Name:-
Kashif Ali

Roll No:-
20P-0648

Section:-
3D

Assignment #01

20P-0698
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→ Propositional Logic:-

Q No 1:- $(\neg P \wedge q) \vee \neg(P \vee q) \equiv \neg P$

$(\neg P \wedge q) \vee (\neg P \wedge \neg q)$

$\neg P \wedge (q \vee \neg q)$

$\neg P \wedge T$

$\neg P$

$\therefore$ L.H.S = R.H.S.

Q No 2:- $\neg(\neg P \to \neg q) \vee (P \wedge q) \equiv q$

$(P \to q) \vee (P \wedge q)$ $\because (P \to q = \neg P \vee q)$

$(\neg P \vee q) \vee (P \wedge q)$

$\neg P \vee (q \vee P) \wedge (q \vee q)$

$\underline{\neg P} \vee q \vee \underline{P} \wedge (q)$

$q \wedge q$ $(\because \neg P \vee P = T)$

$\equiv q$

$\therefore$ L.H.S = R.H.S

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Q No 3:

$(P \to \neg q) \leftrightarrow r$

p	q	r	$\neg q$	$P \to \neg q$	$(P \to \neg q) \leftrightarrow r$
T	T	T	F	F	F
T	T	F	F	F	T
T	F	T	T	T	T
T	F	F	T	T	F
F	T	T	F	T	T
F	T	F	F	T	F
F	F	T	T	T	T
F	F	F	T	T	F

These are the True values.

Q No 4: $q \to (P \to q)$

$q \to (\neg P \vee q)$

$(q \to \neg P) \vee (q \to q)$ $\quad \therefore (P \to q = \neg P \vee q)$

$(\neg q \vee \neg P) \vee (\neg q \vee q)$ $\quad \therefore (\neg q \vee q = T)$

$\neg q \vee \neg P \vee \underline{T}$

$= T$

hence it is proved the given Expression is Tautology.

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Q No 5:- $(P \leftrightarrow \neg q) \rightarrow (\neg P \vee \neg q)$

P	q	$\neg q$	$P \leftrightarrow \neg q$	$(P \leftrightarrow \neg q) \rightarrow (\neg P \vee \neg q)$
T	T	F	F	T
T	F	T	T	T
F	T	F	T	T
T	T	F	F	T

∴ hence it is proved from the truth table that the given Expression is tautology.

→ Quantifiers and written Proofs.

Q No 8:- $A \cup C = B \cup C$ then $A = B$ is false.

Sol:-

Let $A = \emptyset$, $B = \{1\}$ and $C = \{1, 2\}$.

then $A \cup C = \{1, 2\}$.

& $B \cup C = \{1, 2\}$.

∴ $A \cup C = B \cup C$.

But here $A \neq B$,

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QNo 9:- $p \rightarrow$ wearing a Pink tie.
$q \rightarrow$ wearing a red shirt
$r \rightarrow$ It is Saturday.

$$\begin{array}{l} \neg p \vee q \\ \neg r \rightarrow p \\ \neg q \\ \hline \therefore r \end{array}$$

$\therefore p \rightarrow q = \neg p \vee q$ (1)
$p \rightarrow \neg r = \neg p \vee \neg r$ (2)

$$\begin{array}{l} p \vee \neg q \text{ -(Neg)} \\ \neg p \vee \neg r \\ \hline \neg q \vee \neg r \end{array}$$

$$\begin{array}{l} \neg q \vee \neg r \\ \neg q \\ \hline \therefore r \end{array} \rightarrow \begin{array}{l} \neg(\neg q \vee \neg r) \\ \neg q \\ \hline \end{array}$$

$\therefore$ modus Ponens.

$$\begin{array}{l} \neg q \rightarrow r \\ \neg q \\ \hline r \rightarrow \text{Shown.} \end{array}$$

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Set Theory.

(11) $(A\cap B)^c = A^c \cup B^c$

$A^c \cup B^c = A^c \cup B^c$ $\therefore$ De Morgan's Law.

