!! जय श्री गिरिराज जी महाराज जय श्री राधे कृष्णा !!

ULTIMATE MATHEMATICS: BY AJAY MITTAL

Chapter: 3-D | CLASS NO: 5

Ques 1 Find the equation of plane passing through the Intersection of the planes $x+y+z=1$ and $2x+3y+4z=5$ and parallel to the line

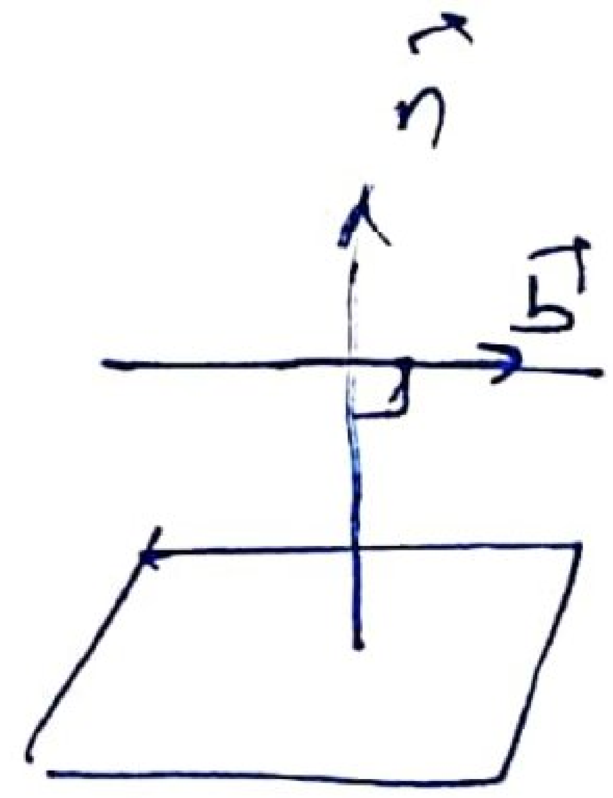
$$\frac{x-2}{1} = \frac{y+1}{-1} = \frac{z-3}{1}$$

Soln Converting given cartesian equations in to vector form

$$\vec{r}\cdot(\hat{i}+\hat{j}+\hat{k}) = 1$$

$$\vec{r}\cdot(2\hat{i}+3\hat{j}+4\hat{k}) = 5$$

line $\quad \vec{r} = (2\hat{i}-\hat{j}+3\hat{k}) + \lambda(\hat{i}-\hat{j}+\hat{k})$



here $\quad \vec{n_1} = \hat{i}+\hat{j}+\hat{k} \quad ; \quad d_1 = 1$

$\vec{n_2} = 2\hat{i}+3\hat{j}+4\hat{k} \quad ; \quad d_2 = 5$

$\vec{b} = \hat{i}-\hat{j}+\hat{k}$

Equation of required plane

$$\vec{r}\cdot(\vec{n_1}+\lambda\vec{n_2}) = d_1+\lambda d_2$$

$$\rightarrow \vec{r}\cdot(\hat{i}+\hat{j}+\hat{k}+2\lambda\hat{i}+3\lambda\hat{j}+4\lambda\hat{k}) = 1+5\lambda$$

$$\rightarrow \vec{r}\cdot(\hat{i}(1+2\lambda)+\hat{j}(1+3\lambda)+\hat{k}(1+4\lambda)) = 1+5\lambda \quad \text{---- (i)}$$

here $\vec{n} = \hat{i}(1+2\lambda) + \hat{j}(1+3\lambda) + \hat{k}(1+4\lambda)$

Since plane is parallel to the line

$\Rightarrow \vec{n} \perp$ line

$\Rightarrow \vec{n} \perp \vec{b}$

$\Rightarrow \vec{n} \cdot \vec{b} = 0$

$\Rightarrow \left(\hat{i}(1+2\lambda) + \hat{j}(1+3\lambda) + \hat{k}(1+4\lambda)\right) \cdot (\hat{i} - \hat{j} + \hat{k}) = 0$

$\Rightarrow 1 + 2\lambda - 1 - 3\lambda + 1 + 4\lambda = 0$

$\Rightarrow 3\lambda = -1 \Rightarrow \lambda = -1/3$ put in eq(1)

$\vec{r} \cdot \left(\hat{i}\left(\frac{1}{3}\right) + \hat{j}(0) + \hat{k}\left(\frac{-1}{3}\right)\right) = \frac{-2}{3}$

$\vec{r} \cdot (\hat{i} - \hat{k}) = -2$

$\Rightarrow (x\hat{i} + y\hat{j} + z\hat{k}) \cdot (\hat{i} - \hat{k}) = -2$

$\Rightarrow \boxed{x - z + 2} = 0$ Ans

Qn. 2 → Find the equation of planes passes through the line of Intersection of the planes $\vec{r} \cdot (2\hat{i} + 6\hat{j}) + 12 = 0$ and $\vec{r} \cdot (3\hat{i} - \hat{j} + 4\hat{k}) = 0$ which are at a unit distance from the origin

Soln given planes $\vec{r} \cdot (2\hat{i} + 6\hat{j}) = -12$ and

$\vec{r} \cdot (3\hat{i} - \hat{j} + 4\hat{k}) = 0$

here $\vec{n_1} = 2\hat{i} + 6\hat{j}$; $d_1 = -12$

$\vec{n_2} = (3\hat{i} - \hat{j} + 4\hat{k})$; $d_2 = 0$

equan of req plane

$\vec{r} \cdot (\vec{n_1} + \lambda \vec{n_2}) = d_1 + \lambda d_2$

$\Rightarrow \vec{r} \cdot (2\hat{i} + 6\hat{j} + 3\lambda\hat{i} - \lambda\hat{j} + 4\lambda\hat{k}) = -12$

