CMPE 257:03 -Homework 1 Learning from data Problem 1.4 and 1.5

(a) Generate a linearly separated data set of size 20. Plot the examples (xn,yn) as well as target function f on a plane. Be sure to mark the examples from different classes differently and add lables to the plot.

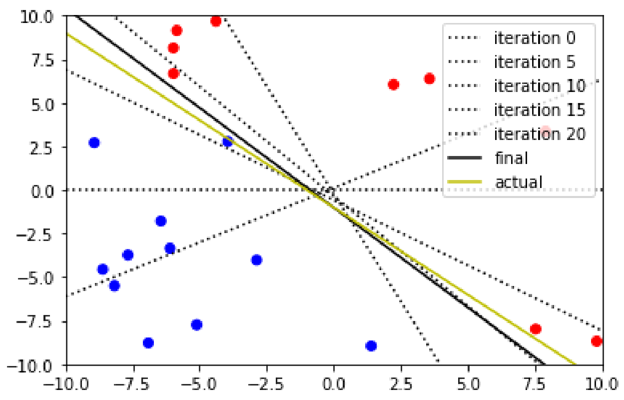
(b) Run the perceptron learning algorithm on the data set above. Report the number of updates that the algorithm takes before converging. plot the examples {(xn,yn)}, the target function f, and the final hypothesis g in the same figure. Comment on wheteher f is close to g.

In [26]:

```
import matplotlib.pyplot as plt
import numpy as np

# generate a linearly separated data of size 20-

w_act = np.array([1, 1, 1])
w_init = np.array([3, -50, 0])
d = 2
num_sample = 20
data_range = 10

X_train = np.vstack([
    np.ones(num_sample),
    np.random.uniform(
        -data_range,
        data_range,
        (d, num_sample),
    ),
])

y_train = evaluate_h(w_act, X_train)
x_axis = X_train[1, :]
y_axis = X_train[2, :]
lables = ['r' if y > 0 else 'b' for y in y_train]
plt.scatter(x_axis, y_axis, c=lables)
x_hypothesis = np.array([-data_range, data_range])

# check if the hypothesis h from the hypothesis set H is following the weight and dimensionality c
ondition
def evaluate_h(w, X):

    '''
    Evaluate the hypothesis, h \in H, which is described by its set of weights,
    w, against 1 or more points in the input space (stored in a matrix).
    '''
    assert len(w.shape) == 1
    assert len(X.shape) == 2
    assert w.shape[0] == X.shape[0]
    return np.sign(w @ X)

# Run perceptron learning algorithm for the training data set
def run_perceptron(w_init, X_train, y_train, iteration_callback=None):
    w = w_init.copy()
    n = 0
    while True:
        y = evaluate_h(w, X_train)
        if iteration_callback:
            iteration_callback(n, w)
        correct = y == y_train
        if np.all(correct):
# Print the total number of iterations that let to the generation of final hypothesis g
            print('Total number of iterations: ',n)
            return w
        else:
            i = np.argmax(~correct)  # indice of first misclassified point
            w = w + y_train[i]*X_train[:, i]
        n = n + 1


def plot_hypothesis(x_axis, w, *plot_args, **plot_kwargs):
    m = -w[1]/w[2] if w[2] != 0 else 0
```

```
    b = -w[0]/w[2] if w[2] != 0 else 0
    y_axis = m*x_axis + b
    plt.plot(x_axis, y_axis, *plot_args, **plot_kwargs)

# Plot the hypothesis for all the iterations
def plot_iteration(n, w):
    if n % 5 == 0:
        label1 = 'iteration {}'.format(n)
        plot_hypothesis(x_hypothesis, w, 'k:', label=label1)

w_final = run_perceptron(w_init, X_train, y_train, plot_iteration)
plot_hypothesis(x_hypothesis, w_final, 'k', label='final')
plot_hypothesis(x_hypothesis, w_act, 'y', label='actual')
plt.legend()
plt.xlim(-data_range, data_range)
plt.ylim(-data_range, data_range)
plt.show()
```

```
Total number of iterations:  24
```



In the above case, we can see that it took 24 iterations for the generation of final hypothesis g. As we can see that the final hyperplane represented by the black line is separating the points perfectly and there are no misclassified points. Hence, f is close to g.

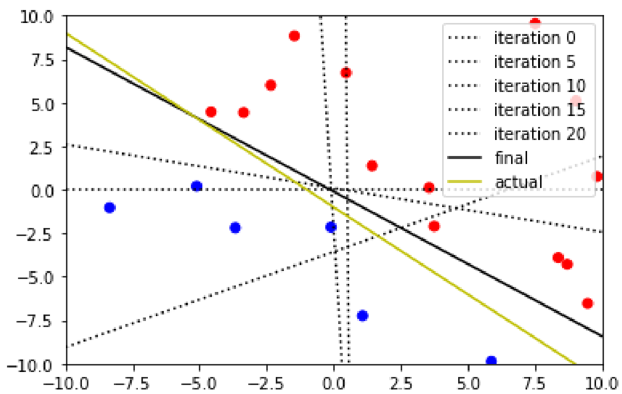
(c) Repeat the step with a different set of randonly generated data of size 20 and compare the results with (b)

In [27]:

```
import matplotlib.pyplot as plt
import numpy as np

# generate a linearly separated data of size 20-

w_act = np.array([1, 1, 1])
w_init = np.array([3, -50, 0])
d = 2
num_sample = 20
data_range = 10

X_train = np.vstack([
    np.ones(num_sample),
    np.random.uniform(
        -data_range,
        data_range,
        (d, num_sample),
    ),
])

y_train = evaluate_h(w_act, X_train)
x_axis = X_train[1, :]
y_axis = X_train[2, :]
lables = ['r' if y > 0 else 'b' for y in y_train]
plt.scatter(x_axis, y_axis, c=lables)
x_hypothesis = np.array([-data_range, data_range])

# check if the hypothesis h from the hypothesis set H is following the weight and dimensionality c
ondition
```

```
def evaluate_h(w, X):
    '''
    Evaluate the hypothesis, h \in H, which is described by its set of weights,
    w, against 1 or more points in the input space (stored in a matrix).
    '''
    assert len(w.shape) == 1
    assert len(X.shape) == 2
    assert w.shape[0] == X.shape[0]
    return np.sign(w @ X)

# Run perceptron learning algorithm for the training data set
def run_perceptron(w_init, X_train, y_train, iteration_callback=None):
    w = w_init.copy()
    n = 0
    while True:
        y = evaluate_h(w, X_train)
        if iteration_callback:
            iteration_callback(n, w)
        correct = y == y_train
        if np.all(correct):
# Print the total number of iterations that let to the generation of final hypothesis g
            print('Total number of iterations: ',n)
            return w
        else:
            i = np.argmax(~correct)  # indice of first misclassified point
            w = w + y_train[i]*X_train[:, i]
        n = n + 1


def plot_hypothesis(x_axis, w, *plot_args, **plot_kwargs):
    m = -w[1]/w[2] if w[2] != 0 else 0
    b = -w[0]/w[2] if w[2] != 0 else 0
    y_axis = m*x_axis + b
    plt.plot(x_axis, y_axis, *plot_args, **plot_kwargs)

# Plot the hypothesis for all the iterations
def plot_iteration(n, w):
    if n % 5 == 0:
        label1 = 'iteration {}'.format(n)
        plot_hypothesis(x_hypothesis, w, 'k:', label=label1)

w_final = run_perceptron(w_init, X_train, y_train, plot_iteration)
plot_hypothesis(x_hypothesis, w_final, 'k', label='final')
plot_hypothesis(x_hypothesis, w_act, 'y', label='actual')
plt.legend()
plt.xlim(-data_range, data_range)
plt.ylim(-data_range, data_range)
plt.show()
```

```
Total number of iterations:  22
```



In the above plot, we can see that with a new set of data, we are able to generate a good approximation of final hypothesis and it takes 22 iterations to classify the points correctly in this case.

