Solutions of I-2

Ans 1 →
$$\tan^{-1}\sqrt{\frac{1-\cos x}{1+\cos x}}$$
$$=\tan^{-1}\sqrt{\frac{\cancel{2}\sin^2\frac{x}{2}}{\cancel{2}\cos^2\frac{x}{2}}}$$
$$=\tan^{-1}\sqrt{\tan^2(x/2)}$$
$$=\tan^{-1}\left(\tan\frac{x}{2}\right) = \frac{x}{2} \quad \text{Ans}$$

Ans 2 →
$$\tan^{-1}\left(\frac{\cos x}{1+\sin x}\right)$$
$$=\tan^{-1}\left(\frac{\sin(\frac{\pi}{2}-x)}{1+\cos(\frac{\pi}{2}-x)}\right)$$
$$=\tan^{-1}\left(\frac{\cancel{2}\sin\left(\frac{\pi}{4}-\frac{x}{2}\right)\cancel{\cos\left(\frac{\pi}{4}-\frac{x}{2}\right)}}{\cancel{2}\cos^{\cancel{2}}\left(\frac{\pi}{4}-\frac{x}{2}\right)}\right)$$
$$=\tan^{-1}\left(\tan\left(\frac{\pi}{4}-\frac{x}{2}\right)\right)$$
$$=\frac{\pi}{4}-\frac{x}{2} \quad \text{Ans}$$

Ans 3 →
$$\tan^{-1}\left(\frac{1-\sin x}{\cos x}\right)$$
$$=\tan^{-1}\left(\frac{1-\cos(\frac{\pi}{2}-x)}{\sin(\frac{\pi}{2}-x)}\right)$$
$$=\tan^{-1}\left(\frac{\cancel{2}\sin^{\cancel{2}}\left(\frac{\pi}{4}-\frac{x}{2}\right)}{\cancel{2}\cancel{\sin\left(\frac{\pi}{4}-\frac{x}{2}\right)}\cos\left(\frac{\pi}{4}-\frac{x}{2}\right)}\right)$$

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$$= \tan^{-1}\left(\tan\left(\frac{\pi}{4}-\frac{x}{2}\right)\right)$$

$$= \frac{\pi}{4}-\frac{x}{2} \quad \underline{\underline{\text{Ans}}}$$

Qn 4 →

$$\tan^{-1}\left(\frac{x}{\sqrt{a^2-x^2}}\right)$$

put $x = a\sin\theta$

$$= \tan^{-1}\left(\frac{a\sin\theta}{\sqrt{a^2-a^2\sin^2\theta}}\right)$$

$$= \tan^{-1}\left(\frac{\cancel{a}\sin\theta}{\cancel{a}\sqrt{1-\sin^2\theta}}\right)$$

$$= \tan^{-1}\left(\frac{\sin\theta}{\cos\theta}\right)$$

$$= \tan^{-1}(\tan\theta)$$

$$= \theta$$

replace θ

$$= \sin^{-1}\left(\frac{x}{a}\right) \quad \underline{\underline{\text{Ans}}}$$

Qn 5 →

$$\tan^{-1}\sqrt{\frac{a-x}{a+x}}$$

put $x = a\cos(2\theta)$

$$= \tan^{-1}\sqrt{\frac{a-a\cos(2\theta)}{a+a\cos(2\theta)}}$$

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$$= \tan^{-1}\sqrt{\frac{1-\cos(2\theta)}{1+\cos(2\theta)}}$$

$$= \tan^{-1}\sqrt{\frac{\cancel{2}\sin^2\theta}{\cancel{2}\cos^2\theta}}$$

$$= \tan^{-1}\sqrt{\tan^2\theta}$$

$$= \tan^{-1}(\tan\theta)$$

$$= \theta$$

replace θ

$$= \frac{1}{2}\cos^{-1}\left(\frac{x}{a}\right)$$

Ans

$$\boxed{\begin{aligned} x &= a\cos(2\theta) \\ \frac{x}{a} &= \cos(2\theta) \\ \cos^{-1}\left(\frac{x}{a}\right) &= 2\theta \end{aligned}}$$

