Single-Qubit Quantum Systems

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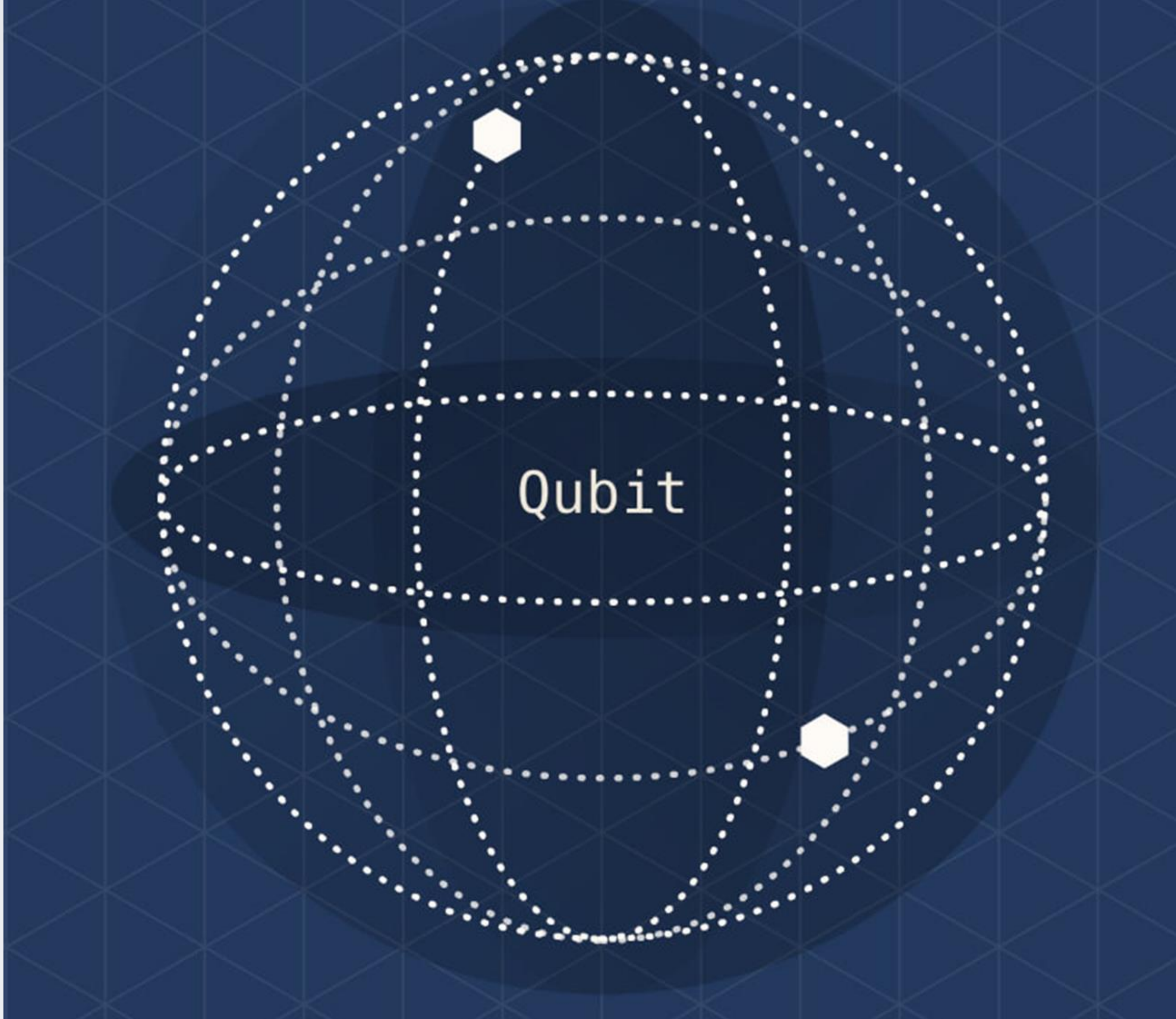


Lecture outline

- Quick review: linear algebra
- Qubits
- Single-qubit quantum gates
- Dirac notation
- Measurements
- First application: random number generation
- Demos

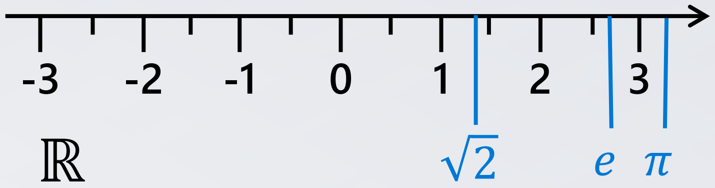
Quick review: linear algebra

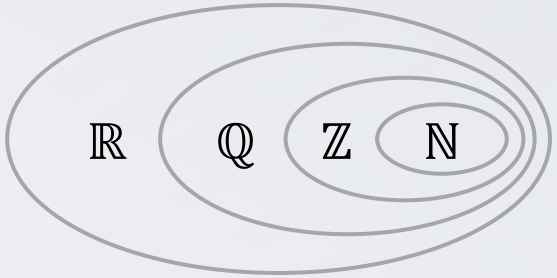
Review: real and complex numbers

Extra material
(not covered in lecture)

Real numbers $\mathbb{R}$

- Can represent distance on a line
- Include integer, rational and irrational numbers





Complex numbers $\mathbb{C}$

Expressed as

$$z = a + bi$$

where $a, b \in \mathbb{R}$, and i is imaginary unit, defined as

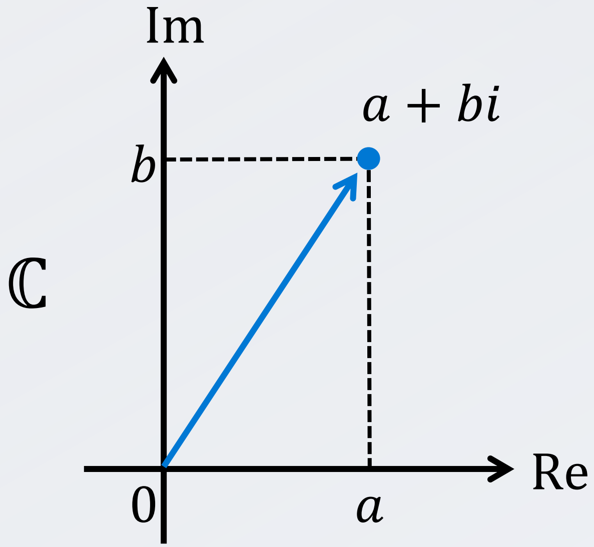
$$i^2 = -1$$

Complex conjugation:

