SOLUTIONS WORKSHEET NO:2 (D.E) ①

(CLASS NO: 2)

Ans 1 → $x\cos\left(\frac{y}{x}\right)\frac{dy}{dx} = y\cos\left(\frac{y}{x}\right) + x$

$$\Rightarrow \frac{dy}{dx} = \frac{y\cos(y/x) + x}{x\cos(y/x)} \quad \text{----}(i)$$

here $f(x,y) = \dfrac{y\cos(y/x) + x}{x\cos(y/x)}$

$$f(\lambda x, \lambda y) = \frac{\lambda y\cdot\cos\left(\frac{\lambda y}{\lambda x}\right) + \lambda x}{\lambda x\cdot\cos\left(\frac{\lambda y}{\lambda x}\right)}$$

$$f(\lambda x, \lambda y) = \frac{\lambda\left(y\cos\left(\frac{y}{x}\right) + x\right)}{\lambda\cos(y/x)} = \lambda^0\cdot f(x,y)$$

∴ given function is a homogeneous function of degree 0

put $y = vx$ in eq(i)

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

$$\therefore \quad v + x\frac{dv}{dx} = \frac{vx\cos v + x}{x\cos v}$$

$$\Rightarrow v + x\frac{dv}{dx} = \frac{v\cos v + 1}{\cos v}$$

$$\Rightarrow x\frac{dv}{dx} = \frac{v\cos v + 1}{\cos v} - v$$

$$\Rightarrow x\frac{dv}{dx} = \frac{\cancel{v\cos v} + 1 - \cancel{v\cos v}}{\cos v}$$

$$\Rightarrow x\frac{dv}{dx} = \frac{1}{\cos v}$$

$$\Rightarrow \cos v\, dv = \frac{dx}{x}$$

$\Rightarrow \int \cos v \, dv = \int \frac{dx}{x}$

$\Rightarrow \sin v = \log|x| + c$

$\Rightarrow \boxed{\sin\left(\frac{y}{x}\right) = \log|x| + C}$ ANS

OR

$\sin\left(\frac{y}{x}\right) = \log|x| + \log c$

$\Rightarrow \boxed{\sin\left(\frac{y}{x}\right) = \log|cx|}$ ANS

Ans 2 →

Slope of tangent $= \frac{dy}{dx}$

given $\frac{dy}{dx} = \frac{x^2+y^2}{2xy}$

$\Rightarrow$ It is a homogeneous D.E

put $y = vx$

$\frac{dy}{dx} = v + x\frac{dv}{dx}$

$\therefore v + x\frac{dv}{dx} = \frac{x^2 + v^2x^2}{2vx^2}$

$\Rightarrow v + x\frac{dv}{dx} = \frac{1+v^2}{2v}$

$\Rightarrow x\frac{dv}{dx} = \frac{1+v^2}{2v} - v$

$\Rightarrow x\frac{dv}{dx} = \frac{1+v^2-2v^2}{2v}$

$\Rightarrow x\frac{dv}{dx} = \frac{-v^2+1}{2v}$

$\Rightarrow x\frac{dv}{dx} = \frac{-(v^2-1)}{2v}$

$\Rightarrow \frac{v}{v^2-1}\,dv = -\frac{1}{2}\frac{dx}{x}$

$$\Rightarrow \int \frac{v}{v^2-1}\, dv = \frac{-1}{2}\int \frac{dx}{x}$$

put $v^2-1=t$

$vdv = \frac{dt}{2}$

$$\Rightarrow \frac{1}{2}\int \frac{dt}{t} = -\frac{1}{2}\log|x|$$

$$\Rightarrow \frac{1}{2}\log|t| = \frac{-1}{2}\log|x| + \log c$$

$$\Rightarrow \frac{1}{2}\log|v^2-1| + \frac{1}{2}\log|x| = \log c$$

$$\Rightarrow \frac{1}{2}\left[\log|(v^2-1)\cdot x|\right] = \log c$$

$$\Rightarrow \log\left|\left(\frac{y^2}{x^2}-1\right)\cdot x\right| = 2\log c$$

$$\Rightarrow \frac{(y^2-x^2)}{x} = c^2$$

$$\Rightarrow y^2-x^2 = c^2x$$

$$\Rightarrow x^2-y^2 = -c^2x$$

let $-c^2 = c'$ (new constant)

