|| जय श्री गिरिराज जी महाराज ||

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XII

← SOLUTIONS →

TEST No: 4 Differentiation & Continuity.

Ans 1 → Continuity at $x=0$

LHL $= \lim_{x \to 0^-} \left(\frac{1-\cos(4x)}{x^2} \right)$ put $x = 0-h = -h$ & $h \to 0$

$\therefore$ LHL $= \lim_{h \to 0} \left[\frac{1-\cos(-4h)}{(-h)^2} \right]$

$= \lim_{h \to 0} \left[\frac{1-\cos(4h)}{h^2} \right]$

$= \lim_{h \to 0} \left[\frac{2\sin^2(2h)}{h^2} \right]$

$= \lim_{h \to 0} \left[\frac{2\sin^2(2h)}{4h^2} \right] \times 4$

$= 2 \times 4$ --- $\left\{ \because \lim_{x \to 0} \left(\frac{\sin^2 x}{x^2} \right) = 1 \right\}$

$\boxed{LHL = 8}$

RHL $\lim_{x \to 0^+} \left(\frac{\sqrt{x}}{\sqrt{16+\sqrt{x}} - 4} \right)$ put $x = 0+h = h$ & $h \to 0$

RHL $= \lim_{h \to 0} \left(\frac{\sqrt{h}}{\sqrt{16+\sqrt{h}} - 4} \right)$

Rationalize RHL $= \lim_{h \to 0} \left(\frac{\sqrt{h}\,(\sqrt{16+\sqrt{h}} + 4)}{\cancel{16} + \sqrt{h} - \cancel{16}} \right)$

RHL $= \lim_{h \to 0} \left(\sqrt{16+\sqrt{h}} + 4 \right)$

$= \sqrt{16} + 4 = 4+4$

$\therefore \boxed{RHL = 8}$

Now $f(0) = a$

Since $f(x)$ is continuous at $x=0$

$\Rightarrow$ LHL = RHL = $f(0)$

$\Rightarrow 8 = 8 = a$

$\Rightarrow \boxed{a = 8}$ (none of these)

$\therefore$ option (D) is Answer.

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Continuity at $x = \pi/2$

Ans 2 → $LHL = \lim_{x \to \frac{\pi}{2}^-} \left(\frac{1 - \sin^3 x}{3\cos^2 x} \right)$ put $x = \frac{\pi}{2} - h$ & $h \to 0$

$\therefore LHL = \lim_{h \to 0} \left(\frac{1 - \sin^3(\frac{\pi}{2} - h)}{3\cos^2(\frac{\pi}{2} - h)} \right)$

$= \lim_{h \to 0} \left(\frac{1 - \cos^3 h}{3\sin^2 h} \right)$

$= \lim_{h \to 0} \left(\frac{\cancel{(1-\cos h)}(1 + \cos^2 h + \cos h)}{3\cancel{(1-\cos h)}(1+\cos h)} \right)$ ... $\{ a^3 - b^3$ ka formula

$= \frac{1+1+1}{3(1+1)} = \frac{1}{2}$

$\therefore \boxed{LHL = \frac{1}{2}}$

$RHL = \lim_{x \to \frac{\pi}{2}^+} \left(\frac{b(1-\sin x)}{(\pi - 2x)^2} \right)$ put $x = \frac{\pi}{2} + h$ & $h \to 0$

$RHL = \lim_{h \to 0} \left[\frac{b(1 - \sin(\frac{\pi}{2} + h))}{(\pi - 2(\frac{\pi}{2} + h))^2} \right]$

$= \lim_{h \to 0} \left[\frac{b(1 - \cos h)}{(\cancel{\pi} - \cancel{\pi} - 2h)^2} \right]$

$= \lim_{h \to 0} \left(\frac{b \cdot 2\sin^2(\frac{h}{2})}{4h^2} \right)$

$= \lim_{h \to 0} \left(\frac{b \cdot 2\sin^2(h/2)}{4\frac{h^2}{4} \times 4} \right)$

$= \frac{2b}{16}$ ... $\{ \because \lim_{x \to 0} \left(\frac{\sin^2 x}{x^2} \right) = 1 \}$

$\therefore \boxed{RHL = \frac{b}{8}}$

$f(\pi/2) = a$

Since $f(x)$ is continuous at $x = \pi/2$

$\Rightarrow \frac{1}{2} = \frac{b}{8} = a \Rightarrow \boxed{a = 1/2 \ \& \ b = 4}$

option (C) is Answer.

