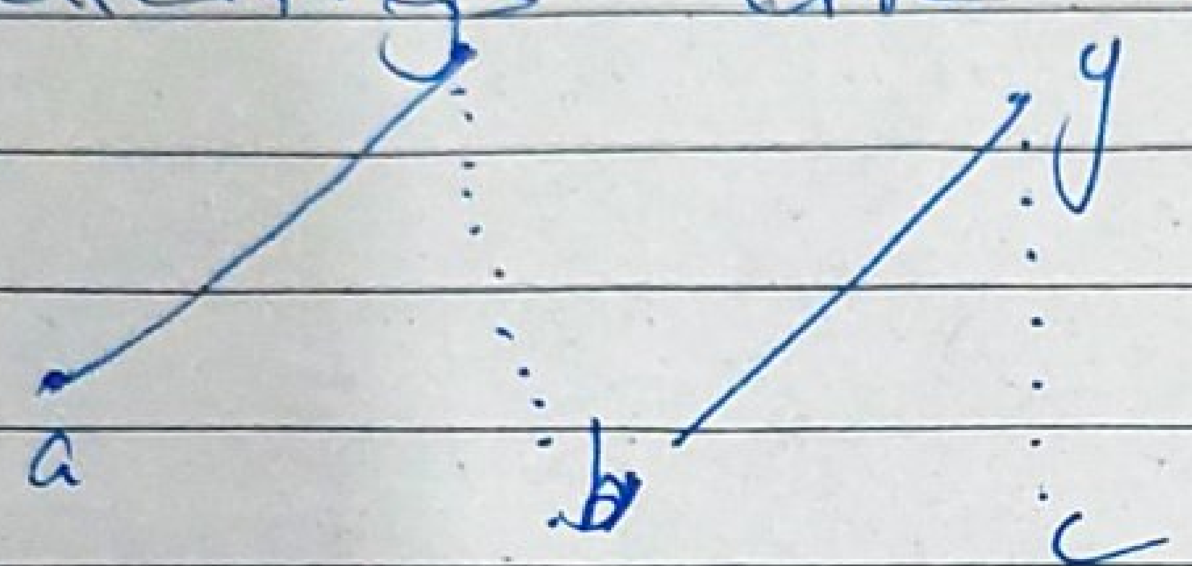
Matching and Factors

Date:

$e_1 = (V, E)$

- Matching $M \subseteq E$ such that no two edges in M share an endpoints
- Matchings are unique



$M = \{xa, yb\}$

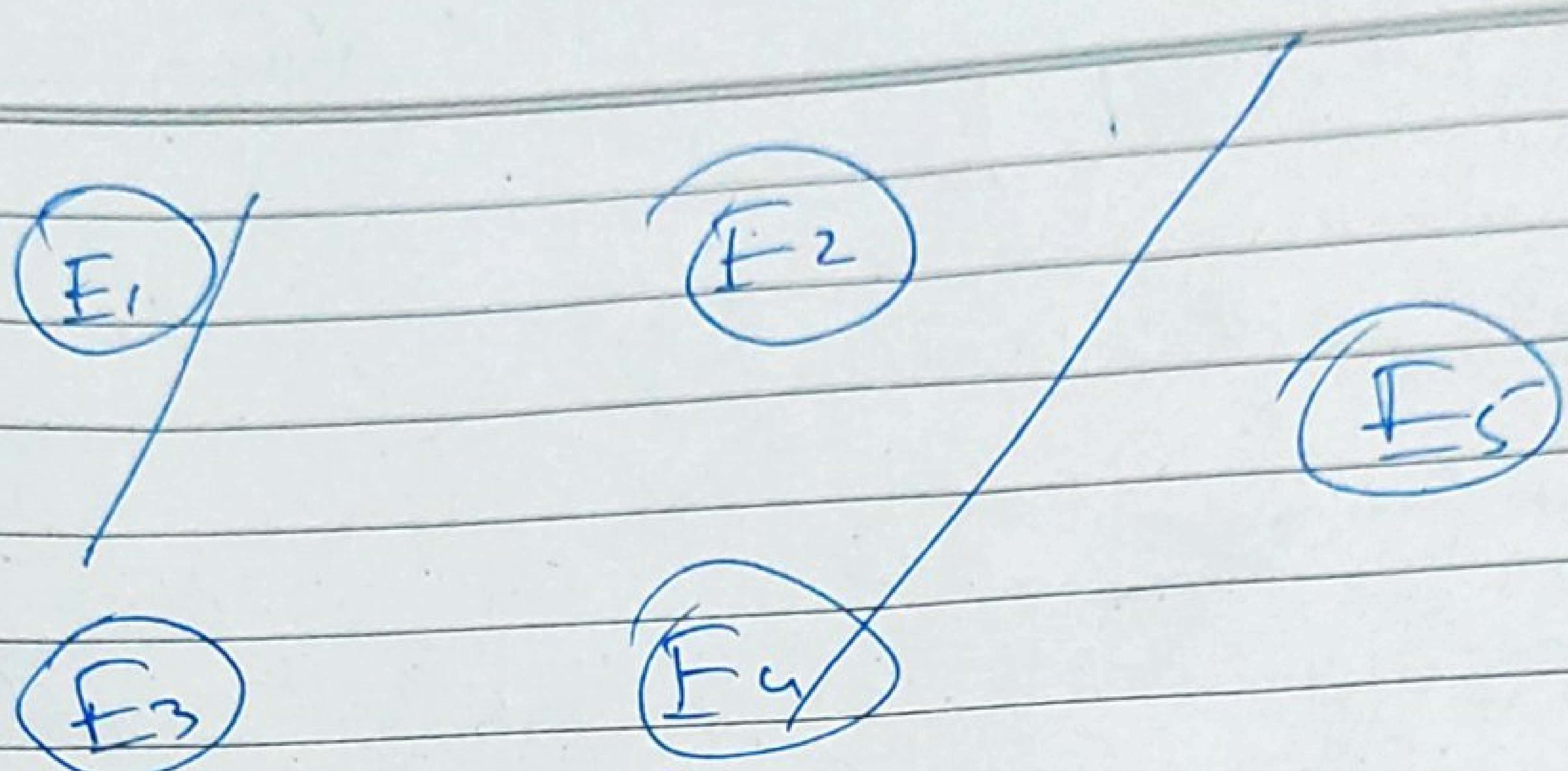
$M_2 = \{xb, yc\}$

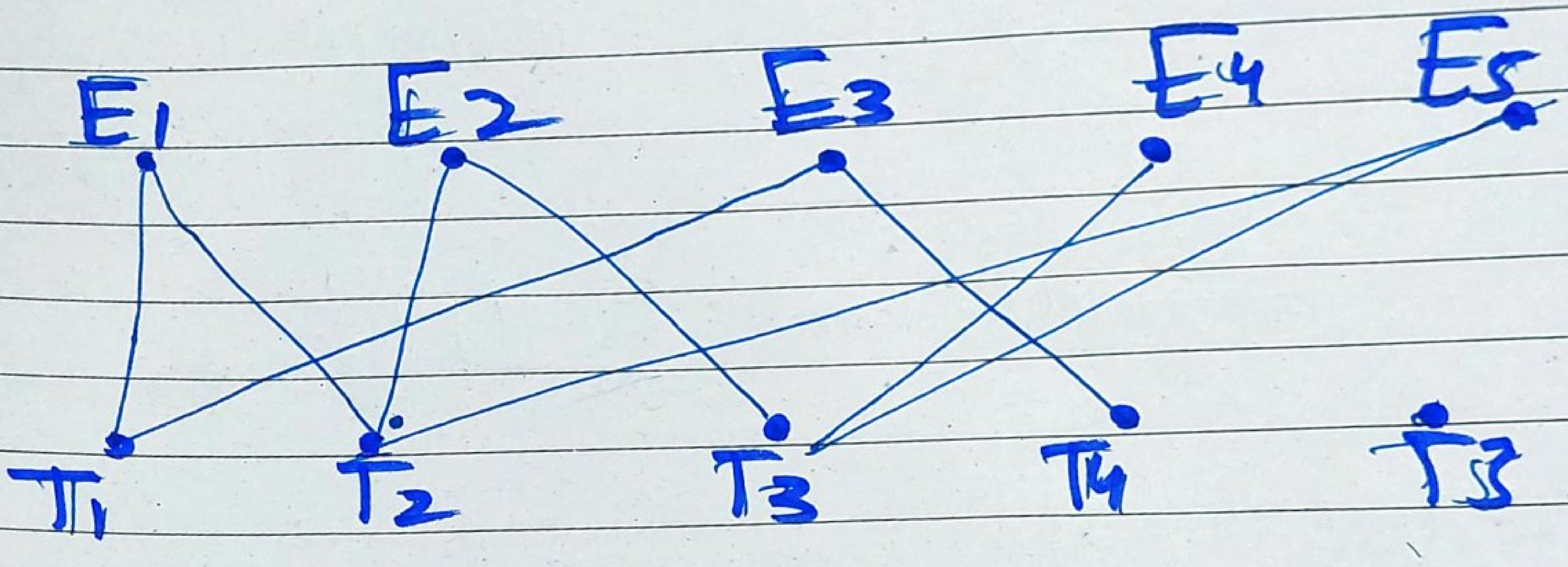
$M_3 = \{xa, yc\}$

Example:

E	T
E_1	T_1, T_2
E_2	T_3, T_2
E_3	T_4, T_1
E_4	T_3
E_5	T_3, T_2

Date:





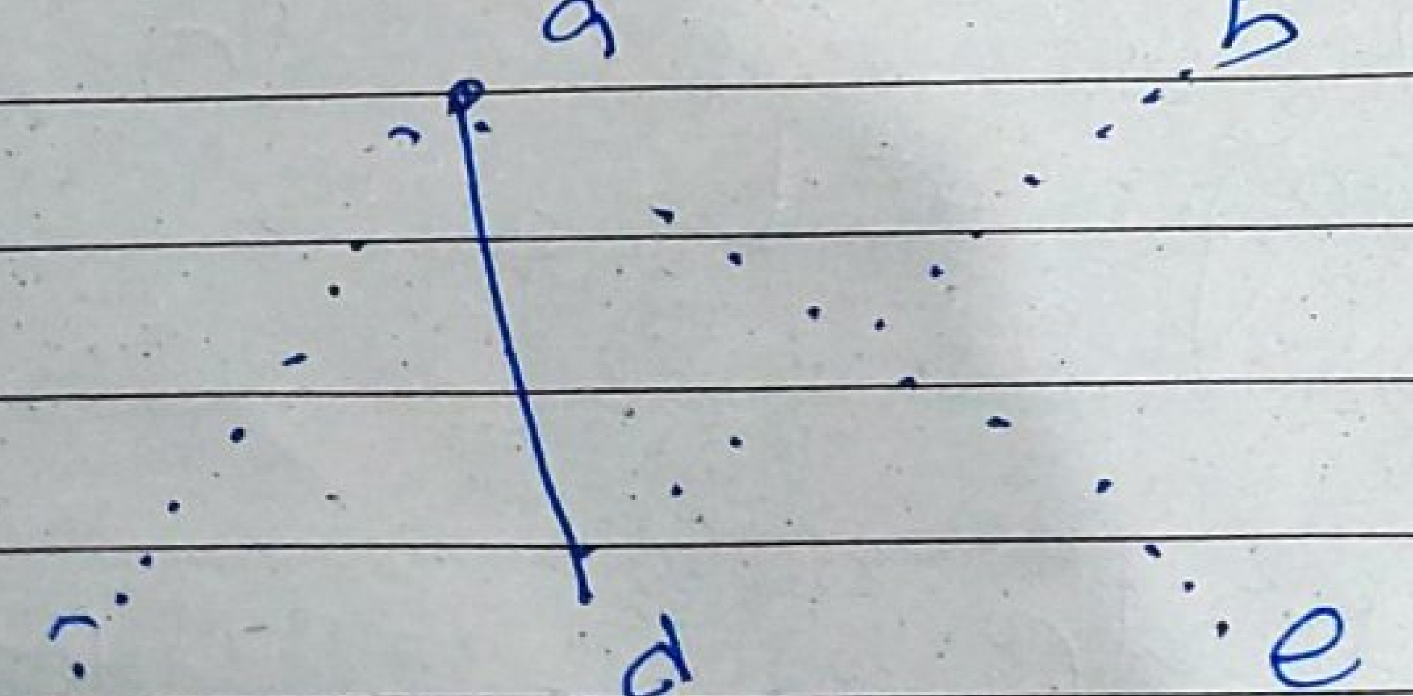
Match = $\{E_1T_2, E_2T_3, ~~E~~\ E_3, T_4\}$ ①
$= 3$
$= \{E_1T_1, E_2T_2, E_3, T_4\}, E_5, T_3\}$ ②

- **Unsaturated** = Edges/paths not in Matching set
- **Saturated**: Edges|path in matching set

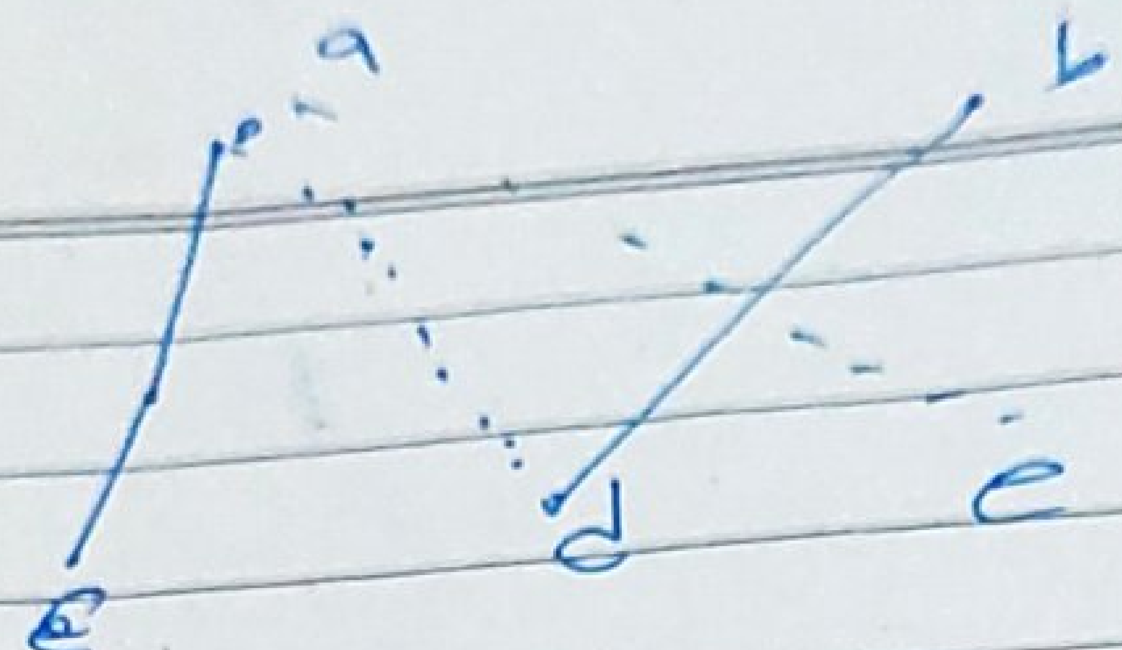
- **Size** = |Match|
  = 3 (in this case (1))
  = 4 (case 2)
- If no more edges can be added then matching is not maximal

Maximum ~~Maximal~~ = max{ |M| }

- **Perfect** → Matching that all edges saturated (included)
- **X-Matching**: The size of match is X.



