

Name : Kashif Ali

Section : 3D

Roll No : 20K-1890

Course : Linear Algebra

Assignment #01

Exercise 1.3

Date: _____

| A (4x5) | B (4x5) | C (5x2) | D (4x2) | E (5x4) |
|------------|------------|------------|------------|------------|
|------------|------------|------------|------------|------------|

1.

(a) $BA = \text{False}$ - B/c the number of columns of B does not equal A.

(b) $AB^T = \text{True}$. \rightarrow when we take the transpose of B then the matrix multiplication is possible.

(c) $AC + D = \text{False} \rightarrow AC = 4 \times 2 \text{ and } D(E) = 5 \times 4$.

(d) $E(AC) = E(4 \times 2) = 5 \times 4 = 4 \times 2 = \text{True}$

(e) $A - 3E^T = \text{True} (4 \times 5)$

(f) $E(5B + A) = \text{True} (5 \times 5)$.

2.

(a) $CD^T = \text{True} (5 \times 4)$

(b) $DC = \text{False}$ num: of $\overset{\text{columns}}{\longleftrightarrow}$ D \neq num: of rows of C

(c) $BC - 3D = \text{True} (4 \times 2)$.

(d) $D^T(BE)$ = True (3×4)

(e) $BTD + ED$ = True (5×2).

(f) $BAT + D$ = False

→ In Exercise 2-6, use the following matrices to compute the indicated expression if it is defined.

$$A = \begin{bmatrix} 3 & 0 \\ -1 & 2 \\ 1 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 4 & -1 \\ 0 & 2 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 4 & 2 \\ 3 & 1 & 5 \end{bmatrix},$$

$$D = \begin{bmatrix} 1 & 5 & 2 \\ -1 & 0 & 1 \\ 3 & 2 & 4 \end{bmatrix}, \quad E = \begin{bmatrix} 6 & 1 & 3 \\ -1 & 1 & 2 \\ 4 & 1 & 3 \end{bmatrix}$$

(3)

a. $D + E$

$$= \begin{bmatrix} 7 & 6 & 5 \\ -2 & 1 & 3 \\ 7 & 3 & 7 \end{bmatrix} \text{ Ans.}$$

b. $D - E$

$$\begin{bmatrix} -5 & 4 & -1 \\ 0 & -1 & -1 \\ -1 & 1 & 1 \end{bmatrix} \text{ Ans.}$$

c. $5A$ d. $-7C$

$$= \begin{bmatrix} 15 & 0 \\ -5 & 10 \\ 5 & 5 \end{bmatrix}$$

$$\begin{bmatrix} -7 & -28 & -14 \\ -21 & -7 & -35 \end{bmatrix}$$

Ans.

Ans.

e. $2B - C$  $2B$:

$$\begin{bmatrix} 8 & -2 \\ 0 & 4 \end{bmatrix} - \begin{bmatrix} 1 & 4 & 2 \\ 3 & 1 & 5 \end{bmatrix}$$

Not Possible (The order is not equal)

(f) $4E - 2D$ 

$$\begin{bmatrix} 24 & 4 & 12 \\ -4 & 4 & 8 \\ 16 & 4 & 12 \end{bmatrix} + \begin{bmatrix} -2 & -10 & -4 \\ +2 & 0 & -2 \\ -6 & -4 & -8 \end{bmatrix} = \begin{bmatrix} 22 & -6 & 8 \\ -2 & 4 & 6 \\ 10 & 0 & 4 \end{bmatrix}$$

Ans.

g. $-3(D + 2E) - (D + 2E)$, first we will compute this.

$$\begin{array}{|ccc|} \hline -3 & 13 & 7 & 8 \\ & -3 & 2 & 5 \\ \hline & 11 & 4 & 10 \\ \hline \end{array}$$

$$\begin{array}{|ccc|} \hline & -39 & -21 & -24 \\ & 9 & -6 & -15 \\ \hline & -33 & -12 & -30 \\ \hline \end{array}$$

Ans.

h. $A - A$

$$= \begin{bmatrix} 3 & 0 \\ -1 & 2 \\ 1 & 1 \end{bmatrix} + \begin{bmatrix} -3 & 0 \\ +1 & -2 \\ -1 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \text{ Ans.}$$

(i) $\text{tr}(D)$ (we will add the values of main/Leading diag)

$$= 5 \text{ Ans.}$$

(j) $\text{tr}(D - 3E)$

$$D - 3E = \begin{bmatrix} -17 & 2 & -7 \\ +2 & -3 & -5 \\ -9 & -1 & -5 \end{bmatrix}$$

∴ Now we will find the Trace:

$$= -25 \text{ Ans.}$$

k. $4 \text{ tr}(7B)$

$$7B = \begin{bmatrix} 28 & -7 \\ 0 & 14 \end{bmatrix}$$

$$= 4 \times 32$$

$$= 128 \text{ Ans.}$$

1. $\text{tr}(A)$

$$= 4 \text{ dm.}$$

(4)

a. $2AT + C$

\swarrow

$$\begin{bmatrix} 6 & -2 & 2 \\ 0 & 4 & 2 \end{bmatrix} + \begin{bmatrix} 1 & 4 & 2 \\ 3 & 1 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 7 & 2 & 4 \\ 3 & 5 & 7 \end{bmatrix} \text{ Ans.}$$

b. $D^T - E^T$

$$\begin{bmatrix} 1 & -1 & 3 \\ 5 & 0 & 2 \\ 2 & 1 & 4 \end{bmatrix} + \begin{bmatrix} -6 & 1 & -4 \\ -1 & -1 & -1 \\ -3 & -2 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} -5 & 0 & -1 \\ 4 & -1 & 1 \\ -1 & -1 & 1 \end{bmatrix} \text{ Ans.}$$

c. $(D-E)^T$: "(D-E) first we will Compute and then transpose"

$$\begin{bmatrix} -5 & 0 & -1 \\ 4 & -1 & 1 \\ -1 & -1 & 1 \end{bmatrix} \text{ Ans.}$$

d. $B^T + S C^T$

$$\begin{bmatrix} 4 & 0 \\ -1 & 2 \end{bmatrix} + \begin{bmatrix} 5 & 15 \\ 20 & 5 \\ 10 & 25 \end{bmatrix}$$

= The order is not equal So therefore the addition is not possible.

e. $\frac{1}{2} C^T - \frac{1}{4} A$.

$$\begin{bmatrix} \frac{1}{2} & \frac{3}{2} \\ 2 & \frac{1}{2} \\ 1 & \frac{5}{2} \end{bmatrix} + \begin{bmatrix} -\frac{3}{4} & 0 \\ \frac{1}{4} & -\frac{1}{2} \\ -\frac{1}{4} & -\frac{1}{4} \end{bmatrix}$$

$$= \begin{bmatrix} -\frac{1}{4} & \frac{3}{2} \\ \frac{9}{4} & 0 \\ \frac{3}{4} & \frac{9}{4} \end{bmatrix} \text{ shw.}$$

f. $B - B^T$.

