D-2

Solution of Determinants Class: 2 (D-2)

ULTIMATE MATHEMATICS

Ans 1 → (i) Do yourself

(ii) $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & \sin\alpha \\ 0 & \sin\alpha & -\cos\alpha \end{bmatrix}$

$|A| = 1(-\cos^2\alpha - \sin^2\alpha)$

$|A| = -(\cos^2\alpha + \sin^2\alpha) = -1$

$C_{11} = -1$, $C_{12} = 0$ $C_{13} = 0$

$C_{21} = 0$ $C_{22} = -\cos\alpha$ $C_{23} = -\sin\alpha$

$C_{31} = 0$ $C_{32} = -\sin\alpha$ $C_{33} = \cos\alpha$

$A^{-1} = \frac{1}{|A|} \text{Adj} A = \frac{1}{-1}\begin{bmatrix} -1 & 0 & 0 \\ 0 & -\cos\alpha & -\sin\alpha \\ 0 & -\sin\alpha & \cos\alpha \end{bmatrix}$

$\Rightarrow A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & \sin\alpha \\ 0 & \sin\alpha & -\cos\alpha \end{bmatrix}$ Ans

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Ans 2 → (i) $A = \begin{bmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{bmatrix}$

$|A| = x(yz) = xyz$

$C_{11} = yz$, $C_{12} = 0$ $C_{13} = 0$

$C_{21} = 0$, $C_{22} = xz$ $C_{23} = 0$

$C_{31} = 0$ $C_{32} = 0$ $C_{33} = xy$

$\text{Adj} A = \begin{bmatrix} yz & 0 & 0 \\ 0 & xz & 0 \\ 0 & 0 & xy \end{bmatrix}$

$A^{-1} = \frac{1}{xyz}\begin{bmatrix} yz & 0 & 0 \\ 0 & xz & 0 \\ 0 & 0 & xy \end{bmatrix} = \begin{bmatrix} 1/x & 0 & 0 \\ 0 & 1/y & 0 \\ 0 & 0 & 1/z \end{bmatrix}$ Ans

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(ii) Do yourself

Ans 3(i) $A = \begin{bmatrix} 1 & -2 & 1 \\ -2 & 3 & 1 \\ 1 & 1 & 5 \end{bmatrix}$

$|A| = 1(14) + 2(-11) + 1(-5)$

$|A| = 14 - 22 - 5 = -13$

$C_{11} = 14$, $C_{12} = 11$ $C_{13} = -5$

$C_{21} = 11$ $C_{22} = 4$ $C_{23} = -3$

$C_{31} = -5$ $C_{32} = -3$ $C_{33} = -1$

$\text{Adj } A = \begin{bmatrix} 14 & 11 & -5 \\ 11 & 4 & -3 \\ -5 & -3 & -1 \end{bmatrix}$

$A^{-1} = \frac{-1}{13}\begin{bmatrix} 14 & 11 & -5 \\ 11 & 4 & -3 \\ -5 & -3 & -1 \end{bmatrix}$

Now we will find Inverse of Adj A (LHS)

$|\text{Adj } A| = 14(-13) - 11(-26) - 5(-13)$

$= -182 + 286 + 65 = 169$

Now Cofactors of Adj A

$C_{11} = -13$, $C_{12} = 26$, $C_{13} = -13$

$C_{21} = +26$ $C_{22} = -39$, $C_{23} = -13$

$C_{31} = -13$ $C_{32} = -13$ $C_{33} = -65$

$\text{Adj}(\text{Adj } A) = \begin{bmatrix} -13 & 26 & -13 \\ 26 & -39 & -13 \\ -13 & -13 & -65 \end{bmatrix}$

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Now $(\text{Adj}\,A)^{-1} = \dfrac{1}{|\text{Adj}\,A|} \cdot \text{Adj}(\text{Adj}\,A)$

$$= \frac{1}{169}\begin{bmatrix} -13 & 26 & -13 \\ 26 & -39 & -13 \\ -13 & -13 & -65 \end{bmatrix}$$

$$= \frac{-13}{169}\begin{pmatrix} 1 & -2 & 1 \\ -2 & 3 & 1 \\ 1 & 1 & 5 \end{pmatrix}$$

$$(\text{Adj}\,A)^{-1} = \frac{-1}{13}\begin{pmatrix} 1 & -2 & 1 \\ -2 & 3 & 1 \\ 1 & 1 & 5 \end{pmatrix}$$

Now we will find $\text{Adj}(A)^{-1}$

$$A^{-1} = \begin{bmatrix} \frac{-14}{13} & \frac{-11}{13} & \frac{5}{13} \\ \frac{-11}{13} & \frac{-4}{13} & \frac{3}{13} \\ 5/13 & 3/13 & 1/13 \end{bmatrix}$$

Its cofactors

$C_{11} = \frac{-4}{169} - \frac{9}{169} = \frac{-13}{169} = \frac{-1}{13}$; $C_{12} = \frac{2}{13}$, $C_{13} = \frac{-1}{13}$

