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5CE2

18DCE115

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DAA Assignment

Q1) Using master theorem:

(A) $T(n) = 9T(n/3) + n$

$\log_3 9 = 2$, $k = 1$, $p = 0$

$\log_b a > k$

$\therefore$ Complexity of function is $O(n^2)$

(B) $T(n) = 9T(2n/3) + 1$

$\therefore \log_{3/2} 9 = 5.9675$, $k = 0$, $p = 0$

$\log_b a > k$

$\therefore O(n^{5.9675}) \simeq O(n^6)$

$\therefore$ Complexity of function is $O(n^6)$.

Using Iterative method:

(A) $T(n) = 9T(n/3) + n$

$= 9\,[9T((n/3)/3) + n/3] + n$
$= 9\,[9T(n/9) + n/3] + n$
$= 81T(n/9) + 3n + n$
$= 3^4 T(n/3^2) + 3n + n$
$= 3^4\,(9T((n/3^2)/3) + n/3^2] + 3n + n$
$= 3^4\,[3^2 + (n/3^3) + n/3^2] + 3n + n$
$= 3^6\,[nT(n/3^3) + 3^2 n] + 3n + n$
$= (3^k)^2\, T(n/3^3) + 3^2 n + 3n + n$

$$= (3^k)^2 T(n/3^k) + (3^{k-1}n + 3^{k-2}n + \dots + 3n)$$

$$\therefore T(n) = (3^k)^2 T(n/3^k) + (3^{k-1}n + 3^{k-2}n + \dots + 3n)$$

Assume that $\frac{n}{3^k} = 1$ (or $3^k = n$)

$$\therefore 3^k = n$$
$$\therefore k = \log_3^n$$

$$\therefore T(n) = (3^k)^2 T(n/3^k) + n\left[3^{k-1} + 3^{k-2} + \dots + 3^1\right]$$

$$= n^2 T(1) + n\left[3^{\log n - 1} + 3^{\log n - 2} + \dots + 3\right]$$

$$= n^2 T(1) + n\,O(3^{\log n - 1})$$ [as $T(1) = 1$]
$$= n^2 + nO(n)$$
$$= O(n^2)$$

Q2) $T(n) = T(n-1) + T(n-2)$ — (1)

$$T(n-1) = T(n-2) + T(n-3)$$
$$T(n-2) = T(n-3) + T(n-4)$$

$$\therefore T(n) = T(n-2) + T(n-3) + T(n-3) + T(n-4)$$
$$= T(n-2) + 2T(n-3) + T(n-4)$$
$$= T(n-3) + T(n-4) + 2T(n-3) + T(n-4)$$
$$= 3T(n-3) + 2T(n-4)$$ — (2)

$$T(n-3) = T(n-4) + T(n-5)$$
$$T(n-4) = T(n-5) + T(n-6)$$
$$\therefore T(n-3) = 2T(n-5) + T(n-6)$$

$$\therefore T(n) = 8T(n-5) + 5T(n-6)$$ — (3)

$T(n-5) = T(n-6) + T(n-7)$

$T(n-6) = T(n-7) + T(n-8)$

$\therefore T(n-5) = 2T(n-7) + T(n-8)$

$\therefore T(n) = 21T(n-7) + 13T(n-8)$ —— (4)

$\therefore T(n) = T(n-1) + T(n-2) + 2T(n-4) + 3T(n-3) + 5T(n-6) + 8T(n-5) \ldots$

The above function is of Fibonnaci series.

$\therefore$ Complexity of the function is $O(2^n)$.

Q3 $T(n) = T(n/3) + T(2n/3) + n$

n $\longrightarrow$ n times

$n/3$ $\quad 2n/3$ $\longrightarrow$ n times

$n/3^2 \quad 2n/3^2 \quad 2n/3^2 \quad 2^2 n/3^2$ $\longrightarrow$ n times

$n/3^3 \quad 2n/3^3 \quad 2n/3^3 \quad 2^2n/3^3$

$\vdots$

$\frac{n}{3^k}, \frac{2^k n}{3^k} = \frac{n}{(3/2)^k}$ $\longrightarrow$ n times

upto k steps
that is total = kn

$n \rightarrow \frac{2n}{3} \rightarrow \frac{2^2 n}{3^2} \rightarrow \frac{2^3 n}{3^3} \ldots \rightarrow \frac{2^k n}{3^k} = \frac{n}{(3/2)^k}$

$$\therefore \frac{n}{(3/2)^k} = 1$$

$$n = (3/2)^k$$

$$k = \log_{3/2}^{n}$$

$\therefore$ complexity of this function is $O(n \log n)$

Q4) $T(n) = T(n-1)+n$ using substitution & iteration method.

