Assignment #02

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Course :- Linear Algebra

Section :- 3D

Exercise 4.1

Q No 2:-

Let V be the set of all ordered pairs, and consider the following addition and scalar multiplication operations.

in

$$u = (u_1, u_2) \text{ and } v = (v_1, v_2)$$

$$u + v = (u_1 + v_1, u_2 + v_2)$$

$$ku = (ku_1, ku_2)$$

(a) Compute $u+v$ and ku for $u = (0, 4)$ and $v = (1, -3)$ and $k = 2$.

Sol:-

Addition:

$$u+v = (0,4) + (1,-3)$$

$$= (0+1+1,\ 4-3+1)$$

$$u+v = (2, 2)$$

Multiplication:-

$$ku = (ku_1, ku_2)$$

$$= (2\times 0,\ 2\times 4).$$

$$= (0, 8)$$

(b) $(0,0) \neq 0$

Sol:-

$$u + (0,0) = (u_1, u_2) + (0,0)$$
$$= (u_1 + 0 + 1, u_2 + 0 + 1)$$
$$= (u_1 + 1, u_2 + 1) \neq (u_1, u_2).$$

Therefore $(0,0)$ is not equal to zero vector 0 required by Axiom 4.

(c) $(-1,-1) = 0$

Sol:-

$$u + (-1,-1) = (u_1, u_2) + (-1,-1).$$
$$= (u_1 - 1 + 1, u_2 - 1 + 1).$$
$$= (u_1, u_2) = (u_1, u_2).$$

$\therefore$ it holds Axiom No 4.

(d) $u + (-u) = 0$ for $u = (u_1, u_2)$.

Sol:-

$$u + (-u) = 0$$

Let $u = (2 + u_1, 2 + u_2)$

$$(2 + u_1, 2 + u_2) + (-2 - u_1, -2 - u_2)$$

$$(2 + u_1 - 2 - u_1 + 1, 2 + u_2 - 2 - u_2 + 1).$$
$$= (-1, -1) \neq 0$$

Since $(-u) + u = 0$ holds as well, Axiom No 5.

Find two vector space that it holds

Sol..

Axiom No 7.

$k\cdot(u+v) = k(u_1+v_1+1,\ u_2+v_2+1)$

$= (ku_1+kv_1+k,\ ku_2+kv_2+k)$

$ku+kv = k(u_1,u_2) + k(v_1,v_2)$

$= (ku_1, ku_2) + (kv_1, kv_2)$

$= (ku_1+kv_1+1),\ ku_2+kv_2+1)$

$k(u+v) \neq ku+kv$

$\rightarrow$ It fails Axiom No 7.

Axiom No 8.

$(k+m)\,u = ((k+m)u_1, (k+m)u_2)$

$= (ku_1+mu_1,\ ku_2+mu_2)$

$ku+mu = k(u_1,u_2) + m(u_1,u_2)$

$=(ku_1, ku_2) + (mu_1, mu_2)$

$= (ku_1+mu_1+1,\ ku_2+mu_2+1)$

$(k+m)\,u \neq (ku+mu)$

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Questions No 9.

Date:

The set of all 2×2 matrices of the form:

$$\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$$

with Standard matrix addition and Scalar multiplication.

Axiom 1:- Sum of two diagonal matrix 2×2 is also the diagonal of 2×2 matrix.

$$\therefore \quad U = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}, \quad V = \begin{bmatrix} a' & 0 \\ 0 & b' \end{bmatrix}.$$

$$U+V = \begin{bmatrix} a+a' & 0 \\ 0 & b+b' \end{bmatrix}.$$

Axiom 2:-

$$U+V = V+U.$$

$$= \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} + \begin{bmatrix} a' & 0 \\ 0 & b' \end{bmatrix} = \begin{bmatrix} a' & 0 \\ 0 & b' \end{bmatrix} + \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}.$$

$$= \begin{bmatrix} a+a' & 0 \\ 0 & b+b' \end{bmatrix} = \begin{bmatrix} a'+a & 0 \\ 0 & b'+b \end{bmatrix}.$$