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Ans 3 → Since $f(x)$ is continuous in interval $[0, \pi]$
$\therefore f(x)$ is also continuous at $x = \pi/4$ & at $x = \pi/2$

Continuity at $x = \frac{\pi}{4}$:

$LHL = \lim_{x \to \frac{\pi}{4}^-} (x + a\sqrt{2}\sin x)$ put $x = \frac{\pi}{4} - h$ & $h \to 0$

$LHL = \lim_{h \to 0} \left(\frac{\pi}{4} - h + a\sqrt{2}\sin\left(\frac{\pi}{4} - h\right)\right)$

$= \left(\frac{\pi}{4} - 0 + a\sqrt{2}\sin\left(\frac{\pi}{4}\right)\right)$

$\therefore LHL = \frac{\pi}{2} + a\cancel{\sqrt{2}} \times \frac{1}{\cancel{\sqrt{2}}} \Rightarrow \boxed{LHL = \frac{\pi}{4} + a}$

$RHL = \lim_{x \to \frac{\pi}{4}^+} (2x\cot x + b)$ put $x = \frac{\pi}{4} + h$ & $h \to 0$

$RHL = \lim_{h \to 0} \left(2\left(\frac{\pi}{4} + h\right)\cot\left(\frac{\pi}{4} + h\right) + b\right)$

$= 2\left(\frac{\pi}{4}\right)\cot\left(\frac{\pi}{4}\right) + b$

$RHL = 2\left(\frac{\pi}{4}\right)(1) + b \Rightarrow \boxed{RHL = \frac{\pi}{2} + b}$

$f\left(\frac{\pi}{4}\right) = 2\left(\frac{\pi}{4}\right)\cot\left(\frac{\pi}{4}\right) + b$

$f\left(\frac{\pi}{4}\right) = \frac{\pi}{2} + b$

Since $f(x)$ is continuous at $x = \pi/4$

$\therefore \frac{\pi}{4} + a = \frac{\pi}{2} + b = \frac{\pi}{2} + b$

$\Rightarrow \boxed{a - b = \frac{\pi}{4}}$ ----- eq (1)

Continuity at $x = \pi/2$

$LHL = \lim_{x \to \frac{\pi}{2}^-} (2x\cot x + b)$ put $x = \frac{\pi}{2} - h$ & $h \to 0$

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$$LHL = \lim_{h \to 0} \left(2\left(\frac{\pi}{2}-h\right) \cot\left(\frac{\pi}{2}-h\right) + b \right)$$

$$= 2\left(\frac{\pi}{2}\right) \cot\left(\frac{\pi}{2}\right) + b$$

$\boxed{LHL = b}$ $\quad ---- \left\{ \because \cot\left(\frac{\pi}{2}\right) = 0 \right\}$

$$RHL = \lim_{x \to \frac{\pi}{2}^+} \left(a\cos(2x) - b\sin x \right)$$

put $x = \frac{\pi}{2} + h$ & $h \to 0$

$$RHL = \lim_{h \to 0} \left[a\cos(\pi + 2h) - b\sin\left(\frac{\pi}{2} + h\right) \right]$$

$$= a\cos(\pi) - b\sin\left(\frac{\pi}{2}\right)$$

$\boxed{RHL = -a - b}$ $\quad ---- \left\{ \because \cos(\pi) = -1 \right\}$

$$f\left(\frac{\pi}{2}\right) = a\cos\left(\cancel{2}\left(\frac{\pi}{\cancel{2}}\right)\right) - b\sin\left(\frac{\pi}{2}\right)$$

$$f(\pi/2) = -a - b$$

Since $f(x)$ is continuous at $x = \pi/2$

$\Rightarrow \quad b = -a - b = -a - b$

$\Rightarrow \quad \boxed{a = -2b} \quad ---- \text{eq}(2)$

Solving eq(1) & eq(2)

$\Rightarrow \quad -2b - b = \pi/4$

$\Rightarrow \quad b = -\frac{\pi}{12}$ and $a = \frac{\pi}{6}$

$\therefore$ option (D) is Answer.

