Lecture 21: Wind Energy – Part II

Course: MECH-422 – Power Plants

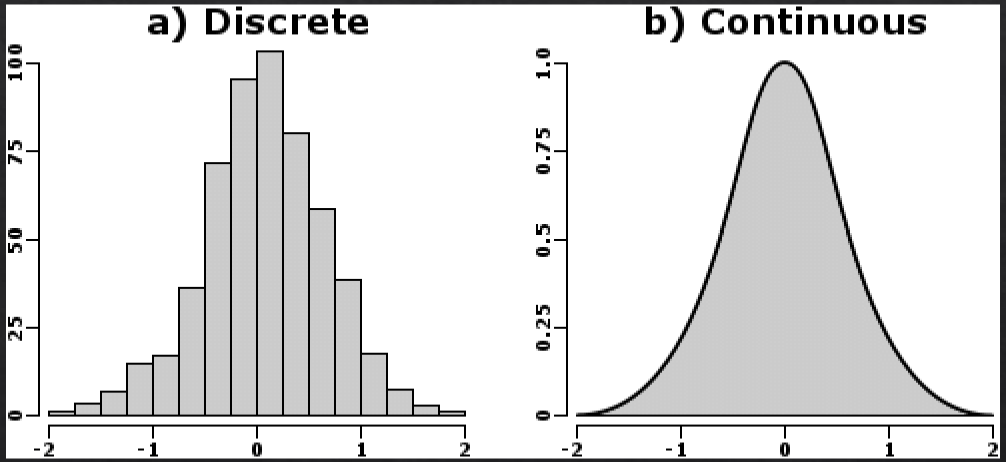
Instructor: Kashif Liaqat

Term: Fall 2021

BUITEMS – DEPARTMENT OF MECHANICAL ENGINEERING

Probability distribution

- The probability distribution for a random variable describes how the probabilities are distributed over the values of the random variable.
- For a discrete random variable, x, the probability distribution is defined by a probability mass function, denoted by $f(x)$



Weibull distribution

- Although wind speed is seemingly random and unpredictable
- In reality to large extent, it follows a predictable pattern that can be expressed by a probability distribution.
- The most common continuous probability distribution used to model wind speed is the **Weibull distribution**

PDF vs CDF

- Probability Density Function (PDF) vs Cumulative Distribution Function (CDF)
- The CDF is the probability that random variable values less than or equal to x
- Whereas the PDF is a probability that a random variable, say X, will take a value exactly equal to x.

Weibull distribution – PDF

The probability density function (PDF) of the Weibull distribution is

$$f(V;k,\lambda)=\left(\frac{k}{\lambda}\right)\left(\frac{V}{\lambda}\right)^{k-1}e^{-\left(\frac{V}{\lambda}\right)^{k}} \text{ for } V>0 \tag{13.21}$$

where V is wind speed (m/s), k is the shape factor (dimensionless), and λ is the scale factor (m/s). In this equation, k and λ depend on the type of wind and its characteristics. For a given k and λ, the probability density function (f) can predict the likelihood of occurrence of a certain wind speed.