Axiom 3:-

$$U+(V+W)=(U+V)+W.$$

Sol.-

Let $W=\begin{bmatrix} a'' & 0 \\ 0 & b'' \end{bmatrix}$.

$$=\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}+\left(\begin{bmatrix} a' & 0 \\ 0 & b' \end{bmatrix}+\begin{bmatrix} a'' & 0 \\ 0 & b'' \end{bmatrix}\right)=\left(\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}+\begin{bmatrix} a' & 0 \\ 0 & b' \end{bmatrix}\right)+\begin{bmatrix} a'' & 0 \\ 0 & b'' \end{bmatrix}$$

$$=\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}+\left(\begin{bmatrix} a'+a'' & 0 \\ 0 & b'+b'' \end{bmatrix}\right)=\left(\begin{bmatrix} a+a' & 0 \\ 0 & b+b' \end{bmatrix}\right)+\begin{bmatrix} a'' & 0 \\ 0 & b'' \end{bmatrix}$$

$$=\begin{bmatrix} a+a'+a'' & 0 \\ 0 & b+b'+b'' \end{bmatrix}=\begin{bmatrix} a+a'+a'' & 0 \\ 0 & b+b'+b'' \end{bmatrix}.$$

$\Rightarrow$ It holds Axiom No 3.

Axiom 4:-

$$U+0=U$$

Sol

$$\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}+\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}=\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}.$$

$$\Rightarrow \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}=\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$$

$\rightarrow$ It also holds Axiom No 4.

Axiom No 5:- $U+(-U)=0$

Sol.-

$$\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}+\left(\begin{bmatrix} -a & 0 \\ 0 & -b \end{bmatrix}\right)=\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}-\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}=\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}.$$

$$=\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}=\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Axiom No 6.

ku

$$= k\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} = \begin{bmatrix} ka & 0 \\ 0 & kb \end{bmatrix}.$$

$\rightarrow$ It holds Axium No 6.

Axiom No 7:-

$$c(u+v) = c(u) + c(v).$$

$$c\left(\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} + \begin{bmatrix} a' & 0 \\ 0 & b' \end{bmatrix}\right) = c\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} + c\begin{bmatrix} a' & 0 \\ 0 & b' \end{bmatrix}.$$

$$c\left(\begin{bmatrix} a+a' & 0 \\ 0 & b+b' \end{bmatrix}\right) = \begin{bmatrix} ca & 0 \\ 0 & cb \end{bmatrix} + \begin{bmatrix} ca' & 0 \\ 0 & cb' \end{bmatrix}.$$

$$\begin{bmatrix} c(a+a') & 0 \\ 0 & c(b+b') \end{bmatrix} = \begin{bmatrix} c(a+a') & 0 \\ 0 & c(b+b') \end{bmatrix}.$$

$\rightarrow$ It holds Axiom No 7.

Axiom No 8:-

Sol:-

$$(c+d)u = c(u) + d(u).$$

$$(c+d)\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} = c\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} + d\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}.$$

$$\begin{bmatrix} (c+d)a & 0 \\ 0 & (c+d)b \end{bmatrix} = \begin{bmatrix} ca & 0 \\ 0 & cb \end{bmatrix} + \begin{bmatrix} da & 0 \\ 0 & db \end{bmatrix}$$

$$\begin{bmatrix} (c+d)a & 0 \\ 0 & (c+d)b \end{bmatrix} = \begin{bmatrix} (c+d)a & 0 \\ 0 & (c+d)b \end{bmatrix}.$$

It holds Axiom No 8.

Axiom No 9:-

$$c(dU) = cd(U).$$

$$c\begin{bmatrix} da & 0 \\ 0 & db \end{bmatrix} = cd\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}.$$

$$\begin{bmatrix} cda & 0 \\ 0 & cdb \end{bmatrix} = \begin{bmatrix} cda & 0 \\ 0 & cdb \end{bmatrix}.$$

It holds Axiom No 9.

