SOLUTIONS: WORKSHEET NO: 5 (VECTORS)

Ans 1 Given $\beta = \pi/4$, $\gamma = \pi/2$; let $\alpha = \theta$

$l = \cos\alpha$; $m = \cos\pi/4 = 1/\sqrt{2}$; $n = \cos\frac{\pi}{2} = 0$

we have $l^2+m^2+n^2=1$

$\Rightarrow \cos^2\alpha + \frac{1}{2} + 0 = 1$

$\Rightarrow \cos^2\alpha = 1/2$

$\Rightarrow \cos\alpha = \pm\frac{1}{\sqrt{2}}$

$\Rightarrow \boxed{l = \pm\frac{1}{\sqrt{2}}}$

Given $|\vec{r}| = 3\sqrt{2}$

→ we know that l, m, n are the components of any unit vector

$\therefore \hat{r} = l\hat{i} + m\hat{j} + n\hat{k}$

$\Rightarrow \hat{r} = \pm\frac{1}{\sqrt{2}}\hat{i} + \frac{1}{\sqrt{2}}\hat{j} + 0\hat{k}$

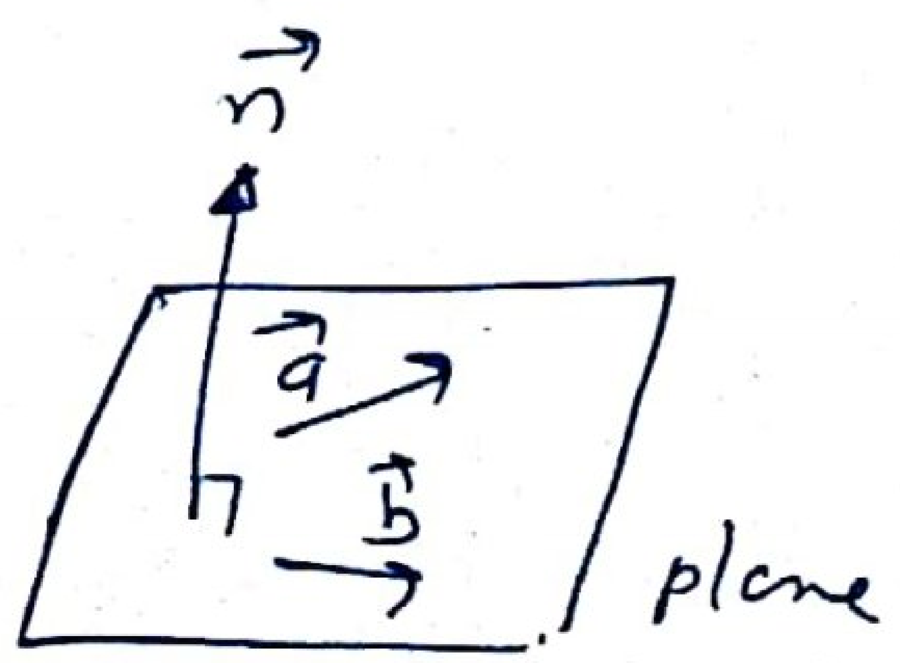
Now $\vec{r} = |\vec{r}|\hat{r}$ --- {vector = (Magnitude)(unit vector)}

$\Rightarrow \vec{r} = 3\sqrt{2}\left(\pm\frac{1}{\sqrt{2}}\hat{i} + \frac{1}{\sqrt{2}}\hat{j} + 0\hat{k}\right)$

$\Rightarrow \boxed{\vec{r} = \pm 3\hat{i} + 3\hat{j}}$ Ans

Ans 2 → let $\vec{a} = \hat{i} + 2\hat{j} + \hat{k}$

$\vec{b} = -\hat{i} + 3\hat{j} + 4\hat{k}$



vector $\perp^r$ to the plane containing $\vec{a}$ & $\vec{b}$ is $\vec{n} = \vec{a}\times\vec{b}$

$$\vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & k \\ 1 & 2 & 1 \\ -1 & 3 & 4 \end{vmatrix}$$

$$\Rightarrow \vec{n} = 5\hat{i} - 5\hat{j} + 5\hat{k}$$

Now $\hat{n} = \pm \frac{\vec{n}}{|\vec{n}|}$ --- {∵ there are two direction possible}

$$\hat{n} = \pm \frac{(5\hat{i} - 5\hat{j} + 5\hat{k})}{\sqrt{25+25+25}} = \pm\left(\frac{1}{\sqrt{3}}\hat{i} - \frac{1}{\sqrt{3}}\hat{j} + \frac{1}{\sqrt{3}}\hat{k}\right)$$

