(1) There are a total '7' places in the license plate.
given that, First 2 places are for letters.
Other 5 places are for numbers.
total no. of letters is 26 (A-Z)
total no. of digits is 10 (0 to 9)

a) total no. of possible ways = $\underset{1^{st}}{26} \times \underset{2^{nd}}{26} \times \underset{3^{rd}}{10} \times \underset{4^{th}}{10} \times \underset{5^{th}}{10} \times \underset{6^{th}}{10} \times \underset{7^{th}}{10}$
= 67600000

therefore, There can be 67600000 different license plates.

b) Assuming that no letter or number can be repeated.
total no. of possible ways without repeating the letter or number = $26 \times 25 \times 10 \times 9 \times 8 \times 7 \times 6$
= 19656000

As, 1^{st} place can have 26 options, 2^{nd} can have 25 options.
3^{rd}, 4^{th}, 5^{th}, 6^{th} and 7^{th} places can have 10, 9, 8, 7 and 6 options respectively.

(7) a) We have a total of 6 people of which 3 are boys and 3 are girls.
The number of ways to arrange 6 people to sit in a row is $6! = 120$ ways.

b) Consider the group of 3 boys as 1 entity.
also consider the group of 3 girls as 1 entity
2 entities can be arranged in $2!$ ways $= 2$ ways.
Within each group, 3 boys can be arranged in $3!$ ways $= 6$ ways
Similarly, 3 girls can be arranged in $3!$ ways $= 6$ ways.

Therefore, 3 boys & 3 girls if sat in a row if the boys and the girls are each to sit together.
the number of ways $= 2 \times 6 \times 6 = 72$ ways

c) Consider, the group of 3 boys as 1 entity.
also consider 3 girls to be 3 different entities
So, objects will be $\{B, G_1, G_2, G_3\}$
total no. of entities $= 4$
4 entities can be arranged in $4!$ ways
3 boys can be arranged in $3!$ ways.
therefore, the no. of ways if only boys must sit together is $4! \times 3! = 144$ ways

d) There can be two arrangements if no two people of same sex are allowed to sit.

i) B G B G B G or ii) G B G B G B

total no. of arrangements = 2.

For each arrangement boys can be arranged = 3!

Girls can be arranged in 3! ways.

therefore, the total no. of ways if no two people of the same sex are allowed to sit together is

$= 2 \times 3! \times 3!$

$= 2 \times 6 \times 6$

$= 72$ ways

(9) given, Total no. of blocks = 12

no. of Black blocks = 6.

no. of red Blocks = 4

no. of white Blocks = 1

no. of Blue Blocks = 1

To calculate the no. of arrangements, if the child puts the blocks in a line can be calculated by:

the formula for permutation with duplicates is

$$P\binom{n}{n_1, n_2 \ldots n_k} = \frac{n!}{n_1! \times n_2! \times \cdots \times n_k!}$$

where, n is total no. of objects to be arranged.

$n_1, n_2, \ldots, n_k$ are the counts of each object of all type.

here, $n = 12$, $n_1 = 6$, $n_2 = 4$, $n_3 = 1$, $n_4 = 1$

Substituting the values in the formula,

$$P\binom{12}{6,4,1,1} = \frac{12!}{6! \times 4! \times 1! \times 1!} = \frac{12 \times 11 \times 10 \times \overset{3}{\cancel{9}} \times \overset{2}{\cancel{8}} \times 7 \times \cancel{6!}}{\cancel{6!} \times \cancel{4} \times \cancel{3} \times \cancel{2} \times 1}$$

$$= 12 \times 11 \times 10 \times 3 \times 7 = 27720$$

Therefore, there are 27720 different arrangements for the 12 blocks with 6 black, 4 red, 1 white & 1 blue block.

(13) given, total no. of people = 20 = n

For every handshake two persons are involved.

$r = 2$.

Using the formula of combinations:

$$^nC_r \text{ (or) } \binom{n}{r} = \frac{n!}{(n-r)!\,r!}$$

$$^{20}C_2 \text{ (or) } \binom{20}{2} = \frac{20!}{(20-2)!\,2!}$$

$$= \frac{\overset{10}{\cancel{20}} \times 19 \times \cancel{18!}}{\cancel{18!} \times \cancel{2}}$$

$$= 190$$

Therefore, there are 190 handshakes that take place among 20 people.

(20) Total no. of friends = 8
no. of friends invited to the party = 5.

a) If 2 friends are feuding and will not attend together.

No. of choices if 5 friends out of 8 are attending without any constraint = $^8C_5$.

$$^nC_r = \frac{n!}{(n-r)!\,r!}$$

$$^8C_5 = \frac{8!}{(8-5)!\,5!} = \frac{8\times7\times6\times\cancel{5!}}{\cancel{3\times2}\times\cancel{5!}} = 56 \text{ choices}$$

If two friends are feuding, then we have to exclude them and it can be calculated as:

$$^8C_5 - {}^6C_3.$$

$$^6C_3 = \frac{6!}{(6-3)!\,3!} = \frac{\cancel{6}\times5\times4\times\cancel{3!}}{3\times\cancel{2}\times1\times\cancel{3!}} = 20 \cdot \text{choices}$$

$$^8C_5 - {}^6C_3 = 56 - 20 = 36.$$

therefore, there are 36 choices if 2 friends are feuding and will not attend together.

b) to calculate the no. of choices if 2 of the friends will only attend together.

no. of choices if 2 friends attend the party together is $^6C_3 = \frac{6!}{(6-3)!\,3!} = 20$ choices.

no. of choices where none of them attend the party is $^6C_5 = \frac{6!}{(6-5)!\,5!} = \frac{6 \times \cancel{5!}}{\cancel{5!}} = 6$ choices.

therefore, No. of choices if 2 of the friends will only attend together is $20+6 = \underline{\underline{26}}$ choices.

theoretical problem.