(d) Repeat the step with a different set of randonly generated data of size 100 and compare the results with (b)

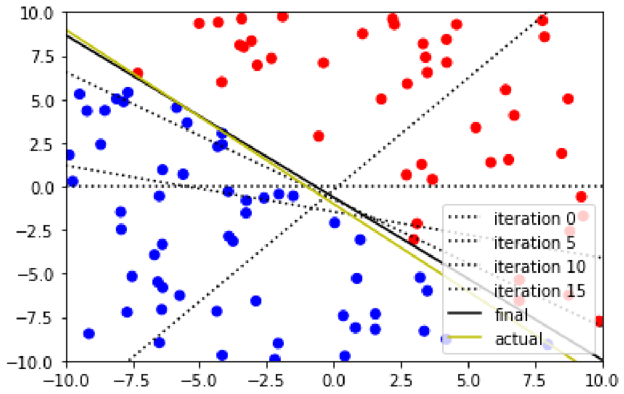
In [31]:

In [31]:

```
import matplotlib.pyplot as plt
import numpy as np

# generate a linearly separated data of size 100-

w_act = np.array([1, 1, 1])
w_init = np.array([3, -50, 0])
d = 2
num_sample = 100
data_range = 10

X_train = np.vstack([
    np.ones(num_sample),
    np.random.uniform(
        -data_range,
        data_range,
        (d, num_sample),
    ),
])

y_train = evaluate_h(w_act, X_train)
x_axis = X_train[1, :]
y_axis = X_train[2, :]
lables = ['r' if y > 0 else 'b' for y in y_train]
plt.scatter(x_axis, y_axis, c=lables)
x_hypothesis = np.array([-data_range, data_range])

# check if the hypothesis h from the hypothesis set H is following the weight and dimensionality c
ondition
def evaluate_h(w, X):

    '''
    Evaluate the hypothesis, h \in H, which is described by its set of weights,
    w, against 1 or more points in the input space (stored in a matrix).
    '''
    assert len(w.shape) == 1
    assert len(X.shape) == 2
    assert w.shape[0] == X.shape[0]
    return np.sign(w @ X)

# Run perceptron learning algorithm for the training data set
def run_perceptron(w_init, X_train, y_train, iteration_callback=None):
    w = w_init.copy()
    n = 0
    while True:
        y = evaluate_h(w, X_train)
        if iteration_callback:
            iteration_callback(n, w)
        correct = y == y_train
        if np.all(correct):
# Print the total number of iterations that let to the generation of final hypothesis g
            print('Total number of iterations: ',n)
            return w
        else:
            i = np.argmax(~correct)  # indice of first misclassified point
            w = w + y_train[i]*X_train[:, i]
        n = n + 1


def plot_hypothesis(x_axis, w, *plot_args, **plot_kwargs):
    m = -w[1]/w[2] if w[2] != 0 else 0
    b = -w[0]/w[2] if w[2] != 0 else 0
    y_axis = m*x_axis + b
    plt.plot(x_axis, y_axis, *plot_args, **plot_kwargs)

# Plot the hypothesis for all the iterations
def plot_iteration(n, w):
    if n % 5 == 0:
        label1 = 'iteration {}'.format(n)
        plot_hypothesis(x_hypothesis, w, 'k:', label=label1)

w_final = run_perceptron(w_init, X_train, y_train, plot_iteration)
plot_hypothesis(x_hypothesis, w_final, 'k', label='final')
plot_hypothesis(x_hypothesis, w_act, 'y', label='actual')
plt.legend()
plt.xlim(-data_range, data_range)
```

```
plt.ylim(-data_range, data_range)
plt.show()
```

```
Total number of iterations:  19
```



In the above plot, we can see that with a new set of 100 data, we are able to generate a good approximation of final hypothesis and it takes 19 iterations to classify the points correctly in this case.

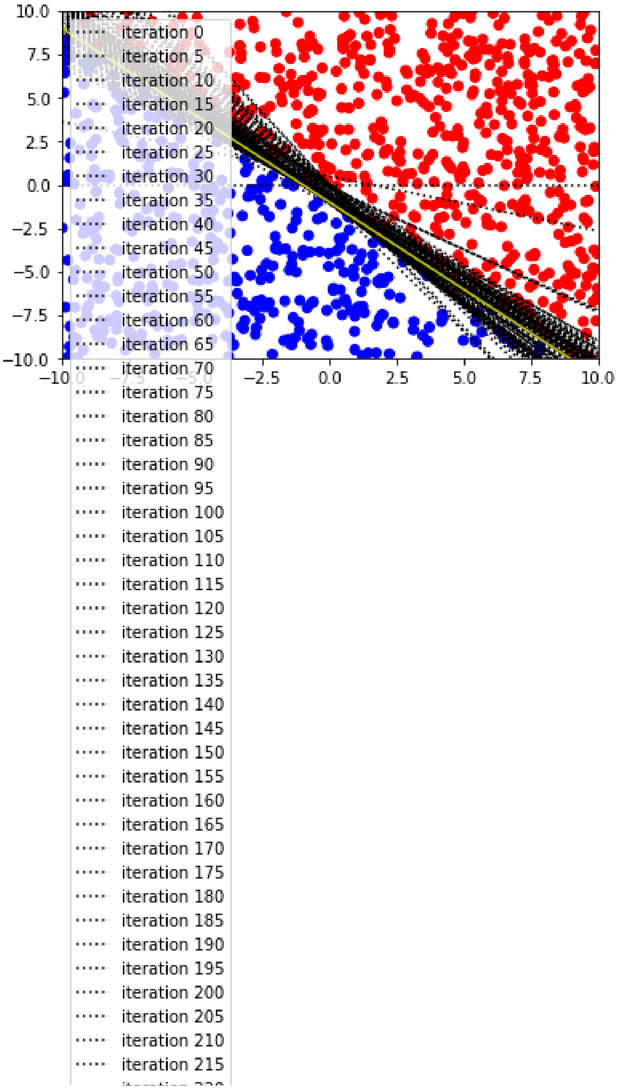
(e)Repeat the step with a different set of randonly generated data of size 1000 and compare the results with (b)