$$(a + bi)^* = a - bi$$

Magnitude (modulus):

$$r = \|a + bi\| = \sqrt{(a + bi)(a + bi)^*} = \sqrt{a^2 + b^2}$$



Review: polar form of a complex number

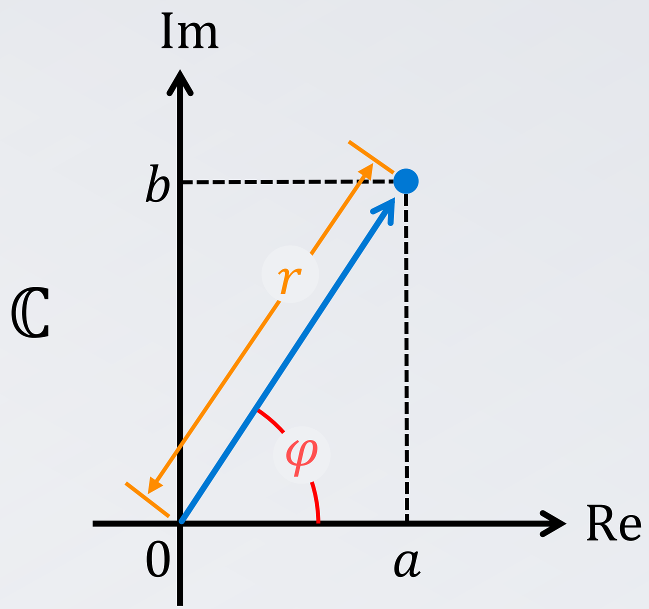
Extra material
(not covered in lecture)

Euler's formula:

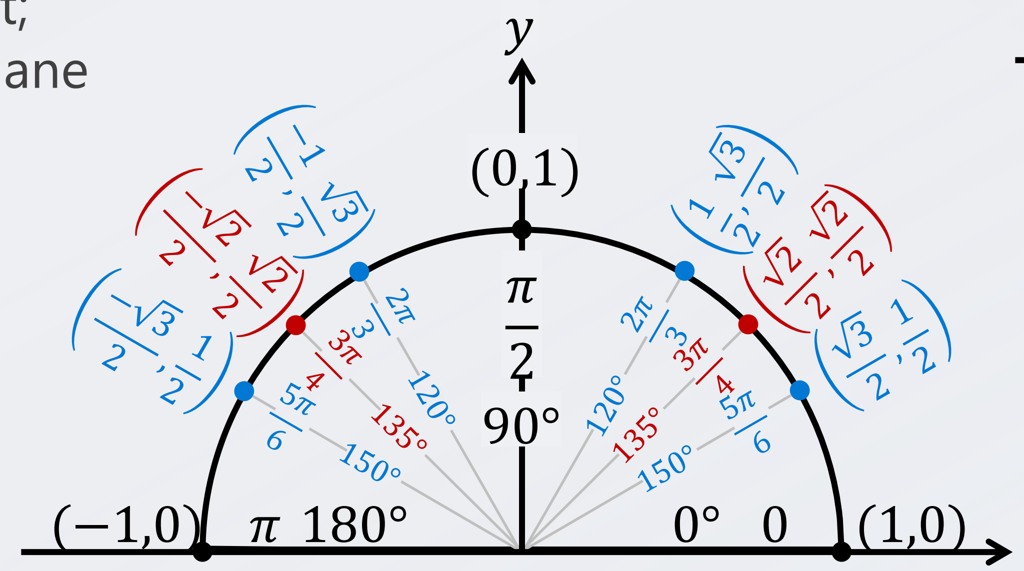
$$e^{i\varphi} = \cos\varphi + i\sin\varphi$$

Representing an arbitrary number:

$$z = a + bi = r(\cos\varphi + i\sin\varphi) = re^{i\varphi}$$



The phase φ is called the argument;
(r, φ) specify a point in complex plane



Review: matrices and vectors of size 2

2 × 2 matrix $A = \begin{pmatrix} a_{00} & a_{01} \\ a_{10} & a_{11} \end{pmatrix}$

Vector of length 2 $x = \begin{pmatrix} x_0 \\ x_1 \end{pmatrix}$

Multiplying a vector by a matrix

$$\begin{pmatrix} a_{00} & a_{01} \\ a_{10} & a_{11} \end{pmatrix} \begin{pmatrix} x_0 \\ x_1 \end{pmatrix} = \begin{pmatrix} a_{00}x_0 + a_{01}x_1 \\ a_{10}x_0 + a_{11}x_1 \end{pmatrix}$$

Review: matrices and vectors

$$n \times m \text{ matrix } A = \begin{pmatrix} a_{00} & a_{01} & \cdots & a_{0,m-1} \\ a_{10} & a_{11} & \cdots & a_{1,m-1} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n-1,0} & a_{n-1,1} & \cdots & a_{n-1,m-1} \end{pmatrix} \quad \text{Vector } x = \begin{pmatrix} x_0 \\ x_1 \\ \vdots \\ x_{m-1} \end{pmatrix}$$

Multiplying a vector by a matrix

$$\begin{pmatrix} a_{00} & a_{01} & \cdots & a_{0,m-1} \\ a_{10} & a_{11} & \cdots & a_{1,m-1} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n-1,0} & a_{n-1,1} & \cdots & a_{n-1,m-1} \end{pmatrix} \begin{pmatrix} x_0 \\ x_1 \\ \vdots \\ x_{m-1} \end{pmatrix} = \begin{pmatrix} a_{00}x_0 + a_{01}x_1 + \ldots + a_{0,m-1}x_{m-1} \\ a_{10}x_0 + a_{11}x_1 + \ldots + a_{1,m-1}x_{m-1} \\ \vdots \\ a_{n-1,0}x_0 + \cdots + a_{n-1,m-1}x_{m-1} \end{pmatrix}$$

Qubits

Qubit: a unit of computation

Classical: *bit*

$$b$$

$b = 0$ or 1

$$b = \begin{pmatrix}1\\0\end{pmatrix} \text{ or } \begin{pmatrix}0\\1\end{pmatrix} \begin{matrix} - \text{"0"} \\ - \text{"1"} \end{matrix}$$

