Hypothesis Testing

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Introduction

- Researchers are interested in answering many types of questions. For example:
- Scientist might want to know whether the earth is warming up.
- A physician might want to know whether a new medication will lower a person's blood pressure.
- An educator might wish to see whether a new teaching technique is better than a traditional one.
- A retail merchant might want to know whether the public prefers a certain color in a new line of fashion.
- Automobile manufacturers are interested in determining whether seat belts will reduce the severity of injuries caused by accidents.
- These types of questions can be addressed through statistical **hypothesis testing,** which is a **decision-making process** for evaluating claims about a population.

Three Methods to test statistical Hypothesis

1. The traditional method
2. The *P*-value method
3. The confidence interval method

Steps in Hypothesis Testing—Traditional Method

A **statistical hypothesis** is a conjecture about a population parameter. This conjecture may or may not be true.

- There are two types of statistical hypotheses for each situation: the **null hypothesis** and the **alternative hypothesis.**

The **null hypothesis,** symbolized by H_0, is a statistical hypothesis that states that there is no difference between a parameter and a specific value, or that there is no difference between two parameters.

The **alternative hypothesis,** symbolized by H_1, is a statistical hypothesis that states the existence of a difference between a parameter and a specific value, or states that there is a difference between two parameters.

Hypothesis Testing (Contd.)

- To state hypotheses correctly, researcher must translate the conjecture or claim from words into mathematical symbols. The basic symbols used are as follows:

Equal to	$=$	Greater than	$>$
Not equal to	$\neq$	Less than	$<$

- The null and alternative hypotheses are stated together, and the null hypothesis contains the equals sign, as shown (where *k* represents a specified number).

Two-tailed test	Right-tailed test	Left-tailed test
H_0: $\mu = k$	H_0: $\mu = k$	H_0: $\mu = k$
H_1: $\mu \neq k$	H_1: $\mu > k$	H_1: $\mu < k$

Hypothesis-Testing Common Phrases

$>$	$<$
Is greater than	Is less than
Is above	Is below
Is higher than	Is lower than
Is longer than	Is shorter than
Is bigger than	Is smaller than
Is increased	Is decreased or reduced from
$=$	$\neq$
Is equal to	Is not equal to
Is the same as	Is different from
Has not changed from	Has changed from
Is the same as	Is not the same as

State the null and alternative hypotheses for each conjecture

- An engineer hypothesizes that the mean number of defects can be decreased in a manufacturing process of compact disks by using robots instead of humans for certain tasks. The mean number of defective disks per 1000 is 18.
- A psychologist feels that playing soft music during a test will change the results of the test. The psychologist is not sure whether the grades will be higher or lower. In the past, the mean of the scores was 73.
- A real estate agent claims that 60% of all private residences being built today are 3bedroom homes. To test the claim, a large sample of new residences are inspected; the proportion of these homes with 3 bedrooms is recorded and used as out test statistic state the null & alternative hypotheses.

9–13. For each conjecture, state the null and alternative hypotheses.

a. The average age of taxi drivers in New York City is 36.3 years.
b. The average income of nurses is $36,250.
c. The average age of disc jockeys is greater than 27.6 years.
d. The average pulse rate of female joggers is less than 72 beats per minute.
e. The average bowling score of people who enrolled in a basic bowling class is less than 100.
f. The average cost of a VCR is $297.75.
g. The average electric bill for residents of White Pine Estates exceeds $52.98 per month.
h. The average number of calories of brand A's low-calorie meals is at most 300.
i. The average weight loss of people who use brand A's low-calorie meals for six weeks is at least 3.6 pounds.

Hypothesis Testing (Contd.)

- After stating the hypothesis, the researcher designs the study. The researcher selects the correct ***statistical test***, chooses an appropriate *level of significance*, and formulates a plan for conducting the study.

A **statistical test** uses the data obtained from a sample to make a decision about whether the null hypothesis should be rejected.

The numerical value obtained from a statistical test is called the **test value.**

4 types of decisions

- In the hypothesis-testing situation, there are four possible outcomes.

