Topic : .... Date. : .... Page No. : ....

॥ जय श्री राधे कृष्णा ॥

1

← ULTIMATE MATHEMATICS →

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Chapter: A.O.D CLASS No: 5

Topic: 3 Maxima - Minima

Basic

- ✓ Given $f(x) =$
- ✓ Diff wrt x
- ✓ get $f'(x) =$
- ✓ Simplify $f'(x) =$
- ✓ put $f'(x) = 0$ (for Max/Min)
- ✓ get value of x

$x = x_1$, $x = x_2$

- ✓ Now find $f''(x)$

(·) $f''(x_1)$

If $f''(x_1) < 0$ then x_1 is the point of Maxima

and Maximum value $= f(x_1)$

(·) If $f''(x_2) > 0$; then x_2 is the point of Minima

and Minimum value $= f(x_2)$

WORD PROBLEMS

(·) Mensuration

- ✓ Given Condition: (Constant)
- ✓ To Max./to Min (Differentiate)
- ✓ to find / to prove

CLASSTIME

Qn. 1 Find two numbers whose sum is 24 and whose product is as large as possible.

Basic

Sol. Let the numbers are x & y

$\because$ $x+y=24$ .... (i) --(given)

Let $P \rightarrow$ product of two no-s

$P = xy$ .... (to be Max)

$P = x(24-x)$ --- {from (1)}

$\Rightarrow$ $P = 24x - x^2$

Diff wrt x

$\frac{dP}{dx} = 24 - 2x$

for ~~For~~ Max/Min

Put $\frac{dP}{dx} = 0$

$\Rightarrow$ $24 - 2x = 0$ $\boxed{x = 12}$

Diff again wrt x

$\frac{d^2P}{dx^2} = -2$

$\left(\frac{d^2P}{dx^2}\right)_{x=12} = -2 < 0$

$\therefore$ Product is Maximum at $x = 12$

put in (i)

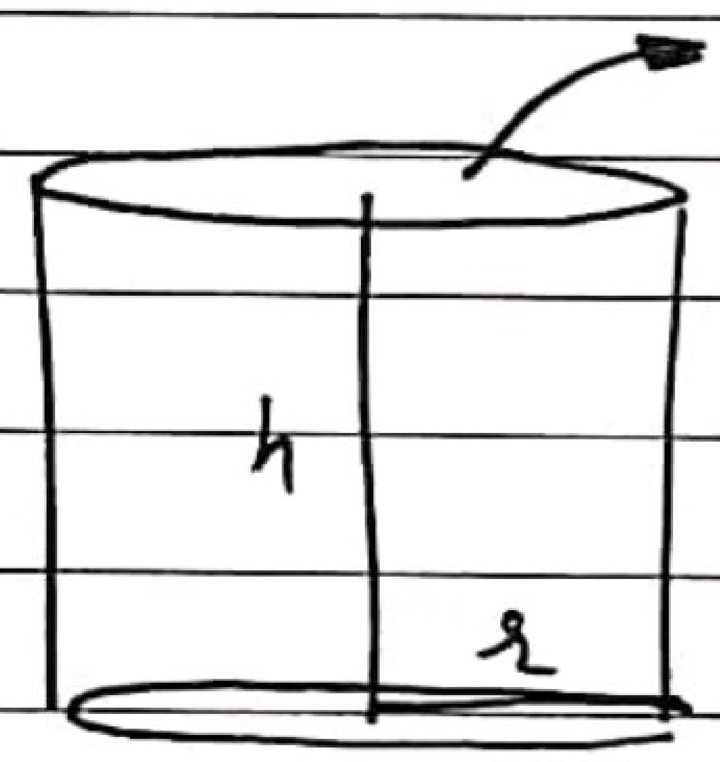
$x + y = 24$

$12 + y = 24$

$y = 12$

$\therefore$ two numbers are 12 & 12 Ans

Que 2 → Show that the height of a cylinder, which is open at the top, having a given surface area and greatest volume, is equal to the radius of its base.