Maximal = { ~~ab~~ ad } → Maximum = { ~~a~~ bd, ae }
= 2



Maximum and Maximal

Neighbour:

$S = \{e, c\}$

$N(S) = \{a\}$

Fact: Hall's theorem

- $G = (X \cup Y, E)$

saturated set ↲ ↳ unsaturated set

if $S \subseteq X$

and X-Match iff $|S| \leq |N(S)|$

- 5.5 - theorem

Date:

Def:

- $G(V,E)$ contains a path then

1) M-alternating (one-saturated, one-unsaturated

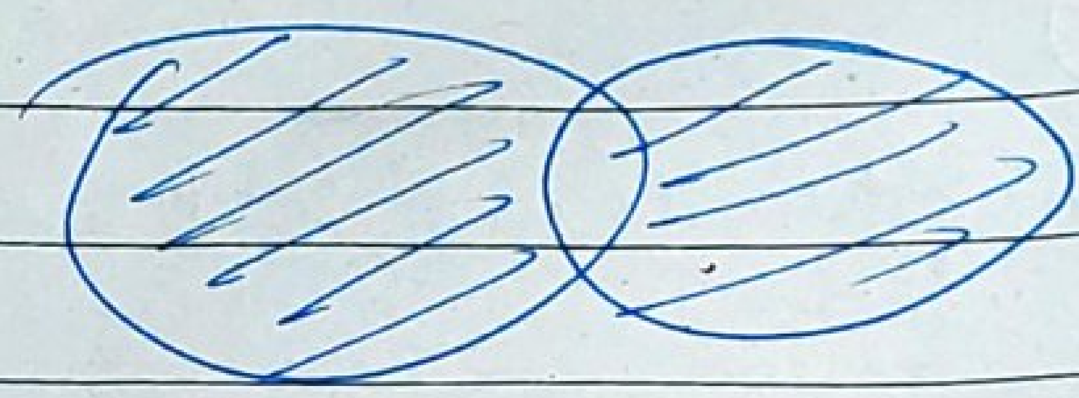
2) M-augmented if

   (a) M-alternating (Y cac)

   (b) first and end vertix is.. an unsaturated

Symmetric Difference

$A \Delta B$



$(A-B) \cup (B-A)$

Examples

Date:

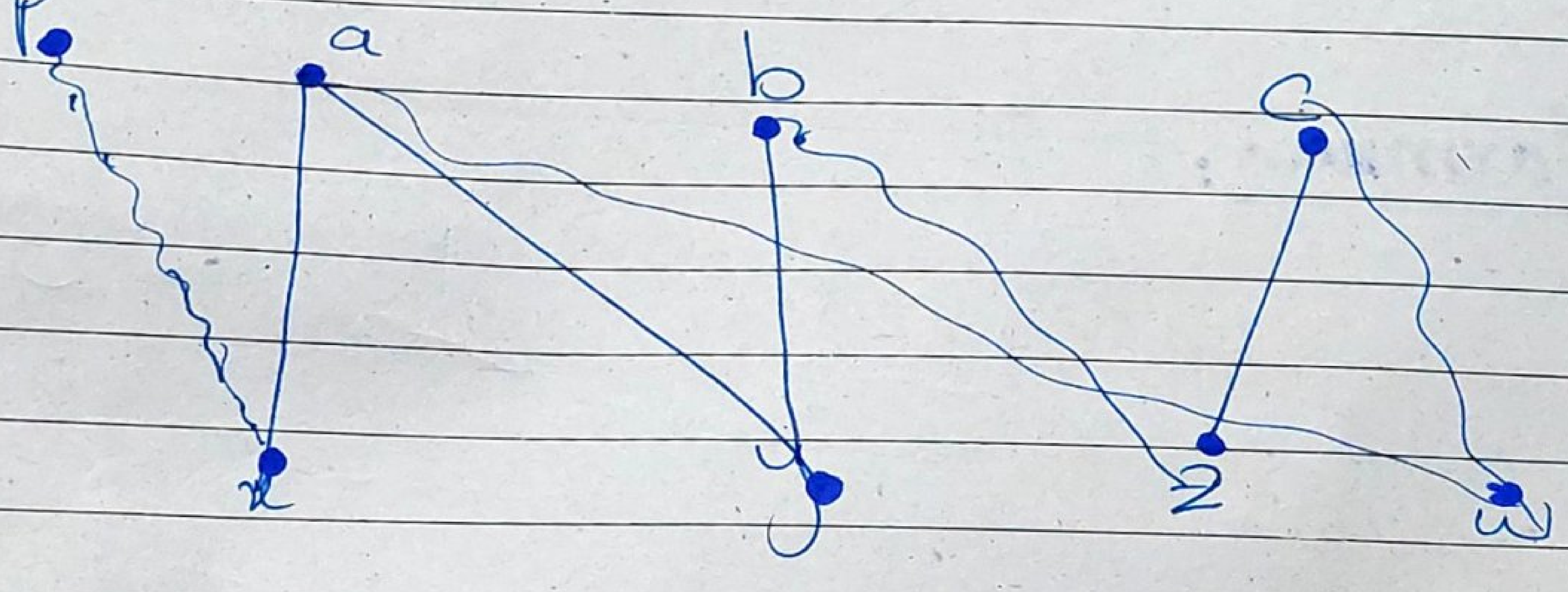
Ex 5.4

M-~~adjancy~~ alternating Path

$V_1 V_2 \quad V_3 V_4 \quad V_5 V_6$

M-augmenting Path

V_1 Z $V_2 V_3 V_4 V_5 V_6$



Matching set = M = { ax, by, cz } → Satrated

M-alternating path

unsatrated   unsatrated

px ay bz cw

saturate   satrated

Date:

M-augment Path :

px. ay bz cw

alternating + starting and ending unsaturated

Maximal : all matching included ie no more edges can be included

Maximum :