P(Type-I) = Level of significance

	Ho True	**Ho False**
Reject Ho	**Error (Type – I)**	Correct Decision
Do not reject Ho	Correct Decision	**Error (Type – II)**

P(Type-II) = β

Hypothesis Testing (Contd.)

The **critical value** separates the critical region from the noncritical region. The symbol for critical value is C.V.

The **critical** or **rejection region** is the range of values of the test value that indicates that there is a significant difference and that the null hypothesis should be rejected.

The **noncritical** or **nonrejection region** is the range of values of the test value that indicates that the difference was probably due to chance and that the null hypothesis should not be rejected.

Hypothesis-testing situation in a Jury Trial

- In a jury trial, there are four possible outcomes. The defendant is either guilty or innocent, and he or she will be convicted or acquitted.

H_0: The defendant is innocent

H_1: The defendant is not innocent (i.e., guilty)

- Next, the evidence is presented in court by the prosecutor, and based on this evidence, the jury decides the verdict, innocent or guilty.

Jury trial (Results of trial)

	H_0 true (innocent)	H_0 false (not innocent)
Reject H_0 (convict)	Type I error 1.	Correct decision 2.
Do not reject H_0 (acquit)	Correct decision 3.	Type II error 4.

Steps in Hypothesis Testing (summary)

In hypothesis testing, the following steps are recommended.

1. State the hypotheses. Be sure to state both the null and the alternative hypotheses.
2. Design the study. This step includes selecting the correct statistical test, choosing a level of significance, and formulating a plan to carry out the study. The plan should include information such as the definition of the population, the way the sample will be selected, and the methods that will be used to collect the data.
3. Conduct the study and collect the data.
4. Evaluate the data. The data should be tabulated in this step, and the statistical test should be conducted. Finally, decide whether to reject or not reject the null hypothesis.
5. Summarize the results.

Z-test for mean

- The ***z* test** is a statistical test for the mean of a population. It can be used when $n > 30$, or when the population is normally distributed and σ is known. The formula for the *z* test is

$$z = \frac{\overline{X} - \mu}{\sigma/\sqrt{n}}$$

where

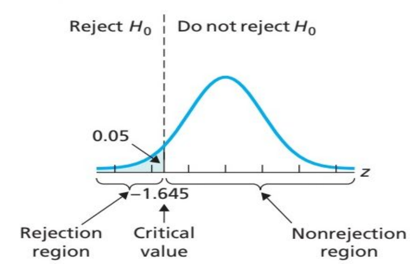
$\overline{X}$ = sample mean
μ = hypothesized population mean
σ = population standard deviation
n = sample size

One-Sample z-test example

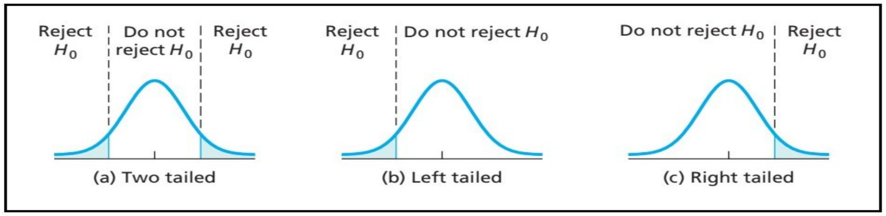
H_0: $\mu = 275$
H_a: $\mu < 275$
$\alpha = 0.05$

- the population standard deviation is 20
- the population is normally distributed

$$z = \frac{\bar{x} - 275}{4} = \frac{264.4 - 275}{4} = -2.65.$$



Interpretation At the 5% significance level, the data provide sufficient evidence to conclude that mean is less than 275



Example # 01 – 02

Level of Significance	0.10	0.05	0.01
$H_1: \mu > \mu_o$ or $\mu < \mu_o$	+1.28, - 1.28	+1.64, -1.64	+2.33, -2.33
H1: $\mu \neq \mu_o$	+1.64, -1.64	+1.96, -1.96	+2.58, - 2.58

- An el gth of life that i 600 hours and a **that μ = 1600** sample of 30 b level of significance.
- A sample of 16 observations is taken from a normal population whose standard deviation σ = 30. The mean is computed as 110. Test the hypothesis that μ = 100 against the alternative μ > 100 at 0.05 level of significance.

