① A) $\frac{d}{dx}\int_{\cos x}^{\sin x} \sqrt{1-t^2}\,.dt$

from Leibniz rule, we know

$$\frac{d}{dx}\int_{a(x)}^{b(x)} f(x,t)\,dt = \int_{a(x)}^{b(x)} \left(\frac{\partial f}{\partial x}\,dt + f(b(x),x).b'(x) - f(a(x),x).a'(x)\right)$$

here, $a(x) = \cos x$ & $b(x) = \sin x$

and $f(t,x) = \sqrt{1-t^2}$

partially derivating the function with respect to 'x'

$$\frac{\partial f}{\partial x} = \frac{\partial}{\partial x}\sqrt{1-t^2}$$

$$\Rightarrow \frac{\partial f}{\partial x} = 0.$$

$$\Rightarrow f(b(x),x) = \sqrt{1-\sin^2 x} = \sqrt{\cos^2 x} = \cos x.$$

$$\Rightarrow f(a(x),x) = \sqrt{1-\cos^2 x} = \sqrt{\sin^2 x} = \sin x$$

$$\Rightarrow a'(x) = \frac{d}{dx}(\cos x) = -\sin x$$

$$\Rightarrow b'(x) = \frac{d}{dx}(\sin x) = \cos x$$

Now, putting all the components in Leibniz rule, we get:

$$F'(x) = \int (0) + \cos x(\cos x) - \sin x(-\sin x)$$

$$= \cos^2 x + \sin^2 x$$

$$= \underline{\underline{1}}$$

B) $\int \frac{x^3}{\sqrt{4-x^2}}\,dx$.

Consider, $4-x^2=u \Rightarrow 4-u=x^2$

$\Rightarrow du = -2x\,dx$

$x\,dx = -\frac{1}{2}\,du$.

Splitting the terms, of x & substituting 'u'

$x^3 dx = x^2 . x\,dx$

$= (4-u)\left(\frac{-1}{2}\right)du$.

$\int \frac{x^3}{\sqrt{4-x^2}}\,dx = \int \frac{1}{\sqrt{u}} \left[\frac{-1}{2}(4-u)\right] du$.

$= \frac{-1}{2}\int\left(\frac{4}{\sqrt{u}} - \sqrt{u}\right)du$.

$= -\frac{1}{2}\int\left(4u^{-1/2} - u^{1/2}\right)du$

$= -\frac{1}{2}\left[\frac{4u^{1/2}}{1/2} - \frac{u^{3/2}}{3/2}\right]$

$= -\frac{1}{2}\left[8\sqrt{u} - \frac{2}{3}u^{3/2}\right]$

$= -\frac{8}{2}\sqrt{4-x^2} - \frac{2}{6}(4-x^2)^{3/2}$.

$\Rightarrow \int \frac{x^3}{\sqrt{4-x^2}}\,dx = -4\sqrt{4-x^2} - \frac{1}{3}(4-x^2)^{3/2}$.

c) $\int x^2 \arctan x \, dx.$

Let $u = \arctan(x)$

$\cancel{\frac{du}{dx} = \frac{1}{1+x^2}} \Rightarrow du = \frac{1}{1+x^2} dx.$

& let $dv = x^2 \cdot \Rightarrow v = \frac{x^3}{3} dx.$

Using Integration by parts,

$\int u \, dv = uv - \int v . du.$

$$\int x^2 \arctan(x) \, dx = \frac{x^3}{3} \arctan x - \int \frac{x^3}{3} \cdot \frac{dx}{1+x^2}$$

$$= \frac{x^3}{3} \arctan x - \frac{1}{3} \int \frac{x^3}{1+x^2} dx \rightarrow (1)$$

$\cancel{\frac{x^3}{1+x^2}}$ We know, Dividend = Divisor × Quotient − Rem.

~~when we~~ here for $\frac{x^3}{1+x^2}$, when we divide

x^3 by $1+x^2$ we get quotient as x &

Remainder as '$-x$'

$$\int \frac{x^3}{1+x^3} dx = \int \frac{(1+x^2)x}{(1+x^2)} dx - \int \frac{x}{1+x^2} dx \rightarrow (2)$$

Substitute (2) in (1)

$$\int x^2 \arctan x \, dx = \frac{x^3}{3} \arctan x - \frac{1}{3}\left(\int x . dx - \int \frac{x}{x^2+1} dx\right)$$

$$= \frac{x^3}{3} \arctan x - \frac{1}{3}\left(\int x . dx - \frac{1}{2} \int \frac{2x}{x^2+1} dx\right)$$

$$= \frac{x^3}{3} \arctan x - \frac{1}{3}\left(\frac{x^2}{2}\right) - \frac{1}{6} \ln(x^2+1) + C$$

$$\int x^2 \arctan x \, dx = \frac{x^3}{3} \arctan x - \frac{1}{3}\left(\frac{x^2}{2}\right) - \frac{1}{6} \ln(x^2+1) + C$$

(2) Given vectors

$\vec{r_1}(t) = \langle 6+t+0.5t^2, t^2+2t, 5t-2t^2 \rangle$

$\vec{r_2}(t) = \langle 7t-0.5t^2, 1+0.5t^2-t, t^2-9t \rangle$

to find the velocity, we need to find the first derivative of vector $\vec{r}(t)$. $\vec{v}(t) = \vec{r}\,'(t)$.

$$\text{So, } \vec{v_1}(t) = \frac{d\vec{r_1}}{dt} = (1+t, 2t+2, 5-4t)$$

$$\vec{v_2}(t) = \frac{d\vec{r_2}}{dt} = (7-t, t-1, 2t-9)$$

Since, it is given that objects were travelling with same velocity, let's assume $\vec{V_1}(t) = \vec{V_2}(t)$

let's take. 't' is time for the object 1 with velocity $\vec{V_1}(t)$.

and 'u' be the time for the object 2 with velocity $\vec{V_2}(t)$

$$\Rightarrow \vec{V_1}(t) = \vec{V_2}(u)$$

$$(1+t, 2t+2, 5-4t) = (7-u, u-1, 2u-9).$$

equating each other vector's components.

