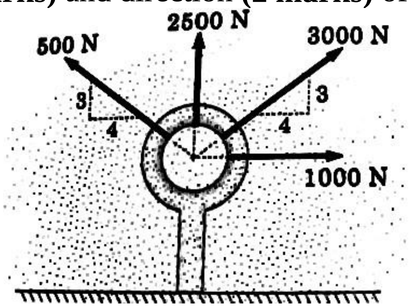
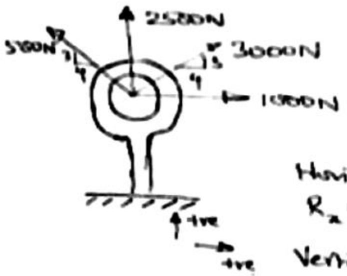
SOMAIYA VIDYAVIHAR UNIVERSITY

Semester: March 2022 – June 2022		
Maximum Marks: 100	**Examination: ESE Examination**	**Duration: 3 hrs**
Programme code: 06 **Programme: B Tech (Common for all)**	**Class:** FY	**Semester:** II **(SVU 2020)**
Name of the Constituent College: **K. J. Somaiya College of Engineering**	**Name of the department:** COMP / IT	
Course Code: 116U05C103	**Name of the Course: Engineering Mechanics**	
Instructions: 1)Draw neat diagrams 2)Assume suitable data if necessary		

<table>
<tr><th>Question No.</th><th></th><th>Max. Marks</th></tr>
<tr><td>Q1 (a)</td><td>

i. The 3000 N force shown in the figure is the resultant of four forces acting on the eyebolt, three of which are shown. Determine the magnitude **(3 marks)** and direction **(2 marks)** of the fourth force





→ 3000N is the resultant

→ ~~Take~~ Assume 'P' to be the fourth force

$\cos\theta = 4/5$, $\sin\theta = 3/5$

Horizontal Component of Resultant

$R_x = 3000 \times \frac{4}{5} =$

Vertical Component of Resultant

$R_y = 3000 \times \frac{3}{5} =$

<u>To find P</u>

$R_x = 1000 - 500 \times \frac{4}{5} + P\cos\theta = 3000 \times 4/5 \quad — (1)$

$R_y = 3000 \times \frac{3}{5} = 2500 + 500 \times 3/5 + P\sin\theta \quad — (2)$

Equation (1)

$2400 = 600 + P\cos\theta \quad — (1)$

$1800 = 2800 + P\sin\theta \quad — (2)$

$\tan\theta = -\frac{1000}{1800} \Rightarrow \theta = -29.054^\circ$ or $\theta = (29.054^\circ)$

From this value

$\boxed{P = 2059.113\ N}$

$\boxed{3\ marks}$ $\boxed{2\ marks}$

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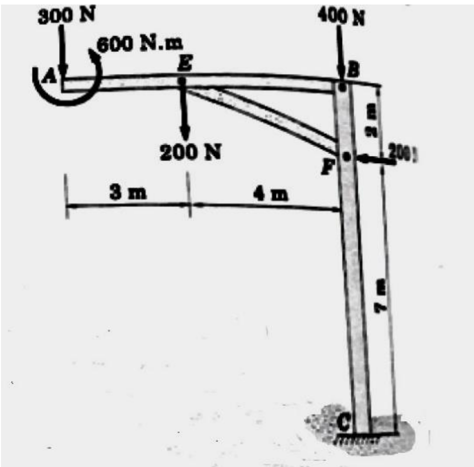
	ii. A frame is loaded as shown in figure. Determine the magnitude **(2 marks)**, direction **(2 marks)** and line of action **(1 mark)** of the equilibrant of all active forces with respect to member AB measured from point A.	



Example 27 **: A frame is loaded as shown in** *figure Ex.27(a).* **(a) Determine the magnitude, direction and line of action of the equilibrant w.r.t. member** *AB* **measured from point** *A*. **(b) Also replace the loading on the frame by a resultant force and couple at point** *C*.

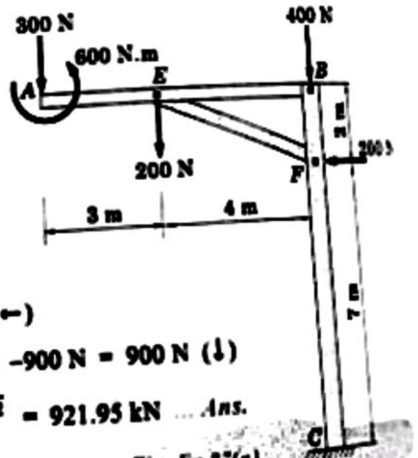


Fig. Ex.27(a)

Solution **: For the entire frame, we have**

$$(+\rightarrow)\ \Sigma F_x = -200\ \text{N} = 200\ \text{kN}\ (\leftarrow)$$

$$(+\uparrow)\ \Sigma F_y = -300 - 200 - 400 = -900\ \text{N} = 900\ \text{N}\ (\downarrow)$$

$$R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2} = \sqrt{(-200)^2 + (-900)^2} = 921.95\ \text{kN} \quad \textit{... Ans.}$$

$$\theta = \tan^{-1}\left|\frac{\Sigma F_y}{\Sigma F_x}\right| = \tan^{-1}\left|\frac{900}{200}\right| = 77.47^\circ \quad \textit{... Ans.}$$

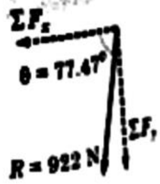


Fig. Ex.27(b)

R lies in third quadrant as both ΣF_x and ΣF_y are negative.
Resultant is represented as shown in *figure Ex.27(b)*.
To locate the position of the resultant from point A, we use Varignon's theorem as

$$|\Sigma M_A| = |\Sigma F_x \times y| = |\Sigma F_y \times x| = |R \times d| \quad \text{..... (1)}$$

Now,

$$(+\circlearrowleft)\ \Sigma M_A = -200 \times 3 + 600 - 400 \times 7 - 200 \times 2$$

$$= -3200\ \text{N.m} = 3200\ \text{N.m}\ (\circlearrowright) \quad \textit{... Ans.}$$

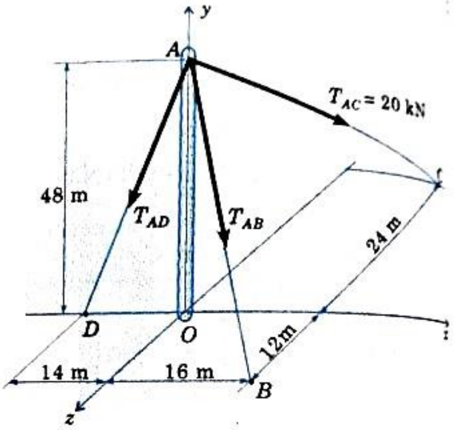
Using equation (1), we get

$$3200 = 200 \times y = 9000 \times x = 922 \times d$$

$$\therefore\ x = 3.56\ \text{m},\ y = 16\ \text{m and } d = 3.471\ \text{m from point } A. \quad \textit{... Ans.}$$

(a) Equilibrant is equal and opposite to the resultant. Hence equilibrant of magnitude E = 922 N at an angle of 77.47° acts at a distance of x = 3.56 m from point A as shown in the *figure Ex.27(c)*.

