National University of Computer & Emerging Sciences, Karachi
MID-Term 2 Examination, Fall-2022, FAST School of Computing
Friday, November 04, 2022 11:30 am to 12:30 pm

Course Code: MT-3001	Course Name: Graph Theory
Instructor Names :	Dr. Nazish Kanwal, Mr. M Abdul Basit Khan
Student Roll No:	Section No:

Instructions:

1. Answer all questions on answer script . Credit will be awarded for correct content and clarity of presentation.
2. There are 3 questions and 2 pages.

Time: 60 minutes. Max Points : 30

Question 1: .... CLO 2 .... *12 points*

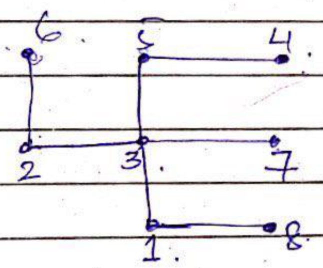
Attempt each part.

(a) $\boxed{\text{4 points}}$ Draw the labelled tree corresponds to the given prüfer sequence.

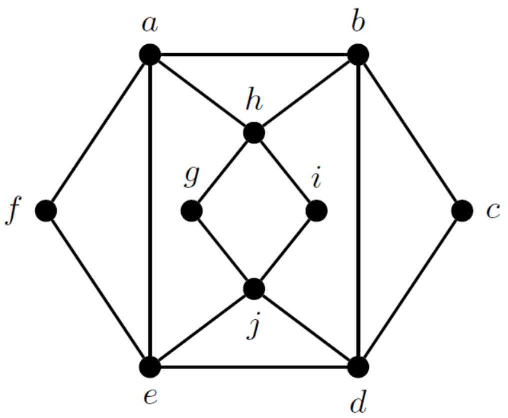
$$(5, 3, 2, 3, 3, 1)$$

Solution:

(5, 3, 2, 3, 3, 1)
(3, 2, 3, 3, 1, 4)
(2, 3, 3, 1, 4, 5)
(3, 3, 1, 4, 5, 6)
(3, 1, 4, 5, 6, 2)
(1, 4, 5, 6, 2, 7)
(4, 5, 6, 2, 7, 3)

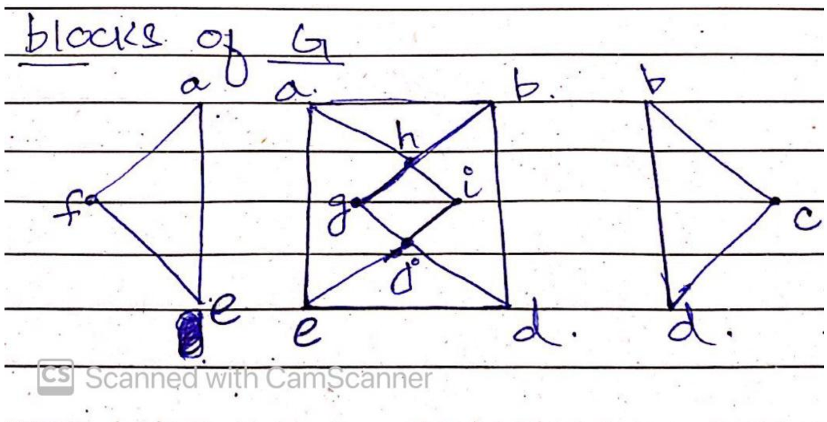


(b) Consider the following graph G, and answer each subparts.



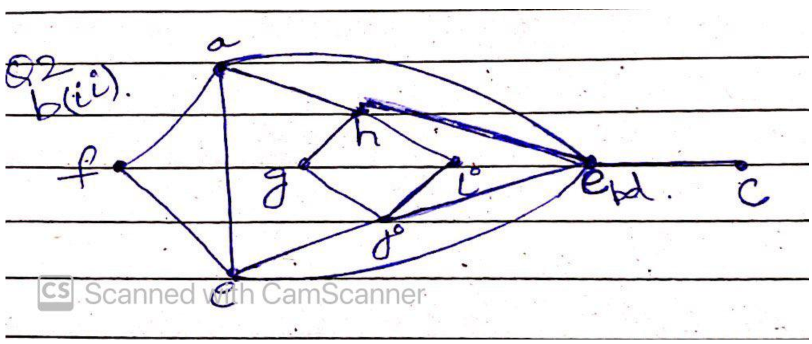
i. $\boxed{\text{2 points}}$ Determine the blocks of G.

Solution:



ii. $\boxed{\text{1 point}}$ Find G/e, the graph obtained by contracting the edge $e = bd$.

Solution:



iii. $\boxed{\text{2 points}}$ Find all the edge-disjoint paths from $h - e$.

Solution: hae(or $hafe$), $hije$(or $hgje$ or $hgjde$), $hbde$ (or $hbcde$ or $habcde$).

iv. $\boxed{\text{2 points}}$ Find all internally disjoint paths from $h - j$.

Solution: hij, hgj, ; $haej$(or $hafej$), $hbdj$(or $hbdcj$).

v. $\boxed{\text{1 point}}$ Find a vertex-separating set for vertex h and j.

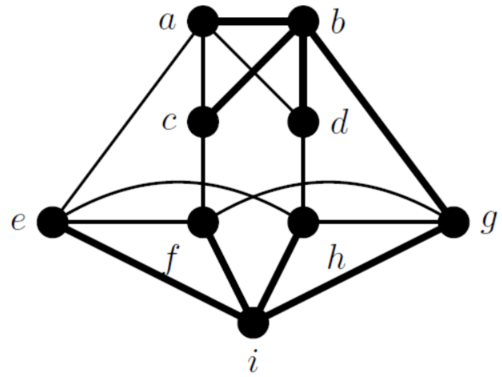
Solution: $\{a, b, i, g\}$.

