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Assignment = 01

Q No1

$(x-y)\,dx + x\,dy = 0$

Let $x = vy$.

$dx = v\,dy + y\,dv$

$\rightarrow (x-y)\,dx + x\,dy = 0$

Substitute $x = vy$.

$(vy - y)(v\,dy + y\,dv) + vy\,dy = 0$

$v^2y\,dy + vy^2\,dv - y^2\,dv = 0$

$y^2(v-1)\,dv + v^2y\,dy = 0$

$y(1-v)\,dv - v^2\,dy = 0$

Separate variables

$y(1-v)\,dv = v^2\,dy$

$\frac{(1-v)}{v^2}\,dv = \frac{dy}{y}$

$\Rightarrow \int \frac{(1-v)}{v^2}\,dv = \int \frac{dy}{y}$

$\int \frac{1}{v^2}\,dv - \int \frac{1}{v}\,dv = \int \frac{1}{y}\,dy$

$-\frac{1}{v} - \ln v\,dv + C = \ln y$

Put back

$-\frac{y}{x} - \ln\frac{x}{y} + C = \ln y$

$y = x\left(\ln y + \ln\frac{x}{y}\right) + Cx$

log Property

$y = -x\ln x + Cx$ Answer.

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(2).

$(x+y)\,dx + x\,dy = 0$

Let $y = ux$:

$\Rightarrow \dfrac{dy}{dx} = u + x\dfrac{du}{dx}.$

$(x+ux)\,dx + x(u\,dx + x\,du) = 0$

$x\,dx + ux\,dx + xu\,dx + x^2\,du -$

$x\,dx + 2ux\,dx + x^2\,du = 0$

$x\,dx(1+2u) + x^2\,du = 0$

Separate Variables.

$\dfrac{x\,dx}{x^2} = \dfrac{-du}{(1+2u)}$

$\dfrac{1}{x\,dx} = -\dfrac{du}{(1+2u)}$

Taking Integration.

$\int \frac{1}{x}\,dx = -\int \dfrac{du}{1+2u}.$

$\ln x = -\frac{1}{2}\ln(1+2u) + c.$

$e^{\ln x} = e^{\ln(1+2u)^{-1/2} + c}$

$x = (1+2u)^{-1/2}$

Back Subsit

$x = c(1+2)(y)^{-1/2}.$

$2\left(\frac{y}{x}\right) + 1 = c\,x^{-2}$

$\boxed{y = \dfrac{c}{2x} - \dfrac{x}{2}}$

Answer.

(iii)

$x\,dx + (y-2x)\,dy = 0$

$\text{let } y = ux \quad \frac{dy}{dx} = u + x\frac{du}{dx}$

$dy = dx\,u + x\,du$.

$x\,dx + (ux-2x)(u\,dx + x\,du) = 0$

$x\,dx + u^2x\,dx - ux^2du - 2x\,dx.u - 2x^2du$

$x(u^2-1)\,dx + (u-2)x^2du = 0$

$$\frac{x\,dx}{x^2} = -\frac{(u-2)}{(u^2-1)}du$$

$$\frac{u-1}{(u-1)^2} - 1\,du = -\int\frac{1}{x}dx$$

$$\int\frac{u-1}{(u-1)^2} - \frac{1}{(u+1)^2}\,du = -\ln x + c$$

$$\ln(u-1) + \frac{1}{u-1} = -\ln x + c$$

$$\ln\ln\left(\frac{y}{x}-1\right) + \frac{x}{y-x} = c$$

$$\ln(y-x) + \frac{x}{(y-x)} = c$$

$(y-x).\ln(y-x) + x = c(y-x)$.