$$\theta = \frac{1}{2}\cos^{-1}\left(\frac{x}{a}\right)$$

$$\cancel{\tan^{-1}}\ \cos^{-1}\left(\frac{x}{\sqrt{x^2+a^2}}\right)$$

put $x = a\tan\theta$

$$= \cancel{\tan^{-1}}\ \cos^{-1}\left(\frac{a\tan\theta}{\sqrt{a^2\tan^2\theta+a^2}}\right)$$

$$= \cancel{\tan^{-1}}\ \cos^{-1}\left(\frac{a\tan\theta}{a\sqrt{\tan^2\theta+1}}\right)$$

$$= \cancel{\tan^{-1}}\ \cos^{-1}\left(\frac{\cancel{a}\tan\theta}{\cancel{a}\sec\theta}\right)$$

$$= \cancel{\tan^{-1}}\ \cos^{-1}\left(\frac{\frac{\sin\theta}{\cancel{\cos\theta}}}{\frac{1}{\cancel{\cos\theta}}}\right)$$

$$= \cos^{-1}(\sin\theta)$$

$$= \cos^{-1}\left(\cos\left(\frac{\pi}{2}-\theta\right)\right)$$

Solutions I.2

$= \frac{\pi}{2} - \theta$

replace θ

$= \frac{\pi}{2} - \tan^{-1}\left(\frac{x}{a}\right)$

(or) $\cot^{-1}(x/a)$ --- $\left\{ \tan^{-1}x + \cot^{-1}x = \pi/2 \right\}$

Ans

Qn 7 - $\tan^{-1}\left(\frac{1}{2}\right) + \tan^{-1}\left(\frac{1}{5}\right) + \tan^{-1}\left(\frac{1}{8}\right)$

$$= \tan^{-1}\left(\frac{\frac{1}{2}+\frac{1}{5}}{1-\frac{1}{2}\times\frac{1}{5}}\right) + \tan^{-1}\left(\frac{1}{8}\right)$$

$$= \tan^{-1}\left(\frac{\frac{7}{10}}{\frac{9}{10}}\right) + \tan^{-1}\left(\frac{1}{8}\right)$$

$$= \tan^{-1}\left(\frac{7}{9}\right) + \tan^{-1}\left(\frac{1}{8}\right)$$

$$= \tan^{-1}\left(\frac{\frac{7}{9}+\frac{1}{8}}{1-\frac{7}{9}\times\frac{1}{8}}\right)$$

$$= \tan^{-1}\left(\frac{\frac{56+9}{72}}{\frac{72-7}{72}}\right)$$

$$= \tan^{-1}\left(\frac{65}{65}\right)$$

$$= \tan^{-1}(1)$$

$$= \pi/4$$ Ans

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Q. 8. please do yourself

Q. 9. $2\tan^{-1}\left(\frac{1}{5}\right) + \tan^{-1}\left(\frac{1}{7}\right) + 2\tan^{-1}\left(\frac{1}{8}\right)$

$$= 2\left[\tan^{-1}\left(\frac{1}{5}\right) + \tan^{-1}\left(\frac{1}{8}\right)\right] + \tan^{-1}\left(\frac{1}{7}\right)$$

$$= 2\tan^{-1}\left(\frac{\frac{1}{5}+\frac{1}{8}}{1-\frac{1}{5}\times\frac{1}{8}}\right) + \tan^{-1}\left(\frac{1}{7}\right)$$

$$= 2\tan^{-1}\left(\frac{8+5}{39}\right) + \tan^{-1}\left(\frac{1}{7}\right)$$

$$= 2\tan^{-1}\left(\frac{1}{3}\right) + \tan^{-1}\left(\frac{1}{7}\right)$$

$$= \tan^{-1}\left(\frac{2\times\frac{1}{3}}{1-\frac{1}{9}}\right) + \tan^{-1}\left(\frac{1}{7}\right)$$

$$= \tan^{-1}\left(\frac{3}{4}\right) + \tan^{-1}\left(\frac{1}{7}\right)$$

$$= \tan^{-1}\left(\frac{\frac{3}{4}+\frac{1}{7}}{1-\frac{3}{4}\times\frac{1}{7}}\right)$$

