Preclass Assignment -13.

① Given, bias vectors.

$$e_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad e_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

A) Reflected across the x-axis
i.e, $T_A(x, y) \rightarrow (x, -y)$

Apply the linear transformation using bias vectors,

hence, $A = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

Fundamental Spaces

→ Column Space (C) = Span of [1,0] & [0,-1]
→ Null Space (N) = Span of [0,1]
→ Row Space (R) = Span of [1,0] and [0,-1]
→ Left Null Space (N^T) = Span of [0,1]

B) Strech horizontally by a factor of 2:
it is given by $T_B(x, y) = (2x, y)$

hence, $A = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$

Fundamental Spaces

→ Column Space (C) = Row Space (R) = Span of [2,0] & [0,1]
→ Null Space (N) = Left null space (N^T) = Span [0,0].

c) Collapse to a single point.

It is given by : $T(x,y) = (0,0)$

Hence, the matrix $A = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

Fundamental Spaces:

→ Column space(C) = Row Space(R) = Span of [0, 0]

→ Null space (N) = left null space(N^T) = span of all vectors in $\mathbb{R}^2$

D) Rotation counter Clockwise by 45°.

It is given by $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$

Hence, the Matrix is

$$A = \begin{bmatrix} \cos 45^\circ & -\sin 45^\circ \\ \sin 45^\circ & \cos 45^\circ \end{bmatrix} = \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix}$$

fundamental Spaces:

→ Column Space(C) = Row space (R) =

= Span of [cos 45°, sin 45°] and [−sin 45°, cos 45°]

→ Null space(N) = left Null space (N^T) =

= span of [0, 0]

e) projected onto the y-axis

It is given by $T(x,y) = (0,y)$

hence, the matrix $A = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$

fundamental space:

$\Rightarrow C = R =$ Span of $[0,0]$ & $[0,1]$

$\Rightarrow N = N^T =$ Span of $[1,0]$

F) Sheared vertically by a factor of 1.

It is given by : ~~(x+y, y)~~ $T(x,y) = (x+y, y)$

hence, the matrix is

$$A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

fundamental space:

$\Rightarrow$ Column Space (C) = Row space (R) = Span of $[1,0]$ & $[1,1]$

$\Rightarrow$ Null Space (N) = Left of Null Space (N^T) = Span of $[0,1]$

(2) Given $A = \begin{bmatrix} 1 & 3 & 5 & 0 & 7 \\ 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

here $m = 3$ & $n = 5$

The rank of the given matrix is $r = 2$.

fundamental Subspaces:

Dimension of Column Space $\Rightarrow r = 2$.

Dimension of row space $\Rightarrow r = 2$

Dimension of Null space $\Rightarrow n - r = 5 - 2 = \underline{\underline{3}}$

Dimension of left null space $= m - r$ ~~= 3-3~~

$= 3 - 2$

$= \underline{\underline{1}}$

for finding the rank, Row reducing Matrix A to its row echelon form (RREF)

$RREF(A) = \begin{bmatrix} 1 & 3 & 5 & 0 & 7 \\ 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

Rank of A = No. of non zero rows in RREF(A)

$= \underline{\underline{2}}$

③ let $S = \{x, y, z, w \in \mathbb{R}^4 \mid x = y = z = w\}$

A) given, $x = y = z = w$

& consider $v = (x, y, z, w)$

$$v = (x, x, x, x)$$

$$v = x(1,1,1,1)$$

it can be written as $v = k(1,1,1,1)$

where k = Scalar

Hence, Basis of all vectors whose components are equal is $(1,1,1,1)$

B) let $S = \{x, y, z, w \in \mathbb{R}^4 \mid x + y + z + w = 0\}$

$$x = -y - z - w.$$

Now,

$$S = \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} = \begin{bmatrix} -y-z-w \\ y \\ z \\ w \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \\ 0 \\ 0 \end{bmatrix} y + \begin{bmatrix} -1 \\ 0 \\ 1 \\ 0 \end{bmatrix} z + \begin{bmatrix} -1 \\ 0 \\ 0 \\ 1 \end{bmatrix} w$$

Hence, the basis of all vector components add to zero is $\begin{bmatrix} -1 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 0 \\ 1 \end{bmatrix}$.

c) $I_4 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

The column space of I_4 is the space spanned by its columns.
Hence, it is spanned by its four standard basis vectors.
Therefore, the basis of the column space of I_4 is $\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$