Ans 4 → given: $f(x)$ is differentiable at $x = c$

$\therefore f(x)$ must be continous at $x = c$

Continuity at $x = c$

$$LHL = \lim_{x \to c^-} (x^2) \quad \text{put } x = c - h \text{ \& } h \to 0$$

$$LHL = \lim_{h \to 0} \left[(c-h)^2 \right]$$

$$\boxed{LHL = c^2}$$

$$RHL = \lim_{x \to c^+} (ax+b) \quad \text{put } x = c+h \ \& \ h \to 0$$

$$RHL = \lim_{h \to 0} \left(a(c+h) + b \right)$$

$$\boxed{RHL = ac+b}$$

$$f(c) = c^2$$

$$\Rightarrow LHL = RHL = f(c)$$

$$\Rightarrow c^2 = ac+b = c^2$$

$$\Rightarrow \boxed{c^2 = ac+b} \text{ ---- eq (i)}$$

Differentiability at $x = c$

$$LHD = \lim_{x \to c^-} \left(\frac{x^2 - f(c)}{x-c} \right)$$

$$= \lim_{x \to c^-} \left(\frac{x^2 - c^2}{x-c} \right)$$

$$= \lim_{x \to c^-} \left(\frac{(x+c)\cancel{(x-c)}}{\cancel{x-c}} \right)$$

put $x = c-h$ & $h \to 0$

$$LHD = \lim_{h \to c} \left(c-h+c \right)$$

$$\boxed{LHD = 2c}$$

$$RHD = \lim_{x \to c^+} \left(\frac{ax+b-f(c)}{x-c} \right)$$

$$RHD = \lim_{x \to c^+} \left(\frac{ax+b-c^2}{x-c} \right)$$

$$= \lim_{x \to c^+} \left(\frac{ax-ac}{x-c} \right) \quad \text{---} \left\{ \begin{array}{l} \text{from eq (i)} \\ c^2 = ac+b \\ \Rightarrow b - c^2 = -ac \end{array} \right.$$

$$= \lim_{x \to c^+} \left(\frac{a\cancel{(x-c)}}{\cancel{x-c}} \right)$$

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$\boxed{RHD = a}$

$\Rightarrow$ LHL = RHD

$2c = a \Rightarrow \boxed{a = 2c}$ put in eq (1)

$c^2 = (2c)c + b \Rightarrow \boxed{b = -c^2}$

$\therefore$ Option (c) is Ans

Qn. 5 $\rightarrow$ equation of curve $y = 12(x+1)(x-2)$; $x \in [-1,2]$

solution we have $y = 12(x+1)(x-2)$

(or) $y = 12(x^2 - x - 2)$

(·) given function is a polynomial function, which is every continuous $\therefore f(x)$ is everywhere continuous on $[-1,2]$

(·) Diff w.r.t x

$$\frac{dy}{dx} = 12(2x-1)$$

Clearly $f'(x)$ exists for all $x \in (-1,2)$

$\therefore f(x)$ is differentiable in $(-1,2)$

(·) $f(-1) = 12(1+1-2) = 0$

$f(2) = 12(4-2-2) = 0$

$\therefore f(-1) = f(2)$

the three conditions of Rolles theorem are satisfied then there exists at least one value $c \in (-1,2)$ such that

$f'(c) = 0$

$\rightarrow 12(2c-1) = 0$

$\rightarrow c = 1/2$

Now $f(c) = f(1/2) = 12(\frac{1}{2}+1)(\frac{1}{2}-2) = 12(\frac{3}{2})(-\frac{3}{2}) = -27$

$\therefore$ Point on the curve, at which tangent is parallel X-axis is $(\frac{1}{2}, -27)$ Ans

$\therefore$ option (B) is Ans

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Ans 6 → given $y=(x-4)^2$; Chord end points $(4,0)$ & $(5,1)$

let $f(x)=(x-4)^2$; here $a=4$, $b=5$

(·) Given function is polynomial function, which is everywhere continuous $\therefore f(x)$ is continuous in $[4,5]$

