National University of computer and emerging sciences

Foundation for Advancement of Science and Technology FAST

Linear Algebra (MT-1004)

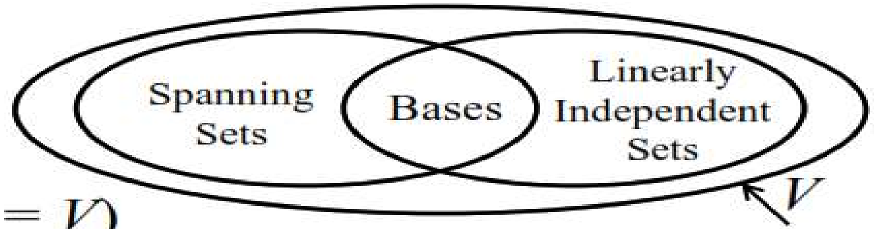
Lecture # 22

Basis

- Basis :

  V: a vector space

  $S = \{\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_n\} \subseteq V$



  1) S spans V (i.e., span(S) = V)

  2) S is linearly independent

  $\Rightarrow$ S is called a basis for V

- Notes:

  A basis S must have enough vectors to span V, but not so many vectors that one of them could be written as a linear combination of the other vectors in S

BASIS COORDINATES

If $S=\{v_1, v_2, \ldots, v_n\}$ is a basis for a vector space V, and

$$v=c_1v_1+ c_2v_2+ \ldots\ldots + c_nv_n$$

Is the expression for a vector v in terms of the basis S, then the scalars c1, c2, ….., cn are called the coordinates of v relative to the basis S. The coordinate vector of v relative to S is denoted by

$$(v)_s=(c_1, c_2, \ldots, c_n)$$

Theorem 4.5.1

Uniqueness of Basis Representation

If $S = \{\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_n\}$ is a basis for a vector space V, then every vector $\mathbf{v}$ in V can be expressed in the form $\mathbf{v} = c_1\mathbf{v}_1 + c_2\mathbf{v}_2 + \cdots + c_n\mathbf{v}_n$ in exactly one way.

National University of computer and emerging sciences

Foundation for Advancement of Science and Technology

Do Question # 1-9, 11-22 & 27 from Ex # 4.5