CMPU2012 **Mathematics 2**
Problem Sheet 03

Q1. For each of the following Diophantine equations find a general solution.

(i). $256x - 64y = 128$

(ii). $751x + 391y = 37$

(iii). $6429x + 23573y = 12858$

(iv). $28761x - 59333y = 311$

Q2. Show the following

(a)	$13 \equiv 1 \pmod{2}$	(b)	$22 \equiv 7 \pmod{5}$	(c)	$91 \equiv 0 \pmod{13}$
(d)	$69 \equiv 62 \pmod{7}$	(e)	$-2 \equiv 1 \pmod{3}$	(f)	$-3 \equiv 30 \pmod{11}$
(g)	$111 \equiv -9 \pmod{40}$	(h)	$666 \equiv 0 \pmod{37}$		

Q3. Which of the following pairs of integers are congruent modulo 7?

(a)	$1, 15$	(b)	$0, 42$	(c)	$2, 99$
(d)	$-1, 8$	(e)	$-9, 5$	(f)	$-1, 699$

Q4. For which $m \in \mathbb{Z}_+$ are the following true?

(a)	$27 \equiv 5 \pmod{m}$	(b)	$1000 \equiv 1 \pmod{m}$	(c)	$1331 \equiv 0 \pmod{m}$

Q5. Show that if $a \in \mathbb{Z}$ is even then $a^2 \equiv 0 \pmod{4}$ and if $a \in \mathbb{Z}$ is odd then $a^2 \equiv 1 \pmod{4}$.

Q6. Show that if $a \in \mathbb{Z}$ is odd then $a^2 \equiv 1 \pmod{8}$.

Q7. Find the residue modulo 28 of

(a)	99	(b)	1100	(c)	12345
(d)	-1	(e)	-1000	(f)	-54321

Q11. Find a solution to the linear congruence equations

(i). $3x \equiv 1 \pmod{5}$

(ii). $5x \equiv 4 \pmod{9}$

(iii). $980x \equiv 1500 \pmod{1600}$

Answers:

Q1.

(i). $x = 1 + k, y = 2 + 4k, k \in \mathbb{Z}$.

(ii). $x = 188 + 391k, y = -361 - 751k, k \in \mathbb{Z}$.

(iii). $x = 2 + 11k, y = 0 - 3k, k \in \mathbb{Z}$.

(iv). $x = 36104 + 59333k, y = 17501 + 28761k, k \in \mathbb{Z}$.

Q3.

(a)	Yes	(b)	Yes	(c)	No
(d)	No	(e)	Yes	(f)	Yes

Q4.

(a) $1, 2, 11, 22$ (b) $1, 3, 9, 27, 37, 111, 333, 999$ (c) $1, 11, 121, 1331$

Q7.

(a)	15	(b)	8	(c)	25
(d)	27	(e)	8	(f)	27

Q11.

(i). 2

(ii). 8

(iii). 75