$$\Rightarrow \boxed{x^2-y^2 = c'x}$$ PROVED

Ans 3 → $(x+y)dy + (x-y)dx = 0$; $y=1$; $x=1$

$$\rightarrow \frac{dy}{dx} = \frac{-(x-y)}{x+y}$$

It is a homogenous D.E

put $y = vx$

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

$$\therefore v + x\frac{dv}{dx} = \frac{-(x-vx)}{x+vx}$$

$$\Rightarrow v + x\frac{dv}{dx} = \frac{-1+v}{1+v}$$

$$\Rightarrow x\frac{dv}{dx} = \frac{-1+v}{1+v} - v$$

$$\Rightarrow x\frac{dv}{dx} = \frac{-1+\cancel{v}-\cancel{v}-v^2}{1+v}$$

$$\Rightarrow x\frac{dv}{dx} = -\frac{(v^2+1)}{v+1}$$

$$\Rightarrow \frac{v+1}{v^2+1} = -\frac{dx}{x}$$

$$\Rightarrow \int \frac{v+1}{v^2+1}dv = -\int\frac{dx}{x}$$

separate

$$\Rightarrow \int \frac{v}{v^2+1}dv + \int\frac{1}{v^2+1}dv = -\log|x|$$

put $v^2+1 = t$

$v\,dv = \frac{dt}{2}$

$$\Rightarrow \frac{1}{2}\int\frac{dt}{t} + \tan^{-1}v = -\log|x|$$

$$\Rightarrow \frac{1}{2}\log|v^2+1| + \tan^{-1}v = -\log|x| + C$$

$$\Rightarrow \frac{1}{2}\log\left|\frac{y^2}{x^2}+1\right| + \tan^{-1}\frac{y}{x} + \log|x| = C$$

$$\Rightarrow \log\left|\frac{x^2+y^2}{x^2}\right| + 2\tan^{-1}\left(\frac{y}{x}\right) + 2\log|x| = 2C$$

$$\Rightarrow \log\left|\left(\frac{x^2+y^2}{\cancel{x^2}}\right)\cdot\cancel{x^2}\right| + 2\tan^{-1}\left(\frac{y}{x}\right) = 2C$$

$$\Rightarrow \log|x^2+y^2| + 2\tan^{-1}\left(\frac{y}{x}\right) = 2C$$

put $x=1$ & $y=1$

$$\Rightarrow \log 2 + 2\tan^{-1}(1) = 2C$$

$\Rightarrow \quad \log 2 + 2\left(\frac{\pi}{4}\right) = 2C$

$\Rightarrow \quad 2C = \log 2 + \frac{\pi}{2}$

$\therefore \boxed{\log|x^2+y^2| + 2\tan^{-1}\left(\frac{y}{x}\right) = \frac{\pi}{2} + \log 2}$ ANS

Ans 4 → $y\,dx + x\log\left(\frac{y}{x}\right)dy - 2x\,dy = 0$

$\Rightarrow \quad dy\left(x\log\left(\frac{y}{x}\right) - 2x\right) = -y\,dx$

$\Rightarrow \quad \frac{dy}{dx} = \frac{-y}{x\log\left(\frac{y}{x}\right) - 2x}$

put $y = vx$

$\frac{dy}{dx} = v + x\frac{dv}{dx}$

$\Rightarrow \quad v + x\frac{dv}{dx} = \frac{-vx}{x\log\left(\frac{vx}{x}\right) - 2x}$