Axiom No 10.

$$1(U) = U.$$

$$= 1\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}.$$

$$\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}.$$

It holds Axiom No 10.

Question No 11 Date:

The set of all pairs of real numbers of form $(1, x)$ with operation $(1, y) + (1, y') = (1, y + y')$

$k(1, y) = (1, ky)$

Sol.

Axiom 1:-

$(1, y) + (1, y') = (1 + y + y')$ is in V

Axiom No 2:-

$(1, y) + (1, y') = (1, y') + (1, y)$

$(1, y + y') = (1, y' + y)$

$\rightarrow$ It holds Axiom No 2.

Axiom No 3.

$u + (v + w) = (u + v) + w$.

L.H.S.

$\Rightarrow (1, y) + ((1, y') + (1, y''))$

$(1, y) + (1, y' + y'')$

$\Rightarrow (1, y + y' + y'')$

R.H.S.

$((1, y) + (1, y')) + (1, y'')$

$(1, y + y') + (1, y'')$

$(1, y + y' + y'')$

It holds Axiom No 3.

Axiom No 4:-

$$u+0=u.$$

$$(1,y)+(0,0)=(1,y).$$

$$=(1,y)=(1,y).$$

It holds Axiom No 4.

Axiom No 5:

$$u+(-u)=0$$

$$(1,y)+(-(1,y))=(0,0).$$

$$(1,y)+(-1,-y).$$

$$(0,0)$$

it holds Axiom No 5.

Axiom No 6.

1 s u is in V.

$$1s(1,y)=$$

$$(1,1sV)$$

→ It holds Axiom No 6.

Axiom No 7.

$$c(u+v)=c(u)+c(v)$$

$$c((1,y)+(1,y'))=c(1,y)+c(1,y').$$

$$c(1,y+y')=(1,cy)+(1,cy')$$

$$(1,cy+cy')=(1,cy+cy')$$

It holds Axiom No 7.

Axiom No 8.

$(c+d)u = c(u) + d(u)$.

$(c+d)(1,y) = c(1,y) + d(1,y)$.

$(1,(c+d)y) = (1,cy) + (1,dy)$.

$(1,(c+d)y) = (1,(c+d)y)$.

It holds Axiom No 8.

Axiom No 9.

$c(du) = cd(u)$.

$c(d(1,y)) = cd(1,y)$.

$c(1,dy) = (1,cdy)$.

$(1,cdy) = (1,cdy)$.

It holds Axiom No 9.

Axiom No 10.

$1(u) = u$.

$1(1,y) = (1,y)$

$(1,y) = (1,y)$.

It holds Axiom No 10.

Question No 12.

The Set of Polynomials of the form $a_0 + a_1 x$ with the operation

$(a_0 + a_1 x) + (b_0 + b_1 x) = (a_0 + b_0) + (a_1 + b_1)x$

and

$K(a_0 + a_1 x) = (Ka_0) + (Ka_1)x.$

$\therefore\ U = (a_0, a_1 x),\ V = (b_0, b_1 x).$

Sol:-

Axiom No 1

$U + V$ is in V

$(a_0, a_1) + (b_0, b_1)$

$= (a_0 + b_0 + a_1 + b_1)$

It holds Axiom No 1

Axiom No 2.

$\Rightarrow (a_0 + b_0 x) + (a_1 + b_1 x) = (a_1 + b_1 x) + (a_0 + b_0)$

$((a_0 + a_1) + (b_0 + b_1))x = (a_1 + a_0) + (b_1 + b_0)x.$

It holds Axiom No 2.

Axiom 3:

$U + (V + W) = (U + V) + W.$

$(a_0 + b_0 x) + ((a_1 + b_1 x) + (a_2 + b_2 x)).$

$= (a_0 + b_0 x) + (a_1 + a_2) + (b_1^x + b_2^x)$

$= (a_0 + a_1 + a_2) + (b_0 + b_1 + b_2)x.$

Date:

R.H.S.

$((a_0+b_0x)+(a_1+b_1x))+(a_2+b_2x)$.

