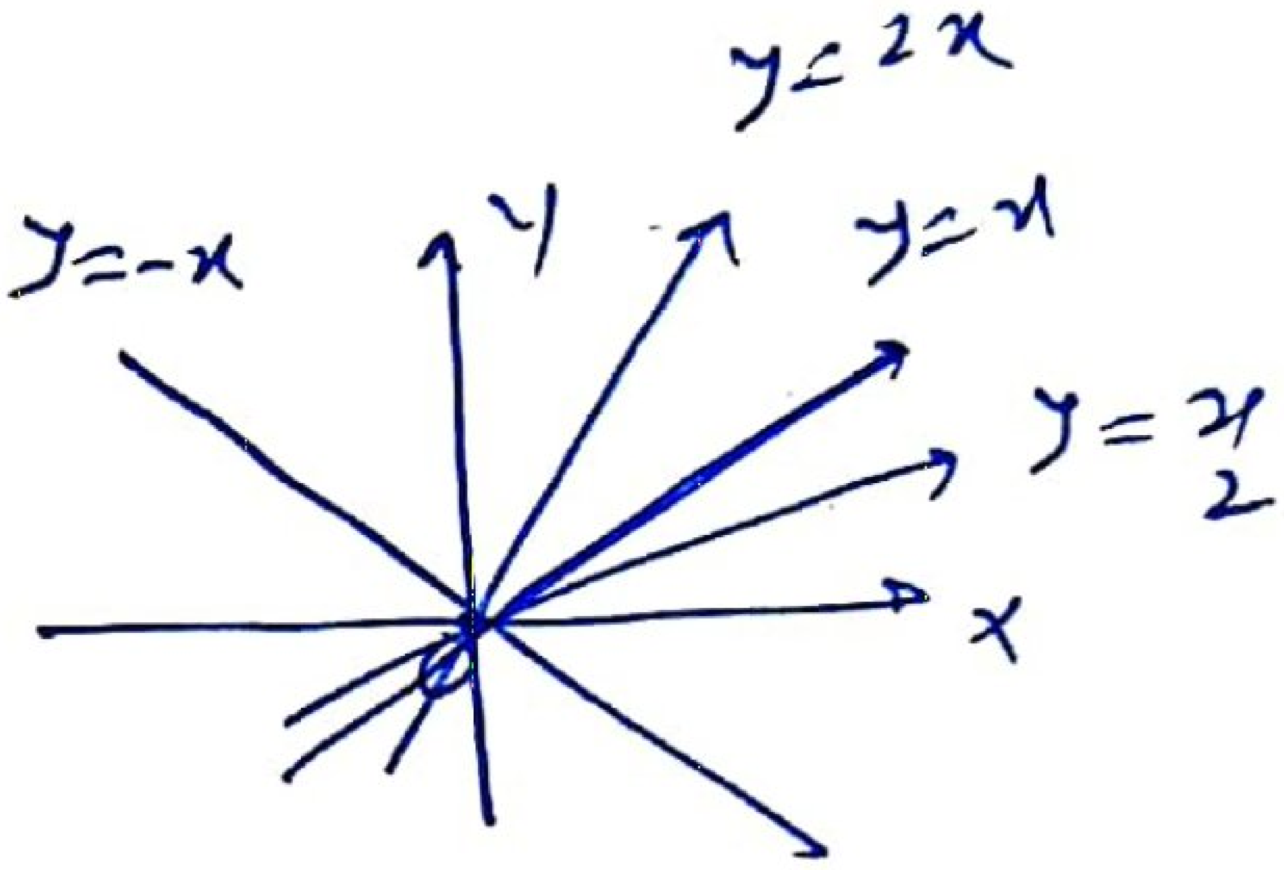
!! जय श्री राधे कृष्णा, जय श्री गिरिराज जी महाराज !! 1

ULTIMATE MATHEMATICS: BY AJAY MITTAL

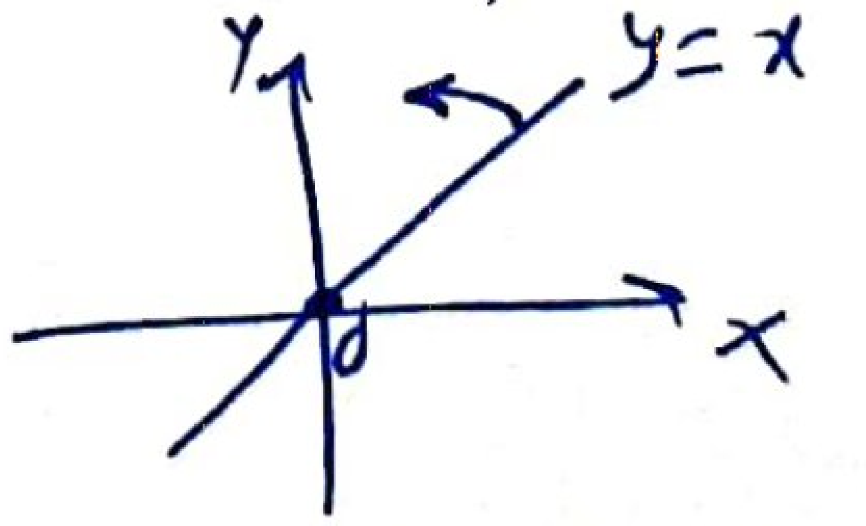
CHAPTER : $\boxed{\text{A}\cdot\text{O}\cdot\text{I}}$ (Areas) CLASS NO: 1

1) Straight-lines

- ✓ $2x+3y=6$ $(0,2)$ $(3,0)$
- ✓ $x=2$; $\|^{l}$ to y-axis
- ✓ $y=3$; line $\|^{l}$ to x-axis
- ✓ $y=x$; pass (0,0)
- ✓ $y=-x$; $\boxed{y=mx}$
- $y=2x$
- $y=\frac{x}{2}$

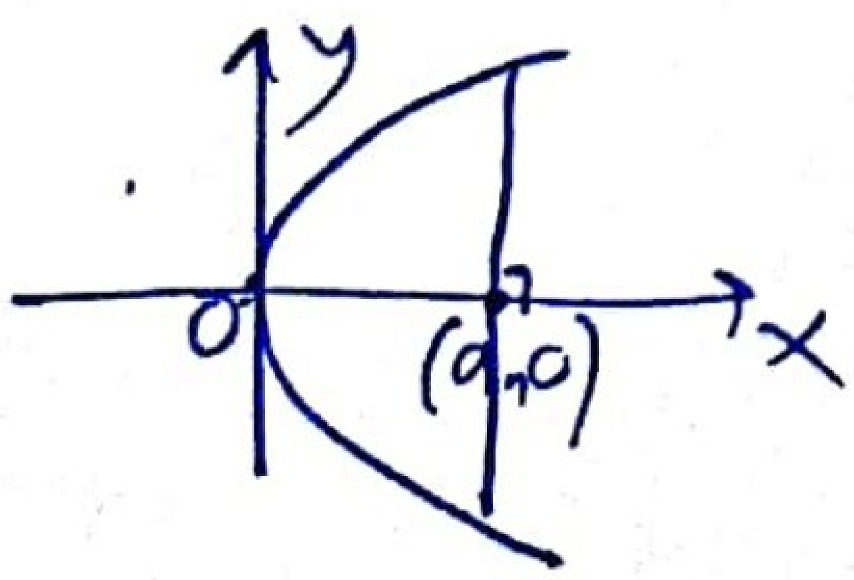


- ✓ $y=0$; equation of x-axis
- ✓ $x=0$; equation of y-axis
- ✓ $2x+3y\le 6$; $0\le 6$ (true) → towards origin
- $2x+3y\ge 6$; $0\ge 6$ (false) → away from (0,0)
- ✓ $y\ge x$

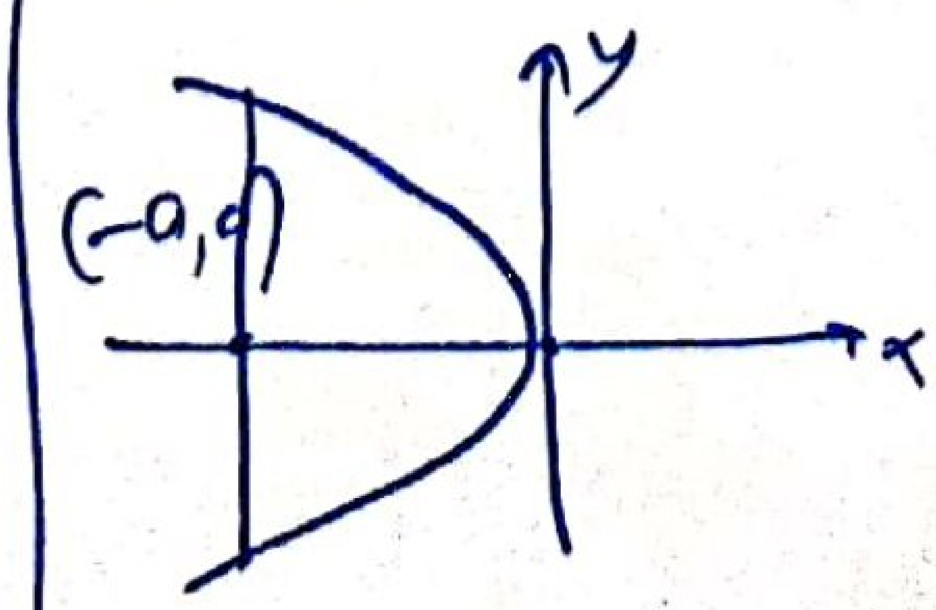


2) PARABOLA (Standard) vertex (0,0)