Q1 (b)	Knowing that the tension in AC is 20 kN, determine the required values of tensions in members AB **(5 marks)** and AD **(5 marks)** so that the resultant of the three forces applied at point A is vertical. Find their resultant.	10



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Example 35 : Knowing that the tension in AC is T_{AC} = 20 kN, determine the required values of tension T_{AB} and T_{AD} so that the resultant of the three forces applied at point A is vertical. Find their resultant.

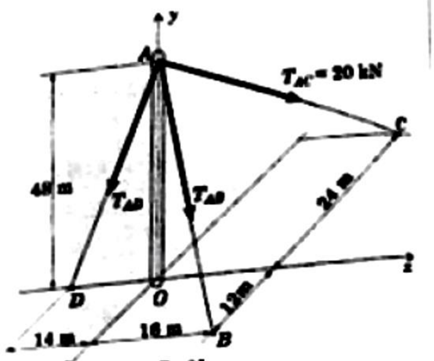


Fig. Ex.35

Solution : Given : $|T_{AC}| = 20$ kN, $R_x = 0$ and $R_z = 0$. **Coordinates of points :** $O = (0, 0, 0)$, $A = (0, 48, 0)$ m, $B = (16, 0, 12)$ m, $C = (16, 0, -24)$ m and $D = (-14, 0, 0)$ m.

From the data given in the problem, we have

$$\bar{R} = \bar{T}_{AB} + \bar{T}_{AC} + \bar{T}_{AD}$$

$$= |\bar{T}_{AB}|\,\lambda_{AB} + |\bar{T}_{AC}|\,\lambda_{AC} + |\bar{T}_{AD}|\,\lambda_{AD}$$

$$= (T_{AB})\left[\frac{(16-0)\,i + (0-48)\,j + (12-0)\,k}{\sqrt{16^2 + (-48)^2 + 12^2}}\right] + (20)\left[\frac{(16-0)\,i + (0-48)\,j + (-24-0)\,k}{\sqrt{16^2 + (-48)^2 + (-24)^2}}\right]$$

$$+ (T_{AD})\left[\frac{(-14-0)\,i + (0-48)\,j + (0-0)\,k}{\sqrt{(-14)^2 + (-48)^2}}\right]$$

$$= \frac{T_{AB}}{\sqrt{2704}}(16\,i - 48\,j + 12\,k) + \frac{20}{\sqrt{3136}}(16\,i - 48\,j - 24\,k) + \frac{T_{AD}}{\sqrt{2500}}(-14\,i - 48\,j)$$

$$= (0.31\,i - 0.92\,j + 0.23\,k)\,T_{AB} + (5.71\,i - 17.14\,j - 8.57\,k) + T_{AD}\,(-0.28\,i - 0.96\,j)$$

$$\bar{R} = (0.31T_{AB} + 5.71 - 0.28T_{AD})\,i + (-0.92T_{AB} - 17.14 - 0.96T_{AD})\,j + (0.23T_{AB} - 8.57)\,k$$

Equating i, j, k components, we get

$$R_x = 0.31T_{AB} + 5.71 - 0.28T_{AD} \qquad \text{..... (I)}$$

$$R_y = -0.92T_{AB} - 17.14 - 0.96T_{AD} \qquad \text{..... (II)}$$

$$R_z = 0.23T_{AB} - 8.57 \qquad \text{..... (III)}$$

It is given that $R_x = 0$ and $R_z = 0$

$$\therefore\ R_z = 0 = 0.23T_{AB} - 8.57$$

$$\therefore\ T_{AB} = 37.26 \text{ kN} \quad \text{... Ans.}$$

$$\text{Also } R_x = 0 = 0.31T_{AB} + 5.71 - 0.28T_{AD}$$

$$0 = 0.31 \times 37.26 + 5.71 - 0.28T_{AD}$$

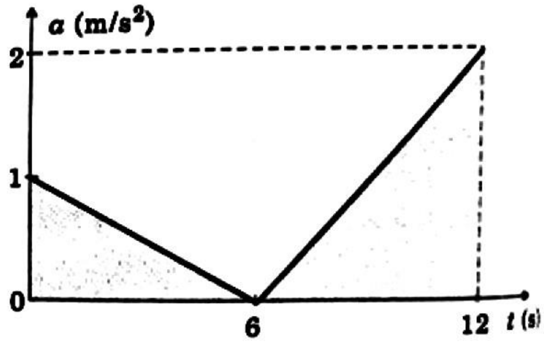
$$\therefore\ T_{AD} = 61.65 \text{ kN} \quad \text{... Ans.}$$

Putting values of T_{AB} and T_{AD} in equation (II), we get

$$R_y = R = -0.92 \times 37.26 - 17.14 - 0.96 \times 61.65$$

$$\therefore\ R = -110.60 \text{ kN} \quad \text{... Ans.}$$

Q2 (a)	The acceleration-time graph for a linear motion is shown in the figure below. Construct the velocity-time **(5 marks)** and displacement-time diagram **(5 marks)** for the motion assuming that the motion starts with Initial velocity of 5 m/s from the starting point.	10



Example 49 : The acceleration - time diagram for the linear motion is shown in *figure Ex.49(a)*. Construct velocity - time diagram and displacement - time diagram for the motion assuming that the motion starts with initial velocity of 5 m/s from the starting point.

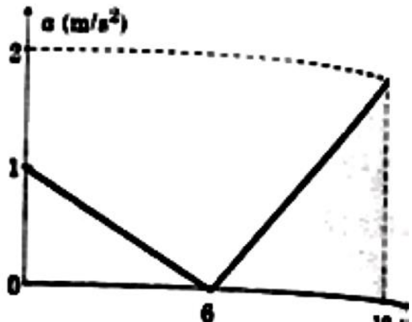


Fig. Ex.49(a) : a-t diagram

Solution : Initial condition : At $t = 0$, $x_0 = 0$, $v_0 = 5$ m/s

Area under a-t diagram = Change in velocity (Δv)
[Refer *figure Ex.49(a)*]

For $0 \le t \le 6$ s, Area $A_1 = \frac{1}{2} \times 6 \times 1 = 3 = v_6 - v_0 = v_6 - 5$ $\quad \therefore v_6 = 8$ m/s

For $6 \le t \le 12$ s, Area $A_2 = \frac{1}{2} \times 6 \times 2 = 6 = v_{12} - v_6 = v_{12} - 8$ $\quad \therefore v_{12} = 14$ m/s

To find position of the particle for x-t diagram (using a-t diagram), we use

$$x_t = x_0 + v_0 t + A\,\bar{t}$$

For $0 \le t \le 6$ s, $\quad x_6 = x_0 + v_0 t_1 + A_1 \bar{t}_1$

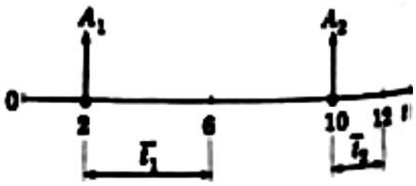
$\therefore x_6 = 0 + 5 \times 6 + 3 \times 4$

$= 42$ m

For $0 \le t \le 12$ s, $\quad x_{12} = x_6 + v_6 t_2 + A_2 \bar{t}_2$

$\therefore x_{12} = 42 + 8 \times 6 + 6 \times 2$

$= 102$ m



$\bar{t}_1 = \frac{2}{3} \times 6 = 4$

$\bar{t}_2 = \frac{1}{3} \times 6 = 2$

Fig. Ex.49(b)

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Plot *v-t* and *x-t* diagram as shown in *figure Ex.49(c)* and *(d)*.

