" जय श्री बिहारी जी महाराज "

→ SOLUTIONS: WORKSHEET No: 8 ←

A.O.D (Increasing - Decreasing)

Qn. 1 $f(x) = 2x^3 + 9x^2 + 12x + 20$

Diff w.r.t x

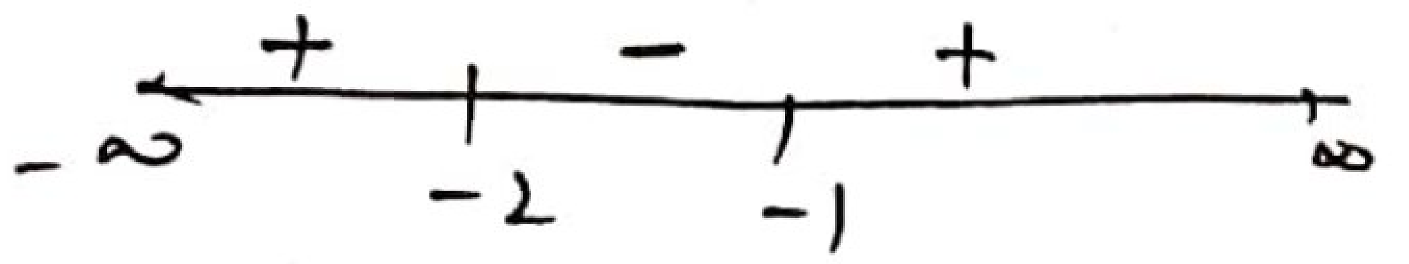
$$f'(x) = 6x^2 + 18x + 12$$

$$f'(x) = 6(x^2 + 3x + 2)$$

$$f'(x) = 6(x+1)(x+2)$$

put $f'(x) = 0$

$$x = -1, \quad x = -2$$



Intervals	Sign of $f'(x)$	Nature of $f(x)$
$(-\infty, -2)$	$(-)(-) = +ve \quad f'(x) > 0$	$f(x)$ is strictly ↑
$(-2, -1)$	$(-)(+) = -ve \quad f'(x) < 0$	$f(x)$ is strictly ↓
$(-1, \infty)$	$(+)(+) = +ve \quad f'(x) > 0$	$f(x)$ is strictly ↑

$\therefore$ $f(x)$ is strictly ↑ in $(-\infty, -2) \cup (-1, \infty)$ and strictly ↓ in $(-2, -1)$ Ans

Qn. 2 → $f(x) = (x+1)^3 (x-3)^3$

Diff w.r.t x

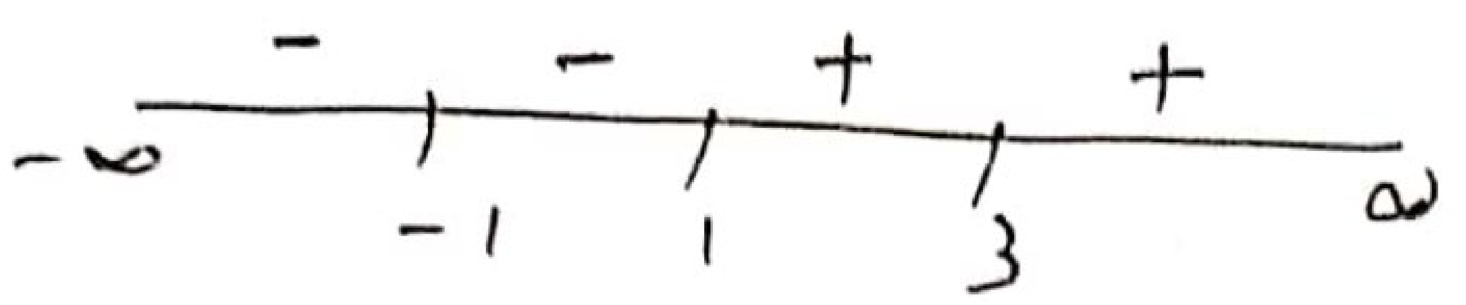
$$f'(x) = (x+1)^3 \cdot 3(x-3)^2 + (x-3)^3 \cdot 3(x+1)^2$$

$$f'(x) = 3(x+1)^2 (x-3)^2 [x+1+x-3]$$

$f'(x) = 3(x+1)^2(x-3)^2(2x-2)$

$f'(x) = 6(x+1)^2(x-3)^2(x-1)$

put $f'(x) = 0 \Rightarrow x = -1,\ x = 3,\ x = 1$



Intervals	Sign of $f'(x)$	Nature of $f(x)$
$(-\infty, -1)$	$(+)(+)(-) = -ve$ $f'(x) < 0$	$f(x)$ is strictly $\downarrow$
$(-1, 1)$	$(+)(+)(-) = -ve$ $f'(x) < 0$	" " " $\downarrow$
$(1, 3)$	$(+)(+)(+) = +ve$ $f'(x) > 0$	" " " $\uparrow$
$(3, \infty)$	$(+)(+)(+) = +ve$ $f'(x) > 0$	" " " $\uparrow$