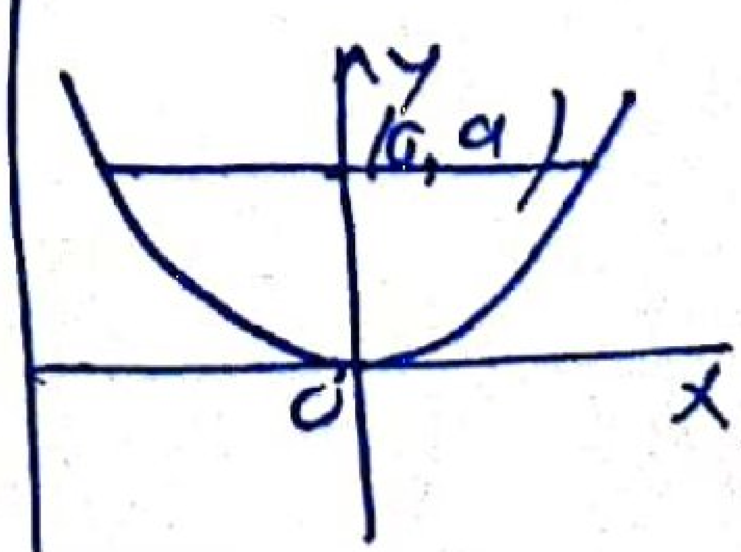
$y^2=4ax$



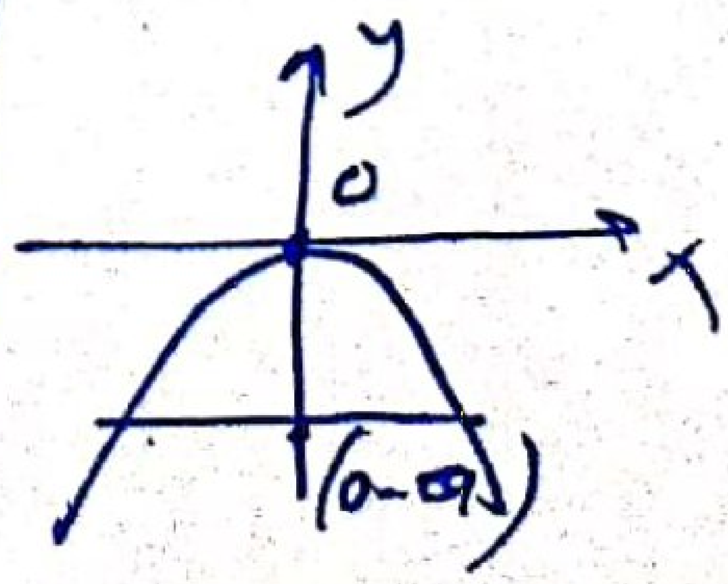
$y^2=-4ax$



$x^2=4ay$



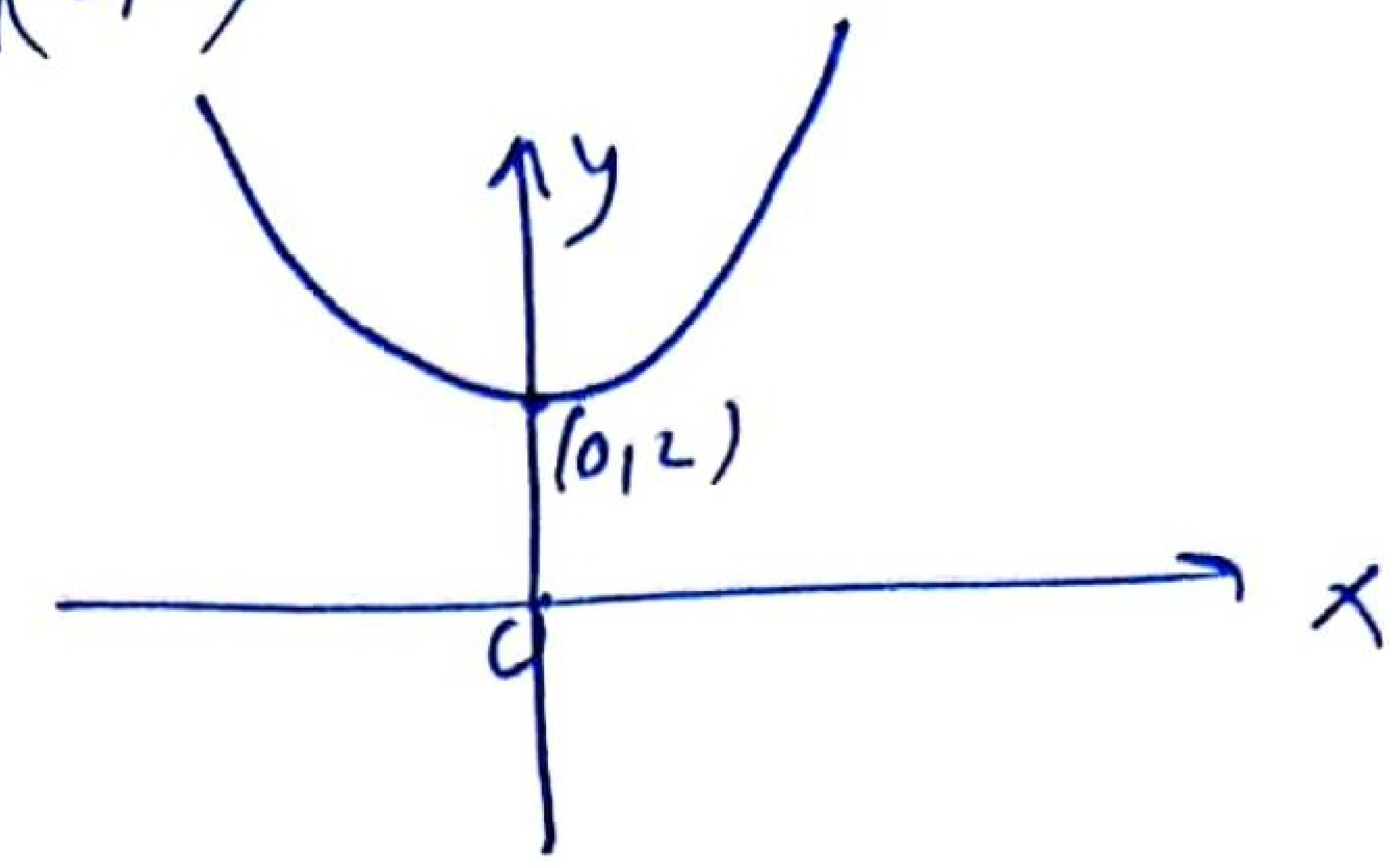
$x^2=-4ay$



Shifting parabola   vertex $\neq (0,0)$

(·) $x^2 = y - 2$

- vertex $(0, 2)$
- face open towards y-axis

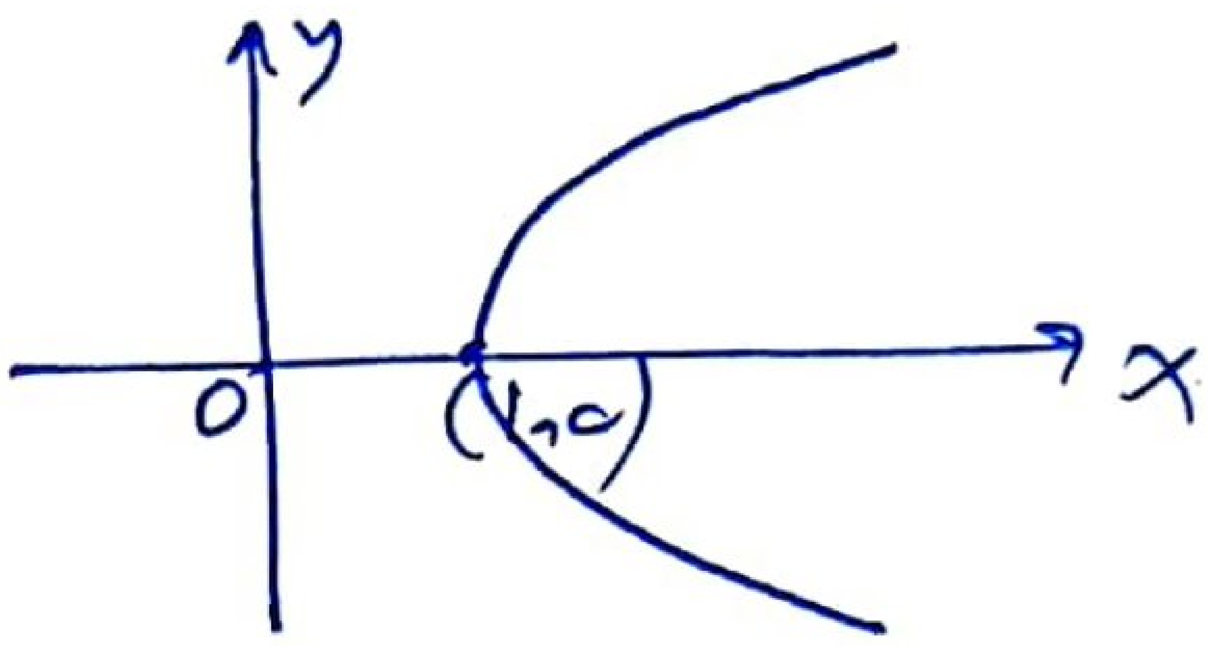


(·) $x = y^2 + 1$

$y^2 = x - 1$

vertex $(1, 0)$

face open +ve X-axis



(·) $\cancel{y^2} \quad y^2 = x + 2y$

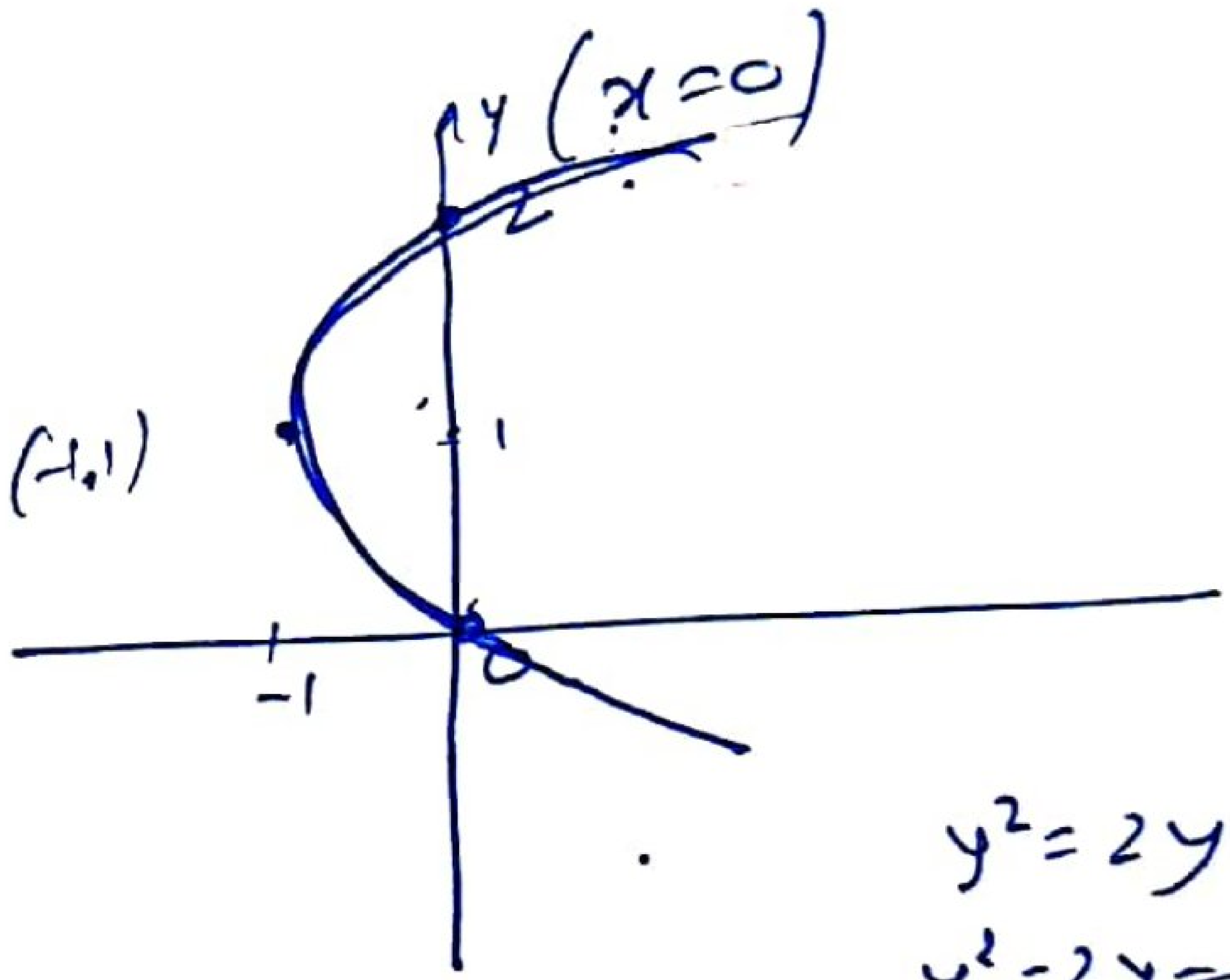