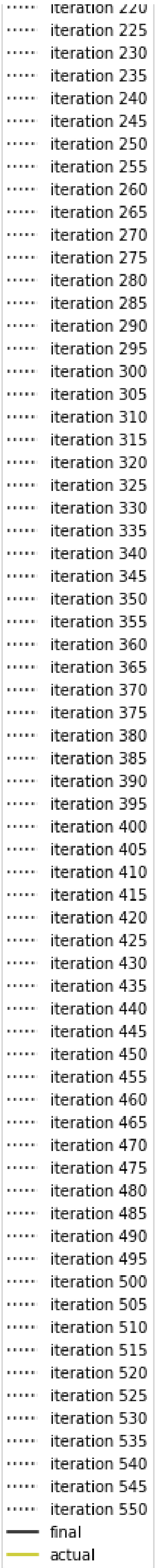
In [32]:

```
import matplotlib.pyplot as plt
import numpy as np

# generate a linearly separated data of size 1000-

w_act = np.array([1, 1, 1])
w_init = np.array([3, -50, 0])
d = 2
num_sample = 1000
data_range = 10

X_train = np.vstack([
    np.ones(num_sample),
    np.random.uniform(
        -data_range,
        data_range,
        (d, num_sample),
    ),
])

y_train = evaluate_h(w_act, X_train)
x_axis = X_train[1, :]
y_axis = X_train[2, :]
lables = ['r' if y > 0 else 'b' for y in y_train]
plt.scatter(x_axis, y_axis, c=lables)
x_hypothesis = np.array([-data_range, data_range])

# check if the hypothesis h from the hypothesis set H is following the weight and dimensionality c
ondition
def evaluate_h(w, X):

    '''
    Evaluate the hypothesis, h \in H, which is described by its set of weights,
    w, against 1 or more points in the input space (stored in a matrix).
    '''
    assert len(w.shape) == 1
    assert len(X.shape) == 2
    assert w.shape[0] == X.shape[0]
    return np.sign(w @ X)

# Run perceptron learning algorithm for the training data set
def run_perceptron(w_init, X_train, y_train, iteration_callback=None):
    w = w_init.copy()
    n = 0
    while True:
```

```python
        y = evaluate_h(w, X_train)
        if iteration_callback:
            iteration_callback(n, w)
        correct = y == y_train
        if np.all(correct):
# Print the total number of iterations that let to the generation of final hypothesis g
            print('Total number of iterations: ',n)
            return w
        else:
            i = np.argmax(~correct)   # indice of first misclassified point
            w = w + y_train[i]*X_train[:, i]
        n = n + 1


def plot_hypothesis(x_axis, w, *plot_args, **plot_kwargs):
    m = -w[1]/w[2] if w[2] != 0 else 0
    b = -w[0]/w[2] if w[2] != 0 else 0
    y_axis = m*x_axis + b
    plt.plot(x_axis, y_axis, *plot_args, **plot_kwargs)

# Plot the hypothesis for all the iterations
def plot_iteration(n, w):
    if n % 5 == 0:
        label1 = 'iteration {}'.format(n)
        plot_hypothesis(x_hypothesis, w, 'k:', label=label1)

w_final = run_perceptron(w_init, X_train, y_train, plot_iteration)
plot_hypothesis(x_hypothesis, w_final, 'k', label='final')
plot_hypothesis(x_hypothesis, w_act, 'y', label='actual')
plt.legend()
plt.xlim(-data_range, data_range)
plt.ylim(-data_range, data_range)
plt.show()
```

```
Total number of iterations:  554
```





In the above plot, we can see that with a new set of 100 data, we are able to generate a good approximation of final hypothesis and it takes 554 iterations to classify the points correctly in this case.

Problem 1.5 - The perceptron learning algorithm works like this: in each iteration t, pick a randon (x(t), y(t)) and compute the 'signal' s(t) = wT(t) x(t). if y(t).s(t) <=0, update w by w(t+1) <- w(t) + y(t). x(t);

One may say that the algorithm does not take the closeness between s(t) and y(t) into account. so if y(t). s(t) <= 1, update w by : w(t+1) <= w(t) + n (y(t) - s(t)).x(t)

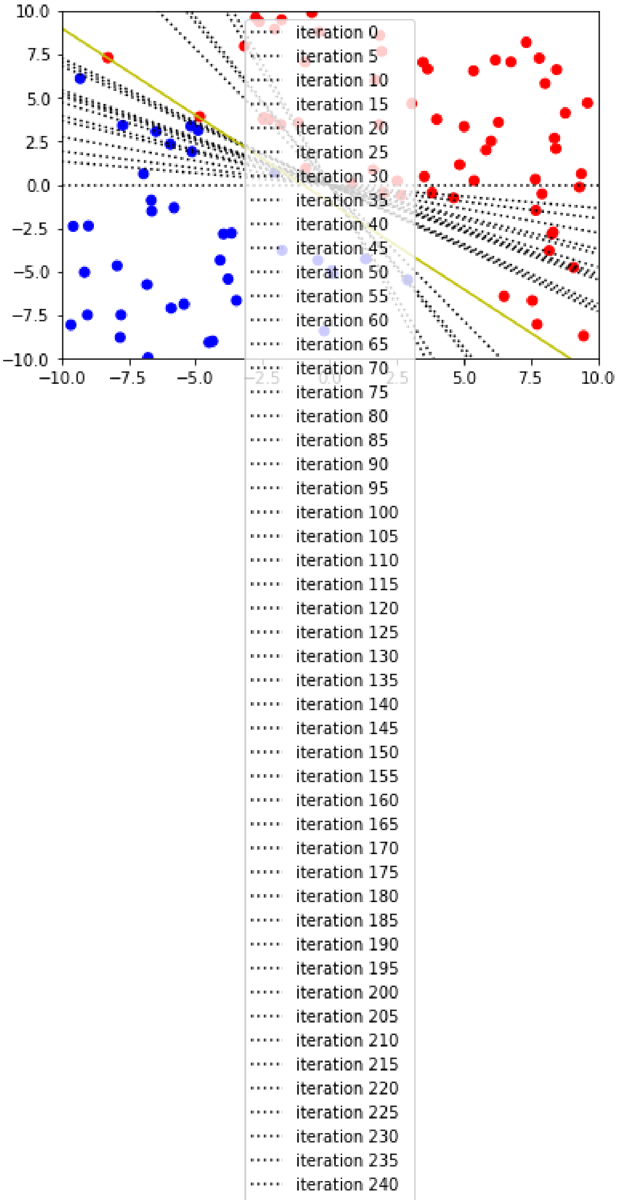
(a) Generate training set of 100 and test set of 1000. run the algorithm by taking n =100 on training set until 1000 updates. plot training set and target function. Report the error on test set

In [104]:

```
import matplotlib.pyplot as plt
import numpy as np
%matplotlib inline

# generate a linearly separated training data of size 100 and test data of size 1000 -
w_act = np.array([1, 1, 1])
w_init = np.array([3, -50, 0])
d = 2
num_sample = 100
data_range = 10
num_sample_test =1000
error = 0
# check if the hypothesis h from the hypothesis set H is following the weight and dimensionality c
ondition
def evaluate_h(w, X):

    '''
    Evaluate the hypothesis, h \in H, which is described by its set of weights,
    w, against 1 or more points in the input space (stored in a matrix).
    '''
    assert len(w.shape) == 1
    assert len(X.shape) == 2
    assert w.shape[0] == X.shape[0]
    return np.sign(w @ X)
X_train = np.vstack([
    np.ones(num_sample),
    np.random.uniform(
        -data_range,
        data_range,
        (d, num_sample),
    ),
])

X_test = np.vstack([
    np.ones(num_sample_test),
    np.random.uniform(
        -data_range,
        data_range,
        (d, num_sample_test),
    ),
])

y_train = evaluate_h(w_act, X_train)
y_test = evaluate_h(w_act, X_test)
x_axis = X_train[1, :]
y_axis = X_train[2, :]
lables = ['r' if y > 0 else 'b' for y in y_train]
plt.scatter(x_axis, y_axis, c=lables)
x_hypothesis = np.array([-data_range, data_range])


# Run perceptron learning algorithm for the training data set
def run_perceptron(w_init, X_train, y_train, iteration_callback=None):
    w = w_init.copy()
    n = 0
    while True:
        y = evaluate_h(w, X_train)
        if iteration_callback:
            iteration_callback(n, w)
        correct = y == y_train
# Calculate error after 1000 updates
        if n == 1000:
            return w

        else:
# calculate weight considering the closeness between s and y_train
            i = np.argmax(~correct)  # indice of first misclassified point
            s = w * X_train[:,i]
            d = y_train[i] * s
            d = d[0] + d[1] + d[2]
            if d <=1:
                w = w + 100 * (y_train[i] - s) * X_train[:, i]
                error = (y_train[i] - s)

        n = n + 1
```

```python
def plot_hypothesis(x_axis, w, *plot_args, **plot_kwargs):
    m = -w[1]/w[2] if w[2] != 0 else 0
    b = -w[0]/w[2] if w[2] != 0 else 0
    y_axis = m*x_axis + b
    plt.plot(x_axis, y_axis, *plot_args, **plot_kwargs)

# Plot the hypothesis for all the iterations
def plot_iteration(n, w):
    if n % 5 == 0:
        label1 = 'iteration {}'.format(n)
        plot_hypothesis(x_hypothesis, w, 'k:', label=label1)

w_final = run_perceptron(w_init, X_train, y_train, plot_iteration)
plot_hypothesis(x_hypothesis, w_final, 'k', label='final')
plot_hypothesis(x_hypothesis, w_act, 'y', label='actual')
plt.legend()
plt.xlim(-data_range, data_range)
plt.ylim(-data_range, data_range)
plt.show()
```

```
C:\Users\kashi\Anaconda3\lib\site-packages\ipykernel_launcher.py:71: RuntimeWarning: overflow
encountered in multiply
C:\Users\kashi\Anaconda3\lib\site-packages\ipykernel_launcher.py:23: RuntimeWarning: invalid value
encountered in sign
C:\Users\kashi\Anaconda3\lib\site-packages\ipykernel_launcher.py:78: RuntimeWarning: invalid value
encountered in double_scalars
C:\Users\kashi\Anaconda3\lib\site-packages\ipykernel_launcher.py:71: RuntimeWarning: invalid value
encountered in add
```