$\Rightarrow \vec{r} \cdot (\hat{i}(2+3\lambda) + \hat{j}(6-\lambda) + 4\lambda\hat{k}) = -12$ -- (1)

here $\vec{n} = \hat{i}(2+3\lambda) + \hat{j}(6-\lambda) + 4\lambda\hat{k}$

$d = -12$

point $(0,0,0)$ PV $\vec{a} = \vec{0} = 0\hat{i} + 0\hat{j} + 0\hat{k}$

distance = 1

$$\text{Distance} = \frac{|\vec{a} \cdot \vec{n} - d|}{|\vec{n}|}$$

$$\Rightarrow 1 = \frac{|\vec{0} \cdot \vec{n} + 12|}{\sqrt{(2+3\lambda)^2 + (6-\lambda)^2 + 16\lambda^2}}$$

$$\Rightarrow \sqrt{4 + 9\lambda^2 + \cancel{12\lambda} + 36 + \lambda^2 - \cancel{12\lambda} + 16\lambda^2} = 12$$

square

$26\lambda^2 + 40 = 144$

$\Rightarrow 26\lambda^2 = 104$ $\Rightarrow \lambda^2 = 4$ $\Rightarrow$ $\lambda = \pm 2$

for $\lambda = 2$ equation of plane is

$$\vec{r} \cdot (8\hat{i} + 4\hat{j} + 8\hat{k}) = -12$$

or $\boxed{\vec{r} \cdot (2\hat{i} + \hat{j} + 2\hat{k}) = -3}$ -- Ans

for $\lambda = -2$ $\vec{r} \cdot (-4\hat{i} + 8\hat{j} - 8\hat{k}) = -12$

or $\boxed{\vec{r} \cdot (\hat{i} - 2\hat{j} + 2\hat{k}) = 3}$ Ans

Ques 3 → If the points $(1, 1, \lambda)$ & $(-3, 0, 1)$ be equidistant from the plane $\vec{r} \cdot (3\hat{i} + 4\hat{j} - 12\hat{k}) + 13 = 0$ find value of λ

Soln Given plane $\vec{r} \cdot (3\hat{i} + 4\hat{j} - 12\hat{k}) = -13$

here $\vec{n} = 3\hat{i} + 4\hat{j} - 12\hat{k}$; $d = -13$

let P.V of given points

$\vec{a} = \hat{i} + \hat{j} + \lambda\hat{k}$ and $\vec{b} = -3\hat{i} + \hat{k}$

Given

$$\frac{|\vec{a} \cdot \vec{n} - d|}{\cancel{|\vec{n}|}} = \frac{|\vec{b} \cdot \vec{n} - d|}{\cancel{|\vec{n}|}}$$

$$\Rightarrow |(\hat{i} + \hat{j} + \lambda\hat{k}) \cdot (3\hat{i} + 4\hat{j} - 12\hat{k}) + 13| = |(-3\hat{i} + \hat{k}) \cdot (3\hat{i} + 4\hat{j} - 12\hat{k}) + 13|$$

$$\Rightarrow |3 + 4 - 12\lambda + 13| = |-9 - 12 + 13|$$

$\Rightarrow |20-12\lambda| = 8$

$\Rightarrow 20-12\lambda = 8$ | $20-12\lambda = -8$

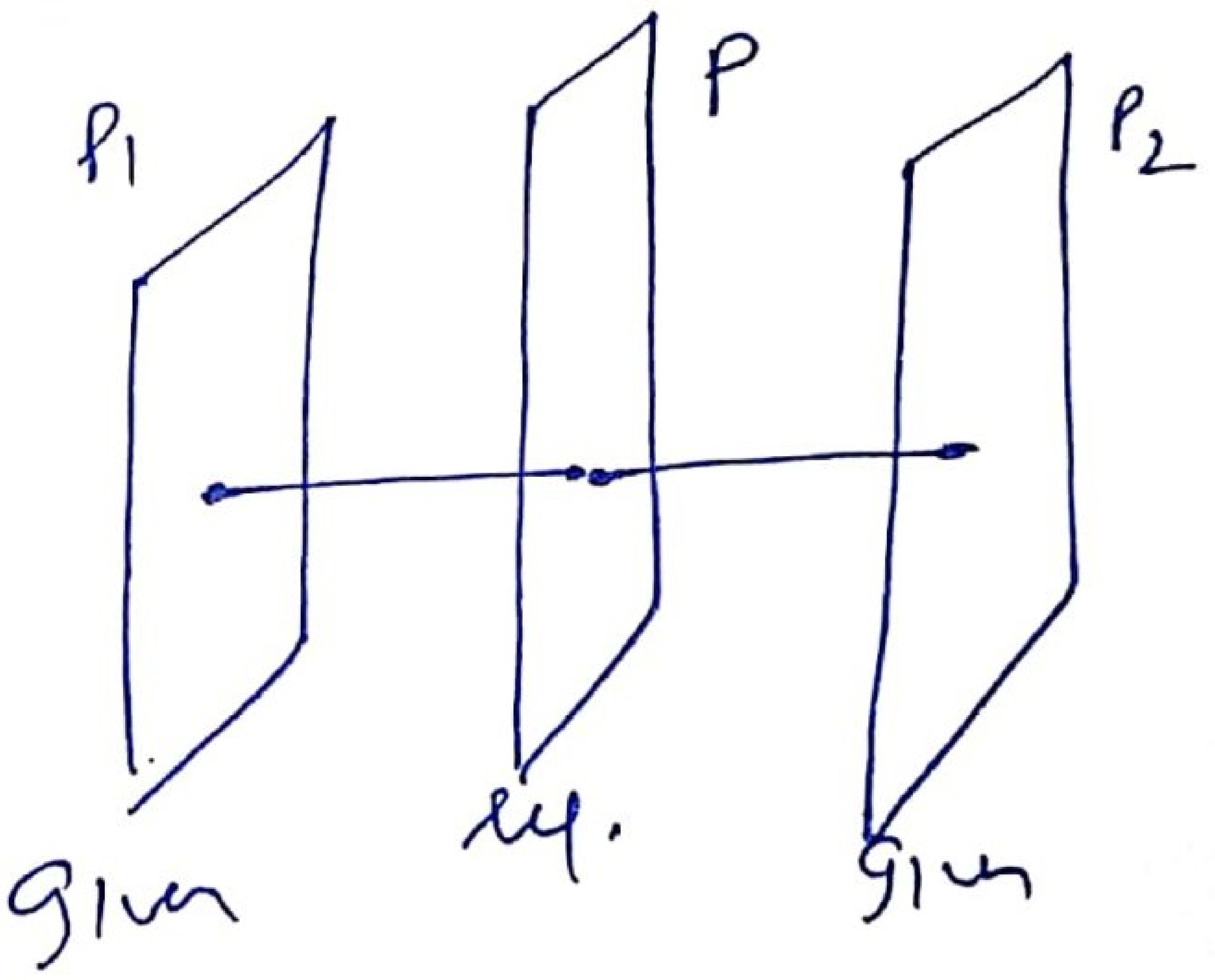
$\Rightarrow 12\lambda = 12$ | $12\lambda = 28$

