Random Number Generators: Implementation and Statistical Analysis Using Dieharder

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***Abstract*—We implement in efficient C++ several kinds of different modern pseudo-random number generators, or PRNGs; a Linear Congruential Generator, a Lagged Fibonacci Generator, an Xorshift type generator, and a Mersenne Twister Generator. We discuss the challenges in testing these kinds of PRNGs, and discuss one of the more common frameworks for testing these random number generators. Finally, we compare our results to what we expect from these implementations, as well as comment on the suitability of these PRNGs for use in simulation of systems.**

***Index Terms*—Random Number Generators, Psuedo Random Number Generators, PRNGs, Statistical Analysis, Dieharder**

I. Introduction

The applications of randomness are far reaching. From statistics, to simulation, to gaming, to cryptography, there is a large need for random number generators. Often time, these random number have to be fast, and deterministic in some cases, like simulations that have to be reproducible and debuggable. Thus, the creation of pseudo random number generators, or PRNGs, has become an important research area in modeling, computer simulation, and mathematics communities.

In the field of analysis of computing systems, simulation is an extremely important part of the modeling process. Today's stochastic models become so complex that solving these models analytically becomes impossible. Simulations driven by randomness then become a far more viable way to study the behavior of systems. This is the sort of need that motivates this project: a study of the theory behind random number generators, how to test them, and their suitability for this purpose.

A. Kinds of Random Number Generators

For the purposes of this paper, there are two kinds of Random Number Generators, or RNGs: Hardware RNGs, and Software RNGs. Hardware RNGs are used not only to output a random bit stream faster than a multipurpose CPU might be able to [1], [2], but also in order to gather entropy in a more unpredictable way, such as measuring thermal noise, detecting air moisture, or using unpredictable user interactions, such as keystrokes and mouse motion , to name a few. Often hardware RNGs are used to seed software RNGs. Readers interested in the seeding of RNGs with external entropy are encouraged to read [3].

Software RNGs, unless they draw upon these sources of entropy from nature for seeds, are in fact PRNGs, because the software that generates these sequences must be deterministic. In this paper, the term RNG will refer to software PRNGs, unless otherwise indicated. These software RNGs use mathematical models to take a finite amount of state and turn this into a sequence of seemingly random output, suitable for use in a variety of purposes. Though special purpose hardware RNG cir-

cuits may be fast, software RNGs are usually faster than repeatedly polling a sensor to gain outside entropy. They are also far more convenient to use, as all is required is a computer, rather than special purpose hardware.

Finally, there are several classes of software RNG based on quality of output. The highest class is those which are suited for cryptography. Generally, this includes passing the next bit test: that if k bits of a random sequence are known, then no polynomial time complexity algorithm should be able to guess the next bit [4]. These cryptographically secure random number generators tend to be slow. They are not designed to be run more than a few times in cryptography, which makes them unsuitable for use in applications in which speed is necessary, like simulation.

This paper deals with RNGs which are not of cryptographic quality, but are nonetheless useful. These RNGs combine speed with memory complexity and quality of output.

B. This Project

In this project, we study and implement a few common random number generators in C++, from those which have been used historically, to those which are most commonly in use today. We test our generators using the Dieharder test suite [5]. Using Dieharder, we can push these RNGs to the point of failure, and therein see their flaws and sutability for various applications. We can also compare our results with the results of the RNGs built into the GNU Scientific Library, or GSL, and thus Dieharder. All of the software written for this project is available on Github [6].

In this paper, we first discuss the methods and challenges of testing random numbers in Section II. Then, we discussed the different classes of RNGs we study in Section III. Finally, we describe our software approach in Section IV and share our results in Section V.

II. Testing Random Number Generators

Testing random number generators can be tricky. An ideal random number generator can be considered to be independently, identically distributed samples of a uniform distribution $U(0, 1)$. However, the RNGs that we consider here are not truly random. So, the crux of the problem is how to determine what set of numbers appear random enough, such that they are suitable for use.

A. The Empirical Approach

Imagine flipping a coin 10 times and recording the number of heads and tails. For a fair coin, one would expect to get roughly the same number of heads and tails. If one got 10 heads, and only 1 tails, one might be inclined to suspect that this coin might not be a fair and truly random coin. However, this event has a distinct nonzero probability of this happening, described by the pdf of the binomial distribution:

$$\binom{10}{9}\frac{1}{2}^{9}\frac{1}{2} = 0.00977$$

In other words, even for a fair coin, in 1000 trials of 10 coin flips, one would expect to get 9 heads and 1 tails 9 or 10 times. Similarly, imagine if one had a different coin, and ran this same test, but always got exactly 5 heads and 5 tails, for all 1000 trials. Though on average we expect to get about the same number of heads and tails, this coin is also suspect, because for all these repeated trials, we should expect them to follow the binomial distribution discussed above.

