

Q1. For each of the following determine if they have solutions and find all the incongruent solutions:

- (i). $2x \equiv 5 \pmod{7}$
- (ii). $3x \equiv 2 \pmod{7}$
- (iii). $3x \equiv 6 \pmod{9}$
- (iv). $6x \equiv 3 \pmod{9}$
- (v). $17x \equiv 14 \pmod{21}$
- (vi). $19x \equiv 30 \pmod{40}$
- (vii). $15x \equiv 9 \pmod{25}$
- (viii). $9x \equiv 5 \pmod{25}$
- (ix). $128x \equiv 833 \pmod{1001}$
- (x). $103x \equiv 444 \pmod{999}$
- (xi). $987x \equiv 610 \pmod{1597}$
- (xii). $980x \equiv 1500 \pmod{1600}$
- (xiii). $6789783x \equiv 2474010 \pmod{28927591}$

Q2. Find the inverse modulo 17 of each of the following

$$(a) \quad 4 \quad (b) \quad 5 \quad (c) \quad 7 \quad (d) \quad 16$$

and hence solve

- (i). $4x \equiv 9 \pmod{17}$
- (ii). $5x \equiv 11 \pmod{17}$
- (iii). $7x \equiv 2 \pmod{17}$
- (iv). $16x \equiv 12 \pmod{17}$
- (v). $16x \equiv 28 \pmod{17}$

Q3. Determine which integers a , with $1 \leq a \leq 11$ have an inverse modulo 12.

Answers:

Q1.

- i. $x \equiv 6 \pmod{7}$
- ii. $x \equiv 3 \pmod{7}$
- iii. $x \equiv 2, 5$ or $8 \pmod{9}$
- iv. $x \equiv 2, 5,$ or $8 \pmod{9}$
- v. $x \equiv 7 \pmod{21}$
- vi. $x \equiv 10 \pmod{40}$
- vii. There are no solutions.
- viii. $x \equiv 20 \pmod{25}$
- ix. $x \equiv 812 \pmod{1001}$
- x. $x \equiv 111 \pmod{999}$
- xi. $x \equiv 1596 \pmod{1597}$
- xii. $x_t \equiv (875 + 80t) \pmod{1600}, t = 0, 1, \dots, 19$
- xiii. $x_t \equiv (247320 + 3157t) \pmod{28927591}, t = 0, 1, \dots, 9162$

Q2. The inverses are:

$$(a) \quad 13 \quad (b) \quad 7 \quad (c) \quad 5 \quad (d) \quad 16$$

The solutions to the equations are:

- i. $x \equiv 15 \pmod{17}$
- ii. $x \equiv 9 \pmod{17}$
- iii. $x \equiv 10 \pmod{17}$
- iv. $x \equiv 5 \pmod{17}$
- v. $x \equiv 6 \pmod{17}$

Q3. Only 1, 5, 7, 11 have inverses modulo 12.