$\cancel{y^2} \quad y^2 - 2y = x$

$(y-1)^2 - 1 = x$

$(y-1)^2 = (x+1)$

vertex $(-1, 1)$

face open +ve x-axis



$y^2 = 2y$

$y^2 - 2y = 0$

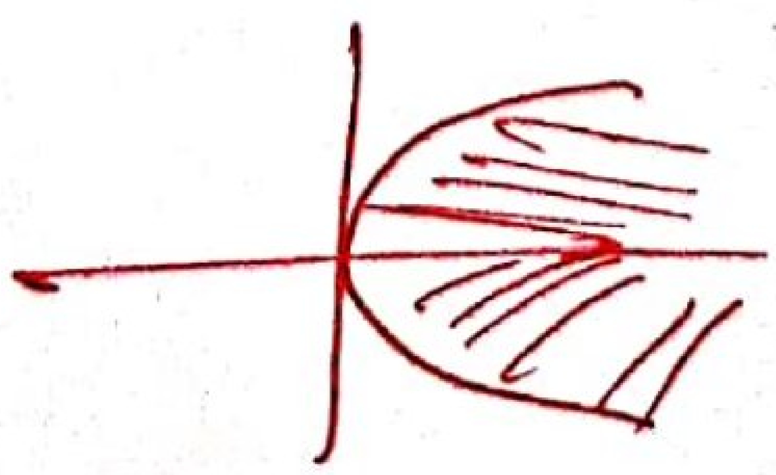
$y(y-2) = 0$

$y = 0, \ y = 2$

(·) Solution

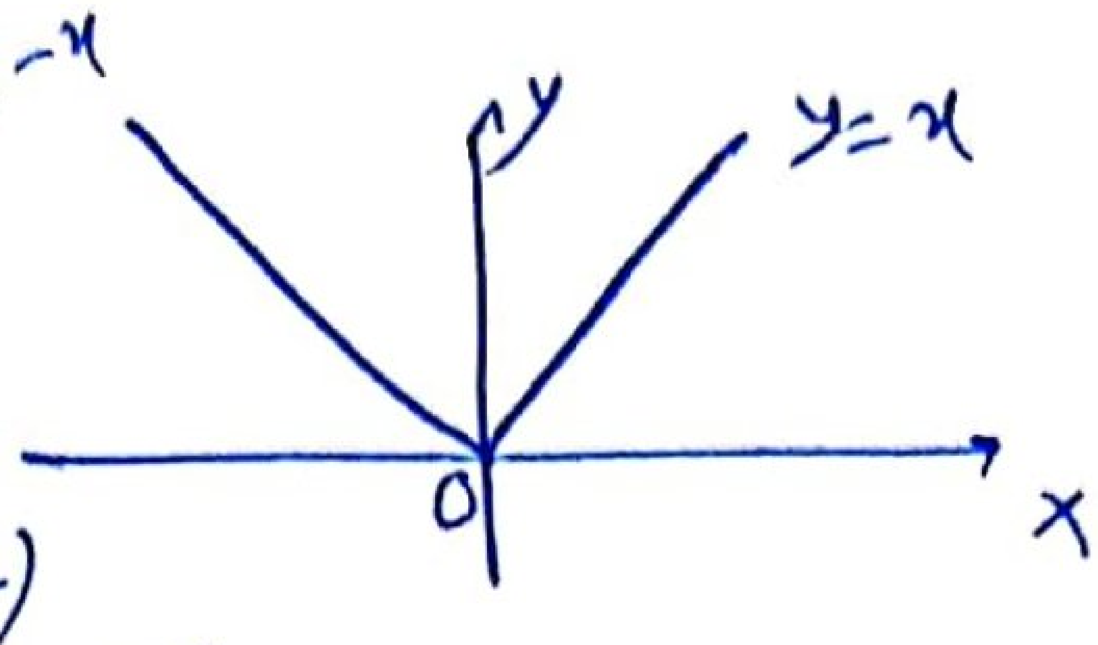
- $y^2 \geq 4ax$ (outside the parabola)
- $y^2 \leq 4ax$ (Inside the parabola)

(·) Symmetry curve

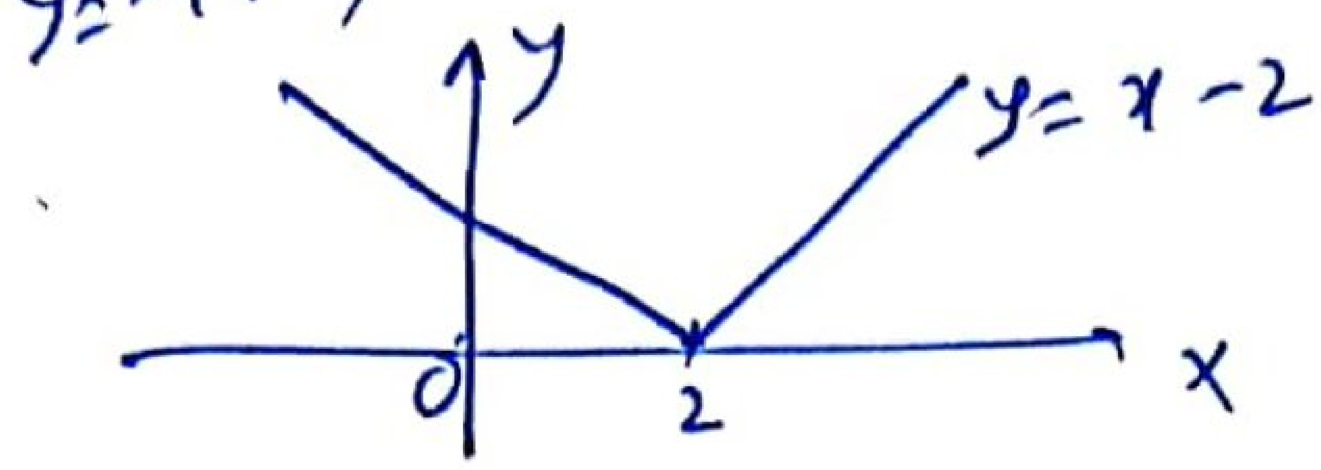


(3) Modulus

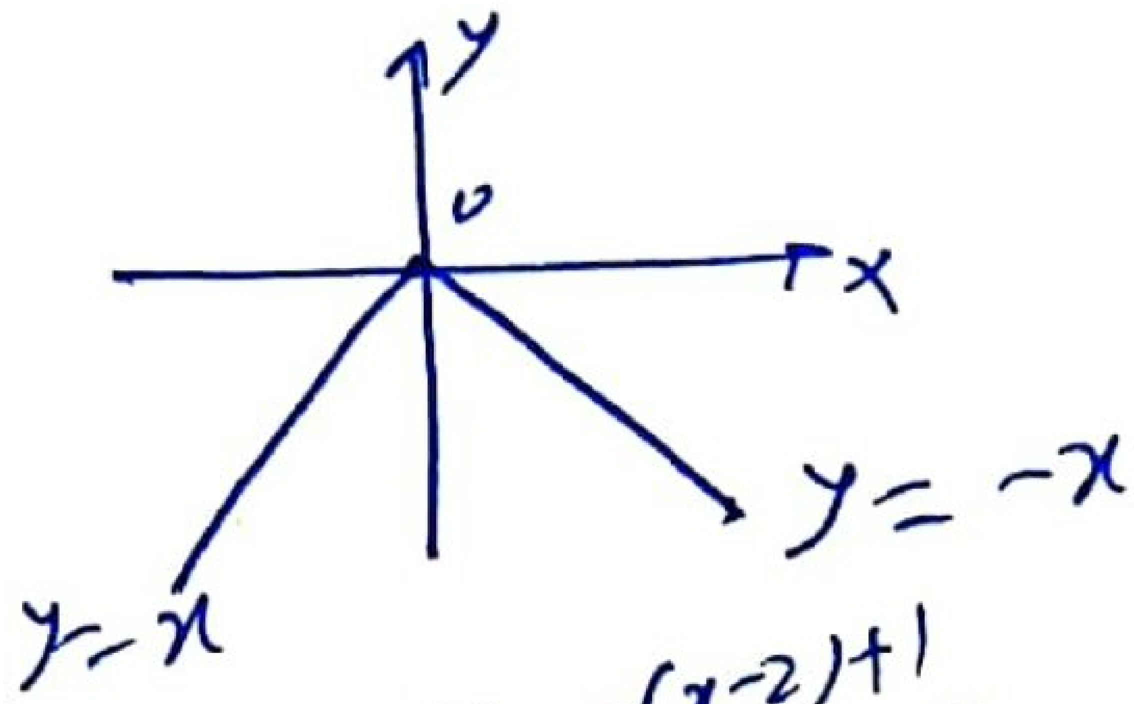
(i) $y = |x|$ (0)



