Topic Date

ULTIMATE MATHEMATICS

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CHAPTER: DETERMINANTS

CLASS: D-1

Symbol = $| \; |$ matrix = $[\;]$

only of a square matrix

is a number/value which is associated only to a square matrix

2×2 matrix

$$A = \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}$$

$|A| = 8 - 3 = 5$

$$A = \begin{bmatrix} 3 & -1 \\ 4 & -5 \end{bmatrix}$$

$|A| = -15 - (-4) = -15 + 4 = -11$

3×3 Matrix

$$A = \begin{matrix} (+) & (-) & (+) \end{matrix} \\ \begin{bmatrix} 3 & 2 & -1 \\ 4 & 2 & 1 \\ 3 & 2 & 6 \end{bmatrix}$$ { expanding along R_1 }

$|A| = 3(12-2) - 2(24-3) - 1(8-6)$

$= 30 - 42 - 2$

$= -14$ Ans

✓ Reason of Sign

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = \begin{vmatrix} (-1)^{1+1} & (-1)^{1+2} & (-1)^{1+3} \\ (-1)^{2+1} & (-1)^{2+2} & (-1)^{1+3} \end{vmatrix}$$

$$= \begin{vmatrix} + & - & + \\ - & + & - \\ + & - & + \end{vmatrix}$$

✓ expanding along C_2

$$A = \begin{bmatrix} 3 & 2 & -1 \\ 4 & 2 & 1 \\ 3 & 2 & 6 \end{bmatrix}$$

$$|A| = -2(24-3) + 2(18+3) - 2(3+4)$$

$$= -42 + 42 - 14$$

$$= -14 \neq 0$$

✓ Singular Matrix

Matrix A is said to be Singular if $\boxed{|A| = 0}$

✓ If $\begin{vmatrix} x & 1 \\ 3 & x \end{vmatrix} = \begin{vmatrix} 2 & -6 \\ 4 & 1 \end{vmatrix}$

find value of x

$\Rightarrow x^2 - 3 = 2 + 24$

$\Rightarrow x^2 = 29 \Rightarrow \boxed{x = \pm\sqrt{29}}$

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Application of determinants

1) Area of Triangle

$A(x_1, y_1)$ $B(x_2, y_2)$ $C(x_3, y_3)$

$$\text{Area} = \frac{1}{2}\left|\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}\right| \text{ sq. units}$$

→ Mod

Collinearity of three points

If Area of triangle = 0

Equation of a line passing through two points

$A(x_1, y_1)$ $B(x_2, y_2)$

$$\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} = 0$$ expand

$ax + by + c = 0$ ✓

If given $\Delta = \begin{vmatrix} 1 & \sin\theta & 1 \\ -\sin\theta & 1 & \sin\theta \\ -1 & -\sin\theta & 1 \end{vmatrix}$ (+) (−) (+)

Find Max value of Δ

Find Min value of Δ

Range of Δ

→ expand

$\Delta = 1(1+\sin^2\theta) - \sin\theta(-\sin\theta + \sin\theta) + 1(\sin^2\theta + 1)$

$\Delta = 2(1+\sin^2\theta)$ ✓

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$\Delta = 2(1+\sin^2\theta)$

We have $-1 \le \sin\theta \le 1$

$0 \le \sin^2\theta \le 1$

$\Rightarrow 1 \le 1+\sin^2\theta \le 2$

$\Rightarrow 2 \le 2(1+\sin^2\theta) \le 4$

$\Rightarrow 2 \le \Delta \le 4$

$\Delta \in [2,4]$

Max value = 4

Min value = 2

simple //

(i) $2 \le x \le 5$

$4 \le x^2 \le 25$

(i) $-5 \le x \le +7$

(X) $25 \le x^2 \le 49$

$0 \le x^2 \le 49$

(i) $-5 \le x \le 2$

$0 \le x^2 \le 25$

(i) $-5 \le x \le -2$

$4 \le x^2 \le 25$

Q15

Area = 35 sq. units

$A(2,-6)$ $B(5,4)$, $C(k,4)$

(+) (−) (+)

$$35 = \frac{1}{2}\begin{vmatrix} 2 & -6 & 1 \\ 5 & 4 & 1 \\ k & 4 & 1 \end{vmatrix} \rightarrow \text{Mod}$$

$$\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} = 0$$

$$70 = \left| 2(0) + 6(5-k) + 1(20-4k) \right|$$

$$70 = \left| 30 - 6k + 20 - 4k \right|$$

$$70 = \left| 50 - 10k \right|$$

$\Rightarrow \pm 70 = 50 - 10k$

$k = \rightarrow$

$k = \rightarrow$

Q.No. 10 → evaluate $\begin{vmatrix} \sin(10^\circ) & -\cos(10^\circ) \\ \sin(80^\circ) & \cos(80^\circ) \end{vmatrix}$

Q.No. 11 → For what value of x the matrix $A = \begin{bmatrix} x-1 & 1 & 1 \\ 1 & x-1 & 1 \\ 1 & 1 & x-1 \end{bmatrix}$ is singular?