Soln: ∴ let $h \rightarrow$ height of cylinder

$r \rightarrow$ radius of cylinder

∴ let $S \rightarrow$ T.S.A of cylinder

$$S = 2\pi r h + \pi r^2 \quad \text{---(given)} \quad \text{--- (i)}$$

∴ let $V \rightarrow$ volume of cylinder

$$V = \pi r^2 h \quad \text{--- (to be Max)}$$

$$V = \cancel{\pi} r^{\cancel{2}}\left(\frac{S - \pi r^2}{2\cancel{\pi}\cancel{r}}\right) \quad \text{--- \{from eq (i)\}}$$

$$V = \frac{1}{2}\left(Sr - \pi r^3\right)$$

Diff w.r.t r

$$\frac{dV}{dr} = \frac{1}{2}\left(S - 3\pi r^2\right)$$

for Max/Min put $\frac{dV}{dr} = 0$

$$\boxed{S = 3\pi r^2} \quad (\text{or}) \quad \boxed{r = \sqrt{\frac{S}{3\pi}}}$$

Diff again w.r.t 'r'

$$\frac{d^2V}{dr^2} = \frac{1}{2}\left(-6\pi r\right) = -3\pi r$$

$$\left(\frac{d^2V}{dr^2}\right)_{r=\sqrt{\frac{S}{3\pi}}} = -3\pi\frac{\sqrt{S}}{\sqrt{3\pi}} = -\sqrt{3\pi}\sqrt{S} < 0$$

$\therefore$ volume of the cylinder is Max at $r = \sqrt{\frac{S}{3\pi}}$

put $S = 3\pi r^2$ in eq(i)

$$3\pi r^2 = 2\pi rh + \pi r^2$$

$$\Rightarrow 2\pi r^2 = 2\pi rh$$

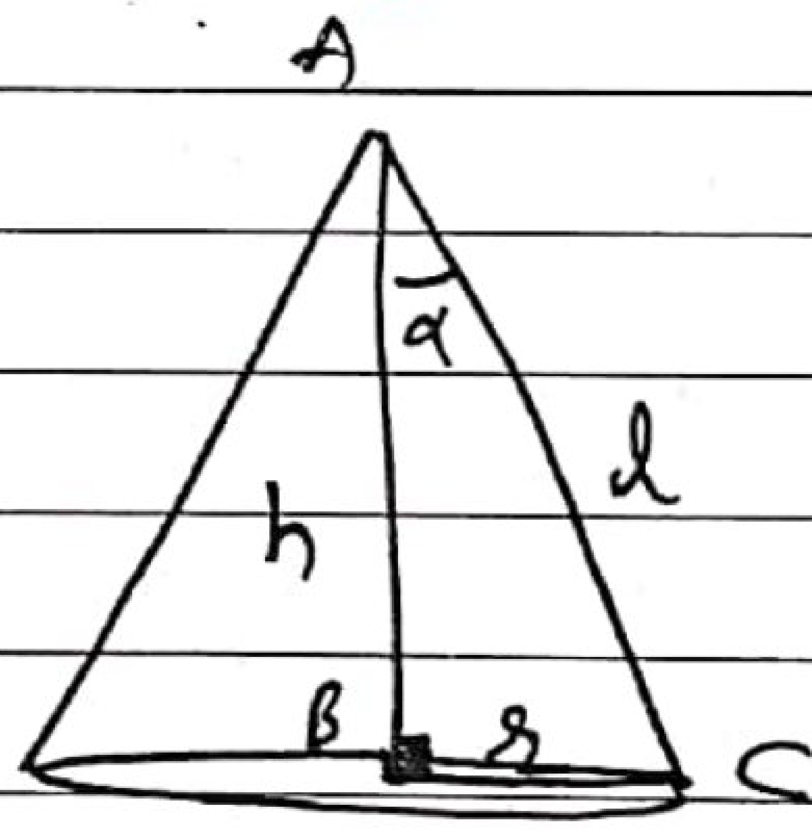
$$\Rightarrow r^2 = rh$$

$$\Rightarrow \boxed{r = h}$$

$\therefore$ hight of cylinder is equal to radius of its base

Ans

Ques 3 → Show that the semi-vertical angle of a cone of maximum volume and given slant height is $\tan^{-1}\sqrt{2}$.