$((a_0+a_1)+(b_0x+b_1x))+(a_2+b_2x)$.

$(a_0+a_1+a_2)+(b_0+b_1+b_2)x$.

$\Rightarrow$ It holds Axiom No 3.

Axiom No 4.

$u+0=u$.

$(a_0+b_0x)+(0,0)=(a_0+b_0x)$

$(a_0+0+b_0x+0) = (a_0+b_0x)$.

$(a_0+b_0x)=(a_0+b_0x)$.

It holds Axiom No 4.

Axiom No 5.

$u+(-u)=0$.

$(a_0+b_1x)+(-a_0-b_1x) = (0,0)$.

$(0,0) = (0,0)$.

It holds Axiom No 5.

Axiom No 6.

ku is in V

$k(a_0+b_1x)$.

$k(a_0)+(kb_1)x$.

It holds Axiom No 6.

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Axiom No 7.

$c(u+v) = c(u)+c(v)$.

$c((a_0+a_1x)+(b_0+b_1x)) = c(a_0+a_1x) + c(b_0+b_1x)$.

$c((a_0+b_0)+(\cancel{\$}a_1+b_1)x) = c((a_0+b_0)+(a_1+b_1)x$.

It holds Axiom No 7.

Axiom No 8.

$(c+d)\,u = c(u)+d(u)$.

$(c+d)(a_0+a_1x) = c(a_0+a_1x) + d(a_0+a_1x)$.

$((c+d)a_0 + (c+d)a_1x) = (ca_0 + ca_1x) + (da_0 + da_1x)$.

$((c+d)a_0 + (c+d)a_1x) = ((c+d)a_0 + (c+d)a_1x)$.

It holds Axiom No 8.

Axiom No 9.

$\cancel{(c+d)\,u} =$

$c(du) = cd(u)$.

$c(d(a_0+a_1x) = cd(a_0+a_1x)$.

$c(da_0+da_1x) = (cda_0+cda_1x)$

$(cda_0 + cda_1x) = (cda_0 + cda_1x)$

It holds Axiom No 9.

Axiom No 10.

$1(u) = u$,

$1(a_0+a_1x) = (a_0+a_1x)$

$(a_0+a_1x) = (a_0+a_1x)$

It also holds Axiom No 10.

Exercise 4.2.

Question - 4

(a) The Set of all $n \times n$ Matrices A such that $A^T = -A$.

Sol:-

Let W be the Set of all $n \times n$ matrices. This set at least contain one matrix e.g. the zero $n \times n$ matrix is W.

Let us assume A and B are both in W. i.e $A^T = -A$ and $B^T = -B$.

According to the Theorem 1.4.8 (b) their Sum satisfies.

$$(A+B)^T = A^T + B^T$$

$$= -A - B = -(A+B).$$

therefore W is closed under Addition

from theorem 1.4.8 (d) we have

$(kA)^T = kA^T = k(-A) = -kA$.

which holds property of multiplicati

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(b) The Set of all $n \times n$ matrices A for which $Ax = 0$ ~~haves~~ has only Trivial solution.

Let W be the Set of all $n \times n$ matrices for which $Ax = 0$ has only Trivial solution. Therefore it follows from theorem 1.5.3 that the set W consist of all $n \times n$ matrices that are invertible this set is not closed under multiplication.

If $K = 0$, So W is not Subspace of $M_{n \times n}$.

(c) The Set of all $n \times n$ matrices A such that $AB = BA$ for some fixed $n \times n$ matrix B.

Let B be some fixed $n \times n$ matrix. and Let W be the set of all $n \times n$ Matrices A such that $AB = BA$, This set contains at least one matrix e.g. In W, Let us assume A and C are both in W. i.e. $AB = BA$ and $CB = BC$

According to theorem their sum satisfies.

$$(A + C)B = AB + BC = BA + BC.$$
$$= B(A + C).$$

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Date:

It holds the property of addition.

$\cancel{\mathbb{R}}$ $(KA)B = K(AB) = K(BA)$.

