Pre-class Assignment - 15

① given, $\vec{b} = \begin{pmatrix} 6 \\ 0 \\ 0 \end{pmatrix}$ Spanned by $a_1 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ & $a_2 = \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix}$

Projection of a vector b onto the Subspace is given by :

$$\text{Proj}_v \vec{b} = \frac{b \cdot a_1}{a_1 \cdot a_1} \cdot a_1 + \frac{b \cdot a_2}{a_2 \cdot a_2} \cdot a_2$$

$b \cdot a_1 = \langle 6, 0, 0 \rangle \cdot \langle 1, 1, 1 \rangle = 6(1) + 0 + 0 = 6.$

$a_1 \cdot a_1 = \langle 1, 1, 1 \rangle \cdot \langle 1, 1, 1 \rangle = 1(1) + 1(1) + 1(1) = 3$

$b \cdot a_2 = \langle 6, 0, 0 \rangle \cdot \langle 0, 1, 2 \rangle = 6(0) + 0(1) + 0(2) = 0$

$a_2 \cdot a_2 = \langle 0, 1, 2 \rangle \cdot \langle 0, 1, 2 \rangle = 0(0) + 1(1) + 2(2) = 5$

$$\text{proj } \vec{b} = \frac{6}{3} \langle 1, 1, 1 \rangle + \frac{0}{5} \langle 0, 1, 2 \rangle$$

$$= 2 \langle 1, 1, 1 \rangle + 0$$

$$= \{2, 2, 2\}$$

Hence, Projection of $\vec{b}$ onto 2-D Subspace $V \subset \mathbb{R}^3$ Spanned by vectors a_1 & a_2 is $[2, 2, 2]$

(2) Given, data set $S = \{(0,6), (1,0), (2,0)\}$

we need to calculate the values of m & b using the formulas for linear regression. the slope (m) & y-intercept (b) can be calculated as:

$$m = \frac{n\sum(xy) - \sum x \cdot \sum y}{n\sum x^2 - (\sum x)^2}$$

$$b = \frac{\sum y - m\sum x}{n}$$

from the given data set, $n = 3$.

$\sum x = 0+1+2 = 3$

$\sum y = 6+0+0 = 6$.

$\sum xy = 0(6) + 1(0) + 2(0) = 0$.

$\sum x^2 = 0^2 + 1^2 + 2^2 = 5$.

Now, $m = \frac{3(0) - 3(6)}{3(5) - 9^2} = \frac{-18}{15-9} = \frac{-18}{6} = -3$

$m = -3$.

$b = \frac{6-(-3)(3)}{3} = \frac{6+9}{3} = \frac{15}{3} = 5$

$b = 5$.

the equation of the line that best fits the data set S is

$y = mx + b$.

$\Rightarrow \boxed{y = -3x + 5}$

③ Given data set $S = \{(-2,0), (-1,0), (0,1), (1,0), (2,0)\}$

Yes, we can find the parabola that best fits the data set S'.

we know, eqn of parabola is $y = ax^2 + bx + c$

for, $(-2,0) \Rightarrow 0 = 4a - 2b + c$

for $(-1,0) \Rightarrow 0 = a - b + c$

for $(0,1) \Rightarrow 1 = c$

for $(1,0) \Rightarrow 0 = a + b + c$

for $(2,0) \Rightarrow 0 = 4a + 2b + c$.

we can write the system of eqn as a vector equation $Ax = b$ where

$$A = \begin{bmatrix} 4 & -2 & 1 \\ 1 & -1 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 1 \\ 4 & 2 & 1 \end{bmatrix} \quad x = \begin{bmatrix} a \\ b \\ c \end{bmatrix} \quad b = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

To compute a least-squares solution,

$\Rightarrow$ Calculate A^TA & A^Tb.

$\Rightarrow$ form an augmented matrix for matrix eqn. $A^TAx = A^Tb$ & write in RREF

$$A^T = \begin{bmatrix} 4 & 1 & 0 & 1 & 4 \\ -2 & -1 & 0 & 1 & 2 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

$$A^T \cdot A = \begin{bmatrix} 4 & 1 & 0 & 1 & 4 \\ -2 & -1 & 0 & 1 & 2 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}_{3\times 5} \begin{bmatrix} 4 & -2 & 1 \\ 1 & -1 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 1 \\ 4 & 2 & 1 \end{bmatrix}_{5\times 3}$$

$$\cancel{= 4(4)+1(1)+0+1(1)+4(4) \quad 4(-2)+1(-1)}$$

$$A^T \cdot A = \begin{bmatrix} 34 & 0 & 10 \\ 0 & 10 & 0 \\ 10 & 0 & 5 \end{bmatrix}$$

$$A^T \cdot b = \begin{bmatrix} 4 & 1 & 0 & 1 & 4 \\ -2 & -1 & 0 & 1 & 2 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}_{3\times 5} \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}_{5\times 1} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

Now, $A^T A x = A^T b$.

$$\begin{bmatrix} 34 & 0 & 10 \\ 0 & 10 & 0 \\ 10 & 0 & 5 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

Now, $\left[\begin{array}{ccc|c} 34 & 0 & 10 & 0 \\ 0 & 10 & 0 & 0 \\ 10 & 0 & 5 & 1 \end{array}\right]$ Apply RREF

$\cancel{R_3 \to R_3 - 10R_1}$, $R_1 \to R_1/34$

$$\left[\begin{array}{ccc|c} 1 & 0 & 5/17 & 0 \\ 0 & 10 & 0 & 0 \\ 10 & 0 & 5 & 1 \end{array}\right]$$

$R_3 \rightarrow R_3 - 10R_1$

$$\left[\begin{array}{ccc|c} 1 & 0 & 5/17 & 0 \\ 0 & 10 & 0 & 0 \\ 0 & 0 & 35/17 & 1 \end{array}\right]$$

$R_2 \Rightarrow R_2/10.$

$$\left[\begin{array}{ccc|c} 1 & 0 & 5/17 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 35/17 & 1 \end{array}\right]$$

$R_3 \rightarrow 17R_3/35$

$$\left[\begin{array}{ccc|c} 1 & 0 & 5/17 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 17/35 \end{array}\right]$$

$R_1 \rightarrow R_1 - \frac{5R_3}{17}$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & -1/7 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 17/35 \end{array}\right]$$

hence, $x = \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} -1/7 \\ 0 \\ 17/35 \end{bmatrix}$

Parabola of best fit in the form

$$y = ax^2 + bx + c.$$

$$\Rightarrow y = \frac{-1}{7}x^2 + (0)x + \frac{17}{35}$$

So, the parabola is