

PLP - 23

TOPIC 23—SET OPERATIONS

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SET OPERATIONS

UNION AND INTERSECTION

DEFINITION:

Let A, B be sets. The **union** of A and B is

$$A \cup B = \{x : x \in A \text{ or } x \in B\}$$

DEFINITION:

The **intersection** of sets A and B , is

$$A \cap B = \{x : x \in A \text{ and } x \in B\}$$

If the intersection $A \cap B = \emptyset$, then we say that A and B are **disjoint**.

NOTES AND EXAMPLES

- Please use **correct notation**

$A \cup B =$ unions of sets

$A \cap B =$ intersection of sets

$P \vee Q =$ disjunction of statements

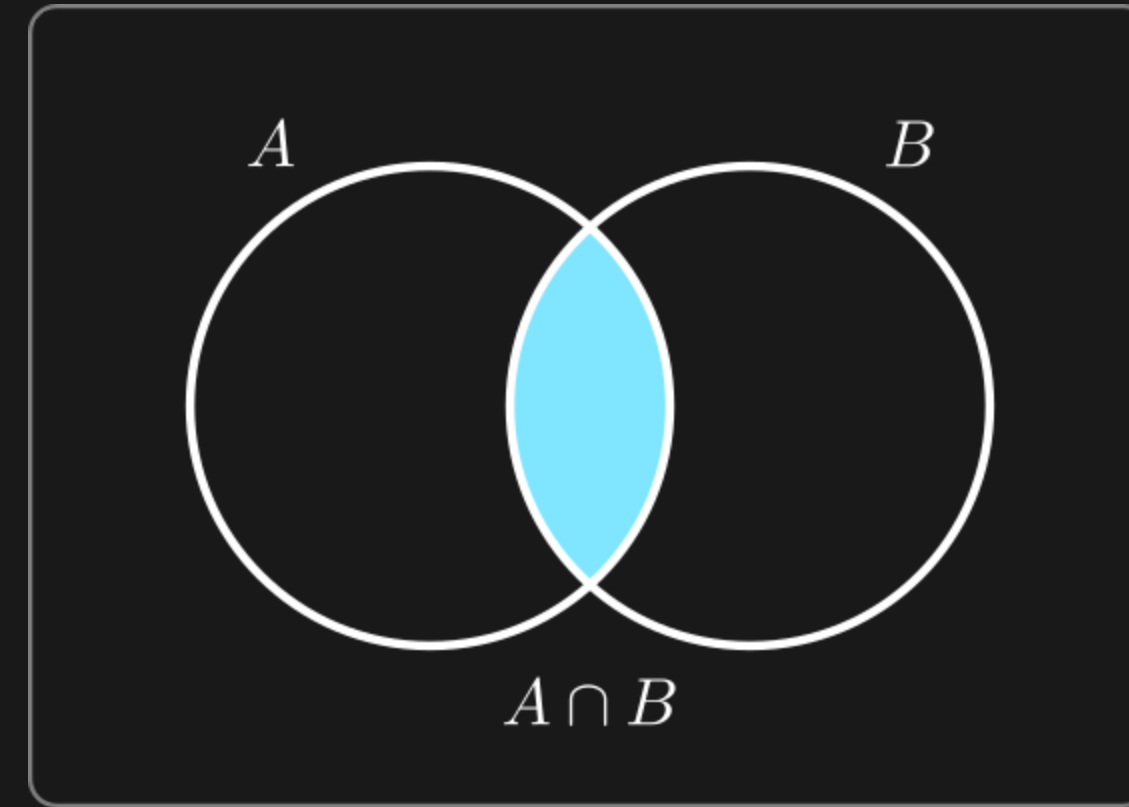
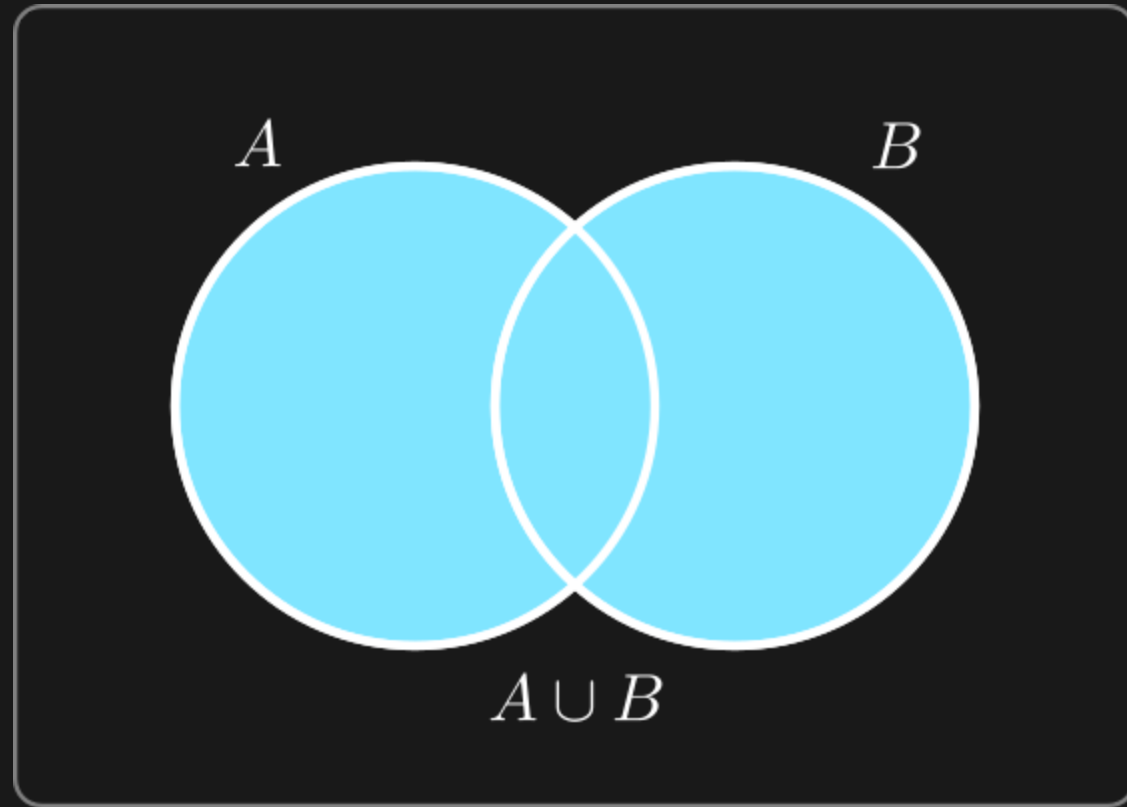
$P \wedge Q =$ conjunction of statements

