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Assignment # 02

Q No 01

$y'' + 3y' + 2y = \sin e^{x}$

$m^2 + 3m + 2 = 0$

$(m+2)(m+1) = 0$

$m_1 = -1, \ m_2 = -2$

$y_c = c_1 e^{-x} + c_2 e^{-2x}$

$$W = \begin{bmatrix} e^{-x} & e^{-2x} \\ -e^{-x} & -2e^{-2x} \end{bmatrix}$$

$W = -2e^{-3x} + e^{-3x}$

$W = -e^{-3x}$

$$v_1' = \frac{-(\sin(e^{x}))(e^{-2x})}{-e^{-3x}}$$

$v_1' = e^{x}\sin(e^{x})$

$\int v_1' = \int e^{x} \sin(e^{x})\,dx$

By v- Substitution

$v_1 = \cos(e^{x})$

$$v_2' = \frac{\sin(e^{x})(e^{-x})}{e^{-3x}}$$

$v_2' = -e^{-2x} \sin(e^{x})$

$\int v_2' = \int(-e^{-2x}(\sin(e^{x})))$

$v_2 = -\int e^{-2x} \sin(e^{x})$

By integration by Parts

$v_2 = e^{x}(\cos e^{x}) - \sin e^{x}$

$y_p = v_1 y_1 + v_2 y_2$

$y_p = \cancel{-e^{-x}\cos(e^{x})} + \cancel{e^{-x}\cos(e^{x})} - e^{-2x}\sin e^{x}$

$y_p = -e^{-2x}\sin(e^{x})$

$y = y_c + y_p$

$(c_1 e^{-x} + c_2 e^{-2x}) + (-e^{-2x}\sin(e^{x}))$

$y = c_1 e^{-x} + c_2 e^{-2x} - e^{-2x}\sin(e^{x})$

Ans

Q No 2. (2)

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$y'' + 2y' + y = e^{-x} \ln t$

Let-

$y = 1$

$y' = x$

$y'' = x^2$

$x^2 + 2x + 1 = 0$

$(x+1)^2$

$(x+1)(x+1) = 0$

$x_1 = -1, x_2 = -1$

$y_1(t) = e^{-t}, \quad y_2(t) = te^{-t}$

$$W = \begin{bmatrix} e^{-t} & te^{-t} \\ -e^{-t} & e^{-t} - te^{-t} \end{bmatrix}$$

$= e^{-2t} - te^{-2t} + te^{-2t}$

$W = e^{-2t}$

$$\int v_1' = \int \frac{(te^{-t})(e^{-t}\ln t)}{e^{-2t}} dt \qquad v_1 = -\frac{t^2 \ln t}{2} + \frac{t^2}{4}$$

$$v_1 = -\int \frac{te^{-2t} \ln t}{e^{-2t}}$$

$$-\ln t \int t\,dt + \int \frac{d(\ln t)}{dt} \int t\,dt \int dt$$

$$-\ln t \cdot \frac{t^2}{2} + \int \frac{1}{t} \cdot \frac{t^2}{2} dt$$

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$$\int v_2' = \int \frac{(e^{-t})(e^{-t}\ln t)}{e^{-2t}}\,dt$$

$$= \int \frac{e^{-2t}\ln t}{e^{-2t}}\,dt$$

$$= \int \ln t\,dt$$

$$= \ln t \int dt - \int \left[\frac{d(\ln t)}{dt}\int dt\right] dt$$

$$= \ln t \cdot t - \int \frac{1}{\cancel{t}}\cdot\cancel{t}\,dt$$

$$= t\cdot\ln t - \int dt$$

$$v_2 = t\cdot\ln t - t$$

$$y_p = v_1 y_1 + v_2 y_2$$

$$\left(-\frac{t^2\ln t}{2} + \frac{t^2}{4}\right)(e^{-t}) + (t\ln t - t)\cdot(te^{-t})$$

$$\left(-\frac{t^2\ln t}{2} + \frac{t^2}{4}\right)(e^{-t}) + (t^2\ln t - t^2)(e^{-t})$$

$$e^{-t}\left(-\frac{t^2\ln t}{2} + \frac{t^2}{4} + t^2\ln t - t^2\right)$$

$$e^{-t}\left(-\frac{t^2\ln t}{2} + t^2\ln t + \frac{t^2}{4} - t^2\right)$$

$$y_p = e^{-t}\left(\frac{t^2\ln t}{2} + \frac{3t^2}{4}\right)$$

$$y = y_p + y_c$$

$$y = (e^{-t})\left(\frac{t^2\ln t}{2} + \frac{3t^2}{4}\right) + c_1 e^{-t} + c_2 t\cdot e^{-t}$$

Q No 3. (4)

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$3y'' - 6y' + 6y = e^x \sec x$.