Example # 03

Level of Significance	0.10	0.05	0.01
$H_1: \mu > \mu_o$ or $\mu < \mu_o$	+1.28, - 1.28	+1.64, -1.64	+2.33, -2.33
H1: $\mu \neq \mu_o$	+1.64, -1.64	+1.96, -1.96	+2.58, - 2.58

- A researcher claims that the average cost of men's athletic shoes is less than $80. He selects a random sample of 36 pairs of shoes from a catalog and finds the following costs (in dollars). (The costs have been rounded to the nearest dollar.) Is there enough evidence to support the researcher's claim at a 0.10? Assume σ =19.2.

60	70	75	55	80	55
50	40	80	70	50	95
120	90	75	85	80	60
110	65	80	85	85	45
75	60	90	90	60	95
110	85	45	90	70	70

Example # 04

A researcher wishes to test the claim that the average cost of tuition and fees at a four-year public college is greater than $5700. She selects a random sample of 36 four-year public colleges and finds the mean to be $5950. The population standard deviation is $659. Is there evidence to support the claim at $\alpha = 0.05$? Use the *P*-value method.

Source: Based on information from the College Board.

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986

Example # 05

A researcher claims that the average wind speed in a certain city is 8 miles per hour. A sample of 32 days has an average wind speed of 8.2 miles per hour. The standard deviation of the population is 0.6 mile per hour. At $\alpha = 0.05$, is there enough evidence to reject the claim? Use the *P*-value method.

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767

Confidence Interval on μ when σ is known

If $\bar{x}$ is the mean of a random sample of size n from a population with known variance σ^2, a $100(1-\alpha)\%$ confidence interval for μ is given by

$$\bar{x} - z_{\alpha/2}\frac{\sigma}{\sqrt{n}} < \mu < \bar{x} + z_{\alpha/2}\frac{\sigma}{\sqrt{n}},$$

where $z_{\alpha/2}$ is the z-value leaving an area of $\alpha/2$ to the right.

Test of Difference between two means

Assumptions for the *z* Test to Determine the Difference Between Two Means

1. Both samples are random samples.
2. The samples must be independent of each other. That is, there can be no relationship between the subjects in each sample.
3. The standard deviations of both populations must be known, and if the sample sizes are less than 30, the populations must be normally or approximately normally distributed.

$$z = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

Test of Difference between two means (Contd.)

Right-tailed			Left-tailed		
$H_0: \mu_1 = \mu_2$ $H_1: \mu_1 > \mu_2$	or	$H_0: \mu_1 - \mu_2 = 0$ $H_1: \mu_1 - \mu_2 > 0$	$H_0: \mu_1 = \mu_2$ $H_1: \mu_1 < \mu_2$	or	$H_0: \mu_1 - \mu_2 = 0$ $H_1: \mu_1 - \mu_2 < 0$

Formula for the *z* Confidence Interval for Difference Between Two Means

$$(\bar{X}_1 - \bar{X}_2) - z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} < \mu_1 - \mu_2 < (\bar{X}_1 - \bar{X}_2) + z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

Example # 06

- **Hotel Room Cost:** A survey found that the average hotel room rate in New Orleans is \$88.42 and the average room rate in Phoenix is \$80.61. Assume that the data were obtained from two samples of 50 hotels each and that the standard deviations of the populations are \$5.62 and \$4.83, respectively. At $\alpha = 0.05$, can it be concluded that there is a significant difference in the rates?
- Find the 95% confidence interval for the difference between the means.