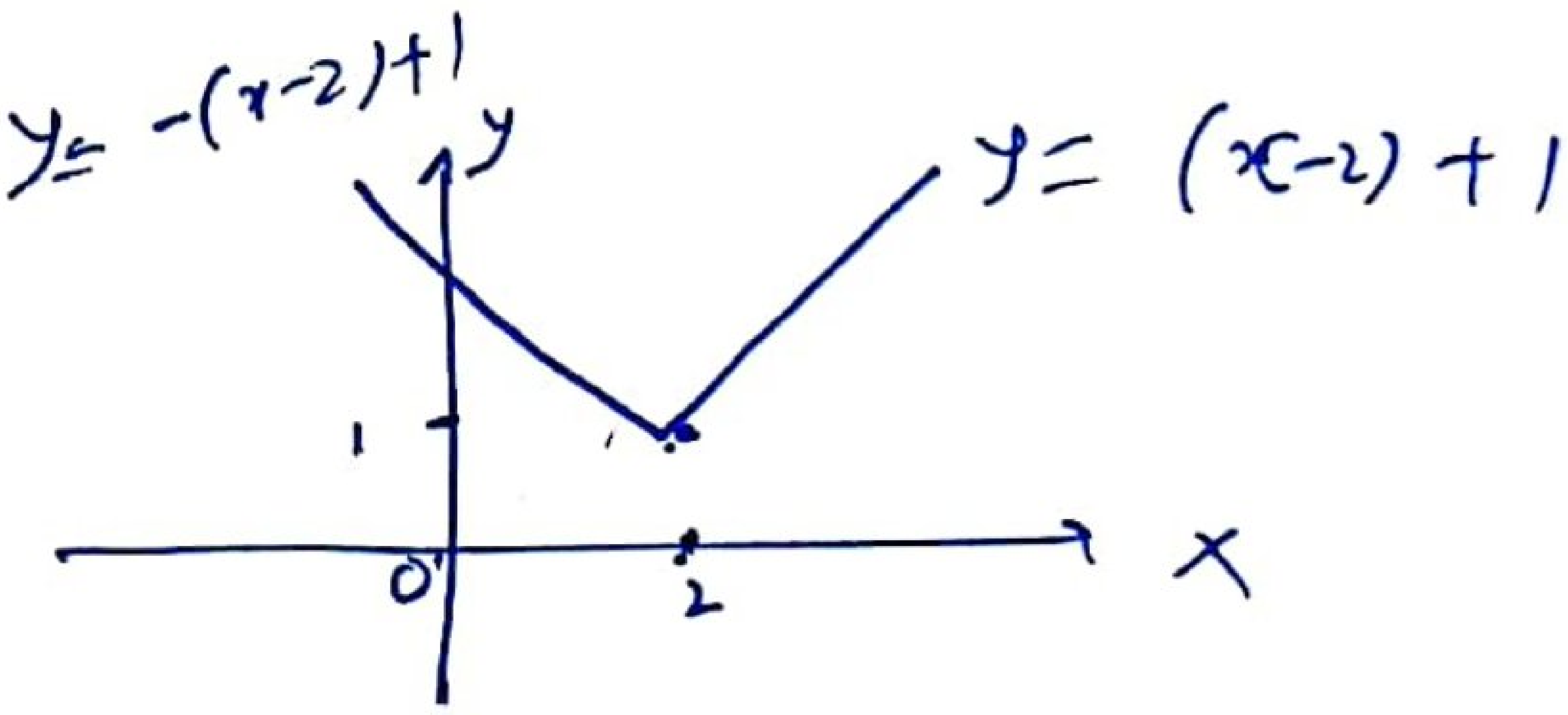
(ii) $y = |x-2|$ (2)



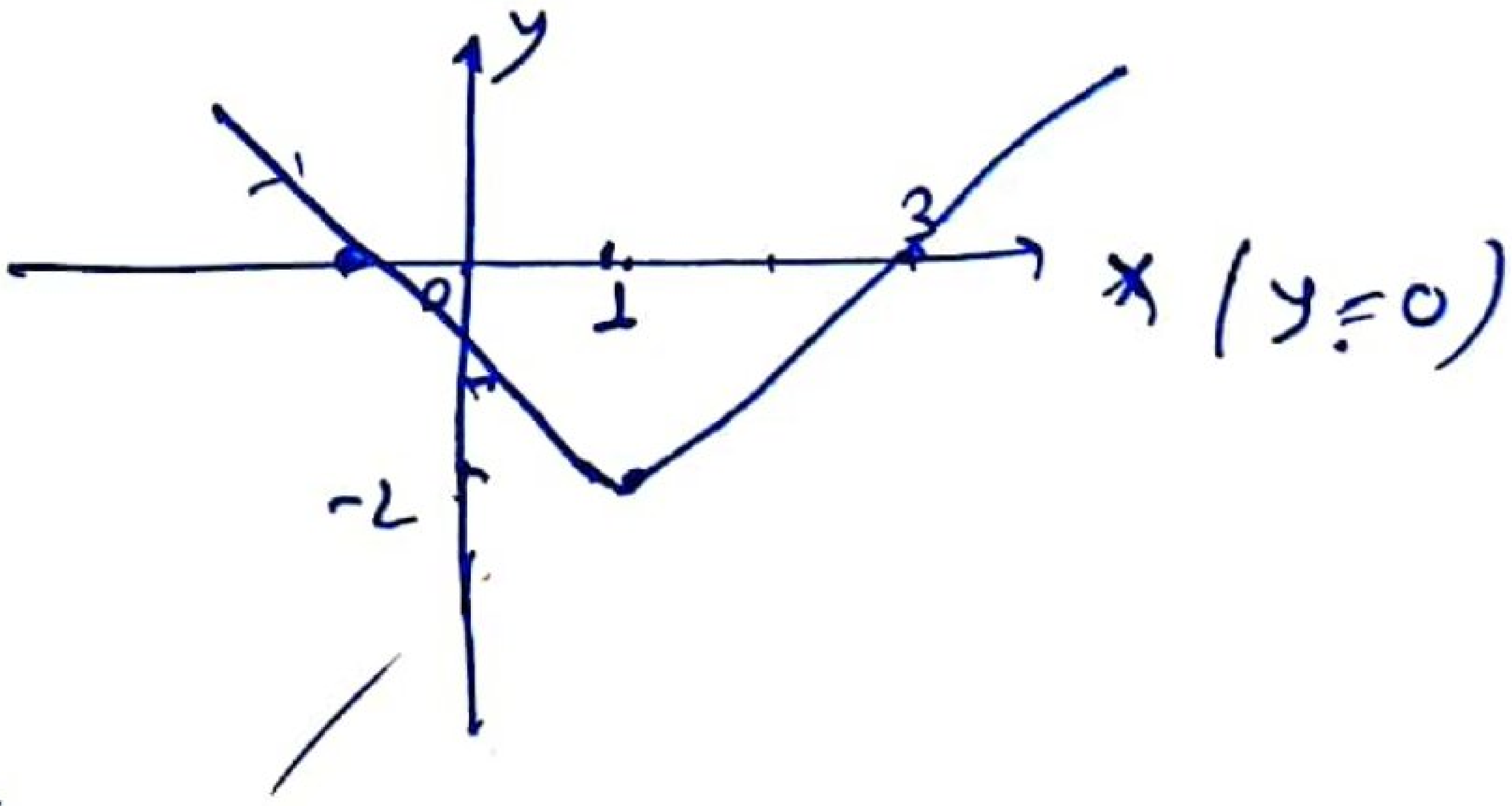
(iii) $y = -|x|$ (0)



(iv) $y = |x-2| + 1$ (2)



