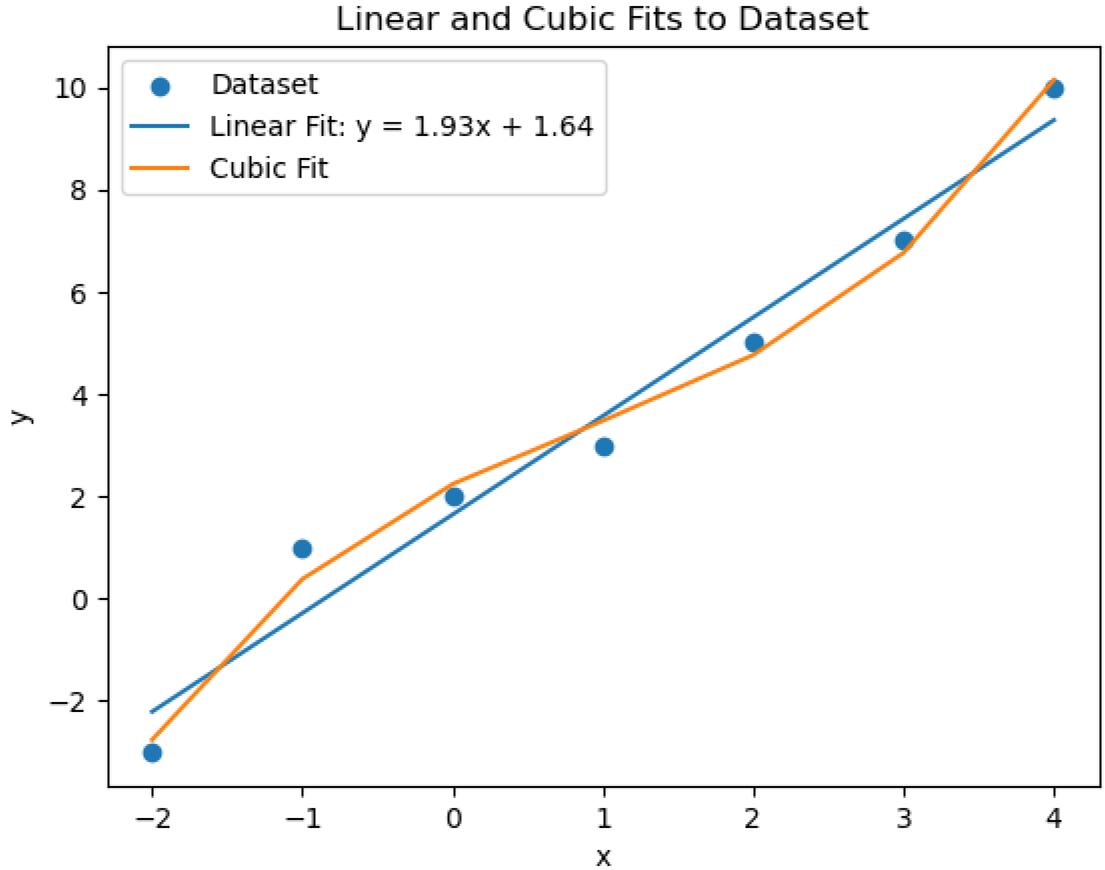
In [1]:

```python
import numpy as np
import matplotlib.pyplot as plt

data_points = np.array([(-2, -3), (-1, 1), (0, 2), (1, 3), (2, 5), (3, 7), (4, 10)])  # Given Da

# For Linear Fit
x_values, y_values = data_points[:, 0], data_points[:, 1]
num_points = len(data_points)
slope = (num_points * np.sum(x_values * y_values) - np.sum(x_values) * np.sum(y_values)) / (
        num_points * np.sum(x_values ** 2) - (np.sum(x_values)) ** 2)
intercept = (np.sum(y_values) - slope * np.sum(x_values)) / num_points

linear_fit = slope * x_values + intercept
cubic_fit_coefficients = np.polyfit(x_values, y_values, 3)  # For Cubic Fit
cubic_fit = np.polyval(cubic_fit_coefficients, x_values)

# SSR Calculations
ssr_linear = np.sum((y_values - linear_fit) ** 2)
ssr_cubic = np.sum((y_values - cubic_fit) ** 2)

print(f"Linear Fit: y = {slope:.2f}x + {intercept:.2f}")
print(f"Cubic Fit Coefficients: {cubic_fit_coefficients}")
print(f"Sum of Squared Residuals (SSR) - Linear Fit: {ssr_linear}")
print(f"Sum of Squared Residuals (SSR) - Cubic Fit: {ssr_cubic}")

# Plotting the obtained values
plt.scatter(x_values, y_values, label='Dataset')
plt.plot(x_values, linear_fit, label=f'Linear Fit: y = {slope:.2f}x + {intercept:.2f}')
plt.plot(x_values, cubic_fit, label=f'Cubic Fit')
plt.legend()
plt.xlabel('x')
plt.ylabel('y')
plt.title('Linear and Cubic Fits to Dataset')
plt.show()
```

```
Linear Fit: y = 1.93x + 1.64
Cubic Fit Coefficients: [ 0.11111111 -0.30952381  1.43650794  2.23809524]
Sum of Squared Residuals (SSR) - Linear Fit: 3.5714285714285703
Sum of Squared Residuals (SSR) - Cubic Fit: 0.857142857142857
```


