HW-2

36) Total number of cards in a deck = 52

2 cards can be chose from 52 cards in $\binom{52}{2}$ ways.

a) probability ~~that~~ of choosing 2 cards that are Aces:

total Aces = 4 cards.

2 Aces can be chose from 4 cards in $\binom{4}{2}$ ways.

probability of choosing 2 cards that are aces

$$= \frac{\binom{4}{2}}{\binom{52}{2}} = \frac{\frac{4!}{2!\,2!}}{\frac{52!}{50!\,2!}} = \frac{\frac{\cancel{4}\times 3\times 2\times 1}{\cancel{2\times 2}}}{\frac{52\times 51\times \cancel{50!}}{\cancel{50!}\times 2}}$$

$$= \frac{6}{1326}$$

$$= \underline{\underline{0.00452}}$$

b) Probability of choosing 2 cards that have the same values is given by:

Since there are 4 suits, there can be 4 cards whose value match in all 4 suits.
Since, there are 13 cards in each suit.

therefore, Choosing 2 cards from 4 cards having same value can be done in $4C_2$ ways. or $\binom{4}{2}$

Hence, the probability of choosing 2 cards that have the same values is $= 13 \times \frac{4C_2}{52C_2}$

$$= 13 \frac{\binom{4}{2}}{\binom{52}{2}} = 13 \times 0.00452$$

$$= \underline{\underline{0.0587}}$$

(41) A die is rolled 4 times.

when a die is rolled once,

probability of getting 6 at least once = 1 − probability of not getting 6.

$P(\text{not getting } 6) = \frac{5}{6}$.

As, in the Sample Space $S = \{1, 2, 3, 4, 5, 6\}$.

we get '6' only once & 'not 6' five times.

when a die is rolled 4 times,

$$P(\text{getting } 6) = 1 - P(\text{not getting } 6)$$

$$= 1 - \left(\frac{5}{6}\right)^4$$

$$= 1 - \frac{5^4}{6^4}$$

$$= 0.5177$$

$$= 51.77\%$$

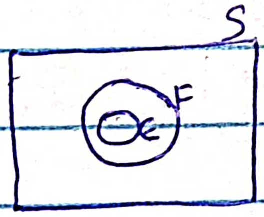
theoretical exercises:

(2) prove: If $E \subset F$ then $F^c \subseteq E^c$

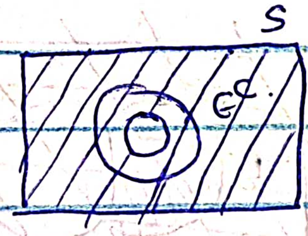
Let us consider $x \in F^c$ then $x \notin F$
Since $E \subset F$ then $x \notin E$.
As, $x \notin E$ it implies that $x \in E^c$
therefore,
$x \in F^c$ and $x \in E^c$

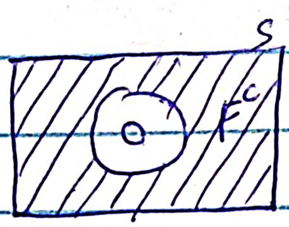
Hence,
$F^c \subseteq E^c$

If 'S' is the sample space for any event 'E', we define a new event 'E^c' to consists of all elements in 'S' that are not in Event 'E'.

⇒ 

$E \subset F$

⇒ 



Hence, $F^c \subseteq E^c$

(13) prove that $P(EF^c) = P(E) - P(EF)$

from Axiom 3 of probability,
for any sequence of mutually exclusive events, the probability of at least one of these events occuring is just sum of their respective probability.

here, $E = EF \cup EF^c$

As, $E = E(F \cup F^c)$

$= E(1)$ $\{$ since, $F \cup F^c = 1 \}$

$= E$

so, $P(E) = P(EF \cup EF^c)$

$P(E) = P(EF) + P(EF^c)$

Rearranging the terms,

hence, $P(EF^c) = P(E) - P(EF)$

(20) Let x_k be the possible values of X.

where $k = 1, 2, 3, \ldots \infty$

* Let the probability of x_k be $p_X(x_k) = p$ &

$p \neq 0$ for all k then,

$$1 = \sum_{k=1}^{\infty} p_X(x_k) = \sum_{k=1}^{\infty} p = \infty$$

which is a contradiction because, the sum of probabilities would be infinite whereas it must be $0 \leq p \leq 1$

* If $p = 0$ for all k then $1 = \sum_{k=1}^{\infty} p_X(x_k)$

$$\Rightarrow \sum_{k=1}^{\infty} p = 0$$

which is again a contradition because, if we sum '0' an infinite number of times as 'k' goes from 1 to infinity, we get $\sum 0 = 0$.

This implies that the total probability would be '0' whereas the probability must sum to '1'.

Let us consider an example:

$p_x(x_k) = \frac{1}{(1+k)^2}$ is always positive

because both the Numerator (1) & the denominator $(1+k)^2$ are positive.

If we check its sum of probabilities:

$$\sum_{k=1}^{\infty} p_x(x_k) = \sum_{k=1}^{\infty}\left(\frac{1}{(1+k)^2}\right) \text{ for all } k \text{ ; } k=1,2,\ldots\infty.$$

this is a convergent series which means that sum is finite.

here,

$$\sum_{k=1}^{\infty}\left(\frac{1}{(1+k)^2}\right) = 1.$$

Therefore, It is possible to have the probabilities without contradiction as long as they satisfy the axioms of probability.