$\Rightarrow$ It also holds Multiplication property

(d) The Set of all invertible $n \times n$ matrices.

Let W be the Set of all invertible $n \times n$ matrices

If $K = 0$ multiplication it possible so, it is not subspace.

Question # 2.

Date:

(a) All 2x2 matrices A such that

$$A\begin{bmatrix}1\\-1\end{bmatrix} = \begin{bmatrix}0\\0\end{bmatrix}.$$

Sol:-

Let W is set of all 2x2 matrices A. This contain one (zero) matrix.

$$A+B: \begin{bmatrix}1\\-1\end{bmatrix} + B\begin{bmatrix}1\\-1\end{bmatrix}.$$

$$= K\begin{bmatrix}0\\0\end{bmatrix} = \begin{bmatrix}0\\0\end{bmatrix}.$$

According to the theorem it is subspace of $M_{2\times 2}$.

(b) All 2x2 matrices A such that

$$A\begin{bmatrix}0 & 2\\-2 & 1\end{bmatrix} = \begin{bmatrix}0 & 2\\-2 & 1\end{bmatrix}A.$$

Sol:-

Let W is set of 2x2 matrix. Adding two matrices in W ~~are~~ result in W.

$$= (A+B)\begin{bmatrix}0 & 2\\-2 & 1\end{bmatrix}$$

$$= A\begin{bmatrix}0 & 2\\-2 & 1\end{bmatrix} + B\begin{bmatrix}0 & 2\\-2 & 1\end{bmatrix}.$$

Date:

$$= \begin{bmatrix} 0 & 2 \\ -2 & 1 \end{bmatrix}(A+B) = \begin{bmatrix} 0 & 2 \\ -2 & 1 \end{bmatrix}A + \begin{bmatrix} 0 & 2 \\ -2 & 1 \end{bmatrix}B$$

Scalar Multiplication of W is in W.

$$(kA)\begin{bmatrix} 0 & 2 \\ -2 & 1 \end{bmatrix} = k\left(A\begin{bmatrix} 0 & 2 \\ -2 & 1 \end{bmatrix}\right)$$

$$k\left(\begin{bmatrix} 0 & 2 \\ -2 & 1 \end{bmatrix}A\right) = \begin{bmatrix} 0 & 2 \\ -2 & 1 \end{bmatrix}(kA).$$

(c) All 2×2 matrices A for which $\det(A) = 0$.

Sol:-

Let W be the set of 2×2 matrices.

Set is not closed under addition for example the matrices.

$\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$ and $\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$ are in W but det of both is 0

but

$$\det\left(\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}\right)$$

$$\det\left(\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}\right) = 1$$

So This is not lies in Subspaces.

Question No 14.

Date:

(a) All vectors x in R^4 such that $Ax = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ where.

$$A = \begin{bmatrix} 0 & -1 & 0 & 2 \\ -1 & +1 & 0 & 1 \end{bmatrix}$$

Sol:-

W be the set of all vectors x in R^4 such that $Ax = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$,

This set is not closed under multiplication when $k = 0$.

So, W is not in Subspace.

(b) All vectors x in R^4 such that $Ax = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$.

Let W be set of all vectors x in R^4 such that $Ax = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

This set is not closed under scalar multiplication, This set contain at least one vector is zero.

Q No 16 Date:

(a) All Polynomials with Even Co-efficient

Soln-

Let W be the set of all even Polynomials (co-efficients) This set is not Empty.

e.g.

$P(x) = 2x^2$ is in W,

Adding two polynomials in W results in another polynomial in W because the Sum of any two corresponding even co-efficients is even. & likewise a scalar multiple of Polynomial of Polynomial with Even Coefficients

According to Theorem 4.2.1 W is subspace of P_∞

(b) All Polynomials whose coefficient Sum to 0.

Sol-

Let W be the Set of All Polynomials whose coefficient Sum to 0. This set is not Empty.

e.g.

Polynomial $P(x) = x^2 - x$ is contained in W. Adding two polynomials in W results in another Polynomial in W.

