Problem Set 2: Bond Markets

Question 1

You have just purchased $10 million par amount of the 2yr, and $8 million par amount of the 30yr US Treasury bonds shown below, at the prices shown. (Assume all bonds are semi-annual compounding.)

Par Amt $millions	Bond Maturity	Coupon	Bond Price	Yield	$Duration (= DV01)
10.00	2yr		109.88	1.00%	1.80
	10yr		129.65	2.50%	7.50
8.00	30yr		146.94	3.50%	13.00

(i) Calculate the market values of the 2yr and 30yr bonds you purchased. [5 points]

The market values are simply the par amounts multiplied by the bond price / 100. See table below.

(ii) Calculate the "DV01 risk" for both of the bonds that you purchased. DV01 risk, for a given bond, is the dollar amount that you would make/lose if the bond's yield moved by 1% instantaneously. [10 points]

DV01 risk is dollar duration multiplied by par amount (*not* by market value). See table below.

(iii) What par amount do you need to buy or sell of the 10yr US Treasury bond, so that the DV01 risk of the entire portfolio is negligable? (Negligable in this context means "less than $100 of DV01 risk over the entire portfolio".) You must calculate the par amount of the 10yr bond to the 3rd decimal place (i.e., to the nearest thousand dollars). [10 points]

DV01 risk from the 2yr and 30yr = –1,220,000, so par amount of 10yr = 1,220,000 / 7.5 = –16.267

Par Amt $millions	Bond Maturity	Coupon	Bond Price	Yield	$Duration (= DV01)	Market Value	DV01 Risk $millions
10.00	2yr		109.88	1.00%	1.80	10.988	-0.18
-16.267	10yr		129.65	2.50%	7.50	-21.091	1.22
8.00	30yr		146.94	3.50%	13.00	11.756	-1.04
						1.653	0.0000

(iv) What are the coupons on the three bonds? [5 points]

2yr: 6.00%; 10yr: 5.87%; 30yr: 6.04% (Use the bond price/yield formula)

(v) Here are the convexity values for the 3 bonds:

2yr convexity = 0.02; 10yr convexity = 0.50; 30yr convexity = 2.30.

Using Duration and Convexity, estimate the change in market value of the portfolio to the nearest $1000 if all interest rates increase by 50bp. [5 points]

Problem Set 2: Bond Markets

Par Amt $millions	Bond Maturity	Coupon	Bond Price	Yield	$Duration (= DV01)	Market Value	DV01 Risk $millions	Price change from DV01	convex	New Price	New MV $millions
10.00	2yr	6.00%	109.88	1.00%	1.80	10.988	-0.18	-0.900	0.003	108.98	10.898
-16.267	10yr	5.87%	129.65	2.50%	7.50	-21.091	1.22	-3.750	0.063	125.97	-20.491
8.00	30yr	6.04%	146.94	3.50%	13.00	11.756	-1.04	-6.500	0.288	140.73	11.259
						1.653	0.0000				1.666
							bp change	0.50%		MV Change:	13,083

The market value of the portfolio will *increase* by $13,083 (= $13,000 to the nearest $1,000).

(iv) Given that your portfolio is duration neutral (i.e., the *estimated* change in value of the portfolio, for small parallel shifts in rates, is supposed to be zero), why do you, in fact, *make* money when the curve moves parallel? [5 points]

You make money from convexity. You are long the 30 year bond, which has much higher convexity than the 2yr and the 10yr. That means that the 30yr's price will not go down as much as duration estimated that it would. (You have some negative convexity which works against you from the 10yr, but its effect is swamped relative to the 30yr positive convexity.)

Question 2

(i) Suppose you have a view that the yield curve will *steepen* over the next few months. You plan to purchase one bond, and *short* sell another bond, so that you will make money if you are right about this view. The two bonds that will be in your portfolio are 2yr US Treasury bonds, and 30 yr US Treasury bonds. Which of these two bonds will you *buy*, and which will you *sell short*, given your view about the likely yield cuve movements over the next few months? [5 points]

If you believe that the yield curve will *steepen*, you need to *purchase* the 2yr bonds and *sell* the 30 year bonds.

(ii) The 2yr bond has a DV01 of 1.8. The 30yr bond has a DV01 of 13. You would like your portfolio to be neutral to small parallel yield curve shifts (i.e., both bonds' yields moving by the same amount in the same direction). That is, you would like your portfolio neither to *make* money nor *lose* money if the yield curve shifts parallel by a few basis points. For the 30yr bond, your position (either long or short, based on your answer to part (i)) will be $5 million par amount. What par amount (to the nearest $10,000) of the 2yr bond should you have in your portfolio so that you will be neutral to small parallel yield curve shifts? Answer this question using duration alone, without considering any convexity differences between the two bonds. [10 points]

The 30yr bond has a DV01 that is (13/1.8 = 7.222) times larger than that of the 2yr bond, so you should have 7.222 times as many 2yrs as 30yrs; so you will have **36.111 million** 2yr bonds in your portfolio.