$\lambda = 1$ | $\lambda = \frac{7}{3}$

$\therefore \boxed{\lambda = 1 \text{ or } \lambda = 7/3}$ Ans

Qn. 4 → Find the equation of the plane mid-parallel to the planes $2x-2y+z+3=0$ and $2x-2y+z+9=0$

Sol. given planes



$P_1: \quad 2x-2y+z = -3$

$P_2: \quad 2x-2y+z = -9$

Since required plane is parallel to the given planes

$\therefore$ let equation of required plane is

$$2x-2y+z = d$$

here $a=2,\ b=-2,\ c=1$

$d_1 = -3,\ d_2 = -9 \ ;\ d_3 = d$

Since distance b/w P_1 & P = distance b/w P & P_2

$$\frac{|d+3|}{\cancel{\sqrt{4+4+1}}} = \frac{|d+9|}{\cancel{\sqrt{4+4+1}}}$$

$\Rightarrow \quad d+3 = \pm(d+9)$

$d+3 = d+9$ | $d+3 = -d-9$

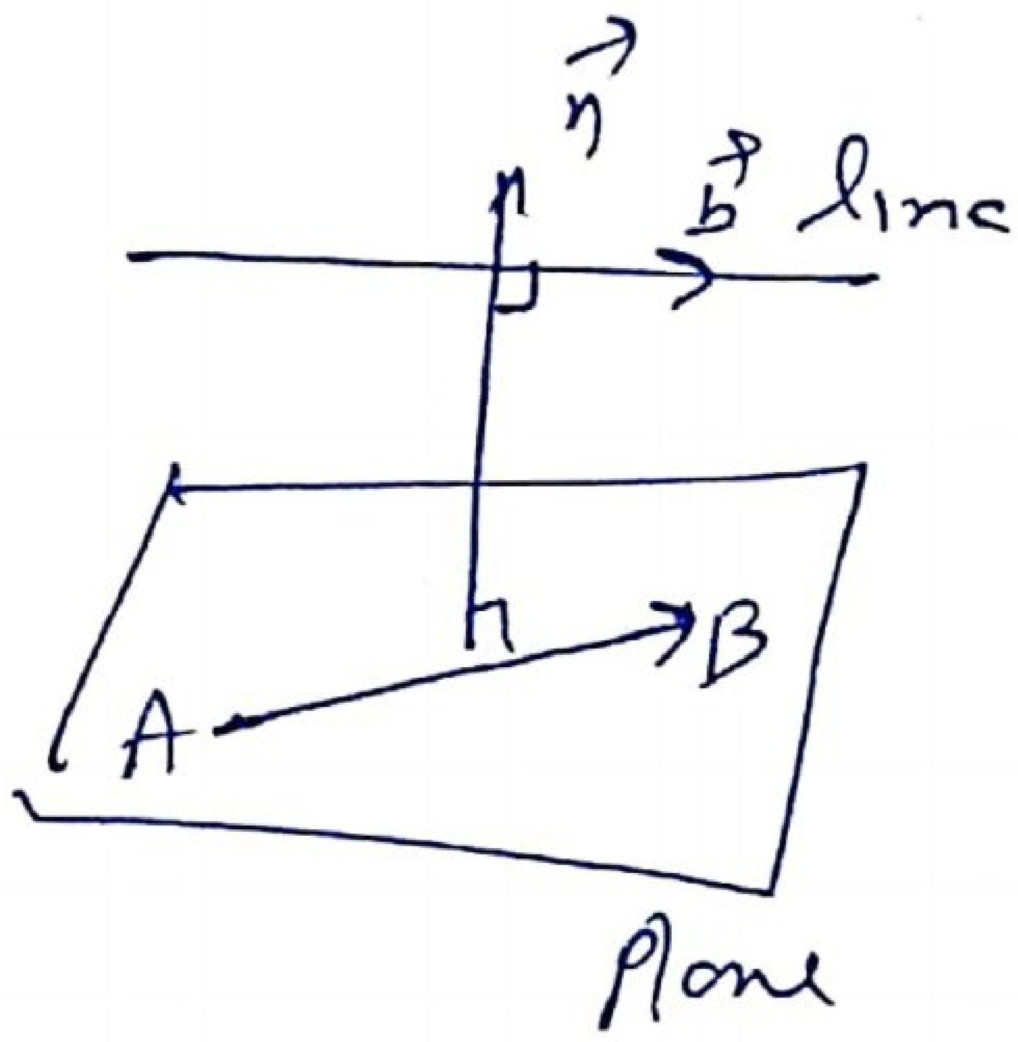
$3 = 9$ ✗ | $2d = -12$

No value of d | $d = -6$

$\therefore$ required eq. of plane is $\boxed{2x-2y+z+6=0}$

ans

Ans 5 → Find the equation of the plane passing through the points $(1,0,-1)$ and $(3,2,2)$ and parallel to the line $\frac{x-1}{1} = \frac{y-1}{-2} = \frac{z-2}{3}$



Soln

$\vec{AB} = 2\hat{i} + 2\hat{j} + 3\hat{k}$

vector equat of line