Now required vector has magnitude $10\sqrt{3}$ and in the direction $\perp^r$ to the plane

∴ Required vector $= (10\sqrt{3})\hat{n}$

$$= \pm(10\sqrt{3})\left(\frac{1}{\sqrt{3}}\hat{i} - \frac{1}{\sqrt{3}}\hat{j} + \frac{1}{\sqrt{3}}\hat{k}\right)$$

$$= \pm 10(\hat{i} - \hat{j} + \hat{k})$$ Ans

Ans 3 → given $\vec{a}$, $\vec{b}$ & $\sqrt{3}\vec{a} - \vec{b}$ are unit vectors

$\Rightarrow |\vec{a}| = 1$; $|\vec{b}| = 1$ and $|\sqrt{3}\vec{a} - \vec{b}| = 1$

let θ be the angle b/w $\vec{a}$ & $\vec{b}$

$$\Rightarrow |\sqrt{3}\vec{a} - \vec{b}|^2 = 1$$

$$\Rightarrow (\sqrt{3}\vec{a} - \vec{b}) \cdot (\sqrt{3}\vec{a} - \vec{b}) = 1$$

$$\Rightarrow 3\vec{a}\cdot\vec{a} - \sqrt{3}\vec{a}\cdot\vec{b} - \sqrt{3}\vec{b}\cdot\vec{a} + \vec{b}\cdot\vec{b} = 1$$

$$\Rightarrow 3|\vec{a}|^2 - 2\sqrt{3}\vec{a}\cdot\vec{b} + |\vec{b}|^2 = 1$$

$$\Rightarrow 3(1) - 2\sqrt{3}|\vec{a}||\vec{b}|\cos\theta + (1) = 1$$

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$\Rightarrow \quad -2\sqrt{3}(1)(1)\cos\theta = -3$

$\Rightarrow \quad \cos\theta = \sqrt{3}/2$

$\Rightarrow \quad \boxed{\theta = \pi/6}$ Ans

Q.No. 4 →

given $|\vec{a}| = 3$

$-1 \leq k \leq 2$

$\Rightarrow \quad 0 \leq |k| \leq 2$

$\Rightarrow \quad 0(|\vec{a}|) \leq |\vec{a}|\,|k| \leq (|\vec{a}|)(2)$ --- {Multiply by $|\vec{a}|$}

$\Rightarrow \quad 0 \leq |k\vec{a}| \leq (3)(2)$ --- {$\because |\lambda\vec{a}| = |\lambda|\,|\vec{a}|$}

$\Rightarrow \quad 0 \leq |k\vec{a}| \leq 6$

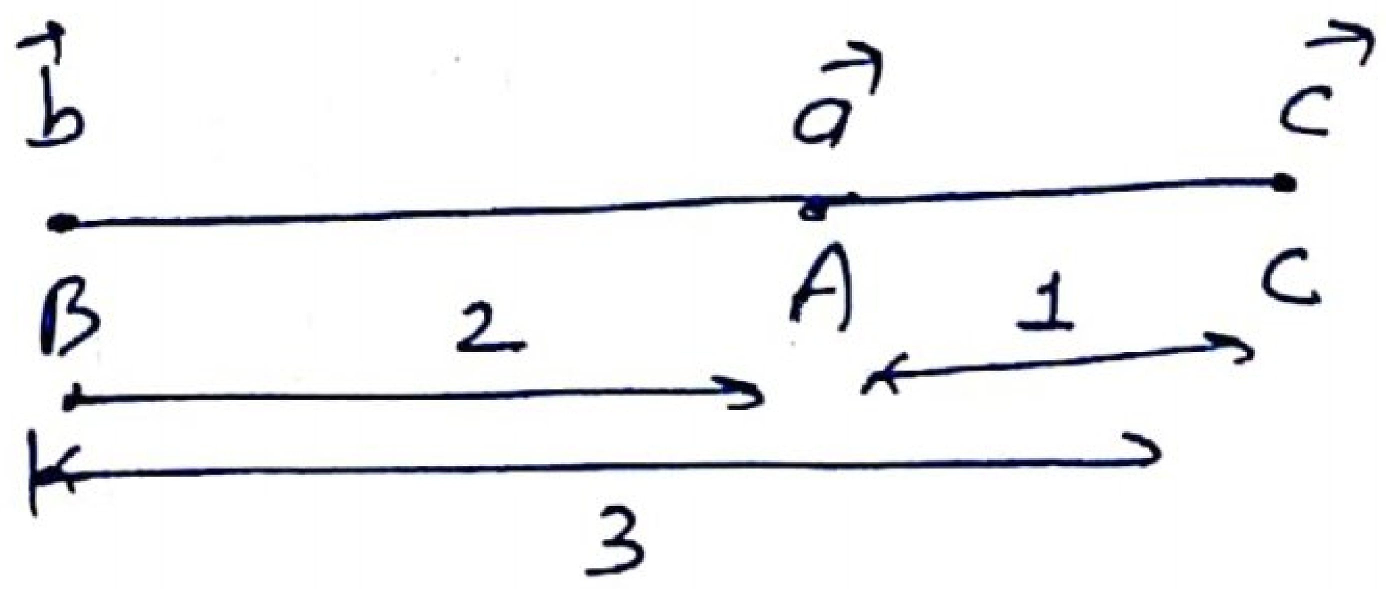
$\therefore \boxed{|k\vec{a}| \in [0, 6]}$ Ans