$\Rightarrow \quad v + x\frac{dv}{dx} = \frac{-v}{\log v - 2}$

$\Rightarrow \quad x\frac{dv}{dx} = \frac{-v}{\log v - 2} - v$

$\Rightarrow \quad x\frac{dv}{dx} = \frac{-v - v\log v + 2v}{\log v - 2}$

$\Rightarrow \quad x\frac{dv}{dx} = \frac{v - v\log v}{\log v - 2}$

$\Rightarrow \quad x\frac{dv}{dx} = \frac{-v(\log v - 1)}{\log v - 2}$

$\Rightarrow \quad \frac{\log v - 2}{v(\log v - 1)}dv = -\frac{dx}{x}$

$$\Rightarrow \int \frac{\log v - 2}{v(\log v - 1)} = -\int \frac{dx}{x}$$

put $\log v - 1 = t$

$\frac{1}{v} dv = dt$

$$\Rightarrow \int \frac{(t+1)-2}{t} \cdot dt = -\log|x|$$

$$\Rightarrow \int \frac{t-1}{t} dt = -\log|x|$$

$$\Rightarrow \int 1 - \frac{1}{t} dt = -\log|x|$$

$$\Rightarrow t - \log|t| = -\log|x| + C$$

$$\Rightarrow \log v - 1 - \log|\log v - 1| = -\log|x| + C$$

$$\Rightarrow \log\left|\frac{y}{x}\right| - 1 - \log\left|\log\left(\frac{y}{x}\right) - 1\right| + \log|x| = C$$

$$\Rightarrow \log\left|\frac{\left(\frac{y}{x}\right) \cdot x}{\log\left|\frac{y}{x}\right| - 1}\right| = C + 1$$

--- { $\log A - \log B + \log C = \log\left|\frac{AC}{B}\right|$

$$\Rightarrow \left|\frac{y}{\log\left|\frac{y}{x}\right| - 1}\right| = e^{C+1}$$