$\vec{r} = (\hat{i} + \hat{j} + 2\hat{k}) + \lambda(\hat{i} - 2\hat{j} + 3\hat{k})$

here $\vec{b} = \hat{i} - 2\hat{j} + 3\hat{k}$

Since plane is $\parallel$ to the line

then $\vec{n} \perp \vec{b}$ and also $\vec{n} \perp \vec{AB}$

$\Rightarrow \vec{n} = \lambda(\vec{b} \times \vec{AB})$

$\Rightarrow \vec{n} = \lambda \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 3 \\ 2 & 2 & 2 \end{vmatrix} = \lambda(-12\hat{i} + 3\hat{j} + 6\hat{k})$

w. Position vector of point $A(1,0,-1)$ is

$$\vec{a} = \hat{i} + 0\hat{j} - \hat{k}$$

Now equation of plane

$$\vec{r}\cdot\vec{n} = \vec{a}\cdot\vec{n}$$

$$\Rightarrow \vec{r}\cdot\lambda(-12\hat{i}+3\hat{j}+6\hat{k}) = (\hat{i}-\hat{k})\cdot\lambda(-12\hat{i}+3\hat{j}+6\hat{k})$$

$$\Rightarrow \vec{r}\cdot(-12\hat{i}+3\hat{j}+6\hat{k}) = -12-6$$

$$\Rightarrow \vec{r}\cdot(-12\hat{i}+3\hat{j}+6\hat{k}) = -18$$

$$\Rightarrow \vec{r}\cdot(4\hat{i}-\hat{j}-2\hat{k}) = 6$$

$$\Rightarrow (x\hat{i}+y\hat{j}+z\hat{k})\cdot(4\hat{i}-\hat{j}-2\hat{k}) = 6$$

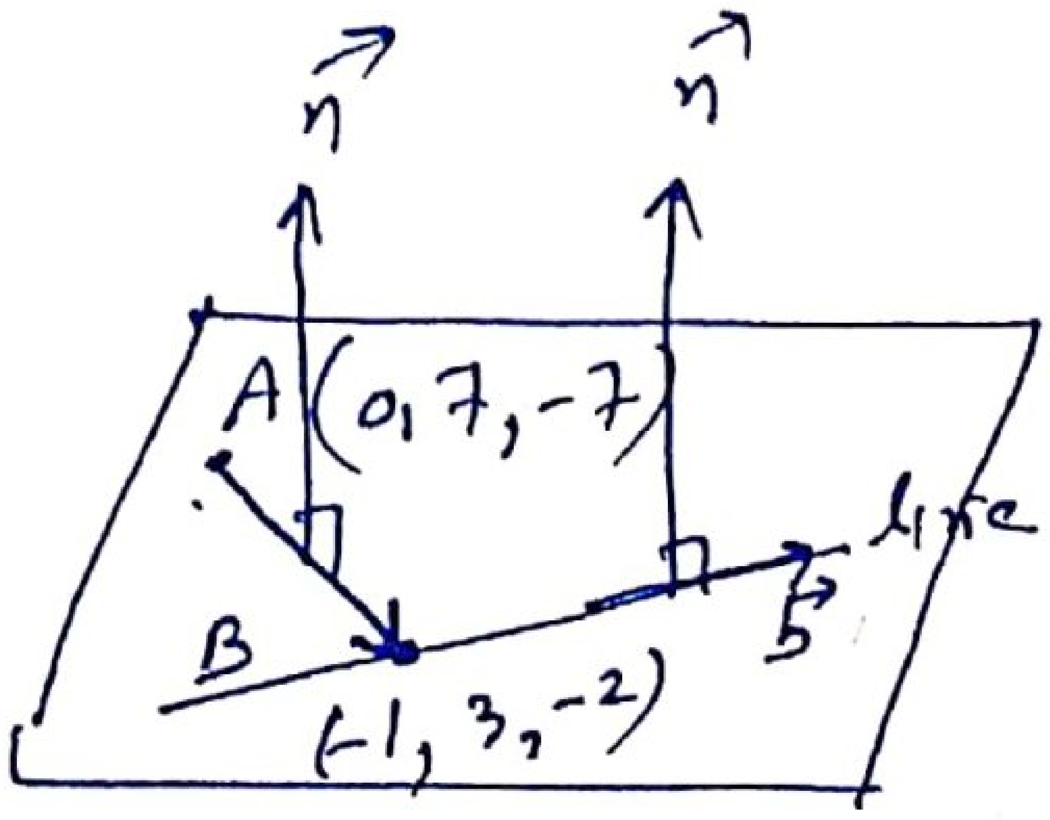
$$\Rightarrow \boxed{4x - y - 2z = 6}$$ Ans