····· iteration 245
····· iteration 250
····· iteration 255
····· iteration 260
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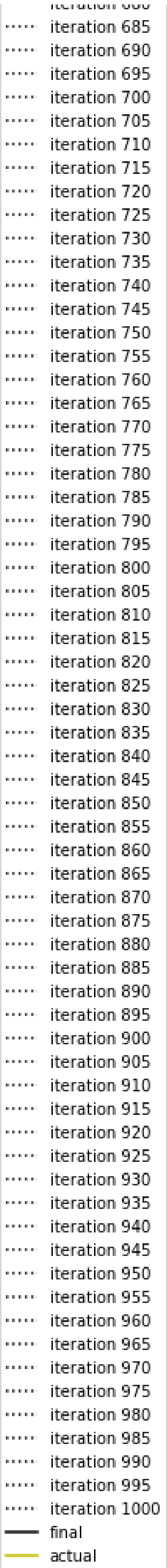
····· iteration 675
····· iteration 680



The plot above is the plot for training data set for n = 100.

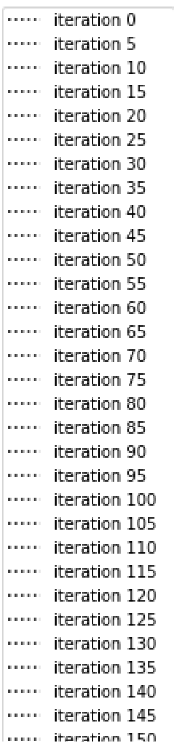
In [111]:

```
# Run perceptron learning algorithm for the test data set
def run_perceptron(w_init, X_train, y_train, iteration_callback=None):
    w = w_init.copy()
    n = 0
    while True:
        y = evaluate_h(w, X_train)
        if iteration_callback:
            iteration_callback(n, w)
        correct = y == y_train
#        if np.all(correct):
# Print the total number of iterations that let to the generation of final hypothesis g
```

```
#             print('Total number of iterations: ',n)
#             return w
        if n == 1000:
            error = np.absolute(error)
            print("Error on test data for n = 100: ",error[0]/1000)
            return w

        else:

            i = np.argmax(~correct)  # indice of first misclassified point
            s = w * X_train[:,i]
            d = y_train[i] * s
            d = d[0] + d[1] + d[2]
            if d <=1:
                w = w + 100 * (y_train[i] - s) * X_train[:, i]
                error = (y_train[i] - s)

        n = n + 1
x_axis = X_test[1, :]
y_axis = X_test[2, :]
lables = ['r' if y > 0 else 'b' for y in y_test]
plt.scatter(x_axis, y_axis, c=lables)
x_hypothesis = np.array([-data_range, data_range])

# Plot the hypothesis for all the iterations
def plot_iteration(n, w):
    if n % 5 == 0:
        label1 = 'iteration {}'.format(n)
        plot_hypothesis(x_hypothesis, w, 'k:', label=label1)
# Run Perceptron learning algorith for test data set

w_final = run_perceptron(w_init, X_test, y_test, plot_iteration)
plot_hypothesis(x_hypothesis, w_final, 'k', label='final test')
plot_hypothesis(x_hypothesis, w_act, 'y', label='actual test')
plt.legend()
plt.xlim(-data_range, data_range)
plt.ylim(-data_range, data_range)
plt.show()
```

```
C:\Users\kashi\Anaconda3\lib\site-packages\ipykernel_launcher.py:27: RuntimeWarning: overflow encountered in multiply
C:\Users\kashi\Anaconda3\lib\site-packages\ipykernel_launcher.py:27: RuntimeWarning: invalid value encountered in add
C:\Users\kashi\Anaconda3\lib\site-packages\ipykernel_launcher.py:23: RuntimeWarning: invalid value encountered in sign
```

```
Error on test data for n = 100:  8.424947340550924e+166
```



····· iteration 155
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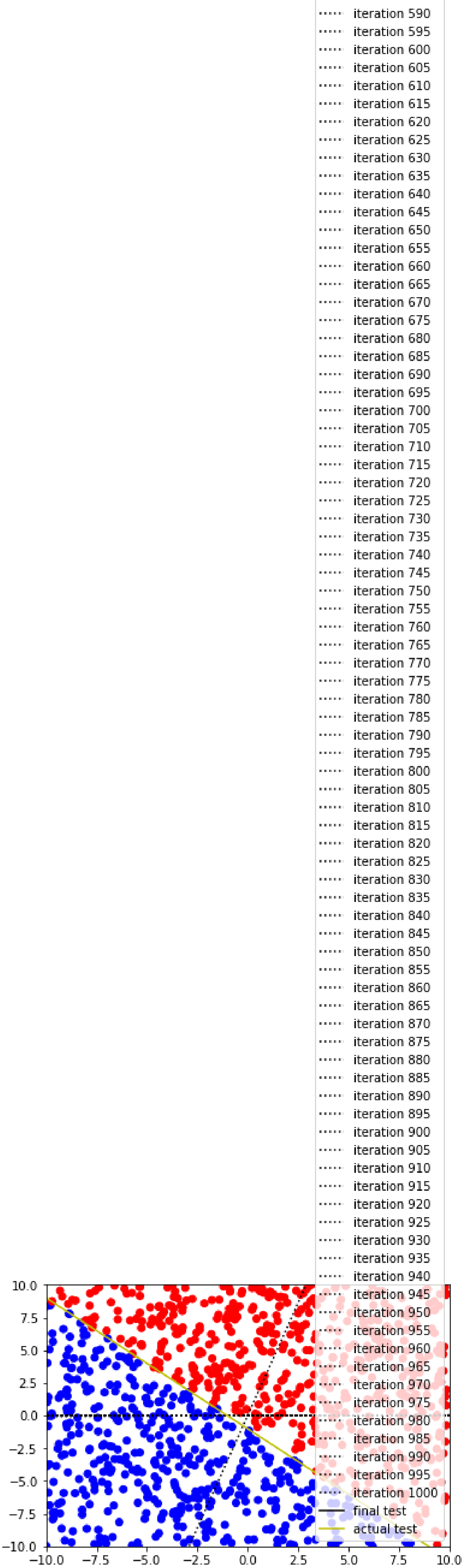
iteration 590
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iteration 955
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iteration 980
iteration 985
iteration 990
iteration 995
iteration 1000
final test
actual test
10.0
7.5
5.0
2.5
0.0
−2.5
−5.0
−7.5
−10.0
−10.0
−7.5
−5.0
−2.5
0.0
2.5
5.0
7.5
10.0