Quantum: *qubit*

$$q$$

$$q = \begin{pmatrix}1\\0\end{pmatrix} \text{ or } \begin{pmatrix}0\\1\end{pmatrix} \text{ or } \begin{pmatrix}c_0\\c_1\end{pmatrix} \begin{matrix} - \text{"0"} \\ - \text{"1"} \end{matrix}$$

with $|c_0|^2 + |c_1|^2 = 1$

Superposition of states 0 and 1

Qubit: a unit of computation

The qubit state is a linear combination of basis states
(not "the qubit is in 0 and 1 state simultaneously!")

$$q = c_0 \begin{pmatrix} 1 \\ 0 \end{pmatrix} + c_1 \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} c_0 \\ c_1 \end{pmatrix}$$

Basis states "0" and "1"

c_0, c_1 are complex: $\begin{pmatrix} c_0 \\ c_1 \end{pmatrix} \in \mathbb{C}^2,\ |c_0|^2 + |c_1|^2 = 1$

Examples

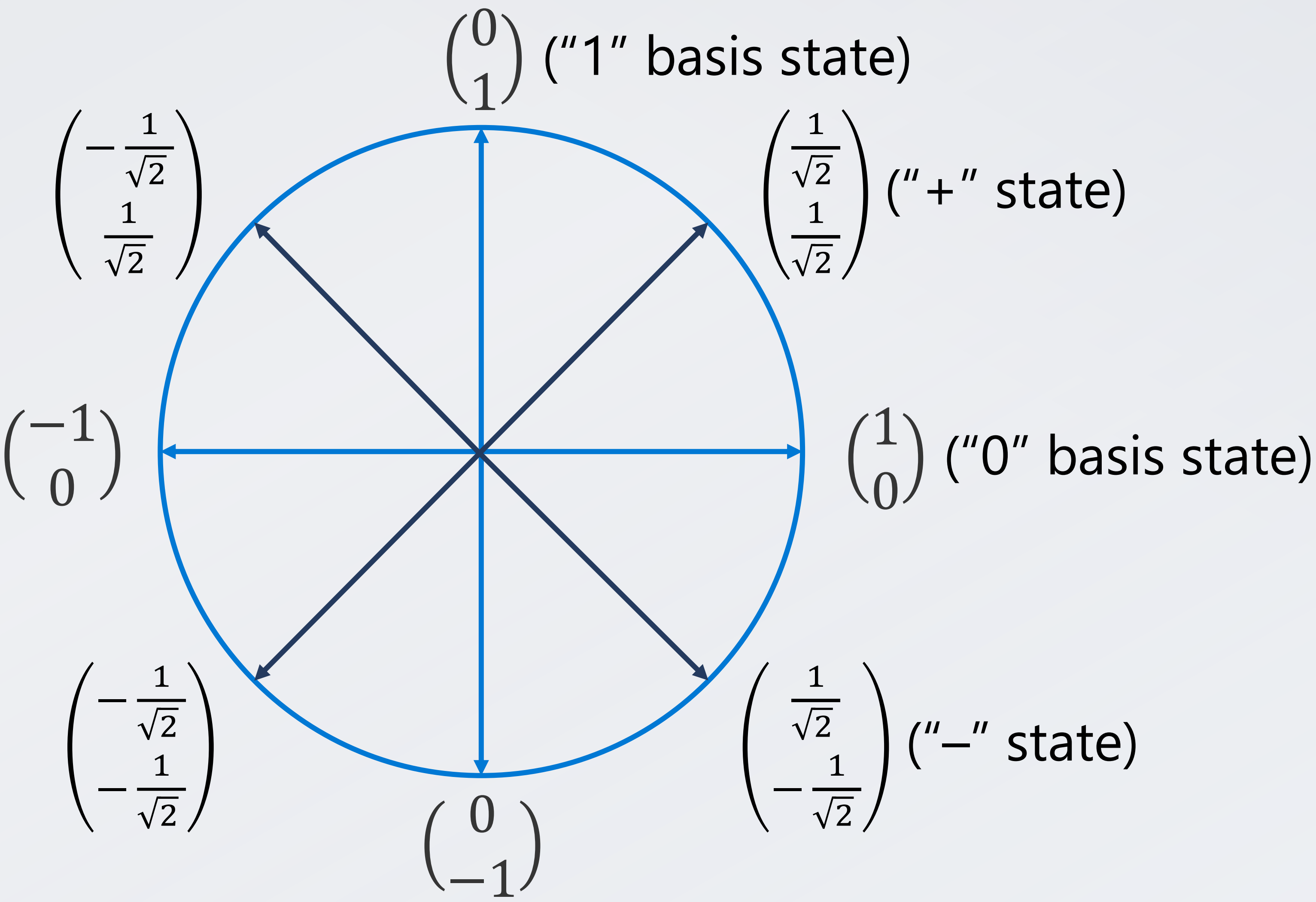
$$\begin{pmatrix} c_0 \\ c_1 \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix}$$

This state is called the "+" state.

$$\begin{pmatrix} d_0 \\ d_1 \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{pmatrix}$$

This state is called the "–" state.

Simple visualization for real coefficients



Dirac notation for single-qubit states

“Ket” notation: $|\cdot\rangle$ denotes a column vector with the given name

$$|0\rangle = \begin{pmatrix}1\\0\end{pmatrix},\ |1\rangle = \begin{pmatrix}0\\1\end{pmatrix},\ |c\rangle = \begin{pmatrix}c_0\\c_1\end{pmatrix} = c_0|0\rangle + c_1|1\rangle$$

Examples

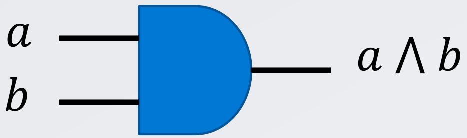
$$|+\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix} = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$

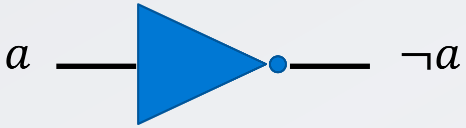
$$|-\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\-1\end{pmatrix} = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$$