$$= \tan^{-1}\left(\frac{21+4}{28-3}\right)$$

$$= \tan^{-1}\left(25/25\right)$$

$$= \tan^{-1}(1)$$

$$= \pi/4 \quad \underline{\underline{\text{Ans}}}$$

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Ans 10 →

$$\tan\left[2\tan^{-1}\left(\frac{1}{5}\right) - \frac{\pi}{4}\right]$$

$$= \tan\left[2\tan^{-1}\left(\frac{1}{5}\right) - \tan^{-1}(1)\right]$$

$$= \tan\left[\tan^{-1}\left(\frac{2\times\frac{1}{5}}{1-\frac{1}{25}}\right) - \tan^{-1}(1)\right]$$

$$= \tan\left[\tan^{-1}\left(\frac{\frac{2}{5}}{\frac{24}{25}}\right) - \tan^{-1}(1)\right]$$

$$= \tan\left[\tan^{-1}\left(\frac{5}{12}\right) - \tan^{-1}(1)\right]$$

$$= \tan\left[\tan^{-1}\left(\frac{\frac{5}{12} - 1}{1+\frac{5}{12}\times 1}\right)\right]$$

$$= \tan\left(\tan^{-1}\left(\frac{-7}{17}\right)\right)$$

$$= \frac{-7}{17}$$ Ans

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(11) $\tan^{-1}\left(\sqrt{1+x^2} - x\right)$

Put $x = \tan\theta$

$$\tan^{-1}\left(\sqrt{1+\tan^2\theta} - \tan\theta\right)$$

$$= \tan^{-1}\left(\sec\theta - \tan\theta\right)$$

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$= \tan^{-1}\left(\frac{1}{\cos\theta} - \frac{\sin\theta}{\cos\theta}\right)$

$= \tan^{-1}\left(\frac{1-\sin\theta}{\cos\theta}\right)$

$= \tan^{-1}\left(\frac{1-\cos(\frac{\pi}{2}-\theta)}{\sin(\frac{\pi}{2}-\theta)}\right)$

$= \tan^{-1}\left(\frac{\cancel{2}\sin^2(\frac{\pi}{4}-\frac{\theta}{2})}{\cancel{2}\sin(\frac{\pi}{4}-\frac{\theta}{2})\cos(\frac{\pi}{4}-\frac{\theta}{2})}\right)$

$= \tan^{-1}\left(\tan(\frac{\pi}{4}-\frac{\theta}{2})\right)$

$= \frac{\pi}{4} - \frac{\theta}{2}$

replace θ by $\tan^{-1}x$

$= \frac{\pi}{4} - \frac{\tan^{-1}x}{2}$

$= \frac{1}{2}\left(\frac{\pi}{2} - \tan^{-1}x\right)$ $\quad \{\because \tan^{-1}x + \cot^{-1}x = \frac{\pi}{2}\}$

$= \frac{1}{2}\cot^{-1}x$ Ans

.12 → $\tan^{-1}\left(\frac{\sqrt{1+x^2}+1}{x}\right)$

put $x = \tan\theta$

$= \tan^{-1}\left(\frac{\sqrt{1+\tan^2\theta}+1}{\tan\theta}\right)$

$= \tan^{-1}\left(\frac{\sec\theta+1}{\tan\theta}\right)$

$= \tan^{-1}\left(\frac{1+\cos\theta}{\sin\theta}\right)$

$$= \tan^{-1}\left(\frac{2\cos^2(\theta/2)}{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}\right)$$

$$= \tan^{-1}\left(\cot\frac{\theta}{2}\right)$$

$$= \tan^{-1}\left(\tan\left(\frac{\pi}{2}-\frac{\theta}{2}\right)\right)$$

$$= \frac{\pi}{2}-\frac{\theta}{2}$$

$$= \frac{\pi}{2} - \frac{\tan^{-1}x}{2} \quad \underline{\text{Ans}}$$

Qn 13 → To prove

$$\cos^{-1}\left(\frac{4}{5}\right) + \cos^{-1}\left(\frac{12}{13}\right) = \cos^{-1}\left(\frac{33}{65}\right)$$