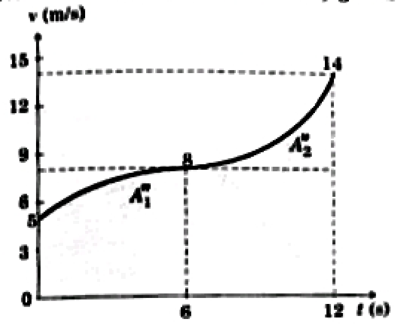

Fig. Ex.49(c) : v-t diagram

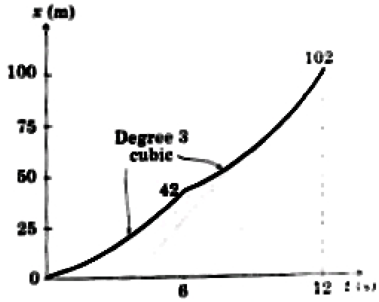

Fig. Ex.49(d) : x-t diagram

Alternative solution for *x-t* diagram. (Using *v-t* diagram)

Area under *v-t* diagram = Change in position (Δx) [Refer *figure Ex.49(c)*]

For $0 \le t \le 6$ s, Area $A_1 = A_1' + A_1'' = 5 \times 6 + \frac{nab}{n+1}$. Here $n = 2$, $a = 6$, $b = 3$

$\therefore\ A_1 = 30 + \frac{2 \times 6 \times 3}{2+1} = 42 = x_6 - x_0 = x_6 - 0 \qquad \therefore\ x_6 = 42$ m

For $6 \le t \le 12$ s, Area $A_2 = A_2' + A_2'' = 6 \times 8 + \frac{ab}{n+1}$. Here $n = 2$, $a = 6$, $b = 6$

$\therefore\ A_2 = 48 + \frac{6 \times 6}{2+1} = 60 = x_{12} - x_6 = x_{12} - 42 \qquad \therefore\ x_{12} = 102$ m

Now, plot *x-t* diagram as shown in *figure Ex.49(d)*.

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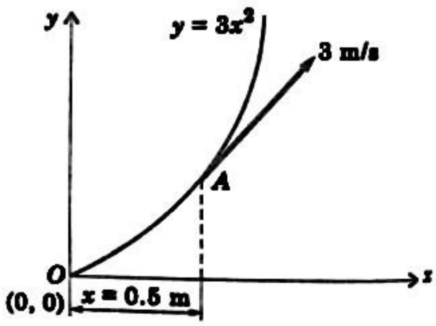
<td>Q2 (b)</td>
<td>

A particle moves with a constant speed of 3 m/s along the path as shown in figure below. What is the resultant acceleration at a position on the path where x=0.5 m **(5 marks)** ?. Also represent the acceleration in the vector form **(5 marks)**.



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Solution : Given data :

Speed of the particle, $v = 3$ m/s [Constant]

Equation of the curve, $y = 3x^2$

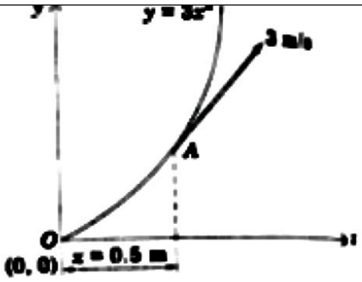

Fig. Ex 61(a)

To find : Acceleration a in vector form = ?

We have, acceleration, $a = \sqrt{a_t^2 + a_N^2}$ ... (i)

Body is moving with constant speed. ... (ii)

Hence $a_t = 0$

Normal acceleration at A, $a_N = \frac{v^2}{\rho} = \frac{3^2}{\rho} = \frac{9}{\rho}$ ... (iii)

To find : Radius of curvature, ρ = ? at $t = 2$ s

Given : $y = 3x^2$

$\therefore \frac{dy}{dx} = 6x$. At point A, $\left(\frac{dy}{dx}\right)_{x=0.5} = 6 \times 0.5 = 3$

$\frac{d^2y}{dx^2} = 6$ [Constant]

We have

$$\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}} \qquad \therefore \rho_{at\,x=0.5} = \frac{\left[1 + (3)^2\right]^{3/2}}{6}$$

$\therefore \rho_{at\,x=0.5} = 5.27$ m. Put this value in equation (iii) to get

$$a_N = \frac{9}{5.27} = 1.708 \text{ m/s}^2$$

Using equation (i) we get

Acceleration, $a = \sqrt{0 + (1.708)^2} = 1.708$ m/s^2 ... *Ans.*

To represent acceleration in vector form

As shown in *figure Ex.61(b)*, tangential acceleration is tangent to the curve at point A and normal acceleration is towards the centre of curvature and is perpendicular to the direction of tangential acceleration.

Fig. Ex 61(b)

From the figure

$\tan\alpha = \left(\frac{dy}{dx}\right)_{x=0.5} = 3 \qquad \therefore \alpha = 71.565°$

We have $a = a_x\,i + a_y\,j$

$a = (-a_N \sin\alpha)\,i + (a_N \cos\alpha)\,j$

$a = (-1.708 \sin 71.565°)\,i + (1.708 \cos 71.565°)\,j$

$a = -1.62\,i + 0.54\,j$ ... *Ans.*

OR

Block D shown in the figure given below, moves with a speed of 3 m/s. Determine the angular velocity of the links BD **(3 marks)** and AB **(3 marks)** and the velocity of point B **(4 marks)** at the instant known. Take length of AB = BD = 0.4 m.

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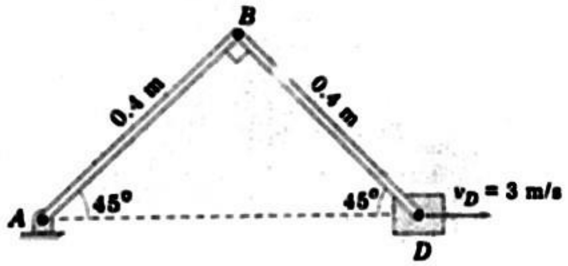

Solution : Rod AB rotates with angular velocity ω_{AB} about point A. Linear velocity of point B is perpendicular to AB. Point D has velocity $v_D = 3$ m/s to the right. ICR is located by drawing lines perpendicular to v_B (extension of AB) and v_D as shown in *figure Ex.6(b)*. Now we have from *figure Ex.6(b)*.

$$v_B = AB \times \omega_{AB} = IB \times \omega$$

$$\therefore\ v_B = 0.4\,\omega_{AB} = IB \times \omega \qquad \text{.....(I)}$$

$$v_D = ID \times \omega$$

$$3 = ID \times \omega \qquad \text{.....(II)}$$

To find IB and ID, we use geometry

From triangle ABD

$$AD = \sqrt{(0.4)^2 + (0.4)^2} = 0.5659 \text{ m}$$

From triangle AID

$$\tan 45° = \frac{ID}{AD} \qquad \therefore\ 1 = \frac{ID}{0.5659} \qquad \therefore\ ID = 0.5659 \text{ m}$$

Also,

$$\cos 45° = \frac{AD}{AI} = \frac{0.5659}{AI} \qquad \therefore\ AI = 0.8 \text{ m} \quad \therefore\ IB = AI - 0.4$$

$$\therefore\ IB = 0.8 - 0.4 = 0.4 \text{ m}$$

Substituting these values in equation (I) and (II)

From (II), $3 = 0.5659 \times \omega$

$\therefore\ \omega = 5.30$ r/s

From (I), $0.4\,\omega_{AB} = IB \times \omega = 0.4 \times 5.30 = v_B$

$\therefore\ \omega_{AB} = 5.3$ r/s and $v_B = 2.12$ m/s

Angular velocity of link BD, $\omega = 5.3$ r/s ... *Ans.*

Angular velocity of link $AB = \omega_{AB} = 5.3$ r/s ... *Ans.*

Velocity of point $B = v_B = 2.12$ m/s (45°, v_B) ... *Ans.*

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<td>Q3 (a)</td>
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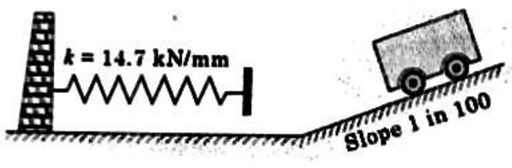
A Wagon weighing 490 kN starts from rest, runs 30 m down on the inclined surface and immediately strikes a post. If the rolling resistance of the track is 5 N/kN, find the velocity of the wagon when it strikes the post **(5 marks)** . If the impact is to be cushioned by means of bumper spring having k = 14.7 kN/mm, determine the maximum compression of the bumper spring **(5 marks)**.