$C_{21} = \frac{2}{13}$, $C_{22} = \frac{-3}{13}$, $C_{23} = \frac{-1}{13}$

$C_{31} = \frac{-1}{13}$, $C_{32} = \frac{-1}{13}$, $C_{33} = -\frac{5}{13}$

$$\therefore \text{Adj}\,A^{-1} = \begin{bmatrix} -1/13 & 2/13 & -1/13 \\ 2/13 & -3/13 & -1/13 \\ -1/13 & -1/13 & -5/13 \end{bmatrix}$$

$\text{Adj}(A)^{-1} = -\frac{1}{13}\begin{bmatrix} 1 & -2 & 1 \\ -2 & 3 & 1 \\ 1 & 1 & 5 \end{bmatrix}$ which is same as $(\text{Adj}\,A)^{-1}$

Proved

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(ii) To prove $(A^{-1})^{-1} = A$

we have (already find) $A^{-1} = \begin{bmatrix} -14/13 & -11/13 & 5/13 \\ -11/13 & -4/13 & 3/13 \\ 5/13 & 3/13 & 1/13 \end{bmatrix}$

$\text{Adj}(A^{-1})$ (already find) $= \begin{bmatrix} -\frac{1}{13} & 2/13 & -1/13 \\ 2/13 & -3/13 & -1/13 \\ -1/13 & -1/13 & -5/13 \end{bmatrix}$

$|A^{-1}| = \frac{-14}{13}\left(\frac{-4}{169} - \frac{9}{169}\right) + \frac{11}{13}\left(\frac{-11}{169} - \frac{15}{169}\right) + \frac{5}{13}\left(\frac{-33}{169} + \frac{20}{169}\right)$

$= \frac{-14}{13}\left(\frac{-13}{169}\right) + \frac{11}{13}\left(\frac{-26}{169}\right) + \frac{5}{13}\left(\frac{-13}{169}\right)$

$= \frac{+14}{169} - \frac{22}{169} - \frac{5}{169} = \frac{-13}{169} = \frac{-1}{13}$

$(A^{-1})^{-1} = \frac{1}{|A^{-1}|} \text{Adj}(A^{-1})$

$= 1/\left(\frac{-1}{13}\right) \begin{bmatrix} -1/13 & 2/13 & -1/13 \\ 2/13 & -3/13 & -1/13 \\ -1/13 & -1/13 & -5/13 \end{bmatrix}$

$= \begin{bmatrix} 1 & -2 & 1 \\ -2 & 3 & 1 \\ 1 & 1 & 5 \end{bmatrix} = A$

Hence proved

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Qn 4 Do yourself

Qn 5 → given $A^{-1} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$

$B = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$

$B^{-1} = \frac{1}{|B|} \text{Adj } B$

$|B| = 1(3) - 2(-1) - 2(2) = 3 + 2 - 4 = 1$

$\text{Adj } B = \begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix}$

$B^{-1} = \frac{1}{1} \begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix}$

To find $(AB)^{-1} = B^{-1} A^{-1}$

$= \begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix} \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$

$= \begin{bmatrix} 9 & -3 & 5 \\ -2 & 1 & 0 \\ 1 & 0 & 2 \end{bmatrix}$ Ans

Qn 6 → $A = \begin{bmatrix} 3 & 7 \\ 2 & 5 \end{bmatrix}$; $B = \begin{bmatrix} 6 & 8 \\ 7 & 9 \end{bmatrix}$

To prove $(AB)^{-1} = B^{-1} A^{-1}$

(i) $|A| = 15 - 14 = 1$

(ii) $\text{Adj } A = \begin{bmatrix} 5 & -7 \\ -2 & 3 \end{bmatrix}$

ULTIMATE MATHEMATICS

(·) $A^{-1} = \frac{1}{1}\begin{bmatrix} 5 & -7 \\ -2 & 3 \end{bmatrix} = \begin{pmatrix} 5 & -7 \\ -2 & 3 \end{pmatrix}$

(·) $|B| = 54 - 56 = -2$

(·) $\text{Adj}\,B = \begin{pmatrix} 9 & -8 \\ -7 & 6 \end{pmatrix}$

(·) $B^{-1} = -\frac{1}{2}\begin{bmatrix} 9 & -8 \\ -7 & 6 \end{bmatrix}$

(·) $AB = \begin{bmatrix} 3 & 7 \\ 2 & 5 \end{bmatrix}\begin{bmatrix} 6 & 8 \\ 7 & 9 \end{bmatrix} = \begin{bmatrix} 67 & 87 \\ 47 & 61 \end{bmatrix}$

(·) $|AB| = 67 \times 61 - 47 \times 87$
$= 4087 - 4089 = -2$

(·) $\text{Adj}(AB) = \begin{bmatrix} 61 & -87 \\ -47 & 67 \end{bmatrix}$

(·) $(AB)^{-1} = -\frac{1}{2}\begin{bmatrix} 61 & -87 \\ -47 & 67 \end{bmatrix}$

(·) $B^{-1}A^{-1} = -\frac{1}{2}\begin{bmatrix} 9 & -8 \\ -7 & 6 \end{bmatrix}\begin{bmatrix} 5 & -7 \\ -2 & 3 \end{bmatrix}$