If we increased the number of flips per trial to a very large n, and increased the number of trials to a very large t, increasingly the distribution of what we expect to see is a normal distribution, with mean heads percentage of $\mu = 0.5$. This is verified by the Central Limit Theorem [7].

So taking again our mystery coin, if we apply the test with reasonably sized n and t, and get a distribution of

our sample means, we expect it to resemble a normal distribution, but reasonable n and t it will be close, but not perfect. There is a certain probability, if we assume that the underlying RNG is perfect, that the result we got would occur; a number on the range $(0, 1)$. We shall call this our p-value. If this p-value is very, very small (on the order of 10^{-6}), we might say with a certain degree of confidence that our assumption that the RNG is good is incorrect. However, we could be more confident if we again repeated this process several times, and looked at the distribution of p-values. Then, we could again find the p-value associated with getting this distribution, which we expect to be uniform on the range $U(0, 1)$.

We can find the probability of getting our p-value distribution using a Kolmogorov-Smirnov test [], which quantifies the difference between the expected distribution and the p-valuedistribution we get. This test yields a second p-value: the probability that the resultant distribution would occur given that the underlying RNG is indeed indistinguishable from a perfect one.

If this second p-value is low, then we can safely decide that the RNG being tested is not good. If this value is high, then, rather than accept that the RNG is 'good', instead we conclude that we cannot reject our hypothesis that it is 'good'.

In fact, this process is exactly what the Dieharder test suite does, and interested readers are encouraged to see [8] for further information. Our implementations of RNGs are used to pipe random numbers to Dieharder, which does this sort of analysis for several different kinds of tests, which are discussed next in Section II-B

B. Dieharder

Dieharder's man page states that "[...]dieharder can eventually contain all rng tests that prove useful in one place[...]" [5]. It aims to be a one stop-shop for testing RNGs. It is the spiritual successor and improvement to George Marsagalia's Diehard testing tool, which in turn is a testing tool which took many tests from the famous Donald Knuth [9]. Though it is not the only set of tests or testing suite of this kind [10], [9], [11], [12], it holds the distinction of being readily available in the Ubuntu package repository, and having good documentation [8].

Dieharder has many tests, listed in Table I. It is worth discussing a few of the tests available in Dieharder to give the reader an idea of what sort of things are tested, and furthermore, what failing one of these tests might indicated about the quality of an RNG. Further information about tests not discussed is available in the Dieharder man pages and help flags.

1) The STS Monobit Test: Taken from the NIST Statistical Test Suite [11], this test is exactly the test described above in Section II-A. The number of ones are counted in a bit stream generated by a random number generator, and the same analysis is done on it. It is not an extremely rigorous test; some bad RNGs will still pass it, this test is good at weeding out extremely poor RNGs.

2) The Diehard Birthdays Test: Taken from Diehard, the Birthdays Test is so named for the Birthday Paradox: That in some set of n randomly chosen people in a room, the probability that two of them share the same birthday becomes high astonishingly quickly, reaching 99.9% at $n = 70$ [13]. If these birthdays are placed in a year then the distance between pairs birthdays should follow a Poisson distribution. The test in Dieharder places 512 birthdays in a 24 bit 'year' and determines if the the resultant distribution is in fact Poisson using the p-value analysis described above.

3) RGB Lagged Sums: Written by the main Dieharder author Robert G. Brown, this test is the only test in the Dieharder suite which tests for lagged correlations. Imagine that overall, an RNG's output seemed very random, but every seven bits, the same pattern emerged throughout the generation. This test tests for that, with several different lag numbers, when running all tests. The lagged bits are taken, used to create single precision numbers between 0 and 1, then t of these numbers are

TABLE I
AVAILABLE TESTS IN DIEHARDER VERSION 3.31.1.