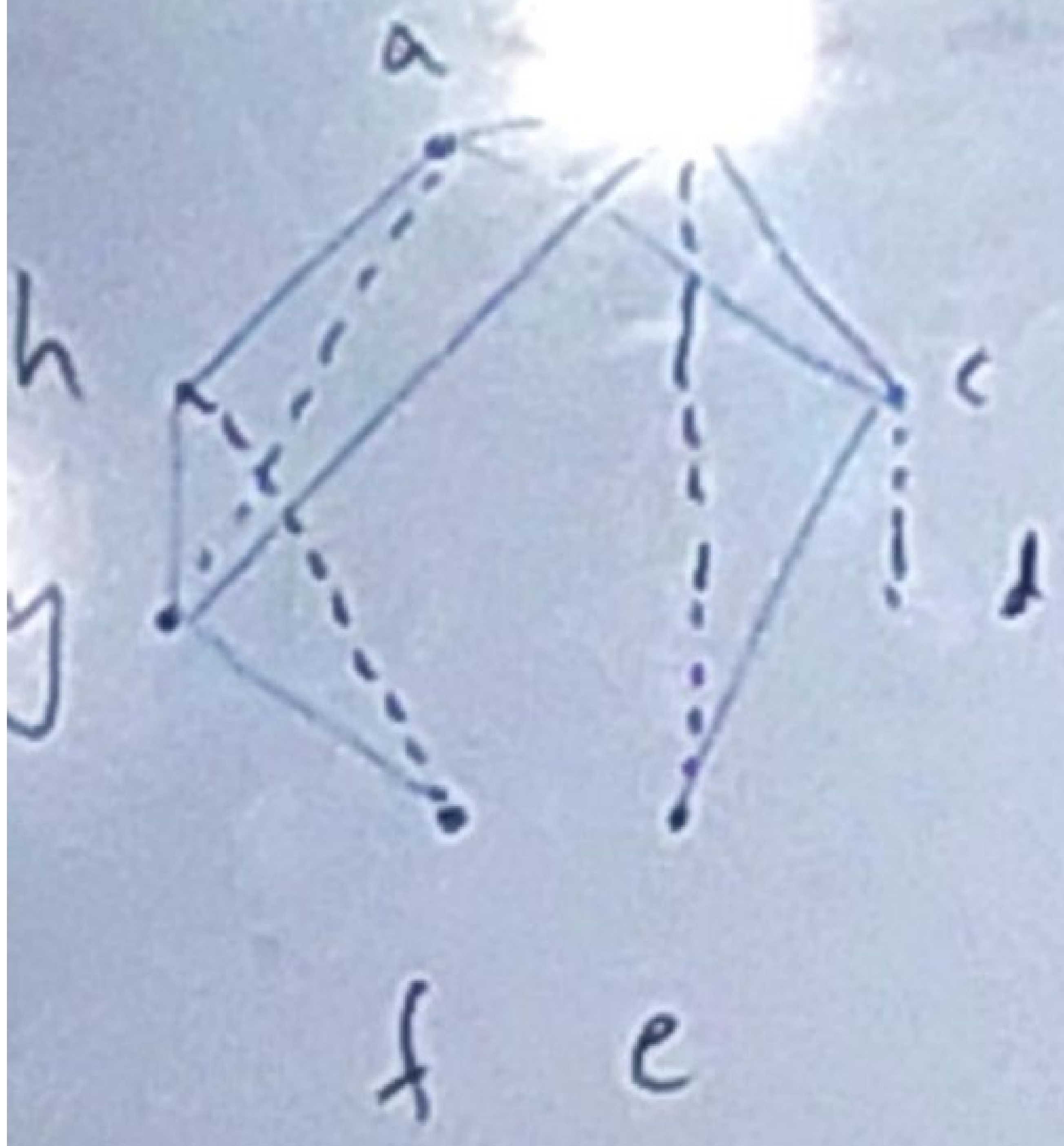
length of maximal matching set

Symmetric Difference

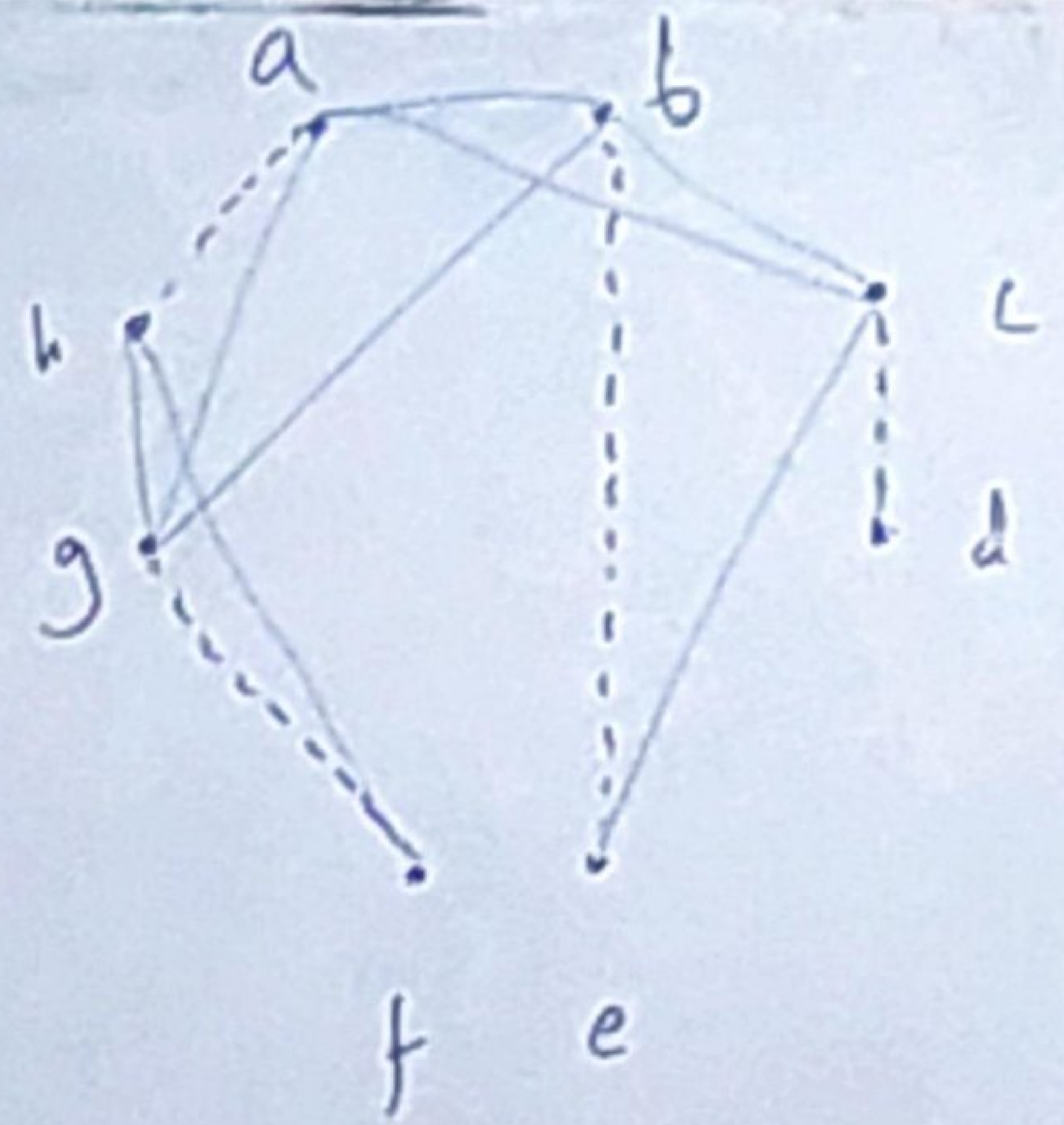
$A \Delta B = (A - B) \cup (B - A)$

$= (A - B) \cup (B - A)$

Symmetric Difference may or may not be matching

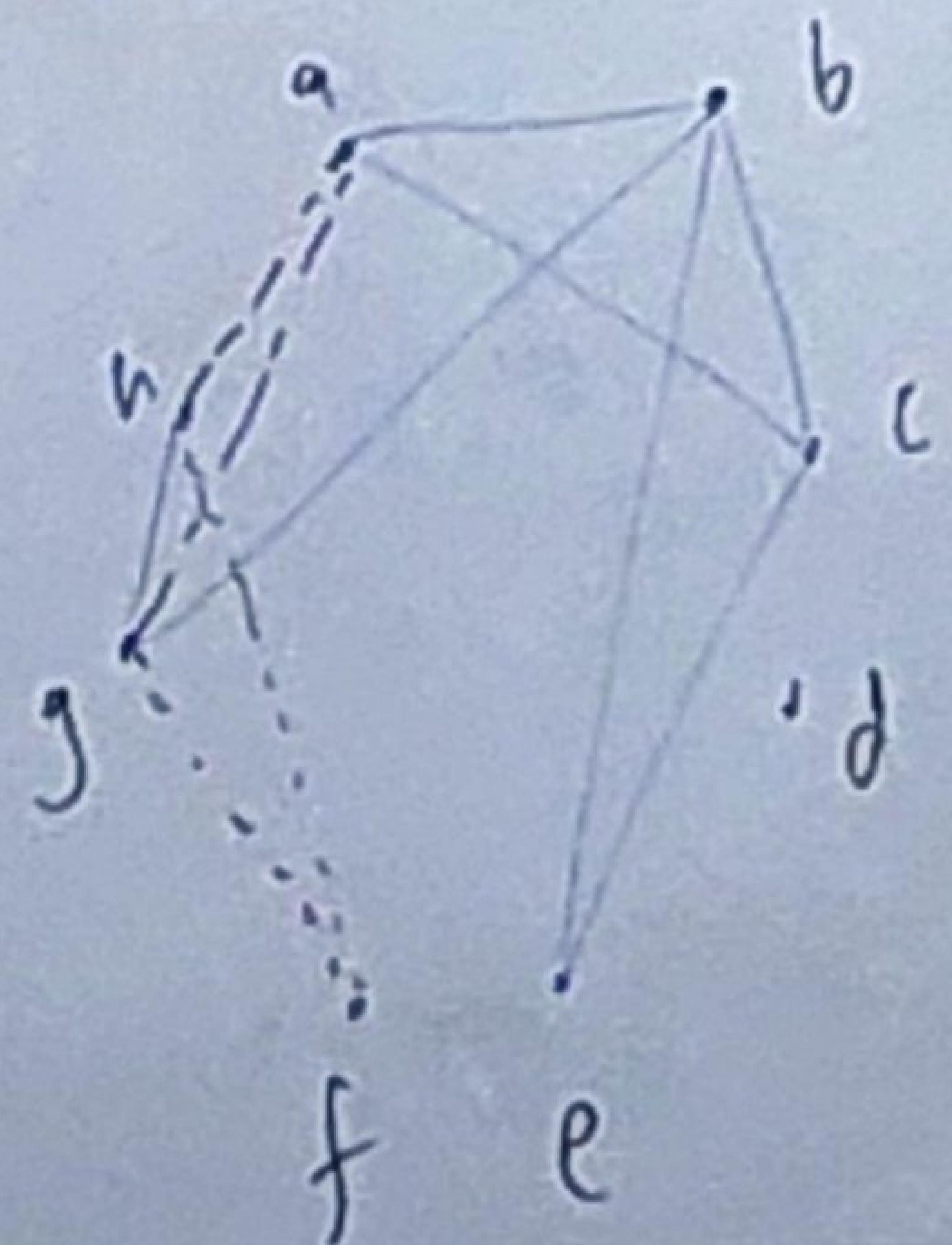


M



M'

$M \triangle M'$



Example

Date:

Fact Let M_i' and M two distinct matching in a graph then each component of $M \Delta M'$ is either M_i-alternating path or M_i alternating cycle

Proof:

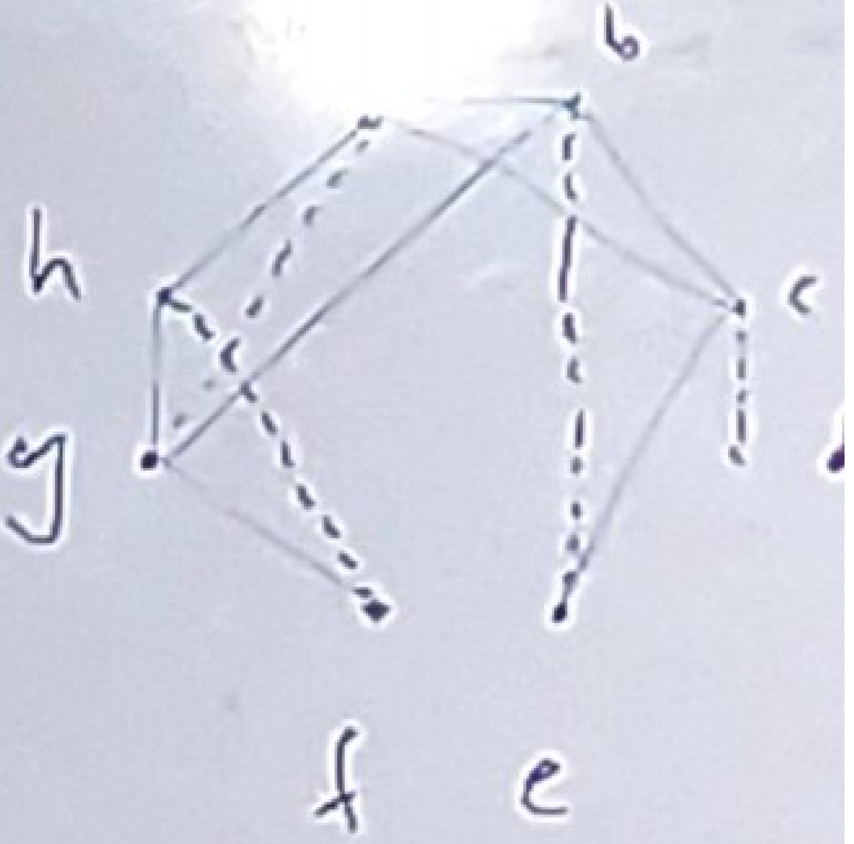
tite graph

n X-matching

for any

Fact 1 Let M_1 and M_2 be two distinct matching in a graph G. Then each component of the graph $[M$ [illegible]$]$ is either an M_i-alternating path or an M_i-alternating cycle.

Proof: Let $H = [M_1 \Delta M_2]$ and v be a vertex in H. Then v is M-saturated or M'-saturated (or both). Thus $1 \le \deg_H(v) \le 2$, and so each component of H is either a path or a cycle. It is clear that the resulting path or cycle must be M_i-alternating for $i = 1, 2$.



M

M

Fact 2: (Berger theorem) let G_1 be a group A maching M in Graph G_1 is maximum if ~~not~~ and only G_1 contains no-~~ma~~-argumenting paths

__Fact 2__ (Berge's theorem) Let G be a graph. A matching M in G is maximum if and only if G contains no M-augmenting path.

__Proof__ ($\Rightarrow$) Suppose there exists an M-augmenting path $v_1 v_2 \ldots v_{2k}$ in G.

Let $M' = \left(M \setminus \{v_2v_3, v_4v_5, \ldots, v_{2k-2}v_{2k-1}\}\right) \cup \{v_1v_2, v_3v_4, \ldots, v_{2k-1}v_{2k}\}$

Then M' is matching in G and $|M'| > |M|$, a contradiction.

($\Leftarrow$) Suppose that M is not maximum. Let M' be a matching in G such that $|M'| > |M|$.

Consider $H = [M \Delta M']$ by (Fact 1), each component of H is either a path or an even cycle, with edges alternately in M and M'. Since $|M'| > |M|$, H must contain a path component say a u-v path, where u and v are M'-s[illegible]

Clearly, this u-v path is an M-augmenting path.

20K-0259

5C

Edmond's Blossom Algo

- ~~previous algo~~ doesn't work on non-bipartite graphs

Steps:

- Graph with matching M.
- Taking start vertex u → unsaturated
  ending vertex v (any vertex)
- M-alternating path
- If two M-~~m~~alternating path and one path is odd and one is even.
- union then its called flower.
- If any vertex split the paths it is called base.
- The cycle after split is blossom
- The path before base is stem

(The length of stem is even)

20K-0259

Date:

- Every vertix of blossom is saturated by M (according to M)
- It give max set/path of saturated vertixes

Example 5.11

- (if no blossom find alternating path) ↳ then go back to orginal path and get max path and do switching
- If blossom (2 m-alternating path) then go back to original and get max path b/w those 2 and do switching.

Stable Matching:

- Should be complete bipartite and having perfect matching.
- Check stable and unstable table

Un-Stable if p un-match in ~~perfect~~ have more preference then the match pair.

Soon........

Example 5.13:

Gale-Shapley Algo

Do it from Book

. Take Bipartite and perfect and gives stable

Example 5.14:

.Steps

- Men propose highest ranking women

R A ~~S~~

S A ✗

T C ~~S~~

W C ✓

If women receives more than one proposal, accept / reject / delay

Initial thoughts

- If more proposal reject lowest ones ✗ other and ~~chose~~ keep highest
  if no conflict choose higher.
- Men proposes next available wom
  if no conflict then choose

Example 5.15:

A T ✓

B S ✓

C W ✓

D R ✓

Date:

5.10

(a)

