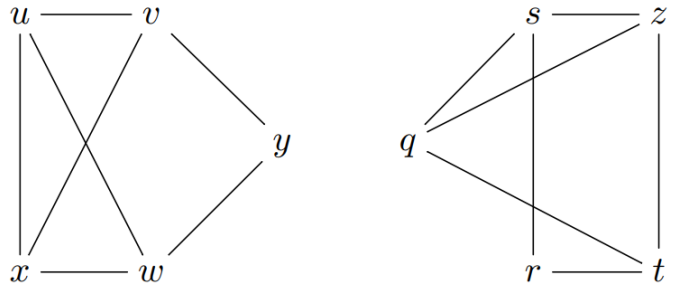
Explain your steps. The calculations and answers should be written neatly on paper which is attached as a single pdf. Submits the solution on GoogleClassroom with in dua date.

Problem 1

The friendship graph F_n has $2n+1$ vertices $x, y_1, \cdots, y_n$, and $z_1, \cdots, z_n$. The vertex x is adjacent to all other vertices; also, vertices y_i and z_i are adjacent for $i = 1, \cdots, n$. There are no other edges. Draw a diagram of the friendship graph F_4.

Problem 2

Find a vertex-bijection that specifies and isomorphism between the two graphs:



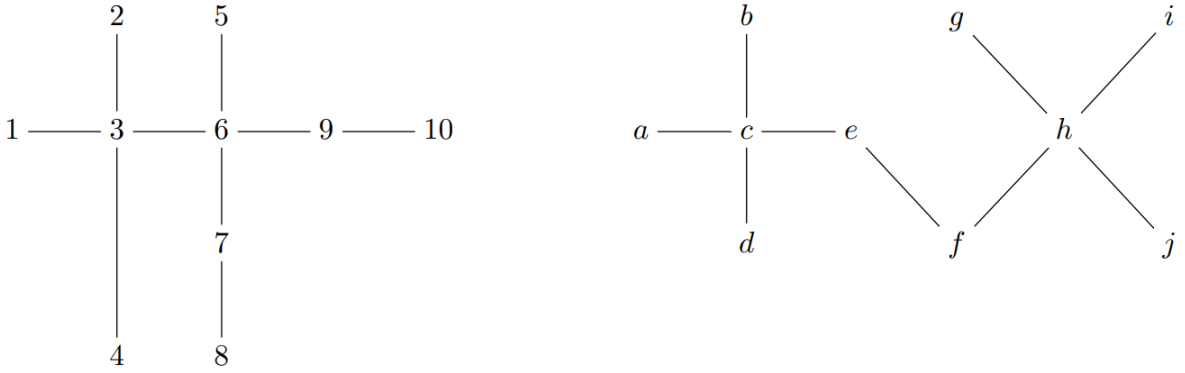
Problem 3

For the graph with the adjacency matrix A. (The vertices are in the order a, b, c, d)

$$A = \begin{pmatrix} 2 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{pmatrix}$$

Problem 4

Isomorphic? produce isomorphism. Non-isomorphic? Explain



Problem 5

A 20-vertex graph has 62-edges. Every vertex has degree 3 or 7. How many vertices have degree 3?

Problem 6

Either draw a 3-regular 7-vertix graph or prove that none exits.

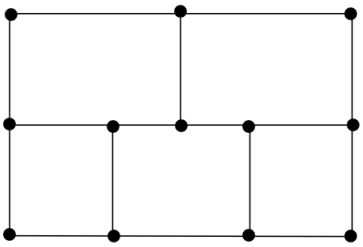
Problem 7

Determine whether or not the sequence $6, 5, 4, 4, 3, 2$ is graphical. If it is, draw a graph with that degree sequence.

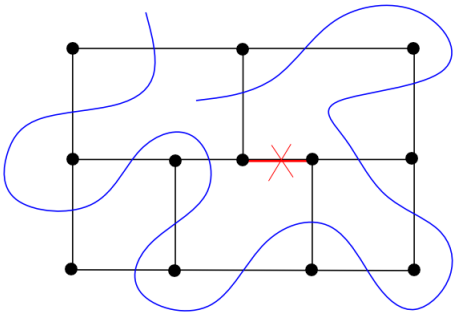
Problem 8

Consider the following blueprint for a house. Each box represents a room, and the lines connecting two dots denote

walls separating adjoining rooms (so there are 16 walls in all).



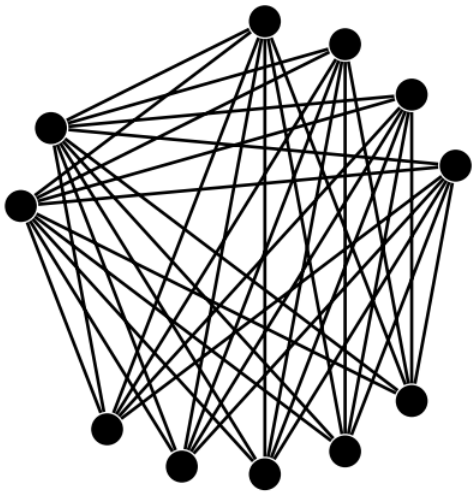
The problem is to draw a curve in the plane that passes through each wall exactly once. The curve may start at any point in the plane and end at any other point (the start and end points don't have to be the same). Here is an example curve that misses a wall

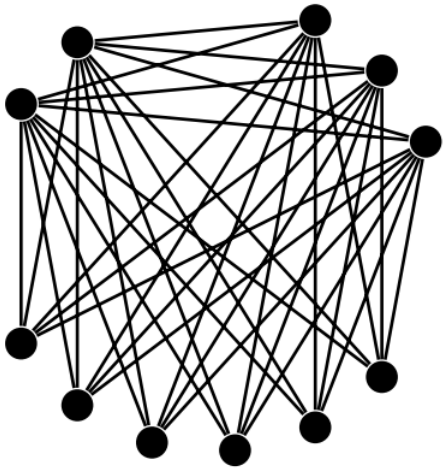


Also, one cannot *kill two walls with one pass*, meaning that you cannot go through a corner and claim that you covered two (or three) walls at the same time. Either find such a curve or show that none exists.

Problem 9

Below are diagrams of $K_{2,4,5}$ (left) and $K_{2,3,6}$ (right).





One of these graphs is Hamiltonian. Find a Hamiltonian cycle in that graph.