Example # 07

- A researcher hypothesizes that the average number of sports that colleges offer for males is greater than the average number of sports that colleges offer for females. A sample of the number of sports offered by colleges is shown. At α = 0.10, is there enough evidence to support the claim? Assume σ_1 and σ_2 = 3.3
-

Males					Females				
6	11	11	8	15	6	8	11	13	8
6	14	8	12	18	7	5	13	14	6
6	9	5	6	9	6	5	5	7	6
6	9	18	7	6	10	7	6	5	5
15	6	11	5	5	16	10	7	8	5
9	9	5	5	8	7	5	5	6	5
8	9	6	11	6	9	18	13	7	10
9	5	11	5	8	7	8	5	7	6
7	7	5	10	7	11	4	6	8	7
10	7	10	8	11	14	12	5	8	5

Source: *USA TODAY.*

Example # 05 – 06

- A random of sample of size n_1 = 50 taken from normal population with a standard deviation σ1 = 7.35 has sample mean 181. A second sample of size n_2 = 72 taken from a different normal population with σ_2 = 4.81 has sample mean 176. Test the hypothesis at 0.05 level of significance that $\mu_1 = \mu_2$, vs. $\mu_1 \neq \mu_2$.
- A farmer claims that the average yield of wheat of variety A exceeds the average yield of variety B by at least 12 bushels per acre. To test this claim, 50 acres of each variety are planted and grown under similar conditions. Variety A yielded on the average, 86.7 bushels per acre with a **standard deviation of 6.28** bushels per acre, while variety B yielded, on the average 77.8 bushels per acre with a **standard deviation of 5.61** bushels per acre. Test the farmer's claim at alpha = 0.01.

Example # 07

- A random sample of size 80 from a non-normal population yielded the sample mean $\bar{x}_1 = 170.4$ and $s_1^2 = 62.80$. Another sample of size 100 from a second non-normal population yielded the sample mean $\bar{x}_1 = 165.3$ and the sample variable $s_2^2 = 89.64$. Test Ho: $\mu_1 - \mu_2 \leq 2$ at alpha = 0.01.

t-test for a Mean
(t-distribution)

- The *t* distribution is similar to the standard normal distribution in the following ways.
  - It is bell-shaped.
  - It is symmetric about the mean.
  - The mean, median, and mode are equal to 0 and are located at the center of the distribution.
  - The curve never touches the *x* axis.

$$h(t) = \frac{\Gamma[(v+1)/2]}{\Gamma(v/2)\sqrt{\pi v}}\left(1+\frac{t^2}{v}\right)^{-(v+1)/2}, \quad -\infty < t < \infty.$$

This is known as the ***t*-distribution** with v degrees of freedom.

t-test for a Mean
(t-distribution)

- The variance is greater than 1.
- The t distribution is a family of curves based on the *degrees of freedom,* which is a number related to sample size.
- As the sample size increases, the t distribution approaches the normal distribution.

t-test for a Mean

- The t-test is defined as:

The ***t* test** is a statistical test for the mean of a population and is used when the population is normally or approximately normally distributed, and σ is unknown.

The formula for the t test is

$$t = \frac{\overline{X} - \mu}{s/\sqrt{n}}$$

The degrees of freedom are d.f. $= n - 1$.

If $\bar{x}$ and s are the mean and standard deviation of a random sample from a normal population with unknown variance σ^2, a $100(1-\alpha)\%$ confidence interval for μ is

$$\bar{x} - t_{\alpha/2}\frac{s}{\sqrt{n}} < \mu < \bar{x} + t_{\alpha/2}\frac{s}{\sqrt{n}},$$

where $t_{\alpha/2}$ is the t-value with $v = n - 1$ degrees of freedom, leaving an area of $\alpha/2$ to the right.

Example # 08 & 09

- **Hospital Infections**: A medical investigation claims that the average number of infections per week at a hospital in southwestern Pennsylvania is 16.3. A random sample of 10 weeks had a mean number of 17.7 infections. The sample standard deviation is 1.8. Is there enough evidence to reject the investigator's claim at a 0.05?
- **Substitute Teachers' Salaries:** An educator claims that the average salary of substitute teachers in school districts in Allegheny County, Pennsylvania, is less than \$60 per day. A random sample of eight school districts is selected, and the daily salaries (in dollars) are shown. Is there enough evidence to support the educator's claim at α 0.10?