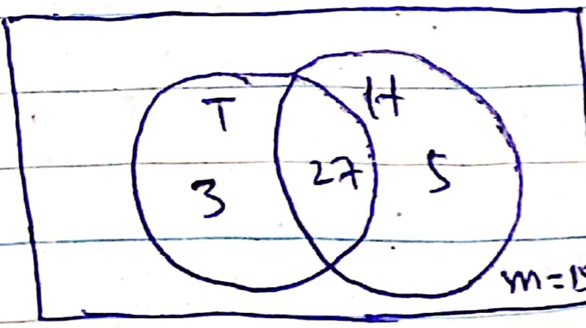
Q No 12:-

Total Motorcycle $= 50$ $(A\cup B)$
Owns Triumph $= 30$ (A)
Owns Honda $= 32$ (B).
neither $= 15$ $(A\cup B)'/\cancel{(A\cup B)} = 35$

$(A\cup B)_n = |A| + |B| - |A\cap B|$

$35 = 30 + 32 - x$

$\boxed{27 = x}$ $\rightarrow$ 27 owns each motorcycle.



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Q No 13:- (A) ex-athletes = 35

(G) Play Golf = 17.

(C) Cycling = 20

(Y) yoga = 19.

$(G \cap C) = 8$

$(G \cap Y) = 8$

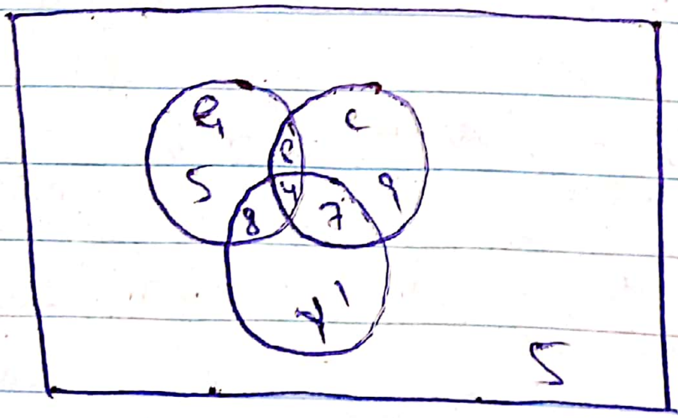
$(C \cap Y) = 7$

$(G \cap C \cap Y) = 4$

5 ex-athletes do none of these activities

$5+4+4+4+3+9+1 = 30$

$\Rightarrow 35-30 = 5$



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→ Number Theory, Combinatorics and Discrete Probability.

(14) $d = \gcd(3142, 900)$

$3142 = 900.3 + 442$
$900 = 442.2 + 16$
$442 = 16.27 + 10$
$16 = 10.1 + 6.$
$10 = 6.1 + 4.$
$6 = 4.1 + \boxed{2}$
$4 = 2.2 + 0$ gcd

Backtrack

$2 = 6 - 4$
$2 = 6 - (10 - 6)$
$2 = 6.2 - 10$
$2 = (16 - 10).2 - 10$
$2 = 16.2 - 10.3$
$2 = 16.82 - 3(442 - 16.27)$
$2 = 16.3 - 442.3.$
$2 = (900 - 442.2).83 - 442.$
$2 = 900.83 - 169(3142 - 900.3)$

$2 = 900.590 - 3142.169$

So

$x = -169$ and $y = 590.$

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Q No 15:

$\gcd(2017, 122) = 1$

$2017 = 122.16 + 65$

$122 = 65.1 + 57$

$65 = 57.1 + 8$

$57 = 8.7 + \boxed{1}$

$8 = 1.8 + 0$ gcd

Back track:

$1 = 57 - 8.7$

$1 = 57 - 7(65 - 57.1)$

$1 = 57.8 - 65.7$

$1 = (122 - 65.1).8 - 65.7$

$1 = 122.8 - 65.15$

$1 = 122.8 - 15(2017 - 122.16)$

$1 = 122.248 - 2017.15$

Sol-

$x = -15$ and $y = 248$.

