① Sketching the domains of the following functions.

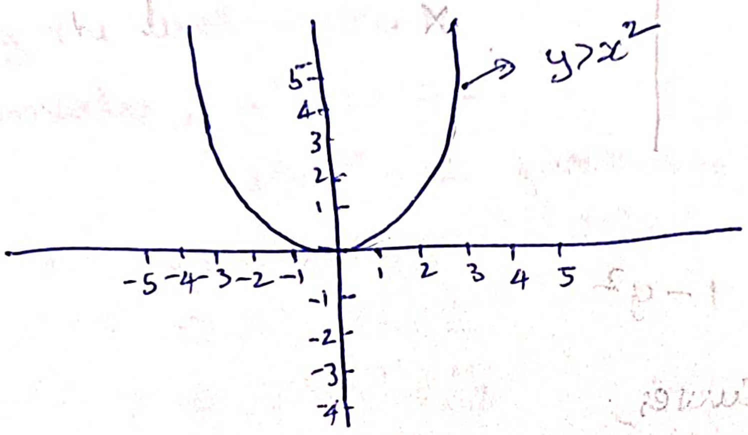
a) $f(x,y) = \ln(y-x^2)$

$\ln(x)$ is only defined for $x>0$ i.e +ve 'x' values.

So, $y-x^2>0$

$y>x^2$

This can be graphically represented as the region above the parabolic curve $y=x^2$ in the xy-plane.



b) $g(x,y) = \sqrt{x} + \sqrt{y} + \sqrt{1-x^2-y^2}$

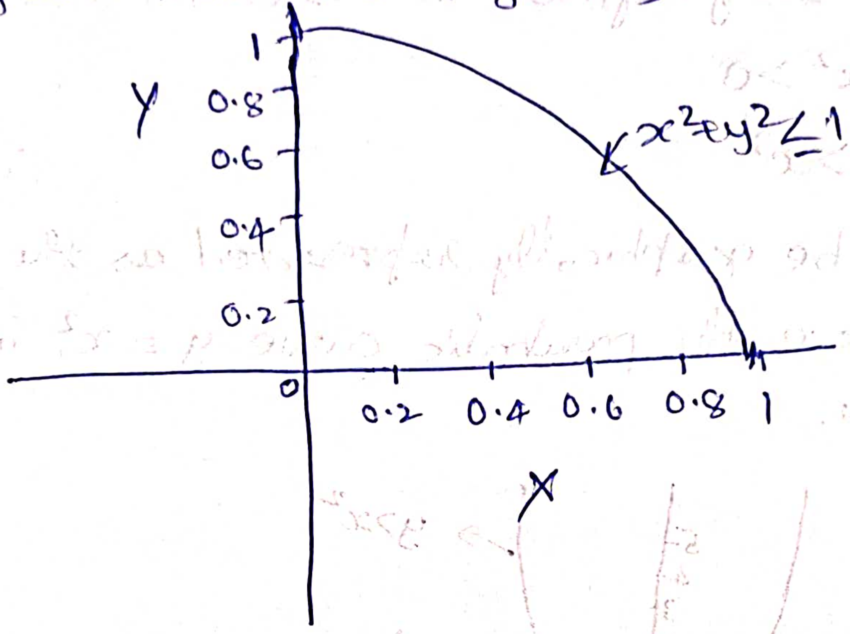
For the square root of a real number to be defined, the number must be non-negative.

i.e., $x \geq 0$, $y \geq 0$, $1-x^2-y^2 \geq 0$

$x^2+y^2 \leq 1$

* for $x \geq 0$, all the points lie on or right of the y-axis which is +ve x-axis
* for $y \geq 0$, all the points lie on or above the x-axis which is +ve y-axis
* for $x^2+y^2 \leq 1$, all the points lie inside or on the unit circle centered at the origin

Therefore, this can be graphically represented as the quarter circle in the I^{st} quadrant of xy-plane along with x-axis & y-axis



