Class - (D-4)

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→ ULTIMATE MATHEMATICS →
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DETERMINANTS CLASS - 4 (D-4)

Properties of Determinants :

(1) $\begin{vmatrix} a & b & c \\ a & b & c \\ 1 & 1 & 2 \end{vmatrix} = 0$

(2) $\begin{vmatrix} a & 1 & a \\ b & 2 & b \\ c & 3 & c \end{vmatrix} = 0$

(3) $\begin{vmatrix} 1 & 2 & 3 \\ a & b & c \\ x & y & z \end{vmatrix}$

$R_1 \to 3R_1$

$= \frac{1}{3}\begin{vmatrix} 3 & 6 & 9 \\ a & b & c \\ x & y & z \end{vmatrix}$

(4) $\begin{vmatrix} 2 & 4 & 6 \\ 10 & 20 & 50 \\ 3 & 9 & 15 \end{vmatrix}$

$= 2 \times 10 \times 3 \begin{vmatrix} 1 & 2 & 3 \\ 1 & 2 & 5 \\ 1 & 3 & 5 \end{vmatrix}$

(5) $\begin{vmatrix} a+b & c+d & e+f \\ 1 & 2 & 3 \\ 4 & 5 & 6 \end{vmatrix}$

(Sum prop)

$= \begin{vmatrix} a & c & e \\ 1 & 2 & 3 \\ 4 & 5 & 6 \end{vmatrix} + \begin{vmatrix} b & d & f \\ 1 & 2 & 3 \\ 4 & 5 & 6 \end{vmatrix}$

(6) $\begin{vmatrix} a+b & 1 & 4 \\ c+d & 2 & 5 \\ e+f & 3 & 6 \end{vmatrix}$

$= \begin{vmatrix} a & 1 & 4 \\ c & 2 & 5 \\ e & 3 & 6 \end{vmatrix} + \begin{vmatrix} b & 1 & 4 \\ d & 2 & 5 \\ f & 3 & 6 \end{vmatrix}$

(7) $\begin{vmatrix} a & b & c \\ 1 & 2 & 3 \\ x & y & z \end{vmatrix}$

$R_1 \leftrightarrow R_2$ (Consecutive)

$= -\begin{vmatrix} 1 & 2 & 3 \\ a & b & c \\ x & y & z \end{vmatrix}$

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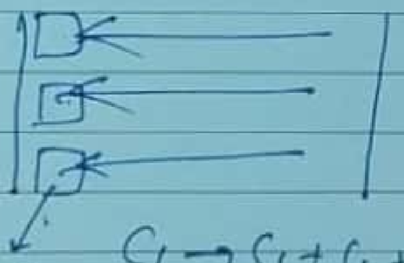
Tricks

(1) $\begin{vmatrix} 1 & - & - \\ 1 & - & - \\ 1 & - & - \end{vmatrix}$

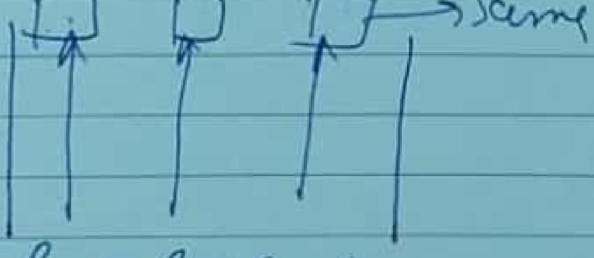
$R_2 \to R_2 - R_1$ & $R_3 \to R_3 - R_1$

(2) $\begin{vmatrix} 1 & 1 & 1 \\ - & - & - \\ - & - & - \end{vmatrix}$

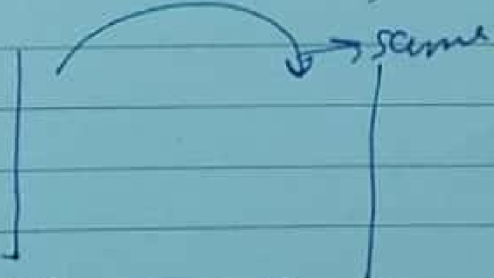
$C_2 \to C_2 - C_1$ & $C_3 \to C_3 - C_1$

(3) 

Same $C_1 \to C_1 + C_2 + C_3$

(4) 

$R_1 \to R_1 + R_2 + R_3$

(5) 

$C_1 \to C_1 + C_3$ (or) $C_2 \to C_2 + C_3$

(or) $R_1 \to R_1 + R_2$

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(6) Common

$$\begin{vmatrix} x & x^2 & x^3 \\ y & y^2 & y^3 \\ z & z^2 & z^3 \end{vmatrix} = xyz \begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix}$$

(7) Sum property

(8) Tricky special

Multiply by a

Multiply by b

Multiply by c

$= \frac{1}{abc}$

Common a b c

$= \frac{ab\cancel{c}}{a\cancel{b}c}$ | Benefit |

(9) See R.h.s

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ULTIMATE MATHEMATICS
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Q(1) Using properties of determinants show that

$$\begin{vmatrix} 1 & a^2 & a^3 \\ 1 & b^2 & b^3 \\ 1 & c^2 & c^3 \end{vmatrix} = (a-b)(b-c)(c-a)(ab+bc+ca)$$

Sol

$$\begin{vmatrix} 1 & a^2 & a^3 \\ 1 & b^2 & b^3 \\ 1 & c^2 & c^3 \end{vmatrix}$$