60 56 60 55 70 55 60 55

Example # 10

- **Jogger's Oxygen Uptake:** A physician claims that joggers' maximal volume oxygen uptake is greater than the average of all adults. A sample of 15 joggers has a mean of 40.6 milliliters per kilogram (ml/kg) and a standard deviation of 6 ml/kg. If the average of all adults is 36.7 ml/kg, is there enough evidence to support the physician's claim at α = 0.05?

Testing Difference between two mean when $\sigma_1 \neq \sigma_2$ (Independent Sample: t-test)

Variances are assumed to be unequal

$$t = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}}}$$

where the degrees of freedom are equal to the smaller of $n_1 - 1$ or $n_2 - 1$.

Assumptions for the t Test for Two Independent Means When σ_1 and σ_2 Are Unknown

1. The samples are random samples.
2. The sample data are independent of one another.
3. When the sample sizes are less than 30, the populations must be normally or approximately normally distributed.

Confidence Intervals for the Difference of Two Means: Independent Samples

Variances assumed to be unequal:

$$(\bar{X}_1 - \bar{X}_2) - t_{\alpha/2}\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} < \mu_1 - \mu_2 < (\bar{X}_1 - \bar{X}_2) + t_{\alpha/2}\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

d.f. = smaller value of $n_1 - 1$ or $n_2 - 1$

Example # 11

- **Farm Sizes:** The average size of a farm in Indiana County, Pennsylvania, is 191 acres. The average size of a farm in Greene County, Pennsylvania, is 199 acres. Assume the data were obtained from two samples with standard deviations of 38 and 12 acres, respectively, and sample sizes of 8 and 10, respectively. Can it be concluded at $\alpha = 0.05$ that the average size of the farms in the two counties is different? Assume the populations are normally distributed.
- Find the 95% confidence interval .

d.f.	One tail, α / Two tails, α	0.10 / 0.20	0.05 / 0.10	0.025 / 0.05	0.01 / 0.02	0.005 / 0.01
1		3.078	6.314	12.706	31.821	63.657
2		1.886	2.920	4.303	6.965	9.925
3		1.638	2.353	3.182	4.541	5.841
4		1.533	2.132	2.776	3.747	4.604
5		1.476	2.015	2.571	3.365	4.032
6		1.440	1.943	2.447	3.143	3.707
7		1.415	1.895	2.365	2.998	3.499
8		1.397	1.860	2.306	2.896	3.355
9		1.383	1.833	2.262	2.821	3.250
10		1.372	1.812	2.228	2.764	3.169
11		1.363	1.796	2.201	2.718	3.106
12		1.356	1.782	2.179	2.681	3.055
13		1.350	1.771	2.160	2.650	3.012
14		1.345	1.761	2.145	2.624	2.977
15		1.341	1.753	2.131	2.602	2.947
16		1.337	1.746	2.120	2.583	2.921
17		1.333	1.740	2.110	2.567	2.898
18		1.330	1.734	2.101	2.552	2.878
19		1.328	1.729	2.093	2.539	2.861
20		1.325	1.725	2.086	2.528	2.845
21		1.323	1.721	2.080	2.518	2.831
22		1.321	1.717	2.074	2.508	2.819
23		1.319	1.714	2.069	2.500	2.807
24		1.318	1.711	2.064	2.492	2.797
25		1.316	1.708	2.060	2.485	2.787
26		1.315	1.706	2.056	2.479	2.779
27		1.314	1.703	2.052	2.473	2.771
28		1.313	1.701	2.048	2.467	2.763
29		1.311	1.699	2.045	2.462	2.756
30		1.310	1.697	2.042	2.457	2.750
32		1.309	1.694	2.037	2.449	2.738
34		1.307	1.691	2.032	2.441	2.728
36		1.306	1.688	2.028	2.434	2.719
38		1.304	1.686	2.024	2.429	2.712
40		1.303	1.684	2.021	2.423	2.704
45		1.301	1.679	2.014	2.412	2.690
50		1.299	1.676	2.009	2.403	2.678
55		1.297	1.673	2.004	2.396	2.668
60		1.296	1.671	2.000	2.390	2.660
65		1.295	1.669	1.997	2.385	2.654
70		1.294	1.667	1.994	2.381	2.648
75		1.293	1.665	1.992	2.377	2.643
80		1.292	1.664	1.990	2.374	2.639
90		1.291	1.662	1.987	2.368	2.632
100		1.290	1.660	1.984	2.364	2.626
500		1.283	1.648	1.965	2.334	2.586
1000		1.282	1.646	1.962	2.330	2.581
$(z)\ \infty$		1.282^a	1.645^b	1.960	2.326^c	2.576^d