$$\begin{bmatrix} 4 & -1 \\ 0 & 2 \end{bmatrix} + \begin{bmatrix} -4 & 0 \\ 1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -1 \\ 1 & 4 \end{bmatrix} \text{ shw.}$$

g. $2 E^T - 3 D^T$

$$\begin{bmatrix} 12 & -2 & 8 \\ 2 & 2 & 2 \\ 6 & 4 & 6 \end{bmatrix} + \begin{bmatrix} -3 & 3 & -9 \\ -15 & 0 & -6 \\ -6 & -3 & -12 \end{bmatrix}$$

$$= \begin{bmatrix} 9 & 1 & -1 \\ -13 & 2 & -4 \\ 0 & 1 & -6 \end{bmatrix} \text{ shw.}$$

h. $(2E^T - 3D^T)^T$: $(2E^T - 3D^T)$ first we will Compute

$$= \begin{bmatrix} 9 & -13 & 0 \\ 1 & 2 & 1 \\ -1 & -4 & -6 \end{bmatrix} \text{ Ans.}$$

(ii) $(CD)E$

$$\begin{bmatrix} 3 & 9 & 14 \\ & & \\ 17 & 25 & 27 \end{bmatrix} * \begin{bmatrix} 6 & 1 & 3 \\ -1 & 1 & 2 \\ 4 & 1 & 3 \end{bmatrix}$$

$$\begin{bmatrix} 56 & 26 & 59 \\ 192 & 69 & 182 \end{bmatrix} \text{ Ans.}$$

j. $C(BA)$

Not possible because the Number of Columns of B does not equal to the Rows of A.

k. $f_6(DET)$

$$= \begin{bmatrix} 17 & 8 & 15 \\ -3 & 3 & 1 \\ 32 & 7 & 26 \end{bmatrix}$$

The Trace of Matrix is
= 46
Ans.

1. To (BC).

$$\begin{bmatrix} 4 & -1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 1 & 4 & 2 \\ 3 & 1 & 5 \end{bmatrix}$$

$$\begin{bmatrix} 4+3 & 16-1 & 8-5 \\ 0+6 & 0+2 & 0+10 \end{bmatrix} = \begin{bmatrix} 1 & 15 & 3 \\ 6 & 2 & 10 \end{bmatrix}$$

→ Trace of matrix is only possible for square matrix.

(5)

a) AB

Sol:-

$$A \cdot B = \begin{bmatrix} 3 & 0 \\ -1 & 2 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 4 & -1 \\ 0 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} (3 \times 4) + (0 \times 0) & (3 \times -1) + (0 \times 2) \\ (-1 \times 4) + (2 \times 0) & (-1 \times -1) + (2 \times 2) \\ (1 \times 4) + (1 \times 0) & (1 \times -1) + (1 \times 2) \end{bmatrix}$$

$$= \begin{bmatrix} 12 & -3 \\ -4 & 5 \\ 4 & 1 \end{bmatrix} \text{ Ans.}$$

(b) BA

Sol:-

undefined (No. of Columns of B ≠ rows of A)

(c) (3E)D.

Sol:-

$$(3E)D = \begin{bmatrix} 3 \times 6 & 3 \times 1 & 3 \times 3 \\ -1 \times 3 & 3 \times 1 & 3 \times 2 \\ 4 \times 3 & 1 \times 3 & 3 \times 3 \end{bmatrix} \begin{bmatrix} 1 & 5 & 2 \\ -1 & 0 & 1 \\ 3 & 2 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 18 & 3 & 9 \\ -3 & 3 & 6 \\ 12 & 3 & 9 \end{bmatrix} \begin{bmatrix} 1 & 5 & 2 \\ -1 & 0 & 1 \\ 3 & 2 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} (18 \times 1) + (3 \times -1) + (9 \times 3) & (18 \times 5) + (3 \times 0) + (9 \times 2) & (18 \times 2) + (3 \times 1) + (9 \times 4) \\ (-3 \times 1) + (3 \times -1) + (6 \times 3) & (-3 \times 5) + (3 \times 0) + (3 \times 2) & (3 \times 2) + (3 \times 1) + (6 \times 4) \\ (12 \times 1) + (12 \times -1) + (12 \times 3) & (12 \times 5) + (12 \times 0) + (12 \times 2) & (12 \times 2) + (12 \times 1) + (12 \times 2) \end{bmatrix}$$

$$= \begin{bmatrix} 42 & 108 & 75 \\ 12 & -3 & 21 \\ 36 & 78 & 63 \end{bmatrix} \text{ Ans.}$$

(d) (AB) C

$$(AB)C = \left(\begin{bmatrix} 3 & 0 \\ -1 & 2 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 4 & -1 \\ 0 & 2 \end{bmatrix} \right) \begin{bmatrix} 1 & 4 & 2 \\ 3 & 15 \end{bmatrix}$$

$$= \begin{bmatrix} 12 & -3 \\ -4 & 5 \\ 4 & 1 \end{bmatrix} \begin{bmatrix} 1 & 4 & 2 \\ 3 & 1 & 5 \end{bmatrix}$$

$$\begin{bmatrix} (12 \times 1) - (3 \times 3) & (12 \times 4) - (3 \times 1) & (12 \times 2) - (3 \times 5) \\ -(4 \times 1) + (5 \times 3) & -(4 \times 4) + (5 \times 1) & -(4 \times 2) + (5 \times 5) \\ (4 \times 1) + (1 \times 3) & (4 \times 4) + (1 \times 1) & (4 \times 2) + (1 \times 5) \end{bmatrix} = \begin{bmatrix} 3 & 45 & 8 \\ 11 & -11 & 17 \\ 7 & 17 & 13 \end{bmatrix}$$

(e) $A(BC)$

$$A(BC) = \begin{bmatrix} 3 & 0 \\ -1 & 2 \\ 1 & 1 \end{bmatrix} \left(\begin{bmatrix} 4 & -1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 1 & 4 & 2 \\ 3 & 1 & 5 \end{bmatrix} \right).$$

$$= \begin{bmatrix} 3 & 0 \\ -1 & 2 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} (4 \times 1) - (1 \times 3) & (4 \times 4) - (1 \times 1) & (4 \times 2) - (1 \times 5) \\ (0 \times 1) + (2 \times 3) & (0 \times 4) + (2 \times 1) & (0 \times 2) + (2 \times 5) \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 0 \\ -1 & 2 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 15 & 3 \\ 6 & 2 & 10 \end{bmatrix}$$

$$= \begin{bmatrix} (3 \times 1) + (0 \times 6) & (3 \times 15) + (0 \times 2) & (3 \times 3) + (0 \times 10) \\ -(1 \times 1) + (2 \times 6) & -(1 \times 15) + (2 \times 2) & -(1 \times 3) + (2 \times 10) \\ (1 \times 1) + (1 \times 6) & (1 \times 15) + (1 \times 2) & (1 \times 3) + (1 \times 10) \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 45 & 9 \\ 11 & -11 & 17 \\ 7 & 17 & 13 \end{bmatrix} \text{ Ans}$$

