EAS 596, Fall 2019, Homework 3
Due Friday 9/27, **12:00 PM**, Box outside Jarvis 326

Work all problems. Show all work, including any M-files you have written or adapted. Make sure your work is clear and readable - if the TA cannot read what you've written, that work will not be graded. All electronic work (m-files, etc.) **must** be submitted through UBLearns by the due time shown above. Electronic files must obey the following naming convention: `ubitname_hw3_p1.m`, replacing ubitname with your ubitname and with the appropriate problem number. Any handwritten work may be submitted in class. Each problem will be graded according to the following scheme:

- 2 Points: Solution is complete and correct,
- 1 Points: Solution is incorrect or incomplete but was using the correct ideas,
- 0 Points: Using incorrect ideas.

1. Consider each of the following linear system of equations. For each, write the system as an augmented matrix, determine the reduced row echlon form (you can use `rref` in MATLAB), and determine the solution for x_0, x_1, x_2, and x_3. If an infinite number of solutions exist, state that and one possible solution. If no solutions exist, state that. Each part is worth 2 pts.

   (a)
$$\begin{aligned} 2x_0 + 4x_1 + x_2 + 6x_3 &= 7 \\ -2x_0 + 2x_2 + x_3 &= 12 \\ -2x_0 + 6x_1 + 2x_2 + x_3 &= 0 \\ -8x_0 - 2x_1 + x_2 + x_3 &= -11 \end{aligned}$$

   (b)
$$\begin{aligned} 2x_0 + 4x_1 + x_2 + 6x_3 &= 7 \\ -2x_0 + 2x_2 + x_3 &= 12 \\ -2x_0 + 6x_1 + 2x_2 + x_3 &= 0 \\ 14x_0 - 14x_1 - 11x_2 &= -29 \end{aligned}$$

   (c)
$$\begin{aligned} 2x_0 + 4x_1 + x_2 + 6x_3 &= 7 \\ -2x_0 + 2x_2 + x_3 &= 12 \\ -2x_0 + 6x_1 + 2x_2 + x_3 &= 0 \\ 14x_0 - 14x_1 - 11x_2 &= 0 \end{aligned}$$

2. Write a matlab function that determines the minimum path length to connect two points ($i \to j$) in a graph. The function header must be the following: `function [length] = ubitname_hw3_p2(A, i, j)`, where `A` is the adjacency matrix, `i` is the departure point and `j` is the goal point, and `length` is the minimum path length. If the path length is larger than 20, return an error using the `error` function. *Make sure that you test your program!*

3. Consider the graphs shown in Figure 1.

   (a) (2 pts) For each of the graphs, give the corresponding adjacency matrix.

   (b) (2 pts) For Graph G_1, how many paths of length 3 are there between points P_1 and P_3? You must justify your answer via operations on the adjacency matrix.

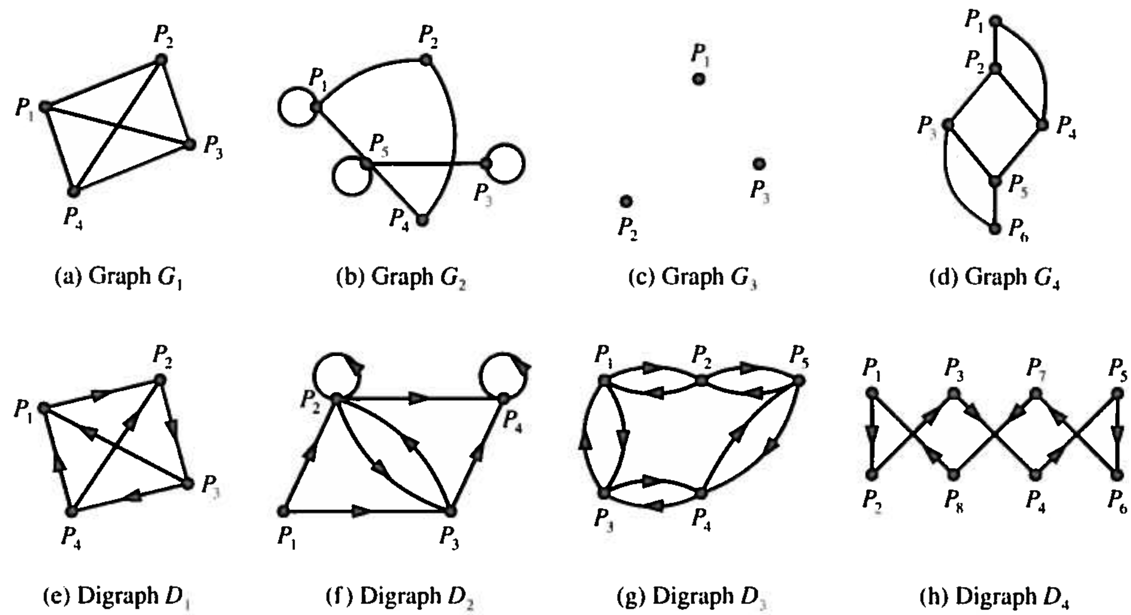

Figure 1: Graphs for Problem 3.

4. Which of the following matrices could be the adjacency matrix for a simple graph or digraph? Draw the corresponding graph and/or digraph when appropriate.