Question 2: .... CLO 2 .... *11 points*

(a) $\boxed{\text{4 points}}$ Prove that, an edge e is a bridge of G if and only if there exist vertices x and y such that e is on every $x - y$ path.

Solution: Suppose that an edge $e = ab$ is a bridge of G, then $G - e$ is disconnected. This implies $G - e$ must have atleast two component. Let x and y be vertices in different component of $G - e$. Since G is connected so x-y path exists in G.
Conversely, let e be an arbitrary edge and there exists vertices x and y in G such that e is on every x-y path, then none of these path exists in $G - e$, this implies e disconnect G. Hence e is a bridge for G.

(b) Consider the following graph G.



i. $\boxed{\text{1 point}}$ State Whitney's Theorem.

Solution: For any graph G, $\kappa(G) \le \kappa^{'}(G) \le \delta(G)$.

ii. $\boxed{\text{4 points}}$ For the graph G, prove the Whitney's theorem.

Solution: We need to prove that G satisfy the inequality: $\kappa(G) \le \kappa^{'}(G) \le \delta(G)$. Since $\delta(G) = 3$ and vertex-cut set of G is $\{a, b, f\}$ and $\{a, b, h\}$, so $\kappa(G) = 3$ and edge-cut set of G is $\{ac, bc, cf\}$ and $\{ad, bd, hd\}$ so $\kappa^{'}(G) = 3$. Hence

$$\kappa(G) \le \kappa^{'}(G) \le \delta(G) \Rightarrow 3 \le 3 \le 3.$$

iii. $\boxed{\text{1 point}}$ Find vertex-cut set for G.

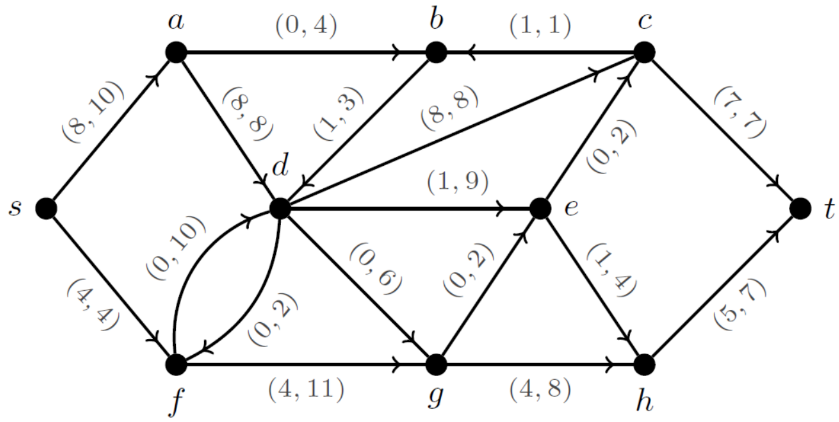
Solution: $\{a, b, f\}$ and $\{a, b, h\}$.

iv. $\boxed{\text{1 point}}$ Find edge-cut set for G.

Solution: $\{ac, bc, cf\}$ and $\{ad, bd, hd\}$.

Question 3: ....CLO 2 .... *7 points*

(a) For the problem below:



i. $\boxed{\text{4 points}}$ Use Augmenting Flow Algorithm to maximize the flow.

Solution:

Augmenting Flow Algorithm.

Step #01

(i) a(s+, 2)

(ii) b(a+, 2)

(iii) d(b+, 2), c(b, 1)
choose d.

(iv) f(d+, 2), e(d+, 2), g(d+, 2)
choose e

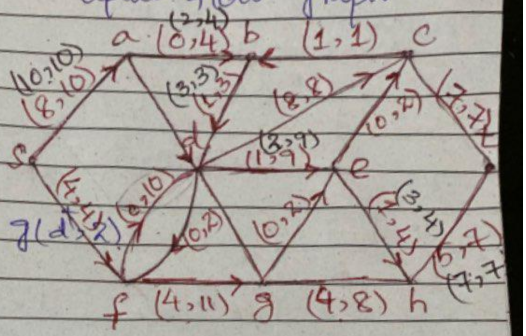
(v) h(e+, 2).

(vi) t(h+, 2)

σ(t) = 2 . , Path: sabdeht

So flow increased by 2 unit on edges.
sa, ab, bd, de, eh, ht.

update flow graph.



Step #02

Neither vertex 'a' nor vertex 'b' possible to label.

So stop here and

Max flow = flow at ct + flow at ht
= 7 + 7 = 14 = In flow.

ii. 1 point Use Min-Cut method to find a minimum cut.

Solution: In the last step of Augmenting flow algorithm, it is not possible to label any vertex. So, $P = \{s\}$ and $\bar{P} = \{a, b, c, d, e, f, g, h, t\}$ and the edges involve in minimum cut are sa and sb. Therefore, capacity of minimum cut is $C = c(sa) + c(sb) = 10 + 4 = 14$, and equal to value of maximum flow.

(b) 2 points The algorithms given below are written for connected graphs as input. Determine the output of each of the algorithms below if the input was a disconnected graph.

(a) Kruskal's Algorithm.
(b) Prim's Algorithm
(c) Depth-First Search
(d) Breadth-First Search

Solution: In above mentioned algorithms the output of the graphs are disconnected graphs of atleast two components in which each component is itself a tree. OR output is the forest.