Quantum gates

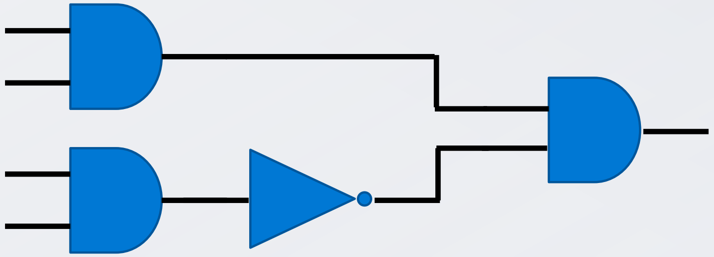
Logical gates

Universal gate set: a set of elementary gates from which any circuit can be built





We compose gates to make larger circuits to express complex computations



Quantum gates are quantum equivalent of logical gates: building blocks of quantum computations

Single-qubit quantum gates

A single-qubit quantum gate is a 2×2 matrix:

$$A = \begin{pmatrix} a_{00} & a_{01} \\ a_{10} & a_{11} \end{pmatrix}$$

Qubit state is a vector of size 2:

$$|\psi\rangle = \begin{pmatrix} c_0 \\ c_1 \end{pmatrix}$$

To apply a gate to a qubit, **multiply the vector by the matrix:**

$$A|\psi\rangle = \begin{pmatrix} a_{00} & a_{01} \\ a_{10} & a_{11} \end{pmatrix} \begin{pmatrix} c_0 \\ c_1 \end{pmatrix} = \begin{pmatrix} a_{00}c_0 + a_{01}c_1 \\ a_{10}c_0 + a_{11}c_1 \end{pmatrix}$$

Single-qubit gates: the X gate (bit flip gate)

Swaps the amplitudes of $|1\rangle$ and $|0\rangle$:

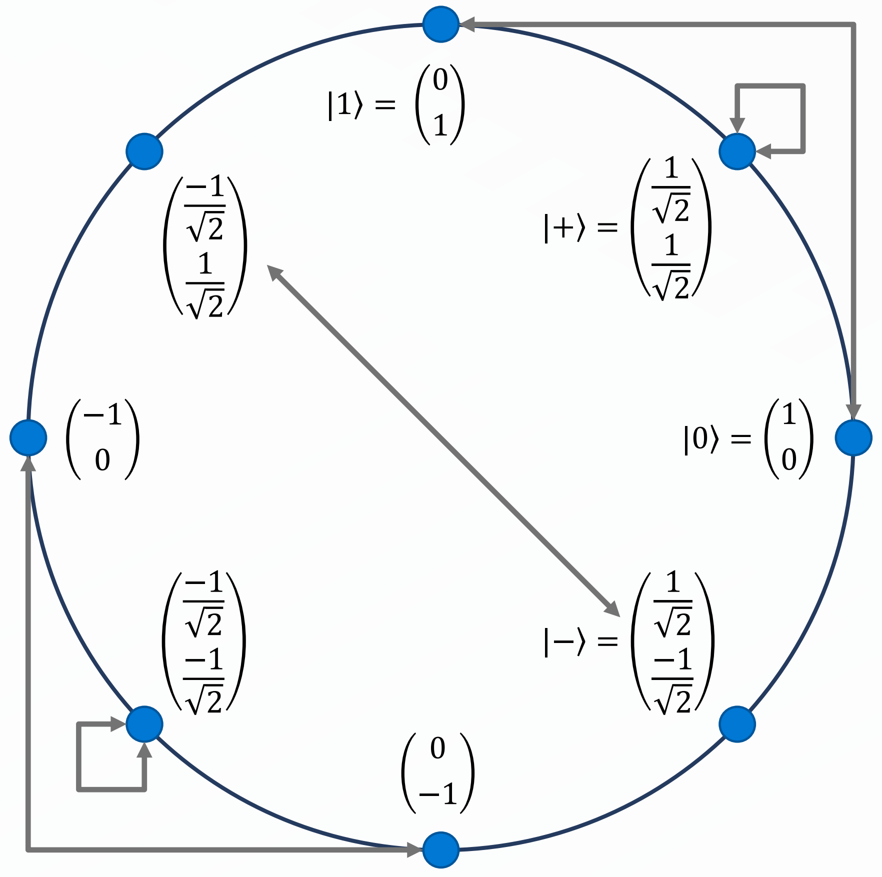
$$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$X\begin{pmatrix} c_0 \\ c_1 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} c_0 \\ c_1 \end{pmatrix} = \begin{pmatrix} c_1 \\ c_0 \end{pmatrix}$$

$$X|\psi\rangle = X(c_0|0\rangle + c_1|1\rangle) =$$
$$= c_0 X|0\rangle + c_1 X|1\rangle = c_0|1\rangle + c_1|0\rangle$$

Classical equivalent: NOT gate

$$|a\rangle \rightarrow |\neg a\rangle$$



Single-qubit gates: the Z gate (phase flip gate)