- There are *parallels* between set operations and logical operations but they are *not* the same
- Let $A = \{1, 2, 3, 4\}$, $B = \{p : p \text{ is prime}\}$ and $C = \{4, 5, 6, 7\}$

$$A \cup C = \{1, 2, 3, 4, 5, 6, 7\} \quad A \cap B = \{2, 3\} \quad B \cap C = \{5, 7\}$$

VISUALISING THINGS

We can picture union and intersection using **Venn diagrams**

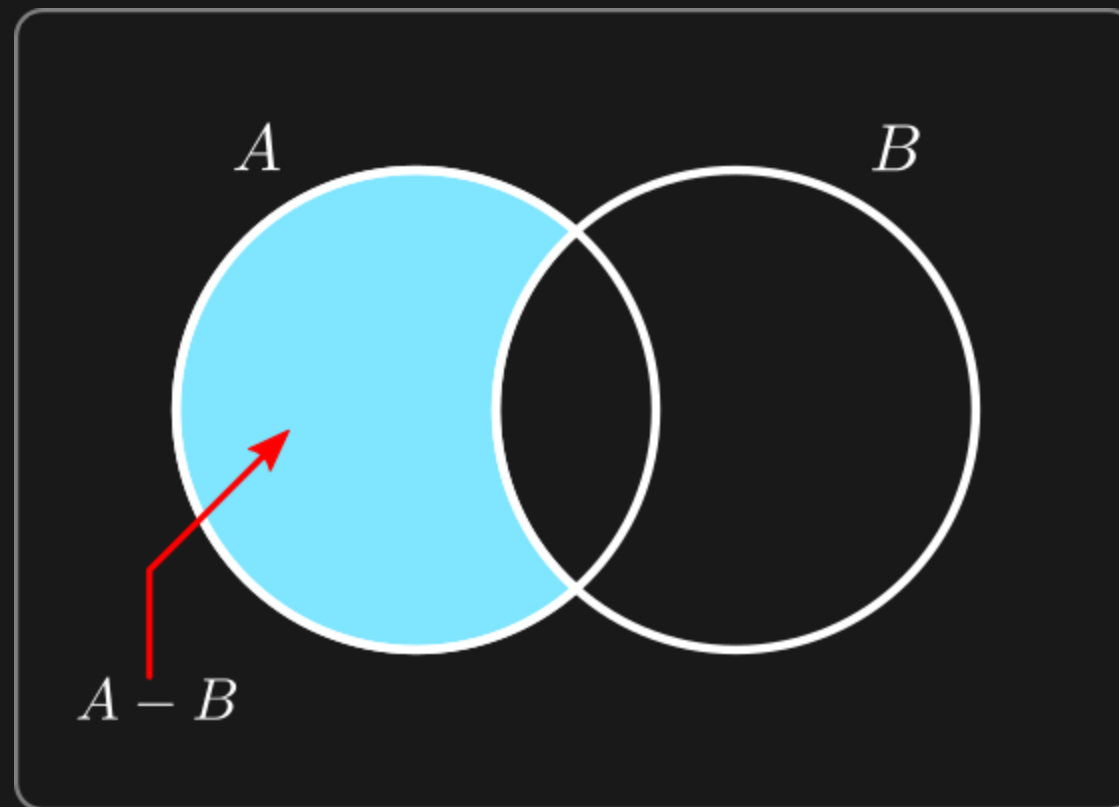


ANOTHER OPERATION

DEFINITION:

Let A and B be sets. Then the **difference**, $A - B$ is

$$A - B = \{x \in A : x \notin B\}$$



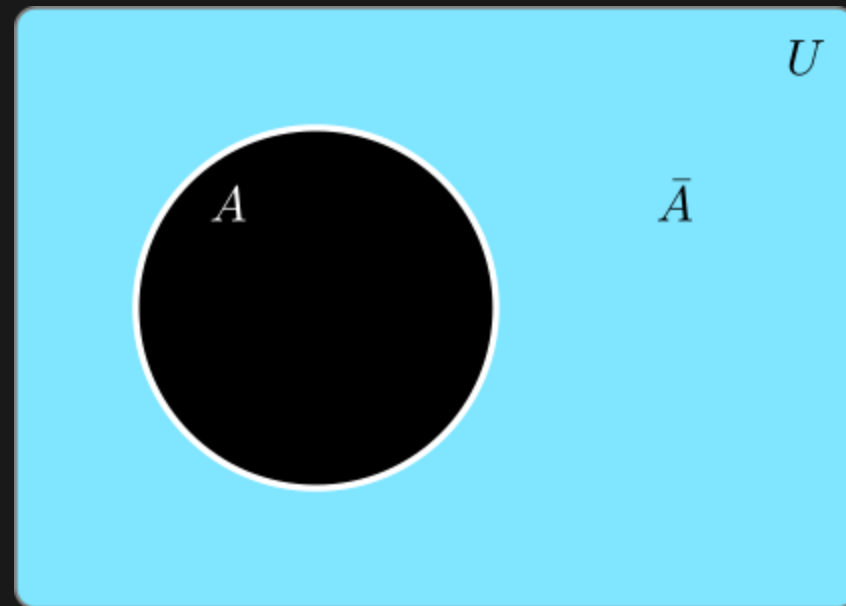
UNIVERSALS AND COMPLEMENTS

DEFINITION:

Given a **universal set** U and $A \subset U$, the **complement** of A is

$$\bar{A} = \{x \in U : x \notin A\} \quad \text{or equivalently} \quad x \in \bar{A} \iff x \notin A$$

The universal set is the set from which we draw elements in the current *context*



NOTES AND EXAMPLES

- Compl-e-ment vs Compl-i-ment
- $A - B$ also written $A \setminus B$ and is called the **relative complement of B in A**
- We have $A - B = A \cap \bar{B}$
