Some useful latex for you to use:

- For sets use the command we defined in the latex source

  $$\left\{1,2,3\right\}, \left\{\emptyset, \left\{4,5,6\right\}\right\}, \left\{\frac{1}{2}, \frac{\alpha}{1+\beta}\right\}$$

  it will format the braces nicely.

- Sometimes it is nice to write ℓ instead of l because it looks nice in formulas.

- For logic, latex defines the symbols we need:

  $$\sim P \qquad P \vee Q \qquad P \wedge Q \qquad P \implies Q \qquad P \iff Q$$

  Unfortunately, we use $\sim$ for negation and not the default negation symbol $\neg$, so it is useful to redefine things in the header of your document (a bit like how we define the set command.)

- For a proof we can (and probably should) use the proof environment. It automatically puts the word "proof" at the start and the little square at the end:

  Proof. This is my proof. It is just missing a few details, but I'll put in an equation

  $$a + b = c$$

  just because I can. □

  Sometimes we want to give the proof a title, and the proof environment helps us do that too. Here is a classic false-proof that $2 = 1$.

  Not-quite-a-proof that two equals one. Let x, y be non-zero real numbers so that $x = y$. Then, multiplying by x gives us

  $$\begin{aligned} x^2 &= xy && \text{now subtract } y^2 \\ x^2 - y^2 &= xy - y^2 && \text{now factor} \\ (x-y)(x+y) &= y(x-y) && \text{divide by common factor of } (x-y) \\ x + y &= y && \text{since } x = y \\ 2y &= y && \text{now divide by y} \\ 2 &= 1 \end{aligned}$$

  □

- For the truth tables you can use the following:

A_1	A_2	A_3	A_4	A_5	A_6
a_{11}	a_{12}	a_{13}	a_{14}	a_{15}	a_{16}
a_{21}	a_{22}	a_{23}	a_{24}	a_{25}	a_{26}
a_{31}	a_{32}	a_{33}	a_{34}	a_{35}	a_{36}
a_{41}	a_{42}	a_{43}	a_{44}	a_{45}	a_{46}

- Remeember to cheque the speeling of your subbmisssion.
- Also remember that you should not include your scratchwork unless a question specifically asks for it.
- Finally, please try to make your work look nice and neat and use 12pt font — think about the reader!

Please do not include the above text in your homework solution — we have just included it here to help you write your homework.

Solutions to homework 1:

1. *Proof.* Let there exist $k \in \mathbf{Z}$ such that,

$$n + 1 = 3k \tag{1}$$

On squaring both sides we get,

$$(n+1)^2 = 9k^2 \tag{2}$$
$$n^2 + 1 + 2k = 9k^2 \tag{3}$$

Now, since (n+1) is divisible by 3, $(n+1)^2$ is also divisible by 3
Adding 3 * Eq (1) on both sides we get,

$$n^2 + 1 + 2k + 3(n+1) = 9k^2 + 9k \tag{4}$$
$$n^2 + 5n + 4 = 3(3k^2 + 3k) \tag{5}$$

Adding 1 on both sides

$$n^2 + 5n + 5 = 3(3k^2 + 3k) + 1 \tag{6}$$

Let there exist $j \in \mathbb{Z}$ where $j = 3k^2 + 3k$,

$$n^2 + 5n + 5 = 3j + 1 \tag{7}$$

Hence $3 \nmid n^2 + 5n + 5$

□

2. *Proof.* We know that 5a+11 is odd so let there exist $k \in \mathbb{Z}$ such that,

$$5a + 11 = 2k + 1 \tag{8}$$
$$5a = 2k - 10 \tag{9}$$
$$5a = 2(k - 5) \tag{10}$$

Adding 4a + 3 on both the sides we get,

$$5a + 4a + 3 = 2(k - 5) + 4a + 3 \tag{11}$$
$$9a + 3 = 2(k - 5 + 2a + 1) + 1 \tag{12}$$

