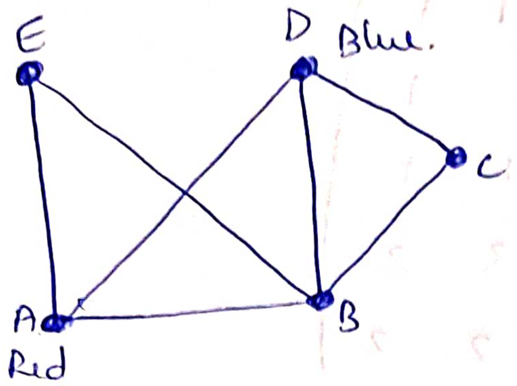
(1)



let us consider,

⇒ if there is an edge between the nodes we take it as 1

⇒ if there is no edge between the nodes we take it as 0.

from the consideration we can construct a matrix as :

$$A = \begin{array}{c c} & \begin{array}{ccccc} A & B & C & D & E \end{array} \\ \begin{array}{c} A \\ B \\ C \\ D \\ E \end{array} & \begin{bmatrix} 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

to find the no. of 3-step paths we need to. multiply the matrix 'A' 3 times.

hence, we need to find A^3

$$A^3 = A^2 \cdot A$$

$$A^2 = \begin{bmatrix} 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 3 & 2 & 2 & 1 & 1 \\ 2 & 4 & 1 & 2 & 1 \\ 2 & 1 & 2 & 1 & 1 \\ 1 & 2 & 1 & 3 & 2 \\ 1 & 1 & 1 & 2 & 2 \end{bmatrix}$$

Now, $A^3 = A^2 \cdot A$

$$= \begin{bmatrix} 3 & 2 & 2 & 1 & 1 \\ 2 & 4 & 1 & 2 & 1 \\ 2 & 1 & 2 & 1 & 1 \\ 1 & 2 & 1 & 3 & 2 \\ 1 & 1 & 1 & 2 & 2 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \end{bmatrix}$$

$$= \begin{array}{c} \\ A \\ B \\ C \\ D \\ E \end{array} \begin{array}{c} \begin{array}{ccccc} A & B & C & D & E \end{array} \\ \begin{bmatrix} 4 & 7 & 3 & 7 & 5 \\ 7 & 6 & 6 & 7 & 6 \\ 3 & 6 & 2 & 5 & 3 \\ 7 & 7 & 5 & 4 & 3 \\ 5 & 6 & 3 & 3 & 2 \end{bmatrix} \end{array}$$

$A \rightarrow D$.

therefore, the no. of 3-step paths from red node (A) to blue node (D) is $\underline{\underline{7}}$.

② given,

$$A = \begin{bmatrix} 1 & 0 & 4 & 1 & 0 & 3 & 1 \\ 0 & 1 & 5 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 8 & 6 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}_{4\times 7}$$

$Ax = 0$

~~A is 4x7 order, x sho~~

$$\begin{bmatrix} 1 & 0 & 4 & 1 & 0 & 3 & 1 \\ 0 & 1 & 5 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 8 & 6 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \cancel{0} \\ \cancel{0} \\ \cancel{0} \end{bmatrix}$$

~~here, any row containing the first element as a non-zero element it is considered as pivot variable. The rest elements variables are free~~

~~Any variables~~.

here, $x_1 + 4x_3 + x_4 + 3x_6 + x_7 = 0$

$x_2 + 5x_3 + 2x_4 = 0$

$x_5 + 8x_6 + 6x_7 = 0$

$x_7 = 0.$

Sub $x_7 = 0$ in all the above eqn.

$$x_1 + 4x_3 + x_4 + 3x_6 = 0$$

$$x_2 + 5x_3 + 2x_4 = 0$$

$$x_5 + 8x_6 = 0$$

$$x_7 = 0.$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \end{bmatrix} = \begin{bmatrix} -4x_3 - x_4 - 3x_6 \\ -5x_3 - 2x_4 \\ x_3 \\ x_4 \\ -8x_6 \\ x_6 \\ 0 \end{bmatrix}$$

It can be re-written as.

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \end{bmatrix} = \begin{bmatrix} -4 \\ -5 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} x_3 + \begin{bmatrix} -1 \\ -2 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} x_4 + \begin{bmatrix} -3 \\ 0 \\ 0 \\ 0 \\ -8 \\ 1 \\ 0 \end{bmatrix} x_6.$$

Hence, this is set of solutions of the homogeneous system $Ax = 0$.

(3) given,

$$A = \begin{pmatrix} 1 & 0 & 4 & 1 & 0 & 3 & 1 \\ 0 & 1 & 5 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 8 & 6 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$B = \begin{bmatrix} 2 & 3 & -1 & 4 \end{bmatrix}^T$$

$$B = \begin{bmatrix} 2 \\ 3 \\ -1 \\ 4 \end{bmatrix}$$

$Ax = b.$

$$\begin{bmatrix} 1 & 0 & 4 & 1 & 0 & 3 & 1 \\ 0 & 1 & 5 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 8 & 6. \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ -1 \\ 4 \end{bmatrix}$$

the system of equation for the above matrix is

$x_1 + 4x_3 + x_4 + 3x_6 + x_7 = 2$

$x_2 + 5x_3 + 2x_4 = 3.$

$x_5 + 8x_6 + 6x_7 = -1$

$x_7 = 4.$

Substitute $x_7 = 4$ in above eqns.