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$= (a_0 + a_1 x + a_2 x^2 + \cdots + a_n x^n) + (b_0 + b_1 x + b_2 x^2 + \cdots + b_m x^m)$

$= (a_0 + b_0) + (a_1 + b_1) x + (a_2 + b_2) x^2 + \cdots + (a_m + b_m) x^m +$
$a_{m+1} x^{m+1} + \cdots + a_n x^n$

where we assume without loss of generality that $n \geq m$.
The sum of this polynomials coefficients is:-

$(a_0 + b_0) + (a_1 + b_1) + \cdots + (a_m + b_m) + a_{m+1} + \cdots$
$\cdots + a_n = a_0 + a_1 + \cdots a_n + b_0 + b_1 + \cdots + b_m$
$\therefore 0 + 0 = 0$

which means it is contained in W likewise a scalar multiplication of a polynomial whose coefficient sum to zero is another polynomial whose coefficients

$k a_0 + k a_1 x + k a_2 x^2 + \cdots + k a_n x^n$, so that
$k a_0 + k a_1 + \cdots + k a_n = k (a_0 + a_1 + \cdots + a_n)$
$, k0 = 0$

According to Theorem 4.2.1 W is subspace of P_∞.

Date:

(c) All Polynomials of even degree

Let W be set of all Polynomials of even degree this is not Empty

e.g.:

$P(x) = x^2$ is contained in W

Adding two polynomials in W results in another Polynomial in W

$$(a_0 + a_1x + a_2x^2 + \dots + a_nx^{2n}) + (b_0 + b_1x + b_2x^2 + \dots + b_mx^{2m})$$

$$= (a_0+b_0) + (a_1+b_1)x + (a_2+b_2)x^2 + \dots + (a_{2m}+b_{2m})x^{2m} + a_{2m+1}x^{2m+1} + \dots + a_nx^{2n}.$$

where we assume without loss of generality that $n \times m$.

This polynomial also even degree which means it is contained in W likewise a Scalar multiple of a Polynomial of even degree is another Polynomial of even degree.

$$k(a_0 + a_1x + \dots + a_nx^{2n}) = ka_0 + ka_1x + ka_2x^2 + \dots + ka_nx^{2n}.$$

According to theorem 4.2.1 W is subspace.

Exercise 4.3.

Question #15

Let W be solution space to system $Ax = 0$. Determine whether set $\{U, V\}$ spans W.

$$A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}$$

(a) $U = (1, 0, -1, 0)$; $V = (0, 1, 0, -1)$.

Sol:-

$$A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}$$

reduced Echelon form is

$$\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$(x, y, z, w) = (s, t, -s, -t).$$
$$= s(1, 0, -1, 0) + t(0, 1, 0, -1).$$

Therefore solution space is spanned by vectors $V_1 = (1, 0, -1, 0)$ and $V_2 = (0, 1, 0, -1)$ we conclude that vectors

$U = (1, 0, -1, 0)$ and $V = (0, 1, 0, -1)$ span solution space W.

Date:

(b) $U = (1, 0, -1, 0)$, $V = (1, 1, -1, -1)$

Sol.

$U = (1, 0, -1, 0)$ and $V = (1, 1, -1, -1)$

$\therefore U_1 \neq V_1$ and $V = V_1 + V_2$.

We conclude vectors.

$U = (1, 0, -1, 0)$, $V = (1, 1, -1, -1)$.

Span Solution Space W.

Question No 16.

Let W be solution space to system $Ax=0$. Determine whether set $\{u,v\}$ spans W.

$$A=\begin{bmatrix} 0 & 1 & -1 & 1 \\ 0 & 2 & -2 & 2 \\ 0 & 3 & -3 & 3 \end{bmatrix}$$

a. $u=(1,1,1,0)$, $v=(0,-1,0,1)$.

Sol:-

reduced Echelon form is

$$\begin{bmatrix} 0 & 1 & -1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

The general solution in vector form is.