(f) $C.CT$

Sol

$$C.CT = \begin{bmatrix} 1 & 4 & 2 \\ 3 & 1 & 5 \end{bmatrix} \begin{bmatrix} 1 & 3 \\ 4 & 1 \\ 2 & 5 \end{bmatrix}$$

$$\begin{bmatrix} (1 \times 1) + (4 \times 4) + (2 \times 2) & (1 \times 3) + (4 \times 1) + (2 \times 5) \\ (3 \times 1) + (1 \times 4) + (5 \times 2) & (3 \times 3) + (1 \times 1) + (5 \times 5) \end{bmatrix}$$

$$= \begin{bmatrix} 21 & 17 \\ 17 & 35 \end{bmatrix} \text{ Ans}$$

(g) $(DA)^T$

$$(DA)^T = \left(\begin{bmatrix} 1 & 5 & 2 \\ -1 & 0 & 1 \\ 3 & 2 & 4 \end{bmatrix} \begin{bmatrix} 3 & 0 \\ -1 & 2 \\ 1 & 1 \end{bmatrix} \right)^T$$

$$= \left(\begin{bmatrix} (1 \times 3) - (5 \times 1) + (2 \times 1) & (1 \times 0) + (5 \times 2) + (2 \times 1) \\ -(1 \times 3) - (0 \times 1) + (1 \times 1) & -1(0) + (0 \times 2) + (1 \times 1) \\ (3 \times 3) - 2(1) + (4 \times 1) & (3 \times 0) + (2 \times 2) + (4 \times 1) \end{bmatrix} \right)^T$$

$$= \begin{bmatrix} 0 & -2 & 11 \\ 12 & 1 & 8 \end{bmatrix} \text{ Ans}$$

(h) $(C^T B) A^T$

Solve

$$(C^T B) A^T = \left(\begin{bmatrix} 1 & 3 \\ 4 & 1 \\ 2 & 5 \end{bmatrix} \begin{bmatrix} 4 & -1 \\ 0 & 2 \end{bmatrix} \right) \begin{bmatrix} 3 & -1 & 1 \\ 0 & 2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} (1 \times 4) + (3 \times 0) & - (1 \times 1) + (3 \times 2) \\ (4 \times 4) + (1 \times 0) & - (4 \times 1) + (1 \times 2) \\ (2 \times 4) + (5 \times 0) & - (2 \times 1) + (5 \times 2) \end{bmatrix} \begin{bmatrix} 3 & -1 & 1 \\ 0 & 2 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 4 & 5 \\ 16 & -2 \\ 8 & 8 \end{bmatrix} \begin{bmatrix} 3 & -1 & 1 \\ 0 & 2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} (4 \times 3) + (5 \times 0) & - (4 \times 1) + (5 \times 2) & (4 \times 1) + (5 \times 1) \\ (16 \times 3) - (2 \times 0) & - (16 \times 1) - (2 \times 2) & (16 \times 1) - (2 \times 1) \\ (8 \times 3) + (8 \times 0) & - (8 \times 1) + (8 \times 2) & (8 \times 1) + (8 \times 1) \end{bmatrix}$$

$$= \begin{bmatrix} 12 & 6 & 9 \\ 48 & -20 & 14 \\ 24 & 8 & 16 \end{bmatrix} \text{ Ans}$$

(j) $\operatorname{tr}(D, DT)$

Soln

$$= \operatorname{tr} \left(\begin{bmatrix} 1 & 5 & 2 \\ -1 & 0 & 1 \\ 3 & 2 & 4 \end{bmatrix} \begin{bmatrix} 1 & -1 & 3 \\ 5 & 0 & 2 \\ 2 & 1 & 4 \end{bmatrix} \right) + (2 \times 4)$$

$$= \operatorname{tr} \left(\begin{bmatrix} (1 \times 1) + (5 \times 5) + (2 \times 2) & -(1 \times 1) + (5 \times 0) + (2 \times 1) & (1 \times 3) + (5 \times 2) \\ -(1 \times 1) + (0 \times 5) + (1 \times 2) & (1 \times 1) + (0 \times 0) + (1 \times 1) & -(3) + (0 \times 2) + (1 \times 1) \\ + (3 \times 1) + (2 \times 5) + (4 \times 2) & -(3 \times 1) + (2 \times 0) + (4 \times 1) & (3 \times 3) + (2 \times 2) + (4 \times 1) \end{bmatrix} \right)$$

$$= \operatorname{tr} \left(\begin{bmatrix} 30 & 1 & 21 \\ 1 & 2 & 1 \\ 21 & 1 & 28 \end{bmatrix} \right) = 30 + 2 + 28 = 61$$

Ans

(j) $\operatorname{tr}(4E^T - D)$

$$\operatorname{tr} \left(4 \begin{bmatrix} 6 & -1 & 4 \\ 1 & 1 & 1 \\ 3 & 2 & 3 \end{bmatrix} - \begin{bmatrix} 1 & 5 & 2 \\ -1 & 0 & 1 \\ 3 & 2 & 4 \end{bmatrix} \right)$$

$$\operatorname{tr} \left(\begin{bmatrix} 24 & -1 & 16 \\ 4 & 4 & 4 \\ 12 & 8 & 12 \end{bmatrix} - \begin{bmatrix} 1 & 5 & 2 \\ -1 & 0 & 1 \\ 3 & 2 & 4 \end{bmatrix} \right)$$

$$\operatorname{tr} \left(\begin{bmatrix} 24-1 & -1-5 & 16-2 \\ 4+1 & 4-0 & 4-1 \\ 12-3 & 8-2 & 12-4 \end{bmatrix} \right) = \operatorname{tr} \left(\begin{bmatrix} 23 & -6 & 14 \\ 5 & 4 & 3 \\ 9 & 6 & 8 \end{bmatrix} \right)$$

$$= 23 + 4 + 8 = 35$$

Ans

$$(k) \rightarrow (CTAT) + 2 ET$$

Sol:-

$$4x \left(\begin{bmatrix} 1 & 3 \\ 4 & 1 \\ 2 & 5 \end{bmatrix} \begin{bmatrix} 3 & -1 & 1 \\ 0 & 2 & 1 \end{bmatrix} + 2 \begin{bmatrix} 6 & -1 & 9 \\ 1 & 1 & 1 \\ 3 & 2 & 3 \end{bmatrix} \right)$$

$$= 4x \left(\begin{bmatrix} (1 \times 3) + (3 \times 0) & -(1 \times 1) + (3 \times 2) & (1 \times 1) + (3 \times 1) \\ (4 \times 3) + (1 \times 0) & -(4 \times 1) + (1 \times 2) & (4 \times 1) + (1 \times 1) \\ (2 \times 3) + (5 \times 0) & -(2 \times 1) + (5 \times 2) & (2 \times 1) + (5 \times 1) \end{bmatrix} + \begin{bmatrix} 6x2 & 2x-1 & 4x2 \\ 2 & 2 & 2 \\ 2x3 & 2x2 & 2x3 \end{bmatrix} \right)$$

$$4x \left(\begin{bmatrix} 3 & 5 & 4 \\ 12 & -2 & 5 \\ 6 & 8 & 7 \end{bmatrix} + \begin{bmatrix} 12 & -2 & 8 \\ 2 & 2 & 2 \\ 6 & 4 & 6 \end{bmatrix} \right).$$

$$4x \left(\begin{bmatrix} 15 & 3 & 12 \\ 14 & 0 & 7 \\ 12 & 12 & 13 \end{bmatrix} \right)$$