Q.No. 5 →

given $\vec{OA} = \vec{a}$

$\vec{OB} = \vec{b}$

let $\vec{OC} = \vec{c}$



given $BC = 1.5\,BA$

$\frac{BC}{BA} = \frac{3}{2}$

$\therefore$ A divides BC internally in the Ratio 2:1

By section formula

$\vec{a} = \frac{2\vec{c} - \vec{b}}{2+1}$

$\Rightarrow 3\vec{a} = 2\vec{c} - \vec{b}$ $\Rightarrow$ $\boxed{\vec{c} = \frac{3\vec{a} + \vec{b}}{2}}$ Ans

(Note = Misprint in worksheet Ans).

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Ans 6 → let given points are

$A(k, -10, 3) \quad B(1, -1, 3) \quad C(3, 5, 3)$

$\vec{AB} = (1-k)\hat{i} + 9\hat{j} + 0\hat{k}$

$\vec{BC} = 2\hat{i} + 6\hat{j} + 0\hat{k}$

Since points A, B, C are collinear (given)

∴ vectors $\vec{AB}$ & $\vec{BC}$ are also collinear

∴ their corresponding components are in equal ratio

$\Rightarrow \quad \frac{1-k}{2} = \frac{9}{6}$

$\Rightarrow \quad 6 - 6k = 18$

$\Rightarrow \quad 6k = -12$

$\Rightarrow \quad \boxed{k = -2}$ Ans

Ans 7 → given $\vec{r}$ makes equal angles with

x, y & z axis

$\Rightarrow \quad \alpha = \beta = \gamma$

$\Rightarrow \quad l = m = n$

we have $l^2 + m^2 + n^2 = 1$

$\Rightarrow \quad 3l^2 = 1$

$\Rightarrow \quad l^2 = 1/3$

$\Rightarrow \quad l = \pm \frac{1}{\sqrt{3}}$

$\therefore l = m = n = \pm \frac{1}{\sqrt{3}}$

we know that $\hat{r} = l\hat{i} + m\hat{j} + n\hat{k}$

$\Rightarrow \hat{r} = \pm\left(\frac{1}{\sqrt{3}}\hat{i} + \frac{1}{\sqrt{3}}\hat{j} + \frac{1}{\sqrt{3}}\hat{k}\right)$