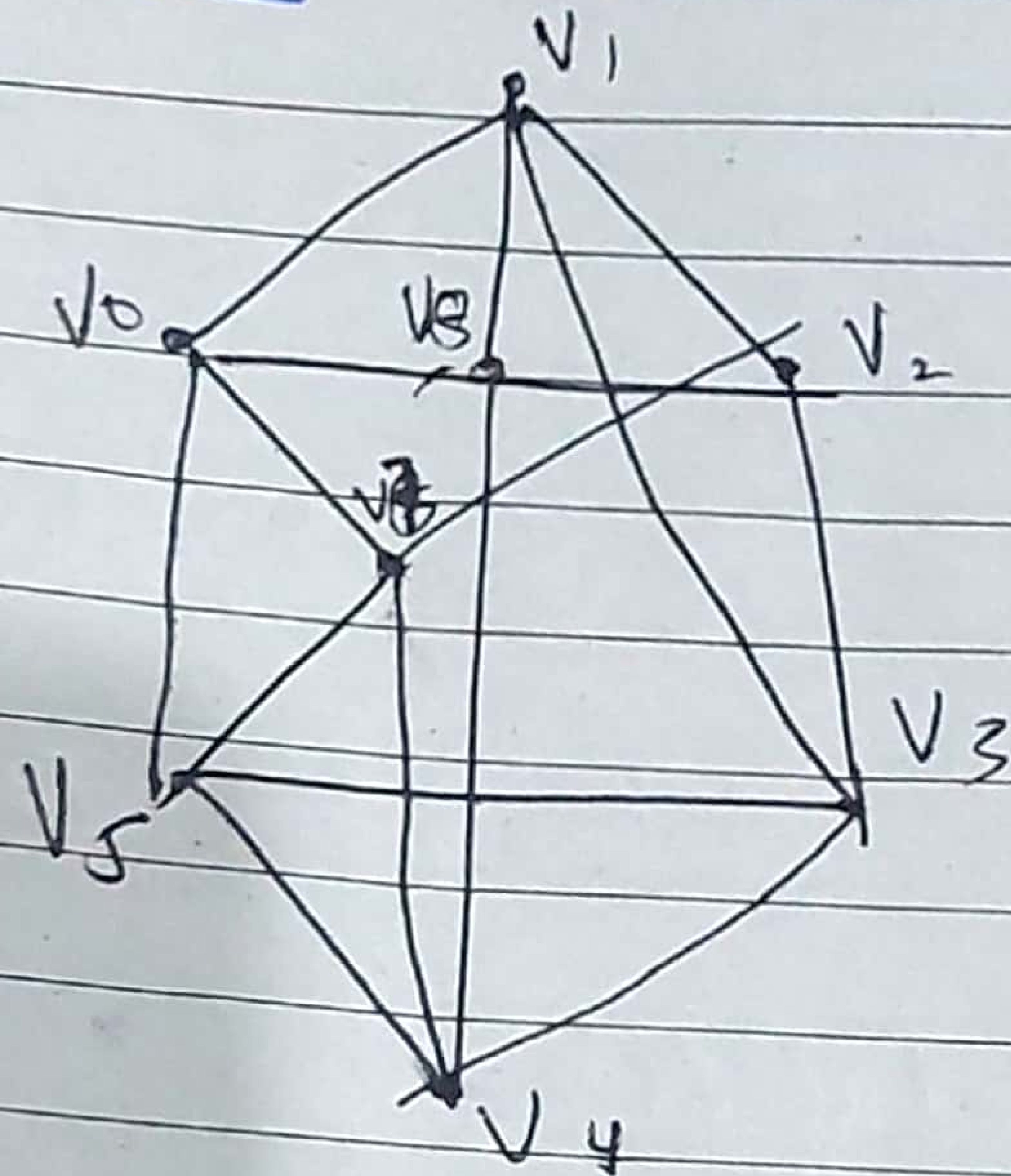
R	A	
S	A	~~2~~
T	C	
V	C	D

R	A	
S	d	
T	b	
V	b	

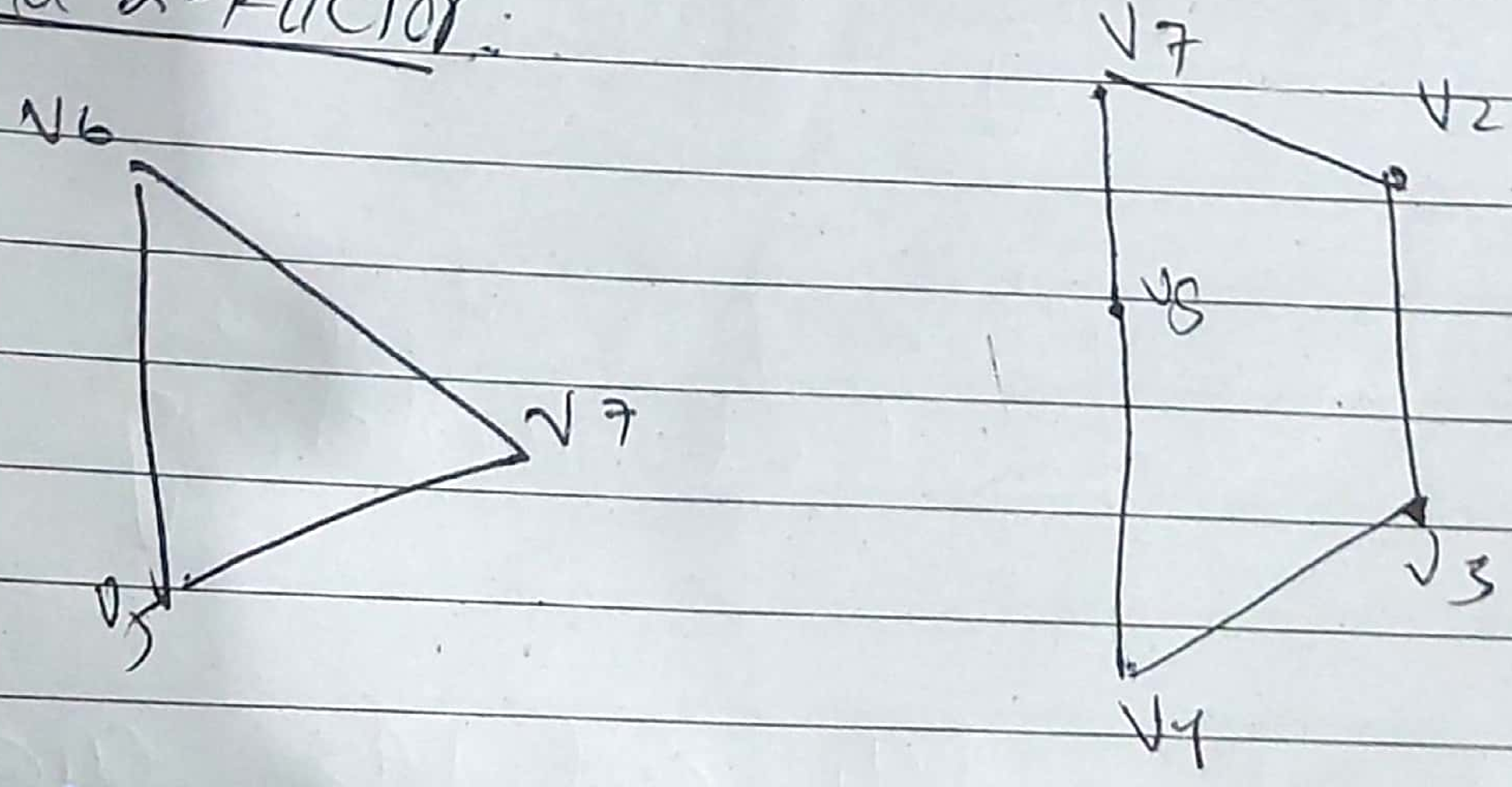
Date:

K-Factor

- Subgraph spanning Graph that k-regular (k degree/same degree) is called K-factor of Graph G.
- **Example.**



Find 2-Factor.



Fact: If a graph is 2-k-regular then G has 2-factor

Def:

A graph is k-factorization of Graph is collection of disjoint k factors and make edges partition

Q 5.19

Every tree has at most one perfect matching

Sol:

Suppose M_1 and M_2 are perfect matching

Q 5.19 Every tree has at most one perfect matching.

Sol Suppose M_1 and M_2 are two perfect matching in a tree such that there is an edge e_1 joining V_1 and V_2 in $M_1 - M_2$. Edge e_1 cannot be terminal edge of any M_i-alternating path $(i = 1,2)$, since terminal edge of a tree has to be an edge in every perfect matching. So there is a nonterminal vertex V_3 and edge e_2 joining V_2 and V_3 such that e_2 is in $M_1 - M_2$. If this process continues, we get a cycle that will terminate at V_1.

But the graph is acyclic.

Fact: If C

Def A

ed

$Q_{5.18}$ Every K-regular bipartite graph has a perfect matching.

Sol Let G be a K-regular bipartite graph with bipartition (A, B)

Let $X \subseteq A$ and let t be the number of edges with one end in X.

Since every vertex in X has degree K, it follows that $K|X| = t$.

Similarly, every vertex in N(X) has degree K, so t is less than or equal to $K|N(X)|$

It follows that $|X|$ is atmost $|N(X)|$. Thus by Hall's theorem there is a matching covering in A

or equivalently, every maximum matching covers A. By similar argument, we find that every maximum

matching covers B, and this complets the proof

Chap 6

Date:

K-coloring

The K colors used such that adjacent vertices has different colors.

color-class:

$S_1 = \{\text{set of vertices of color } 1\}$
$S_2 = \{\text{set of vertices of color } 2\}$
$\vdots$
$S_n = \{\text{set of vertices of color}\}$

Index-Number:

~~Pro~~ **Chromating Color:**

smalles value of K.

- ~~Colors for K-graph is Kare K.~~
- Chromatic number for cyclic odd number = 3
- Chromatic number for cycle of even no. = 2

Date:

Chap 6

k-coloring

The k colors used such that adjacent vertices has different colors.

color-class:

$S_1 = \{$ set of vertices of color 1 $\}$
$S_2 = \{$ set of vertices of color 2 $\}$
$\vdots$
$S_n = \{$ set of vertices of color n $\}$

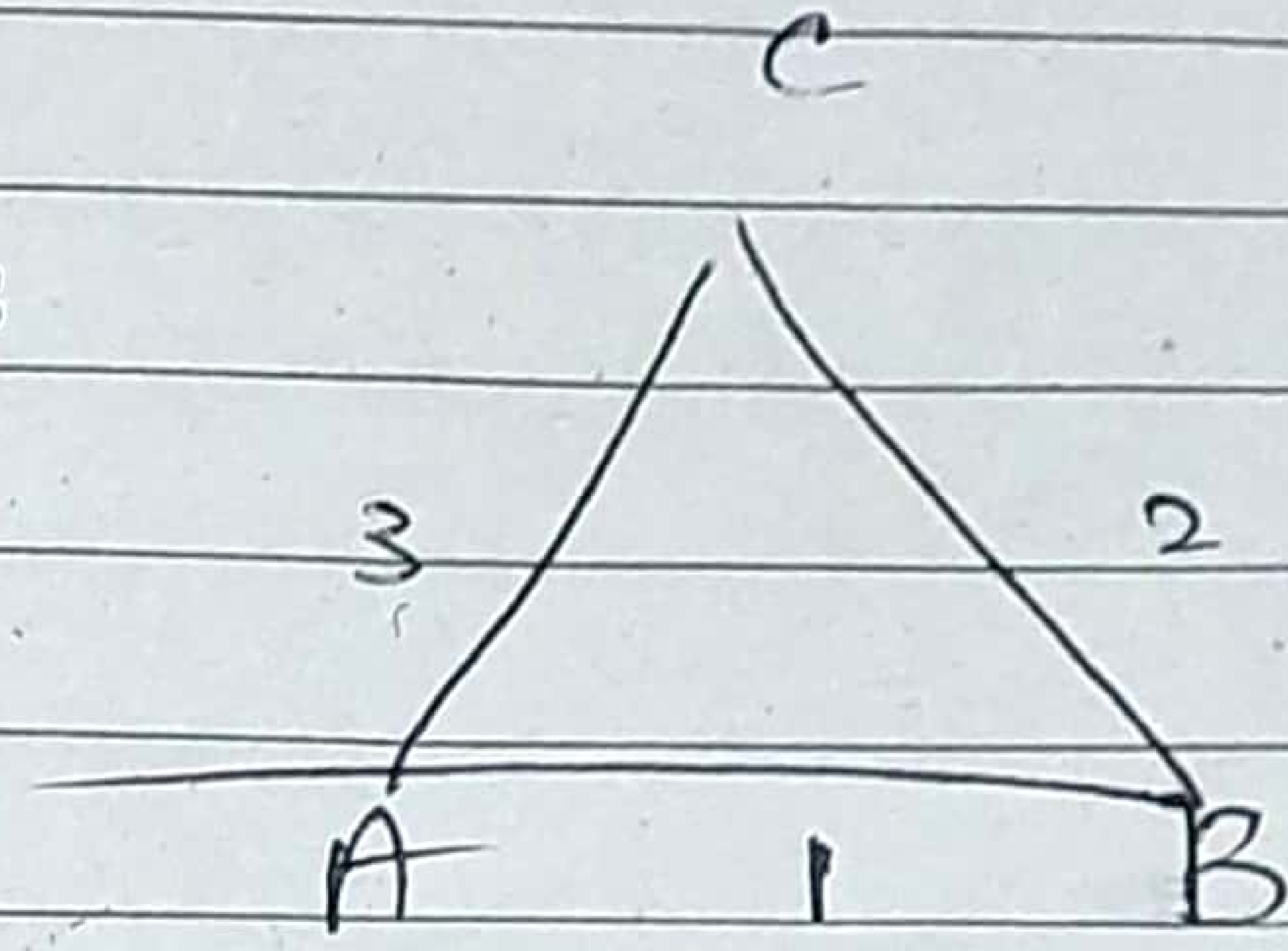
Index-Number:

~~Pro~~ Chromating Color:

smalles value of k.

- ~~Colors for k-graph is are k.~~
- Chromatic number for cyclic odd number = 3
- Chromatic number for cycle of even no. = 2
- **Wheels: from book** (Chromati for $W_n = 4$ if it is odd
- Chromatic number of k-complete = k
-

Edge Coloring:



- No two adjacent edges has same color.

