National University of computer and emerging sciences

Foundation for Advancement of Science and Technology

Linear Algebra (MT-1004)

Lecture # 02

Dependent Linear System & Parametric Equations

We can describe the solution set of dependent linear system by the following steps

i. Taking the main equation and make x as subject in terms of y *(Note: you can make y as a subject as well)*

ii. Assign an arbitrary value t (called a parameter) to y.

For example; if we have linear system of two variables x & y

$$\begin{gathered} x + y = 6 \\ 2x + 2y = 12 \end{gathered}$$

We can eliminate x from the second equation by adding -2 times the first equation to the second. This yields the simplified system

$$\begin{gathered} x + y = 6 \\ 0 = 0 \end{gathered}$$

Dependent Linear System & Parametric Equations

The second equation does not impose any restrictions on x and y and hence can be omitted. Thus, the solutions of the system are those values of x and y that satisfy the single equation

$$x + y = 6$$

Now make x as a subject i.e. $x = 6 - y$

and then assign an arbitrary value t (called a parameter) to y i.e. $y = t$

This allows us to express the solution by the pair of equations (called parametric equations)

$$x = 6 - t \quad \& \quad y = t$$

We can obtain specific numerical solutions from these equations by substituting numerical values for the parameter t.

i.e. when $t = 0$, $y = 0$ & $x = 6$ *i.e. yields solution* $(6,0)$ *etc*

Augmented Matrix

The essential information of a linear system can be recorded compactly in a rectangular array called a ***"Matrix"***.
Given the system,

$$\begin{aligned} x_1 - 2x_2 + x_3 &= 0 \\ 2x_2 - 8x_3 &= 8 \\ 5x_1 \qquad - 5x_3 &= 10 \end{aligned}$$

with the coefficients of each variable aligned in columns, the matrix. In terms of ***Ax = b***

A= $\begin{bmatrix} \mathbf{1} & \mathbf{-2} & \mathbf{1} \\ \mathbf{0} & \mathbf{2} & \mathbf{-8} \\ \mathbf{5} & \mathbf{0} & \mathbf{-5} \end{bmatrix}$ is called the coefficient matrix (or matrix of coefficients) ; $x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ *for variables* ; $b = \begin{bmatrix} 0 \\ 8 \\ 10 \end{bmatrix}$ *for constants (RHS)*

If we write the same system like $\begin{bmatrix} \mathbf{1} & \mathbf{-2} & \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{2} & \mathbf{-8} & \mathbf{8} \\ \mathbf{5} & \mathbf{0} & \mathbf{-5} & \mathbf{10} \end{bmatrix}$ It is called the ***Augmented matrix*** of the system.

"An augmented matrix of a system consists of the coefficient matrix with an added column containing the constants from the right sides of the equations".

Elementary Row Operations (*ERO*)

ERO are of three types:

i. Multiply a given row by a non-zero number (i.e. kR_i which means multiply row R_i by a constant k)

ii. Interchanging any two rows of a matrix (i.e. R_{ij} which means interchange row R_i with R_j)

iii. Addition of any multiple of one row to another row (i.e. $kR_i + R_j$ which means multiply row R_i by a constant k and result so obtained is added in R_j)

Row Equivalent Matrices

The matrices ***A & B*** are called ***Row Equivalent Matrices***, written as $A \approx B$, if ***one can be obtained from the other*** by performing a finite sequence of ***ERO***

For example;

$$A = \begin{bmatrix} 1 & 5 & 2 & 3 \\ 3 & 1 & 8 & -1 \\ 2 & 5 & 1 & 6 \end{bmatrix}$$

*(Apply an ERO on $\boldsymbol{R_2}$ $i.e.\ \boldsymbol{3R_2}$, after multiplication, name the new matrices as **B** which is equivalent to **A**)*

$$B = \begin{bmatrix} 1 & 5 & 2 & 3 \\ 9 & 3 & 24 & -3 \\ 2 & 5 & 1 & 6 \end{bmatrix}$$

Here $B \approx A$

Row Equivalent Matrices

$$A = \begin{bmatrix} 1 & 5 & 2 & 3 \\ 3 & 1 & 8 & -1 \\ 2 & 5 & 1 & 6 \end{bmatrix}$$

*(Apply an ERO as R_{13} $i.e. interchange$ R_1 to R_3 then name the new matrices as **C** which is equivalent to **A**)*

$$C = \begin{bmatrix} 2 & 5 & 1 & 6 \\ 3 & 1 & 8 & -1 \\ 1 & 5 & 2 & 3 \end{bmatrix}$$

Here $C \approx A$

Row Equivalent Matrices

$$A = \begin{bmatrix} 1 & 5 & 2 & 3 \\ 3 & 1 & 8 & -1 \\ 2 & 5 & 1 & 6 \end{bmatrix}$$

*(Apply an ERO as $R_2 + 4R_1$ then name the new matrices as **D** which is equivalent to **A**)*

$$D = \begin{bmatrix} 1 & 5 & 2 & 3 \\ 7 & 21 & 16 & 11 \\ 2 & 5 & 1 & 6 \end{bmatrix}$$

*Here **D** ≈ A*