Sol: LHS

Conversion

$\cos^{-1}\left(\frac{4}{5}\right)$ here $B=4$, $H=5$

$P=\sqrt{25-16}=3$

$$\cos^{-1}\left(\frac{4}{5}\right) = \tan^{-1}\left(\frac{3}{4}\right)$$

$\cos^{-1}\left(\frac{12}{13}\right)$ here $B=12$, $H=13$

$P=\sqrt{169-144}=5$

$$\cos^{-1}\left(\frac{12}{13}\right) = \tan^{-1}\left(\frac{5}{12}\right)$$

$$= \tan^{-1}\left(\frac{3}{4}\right) + \tan^{-1}\left(\frac{5}{12}\right)$$

$$= \tan^{-1}\left(\frac{\frac{3}{4}+\frac{5}{12}}{1-\frac{3}{4}\times\frac{5}{12}}\right)$$

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Solution I-2

$$= \tan^{-1}\left(\frac{\frac{36+20}{48}}{\frac{48-15}{48}}\right)$$

$$= \tan^{-1}\left(\frac{56}{33}\right)$$

here $P = 56$, $B = 33$

$H = \sqrt{(56)^2 + (33)^2} = 65$

$$= \cos^{-1}\left(\frac{33}{65}\right) = \underline{\text{Ans}}.$$

Ques 14 Do yourself.

Ques 15 → $\sin^{-1}\left(\frac{4}{5}\right) + \sin^{-1}\left(\frac{5}{13}\right) + \sin^{-1}\left(\frac{16}{65}\right)$

Ans

Use conversion

$\sin^{-1}\left(\frac{4}{5}\right)$ here $P = 4$, $H = 5$, $B = \sqrt{25-16} = 3$

$\to \tan^{-1}\left(\frac{4}{3}\right)$

$\sin^{-1}\left(\frac{5}{13}\right)$ here $P = 5$, $H = 13$, $B = \sqrt{169-25} = 12$

$\to \tan^{-1}\left(\frac{5}{12}\right)$

$\sin^{-1}\left(\frac{16}{65}\right)$ here $P = 16$, $H = 65$

$H = \sqrt{65^2 - 16^2} = 63$

$\to \tan^{-1}\left(\frac{16}{63}\right)$

$$= \tan^{-1}\left(\frac{4}{3}\right) + \tan^{-1}\left(\frac{5}{12}\right) + \tan^{-1}\left(\frac{16}{63}\right)$$

Solution I-2

$$= \tan^{-1}\left(\frac{\frac{4}{3}+\frac{5}{12}}{1-\frac{4}{3}\times\frac{5}{12}}\right) + \tan^{-1}\left(\frac{16}{63}\right)$$

$$= \tan^{-1}\left(\frac{48+15}{36-20}\right) + \tan^{-1}\left(\frac{16}{63}\right)$$

$$= \tan^{-1}\left(\frac{63}{16}\right) + \tan^{-1}\left(\frac{16}{63}\right)$$

perty $\cot^{-1}x = \tan^{-1}(1/x)$

$$= \cot^{-1}\left(\frac{16}{63}\right) + \tan^{-1}\left(\frac{16}{63}\right)$$

$$= \frac{\pi}{2}$$ { By property $\tan^{-1}x + \cot^{-1}x = \pi/2$

Ans

$$\sin^{-1}\left(\frac{4}{5}\right) + 2\tan^{-1}\left(\frac{1}{3}\right)$$

onversion

$$\tan^{-1}\left(\frac{4}{3}\right) + \tan^{-1}\left(\frac{2\times 1/3}{1-\frac{1}{9}}\right)$$

$$= \tan^{-1}\left(\frac{4}{3}\right) + \tan^{-1}\left(\frac{\frac{2}{3}}{\frac{8}{9}}\right)$$

$$= \tan^{-1}\left(\frac{4}{3}\right) + \tan^{-1}\left(\frac{3}{4}\right)$$

$$= \cot^{-1}\left(\frac{3}{4}\right) + \tan^{-1}(3/4)$$

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$$= \frac{\pi}{2}$$ { By property $\tan^{-1}x + \cot^{-1}x = \pi/2$ }

Ans