PLP - 5

TOPIC 5 —MODUS PONENS AND PROOFS

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MODUS PONENS — TOWARDS PROOFS

MODUS PONENS

Once we know an implication $P \implies Q$ is true, we want to use it.

- If we also know that P is true
- Then (by our truth table) we must have that Q is true

This deduction very useful and is precisely why we want to prove implications.

DEFINITION: (MODUS PONENS).

The deduction:

$(P \implies Q)$ *and* P *are true so* Q *must be true*

is called **modus ponens**.

MODUS PONENS AND PROOFS

Say we need to prove $P \implies Q$ but cannot directly. Instead show that

- $P \implies P_1$ is true
- $P_1 \implies P_2$ is true
- $P_2 \implies P_3$ is true
- $\ldots$
- $P_n \implies Q$ is true

Consider what happens when P is true (we can ignore P is false).

- When P is true, we can use *modus ponens*:
  - Since P is true and $P \implies P_1$ is true, we know P_1 is true
  - Since P_1 is true, we know that P_2 is true
  - $\ldots$
  - Since P_n is true, we know that Q is true
- Hence Q cannot be false, so the implication is true!

TYPICAL PROOF STRUCTURE

PROPOSITION:

If P then Q

PROOF.

- Assume that the hypothesis P is true
- We see that P implies P_1
- From this we know that P_1 implies P_2
- $\vdots$
- From P_n we know that Q as required.