$$= 15 + 0 + 13 = 28$$

$$(l) \rightarrow (ECT)^T A.$$

$$4x \left(\left(\begin{bmatrix} 6 & 1 & 3 \\ -1 & 1 & 2 \\ 1 & 1 & 3 \end{bmatrix} \begin{bmatrix} 1 & 3 \\ 4 & 1 \\ 2 & 5 \end{bmatrix} \right)^T \begin{bmatrix} 3 & 0 \\ -1 & 2 \\ 1 & 1 \end{bmatrix} \right)$$

$$4x \left(\left(\begin{bmatrix} (6 \times 1) + (1 \times 4) + (3 \times 2) & (6 \times 3) + (1 \times 1) + (3 \times 5) \\ -(1 \times 1) + (1 \times 4) + (2 \times 2) & -(1 \times 3) + (1 \times 1) + (2 \times 5) \\ (4 \times 1) + (1 \times 4) + (3 \times 2) & (4 \times 3) + (1 \times 1) + (3 \times 5) \end{bmatrix} \right)^T \begin{bmatrix} 3 & 0 \\ -1 & 2 \\ 1 & 1 \end{bmatrix} \right)$$

$$4x \left(\begin{bmatrix} 16 & 34 \\ 7 & 8 \\ 14 & 28 \end{bmatrix} \right)^T \left(\begin{bmatrix} 3 & 0 \\ -1 & 2 \\ 1 & 1 \end{bmatrix} \right)$$

$$s + r \begin{pmatrix} (16 \times 3) - (7 \times 1) + (14 \times 1) & (16 \times 0) + (7 \times 2) + (14 \times 1) \\ (34 \times 3) - (8 \times 1) + (28 \times 1) & (34 \times 0) + (8 \times 2) + 28 \end{pmatrix}$$

$$= s + r \begin{pmatrix} 55 & 28 \\ 122 & 44 \end{pmatrix}.$$

=

$$55 + 44 = 99 \text{ dm.}$$

$$\textcircled{6} \quad \textcircled{6} (2DT - E) A.$$

Sol:

$$= \left(2 \begin{bmatrix} 1 & -1 & 3 \\ 5 & 0 & 2 \\ 2 & 1 & 4 \end{bmatrix} - \begin{bmatrix} 6 & 1 & 3 \\ -1 & 1 & 2 \\ 4 & 1 & 3 \end{bmatrix} \right) \begin{bmatrix} 2 & 0 \\ -1 & 2 \\ 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 \times 1 - 6 & 2 \times (-1) - 1 & 2 \times 3 - 3 \\ 2 \times 5 - (-1) & 2 \times 0 - 1 & 2 \times 2 - 2 \\ 2 \times 2 - 4 & 2 \times 1 - 1 & 2 \times 4 - 3 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ -1 & 2 \\ 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -4 & -3 & 3 \\ 11 & -1 & 2 \\ 0 & 1 & 5 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ -1 & 2 \\ 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -(4 \times 3) + (3 \times 1) + (3 \times 1) & -(4 \times 0) - (3 \times 2) + (3 \times 1) \\ (11 \times 3) + (1 \times 1) + (2 \times 1) & (11 \times 0) - (1 \times 2) + (2 \times 1) \\ (0 \times 3) - (1 \times 1) + (5 \times 1) & (0 \times 0) + (1 \times 2) + (5 \times 1) \end{bmatrix}$$

$$= \begin{bmatrix} -6 & -3 \\ 36 & 0 \\ 4 & 7 \end{bmatrix}. \text{ dm.}$$

$$\textcircled{b} \quad (4B)C + 2B$$

Sol:

undefined (a 2×3 matrix $(4B)C$ cannot be added to a 2×2 matrix $2B$).

$$\textcircled{c} \quad (-AC)^T + 5DT$$

$$= \left(- \begin{bmatrix} 3 & 0 \\ -1 & 2 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 4 & 2 \end{bmatrix} \right)^T + 5 \begin{bmatrix} 1 & -1 & 3 \\ 5 & 0 & 2 \\ 2 & 1 & 4 \end{bmatrix}$$

$$= 5 \begin{pmatrix} (3 \times 1) + (0 \times 3) & (3 \times 4) + (0 \times 1) & (3 \times 2) + (0 \times 5) \\ -(1 \times 1) + (2 \times 3) & -(1 \times 4) + (2 \times 1) & -(1 \times 2) + (2 \times 5) \\ (1 \times 1) + (1 \times 3) & (1 \times 4) + (1 \times 1) & (1 \times 2) + (1 \times 5) \end{pmatrix}^T + \begin{bmatrix} 5 & -5 & 15 \\ 25 & 0 & 10 \\ 10 & 5 & 20 \end{bmatrix}$$

$$= \begin{bmatrix} -3+5 & -5-5 & -4+15 \\ -12+25 & 2+0 & -5+10 \\ -6+10 & -8+5 & -7+20 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & -10 & 11 \\ 13 & 2 & 5 \\ 4 & -3 & 13 \end{bmatrix}$$

(d) $(BAT - 2C)^T$

$$= \left(\begin{bmatrix} 4 & -1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 3 & -1 & 1 \\ 0 & 2 & 1 \end{bmatrix} - \begin{bmatrix} 2x1 & 2x4 & 2x2 \\ 2x3 & 2x1 & 2x5 \end{bmatrix} \right)$$

$$= \left\{ \begin{bmatrix} (4 \times 3) - (1 \times 0) & -(4 \times 1) - (1 \times 2) & (4 \times 1) - (1 \times 1) \\ (0 \times 3) + (2 \times 0) & -(0 \times 1) + (2 \times 2) & (0 \times 1) + (2 \times 1) \end{bmatrix} \right\}$$

$$= \begin{bmatrix} 2 & 8 & 4 \\ 0 & 2 & 10 \end{bmatrix}^T$$

$$= \begin{bmatrix} 12-2 & -6-8 & 3-4 \\ 0-6 & 4-6 & 2-10 \end{bmatrix}^T$$

$$= \begin{bmatrix} 10 & -6 \\ -14 & -2 \\ -1 & -8 \end{bmatrix} \text{ Ans}$$

(e) $B^T (CCT - A^T A)$

Solve.