(v) $y = |x-1| - 2$



$0 = |x-1| - 2$

$|x-1| = 2$

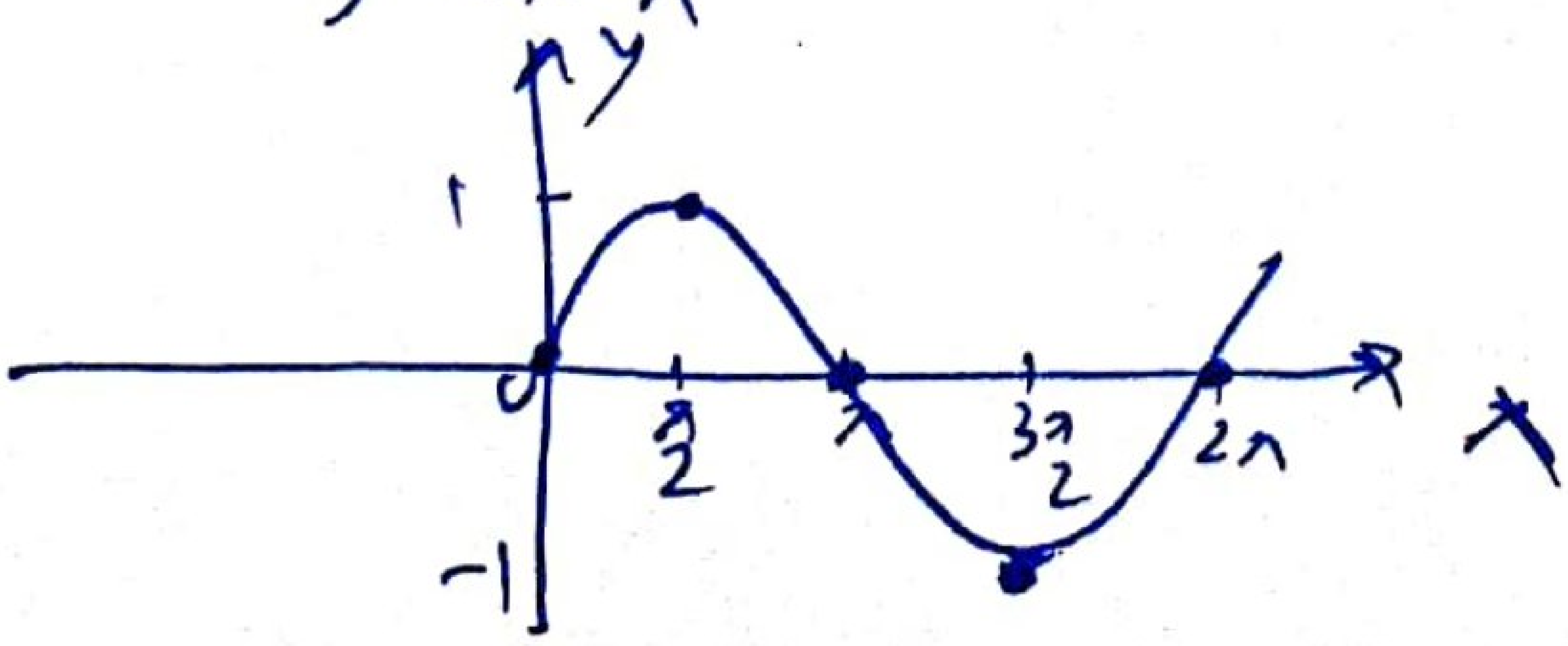
$x - 1 = \pm 2$

$x-1=2 \quad | \quad x-1=-2$

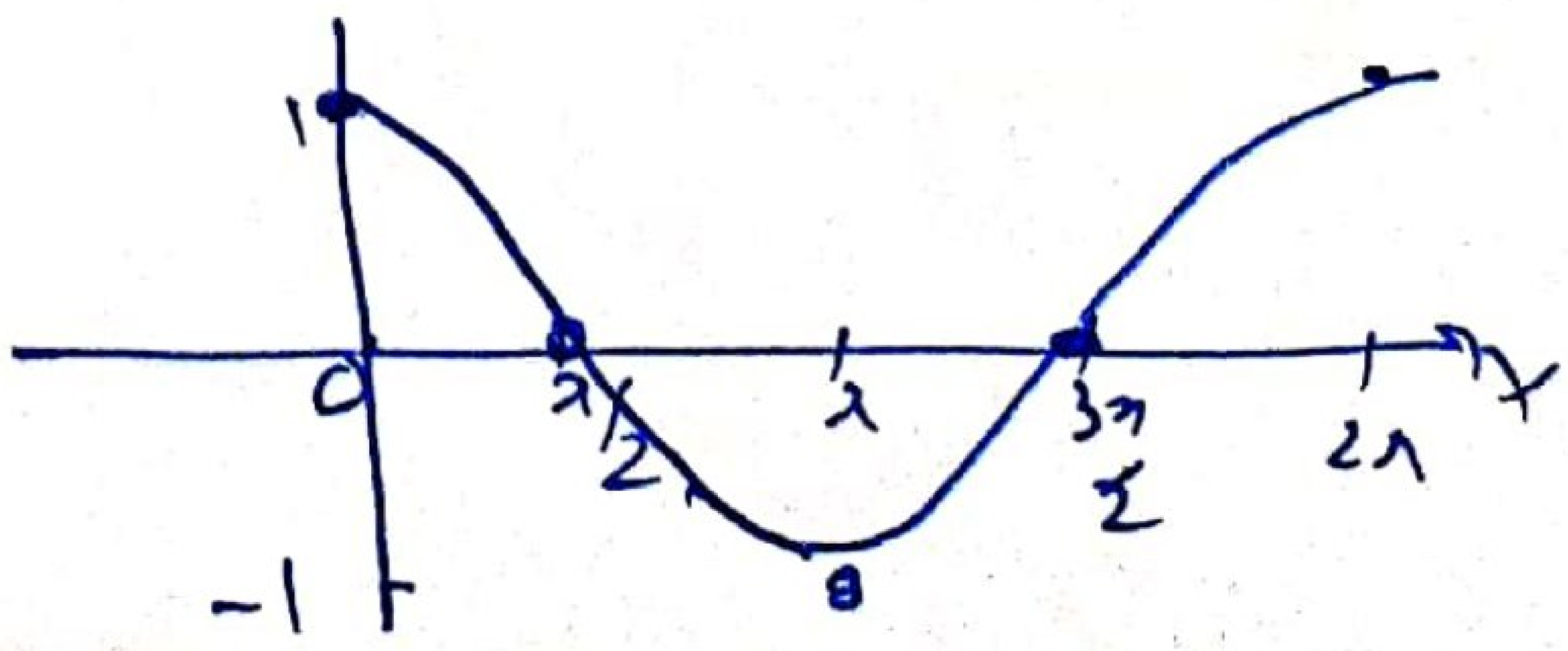
$x=3 \quad | \quad x=-1$

(4) Trigo graph

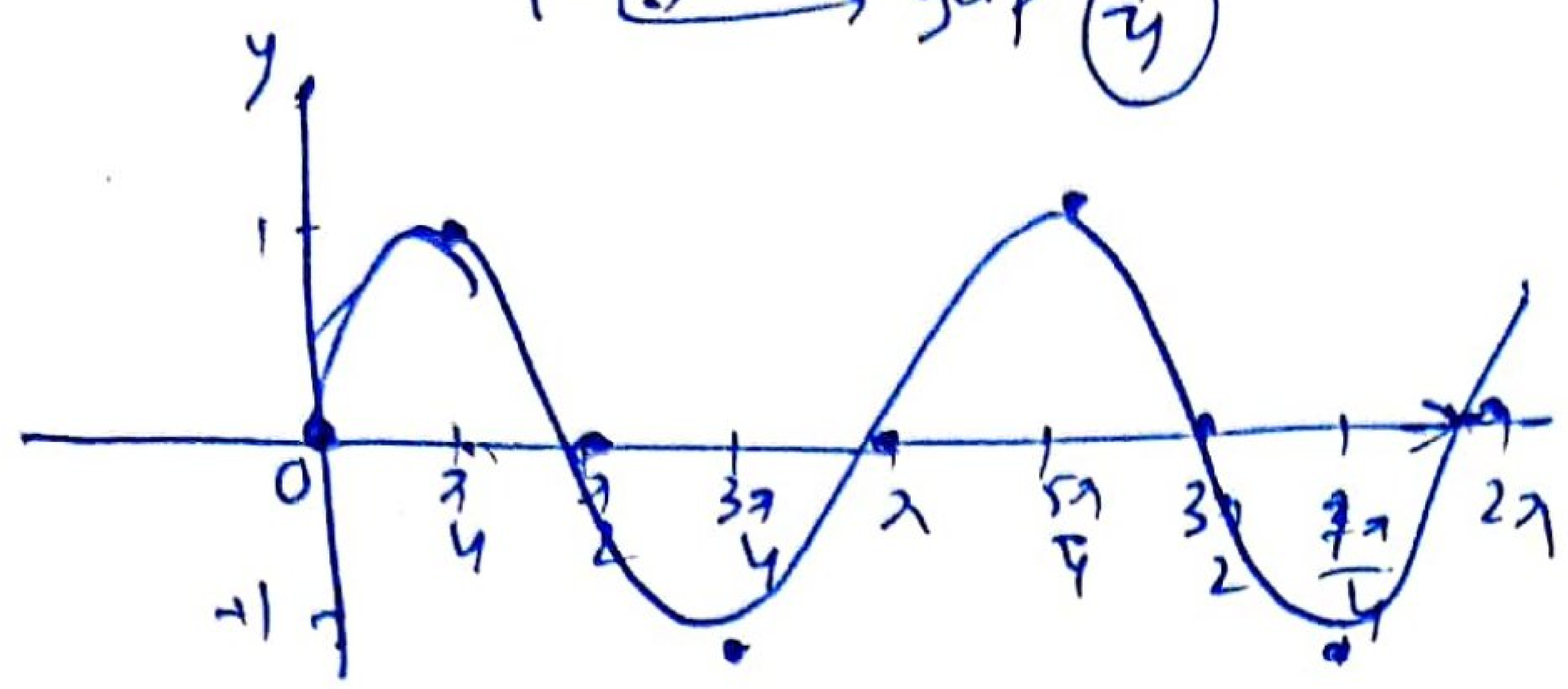
$y = \sin x$



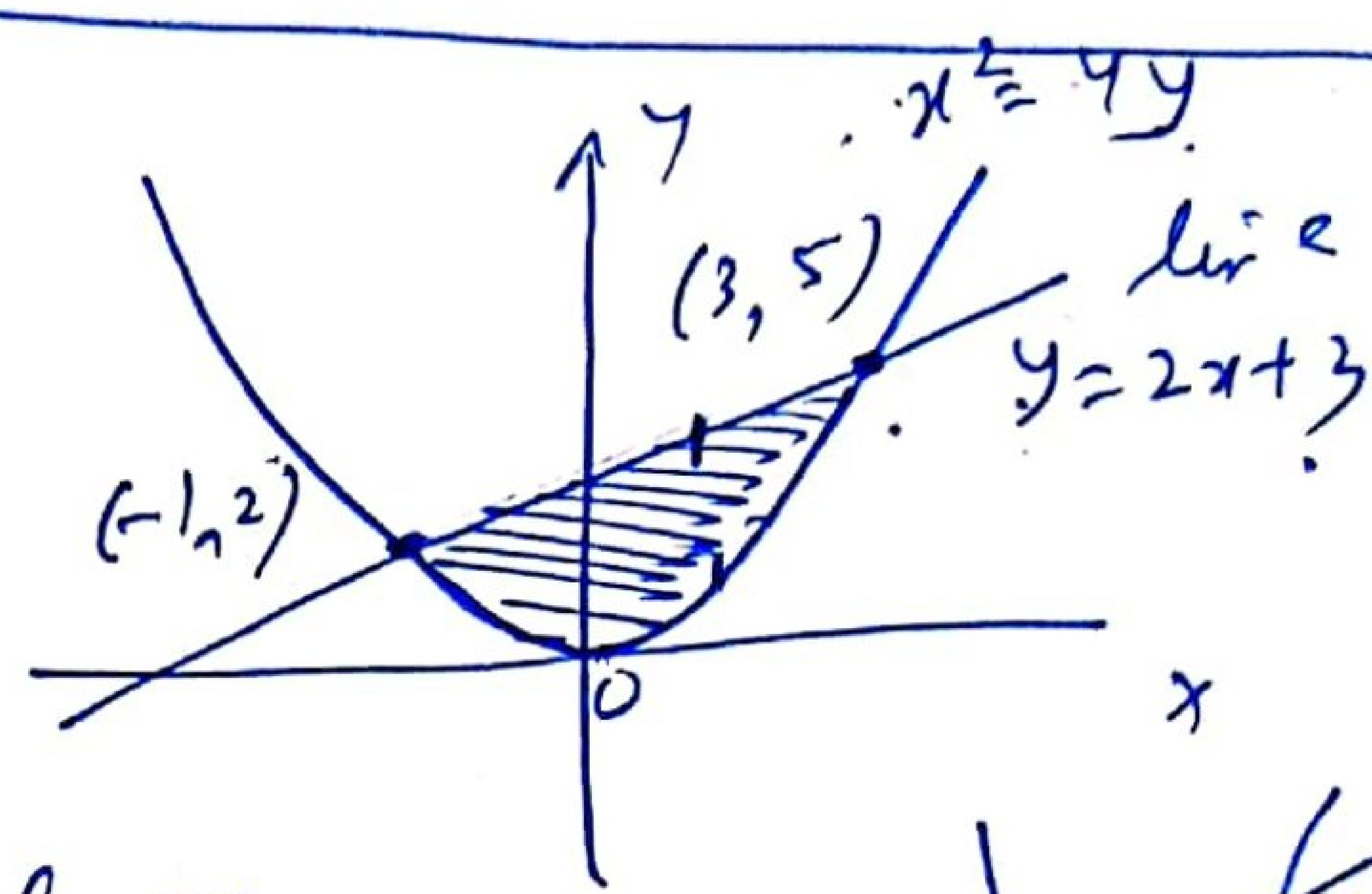
$y = \cos x$



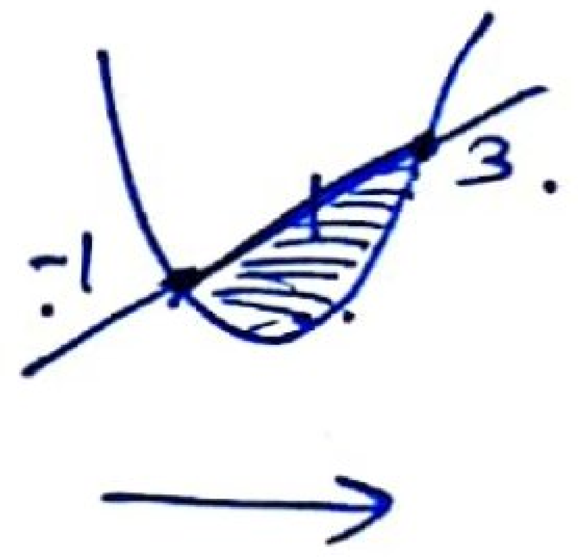
$y = \sin(2x) \longrightarrow$ gap $\left(\frac{\pi}{4}\right)$



· Basic given curves
 ✓ parabola
 ✓ line



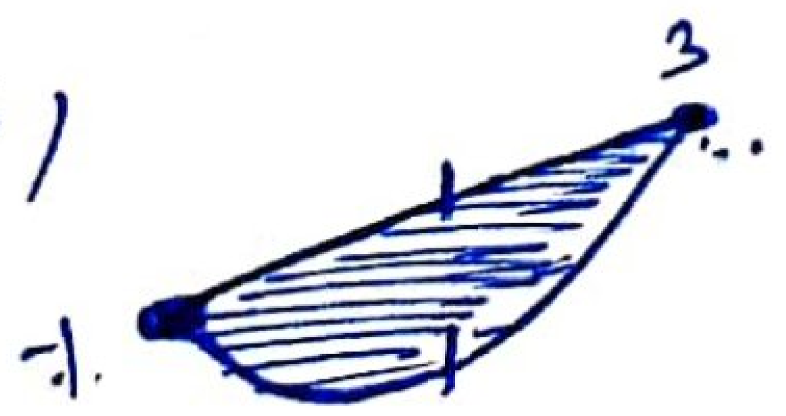