given $|\vec{r}| = 2\sqrt{3}$

Now $\vec{r} = (2\sqrt{3})\hat{r}$

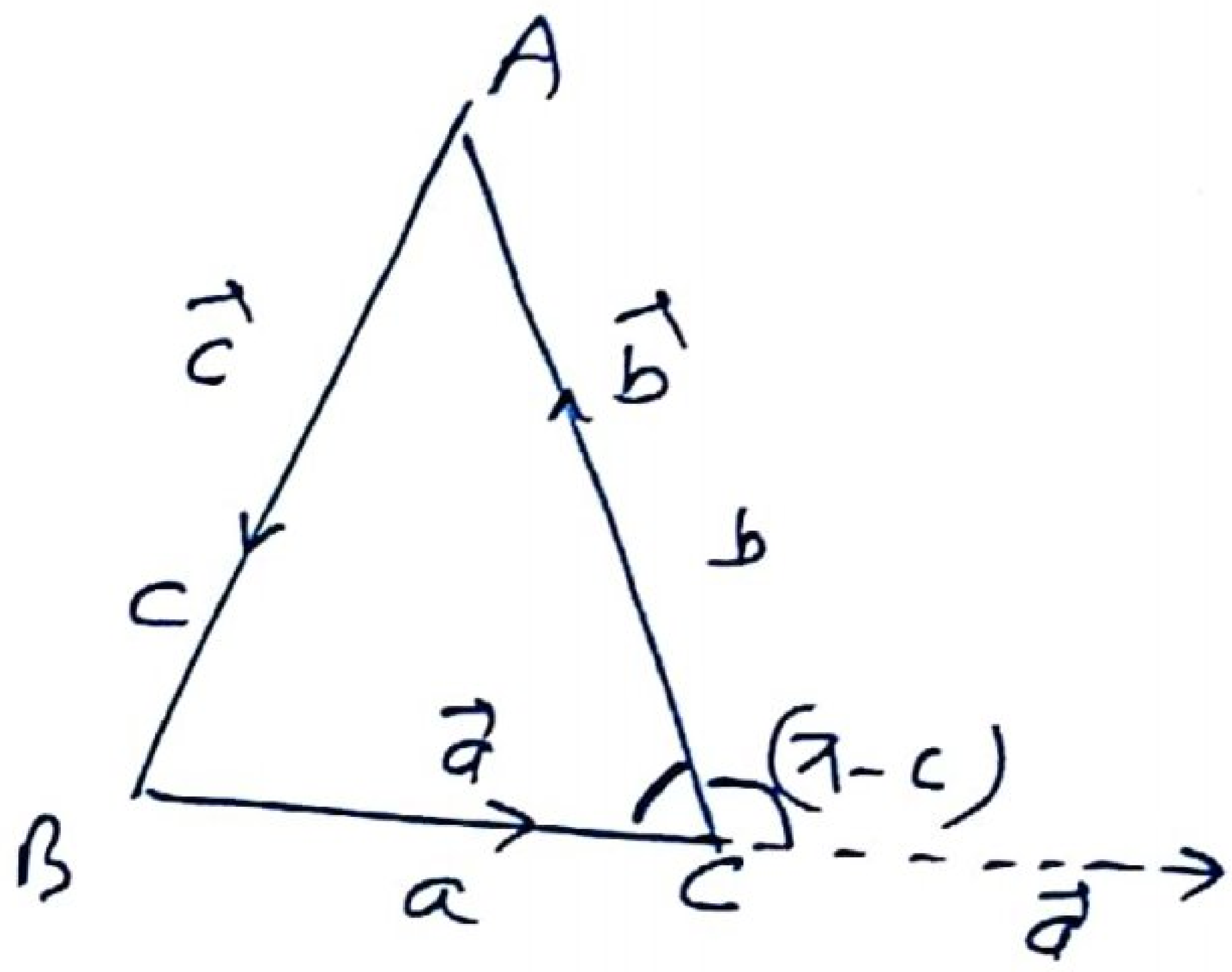
$\Rightarrow \boxed{\vec{r} = \pm 2(\hat{i} + \hat{j} + \hat{k})}$ Ans

Ans. 8 →

In ΔABC

$\vec{a} + \vec{b} + \vec{c} = \vec{0}$

$\vec{a} + \vec{b} = -\vec{c}$



$\Rightarrow |\vec{a} + \vec{b}| = |-\vec{c}| = |\vec{c}|$

$\Rightarrow |\vec{a} + \vec{b}|^2 = |\vec{c}|^2$

$\Rightarrow (\vec{a} + \vec{b}) \cdot (\vec{a} + \vec{b}) = |\vec{c}|^2$

$\Rightarrow |\vec{a}|^2 + |\vec{b}|^2 + 2\vec{a}\cdot\vec{b} = |\vec{c}|^2$

$\Rightarrow a^2 + b^2 + 2|\vec{a}||\vec{b}|\cos(\pi - c) = c^2$

$\Rightarrow$

$a^2 + b^2 + 2ab(-\cos C) = c^2$

$\Rightarrow \boxed{\cos C = \frac{a^2 + b^2 - c^2}{2ab}}$ Proved

Ans. 9 → (i) False

If $|\vec{a}| = |\vec{b}|$ then it is not necessary that $\vec{a} = \pm\vec{b}$

For example $\vec{a} = 2\hat{i} + \hat{j} + \hat{k}$ & $\vec{b} = \hat{i} + 2\hat{j} + \hat{k}$

clearly $|\vec{a}| = |\vec{b}| = \sqrt{6}$

but $\vec{a} \neq \vec{b}$ also $\vec{a} \neq -\vec{b}$

(ii) False

If $|\vec{a}+\vec{b}| = |\vec{a}-\vec{b}|$ then it is not necessary that $\vec{a} \perp \vec{b}$