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... determine the maximum compression of the bumper spring.

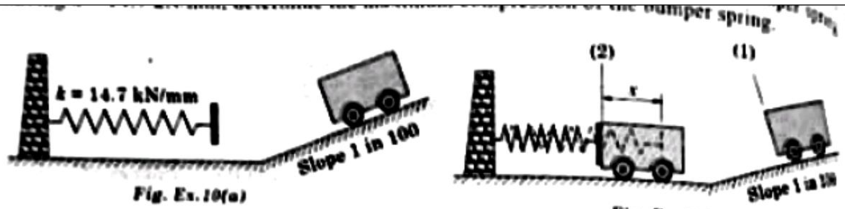


Fig. Ex. 10(a) Fig. Ex. 10(b)

Solution : Let x be the maximum compression of the spring in meters. At maximum compression of spring, velocity of wagon will be zero. Choose position (1) and (2) as shown in *figure Ex. 10(b)* and apply work energy principle for these two positions.

$$\text{K.E. at position 1} = \frac{1}{2}mv_1^2 = 0 \quad [\because \text{wagon starts from rest}]$$

$$\text{K.E. at position 2} = \frac{1}{2}mv_2^2 = 0 \quad [\because \text{wagon comes to rest}]$$

$$\text{Change in K.E.} = \text{K.E. at position 2} - \text{K.E. at position 1} = 0$$

$$\text{Work done by weight } U_{mg} = mgh = 490 \times 10^3 \times 30 \sin\theta$$

$$= 490 \times 10^3 \times 30 \times \frac{1}{100} \quad .....\left[\sin\theta \approx \tan\theta = \frac{1}{100}\right]$$

$$= 147000 \text{ J}$$

$$\text{Work done by rolling resistance} = -F_{rolling} \times S = -5 \times 490 \times 30$$

[Refer F.B.D. of wagon in motion]

$$U_{F_{rolling}} = -73500 \text{ J}$$

$$\text{Work done on spring}, \quad U_{spring} = \text{Initial spring energy} - \text{Final spring energy}$$

$$= \frac{1}{2}kx_1^2 - \frac{1}{2}kx_2^2$$

$$= 0 - \frac{1}{2} \times 14.7 \times 10^6 x^2 = -7.35 \times 10^6 x^2$$

By work energy principle

$$\text{Total work done} = \text{Change in K.E.}$$

$$U_{mg} + U_{F_{rolling}} + U_{spring} = 0$$

$$147000 - 73500 - 7.35 \times 10^6 x^2 = 0$$

$$7.35 \times 10^6 x^2 = 73500$$

$$\therefore x = 0.1 \text{ m} = 100 \text{ mm} \quad \textit{... Ans.}$$

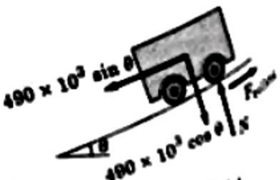


Fig. Ex. 10(c) F.B.D. of wagon in motion

To find the velocity of wagon when it strikes the bumper

Let v be the velocity of wagon when it just strikes the bumper. There is no compression of the spring.

By work energy principle

$$\text{Total work done} = \text{Change in K.E.}$$

$$U_{mg} + U_{F_{rolling}} = \text{Final K.E.} - \text{Initial K.E.}$$

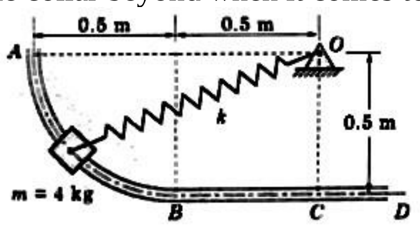
$$147000 - 73500 = \frac{1}{2} \times \frac{490 \times 10^3 v^2}{9.81} - 0$$

$$v = 1.715 \text{ m/s} \quad \textit{... Ans.}$$

OR

A 4 kg collar is attached to a spring and slides on a smooth bent rod ABCD (see figure given below). The spring has a constant k = 500 N/m and is undeformed when the collar is at C. If the collar is released from rest at A, determine

1. Velocity of the collar when it passes through B **(5 marks)**
2. Distance moved by the collar beyond when it comes to rest again.**(5 marks)**



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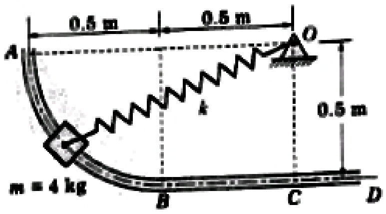

Fig. Ex.16(a)

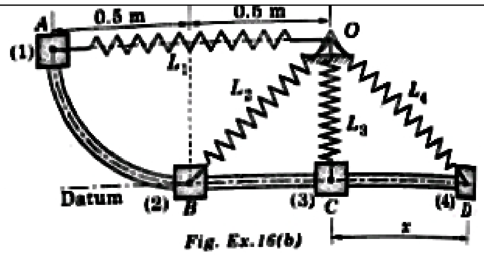

Fig. Ex.16(b)

Solution : Mark position (1), (2), (3) and (4). Datum coincides with position (2) (3) and (4). Let x be the distance moved by collar beyond C. Apply energy principle for position (1), (2), (3) and (4) [refer *figure Ex. 16(b)*].

Undeformed length of spring L_0 = Length OC = 0.5 m ... (given)

Length of spring at position (1), $L_1 = 1$ m

Length of spring at position (2), $L_2 = \sqrt{(0.5)^2 + (0.5)^2} = 0.707$ m

Length of spring at position (3), $L_3 = 0.5$ m

Length of spring at position (4), $L_4 = \sqrt{(0.5)^2 + x^2}$

By energy principle

Total energy at position (1) = Total energy at position (2) = Total energy at position (3) = Total energy at position (4)

$$[K.E. + P.E + S.E.]_1 = [K.E. + P.E + S.E.]_2 = [K.E. + P.E + S.E.]_3 = [K.E. + P.E + S.E.]_4$$

$$\frac{1}{2}mv_A^2 + mgh_A + \frac{1}{2}kx_1^2 = \frac{1}{2}mv_B^2 + mgh_B + \frac{1}{2}kx_2^2 = \frac{1}{2}mv_C^2 + mgh_C + \frac{1}{2}kx_3^2$$

$$= \frac{1}{2}mv_D^2 + mgh_D + \frac{1}{2}kx_4^2$$

$$0 + 4 \times 9.81 \times 0.5 + \frac{1}{2} \times 500\,(1 - 0.5)^2 = \frac{1}{2} \times 4 \times v_B^2 + 0 + \frac{1}{2} \times 500\,(0.707 - 0.5)^2$$

$$= \frac{1}{2} \times 4 \times v_C^2 + 0 + \frac{1}{2} \times 500 \times (0.5 - 0.5)^2 = 0 + 0 + \frac{1}{2} \times 500 \times \left[\sqrt{(0.5)^2 + x^2} - 0.5\right]^2$$

$$19.62 + 62.5 = 2v_B^2 + 10.712 = 2v_C^2 = 250\left[\sqrt{(0.5)^2 + x^2} - 0.5\right]^2$$

