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← ULTIMATE MATHEMATICS →

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DETERMINANTS : CLASS-2 (D-2)

(·) $|A| =$ number ✓

(·) Adjoint of a matrix
denoted by $Adj(A) = [\ \ ]$

(·) Cofactors

eg A $A = \begin{bmatrix} 3 & 2 & -1 \\ 1 & 2 & 4 \\ 0 & 1 & 3 \end{bmatrix}_{3\times 3}$

$C_{11} = 2$; $C_{12} = -3$ $C_{13} = 1$

$C_{21} = -7$ $C_{22} = 9$ $C_{23} = -3$

$C_{31} = 10$ $C_{32} = -13$ $C_{33} = 4$

(even → no change & odd → change)

(·) $Adj A = \begin{bmatrix} 2 & -7 & 10 \\ -3 & 9 & -13 \\ 1 & -3 & 4 \end{bmatrix}$

(·) $\boxed{2\times 2 \text{ Matrix}}$ $A = \begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix}$ → Sign change → interchange

$Adj A = \begin{bmatrix} 4 & 1 \\ -3 & 2 \end{bmatrix}$

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(·) Inverse of a Matrix

$A^{-1} = \frac{1}{|A|} . Adj A$; $|A| \neq 0$

Matrix A is Invertible when $|A| \neq 0$

(·) Properties of Inverse

(1) $AA^{-1} = I = A^{-1}A$

(2) $A^{-1}I = A^{-1} = IA^{-1}$

(3) (MCQ) $(AB)^{-1} = B^{-1}A^{-1}$

(4) $(A^{-1})^{-1} = A$

Qn 1 given $A = \begin{bmatrix} \quad \end{bmatrix}_{3\times3}$

(i) Show that $A^3 - 3A^2 + 4A + 7I = 0$
(ii) hence find A^{-1}

(·) we have $A^3 - 3A^2 + 4A + 7I = 0$

$A^{-1}A^3 - 3A^{-1}A^2 + 4A^{-1}A + 7A^{-1}I = 0$

$(A^{-1}A)A^2 - 3(A^{-1}A)A + 4(A^{-1}A) + 7A^{-1} = 0$

$IA^2 - 3IA + 4I + 7A^{-1} = 0$

$A^2 - 3A + 4I + 7A^{-1} = 0$

$\Rightarrow 7A^{-1} = -A^2 + 3A - 4I$

$\rightarrow 7A^{-1} = \begin{bmatrix} \quad \end{bmatrix}$

$\rightarrow A^{-1} = \frac{1}{7}\begin{bmatrix} \quad \end{bmatrix}$ Ans

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(1) $AB = C$ (only square matrix given)

$ABB^{-1} = CB^{-1}$ (post ~~Mult~~ Multiplication)

$AI = CB^{-1}$

$\boxed{A = CB^{-1}}$

(2) $BA = C$

$B^{-1}BA = B^{-1}C$ (Pre-mult)

$IA = B^{-1}C$

$\boxed{A = B^{-1}C}$

Qns $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ $B = \begin{bmatrix} 2 & 0 \\ 1 & 4 \end{bmatrix}$

Show $(AB)^{-1} = B^{-1}A^{-1}$

(1) (i) $AB = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 1 & 4 \end{bmatrix} = \begin{bmatrix} 4 & 8 \\ 10 & 16 \end{bmatrix}$

(2) $|AB| = 64 - 80 = -16$

(3) $\text{Adj}(AB) = \begin{bmatrix} 16 & -8 \\ -10 & 4 \end{bmatrix}$

(4) $(AB)^{-1} = \frac{-1}{16}\begin{bmatrix} 16 & -8 \\ -10 & 4 \end{bmatrix}$

(5) $|B| = 8$

(6) $\text{Adj}\,B = \begin{bmatrix} 4 & 0 \\ -1 & 2 \end{bmatrix}$

(7) $B^{-1} = \frac{1}{8}\begin{bmatrix} 4 & 0 \\ -1 & 2 \end{bmatrix}$

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Q.1 $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

(8) ✓ $|A| = 4 - 6 = -2$

(9) ✓ $\text{Adj } A = \begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}$

(10) ✓ $A^{-1} = -\frac{1}{2}\begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}$

(11) ✓ $B^{-1}A^{-1} = \frac{1}{8}\begin{bmatrix} 4 & 0 \\ -1 & 2 \end{bmatrix} \times \frac{-1}{2}\begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}$

$= -\frac{1}{16}\begin{bmatrix} 16 & -8 \\ -10 & 4 \end{bmatrix}$ ✓

$(AB)^{-1} = B^{-1}A^{-1}$

—x—

Q1.