Q No 16: $d = \gcd(558, 442)$

$558 = 442.1 + 116$

$442 = 116.3 + 94$

$116 = 94.1 + 22$

$94 = 22.4 + 6$

$22 = 6.3 + 4$

$6 = 4.1 + \boxed{2 \text{ gcd}}$

$4 = 2.2 + 0$

$d = \gcd(558, 442) = 2$

Back track

$2 = 6 - 4$

$2 = 6 - (22 - 6.3)$

$2 = 6.4 - 22$

$2 = (94 - 22.4).4 - 22$

$2 = 94.4 - 22.17$

$2 = 94.4 - 17(116 - 94.1)$

$2 = 94.21 - 116.17$

$2 = (442 - 116.3).21 - 116.17$

$2 = 442.21 - 116.63 - 116.17$

$2 = 442.21 - 116.80$

$2 = 442.21 - 80(558 - 442.1)$

$2 = 442.101 - 558.80$

Sol-

$x = -80$ and $y = 101$

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Q (17) $m = 900$, $n = 189 \Rightarrow 3^3 . 7$

a) Prime factorization of $m = 2^2 . 3^2 . 5^2$

b) $gcd(m,n) = 2^0 . 3^0 5^0 . 7^0$ and

$LCM(m,n) = 2^2 . 3^2 . 5^2 . 7^1$

$\therefore gcd = (m \rightarrow (2^2 3^2 5^2 7^0), n \rightarrow (3^3 7^0 2^0 5^0)$ min

$\therefore LCM(m \rightarrow$ " ") max

c) Smallest multiple u of 189 such that $gcd(m,u) = 45$ is

$\Rightarrow 5 \times 189$ "

Q No 19:-

$a_1 = 4$

$a_n = 7a_{n-1} + 4 \quad \therefore a_2, a_3, a_4 = ?$

$a_1 = 7a_0 + 4$

$4 = 7a_0 + 4$

$a_0 = 0$

$a_2 = 7a_1 + 4 = 7(4) + 4 = 32$

$a_3 = 7a_2 + 4 = 7(32) + 4 = 228$

$a_4 = 7a_3 + 4 = 7(228) + 4 = 1600$

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Q No 2) Total customer = 50
$M = 30$
$R = 35$
$(M \cup B)' = 15$

$|M \cup R| = |M| + |R| + |M \cup B|' - |M \cap R|$
$50 = 30 + 35 + 15 - x$
$\boxed{x = 30}$

∴ 30 have both a mountain bike and a round bike

Q6) ISBN-10 :- 125973128

$1.1 + 2.2 + 3.5 + 4.9 + 5.7 + 6.3 + 7.1 + 8.2 + 9.8 \pmod{11}$
$= x_{10} \rightarrow$ Check digit

$264 \pmod{11} = x_{10}$

$x_{10} = 4 \rightarrow$ is the check digit.

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→ Functions and Relations.

Q NO 28:- Suppose that $f(x) = f(y)$

$$g(f(x)) = g(f(y))$$

$$g \circ f(x) = g \circ f(y)$$

$$x = y.$$

Thus f is one-one

Q NO 30:- Reflexive :- $\frac{n}{n+2} \geq \frac{n}{n+2}$ $\therefore n \sim n$

$\therefore$ it is reflexive

antisymmetric suppose $a \sim b \wedge b \sim a$

$a = b$, Since $a \sim b$, $\frac{a}{b+2} \geq \frac{b}{a+2}$

Since $b \sim a$, $\frac{b}{a+2} \geq \frac{a}{b+2}$

$\therefore \frac{a}{b+2} = \frac{b}{a+2}$ So $a(a+2) = b(b+2)$

$$a^2+2a = b^2+2b$$

$$\therefore a^2+2a+1 = b^2+2b+1$$

$$(a+1)^2 = (b+1)^2$$

Since $a, b \in N$, $a+1 = b+1$ $\therefore$

$\therefore a = b$ → it is antisymmetri

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Q No 3) reflexive:- for any $x \subseteq \{1,2,3,4\}$,
The Smallest element of x
equals the Smallest element of x
(even at $x \neq \phi$)