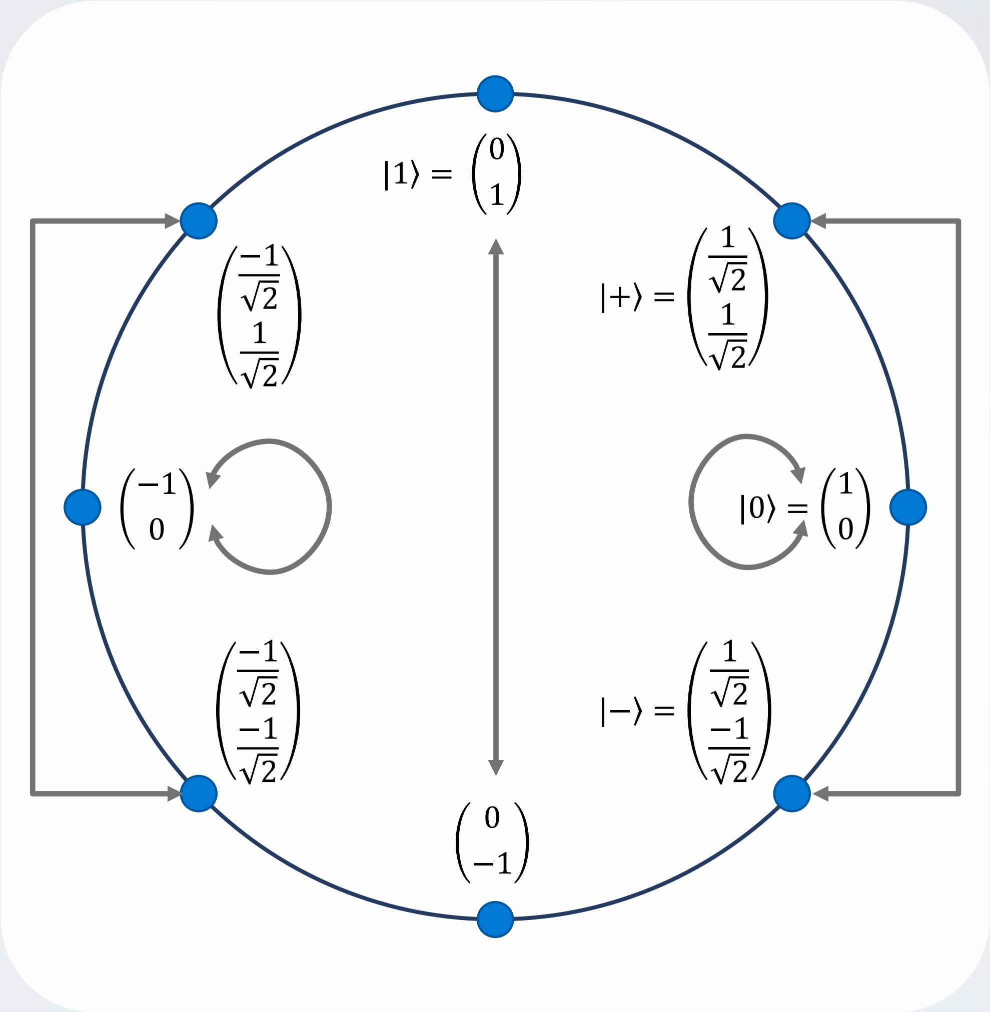
Multiplies the amplitude of $|1\rangle$ by -1

$$Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$Z \begin{pmatrix} c_0 \\ c_1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} c_0 \\ c_1 \end{pmatrix} = \begin{pmatrix} c_0 \\ -c_1 \end{pmatrix}$$

$$\begin{aligned} Z|\psi\rangle = Z(c_0|0\rangle + c_1|1\rangle) = \\ = c_0 Z|0\rangle + c_1 Z|1\rangle = \\ = c_0|0\rangle - c_1|1\rangle \end{aligned}$$

No classical equivalent!



Single-qubit gates: the H gate (Hadamard gate)

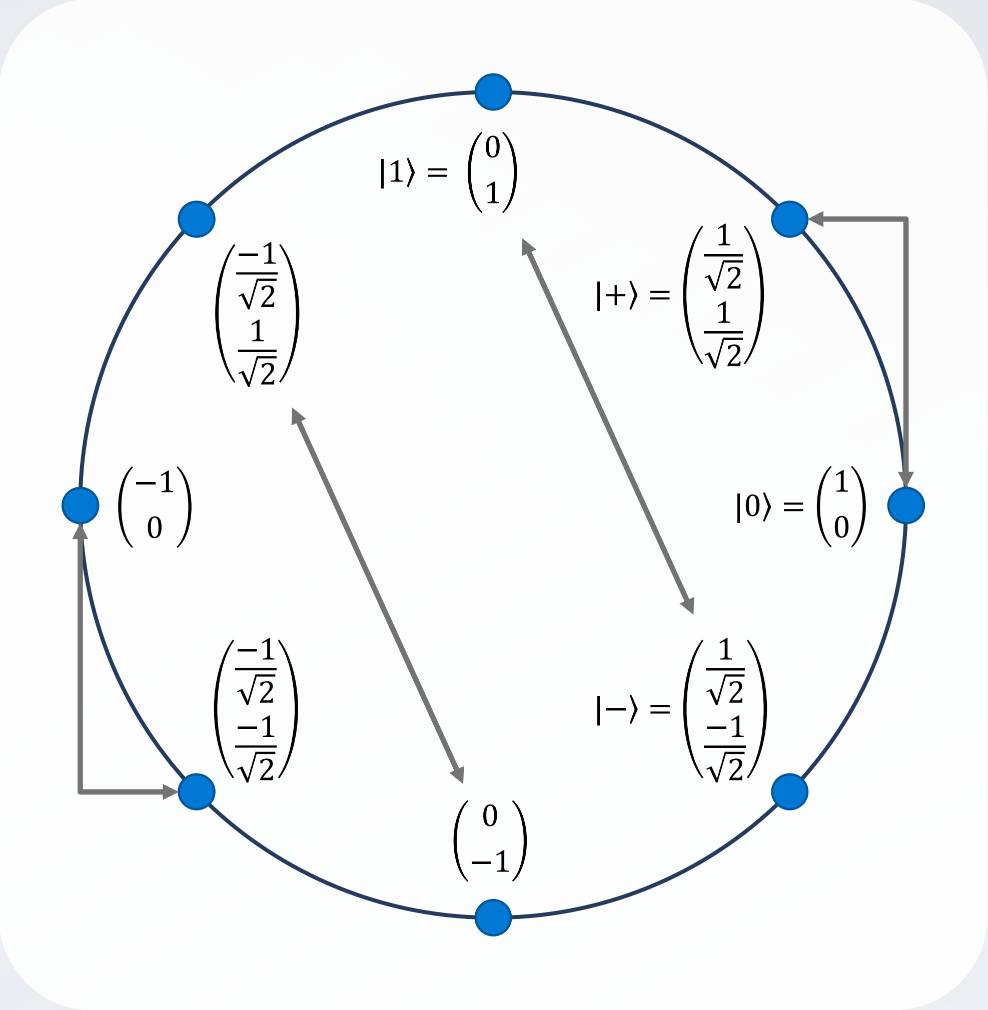
Converts a basis state to superposition (and vice versa)

$$H = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

$$H\begin{pmatrix} c_0 \\ c_1 \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} c_0 + c_1 \\ c_0 - c_1 \end{pmatrix}$$

$$H|0\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) = |+\rangle, H|1\rangle = |-\rangle$$
$$H|+\rangle = |0\rangle, H|-\rangle = |1\rangle$$

No classical equivalent!