Linear and Cubic Fits to Dataset
Dataset
Linear Fit: y = 1.93x + 1.64
Cubic Fit
10
8
6
4
2
0
−2
y
−2
−1
0
1
2
3
4
x

(2) given, $A = \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}$

$$|A - \lambda I| = 0$$

$$\begin{vmatrix} 2-\lambda & 3 \\ 1 & 4-\lambda \end{vmatrix} = 0$$

$$(2-\lambda)(4-\lambda) - 3 = 0$$

$$8 - 2\lambda - 4\lambda + \lambda^2 - 3 = 0$$

$$\Rightarrow \lambda^2 - 6\lambda + 5 = 0$$

from Cayley-Hamilton theorem,

$$A^2 - 6A + 5 = 0 \rightarrow (1)$$

To find A^4: Multiply A^2 to both side of eq (1)

$$A^4 - 6A^3 + 5A^2 = 0$$

$$\Rightarrow \boxed{A^4 = 6A^3 - 5A^2}$$

To find A^{-1}: from eqn (1)

$$A^2 - 6A + 5I = 0 \Rightarrow A(A - 6I) = -5I$$

$$A - 6I = \frac{-5I}{A}$$

$$A - 6I = -5IA^{-1} \Rightarrow A - 6I = -5A^{-1}$$

$$\Rightarrow \boxed{A^{-1} = \frac{1}{5}(6I - A)}$$

③ given matrix $A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$

$$|A - \lambda I| = 0$$

$$\begin{vmatrix} 1-\lambda & 1 & 0 \\ 0 & 1-\lambda & 0 \\ 0 & 0 & 2-\lambda \end{vmatrix} = 0$$

$$= (1-\lambda)\left[(1-\lambda)(2-\lambda)\right] - 1\left[(2-\lambda) - 0\right] + 0$$

$$= 2\lambda^2 - 4\lambda + 2 - \lambda^3 + 2\lambda - \lambda - 2 + \lambda$$

$$\Rightarrow -\lambda^3 + 4\lambda^2 - 4\lambda = 0$$

$$\lambda^3 - 4\lambda^2 + 4\lambda = 0$$

$$\lambda = 0, 2, 2$$

To find the eigen vectors,

for $\lambda_1 = 0$, let v_1 be the eigen vector

$$\begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

On solving, $V_1 = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$

for $\lambda_2 = 2$, let v_2 be the eigen vector,

$$\begin{bmatrix} -1 & 1 & 0 \\ 1 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_2 \\ y_2 \\ z_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$v_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$$

therfore, Eigen vectors are :

$$\begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \Rightarrow \quad P = \begin{bmatrix} 1 & 0 & 1 \\ -1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

Normalizing the eigen vectors,

$$u_1 = \frac{\begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}}{\sqrt{2}} \quad u_2 = \frac{\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}}{\sqrt{1}} \quad u_3 = \frac{\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}}{\sqrt{2}}$$

$$u = \begin{bmatrix} 1/\sqrt{2} & 0 & 1/\sqrt{2} \\ -1/\sqrt{2} & 0 & 1/\sqrt{2} \\ 0 & 1 & 0 \end{bmatrix} \quad \Sigma = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