Example # 12

- **Too Long on the Telephone:** A company collects data on the lengths of telephone calls made by employees in two different divisions. The mean and standard deviation for the sales division are 10.26 and 8.56, respectively. The mean and standard deviation for the shipping and receiving division are 6.93 and 4.93, respectively. A hypothesis test was run, and the computer output follows.

Degrees of freedom = 56
Confidence interval limits = −0.18979, 6.84979
Test statistic t = 1.89566
Critical value t = −2.0037, 2.0037
P-value = 0.06317
Significance level = 0.05

Example # 12 (Contd.)

1. Are the samples independent or dependent? **Independent**
2. Which number from the output is compared to the significance level to check if the null hypothesis should be rejected? **We compare the p-value**
3. Which number from the output gives the probability of a type I error that is calculated from the sample data? **P-value = P(Type – I)**
4. Was a right-, left-, or two-tailed test done? Why? **Two-tailed, since two critical values are given**
5. What are your conclusions? **we fail to reject the null hypothesis & conclude that there is a difference in the lengths of telephone calls.**
6. What would your conclusions be if the level of significance were initially set at 0.10?

If the significance level had been 0.10, we would have rejected the null hypothesis,

Example # 13

- Test the claim that there is no difference between population means based on these sample data.

Set A	32	38	37	36	36	34	39	36	37	42
Set B	30	36	35	36	31	34	37	33	32	

t-Test: Two-Sample Assuming Unequal Variances

	Variable 1	*Variable 2*
Mean	36.7	33.77777778
Variance	7.344444444	5.944444444
Observations	10	9
Hypothesized Mean Difference	0	
df	17	
t Stat	2.474205364	
P(T<=t) one-tail	0.012095	
t Critical one-tail	1.739606716	
P(T<=t) two-tail	0.024189999	
t Critical two-tail	2.109815559	

Testing Difference between two mean when $\sigma_1 = \sigma_2$ (Independent Sample: t-test)

- When the variances are assumed to be equal, this formula is used:

$$t = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}}\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

Example # 14

- Test $H_o: \mu_1 - \mu_2 \le 3$ against $H_1: \mu_1 - \mu_2 > 3$. Let $\alpha = 0.10$, $\sigma_1 = \sigma_2$ but unknown & normally distributed populations.
- **Sample I:** 51, 42, 49, 55, 46, 63, 56, 58, 47, 39, 47.
- **Sample II:** 38, 49, 45, 29, 31, 35.

Testing the Difference Between Two Means: Dependent Samples

- Samples are considered to be **dependent samples** when the subjects are paired, matched or related in some way.
- Here are some other examples of dependent samples:
- A researcher may want to design an SAT preparation course to help students raise their test scores the second time they take the SAT. Hence, the differences between the two exams are compared.
- A medical specialist may want to see whether a new counseling program will help subjects lose weight. Therefore, the preweights of the subjects will be compared with the postweights.

Testing the Difference Between Two Means: Dependent Samples (Contd.)

