

TECHNOLOGICAL UNIVERSITY DUBLIN

School of Mathematics & Statistics

TU856 BSc Computer Science

TU858 BSc Computer Science (International)

Year 2

Semester 1 Examination Session 2023/2024

CMPU2012: MATHEMATICS 2

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HEAD OF SCHOOL: DR C HILLS

EXTERNAL EXAMINER: MS PAMELA O'BRIEN

EXAMINATION DURATION: 2 HOURS

Answer question 1 and any two other questions

All questions carry equal marks

Approved calculators may be used

Mathematical tables are provided

1. a) Find the residue modulo 601, of 2^{518} i.e.

$$2^{528} \pmod{601}.$$

(8 marks)

- b) In a class, 27% of the students failed Mathematics, 22% failed Programming and 14% failed both. A student is selected at random from the class. Calculate the following probabilities:

- i) The student failed Mathematics if they failed Programming.
- ii) The student failed Programming if they failed Mathematics.
- iii) The student failed Mathematics *or* Programming.
- iv) The student failed neither Mathematics or Programming.

(8 marks)

- c) Using the Chinese Remainder Theorem, solve the following system of simultaneous congruence equations:

$$x \equiv 2 \pmod{5}$$

$$x \equiv 3 \pmod{7}$$

$$x \equiv 10 \pmod{11}$$

(8 marks)

- d) Using proof by induction and showing all steps, prove that for all $n \in \mathbb{Z}^+$,

$$1 + 2 + \cdots + n = \frac{n(n+1)}{2}$$

(8 marks)

- e) Let the domain of discourse be the set $D = \{x : x \text{ is an animal}\}$. Given the following predicates:

$F(x) = x \text{ is furry}$

$W(x) = x \text{ has wings}$

$C(x) = x \text{ climbs trees}$

express the following symbolic statements in English:

i)

$$\forall x (F(x) \longrightarrow \neg W(x)) ,$$

ii)

$$\forall x (W(x) \longrightarrow C(x)) ,$$

iii)

$$F(Koala) \wedge C(Koala).$$

(8 marks)

[40 marks]

- 2.a) The ciphertext

APADJTFTWLFJ

was encrypted by means of a Hill digraph cipher, using the matrix

$$A = \begin{pmatrix} 7 & 8 \\ 11 & 11 \end{pmatrix}$$

modulo 26 where $A = 0, B = 1, C = 2, Z = 25$. Find the inverse of A modulo 26 and hence retrieve the message plaintext. (15 marks)

- b) Find the multiplicative inverse of

$$51 \pmod{176}.$$

(5 marks)

- c) Find all integer solutions of the following Diophantine equation

$$56x + 72y = 8.$$

(10 marks)

[30 marks]

3.a) Consider the following table.

	Right-handed	Left-handed	Total
Male	0.41	0.08	0.49
Female	0.45	0.06	0.51
Total	0.84	0.14	1

Find the probability that a randomly selected person is:

- i) Right-handed, (3 marks)
 - ii) A male given that he is right-handed, (3 marks)
 - iii) Left-handed given that she is a female, (3 marks)
 - iv) A female given that she is left-handed, (3 marks)
 - v) Are the events *being a female* and *being left-handed* independent? Explain your answer. (3 marks)
- b)** A computer is broken 14 times over 95 days. The computer is checked daily for 5 days. Assuming that each day is an independent Bernoulli trial with a broken computer indicating success, find:
- i) The probability that it is broken on exactly 2 of the days.
 - ii) The probability that it is broken on all 5 of the days.
 - iii) The probability that it is broken on at least 3 of the days.
 - iv) The probability that it is broken on at most 4 of the days.

Give your answers to four decimal places.

(15 marks)

[30 marks]

4.a) Explain what is meant by each of the following terms from graph theory, restricting your attention to simple, undirected graphs:

i) The *degree* of a vertex in a graph,

ii) A *Spanning Tree* in a graph.

(4 marks)

b) For the graph G shown below in Figure 1, answer each of the following questions. Carefully justify your answers.

i) Is G a bipartite graph?

(3 marks)

ii) Does G have an Euler path?

(3 marks)

iii) Does G have an Euler cycle?

(3 marks)

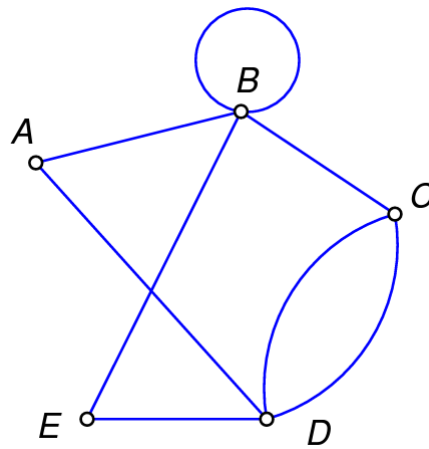


Figure 1 : Graph G

- c) Construct the *adjacency and incidence* matrices for the graph H in Figure 2 below.

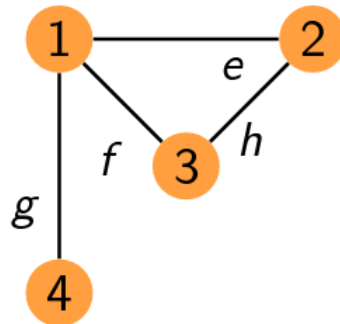


Figure 2 : Graph H

(6 marks)

- d) i) Write down the steps of Dijkstra's algorithm for finding the shortest (i.e. minimal weight) path between nodes on a weighted graph. (3 marks)
- ii) Use Dijkstra's algorithm to find the shortest path from node 0 to all other nodes for graph I in Figure 3 below. What is the total weight of this minimal spanning tree?

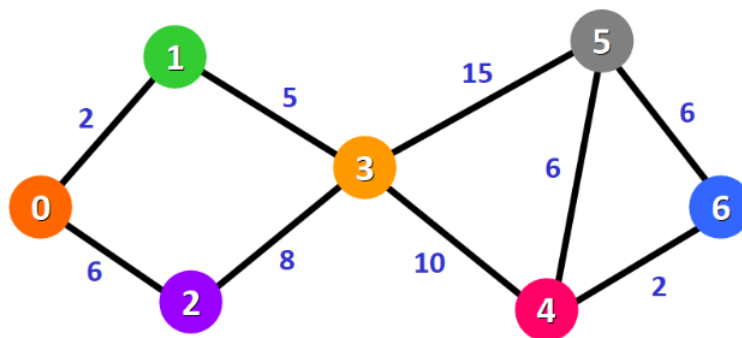


Figure 3 : Graph I

(8 marks)

[30 marks]