$\therefore \sim$ is reflexive

Symmetric:- Suppose $x \sim y$
The Smallest element of x
equals the Smallest element of y
→The Smallest element of y equals
the Smallest element of x
$\therefore y \sim x \Rightarrow$ "$\sim$" is Symmetric

Transitive:- Suppose $x \sim y$ & $y \sim z$
→ The Smallest element of x equals the
Smallest element of y and the Smallest
element of y equals the Smallest element
of z
Hence, the smallest element element of
x equals the Smallest of z

$\Rightarrow x \sim z$:- "$\sim$" is transitive
→ So "$\sim$" is an Equavalence relation
$[\{2,4\}] = [\{2,3\}, \{2,4\}, \{2,3,4\}, \{2\}]$

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QNO 32:- Let $R = \{(1,2)\}$ and $S = \{(2,1)\}$
Then R and S are anti-Symmetric

But $R \cup S = \{(1,2),(2,1)\}$ is not anti-Symmetric

Because $1 \neq 2$

→ Binomial Theorem, Trees and Proofs.

QNO-36:- $1.1! + 2.2! + \dots + n.n! = (n+1)!$

Basic Step $P(1)$ is true since

$1.1! = (1+1)! - 1$

$1 = 2 - 1$

$1 = 1$ → Show (R.H.S = L.H.S)

Inductive Step:-

Prove for $n = k$;

$1.1! + 2.2! + \dots + k.k! - 1$ → ①

Prove for $n = k+1$

$1.1! + 2.2! + \dots + (k+1)(k+1)! - 1$ → ②

$1.1! + 2.2! + \dots + (k+1)(k+1)! = (k+1)! - 1 + (k+1)(k+1)!$

$(k+1)(k+1)! + 1 = (x+1)!(x+2)$.

through property of factorial.

$(x+2)! = (x+2)(x+1)!$

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$(u+1)(u+1)! = (u+2)! - 1$

now substituting $u = n-1$ (Reverse)

$(n)(n)! = (n+1)! - 1 \Rightarrow$ Shown proved.

QNo 38:- $\left[\frac{2}{3}x+1\right]^3 = (1)\left(\frac{2}{3}\right)^3 (1)^0 + (1)^1(3)\left(\frac{2}{3}x\right)^2 + (3)\left(\frac{2}{3}x\right)$

$(1)^2 + (1)\left(\frac{2}{3}x\right)^0 (1)$

$\Rightarrow \frac{8}{27}x^3 + \frac{4}{3}x^2 + 2x + 1$

QNo 39:- Co-efficient of $1/y^{10}$ in $\left(y^3 + \frac{a^7}{y^5}\right)^{10}$

at term = 5

$T5 = (y^3)^5 \times \left(\frac{a^7}{y^5}\right)^5 \times {}^{10}C_5$

$\Rightarrow \boxed{\text{Coefficient} = 252}$

T_5 is $\rightarrow \frac{252a^{35}}{y^{10}}$

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QNo 40:- $(x-y)^{15} \Rightarrow x^3y^{12} = ?,\ x^2y^{13} = ?$

$\cdot\ T_{12} = {}^{15}C_{12} \times (x)^{15-12} \times (-y)^{12} = 455x^3y^{12}$