$\therefore$ Velocity of collar at B, $v_B = 5.975$ m/s ... *Ans.*

Velocity of collar at C, $v_C = 6.408$ m/s ... *Ans.*

(ii) To find distance moved by collar beyond point C

Consider $19.62 + 62.5 = 250\left[\sqrt{(0.5)^2 + x^2} - 0.5\right]^2$

$\therefore \sqrt{0.5^2 + x^2} = \sqrt{\frac{82.12}{250}} + 0.5$

$\therefore \sqrt{0.5^2 + x^2} = 1.073$ Squaring both sides

$0.5^2 + x^2 = (1.073)^2$

$\therefore$ Distance moved by collar beyond point C is $x = 0.95$ m ... *Ans.*

A and travels along the smooth guide.

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<td>Q3 (b)</td>
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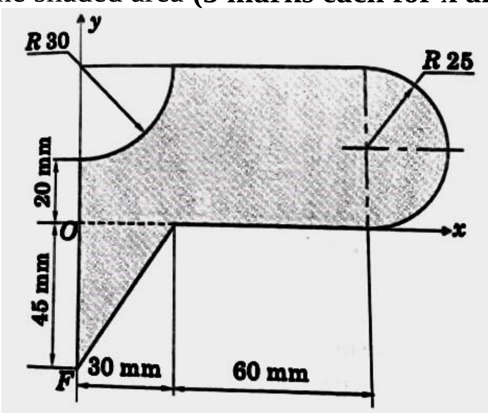
Find the centroid of the shaded area **(5 marks each for x and y coordinates)**



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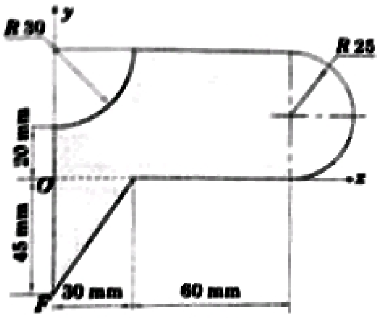


Fig. Ex.4(a)

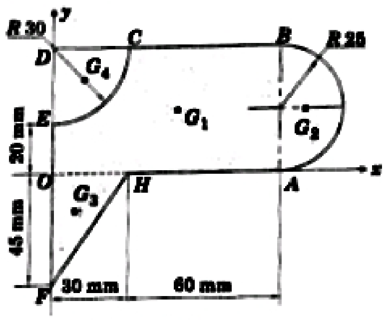


Fig. Ex.4(b)

Solution **: Divide the shaded areas into 4 parts and mark the centriod of the respective area. Prepare the table as follows.**

Component	Area A_i (mm²)	$\bar{x}_{Gi}$ (mm)	$\bar{y}_{Gi}$ (mm)	$A_i\bar{x}_{Gi}$ (mm³)	$A_i\bar{y}_{Gi}$ (mm³)
(1) Rectangle *OABD*	$90 \times 50 = 4500$	$\frac{90}{2} = 45$	$\frac{50}{2} = 25$	202500	112500
(2) Semicircle *ABA*	$\frac{\pi}{2} \times 25^2 = 981.747$	$90 + \frac{4 \times 25}{3\pi} = 100.61$	25	98773.566	24543.675
(3) Triangle *OHF*	$\frac{1}{2} \times 30 \times 45 = 675$	$\frac{1}{3} \times 30 = 10$	$-\frac{1}{3} \times 45 = -15$	6750	−10125
(4) Quarter Circle *DCE*	$-\frac{\pi \times 30^2}{4} = -706.858$	$\frac{4 \times 30}{3\pi} = 12.733$	$50 - \frac{4 \times 30}{3\pi} = 37.267$	−9000.423	−26342.477

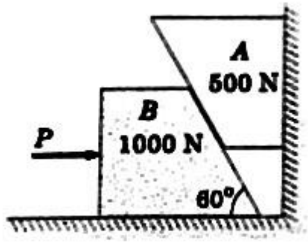
$\Sigma A_i = 5449.889 \text{ mm}^2$, $\Sigma A_i\bar{x}_{Gi} = 299039 \text{ mm}^3$, $\Sigma A_i\bar{y}_{Gi} = 100576.198 \text{ mm}^3$

$$\bar{x} = \frac{\Sigma A_i\bar{x}_{Gi}}{\Sigma A_i} = \frac{299039}{5449.889} = 54.868 \text{ mm}, \quad \bar{y} = \frac{\Sigma A_i\bar{y}_{Gi}}{\Sigma A_i} = \frac{100576.198}{5449.889} = 18.455 \text{ mm}$$

Centroid $G[\bar{x}, \bar{y}] = [54.868, 18.455]$ **mm** ... *Ans.*

Q4 (a) — Marks: 10

Assuming the values for μ as 0.25 for the floor and 0.3 at the wall and 0.2 between the blocks, find the minimum value of a horizontal force P applied to the lower block that will hold the system in equilibrium. **[FBD – 3 marks each for both bodies; result after solving equilibrium – 4 marks]**



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Example 28 : Assuming the values for $\mu = 0.25$ at the floor and 0.3 at the wall and 0.2 between the blocks, find the minimum value of a horizontal force P applied to the lower block that will hold the system in equilibrium.

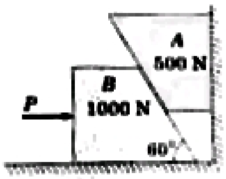
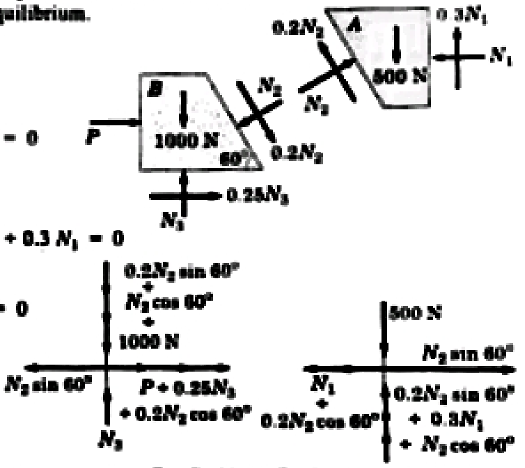


Fig. Ex 28(a)

Solution : In the absence of external force P, body B will be moving to the left and body A will be sliding down the wall. When P is gradually applied, motion gets retarded and at some minimum value of P, the body will come to static equilibrium. At P_{min} 500 N will show maximum tendency to move down and 1000 N will show maximum tendency to move left. Draw the F.B.D. and apply the conditions of equilibrium.



