PLP - 26

TOPIC 26—RELATIONS

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RELATIONS

RELATIONSHIPS

- Many expressions in mathematics describe *relationships* between objects
  - $a = b$ — the objects a and b are equal.
  - $a < b$ — the number a is strictly less than the number b.
  - $a \in B$ — the object a is a member of the set B.
  - $A \subseteq B$ — the set A is a subset of the set B.
  - $a \mid b$ — the number a is a divisor of the number b.
- Focus on (say) divisibility — we can think of the symbol "$|$" as an operator on *pairs* of integers.
  - we write $a \mid b$ when a divides b
  - and write $a \nmid b$ when a does not divide b
- Divisibility naturally defines a subset of $\mathbb{N} \times \mathbb{N}$:

$$R = \{(a, b) \in \mathbb{N} \times \mathbb{N} \ : \ a \text{ divides } b\}$$

RELATION AS SUBSET OF CARTESIAN PRODUCT

Consider divisibility on the set $A = \{1, 2, 4, 8\}$

$$
\begin{array}{cccc}
1 \mid 1 & 1 \mid 2 & 1 \mid 4 & 1 \mid 8 \\
2 \nmid 1 & 2 \mid 2 & 2 \mid 4 & 2 \mid 8 \\
4 \nmid 1 & 4 \nmid 2 & 4 \mid 4 & 4 \mid 8 \\
8 \nmid 1 & 8 \nmid 2 & 8 \nmid 4 & 8 \mid 8
\end{array}
$$

Can *define* the relation as subset of $A \times A$:

$$R = \{(1,1), (1,2), (1,4), (1,8), (2,2), (2,4), (2,8), (4,4), (4,8), (8,8)\}$$

And we can write $x \; R \; y$ when $(x, y) \in R$

RELATIONS

DEFINITION:

Let A be a set.

- A **relation**, R, on A is a subset $R \subseteq A \times A$.
- If $(x, y) \in R$ we write $x \; R \; y$, and otherwise write $x \not R \; y$

Examples

- $R = \{(x, x) \; : \; x \in \mathbb{R}\}$ is "$=$" on the reals
- $S = \left\{(x, y) \in \mathbb{Z}^2 \; : \; x - y \in \mathbb{N}\right\}$ is "$>$" on integers.

- Let B be a set, then
  - $R = \varnothing$ is the **trivial relation** on B
  - $S = B \times B$ is the **universal relation** on B

DRAW THE RELATION

- Consider the set $A = \{1, 2, 4, 8\}$ and divisibility.
- Draw node for each $a \in A$.
- If $a \ R \ b$ then draw arrow $a \to b$