(1) A vector is a mathematical object that has both magnitude and direction.

A) Directed line segment in $\mathbb{R}^2$

→ It is a vector as it has both magnitude and direction defined.

B) Polynomial function $P: R \to R$.

→ It is not a vector as they are not defined with directions.

c) Elements of R^n

→ vectors are represented as n-dimentional arrays, Hence, they have both magnitude & direction. in ~~n-Dimentied.~~ n-Dimensional Space.

d) Continuous functions $f: R \to R$.

→ As they are not vectors, they do not have direction.

(2) ~~given~~

let 'x' represent the price for one sheaf of a good crop

'y' represent the price for one sheaf of a mediocre crop.

'z' represent the price for one sheaf of a bad crop.

According to the given data, there can be '3' Scenarios.

Scenario 1 $\Rightarrow$ $3x+2y+z=39$ $\rightarrow$ (1)

Scenario 2 $\Rightarrow$ $2x+3y+z=34$ $\rightarrow$ (2)

Scenario 3 $\Rightarrow$ $x+2y+3z=26.$ $\rightarrow$ (3)

This gives us a system of 3 linear equations with 3 unknown Variables x, y & z.

Yes, we can solve the above equation for x, y, z values.

Subtract (1) ~~&~~ & (2).

$$
\begin{array}{r}
3x+2y+z=39 \\
2x+3y+z=34 \\
(-)\quad(-)\quad(-)\quad(-) \\
\hline
x-y=5
\end{array}
\rightarrow (4)
$$

$\Rightarrow x = 5+y \rightarrow$ (5)

Sub ~~the~~ (5) in (3)

$(5+y)+2y+3z=26$

$5 + y + 2y + 3z = 26.$

$3y + 3z = 21$

$y + z = 7 \rightarrow 6$

$\Rightarrow y = 7 - z \rightarrow 7$

Sub eq 7 in 4

$x - y = 5.$

$x - (7 - z) = 5$

$x - 7 + z = 5$

$x + z = 12. \Rightarrow x = 12 - z \rightarrow 8$

Sub eq 7 & 8 in eq 1

$3(12 - z) + 2(7 - z) + z = 39$

$36 - 3z + 14 - 2z + z = 39$

$50 - 4z = 39.$

$4z = 11$

$\boxed{z = \frac{11}{4}}$

Sub 'z' in 8

$x = 12 - \frac{11}{4}$

$x = \frac{48 - 11}{4}$

$\boxed{x = \frac{37}{4}}$

Sub 'z' in (7)

$y = 7 - z$

$y = 7 - \frac{11}{4}$

$y = \frac{28-11}{4}$

$\boxed{y = \frac{17}{4}}$

hence, the values of x, y, z are

$x = \frac{37}{4}$, $y = \frac{17}{4}$ & $z = \frac{11}{4}$

(3) The objective is to find an optimal production plan i.e, a plan how many units of x_j of product N_j should be produced if a total of b_i units of Resource 'R_i' are available & no resources are left over.

If x_1, x_2 units of the corresponding products are produced, we need a total of $a_{i1}x_1 + a_{i2}x_2$ units of Resource R_i. The desired optimal production plan x_1, x_2 has to satisfy the following equations:

$$a_{11}x_1 + a_{12}x_2 = b_1$$
$$a_{21}x_1 + a_{22}x_2 = b_2$$

to solve the linear equations, it can be written in the following form,

$$x_1 \begin{bmatrix} a_{11} \\ a_{21} \end{bmatrix} + x_2 \begin{bmatrix} a_{12} \\ a_{22} \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$

Yes, there exists an optimal production plan.

④ given, A is a 3×4 matrix

B is a 4×3 matrix

C is a 3×3 matrix

Multiplication of 2 matrices is possible only when the no. of columns of first matrix is equal to no. of row of second matrix

It can be represented as.

$A_{m \times n} \times B_{n \times p} = C_{m \times p}$.

A) $AB = A_{3 \times 4} \times B_{4 \times 3}$

The multiplication can be performed and results in a matrix of 3×3.

B) $BA = B_{4 \times 3} \times A_{3 \times 4}$

here, the multiplication of B & A can be performed and results in a matrix of 4×4.

C) $BB = B_{4 \times 3} \times B_{4 \times 3}$

Since, ~~be~~ the no. of rows is not equal to no. of column, the multiplication cannot be performed.