(·) Diff wrt x

$$f'(x)=2(x-4)$$

clearly $f'(x)$ exists for all $x\in[4,5)$

$\therefore f(x)$ is differentiable in $(4,5)$

(·) The two conditions of L-MV are satisfied, then there exists ~~for all~~ atleast one value $c\in(4,5)$ such that

$$f'(c)=\frac{f(5)-f(4)}{5-4}$$

$$\Rightarrow 2(c-4)=\frac{(5-4)^2-(4-4)^2}{1}$$

$$\Rightarrow 2c-8=1-0=1$$

$$2c=9 \Rightarrow \boxed{c=9/2}$$

Now $f(c)=f\left(\frac{9}{2}\right)=\left(\frac{9}{2}-4\right)^2=\left(\frac{1}{2}\right)^2=\frac{1}{4}$

$\therefore$ points on the curve $\left(\frac{9}{2},\frac{1}{4}\right)$

$\therefore$ option (D) is the answer

Ans 7 → let $y=\log\left[\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)\right]$

Diff wrt x

$$\frac{dy}{dx}=\frac{1}{\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)}\cdot\sec^2\left(\frac{\pi}{4}+\frac{x}{2}\right)\cdot\left(\frac{1}{2}\right)$$

$$=\frac{\frac{1}{\cos^2\left(\frac{\pi}{4}+\frac{x}{2}\right)}}{\frac{\sin\left(\frac{\pi}{4}+\frac{x}{2}\right)}{\cos\left(\frac{\pi}{4}+\frac{x}{2}\right)}}\cdot\frac{1}{2}$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{2\sin\left(\frac{\pi}{4}+\frac{x}{2}\right)\cos\left(\frac{\pi}{4}+\frac{x}{2}\right)}$$

$$\frac{dy}{dx} = \frac{1}{\sin\left(\frac{\pi}{2}+x\right)} \quad \text{-----} \{2\sin\theta\cos\theta = \sin(2\theta)\}$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{\cos x} \quad \text{---} \{\sin(90+\theta) = \cos\theta\}$$

$$\Rightarrow \frac{dy}{dx} = \sec x$$

$\therefore$ option (B) is Ans

Ans 8 $\rightarrow$ $y = [x + \sqrt{x^2+a^2}]^n$

Diff wrt x

$$\frac{dy}{dx} = n\left(x+\sqrt{x^2+a^2}\right)^{n-1} \cdot \left(1 + \frac{2x}{2\sqrt{x^2+a^2}}\right)$$

$$= n\left(x+\sqrt{x^2+a^2}\right)^{n-1} \cdot \frac{\left(\sqrt{x^2+a^2}+x\right)}{\sqrt{x^2+a^2}}$$

$$= \frac{n\left(x+\sqrt{x^2+a^2}\right)^n}{\sqrt{x^2+a^2}}$$

$$\frac{dy}{dx} = \frac{ny}{\sqrt{x^2+a^2}}$$

$\therefore$ option (D) is Ans

Ans 9 $\rightarrow$ let $y = \log\sqrt{\frac{1+\tan x}{1-\tan x}}$

$$y = \frac{1}{2}\left[\log(1+\tan x) - \log(1-\tan x)\right]$$

Diff wrt x

$$\frac{dy}{dx} = \frac{1}{2}\left[\frac{\sec^2 x}{1+\tan x} - \frac{(-\sec^2 x)}{1-\tan x}\right]$$

$$= \frac{1}{2}\sec^2 x\left[\frac{1}{1+\tan x} + \frac{1}{1-\tan x}\right]$$

$$= \frac{1}{\cancel{2}}\sec^2 x\left(\frac{\cancel{2}}{1-\tan^2 x}\right)$$

$$\frac{dy}{dx} = \frac{1+\tan^2 x}{1-\tan^2 x}$$

$$\frac{dy}{dx} = \frac{1}{\cos(2x)}$$

$$\frac{dy}{dx} = \sec(2x)$$ $\therefore$ option (C) is Ans

Ans. 10 → $y = \tan^{-1}\left(\frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}\right)$; $-1 < x < 1$

