National University of Computer & Emerging Sciences, Karachi
Mid-Term 1 Examination, Fall-2022, FAST School of Computing
Friday, September 30, 2022 11:30am to 12:30pm

Course Code: MT-3001	Course Name: Graph Theory
Instructor Names :	Dr. Nazish Kanwal, Mr. M Abdul Basit Khan
Student Roll No:	Section No:

Instructions:

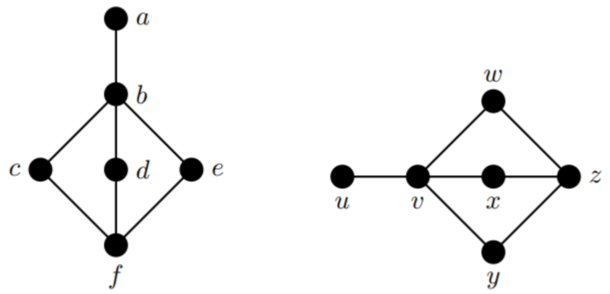
1. Answer all questions on answer script . Credit will be awarded for correct content and clarity of presentation.
2. There are 3 questions and 2 pages.

Time: 60 minutes. Max Points : 30

Question 1: .... CLO 1 .... *10 points*

Attempt each part.

(a) $\boxed{\text{3 points}}$ The following two graphs G and H are isomorphic. List all the isomorphisms from G to H.



(a) G (b) H

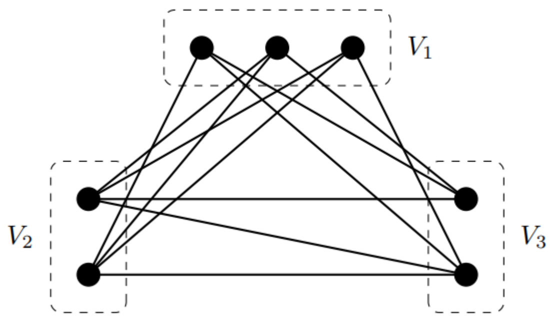
Solution:

Degrees of Vertices
$deg(a) = 1, deg(b) = 3, deg(c) = 2, deg(d) = 2, deg(e) = 2, deg(f) = 3$
$deg(u) = 1, deg(v) = 3, deg(w) = 2, deg(x) = 2, deg(y) = 2, deg(z) = 3$

$V(G)$	a	b	c	d	e	f
$V(H)$	u	v	w	x	y	z

$E(G)$	ab	bc	cf	fe	be	bd	df
$E(H)$	uv	vw	wz	yz	vy	vx	xz

(b) Consider the following graph G, and answer each subparts.



i. $\boxed{\text{1/2 point}}$ Is the graph G is k-partite, if so find k.

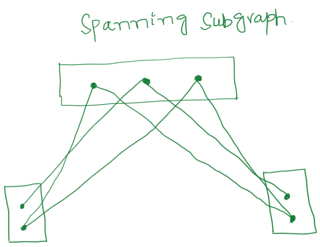
Solution: yes. $k = 3$.

ii. $\boxed{\text{2 points}}$ Find the adjacency matrix of G.

Solution:

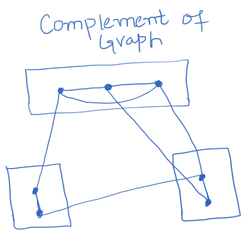
$$\begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 1 & 0 & 0 \end{bmatrix}$$

iii. $\boxed{\text{2 points}}$ Find any spanning subgraph of G.



Solution:

iv. $\boxed{\text{2 points}}$ Find the complement of G.



Solution:

v. $\boxed{\text{1/2 point}}$ How many edges needed to make the graph G complete.

Solution: 4 edges are required.

Question 2: . . . . CLO 1 . . . . *10 points*

(a) $\boxed{\text{4 points}}$ Prove that a complete graph K_n has $\frac{n(n-1)}{2}$ edges.

Solution:

Proof: Argue by induction on n. If $n = 1$ then K_1 is just a single vertex and has $0 = \frac{1(0)}{2}$ edges.

Suppose for some $n \geq 1$ that K_n has $\dfrac{n(n-1)}{2}$ edges. We can form K_{n+1} by adding a new vertex v to K_n and adjoining v to all the vertices from K_n. Thus K_{n+1} has n more edges than K_n and so by the induction hypothesis has

$$n + \frac{n(n-1)}{2} = \frac{2n + n(n-1)}{2} = \frac{n(2+n-1)}{2} = \frac{n(n+1)}{2}$$

edges.

