← ULTIMATE MATHEMATICS →

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MATRICES [CLASS NO: 3] : M3

Qn.1 Show that all the diagonal elements of a Skew-symm matrix are always zero

Soln for skew-symm matrix
we have $a_{ij} = -a_{ji}$

$\Rightarrow a_{ij} + a_{ji} = 0$

for diagonal elements $i = j$

$\Rightarrow a_{ii} + a_{ii} = 0$

$\Rightarrow 2a_{ii} = 0$

$\Rightarrow a_{ii} = 0$

put $i = 1 \Rightarrow a_{11} = 0$

$i = 2 \Rightarrow a_{22} = 0$

$i = 3 \Rightarrow a_{33} = 0$

hence all the diagonal elements of a Skew-Symm matrix are always zero

Qn.2 If A and B are Symm. matrices Show that AB+BA is also a Symm Matrix

Soln given: $A' = A$ and $B' = B$

To prove $AB + BA \rightarrow$ is a symm Matrix

let $P = AB + BA$

$\Rightarrow P' = (AB + BA)'$

$\Rightarrow P' = (AB)' + (BA)'$

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$P' = B'A' + A'B'$

$P' = BA + AB$ .... (given)

$P' = AB + BA$

$P' = P$

$\therefore$ P is a symm Matrix

Qu. 3 Show that $B'AB$ is a symmetric Matrix or Skew-symm Matrix as according to A is symm or skew symmetric

Sol: Case II given: $A \to$ skew symm Matrix $A' = -A$

T.P $B'AB \to$ skew symm Matrix

let $P = B'AB$

$\Rightarrow P' = (B'AB)'$

$\Rightarrow P' = B'A'B$

$\Rightarrow P' = B'(-A)B$ --- (given)

$P' = -B'AB$

$P' = -P$

$\therefore$ P is a skew symm Matrix

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Q. 4 A and B are Symm Matrices Show that AB is a symmetric Matrix if and only if ~~Af~~ A and B commute i.e $AB = BA$

Sol. (1) given $\Leftrightarrow$ T.P

(2) given $\Leftrightarrow$ T.P

$\Rightarrow$.

Conversely

Sol. given $A' = A$ & $B' = B$

Case I given. $AB = BA$

T.P $AB \rightarrow$ Symm Matrix

let $P = AB$

$\Rightarrow P' = (AB)'$

$\Rightarrow P' = B'A'$

$\Rightarrow P' = BA$ .. (given)

$\Rightarrow P' = AB$ .. (given)

$\Rightarrow P' = P$

$\therefore$ P is a Symm Matrix

Conversely

given : $AB \rightarrow$ Symm.

T.P : $AB = BA$

we have

$(AB)' = AB$

$B'A' = AB$

$BA = AB$ .. (given)

$\therefore$ A & B commute

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Q.No.5 (theorem) : Show that every square Matrix can be uniquely expressed as the sum of Symm matrix and Skew-Symm Matrix.

Sol. $\begin{bmatrix} \text{square} \end{bmatrix} = \begin{pmatrix} \text{Symm} \\ \text{Matrix} \end{pmatrix} + \begin{pmatrix} \text{Skew} \\ \text{Symm} \\ \text{Matrix} \end{pmatrix}$

Proof: Let $A \to$ be any square Matrix

then $\boxed{A = \frac{1}{2}(A+A') + \frac{1}{2}(A-A')}$

Let $P = \frac{1}{2}(A+A')$

$P' = \frac{1}{2}(A+A')'$

$P' = \frac{1}{2}(A'+A)$

$P' = \frac{1}{2}(A+A')$

$P' = P$

$\therefore P \to$ symm Matrix

Let $Q = \frac{1}{2}(A-A')$

$Q' = \frac{1}{2}(A-A')'$

$Q' = \frac{1}{2}(A'-A)$

$Q' = -\frac{1}{2}(A-A')$

$Q' = -Q$

$\therefore Q \to$ Skew Sym Matrix

Since $A = P + Q$

$\therefore A$ can be uniquely - - - - -

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Qn.6 express the Matrix $A = \begin{bmatrix} 2 & 3 \\ -1 & 4 \end{bmatrix}$ as the sum of symmetric Matrix & skew symm Matrix.