Qn 6 → Find the equation of the plane passing through the point $(0,7,-7)$ and containing the line

$$\frac{x+1}{-3} = \frac{y-3}{2} = \frac{z+2}{1}$$

Soln Vector equation of given line

$$\vec{r} = (-\hat{i}+3\hat{j}-2\hat{k}) + \lambda(-3\hat{i}+2\hat{j}+\hat{k})$$



here $\vec{b} = -3\hat{i}+2\hat{j}+\hat{k}$

fixed point on line $B(-1,3,-2)$

given $A(0,7,-7)$ on the plane

Now $\vec{AB} = -\hat{i} - 4\hat{j} + 5\hat{k}$

$\vec{n} \perp \vec{AB}$ and $\vec{n} \perp \vec{b}$

$\Rightarrow \vec{n} = \lambda(\vec{AB} \times \vec{b})$

$$\Rightarrow \vec{n} = \lambda \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & -4 & 5 \\ -3 & 2 & 1 \end{vmatrix} = \lambda(-14\hat{i} - 14\hat{j} - 14\hat{k})$$

P.V of point A is $\vec{a} = 7\hat{j} - 7\hat{k}$

Equation of plane $\vec{r}\cdot\vec{n} = \vec{a}\cdot\vec{n}$

$$\Rightarrow \vec{r}\cdot\cancel{\lambda}(-14\hat{i} - 14\hat{j} - 14\hat{k}) = (7\hat{j} - 7\hat{k})\cdot\cancel{\lambda}(-14\hat{i} - 14\hat{j} - 14\hat{k})$$

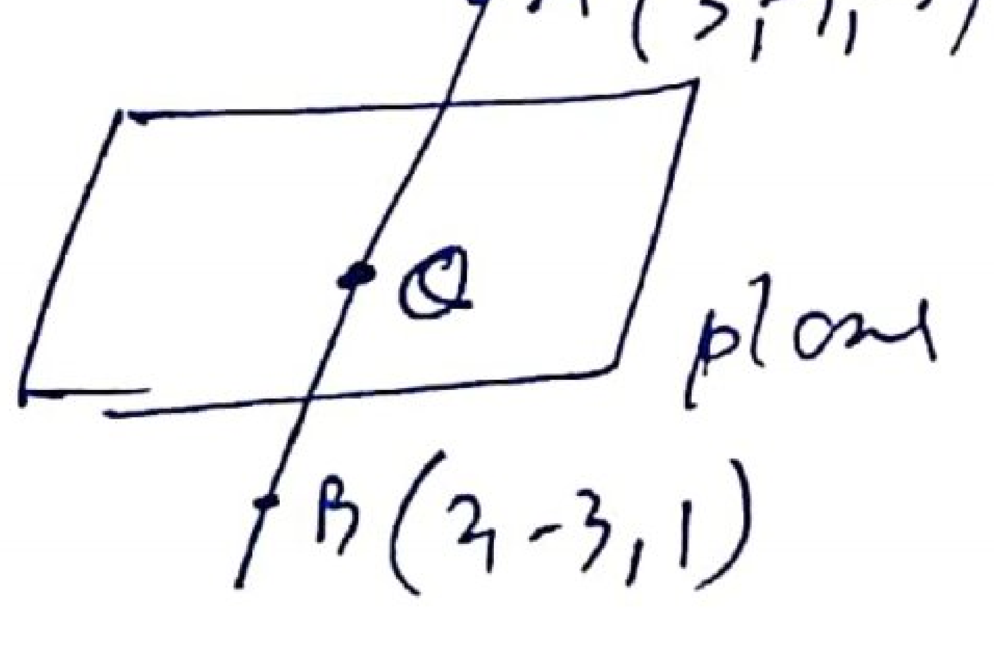
$$\Rightarrow \vec{r}\cdot(-14\hat{i} - 14\hat{j} - 14\hat{k}) = -14(7\hat{j} - 7\hat{k})\cdot(0\hat{i} + \hat{j} + \hat{k})$$

$$\Rightarrow \vec{r}\cdot(\hat{i} + \hat{j} + \hat{k}) = 7 - 7$$

$$\Rightarrow \boxed{\vec{r}\cdot(\hat{i} + \hat{j} + \hat{k}) = 0}$$ Ans

Qn. 7 → Find the coordinates of the point where the line through $(3,-4,-5)$ and $(2,-3,1)$ crosses the plane $2x + y + z = 7$