Reason If $\vec{a} = \vec{0}$ or $\vec{b} = \vec{0}$ or both

then $|\vec{a}+\vec{b}|$ can be equal to $|\vec{a}-\vec{b}|$

Qu 10 → let $\vec{a} = x\hat{i} + y\hat{j} + z\hat{k}$

LHS $(\vec{a}\cdot\hat{i})\hat{i} + (\vec{a}\cdot\hat{j})\hat{j} + (\vec{a}\cdot\hat{k})\hat{k}$

$\Rightarrow ((x\hat{i}+y\hat{j}+z\hat{k})\cdot\hat{i})\hat{i} + ((x\hat{i}+y\hat{j}+z\hat{k})\cdot\hat{j})\hat{j}$
$+ ((x\hat{i}+y\hat{j}+z\hat{k})\cdot\hat{k})\hat{k}$

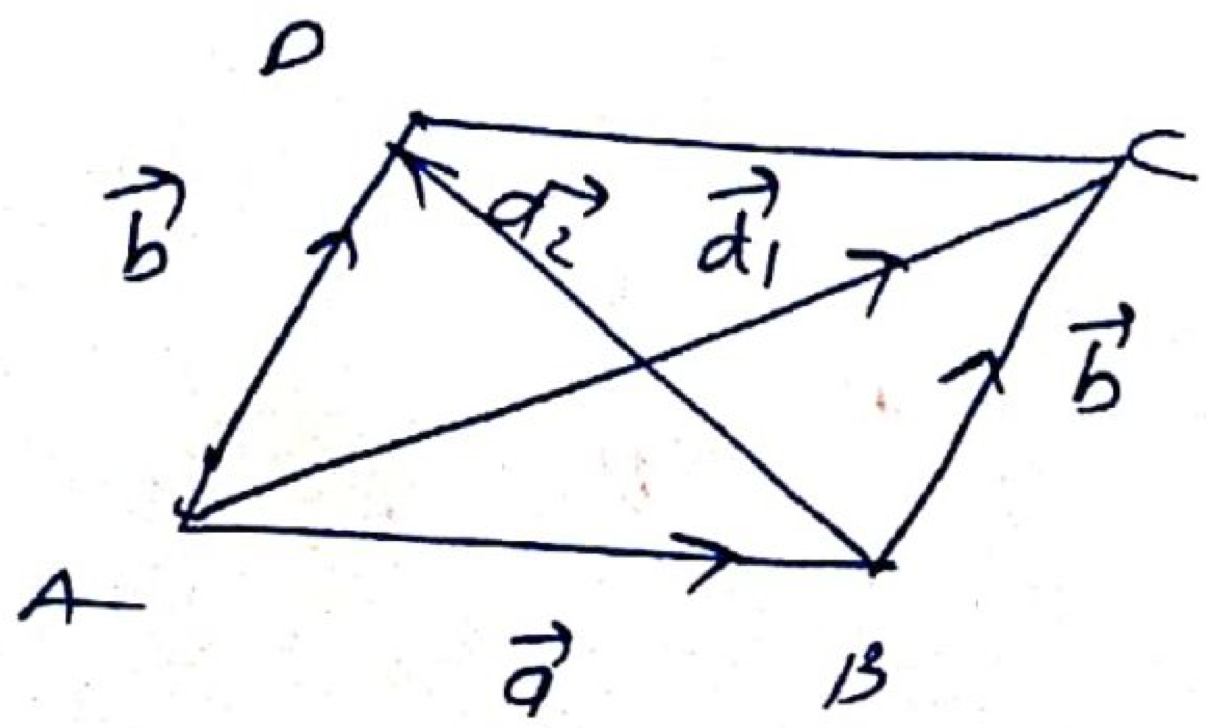
$\Rightarrow (x)\hat{i} + (y)\hat{j} + (z)\hat{k}$

$= x\hat{i} + y\hat{j} + z\hat{k}$

$= \vec{a}$ proved

Qu. 11 → let $\vec{a} = 3\hat{i} - 2\hat{j} + 2\hat{k}$ and $\vec{b} = -\hat{i} - 2\hat{k}$