② a) $f(x,y) = 1-y^2$

finding level curves

let consider $f(x,y) = C$

$$\Rightarrow 1-y^2 = C.$$

when, $C=0 \Rightarrow 1-y^2=0 \Rightarrow y^2=1 \Rightarrow y=\pm 1$

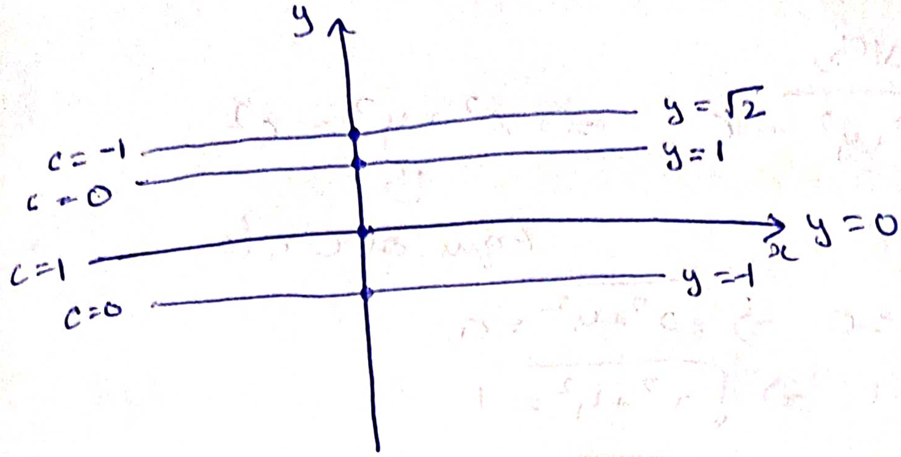
$C=1 \Rightarrow 1-y^2=1 \Rightarrow y^2=0 \Rightarrow y=0$

$C=-1 \Rightarrow 1-y^2=-1 \Rightarrow y^2=2 \Rightarrow y=\sqrt{2}$

$C=2 \Rightarrow 1-y^2=2 \Rightarrow y^2=-1$

from $C=2,3,4\ldots n$ they have no solutions because the square of a real number cannot be negative.