Answer

(4) $y\,dx = 2(x+y)\,dy$

$y\,dx - 2(x+y)\,dy = 0$

Let $y = ux$

$$\frac{dy}{dx} = u + x\frac{du}{dx}$$

$dy = dx\,u + x\,du$

Put

$ux\,dx - 2(x+ux)(u\,dx + x\,du) = ux\,dx - 2[x\,dx + x^2\,du + u^2x\,dx + ux^2\,du]$

$ux\,dx - 2\,ux\,dx - 2x^2\,du - 2u^2x\,dx - 2ux^2\,du$

$x\,dx(u - 2u - 2u^2) - 2x^2\,du(1-u)$

$x\,dx(-u - 2u^2) - 2x^2\,du(1-u)$

$x\,dx/x \cdot (-1 - 2u) - 2x^2\,du(1-u)$

Separate the variables.

$$\frac{1}{2}x^{-1}\,dx = \frac{u+1}{-u-2u^2}\,du$$

Take Integration.

$$+\frac{1}{2}\ln x = -\int \frac{u+1}{2u^2+u}$$

do Partial fraction.

$$\int\left(\frac{A}{u}+\frac{B}{(2u+1)}\right)du$$

$$A(2u+1)+B(u)=u+1$$

$$2Au+A+Bu=u+1$$

$$2A+B=1$$

$$A=1 \qquad B=-1$$

$$\frac{1}{2}\ln(x)=\int\frac{1}{2u+1}du-\int\frac{1}{u}du$$

$$\int\frac{1}{u}+\frac{-1}{2u+1}$$

Let $v=2u+1$ $dv=2du$

$$\frac{1}{2}dv=du$$

$$\frac{1}{2}\ln(x)=\frac{1}{2}\ln|2u+1)-\ln|u|$$

$$y\,dx-2(x+y)dy=0$$

$$y(v\,dy+y\,dv)-2(vy+y)dy=0$$

$$y^2dv=(v+2)y\,dy$$

$$\int\frac{1}{v+2}dv=\int\frac{1}{y}dy$$

$$\ln|y|=\ln|v+2|+c$$

$y = (v+2) \cdot e^{c}$

$e^{c} = C_2$

$x = vy$

$v = \frac{x}{y}$

$y = C_2\left(\frac{x}{y} + 2\right)$ Ans

(5) $(y^2 + yx)\,dx - x^2 dy = 0$

Let $y = (u(x)$

$dy = u\,dx + x\,du$

$((ux))^2 + ux^2)\,dx - x^2(u\,dx + x\,du) = 0$

$(u^2x^2 + ux^2)\,dx - x^2(u\,dx + x\,du) = 0$

$u^2x^2\,dx + u.x^2\,dx - x^2u\,dx - x^3\,du = 0$

$u^2x^2\,dx = +x^3\,du$

$u^2\,dx = x\,du$

$\frac{dx}{x} = \frac{du}{u^2}$

$\int \frac{dx}{x} = \int \frac{du}{u^2}$

$\ln(x) + c = -u^{-1}$

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$$\ln(x) + c = -\frac{x}{y}$$

$$\frac{y}{x} = \frac{1}{c - \ln(x)} \text{ Ans.}$$

(6) $(y^2 + yx)\,dx + x^2\,dy = 0$.

Let $y = ux$.

$$\frac{dy}{dx} = u + \frac{x\,du}{dx} \Rightarrow dy = u\,dx + x\,du.$$

$$(y^2 + yx)\,dx + x^2\,dy = 0$$

$$((ux)^2 + (ux))\,dx + x^2\,[u\,dx + x\,du] = 0$$

$$u^2x^2 + ux^2\,dx + x^2u\,dx + x^3\,du = 0$$

$$(u^2x^2 + 2ux^2)\,dx = -x^3\,du$$

$$u\cancel{x^2}(u+2)\,dx = -x^3\,du$$

$$u(u+2)\,dx = -x\,du$$

$$-\frac{dx}{x} = \frac{du}{u(u+2)}$$

$$-\int \frac{dx}{x} = \int \frac{du}{u(u+2)}$$

$$-\ln(x) + c = \int \left(\frac{\frac{1}{2}}{u} - \frac{\frac{1}{2}}{(u+2)}\right) du$$