- If n is even $\chi'(K_n) = n-1$
- If n is odd $\chi'(K_n) = n$

no of vertices

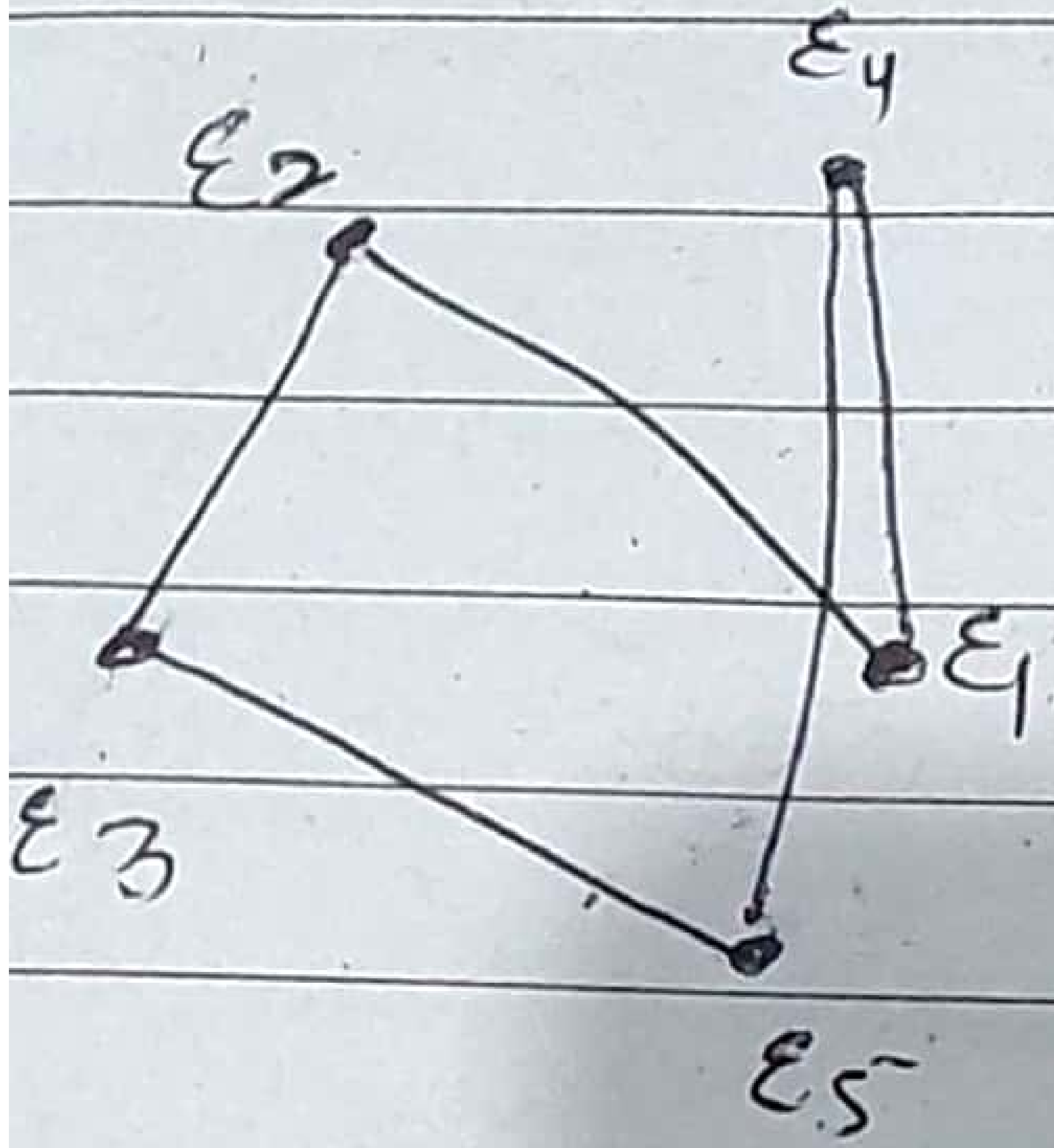
Theorem 6.19

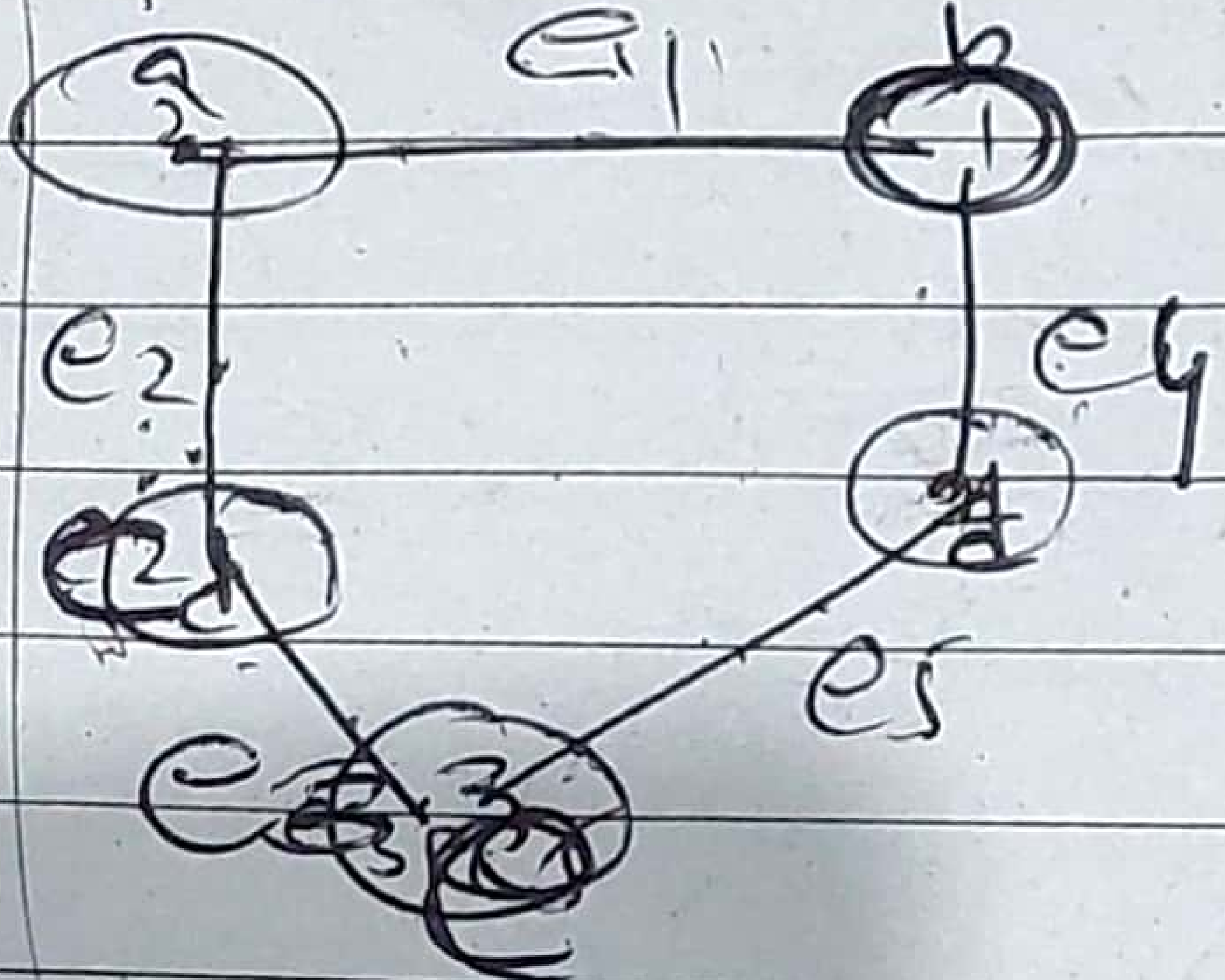
$$\Delta(G) \le \chi'(G) \le \Delta(G)+1$$

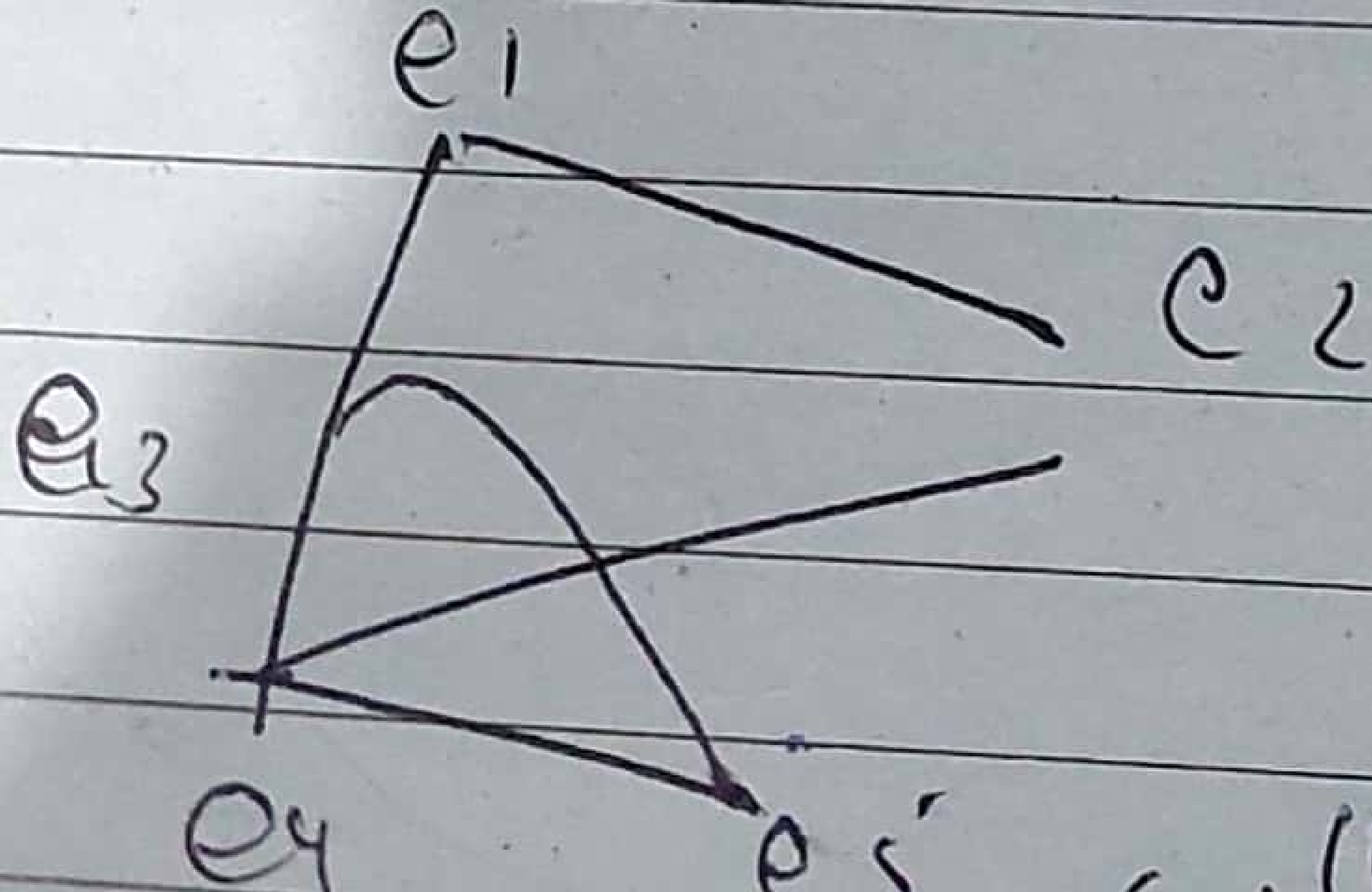
for all simple graphs

Def 6.20

~~A graph of G~~







Example 6.11

Line Graph

Ramsey Number

All graphs of min vertices of r

of all simple graph such that it contains clique of size m or an independent of size n

Example 6.12

Color Variation

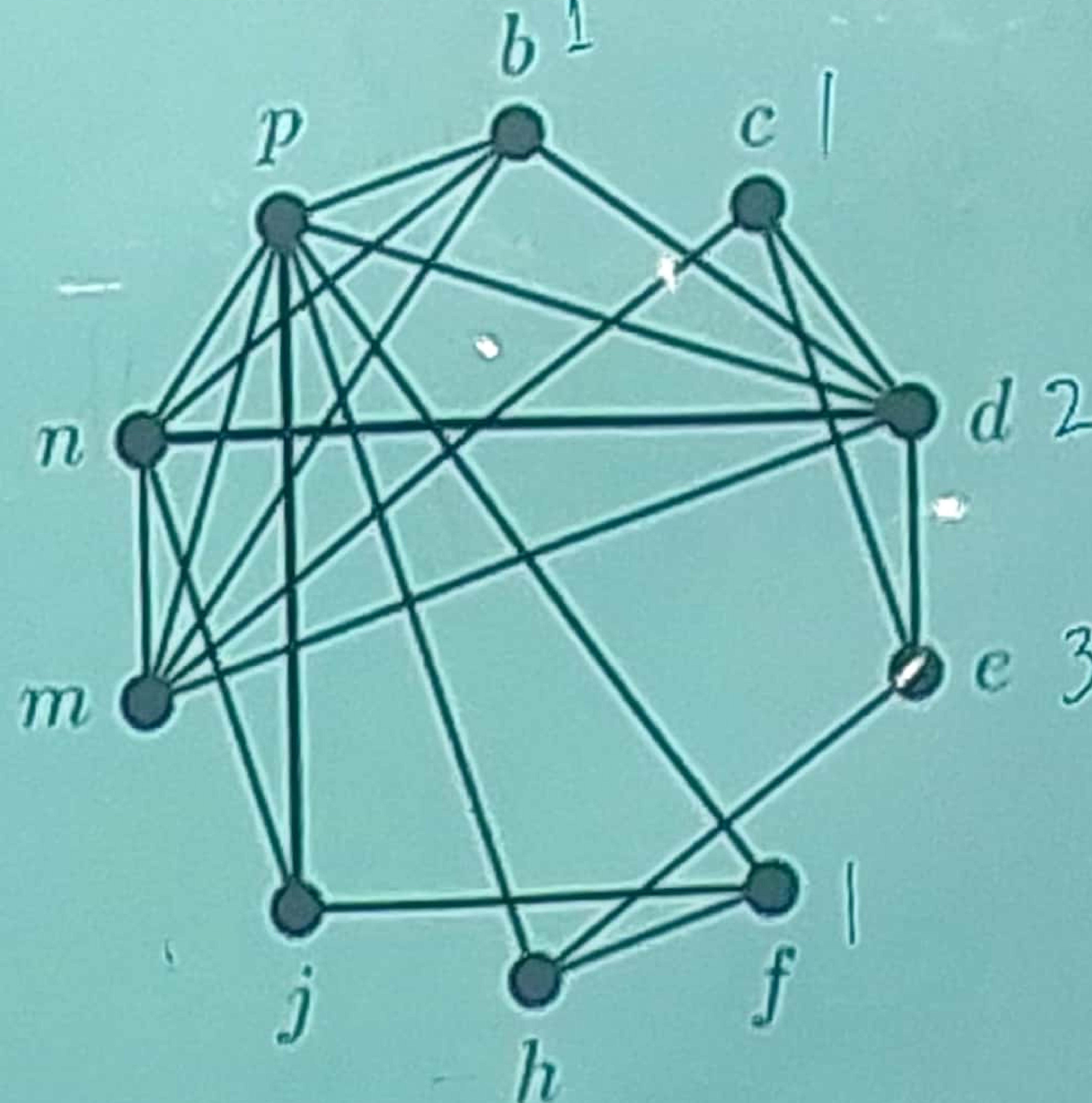
(online colors):

Graph of vertices m of their orders. An online graph colors a subgraph (induced from $x_i \cdots x_j$)

First fit coloring Algo

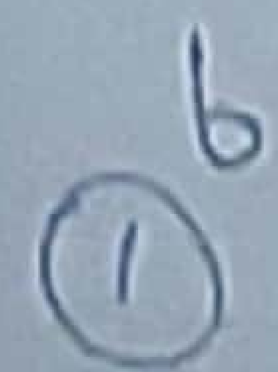
1°WP

rtex Coloring

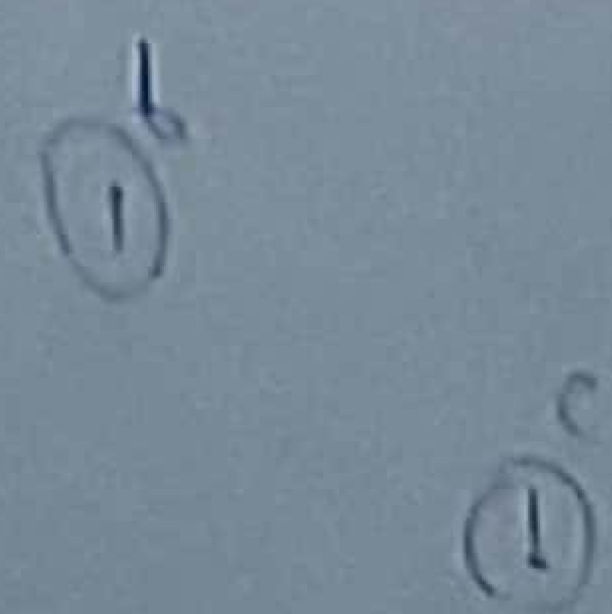


At our initial step, we want to find a vertex of highest degre
give it color 1. Once p has been assigned a color, we look at its
with high degree as well, namely d (degree 6), m and n (both,