#	Test Name	Reliability
0	Diehard Birthdays	Good
1	Diehard OPERM5	Good
2	Diehard 32x32 Binary Rank	Good
3	Diehard 6x8 Binary Rank	Good
4	Diehard Bitstream	Good
5	Diehard OPSO	Suspect
6	Diehard OQSO	Suspect
7	Diehard DNA	Suspect
8	Diehard Count the 1s (stream)	Good
9	Diehard Count the 1s (byte)	Good
10	Diehard Parking Lot	Good
11	Diehard Minimum Distance (2d Circle)	Good
12	Diehard 3d Sphere (Minimum Distance)	Good
13	Diehard Squeeze	Good
14	Diehard Sums	Don't Use
15	Diehard Runs	Good
16	Diehard Craps	Good
17	Marsaglia and Tsang GCD	Good
100	STS Monobit	Good
101	STS Runs	Good
102	STS Serial (Generalize)	Good
200	RGB Bit Distribution	Good
201	RGB Generalized Minimum Distance	Good
202	RGB Permutations	Good
203	RGB Lagged Sum	Good
204	RGB Kolmogorov-Smirnov	Good
205	Byte Distribution	Good
206	DAB DCT	Good
207	DAB Fill Tree	Good
208	DAB Fill Tree 2	Good
209	DAB Monobit 2	Good

summed. A single trial should have a mean $\mu = 0.5t$. Then, the p-values are determined from several trials of t sample sums. Interested readers are encouraged to see [8] for a more discussion.

C. Interpreting Dieharder Results

The authors of Dieharder are careful to avoid saying that Dieharder can verify that a random number generator appears truly random. Rather, Dieharder serves as a set of tests which can weed out poor RNGs, and say with certainty that they do not appear truly random under scrutiny. This is not to say that they are not useful; for instance, if one is simply creating a website which provides a virtual die for a user to roll, then the RNG powering the die need not necessarily pass every Dieharder test (perhaps only the Diehard Craps Test, which specifically tests using an RNG to power dice for a game), but rather pass a majority of them, and still fulfill the requirements. However, if one is running Monte Carlo simulations, an RNG of a higher quality may be desired, which passes more tests.

A system designer must take into account the trade offs between quality of RNG output, memory efficiency, speed, and complexity of implementation when choosing an RNG for a system. These days, with computer being as fast as they are, and many programming languages having already chosen a de facto RNG, this choice is hidden from most users, but is still relevant to those who work in large simulations, mathematicians, and those concerned with extremely high quality of randomness and speed.

What does failing a Dieharder test look like? We look back to the p-value analysis; This final p-value represents the probability that, given that the underlying PRNG resembles a true RNG, the series of our tests would yeild the results that they did. The final p-value is a probability on the range $(0, 1)$. If this probability is extremely low or extremely high, then the result is considered to be a test failure.

In Dieharder specifically, if the p-value p is $p > \%99.95$ or $p < \%0.05$, then the test is regarded as failed, which is an extremely high threshold. If $p > \%99.5$ or $p < \%0.5$, then the test is marked as weak performance on the part of the RNG. In either the case of the failure or weakness result, it may be worth re-running the test using a different seed to see if this result is consistent. After all, there is some degree of randomness involved,

and finding an area of zero ambiguity is nigh impossible.

D. The Theoretical Approach

Apart from the empirical approach described above and used in Dieharder, there is also theoretical analysis that can be done with a priori knowledge of the RNG's implementation. For instance, the period of an RNG can be time consuming and memory inefficient to measure empirically, but with knowledge of the generating algorithm, can be deduced. Additionally some RNG algorithms (specifically the Linear Congruential Generator discussed in Section III-A) have a property where their output falls in planes which can be determined easily, if one knows where to look.

This sort of analysis exposes flaws of specific generator algorithms, but doesn't always provide an even footing on which to analyze the RNGs. The periods of the generators are discussed in each RNG class's section, along with other strengths and weaknesses, but the focus of our project is on the performance of these RNGs in Dieharder.

III. Classes of Random Number Generators

Here we describe a few different kinds of random number generators, the math that makes them work, give examples of each kind, and give their strengths and weaknesses.

A. Linear Congruential Generators

The Linear Congruential class of RNGs, or LCG for short, is a very old and relatively simple random number generator. They are based on the formula for a line, modulo another number. The following recurrence relation defines the LCG.

$$X_{n+1} = (aX_n + c) \mod m \tag{1}$$

TABLE II
TABLE OF COMMON LCG PARAMETERS.