Soln

let $h \rightarrow$ hyht of ~~cy~~ cone

$r \rightarrow$ radius of cone

let $l \rightarrow$ slant hyht of cone

$$l^2 = h^2 + r^2 \quad \text{-----(i)} \quad (\text{given})$$

$V \rightarrow$ volume of cone

$$V = \frac{1}{3}\pi r^2 h \quad \text{---} \quad (\text{to be Max})$$

$$V = \frac{1}{3}\pi(l^2 - h^2)h \quad \text{---} \{\text{from eq(i)}\}$$

$$V = \frac{1}{3}\pi(l^2 h - h^3)$$

Diff wrt h

$$\frac{dV}{dh} = \frac{1}{3}\pi(l^2 - 3h^2)$$

CLASSTIME

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for Max/Min put $\frac{dv}{dh} = 0$

$$\Rightarrow \frac{1}{3}\pi(l^2 - 3h^2) = 0$$

$$\Rightarrow l^2 = 3h^2$$

$$\boxed{l = \sqrt{3}h} \quad (or) \quad \boxed{h = \frac{l}{\sqrt{3}}}$$

Diff again w.r.t h

$$\frac{d^2v}{dh^2} = \frac{1}{3}\pi(-6h) = -2\pi h$$

$$\left(\frac{d^2v}{dh^2}\right)_{h=\frac{l}{\sqrt{3}}} = -2\pi\left(\frac{l}{\sqrt{3}}\right) < 0$$

$\therefore$ volume of cone is Max. at $h = \frac{l}{\sqrt{3}}$

from eq (1)

put $l = \sqrt{3}h$ in eq (1)

