PLP - 4

TOPIC 04 — THE CONDITIONAL

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CONDITIONAL

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Many interesting mathematical statements are **conditionals**

If $f(x)$ is differentiable then $f(x)$ is continuous

DEFINITION: CONDITIONAL.

Given P and Q, the **conditional** is the statement "if P then Q" and is denoted "$P \implies Q$".

Also called **implication** and the **hypothesis** is P, and the **conclusion** is Q.

Implication $P \implies Q$ is true except when $(P, Q) = (T, F)$.

- please use correct notation — "$\to$" is *not* "$\implies$"
- order matters "$Q \implies P$" is *not* "$P \implies Q$"
- Read "$P \implies Q$" as "If P then Q", "P implies Q", "Whenever P then also Q".

TRUTH TABLE OF THE CONDITIONAL

Important to *memorise* this table

P	Q	$P \implies Q$
T	T	T
T	F	F
F	T	T
F	F	T

Note that

- When Q is true, the implication is always true
- When P is false, the implication is always true

EXAMPLES

P	Q	$P \implies Q$
T	T	T
T	F	F
F	T	T
F	F	T

- If 8 is even then 17 is prime — true
- If 8 is even then 4 is prime — false
- If 4 is prime then 8 is even — true (but…)
- If 6 is prime then 19 is even — true (but…)

EXPLAINING THE TABLE

it rains	roads get wet	If it rains then roads get wet
T	T	T
T	F	F
F	T	T
F	F	T

Do one by one:

- (T,T): it rained and roads got wet — implication is true.
- (T,F): it rained and but roads are dry — *it is false!*
- (F,T): it is sunny and roads got wet — implication is not false
- (F,F): it is sunny and roads are dry — implication is not false

Last two mean that implication *is true unless you prove it false*.

WHAT DO WE NEED TO PROVE?

When we *prove* an implication "$P \implies Q$" we want to show it is *always true and never false*.

- When P is false *no work needed* — we know "$P \implies Q$" is true
- When P is true *work required* — truth of "$P \implies Q$" depends on truth of Q

In a proof we do not have to consider the case "P is false".

Structure of most proofs:

- Assume the **hypothesis** is true
- Do "stuff"
- Show that the **conclusion** must also be true
- So the case $\mathsf{T} \implies \mathsf{F}$ cannot happen

Since the implication cannot be false, it must be true!

WHAT ABOUT OPEN SENTENCES?

A note on proofs of conditionals containing open sentences:

If $f(x)$ is continuous then $f(x)$ is differentiable.

We still want this true no matter what, so

- we assume the hypothesis is true — assume that $f(x)$ is *any* continuous function
- then work our way to showing that $f(x)$ must be differentiable

This example is false: $f(x) = |x|$ is continuous, but it is not differentiable.

3) (a) $P = \{x, y \in \mathbb{Z} : 3x + 7y\}$ ← Assume we introduced new coins worth 3 & 7. Write in set builder notation the set of prices obj can be charged

(b) All numbers ← what nos. in this set

Given $m \in \mathbb{Z}$, let $k = -2m$ & $j = m$

Then, $3k + 7j = -6m + 7m = m$

so, $m \in C$

$\therefore C = \mathbb{Z}$

(c) $P = \{x, y \in \mathbb{Z} : 2x + 6y\}$ ← Prove it like (b)

Even nos.

New coins of 2 & 6

(d) $\gcd(3, 7) = 1$

$\gcd(2, 6) = 2$