
In the following questions the set D is the domain of discourse.

Q1. If $P(x) = x$ spends more than five hours in class each weekday and $D = \{\text{Students in DIT}\}$ then express the following in English:

(a). $\exists x P(x)$

(b). $\forall x P(x)$

(c). $\exists x(\neg P(x))$

(d). $\forall x(\neg P(x))$

Q2. If $P(x) = x$ is happy and $D = \{\text{Students in DIT}\}$ then express the following in English:

(a). $\exists x P(x)$

(b). $\forall x(\neg P(x))$

(c). $\exists x(\neg P(x))$

(d). $\neg\forall x(\neg P(x))$

Q3. Let $C(x) = x$ is a comedian, $F(x) = x$ is funny, $D = \{\text{All people}\}$ then express in English:

(a). $\forall x(C(x) \rightarrow F(x))$

(b). $\forall x(C(x) \wedge F(x))$

(c). $\exists x(C(x) \rightarrow F(x))$

(d). $\exists x(C(x) \wedge F(x))$

Q4. Let $P(x)$ be the predicate $x > x^2, D = \mathbb{R}$. Determine

(a). $P(0)$

(b). $P(1/2)$

(c). $P(2)$

(d). $P(-1)$

(e). $\exists x P(x)$

(f). $\forall x P(x)$

Q5. Let $Q(x)$ be the predicate $x + 1 > 2x, D = \mathbb{Z}$. Determine

(a). $Q(0)$

(b). $Q(-1)$

(c). $Q(1)$

(d). $\exists x Q(x)$

(e). $\forall x Q(x)$

(f). $\exists x(\neg Q(x))$

(g). $\forall x(\neg Q(x))$

Q6. Determine the truth value of:

(a). $\exists x(x^3 = -1)$

(b). $\exists x(x^4 < x^2)$

(c). $\forall x((-x)^2 = x^2)$

(d). $\forall x(2x > x)$

Q7. Express using quantifiers:

(a). All dogs have fleas.

(b). There is a horse that can add.

(c). Every Koala can climb.

(d). No monkey can speak French.

(e). There exists a pig that can swim and and catch fish.

Q8. Let $E(x) = x$ is expensive, $G(x) = x$ is good, $D = \{\text{All books}\}$.

(a). No books are expensive.

(b). All expensive books are good.

(c). No books are good.

Q9. $C(x) = x$ has a cat, $D(x) = x$ has a dog, $B(x) = x$ has a budgie, $D = \{\text{All students in the class}\}$.

(a). A student has a cat, a dog and a budgie.

(b). All students have a cat, a dog or a budgie.

(c). Some student has a cat and a budgie but not a dog.

(d). No student has a cat, a dog and a budgie.

(e). For each animal type there is a student in the class who has a pet of that type.

Q10. If $P(x) = x$ is a professor, $Q(x) = x$ talks too fast, $R(x) = x$ is hard to understand, and $D = \{\text{All people}\}$, then express in English:

(a). $\forall x(P(x) \rightarrow \neg Q(x))$

(b). $\forall x(Q(x) \rightarrow R(x))$

(c). $\forall x(P(x) \rightarrow \neg R(x))$

Q11. Let $L(x, y) = x$ likes y, $D = \{\text{All people}\}$.

(a). Everyone likes everyone.

(b). Everyone likes someone.

(c). Someone does not like anyone.

(d). Everyone likes George.

(e). There is someone whom everyone likes.

(f). There is no one whom everyone likes.

(g). Everyone does not like someone.

(h). There is no one whom everyone dislikes.

Answers

Q1.

(a). At least one student in DIT spends more than five hours in class each weekday.

(b). Every student in DIT spends more than five hours in class each weekday.

(c). At least one student in DIT does not spend more than five hours in class each weekday.

(d). Every student in DIT spends less than five hours in class each weekday.

Q2.

(a). At least one student in DIT is happy.

(b). Every student in DIT is not happy.

(c). At least one student in DIT is not happy.

(d). Not every student in DIT is not happy. Alternately, at least one student in DIT is happy.

Q3.

(a). Everyone who is a comedian is funny.

(b). Everyone is a comedian and is funny.

(c). At least one person who is a comedian is funny.

(d). At least one person is a comedian and is funny.

Q4.

(a). False since $0 = 0^2$.

(b). True since $1/2 > 1/4$.

(c). False since $2 < 2^2$.

(d). False since $-1 < (-1)^2$

(e). True. Take $x = 1/2$.

(f). False. $x = 2$ is a counter-example.

Q5.

(a). True since $0 + 1 > 2(0)$.

(b). True since $-1 + 1 = 0$ which is greater than $2(-1) = -2$.

(c). False since $1 + 1 = 2 = 2(1)$.

(d). True. Take $x = 0$.

(e). False. $x = 1$ is a counter-example.

(f). True. Take $x = 1$.

(g). False. $x = 0$ is a counter-example.

Q6.

(a). True since $(-1)^3 = -1$.

(b). True since $(1/2)^4 < (1/2)^2$

(c). True since the square of a negative number is the same as the square of the corresponding positive number.

(d). False since $2(-1) = -2$ and $-2 < -1$.

Q7.

(a). Let $D = \{\text{All dogs}\}$, $F(x) = \text{has fleas}$, then $\forall x F(x)$ is the required statement.

(b). Let $D = \{\text{All horses}\}$, $A(x) = \text{can add}$, then $\exists x A(x)$ is the required statement.

(c). Let $D = \{\text{All koalas}\}$, $C(x) = \text{can climb}$, then $\forall x C(x)$ is the required statement.

(d). Let $D = \{\text{All monkeys}\}$, $F(x) = \text{can speak French}$, then $\forall x(\neg F(x))$ is the required statement.

(e). Let $D = \{\text{All pigs}\}$, $S(x) = \text{can swim}$, $F(x) = \text{can catch fish}$, then $\exists x(S(x) \wedge F(x))$ is the required statement.

Q8.

(a). $\forall x(\neg E(x))$.

(b). $\forall x(E(x) \rightarrow G(x))$.

(c). $\forall x(\neg G(x))$

Q9.

(a). $\exists x(C(x) \wedge D(x) \wedge B(x))$.

(b). $\forall x(C(x) \vee D(x) \vee B(x))$.

(c). $\exists x(C(x) \wedge B(x) \wedge (\neg D(x)))$.

(d). $\neg\exists x(C(x) \wedge D(x) \wedge B(x))$.

(e). $(\exists x C(x)) \wedge (\exists y D(y)) \wedge (\exists z B(z))$.

Q10.

(a). No professor talks too fast.

(b). All people who talk too fast are hard to understand.

(c). No professors are hard to understand.

Q11.

(a). $\forall x \forall y L(x, y)$.

(b). $\forall x \exists y L(x, y)$.

(c). $\exists x \forall y(\neg L(x, y))$.

(d). $\forall x L(x, George)$.

(e). $\forall x \exists y L(x, y)$.

(f). $\forall x(\neg\exists y)L(x,y)$.

(g). $\forall x\exists y(\neg L(x,y)))$.

(h). $\forall x(\neg\exists y)(\neg L(x,y))$.