$A = PDP^{-1}$, $A^K = PD^K P^{-1}$

$$A = \begin{bmatrix} 1 & 0 & 1 \\ -1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} 1/2 & -1/2 & 0 \\ 0 & 0 & 1 \\ 1/2 & 1/2 & 0 \end{bmatrix}$$

$$A = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

Now, $A^5 - 10A^3 - 5A + 8I$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}^5 - 10 \begin{bmatrix} 0 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}^3 - 5 \begin{bmatrix} 0 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix} + \begin{bmatrix} 8 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & 8 \end{bmatrix}$$

$$= P \begin{bmatrix} 8 & 0 & 0 \\ 0 & -50 & 0 \\ 0 & 0 & -50 \end{bmatrix} P^{-1}$$

$$= \begin{bmatrix} 1 & 0 & 1 \\ -1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 8 & 0 & 0 \\ 0 & -50 & 0 \\ 0 & 0 & -50 \end{bmatrix} \begin{bmatrix} 1/2 & -1/2 & 0 \\ 0 & 0 & 1 \\ 1/2 & 1/2 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} -21 & -29 & 0 \\ -29 & -21 & 0 \\ 0 & 0 & -50 \end{bmatrix}$$

④ given Stochastic matrix $A = \begin{bmatrix} 0.9 & 0.4 \\ 0.1 & 0.6 \end{bmatrix}$

a) The matrix given here is a probability matrix used to describe the transition of Markov chain since by adding the column value, we get '1'.

b) $\lim_{k \to \infty} A^k \Rightarrow |A - \lambda I| = 0$

$$\begin{vmatrix} 0.9-\lambda & 0.4 \\ 0.1 & 0.6-\lambda \end{vmatrix} = 0$$

$$(0.9-\lambda)(0.6-\lambda) - (0.4)(0.1) = 0$$

$$(2\lambda - 1)(\lambda - 1) = 0$$

$$\lambda_1 = 1/2 , \lambda_2 = 1$$

for $\lambda_1 = 1/2$ the eigen vector, $v_1 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$

for $\lambda_2 = 1$ the eigen vector, $v_2 = \begin{bmatrix} 4 \\ 1 \end{bmatrix}$

$P = [v_2 \ v_1]$ $\quad P = \begin{bmatrix} 4 & -1 \\ 1 & 1 \end{bmatrix}$

Now, $A = PDP^{-1}$

$$A = \begin{bmatrix} 4 & -1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 0.5 \end{bmatrix} \begin{bmatrix} 4 & -1 \\ 1 & 1 \end{bmatrix}^{-1}$$

$$A^k = PD^kP^{-1}$$

$$= \begin{bmatrix} 4 & -1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1^k & 0 \\ 0 & (0.5)^k \end{bmatrix} \begin{bmatrix} 1/5 & 1/5 \\ -1/5 & 4/5 \end{bmatrix}$$

$$= \frac{1}{5} \begin{bmatrix} 4 & -1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & (1/2)^k \end{bmatrix} \begin{bmatrix} 1 & 1 \\ -1 & 4 \end{bmatrix}$$

$$\lim_{k\to\infty} A^k = \frac{1}{5} \begin{bmatrix} 4 & -1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & \lim_{k\to\infty} \frac{1}{2^k} \end{bmatrix} \begin{bmatrix} 1 & 1 \\ -1 & 4 \end{bmatrix}$$

On solving, $\lim_{k\to\infty} A^k = \frac{1}{5} \begin{bmatrix} 4 & 4 \\ 1 & 1 \end{bmatrix}$

$$A^k = \frac{1}{5} \begin{bmatrix} 4 & -1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1/2^k & 4/2^k \end{bmatrix}$$

for $k=2$, $A^2 = \frac{1}{5} \begin{bmatrix} 4 & -1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ -1/2^2 & 4/2^2 \end{bmatrix}$

$$A^2 = \begin{bmatrix} 17/20 & 3/5 \\ 3/20 & 2/5 \end{bmatrix}$$

Similarly $k=3$,

$$A^3 = \begin{bmatrix} 33/40 & 7/10 \\ 7/40 & 3/10 \end{bmatrix}$$

It is evident that, as $K \uparrow$ the 1^{st} row $\downarrow$ & 2^{nd} row $\uparrow$. whereas, the Sum of the Columns still equals to '1'. Therefore, $A, A^2, A^3, \ldots, A^K$ represents probability Matrix.

$$\lim_{k\to\infty} A^K = \frac{1}{5}\begin{bmatrix} 4 & 4 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 0.8 & 0.8 \\ 0.2 & 0.2 \end{bmatrix}$$

c) Solution of B, indicates that each value of 'k' represents the prob. matrix in the transition of Markov chain.

