Pre-class Assignment - 18

①
$$\begin{bmatrix} B & B & B & W & W & W & R & R & R \\ B & B & B & W & W & W & R & R & R \\ B & B & B & W & W & W & R & R & R \\ B & B & B & W & W & W & R & R & R \\ B & B & B & W & W & W & R & R & R \\ B & B & B & W & W & W & R & R & R \\ B & B & B & W & W & W & R & R & R \end{bmatrix}_{7\times 9}$$

To Express it in uV^T form, given that
u & V are column vectors
To form a 7x9 Matrix, column vectors
u & v must be of index 7x1.
In the given matrix, the same pattern is getting repeated in each row, so all the elements in vector 'u' = 1 & Column vector 'v' is same as a row in the given matrix.

So, $u = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}_{7\times 1}$ & $V = \begin{bmatrix} B \\ B \\ B \\ W \\ W \\ W \\ R \\ R \\ R \end{bmatrix}_{9\times 1}$ $\Rightarrow$ $V^T = \begin{bmatrix} B & B & B & W & W & W & R & R & R \end{bmatrix}_{1\times 9}$

Hence, $uv^T = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}_{7\times 1} \begin{bmatrix} B & B & B & W & W & W & R & R & R \end{bmatrix}_{1\times 9}$

Hence, the resultant matrix of the flag is of the order $\underline{\underline{7\times 9}}$.

(2) given, $A_0 = \begin{pmatrix} 5 & 4 & 3 & 2 & 1 \\ -1 & 1 & 0 & 1 & -1 \end{pmatrix}$

a)

Mean of 1^{st} row $= \dfrac{5+4+3+2+1}{5} = 3$

Mean of 2^{nd} row $= \dfrac{-1+1+0+1-1}{5} = 0$

Subtracting these row means gives us the centered matrix A:

$$A = \begin{bmatrix} 5-3 & 4-3 & 3-3 & 2-3 & 1-3 \\ -1-0 & 1-0 & 0-1 & 1-0 & -1-0 \end{bmatrix}$$

$$A = \begin{bmatrix} 2 & 1 & 0 & -1 & -2 \\ -1 & 1 & 0 & 1 & -1 \end{bmatrix}$$

b) Sample Covariance matrix

$$S = \frac{AA^T}{n-1} \quad ; n \rightarrow \text{no. of columns.}$$

$$A.A^T = \begin{bmatrix} 2 & 1 & 0 & -1 & -2 \\ -1 & 1 & 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} 2 & -1 \\ 1 & 1 \\ 0 & 0 \\ -1 & 1 \\ -2 & -1 \end{bmatrix}$$

$$A.A^T = \begin{bmatrix} 4+1+0+1+4 & -2+1+0-1+2 \\ -2+1+0-1+2 & 1+1+0+1+1 \end{bmatrix} = \begin{bmatrix} 10 & 0 \\ 0 & 4 \end{bmatrix}$$

$$S = \frac{1}{4}\begin{bmatrix} 10 & 0 \\ 0 & 4 \end{bmatrix} \Rightarrow S = \begin{bmatrix} 2.5 & 0 \\ 0 & 1 \end{bmatrix}$$

c) The Eigen Values of Matrix 'S' are

$$\lambda_1 = 2.5 \ \& \ \lambda_2 = 1$$

d) corresponding eigen vectors are:

$[1,0]$ for $\lambda_1 = 2.5$ & $[0,1]$ for $\lambda_2 = 1$

Since $\lambda_1 = 2.5$ is the largest eigen value, its eigen vector $[1,0]$ spans the one-dimensional subspace closest to the colums of A.

③ given, $A_0 = \begin{bmatrix} 1 & 2 & 3 \\ 5 & 2 & 2 \end{bmatrix}$

A) $S = \frac{A_0 A_0^T}{n-1}$; $n = 3$

$$A_0 A_0^T = \begin{bmatrix} 1 & 2 & 3 \\ 5 & 2 & 2 \end{bmatrix} \begin{bmatrix} 1 & 5 \\ 2 & 2 \\ 3 & 2 \end{bmatrix} = \begin{bmatrix} 1+4+9 & 5+4+6 \\ 5+4+6 & 25+4+4 \end{bmatrix}$$

$$A_0 A_0^T = \begin{bmatrix} 14 & 15 \\ 15 & 33 \end{bmatrix}$$

$$\Rightarrow S = \frac{1}{2} \begin{bmatrix} 14 & 15 \\ 15 & 33 \end{bmatrix} = \begin{bmatrix} 7 & 7.5 \\ 7.5 & 16.5 \end{bmatrix}$$

B) The entries of S are the sample variances S_1^2 & S_2^2 and the sample covariance S_{12}. These are given by:

$s_1^2 = S_{11}$, $s_2^2 = S_{22}$, $S_{12} = S_{12}$

$s_1^2 = 7 = \sqrt{7}$

$S_2^2 = 16.5 = \sqrt{16.5}$

$S_{12} = 7.5$

c)

$\sigma_1 = \sqrt{s_1^2} = 7$

4) Given that A_0 matrix is of the order 5×10

* To find the Centered matrix 'A',
  - → Calculate the mean of each row in A_0
  - → Mean will be a 5×1 column vector containing the mean grade for each of the 5 courses.
  - → Subtract mean from each row of A_0 to center each row.

  This is called as Centered Matrix 'A'.

* To find the sample covariance matrix 'S':
  - → $S = \frac{AA^T}{n-1}$; where n = no. of columns.

  Since, we have 10 columns, $n = 10$

  $$S = \frac{A.A^T}{9}$$

* The leading vector (v) of 'S' will tell us:
  - → 'v' is the direction of greatest variance in the centrered data 'A'.
  - → It shows which courses had grades that covaried together over the 10 years.