(Topic : Intersection point of line & plane)



equation of line (two point form)

$$\frac{x-3}{-1} = \frac{y+4}{1} = \frac{z+5}{6} = \lambda$$

$\Rightarrow x = -\lambda+3$; $y = \lambda-4$, $z = 6\lambda-5$

let coordinate of Q is $(-\lambda+3, \lambda-4, 6\lambda-5)$

point Q also lies on the plane $2x+y+z=7$

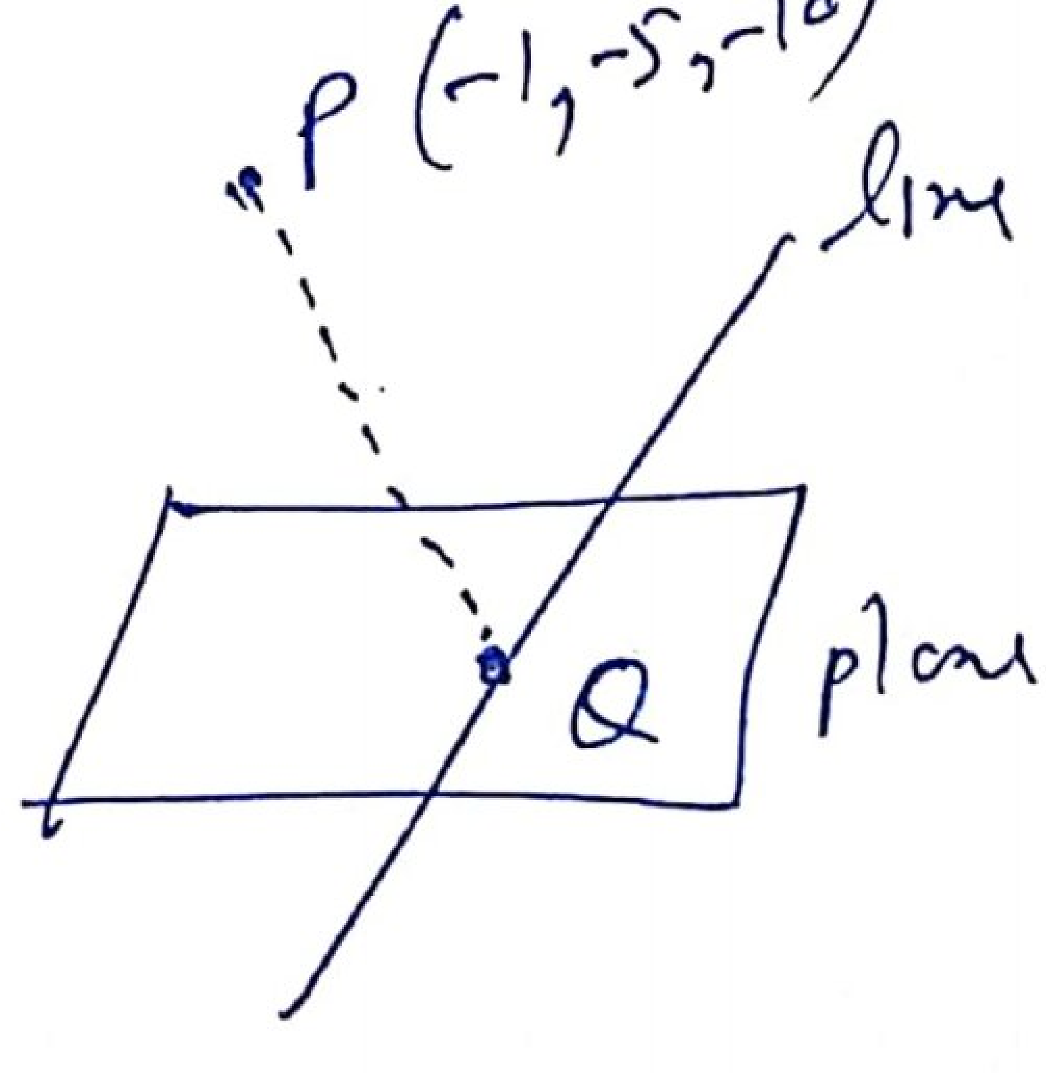
$\Rightarrow -2\lambda+6+\lambda-4+6\lambda-5=7$

$\Rightarrow 5\lambda = 10$

$\lambda = 2$

$\therefore$ $\boxed{\text{Int point is } Q(1,-2,7)}$ Ans

Que 8 $\Rightarrow$ Find the distance b/w the point $-\hat{i}-5\hat{j}-10\hat{k}$ and the point of Intersection of the line $\vec{r} = (2\hat{i}-\hat{j}+2\hat{k})+\lambda(3\hat{i}+4\hat{j}+12\hat{k})$ and the plane $\vec{r}\cdot(\hat{i}-\hat{j}+\hat{k})=5$