Contour Diagram,



b) $g(x,y) = x^2+y^2$

finding the level curves,

lets consider, $g(x,y) = c$

$x^2+y^2 = c.$ {we know $x^2+y^2=r^2$ eqn of circle}

$r^2 = c.$

when, $c=1 \Rightarrow x^2+y^2=1$

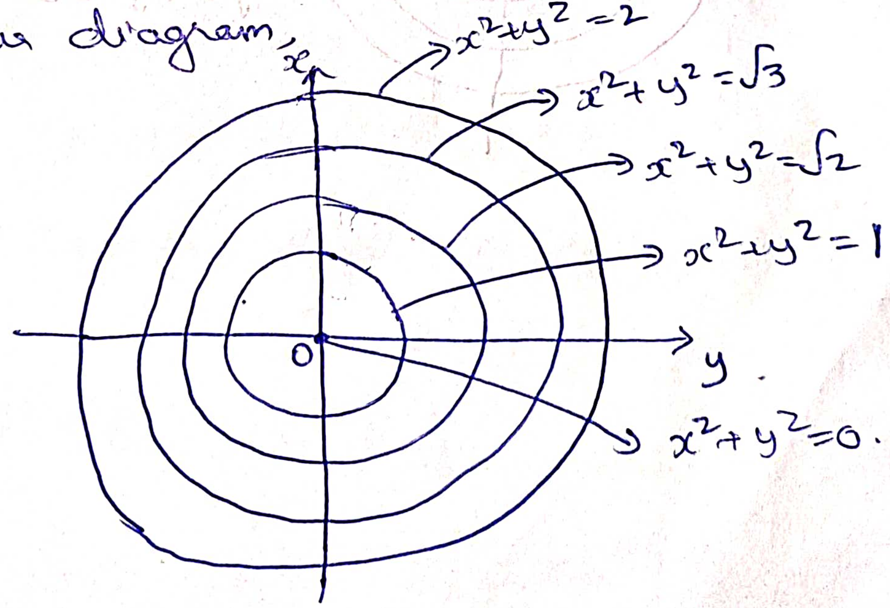
$c=2 \Rightarrow x^2+y^2=\sqrt{2}$

$c=3 \Rightarrow x^2+y^2=\sqrt{3}$

$c=4 \Rightarrow x^2+y^2=2.$

$c=0 \Rightarrow x^2+y^2=0$

Contour diagram,



c) $h(x, y) = \sqrt{x^2+y^2}$

Level curves,

Let $\sqrt{x^2+y^2} = c$ $\Rightarrow x^2+y^2 = c^2$

$\Downarrow$

Eqn. of circle.

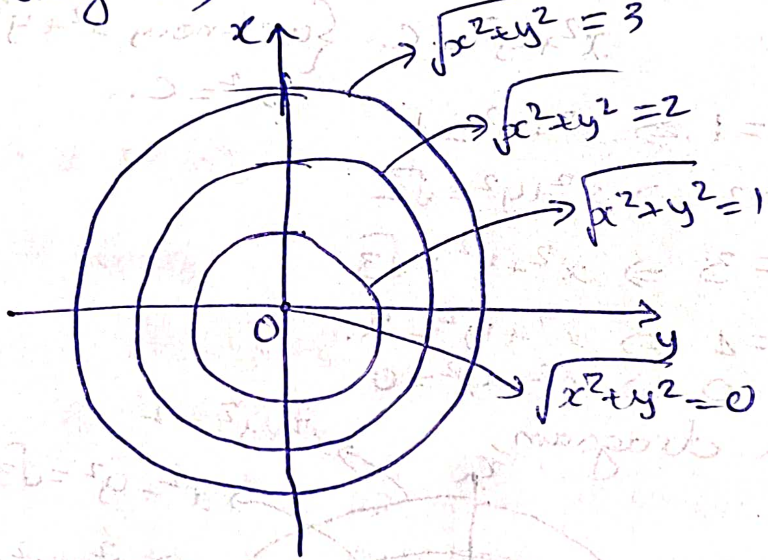
when $c = 0 \Rightarrow \sqrt{x^2+y^2} = 0$

$c = 1 \Rightarrow \sqrt{x^2+y^2} = 1$

$c = 2 \Rightarrow \sqrt{x^2+y^2} = 2$

$c = 3 \Rightarrow \sqrt{x^2+y^2} = 3$

Contour diagram,



(3) In the given map, isochrone map equal travel time or travel distance from a specific location. Here, the different colors represents the travel ~~Green color represents bro very less travel~~ distance

~~Blue color represents~~

It can be represented as:

Dark Green $<$ Blue $<$ Red $<$ light green $<$ Orange $<$ pink.

Since, we are considering distance as the only variable, in the given isochrone map the contour diagram is not multi variable function

~~the function~~

here, the function cannot be determined.

(1)

a) $\frac{\partial}{\partial v}\left(\frac{2\pi r}{v}\right)$

$$\frac{\partial u}{\partial v} = \frac{v'u - u'v}{u^2}$$; derivative of u with respect to v.

here, $v = 2\pi r$. & $u = v$

$$\frac{\partial}{\partial v}(2\pi r) = 0 \quad \& \quad \frac{\partial}{\partial v}(v) = 1$$

Now,

$$\frac{\partial u}{\partial v} = \frac{0.v - 1.2\pi r}{v^2} = \frac{-2\pi r}{v^2}$$

b) f_x, where $f(x,y) = \sin(5x^3y - 3xy^2)$.