By triangle law in ΔABC



$\vec{d_1} = \vec{b} + \vec{a}$

By triangle law in ΔABD

$\vec{d_2} = \vec{b} - \vec{a}$

$\therefore \vec{d_1} = 2\hat{i} - 2\hat{j} + 0\hat{k}$

$\vec{d_2} = -4\hat{i} + 2\hat{j} - 4\hat{k}$

let θ be the angle b/w $\vec{d_1}$ & $\vec{d_2}$

$$\cos\theta = \left| \frac{\vec{d_1} \cdot \vec{d_2}}{|\vec{d_1}||\vec{d_2}|} \right| \quad \text{---} \left\{ \text{modulus is taken since } \theta \text{ is acute} \right\}$$

$$\Rightarrow \cos\theta = \frac{|-8-4-0|}{\sqrt{4+4}\ \sqrt{16+4+16}}$$

$$\Rightarrow \cos\theta = \frac{12}{(2\sqrt{2})(6)} = \frac{1}{\sqrt{2}}$$

$$\Rightarrow \boxed{\theta = \pi/4} \text{ Ans}$$

Ans. 12 → (i) $|\vec{a}| = |\vec{b}|$ Ans

Reason given By triangle law $\vec{a}+\vec{b}$ is the diagonal of a parallelogram with adjacent sides $\vec{a}$ & $\vec{b}$

But $\vec{a}+\vec{b}$ also bisect the angle b/w $\vec{a}$ & $\vec{b}$ this is possible only when it is a square

$\Rightarrow |\vec{a}| = |\vec{b}|$

(ii) $|\vec{a} \times \vec{b}|^2 + (\vec{a} \cdot \vec{b})^2$

$= (|\vec{a}||\vec{b}|\sin\theta)^2 + (|\vec{a}||\vec{b}|\cos\theta)^2$

$= |\vec{a}|^2 |\vec{b}|^2 \sin^2\theta + |\vec{a}|^2 |\vec{b}|^2 \cos^2\theta$

$$= |\vec a|^2 |\vec b|^2 (\sin^2\theta + \cos^2\theta)$$

$$= |\vec a|^2 |\vec b|^2 \quad \underline{\text{Ans}}$$

(3) given

$$|\vec a \times \vec b|^2 + |\vec a \cdot \vec b|^2 = 144$$

$$\Rightarrow |\vec a|^2 |\vec b|^2 \sin^2\theta + |\vec a|^2 |\vec b|^2 \cos^2\theta = 144$$

$$\Rightarrow |\vec a|^2 |\vec b|^2 (\sin^2\theta + \cos^2\theta) = 144$$

$$\Rightarrow |\vec a|^2 |\vec b|^2 = 144$$

$$\Rightarrow (4)^2 |\vec b|^2 = 144 \quad \ldots \{\text{given } |\vec a| = 4\}$$

