Q1. For each of the following Diophantine equations determine if there are solutions and if there are find the general solution.

(i). $21x + 13y = 1$

(ii). $21x + 35y = 12$

(iii). $56x + 138y = 2$

(iv). $71x + 50y = 1$

(v). $84x + 438y = 6$

(vi). $123x + 360y = 3$

(vii). $325x + 26y = 13$

(viii). $966x + 686y = 71$

Answers:

Q1.

(i). Solutions exist as $\gcd(21,13) = 1$ and $1|1$.
The general solution is $x = 5 + 13k, y = -8 - 21k, k \in \mathbb{Z}$.

(ii). No solutions as $\gcd(21,35) = 7$ and $7 \nmid 12$.

(iii). Solutions exist as $\gcd(56,138) = 2$ and $2|2$.
The general solution is $x = -32 + 69k, y = 13 - 28k, k \in \mathbb{Z}$.

(iv). Solutions exist as $\gcd(71,50) = 1$ and $1|1$.
The general solution is $x = -19 + 50k, y = 27 - 71k, k \in \mathbb{Z}$.

(v). Solutions exist as $\gcd(84,438) = 6$ and $6|6$.
The general solution is $x = -26 + 73k, y = 5 - 14k, k \in \mathbb{Z}$.

(vi). Solutions exist as $\gcd(123,360) = 3$ and $3|3$.
The general solution is $x = 41 + 120k, y = -14 - 41k, k \in \mathbb{Z}$.

(vii). Solutions exist as $\gcd(325,26) = 13$ and $13|13$.
The general solution is $x = 1 + 2k, y = -12 - 25k, k \in \mathbb{Z}$.

(viii). No solutions as $\gcd(966,686) = 14$ and $14 \nmid 71$.