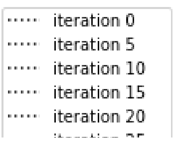
The above plot is the output of test data for the previously trained data set with n = 100. The error rate for this case is 8.42e+166 which is equivalent to 10.3

(b) The previous problem is now solved for n=1

In [121]:

```
y_train = evaluate_h(w_act, X_train)
y_test = evaluate_h(w_act, X_test)
x_axis = X_train[1, :]
y_axis = X_train[2, :]
lables = ['r' if y > 0 else 'b' for y in y_train]
plt.scatter(x_axis, y_axis, c=lables)
x_hypothesis = np.array([-data_range, data_range])
# Run perceptron learning algorithm for the training data set
def run_perceptron(w_init, X_train, y_train, iteration_callback=None):
    w = w_init.copy()
    n = 0
    while True:
        y = evaluate_h(w, X_train)
        if iteration_callback:
            iteration_callback(n, w)
        correct = y == y_train
# Calculate error after 1000 updates
        if n == 1000:
            return w

        else:
# calculate weight considering the closeness between s and y_train
            i = np.argmax(~correct)  # indice of first misclassified point
            s = w * X_train[:,i]
            d = y_train[i] * s
            d = d[0] + d[1] + d[2]
            if d <=1:
                w = w + 1 * (y_train[i] - s) * X_train[:, i]
                error = (y_train[i] - s)

        n = n + 1


def plot_hypothesis(x_axis, w, *plot_args, **plot_kwargs):
    m = -w[1]/w[2] if w[2] != 0 else 0
    b = -w[0]/w[2] if w[2] != 0 else 0
    y_axis = m*x_axis + b
    plt.plot(x_axis, y_axis, *plot_args, **plot_kwargs)

# Plot the hypothesis for all the iterations
def plot_iteration(n, w):
    if n % 5 == 0:
        label1 = 'iteration {}'.format(n)
        plot_hypothesis(x_hypothesis, w, 'k:', label=label1)

w_final = run_perceptron(w_init, X_train, y_train, plot_iteration)
plot_hypothesis(x_hypothesis, w_final, 'k', label='final')
plot_hypothesis(x_hypothesis, w_act, 'y', label='actual')
plt.legend()
plt.xlim(-data_range, data_range)
plt.ylim(-data_range, data_range)
plt.show()
```

```
C:\Users\kashi\Anaconda3\lib\site-packages\ipykernel_launcher.py:28: RuntimeWarning: overflow encountered in multiply
C:\Users\kashi\Anaconda3\lib\site-packages\ipykernel_launcher.py:28: RuntimeWarning: invalid value encountered in add
C:\Users\kashi\Anaconda3\lib\site-packages\ipykernel_launcher.py:23: RuntimeWarning: invalid value encountered in sign
```



```
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```
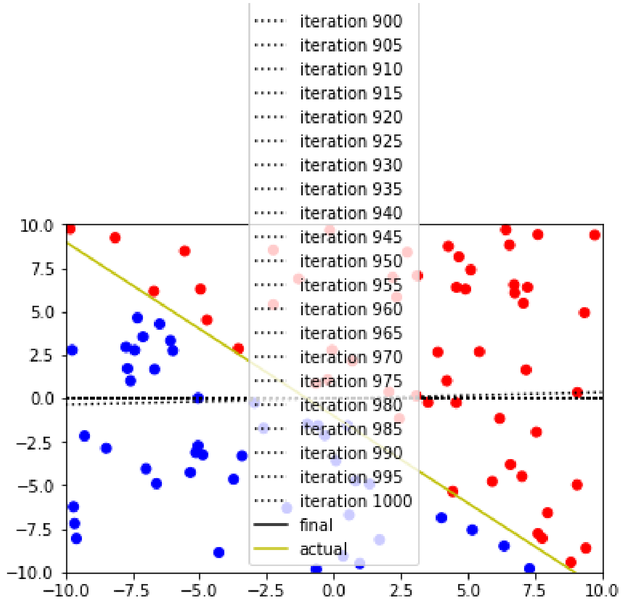



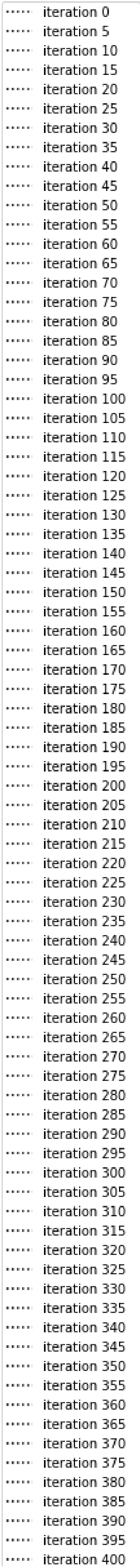
The plot above takes into account n =1 for training data

In [141]:

```
# Run perceptron learning algorithm for the test data set
def run_perceptron(w_init, X_train, y_train, iteration_callback=None):
    w = w_init.copy()
    n = 0
    while True:
        y = evaluate_h(w, X_train)
        if iteration_callback:
            iteration_callback(n, w)
        correct = y == y_train
#         if np.all(correct):
# Print the total number of iterations that let to the generation of final hypothesis g
#             print('Total number of iterations: ',n)
#             return w
        if n == 1000:
            error = np.absolute(error)
            print("Error on test data for n = 1: ",error[0]/1000)
            return w

        else:

            i = np.argmax(~correct)  # indice of first misclassified point
            s = w * X_train[:,i]
            d = y_train[i] * s
            d = d[0] + d[1] + d[2]
            if d <=1:
                w = w + 1 * (y_train[i] - s) * X_train[:, i]
                error = (y_train[i] - s)

        n = n + 1
x_axis = X_test[1, :]
y_axis = X_test[2, :]
lables = ['r' if y > 0 else 'b' for y in y_test]
plt.scatter(x_axis, y_axis, c=lables)
x_hypothesis = np.array([-data_range, data_range])

# Plot the hypothesis for all the iterations
def plot_iteration(n, w):
    if n % 5 == 0:
        label1 = 'iteration {}'.format(n)
        plot_hypothesis(x_hypothesis, w, 'k:', label=label1)
# Run Perceptron learning algorith for test data set

w_final = run_perceptron(w_init, X_test, y_test, plot_iteration)
plot_hypothesis(x_hypothesis, w_final, 'k', label='final test')
plot_hypothesis(x_hypothesis, w_act, 'y', label='actual test')
plt.legend()
plt.xlim(-data_range, data_range)
plt.ylim(-data_range, data_range)
```