$= -\frac{1}{2}\begin{bmatrix} 61 & -87 \\ -47 & 67 \end{bmatrix}$

Clearly LHS = RHS proved

QN 7 → Do yourself

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Ans. 8 → $A = \begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix}$

$|A| = -4 - 15 = -19$

$\text{Adj}\, A = \begin{bmatrix} -2 & -3 \\ -5 & 2 \end{bmatrix}$

$A^{-1} = \frac{-1}{19}\begin{bmatrix} -2 & -3 \\ -5 & 2 \end{bmatrix}$

$A^{-1} = \frac{1}{19}\begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix}$

$\boxed{A^{-1} = \frac{1}{19} A}$ Ans

Ans 9 → $A = \begin{bmatrix} 1 & \tan x \\ -\tan x & 1 \end{bmatrix}$

$A' = \begin{bmatrix} 1 & -\tan x \\ \tan x & 1 \end{bmatrix}$

$|A| = 1 + \tan^2 x$

$\text{Adj}\, A = \begin{bmatrix} 1 & -\tan x \\ \tan x & 1 \end{bmatrix}$

$A^{-1} = \frac{1}{1+\tan^2 x}\begin{bmatrix} 1 & -\tan x \\ \tan x & 1 \end{bmatrix}$

Taking LHS $A'A^{-1} = \begin{bmatrix} 1 & -\tan x \\ \tan x & 1 \end{bmatrix} \times \frac{1}{1+\tan^2 x}\begin{bmatrix} 1 & -\tan x \\ \tan x & 1 \end{bmatrix}$

$= \frac{1}{1+\tan^2 x}\begin{bmatrix} 1-\tan^2 x & -2\tan x \\ 2\tan x & 1-\tan^2 x \end{bmatrix}$

$= \begin{bmatrix} \frac{1-\tan^2 x}{1+\tan^2 x} & \frac{-2\tan x}{1+\tan^2 x} \\ \frac{2\tan x}{1+\tan^2 x} & \frac{1-\tan^2 x}{1+\tan^2 x} \end{bmatrix}$

$$= \begin{bmatrix} \cos(2x) & -\sin(2x) \\ \sin(2x) & \cos(2x) \end{bmatrix}$$

$= \underline{RHS}$ PROVED

Qn 10 → $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$

(i) Do yourself show $A^2 - 4A - 5I = 0$

(ii) we have $A^2 - 4A - 5I = 0$

pre-multiply by A^{-1}

$\Rightarrow A^{-1}A^2 - 4A^{-1}A - 5A^{-1}I = 0$

$\Rightarrow (A^{-1}A)A - 4A^{-1}A - 5A^{-1} = 0$

$\Rightarrow IA - 4I - 5A^{-1} = 0$

$\Rightarrow A - 4I - 5A^{-1} = 0$

$\Rightarrow 5A^{-1} = A - 4I$

$$\Rightarrow 5A^{-1} = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix} - \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix}$$

$$5A^{-1} = \begin{bmatrix} -3 & 2 & 2 \\ 2 & -3 & 2 \\ 2 & 2 & -3 \end{bmatrix}$$

$$\Rightarrow A^{-1} = \frac{1}{5}\begin{bmatrix} -3 & 2 & 2 \\ 2 & -3 & 2 \\ 2 & 2 & -3 \end{bmatrix}$$ Ans

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Qn 11 → there is a "correction" in LHS

It should be $x^2 - 3x - 7$

(i) To prove $A^2 - 3A - 7I = 0$

Do same as Qn No: 10

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Qn 13 → $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$

(i) Show $A^3 - 6A^2 + 9A - 4I = 0$ (Do yourself)

(ii) we have $A^3 - 6A^2 + 9A - 4I = 0$

Pre-multiply by A^{-1}

$\Rightarrow A^{-1}A^3 - 6A^{-1}A^2 + 9A^{-1}A - 4A^{-1}I = 0$

$\Rightarrow (A^{-1}A)A^2 - 6(A^{-1}A)A + 9A^{-1}A - 4A^{-1} = 0$

$\Rightarrow IA^2 - 6IA + 9I - 4A^{-1} = 0$

$\Rightarrow A^2 - 6A + 9I - 4A^{-1} = 0$

$\Rightarrow 4A^{-1} = A^2 - 6A + 9I$

$$\Rightarrow 4A^{-1} = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix} - 6\begin{pmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{pmatrix} + 9\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow 4A^{-1} = \begin{bmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{bmatrix} - \begin{bmatrix} 12 & -6 & 6 \\ -6 & 12 & -6 \\ 6 & -6 & 12 \end{bmatrix} + \begin{bmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{bmatrix}$$

$$= \begin{bmatrix} -6 & 1 & -1 \\ 1 & -6 & 1 \\ -1 & 1 & -6 \end{bmatrix} + \begin{pmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{pmatrix}$$

$$A^{-1} = \frac{1}{4}\begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & 1 \\ -1 & 1 & 3 \end{bmatrix}$$ Ans