$\therefore f(x)$ is strictly $\uparrow$ in $(1,3) \cup (3, \infty)$ and strictly $\downarrow$ in $(-\infty, -1) \cup (-1, 1)$ ANS

Qn 3 $\rightarrow$ $f(x) = \frac{3}{10}x^4 - \frac{4}{5}x^3 - 3x^2 + \frac{36}{5}x + 11$

Diff. w.r.t x $\quad f'(x) = \frac{12x^3}{10} - \frac{12x^2}{5} - 6x + \frac{36}{5} + 0$

$= \frac{1}{10}\left(12x^3 - 24x^2 - 60x + 72\right)$

$f'(x) = \frac{12}{10}\left(x^3 - 2x^2 - 5x + 6\right)$

Hit and trial method

$$
\begin{array}{r|l}
 & x^2 - x - 6 \\
\hline
x - 1 & x^3 - 2x^2 - 5x + 6 \\
 & -(x^3 - x^2) \\
\hline
 & -x^2 - 5x + 6 \\
 & -(-x^2 + x) \\
\hline
 & -6x + 6 \\
 & -6x + 6 \\
\hline
 & \times
\end{array}
$$

$f'(x) = (x-1)(x^2 - x - 6)$

$f'(x) = (x-1)(x-3)(x+2)$

put $f'(x) = 0$

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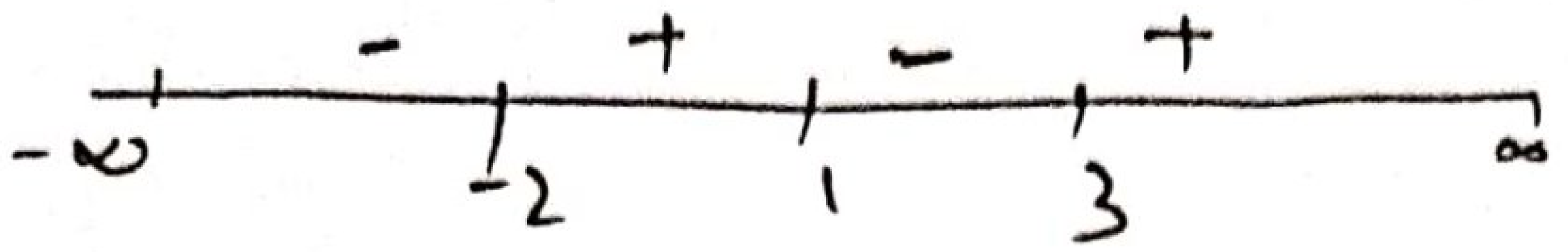
soln A.O.D (w.s 8)

$x = 1, \quad x = 3, \quad x = -2$



Intervals	Sign of $f'(x)$	Nature of $f(x)$
$(-\infty, -2]$	$f'(x) \le 0$	$f(x)$ is $\downarrow$
$[-2, +1]$	$f'(x) \ge 0$	$f(x)$ is $\uparrow$
$[1, 3]$	$f'(x) \le 0$	$f(x)$ is $\downarrow$
$[3, \infty)$	$f'(x) \ge 0$	$f(x)$ is $\uparrow$

$\therefore f(x)$ is Increasing in $[-2, 1] \cup [3, \infty)$ and decreasing in $(-\infty, -2] \cup [1, 3]$ Ans

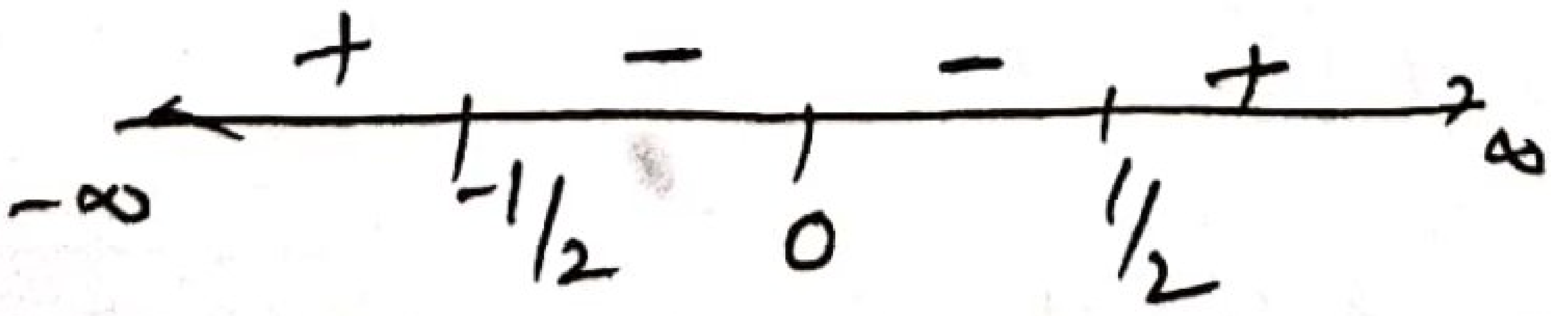
Qn 4 $\rightarrow$ $f(x) = \dfrac{4x^2+1}{x}$