Q.No. 12 → let $\begin{vmatrix} 3 & y \\ x & 1 \end{vmatrix} = \begin{vmatrix} 3 & 2 \\ 4 & 1 \end{vmatrix}$ Find possible values of x and y if x and y are natural numbers

Q.No. 13 → let $A = \begin{bmatrix} 1 & \sin\theta & 1 \\ -\sin\theta & 1 & \sin\theta \\ -1 & -\sin\theta & 1 \end{bmatrix}$ where $0 \le \theta \le 2\pi$ then Show that $\det(A) \in [2, 4]$

Q.No. 14 → Find the area of the triangle whose vertices are
(i) A(2,7) B(1,1) C(10,8)
(ii) A(-2,-3) B(3,2) C(-1,-8)

Q.No. 15 → If area of triangle is 35 square units with vertices (2,-6), (5,4), (k,4) find value of k

Q.No. 16 → If the points (2,-3), (1,-1), (0,4) are collinear. find value of λ

Q.No. 17 → Find equation of line passing through (1,2) & (3,6) using determinants

Q.No. 18 → If points $(k, 2-2k)$, $(-k+1, 2k)$ & $(-4-k, 6-2k)$ are collinear. find value of k

→ ANSWERS ←

(2) $x = \pm 6$	(7) $x = -1$	(10) 1	(14) $\frac{49}{2}$, 15
(3) 0	(8) $x = 0$, $x = \frac{1}{4}(3 \pm \sqrt{205})$	(11) $x = -1, 2$	(15) $k = 12, -2$
(5) 1		(12) (1,8) (8,1) (2,4) (4,2)	(16) $\lambda = 10/7$
(6) -37	(9) $a^2+b^2+c^2+d^2$		(17) $y = 2x$
			(18) $k = -1, 4/2$

!! जय श्री गिरिराज जी महाराज !!

Q No 1 → If $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 4 \end{bmatrix}$ then show that $|3A| = 27|A|$

Q No 2 → If $\begin{vmatrix} x & 2 \\ 18 & x \end{vmatrix} = \begin{vmatrix} 6 & 2 \\ 18 & 6 \end{vmatrix}$, then find value of x

Q No 3 → Evaluate $\Delta = \begin{vmatrix} 0 & \sin\alpha & -\cos\alpha \\ -\sin\alpha & 0 & \sin\beta \\ \cos\alpha & -\sin\beta & 0 \end{vmatrix}$

Q No 4 → Prove that the determinant $\begin{vmatrix} x & \sin\theta & \cos\theta \\ -\sin\theta & -x & 1 \\ \cos\theta & 1 & x \end{vmatrix}$ is independent of θ (Ans: $-x^3$)

Q No 5 → Evaluate $\begin{vmatrix} \cos\alpha\cos\beta & \cos\alpha\sin\beta & -\sin\alpha \\ -\sin\beta & \cos\beta & 0 \\ \sin\alpha\cos\beta & \sin\alpha\sin\beta & \cos\alpha \end{vmatrix}$

Q No 6 → Expand $\Delta = \begin{vmatrix} 2 & 3 & -2 \\ 1 & 2 & 3 \\ -2 & 1 & -3 \end{vmatrix}$ by expanding along I^{st} column and 3^{rd} Row

Q No 7 → For what value of x the matrix $A = \begin{bmatrix} 1 & -2 & 3 \\ 1 & 2 & 1 \\ x & 2 & -3 \end{bmatrix}$ is Singular?

Q No 8 → Determine the value of x for which the matrix $A = \begin{bmatrix} x+1 & -3 & 4 \\ -5 & x+2 & 2 \\ 4 & 1 & x-6 \end{bmatrix}$ is singular?

Q No 9 → Evaluate $\begin{vmatrix} a+ib & c+id \\ -c+id & a-ib \end{vmatrix}$ $(i^2 = -1)$