Pre-class Assignment -14

(1) given vectors:

A) $\begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} -4 \\ 0 \\ 8 \end{pmatrix}$

let us consider

$\vec{a} = 2i - 3j + k$ and $\vec{b} = -4i + 0j + 8\hat{k}$

$\vec{a} \cdot \vec{b} = (2i - 3j + k) \cdot (-4\hat{i} + 0\hat{j} + 8\vec{k})$

$= -8 + 0 + 8$

$= 0$

$\vec{a} \cdot \vec{b} = 0$

Since, the dot product of both $\vec{a}$ & $\vec{b}$ vectors is zero. Hence, they are orthogonal.

B) let $u = Ui$

$v = Vj$

$w = Wk$

and are mutually perpendicular.

when u is orthogonal to v

~~$u \cdot v = ui \cdot Vj$~~ $u \cdot v = ui \cdot V\hat{j} = 0$

when v is orthogonal to w

$v \cdot w = Vj \cdot Wk = 0$

~~When~~ Hence, $u \cdot w = ui \cdot WK = 0$.

$\Rightarrow$ u is orthogonal to w.

c) Let $V = V\hat{i}$, $W_1 = W_1\hat{j}$, $W_2 = W_2\hat{k}$

Since, all three vectors, meet at right angles, it can bee understood that, they are orthogonal to each other aswell.

Hence, ~~$V.W_1 = V\hat{i}.W$~~

$$V.W_1 = V_i \cdot W_j = 0$$

As, the dot product of V and W_1 vector is zero.

$\Rightarrow$ V & W_1 are orthogonal ~~to~~ surfaces

(2) A) 3x3 Matrix that projects vectors in R^3 onto the z-axis

$$P_x = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

This matrix will tranform any vector in R^3 by setting the x and y plane components to zero and leaving the z component unchanged, which is effectively projecting it onto the z-axis

B) 3x3 matrix that projects vectors in R^3 onto the xy plane

$$P_{xy} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

This matrix will transform any vector in R^3 by keeping the x & y components the same and setting the z-component to zero, which is effectively projecting it onto the ~~xy~~ XY-plane.

(3) Given $u = \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix}$ $v = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ $w = ?$

to find the projection.

$u = 1i + 2j + 2j$ & $v = 1i + 1j + 1k$

$$\vec{p} = \frac{\vec{u} \cdot \vec{v}}{\|\vec{v}\|} \vec{v} = \frac{(1i + 2j + 2j) \cdot (1i + 1j + 1k)}{\left(\sqrt{1^2 + 1^2 + 1^2}\right)^2}$$

$$\vec{p} = \frac{(1 + 2 + 2)}{\sqrt{3}} = \frac{5}{3} (1i + 1j + 1k)$$

$$\vec{p} = \frac{5}{3} i + \frac{5}{3} j + \frac{5}{3} k$$

$1 - \frac{5}{\sqrt{3}}$

$\frac{\sqrt{3} - 5}{\sqrt{3}}$

Now error $\Rightarrow e = u - p$ is given by

$$\vec{e} = \vec{u} - \vec{p}$$

$$\vec{e} = (1i + 2j + 2j) - \left(\frac{5}{3} i + \frac{5}{3} j + \frac{5}{3} k\right)$$

$$\vec{e} = -\frac{2}{3} i - \frac{1}{3} j - \frac{1}{3} k$$

Now, to check the orthogonality of $\vec{e}$ & $\vec{v}$

$$\vec{e} \cdot \vec{v} = \left(-\frac{2}{3} i - \frac{1}{3} j - \frac{1}{3} k\right) \cdot (1i + 1j + 1k)$$

$$= \left(-2/3 - 1/3 - 1/3\right)$$

$$\Rightarrow \vec{e} \cdot \vec{v} = 0$$

Since, the dot product of $\vec{e}$ & $\vec{v}$ is zero, error vetor $\vec{e}$ is orthogonal to vector $\vec{v}$.