(1) plot the given curve
(2) shade the Required / bounded area
(3) Get the Intersection points
(4) decide $? \longleftrightarrow ?$
 $-1 \longleftrightarrow 3$



(5) Integrate
$$\int_{-1}^{3} (\text{upper curve ka } y) - (\text{lower curve ka } y)\, dx$$

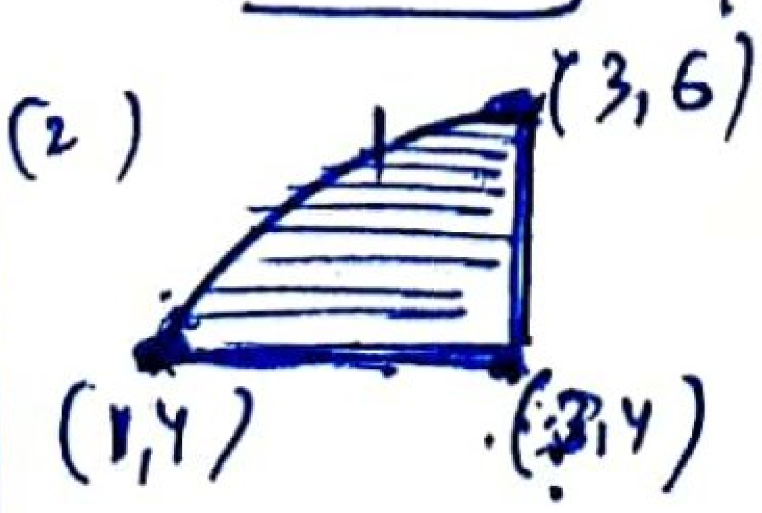
$$= \int_{-1}^{3} (2x+3) - \frac{x^2}{4}\, dx$$

$$= \boxed{50} \text{ square unit}$$

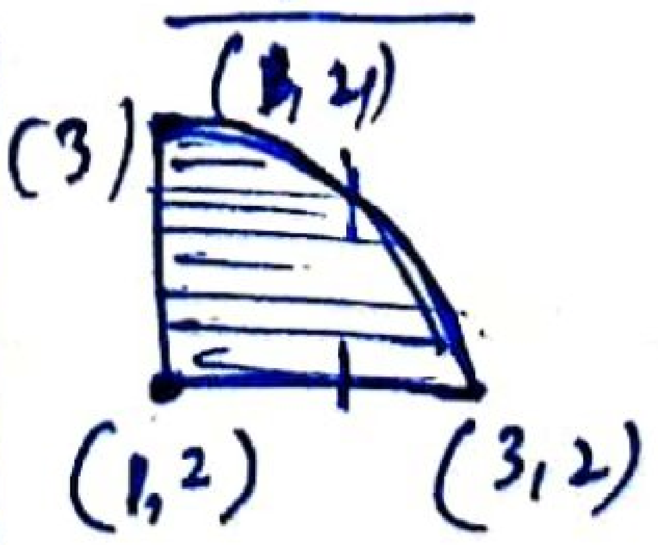
(1)


$\int_{-1}^{3}$

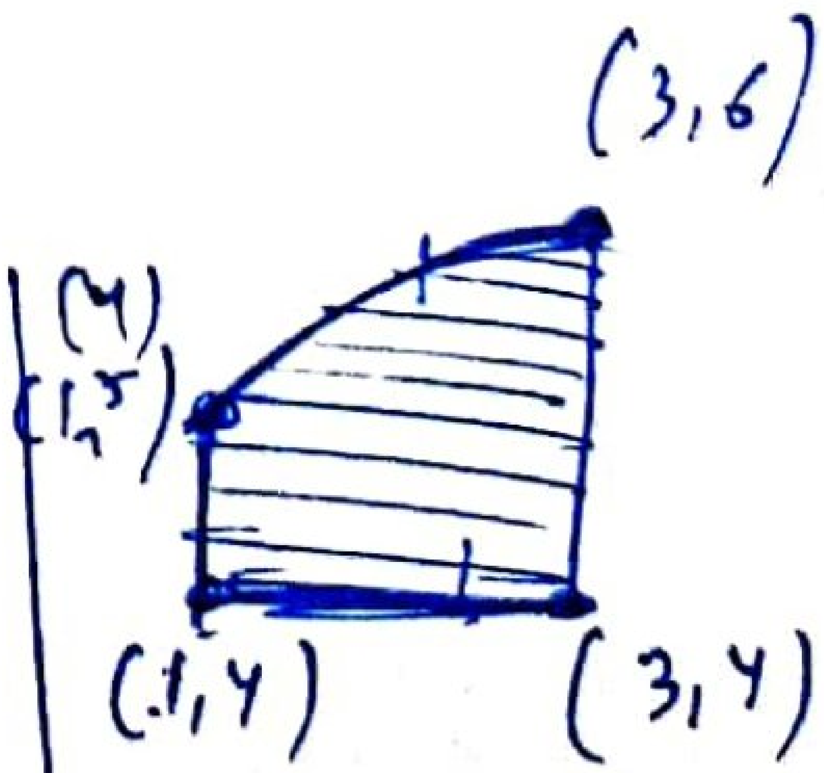
$\int_{3}^{3}$

(2)


