SOLUTIONS VECTORS | WORKSHEET No: 2 | (1)

Ans: 1 → let $\vec{a} = 2\hat{i} + 3\hat{j} + 2\hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} + \hat{k}$

projection of $\vec{a}$ on $\vec{b}$ = $\dfrac{\vec{a}\cdot\vec{b}}{|\vec{b}|}$

Now $\vec{a}\cdot\vec{b} = (2\hat{i} + 3\hat{j} + 2\hat{k})\cdot(\hat{i} + 2\hat{j} + \hat{k}) = 2 + 6 + 2 = 10$

$|\vec{b}| = \sqrt{1+4+1} = \sqrt{6}$

$\therefore$ projection of $\vec{a}$ on $\vec{b}$ = $\dfrac{10}{\sqrt{6}} = \dfrac{10}{\sqrt{6}} \times \dfrac{\sqrt{6}}{\sqrt{6}} = \dfrac{5}{3}\sqrt{6}$ Ans

Ans 2 → given $\vec{a} = 5\hat{i} - \hat{j} + 3\hat{k}$ & $\vec{b} = \hat{i} + 3\hat{j} + \lambda\hat{k}$

$\vec{a} + \vec{b} = 6\hat{i} + 2\hat{j} + \hat{k}(\lambda - 3)$

$\vec{a} - \vec{b} = 4\hat{i} - 4\hat{j} + \hat{k}(-3 - \lambda)$

Since $(\vec{a}+\vec{b})$ and $(\vec{a}-\vec{b})$ are orthogonal ($\perp$)

$\therefore (\vec{a}+\vec{b})\cdot(\vec{a}-\vec{b}) = 0$

$\Rightarrow 24 - 8 - (\lambda^2 + 9) = 0$

$\Rightarrow 25 - \lambda^2 = 0$

$\Rightarrow \lambda^2 = 25$

$\Rightarrow \boxed{\lambda = \pm 5}$ Ans (Note: Misprint in worksheet answer)

Ans 3 → given $|\vec{a}| = 2$; $|\vec{b}| = 3$ and $\vec{a}\cdot\vec{b} = 4$

we have, $|\vec{a} - \vec{b}|^2 = (\vec{a} - \vec{b})\cdot(\vec{a} - \vec{b})$

$= \vec{a}\cdot\vec{a} - \vec{a}\cdot\vec{b} - \vec{b}\cdot\vec{a} + \vec{b}\cdot\vec{b}$

$|\vec{a} - \vec{b}|^2 = |\vec{a}|^2 - 2\vec{a}\cdot\vec{b} + |\vec{b}|^2$

$\Rightarrow |\vec{a}-\vec{b}|^2 = 4-8+9$

$\Rightarrow |\vec{a}-\vec{b}|^2 = 5$

$\Rightarrow \boxed{|\vec{a}-\vec{b}| = \sqrt{5}}$ Ans

Qn. 4 → given $\vec{a}$ is a unit vector

$\Rightarrow |\vec{a}| = 1$

given $(\vec{x}-\vec{a})\cdot(\vec{x}+\vec{a}) = 8$

$\Rightarrow |\vec{x}|^2 - |\vec{a}|^2 = 8$

$\Rightarrow |\vec{x}|^2 - 1 = 8$

$\Rightarrow |\vec{x}|^2 = 9$

$\Rightarrow \boxed{|\vec{x}| = 3}$ Ans

Qn. 5 → given $|\vec{a}| = \sqrt{3}$ and $|\vec{b}| = 2$

$\vec{a}\cdot\vec{b} = \sqrt{6}$

let θ be the angle b/w $\vec{a}$ & $\vec{b}$

$\Rightarrow \cos\theta = \dfrac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|}$

$\Rightarrow \cos\theta = \dfrac{\sqrt{6}}{2\sqrt{3}}$

$\Rightarrow \cos\theta = \dfrac{\sqrt{2}\,\cancel{\sqrt{3}}}{2\,\cancel{\sqrt{3}}} = \dfrac{1}{\sqrt{2}}$

$\Rightarrow \boxed{\theta = \pi/4}$ Ans

Q.N. 6 → Let $\vec{a} = \frac{2}{7}\hat{i} + \frac{3}{7}\hat{j} + \frac{6}{7}\hat{k}$