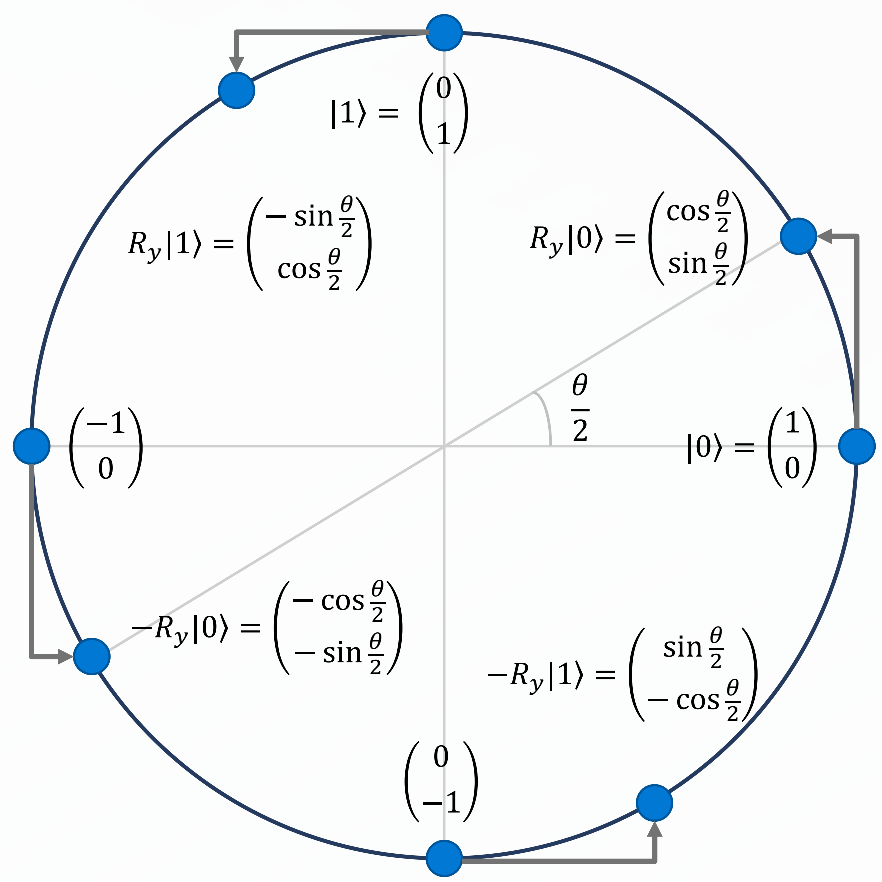
Single-qubit gates: the Ry gate (rotation gate)

Arbitrary rotation of the quantum state

$$R_y(\theta) = \begin{pmatrix} \cos\frac{\theta}{2} & -\sin\frac{\theta}{2} \\ \sin\frac{\theta}{2} & \cos\frac{\theta}{2} \end{pmatrix}$$

$$R_y|0\rangle = \cos\frac{\theta}{2}|0\rangle + \sin\frac{\theta}{2}|1\rangle$$
$$R_y|1\rangle = -\sin\frac{\theta}{2}|0\rangle + \cos\frac{\theta}{2}|1\rangle$$

Counter-clockwise rotation on the unit circle
Not self-adjoint!



Dirac notation for inner product

Extra material
(not covered in lecture)

"Bra" notation: $\langle \cdot |$ denotes a row vector (**adjoint** of ket $| \cdot \rangle$)

$$\langle 0| = \begin{pmatrix} 1 & 0 \end{pmatrix} = |0\rangle^\dagger$$

$$\langle 1| = \begin{pmatrix} 0 & 1 \end{pmatrix} = |1\rangle^\dagger$$

$$\langle c| = \begin{pmatrix} c_0 & c_1 \end{pmatrix} = c_0\langle 0| + c_1\langle 1| = |c\rangle^\dagger$$

Inner product of vectors $|\varphi\rangle$ and $|\psi\rangle$: **bra-ket**

$$\langle \varphi|\psi\rangle = \begin{pmatrix} \varphi_0^* & \varphi_1^* \end{pmatrix} \begin{pmatrix} \psi_0 \\ \psi_1 \end{pmatrix} = \varphi_0^*\psi_0 + \varphi_1^*\psi_1$$

We will use bra-ket notation when discussing both quantum gates and measurements

Dirac notation for gates

Extra material
(not covered in lecture)

Outer product of vectors $|\varphi\rangle$ and $|\psi\rangle$: **ket-bra**

$$|\psi\rangle\langle\varphi| = \begin{pmatrix}\psi_0 \\ \psi_1\end{pmatrix}\begin{pmatrix}\varphi_0^* & \varphi_1^*\end{pmatrix} = \begin{pmatrix}\psi_0\varphi_0^* & \psi_0\varphi_1^* \\ \psi_1\varphi_0^* & \psi_1\varphi_1^*\end{pmatrix}$$

We can use ket-bra notation to write gates

$$\begin{pmatrix}a_{00} & a_{01} \\ a_{10} & a_{11}\end{pmatrix} = a_{00}|0\rangle\langle 0| + a_{01}|0\rangle\langle 1| + a_{10}|1\rangle\langle 0| + a_{11}|1\rangle\langle 1|$$

Convenient for sparse gates (gates with few non-zero elements)