Divide by $\sqrt{1+x^2}$

$$y = \tan^{-1}\left(\frac{1+\sqrt{\frac{1-x^2}{1+x^2}}}{1-\sqrt{\frac{1-x^2}{1+x^2}}}\right)$$

$$y = \tan^{-1}(1) + \tan^{-1}\left(\sqrt{\frac{1-x^2}{1+x^2}}\right)$$

put $x^2 = \cos(2\theta)$

$$y = \frac{\pi}{4} + \tan^{-1}\sqrt{\frac{1-\cos(2\theta)}{1+\cos(2\theta)}}$$

$$y = \frac{\pi}{4} + \tan^{-1}\sqrt{\frac{\cancel{2}\sin^2\theta}{\cancel{2}\cos^2\theta}}$$

$$y = \frac{\pi}{4} + \tan^{-1}|\tan\theta|$$

$$y = \frac{\pi}{4} + \tan^{-1}(\tan\theta)$$

$$y = \frac{\pi}{4} + \theta$$

$\left\{ \begin{array}{l} -1 < x < 1 \\ 0 < x^2 < 1 \\ 0 < \cos(2\theta) < 1 \\ 0 < 2\theta < \frac{\pi}{2} \\ 0 < \theta < \frac{\pi}{4} \end{array} \right\}$

Ist quad and within Range

replace θ

$$y = \frac{\pi}{4} + \frac{1}{2}\cos^{-1}(x^2)$$

Diff w.r.t x

$$\frac{dy}{dx} = 0 - \frac{1}{\cancel{2}}\cdot\frac{1}{\sqrt{1-x^4}}\cdot(\cancel{2}x)$$

$$\frac{dy}{dx} = \frac{-x}{\sqrt{1-x^4}}$$

$\therefore$ option (A) is Ans

Qn. 11 → Let $y = \cot^{-1}\left(\frac{\sqrt{1+\sin x} - \sqrt{1-\sin x}}{\sqrt{1+\sin x} + \sqrt{1-\sin x}}\right)$; $0 < x < \frac{\pi}{2}$

$\Rightarrow y = \tan^{-1}\left(\frac{\sqrt{1+\sin x} + \sqrt{1-\sin x}}{\sqrt{1+\sin x} - \sqrt{1-\sin x}}\right)$ ..... $\left\{\tan^{-1}\left(\frac{1}{x}\right) = \cot^{-1} x\right\}$

Divide by $\sqrt{1+\sin x}$

$\Rightarrow y = \tan^{-1}\left(\frac{1 + \sqrt{\frac{1-\sin x}{1+\sin x}}}{1 - \sqrt{\frac{1-\sin x}{1+\sin x}}}\right)$

$\Rightarrow y = \tan^{-1}(1) + \tan^{-1}\left(\sqrt{\frac{1-\sin x}{1+\sin x}}\right)$

$\Rightarrow y = \tan^{-1}(1) + \tan^{-1}\left(\sqrt{\frac{1-\cos(\frac{\pi}{2}-x)}{1+\cos(\frac{\pi}{2}-x)}}\right)$

$\Rightarrow y = \frac{\pi}{4} + \tan^{-1}\sqrt{\frac{\cancel{2}\sin^2(\frac{\pi}{4}-\frac{x}{2})}{\cancel{2}\cos^2(\frac{\pi}{4}-\frac{x}{2})}}$

$\Rightarrow y = \frac{\pi}{4} + \tan^{-1}\sqrt{\tan^2(\frac{\pi}{4}-\frac{x}{2})}$

$\Rightarrow y = \frac{\pi}{4} + \tan^{-1}\left|\tan(\frac{\pi}{4}-\frac{x}{2})\right|$

$\Rightarrow y = \frac{\pi}{4} + \tan^{-1}\left(\tan(\frac{\pi}{4}-\frac{x}{2})\right)$

$\Rightarrow y = \frac{\pi}{4} + \frac{\pi}{4} - \frac{x}{2}$

$$\left\{\begin{array}{l} 0 < x < \frac{\pi}{2} \\ 0 > -x > -\frac{\pi}{2} \\ 0 > -\frac{x}{2} > -\frac{\pi}{4} \\ \frac{\pi}{4} > (\frac{\pi}{4}-\frac{x}{2}) > 0 \end{array}\right.$$

or $0 < (\frac{\pi}{4}-\frac{x}{2}) < \frac{\pi}{4}$

Ist Quad and within Range

diff w.r.t x

$\frac{dy}{dx} = -\frac{1}{2}$

$\therefore$ option (c) is the answer.