$$\Rightarrow 3h^2 = h^2 + r^2$$

$$\Rightarrow 2h^2 = r^2$$

$$\Rightarrow \boxed{\sqrt{2}h = r}$$

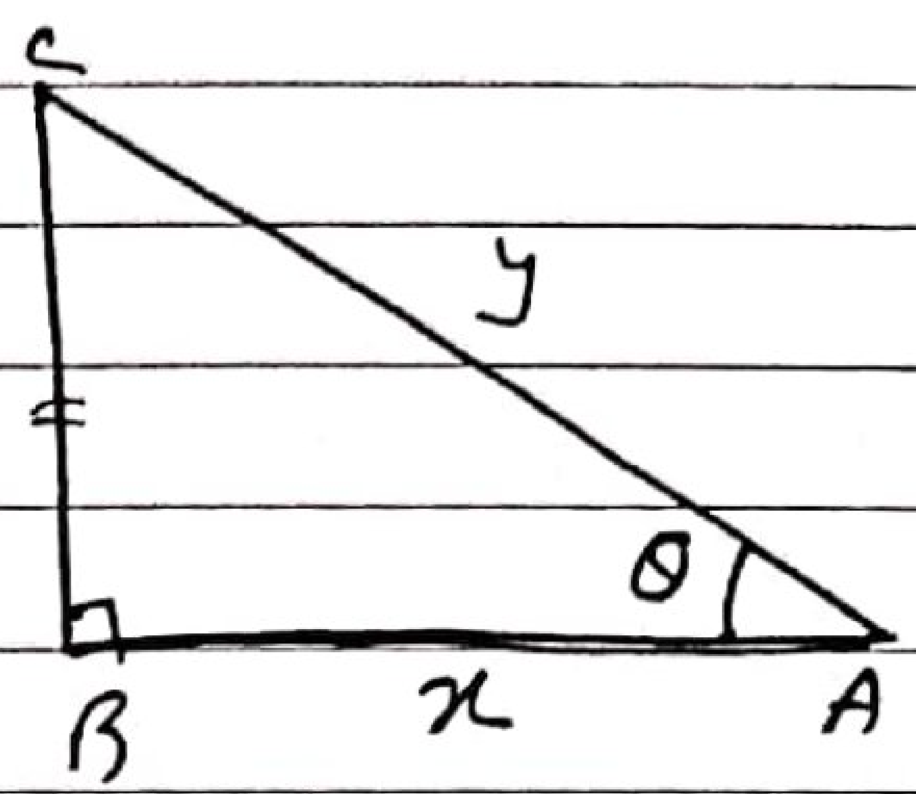
for ΔABC $\tan\alpha = \frac{r}{h}$

$$\tan\alpha = \frac{\sqrt{2}h}{h}$$

$$\Rightarrow \boxed{\alpha = \tan^{-1}\sqrt{2}} \quad \underline{\text{Ans}}$$

Ques 4 → If the sum of the lengths of the hypotenuse and a side of a right angled triangle is given, show that the area of the triangle is maximum when the angle between them is $\pi/3$.

Solution



let $x \to$ Base

$y \to$ hypotenuse

(i) let $S \to$ Sum

$\therefore S = x + y$ --- (given) --- (i)

(ii) $A \to$ Area of ΔABC

$$A = \frac{1}{2} x \sqrt{y^2 - x^2} \quad \text{--- (to be Max)}$$

$$A = \frac{1}{2} x \sqrt{(S-x)^2 - x^2} \quad \text{--- \{from eq (i)\}}$$

$$A = \frac{1}{2} x \sqrt{S^2 - 2Sx}$$

Squaring $A^2 = \frac{1}{4} x^2 (S^2 - 2Sx)$

Imp: let $A^2 = Z$

then Area is Max/Min as according to Z is Max/Min

$$Z = \frac{1}{4}(x^2 S^2 - 2S x^3)$$

Diff wrt x

$$\frac{dZ}{dx} = \frac{1}{4}(2xS^2 - 6Sx^2)$$

for Max/Min put $\frac{dZ}{dx} = 0$

$$2xS^2 = 6Sx^2$$

$\boxed{S = 3x}$ or $\boxed{x = S/3}$

Diff again wrt x

$$\frac{d^2Z}{dx^2} = \frac{1}{4}(2S^2 - 12Sx)$$

$$\left(\frac{d^2Z}{dx^2}\right)_{x=S/3} = \frac{1}{4}(2S^2 - 4S^2) = -\frac{S^2}{2} < 0$$

Topic : .... Date. : .... Page No. : 7

$\therefore$ Z is Max. at $x = 5/3$

Since $Z = A^2$

$\therefore$ Area of triangle is Max at $x = 5/3$

Put $y = 3x$ in eq (1)

