① To find three planes in 3-D space that are pairwise orthogonal (perpendicular) & intersect at $(5,7,4) \in R^3$

Let consider three normal vectors $\vec{n_1}, \vec{n_2}$ & $\vec{n_3}$ & the point of intersection as $P(5,7,4) = (a,b,c)$

Since, it is given ~~as~~ that the planes are pairwise orthogonal, so, the dot products between the normal vectors are zero.

i.e., $\vec{n_1} \cdot \vec{n_2} = 0$

$\vec{n_1} \cdot \vec{n_3} = 0$

$\vec{n_2} \cdot \vec{n_3} = 0$.

* for plane 1; Consider a simple plane whose normal vector is along x-axis.

normal vector $\vec{n_1} = (1,0,0) = (A,B,C)$

from point-normal equation.

$$A(x-a) + B(y-b) + C(z-c) = 0.$$

$$1(x-5) + 0 + 0 = 0$$

$$x - 5 = 0$$

$\therefore$ Equation of plane 1 is $x-5=0$.

* ~~For plane~~ $(1,0,0)$ is taken as the normal vector because, any nonzero vector that is orthogonal (perpendicular) to the plane can be considered as the normal vector.

* for plane 2; we need to consider a normal vector that is orthogonal to $\vec{n_1} = (1,0,0)$ and along the y-axis.

normal vector $\vec{n_2} = (0,1,0) = (A, B, C)$

from point-normal equation,

$$A(x-a) + B(y-b) + C(z-c) = 0$$

$$0 + 1(y-7) + 0 = 0$$

$$y - 7 = 0$$

$\therefore$ Equation of plane 2 is $y - 7 = 0$

* for plane 3, we need to consider a normal vector that is orthogonal to $\vec{n_1} = (1,0,0)$ & $\vec{n_2} = (0,1,0)$ and along z-axis.

~~we~~ take the ~~for~~ cross product of $\vec{n_1}$ & $\vec{n_2}$ to get $\vec{n_3}$.

$$\vec{n_3} = \vec{n_1} \times \vec{n_2}$$

$$\vec{n_3} = (1,0,0) \times (0,1,0)$$

$$\vec{n_3} = (0,0,1) = (A, B, C)$$

from point-normal equation,

$$0 + 0 + 1(z-4) = 0$$

$$z - 4 = 0$$

$\therefore$ The equation of plane 3 is $z - 4 = 0$

$\therefore$ The equation are; $x - 5 = 0$, $y - 7 = 0$ & $z - 4 = 0$

② If the component function of a parametric curve

a) are linear, then the parametric curve describes a straight line in space.

The above statement is true.

If the component function of a parametric curve are linear, each component has the form

$f(t) = At + B$, $g(t) = Ct + D$, $h(t) = Et + F$

when these ~~for~~ linear components are used as the components of a parametric curve

$\vec{r}(t) = \langle f(t), g(t), h(t) \rangle$ the resulting curve is a straight line.

~~the symmetric equation of a line is given by.~~

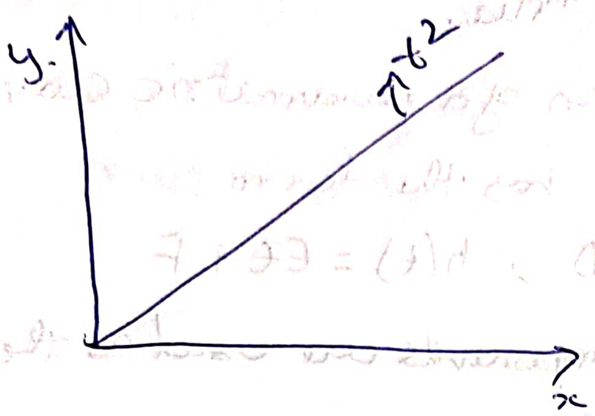
the combination of all three linear equations describes a straight line in 3-D space.

b) If a parametric curve describes a straight line in space, then its component functions are linear.

