

TECHNOLOGICAL UNIVERSITY DUBLIN

School of Mathematics & Statistics

TU856 BSc Computer Science
TU858 BSc Computer Science (International)

Year 2

Semester 1 Examination Session 2022/2023

CMPU2012: MATHEMATICS 2

INTERNAL EXAMINER: DR BLATHNAID SHERIDAN

HEAD OF SCHOOL: DR C HILLS

EXTERNAL EXAMINER: MS PAMELA O'BRIEN

EXAMINATION DURATION: 2 HOURS

Answer question 1 and any two other questions

Question 1 carries 40 marks. All other questions carry 20 marks each.

Approved calculators may be used

Mathematical tables are provided

1. a) i) Find the residue modulo 705, of 3^{702} i.e.

$$3^{702} \pmod{705}.$$

(8 marks)

- b) In a class, 30% failed Mathematics, 17% failed Physics and 11% failed both. A student is selected at random from the class. Calculate the following probabilities:

- i) The student failed Mathematics if he failed Physics.
- ii) The student failed Physics if he failed Mathematics.
- iii) The student failed Mathematics *or* Physics.
- iv) The student failed neither Mathematics or Physics.

(8 marks)

- c) Using the Chinese Remainder Theorem, solve the following system of simultaneous congruence equations:

$$x \equiv 2 \pmod{3}$$

$$x \equiv 3 \pmod{5}$$

$$x \equiv 4 \pmod{11}$$

(8 marks)

- d) Using proof by induction and showing all steps, prove that the sum of the first n odd numbers is n^2 i.e.

$$1 + 3 + 5 + 7 + \dots(2n - 1) = n^2$$

(8 marks)

- e) Let the domain of discourse be the set $D = \{x : x \text{ is a person}\}$. Given the following predicates: $T(x) = x \text{ is a teacher}$,
 $F(x) = x \text{ talks too fast}$,
 $H(x) = x \text{ is hard to understand}$, express the following symbolic statements in English:

i)

$$\forall x (T(x) \longrightarrow \neg F(x)) ,$$

ii)

$$\forall x (F(x) \longrightarrow H(x)) ,$$

iii)

$$\forall x (T(x) \longrightarrow \neg H(x)) .$$

(8 marks)

[40 marks]

- 2.a)** The ciphertext

BSFAGNLJ

was encrypted by means of a Hill digraph cipher, using the matrix

$$A = \begin{pmatrix} 3 & 5 \\ 6 & 3 \end{pmatrix}$$

modulo 26 where $A = 0, B = 1, C = 2, Z = 25$. Find the inverse of A modulo 26 and hence retrieve the message plaintext. (15 marks)

- b) Find the multiplicative inverse of $23 \bmod 491$. (5 marks)

- c) Find all integer solutions of the following Diophantine equation

$$692x + 1246y = 84.$$

(10 marks)

[30 marks]

- 3.a)** A card is drawn at random from a well-shuffled standard deck of 52 cards. Define the events:

A : A face card is selected i.e. any of the Kings, Queens or Jacks,

B : A queen is selected,

C : A spade is selected.

- i) Calculate the following probabilities, giving your answer as a fraction in each case:

A. $P(B)$,

B. $P(B | A)$,

C. $P(B | C)$,

D. $P(C | A^c)$, where A^c is the complement of A . (12 marks)

- ii) Is event B independent of event C ? Give a reason for your answer.

(3 marks)

- b)** A printer breaks down 22 times over 100 days. The printer is then checked daily for 6 days. Assuming that each day is an independent Bernoulli trial with a broken printer indicating success, find:

i) The probability that it is broken on exactly 2 of the days.

ii) The probability that it is broken on exactly 5 of the days.

iii) The probability that it is broken on at least 2 of the days.

iv) The probability that it is broken on at most 4 of the days.

Leave your answers to three decimal places.

(15 marks)

[30 marks]

4.a) Explain what is meant by each of the following terms from graph theory, restricting your attention to simple, undirected graphs:

i) A Bipartite graph,

ii) A Complete Bipartite graph.

(4 marks)

b) For the graph G shown below in Figure 1, answer each of the following questions. Carefully justify your answers.

i) Is G a bipartite graph?

ii) Does G have an Euler path?

iii) Does G have an Euler cycle?

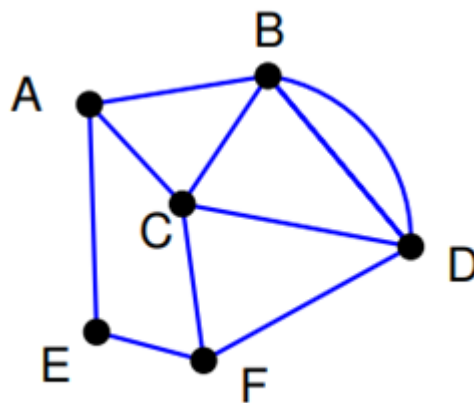


Figure 1 : Graph G

(9 marks)

c) Construct the *incidence* matrix for the graph H in Figure 2 below.

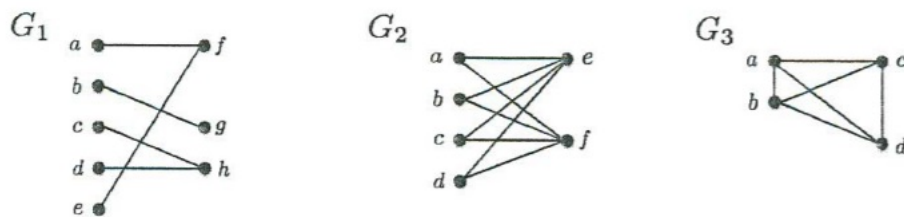


Figure 2 : Graph H

(6 marks)

- d) i) Write down the steps of Kruskal's algorithm for constructing a minimal weight spanning tree for a graph. (3 marks)
- ii) Use Kruskal's algorithm to construct a minimal weight spanning tree for the weighted graph *I* shown in Figure 3 below. What is the total weight of this minimal spanning tree?

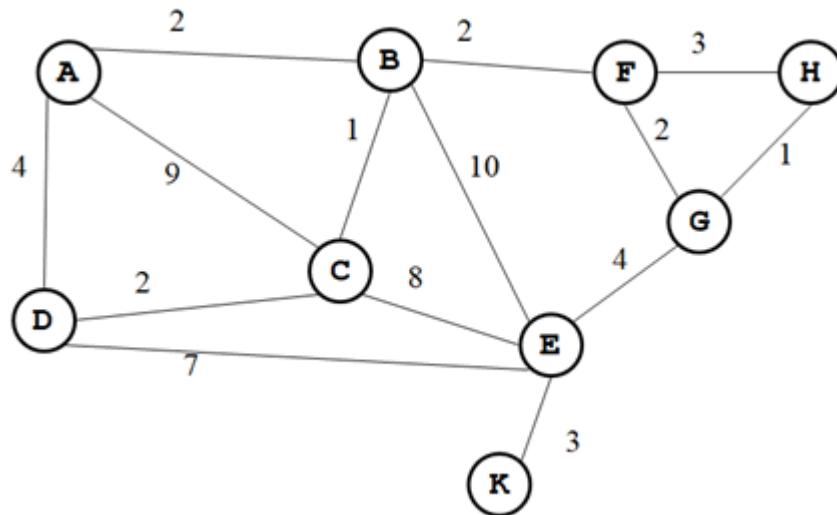


Figure 3 : Graph *I*

(8 marks)

[30 marks]