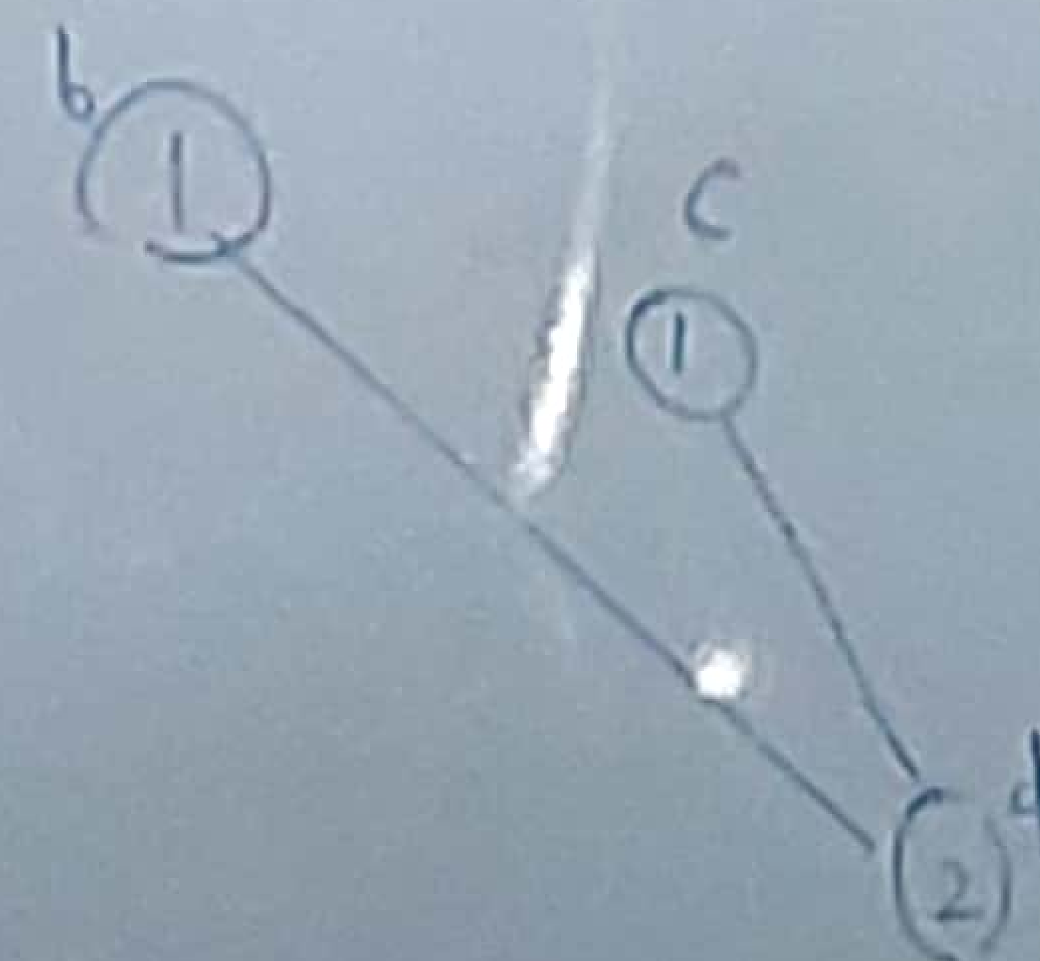
$b < c < d < e < f < h < j < m < n < p$



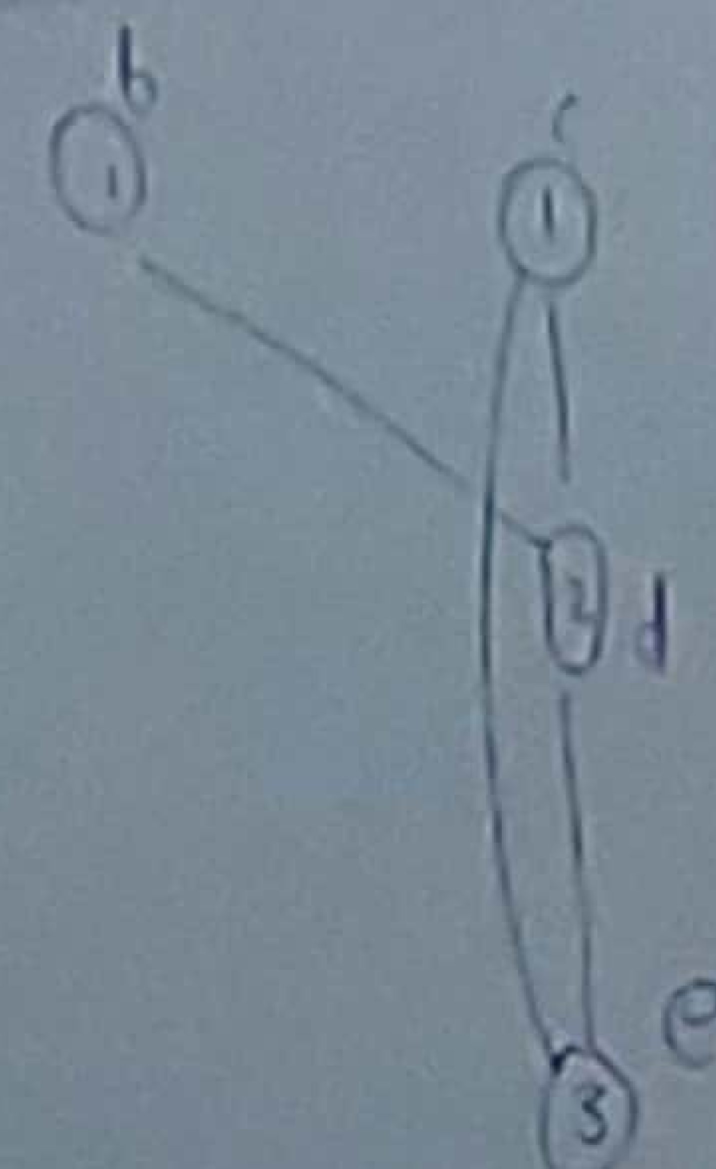
G_1



G_2



G_3



G_4

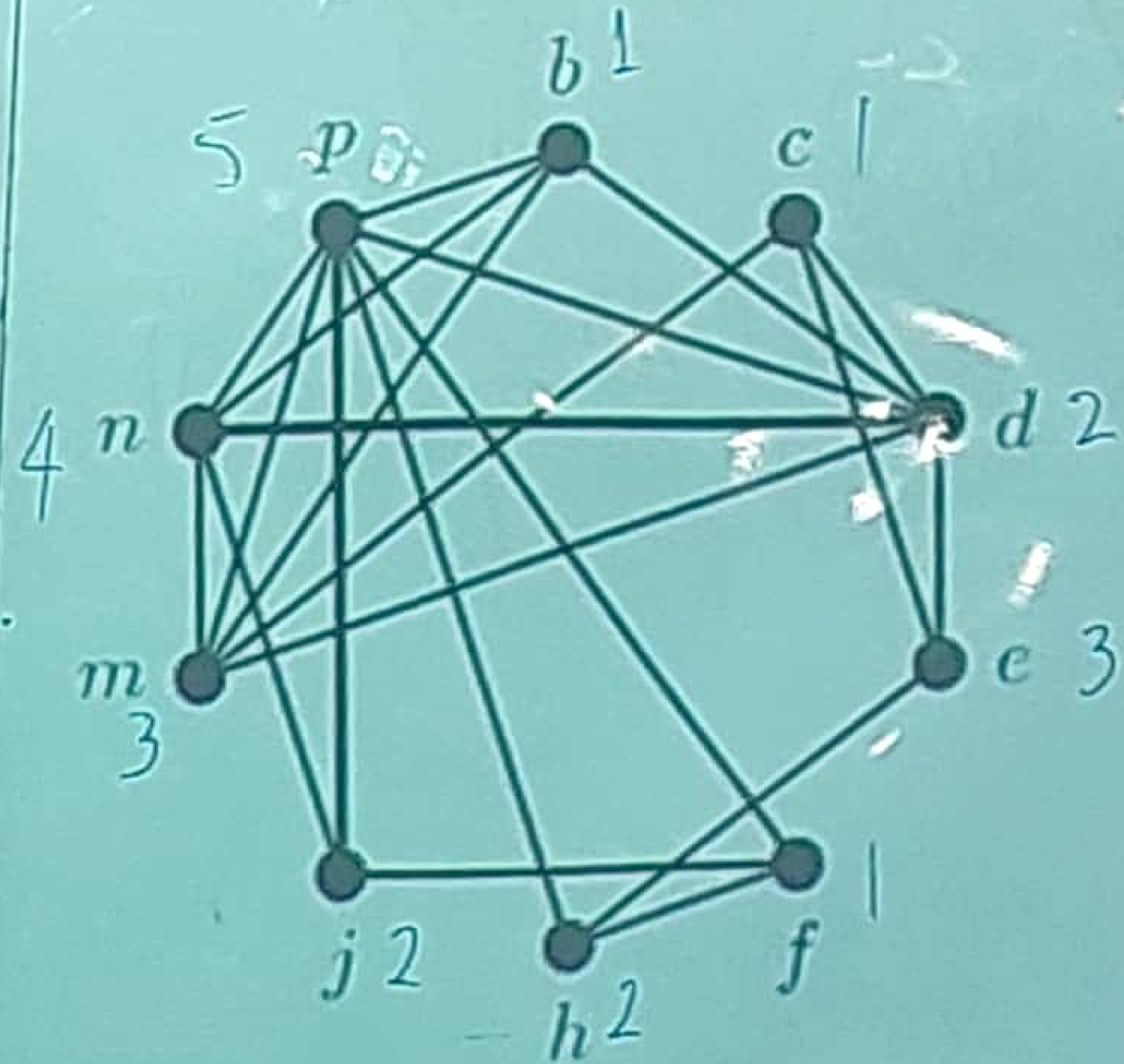
View Document Tools Window Help

285 (302 of 439) Find

Vertex Coloring

$\chi_A(G)$

$\chi(G$



At our initial step, we want to find a vertex of highest degre
give it color 1. Once p has been assigned a color, we look at its
with high degree as well, namely d (degree 6), m and n (both,

6.14 x 9.21 in

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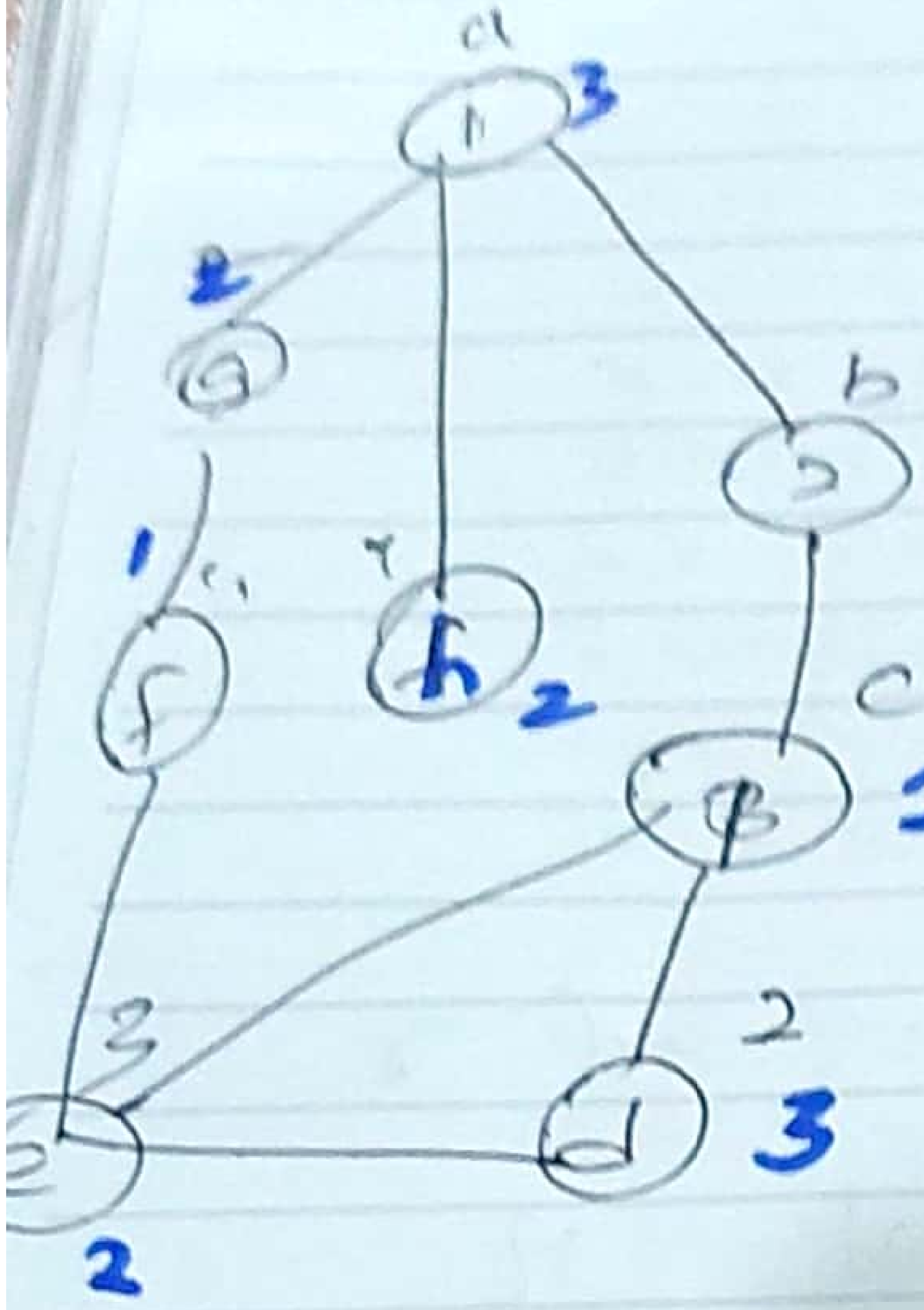
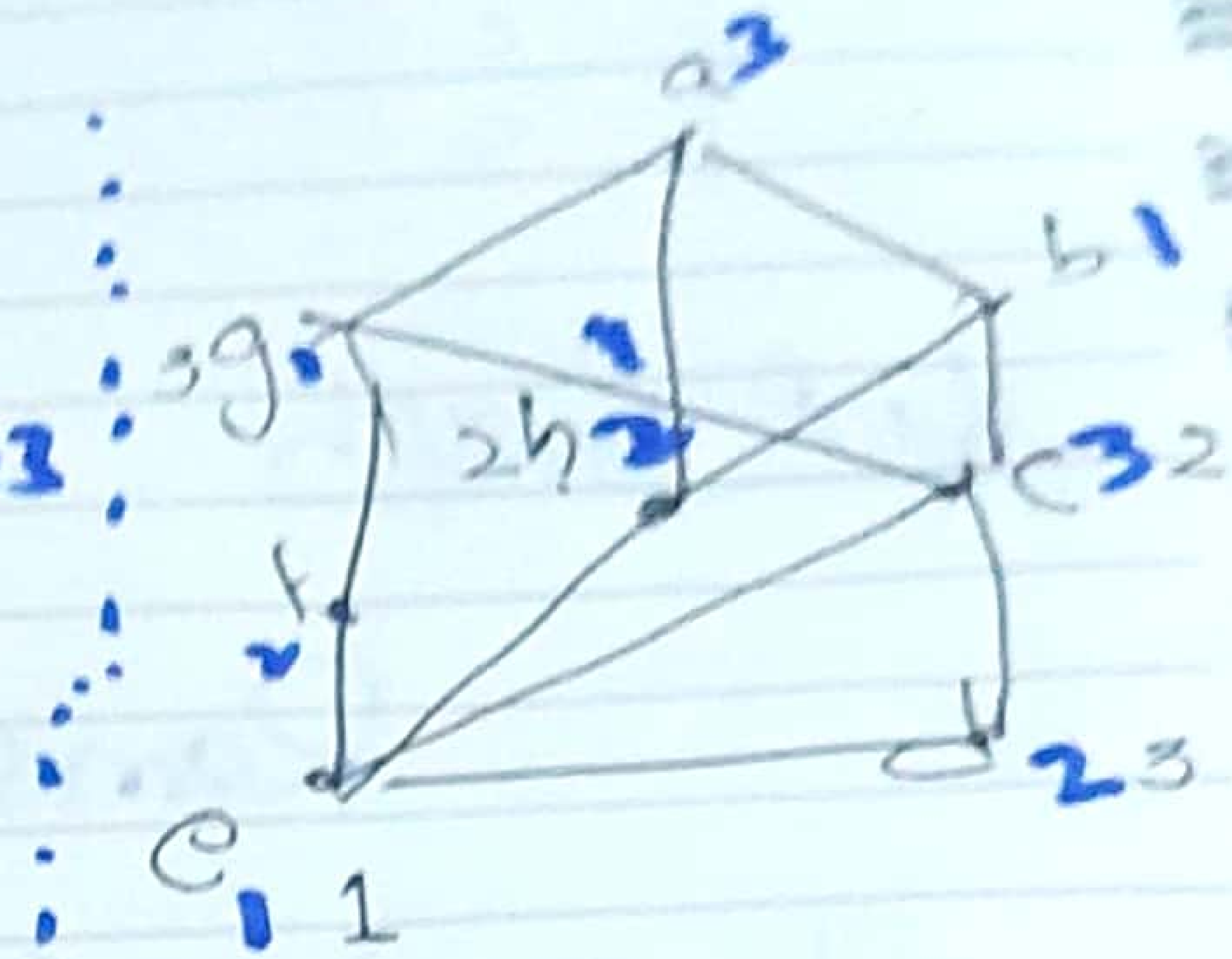
a b c d e f g h