Sol: (8 steps)

(·) $A' = \begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix}$

(·) $A + A' = \begin{bmatrix} 4 & 2 \\ 2 & 8 \end{bmatrix}$

(·) $A - A' = \begin{bmatrix} 0 & 4 \\ -4 & 0 \end{bmatrix}$

(·) let $P = \frac{1}{2}(A + A') = \begin{bmatrix} 2 & 1 \\ 1 & 4 \end{bmatrix}$

(·) $P' = \begin{bmatrix} 2 & 1 \\ 1 & 4 \end{bmatrix} = P$

$\therefore$ P is a symm Matrix

(·) let $Q = \frac{1}{2}(A - A') = \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix}$

(·) $Q' = \begin{bmatrix} 0 & -2 \\ 2 & 0 \end{bmatrix} = -\begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix} = -Q$

$\therefore$ Q → Skew Sym Matrix

(·) $P + Q = \begin{bmatrix} 2 & 1 \\ 1 & 4 \end{bmatrix} + \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ -1 & 4 \end{bmatrix} = A$

$\therefore$ A can be express - - - - - - - -

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Ques. 1 → If $A = \begin{bmatrix} -1 & 2 & 3 \\ 5 & 7 & 9 \\ -2 & 1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} -4 & 1 & -5 \\ 1 & 2 & 0 \\ 1 & 3 & 1 \end{bmatrix}$ verify that

(i) $(A+B)' = A' + B'$

(ii) $(A-B)' = A' - B'$

(iii) $(AB)' = B'A'$

Ques. 2 → If $A = \begin{bmatrix} 1 \\ -4 \\ 3 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 2 & 1 \end{bmatrix}$ verify that

$(AB)' = B'A'$

Ques. 3 → for the matrix $A = \begin{bmatrix} 1 & 5 \\ 6 & 7 \end{bmatrix}$ verify that

(i) $A + A'$ is a symmetric matrix

(2) $A - A'$ is a skew symm. matrix

Ques. 4 → If $A = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$ and $A + A' = I$. Find value of α.

Ques. 5 → Express the matrix $A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$ as the sum of symmetric matrix and skew-symmetric matrix

Ques. 6 → Express the matrix $B = \begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix}$ as the sum of symmetric matrix and skew-symmetric matrix

Ques 7 → If A and B are symmetric matrices, then show that AB + BA is a symmetric matrix and AB − BA is a skew-symmetric matrix

Ques: 8 → Show that the matrix $B'AB$ is symmetric or Skew-Symmetric according as A is Symmetric or skew-symmetric.

Qns: 9 → Find the values of x, y, z if the matrix

$$A = \begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{bmatrix}$$ Satisfy the equation $A'A = I$

Qns 10 → If $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ a & 2 & b \end{bmatrix}$ satisfy the equation $AA' = 9I$. Find the values of a and b.

Qns 11 → Show that all the diagonal elements of a Skew-symmetric matrix are always zero.

Qns: 12 → If A and B are symmetric matrices of the same order, then show that AB is symmetric if and only if A and B Commute i.e $AB = BA$

Qns 13 → (i) If the matrix A is both Symmetric and Skew Symmetric, then A is ______

(ii) If A is a Square matrix such that $A^2 = A$, then find $(I+A)^3 - 7A$

Qns 14 → A is a Square matrix such that $A^2 = I$, then $(A-I)^3 + (A+I)^3 - 7A$ is equal to ______

Qns 15 → If matrix $\begin{bmatrix} 0 & a & 3 \\ 2 & b & -1 \\ c & 1 & 0 \end{bmatrix}$ is a Skew-symmetric matrix find the value of a, b and c

← ANSWERS →

(4). $\alpha = 2n\pi \pm \pi/3$; $n \in Z$	(10). $a = -2$, $b = -1$	(11) I	(15) $a = -2$
(9). $x = \pm \frac{1}{\sqrt{2}}$, $y = \pm \frac{1}{\sqrt{6}}$; $z = \pm \frac{1}{\sqrt{3}}$	(13) (i) Null matrix	(14). A	$b = 0$ $c = -3$