

# Linear Algebra (MT-1004)

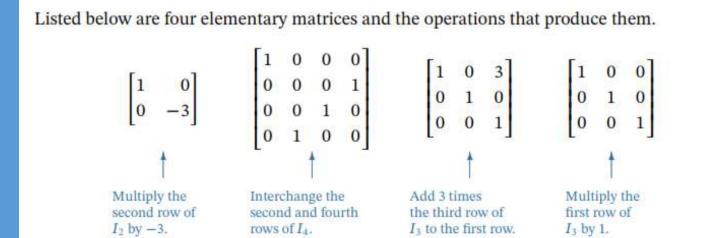
Lecture # 06



## **Elementary Matrix:**

"A matrix (E) is called Elementary Matrix if it is obtained from an identity matrix by performing a single elementary row operation"

#### **Examples:**







## **Elementary Matrix:**

### { Theorem 1.5.1 }

Row Operations by Matrix Multiplication: If the elementary matrix E results from performing a certain row operation on  $I_m$  and if A is an m  $\times$  n matrix, then the product EA is the matrix that results when this same row operation is performed on A.

**Example:** 

Consider the matrix

$$A = \begin{bmatrix} 1 & 0 & 2 & 3 \\ 2 & -1 & 3 & 6 \\ 1 & 4 & 4 & 0 \end{bmatrix}$$

and consider the elementary matrix

$$E = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 3 & 0 & 1 \end{bmatrix}$$

which results from adding 3 times the first row of  $I_3$  to the third row. The product EA is

$$EA = \begin{bmatrix} 1 & 0 & 2 & 3 \\ 2 & -1 & 3 & 6 \\ 4 & 4 & 10 & 9 \end{bmatrix}$$

which is precisely the matrix that results when we add 3 times the first row of A to the third row.





## **Elementary Matrix:**

## { Theorem 1.5.2 }

"Every elementary matrix is invertible, and the inverse is also an elementary matrix"

i.e. if E is an elementary matrix and  $E_0$  is its inverse then It always follows:

$$E_0E = I$$
 and  $EE_0 = I$ 



## **Method for Inverting Matrices:**

Assume that the reduced row echelon form of A is  $I_n$ , so that A can be reduced to  $I_n$  by a finite sequence of elementary row operations. By Theorem 1.5.1, each of these operations can be accomplished by multiplying on the left by an appropriate elementary matrix. Thus we can find elementary matrices  $E_1, E_2, \ldots, E_k$  such that

$$E_k \cdot \cdot \cdot E_2 E_1 A = I_n \tag{3}$$

assume for the moment, that A is an invertible  $n \times n$  matrix. In Equation (3), the elementary matrices execute a sequence of row operations that reduce A to  $I_n$ . If we multiply both sides of this equation on the right by  $A^{-1}$  and simplify, we obtain

$$A^{-1} = E_k \cdot \cdot \cdot E_2 E_1 I_n$$

But this equation tells us that the same sequence of row operations that reduces A to  $I_n$  will transform  $I_n$  to  $A^{-1}$ . Thus, we have established the following result.





## **Method for Inverting Matrices:**

**Inversion Algorithm** To find the inverse of an invertible matrix A, find a sequence of elementary row operations that reduces A to the identity and then perform that same sequence of operations on  $I_n$  to obtain  $A^{-1}$ .

Solved examples/pattern of inverse of a 3x3 & 4x4 matrices are from next slides

















$$A^{-1} = \begin{bmatrix} 2 & 3 & -3 \\ -1 & -2 & 2 \\ 4 & 6 & -7 \end{bmatrix}$$





Entt. 1.5 Q.16 





42-41 43-41 Au-R,

 $R_4-R_2$   $R_4-R_3$ 





K2/3 K2/5 K4/7