$\vec{b} = \frac{3}{7}\hat{i} - \frac{6}{7}\hat{j} + \frac{2}{7}\hat{k}$

$\vec{c} = \frac{6}{7}\hat{i} + \frac{2}{7}\hat{j} - \frac{3}{7}\hat{k}$

$|\vec{a}| = \sqrt{\frac{4}{49} + \frac{9}{49} + \frac{36}{49}} = \sqrt{\frac{49}{49}} = 1$

$|\vec{b}| = \sqrt{\frac{9}{49} + \frac{36}{49} + \frac{4}{49}} = 1$

$|\vec{c}| = \sqrt{\frac{36}{49} + \frac{4}{49} + \frac{9}{49}} = 1$

Clearly $|\vec{a}| = |\vec{b}| = |\vec{c}| = 1$

$\therefore$ $\vec{a}$, $\vec{b}$ & $\vec{c}$ are unit vectors

Now $\vec{a}\cdot\vec{b} = \frac{6}{49} - \frac{18}{49} + \frac{12}{49} = 0 \Rightarrow \vec{a} \perp \vec{b}$

$\vec{b}\cdot\vec{c} = \frac{18}{49} - \frac{12}{49} - \frac{6}{49} = 0 \Rightarrow \vec{b} \perp \vec{c}$

$\vec{c}\cdot\vec{a} = \frac{12}{49} + \frac{6}{49} - \frac{18}{49} = 0 \Rightarrow \vec{c} \perp \vec{a}$

$\therefore$ $\vec{a} \perp \vec{b} \perp \vec{c}$

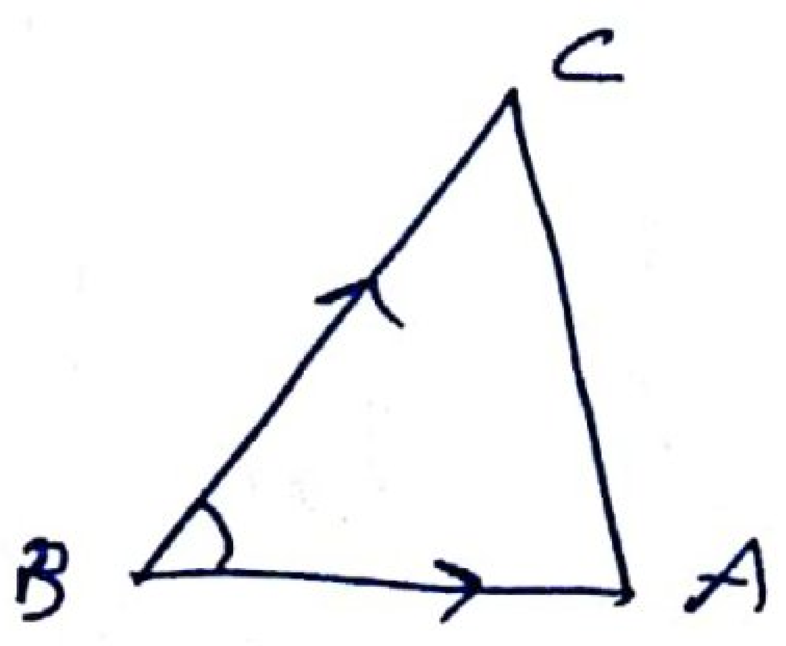
$\Rightarrow$ $\vec{a}, \vec{b}, \vec{c}$ are mutually $\perp^r$ vectors **PROVED**

Q.N. 7 → $A(1,2,3)$; $C(0,1,2)$; $B(-1,0,0)$

$\vec{OA} = \hat{i} + 2\hat{j} + 3\hat{k}$

$\vec{OB} = -\hat{i} + 0\hat{j} + 0\hat{k}$

$\vec{OC} = 0\hat{i} + \hat{j} + 2\hat{k}$



to find $\angle ABC$ or $\angle B$

Prepare $\vec{BA}$ & $\vec{BC}$ (Coinitial vectors)