$(x, y, z, w) = (r, s, -t, s, t)$

$= r(1, 0, 0, 0) + s(0, 1, 1, 0) + t(0, -1, 0, 1)$

∴ Solution space is spanned vectors

$(1,0,0,0) = a(1,1,1,0) + b(0,-1,0,1)$

$(1,0,0,0) = (a,a,a,0) + (0,-b,0,b)$.

$a+0=1$ ∴ $a=1$

$a-b=0$ $\Rightarrow 0-0=0$

$a+0=0$ $\Rightarrow a=0$

$0+b=0$ $\Rightarrow b=0$.

therefore Theorem 4.3.2 they don't span solution space W.

Pg No.

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(b) $u = (0,1,1,0)$, $V = (1,0,1,1)$

Solr

$(1,0,0,0) = a(0,1,1,0) + b(1,0,1,1)$

$(1,0,0,0) = (0,a,a,0) + (b,0,b,b)$

$b = 1 \Rightarrow b = 1$

$a + 0 = 0 \Rightarrow a = 0$

$a + b = 0 \Rightarrow a = 0 - 1$

$b = 0 \Rightarrow b = 0$

According to theorem 4.3.2

They don't span in solution space of W.

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Question 18.

Date:

In each part, let $T_A = R^3 \to R^2$ be multiplication ~~eg~~ by A, and let $u_1 = (0,1,1)$ and $u_2 = (0,-1,1)$ $u_3 = (1,1,-2)$

Determine whether set $\{T_A(u_1), T_A(u_2), T_A(u_3)\}$ span R^2.

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & -1 \end{bmatrix}$$

The vectors. $TA(0,1,1) = (1,1,0)$, $TA(0,-1,-1) = (1,-$ and $TA = (1,1,-2) = (2,3)$ Span R^2.

if an arbitary vectors. $b = (b_1, b_2)$ can be expressed as a linear combination.

$$(b_1, b_2) = k_1(1,0) + k_2(1,-2) + k_3(2,3).$$

$$(b_1, b_2) = (k_1, 0) + (k_2, -2k_2) + (2k_3, 3k_3).$$

$$(b_1, b_2) = (k_1 + k_2 + 2k_3, -2k_2 + 3k_3).$$

$$k_1 + k_2 + 2k_3 = b_1.$$
$$-2k_2 + 3k_3 = b_2$$

Reduced Echelon form.

$$\begin{bmatrix} 1 & 0 & 7/2 \\ 0 & 1 & -3/2 \end{bmatrix}.$$

System is consistant for all right hand side vectors B we conclude that $TA(u_1) + TA(u_2)$ $TA(u_4)$ span R^2.

Date:

(b)

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & -3 \end{bmatrix}.$$

Soln:-

The vectors $T_A(0,1,1) = (1,4)$, $T_4(2,-1,1) = (-1,4)$ and $T_A(1,1,-2) = (1,-4)$ span R^2 if an arbitary vector.

$b = (b_1, b_2)$ can be expressed as linear combination.

$$(b_1, b_2) = k_1(1,4) + k_2(-1,4) + k_3(1,-4).$$

$$(b_1, b_2) = (k_1, 4k_1) + (-k_2, 4k_2) + (k_3, -4k_3)$$

$$\therefore (b_1, b_2) = (k_1 - k_2 + k_3,\ 4k_1 + 4k_2 - 4k_3).$$

$\therefore$

$$k_1 - k_2 + k_3 = b_1.$$

$$\Rightarrow \quad 4k_1 + 4k_2 - 4k_3 = b_2.$$

$$\begin{bmatrix} 1 & -1 & 1 \\ 4 & 4 & -4 \end{bmatrix}.$$

The reduced row echelon form of coefficients matrix of this system is

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \end{bmatrix}$$

The system is consistant for all right hand side vector B.

We conclude that $T_A(u_1)$, $T_A(u_2)$ and $T_A(u_3)$ span R^2.

Date:

Exercise 4.4

Question 11

For which values of λ do following vectors form a linearly independent set in R^3?