D) $CA = C_{3 \times 3} \times A_{3 \times 4}$

~~so~~ the multiplication of C & A can be performed and results in a 3×4 matrix.

E) $ABC = (A_{3\times4} \times B_{4\times3}) \times C_{3\times3}$

$= AB_{3\times3} \times C_{3\times3}$

$= ABC_{3\times3}$.

The multiplication of ABC can be performed and results in a 3×3 matrix

F) $ACB = (A_{3\times4} \times C_{3\times3}) \times B_{4\times3}$

$\Rightarrow$ As the no. of rows of 'A' is not equal to no. of column of 'c' the multiplication. Cannot be performed.

Q5) A) $\begin{pmatrix} 1 & 2 \\ 4 & 5 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$

$C = \begin{pmatrix} 1\times1 + 2\times0 & 1\times1+2\times1 \\ 4\times1 + 5\times0 & 4\times1+5\times1 \end{pmatrix}$

$C = \begin{pmatrix} 1 & 3 \\ 4 & 9 \end{pmatrix}$

B) $\begin{pmatrix} 7 & -1 \\ 3 & 2 \\ 6 & 0 \end{pmatrix}_{3\times2} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}_{3\times3}$

As the order of both matrix does not match, hence the multiplication is not possible.

(6) $A = \begin{pmatrix} 1 & 2 & 1 \\ 4 & 4 & 5 \\ 6 & 7 & 7 \end{pmatrix}$ $B = \begin{pmatrix} -7 & -7 & 6 \\ 2 & 1 & -1 \\ 4 & 5 & -4 \end{pmatrix}$

Inverse of two matrices can be defined as

$I = AB$

$$I = \begin{pmatrix} 1 & 2 & 1 \\ 4 & 4 & 5 \\ 6 & 7 & 7 \end{pmatrix} \begin{pmatrix} -7 & -7 & 6 \\ 2 & 1 & -1 \\ 4 & 5 & -4 \end{pmatrix}$$

$$= \begin{bmatrix} -7+4+4 & -7+2+5 & 6-2-4 \\ -28+8+20 & -28+4+25 & 24-4-20 \\ -42+14+28 & -42+7+35 & 36-7-28 \end{bmatrix}$$

$$I = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

here, I is the Identity matrix

hence, A & B are inverse of one another.

(7) Let A be an $m \times n$ matrix, where $m \neq n$

for a matrix 'A' to have an inverse matrix 'A^{-1}'

it must comply with the following conditions:

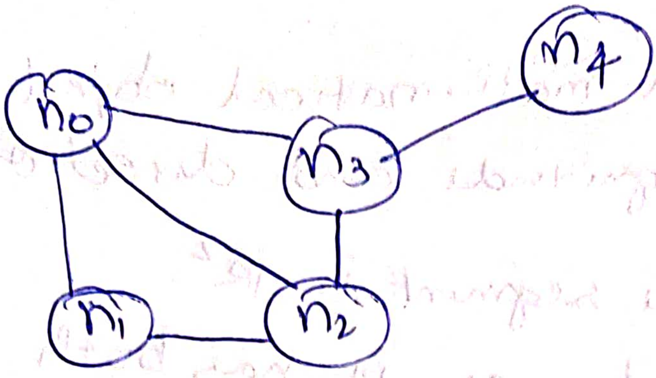
* It must be full rank i.e, its ~~values~~ rows or columns are linearly independent.
* It's determinant must be non-zero, as the determinant is only defined for square matrices.

Since, the given matrix with $m \times n$ does not have a full rank, so the linear independence condition fails.

* The determinant is undefined, so we cannot check if it is non-zero.

Hence, the given $m \times n$ matrix cannot have its inverse.

8)



given that, the matrix's ~~whose~~ $(i,j)^{th}$ entry $=1$ if there is an edge joining the nodes. else 0.

A) Since there are 5 nodes, the adjacency matrix would be a 5×5 matrix

hence, the matrix is given by:

$$A = \begin{bmatrix} 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

B) Yes, it holds true for $A = A^T$. Because, A is symmetric across the diagonal of the adjacency matrix. For any undirected network, the matrix is always symmetric across the diagonal.

c) Multiplying the adjacency matrix by itself has the useful interpretation of counting the no. of paths of length 2 between nodes. and taking higher powers counts longer paths. This provides insight into the overall structure and connectivity of the network.