Soln Convert given vector equations in to Cartesian form

line $\dfrac{x-2}{3} = \dfrac{y+1}{4} = \dfrac{z-2}{12} = \lambda$

plane $x-y+z=5$

$\Rightarrow x = 3\lambda+2$; $y = 4\lambda-1$, $z = 12\lambda+2$

let Coordinate of point Q is $(3\lambda+2, 4\lambda-1, 12\lambda+2)$

point Q also lies on the plane

$3\lambda+2 - 4\lambda+1 + 12\lambda+2 = 5$

$11\lambda = 0$

$\lambda = 0$

$\therefore$ $Q(2,-1,2)$

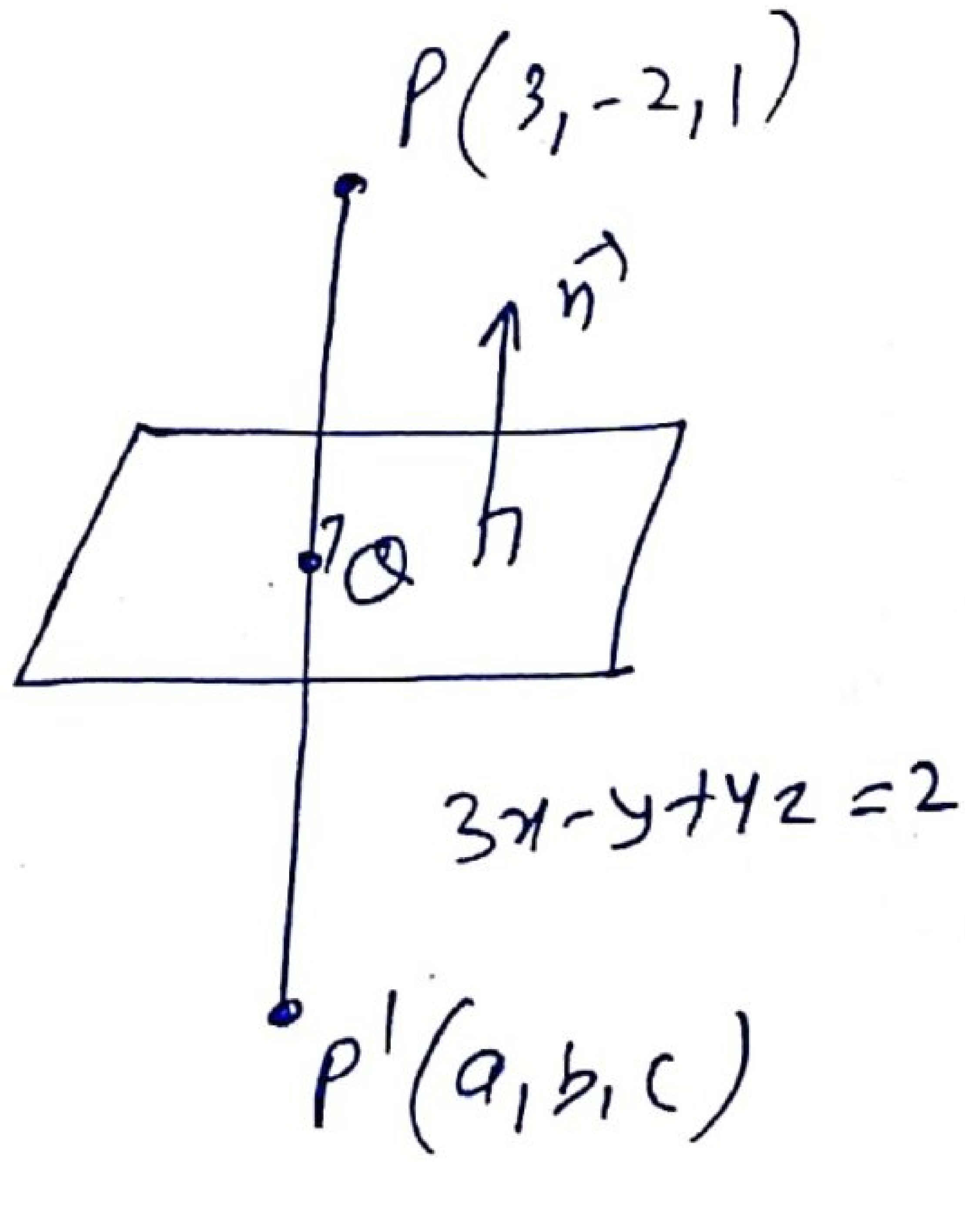
Now required distance

$$PQ = \sqrt{(2+1)^2 + (-1+5)^2 + (2+10)^2}$$

$$= \sqrt{9+16+144}$$

$$\boxed{PQ = 13 \text{ units}}$$ Ans

Qu. 9 → Find the image of the point $(3,-2,1)$ in the plane $3x-y+4z=2$



Soln

given plane $3x-y+4z=2$

D.R's of normal vector $\vec{n}$ to this plane $= 3,-1,4$

Since $PQ \perp$ plane & $\vec{n} \perp$ plane

$\therefore PQ \parallel \vec{n}$

$\therefore$ DR's of $PQ = 3,-1,4$

equation of PQ : $\dfrac{x-3}{3} = \dfrac{y+2}{-1} = \dfrac{z-1}{4}$

let $\frac{x-3}{3} = \frac{y+2}{-1} = \frac{z-1}{4} = \lambda$

$\Rightarrow x = 3\lambda+3,\ y = -\lambda-2,\ z = 4\lambda+1$

let coordinate of Q is $(3\lambda+3, -\lambda-2, 4\lambda+1)$

point Q also lies on the plane

$9\lambda+9 + \lambda+2 + 16\lambda+4 = 2$

$\Rightarrow 26\lambda = -13$

$\lambda = -1/2$

foot of $\perp$: $Q\left(\frac{3}{2}, -\frac{3}{2}, -1\right)$

Q is the mid point of PP'

$\frac{3+a}{2} = \frac{3}{2}$	$\frac{-2+b}{2} = \frac{-3}{2}$	$\frac{1+c}{2} = -1$
$a = 0$	$b = -1$	$c = -3$

