



# National University

Of Computer & Emerging Sciences, Karachi-Campus



## GRAPH THEORY QUIZ-1 [Max Marks:15]

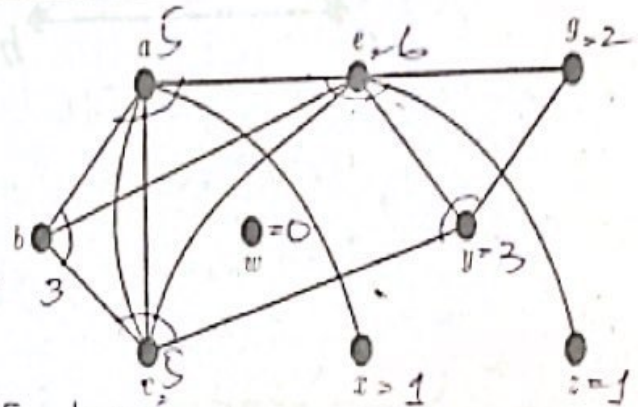
Instructor: Dr. Nazish Kanwal

Sections: 5B, 5D & 5F

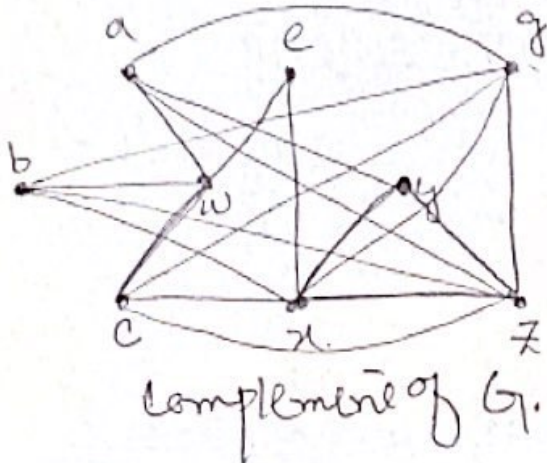
Date: September 23<sup>rd</sup>, 2022.

Q.1 [7marks] In the following multigraph  $G$ , find

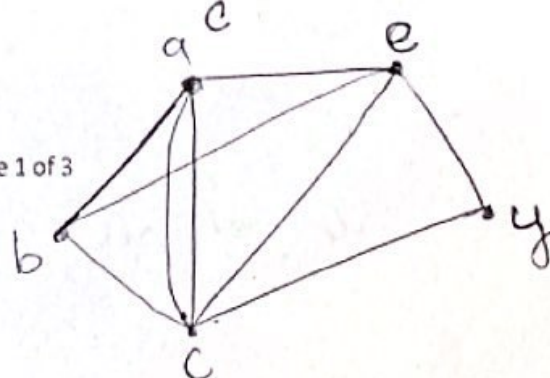
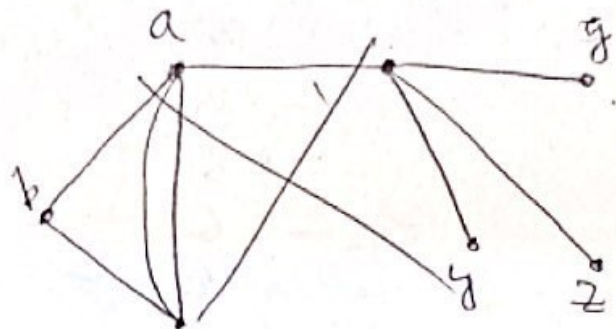
- the size of  $G$ ,  $= (9, 26)$
- the degree of each vertex,  $\begin{matrix} a=5 \\ b=3 \\ c=5 \\ d=6 \\ e=1 \\ f=1 \\ g=2 \\ h=3 \\ i=0 \end{matrix}$
- the sum  $\sum_{v \in V(G)} d(v) = 26$
- the number of odd vertices,  $= 6$
- the  $|E(G)| = 26$
- the complement of  $G$ .
- any spanning subgraph of  $G$ .