Qn. 12 → given $\sqrt{1-x^6} + \sqrt{1-y^6} = a(x^3-y^3)$

put $x^3 = \sin A$ and $y^3 = \sin B$

$\therefore \sqrt{1-\sin^2 A} + \sqrt{1-\sin^2 B} = a(\sin A - \sin B)$

$\Rightarrow \cos A + \cos B = a(\sin A - \sin B)$

$\Rightarrow \cancel{2\cos\left(\frac{A+B}{2}\right)} \cdot \cos\left(\frac{A-B}{2}\right) = a \cdot \cancel{2\cos\left(\frac{A+B}{2}\right)} \sin\left(\frac{A-B}{2}\right)$

$\Rightarrow \cot\left(\frac{A-B}{2}\right) = a$

$\Rightarrow \frac{A-B}{2} = \cot^{-1} a$

$\Rightarrow A - B = 2\cot^{-1} a$

replace A and B

$\Rightarrow \sin^{-1}(x^3) - \sin^{-1}(y^3) = 2\cot^{-1} a$

Diff w.r.t x

$\frac{1}{\sqrt{1-x^6}} \cdot 3x^2 - \frac{1}{\sqrt{1-y^6}} \cdot 3y^2 \cdot \frac{dy}{dx} = 0$

$\Rightarrow \cancel{3}y^2 \frac{dy}{dx} \cdot \frac{1}{\sqrt{1-y^6}} = \frac{\cancel{3}x^2}{\sqrt{1-x^6}}$

$\Rightarrow \frac{dy}{dx} = \frac{x^2}{y^2}\sqrt{\frac{1-y^6}{1-x^6}}$

$\therefore$ option (B) is the answer

Ans 13 → given $x^m \cdot y^n = (x+y)^{m+n}$

taking log on both sides

$m\log x + n\log y = (m+n)\cdot\log(x+y)$

Diff. w.r.t x

$\frac{m}{x} + \frac{n}{y}\cdot\frac{dy}{dx} = (m+n)\cdot\frac{1}{x+y}\left(1+\frac{dy}{dx}\right)$

$\Rightarrow \frac{dy}{dx}\left(\frac{n}{y} - \frac{m+n}{x+y}\right) = \frac{m+n}{x+y} - \frac{m}{x}$

$\Rightarrow \frac{dy}{dx}\left(\frac{nx + \cancel{ny} - my - \cancel{ny}}{y(\cancel{x+y})}\right) = \frac{\cancel{mx} + nx - \cancel{mx} - my}{x(\cancel{x+y})}$

$\Rightarrow \frac{dy}{dx}\left(\frac{\cancel{nx-my}}{y}\right) = \frac{\cancel{nx-my}}{x}$

$\Rightarrow \frac{dy}{dx} = \frac{y}{x}$

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Now $\left(\frac{dy}{dx}\right)_{(4,2)} = \frac{2}{4} = \frac{1}{2}$ ∴ option (B) is Ans

Qn. 14 →

$$y = (\tan x)^{(\tan x)^{\cdots \infty}}$$

$$\Rightarrow y = (\tan x)^y \quad \text{-----} (i)$$

taking log on both sides

$$\log y = y \log(\tan x)$$

Diff w.r.t. x

$$\frac{1}{y} \cdot \frac{dy}{dx} = y \cdot \frac{1}{\tan x} \cdot \sec^2 x + \log(\tan x) \cdot \frac{dy}{dx}$$

$$\Rightarrow \frac{dy}{dx}\left(\frac{1}{y} - \log(\tan x)\right) = \frac{y \sec^2 x}{\tan x}$$

$$\Rightarrow \frac{dy}{dx}\left(\frac{1 - y\log(\tan x)}{y}\right) = \frac{y \sec^2 x}{\tan x}$$

$$\Rightarrow \frac{dy}{dx} = \frac{y^2 \sec^2 x}{\tan x\,(1 - y\log(\tan x))}$$

when $x = \frac{\pi}{4}$

$$\left(\frac{dy}{dx}\right)_{x=\frac{\pi}{4}} = \frac{y^2 \sec^2\left(\frac{\pi}{4}\right)}{\tan\left(\frac{\pi}{4}\right)\left(1 - y\log\left(\tan\frac{\pi}{4}\right)\right)}$$

$$= \frac{y^2(2)}{1(1 - y(0))} \quad \cdots \{\text{since } \log(1) = 0\}$$

$$\left(\frac{dy}{dx}\right)_{x=\pi/4} = 2y$$

from eq (i) when $x = \frac{\pi}{4} \Rightarrow y = \left(\tan\frac{\pi}{4}\right)^y = (1)^y = 1$

$$\therefore y = 1$$

$$\Rightarrow \left(\frac{dy}{dx}\right)_{x=\frac{\pi}{4}} = 2(1) = 2$$

∴ option (D) is the Ans

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Qn. 15 → $x = \sqrt{a^{\sin^{-1}t}}$; $y = \sqrt{a^{\cos^{-1}t}}$