$$-\ln(x)+c=\frac{1}{2}\ln(u)-\frac{1}{2}\ln(u+2)$$

$$-\ln(x)+c=\frac{1}{2}\ln\left(\frac{u}{u+2}\right)$$

$$-2\ln(x)+c=\ln\left(\frac{u}{u+2}\right)$$

$$\ln(x)^{-2}+c=\ln\left(\frac{u}{u+2}\right)$$

$$\ln\left(\frac{1}{x^2}\right)+c=\ln\left(\frac{u}{u+2}\right)$$

$$\ln\left(\frac{c}{x^2}\right)=\ln\left(\frac{u}{u+2}\right)$$

$$\frac{c}{x^2}=\frac{u}{u+2}$$

$$\frac{c}{x^2}=\frac{y/x}{(y/x)+2}$$

$$\frac{c}{x^2}=\frac{y}{y+2x}=c(y+2x)=yx^2$$

$$cy+2cx=yx^2=2cx=yx^2-cy$$

$$2cx=y(x^2-c)$$

$$y=\frac{2cx}{x^2-c}$$ Ans.

(7) $\dfrac{dy}{dx} = \dfrac{y-x}{y+x}$

Let $y = ux$

$\dfrac{dy}{dx} = u + x\dfrac{du}{dx}$

$\dfrac{dy}{dx} = \dfrac{y-x}{y+x}$

$u + x\dfrac{du}{dx} = \dfrac{ux-x}{ux+x}$

$x\dfrac{du}{dx} = \dfrac{u-1}{u+1} - u$

$x\dfrac{du}{dx} = \dfrac{\cancel{u}-1-u^2\cancel{-u}}{u+1}$

$x\dfrac{du}{dx} = \dfrac{-(u^2+1)}{u+1}$

$\dfrac{(u+1)\,du}{u^2+1} = -\dfrac{dx}{x}$

$\displaystyle\int \frac{(u+1)\,du}{(u^2+1)} = -\int\frac{dx}{x}$

$\displaystyle\int \frac{u}{u^2+1}\,du + \int\frac{1}{u^2+1}\,du = -\ln|x| + c$

$\displaystyle\frac{1}{2}\int\frac{2u}{u^2+1}\,du + \tan^{-1}u = -\ln(x) + c$

$$\frac{1}{2}\ln(u^2+1) + \tan^{-1} u = -\ln(x) + c$$

$$\frac{1}{2}\ln\left(y^2/x^2 + 1\right) + \tan^{-1}(y/x) = -\ln|x| + c$$

$$\frac{1}{2}\ln\left(y^2/x^2+1\right) + \tan^{-1}\left(\frac{y}{x}\right) = -\ln x + c$$

$$\frac{1}{2}\ln\left(\frac{y^2+x^2}{x^2}\right) - \frac{1}{2}\ln(x^2) + \tan^{-1}\left(\frac{y}{x}\right) = -\ln|x| + c$$

$$\frac{1}{2}\ln(y^2+x^2) - \ln(x) + \cancel{\ln(x)} + \tan^{-1}(y/x) - c$$

$$\frac{1}{2}\ln(y^2+x^2) + \tan^{-1}(y/x) = c$$

Ans

(8) $dy/dx = x+3y / 3x+y$

Let $y = ux$.

$$\frac{dy}{dx} = u + \frac{x\,du}{dx}$$

$$\frac{dy}{dx} = \frac{x+3y}{3x+y}$$

$$u + \frac{x\,du}{dx} = \frac{x+3ux}{3x+ux}$$

$$u + \frac{x\,du}{dx} = \frac{1+3u}{3+u}$$

$$x\frac{dv}{dx} = \frac{1+3v}{3+v} - v.$$

$$x\frac{dv}{dx} = \frac{1-v^2}{3+v}.$$

$$\frac{(3+v)}{1-v^2}\,dv = \frac{dx}{x}.$$

$$\int \frac{3+v}{(1-v^2)}\,dv = \int \frac{dx}{x}.$$

$$\int \left(\frac{2}{(1-v)} + \frac{1}{(1+v)}\right) dv = \int \frac{dx}{x}$$

$$\int \frac{2}{1-v}\,dv + \int \frac{1}{1+v}\,dv = \ln x + \ln c.$$

$$-2\ln(1-v) + \ln(1+v) = \ln(x)$$

$$\ln(1-v)^{-2} + \ln(1+v) = \ln cx$$

$$\ln\frac{(1+v)}{(1-v)^2} = \ln cx$$

$$\frac{1+v}{(1-v)^2} = cx$$

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$$\frac{1+(y/x)}{(1-(y/x))^2} = cx$$

