Programme Code: TU856TU858
Module Code: CMPU2012

TECHNOLOGICAL UNIVERSITY DUBLIN

School of Mathematics & Statistics

TU856 BSc Computer Science
TU858 BSc Computer Science (International)

Year 2

Semester 1 Examination Session 2024/2025

CMPU2012: MATHEMATICS 2

INTERNAL EXAMINER: DR BLATHNAID SHERIDAN
HEAD OF SCHOOL: DR C HILLS
EXTERNAL EXAMINER: DR. COLM O RIORDAN

EXAMINATION DURATION: 2 HOURS

Answer question 1 and any two other questions

Question 1 carries 40 marks. All other questions carry 30 marks each.

Approved calculators may be used

Mathematical tables are provided

1. a) Compute the following numbers and show all of your work.

i) $2^{12} \pmod{11}$,

ii) $20^{40} \pmod{21}$.

(8 marks)

b) Solve the following system of congruences using the Chinese Remainder Theorem:

$$x \equiv 3 \bmod 4$$

$$x \equiv 5 \bmod 7$$

$$x \equiv 6 \bmod 9$$

(8 marks)

c) Given that $D(x) = \{\text{All students in TUD}\}$ is the domain of discourse and the predicates: $H(x)$;x is happy and $S(x)$; x is studious

i) Express the following statements using logical notation:

A. No students are happy,

B. Some students are happy and studious.

(4 marks)

ii) Express in English

A. $\exists x \left(H(x) \longrightarrow S(x)\right)$,

B. $\forall x \left(H(x) \land S(x)\right)$.

(4 marks)

d) Solve the following linear congruence equations

i)

$$5x \equiv 1(mod\ 7)$$

(4 marks)

ii)

$$7x \equiv 4(mod\ 11)$$

(4 marks)

e) Two dice are rolled. Calculate the probability that

i) The number on each die is even,

ii) The sum of the numbers rolled is 8 or 11,

iii) The number on one dice is even and on the other is odd.

(8 marks)

[40 marks]

2. a) A card is drawn from a well-shuffled standard deck of 52 cards. Define the events

A: a face card is drawn (King, Queen or Jack card),

B: a Queen is drawn,

C: a spade is drawn, Calculate the following probabilities:

i) $P(B)$

ii) $P(B \mid C)$

iii) $P(B \mid A^c)$

iv) $P(A \mid B^c)$ (12 marks)

b) A football team has a probability of $\frac{5}{7}$ of winning whenever it plays. Suppose they play 5 matches. Find the probability that

i) The team wins exactly 2 matches,

ii) The team wins exactly 4 matches,

iii) The team wins at most 3 matches,

iv) The team wins at least 2 matches.

(18 marks)

[30 marks]

3. a) Find all integer solutions of the following Diophantine equation

$$84x + 438y = 6.$$

(12 marks)

b) Find the modular inverse (mod 26) of the matrix

$$\begin{pmatrix} 2 & 3 \\ 3 & 6 \end{pmatrix}$$

and use it to decrypt the ciphertext "FKMFIO". Assume that a 26 letter alphabet with numerical equivalents $A = 0, B = 1, ...Z = 25$ is being used.

(18 marks)

[30 marks]

4. a) Given the graph G below, sketch the following:

i) Two different subgraphs of G,

ii) A **spanning** subgraph of G. (3 marks)

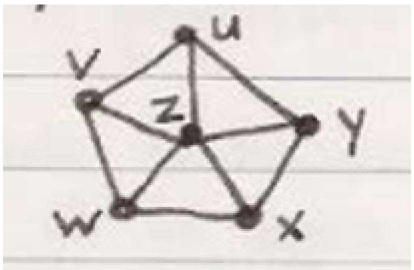

Figure 1: Graph G

b) Construct the *adjacency* matrix for the graph H shown in Fig 2. (5 marks)

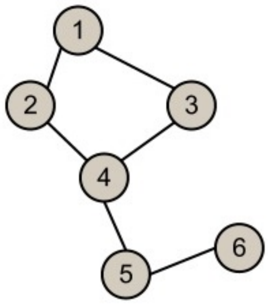

Figure 2: Graph H

c) For graph I shown below in Fig 3:

i) Write down the steps of Dijkstra's algorithm for finding the shortest path from a starting node to all other nodes. (6 marks)

ii) Use Disjkstra's algorithm to find the shortest path from node 1 to all other nodes. Show all you workings. (16 marks)

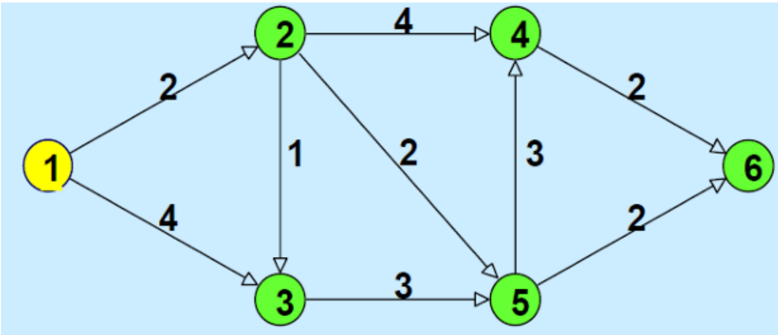

Figure 3: Graph I

[30 marks]