--- {∵ If $\log x = y$ then $x = e^y$ }

$$\Rightarrow \frac{y}{\log\left|\frac{y}{x}\right| - 1} = \pm e^{C+1}$$

→ new constant $= c'$

$$\Rightarrow \boxed{y = c'\left(\log\left|\frac{y}{x}\right| - 1\right)}$$ Ans

OR

$$\frac{1}{c'} \cdot y = \log\left|\frac{y}{x}\right| - 1$$

$$\Rightarrow \boxed{ky = \log\left|\frac{y}{x}\right| - 1}$$ Ans

{ $\frac{1}{c'}$ = new constant $= k$ }

Ans 5 →

$$2xy + y^2 - 2x^2\frac{dy}{dx} = 0 \quad ; \; y = 2 \; ; \; x = 1$$

$$\Rightarrow \frac{dy}{dx} = \frac{2xy + y^2}{2x^2}$$

It is a homogenous D·E

put $y = vx$

$$\Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$$

$$\therefore \; v + x\frac{dv}{dx} = \frac{2vx^2 + v^2x^2}{2x^2}$$

$$\Rightarrow v + x\frac{dv}{dx} = \frac{2v + v^2}{2}$$

$$\Rightarrow x\frac{dv}{dx} = \frac{2v + v^2}{2} - v$$

$$\Rightarrow x\frac{dv}{dx} = \frac{\cancel{2v} + v^2 - \cancel{2v}}{2}$$

$$\Rightarrow \frac{dv}{v^2} = \frac{1}{2} \cdot \frac{1}{x}dx$$

$$\Rightarrow \int \frac{1}{v^2}dv = \frac{1}{2}\int \frac{1}{x}dx$$

$$\Rightarrow \frac{-1}{v} = \frac{1}{2}\log|x| + c$$

$$\Rightarrow -\frac{x}{y} = \frac{1}{2}\log|x| + c$$

put $x = 1 \; ; \; y = 2$

$$\Rightarrow \frac{-1}{2} = \frac{1}{2}\log|1| + c$$

$$\Rightarrow \boxed{c = \frac{-1}{2}}$$

$$\therefore \; -\frac{x}{y} = \frac{1}{2}\log|x| - \frac{1}{2}$$

Scanned with CamScanner

$\Rightarrow \quad \frac{-2x}{y} = \log|x| - 1$

$\Rightarrow \quad \boxed{y = \frac{2x}{1-\log|x|}}$ Ans

Ques 6 → $\frac{dy}{dx} + y\cot x = 4x\,\mathrm{cosec}\,x \quad ; \quad y\left(\frac{\pi}{2}\right) = 0$

Comparing with $\frac{dy}{dx} + Py = Q$

here $P = \cot x \quad ; \quad Q = 4x\,\mathrm{cosec}\,x$

$I.F = e^{\int P dx} = e^{\int \cot x\, dx} = e^{\log(\sin x)} = \sin x$

$\therefore \boxed{I.F = \sin x}$

Solution

$y \times IF = \int (Q \times IF)\, dx + C$

$\Rightarrow y(\sin x) = \int 4x\, \cancel{\mathrm{cosec}\, x} \cdot \cancel{\sin x}\, dx + C$

$\Rightarrow y \sin x = 4\frac{x^2}{2} + C$

$\Rightarrow y \sin x = 2x^2 + C$

put $x = \frac{\pi}{2}$ & $y = 0$

$\Rightarrow \quad 0 = 2\left(\frac{\pi^2}{4}\right) + C$

$\Rightarrow \quad C = -\frac{\pi^2}{2}$

$\therefore \quad \boxed{y \sin x = 2x^2 - \frac{\pi^2}{2}}$ Ans

Qn 7 →
$$\frac{dy}{dx} + \sqrt{\frac{1-y^2}{1-x^2}} = 0$$

$$\Rightarrow \frac{dy}{dx} = -\frac{\sqrt{1-y^2}}{\sqrt{1-x^2}}$$

$$\Rightarrow 1/\sqrt{1-y^2}\,dy = -\frac{1}{\sqrt{1-x^2}}\,dx$$

$$\Rightarrow \int \frac{1}{\sqrt{1-y^2}}\,dy = -\int \frac{1}{\sqrt{1-x^2}}\,dx$$