$$\begin{bmatrix} 4 & 0 \\ -1 & 2 \end{bmatrix} \left(\begin{bmatrix} 1 & 4 & 2 \\ 3 & 1 & 5 \end{bmatrix} \begin{bmatrix} 1 & 3 \\ 4 & 1 \\ 2 & 5 \end{bmatrix} - \begin{bmatrix} 3 & -1 & 1 \\ 0 & 2 & 1 \end{bmatrix} \begin{bmatrix} 3 & 0 \\ -1 & 2 \\ 1 & 1 \end{bmatrix} \right)$$

$$= \begin{bmatrix} 4 & 0 \\ -1 & 2 \end{bmatrix} \left[\begin{array}{cc} (1 \times 1) + (4 \times 4) + (2 \times 2) & (1 \times 3) + (4 \times 1) + (2 \times 5) \\ (3 \times 1) + (1 \times 4) + (5 \times 2) & (3 \times 3) + (1 \times 1) + (5 \times 5) \end{array} \right]$$

$$= \begin{bmatrix} 4 & 0 \\ -1 & 2 \end{bmatrix} \left(\begin{bmatrix} 21 & 17 \\ 17 & 35 \end{bmatrix} - \begin{bmatrix} 11 & -1 \\ -1 & 5 \end{bmatrix} \right)$$

$$= \begin{bmatrix} 4 & 0 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 10 & 18 \\ 18 & 30 \end{bmatrix}$$

$$= \begin{bmatrix} 40 & 72 \\ 26 & 42 \end{bmatrix} \text{ Ans}$$

(f) $DTE T - (ED)^T$

Sol:-

$$= \begin{vmatrix} 1 & -1 & 3 \\ 5 & 0 & 2 \\ 2 & 1 & 4 \end{vmatrix} \begin{vmatrix} 6 & -1 & 4 \\ 1 & 1 & 1 \\ 3 & 2 & 3 \end{vmatrix} - \left(\begin{vmatrix} 6 & 1 & 3 \\ -1 & 1 & 2 \\ 4 & 1 & 3 \end{vmatrix} \begin{vmatrix} 1 & 5 & 2 \\ -1 & 0 & 1 \\ 3 & 2 & 4 \end{vmatrix} \right)^T$$

$$= \begin{vmatrix} (1 \times 6) - (1 \times 1) + (3 \times 3) & (1 \times 1) + (-1 \times 1) + (3 \times 2) & (1 \times 4) - (1 \times 1) + (3 \times 3) \\ (5 \times 6) + (0 \times 1) + (2 \times 3) & -(5 \times 1) + (0 \times 1) + (2 \times 2) & (5 \times 4) + (0 \times 1) + (2 \times 3) \\ (2 \times 6) + (1 \times 1) + (4 \times 3) & -(2 \times 1) + (1 \times 1) + (4 \times 2) & (2 \times 4) + (1 \times 1) + (4 \times 3) \end{vmatrix}$$

$$= \begin{vmatrix} (6 \times 1) - (1 \times 1) + (3 \times 3) & (6 \times 5) + (1 \times 0) + (3 \times 2) & (6 \times 2) + (1 \times 1) + (3 \times 4) \\ -(1 \times 1) - (1 \times 1) + (2 \times 3) & -(1 \times 5) + (1 \times 0) + (2 \times 2) & -(1 \times 2) + (1 \times 1) + (2 \times 4) \\ (4 \times 1) - (1 \times 1) + (3 \times 3) & (4 \times 5) + (1 \times 0) + (3 \times 2) & (4 \times 2) + (1 \times 1) + (3 \times 4) \end{vmatrix}$$

$$= \begin{vmatrix} 6-1+9 & -1-1+6 & 4-1+9 \\ 20+1+6 & -5+0+4 & 26+0+6 \\ 19+1+12 & -2+1+8 & 8+1+12 \end{vmatrix} - \begin{vmatrix} 6-1+9 & 30+0+6 & 12+1+12 \\ -1-1+6 & -5+0+4 & -2+1+8 \\ 4-1+9 & 26+0+6 & 8+1+12 \end{vmatrix}$$

$$= \begin{vmatrix} 2 & 4 & 19 \\ 37 & -1 & 26 \\ 25 & 7 & 21 \end{vmatrix} - \begin{vmatrix} 14 & 36 & 25 \\ 4 & 1 & 6 \\ 19 & 26 & 21 \end{vmatrix}$$

$$= \begin{vmatrix} 2-14 & 4-36 & 19-25 \\ 37-4 & -1-1 & 26-6 \\ 25-12 & 7-26 & 21-21 \end{vmatrix}$$

$$= \begin{vmatrix} -12 & -32 & -13 \\ 33 & -2 & 20 \\ 13 & 19 & 0 \end{vmatrix} \text{ skew}$$

In Ex-7-8, we following matrices and either the row method or column matrix method as appropriate to find the indicated row or column.

$$A = \begin{bmatrix} 3 & -2 & 7 \\ 6 & 5 & 4 \\ 0 & 4 & 9 \end{bmatrix} \text{ and } B = \begin{bmatrix} 6 & -2 & 4 \\ 0 & 1 & 3 \\ 7 & 7 & 5 \end{bmatrix}$$

(7)

(a) The first row of AB .

$$= [\text{First row of } A] B.$$

$$= [0 \ 4 \ 9] \begin{bmatrix} 6 & -2 & 4 \\ 0 & 1 & 3 \\ 7 & 7 & 5 \end{bmatrix}$$

$$= [(0 \times 6) + (4 \times 0) + (9 \times 7) \quad (0 \times -2) + (1 \times 4) + (9 \times 7) \quad (0 \times 4) + (4 \times 3) + (9 \times 5)]$$

$$= [63 \ 41 \ 57]$$

b) third row of AB.

Soln
= [third row of A] B

$$= \begin{bmatrix} 0 & 4 & 9 \end{bmatrix} \begin{bmatrix} 6 & -2 & 4 \\ 0 & 1 & 3 \\ 7 & 7 & 5 \end{bmatrix}$$

$$[(0 \times 6) + (4 \times 0) + (9 \times 7), (0 \times -2) + (4 \times 1) + (9 \times 7), (0 \times 4) + (4 \times 3) + (9 \times 5)]$$

$$= \begin{bmatrix} 63 & 67 & 57 \end{bmatrix}. \text{ Ans}$$

c) Second Column of AB.

= A [Second column of B]

$$= \begin{bmatrix} 3 & -2 & 7 \\ 6 & 5 & 4 \\ 0 & 4 & 9 \end{bmatrix} \begin{bmatrix} -2 \\ 1 \\ 7 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} (3 \times -2) + (-2 \times 1) + (7 \times 7) \\ (6 \times -2) + (5 \times 1) + (4 \times 7) \\ (0 \times -2) + (4 \times 1) + (9 \times 7) \end{bmatrix}$$

$$= \begin{bmatrix} 41 \\ 21 \\ 67 \end{bmatrix} \text{ Ans}$$

d) Second column of BA.

Sol:-

$\Rightarrow B [$ First column of A].

$$\begin{matrix} & \left[\begin{matrix} 6 & -2 & 4 \\ 0 & 1 & 3 \\ 7 & 7 & 5 \end{matrix} \right] & \left[\begin{matrix} 3 \\ 6 \\ 6 \end{matrix} \right] \end{matrix}$$

$$\Rightarrow \left[\begin{matrix} (6 \times 3) + (2 \times -6) + (4 \times 0) \\ (0 \times 3) + (1 \times 6) + (3 \times 0) \\ (7 \times 3) + (7 \times 6) + (5 \times 0) \end{matrix} \right] = \left[\begin{matrix} 6 \\ 6 \\ 63 \end{matrix} \right] \text{ Ans.}$$

e) Third row of AA.

