14.01.20

09.30 - 11.30am

CMPU 2012 Mathematics 2

Basement 2, Kevin Street, Basement 3, Kevin Street

DT228/DT282
CMPU 2012
CRN: 22501,22393, 26461

TECHNOLOGICAL UNIVERSITY DUBLIN

KEVIN STREET, CITY CAMPUS

BSc (Hons) Computer Science
BSc (Hons) Computer Science (International)

Year 2

WINTER EXAMINATION SESSION 2019/20

Mathematics 2

Internal Examiner: Dr B Sheridan

Head of School: Dr C Hills

External Examiner:

Attempt question 1 and any two other questions
Question 1 carries 40 marks. All other questions carry 30 marks
Mathematical tables are provided
Approved calculators may be used

1. **a)** i) Find the residue modulo 705, of 3^{702} i.e.

$$3^{702} \,(\text{mod}705)\,.$$

(8 marks)

b) In a class, 30% failed Mathematics, 17% failed Physics and 11% failed both. A student is selected at random from the class. Calculate the following probabilities:

i) The student failed Mathematics if he failed Physics.

ii) The student failed Physics if he failed Mathematics.

iii) The student failed Mathematics *or* Physics.

iv) The student failed neither Mathematics or Physics.

(8 marks)

c) Using the Chinese Remainder Theorem, solve the following system of simultaneous congruence equations:

$$x \equiv 2 \bmod 3$$

$$x \equiv 3 \bmod 5$$

$$x \equiv 4 \bmod 11$$

(8 marks)

d) Using proof by induction and showing all steps, prove that the sum of the first n odd numbers is n^2 i.e.

$$1 + 3 + 5 + 7 + ...(2n - 1) = n^2$$

(8 marks)

e) Let the domain of discourse be the set $D = \{x : x \text{ is a person}\}$. Given the following predicates: T(x) = x is a teacher,
F(x) = x talks too fast,
H(x) = x is hard to understand, express the following symbolic statements in English:

i)

$$\forall x\,(T(x) \longrightarrow \neg F(x))\,,$$

ii)

$$\forall x\,(F(x) \longrightarrow H(x))\,,$$

iii)

$$\forall x\,(T(x) \longrightarrow \neg H(x))\,.$$

(8 marks)

[40 marks]

2.a) The ciphertext

$$BSFAGNLJ$$

was encrypted by means of a Hill digraph cipher, using the matrix

$$A = \begin{pmatrix} 3 & 5 \\ 6 & 3 \end{pmatrix}$$

modulo 26 where $A = 0, B = 1, C = 2, Z = 25$. Find the inverse of A modulo 26 and hence retrieve the message plaintext. (15 marks)

b) Find the multiplicative inverse of 23491. (5 marks)

c) Find all integer solutions of the following Diophantine equation

$$692x + 1246y = 84.$$

(10 marks)

[30 marks]

3.a) A card is drawn at random from a well-shuffled standard deck of 52 cards. Define the events:

A : A face card is selected i.e. any of the Kings, Queens or Jacks,

B : A queen is selected,

C : A spade is selected.

i) Calculate the following probabilities, giving your answer as a fraction in each case:

A. $P(B)$,

B. $P(B \mid A)$,

C. $P(B \mid C)$,

D. $P(C \mid A^c)$, where A^c is the complement of C. (12 marks)

ii) Is event B independent of event C? Give a reason for your answer.

(3 marks)

b) A printer breaks down 22 times over 100 days. The printer is then checked daily for 6 days. Assuming that each day is an independent Bernoulli trial with a broken printer indicating success, find:

i) The probability that it is broken on exactly 2 of the days.

ii) The probability that it is broken on exactly 5 of the days.

iii) The probability that it is broken on at least 2 of the days.

iv) The probability that it is broken on at most 4 of the days.

Leave your answers to three decimal places. (15 marks)

[30 marks]

4. a) Explain what is meant by each of the following terms from graph theory, restricting your attention to simple, undirected graphs:

i) A Bipartite graph,

ii) A Complete Bipartite graph.

(4 marks)

b) For the graph G shown below in Figure 1, answer each of the following questions. Carefully justify your answers.

i) Is G a bipartite graph?

ii) Does G have an Euler path?

iii) Does G have an Euler cycle?

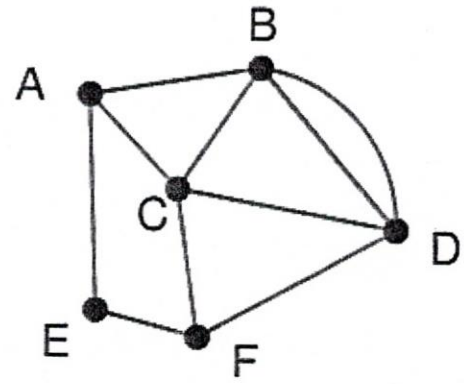

Figure 1 : Graph G

(9 marks)

c) Construct the *incidence* matrix for the graph H in Figure 2 below.

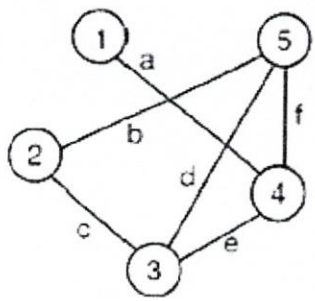

Figure 2 : Graph H

(6 marks)

d) i) Write down the steps of Kruskal's algorithm for constructing a minimal weight spanning tree for a graph. (3 marks)

ii) Use Kruskal's algorithm to construct a minimal weight spanning tree for the weighted graph I shown in Figure 3 below. What is the total weight of this minimal spanning tree?

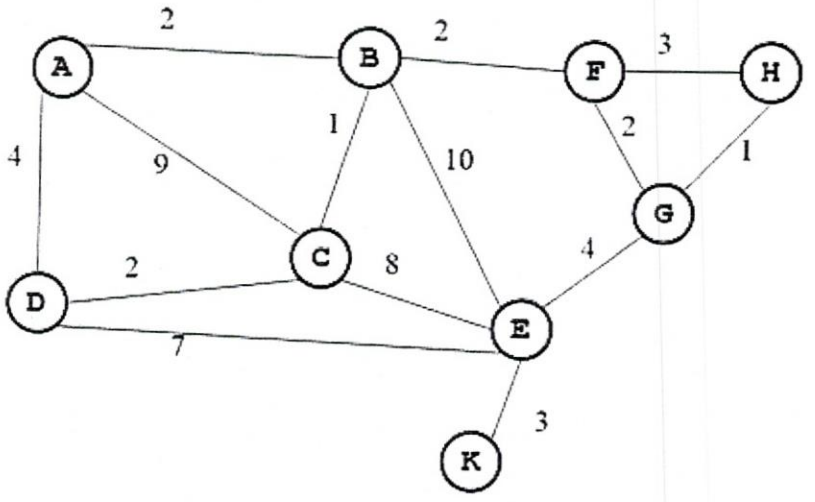

Figure 3 : Graph I

(8 marks)

[30 marks]