$$X = |1\rangle\langle 0| + |0\rangle\langle 1|$$

More single-qubit quantum gates

Single Qubit gates			
Gate	Matrix representation	Ket-bra representation	Applying to $\|\psi\rangle = \alpha\|0\rangle + \beta\|1\rangle$
X	$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$	$\|0\rangle\langle 1\| + \|1\rangle\langle 0\|$	$\|\psi\rangle = \alpha\|1\rangle + \beta\|0\rangle$
Y	$\begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$	$i(\|1\rangle\langle 0\| - \|0\rangle\langle 1\|)$	$Y\|\psi\rangle = i(\alpha\|1\rangle - \beta\|0\rangle)$
Z	$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$	$\|0\rangle\langle 0\| - \|1\rangle\langle 1\|$	$Z\|\psi\rangle = \alpha\|0\rangle - \beta\|1\rangle$
I	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\|0\rangle\langle 0\| + \|1\rangle\langle 1\|$	$I\|\psi\rangle = \|\psi\rangle$
H	$\frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$	$\|0\rangle\langle +\| + \|1\rangle\langle -\|$	$H\|\psi\rangle = \alpha\|+\rangle + \beta\|-\rangle = \frac{\alpha+\beta}{\sqrt{2}}\|0\rangle + \frac{\alpha-\beta}{\sqrt{2}}\|1\rangle$
S	$\begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix}$	$\|0\rangle\langle 0\| + i\|1\rangle\langle 1\|$	$S\|\psi\rangle = \alpha\|0\rangle + i\beta\|1\rangle$
T	$\begin{bmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{bmatrix}$	$\|0\rangle\langle 0\| + e^{i\pi/4}\|1\rangle\langle 1\|$	$T\|\psi\rangle = \alpha\|0\rangle + e^{i\pi/4}\beta\|1\rangle$
$R_x(\theta)$	$\begin{bmatrix} \cos\frac{\theta}{2} & -i\sin\frac{\theta}{2} \\ -i\sin\frac{\theta}{2} & \cos\frac{\theta}{2} \end{bmatrix}$	$\cos\frac{\theta}{2}\|0\rangle\langle 0\| - i\sin\frac{\theta}{2}\|1\rangle\langle 0\| -$ $- i\sin\frac{\theta}{2}\|0\rangle\langle 1\| + \cos\frac{\theta}{2}\|1\rangle\langle 1\|$	$R_x(\theta)\|\psi\rangle = (\alpha\cos\frac{\theta}{2} - i\beta\sin\frac{\theta}{2})\|0\rangle +$ $+ (\beta\cos\frac{\theta}{2} - i\alpha\sin\frac{\theta}{2})\|1\rangle$
$R_y(\theta)$	$\begin{bmatrix} \cos\frac{\theta}{2} & -\sin\frac{\theta}{2} \\ \sin\frac{\theta}{2} & \cos\frac{\theta}{2} \end{bmatrix}$	$\cos\frac{\theta}{2}\|0\rangle\langle 0\| + \sin\frac{\theta}{2}\|1\rangle\langle 0\| -$ $- \sin\frac{\theta}{2}\|0\rangle\langle 1\| + \cos\frac{\theta}{2}\|1\rangle\langle 1\|$	$R_y(\theta)\|\psi\rangle = (\alpha\cos\frac{\theta}{2} - \beta\sin\frac{\theta}{2})\|0\rangle +$ $+ (\beta\cos\frac{\theta}{2} + \alpha\sin\frac{\theta}{2})\|1\rangle$
$R_z(\theta)$	$\begin{bmatrix} e^{-i\theta/2} & 0 \\ 0 & e^{i\theta/2} \end{bmatrix}$	$e^{-i\theta/2}\|0\rangle\langle 0\| + e^{i\theta/2}\|1\rangle\langle 1\|$	$R_z(\theta)\|\psi\rangle = \alpha e^{-i\theta/2}\|0\rangle + \beta e^{i\theta/2}\|1\rangle$
$R_1(\theta)$	$\begin{bmatrix} 1 & 0 \\ 0 & e^{i\theta} \end{bmatrix}$	$\|0\rangle\langle 0\| + e^{i\theta}\|1\rangle\langle 1\|$	$R_1(\theta)\|\psi\rangle = \alpha\|0\rangle + \beta e^{i\theta}\|1\rangle$

https://github.com/microsoft/QuantumKatas/blob/main/quickref/qsharp-quick-reference.pdf

Measurements

Information readout: quantum vs classical

Classical

$$\begin{pmatrix}1\\0\end{pmatrix} \rightarrow \begin{pmatrix}1\\0\end{pmatrix} \text{ "0"}$$

$$\begin{pmatrix}0\\1\end{pmatrix} \rightarrow \begin{pmatrix}0\\1\end{pmatrix} \text{ "1"}$$

Quantum

$$\begin{pmatrix}c_0\\c_1\end{pmatrix} \rightarrow \begin{pmatrix}1\\0\end{pmatrix} \text{ "0"} \quad \text{Prob } |c_0|^2$$

or

$$\begin{pmatrix}c_0\\c_1\end{pmatrix} \rightarrow \begin{pmatrix}0\\1\end{pmatrix} \text{ "1"} \quad \text{Prob } |c_1|^2$$

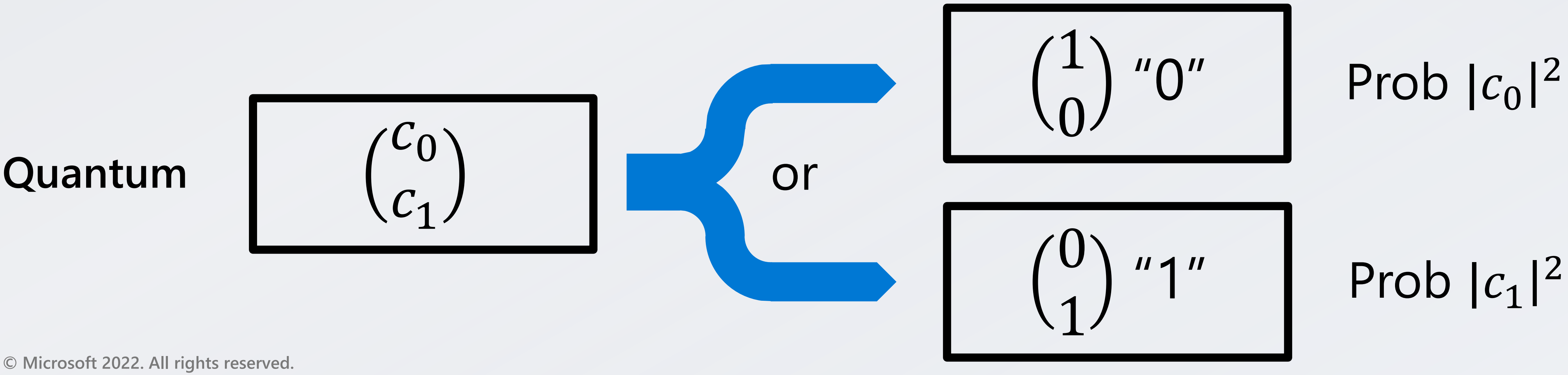
Information readout: quantum measurement

We need measurement to extract information out of the system

Measurement **limits** the power of quantum computing (we can not directly learn c_0 and c_1)

Measurement destroys (**collapses**) superposition (but not the qubit itself!)