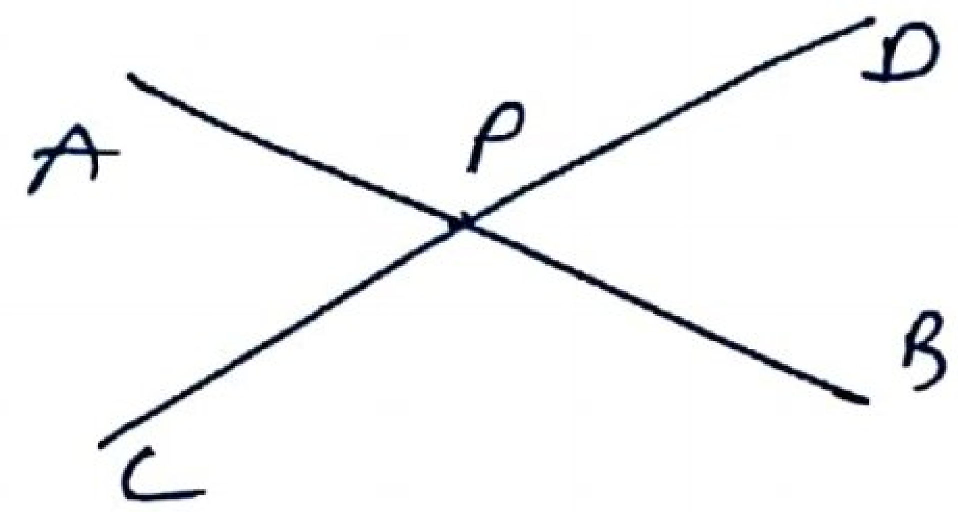
$$\Rightarrow |\vec b|^2 = 9$$

$$\Rightarrow \boxed{|\vec b| = 3} \quad \underline{\text{Ans}}$$

Qu 13 → $A(6,-7,0)$ $B(16,-19,-4)$

$C(0,3,-6)$ $D(2,-5,10)$

$P(1,-1,2)$



Now $\overrightarrow{AP} = -5\hat i + 6\hat j + 2\hat k$

$$\overrightarrow{PB} = 15\hat i - 18\hat j - 6\hat k$$

$$\overrightarrow{PB} = -3(-5\hat i + 6\hat j + 2\hat k)$$

$$\Rightarrow \overrightarrow{PB} = -3(\overrightarrow{AP})$$

$\Rightarrow$ $\overrightarrow{AP}$ & $\overrightarrow{PB}$ are collinear

$\Rightarrow$ points A, P, B are collinear --(1)

Now $\vec{CP} = \hat{i} - 4\hat{j} + 8\hat{k}$

$\vec{PD} = \hat{i} - 4\hat{j} + 8\hat{k}$

$\Rightarrow \vec{CP} = \vec{PD}$

$\Rightarrow \vec{CP}$ & $\vec{PD}$ are collinear

but point P is common

$\therefore$ C, P, D points are collinear --- (2)

from (1) & (2)

P is the common point of AB & CD

$\therefore$ AB & CD intersect at point P PROVED

Ans. 14 $\rightarrow$

given $\theta = 120°$

$|\vec{a}| = 1$, $|\vec{b}| = 2$

Now $|(\vec{a} + 3\vec{b}) \times (3\vec{a} - \vec{b})|^2$

$= |3\vec{a}\times\vec{a} - \vec{a}\times\vec{b} + 9\vec{b}\times\vec{a} - 3\vec{b}\times\vec{b}|^2$

$= |\vec{b}\times\vec{a} + 9\vec{b}\times\vec{a}|^2$ --- $\{\because \vec{a}\times\vec{a} = \vec{0}$, $\vec{a}\times\vec{b} = -\vec{b}\times\vec{a}\}$

$\Rightarrow |10\vec{b}\times\vec{a}|^2$

$\Rightarrow (10|\vec{b}||\vec{a}|\sin(120°))^2$

$\Rightarrow 100(\cancel{4})(1) \times \frac{3}{\cancel{4}}$ --- $\{\sin(120°) = \sqrt{3}/2\}$

$= 300$

$\therefore$ (A) Ans

Ans 15 →

given
$$|\vec{p}| = 1$$
$$|\vec{q}| = 1$$
$$\theta = 30^\circ$$

given $\vec{a} = \vec{p} + 2\vec{q}$ & $\vec{b} = 2\vec{p} + \vec{q}$ are diagonals

Now Area of parallelogram in terms of diagonals

$$= \frac{1}{2} |\vec{a} \times \vec{b}|$$

$$= \frac{1}{2} |(\vec{p} + 2\vec{q}) \times (2\vec{p} + \vec{q})|$$

$$= \frac{1}{2} |2\vec{p} \times \vec{p} + \vec{p} \times \vec{q} + 4\vec{q} \times \vec{p} + 2\vec{q} \times \vec{q}|$$

$$= \frac{1}{2} |\vec{0} + \vec{p} \times \vec{q} - 4\vec{p} \times \vec{q} + \vec{0}|$$