- When the samples are dependent, a special *t* test for dependent means is used. This test employs the difference in values of the matched pairs. The hypotheses are as follows:

Two-tailed	Left-tailed	Right-tailed
H_0: $\mu_D = 0$	H_0: $\mu_D = 0$	H_0: $\mu_D = 0$
H_1: $\mu_D \neq 0$	H_1: $\mu_D < 0$	H_1: $\mu_D > 0$

Step 1 State the hypotheses and identify the claim.

Step 2 Compute the test value.

a. Make a table, as shown.

X_1	X_2	A $D = X_1 - X_2$	B $D^2 = (X_1 - X_2)^2$
$\vdots$	$\vdots$		
		$\Sigma D =$ ______	$\Sigma D^2 =$ ______

b. Find the differences and place the results in column A. $D = X_1 - X_2$

c. Find the mean of the differences. $\overline{D} = \dfrac{\Sigma D}{n}$

d. Square the differences and place the results in column B. Complete the table

$D^2 = (X_1 - X_2)^2$

e. Find the standard deviation of the differences.

$$s_D = \sqrt{\frac{n\Sigma D^2 - (\Sigma D)^2}{n(n-1)}}$$

f. Find the test value.

$$t = \frac{\overline{D} - \mu_D}{s_D/\sqrt{n}} \quad \text{with d.f.} = n - 1$$

Step 3 Make the decision.

Step 4 Summarize the results.

Assumptions for the *t* Test for Two Means When the Samples Are Dependent

1. The sample or samples are random.
2. The sample data are dependent.
3. When the sample size or sample sizes are less than 30, the population or populations must be normally or approximately normally distributed.

Example # 15

- **Bank Deposits:** A sample of nine local banks shows their deposits (in billions of dollars) 3 years ago and their deposits (in billions of dollars) today. At a 0.05, can it be concluded that the average in deposits for the banks is greater today than it was 3 years ago? Use a 0.05.

Bank	1	2	3	4	5	6	7	8	9
3 years ago	11.42	8.41	3.98	7.37	2.28	1.10	1.00	0.9	1.35
Today	16.69	9.44	6.53	5.58	2.92	1.88	1.78	1.5	1.22

Example # 15 (contd.)

3 years ago (X_1)	Now (X_2)	A $D = X_1 - X_2$	B $D^2 = (X_1 - X_2)^2$
11.42	16.69	−5.27	27.7299
8.41	9.44	−1.03	1.0609
3.98	6.53	−2.55	6.5025
7.37	5.58	+1.79	3.2041
2.28	2.92	−0.64	0.4096
1.10	1.88	−0.78	0.6084
1.00	1.78	−0.78	0.6084
0.90	1.50	−0.60	0.3600
1.35	1.22	+0.13	0.1690
		9.73	**40.5437**

Subject	1	2	3	4	5	6
Before (X_1)	210	235	208	190	172	244
After (X_2)	190	170	210	188	173	228

I have calculated mean and variance for their differences above in excel.

t-critical = 2.571

t-statistics = 0.0126

we are unable to reject Ho because t-statistics is less than t-critical.

the supplement has no effect in changing cholesterol level.

Example # 16

- **Cholesterol Levels:** A dietitian wishes to see if a person's cholesterol level will change if the diet is supplemented by a certain mineral. Six subjects were pretested, and then they took the mineral supplement for a 6-week period. The results are shown in the table. (Cholesterol level is measured in milligrams per deciliter.) Can it be concluded that the cholesterol level has been changed at a 0.10? Assume the variable is approximately normally distributed.

Subject	1	2	3	4	5	6
Before (X_1)	210	235	208	190	172	244
After (X_2)	190	170	210	188	173	228

How to select correct test for testing mean

- Students sometimes have difficulty deciding whether to use the *z* test or *t* test.

1. If σ is known, use the z test. The variable must be normally distributed if $n < 30$.

2. If σ is unknown but $n \geq 30$, use the t test.

3. If σ is unknown and $n < 30$, use the t test. (The population must be approximately normally distributed.)