



TECHNOLOGICAL UNIVERSITY DUBLIN – CITY CAMPUS

School of Mathematical Sciences

DT228 (TU856) BSc Computer Science

DT282 (TU858) BSc Computer Science (International)

Year 1

WINTER EXAMINATIONS 2020/2021

CMPU1018 MATHEMATICS 1

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DR C HILLS

14.00 – 16.00 pm, Tuesday, 05 January 2021

Duration: 2 hours

Answer question 1 and any two other questions

Question 1 carries 40 marks. All other questions carry 30 marks each.

Approved calculators may be used

Mathematical tables are provided

New Cambridge Statistical Tables are NOT permitted

1. a) Using the rules of logarithms, solve the following log equation:

$$\log_3(x) - \log_3(x + 6) = 1.$$

(5 marks)

- b) Draw up the modular arithmetic (addition and multiplication) tables for \mathbf{Z}_7 .

(5 marks)

- c) Using the modular multiplication tables in part (b) above, answer the following questions:

i) What is the inverse of 5 mod 7,

ii) Are there any *zero divisors* in \mathbf{Z}_7 ? Give a reason for your answer.

(5 marks)

- d) Use *prime factorisation* to find the highest common factor (hcf) and the lowest common multiple (lcm) of 1350 and 14700.

(5 marks)

- e) The data set below represents the monthly rainfall statistics (mm) for Ireland during 2019:

512, 416, 411, 519, 523, 423, 324, 320, 422, 411, 515, 524.

Calculate the mean, mode and median of the data, giving your answers to 2 decimal places.

(5 marks)

- f) Find the inverse relation f^{-1} for the function $f(x) = \frac{1}{2}x + 3$.

(5 marks)

- g) Find the sum of the following series:

$$1 + 3.5 + 6 + 8.5 + \dots + 101.$$

(5 marks)

- h) A survey was carried out in TU Dublin during 2019 in which students were asked "Which of the online platforms (Instagram, Facebook, Both) have you used in the last week"?

The results showed that 45% of those surveyed had used Instagram, 70% had used Facebook and 35% had used both. What percentage of those surveyed had used *neither* Instagram or Facebook?

(5 marks)

[40 marks]

2.a) Let

$$A = \begin{pmatrix} -3 & 7 \\ 10 & 8 \end{pmatrix}, B = \begin{pmatrix} 4 & -2 & 6 \\ 3 & 0 & -1 \end{pmatrix}, C = \begin{pmatrix} 5 & 11 \\ 0 & -4 \\ -3 & 7 \end{pmatrix}, D = \begin{pmatrix} 4 & 2 & 5 \\ 0 & 2 & -4 \\ 0 & -5 & 1 \end{pmatrix}.$$

Evaluate the following (if possible), or say why it is not possible:

- i) $B - C^T$,
- ii) AB ,
- iii) $5AB$,
- iv) $2BD$,
- v) $A + B$.

(12 marks)

b) Find the inverse of the matrix:

$$A = \begin{pmatrix} 5 & -7 \\ 12 & \frac{1}{2} \end{pmatrix}$$

Hence or otherwise, solve the following system of linear equations:

$$5x - 7y = 58,$$

$$12x + \frac{1}{2}y = 70.$$

(6 marks)

c) A triangle has vertices A , B and C given in homogenous coordinates as follows:

$$A = \begin{pmatrix} 6 \\ 2 \\ -2 \end{pmatrix}, B = \begin{pmatrix} -1 \\ 3 \\ 7 \end{pmatrix} \text{ and } C = \begin{pmatrix} 1 \\ 5 \\ -4 \end{pmatrix}$$

Find the image of the triangle under each of the individual transformations below:

- i) Translate the triangle by -14 in the x -direction and 11 in the y -direction.

(3 marks)

- ii) Scale the triangle by 6 in the x -direction, -4 in the y -direction and -1 in the z -direction.

Note: The scaling matrix S , is given by

$$S = \begin{pmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & s_z \end{pmatrix}.$$

(3 marks)

- iii) Rotate the triangle through an angle of $\frac{4\pi}{9}$ radians (80°) **counter-clockwise** about the origin in the $x-y$ plane. Leave your answers to two decimal places.
Note: The rotation matrix R_θ , that rotates points counter-clockwise about the origin, in the $x-y$ plane through an angle θ , is given by

$$R_\theta = \begin{pmatrix} \cos \theta & -\sin \theta & 0 \\ \sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

(6 marks)

[30 marks]

3. a) Let $f: \mathbb{N} \rightarrow \mathbb{N}$ be given by $f(x) = \frac{1}{2x^2+5x-42}$ and let $g: \mathbb{R}_+ \rightarrow \mathbb{R}_+$ be given by $g(x) = 2x + 10$. Find

- i) $f(7)$,
- ii) $g(y - 1)$,
- iii) $(g \circ f)(x)$,
- iv) $(f \circ g)(5)$.

(8 marks)

- b) Let $S = \{1, 2, 3, 4, \dots, 20\}$ be the universal set and let $A = \{2, 4, 6, 8, 10, 11, 12, 13\}$, $B = \{3, 7, 9, 11, 13, 15, 16, 17\}$ be subsets of S . Given that A^c is the complement of the set A , list the elements in the following sets:

- i) $(A \cup B)^c$,
- ii) $(B \cup A^c) \cap (A \cup B^c)$.

(6 marks)

- c) Let $A = \{a, b, c, d\}$ and $B = \{x, y, z\}$. List the elements of the following sets:

- i) The *Cartesian Product* of A and B , $(A \times B)$.
- ii) The *Power Set* of B , $P(B)$.

(8 marks)

- d) Which of the following relations are *equivalence* relations over $A = \{a, b, c, d\}$?
Give an explanation for your answers.

i) $R_1 = \{(a, a), (b, b), (b, c), (c, b)\}$

ii) $R_2 = \{(a, a), (a, b), (b, a), (b, b), (c, c), (d, d)\}$

iii) $R_3 = \{(a, a), (a, b), (b, a), (b, b), (c, a), (c, b), (a, c), (b, c), (c, c), (d, d)\}$

(8 marks)

[30 marks]

- 4.a) i) Use *Euclid's Algorithm* to calculate the highest common factor of 23100 and 825000:

$$hcf(23100, 825000)$$

(6 marks)

- ii) Using the reverse of Euclid's Algorithm, find integers x and y which satisfy the following equation:

$$23100x + 825000y = hcf(23100, 825000)$$

(6 marks)

- b) Use Fermat's Little Theorem to evaluate

i) The multiplicative inverse of 5 modulo 13.

ii) The multiplicative inverse of 2 modulo 11.

(6 marks)

- c) i) Use Caesar's Cipher with key $k = 11$ to encrypt the following message:

"I HOPE TO SEE SANTA"

(6 marks)

- ii) Use Caesar's Cipher with key $k = 10$ to decrypt the following message:

"IYE KBO NYSXQ QBOKD"

(6 marks)

[30 marks]