```
plt.show()
```

```
Error on test data for n = 1:  0.0
```



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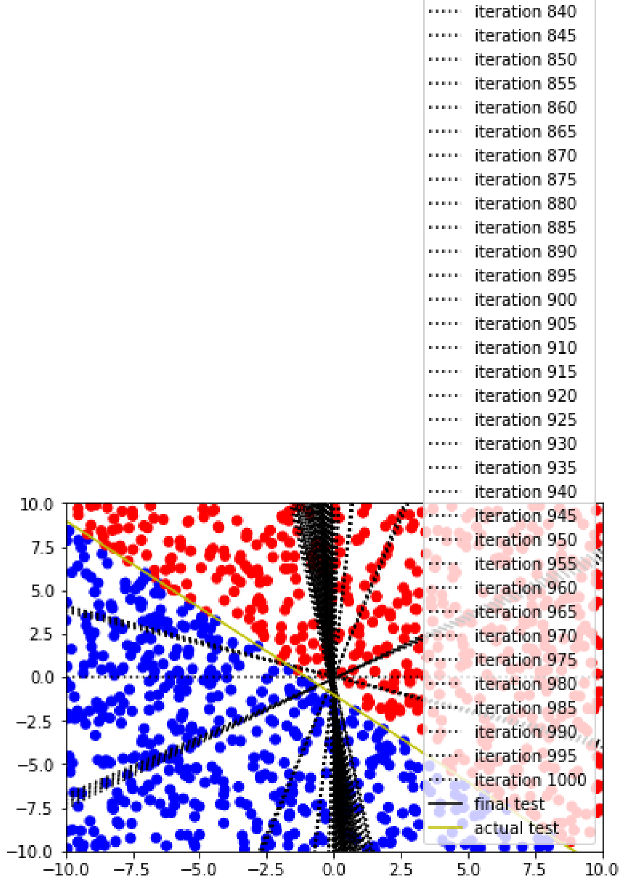


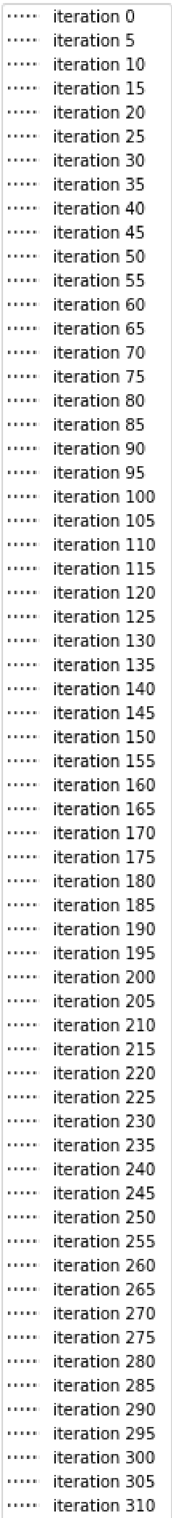
In the above plot the test data is plotted for n =1 and error is measured. The error rate in this case is 0

(c) The problem is now run for n = 0.01

In [122]:

```
y_train = evaluate_h(w_act, X_train)
y_test = evaluate_h(w_act, X_test)
x_axis = X_train[1, :]
y_axis = X_train[2, :]
lables = ['r' if y > 0 else 'b' for y in y_train]
plt.scatter(x_axis, y_axis, c=lables)
x_hypothesis = np.array([-data_range, data_range])


# Run perceptron learning algorithm for the training data set
def run_perceptron(w_init, X_train, y_train, iteration_callback=None):
    w = w_init.copy()
    n = 0
    while True:
        y = evaluate_h(w, X_train)
        if iteration_callback:
            iteration_callback(n, w)
        correct = y == y_train
# Calculate error after 1000 updates
        if n == 1000:
            return w

        else:
# calculate weight considering the closeness between s and y_train
            i = np.argmax(~correct)   # indice of first misclassified point
            s = w * X_train[:,i]
            d = y_train[i] * s
            d = d[0] + d[1] + d[2]
            if d <=1:
                w = w + 0.01 * (y_train[i] - s) * X_train[:, i]
                error = (y_train[i] - s)

        n = n + 1
```

```python
def plot_hypothesis(x_axis, w, *plot_args, **plot_kwargs):
    m = -w[1]/w[2] if w[2] != 0 else 0
    b = -w[0]/w[2] if w[2] != 0 else 0
    y_axis = m*x_axis + b
    plt.plot(x_axis, y_axis, *plot_args, **plot_kwargs)

# Plot the hypothesis for all the iterations
def plot_iteration(n, w):
    if n % 5 == 0:
        label1 = 'iteration {}'.format(n)
        plot_hypothesis(x_hypothesis, w, 'k:', label=label1)

w_final = run_perceptron(w_init, X_train, y_train, plot_iteration)
plot_hypothesis(x_hypothesis, w_final, 'k', label='final')
plot_hypothesis(x_hypothesis, w_act, 'y', label='actual')
plt.legend()
plt.xlim(-data_range, data_range)
plt.ylim(-data_range, data_range)
plt.show()
```



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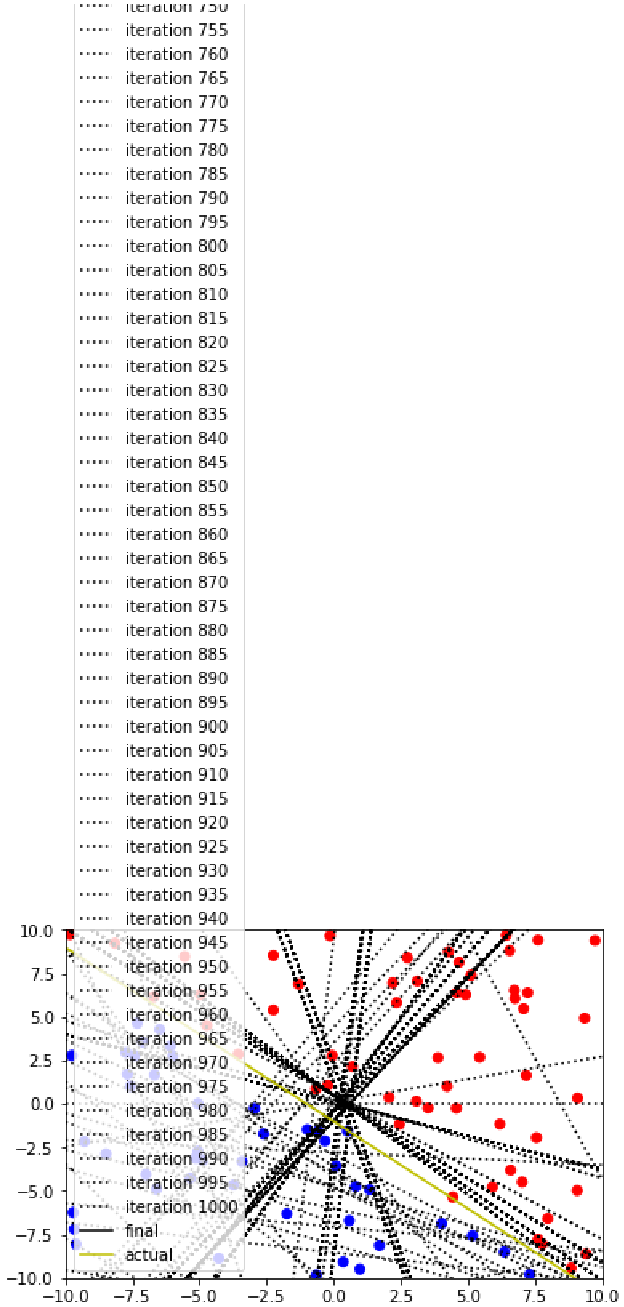
iteration 750



The plot above takes into account n =1 for training data

In [132]:

```
# Run perceptron learning algorithm for the test data set
def run_perceptron(w_init, X_train, y_train, iteration_callback=None):
    w = w_init.copy()
    n = 0
    while True:
        y = evaluate_h(w, X_train)
        if iteration_callback:
            iteration_callback(n, w)
        correct = y == y_train
#         if np.all(correct):
# Print the total number of iterations that let to the generation of final hypothesis g
#             print('Total number of iterations: ',n)
#             return w
        if n == 1000:
            error = np.absolute(error)
            print("Error on test data for n = 0.01: ",error[0]/1000)
            return w

        else:

            i = np.argmax(~correct)   # indice of first misclassified point
            s = w * X_train[:,i]
```

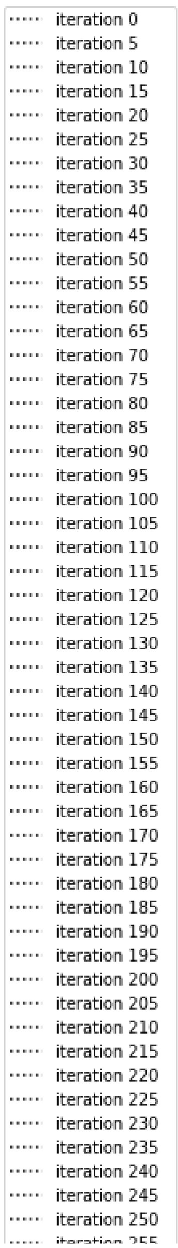
```
            d = y_train[i] * s
            d = d[0] + d[1] + d[2]
            if d <=1:
                w = w + 0.01 * (y_train[i] - s) * X_train[:, i]
                error = (y_train[i] - s)

        n = n + 1
x_axis = X_test[1, :]
y_axis = X_test[2, :]
lables = ['r' if y > 0 else 'b' for y in y_test]
plt.scatter(x_axis, y_axis, c=lables)
x_hypothesis = np.array([-data_range, data_range])

# Plot the hypothesis for all the iterations
def plot_iteration(n, w):
    if n % 5 == 0:
        label1 = 'iteration {}'.format(n)
        plot_hypothesis(x_hypothesis, w, 'k:', label=label1)
# Run Perceptron learning algorith for test data set

w_final = run_perceptron(w_init, X_test, y_test, plot_iteration)
plot_hypothesis(x_hypothesis, w_final, 'k', label='final test')
plot_hypothesis(x_hypothesis, w_act, 'y', label='actual test')
plt.legend()
plt.xlim(-data_range, data_range)
plt.ylim(-data_range, data_range)
plt.show()
```

```
Error on test data for n = 0.01:  0.0009270516925850184
```



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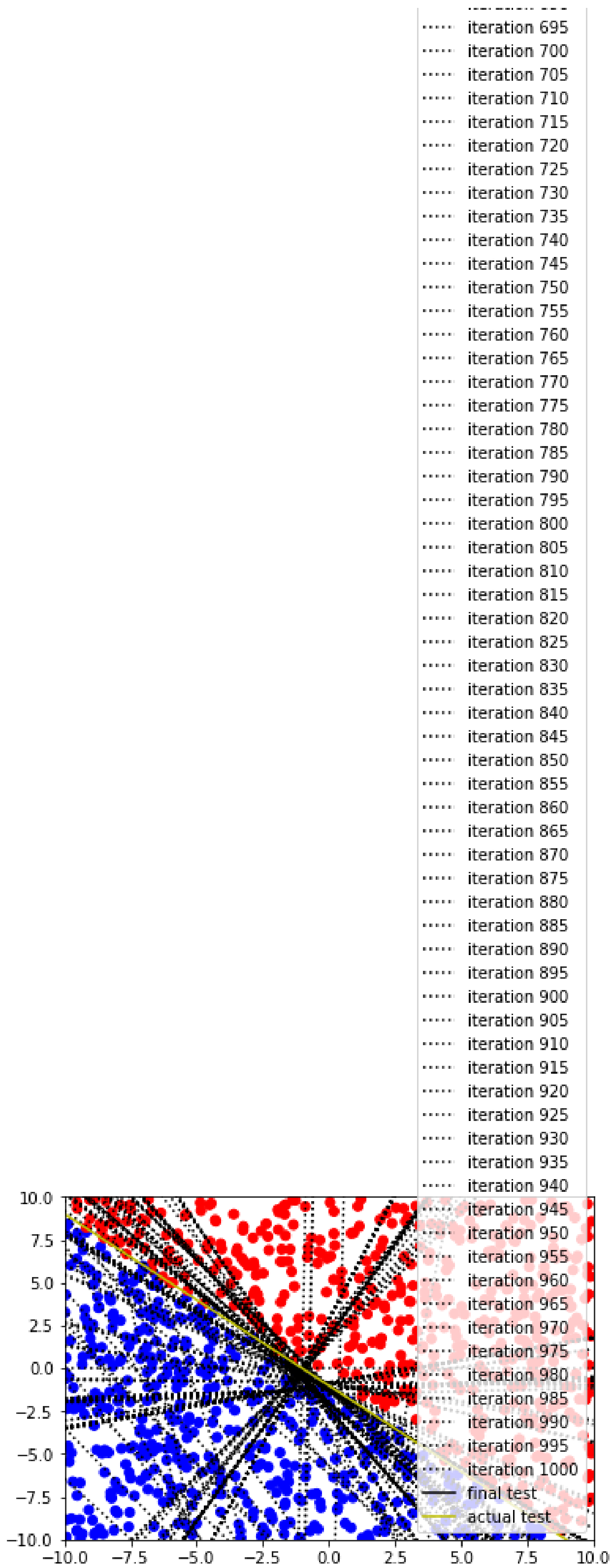

In the above plot the test data is plotted for n =0.01 and error is measured. The error rate in this case is 0.00092

(d)The problem is now run for n = 0.0001

In [117]:

```
import matplotlib.pyplot as plt
import numpy as np
%matplotlib inline

# generate a linearly separated training data of size 100 and test data of size 1000 -
w_act = np.array([1, 1, 1])
w_init = np.array([3, -50, 0])
d = 2
num_sample = 100
```

```
data_range = 10
num_sample_test =1000
error = 0
# check if the hypothesis h from the hypothesis set H is following the weight and dimensionality c
ondition
def evaluate_h(w, X):

    '''
    Evaluate the hypothesis, h \in H, which is described by its set of weights,
    w, against 1 or more points in the input space (stored in a matrix).
    '''
    assert len(w.shape) == 1
    assert len(X.shape) == 2
    assert w.shape[0] == X.shape[0]
    return np.sign(w @ X)
X_train = np.vstack([
    np.ones(num_sample),
    np.random.uniform(
        -data_range,
        data_range,
        (d, num_sample),
    ),
])

X_test = np.vstack([
    np.ones(num_sample_test),
    np.random.uniform(
        -data_range,
        data_range,
        (d, num_sample_test),
    ),
])

y_train = evaluate_h(w_act, X_train)
y_test = evaluate_h(w_act, X_test)
x_axis = X_train[1, :]
y_axis = X_train[2, :]
lables = ['r' if y > 0 else 'b' for y in y_train]
plt.scatter(x_axis, y_axis, c=lables)
x_hypothesis = np.array([-data_range, data_range])


# Run perceptron learning algorithm for the training data set
def run_perceptron(w_init, X_train, y_train, iteration_callback=None):
    w = w_init.copy()
    n = 0
    while True:
        y = evaluate_h(w, X_train)
        if iteration_callback:
            iteration_callback(n, w)
        correct = y == y_train
# Calculate error after 1000 updates
        if n == 1000:
            return w

        else:
# calculate weight considering the closeness between s and y_train
            i = np.argmax(~correct)  # indice of first misclassified point
            s = w * X_train[:,i]
            d = y_train[i] * s
            d = d[0] + d[1] + d[2]
            if d <=1:
                w = w + 0.0001 * (y_train[i] - s) * X_train[:, i]
                error = (y_train[i] - s)

        n = n + 1


def plot_hypothesis(x_axis, w, *plot_args, **plot_kwargs):
    m = -w[1]/w[2] if w[2] != 0 else 0
    b = -w[0]/w[2] if w[2] != 0 else 0
    y_axis = m*x_axis + b
    plt.plot(x_axis, y_axis, *plot_args, **plot_kwargs)