The above statement is false. As previously we saw it is true that a parametric curve with linear ~~fun~~ component function describes a straight line in space, the vice-versa may not be necessarily true because the parametric curve that describes a straight line in space can have a non-linear components functions.

such as $\vec{r}(t) = \langle t^2, t^3, t \rangle$.

a straight line ~~has~~ has slope equal to zero but the component functions are quadratic & cubic with different slopes.



c) If the component functions of a parametric curve are quadratic, then the parametric curve describes a parabola in space.

The Above Statement is True. Each component function has form $f(t) = At^2 + Bt + C$, $g(t) = Dt^2 + Et + F$ & $h(t) = Gt^2 + Ht + I$ in the parametric curve $\vec{r} = \langle f(t), g(t), h(t) \rangle$ where, A, B, C, D, E, F, G, H, & I are constants.

the resulting curve is a parabola in 3-D space. Here, Each components quadratic equation represents the point's position on the parabola as function of 't'.

③ To find the tangent line to the parametric curve $\vec{r}(t) = \langle \cos t, t+1, \sin t \rangle$ at the point $(1,1,0)$.

derivative of $\vec{r}(t)$ is

$$\vec{r}\,'(t) = \langle -\sin t, 1, \cos t \rangle$$

to find the value of t:

we know, $\vec{r}(t) = \langle \cos t, t+1, \sin t \rangle$

equate $\vec{r}(t) = (1,1,0)$

so, $\cos t = 1$

$t+1 = 1$

$\sin t = 0$

Solving for 't' from the above equation,

$\cos t = 1 \Rightarrow t = 0$

$t+1 = 1 \Rightarrow t = 0$

$\sin t = 0 \Rightarrow t = 0$

Evaluating $\vec{r}(t)$ & $\vec{r}\,'(t)$ at $t=0$

$\vec{r}(0) = \langle \cos 0, 0+1, \sin 0 \rangle = (1,1,0)$

$\vec{r}\,'(t) = \langle -\sin 0, 1, \cos 0 \rangle = (0,1,1)$

Equation of the tangent line in vector form, can be written using $\vec{r}(0)$ & $\vec{r}\,'(0)$

$$\vec{L}(t) = \vec{r}(0) + t\,\vec{r}\,'(0)$$

Substitute the values of $\vec{r}(0)$ & $\vec{r}\,'(0)$

$$\vec{L}(t) = (1,1,0) + t(0,1,1)$$

$$\vec{L}(t) = (1, 1+t, t)$$

So, the equation of the tangent line to the parametric curve $\vec{x}(t) = \langle \cos t, t+1, \sin t \rangle$ at point $(1,1,0)$ is given by :

$$\vec{L}(t) = (1, 1+t, t)$$

where, $x(t) = 1$, $y(t) = 1+t$ & $z(t) = t$.

④ given two parametric curves

$\vec{r}(t) = \langle t, 1+2t, 3-2t \rangle$

$\vec{s}(t) = \langle -2-2t, 1-2t, 1+t \rangle$

a) To check for collision of the points, we need to equate the points of the two curves, then solve for 't'. If there are values of 't' that make both equation true, then the two points collide else they do not collide.

Now, let equate the curve's components & check for collision

$t = -2-2t \quad , \quad 1+2t = 1-2t \quad , \quad 3-2t = 1+t.$

$3t = -2 \quad , \quad 4t = 0 \quad , \quad 3t = 2$

$\boxed{t = \frac{-2}{3}} \quad , \quad \boxed{t = 0} \quad , \quad \boxed{t = \frac{2}{3}}$

So, there are no values of t where the positions of a points are equal ~~irrespective of their magnitude~~.
Hence, the points do not collide at any of the above mentiond time instances.

b) To check for crossing of their paths, equate the components of the curves, then solve for 't'
If there are value of 't' where the position of two curves are equal, then the paths cross else they do not cross.

here, we found out 't' values to be

$t = -\frac{2}{3}$, $t = 0$, $t = \frac{2}{3}$

therfore, their paths also donot cross, as they never occupy the same point simultaneously inall three dimensions