$R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$

$$= \begin{vmatrix} 1 & a^2 & a^3 \\ 0 & (b+a)(b-a) & (b-a)(b^2+a^2+ab) \\ 0 & (c+a)(c-a) & (c-a)(c^2+a^2+ac) \end{vmatrix}$$

taking common $(b-a)$ & $(c-a)$ from R_2 & R_3 resp.

$$= (b-a)(c-a)\begin{vmatrix} 1 & a^2 & a^3 \\ 0 & b+a & b^2+a^2+ab \\ 0 & c+a & c^2+a^2+ac \end{vmatrix}$$

$R_3 \to R_3 - R_2$

$$= (b-a)(c-a)\begin{vmatrix} 1 & a^2 & a^3 \\ 0 & b+a & b^2+a^2+ab \\ 0 & c-b & (c+b)(c-b)+a(c-b) \end{vmatrix}$$

Taking $(c-b)$ common from R_3

$$= (b-a)(c-a)(c-b)\begin{vmatrix} 1 & a^2 & a^3 \\ 0 & b+a & b^2+a^2+ab \\ 0 & 1 & c+b+a \end{vmatrix}$$

expanding along R_1

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ULTIMATE MATHEMATICS

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$= (b-a)(c-a)(c-b)\left\{ \left((b+a)(c+b+a) - b^2 - a^2 - ab \right) \right\}$

$= (b-a)(c-a)(c-b)\left(bc + \cancel{b^2} + ab + ac + \cancel{ab} + \cancel{a^2} - \cancel{b^2} - \cancel{a^2} - \cancel{ab} \right)$

$= \overset{(-)}{(b-a)}(c-a)\overset{(-)}{(c-b)}(ab + bc + ca)$

$= (a-b)(b-c)(c-a)(ab+bc+ca) = \underline{RHS}$

DETERMINANTS

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Que: Using properties of determinants show that:

Que 1 → $\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = (a-b)(b-c)(c-a)$

Que 2 → Show that $\begin{vmatrix} 1 & 1 & 1 \\ x^2 & y^2 & z^2 \\ x & y & z \end{vmatrix} = -(x-y)(y-z)(z-x)$

Que 3 → Show that $\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix} = (a-b)(b-c)(c-a)(a+b+c)$

Que 4 → Show that $\begin{vmatrix} 1 & a^2 & a^3 \\ 1 & b^2 & b^3 \\ 1 & c^2 & c^3 \end{vmatrix} = (a-b)(b-c)(c-a)(ab+bc+ca)$

Que 5 → Show that $\begin{vmatrix} x & y & z \\ x^2 & y^2 & z^2 \\ x^3 & y^3 & z^3 \end{vmatrix} = xyz(x-y)(y-z)(z-x)$

Que 6 → Show that $\begin{vmatrix} x+4 & 2x & 2x \\ 2x & x+4 & 2x \\ 2x & 2x & x+4 \end{vmatrix} = (5x+4)(4-x)^2$

Que 7 → Show that $\begin{vmatrix} y+k & y & y \\ y & y+k & y \\ y & y & y+k \end{vmatrix} = k^2(3y+k)$

Que 8 Show that $\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix} = (a+b+c)^3$

Que 9 → Show that $\begin{vmatrix} x+y+2z & x & y \\ z & y+z+2x & y \\ z & x & z+x+2y \end{vmatrix} = 2(x+y+z)^3$

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Q.N 10 → Show that $\begin{vmatrix} 3a & -a+b & -a+c \\ -b+a & 3b & -b+c \\ -c+a & -c+b & 3c \end{vmatrix} = 3(a+b+c)(ab+bc+ca)$

Q.N 11 → Show that $\begin{vmatrix} a & a+b & a+2b \\ a+2b & a & a+b \\ a+b & a+2b & a \end{vmatrix} = 9(a+b)\,b^2$

Q.N 12 → Show that $\begin{vmatrix} 1 & x & x^2 \\ x^2 & 1 & x \\ x & x^2 & 1 \end{vmatrix} = (1-x^3)^2$

Q.N 13 → Show that $\begin{vmatrix} \alpha & \alpha^2 & \beta+\gamma \\ \beta & \beta^2 & \gamma+\alpha \\ \gamma & \gamma^2 & \alpha+\beta \end{vmatrix} = (\alpha-\beta)(\beta-\gamma)(\gamma-\alpha)(\alpha+\beta+\gamma)$

Q.N 14 → Show that $\begin{vmatrix} a & b-c & c-b \\ a-c & b & c-a \\ a-b & b-a & c \end{vmatrix} = (a+b-c)(b+c-a)(c+a-b)$

Q.N 15 → Solve the equation (find x) if $\begin{vmatrix} x+a & x & x \\ x & x+a & x \\ x & x & x+a \end{vmatrix} = 0$

ANS $x = -a/3$

Q.N 16 → Solve the equation: if $\begin{vmatrix} 3x-8 & 3 & 3 \\ 3 & 3x-8 & 3 \\ 3 & 3 & 3x-8 \end{vmatrix} = 0$

ANS $x = \frac{2}{3}, \frac{11}{3}$

Q.N 17 → Show that $\begin{vmatrix} 1 & 1+p & 1+p+q \\ 2 & 3+2p & 1+3p+2q \\ 3 & 6+3p & 1+6p+3q \end{vmatrix} = 1$

Q.N 18 → Show that $\begin{vmatrix} a & a+b & a+b+c \\ 2a & 3a+2b & 4a+3b+2c \\ 3a & 6a+3b & 10a+6b+3c \end{vmatrix} = a^3$