Diff. wrt t

$\frac{dx}{dt} = \frac{1}{2\sqrt{a^{\sin^{-1}t}}} \cdot a^{\sin^{-1}t} \cdot \frac{\log a}{\sqrt{1-t^2}}$

$\frac{dx}{dt} = \frac{\sqrt{a^{\sin^{-1}t}} \cdot \log a}{2\sqrt{1-t^2}}$

$\Rightarrow \frac{dx}{dt} = \frac{x \log a}{2\sqrt{1-t^2}}$

$\frac{dy}{dt} = \frac{-1}{2\sqrt{a^{\cos^{-1}t}}} \cdot a^{\cos^{-1}t} \cdot \frac{\log a}{\sqrt{1-t^2}}$

$\frac{dy}{dt} = \frac{-\sqrt{a^{\cos^{-1}t}} \cdot \log a}{2\sqrt{1-t^2}}$

$\frac{dy}{dt} = \frac{-y \log a}{2\sqrt{1-t^2}}$

Now $\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{\frac{-y\log a}{2\sqrt{1-t^2}}}{\frac{x\log a}{2\sqrt{1-t^2}}} = \frac{-y}{x}$

$\therefore$ option (C) is the Ans

Qn 16 → let $u = \cos^{-1}(4x^3 - 3x)$

Put $x = \cos\theta$

$\therefore u = \cos^{-1}(4\cos^3\theta - 3\cos\theta)$

$\Rightarrow u = \cos^{-1}(\cos(3\theta))$

$\Rightarrow u = 3\theta$

$\Rightarrow u = 3\cos^{-1}x$

Diff. wrt x

$\frac{du}{dx} = \frac{-3}{\sqrt{1-x^2}}$

let $v = \tan^{-1}\frac{\sqrt{1-x^2}}{x}$

Put $x = \cos\theta$

$v = \tan^{-1}\frac{\sqrt{1-\cos^2\theta}}{\cos\theta}$

$v = \tan^{-1}\left(\frac{\sin\theta}{\cos\theta}\right)$

$v = \tan^{-1}(\tan\theta)$

$v = \theta$

$v = \cos^{-1}x$

Diff $\frac{dv}{dx} = \frac{-1}{\sqrt{1-x^2}}$

Now $\frac{dy}{dx} = \frac{du/dx}{dv/dx}$

$= \frac{\frac{-3}{\sqrt{1-x^2}}}{\frac{-1}{\sqrt{1-x^2}}} = 3$ $\therefore$ option (D) is Ans

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Qn 17 → $y = e^{a\cos^{-1}x}$ ....(1)

taking log

$\Rightarrow \log y = a\cos^{-1}x . \log e$

$\Rightarrow \log y = a\cos^{-1}x$ .... $\{\log e = 1\}$

Diff. w.r.t. x

$\frac{1}{y} . \frac{dy}{dx} = \frac{-a}{\sqrt{1-x^2}}$

$\Rightarrow \frac{dy}{dx} = \frac{-ay}{\sqrt{1-x^2}}$

$\Rightarrow \sqrt{1-x^2}\frac{dy}{dx} = -ay$ ...(2)

Diff. w.r.t. x

$\sqrt{1-x^2} . \frac{d^2y}{dx^2} + \frac{dy}{dx} . \frac{1}{\cancel{2}\sqrt{1-x^2}}(-\cancel{2}x) = -a\frac{dy}{dx}$

L.C.M

$(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = -a\frac{dy}{dx}\sqrt{1-x^2}$

$\Rightarrow (1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = -a(-ay)$ --- {from eq (2)}

$\Rightarrow (1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - a^2y = 0$ ∴ option (C) is Ans

