Name:- Kashif Ali

Section:- D.

①

Roll No:- 20P-0648

Assignment #04.

$y'' - (x+1)y' - y = 0$.

Solution.

We have differential equation.

$$y = \sum_{n=0}^{\infty} C_n x^n \rightarrow (i)$$

$$y' = \sum_{n=0}^{\infty} C_n n x^{n-1} \rightarrow (ii)$$

$$y'' = \sum_{n=0}^{\infty} C_n n(n-1) x^{n-2} \rightarrow (iii)$$

Putting values.

$$\sum_{n=2}^{\infty} C_n n(n-1) x^{n-2} - (x+1) \sum_{n=1}^{\infty} C_n n x^{n-1} - \sum_{n=0}^{\infty} C_n x^n = 0.$$

By making x^n.

$$\sum_{n=0}^{\infty} C_{n+2} (n+2)(n+1) x^n - \sum_{n=0}^{\infty} C_n . n x^n - \sum_{n=0}^{\infty} C_{n+1} (n+1) x^n - \sum_{n=0}^{\infty} C_n x^n = 0$$

$$= C_2(2)(4) x^0 - \sum_{n=0}^{\infty} C_n x^n - C_1(0) x^{\infty} - C_0(1)$$

$$= C_2(2)(1) x^0 + \sum_{n=1}^{\infty} C_{n+2} (n+2)(n+1) x^n - \sum_{n=1}^{\infty} C_n n x^n - C_1(1) x^n$$

$$- C_1(1) x^0 - \sum_{n=1}^{\infty} C(n+1)(n+1) x^n - C_0(1) - \sum_{n=1}^{\infty} C_n x^n = 0$$

$$2C_2 - C_1 - C_0 + \sum_{n=1}^{\infty} [C_{n+2}(n+2)(n+1)$$

$$- C_n n x^n - C_{n+1}(n+1)x^n - C_n x^n]$$

$$\Rightarrow 2C_2 - C_1 - C_0 = 0.$$

$$C_2 = \frac{C_1 + C_0}{2}$$

$$\Rightarrow C_{n+2}(n+2)(n+1) - C_n \cdot n - C_{n+1}(n+1) - C_n = 0$$

$$C_{n+2} = \frac{C_n(n+1) + (n+1)C_{n+1}}{(n+2)(n+1)}$$

$$C_{n+2} = \frac{C_n + C_{n+1}}{n+1}$$

when $n = 0$,

$C_2 = \frac{C_0 + C_1}{2}$, So both are Same.

$$C_3 = \frac{C_1 + \left(\frac{C_1 + C_0}{2}\right)}{3}$$

$$C_4 = \frac{C_2 + C_3}{4} = \frac{3C_1 + C_0}{6} = \frac{C_2 + C_3}{4}$$

$$C_4 = \left(\frac{C_1 + C_0}{2} + \frac{3C_1 + C_0}{6}\right) * 4$$

$$\frac{6C_1 + 4C_0}{24} = \frac{3C_1 + C_0}{12}.$$

Now,

$$y = C_n x^n \sum_{n=0}^{\infty}.$$

$$y = C_0 + C_1 x + C_2 x^2 + C_3 x^3 + C_4 x^4 \cdots$$

$$y = C_0 + C_1 x + \left(\frac{C_1 + C_0}{2}\right) x^2 + \left[\frac{3C_1 + C_0}{6}\right] x^3 + \left[\frac{3C_1 + 2C_0}{12}\right] x^4$$

$$= C_0\left(1 + \frac{1}{2}x^2 + \frac{1}{6}x^3 + \frac{1}{6}x^4\right) + C_1\left(x + \frac{1}{2}x^2 + \frac{1}{2}x^3 + \frac{1}{4}x^4 + \cdots\right)$$

Q No 2:- $(x^2+2)y'' + 3xy' - y = 0$.

Sol:-

$$y = \sum_{n=0}^{\infty} C_n x^n \Rightarrow y' = \sum_{n=1}^{\infty} C_n n x^{n-1}$$

$$y'' = \sum_{n=2}^{\infty} C_n n(n-1) x^{n-2}$$

$$= (x^2+2)\sum_{n=2}^{\infty} C_n n(n-1) x^{n-2} + 3x\sum_{n=1}^{\infty} C_n (n-1) x^{n-1} - \sum_{n=0} C_n x^n$$

$$= \sum_{n=2}^{\infty} C_n n(n-1) x^n + 2\sum_{n=1}^{\infty} C_n n (n-1) x^{n-2} + 3\sum_{n=1}^{\infty} C_n n x^n - \sum_{n=0} C_n x^n = 0$$

$$= \sum_{n=2}^{\infty} C_n n(n-1) x^n + 2C_2(2)(1)x^0 + 2(C_3 3(3-1))x^1 + 2\sum_{n=4}^{\infty} C_n n(n-1) x^{n-2} + 3C_1 x^1 + 3\sum_{n=2}^{\infty} C_n n x^n$$

$$- C_0 x^0 - C_1 x^1 - \sum_{n=2}^{\infty} C_n x^n = 0.$$

$$= \sum_{n=2}^{\infty} n(n-1) C_n x^n + 4C_2 + 12C_3 x + 2\sum_{n=2}^{\infty} C_{n+2}(n+2)(n+1) x^n + 3C_1 x + 3\sum_{n=2}^{\infty} C_n n x^n - C_0 x^0$$

$$= 4C_2 + 12C_3 x + 3C_1 x - C_0 x^0 - C_1 x^1 = 0.$$

$$4C_2 - C_0 = 0 \longrightarrow 1$$

$$12C_3 x + 2C_1 = 0 \longrightarrow 2$$

$$C_2 = \frac{C_0}{4}$$

$$C_3 = -\frac{C_1}{6}.$$

$$= x^n \sum_{n=2}^{\infty} C_n(n-1)C_n + C_{n+2}(n+1)C_{n+2} + C_n \cdot n x^{n-1}$$

$$= C_{n+2} = \frac{C_n(n^2-n)\,C_n(n-1)\,C_n}{2(n+2)(n+1)}$$

$C=2.$

$$= C_4 = \frac{C_2(1-2(1)-3(2))}{2(4)(3)} = \frac{-7C_2}{24}$$

$$= \frac{-7(C_0/4)}{24} = \frac{-7C_0}{96}.$$

$C=3.$

$$C_5 = \frac{C_3(1-3(2)-3(3))}{2(5)(4)} = \frac{-14C_3}{40} = \frac{-14(-C_1/6)}{60} = \frac{+7C_1}{180}.$$

$$y(x) = C_0 + C_1 x + \frac{1}{4}C_2 x^2 - \frac{1}{6}C_1 x^3 - \frac{7}{4.41}C_0 x^4 + \frac{14}{2.51}$$

$$= C_0\left[1 + \frac{1}{4}x^2 - \frac{7}{4.41}x^4 + \frac{23.7}{18.61}x^6\right].$$

$$y_2(x) = C_1\left[x - \frac{1}{6}x^3 + \frac{14}{2.51}x^5 - \frac{34.14}{71}x^7\right].$$