$\Rightarrow \vec{BA} = \vec{OA} - \vec{OB} = 2\hat{i} + 2\hat{j} + 3\hat{k}$

$\vec{BC} = \vec{OC} - \vec{OB} = \hat{i} + \hat{j} + 2\hat{k}$

$$\cos B = \frac{\vec{BA} \cdot \vec{BC}}{|\vec{BA}|\,|\vec{BC}|}$$

$$= \frac{2 + 2 + 6}{\sqrt{4+4+9}\,\sqrt{1+1+4}}$$

$$= \frac{10}{\sqrt{17}\,\sqrt{6}}$$

$$\cos B = \frac{10}{\sqrt{102}}$$

$$\Rightarrow \boxed{\angle B = \cos^{-1}\left(\frac{10}{\sqrt{102}}\right)}$$ Ans

Qn. 8 → given $\vec{a} = 2\hat{i} + 2\hat{j} + 3\hat{k}$; $\vec{b} = -\hat{i} + 2\hat{j} + \hat{k}$ and $\vec{c} = 3\hat{i} + \hat{j}$

$\vec{a} + \lambda\vec{b} = (2\hat{i} + 2\hat{j} + 3\hat{k}) + (-\lambda\hat{i} + 2\lambda\hat{j} + \lambda\hat{k})$

$\vec{a} + \lambda\vec{b} = \hat{i}(2-\lambda) + \hat{j}(2+2\lambda) + \hat{k}(3+\lambda)$

given $(\vec{a} + \lambda\vec{b}) \perp \vec{c}$

$\Rightarrow (\vec{a} + A\vec{B}) \cdot \vec{c} = 0$

$\Rightarrow \left(\hat{i}(2-\lambda) + \hat{j}(2+2\lambda) + \hat{k}(3+\lambda)\right) \cdot (3\hat{i} + \hat{j}) = 0$

$\Rightarrow 6 - 3\lambda + 2 + 2\lambda = 0$

$\Rightarrow \boxed{\lambda = 8}$ Ans (Note. Misprint in worksheet answer)

Ans 9 →

given $\vec{a}, \vec{b}, \vec{c}$ are unit vectors

$\Rightarrow |\vec{a}| = |\vec{b}| = |\vec{c}| = 1$

given $\vec{a}+\vec{b}+\vec{c} = \vec{0}$

$\Rightarrow |\vec{a}+\vec{b}+\vec{c}| = 0$

$\Rightarrow |\vec{a}+\vec{b}+\vec{c}|^2 = 0$

$\Rightarrow (\vec{a}+\vec{b}+\vec{c}) \cdot (\vec{a}+\vec{b}+\vec{c}) = 0$

$\Rightarrow |\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2\vec{a}\cdot\vec{b} + 2\vec{b}\cdot\vec{c} + 2\vec{c}\cdot\vec{a} = 0$

$\Rightarrow 1+1+1 + 2(\vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a}) = 0$

$\Rightarrow \boxed{\vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a} = -\frac{3}{2}}$ Ans

Ans 10 →

given $|\vec{a}| = |\vec{b}|$

$\theta = 60°$ & $\vec{a}\cdot\vec{b} = \frac{1}{2}$

we have $\cos\theta = \dfrac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|}$

$\Rightarrow \cos(60°) = \dfrac{1/2}{|\vec{a}||\vec{a}|}$ --- {since $|\vec{a}| = |\vec{b}|$}

$\Rightarrow$

$\cancel{1/2} = \dfrac{\cancel{1/2}}{|\vec{a}|^2}$

$\Rightarrow |\vec{a}|^2 = 1$

$\Rightarrow |\vec{a}| = 1$

$\therefore |\vec{a}| = |\vec{b}| = 1$ Ans

Q.N. 11 → given $|\vec{a}| = 3$, $|\vec{b}| = 4$, $|\vec{c}| = 5$

given $\vec{a} \perp (\vec{b}+\vec{c}) \Rightarrow \vec{a}\cdot(\vec{b}+\vec{c}) = 0$

$\vec{b} \perp (\vec{c}+\vec{a}) \Rightarrow \vec{b}\cdot(\vec{c}+\vec{a}) = 0$

$\vec{c} \perp (\vec{a}+\vec{b}) \Rightarrow \vec{c}\cdot(\vec{a}+\vec{b}) = 0$

we have $|\vec{a}+\vec{b}+\vec{c}|^2 = (\vec{a}+\vec{b}+\vec{c})\cdot(\vec{a}+\vec{b}+\vec{c})$