⑤

$$A = \begin{bmatrix} 3 & -2 \\ -2 & 3 \\ 2 & 2 \end{bmatrix}$$

$$A^T A = P D P^T$$

$$A^T A = \begin{bmatrix} 3 & -2 & 2 \\ -2 & 3 & 2 \end{bmatrix} \begin{bmatrix} 3 & -2 \\ -2 & 3 \\ 2 & 2 \end{bmatrix}$$

$$A^T A = \begin{bmatrix} 9+4+4 & -6-6+4 \\ -6-6+2 & 4+9+4 \end{bmatrix} = \begin{bmatrix} 17 & -8 \\ -8 & 17 \end{bmatrix}$$

Now, $\begin{vmatrix} 17-\lambda & -8 \\ -8 & 17-\lambda \end{vmatrix} = 0$

$$(17-\lambda)(17-\lambda) - 64 = 0$$

$$289 - 17\lambda - 17\lambda - \lambda^2 = 0$$

$$\lambda^2 + 34\lambda - 225 = 0$$

$$\lambda^2 + 25\lambda + 9\lambda - 225 = 0$$

$$\lambda(\lambda+25) - 9(\lambda+25) = 0$$

$$\lambda_1 = -25, \ \lambda_2 = 9.$$

$$AA^T = \begin{bmatrix} 3 & -2 \\ -2 & 3 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} 3 & -2 & 2 \\ -2 & 3 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 9+4 & -6-6 & 6-4 \\ -6-6 & 4+9 & -4+6 \\ 6-4 & -4+6 & 4+4 \end{bmatrix}$$

$$= \begin{bmatrix} 13 & -12 & 2 \\ -12 & 13 & 2 \\ 2 & 2 & 8 \end{bmatrix}$$

$$|AA^T - \lambda I| = 0$$

$$\begin{vmatrix} 13-\lambda & -12 & 2 \\ -12 & 13-\lambda & 2 \\ 2 & 2 & 8-\lambda \end{vmatrix} = 0$$

$$(13-\lambda)\left[(13-\lambda)(8-\lambda)-4\right] + 12\left[-12(8-\lambda)-4\right]$$

$$+ 2\left[-24-2(13-\lambda)\right] = 0$$

$$-\lambda(\lambda-25)(\lambda-9) = 0$$

Hence, the eigen values are

$$\lambda_1 = 25, \lambda_2 = 9, \lambda_3 = 0$$

the eigen vectors are:

$$v_1 = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} \quad v_2 = \begin{bmatrix} 1/4 \\ 1/4 \\ 1 \end{bmatrix} \quad v_3 = \begin{bmatrix} -2 \\ -2 \\ 1 \end{bmatrix}$$

$$\sigma_1 = \sqrt{25} = 5 \qquad \sigma_2 = \sqrt{9} = 3$$

$$\Sigma = \begin{bmatrix} 5 & 0 \\ 0 & 3 \\ 0 & 0 \end{bmatrix}$$

to find the orthogonal matrix u, arrange eigen vector in column in descending order of eigen values.

$$u = \begin{bmatrix} -1/\sqrt{2} & \sqrt{2}/6 & -2/3 \\ 1/\sqrt{2} & \sqrt{2}/6 & -2/3 \\ 0 & 2\sqrt{2}/3 & 1/3 \end{bmatrix}$$

To find v,

$$v_i = \frac{A^T u_i}{\sigma_i}$$

$$v_1 = \frac{1}{5}\begin{bmatrix} 3 & -2 & 2 \\ -2 & 3 & 2 \end{bmatrix}\begin{bmatrix} -1/\sqrt{2} \\ 1/\sqrt{2} \end{bmatrix} = \begin{bmatrix} -1/\sqrt{2} \\ 1/\sqrt{2} \end{bmatrix}$$

$$V_2 = \frac{1}{3}\begin{bmatrix} 3 & -2 & 2 \\ -2 & 3 & 2 \end{bmatrix}\begin{bmatrix} \sqrt{2}/6 \\ \sqrt{2}/6 \\ 2\sqrt{2}/3 \end{bmatrix} = \begin{bmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{bmatrix}$$

$$V = \begin{bmatrix} -1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix}$$

$$A = U \Sigma V^T$$

$$\Rightarrow A = \begin{bmatrix} -1/\sqrt{2} & \sqrt{2}/6 & -2/3 \\ 1/\sqrt{2} & \sqrt{2}/6 & -2/3 \\ 0 & 2\sqrt{2}/3 & 1/3 \end{bmatrix}\begin{bmatrix} 5 & 0 \\ 0 & 3 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} -1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix}$$