(10) a) Total no. of people $= n$.

No. of committee members to be selected $= k$.

Selecting 'k' committee members from 'n' people is ${}^{n}C_{k}$ or $\binom{n}{k}$ ways

Once, 'k' committee members are selected, the chairperson has to be selected from 'k' committee members and it can be done in $\binom{k}{1} = k$ ways

$\Rightarrow$ No. of possible choices for choosing a committee of $k \leq n$ is $\binom{n}{k}(k)$

b) Total no. of people $= n$.

No. of committee memberes to be selected $= k$.

So, no. of non-chair committee members to be selected $= k-1$

as '1' person from 'k' member will be the chairperson selecting $(k-1)$ non-committee members from 'n' people is $\binom{n}{k-1}$

Once $(k-1)$ non-committee members are selected, the chair person can be selected from the remaining is $n-(k-1) = n-k+1$ people

Therefore, No. of possible choices for choosing a committee of $k \leq n$ is $\binom{n}{k-1}(n-k+1)$

c) Out of 'n' people, chairperson can be selected in $\binom{n}{1} = n$ ways.

Once, chairperson is selected, $(k-1)$ non-chairperson who belong to the committee can be selected from remaining $(n-1)$ people in $\binom{n-1}{k-1}$ ways.

Therefore, No. of possible choices for choosing a committee of $k \leq n$ is $n\binom{n-1}{k-1}$.

Q) $\int_1^2 \frac{1}{x}\, dx$.

$= \left[\ln(x)\right]_1^2$

Substituting upper bound & lower bound.

$= \ln(2) - \ln(1)$

$= \ln(2) - 0$

$\Rightarrow \int_1^2 \frac{1}{x} dx = \ln(2)$

Q) $\iint_{\substack{x+y<1 \\ x>0,\, y>0}} (x+y)^2\, dx\, dy$

Since, $x+y<1 \Rightarrow y<1-x$

So, $0<x<1$ & $0<y<1-x$.

consider the limits of x as $0 \to 1$

and y as $0 \to 1-x$.

$\int_0^1 \int_0^{1-x} (x+y)^2\, dx\, dy$.

Solving $\int_0^{1-x} (x+y)^2\, dy$.

Integrate with respect to 'y' keeping 'x' constant

$$\int_0^{1-x} (x^2+y^2+2xy)\,dy = \left[x^2y + \frac{y^3}{3} + \frac{2xy^2}{2}\right]_0^{1-x}$$

$$= \left[x^2y + \frac{y^3}{3} + xy^2\right]_0^{1-x}$$

$$= x^2(1-x) + \frac{(1-x)^3}{3} + x(1-x)^2 - 0.$$

$$\int_0^{1-x} (x^2+y^2+2xy)\,dy = x^2(1-x) + \frac{(1-x)^3}{3} + x(1-x)^2$$

Now, Integrate with respect to 'x'.

Let's integrate each term separately & finally sum it.

$$\int_0^1 \left(x^2(1-x) + \frac{(1-x)^3}{3} + x(1-x)^2\right) dx =$$

$$\int_0^1 (x^2(1-x))\,dx + \frac{1}{3}\int_0^1 (1-x)^3\,dx + \int_0^1 x(1-x)^2\,dx.$$

$$\Rightarrow \int_0^1 (x^2(1-x))\,dx = \int_0^1 (x^2-x^3)\,dx$$

$$= \left[\frac{x^3}{3} - \frac{x^4}{4}\right]_0^1$$

$$= \frac{1}{3} - \frac{1}{4} - 0$$

$$= \frac{1}{12}.$$

$$\Rightarrow \frac{1}{3}\int_0^1 (1-x)^3 dx = \frac{1}{3}\int_0^1 (1^3 - 3x + 3x^2 - x^3)\,dx$$

$$= \frac{1}{3}\left[x - \frac{3x^2}{2} + \frac{\cancel{3}x^3}{\cancel{3}} - \frac{x^4}{4}\right]_0^1$$

$$= \frac{1}{3}\left[\left[1 - \frac{3}{2} + 1 - \frac{1}{4}\right] - 0\right]$$

$$= \frac{1}{3}\left[\frac{8-6-2}{4}\right]$$

$$= \frac{1}{12}$$

$$\Rightarrow \int_0^1 (x(1-x)^2)\,dx = \int_0^1 x(1 - 2x + x^2)\,dx$$

$$= \int_0^1 (x - 2x^2 + x^3)\,dx$$

$$= \left[\frac{x^2}{2} - \frac{2x^3}{3} + \frac{x^4}{4}\right]_0^1$$

$$= \left[\frac{1}{2} - \frac{2}{3} + \frac{1}{4}\right] - 0$$

$$= \frac{6-8+3}{12}$$

$$= \frac{1}{12}$$

Hence, $\frac{1}{12} + \frac{1}{12} + \frac{1}{12} = \frac{3}{12} = \frac{1}{4}$

$$\boxed{\int_0^1 \int_0^{1-x} (x+y)^2 \, dx \, dy = \frac{1}{4}}$$