$V_1 = (\lambda, -1/2, -1/2)$, $V_2 = (-1/2, \lambda, -1/2)$

$V_3 = (-1/2, -1/2, \lambda_3)$.

Soln:

$U = C_1V_1 + C_2V_2 + C_3V_3$.

$(0,0,0) = C_1(\lambda, -1/2, -1/2) + C_2(-1/2, \lambda, -1/2) + C_3(-1/2, -1/2, \lambda)$

$(0,0,0) = (\lambda C_1, -1/2 C_1, -1/2 C_1) + (-1/2 C_2, \lambda C_2, -1/2 C_2) + (-1/2 C_3, -1/2 C_3, \lambda C_3)$.

$\lambda C_1 + (-1/2) C_2 - 1/2 C_3 = 0$

$-1/2 C_2 + \lambda C_2 - 1/2 C_3 = 0$

$-1/2 C_1 - 1/2 C_2 - \lambda C_3$

$$\begin{bmatrix} \lambda & -1/2 & -1/2 \\ -1/2 & \lambda & -1/2 \\ -1/2 & -1/2 & \lambda \end{bmatrix}$$

$= \lambda(\lambda^2 - 1/4) + 1/2\left(-\frac{1}{2}\lambda - \frac{1}{4}\right) - \frac{1}{2}\left(\frac{1}{4} + \lambda/2\right)$

$\lambda^3 - \frac{1}{4}\lambda - \frac{1}{4}\lambda - \frac{1}{8} - \frac{1}{8} - \frac{\lambda}{4} = 0$

$= \lambda^3 - \frac{3}{4}\lambda - \frac{1}{4} = 0$

Date:

$$\left(\lambda+\frac{1}{2}\right)\left(\lambda^2-\frac{3}{2}\lambda-\frac{1}{2}\right)=0.$$

$$=\left(\lambda+\frac{1}{2}\right)\left(\lambda+\frac{1}{2}\right)(\lambda-1)=0.$$

$$\lambda+\frac{1}{2}=0,\quad \lambda-1=0.$$

$$\boxed{\lambda=-1/2}\;;\;\boxed{\lambda=1}$$

Question 14.

Date:

$T_A : R^3 \to R^3$ be multiplication by A and let $u_1 = (1,0,0)$, $u_2 = (2,-1,0)$ and $u_3 = (0,1,1)$.

determine whether it is linearly independant.

$$\begin{bmatrix} 1 & 1 & 2 \\ 1 & 0 & -3 \\ 2 & 2 & 0 \end{bmatrix}$$

Solve

$T_A(1,0,0) = (1,1,2)$, $T_A(2,-1,1) = (3,-1,2)$, $T_A(0,1,1) = (3,-3,2)$

$k_1 u_1 + k_2 u_2 + k_3 u_3 = (0,0,0)$.

$k_1(1,1,2) + k_2(3,-1,2) + k_3(3,-3,2)$.

$(k_1, k_1, 2k_1) + (3k_2, k_2, 2k_2) + (3k_3, -3k_3, 2k_3)$.

$= (k_1 + 3k_2 + 3k_3,\ k_1 + k_2 - 3k_3,\ 2k_1 + 2k_2 + 2k_3)$

$$\begin{bmatrix} 1 & 3 & 3 \\ 1 & -1 & -3 \\ 2 & 2 & 2 \end{bmatrix}.$$

determinant

$$\begin{vmatrix} 1 & 3 & 3 \\ 1 & -1 & -3 \\ 2 & 2 & 2 \end{vmatrix} = -8 \neq 0.$$

it is linearly independant

Date:

(b) $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & -3 \\ 2 & 2 & 0 \end{bmatrix}$.

Soln.

We calculate

$T_A(1,0,0) = (1,1,2)$, $T_A(2,-1,1) = (2,-2,2)$

$T_A(0,1,1) = (2,-2,2)$.

Since

$T_A(u_2) = 1\, T_A(u_3)$, it follows that the set $\{T_A(u_1), T_A(u_2), T_A(u_3)\}$.

is linearly independant

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