$3(y'' - 2y' + 2) = e^x \sec x$

$y'' - 2y' + 2 = \frac{1}{3} e^x \sec x$

let-

$y = 1$

$y' = x$

$y'' = x^2$

$\sqrt{b^2 - 4ac}$

$\sqrt{(2)^2 - 4 \times 1 \times 2}$

$\sqrt{4-8}$

$\sqrt{-4}$

$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\frac{+2 + \sqrt{-4}}{2}$

$\frac{+2 \pm 2\sqrt{-1}}{2}$

$\frac{+\cancel{2}(1 \pm \sqrt{-1})}{\cancel{2}}$

$x^2 - 2x + 2 = 0$

$x = 1 \pm i$

$y = C_1 (e^{1+i})x + C_2 e^{(1-i)x}$

$C_1 e^x (\cos x + \sin x) + C_2 e^x (\cos x - \sin x)$

$(C_1 + C_2) e^x \cos x + (C_1 - C_2) e^x \sin x$.

$C_1 + C_2 = C_1$ and $C_1 - C_2 = C_2$.

$y_c = e^x (C_1 \cos x + C_2 \sin x)$.

$$w = \begin{bmatrix} e^x \cos(x) & e^x \sin(x) \\ e^x \cos(x) - e^x \sin(x) & e^x \sin(x) + e^x \cos(x) \end{bmatrix}$$

$= e^{2x} \sin(x) \cos(x) + e^{2x} \cos^2(x) - e^{2x} \sin(x) \cos(x) + e^{2x} \sin^2(x)$.

$= e^{2x} (\cos^2(x) + \sin^2(x))$

$w = e^{2x}(1) = e^{2x}$.

$\int v_1' = \int -\frac{1}{3} e^{2x} \frac{\sec(x) \cdot \sin(x)}{e^{2x}} dx$.

$= -\frac{1}{3} \int \tan(x) dx$.

$-\frac{1}{3} \ln(\sec(x))$.

$V_1 = \frac{1}{3} \ln(\cos(x))$. (5)

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$$\int V_2' = \int \frac{1}{3} \frac{e^{2x} \sec(x) \cos(x)}{e^{2x}}$$

$$= \frac{1}{3} \int \frac{e^{2x} \sec(x) . \cos(x)}{e^{2x}}$$

$$V_2 = \frac{1}{3} x.$$

$$y_p = V_1 y_1 + V_2 y_2.$$

$$y_p = \left(\frac{1}{3} e^x \cos x\right) \left(\ln \cos(x) + \frac{1}{3} x e^x \sin(x)\right).$$

$$y = y_p + y_c$$

$$\left(\frac{1}{3} e^x \cos(x)\right)\left(\ln \cos(x)\right) + \frac{1}{3} x e^x \sin(x) + e^x \left(c_1 \cos(x) + c_2 \sin(x)\right)$$

Ans

Q NO4.

$$4y'' - y = x e^{x/2}.$$

$$y'' - \frac{1}{4} y = \frac{x e^{x/2}}{4}$$

$$y'' - \frac{1}{4} y = 0$$

$$y = e^{mx}.$$

$$y' = m e^{mx}.$$

$$y'' = m^2 e^{mx}$$

$$m^2 e^{mx} - \frac{1}{4} e^{mx} = 0.$$

$$e^{mx} \left(m^2 - \frac{1}{4}\right) = 0$$

$$m^2 - \frac{1}{4} = 0$$

$$m = \pm 1/2.$$

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$y_c = c_1 y_1 + c_2 y_2$

$y_c = c_1 e^{x/2} + c_2 e^{-x/2}$

$$W = \begin{bmatrix} e^{x/2} & e^{-x/2} \\ \frac{1}{2}e^{x/2} & -\frac{1}{2}e^{-x/2} \end{bmatrix}$$

$-\frac{1}{2} - \frac{1}{2} = -1$

$$W(e^{-x/2}) = \begin{bmatrix} e^{x/2} & 0 \\ \frac{1}{2}e^{x/2} & \frac{1}{4}xe^{x/2} \end{bmatrix}$$