Qn 18 → $x = 2\cos t - \cos(2t)$

$y = 2\sin t - \sin(2t)$

Diff w.r.t. t

$\frac{dx}{dt} = -2\sin t + 2\sin(2t)$

$\frac{dy}{dt} = 2\cos t - 2\cos(2t)$

$\frac{dy}{dx} = \frac{2\cos t - 2\cos(2t)}{2\sin(2t) - 2\sin t}$

CLASSTIME

$$\frac{dy}{dx} = \frac{\cos t - \cos(2t)}{\sin(2t) - \sin t}$$

$$\frac{dy}{dx} = \frac{-\cancel{2}\sin\left(\frac{3t}{2}\right) \cdot \cancel{\sin\left(-\frac{t}{2}\right)}}{\cancel{2}\cos\left(\frac{3t}{2}\right) \cdot \cancel{\sin\left(\frac{t}{2}\right)}}$$

$$\frac{dy}{dx} = \tan\left(\frac{3t}{2}\right)$$

Diff wrt x

$$\frac{d^2y}{dx^2} = \sec^2\left(\frac{3t}{2}\right) \cdot \frac{3}{2} \frac{dt}{dx}$$

$$= \sec^2\left(\frac{3t}{2}\right) \cdot \frac{3}{2} \cdot \frac{1}{(2\sin(2t) - 2\sin t)}$$

$$\left(\frac{d^2y}{dx^2}\right)_{t=\frac{\pi}{2}} = \sec^2\left(\frac{3\pi}{4}\right) \cdot \frac{3}{2} \cdot \frac{1}{2\sin(\pi) - 2\sin\frac{\pi}{2}}$$

$$= (-\sqrt{2})^2 \cdot \frac{3}{2} \cdot \frac{1}{2(0) - 2(1)}$$

$$= \cancel{2} \times \frac{3}{\cancel{2}} \times \left(\frac{1}{-2}\right)$$

$$= -\frac{3}{2}$$ $\therefore$ option (A) is Ans

Qn 19 → $x = a(\cos\theta + \theta\sin\theta)$; $y = a(\sin\theta - \theta\cos\theta)$

Diff wrt θ

$$\frac{dx}{d\theta} = a(\cancel{-\sin\theta} + \theta\cos\theta \cancel{+\sin\theta})$$

$$\frac{dx}{d\theta} = a\theta\cos\theta$$

$$\frac{dy}{d\theta} = a(\cos\theta - (-\theta\sin\theta + \cos\theta))$$

$$\frac{dy}{d\theta} = a\theta\sin\theta$$

$$\frac{dy}{dx} = \frac{a\theta\sin\theta}{a\theta\cos\theta} = \tan\theta$$

Diff wrt x

$$\frac{d^2y}{dx^2} = \sec^2\theta \cdot \frac{d\theta}{dx}$$

$$= \sec^2\theta \cdot \frac{1}{a\theta\cos\theta}$$

$$\frac{d^2y}{dx^2} = \frac{1}{a\theta\cos^3\theta}$$

$$\left(\frac{d^2y}{dx^2}\right)_{\theta=\pi/3} = \frac{1}{a\left(\frac{\pi}{3}\right)\cos^3\left(\frac{\pi}{3}\right)}$$

$$= \frac{1}{a\left(\frac{\pi}{3}\right)\left(\frac{1}{8}\right)}$$

$$= \frac{24}{a\pi}$$ $\therefore$ option (C) is Ans

Qu. 20 → $f(x) = (1+x)(1+x^2)(1+x^4)(1+x^8)$

taking log

$$\log(f(x)) = \log(1+x) + \log(1+x^2) + \log(1+x^4) + \log(1+x^8)$$

Diff wrt x

$$\frac{1}{f(x)} f'(x) = \frac{1}{1+x} + \frac{2x}{1+x^2} + \frac{4x^3}{1+x^4} + \frac{8x^7}{1+x^8}$$

$$f'(x) = (1+x)(1+x^2)(1+x^4)(1+x^8)\left[\frac{1}{1+x} + \frac{2x}{1+x^2} + \frac{4x^3}{1+x^4} + \frac{8x^7}{1+x^8}\right]$$

put $x=1$

$$f'(1) = (2)(2)(2)(2)\left[\frac{1}{2} + \frac{2}{2} + \frac{4}{2} + \frac{8}{2}\right]$$

$$= 16\left(\frac{1}{2} + 1 + 2 + 4\right)$$

$$= 16 \times \frac{15}{2} = 8 \times 15 = 120$$ $\therefore$ option (D) is Ans