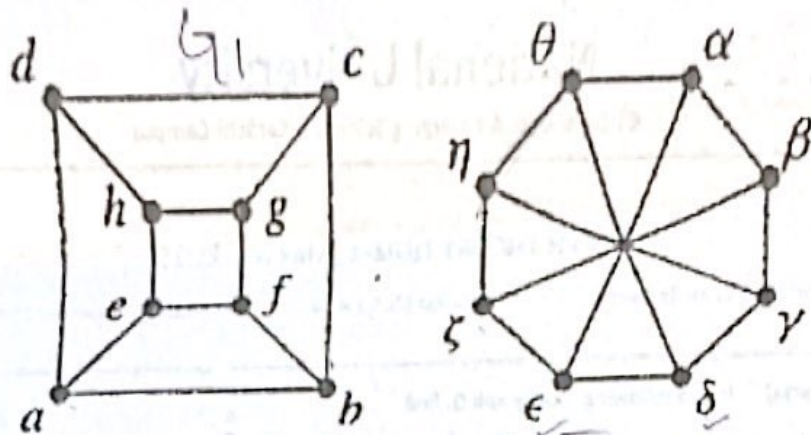
ii)  $\deg(a) = 5, \deg(b) = 3, \deg(c) = 5, \deg(d) = 6, \deg(e) = 1, \deg(f) = 1, \deg(g) = 2, \deg(h) = 3, \deg(i) = 0$



Spanning subgraph of  $G$  (one of the possible)



Q.2 [4 marks] Determine if the following pair of graphs are isomorphic. If so, give the vertex bijection map, if not, explain why?



degree of each vertex of both graphs is 3  
# of vertices of both graphs are 8 & edges are 12

$E(G_1) = \{ae, ab, ad, bc, bf, cg, cd, dh, he, ef, fg, gh\}$

$E(G_2) = \{\alpha\beta, \alpha\theta, \alpha\epsilon, \beta\gamma, \beta\theta, \beta\delta, \gamma\delta, \gamma\theta, \delta\epsilon, \delta\theta, \epsilon\zeta, \epsilon\theta, \zeta\eta, \zeta\theta, \eta\theta\}$

$a \rightarrow \beta$   
 $b \rightarrow \delta$   
 $c \rightarrow \eta$   
 $d \rightarrow \zeta$   
 $e \rightarrow \alpha$   
 $f \rightarrow \delta$   
 $g \rightarrow \theta$   
 $h \rightarrow \epsilon$

Edge.

$ae \rightarrow \alpha\beta$   
 $ab \rightarrow \beta\delta$   
 $ad \rightarrow \beta\zeta$   
 $bc \rightarrow \eta\gamma$   
 $bf \rightarrow \delta\delta$   
 $cg \rightarrow \eta\theta$   
 $cd \rightarrow \eta\zeta$   
 $dh \rightarrow \zeta\epsilon$   
 $he \rightarrow \alpha\epsilon$   
 $ef \rightarrow \alpha\delta \quad \times$   
 $fg \rightarrow \delta\theta$   
 $gh \rightarrow \theta\epsilon \quad \times$

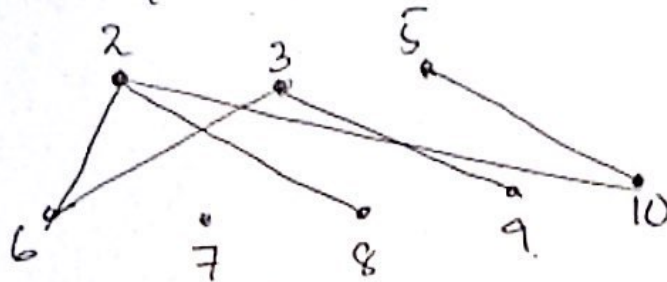
Not isomorphic.



Q.3 [4 marks] A graph  $G$  is defined as follows:  $V(G) = X \cup Y$ , where  $X = \{2, 3, 5\}$  and  $Y = \{6, 7, 8, 9, 10\}$  and  $E(G) = \{xy \mid x \in X, y \in Y \text{ and } y \text{ is divisible by } x\}$ . Draw the graph, is  $G$  bipartite?

$$V(G) = \{2, 3, 5, 6, 7, 8, 9, 10\}$$

$$E(G) = \{26, 28, 210, 36, 39, 510\}$$



~~is~~ Bipartite —

b/c

$$V(G) = \{2, 3, 5\} \cup \{6, 8, 9, 10\} \cup \{7\}$$

and we can add 7 into any set.

$$\text{so } V(G) = \{2, 3, 5, 7\} \cup \{6, 8, 9, 10\}$$

OR

$$V(G) = \{2, 3, 5\} \cup \{6, 7, 8, 9, 10\}$$