$3x = x + y$

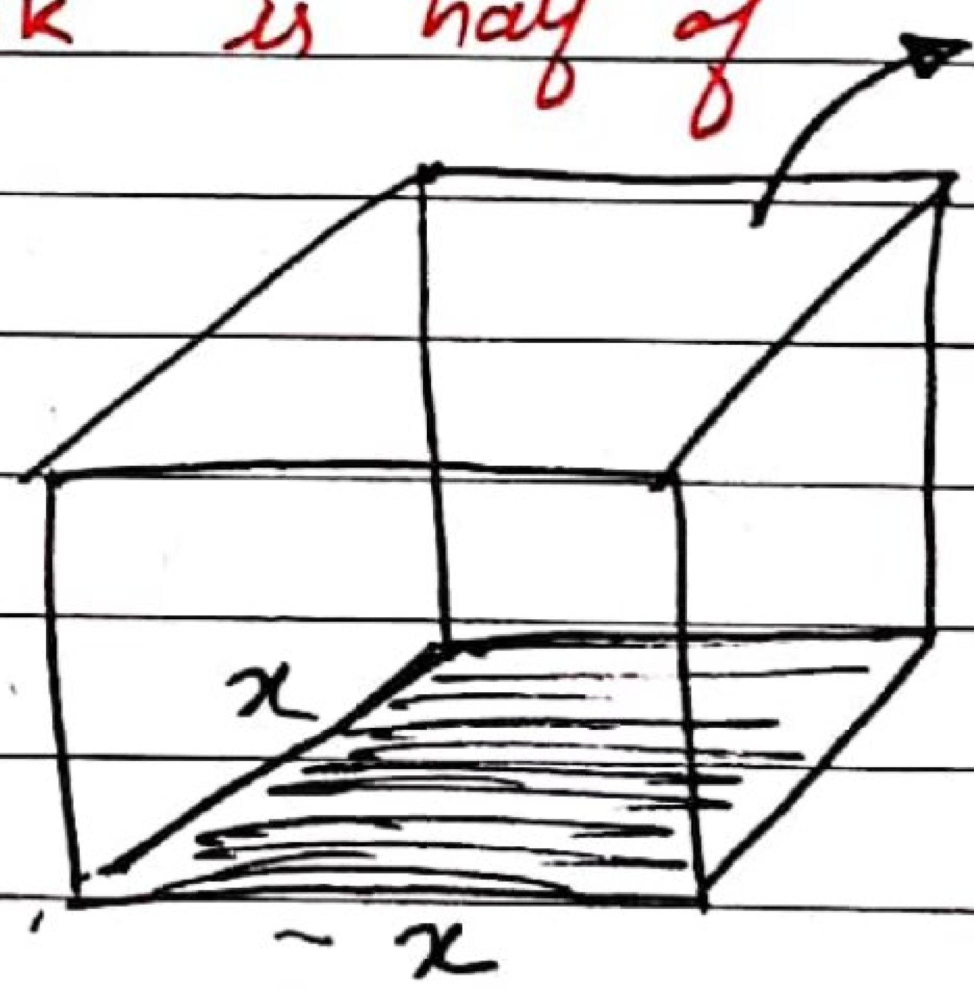
$\boxed{2x = y}$

ΔABC $\cos\theta = \frac{x}{y} = \frac{x}{2x} = \frac{1}{2}$

$\Rightarrow \cos\theta = 1/2$

$\boxed{\theta = \pi/3}$ Ans

Qn 5 → An open tank with a square base and vertical sides is to be constructed from a metal sheet so as to hold a given quantity of water. Show that the cost of the material will be "least" when depth of the tank is half of its width.



Soln Let $x \to$ length & breadth

$y \to$ height of the tank

(i) $V \to$ volume of tank / cuboid

$V = x^2 y$ --- (given) -- (i)

(ii) $S \to$ S.A of tank / cuboid

$S = lb + 2bh + 2hl$

$S = x^2 + 2xy + 2xy$

CLASSTIME

$\Rightarrow \quad S = x^2 + 4xy \quad \ldots$ (to be Min)

$S = x^2 + 4x\left(\frac{V}{x^2}\right) \quad \ldots$ (from (i))

$S = x^2 + \frac{4V}{x}$

Diff w.r.t x

$\frac{dS}{dx} = 2x - \frac{4V}{x^2}$

for Max/Min, put $\frac{dS}{dx} = 0$

$2x = \frac{4V}{x^2}$

$\boxed{x^3 = 2V} \quad \boxed{x = (2V)^{1/3}}$

diff again w.r.t x

$\frac{d^2S}{dx^2} = 2 + \frac{8V}{x^3}$

$\left(\frac{d^2S}{dx^2}\right)_{x^3 = 2V} = 2 + \frac{8V}{2V} = 6 > 0$

$\therefore$ Surface area of ~~the~~ the tank is Min

$\therefore$ Cost of the tank is least at $x^3 = 2V$

put ~~at~~ $V = \frac{x^3}{2}$ in eq (i)

$\frac{x^3}{2} = x^2 y$

$\frac{x}{2} = y$

$\therefore$ ~~depth~~ depths of the tank $= \frac{1}{2}$ of its width

Ans