$\int_{1}^{3}$

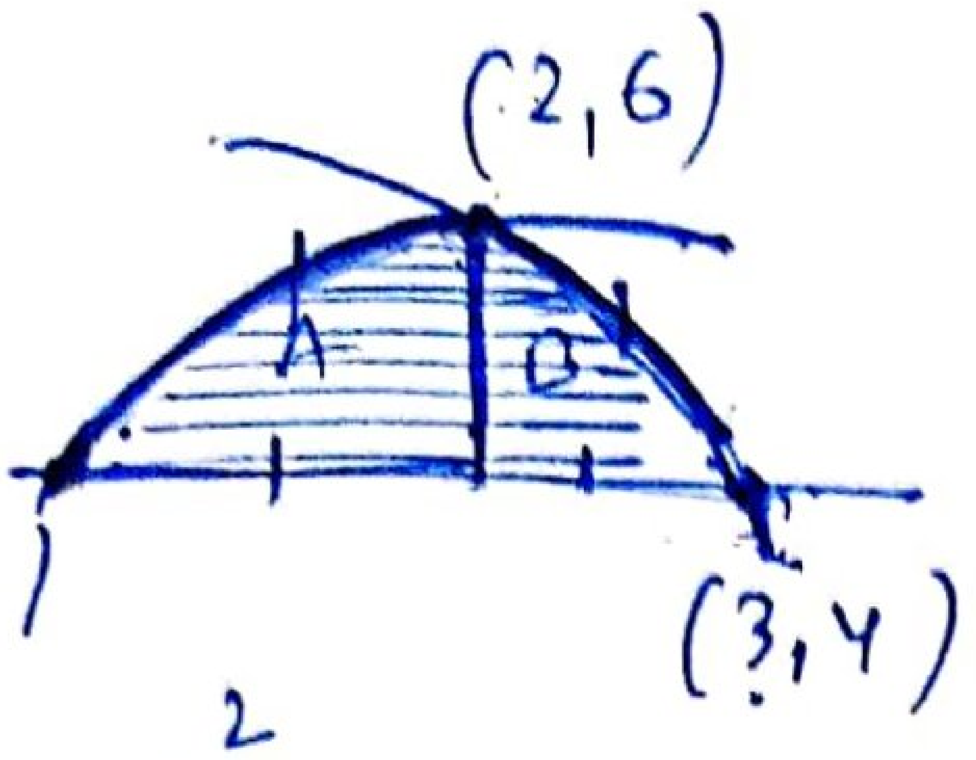
(3)


$\int_{1}^{3}$

(4)


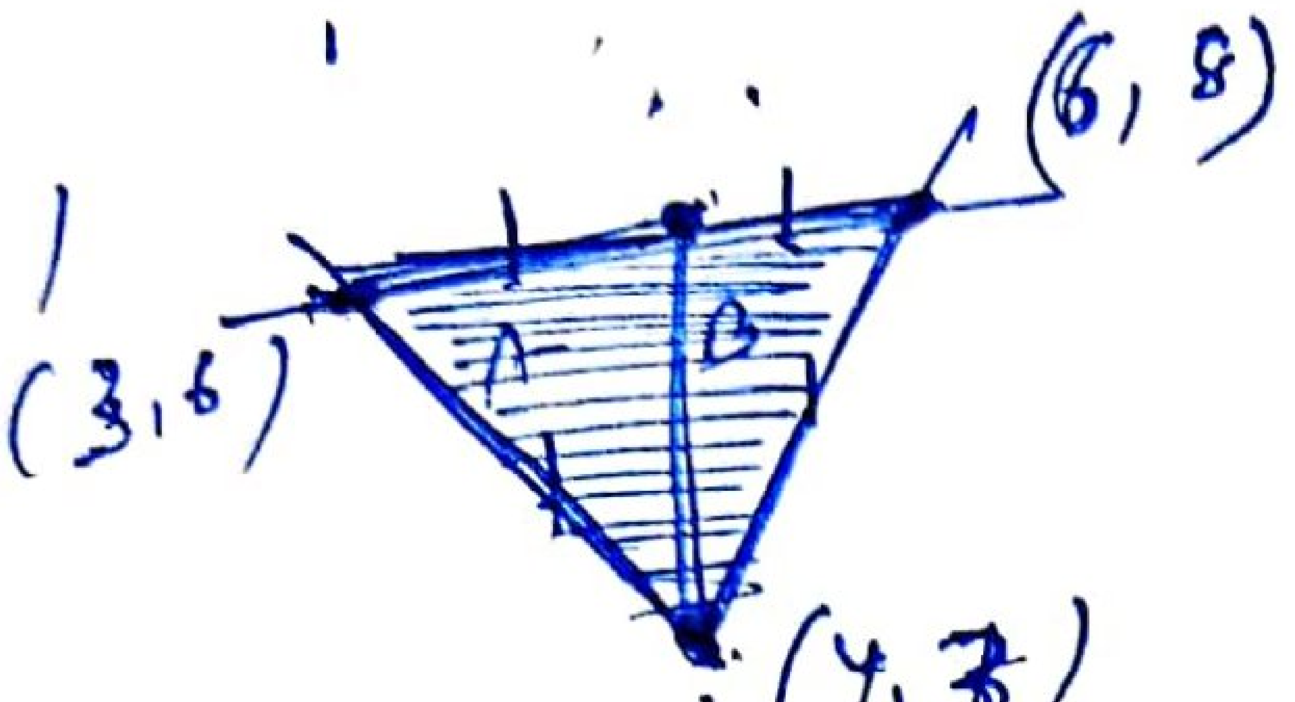
$\int_{1}^{3}$

(3)



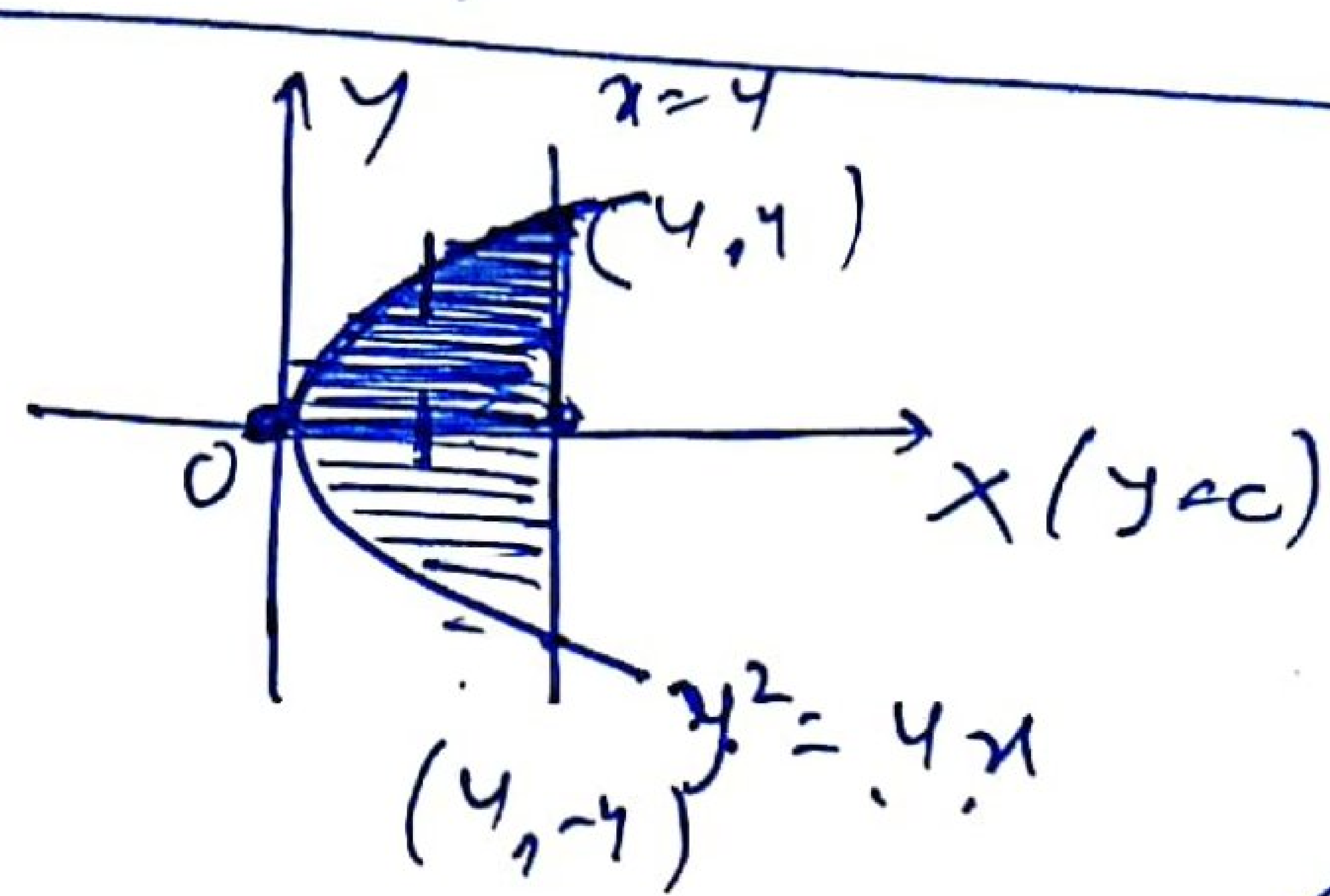
$$= \int_1^2 \square - \square \, dx + \int_2^3 \square - \square \, dy$$

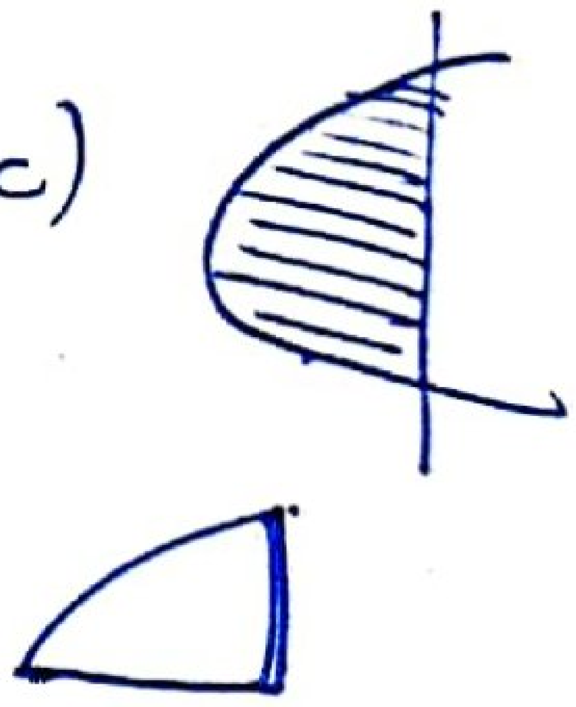
(6)