$$\frac{(x+y)\,x}{(x^2-y^2)^2 x^2} = cx$$

$$(x+y) = (x-y)^2 c$$

$$c(x+y) = (x-y)^2 \text{ sh}$$

(9) $-y\,dx + (x + \sqrt{xy})\,dy = 0$

Let $x = vy$

$$\frac{dx}{dy} = v + y\frac{dv}{dy}$$

$$dx = v\,dy + y\,dv$$

$$-y\,dx + (x + \sqrt{xy})\,dy = 0$$

$$-y\,[v\,dy + y\,dv] + (\cdot(vy) + \sqrt{(vy)\,y})\,dy = 0$$

$$-yv\,dy - y^2\,dv + vy\,dy + \sqrt{vy^2}\,dy = 0$$

$$-y^2\,dv = y\sqrt{v}\,dy$$

$$\frac{dv}{\sqrt{v}} = \frac{dy}{y}$$

$$\int \frac{du}{\sqrt{v}} = \int \frac{dy}{y}$$

$$2\sqrt{v} = \ln y + c.$$

$$2\sqrt{x/y} = \ln y + c.$$

$$4(x/y) = (\ln y + c)^2$$

$$4x = y(\ln y + c)^2$$

(10) $$x\frac{dy}{dx} = y + \sqrt{x^2 - y^2}.$$

$$y = ux.$$

$$dy = u\,dx + x\,du.$$

$$x\,dy = (y + \sqrt{x^2-y^2})\,dx$$

$$x\left[u\,dx + x\,du\right] = \left[ux + \sqrt{x^2 - u^2x^2}\right]dx,$$

$$ux\,dx + x^2\,du = ux\,dx + x\sqrt{1-u^2}\,dx.$$

$$x^2\,du = x\sqrt{1-x^2}\,dx.$$

$$\frac{du}{\sqrt{1-u^2}} = \frac{dx}{x}.$$

$$\int \frac{du}{\sqrt{1-u^2}} = \int \frac{dx}{x^2}$$

$\sin^{-1} u = \ln|x| + c$

$\sin^{-1}(y/x) = \ln x + c$

$\frac{y}{x} = \sin(\ln x + c)$.

$y = x \cdot \sin(\ln x + c)$.

(11) $xy^2 \frac{dy}{dx} = y^3 - x^3 \quad y(1) = 2$.

$xy^2 dy = (y^3 - x^3) dx$.

Let $y = ux$.

$dy = u dx + x du$.

$x(ux)^2 [u dx + x du] = ((ux)^3 - x^3) dx$.

$u^2 x^3 (u dx + x du) = (u^3 x^3 - x^3) dx$.

$x^3 (u^3 dx + u x^2 du) = x^3 (u^3 - 1) dx$

$u^3 dx + u^2 x du = (u^3 - 1) dx$.

$u^3 dx + (1 - u^3) dx = -u^2 x du$.

$(u^3 - u^3 + 1) dx = -u^2 x du$.

$dx = -u^2 x du$.

$$-\frac{du}{u} = u^2 dv$$

$$-\int \frac{du}{u} = \int u^2 du$$

$$-\ln u + c = \frac{u^3}{3}$$

$$-\ln(u) + c = \left(\frac{(y/x)^3}{3}\right)$$

$$-\ln x + c = \frac{y^3}{3x^3}$$

~~$\ln(x)$~~

$$y^3 = -3x^3 \ln(x) + 3x^3 c$$

$$(2)^3 = -3(1)^3 \ln(1) + 3(1)^3 c$$

$$8 = 3c$$

$$\boxed{c = 8/3}$$

$$y^3 = -3x^3 \ln(x) + 8x^3$$

$$y^3 + 3x^3 \ln(x) = 8x^3 \text{ Ans}$$

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