CMPU2012 **Mathematics 2**
Problem Sheet 05

In the following questions, all dice, cards, coins etc. are assumed to be fair and unbiased.

Q1. How many ways can three books be arranged side by side on a shelf?

Q2. A bag contains 1 red, 1 black and 1 green marble. A single marble is drawn randomly from the bag three times in succession and is replaced after each draw before the next is drawn. How many different samples of three marbles can occur?

Q3. In how many ways can two teams be chosen to play each other in an 8-team league? How many games are necessary if each team is to play every other team once?

Q4. How many committees of 3 people can be chosen from a panel of 10 people?

Q5. A 3-digit number is formed at random from the digits $2, 3, 5, 6, 7, 9$ without repetition.

(a). How many such numbers can be formed?

(b). How many are less than 400?

(c). How many are even?

(d). How many are odd?

(e). How many are multiples of 5?

Q6. 3 boys and 2 girls sit in a row.

(a). How many ways can they be arranged.

(b). How many ways can they be arranged if the boys must sit together and the girls must sit together.

(c). how many ways if only the girls must sit together.

Q7. An urn contains 8 balls. How many ordered samples of three balls can be chosen with replacement? Without replacement?

Q8. Two dice are rolled. Calculate the probability that

(a). The number on each die is odd?

(b). The sum of the numbers rolled is 10?

(c). The sum of the numbers rolled is even?

(d). The sum of the numbers rolled is 7 or 11?

(e). The sum of the numbers rolled is $2, 3$ or 12?

(f). The number on one die is even and on the other is odd?

Q9. A coin is tossed three times. Calculate the probability that

(a). There are exactly two heads?

(b). The last two tosses are tails?

(c). All three tosses are the same?

(d). The second toss is heads?

Q10. A card is chosen at random from a standard deck of cards. Which pairs of the following events are mutually exclusive?

(a). The card is a heart?

(b). The card is a face card (J, Q or K)?

(c). The card is an Ace?

Q11. A bowl contains 10 marbles, 3 red, 2 white and 5 blue. A marble is selected at random from the bowl. What is the probability that the marble chosen is white?

Q12. A bag contains 10 tickets numbered from 1 to 10. Three tickets are chosen at random without replacement. Calculate the probability that

(a). The first ticket drawn is 10?

(b). The second ticket drawn is 10?

(c). The first two tickets are drawn in order?

(d). All three tickets are drawn in order?

Q13. An integer is selected at random from the integers $1 \ldots 100$. Calculate the probability that

(a). The number is divisible by 11?

(b). The number is greater than 90?

(c). The number is less than or equal to 3?

(d). The number is a perfect square?

Q14. An urn contains 4 balls numbered $1, 2, 3, 4$. Two balls are drawn without replacement. Calculate the probability that

(a). The first ball is a 4?

(b). The first ball is a 5?

(c). The sum of the balls drawn is 5?

(d). The sum of the balls is less than 5?

Q15. A fair die is rolled. Define the events:

F : a 5 is rolled.

O : an odd number is rolled.

Calculate:

(a). $P(F|O)$.

(b). $P(O|F^C)$.

Q16. Three fair coins, a 1 cent, 5 cent and 10 cent, are tossed. Calculate the probability that all three coins come up heads given that:

(a). The 1 cent coin is heads.

(b). At least one of the coins is heads.

Q17. Two fair dice are thrown. Given that the two numbers appearing are different calculate the probability that:

(a). The sum is 6.

(b). A 1 appears.

(c). The sum is 4 or less.

Q18. A card is drawn at random from a well-shuffled standard deck of 52 cards. Define the events:

A : A face card is drawn.

B : A King is selected.

C : A heart is selected.

Calculate the following probabilities:

(a). $P(B)$

(b). $P(B|A)$

(c). $P(B|C)$

(d). $P(B|A^C)$

(e). $P(A)$

(f). $P(A|B)$

(g). $P(A|C)$

(h). $P(A|B^C)$

Answers:

Q1. 3!

Q2. 3^3

Q3. $\binom{8}{2}, 28$

Q4. $\binom{10}{3}$

Q5. (a). 120, (b). 40, (c). 40, (d). 80, (e). 20

Q6. (a). 120, (b). 24, (c). 48

Q7. 512, 336

Q8. (a). 1/4, (b). 1/12, (c). 1/2, (d). 2/9, (e). 1/9, (f). 1/2

Q9. (a). 3/8, (b). 1/4, (c). 1/4, (d). 1/2

Q10. Only (b). and (c) are mutually exclusive.

Q11. 1/5

Q12. (a). 1/10, (b). 1/10, (c). 1/10, (d). 1/90

Q13. (a). 9/100, (b). 1/10, (c). 3/100, (d). 1/10

Q14. (a). 1/4, (b). 0, (c). 1/3, (d). 1/3

Q15. (a). 1/3, (b). 2/5.

Q16. (a). 1/4, (b). 1/7.

Q17. (a). 2/15, (b). 1/3, (c). 2/15.

Q18. (a). 4/52, (b), 1/3, (c). 4/52, (d). 0, (e). 3/13, (f). 1, (g). 3/13, (h). 1/6.