$f(x) = 4x + \dfrac{1}{x}$

Diff $f'(x) = 4 - \dfrac{1}{x^2}$

$f'(x) = \dfrac{4x^2-1}{x^2} = \dfrac{(2x+1)(2x-1)}{x^2}$

Critical points $\quad x = -1/2, \quad x = 1/2, \quad x = 0$



$f(x)$ is strictly $\uparrow$ in $(-\infty, -1/2) \cup (1/2, \infty)$ and strictly $\downarrow$ in $(-1/2, 0) \cup (0, 1/2)$ Ans

Qn. 5 $\quad f(x) = \dfrac{x}{2} + \dfrac{2}{x}$

Diff w.r.t x.

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$$f'(x) = \frac{1}{2} - \frac{2}{x^2}$$

$$= \frac{x^2 - 4}{2x^2}$$

$$f'(x) = \frac{(x+2)(x-2)}{2x^2}$$

Critical points $x = -2,\ x = 2,\ x = 0$

{but we cannot take/include $x = 0$
as $f(x)$ is not defined at $x = 0$}

$$\begin{array}{ccccccc} & + & & - & & - & & + & \\ -\infty & & -2 & & 0 & & 2 & & \infty \end{array}$$

Intervals	Sign of $f'(x)$	Sign of $f(x)$
$(-\infty, -2]$	$f'(x) \geq 0$	$f(x)$ is $\uparrow$
$[-2, 0)$	$f'(x) \leq 0$	$f(x)$ is $\downarrow$
$(0, 2]$	$f'(x) \leq 0$	$f(x)$ is $\downarrow$
$[2, \infty)$	$f'(x) \geq 0$	$f(x)$ is $\uparrow$

$\therefore f(x)$ is increasing in $(-\infty, -2] \cup [2, \infty)$

$f(x)$ is decreasing in $[-2, 0) \cup (0, 2]$ Ans

Qn. 6 → $f(x) = 2\log(x-2) - x^2 + 4x + 1$

$$f'(x) = \frac{2}{x-2} - 2x + 4$$

$$f'(x) = \frac{2 - 2x^2 + 4x + 4x - 8}{x-2}$$

$$f'(x) = \frac{-2x^2 + 8x - 6}{x-2}$$

$$f'(x) = \frac{-2(x^2 - 4x + 3)}{(x-2)}$$

IMP..
$f(x)$ is defined for $x > 2$
as $\log(x-2)$ it exists when
$x - 2 > 0$ $x > 2$

$$f'(x) = \frac{-2(x-3)(x-1)}{x-2}$$

Critical points

$x = 3,\ x = 1,\ x = 2$

$x = 1 \to$ Rejected (as $x > 2$)

line
($x > 2$)

$$\underset{2}{\cdot}\xrightarrow{\quad + \quad}\underset{3}{\cdot}\xrightarrow{\quad - \quad}\infty$$

$\therefore f(x)$ is strictly $\uparrow$ in $(2,3)$ and strictly $\downarrow$ in $(3,\infty)$ Ans

Qn. 7 $\Rightarrow$ $f(x) = x^3 - 3x^2 + 4x$ (Type = 2)

Diff w.r.t x $f'(x) = 3x^2 - 6x + 4$

$f'(x) = 3x^2 - 6x + 3 + 1$

$= 3(x^2 - 2x + 1) + 1$

$f'(x) = 3(x-1)^2 + 1$

for $x \in R$ $(-\infty, \infty)$

$(x-1)^2 \geq 0$

$\Rightarrow 3(x-1)^2 \geq 0$

$\Rightarrow 3(x-1)^2 + 1 > 0$ ... $\{\because$ when $x \geq 0$ then $x+1 > 0\}$

$\Rightarrow f'(x) > 0$

$\therefore f(x)$ is strictly Increasing in R Ans

Qn. 8 $f(x) = \tan^{-1}(\sin x + \cos x)$ (type - 2)

given $x \in (\pi/4, \pi/2)$

Diff w.r.t x

$$f'(x) = \frac{1}{1+(\sin x+\cos x)^2} \cdot (\cos x - \sin x)$$

$$f'(x) = \frac{\cos x(1-\tan x)}{1+(\sin x+\cos x)^2}$$

for $x \in \left(\frac{\pi}{4}, \frac{\pi}{2}\right)$

$\tan x > 1$

$\Rightarrow \quad 1 - \tan x < 0$

also $1+(\sin x+\cos x)^2 > 0$

and $\cos x > 0$

$$\Rightarrow \quad \frac{\cos x(1-\tan x)}{1+(\sin x+\cos x)^2} < 0$$

$\Rightarrow \quad f'(x) < 0$

$\therefore f(x)$ is strictly decreasing in $\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$ Ans

