

# PLP - 2

## TOPIC 2 — LOGICAL STATEMENTS

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# LOGICAL STATEMENTS

# PROVING THINGS FROM SMALLER THINGS

A big part of mathematics is proving that **statements** are **true**.

*The square of an even number is even.*

We do this by

- starting from known *facts* — **axioms, lemmas, theorems**
- *combine* those facts using **logic** to build new facts

So we need to introduce and understand

- mathematical sentences that can carry a **truth value**
- the rules for combining those sentences

# STATEMENTS

## DEFINITION: (A NOT SO FORMAL DEFINITION OF STATEMENTS).

A **statement** is a sentence that is either **true** or **false**.

A statement has exactly one of those **truth values**.

Note:

- A rigorous definition is way too hard for this course.
- Typically use  $P, Q, R$  to denote statements

*The number  $\sqrt{2}$  is not a rational number.*

*The number 17 is even.*

# MORE STATEMENTS

Sentences like

*The 100th decimal digit of  $\pi$  is 7.*

and

*Every even integer greater than 2 is the sum of two primes.*

are statements since we can determine their truth value

1.  $\pi = 3.1415\dots$  — we can just keep computing and check.
2. This is **Goldbach's conjecture** — truth value unknown, but it must be true or false.

# NON-STATEMENTS

Sentences like

*I am tall*

and

*This sentence is false*

are *not* statements since we cannot decide their truth value

1. Who is “I”? What is “tall”?

2. This cannot be true or false

- If it is true, then it must be false!
- If it s false, then it must be true!

# OPEN SENTENCES

Some sentences contain variables

*If the integer  $x$  is a multiple of 6 then it is even.*

*The integer  $x$  is even.*

1. This is a statement since it is always true
2. The truth value depends on the variable  $x$

## DEFINITION:

An **open sentence** is a sentence whose truth value depends on the variable(s) it contains.

Typically denote it by  $P(x)$  and similar.

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