$$x_1 + 4x_3 + x_4 + 3x_6 = -2$$

$$x_2 + 5x_3 + 2x_4 = 3$$

$$x_5 + 8x_6 = -25.$$

$$x_7 = 4$$

hence,
$$x_1 = -2 - 4x_3 - x_4 - 3x_6$$
$$x_2 = 3 - 5x_3 - 2x_4$$
$$x_5 = -25 - 8x_6.$$
$$x_7 = 4.$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \end{bmatrix} = \begin{bmatrix} -2 - 4x_3 - x_4 - 3x_6. \\ 3 - 5x_3 - 2x_4 \\ x_3 \\ x_4 \\ -25 - 8x_6. \\ x_6 \\ 4 \end{bmatrix}$$

It can be written as:

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \end{bmatrix} = \begin{bmatrix} -2 \\ 3 \\ 0 \\ 0 \\ -25 \\ 0 \\ 4 \end{bmatrix} + \begin{bmatrix} -4 \\ -5 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} x_3 + \begin{bmatrix} -1 \\ -2 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} x_4 + \begin{bmatrix} -3 \\ 0 \\ 0 \\ 0 \\ -8 \\ 0 \\ 0 \end{bmatrix} x_6.$$

hence, this is the general solution of the non-homogeneous system $Ax = b$..

(4) given, $A = \begin{pmatrix} -2 & 2 & -1 \\ 3 & -5 & 4 \\ 5 & -6 & 4 \end{pmatrix}$; $A^{-1} = ?$

$$= \left[\begin{array}{ccc|ccc} -2 & 2 & -1 & 1 & 0 & 0 \\ 3 & -5 & 4 & 0 & 1 & 0 \\ 5 & -6 & 4 & 0 & 0 & 1 \end{array}\right]$$

$R_2 \to 3R_1 + 2R_2$ & $R_3 \to 2R_3 + 5R_1$

$$= \left[\begin{array}{ccc|ccc} -2 & 2 & -1 & 1 & 0 & 0 \\ 0 & -4 & 5 & 3 & 2 & 0 \\ 0 & -2 & 3 & 5 & 0 & 2 \end{array}\right]$$

$R_3 \to R_2 - 2R_3$

$$= \left[\begin{array}{ccc|ccc} -2 & 2 & -1 & 1 & 0 & 0 \\ 0 & -4 & 5 & 3 & 2 & 0 \\ 0 & 0 & -1 & -7 & 2 & -4 \end{array}\right]$$

$R_2 \to 5R_3 + R_2$

$$\left[\begin{array}{ccc|ccc} -2 & 2 & -1 & 1 & 0 & 0 \\ 0 & -4 & 0 & -32 & 12 & -20 \\ 0 & 0 & -1 & -7 & 2 & -4 \end{array}\right]$$

$R_1 \to R_1 - R_3$

$$\left[\begin{array}{ccc|ccc} -2 & 2 & 0 & 8 & -2 & 4 \\ 0 & -4 & 0 & -32 & 12 & -20 \\ 0 & 0 & -1 & -7 & 2 & -4 \end{array}\right]$$

$R_1 \to 2R_1 + R_2$

$$= \left[\begin{array}{ccc|ccc} -4 & 0 & 0 & -16 & 8 & -12 \\ 0 & -4 & 0 & -32 & 12 & -20 \\ 0 & 0 & -1 & -7 & 2 & -4 \end{array}\right]$$

$R_1 \to \frac{R_1}{-4}$; $R_2 \to \frac{R_2}{-4}$; $R_3 \to -R_3$

$$= \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 4 & -2 & 3 \\ 0 & 1 & 0 & 8 & -3 & 5 \\ 0 & 0 & 1 & 7 & -2 & 4 \end{array}\right]$$

Hence, $A^{-1} = \begin{bmatrix} 4 & -2 & 3 \\ 8 & -3 & 5 \\ 7 & -2 & 4 \end{bmatrix}$

(5) given, A and B are 10×10 matrices

$\det(A) = 4$ and $\det(B) = 5$.

→ Exchange row 5 & 7 of matrix 'A' & scale row 9 by 3 to get Matrix 'C'.

we know,

If 'A' is $n \times n$ matrix & B is formed by interchanging 2 rows then

$$\det(A) = -\det(B)$$

hence, $\det(C) = -\det(A)$

$= -4$

Also, if any row is multiplied with scalar 'c' to $n \times n$ matrix, then determinant of newly formed matrix is $c \times \det(A)$

hence, $\det(C) = 3 \times (-4)$

$= -12$

If A is $n \times n$ matrix & $\det(A^{-1}) = \frac{1}{\det(A)}$

here, $\det(C^{-1}) = \frac{-1}{12}$

→ Exchange 1 & 3 columns & 6 & 7 rows, then Scale entire matrix by 2 in matrix B to form matrix 'D'.

we know, if any two columns of $n \times n$ matrix 'A' is interchanged then the sign of determinant changes i.e $\det(A) = -\det(A)$

hence, $\det(D) = -\det(B)$

$= -5$

Since, the rows are being interchanged the sign of determinant changes again

$\det(D) = -(-5)$

$= 5.$

when scaled the entire matrix by 2.

$\det(D) = 2^{10} \times 5$ $\{k^n \det(B)\}$

$= 1024 \times 5$

$= 5120$

Now, $\det(A^{-1}BC^{-1}D)$.

$$\det(A^{-1}) = \frac{1}{\det(A)} = \frac{1}{4}$$

$$\det(B) = 5$$

$$\det(C^{-1}) = \frac{1}{\det(C)} = \frac{-1}{12}$$

$$\det(D) = 5120.$$

$$\det(A^{-1}BC^{-1}D) = \frac{1}{4} \times 5 \times \frac{-1}{12} \times 5120$$

$$= \underline{\underline{\frac{-1600}{3}}}$$