National University of Computer & Emerging Sciences, Karachi

GRAPH THEORY- Assignment#03

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Submission Date - December 05, 2022

Question 1.[2 marks]

Recall that $\delta(G)$ denotes the minimum degree of G. Prove that if G is a graph with $2n$ vertices and $\delta \geq n$ then G has a perfect matching.

Question 2.[4 marks]

Consider the 3×5 Latin rectangle $L = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 5 & 1 & 2 & 3 & 4 \\ 4 & 5 & 1 & 2 & 3 \end{pmatrix}$.

Define a bipartite graph G with bipartition (X, Y) associated with L as follows:

(i) $X = \{1, 2, 3, 4, 5\}$,
(ii) $Y = \{C_1, C_2, C_3, C_4, C_5\}$, where C_i is the ith column of L,
(iii) 'i' in X is adjacent to 'C_j' in Y if and only if 'i' does not appear in 'C_j'.

(a) Draw the diagram of G.
(b) What is the degree of each vertex in X?
(c) What is the degree of each vertex in Y?
(d) Is G 2-regular?
(e) Does G contain a perfect matching?
(f) Display a perfect matching in G if your answer to (e) is 'yes'.

Question 3.[2 marks]

Consider the following graph G:

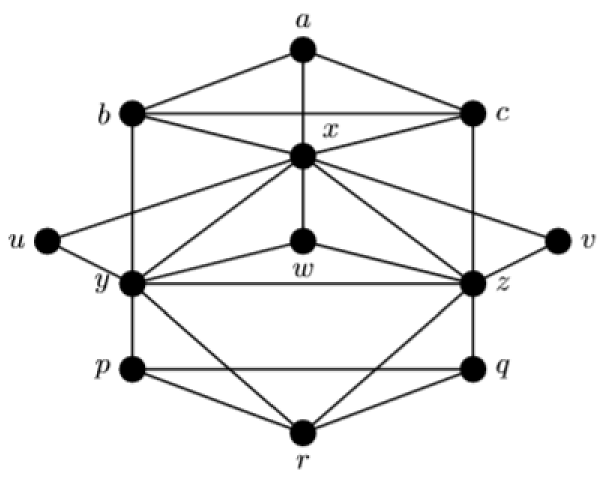


Fig. 7.5.10

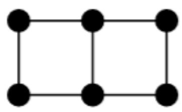
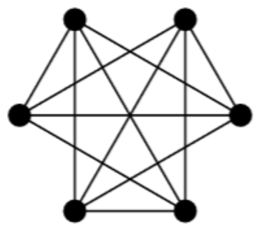
(i) Does G have a perfect matching?
(ii) What is the size of a maximum matching in G?

Question 4.[4 marks]

Determine the stable matchings resulting from the Proposal Algorithm run with men proposing and with women proposing, given the preference lists below.

Men $\{u, v, w, x, y, z\}$	**Women** $\{a, b, c, d, e, f\}$
$u: a > b > d > c > f > e$	$a: z > x > y > u > v > w$
$v: a > b > c > f > e > d$	$b: y > z > w > x > v > u$
$w: c > b > d > a > f > e$	$c: v > x > w > y > u > z$
$x: c > a > d > b > e > f$	$d: w > y > u > x > z > v$
$y: c > d > a > b > f > e$	$e: u > v > x > w > y > z$
$z: d > e > f > c > b > a$	$f: u > w > x > v > z > y$

Question 5.[3 marks]
For each of the following graphs, find its chromatic number, and equitable coloring.

Question 6.[5 marks]

Let G be the graph shown in Fig. 8.3.7.

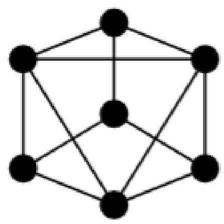


Fig. 8.3.7

(i) Find $\chi(G)$.
(ii) Verify that $\chi(G - e) = \chi(G) - 1$ for each edge e in G.

(iii) Find $\chi^{'}(G)$, and verify vizing theorem for graph G.

(iv) Draw the line graph $L(G)$.

(v) Verify that $\chi^{'}(G) = \chi\left(L(G)\right)$.

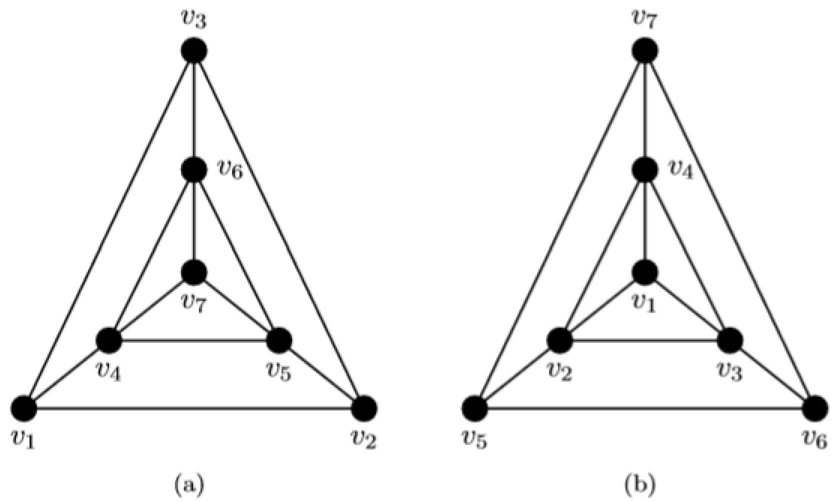
Question 7.[3 marks]

A school is preparing a timetable for exams in 7 different subjects, labeled A to G. It is understood that if there is a pupil taking two of these subjects, their exams must be held in different time slots. The table below shows (by crosses) the pairs of subjects which are taken by at least one pupil in common. The school wants to find the minimum number of time slots necessary and also to allocate subjects to the time slots accordingly. Interpreting this problem as a vertex-coloring problem, find the minimum number of time slots needed and a suitable time allocation of the subjects.

	A	B	C	D	E	F	G
A		x	x	x		x	
B	x		x			x	x
C	x	x		x			x
D	x		x		x		
E				x		x	x
F	x	x			x		x
G		x	x		x	x	

Question 8.[2 marks]

Let G be the graph shown in Figure below with two different ways of ordering its vertices. Apply First-Fit Algorithm (greedy coloring algorithm) to color G and find the number of colors produced in each case.



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