# Plot the hypothesis for all the iterations
def plot_iteration(n, w):
    if n % 5 == 0:
```

```
        label1 = 'iteration {}'.format(n)
        plot_hypothesis(x_hypothesis, w, 'k:', label=label1)

w_final = run_perceptron(w_init, X_train, y_train, plot_iteration)
plot_hypothesis(x_hypothesis, w_final, 'k', label='final')
plot_hypothesis(x_hypothesis, w_act, 'y', label='actual')
plt.legend()
plt.xlim(-data_range, data_range)
plt.ylim(-data_range, data_range)
plt.show()
```

····· iteration 365
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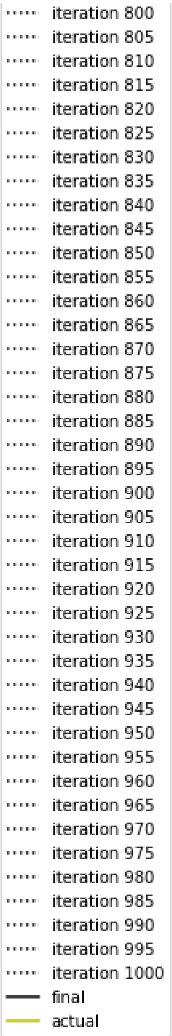
····· iteration 785
····· iteration 790
····· iteration 795



In [ ]:

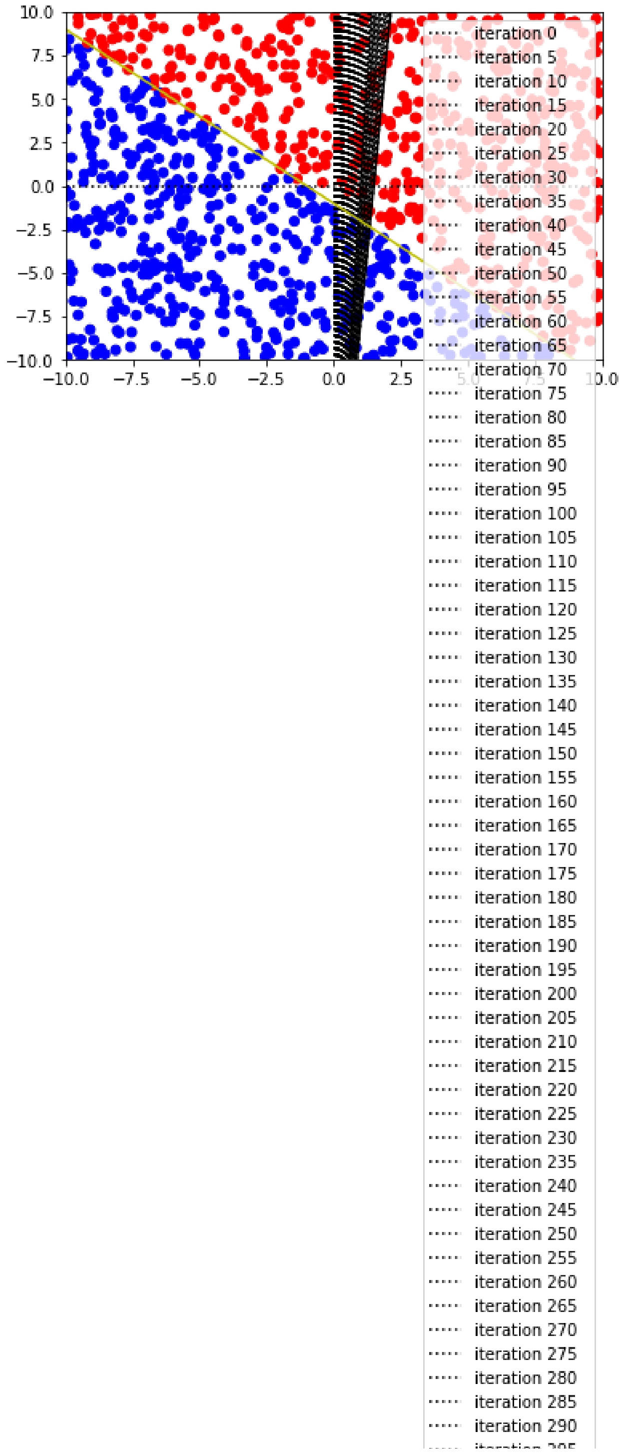
```
The plot above takes into account n =1 for training data
```

In [133]:

```
# Run perceptron learning algorithm for the test data set
def run_perceptron(w_init, X_train, y_train, iteration_callback=None):
    w = w_init.copy()
    n = 0
    while True:
        y = evaluate_h(w, X_train)
        if iteration_callback:
            iteration_callback(n, w)
        correct = y == y_train
#         if np.all(correct):
# Print the total number of iterations that let to the generation of final hypothesis g
#             print('Total number of iterations: ',n)
#             return w
        if n == 1000:
            error = np.absolute(error)
            print("Error on test data for n = 0.0001: ",error[0]/1000)
            return w

        else:

            i = np.argmax(~correct)  # indice of first misclassified point
            s = w * X_train[:,i]
            d = y_train[i] * s
            d = d[0] + d[1] + d[2]
            if d <=1:
                w = w + 0.0001 * (y_train[i] - s) * X_train[:, i]
                error = (y_train[i] - s)

        n = n + 1
x_axis = X_test[1, :]
```

```
x_axis = X_test[1, :]
y_axis = X_test[2, :]
lables = ['r' if y > 0 else 'b' for y in y_test]
plt.scatter(x_axis, y_axis, c=lables)
x_hypothesis = np.array([-data_range, data_range])

# Plot the hypothesis for all the iterations
def plot_iteration(n, w):
    if n % 5 == 0:
        label1 = 'iteration {}'.format(n)
        plot_hypothesis(x_hypothesis, w, 'k:', label=label1)
# Run Perceptron learning algorith for test data set

w_final = run_perceptron(w_init, X_test, y_test, plot_iteration)
plot_hypothesis(x_hypothesis, w_final, 'k', label='final test')
plot_hypothesis(x_hypothesis, w_act, 'y', label='actual test')
plt.legend()
plt.xlim(-data_range, data_range)
plt.ylim(-data_range, data_range)
plt.show()
```

```
Error on test data for n = 0.0001:  0.0036196935435885438
```



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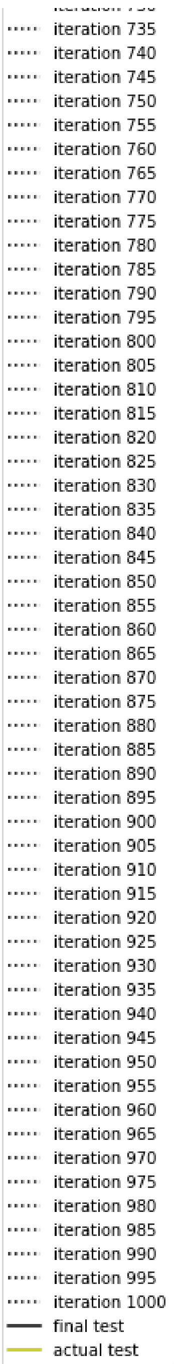

In the above plot the test data is plotted for n =0.01 and error is measured. The error rate in this case is 0.0036

Comparing all the 4 scenarios, we see that the error rate for n = 100 is 10.3 n = 1 is 0 n = 0.01 is 0.00092 n = 0.0001 is 0.0036