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**Q1.** For each of the following determine if they have solutions and find all the incongruent solutions:

- (i).  $2x \equiv 5 \pmod{7}$
- (ii).  $3x \equiv 2 \pmod{7}$
- (iii).  $3x \equiv 6 \pmod{9}$
- (iv).  $6x \equiv 3 \pmod{9}$
- (v).  $17x \equiv 14 \pmod{21}$
- (vi).  $19x \equiv 30 \pmod{40}$
- (vii).  $15x \equiv 9 \pmod{25}$
- (viii).  $9x \equiv 5 \pmod{25}$
- (ix).  $128x \equiv 833 \pmod{1001}$
- (x).  $103x \equiv 444 \pmod{999}$
- (xi).  $987x \equiv 610 \pmod{1597}$
- (xii).  $980x \equiv 1500 \pmod{1600}$
- (xiii).  $6789783x \equiv 2474010 \pmod{28927591}$

**Q2.** Find the inverse modulo 17 of each of the following

$$(a) \quad 4 \quad (b) \quad 5 \quad (c) \quad 7 \quad (d) \quad 16$$

and hence solve

- (i).  $4x \equiv 9 \pmod{17}$
- (ii).  $5x \equiv 11 \pmod{17}$
- (iii).  $7x \equiv 2 \pmod{17}$
- (iv).  $16x \equiv 12 \pmod{17}$
- (v).  $16x \equiv 28 \pmod{17}$

**Q3.** Determine which integers  $a$ , with  $1 \leq a \leq 11$  have an inverse modulo 12.

**Answers:**

**Q1.**

- i.  $x \equiv 6 \pmod{7}$
- ii.  $x \equiv 3 \pmod{7}$
- iii.  $x \equiv 2, 5 \text{ or } 8 \pmod{9}$
- iv.  $x \equiv 2, 5, \text{ or } 8 \pmod{9}$
- v.  $x \equiv 7 \pmod{21}$
- vi.  $x \equiv 10 \pmod{40}$
- vii. There are no solutions.
- viii.  $x \equiv 20 \pmod{25}$
- ix.  $x \equiv 812 \pmod{1001}$
- x.  $x \equiv 111 \pmod{999}$
- xi.  $x \equiv 1596 \pmod{1597}$
- xii.  $x_t \equiv (875 + 80t) \pmod{1600}, t = 0, 1, \dots, 19$
- xiii.  $x_t \equiv (247320 + 3157t) \pmod{28927591}, t = 0, 1, \dots, 9162$

**Q2.** The inverses are:

$$(a) \quad 13 \quad (b) \quad 7 \quad (c) \quad 5 \quad (d) \quad 16$$

The solutions to the equations are:

- i.  $x \equiv 15 \pmod{17}$
- ii.  $x \equiv 9 \pmod{17}$
- iii.  $x \equiv 10 \pmod{17}$
- iv.  $x \equiv 5 \pmod{17}$
- v.  $x \equiv 6 \pmod{17}$

**Q3.** Only 1, 5, 7, 11 have inverses modulo 12.