$$= \frac{1}{2} |-3\vec{p} \times \vec{q}|$$

$$= \frac{3}{2} |\vec{p}||\vec{q}| \sin\theta$$

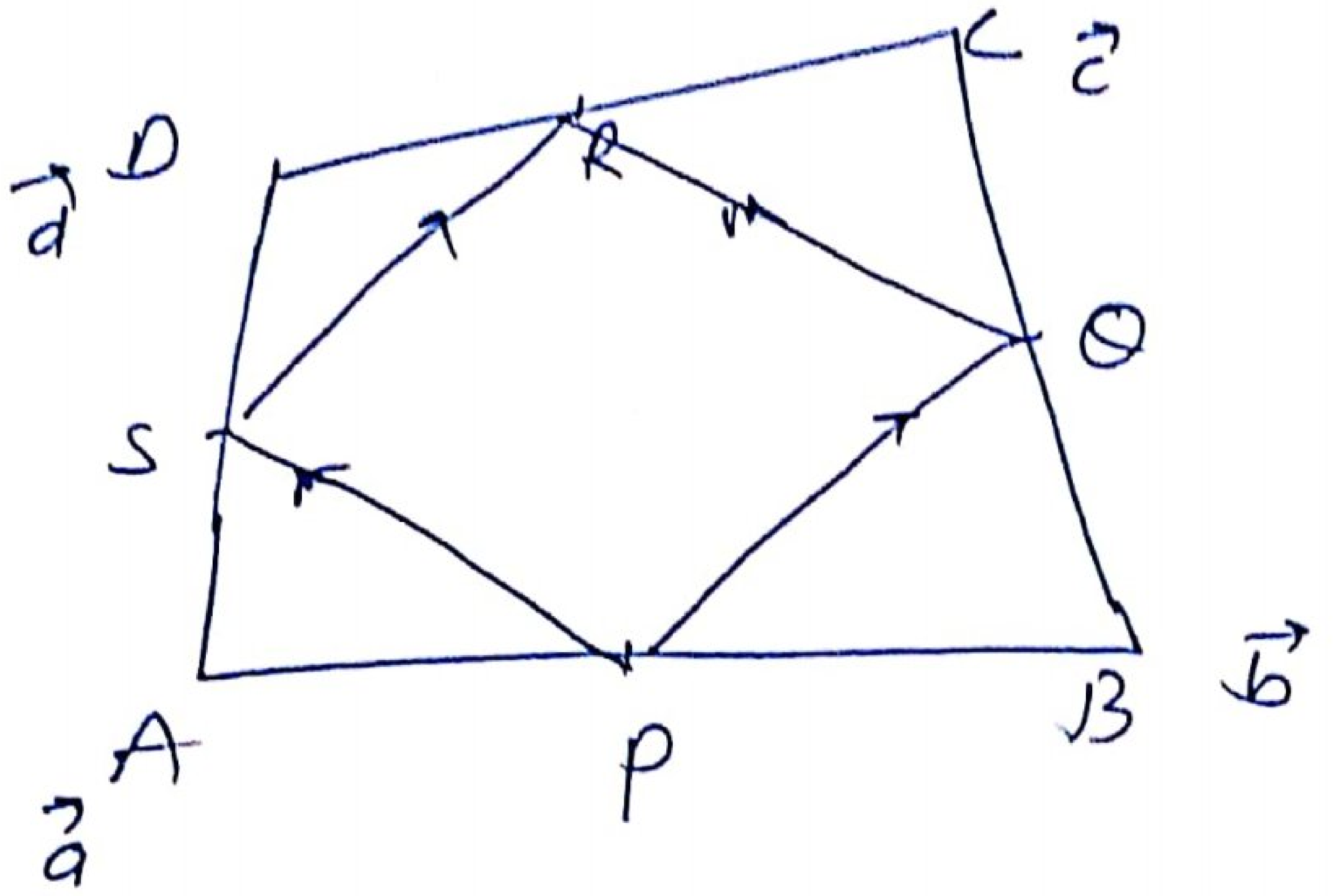
$$= \frac{3}{2} (1)(1) \sin(30^\circ)$$

$$= \frac{3}{2} \times \frac{1}{2}$$

$$= \frac{3}{4}$$ square units Ans

Ans 16 →

let ABCD is a quadrilateral and $\vec{a}, \vec{b}, \vec{c}, \vec{d}$ are position vectors of points A, B, C, D resp let P, Q, R, S are the midpoints ~~wrty~~



p.v. of point $P = \frac{\vec{a}+\vec{b}}{2}$

p.v of point $Q = \frac{\vec{b}+\vec{c}}{2}$

p.v of point $R = \frac{\vec{c}+\vec{d}}{2}$

p.v of point $S = \frac{\vec{a}+\vec{d}}{2}$

Now $\vec{PQ} = \vec{OQ} - \vec{OP} = \left(\frac{\vec{b}+\vec{c}}{2}\right) - \left(\frac{\vec{a}+\vec{b}}{2}\right)$

$= \frac{\vec{c}-\vec{a}}{2}$

and $\vec{SR} = \vec{OR} - \vec{OS} = \left(\frac{\vec{c}+\vec{d}}{2}\right) - \left(\frac{\vec{a}+\vec{d}}{2}\right)$

$= \frac{\vec{c}-\vec{a}}{2}$

Clearly $\vec{PQ} = \vec{SR}$

$\Rightarrow |\vec{PQ}| = |\vec{SR}|$

Similarly $\vec{PS} = \vec{QR}$

$\Rightarrow |\vec{PS}| = |\vec{QR}|$

$\therefore$ opposite sides are equal $\therefore$ PQRS is a parallelogram Ans