Let there exist $j \in \mathbb{Z}$ where $j = (k - 5 + 2a + 1)$ so,

$$9a + 3 = 2j + 1 \tag{13}$$

Hence, 9a + 3 is odd □

3. *Proof.* We know that $-1 < x < 2$ So, on squaring we get,

$$0 < x^2 < 4 \qquad \text{Since } -1^2 = 1 \text{ so 0 is minimum} \tag{14}$$

Subtracting the two equations,

$$0 - (-1) < x^2 - x < 4 - 2 \tag{15}$$
$$1 < x^2 - x < 2 \tag{16}$$

We then subtract 2 to the equation thus giving us,

$$1 - 2 < x^2 - x - 2 < 2 - 2 \tag{17}$$
$$-1 < x^2 - x - 2 < 0 \tag{18}$$

Thus $x^2 - x - 2 < 0$ □

4. *Proof.* Let there exist $i, j, k \in \mathbb{Z}$ such that,

$$a = 2i + 1 \tag{19}$$
$$c = 2j + 1 \tag{20}$$
$$b + d = 2k + 1 \tag{21}$$
$$\text{So,} \;\; b = 2k + 1 - d \tag{22}$$

We need to prove that ab + cd is odd , so on substituting values of a,b,c and d in ab + cd we get,

$$(2i + 1) * (2k + 1 - d) + (2j + 1) * d \tag{23}$$
$$(4ik + 2i - 2id + 2k + 1 - d) + (2jd + d) \tag{24}$$
$$4ik + 2i - 2id + 2k + 2jd + 1 \tag{25}$$
$$2 * (2ik + i - id + k + jd) + 1 \tag{26}$$

Let there exist $m \in \mathbb{Z}$ such that $m = 2ik + i - id + k + jd$
So, ab + cd = 2m + 1
Hence, ab + cd is odd

□

5. *Proof.* x and y are real numbers and
we know that the square of any number is positive i.e greater than 0 so,

$$(x-y)^2 \geqq 0 \tag{27}$$
$$\text{So,}\ \ x^2+y^2-2xy \geqq 0 \tag{28}$$
$$x^2+y^2 \geqq 2xy \tag{29}$$
$$\frac{(x^2+y^2)}{2} \geqq xy \tag{30}$$

□

6. *Proof.* We know that,

$$y^2 < x^2 \tag{31}$$
$$x < y \tag{32}$$

On further analysis of the inequities,

$$y^2 - x^2 < 0 \tag{33}$$
$$y - x > 0 \tag{34}$$
$$\tag{35}$$

Hence,

$$(y-x)(y+x) < 0 \tag{36}$$
$$\text{We know that } y-x>0 \quad \text{so,}\ \ y+x<0 \tag{37}$$

Therefore $x+y<0$ □

7. *Proof.* Let there exist $k \in \mathbb{Z}$ such that,

$$n+7 = 5k \tag{38}$$
$$n = 5k-7 \tag{39}$$

On subtracting 5 from both sides,

$$n+2 = 5k-5 \tag{40}$$

Squaring the previous equation we get,

$$n^2+4n+4 = 25k^2+25-50k \tag{41}$$

We then subtract (4n + 3),

$$n^2+1 = 25k^2-50k-4n+22 \tag{42}$$
$$n^2+1 = 25k^2-50k-4(5k-7)+22 \tag{43}$$
$$n^2+1 = 25k^2-50k-20k+28+22 \tag{44}$$
$$n^2+1 = 25k^2-50k-20k+50 \tag{45}$$
$$n^2+1 = 5(5k^2-10k-4k+10) \tag{46}$$
$$\tag{47}$$

There exists $m \in \mathbb{Z}$ such that, $n^2 + 1 = 5m$
Hence $5 \mid n^2 + 1$ $\square$

8. *Proof.* Let there exist $j, k \in \mathbb{Z}$ such that,

$$a = jn \tag{48}$$
$$b = kn \tag{49}$$

Now, for the equation $ax + by$

$$jn * x + kn * y \tag{50}$$
$$n * (jx + ky) \tag{51}$$

Let there exist $p \in \mathbb{Z}$, where $p = jx + ky$
So, $ax + by = n * p$
Hence $n \mid ax + by$ $\square$