[third row of A] A.

$$\left[\begin{matrix} 0 & 4 & 9 \end{matrix} \right] \left[\begin{matrix} 3 & -2 & 7 \\ 6 & 5 & 4 \\ 0 & 4 & 9 \end{matrix} \right]$$

$$[(0 \times 3) + (4 \times 6) + (9 \times 0) \quad -(0 \times 2) + (4 \times 5) + (9 \times 4) \quad (0 \times 7) + (4 \times 4) + (9 \times 8)].$$

$$\left[\begin{matrix} 24 & 56 & 97 \end{matrix} \right] \text{ Ans.}$$

(8)

a) First column of AB.

Sol:-

$$= A \begin{bmatrix} \text{Third column of } A \end{bmatrix}.$$

$$= \begin{bmatrix} 3 & -2 & 7 \\ 6 & 5 & 4 \\ 0 & 4 & 9 \end{bmatrix} \begin{bmatrix} 6 \\ 0 \\ 7 \end{bmatrix}.$$

$$= \begin{bmatrix} (3 \times 6) + (-2 \times 0) + (7 \times 7) \\ (6 \times 6) + (5 \times 0) + (4 \times 7) \\ (0 \times 6) + (4 \times 0) + (9 \times 7) \end{bmatrix} = \begin{bmatrix} 67 \\ 64 \\ 63 \end{bmatrix}.$$

b) Third column of BB.

Sol:-

$$= B \begin{bmatrix} \text{Third column of } B \end{bmatrix}.$$

$$= \begin{bmatrix} 6 & -2 & 4 \\ 0 & 1 & 3 \\ 7 & 7 & 5 \end{bmatrix} \begin{bmatrix} 4 \\ 3 \\ 5 \end{bmatrix}.$$

$$= \begin{bmatrix} (6 \times 4) - (2 \times 3) + (7 \times 7) \\ (6 \times 6) + (5 \times 0) + (4 \times 7) \\ (0 \times 6) + (4 \times 4) + (9 \times 5) \end{bmatrix}$$

$$= \begin{bmatrix} 38 \\ 18 \\ 74 \end{bmatrix} *$$

c) Second row of BB.

$$= [\text{Second row of } B] \times B$$

$$= [0 \ 1 \ 3] \begin{bmatrix} 6 & -2 & 4 \\ 0 & 1 & 3 \\ 7 & 7 & 5 \end{bmatrix}$$

$$= [(0 \times 6) + (1 \times 0) + (3 \times 7) \quad -(0 \times 2) + (1 \times 1) + (3 \times 7) \quad (0 \times 4) + (1 \times 3) + (3 \times 5)].$$

$$= [21 \ 22 \ 18] \text{ Ans.}$$

d) first column of AA.

Sol:

$$= \begin{bmatrix} 3 & -2 & 7 \\ 6 & 5 & 4 \\ 0 & 4 & 9 \end{bmatrix} \begin{bmatrix} 3 \\ 6 \\ 6 \end{bmatrix}$$

$$= \begin{bmatrix} (3 \times 3) - (2 \times 6) + (7 \times 0) \\ (6 \times 3) + (5 \times 6) + (4 \times 0) \\ (0 \times 3) + (4 \times 6) + (9 \times 0) \end{bmatrix}$$

$$= \begin{bmatrix} -3 \\ 48 \\ 24 \end{bmatrix} \text{ Ans.}$$

e) third column of AB.

$$\begin{bmatrix} 3 & -2 & 7 \\ 6 & 5 & 4 \\ 0 & 4 & 9 \end{bmatrix} \begin{bmatrix} 4 \\ 3 \\ 5 \end{bmatrix} = \begin{bmatrix} (3 \times 4) - (2 \times 3) + (7 \times 0) \\ (6 \times 4) + (5 \times 3) + (4 \times 0) \\ (0 \times 4) + (4 \times 3) + (9 \times 5) \end{bmatrix} = \begin{bmatrix} 41 \\ 59 \\ 57 \end{bmatrix} \text{ Ans.}$$

f) First row of BA.

Sol:-

[First row of B] A.

$$= \begin{bmatrix} 6 & -2 & 4 \end{bmatrix} \begin{bmatrix} 3 & -2 & 7 \\ 6 & 5 & 4 \\ 0 & 4 & 9 \end{bmatrix}$$

$$[(6 \times 3) + (-2 \times 6) + (4 \times 0) \quad (6 \times -2) + (-2 \times 5) + (4 \times 4) \quad (6 \times 7) + (-2 \times 4) (4 \times 9)]$$

$$= \begin{bmatrix} 6 & -6 & 70 \end{bmatrix} \text{ Ans.}$$

9) In Ex-9-10, use matrices A and B from Ex 7-8-

9) Express each column vector of AA as a linear combination of column vectors of A.

Soln

$$\text{First column of } AA = 3 \begin{bmatrix} 3 \\ 6 \\ 6 \end{bmatrix} + 6 \begin{bmatrix} -2 \\ 5 \\ 4 \end{bmatrix} + 0 \begin{bmatrix} 7 \\ 4 \\ 9 \end{bmatrix},$$

$$= \begin{bmatrix} 9 \\ 18 \\ 0 \end{bmatrix} + \begin{bmatrix} -12 \\ 30 \\ 24 \end{bmatrix} = \begin{bmatrix} -3 \\ 48 \\ 24 \end{bmatrix} \text{ from}$$

$$\text{Second column of } AA = -2 \begin{bmatrix} 3 \\ 6 \\ 6 \end{bmatrix} + 5 \begin{bmatrix} -2 \\ 5 \\ 4 \end{bmatrix} + 4 \begin{bmatrix} 7 \\ 4 \\ 9 \end{bmatrix}.$$

$$= \begin{bmatrix} -6 \\ -12 \\ 0 \end{bmatrix} + \begin{bmatrix} -10 \\ 25 \\ 20 \end{bmatrix} + \begin{bmatrix} 28 \\ 16 \\ 36 \end{bmatrix} = \begin{bmatrix} 12 \\ 29 \\ 56 \end{bmatrix} \text{ from}$$

$$\text{Third column of } AA = 4 \begin{bmatrix} 6 \\ 0 \\ 7 \end{bmatrix} + 3 \begin{bmatrix} -2 \\ 1 \\ 7 \end{bmatrix} + 5 \begin{bmatrix} 4 \\ 3 \\ 5 \end{bmatrix}.$$

$$= \begin{bmatrix} 24 \\ 0 \\ 28 \end{bmatrix} + \begin{bmatrix} -6 \\ 3 \\ 21 \end{bmatrix} + \begin{bmatrix} 20 \\ 15 \\ 25 \end{bmatrix}.$$

$$= \begin{bmatrix} 76 \\ 98 \\ 97 \end{bmatrix} \text{ from}$$

b) Express each column vector of BB as a linear combination of column vectors of B .