→ substitution method

$$T(n) = T(n-1)+n \quad \text{—— (1)}$$
$$T(n-1) = T(n-2) + n-1$$

$$\therefore T(n) = T(n-2) + n-1+n$$
$$= T(n-2) + 2n - 1$$
$$= T(n-3) + 3n - 2$$
$$= T(n-k) + kn - (k-1) \rightarrow \text{constant}$$

let, $n-k=1$
$k = n-1$

$$\therefore T(n-(n-1)) + n(n-1) \Rightarrow T(1)=1$$

$$= T(1) + n^2 - 1$$
$$= 1 + n^2 - 1$$
$$= n^2$$

$\therefore$ complexity is $O(n^2)$

→ Iteration method

$T(n) = T(n-1) + n$

let

$T(1) = 1$

$T(2) = T(2-1) + 2$

$= T(1) + 2$

$= 1 + 2$

$T(3) = T(3-1) + 3$

$= T(2) + 3$

$= 1 + 2 + 3$

$T(4) = T(3) + 4$

$= 1 + 2 + 3 + 4$

$\vdots$

$T(n) = 1 + 2 + 3 + \dots + (n-1) + n$

$\therefore T(n) = \frac{n(n+1)}{2} = \frac{n^2}{2} + \frac{n}{2}$

$\therefore$ Complexity is $O(n^2)$

Q5) Find the recurrence relation of recursive factorial algorithm & solve using recursion tree method

```
T(n)   <-  fun Factorial (n)
           {
              if (n==1)
1      <-        return 1;
              else
T(n-1) <-        return n * Factorial (n+1)
           }
```

$\therefore T(n) = T(n-1) + 1$

$\therefore T(n) = (n + (n-1) + (n-2) + \dots + 2) + 1$

$= 1 + 2 + \dots + (n-1) + n$

$= \frac{n(n+1)}{2}$

$= \frac{n^2}{2} + \frac{n}{2}$

$= O(n^2)$

$n \rightarrow n$

$n-1 \quad 1 \rightarrow n-1$

$n-2 \quad 1 \rightarrow n-2$

$n-3 \quad 1 \rightarrow n-3$

$\vdots$

$2 \quad 1 \rightarrow 2$

Q6) $T(n) = 5T(n/4) + n$

$\longrightarrow$ Recursion Tree

$n \longrightarrow$ n times

$5n/4 \quad n \longrightarrow$ n times

$5n/4^2 \quad n/4 \longrightarrow$ n times

$5n/4^3 \quad n/4^2 \longrightarrow$ n times

$\vdots$

$n/4^k$

upto k steps that is nk times

$\therefore \frac{n}{4^k} = 1$

$n = 4^k$

$k = \log_4 n$

$\therefore$ Complexity is $O(n \log n)$

→ Iteration method

$T(n) = 5T(n/4) + n$

$= 5\left[5T + \left(\frac{n}{4^2}\right) + n\right] + n$

$= 5^2 T\left[\frac{n}{4^2}\right] + 5n + 5$

$= 5^3 T\left(\frac{n}{4^3}\right) + 5^2 n + 5n + 5$

$= 5^k T\left(\frac{n}{4^k}\right) + n\left(5^{k-1} + 5^{k-2} + \dots + 5\right)$

Assume $\frac{n}{4^k} = 1$ ① $\therefore k = \log_4^n$

$\therefore T(n) = 5^{\log n} T(1) + n\left[5^{\log n - 1} + 5^{\log n - 2} + \dots + 5\right]$

$= 5^{\log n} + n o(n)$, $T(1) = 1$

$= o(\log n) + o(n)$

$\therefore$ Complexity is $O(n \log n)$

Q7) $T(n) = 49T\left(\frac{n}{25}\right) + n^{3/2}\log n$

Using master theorm

$\log_{25}^{49} = 1.2090$, $k = \frac{3}{2}$, $p = 1$

$\therefore \log_b^a < k$, $p > 0$

$\therefore O(n^k \log n^p)$

$= O(n^{3/2}\log n) \approx O(n^2 \log n)$

Q8) Worst case analysis of Quick sort

Recurrence Relation:

$T(n) = T(n-1) + n$, $T(0) = T(1) = 0$

$\therefore T(n) = T(n-1) + n$ — ①

$T(n-1) = T(n-2) + n-1$

$T(n-2) = T(n-3) + n-2$

$\therefore T(n) = n + (n-1) + (n-2) + \dots + 3 + 2$

$\approx \frac{n^2}{2}$

$\therefore$ Complexity is $O(n^2)$