(iii) Now bear in mind that these two bonds have very different convexities. Right after you created your portfolio as determined by parts (i) and (ii) above, the yield on both the 2yr bond and the 30yr bond *decrease* by 20bp that same day. Will your portfolio's value in fact remain the same? If not, do you anticipate that you will *make* money or *lose* money following these yield changes? You should answer this part of the question by including convexity, assuming that the 2 yr bond has a convexity of 0.02, while the 30 year's convexity is 2.3. [10 points]

The 2yr bond has very low convexity: duration alone will be a fairly accurate measure of MV change here:
The price change will be 1.8 * 0.2 = $0.36 (i.e. DV01 x 20/100 for a 20bp change).
The price change from convexity will be $0.02 * (20/100)^2 * 0.5 = 0.0004$ (i.e. negligable)

Hence the market value increase will be 0.36 * 36,111,000 / 100 = $130,000

The 30yr bond, however, has much higher convexity.
The price change from duration alone will be 13 * 0.2 = $2.60

Problem Set 2: Bond Markets

The price change from convexity will be $2.3 * (20/100)^2 * 0.5 = \$0.046$

Hence the market value *decrease* (since we shorted this bond) will be \$2.646 * –5,000,000 / 100 = –\$132,300

Hence we will **lose** around \$2,300

(iv) Again taking convexity into account, will your portfolio make or lose money if both bonds' yields *increase* by 20bp that same day? [5 points]

The answer is the same as in part (iii).

Question 3

Qintex Ltd. has issued a prospectus for their forthcoming bond issue. The Qintex bonds have a 16% coupon (semi-annual) and mature in 10 years. The CFO of Qintex is fully aware that his company's reputation in the debt market leaves a lot to be desired. He is not, however, worried about attracting buyers. "Over a ten-year horizon, the market-determined yield for high-risk companies, like ourselves, is only 12%" he tells you. "Hence, by paying a coupon of 16% per annum, investors will find our bonds very attractive." He continues "One of our major competitors, who is of similar risk to ourselves, is selling 10-year bonds and are offering a coupon of only 10%. Investors would be crazy to buy their bonds rather than ours."

(i) Evaluate the merit of the managing director's argument. Does the fact that the coupon rate (16%) exceeds the market-determined yield (12%) make the bonds particularly attractive to investors? Do you think investors will favor Qintex's bonds over the competitor's because they offer a higher coupon? No calculations are necessary to answer this question; however, you need to be very clear about your reasons for why investors would or wouldn't disagree with the Qintex CFO's comments. [10 points]

There is no merit in this argument. Investors will simply value the bonds so that the yield to maturity is 12% per annum. Qintex bonds will sell at a premium to face value since the coupon rate (16%) is greater than the required yield (12%). Conversely, the competitor's bonds will sell at a discount to face value since the coupon rate (10%) is less than the required yield (12%). The market will price the bonds to yield 12% per annum regardless of the coupon rate. If the risk of the two bonds is identical, investors will be indifferent between them.

(ii) Calculate the market price at which the 10-year Qintex bonds will sell. [5 points]

The Qintex bonds will pay a coupon of \$8 (per 100 face amount) every six months. Since the bonds have a ten-year life, there will be 20 coupon payments. Since the yield to maturity is quoted as 12% per annum this implies a semi-annual yield of 6% per six months. Thus the price of the bond will be the present value of coupon and face value payments:

$$\text{Price} = 8\left[\frac{1 - (1.06)^{-20}}{0.06}\right] + \frac{100}{(1.06)^{20}}$$

$$= \$122.94$$

(iii) Calculate the market price at which the major competitor's bonds will sell. [5 points]

Problem Set 2: Bond Markets

The only difference here is that the competitor's bonds pay a semi-annual coupon of $5:

$$\text{Price} = 5\left[\frac{1 - (1.06)^{-20}}{0.06}\right] + \frac{100}{(1.06)^{20}}$$
$$= \$88.53$$

(iv) Which bond's price is more sensitive to changes in yield on a *percentage* basis? [10 points]

The easiest way to answer this is to test what happens to both bonds if their yields move by the same amount. For example, if we shift the yield on both bonds to 6.1%, the new prices move to 121.62 and 87.48. The percentage changes are 1.08% and 1.18% respectively. Thus the lower coupon bond has higher percentage price sensitivity.