Soln.

$$\text{First column of } BB = 6 \begin{bmatrix} 6 \\ 0 \\ 7 \end{bmatrix} + 6 \begin{bmatrix} -2 \\ 5 \\ 7 \end{bmatrix} + 7 \begin{bmatrix} 4 \\ 3 \\ 5 \end{bmatrix}.$$

$$= \begin{bmatrix} 36 \\ 0 \\ 42 \end{bmatrix} + \begin{bmatrix} 28 \\ 21 \\ 35 \end{bmatrix} = \begin{bmatrix} 64 \\ 21 \\ 77 \end{bmatrix} \text{ dm}$$

Second column of BB

$$= -2 \begin{bmatrix} 6 \\ 0 \\ 7 \end{bmatrix} + 1 \begin{bmatrix} -2 \\ 1 \\ 7 \end{bmatrix} + 7 \begin{bmatrix} 4 \\ 3 \\ 5 \end{bmatrix}.$$

$$= \begin{bmatrix} -12 \\ 0 \\ -4 \end{bmatrix} + \begin{bmatrix} -2 \\ 1 \\ 7 \end{bmatrix} + \begin{bmatrix} 28 \\ 21 \\ 35 \end{bmatrix}.$$

$$= \begin{bmatrix} 14 \\ 22 \\ 28 \end{bmatrix} \text{ dm}$$

Third column of BB .

$$= 4 \begin{bmatrix} 6 \\ 0 \\ 7 \end{bmatrix} + 3 \begin{bmatrix} -2 \\ 1 \\ 7 \end{bmatrix} + 5 \begin{bmatrix} 4 \\ 3 \\ 5 \end{bmatrix}.$$

$$= \begin{bmatrix} 24 \\ 0 \\ 28 \end{bmatrix} + \begin{bmatrix} -6 \\ 3 \\ 21 \end{bmatrix} + \begin{bmatrix} 20 \\ 15 \\ 25 \end{bmatrix}$$

$$= \begin{bmatrix} 38 \\ 18 \\ 28 \end{bmatrix} \text{ dm}$$

(10) Express each column of vector of AB as linear combination of column vectors A.

$$\text{First column of } AB = 6 \begin{bmatrix} 3 \\ 6 \\ 6 \end{bmatrix} + 0 \begin{bmatrix} -2 \\ 5 \\ 4 \end{bmatrix} + 7 \begin{bmatrix} 7 \\ 4 \\ 9 \end{bmatrix}.$$

$$= \begin{bmatrix} 18 \\ 36 \\ 0 \end{bmatrix} + \begin{bmatrix} 49 \\ 28 \\ 63 \end{bmatrix} = \begin{bmatrix} 67 \\ 64 \\ 63 \end{bmatrix} \text{ Ans.}$$

$$\text{Second Column of } AB = -2 \begin{bmatrix} 3 \\ 6 \\ 6 \end{bmatrix} + 1 \begin{bmatrix} -2 \\ 5 \\ 4 \end{bmatrix} + 7 \begin{bmatrix} 7 \\ 4 \\ 9 \end{bmatrix}.$$

$$= \begin{bmatrix} -6 \\ -12 \\ 0 \end{bmatrix} + \begin{bmatrix} -2 \\ 5 \\ 4 \end{bmatrix} + \begin{bmatrix} 49 \\ 28 \\ 63 \end{bmatrix}$$

$$= \begin{bmatrix} -8 \\ -7 \\ 4 \end{bmatrix} + \begin{bmatrix} 49 \\ 28 \\ 63 \end{bmatrix} =$$

$$= \begin{bmatrix} 41 \\ 21 \\ 67 \end{bmatrix} \text{ Ans.}$$

$$\text{Third Column of } AB = 4 \begin{bmatrix} 3 \\ 6 \\ 6 \end{bmatrix} + 3 \begin{bmatrix} -2 \\ 5 \\ 4 \end{bmatrix} + 5 \begin{bmatrix} 7 \\ 4 \\ 9 \end{bmatrix}$$

$$= \begin{bmatrix} 12 \\ 24 \\ 0 \end{bmatrix} + \begin{bmatrix} -6 \\ 15 \\ 12 \end{bmatrix} + \begin{bmatrix} 35 \\ 20 \\ 45 \end{bmatrix}$$

$$= \begin{bmatrix} 6 \\ 9 \\ 12 \end{bmatrix} + \begin{bmatrix} 35 \\ 20 \\ 45 \end{bmatrix} = \begin{bmatrix} 41 \\ 29 \\ 57 \end{bmatrix}.$$

(11)

$$a) \begin{aligned} 2u_1 - 3u_2 + 5u_3 &= 7 \\ 9u_1 - u_2 + u_3 &= -1 \\ u_1 + 5u_2 + 4u_3 &= 0 \end{aligned}$$

Sol 2-

$$Ax = b.$$

the matrix eqn is

$$A = \begin{bmatrix} 2 & -3 & 5 \\ 9 & -1 & 1 \\ 1 & 5 & 4 \end{bmatrix}, x = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}, b = \begin{bmatrix} 7 \\ -1 \\ 0 \end{bmatrix}.$$

$$b) \begin{aligned} 4u_1 - 3u_2 + u_3 &= 1 \\ 5u_1 + u_2 - 8u_3 &= 3 \\ 2u_1 - 5u_2 + 9u_3 - u_4 &= 0 \\ + 3u_2 - u_3 + 7u_4 &= 2 \end{aligned}$$

Sol 2-

$$A = \begin{bmatrix} 4 & 0 & -3 & 1 \\ 5 & 1 & 0 & -8 \\ 2 & -5 & 9 & -1 \\ 0 & 3 & -1 & 7 \end{bmatrix}, x = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{bmatrix}, b = \begin{bmatrix} 1 \\ 3 \\ 0 \\ 2 \end{bmatrix}.$$

The equation of Matrix is

$$\begin{bmatrix} 4 & 0 & -3 & 1 \\ 5 & 1 & 0 & -8 \\ 2 & -5 & 9 & -1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ 0 \\ 2 \end{bmatrix}.$$

(19)

$$a. \quad u_1 - 2u_2 + 3u_3 = -3$$

$$2u_1 + u_2 = 0$$

$$-3u_2 + 4u_3 = 1$$

$$u_1 + u_3 = 5.$$

Soln-

$$Ax = b.$$

$$A = \begin{bmatrix} 1 & -2 & 3 \\ 2 & 1 & 0 \\ 0 & -3 & 4 \\ 1 & 0 & 1 \end{bmatrix}, \quad x = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}, \quad b = \begin{bmatrix} -3 \\ 0 \\ 1 \\ 5 \end{bmatrix}$$

The equation of matrix is:

$$\Rightarrow \begin{bmatrix} 1 & -2 & 3 \\ 2 & 1 & 0 \\ 0 & -3 & 4 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} -3 \\ 0 \\ 1 \\ 5 \end{bmatrix}$$

Ans

$$b) \quad 3u_1 + 3u_2 + 3u_3 = -3$$

$$-u_1 - 5u_2 - 2u_3 = 3$$

$$-4u_2 + u_3 = 0.$$

Soln

The equation of matrix is:-

$$\begin{bmatrix} 3 & 3 & 3 \\ -1 & -5 & -2 \\ 0 & -4 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} -3 \\ 3 \\ 0 \end{bmatrix}$$

(13) In each of Ex 13-14, express the matrix equation as a system of linear equations.

a) $\begin{bmatrix} 5 & 6 & -7 \\ -1 & -2 & 3 \\ 0 & 4 & -1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ 3 \end{bmatrix}$.