Name	m	a	c
MINSTD	$2^{32}-1$	48271	0
glibc	2^{31}	1103515245	12345
RANDU	2^{31}	65539	0

Where the parameters are chosen with the following restraints:

$$\begin{aligned} m, &\quad 0 < m \\ a, &\quad 0 < a < m \\ c, &\quad 0 \leq c < m \\ X_0, &\quad 0 \leq X_0 < m \end{aligned}$$

If $c = 0$, this particular class of of RNG is called a Lehmer, or Park-Miller RNG [14], [15].

The period of such generators depends heavily on the chosen parameters. It can be at most m, and as a result, m is often chosen to be the largest number which can fit in the number of bits of the output; usually 2^{32} or 2^{64}. Some common parameter sets in use include MINSTD, and glibc. A set of parameters notoriously deemed unfit for use is called RANDU, and is discussed below. Values for these parameters can be found in Table II.

1) Strengths: LCG generators are tiny. They store a single word of state, are easy to seed, and with proper parameter choosing can be quite effective RNGs. However, some of the flaws greatly outweigh the lightweight and speed benefits of LCGs for all but the most basic of platforms, discussed next.

2) Weaknesses: LCGs have a fatal flaw, that is amplified by bad parameter choosing: their outputs fall in easily identifiable planes, as proved by Marsaglia [16]. This is most easily demonstrated by viewing the output of a notoriously bad set of parameters: RANDU. In Figure 1, the output of RANDU is treated as (x, y, z) pairs as it is generated, then these points are plotted.

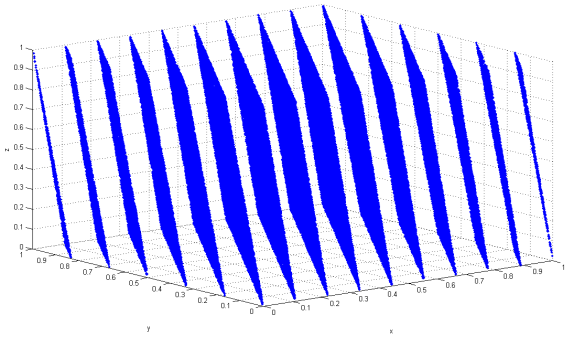

Fig. 1. The problem with LCGs demonstrated by RANDU; a MATLAB plot of consecutive outputs from RANDU used as (x, y, z) points and plotted [18].

It is obvious that all of the output falls in the same set of planes, and is not even an approximation of a truly random number generator. Better parameter choices alleviate this phenomenon by adding the number of planes that would come up in a test like this, and making them closer together, but the basic problem is unavoidable.

Another issue is the period length; at most the period length is m, which cannot be easily increased without increasing the number of bits in the word being operated on, which is not practical on modern architectures. In fact, experts recommend not even considering generators with periods $< 2^{50}$ for anything but the most trivial uses [17].

Due to these problems, most LCGs pass some tests in Dieharder, but do not stand up to more intense scrutiny, like the lagged sums tests. RANDU's performance is patently awful, and stands up to its infamy. MINSTD fares better, but still does not pass as many tests as more sophisticated generators.

Despite these flaws, these extremely small and fast RNGs may still be suited to small embedded architectures and applications where speed is significantly more important than quality.

B. Xorshift Generators

A far more attractive choice than LCGs for those trying to implement RNGs with a lot of memory constraints, speed requirements, or a combination of the two on an embedded platform, is the Xorshift generator. Discovered and described by George Marsaglia [19], they are significantly more adept at creating sufficiently long periods for more applications.

Xorshift RNGs work by keeping a small amount of state (in many example implementations, four numbers), then repeatedly XORing these numbers with shifted versions of themselves. By doing this, the period of the generation can be $2^{32*4} = 2^{128}$ with very little code. The following is a short example for how easily Xorshift generators can be implemented. Let `t`, `x`, `y`, and `w` be seed values. Then, let `a`, `b`, and `c` be the parameters of the RNG. Then, the C style code is as simple as the following:

```
// Random seed numbers
uint32_t t, x, y, w;
// Fixed constants for shifting
const uint32_t a, b, c;
uint32_t get_rand(void){
    t = x ^ (x << a);
    x = y;
    y = z;
    z = w;
    w = w ^ (x >> b) ^ t ^ t( >> c);
    return w;
}
```

Picking parameters for the directions and amounts to shift each state variable by is also relatively easy, and is discussed in [19], along with every set of appropriate parameters a, b, and c for 32 bit and 64 bit words.