Thus by induction we know K_n has $\dfrac{n(n-1)}{2}$ edges for all $n \geq 1$.

(b) $\boxed{\text{6 points}}$ Determine if the sequence $(4, 4, 3, 2, 2, 1)$ is graphical. If yes construct a graph representing it.

Solution: The following is what we have obtained by applying "Havel Hakimi Theorem.

4	4	3	2	2	1
	3	2	1	1	1
		1	0	0	1
		1	1	0	0

It is clear that $(1, 1, 0, 0)$ is representable by the graph G_1 shown in Fig. 2.5.3.

Fig. 2.5.3: G_1

A graph representing $(3, 2, 1, 1, 1)$, shown in Fig. 2.5.4, can then be obtained using the method mentioned above.

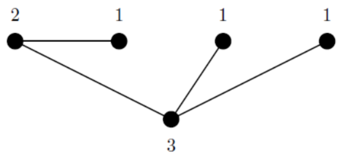
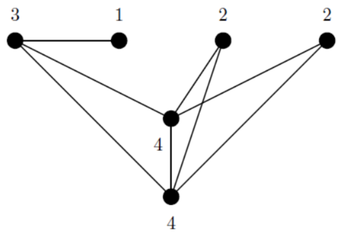


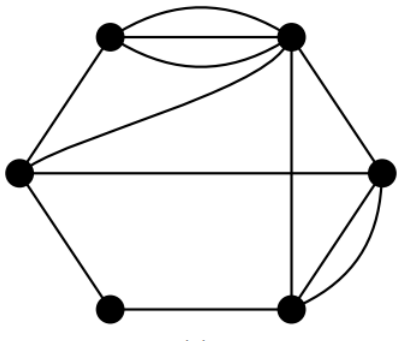
Fig. 2.5.4: G_2

Repeating this procedure, we finally obtain the following graph G representing the sequence $(4, 4, 3, 2, 2, 1)$



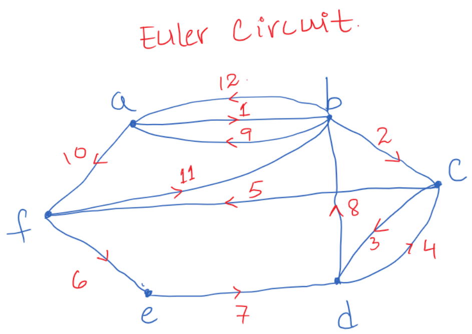
Question 3: .... CLO 1 .... *10 points*

Answer each part for the following graph.



(a) $\boxed{\text{4 points}}$ Find an Eulerian circuit using Fluery's Algorithm. Also draw labeled Eulerian graph.

Solution: Circuit is: $abcdcfedbafba$.



(b) $\boxed{\text{3 points}}$ Find the closure of the given graph.

Solution: The number of vertices $n = 6$ and the pair of non-adjacent vertices are:
a & c, a & d, a & e, b & e, c & e, and d & f.

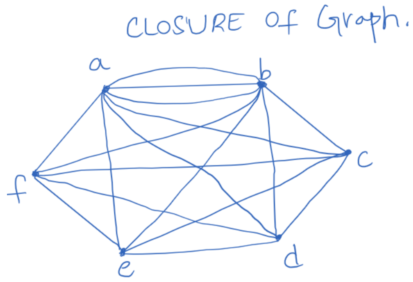
The degree of vertices are given below:

a	b	c	d	e	f
4	6	4	4	2	4

Also,

$$deg(a) + deg(c) = 8 > 6 \quad deg(a) + deg(d) = 8 > 6 \quad deg(a) + deg(e) = 6 \geq 6$$
$$deg(b) + deg(e) = 8 > 6 \quad deg(c) + deg(e) = 6 \geq 6 \quad deg(d) + def(f) = 8 > 6$$

So, we can add the edges b/w the non-adjacent pair vertices of mentioned above, the closure of given graph is of the form:



(c) $\boxed{\text{3 points}}$ Use Ore's theorem to check given graph has a Hamiltonian cycle.

Solution: From part b we can see that, all pair of non-adjacent vertices satisfy the condition of Ore's theorem:

$$\begin{array}{cc} deg(a) + deg(c) = 8 > 6 & deg(a) + deg(d) = 8 > 6 \\ deg(a) + deg(e) = 6 \geq 6 & deg(b) + deg(e) = 8 > 6 \\ deg(c) + deg(e) = 6 \geq 6 & deg(d) + def(f) = 8 > 6 \end{array}$$

Thus, graph has a Hamiltonian cycle.