\frac{dy}{dx}$

Qu. 6 → Prove ~~that~~ that the greatest Integer function

$f(x) = [x]$ is not differentiable at $x = 1$ & $x = 2$

where $0 < x < 3$

Qn. 7 → Solve (find x)

$$\sin^{-1}(1-x) - 2\sin^{-1}x = \frac{\pi}{2}$$

Qn. 8 → Show that $\sin^{-1}\left(\frac{12}{13}\right) + \cos^{-1}\left(\frac{4}{5}\right) + \tan^{-1}\left(\frac{63}{16}\right) = \pi$

Qns 9 → If $f(x) = \dfrac{\sqrt{2}\cos x - 1}{\cot x - 1}$; $x \neq \frac{\pi}{4}$

find the value of $f(\pi/4)$ so that $f(x)$ becomes continuous at $x = \pi/4$

Qn. 10 → Differentiate $\tan^{-1}\left(\frac{\sqrt{1-x^2}}{x}\right)$ with respect to $\cos^{-1}(2x\sqrt{1-x^2})$ where $x \in \left(\frac{1}{\sqrt{2}}, 1\right)$

Qns 11 → If $x^m y^n = (x+y)^{m+n}$ Show that $\frac{d^2y}{dx^2} = 0$

Qn. 12 → Find the value of p and q so that

$$f(x) = \begin{cases} x^2 + 3x + p & ; \quad x \leq 1 \\ 2x + 2 & ; \quad x > 1 \end{cases}$$

is differentiable at $x = 1$

Qn. 13 → If $x\sin(a+y) + \sin a \cdot \cos(a+y) = 0$ Show that

$$\frac{dy}{dx} = \frac{\sin^2(a+y)}{\sin a}$$

Qn. 14 → Show that $\tan^{-1}\left(\dfrac{\sqrt{1+x^2} + \sqrt{1-x^2}}{\sqrt{1+x^2} - \sqrt{1-x^2}}\right) = \frac{\pi}{4} + \frac{1}{2}\cos^{-1}(x^2)$

Qn. 15 → Find the greatest and least value of

$(\sin^{-1}x)^2 + (\cos^{-1}x)^2$

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