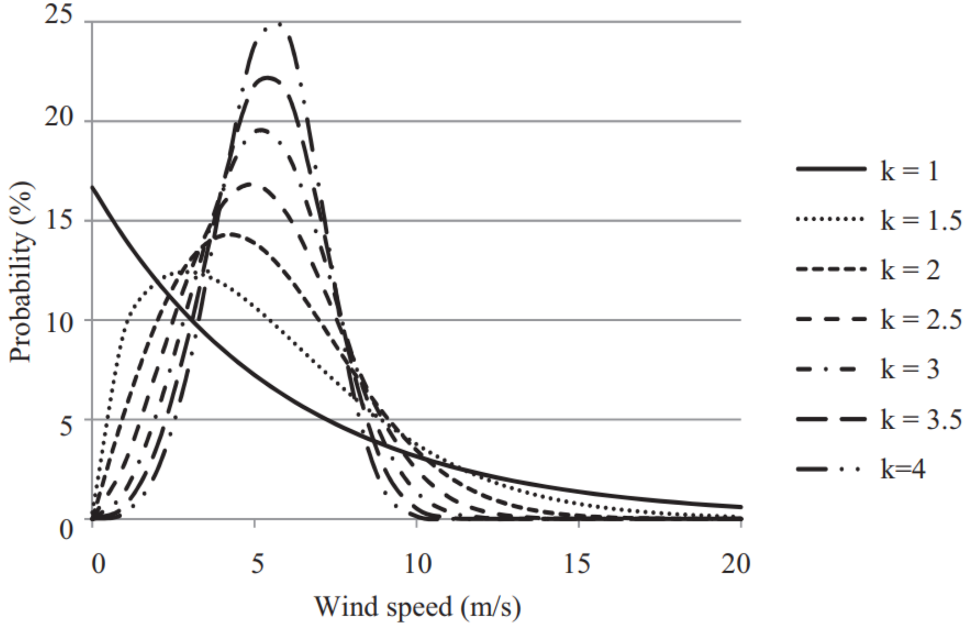


FIGURE 13.11
Weibull probability density function for $\lambda = 6\,\text{m/s}$.

Weibull distribution – CDF

- Wind turbines can produce their full power (the rated power) above a certain wind speed (the rated wind speed).

- So it is vital to be able to determine what fraction of the wind distribution occurs above the rated wind speed.

- The area under the curve between the upper and lower limits in this curve is equal to the probability of having wind at the specified range.

- A simpler method is to use the cumulative distribution function (CDF) of the Weibull distribution:

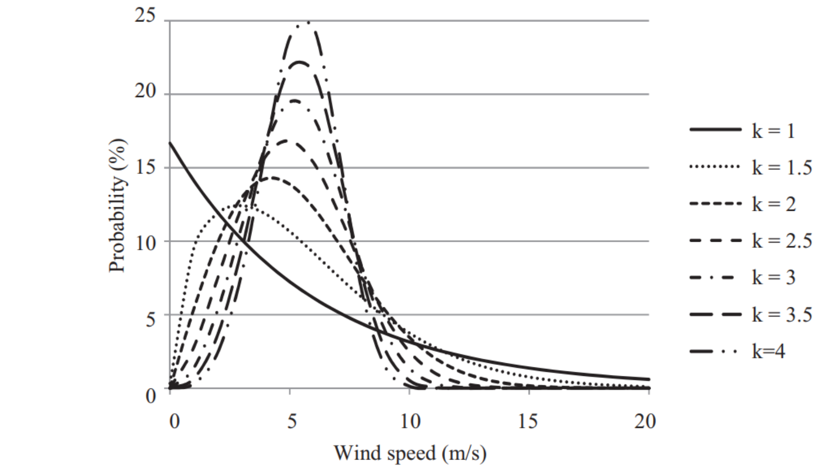


FIGURE 13.11
Weibull probability density function for $\lambda = 6$ m/s.

Weibull distribution – CDF

A simpler method is to use the cumulative distribution function (CDF) of the Weibull distribution:

$$F(V;k,\lambda) = 1 - e^{-\left(\frac{V}{\lambda}\right)^k} \qquad (13.22)$$

where F is the probability of wind with a speed of smaller than or equal to V.

Example 13.4

For wind in a region, the shape factor and scale factor of the Weibull distribution that fits the distribution of the wind are 2 and 6 m/s, respectively. Determine the probability of having the wind speed of 10 m/s and how many hours a year the wind with the speed of 10 m/s is likely to blow.

Example 13.4

For wind in a region, the shape factor and scale factor of the Weibull distribution that fits the distribution of the wind are 2 and 6 m/s, respectively. Determine the probability of having the wind speed of 10 m/s and how many hours a year the wind with the speed of 10 m/s is likely to blow.

In this case $V = 10\,\text{m/s}, k = 2, \lambda = 6\,\text{m/s}$, substituting these values in the probability distribution function in Equation 13.21

$$f(10;2,6) = \left(\frac{2}{6}\right)\left(\frac{10}{6}\right)^{2-1} e^{-\left(\frac{10}{6}\right)^{2}} = 0.0345$$

Hours a year with the wind speed of $10\,\text{m/s} = 0.0345 \times 24 \times 365 = 302.22\,\text{h}$

That means for this wind there is only 3.45% chance of having wind with the speed of 10 m/s or about 302 hours a year (about 12.5 days a year).