(6) Let's consider a matrix of order 2×5, with rank = 2

$$A_0 = \begin{pmatrix} 5 & 4 & 3 & 2 & 1 \\ -1 & 1 & 0 & 1 & -1 \end{pmatrix}$$

a)

$$\text{Mean of } 1^{st} \text{ row} = \frac{5+4+3+2+1}{5} = 3$$

$$\text{Mean of } 2^{nd} \text{ row} = \frac{-1+1+0+1-1}{5} = 0$$

Subtracting these row means gives us the centered matrix A:

$$A = \begin{bmatrix} 5-3 & 4-3 & 3-3 & 2-3 & 1-3 \\ -1-0 & 1-0 & 0-1 & 1-0 & -1-0 \end{bmatrix}$$

$$A = \begin{bmatrix} 2 & 1 & 0 & -1 & -2 \\ -1 & 1 & 0 & 1 & -1 \end{bmatrix}$$

b) Sample Covariance matrix

$$S = \frac{AA^T}{n-1} \quad ; n \to \text{no. of columns.}$$

$$A.A^T = \begin{bmatrix} 2 & 1 & 0 & -1 & -2 \\ -1 & 1 & 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} 2 & -1 \\ 1 & 1 \\ 0 & 0 \\ -1 & 1 \\ -2 & -1 \end{bmatrix}$$

$$A \cdot A^T = \begin{bmatrix} 4+1+0+1+4 & -2+1+0-1+2 \\ -2+1+0-1+2 & 1+1+0+1+1 \end{bmatrix} = \begin{bmatrix} 10 & 0 \\ 0 & 4 \end{bmatrix}$$

$$S = \frac{1}{4}\begin{bmatrix} 10 & 0 \\ 0 & 4 \end{bmatrix} \Rightarrow S = \begin{bmatrix} 2.5 & 0 \\ 0 & 1 \end{bmatrix}$$

find eigen values (Conventionally have diagonal matrix) $\Rightarrow$ eigen values.

for $\lambda_1 = 5/2$ $\left[\begin{array}{cc|c} 0 & 0 & 0 \\ 0 & -3/2 & 0 \end{array}\right] \rightarrow \left[\begin{array}{cc|c} 0 & 1 & 0 \\ 0 & 0 & 0 \end{array}\right]$

Soln: $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} t \\ 0 \end{bmatrix}$ unit vecto $u_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

for $\lambda_2 = 1$ $\left[\begin{array}{cc|c} 3/2 & 0 & 0 \\ 0 & 0 & 0 \end{array}\right] \rightarrow \left[\begin{array}{cc|c} 1 & 0 & 0 \\ 0 & 0 & 0 \end{array}\right]$

Soln:

$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ t \end{bmatrix}$ $\vec{u_2} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

Correlation matrix is given by:

$$Corr(A) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

c) To find the 1-D Subspace of $\mathbb{R}^2$ that best approximates, we need to find SVD.

$$A \cdot A^T = \begin{bmatrix} 5 & 4 & 3 & 2 & 1 \\ -1 & 1 & 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} 5 & -1 \\ 4 & 1 \\ 3 & 0 \\ 2 & 1 \\ 1 & -1 \end{bmatrix} = \begin{bmatrix} 55 & 0 \\ 0 & 4 \end{bmatrix}$$

hence the eigen values are

$$\lambda_1 = 55, \lambda_2 = 4$$

Eigen vector for $\lambda_1 = 55 \Rightarrow v_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

Eigen vector for $\lambda_2 = 4 \Rightarrow v_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$$A^T \cdot A = \begin{bmatrix} 5 & -1 \\ 4 & 1 \\ 3 & 0 \\ 2 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 5 & 4 & 3 & 2 & 1 \\ -1 & 1 & 0 & 1 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 26 & 19 & 15 & 9 & 6 \\ 19 & 17 & 12 & 9 & 3 \\ 15 & 12 & 9 & 6 & 3 \\ 9 & 9 & 6 & 5 & 1 \\ 6 & 3 & 3 & 1 & 2 \end{bmatrix}$$

