① given, $x - 2y = 1 \rightarrow ①$ $\Rightarrow x = 2y + 1$

$3x + 2y = 11 \rightarrow ②$

for ①, when $x = 0 \Rightarrow y = -0.5 \Rightarrow (0, -0.5)$

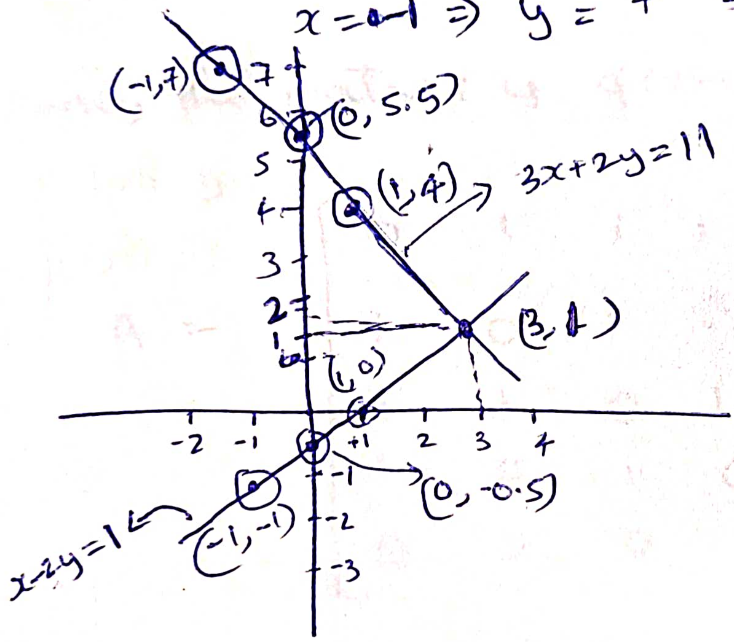
when $x = 1 \Rightarrow y = 0 \Rightarrow (1, 0)$

when $x = -1 \Rightarrow y = -1 \Rightarrow (-1, -1)$

for ②, when $x = 0 \Rightarrow y = 5.5 \Rightarrow (0, 5.5)$

$x = 1 \Rightarrow y = 4 \Rightarrow (1, 4)$

$x = -1 \Rightarrow y = 7 \Rightarrow (-1, 7)$



the two lines intersect at (3, 1) hence the given system of equations have a unique solution.

Now, convert the given system of equations in coefficient matrix form.

$$\begin{bmatrix} 1 & -2 & 1 \\ 3 & 2 & 11 \end{bmatrix}$$

echelon form

$$\begin{bmatrix} 1 & 0 & a_{1,3} \\ 0 & 1 & a_{2,3} \end{bmatrix}$$

$R_2 \rightarrow R_2 - 3R_1$

$$\begin{bmatrix} 1 & -2 & 1 \\ 0 & 8 & 8 \end{bmatrix}$$

~~$R_1 \Rightarrow R_1 + 2R_2$~~

$R_2 \rightarrow \frac{R_2}{8}$

$$\begin{bmatrix} 1 & -2 & 1 \\ 0 & 1. & 1 \end{bmatrix}$$

$R_1 \rightarrow R_1 + 2R_2$

$$\begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 1 \end{bmatrix}$$

In the reduced echelon form, we get $a_{1,3} = 3$ & $a_{2,3} = 1$ which same as the point of intersection of the two lines shown in the graph. Hence, the given system of equations has a unique solution.

Q2) given,
$$2x + 3y + z = 8$$
$$4x + 7y + 5z = 20$$
$$-2y + 2z = 0.$$

Solve using Gaussian elimination.

Convert the given system of equation in coefficient matrix form:

$$A = \left[\begin{array}{ccc|c} 2 & 3 & 1 & 8 \\ 4 & 7 & 5 & 20 \\ 0 & -2 & 2 & 0 \end{array}\right]$$

$R_1 \Rightarrow R_1/2.$

$$A = \left[\begin{array}{ccc|c} 1 & 1.5 & 0.5 & 4 \\ 4 & 7 & 5 & 20 \\ 0 & -2 & 2 & 0 \end{array}\right]$$

$R_2 \to R_2 - 4R_1$

$$\left[\begin{array}{ccc|c} 1 & 1.5 & 0.5 & 8 \\ 0 & 1 & 3 & 4 \\ 0 & -2 & 2 & 0 \end{array}\right]$$

$R_1 \to R_1 - 1.5R_2$; $R_3 \to R_3 + 2R_2$.

$$\left[\begin{array}{ccc|c} 1 & 0 & -4 & -2 \\ 0 & 1 & 3 & 4 \\ 0 & 0 & 8 & 8 \end{array}\right]$$

$R_3 \to R_3 / 8$.

$$\left[\begin{array}{ccc|c} 1 & 0 & -4 & -2 \\ 0 & 1 & 3 & 4 \\ 0 & 0 & 1 & 1 \end{array}\right]$$

$R_1 \to R_1 + 4R_3$; $R_2 \to R_2 - 3R_3$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{array}\right]$$

hence, $x=2$, $y=1$, & $z=1$ are the solution for the given system of equations.

(3) given quadratic equation

$$q(x) = ax^2 + bx + c \rightarrow (1)$$

Set of data points $D = \{(1,4),(2,8),(3,16)\}$

Substitute the data point $(1,4)$ in eq (1)

$$4 = a(1)^2 + b(1) + c$$

the matrix is

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 4 \end{array}\right]$$

for $(2,8) \Rightarrow 8 = a(2)^2 + b(2) + c$

matrix is $\left[\begin{array}{ccc|c} 4 & 2 & 1 & 8 \end{array}\right]$

for $(3,16) \Rightarrow 16 = a(3)^2 + b(3) + c$

matrix $\Rightarrow \left[\begin{array}{ccc|c} 9 & 3 & 1 & 16 \end{array}\right]$

Overall matrix is

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 4 \\ 4 & 2 & 1 & 8 \\ 9 & 3 & 1 & 16 \end{array}\right]$$

$R_2 \Rightarrow R_2 - 4R_1$ & $R_3 \Rightarrow R_3 - 9R_1$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 4 \\ 0 & -2 & -3 & -8 \\ 0 & -6 & -8 & -20 \end{array}\right]$$

$R_2 \Rightarrow \frac{-R_2}{2}$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 4 \\ 0 & 1 & 1.5 & 4 \\ 0 & -6 & -8 & -20 \end{array}\right]$$

$R_1 \to R_1 - R_2$ & $R_3 \to R_3 + 6R_2$

$$\left[\begin{array}{ccc|c} 1 & 0 & -0.5 & 0 \\ 0 & 1 & 1.5 & 4 \\ 0 & 0 & 1 & 4 \end{array}\right]$$

$R_1 \to R_1 + 0.5R_3$ & $R_2 \to R_2 - 1.5R_3$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 1 \end{array}\right]$$

hence, ~~the~~ $a = 2$, $b = -2$, & $c = 1$ are the coefficients ~~of the matrix~~ e.