$\hookrightarrow$ $\boxed{\text{Coefficient} = 455}$

$-\ T_{13} = {}^{13}C_{13}\ (x)^{15-12}\ (-y)^{12} = 455x^3y^{12}$

$\hookrightarrow$ $\boxed{\text{Coefficient} = -105}$

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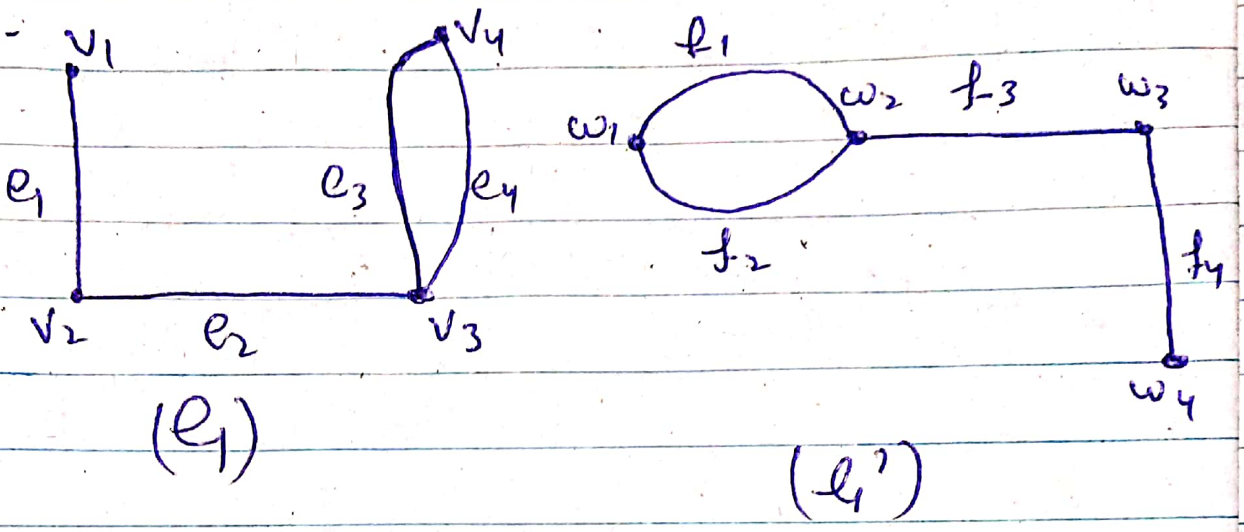
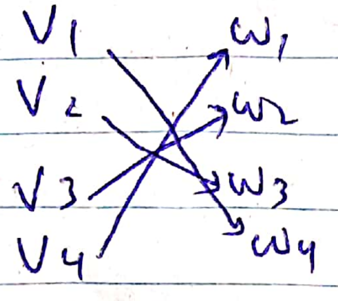
→ Relations and Graph Theory.

Q No.43:-



$G_1(\text{vertex}) \equiv G_1'(\text{vertex})$
$G_1(\text{edge}) \equiv G_1'(\text{edge})$
$G_1(\text{degree}) \equiv G_1'(\text{degree})$ degree
3, 2, 2, 1

now mapping :- degree



Hence,
it is isomorphic

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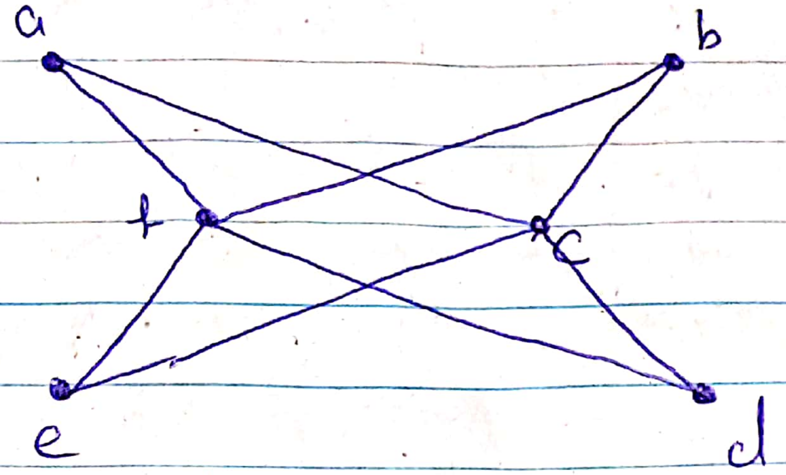
Q No 44:-