From F.B.D. of 500 N block
[Refer *figure Ex.28(b)*]

$\Sigma F_x = 0$

$-0.2\,N_2 \cos 60° + N_2 \sin 60° - N_1 = 0$

$\therefore\ N_1 = 0.766\,N_2$ ..... (I)

$\Sigma F_y = 0$

$0.2\,N_2 \sin 60° + N_2 \cos 60° - 500 + 0.3\,N_1 = 0$

Put value of N_1 from equation (I)

$\therefore\ 0.673 N_2 - 500 + 0.3 \times 0.766\,N_2 = 0$

$\therefore\ N_2 = 553.706$ N ..... (II)

From F.B.D. of 1000 N block
[Refer *figure Ex.31(b)*]

$\Sigma F_x = 0$

$P + 0.25\,N_3 + 0.2\,N_2 \cos 60° - N_2 \sin 60° = 0$

Fig. Ex.28(b) : F.B.D. of 1000 N block and 500 N block

Put value of N_2 from equation (II)

$\therefore\ P + 0.25\,N_3 + 0.2 \times 553.706 \cos 60° - 553.706 \sin 60° = 0$

$\therefore\ P + 0.25\,N_3 - 424.153 = 0$ ..... (III)

$\Sigma F_y = 0$: $N_3 - 0.2\,N_2 \sin 60° - N_2 \cos 60° - 1000 = 0$

Put value of N_2 from equation (I)

$\therefore\ N_3 - 0.2 \times 553.706 \sin 60° - 553.706 \cos 60° - 1000 = 0$

$\therefore\ N_3 = 1372.76$ N ..... (IV)

Equation (IV) in (III) gives

$P + 0.25 \times 1372.76 - 424.153 = 0$

$P = 80.964$ N ... *Ans.*

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<td>Q4 (b)</td>
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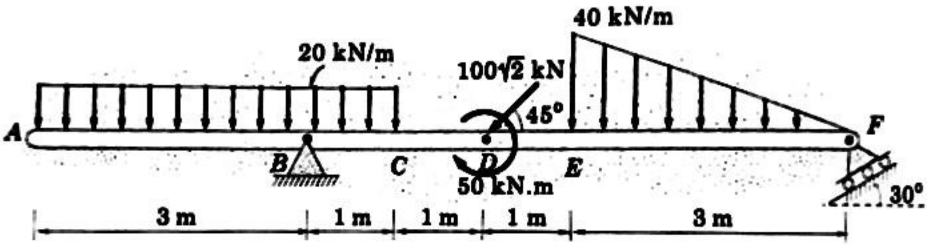
Determine the support reaction for the beam loaded as shown in the figure given below. **[FBD – 5 Marks ; 5 marks for solution]**



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Example 66 : ...
Ex.66(a).

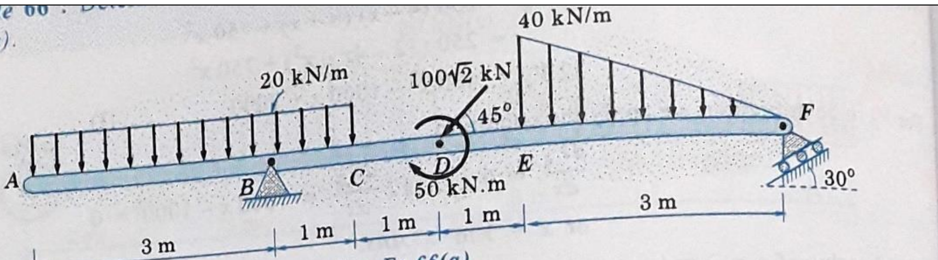


Fig. Ex.66(a)

Solution : Draw F.B.D. of beam as shown in *figure Ex.66(b)*. V_F is perpendicular to the surface of the roller.

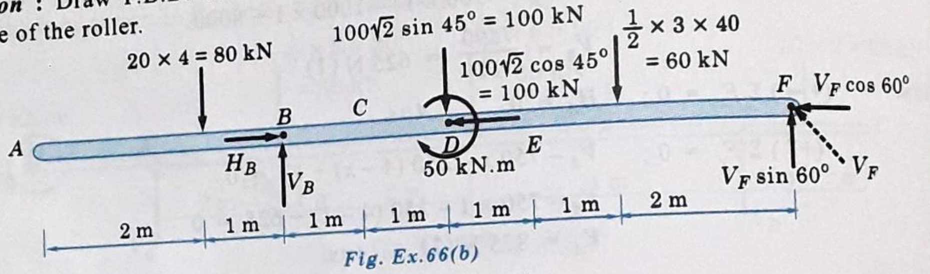


Fig. Ex.66(b)

Applying conditions of equilibrium to the beam, we get

$(+\rightarrow)\ \Sigma F_x = 0;\qquad H_B - V_F \cos 60^\circ - 100 = 0$

$$H_B = 100 + \frac{V_F}{2} \qquad \text{.... (I)}$$

$(+\uparrow)\ \Sigma F_y = 0;\qquad V_B + V_F \sin 60^\circ - 100 - 60 - 80 = 0$

$$V_B + 0.866\, V_F = 240 \qquad \text{.... (II)}$$

$(+\circlearrowleft)\ \Sigma M_B = 0;\qquad 80 \times 7 + 100 \times 4 + 60 \times 2 - V_B \times 6 - 50 = 0$

$$V_B = 171.666 \text{ kN}$$

Putting value of V_B in equation (II), we get

$$V_F = \frac{240 - 171.666}{0.866} = 78.90 \text{ kN} \qquad \textit{... Ans.}$$

Putting value of V_F in equation (I), we get

$$H_B = 100 + \frac{78.90}{2} = 139.45 \text{ kN}$$

Adding vectorially the components V_B and H_B, we get

$$R_B = \sqrt{(V_B)^2 + (H_B)^2} = \sqrt{(171.666)^2 + (139.45)^2}$$

$$= 221.169 \text{ kN} \quad \textit{... Ans.}$$

and $\theta = \tan^{-1}\left(\frac{V_B}{H_B}\right) = \tan^{-1}\left(\frac{171.666}{139.45}\right) = 50.91^\circ$ *... Ans.*

Reaction at B is as shown in *figure Ex.66(c)*.

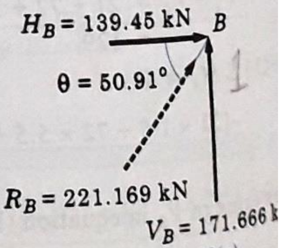
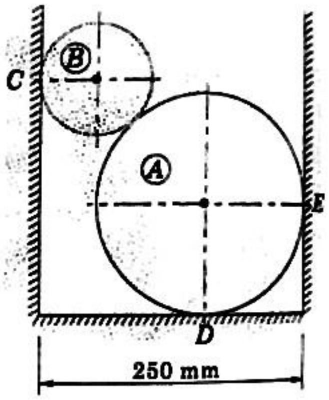


Fig. Ex.66(c)

OR

Two smooth spheres A and B of weight 200 N and 100 N respectively are resting against two smooth vertical walls and smooth horizontal floor as shown in figure below. The radius of sphere A is 100 mm and radius of sphere B is 50 mm. Find the reaction from the vertical wall and horizontal floor. Also find the reaction exerted by each sphere on the other. **[FBD – 3+2 Marks ; Solution – 5 marks]**



sphere on the ...

Solution : *Figure Ex.54(b)* shows F.B.D. of combined spheres. Reactions R_C, R_D and R_E are perpendicular to their respective surfaces.