(a) $\mathbf{A} = \begin{bmatrix} -1 & 4 \\ 0 & 1 \\ 6 & 0 \end{bmatrix}$

(b) $\mathbf{B} = \begin{bmatrix} 2 & 0 \\ 0 & -1 \end{bmatrix}$

(c) $\mathbf{C} = \begin{bmatrix} 6 & 0 & 0 & 0 \\ 0 & 6 & 0 & 0 \\ 0 & 0 & 6 & 0 \\ 0 & 0 & 0 & 6 \end{bmatrix}$

(d) $\mathbf{D} = \begin{bmatrix} -1 \\ 4 \\ 2 \end{bmatrix}$

(e) $\mathbf{E} = \begin{bmatrix} 0 & 0 & 0 & 6 \\ 0 & 0 & 6 & 0 \\ 0 & -6 & 0 & 0 \\ -6 & 0 & 0 & 0 \end{bmatrix}$

(f) $\mathbf{F} = \begin{bmatrix} 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 1 \end{bmatrix}$

(g) $\mathbf{G} = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$

(h) $\mathbf{H} = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix}$

(i) $\mathbf{I} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

(j) $\mathbf{J} = \begin{bmatrix} 1 & 2 & 3 & 4 \\ -2 & 1 & 5 & 6 \\ -3 & -5 & 1 & 7 \\ -4 & -6 & -7 & 1 \end{bmatrix}$

(k) $\mathbf{K} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

(l) $\mathbf{L} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}$

(m) $\mathbf{M} = \begin{bmatrix} -2 & 0 & 0 \\ 4 & 0 & 0 \\ -1 & 2 & 3 \end{bmatrix}$

5. Suppose that each of the following represents the transition matrix $\mathbf{M}$ and the initial probability vector $\mathbf{p}$ for a Markov chain. Find the probability vectors $\mathbf{p}_1$ and $\mathbf{p}_2$. If you use MATLAB to perform the matrix-vector multiplication you must state so.

   (a) $\mathbf{M} = \begin{bmatrix} \frac{1}{4} & \frac{1}{3} \\ \frac{3}{4} & \frac{2}{3} \end{bmatrix}$, $\mathbf{p} = \begin{bmatrix} \frac{2}{3} \\ \frac{1}{3} \end{bmatrix}$

   (b) $\mathbf{M} = \begin{bmatrix} \frac{1}{2} & \frac{1}{3} & 0 \\ 0 & \frac{2}{3} & \frac{1}{2} \\ \frac{1}{2} & 0 & \frac{1}{2} \end{bmatrix}$, $\mathbf{p} = \begin{bmatrix} \frac{1}{3} \\ \frac{1}{6} \\ \frac{1}{2} \end{bmatrix}$

   (c) $\mathbf{M} = \begin{bmatrix} \frac{1}{4} & \frac{1}{3} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{3} & \frac{1}{6} \\ \frac{1}{4} & \frac{1}{3} & \frac{1}{3} \end{bmatrix}$, $\mathbf{p} = \begin{bmatrix} \frac{1}{4} \\ \frac{1}{2} \\ \frac{1}{4} \end{bmatrix}$

6. Suppose that citizens in a certain community tend to switch their votes among political parties, as shown in the following transition matrix. The ordering is Party A, Party B, Party C, and Nonvoting. Hint: Don't do this by hand!

$$\begin{bmatrix} 0.7 & 0.2 & 0.2 & 0.1 \\ 0.1 & 0.6 & 0.1 & 0.1 \\ 0.1 & 0.2 & 0.6 & 0.1 \\ 0.1 & 0 & 0.1 & 0.7 \end{bmatrix}$$

   (a) Suppose that in the last election 30% of the citizens voted for Party A, 15% voted for Part B, and 45% voted for Party C. What is the likely outcome of the next election? What is the likely outcome of the election after that?

   (b) If current trends continue, what percentage of the citizens will vote for Party A one century from now? Party C?

7. Which of the following subsets of $\mathbb{R}^3$ are subspaces?

   (a) All vectors of the form $[a, b, a]$.

   (b) All linear combinations of $[1, 4, 0]$ and $[2, 2, 2]$

   (c) All vectors of the form $[a, b, c]$ where $a \leq b \leq c$.

8. What is the smallest set that will span all 3×3 upper triangular matrices?

9. Let the following vectors be defined.

$$\boldsymbol{a} = \begin{bmatrix} 4 \\ 1 \\ 5 \end{bmatrix}, \quad \boldsymbol{b} = \begin{bmatrix} 3 \\ 4 \\ 5 \\ -5 \\ 4 \end{bmatrix}, \quad \boldsymbol{c} = \begin{bmatrix} -4 \\ 6 \\ -13 \\ 8 \\ -19 \end{bmatrix}, \quad \boldsymbol{d} = \begin{bmatrix} -8 \\ -7 \\ -5 \end{bmatrix}, \quad \boldsymbol{e} = \begin{bmatrix} 16 \\ 9 \\ 15 \end{bmatrix}$$

Answer the following questions regarding these vectors:

(a) $\text{span}(\{\boldsymbol{d}, \boldsymbol{e}\})$ is a subset of what vector space?

(b) Are the vectors $\boldsymbol{b}$ and $\boldsymbol{c}$ sufficient to span all of $\mathbb{R}^5$?

(c) Are the vectors $\boldsymbol{a}$, $\boldsymbol{d}$, $\boldsymbol{e}$ sufficient to span all of $\mathbb{R}^3$?

(d) Write three vectors which span a subspace of $\mathbb{R}^4$