Example 13.5

In Example 13.4, determine the probability of having the wind speed between 3 and 10 m/s and how many hours a year wind with the speed of 3 and 10 m/s is likely to blow.

Example 13.5

In Example 13.4, determine the probability of having the wind speed between 3 and 10 m/s and how many hours a year wind with the speed of 3 and 10 m/s is likely to blow.

$$F(3;2,6) = 1 - e^{-\left(\frac{3}{6}\right)^2} = 0.221$$

$$F(10;2,\ 6) = 1 - e^{-\left(\frac{10}{6}\right)^2} = 0.938$$

Probability of having wind speed between 3 and 10 m/s

$$= F(10;2,\ 6) - F(3;2,\ 6) = 0.938 - 0.221 = 0.717$$

Hours a year with the wind speed of between 3 and 10 m/s

$$= 0.717 \times 24 \times 365 = 6280.92\,\text{h}$$

That means it is likely that about 262 days a year, the wind speed would be between 3 and 10 m/s.

Variation of Wind Speed as a Function of Height

- On many occasions, wind measurement elevation is not at the level of the hub of the intended turbine.
- How wind speed information in one elevation can be used to predict the same properties in other?
- The variation of wind speed as a function of height is known as wind shear.
- Generally speaking, the wind speed increases with an increase in height, but the magnitude of this variation depends on the ground roughness and obstacles, such as buildings and trees.
- The most widely used method for this purpose is the wind power law.

Wind Power Law

$$\frac{V_2}{V_1} = \left(\frac{h_2}{h_1}\right)^{\gamma} \qquad (13.23)$$

where V_2 and V_1 are the wind speeds at the heights of h_2 and h_1, respectively, and γ is the wind shear exponent (also known as the Hellmann exponent). This exponent depends on the roughness of the terrain and its obstacles. The most commonly used wind shear exponent when wind speed is available at a single height in open flat terrains is 0.14 $\left(\text{or } \frac{1}{7}\right)$. This exponent can be as high as 0.6 for rough terrains with many obstacles

Example 13.6

A wind turbine can be installed at the hub height of 15 m or 60 m. If the location of the installation is open flat terrain, determine the power ratio of the turbine at two elevations.

Turbine Blade

$$C_D = \frac{F_D}{\frac{1}{2}\rho A V_{rel}^2} \quad (13.25)$$

$$C_L = \frac{F_L}{\frac{1}{2}\rho A V_{rel}^2}$$

where C_D and C_L are **coefficients of drag and lift**, respectively, F_D and F_L are the drag and lift forces, respectively, and V_{rel} is the velocity of the fluid relative to the object. For a stationary object, V_{rel} will be equal to the velocity of the flow (wind).

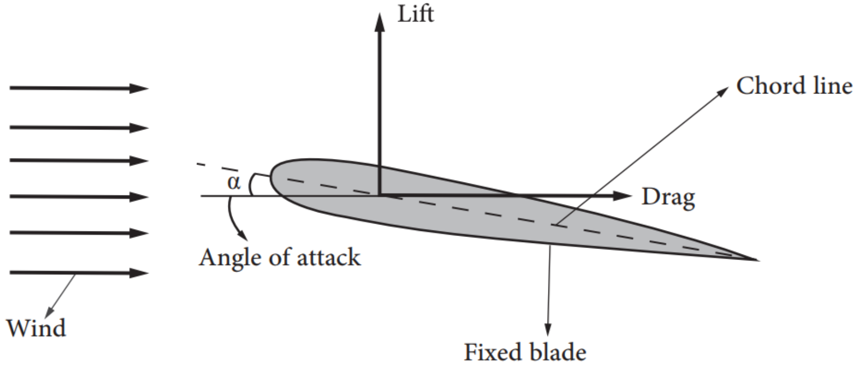


FIGURE 13.15
Schematic of lift and drag forces.

End of Lecture!