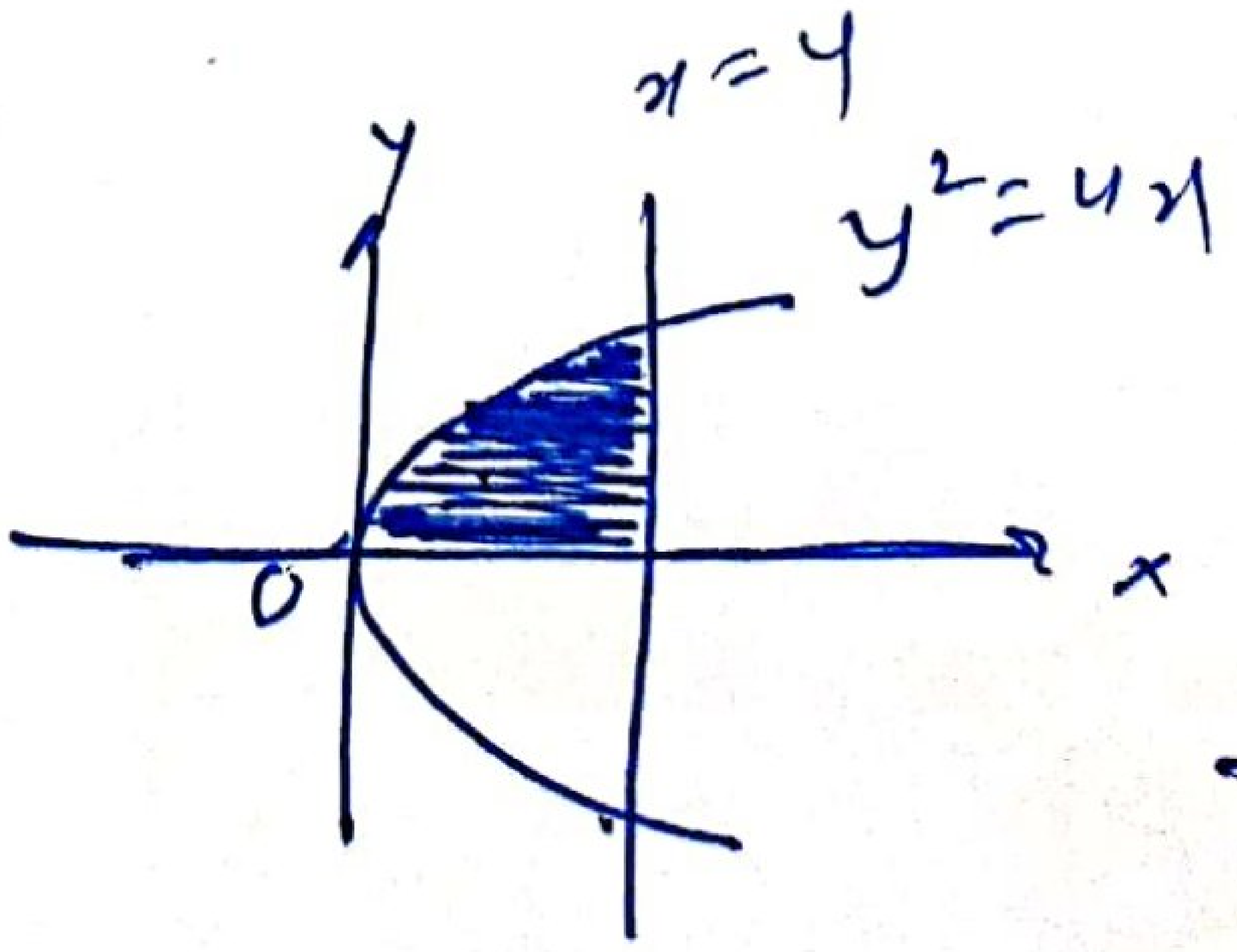
$$\int_3^4 \quad + \int_4^6$$

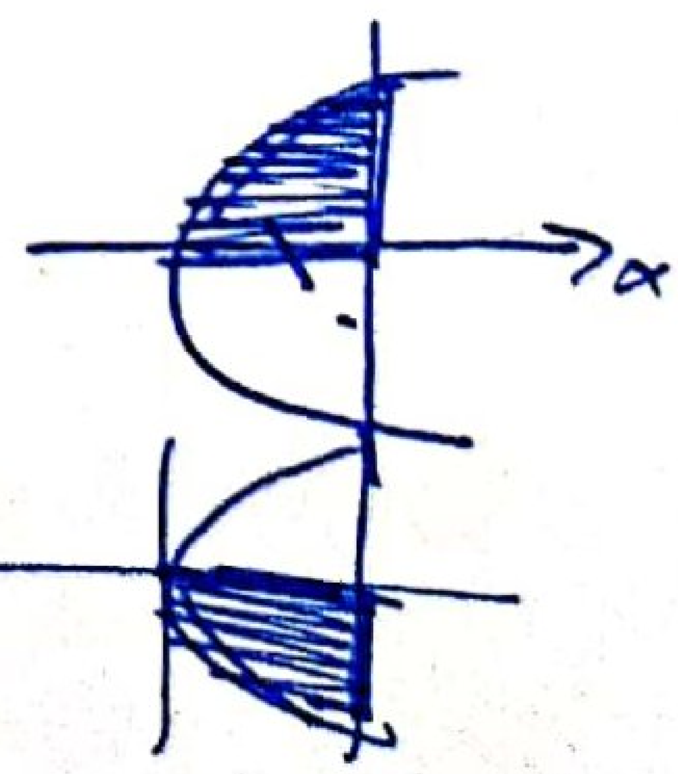
Symmetric





① ✓ parabola
✓ line
✓ $2 \int_0^4 2\sqrt{x} - 0 \, dx$





② parabola
✓ line
✓ $\boxed{\text{X-axis}}$ (y=0)

$$= \int_0^4 2\sqrt{x} - 0 \, dx$$

Que 1 Find the area bounded by the curves using Integration.

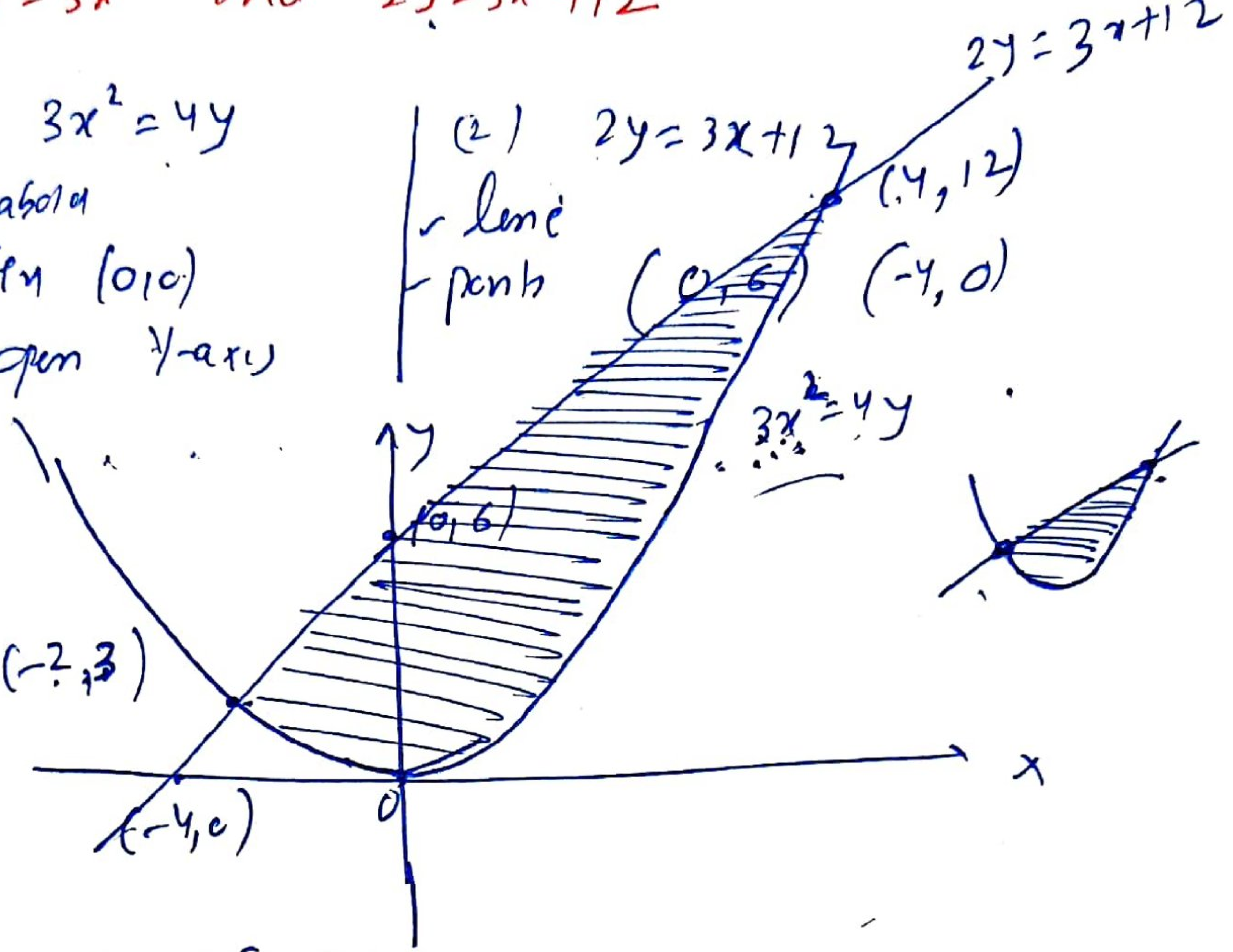
$4y = 3x^2$ and $2y = 3x + 12$

Sol (1) $3x^2 = 4y$

- ✓ parabola
- ✓ vertex $(0,0)$
- ✓ face open y-axis

(2) $2y = 3x + 12$

- ✓ line
- points $(0,6)$ $(-4,0)$



Int point $2\left(\frac{3x^2}{4}\right) = 3x + 12$

$3x^2 = 6x + 24 \Rightarrow x^2 - 2x - 8 = 0$

$(x-4)(x+2) = 0$

$x = 4, \quad x = -2$

Req area $= \int_{-2}^{4} \frac{3x+12}{2} - \frac{3x^2}{4}\, dx$

$= \frac{1}{4}\int_{-2}^{4} 6x + 24 - 3x^2\, dx$

$= \frac{3}{4}\int_{-2}^{4} 2x + 8 - x^2\, dx$

$= \frac{3}{4}\left[x^2 + 8x - \frac{x^3}{3}\right]_{-2}^{4}$

$= \frac{3}{4}\left[(\quad) - (\quad)\right]$

$= \boxed{\quad}$ square units