Qn. 9 $\rightarrow$ $f(x) = \log(1+x) - \frac{x}{1+x}$

Diff w.r.t x

$$f'(x) = \frac{1}{1+x} - \left\{\frac{(1+x)(1) - x(1)}{(1+x)^2}\right\}$$

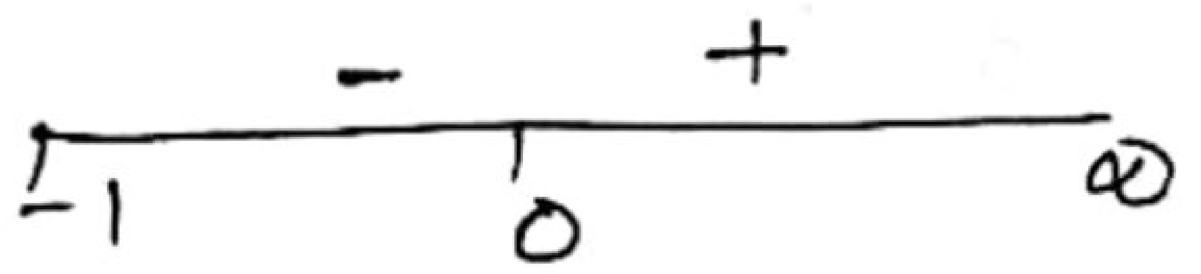
$$f'(x) = \frac{1}{1+x} - \left(\frac{1+x-x}{(1+x)^2}\right)$$

$$= \frac{1+x-1}{(1+x)^2}$$

$$f'(x) = \frac{x}{(1+x)^2}$$

Note $f(x)$ exists only when $x+1>0$
$\Rightarrow \underline{x>-1}$

Critical points $x=0,\ x=-1$



$f(x)$ is increasing in $[0,\infty)$ and
$f(x)$ is decreasing in $(-1,0]$ **ANS**

(-1 cannot be included as $x>-1$)

QN 10 → $f(x) = \cos(2x+\pi/4)$ --- (Type = 2)

Given $x \in (3\pi/8, 7\pi/8)$

Diff wrt x

$$f'(x) = -2\sin(2x+\pi/4)$$

→ we have to check in which quadrant this angle lies?

~~for~~ we have $\frac{3\pi}{8} < x < \frac{7\pi}{8}$

$$\Rightarrow \frac{3\pi}{8} < 2x < \frac{7\pi}{8}$$

$$\Rightarrow \left(\frac{3\pi}{8} - \frac{\pi}{4}\right) < \left(2x - \frac{\pi}{4}\right) < \frac{7\pi}{8} - \frac{\pi}{4}$$

$\Rightarrow \quad \frac{3\pi}{4} < 2x < \frac{7\pi}{4}$

$\Rightarrow \quad \frac{3\pi}{4} + \frac{\pi}{4} < 2x + \frac{\pi}{4} < \frac{7\pi}{4} + \frac{\pi}{4}$

$\Rightarrow \quad \pi < \left(2x + \frac{\pi}{4}\right) < 2\pi$

$\therefore$ angle lies in 3^{rd} & 4^{th} quadrant

and $\sin\theta$ is $-ve$ in 3^{rd} & 4^{th} quadrant

$\Rightarrow \quad \sin\left(2x + \frac{\pi}{4}\right) < 0$

$\Rightarrow \quad -2\sin\left(2x + \frac{\pi}{4}\right) > 0$

$\Rightarrow \quad f'(x) > 0$

$\therefore$ $f(x)$ is strictly $\uparrow$ on $\left(\frac{3\pi}{8}, \frac{7\pi}{8}\right)$ Ans

Q No. 11 → $f(x) = [x(x-2)]^2$

$f(x) = x^2(x-2)^2$

$f'(x) = 2x^2(x-2) + (x-2)^2 \cdot 2x$

$= 2x(x-2)(x + x - 2)$

$= 2x(x-2)(2x-2)$

$f'(x) = 4x(x-2)(x-1)$

put $f'(x) = 0$

$x = 0, \; x = 2, \; x = 1$

$-\infty \quad - \quad 0 \quad + \quad 1 \quad - \quad 2 \quad + \quad \infty$

Clearly $f(x)$ is $\uparrow$ in $[0,1] \cup [2, \infty)$

Ans