Calculating eigen values, we get

$$\lambda_1 = 55, \ \lambda_2 = 4, \ \lambda_3 = 0, \lambda_4 = 0, \ \lambda_5 = 0$$

Calculating the eigen vectors, we get

<u>for $\lambda_1 = 55$:-</u> <u>for $\lambda_2 = 4$:-</u> <u>for $\lambda_3 = 0$:-</u>

$$v_1 = \begin{bmatrix} 5 \\ 4 \\ 3 \\ 2 \\ 1 \end{bmatrix} \quad v_2 = \begin{bmatrix} -1 \\ 1 \\ 0 \\ 1 \\ -1 \end{bmatrix} \quad v_3 = \begin{bmatrix} -1/3 \\ -1/3 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

<u>for $\lambda_4 = 0$:</u> <u>for $\lambda_5 = 0$:-</u>

$$v_4 = \begin{bmatrix} 2/9 \\ -7/9 \\ 0 \\ 1 \\ 0 \end{bmatrix} \quad v_5 = \begin{bmatrix} -5/9 \\ 4/9 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

finding the singular values,

$$\sigma_1 = \sqrt{55} \quad , \quad \sigma_2 = 2$$

The Σ matrix is a zero matrix with σ_i on its diagonal

$$\text{Hence, } \Sigma = \begin{bmatrix} \sqrt{55} & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 \end{bmatrix}$$

Matrix V is given by:

$$V = \begin{bmatrix} \frac{\sqrt{55}}{11} & \frac{1}{2} & -\frac{\sqrt{11}}{11} & \frac{9\sqrt{1771}}{1771} & \frac{-15\sqrt{8855}}{3542} \\ \frac{4\sqrt{55}}{55} & \frac{-1}{2} & -\frac{\sqrt{11}}{11} & \frac{-24\sqrt{1771}}{1771} & \frac{39\sqrt{8855}}{17710} \\ \frac{3\sqrt{55}}{55} & 0 & \frac{3\sqrt{11}}{11} & \frac{-5\sqrt{1771}}{1771} & \frac{-6\sqrt{8855}}{8855} \\ \frac{2\sqrt{55}}{55} & \frac{-1}{2} & 0 & \frac{3\sqrt{1771}}{161} & \frac{47\sqrt{8855}}{17710} \\ \frac{\sqrt{55}}{55} & \frac{1}{2} & 0 & 0 & \frac{\sqrt{8855}}{110} \end{bmatrix}$$

$$u_i = \frac{1}{\sigma_i} \begin{bmatrix} 5 & 4 & 3 & 2 & 1 \\ -1 & 1 & 0 & 1 & -1 \end{bmatrix}$$

hence, $u_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ & $u_2 = \begin{bmatrix} 0 \\ -1 \end{bmatrix}$

therfore, $U = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

$$\Sigma = \begin{bmatrix} \sqrt{55} & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 \end{bmatrix}$$

$$V^T = \begin{bmatrix} \frac{\sqrt{55}}{11} & \frac{4\sqrt{55}}{55} & \frac{3\sqrt{55}}{55} & \frac{2\sqrt{55}}{55} & \frac{\sqrt{55}}{55} \\ \frac{1}{2} & \frac{-1}{2} & 0 & \frac{-1}{2} & \frac{1}{2} \\ -\frac{\sqrt{11}}{11} & -\frac{\sqrt{11}}{11} & \frac{3\sqrt{11}}{11} & 0 & 0 \\ \frac{9\sqrt{1771}}{1771} & \frac{-24\sqrt{1771}}{771} & \frac{-5\sqrt{1771}}{1771} & \frac{3\sqrt{1771}}{161} & 0 \\ \frac{-15\sqrt{8855}}{3542} & \frac{39\sqrt{885}}{7710} & \frac{-6\sqrt{8855}}{8855} & \frac{47\sqrt{8855}}{17710} & \frac{\sqrt{8855}}{110} \end{bmatrix}$$

u_1 gives the direction of the best 1-D Subspace:

$$u_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

the 1-D Subspace is the line passing through the centroid (μ_1, μ_2) of the data A, with direction u_1.

The subspace is represented by:

$$x = \mu 1 + t(u11) \quad y = \mu 2 + t(u21)$$

$$x = 3 + t(1), \quad y = 0 + t(0)$$

$$x = 3 + t, \quad y = 0$$

$$\Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3+t \\ 0 \end{bmatrix}$$

d)