Q15 of D-1

$[0.5] = 0$

$[1.2] = 1$

$[1] = 1$

$[-0.5] = -1$

$[-3.999] = -4$

$[-3] = -3$

$[1 \text{ to } 2] = 1$

Included excluded

$[-9 \text{ to } -8] = -9$

$[99 \text{ to } 100] = 99$

॥ जय श्री गिरिराज जी महाराज ॥

Q No. 1 → Find the inverse of the following matrices

(i) $A = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}$ (ii) $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & \sin\alpha \\ 0 & \sin\alpha & -\cos\alpha \end{bmatrix}$

Q No. 2 → Find the inverse of the following matrices

(i) $A = \begin{bmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{bmatrix}$ (ii) $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$

Q No. 3 → Let $A = \begin{bmatrix} 1 & -2 & 1 \\ -2 & 3 & 1 \\ 1 & 1 & 5 \end{bmatrix}$ verify that

NCERT

(i) $[\operatorname{Adj} A]^{-1} = \operatorname{Adj}(A^{-1})$ (ii) $(A^{-1})^{-1} = A$

Q No. 4 → Let $A = \begin{bmatrix} 1 & -1 & 2 \\ 3 & 0 & 2 \\ 1 & 0 & 3 \end{bmatrix}$ verify that $A(\operatorname{Adj} A) = (\operatorname{Adj} A)A = |A|\,I$

Q No. 5 → If $A^{-1} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$ & $B = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$ find $(AB)^{-1}$

Q No. 6 → Let $A = \begin{bmatrix} 3 & 7 \\ 2 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 6 & 8 \\ 7 & 9 \end{bmatrix}$ verify that

$(AB)^{-1} = B^{-1}A^{-1}$,

Q No. 7 → Let $A = \begin{bmatrix} 2 & 3 \\ 1 & -4 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -2 \\ -1 & 3 \end{bmatrix}$ verify that

$(AB)^{-1} = B^{-1}A^{-1}$

Q No. 8 → If $A = \begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix}$ write A^{-1} in terms of A

Q No. 9 → If $A = \begin{bmatrix} 1 & \tan x \\ -\tan x & 1 \end{bmatrix}$ show that $A'A^{-1} = \begin{bmatrix} \cos(2x) & -\sin(2x) \\ \sin(2x) & \cos(2x) \end{bmatrix}$

$\begin{bmatrix} \cos(2x) & -\sin(2x) \\ \sin(2x) & \cos(2x) \end{bmatrix}$

Qn. 10 → Show that $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$ satisfies the equation $A^2 - 4A - 5I = 0$ and Hence find A^{-1}

Qn. 11 → Show that $A = \begin{bmatrix} 5 & 3 \\ -1 & -2 \end{bmatrix}$ satisfies the equation $x^2 - 3x - 7 = 0$. Thus find A^{-1}

Qn. 12 → Show that $A = \begin{bmatrix} 1 & 0 & -2 \\ -2 & -1 & 2 \\ 3 & 4 & 1 \end{bmatrix}$ is a root of the polynomial $f(x) = x^3 - x^2 - 3x - 1$ Hence find A^{-1}

Qn. 13 → If $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$. Verify that $A^3 - 6A^2 + 9A - 4I = 0$ and hence find A^{-1}

Qn. 14 → If $A = \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix}$ find the numbers a and b such that $A^2 + aA + bI = 0$. Hence find A^{-1}

Qn. 15 → If $-1 \le x < 0$, $0 \le y < 1$, $1 \le z < 2$ and [ ] is a greatest integer function. Find the value of the determinant $\begin{vmatrix} [x]+1 & [y] & [z] \\ [x] & [y]+1 & [z] \\ [x] & [y] & [z]+1 \end{vmatrix}$

1M

← ANSWERS →

(1) (i) $A^{-1} = \begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix}$

(ii) $A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & \sin\alpha \\ 0 & \sin\alpha & -\cos\alpha \end{bmatrix}$

(2) (i) $A^{-1} = \begin{bmatrix} x^{-1} & 0 & 0 \\ 0 & y^{-1} & 0 \\ 0 & 0 & z^{-1} \end{bmatrix}$

(ii) $A^{-1} = \frac{1}{5}\begin{bmatrix} -3 & 2 & 2 \\ 2 & -3 & 2 \\ 2 & 2 & -3 \end{bmatrix}$

(5) $\begin{bmatrix} 9 & -3 & 5 \\ -2 & 1 & 0 \\ 1 & 0 & 2 \end{bmatrix}$

(8) $\frac{1}{19} A$

(10) $A^{-1} = \frac{1}{5}\begin{bmatrix} -3 & 2 & 2 \\ 2 & -3 & 2 \\ 2 & 2 & -3 \end{bmatrix}$

(11) $A^{-1} = \frac{1}{7}\begin{bmatrix} 2 & 3 \\ -1 & -5 \end{bmatrix}$

(12) $A^{-1} = \begin{bmatrix} -9 & -8 & -2 \\ 8 & 7 & 2 \\ -5 & -4 & -1 \end{bmatrix}$

(14) $a = -4$, $b = 1$

$A^{-1} = \begin{bmatrix} 1 & -2 \\ -1 & 3 \end{bmatrix}$

(15) 1