1) Strengths: Xorshift RNGs are very fast on modern architectures, and use very little state. They have longer periods than LCGs, and are easier to choose parameters

for. These RNGs also do far better on the Dieharder tests than LCGs, and pass the majority of them rather convincingly.

2) Weaknesses: However, the periods of Xorshift generators are not as large as some other generators. They are suitable for most uses, but in large simulations and Monte Carlo analysis, which both use massive amounts of random numbers, the relatively small period could affect results.

C. Lagged Fibonacci Generators

Lagged Fibonacci Generators, or LFGs, owe their name to the general form of the recurrence relation that defines them, which is reminiscent of the Fibonacci recurrence relation:

$$X_n \equiv (X_{n-r} \star X_{n-s}) \mod m, \quad 0 < r < s \qquad (2)$$

Here, $\star$ can be any operation, usually addition, subtraction, or logical XOR, and m is a modulus which ensures the output is in a proper range. Usually, $m = 2^{32} - 1$, to give maximum period and avoid correlation issues that can arise when $m = 2^32$. The generator stores the past s items of state, then uses the recurrence relation above in Equation 2 to generate the next output. The two lag constants r and s are specially chosen to avoid correlations.

For example, in a popular implementation called R250, $\star$ is defined as XOR, $r = 147$, and $s = 250$ [20]. Another popular implementation is RANLUX, which adds a few wrinkles to a simpler recurrence style generator, called Subtract with Carry [21], [22]:

$$X_n \equiv (X_{n-r} - X_{n-s} - c_{n-1}) \mod m \qquad (3)$$

Here, r and s are the usual lag constants where $0 < r < s$, and c is defined as:

$$c_n = 1 \qquad \text{if } X_{n-r} - X_{n-s} - c_{n-1} \geq 0,$$
$$c_n = 0 \qquad \text{otherwise.}$$

RANLUX takes this generator and adds skipping element parameter p, designed to remove unwanted correlations at the expense of speed. After 24 random numbers are generated, $p - 24$ random numbers are thrown away [23], [24]. The default C++11 parameters for the RANLUX 24 bit engine are $r = 10$, $s = 24$, $p = 24$, which results in no numbers being thrown away, increasing speed, but reducing effectiveness.

1) Strengths: Lagged Fibonacci generators can store a wide range of amount of state, but their flexibility results in many generators of this type being created and seeing common use. The operations are relatively fast, and have larger periods when a few different twist like the subtract with carry has been applied [22]. As a result, good implementations generally do well on the Dieharder tests, passing most, and only getting weak on a some, sporadically.

2) Weaknesses: LFGs have a wrinkle in understanding their simplicity; they are difficult to seed properly, and require special initialization to make sure that there are no linear correlations in the initial seed data, lest the entire generator become extremely predictable [25]. They are in general, slower and require more state than Xorshift generators, and do not have periods as large as Xorshift or Multiply with Carry Generators, discussed next.

D. Multiply With Carry Generators

1) Strengths:

2) Weaknesses:

E. Mersenne Twister Generators

This section will describe the Mersenne Twister class of generators.

1) Strengths:

2) Weaknesses: http://en.wikipedia.org/wiki/Mersenne_twister

IV. Software Implementation

For this project, we use the Dieharder binary on Ubuntu in its raw input mode. This allows us to use a shell pipe to send raw random numbers from an arbitrary generator into Dieharder for analysis.

There are two major components included in our code: C++ source to build an RNG binary, and a small Python script to call the RNG binary and pipe it into Dieharder appropriately. We chose C++ because these tests can take a long time if the generator is slow, and C++ is a good compromise between its ability to allow for modular code and its speed.

The specific RNG algorithms implemented are as follows:

- Mersenne Twister
- RANDU
- MINSTD
- R250
- RANLUX
- Xorshift
- CMWC

Each RNG algorithm inherits from a common base class, which allows for the RNG binary to choose the RNG in use via a command line argument. Random 32 bit numbers are written to `stdout` in raw binary, which is in turned piped to Dieharder for analysis. Finally, the output of Dieharder can be optionally piped to file.

All code for this project is available on Github [6], and was tested using Ubuntu 14.04, gcc version 4.9.1, and Python version 3.4.2. Note that Dieharder is required to run examples, and can be installed via your package manager if available, or built from source [5].

V. Results and Analysis

Describe the results of the dieharder tests for our implementations. Discuss speed, and suitability for simulation.

VI. Conclusion

Talk about the generators.

[26] Marsaglia giving a better MCCSFWCW gen than twister.

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