$= \vec{a}\cdot\vec{a} + \vec{a}\cdot(\vec{b}+\vec{c}) + \vec{b}\cdot\vec{b} + \vec{b}\cdot(\vec{a}+\vec{c})$
$+ \vec{c}\cdot\vec{c} + \vec{c}\cdot(\vec{a}+\vec{b})$

$= |\vec{a}|^2 + 0 + |\vec{b}|^2 + 0 + |\vec{c}|^2 + 0$

$= 9 + 16 + 25$

$\Rightarrow |\vec{a}+\vec{b}+\vec{c}|^2 = 50$

$\Rightarrow \boxed{|\vec{a}+\vec{b}+\vec{c}| = 5\sqrt{2}}$ Ans

Q.N. 12 → Given $\vec{a} = \hat{i} + 4\hat{j} + 2\hat{k}$; $\vec{b} = 3\hat{i} - 2\hat{j} + 7\hat{k}$

and $\vec{c} = 2\hat{i} - \hat{j} + 4\hat{k}$

let $\vec{d} = x\hat{i} + y\hat{j} + z\hat{k}$

given $\vec{d}$ is $\perp$ to both $\vec{a}$ & $\vec{b}$ & $\vec{c}\cdot\vec{d} = 15$

$\Rightarrow \vec{d}\cdot\vec{a} = 0$ & $\vec{d}\cdot\vec{b} = 0$ & $\vec{c}\cdot\vec{d} = 15$

$\Rightarrow x + 4y + 2z = 0$
$3x - 2y + 7z = 0$
$2x - y + 4z = 15$

Solving these equations, we get

$x = \frac{160}{3}$; $y = -\frac{5}{3}$, $z = -\frac{70}{3}$

$\therefore \vec{d} = \frac{1}{3}(160\hat{i} - 5\hat{j} - 70\hat{k})$ Ans (Note: Misprint in worksheet Ans)

Qn 13 →

let $\vec{\beta} = 6\hat{i} - 3\hat{j} - 6\hat{k}$

and $\vec{\alpha} = \hat{i} + \hat{j} + \hat{k}$

let $\vec{\beta} = \vec{\beta_1} + \vec{\beta_2}$

given $\vec{\beta_1}$ parallel to $\vec{\alpha}$ & $\vec{\beta_2}$ $\perp^r$ to $\vec{\alpha}$

we have $\vec{\beta_1} = \lambda\vec{\alpha}$ --- {since parallel}

$\Rightarrow \vec{\beta_1} = \lambda\hat{i} + \lambda\hat{j} + \lambda\hat{k}$

we have $\vec{\beta} = \vec{\beta_1} + \vec{\beta_2}$

$\Rightarrow \vec{\beta_2} = \vec{\beta} - \vec{\beta_1}$

$\Rightarrow \vec{\beta_2} = (6\hat{i} - 3\hat{j} - 6\hat{k}) - (\lambda\hat{i} + \lambda\hat{j} + \lambda\hat{k})$

$\Rightarrow \vec{\beta_2} = \hat{i}(6-\lambda) + \hat{j}(-3-\lambda) + \hat{k}(-6-\lambda)$

we have $\vec{\beta_2} \perp \vec{\alpha}$

$\Rightarrow \vec{\beta_2} \cdot \vec{\alpha} = 0$

$\Rightarrow (\hat{i}(6-\lambda) + \hat{j}(-3-\lambda) + \hat{k}(-6-\lambda)) \cdot (\hat{i} + \hat{j} + \hat{k}) = 0$

$\Rightarrow 6 - \lambda - 3 - \lambda - 6 - \lambda = 0$

$\Rightarrow \quad -3 - 3\lambda = 0$

$\Rightarrow \quad \boxed{\lambda = -1}$

$\therefore \vec{\beta_1} = -\hat{i} - \hat{j} - \hat{k}$

and $\vec{\beta_2} = 7\hat{i} - 2\hat{j} - 5\hat{k}$

Now $\quad \vec{\beta_1} + \vec{\beta_2} = (-\hat{i} - \hat{j} - \hat{k}) + (7\hat{i} - 2\hat{j} - 5\hat{k})$

$= 6\hat{i} - 3\hat{j} - 6\hat{k}$

$= \vec{\beta}$ verified

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