$1+t = 7-u \Rightarrow t+u-6=0 \rightarrow (1)$

$2t+2 = u-1 \Rightarrow 2t-u+3=0 \rightarrow (2).$

$5-4t = 2u-9 \Rightarrow 4t+2u-14=0 \rightarrow (3)$

Solving (1) & (2) to find 't'

$$\begin{aligned} t+\cancel{u}-6&=0 \\ 2t-\cancel{u}+3&=0 \\ \hline 3t\cancel{+}3&=0 \end{aligned} \Rightarrow \boxed{t=1}$$

Substituting $t=1$ in (1).

$1+u-6=0$

$\boxed{u=5}$

So, $\vec{V_1}$ at 1 unit of time is equal to $\vec{V_2}$ at 5 unit of time.

put $t=1$ in $\vec{V_1}$

$\vec{V_1} = (1+1, 2+2, 5-4) = (2,4,1)$

Put $u=5$ in $\vec{V_2}$

$\vec{V_2} = (7-5, 5-1, 10-9) = (2,4,1)$

Speed of the object is given by

$V_1 = \sqrt{2^2+4^2+1^2} = \sqrt{21}$ units.

Hence, the speed of objects are $\sqrt{21}$ units.

④ given,

domain of $f = (-\infty, 1)$

domain of $g = [0, \pi]$

domain of $h = (-1, 1)$

to find the domain of function

$$F(x,y) = f(x) - g(x)h(y)$$

· x has to lie between $(-\infty, 1)$ and $[0, \pi]$

so, $x \in \{(-\infty, 1) \cap [0, \pi]\}$

$x \in \{[0, 1)\}$ & $y \in \{(-1, 1)\}$.

Hence,

Domain of $F = \{(x,y) \in \mathbb{R}^2 \mid x \in [0,1), y \in (-1,1)\}$

⑤ ~~A) In the given blue curve~~

(5) A) In the given Blue curve, At 'A' & 'B' the Contour value starts decreasing as we move along x-axis. At 'C' the contour value ~~is~~ starts increasing as we move along x-axis. Since, f_x is negative at 'A' & 'B', so f_x is greatest ~~highest~~ at 'C'

B) In the given Blue curve, At 'A' the Contour value ~~remains constant~~ increases along +ve y-axis, At 'B', as we move along positive y-axis, the contour value increases. At 'C', the contour value decreases as we move along positive y-axis.
Since, A is +ve, so f_y is greatest at 'A'

c) we know that a level curve has a single value. But here, the red curve cannot be considered as a level curve as it corresponds to two different ~~level~~ values at two different points.

(3) Let us consider a triangle ABC

A(-1, 0; 0) & B(1, 0; 0)

from the given points, the distance between A & B is 2 units since a tetrahedron is made of equilateral Δles, distance of CA & CB is 2 units

$$CA = \sqrt{(x_A - x_c)^2 + (y_A - y_c)^2 + (z_A - z_c)^2}$$

$$2 = \sqrt{(-1-x)^2 + (0-y)^2 + (0)^2}$$

$$4 = (-1-x)^2 + y^2$$

Similarly,

$$2 = \sqrt{(x_B - x_c)^2 + (y_B - y_c)^2 + (z_B - z_c)^2}$$

$$2 = \sqrt{(1-x)^2 + y^2} \Rightarrow 4 = (1-x)^2 + y^2$$

$$(-1-x)^2 + y^2 = 4 \rightarrow (1)$$

$$(1-x)^2 + y^2 = 4 \rightarrow (2)$$

$$x^2 + 2x + 1 + y^2 = 4$$

$$x^2 - 2x + 1 + y^2 = 4$$

Solving the eqns,

$$x^2 + 2x + 1 + y^2 - x^2 - 2x + 1 + y^2 = 0$$

$$x = 0$$

Sub $x = 0$ in (1)

$$0^2 + 2(0) + 1 + y^2 = 4$$

$$1 + y^2 = 4$$

$$\Rightarrow \boxed{y = \sqrt{3}}$$

so the co-ordinates of point C are $(0, \sqrt{3}, 0)$

$A(-1, 0, 0)$, $B(1, 0, 0)$ $C(0, \sqrt{3}, 0)$

centroid of Δle

$C_x = (-1+1+0)/3 = 0$

$C_y = (0+0+\sqrt{3})/3 = \frac{\sqrt{3}}{3} = \frac{1}{\sqrt{3}}$

$C_z = (0+0+0)/3 = 0$

similarly we can take the co-ordinates of E as $(0,0,0)$

Co-ordinates of D $= (0, 1/\sqrt{3}, 0)$.

edge AC $= C - A = (1, 1/\sqrt{3}, 0)$

edge BC $= C - B = (-1, 1/\sqrt{3}, 0)$

edge DC $= D - C = (0, 0, \sqrt{-3/2})$

The Angle at the centroid. is 90°.