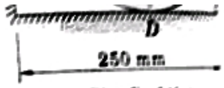


Fig. Ex.54(a)

From the geometry of the figures we have

$$\text{Length } NM = 250 - r_B - r_A = 250 - 50 - 100 = 100 \text{ mm}$$

$$\text{Length } ML = r_A + r_B = 100 + 50 = 150 \text{ mm}$$

From triangle *LMN*

$$\cos\theta = \frac{NM}{LM} = \frac{100}{150} \quad \text{or } \theta = 48.189°$$

$$\sin\theta = \frac{LN}{LM} \quad \text{or } \sin 48.189° = \frac{LN}{150}$$

$$\therefore LN = 111.80 \text{ mm}$$

Applying conditions of equilibrium, we have

$(\overset{+}{\rightarrow})\ \Sigma F_x = 0$

$$R_C - R_E = 0$$

$$\therefore R_C = R_E \quad \text{......... (1)}$$

$(+\uparrow)\ \Sigma F_y = 0$

$$R_D - 100 - 200 = 0$$

$$R_D = 300 \text{ N } (\uparrow) \quad \text{... } Ans.$$

$(+\circlearrowleft)\ \Sigma M = 0$

$$100 \times (MN) - R_C \times LN = 0$$

$$100 \times 100 - R_C \times 111.80 = 0$$

$$R_C = 89.45 \text{ N } (\rightarrow) \quad \text{... } Ans.$$

Substitute $R_C = 89.45$ N in equation (1) to get

$$R_E = 89.45 \text{ N } (\leftarrow) \quad \text{... } Ans.$$

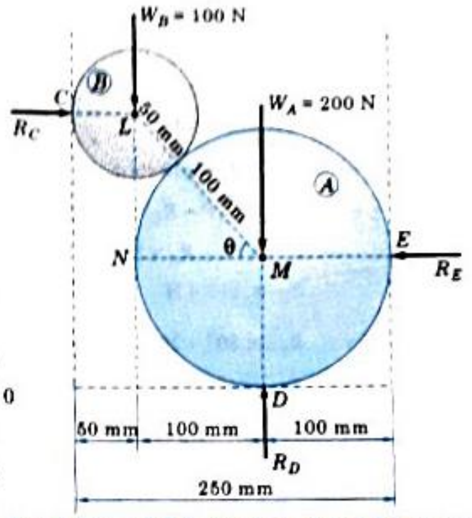


Fig. Ex.54(b) : F.B.D. of spheres A and B together

To find reaction exerted by each sphere.

Draw F.B.D. of cylinder *B* as shown in *figure Ex.54(c)* and apply conditions of equilibrium.

$(+\uparrow)\ \Sigma F_y = 0$

$$R_{AB} \sin\theta - W_B = 0$$

Here $\theta = 48.189°$ and $W_B = 100$ N

$$R_{AB} \sin 48.189° - 100 = 0$$

$$R_{AB} = 134.165 \text{ N} \quad \text{... } Ans.$$

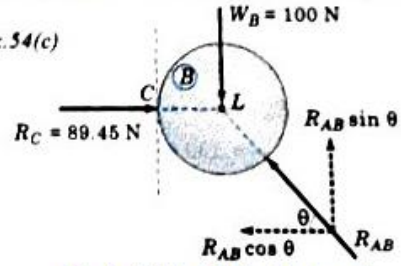
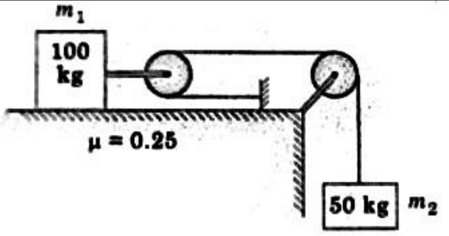


Fig. Ex.54(c) : F.B.D. of sphere B

Q5 (a)	Two blocks shown in figure below start from rest. If the string is inextensible, friction and inertia of the pulley is negligible. Calculate the acceleration of each block and tension in each string**. [FBD – 5 Marks ; acceleration and tension – 5 marks]**	10



friction
in each string.

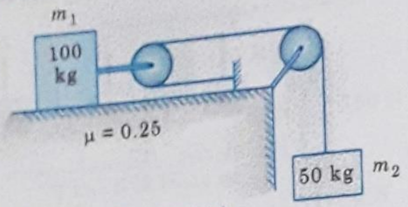


Fig. Ex.8(a)

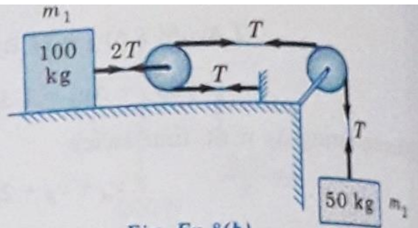


Fig. Ex.8(b)

Solution : Mass m_1 is subjected to tension $2T$ and mass m_2 is subjected to tension T. Let Δx_1 and Δy_2 be the small displacements of m_1 and m_2 respectively.

By principle of virtual work

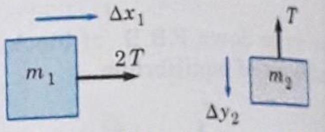


Fig. Ex.8(c) : Showing internal forces

$$2T\,\Delta x_1 - T\,\Delta y_2 = 0$$

$$2\,\Delta x_1 = \Delta y_2 \qquad \text{Differentiating } \Delta x \text{ and } \Delta y \text{ w.r.t time } t$$

$$2v_1 = v_2 \qquad \text{Differentiating } v \text{ w.r.t time } t$$

$$2a_1 = a_2 \qquad \dots \text{(I)}$$

Draw F.B.D. of masses m_1 and m_2 in dynamic equilibrium and apply conditions of dynamic equilibrium. [*figure Ex.8(d)*]

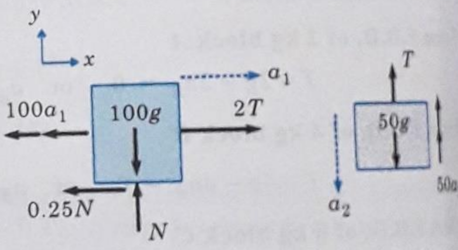


Fig. Ex.8(d) : F.B.D. of mass m_1 and m_2 in dynamic equilibrium

From F.B.D. of mass m_1

$\Sigma F_y = 0$; $\quad N - 100g = 0$

$$N = 100g \qquad \dots \text{(II)}$$

$\Sigma F_x = 0$; $\quad 2T - 100a_1 - 0.25N = 0$

$$2T - 100a_1 - 0.25 \times 100g = 0$$

$$2T - 100a_1 = 25g \qquad \dots \text{(III)}$$

From F.B.D. of mass m_2 [*figure Ex.8(d)*]

$\Sigma F_y = 0$; $\quad -50a_2 + 50g - T = 0$

$$\therefore 50a_2 + T = 50g$$

But from equation (I), $a_2 = 2a_1$

$$\therefore T + 100a_1 = 50g \qquad \dots \text{(IV)}$$

Solving equation (III) and (IV), we get

$$T = 245.25 \text{ N} \quad \text{and} \quad a_1 = 2.4525 \text{ m/s}^2 \ (\rightarrow)$$

$$\therefore a_2 = 2a_1 = 2 \times 2.4525 = 4.905 \text{ m/s}^2 \ (\downarrow)$$

$\therefore$ Acceleration of mass $m_1 = a_1 = 2.4525$ m/s^2 ... ***Ans.***

Tension in the string connecting mass, $m_1 = 2T = 490.5$ N ... ***Ans.***

Acceleration of mass, $m_2 = a_2 = 4.905$ m/s^2 ... ***Ans.***

Tension in the string connecting mass, $m_2 = T = 245.25$ N ... ***Ans.***

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<td>Q5 (b)</td>
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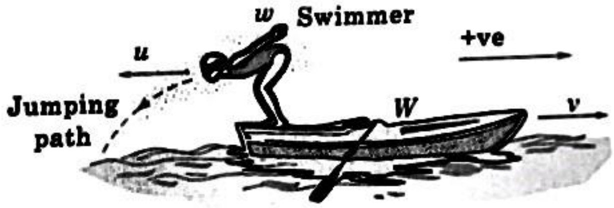
A swimmer of weight w and boat of weight W are moving together with a velocity of v towards the right as shown in the figure below. Find the increase in velocity of the boat when the swimmer starts walking towards left with a speed of u relative to the boat and then dives off from the boat to the left in the water with velocity u. **[5 marks for the derivation]**