Sol:-

$$= 5u_1 + 6u_2 - 7u_3 = 2$$

$$-u_1 - 2u_2 + 3u_3 = 0$$

$$4u_2 - u_3 = 3 \text{ from}$$

b) $\begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 0 \\ 5 & -3 & -6 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \\ -9 \end{bmatrix}$

Sol:-

$$= x + y + z = 2$$

$$2u_1 + 3u_2 = 2$$

$$5u_1 - 3u_2 - 6u_3 = -9 \text{ from}$$

(19)

$$9) \begin{bmatrix} 3 & -1 & 2 \\ 4 & 3 & 7 \\ -3 & 1 & 5 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix}$$

Sol:

$$\therefore 3u_1 - u_2 + 2u_3 = 2.$$

$$4u_1 + 3u_2 + 7u_3 = -1$$

$$-3u_1 + u_2 + 5u_3 = 4 \text{ given}$$

$$b) \begin{bmatrix} 3 & -2 & 0 & 1 \\ 5 & 0 & 2 & -3 \\ 3 & 1 & 4 & 7 \\ -2 & 5 & 1 & 6 \end{bmatrix} \begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Sol:

$$3w - 2x + z = 0$$

$$5w + 2y - 3z = 0$$

$$3w + x + 4y + 7z = 0$$

$$-2w + 5x + y + 6z = 0$$

given

In Ex-15-16 find all values of k if any that satisfy the equation.

$$(15) \begin{bmatrix} k & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 2 \\ 0 & 2 & -3 \end{bmatrix} \begin{bmatrix} k \\ 1 \\ 1 \end{bmatrix} = 0$$

Soln

$$= \begin{bmatrix} k & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 2 \\ 0 & 2 & -3 \end{bmatrix} \begin{bmatrix} k+1 \\ k+2 \\ 2 \end{bmatrix}$$

$$= \begin{bmatrix} k & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 2 \\ 0 & 2 & -3 \end{bmatrix} \begin{bmatrix} k+1 \\ k+2 \\ 2 \end{bmatrix}$$

$$= k^2 + k + k + 2 - 1 = k^2 + 2k + 1$$

$$= (k+1)^2$$

\therefore value of k satisfy the equation.

$$k = -1$$

$$\textcircled{16} \quad \left[\begin{matrix} 2 & 2 & k \end{matrix} \right] \left[\begin{matrix} 1 & 2 & 0 \\ 2 & 0 & 3 \\ 0 & 3 & 1 \end{matrix} \right] \left[\begin{matrix} 2 \\ 2 \\ k \end{matrix} \right]$$

Sol:-

$$\rightarrow \left[\begin{matrix} 2 & 2 & k \end{matrix} \right] \left[\begin{matrix} 2+4 \\ 4+0+3k \\ 0+6+k \end{matrix} \right].$$

$$\rightarrow \left[\begin{matrix} 2 & 2 & k \end{matrix} \right] \left[\begin{matrix} 6 \\ 3k+4 \\ 6+k \end{matrix} \right].$$

$$= 12 + 6k + 6k + k^2 + 8.$$

$$= k^2 + 12k + 20.$$

$$\Rightarrow (k+10)(k+2).$$

The only value of k that satisfy equation are $k = -10$ and $k = -2$.

In Ex-17-20 use column-row expansion of AB to express this product as sum of matrix products.

$$\textcircled{17} \quad A = \left[\begin{matrix} 4 & -3 \\ 2 & 1 \end{matrix} \right], \quad B = \left[\begin{matrix} 0 & 1 & 2 \\ -2 & 3 & 1 \end{matrix} \right].$$

$$\text{Sol:- } \left[\begin{matrix} 4 \\ 2 \end{matrix} \right] [0 \ 1 \ 2] + \left[\begin{matrix} -3 \\ -1 \end{matrix} \right] [-2 \ 3 \ 1]$$

$$= \left[\begin{matrix} 0 & 4 & 8 \\ 0 & 2 & 4 \end{matrix} \right] + \left[\begin{matrix} -6 & -9 & -3 \\ 2 & -3 & -1 \end{matrix} \right].$$

$$= \left[\begin{matrix} -6 & -5 & 5 \\ 2 & -1 & 3 \end{matrix} \right].$$

$$(18) \quad A = \begin{bmatrix} 0 & -2 \\ 4 & -3 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 4 & 1 \\ -3 & 0 & 2 \end{bmatrix}$$

Soln

$$\Rightarrow \begin{bmatrix} 0 \\ 4 \end{bmatrix} \begin{bmatrix} 1 & 4 & 1 \end{bmatrix} + \begin{bmatrix} -2 \\ -3 \end{bmatrix} \begin{bmatrix} -3 & 0 & 2 \end{bmatrix}.$$

$$\begin{bmatrix} 0 & 0 & 0 \\ 4 & 16 & 4 \end{bmatrix} + \begin{bmatrix} 6 & 0 & -4 \\ 9 & 0 & -6 \end{bmatrix}.$$

$$= \begin{bmatrix} 6 & 0 & -4 \\ 13 & 16 & -2 \end{bmatrix} \text{ Ans}$$

$$(19) \quad A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}.$$

Soln

$$\Rightarrow \begin{bmatrix} 1 \\ 4 \end{bmatrix} \begin{bmatrix} 1 & 2 \end{bmatrix} + \begin{bmatrix} 2 \\ 5 \end{bmatrix} \begin{bmatrix} 3 & 4 \end{bmatrix} + \begin{bmatrix} 3 \\ 6 \end{bmatrix} \begin{bmatrix} 5 & 6 \end{bmatrix}.$$

$$= \begin{bmatrix} 1 & 2 \\ 4 & 8 \end{bmatrix} + \begin{bmatrix} 6 & 8 \\ 15 & 8 \end{bmatrix} + \begin{bmatrix} 15 & 18 \\ 30 & 36 \end{bmatrix}$$

$$= \begin{bmatrix} 7 & 10 \\ 19 & 16 \end{bmatrix} + \begin{bmatrix} 15 & 18 \\ 30 & 36 \end{bmatrix} = \begin{bmatrix} 22 & 28 \\ 49 & 64 \end{bmatrix} \text{ Ans}$$

(20)

$$A = \begin{bmatrix} 0 & 4 & 2 \\ 1 & -2 & 5 \end{bmatrix}, \quad B = \begin{bmatrix} 2 & -1 \\ 4 & 0 \\ 1 & -1 \end{bmatrix}.$$

Sol~

$$\begin{bmatrix} 0 \\ 1 \end{bmatrix} [2 \ -1] + \begin{bmatrix} 4 \\ -2 \end{bmatrix} [4 \ 0] + \begin{bmatrix} 2 \\ 5 \end{bmatrix} [1 \ -1].$$

$$= \begin{bmatrix} 0 & 0 \\ 2 & -1 \end{bmatrix} + \begin{bmatrix} 16 & 0 \\ -8 & 0 \end{bmatrix} + \begin{bmatrix} 2 & -2 \\ 5 & -5 \end{bmatrix}.$$

$$= \begin{bmatrix} 16 & 0 \\ -6 & -1 \end{bmatrix} + \begin{bmatrix} 2 & -2 \\ 5 & -5 \end{bmatrix}.$$

$$\Rightarrow \boxed{\begin{bmatrix} 18 & -2 \\ -1 & -6 \end{bmatrix}} \text{ Ans}$$

The End,