$= \frac{1}{4}xe^{x}$

$$V_1' = \frac{W(e^{x/2})}{W} = \frac{-\frac{1}{4}x}{-1} = \frac{1}{4}x$$

$\int V_1' = \int \frac{1}{4}x\,dx$

$V_1 = \frac{1}{8}x^2$

$$V_2' = \frac{W(e^{-x/2})}{W} = \frac{\frac{1}{4}x}{-1} = -\frac{1}{4}x$$

$\int V_2' = \int -\frac{1}{4}xe^{x}dx$

$-\frac{1}{4}\int xe^{x}dx$

$V_2 = -\frac{1}{4}e^{x}(x-1)$

$y_p = V_1y_1 + V_2y_2$

$\left(\frac{1}{8}x^2\right)(e^{x/2}) - \frac{1}{4}e^{x}(x-1)(e^{-x/2})$

$y_p = \frac{1}{8}x^2e^{x/2} - \frac{1}{4}xe^{x/2} + \frac{1}{4}e^{x/2}$

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$y = y_p + y_c$

$= \frac{1}{8} x^2 e^{x/2} - \frac{1}{4} x e^{x/2} + \frac{1}{4} e^{x/2} + c_1 e^{x/2} + c_2 e^{-x/2}$.

Q NO 5

$y'' + 2y' - 8y = 2e^{-2x} - e^{-x}$.

$y = e^{mx}$.

$y' = m e^{mx}$.

$y'' = m^2 e^{mx}$.

$m^2 e^{mx} + 2(m e^{mx}) - 8(e^{mx}) = 0$

$e^{mx}(m^2 + 2m - 8) = 0$.

$m^2 + 2m - 8 = 0$.

$(m-2)(m+4) = 0$.

$m_1 = 2, \; m_2 = -4$

$y_c = c_1 e^{2x} + c_2 e^{-4x}$.

$$W = \begin{bmatrix} e^{2x} & e^{-4x} \\ 2e^{2x} & -4e^{-4x} \end{bmatrix}$$

$W = -4e^{-2x} - 2e^{-2x} = -6e^{-2x}$.

$$W(e^{2x}) = \begin{bmatrix} 0 & e^{-4x} \\ 2e^{-2x} - e^{-x} & -4e^{-4x} \end{bmatrix}$$

$W(e^{2x}) = -2e^{-6x} + e^{-5x}$.

$$W(e^{-4x}) = \begin{bmatrix} e^{2x} & 0 \\ 2e^{2x} & 2e^{-2x} - e^{-x} \end{bmatrix}$$

$W(e^{-4x}) = 2 - e^{x}$.

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$$V_1' = \frac{W(e^{2x})}{W} = \frac{-2e^{-6x} + e^{-5x}}{-6e^{-2x}} = \frac{1}{3}e^{-4x} - \frac{1}{6}e^{-3x}$$

$$\int V_1' = \int \left(\frac{1}{3}e^{-4x} - \frac{1}{6}e^{-3x}\right) dx$$

$$V_1 = -\frac{1}{12}e^{-4} + \frac{1}{18}e^{-3x}$$

$$V_2' = \frac{W(e^{-4x})}{W} = \frac{2 - e^{x}}{-6e^{-2x}} = \frac{1}{3}e^{2x} + \frac{1}{6}e^{3x}$$

$$\int V_2' = \int \left(\frac{1}{3}e^{2x} + \frac{1}{6}e^{3x}\right) dx$$

$$V_2 = -\frac{1}{6}e^{2x} + \frac{1}{18}e^{3x}$$

$$y_p = V_1 y_1 + V_2 y_2$$

$$\left(-\frac{1}{12}e^{-4x} + \frac{1}{18}e^{-3x}\right)(e^{2x}) + \left(-\frac{1}{6}e^{2x} + \frac{1}{18}e^{3x}\right)(e^{-4x})$$

$$= -\frac{1}{12}e^{-2x} + \frac{1}{18}e^{-x} - \frac{1}{16}e^{-2x} + \frac{1}{18}e^{-x}$$

$$y_p = -\frac{1}{4}e^{-2x} + \frac{1}{9}e^{-x}$$

$$y = y_c + y_p$$

$$c_1 e^{2x} + c_1 e^{-4x} + \left(-\frac{1}{4}e^{-2x} + \frac{1}{9}e^{-x}\right)$$