(1) given,

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 4 & 4 & 5 \\ 6 & 7 & 7 \end{bmatrix} ; \quad A^{-1} = \begin{bmatrix} -7 & -7 & 6 \\ 2 & 1 & -1 \\ 4 & 5 & -4 \end{bmatrix}$$

$$B = \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}$$

we know, $AX = B$

$$\Rightarrow X = A^{-1} B$$

$$X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -7 & -7 & 6 \\ 2 & 1 & -1 \\ 4 & 5 & -4 \end{bmatrix} \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}$$

$$X = \begin{bmatrix} (-7\times 2) + (-7\times 0) + (6\times 1) \\ (2\times 2) + (1\times 0) + (-1\times 1) \\ (4\times 2) + (5\times 0) + (-4\times 1) \end{bmatrix}$$

$$X = \begin{bmatrix} -14+6 \\ 4-1 \\ 8-4 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -8 \\ 3 \\ 4 \end{bmatrix}$$

(2) $A = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 3 \\ 0 & 0 & 1 \end{pmatrix}$ find A^{-1}

$$= \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 2 & 1 & 3 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right]$$

$R_2 : R_2 - 2R_1$

$$= \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 3 & -2 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right]$$

$R_2 : R_2 - 3R_3$

$$= \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & -2 & 1 & -3 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right]$$

hence,

$$A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & -3 \\ 0 & 0 & 1 \end{bmatrix}$$

③ a) $\begin{pmatrix} 4 & 3 \\ 8 & 6 \end{pmatrix}$

$$\text{Determinant} = 4(6) - 3(8)$$
$$= 24 - 24$$
$$= 0$$

$\Rightarrow$ Since, Determinant is zero, the matrix is not invertible.

b) $\begin{pmatrix} 4 & 3 \\ 8 & 7 \end{pmatrix}$

$$\text{Determinant} = 4(7) - 3(8)$$
$$= 28 - 24$$
$$= 4$$

$\Rightarrow$ the matrix is invertible.

c) $\begin{pmatrix} 6 & 6 \\ 6 & 0 \end{pmatrix}$ Determinant $= 6(0) - 6(6)$
$$= -36$$

$\Rightarrow$ the matrix is invertible.

d) $\begin{pmatrix} 6 & 6 \\ 6 & 6 \end{pmatrix}$ Determinant $= 6(6) - 6(6)$
$$= 0$$

$\Rightarrow$ The matrix is not invertible.

e) $\begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{pmatrix}$ Determinant $= 1(1(1) - 0) + 0 + 0$
$$= 1$$

$\Rightarrow$ The matrix is invertible.

f) $\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{pmatrix}$ Determinant $= 1(1-0) - 1(1-0) + 1(1-1)$

$= 1 - 1 + 0$

$= 0$

$\Rightarrow$ The matrix is not invertible.

④ given, $A = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \\ 3 & 2 & 1 \end{pmatrix}$

The given matrix 'A' is invertible only if the determinant of the matrix is non-zero.

Determinant $= 1(1 - 2(2)) - 2(3(1) - 3(2)) + 3(3(2) - 3(1))$

$= 1(-3) - 2(-3) + 3(3)$

$= -3 + 6 + 9$

$= 12$

Hence, the given matrix 'A' is invertible.

(5) we know that, If A & B are invertible matrices then AB is invertible and

$$(AB)^{-1} = B^{-1}A^{-1}$$

This is also known as shoes & socks rule.

Since, A, B, C are invertible $n \times n$ matrices,

$\Rightarrow$ ABC is also invertible.

$$(ABC)^{-1} = C^{-1} \cdot B^{-1} A^{-1}$$

The product $B^{-1}CA^{-1}B$ is also invertible & its inverse is given by

$$(B^{-1}CA^{-1}B)^{-1} = B^{-1} \cdot A \cdot C^{-1} \cdot B.$$

(6) given, $P = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$

the inverse of a permutation matrix can be found out by taking the transpose of the matrix itself. As, Permutation matrix is orthogonal hence, $P^{-1} = P^T$

$$\Rightarrow P^{-1} = P^T = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

Here, the 1's indicates which columns of the identity matrix get moved where.

It is called as permutation matrix because it represents a permutation or re-ordering of columns of the identity matrix.

(7) for a matrix to be invertible it should satisfy the below conditions:

* the matrix must be a Square matrix i.e, $m = n$.
* The determinate of the matrix must be non zero.

Here, It is given as the matrix is a 300 x 300 matrix. hence, it is a Square matrix and one condition is Satisfied.
however, for the Second condition, as this matrix is made up of 0's and 1's, is more likely that it might ~~be~~ result the determinant to be zero. (assuming the 1's & 0's are arranged according to the given figure). Hence, making the matrix non-invertible.
If the 0's & 1's are arranged in a specific pattern, the matrix could be invertible. as it gives non-zero as determinant.