National University of computer and emerging sciences

Foundation for Advancement of Science and Technology

Linear Algebra (MT-1004)

Lecture # 03

Zero Row

"A row of any matrix is called zero if its entries are zeros"

Examples;

i. $\begin{bmatrix} \mathbf{1} & \mathbf{3} & \mathbf{0} \\ \mathbf{1} & \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix}$

ii. $\begin{bmatrix} \mathbf{1} & \mathbf{5} & \mathbf{2} & \mathbf{4} \\ \mathbf{0} & \mathbf{1} & \mathbf{6} & \mathbf{4} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix}$

Echelon Form OR Row Echelon Form

A matrix is in echelon form if :

- If a row does not consist entirely of zeros, then ***the first non-zero number in the row is a 1***. We call this a ***leading 1***
- If there are any rows that consist ***entirely of zeros***, then ***they are grouped together at the bottom of the matrix***
- In any two ***successive rows*** that do not consist entirely of zeros, ***the leading 1 in the lower row*** occurs ***farther to the right*** than the leading 1 in ***the higher/above row.***

Examples; $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$ $\begin{bmatrix} 1 & 3 & -1 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{bmatrix}$ $\begin{bmatrix} 1 & 0 & 3 & 4 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

Reduced Echelon Form

If an echelon matrix also possess the additional property that:

- ***Each column*** that contains ***a leading 1*** ***has zeros everywhere else in that column***

Then it becomes ***Reduced Echelon Form***. *(Thus, a matrix in reduced row echelon form is of necessity in row echelon form, but not conversely)*

Examples; $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 2 \end{bmatrix}$ $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ $\begin{bmatrix} 1 & 3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$