$\therefore$ Image $P'(0,-1,-3)$ Ans

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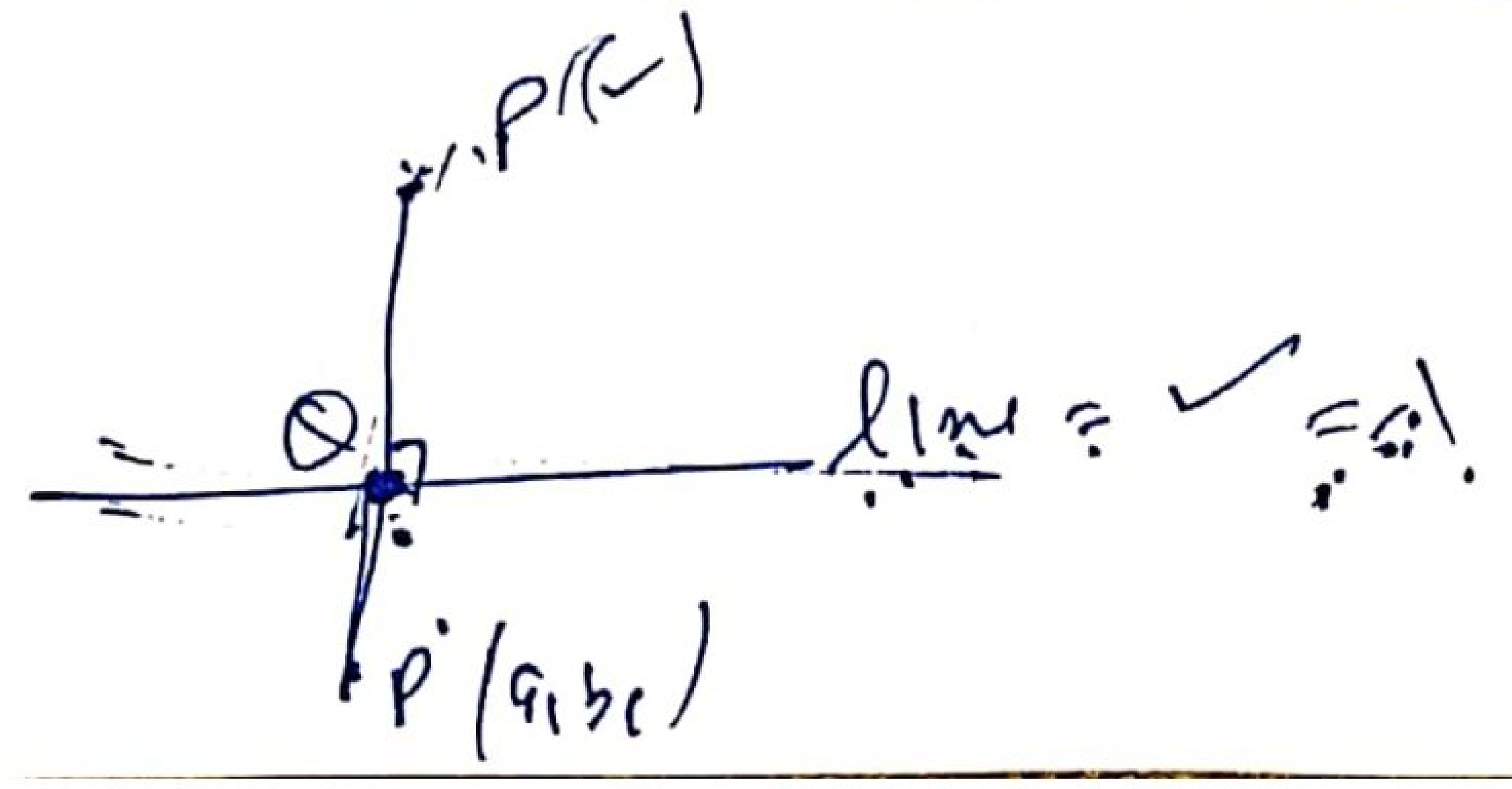

WORKSHEET No: 5 3-D

Qns 1 → Find the coordinates of the foot of the perpendicular and the perpendicular distance of the point P(3, 2, 1) from the plane $2x - y + z + 1 = 0$

Ans $(-1, 3, 0)$; $\sqrt{6}$

Qns 2 → Find the image of the point with position vector $3\hat{i} + \hat{j} + 2\hat{k}$ in the plane $\vec{r} \cdot (2\hat{i} - \hat{j} + \hat{k}) = 4$

Ans $(-1, 2, -1)$

Qns 3 → Find the vector equation of the plane passing through the points (3, 4, 2) and (7, 0, 6) and perpendicular to the plane $2x - 5y - 15 = 0$. Also show that the plane thus obtained contains the line $\vec{r} = (\hat{i} + 3\hat{j} - 2\hat{k}) + \lambda(\hat{i} - \hat{j} + \hat{k})$

Ans $\vec{r} \cdot (5\hat{i} + 2\hat{j} - 3\hat{k}) = 17$

Qn. 4 → Find the coordinates of the point where the line $\frac{x-2}{3} = \frac{y+1}{4} = \frac{z-2}{2}$ intersect the plane $x - y + z - 5 = 0$. Also, find the angle b/w the line and the plane

Ans $(2, -1, 2)$, $\sin^{-1}\left(\frac{1}{\sqrt{87}}\right)$

QN.5 → If $4x + 4y - \lambda z = 0$ is the equation of plane and contains the line $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z}{4}$ find the value of λ

Ans $\lambda = 5$

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Qn. 6 → Find the coordinates of the point where the line through the points $A(3,4,1)$ and $B(5,1,6)$ crosses the XY- plane **Ans** $\left(\frac{13}{5}, \frac{23}{5}, 0\right)$

Qn. 7 → Find the distance of the point $(2,2,-2)$ from the point of Intersection of the line

$\vec{r} = 3\hat{i}-\hat{j}-\hat{k} + \lambda(2\hat{i}-2\hat{j}+\hat{k})$ and the plane

$\vec{r}\cdot(9\hat{i}+3\hat{j}-\hat{k}) = 14$ **Ans** $\sqrt{2}$ units

Qn. 8 → Find the equation of the plane passing through the intersection of the planes $3x-4y+5z=10$ and $2x+2y-3z=4$ and parallel to the line $x=2y=3z$ **Ans** $x-20y+27z=14$

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