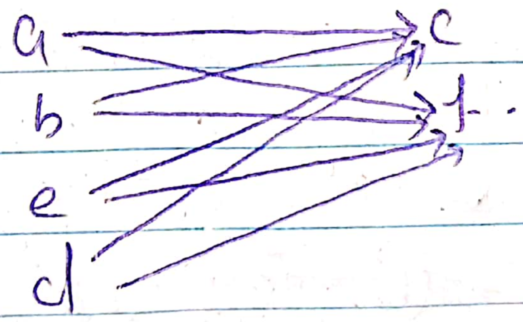


$S = \{a, b, c, d, e, f\}$

$S_1 = \{a, b, e, d\}$, $S_2 = \{c, f\}$.

Two disjoint sets.

S_1 or V_1 $\quad$ S_2 or V_2



'V_1' vertices are not connected with eachother so it is Bipartile graph. Same with V_2

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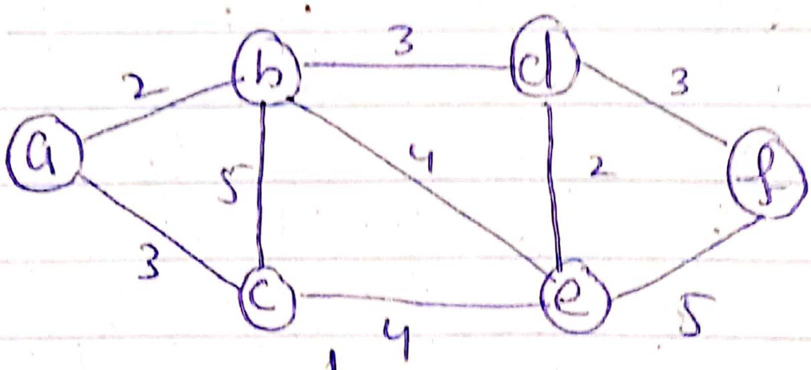
Q No 45:

$$
\begin{array}{c|ccccccc}
 & e_1 & e_2 & e_3 & e_4 & e_5 & e_6 & e_7 \\
\hline
v_1 & 1 & 0 & 0 & 0 & 0 & 1 & 0 \\
v_2 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\
v_3 & 0 & 0 & 0 & 1 & 1 & 0 & 1 \\
v_4 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\
v_5 & 0 & 1 & 1 & 0 & 0 & 0 & 1
\end{array}
$$

↓

Incidence Matrix

Q No:
(47)



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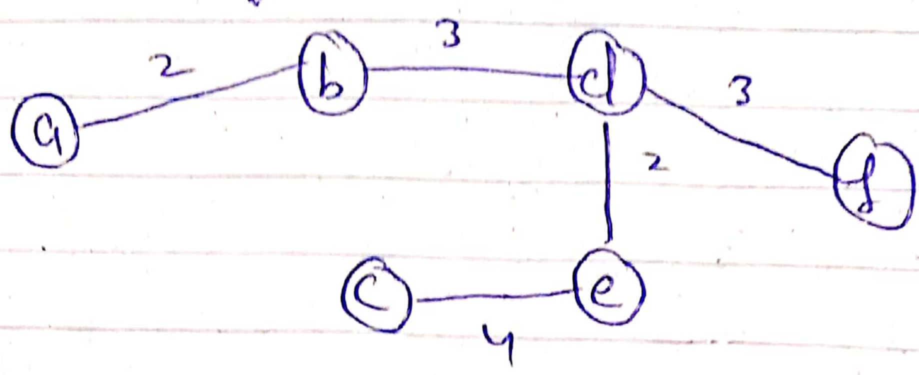

Total Cost

2 + 3 + 3 + 2 + 4

= 14

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Q NO 48 :-

Hamiltonian Circuit
(All nodes to be cover)

ABCDA = 125, ABDCA = 140, ACBDA = 155

So,
ABCDA = 125 is minimum distance travelled