Quantum system must be protected from unwanted measurement (known as **decoherence**)



Measurement in computational basis

Computational basis is vectors $|0\rangle$ and $|1\rangle$

We can always represent a qubit state as superposition of these two vectors:

$$|q\rangle = c_0|0\rangle + c_1|1\rangle$$

If we **measure** the qubit **in the basis** $\{|0\rangle, |1\rangle\}$, we get:

- Outcome "0" with probability $|c_0|^2$ (qubit state collapses to $|0\rangle$)
- Outcome "1" with probability $|c_1|^2$ (qubit state collapses to $|1\rangle$)

Examples

$$\begin{pmatrix}1\\0\end{pmatrix} \qquad \begin{pmatrix}i\\0\end{pmatrix} \qquad \frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix} \qquad \frac{1}{\sqrt{2}}\begin{pmatrix}1\\-i\end{pmatrix}$$

$$\begin{pmatrix}0\\1\end{pmatrix} \qquad \begin{pmatrix}0.6\\0.8\end{pmatrix}$$

Measurement is why we normalize state vectors

We previously mentioned global phase (multiplying quantum state by a complex number) – we can not observe it using measurement

Can we observe relative phase?

Review: orthogonality

Extra material
(not covered in lecture)

Two vectors are **orthogonal** if their inner product is 0: $\langle \varphi | \psi \rangle = 0$.

Example:

- Computational basis states are orthogonal: $|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$, $|1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$
- Hadamard basis states are orthogonal: $|+\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$, $|-\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \end{pmatrix}$

The **norm** of a vector is $\| |q\rangle \| = \sqrt{\langle q | q \rangle} = (\sum_i c_i^* c_i)^{1/2}$

A vector is a **unit vector** or **normalized** if $\| |q\rangle \| = 1$

We can normalize a vector by dividing it by its norm: $\frac{|q\rangle}{\| |q\rangle \|}$

Orthogonal measurement in another basis

Extra material
(not covered in lecture)

Consider orthogonal basis vectors $|b_0\rangle$ and $|b_1\rangle$.

We represent a qubit state as superposition:

$$|q\rangle = c_0|b_0\rangle + c_1|b_1\rangle$$

If we measure the qubit in the basis $\{|b_0\rangle, |b_1\rangle\}$, we get:

- Outcome b_0 with probability $|c_0|^2$ (qubit state collapses to $|b_0\rangle$)
- Outcome b_1 with probability $|c_1|^2$ (qubit state collapses to $|b_1\rangle$)

Measurement in Dirac notation

Extra material
(not covered in lecture)

We can represent measurement in the basis $\{|b_0\rangle, |b_1\rangle\}$ as a pair of **projection operators** $|b_0\rangle\langle b_0|$ and $|b_1\rangle\langle b_1|$

Measuring a qubit in state $|q\rangle$ is done by picking one of these projection operators at random and applying it

- Operator $|b_i\rangle\langle b_i|$ is picked with probability $|\langle b_i|q\rangle|^2$
- The outcome of applying operator $|b_i\rangle\langle b_i|$ is b_i
- Qubit state collapses to $|b_i\rangle\langle b_i||q\rangle$ (i.e., $|b_i\rangle$), renormalized

Example: measurement in Hadamard basis

Extra material
(not covered in lecture)

Consider the state $|q\rangle = \begin{pmatrix} c_0 \\ c_1 \end{pmatrix}$. Measure $|q\rangle$ in the $\{|+\rangle, |-\rangle\}$ basis.

Recall that $|+\rangle = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix} = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$, and $|-\rangle = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{-1}{\sqrt{2}} \end{pmatrix} = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$.

Rewrite $|q\rangle$ in this basis:

$$|q\rangle = \begin{pmatrix} c_0 \\ c_1 \end{pmatrix} = c_0|0\rangle + c_1|1\rangle = c_0\frac{1}{\sqrt{2}}(|+\rangle + |-\rangle) + c_1\frac{1}{\sqrt{2}}(|+\rangle - |-\rangle) =$$

$$= \frac{1}{\sqrt{2}}((c_0 + c_1)|+\rangle + (c_0 - c_1)|-\rangle)$$

Probability of measuring $|+\rangle$: $\left|\frac{1}{\sqrt{2}}(c_0 + c_1)\right|^2 = \frac{|c_0+c_1|^2}{2} = |\langle +|q\rangle|^2$

Probability of measuring $|-\rangle$: $\left|\frac{1}{\sqrt{2}}(c_0 - c_1)\right|^2 = \frac{|c_0-c_1|^2}{2} = |\langle -|q\rangle|^2$

Observing relative phase

How to distinguish orthogonal states $|+\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix}$ and $|-\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\-1\end{pmatrix}$?

Measuring in computational basis gives 50% “0” and 50% “1” for both

Measure in a different basis! (in this case, Hadamard basis)

Note that you always can represent measurement in a different basis as:

- applying some unitary to transform the measurement basis into the computational basis,
- doing measurement in computational basis, and
- applying the adjoint of that unitary to make sure the quantum state after the measurement matches the measurement outcome

For Hadamard basis, the unitary to apply is the Hadamard gate H

$$H|+\rangle = |0\rangle, H|-\rangle = |1\rangle$$

Things you can *not* do with measurement

If you are given a single copy of a state that is guaranteed to be $|\varphi\rangle$ or $|\psi\rangle$, and $\langle\varphi|\psi\rangle \neq 0$, you can not do a measurement to say which state it is

- You can try to maximize the probability that you're right,
- or you can do a measurement that allows you to never be wrong if you're allowed to also say "I don't know"

You can not find out the amplitudes from a single copy of a state

- Given multiple copies of this state, you can do an estimate

First application: random number generation

True (hardware) random number generation

TRNG generates random numbers from a physical process

- As opposed to pseudorandom number generators (PRNG) which generate deterministic sequences of numbers whose properties approximate the properties of truly random numbers

Quantum mechanics is fundamentally random

- Measuring a qubit in superposition produces a random result with configurable probabilities (not limited to 50-50 coin toss) physically

Allows to implement "true" random number generators

- Used in cryptography, sampling, gambling and lotteries, etc.

Already offered by some companies

- Just search for "quantum random number generator"

Demos

Microsoft Quantum Development Kit

The Quantum Katas

Automatically graded programming assignments

Running code on Azure Quantum