$$\frac{\partial f}{\partial x} = \frac{\partial}{\partial x}(\sin(5x^3y - 3xy^2))$$ diff w.r.t 'x'

let consider $5x^3y - 3xy^2 = u$

$$\frac{\partial}{\partial x}(\sin(5x^3y - 3xy^2)) = \cos(5x^3y - 3xy^2)\frac{\partial}{\partial x}(5x^3y - 3xy^2)$$

$$\frac{\partial}{\partial x}(5x^3 - 3xy^2) = 5y\frac{\partial}{\partial x}(x^3) - 3y^2\frac{\partial}{\partial x}(x).$$

$$\frac{\partial}{\partial x}(5x^3y - 3xy^2) = 15x^2y - 3y^2$$

Now,

$$\therefore \frac{\partial f}{\partial x} = \cos(5x^3y - 3xy^2)(15x^2y - 3y^2)$$

~~partial diff. with respect~~

$$\cancel{\frac{\partial f}{\partial y} = \frac{\partial}{\partial y}(\sin(5x^3y - 3xy^2))}$$

$$\cancel{\frac{\partial}{\partial y}(\sin(5x^3y - 3xy^2)) = \cos(5x^3y - 3xy^2) \cdot \frac{\partial}{\partial x}(}$$

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c) $g(x,y,z) = x \cdot e^{1-z^2}$

find $\frac{\partial g}{\partial y}$.

to fing $\frac{\partial g}{\partial y}$, treat x & z as constants

Since, 'y' does not appear in the given function.

Hence, $\frac{\partial g}{\partial y} = 0$

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5) given, $\frac{dr}{dt} = 1.8$ in/s , $r = 120$, $h = 140$

$\frac{dh}{dt} = -2.5$ in/s. (Since height is decreasing)

Volume of a right circular cone is given by

$$V = \frac{1}{3}\pi r^2 h.$$

here, $r \rightarrow$ radius & $h \rightarrow$ height

we need to find $\frac{dV}{dt}$

$$\frac{dV}{dt} = \frac{\partial V}{\partial r}\frac{dr}{dt} + \frac{\partial V}{\partial h}\frac{dh}{dt}.$$

$$\frac{\partial V}{\partial r} = \frac{\pi}{3} h \frac{\partial}{\partial r}(r^2) \qquad \frac{\partial V}{\partial h} = \frac{1}{3} r^2 \pi . \frac{\partial}{\partial h}(h)$$

$$\frac{\partial V}{\partial r} = \frac{2}{3} \pi r h. \qquad \frac{\partial V}{\partial h} = \frac{\pi r^2}{3}.$$

Now, $\frac{dV}{dt} = \frac{2}{3} \pi r h . \frac{dr}{dt} + \frac{\pi r^2}{3} . \frac{dh}{dt}$

$$= \frac{2}{3} \times 1.8 \pi r h + \frac{1}{3} \times (-2.5) \pi r^2$$

$$= \frac{2}{3} \times 1.8 \times 120 \times 140 \times \pi . + \frac{1}{3} (-2.5) \pi (120)^2$$

$$= 63334.5 - 37699.1$$

$$\frac{dV}{dt} = 25635.3 \text{ in}^3/s.$$

(6) to calculate the intensity of red color at a specific pixel,

Calculate: $\frac{\partial R}{\partial x}$ & $\frac{\partial R}{\partial y}$.

Let us assume the pixel is at (x, y) point.

we have to use the neighbouring pixels to compute partial derivatives.

So, $$\frac{\partial R}{\partial x} = \frac{R(x+1, y) - R(x-1, y)}{2}.$$

$$\frac{\partial R}{\partial y} = \frac{R(x, y+1) - R(x, y-1)}{2}.$$

here, +ve values ~~of R~~ indicate increase in Red intensity

-ve value indicate decrease in Red intensity.

Neighbouring pixels with respect to x-axis is $(x+1, x-1)$ similarly for y-axis it is $(y+1, y-1)$