- Let $U = \mathbb{N}$, $A = \{1, 2, 3, 4\}$, $B = \{p : p \text{ is prime}\}$ and $C = \{4, 5, 6, 7\}$

$$A - C = \{1, 2, 3\} \quad A - B = \{1, 4\} \quad \bar{A} = \{n \in \mathbb{N} : n \geq 5\}$$

ORDERED PAIRS

Sets don't care about order, but many applications need *pairs* of objects.

DEFINITION:

An **ordered pair** of elements is an *ordered* list of two elements.

The ordered pair of two elements a, b is written (a, b) and satisfies

- $(a, b) = (c, d)$ only when $(a = c)$ and $(b = d)$, and
- $(a, b) \neq (b, a)$ unless $(a = b)$.

Warning

- use correct notation: $\{1, 3\}$ is a set, $(1, 3)$ is an ordered pair
- we sometimes use $(1, 3) = \{x \in \mathbb{R} : 1 < x < 3\}$ — give your **reader** context

CARTESIAN PRODUCT

DEFINITION: CARTESIAN PRODUCT.

The **Cartesian product** of sets A, B is

$$A \times B = \{(a, b) : a \in A, b \in B\}$$

Note for $A, B \neq \emptyset$, $A \times B \neq B \times A$ unless $A = B$.

Let $A = \{a, b, c\}$ and $B = \{1, 2\}$ then

$$A \times B = \{(a, 1), (a, 2), (b, 1), (b, 2), (c, 1), (c, 2)\}$$