$$\Rightarrow \sin^{-1}y = -\sin^{-1}x + c$$

$$\Rightarrow \boxed{\sin^{-1}y + \sin^{-1}x = c}$$ Ans

Qn 8 →
$$\sin x \cdot \cos y\,dx + \cos x \cdot \sin y\,dy = 0$$

$$\Rightarrow \frac{dy}{dx} = \frac{-\sin x \cos y}{\cos x \sin y}$$

$$\Rightarrow \frac{dy}{dx} = \frac{-\tan x}{\tan y}$$

$$\Rightarrow \tan y\,dy = -\tan x\,dx$$

$$\Rightarrow \int \tan y\,dy = -\int \tan x\,dx$$

$$\Rightarrow \log|\sec y| = -\log|\sec x| + \log c$$

$$\Rightarrow \log|\sec y \cdot \sec x| = \log c$$

$$\Rightarrow |\sec y \cdot \sec x| = c$$

thus curve/solution passes through $(0, \pi/4)$
put $x=0$ & $y = \pi/4$

$$\Rightarrow |\sec\tfrac{\pi}{4} \cdot \sec 0| = c$$

$\Rightarrow \quad \sqrt{2} \times 1 = c$

$\Rightarrow \quad c = \sqrt{2}$

$\therefore \quad |\sec y \cdot \sec x| = \sqrt{2}$

$$\Rightarrow \quad \frac{\sec x}{\cos y} = \sqrt{2}$$

$\Rightarrow \quad \boxed{\sec x = \sqrt{2} \cos y}$ Ans

Ans 9 → $(x^2+xy)\,dy = (x^2+y^2)\,dx$

$$\frac{dy}{dx} = \frac{x^2+y^2}{x^2+xy}$$

It is a homogeneous D.E

put $y = vx$

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

$$\Rightarrow \quad v + x\frac{dv}{dx} = \frac{x^2+v^2x^2}{x^2+vx^2}$$

$$\Rightarrow \quad v + x\frac{dv}{dx} = \frac{1+v^2}{1+v}$$

$$\Rightarrow \quad x\frac{dv}{dx} = \frac{1+v^2}{1+v} - v$$

$$\Rightarrow \quad x\frac{dv}{dx} = \frac{1+\cancel{v^2}-v-\cancel{v^2}}{1+v}$$

$$\Rightarrow x\frac{dv}{dx} = \frac{1-v}{v+1}$$

$$\Rightarrow x\frac{dv}{dx} = -\frac{(v-1)}{v+1}$$

$$\Rightarrow \frac{v+1}{v-1}\,dv = -\frac{dx}{x}$$

$$\Rightarrow \int\frac{v+1}{v-1}\,dv = -\int\frac{dx}{x}$$

$$\Rightarrow \int\frac{v+1-1+1}{v-1}\,dv = -\log|x|$$

$$\Rightarrow \int\frac{(v-1)+2}{v-1}\,dv = -\log|x|$$

$$\Rightarrow \int 1+\frac{2}{v-1}\,dv = -\log|x|$$

$$\Rightarrow v + 2\log|v-1| = -\log|x| + c$$

$$\Rightarrow \frac{y}{x} + 2\log\left|\frac{y}{x}-1\right| = -\log|x| + c$$

$$\Rightarrow \frac{y}{x} + \log\left|\frac{(y-x)^2}{x^2}\right| + \log|x| = c$$

$$\Rightarrow \frac{y}{x} + \log\left|\frac{(y-x)^2}{x^2}\cdot x\right| = c$$

$$\Rightarrow \log\left|\frac{(x-y)^2}{x}\right| = c-\frac{y}{x} \quad \text{-- } \{(a-b)^2 = (b-a)^2\}$$

$$\Rightarrow \left|\frac{(x-y)^2}{x}\right| = e^{c-\frac{y}{x}} \quad \text{--- } \{\text{If } \log x = y \text{ then } x = e^y\}$$

$$\Rightarrow \left|\frac{(x-y)^2}{x}\right| = e^c\cdot e^{-y/x}$$

$$\Rightarrow \frac{(x-y)^2}{x} = \pm e^c\cdot e^{-y/x}$$

$$\Rightarrow \frac{(x-y)^2}{x} = c_1 \cdot e^{-y/x} \quad --- \left\{ \pm e^c = \text{new constant} = c_1 \right\}$$

$$\Rightarrow \boxed{(x-y)^2 = c_1 x e^{-y/x}}$$ Ans

Ques 11 → $\sec^2 x \tan y \, dx + \sec^2 y \tan x \, dy = 0$

$$\Rightarrow \frac{dy}{dx} = \frac{-\sec^2 x \tan y}{\sec^2 y \tan x}$$

$$\Rightarrow \frac{\sec^2 y}{\tan y} dy = -\frac{\sec^2 x}{\tan x} dx$$

$$\Rightarrow \int \frac{\sec^2 y}{\tan y} dy = -\int \frac{\sec^2 x}{\tan x} dx$$

put $\tan y = t$ | put $\tan x = z$

$\sec^2 y \, dy = dt$ | $\sec^2 x \, dx = dz$

$$\Rightarrow \int \frac{dt}{t} = -\int \frac{dz}{z}$$

$$\Rightarrow \log|t| = -\log|z| + \log c$$

$$\Rightarrow \log|tz| = \log c$$

$$\Rightarrow |tz| = c$$

$$\Rightarrow tz = \pm c = \text{new constant} = c_1$$

$$\Rightarrow \boxed{\tan y \cdot \tan x = c_1}$$ Ans

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