If two swimmers A and B of mass 75 kg and 50 kg respectively, dive off from the rear end of a boat of 250 kg weight which is moving at a velocity of 5 ms^{-1} and the horizontal velocity of both swimmers relative to the boat is 4 ms^{-1}. Find the final velocity of the boat when B jumps first into the water and then followed by A. **[5 marks]**

Solution : Initial momentum when swimmer is standing in the boat which is moving with velocity v is given by

Initial momentum of boat + swimmer $= \left(\frac{W+w}{g}\right)v$ ... (I)

When swimmer starts walking towards left, there will be an increase in velocity of boat by Δv.

$\therefore$ Absolute velocity of boat $= (v + \Delta v)$ and

Absolute velocity of swimmer $= v + \Delta v - u$

Final momentum of boat + swimmer $= \frac{W}{g}(v+\Delta v) + \frac{w}{g}[v + \Delta v - u]$ ... (II)

As per momentum principle

Initial momentum = Final momentum

$$\left(\frac{W+w}{g}\right)v = \frac{W}{g}(v+\Delta v) + \frac{w}{g}[v+\Delta v - u]$$

$\therefore \Delta v = \left(\frac{wu}{W+w}\right)$ But $w = mg$ where m is the mass of swimmer and $W = Mg$ where M is the mass of the boat

$\therefore \Delta v = \left(\frac{mu}{M+m}\right)$ where Δv = increase in velocity of boat ... Ans.

(III) When swimmer B dives first followed by swimmer A

Using the formula

$$\Delta v = \frac{mu}{m+M}$$

When B jumps off, $\Delta v = \Delta v_1$, $m = 50$ kg, $u = 4$ m/s, $M = [75 + 250]$ kg

$$\therefore \Delta v_1 = \frac{50 \times 4}{50 + 325} = 0.5333 \text{ m/s } (\rightarrow)$$

When A jumps off, $\Delta v = \Delta v_2$, $m = 75$ kg, $u = 4$ m/s, $M = 250$ kg

$$\therefore \Delta v_2 = \frac{75 \times 4}{75 + 250} = 0.923 \text{ m/s } (\rightarrow)$$

$\therefore$ Final velocity of boat after both swimmers dive off

$= v + \Delta v_1 + \Delta v_2 = 5 + 0.533 + 0.923$

$= 6.456$ m/s $(\rightarrow)$ ... Ans.

1

OR

Two smooth balls collide as shown in the figure given below. Find the velocities after impact. Take $m_A = 1$ kg, $m_B = 2$ kg and e = 0.75 **[FBD – 3 marks ; Equations – 2 marks ; solving to get final velocities – 2 marks ; resultant**

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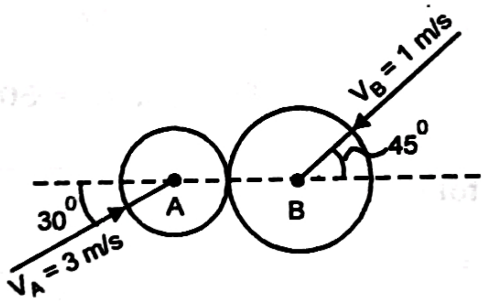
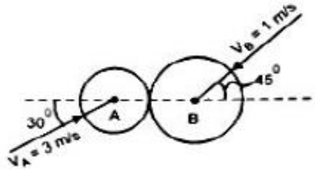
velocity and direction – 3 marks]

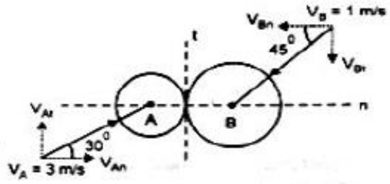


Ex. 15.10 Two smooth balls collide as shown. Find the velocities after impact.
Take $m_A = 1$ kg, $m_B = 2$ kg and $e = 0.75$



Solution: This is a case of Oblique Central Impact
Let the line of impact be the n direction and a perpendicular to it be the t direction.
Resolving the velocities along n and t direction.

$v_{An} = 2.6$ m/s $\rightarrow$, $v_{At} = 1.5$ m/s $\uparrow$
$v_{Bn} = 0.707$ m/s $\leftarrow$, $v_{Bt} = 0.707$ m/s $\downarrow$



Working in n direction

Using Conservation of Momentum Equation $\rightarrow$ + ve

$$m_A v_{An} + m_B v_{Bn} = m_A v_{A'n} + m_B v_{B'n}$$
$$1 \times 2.6 + 2 \times (-0.707) = 1 \times v_{A'n} + 2 v_{B'n}$$
$$1.186 = v_{A'n} + 2 v_{B'n} \quad \text{............ (1)}$$

Using Coefficient of Restitution Equation $\rightarrow$ + ve

$$v_{B'n} - v_{A'n} = e\left[v_{An} - v_{Bn}\right]$$
$$v_{B'n} - v_{A'n} = 0.75\left[2.6 - (-0.707)\right]$$
$$v_{B'n} = v_{A'n} + 2.48 \quad \text{............ (2)}$$

Solving equations (1) and (2), we get

$$v_{A'n} = -\ 1.26 \text{ m/s} = 1.26 \text{ m/s} \leftarrow$$
$$v_{B'n} = 1.22 \text{ m/s} = 1.22 \text{ m/s} \rightarrow$$

Working in t direction

Since velocities don't change in t direction

$$v_{A't} = v_{At} = 1.5 \text{ m/s} \uparrow$$
$$v_{B't} = v_{Bt} = 0.707 \text{ m/s} \downarrow$$

$\therefore$ **Total velocity** $v_A' = \sqrt{(v_{A'n})^2 + (v_{A't})^2}$

$$= \sqrt{(1.26)^2 + (1.5)^2} = 1.96 \text{ m/s}$$

at angle $\alpha' = \tan^{-1}\left(\frac{v_{A't}}{v_{A'n}}\right) = \tan^{-1}\left(\frac{1.5}{1.26}\right) = 50°$

$\therefore$ $v_A' = 1.96$ m/s, $\alpha' = 50°$ .... **Ans.**

Similarly total velocity $v_B' = \sqrt{(v_{B'n})^2 + (v_{B't})^2}$

$$= \sqrt{(1.22)^2 + (0.707)^2} = 1.41 \text{ m/s}$$

at angle $\beta' = \tan^{-1}\left(\frac{v_{B't}}{v_{B'n}}\right) = \tan^{-1}\left(\frac{0.707}{1.22}\right) = 30.1°$

$\therefore$ $v_B' = 1.41$ m/s, $\beta' = 30.1°$ .... **Ans.**