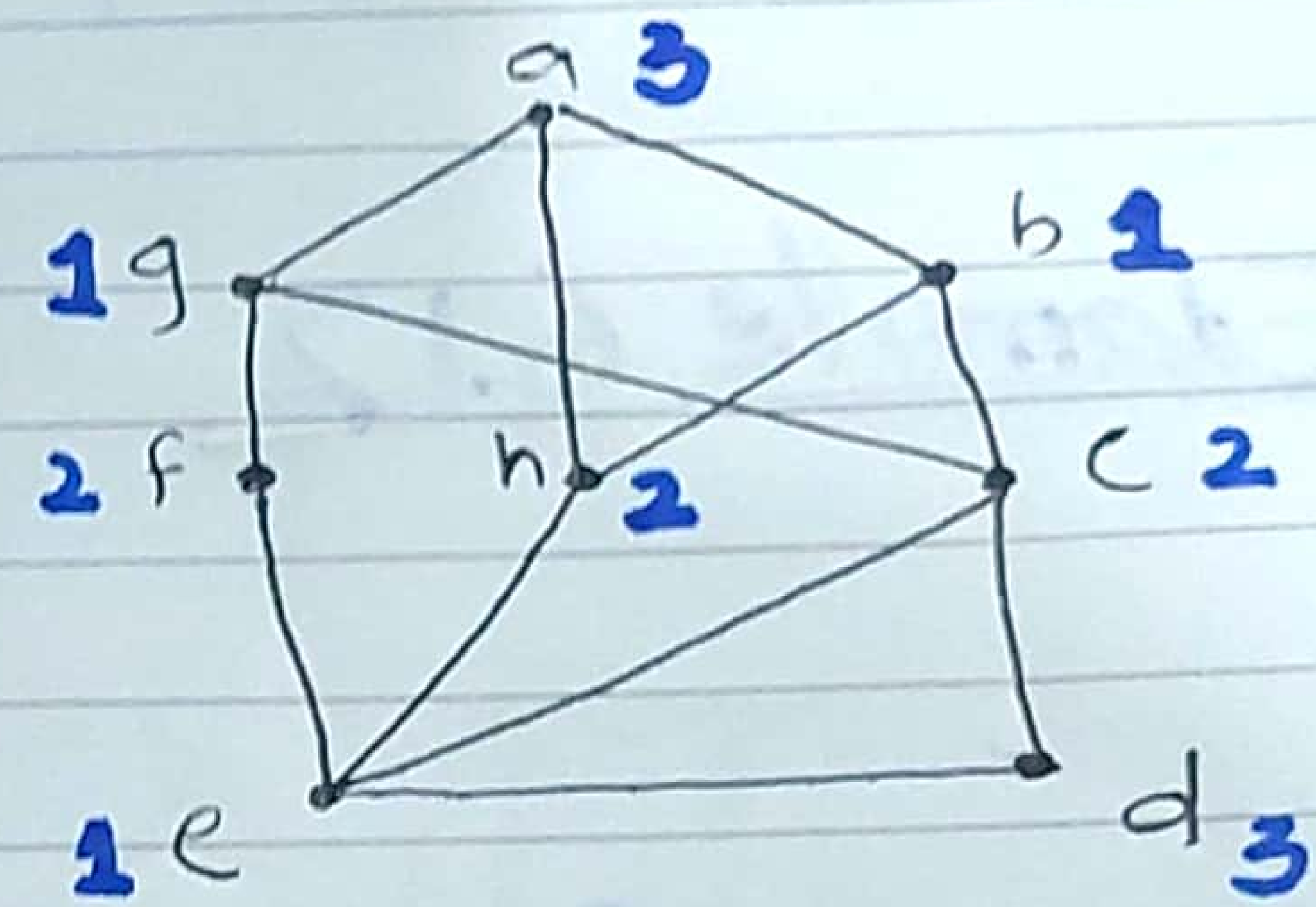


$\chi(G)$ (4)

Chromatic = 3



$\chi_A(G) = 3$



chromatic = $\chi(G) = 3$

Def Weighted coloring

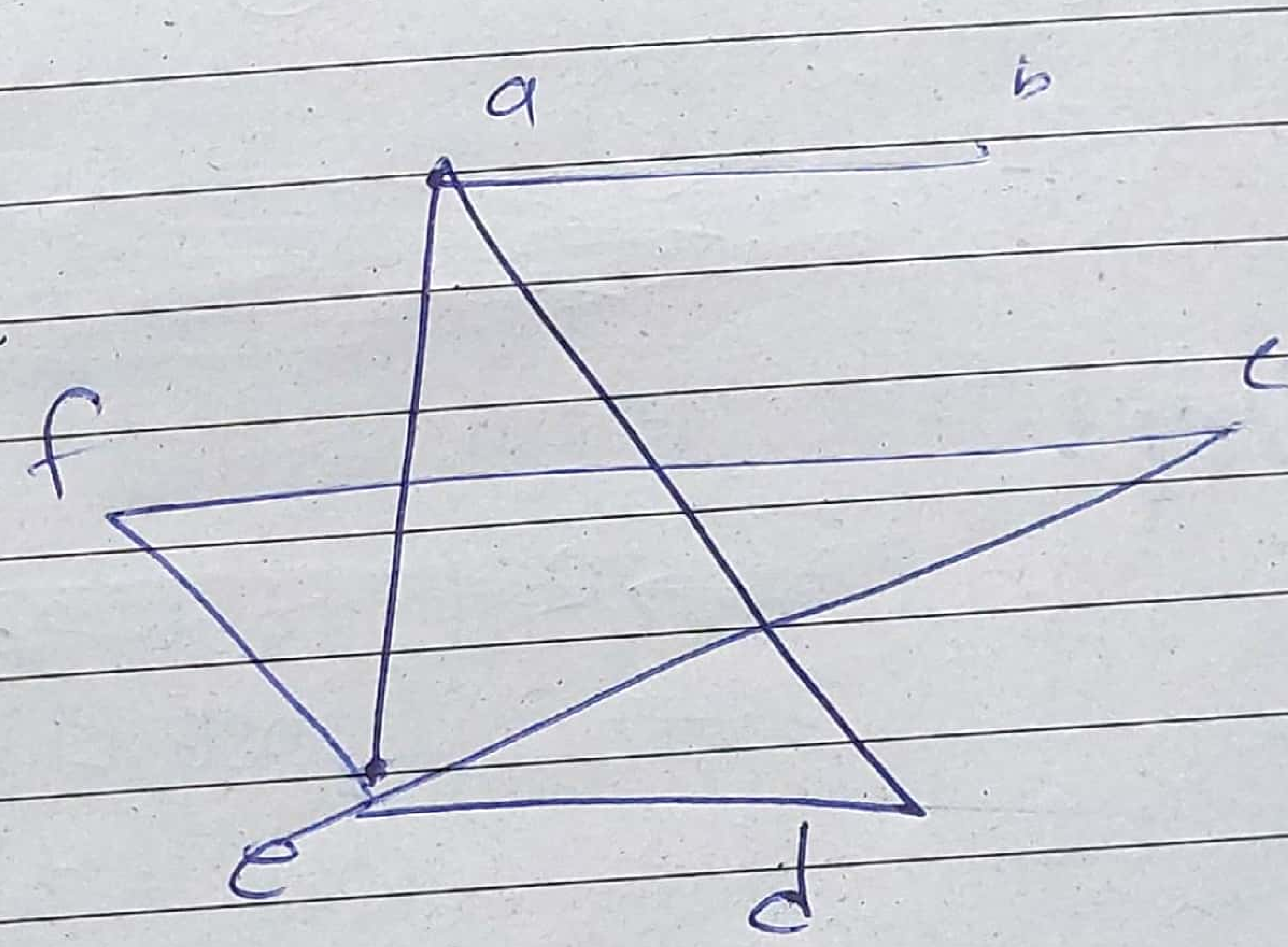
Date:

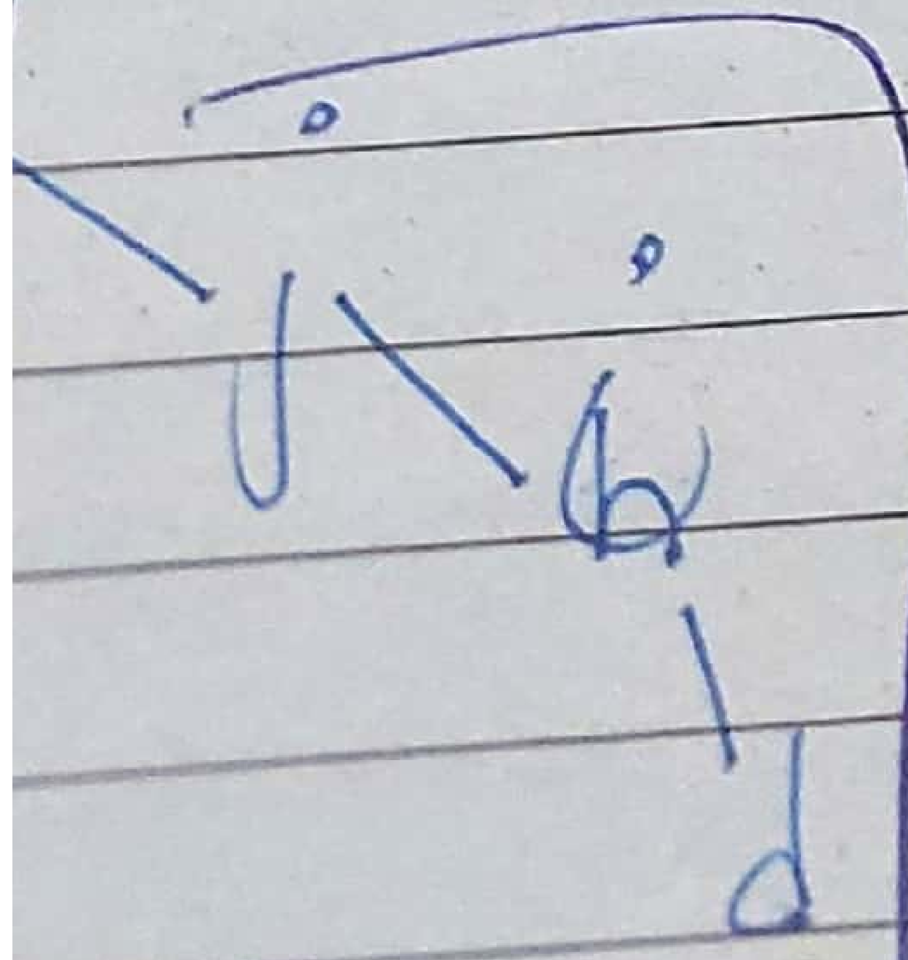
The weight coloring of graph G assign each vertex a set of color such that

(1) The set consist of consecutive colors

(2) The color assign to an vertex equal to weight

(3) If two vertices are adjacent then their colors must be disjoint





$w(a)=2$ $w(b)=1$

$w(c)=4$ $w(d)=2$

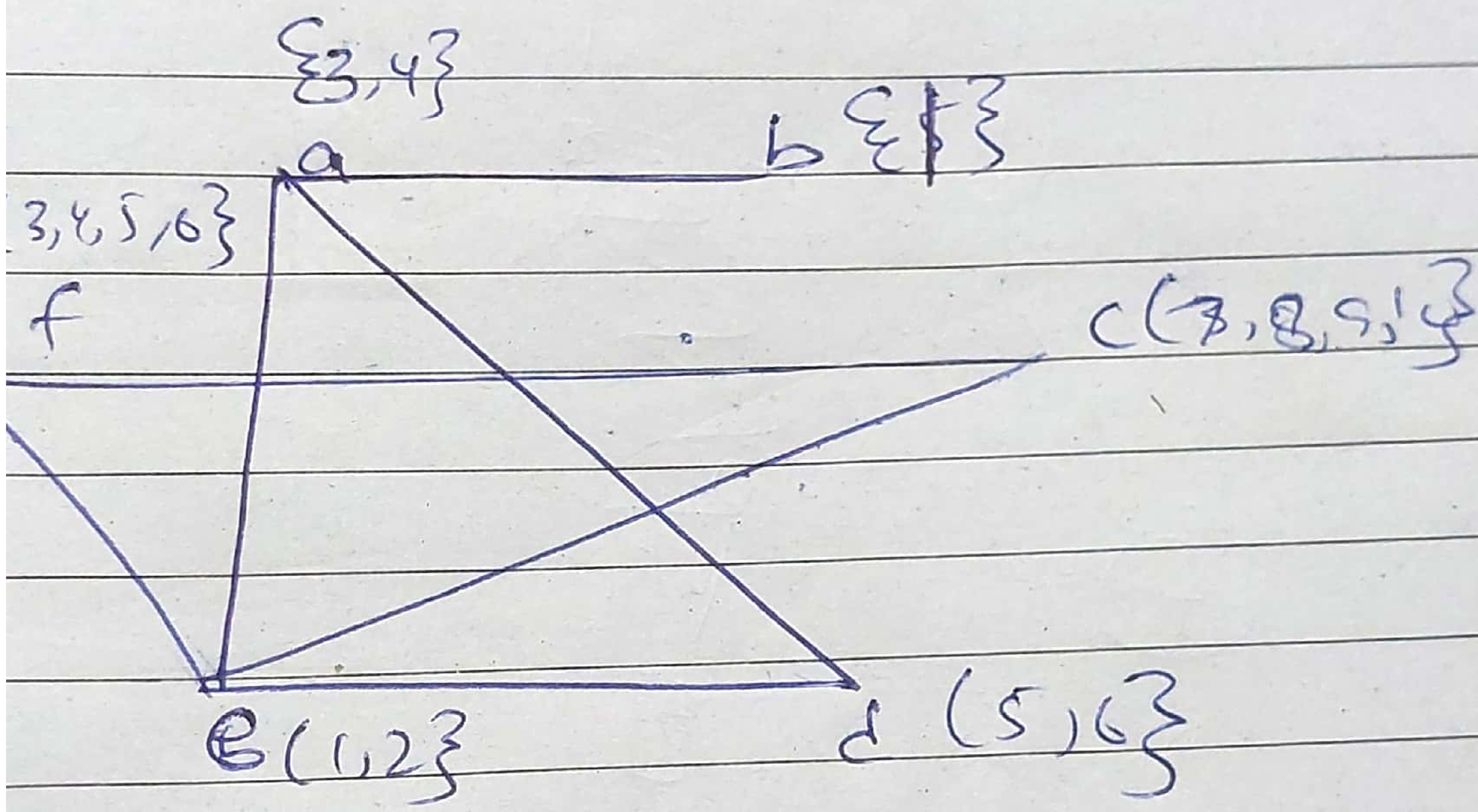
$w(e)=2$ $w(f)=4$

Date:

Choose bigger weight clique. (max)

ie. fce.

chose 'e' (coloring in both cliques)



choosing first vertex is common then proceed independently


{1,2}
{1,2,3,4,5}
{3,4,5,6,7}
{1,2,3,4}
{1,2}
{8,9,10}
{7}
{3}
{3,4,5,6}
{7,8,9}
△ abh 10
△ afe 9
△ ahe 9
△ cfe 9
△ ade 7
△ cfg 6

Do list coloring

Q 6.26

Do list coloring

Def 6.26

Do 6.27

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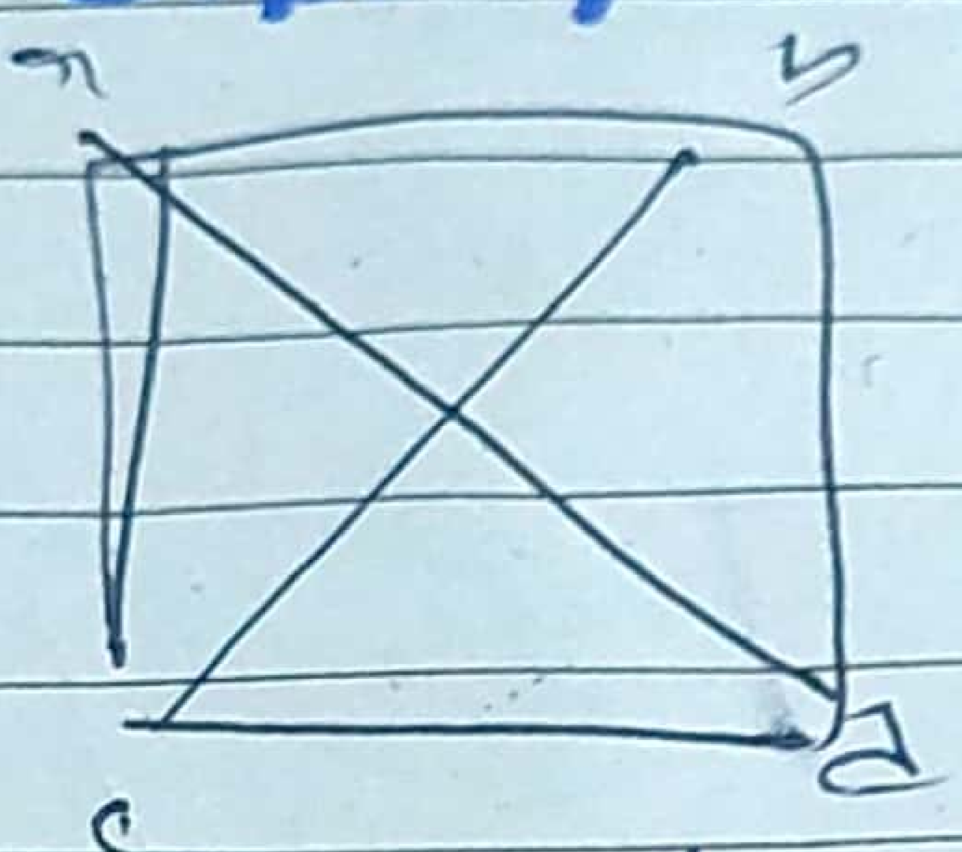
Chapter 07

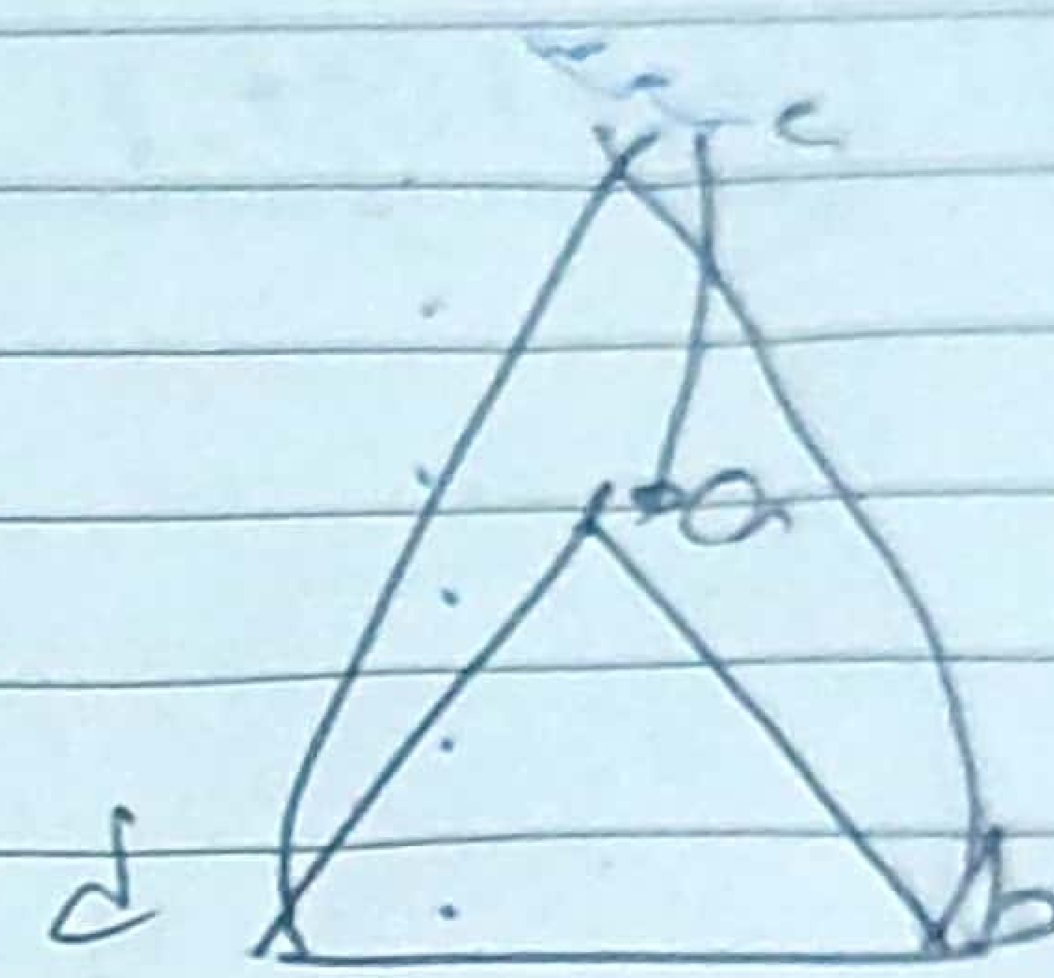
Planner :

1. Vertices are arranged no vedges cross each othe

Example



no planner



Yes

example 7.3

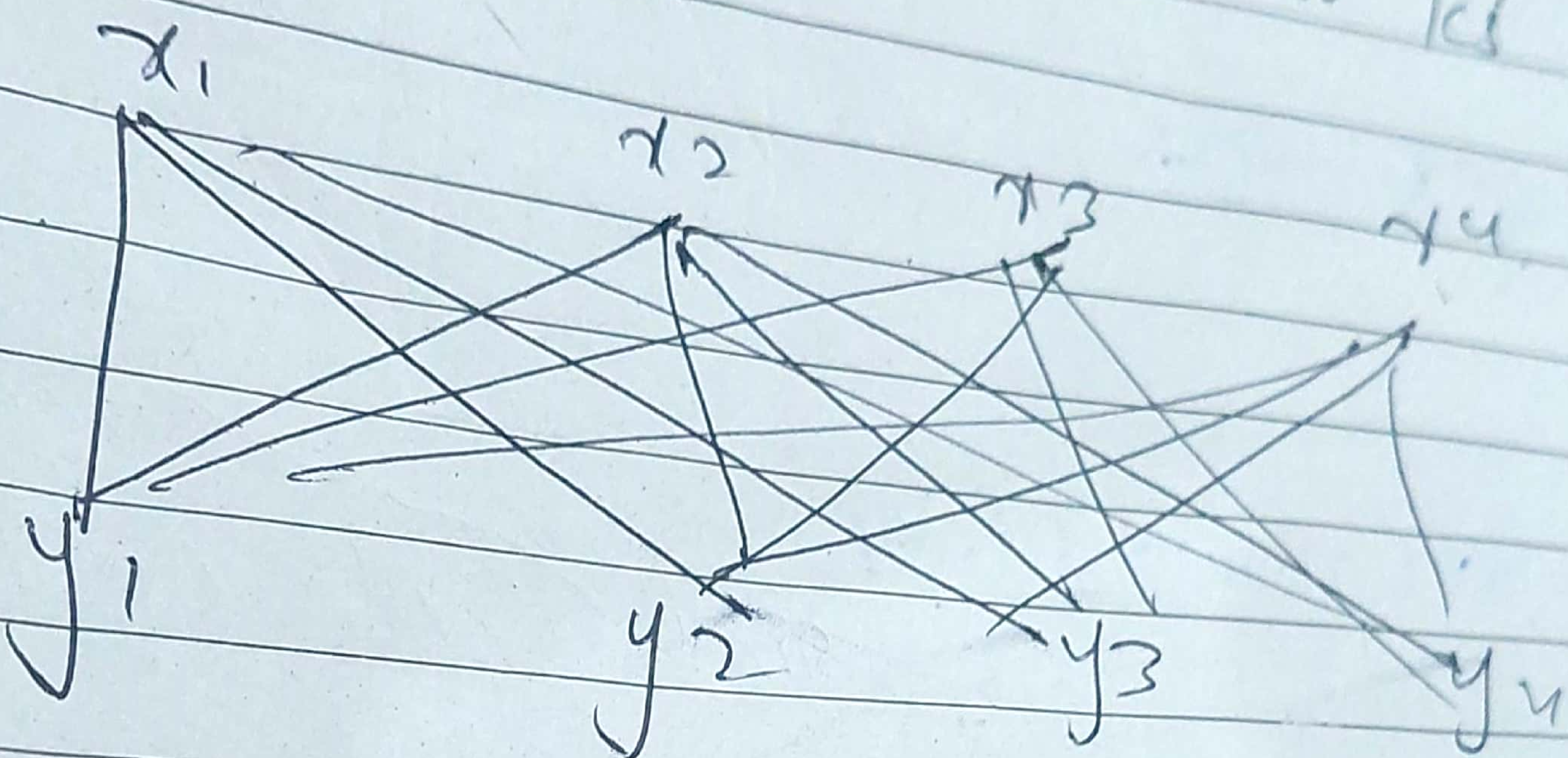
$K_{3,3}$ not planner

Kuratowsk's theorem

Example 7.2 :

K_5 not plan

If graph is planes if no subgraph of $K_{3,3}$ or K_5 exist.



K_{4}, K_{4}

It has a subgraph of $K_{3,3}$

Region:

Area completly bunded by graph

Earlier formula

a graph of connected and planer has $n - m + r = 2$

vertices edges region

with proof

Maximaly planar

If Graph G+e in non adjacent vertices and produces G the with planar then not maximally planner