REPORT FOR MATHEMATICS ASSESSMENT TASK

Error of Estimation

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Table of Contents

1. Introduction

1.1 Interested Topics:

- Human Error of Estimation (Chosen)
- Skipping Stones
- Angle of Computer and Reading Speed

1.2 Justification:

Research has shown that one of the main reasons why humans procrastinate is the fact that we tend to underestimate how long it will take you to complete the task at hand, and you also underestimate how quickly you'll get it done. Experts have been suggesting people to break down the large work into smaller subtasks and reevaluate the time of each subtask as our estimations will become more accurate.

Why do these experts recommend this particular tip? Does the size of work really affect how we estimate the time needed?

1.3 Aim:

To prove the technique by finding out whether the reduction of sizes of input (i.e. the reduction of time intervals) increases the accuracy of estimations (i.e. percentage errors of time estimations).

2. Content Knowledge

2.1 Percentages

Percentage Error: $\%\ \text{Error} = \dfrac{\text{Experimental} - \text{Theoretical}}{|\text{Theoretical}|} \times 100$

(More explanation about error and percentage error is in **3.2 Experimental Design** under **Operational Definitions**)

2.2 Statistics

2.2.1 Arithmetic Mean:

The arithmetic mean, also called the average, is the sum of the values divided by the number of values.

The formula for mean is $\bar{x} = \frac{1}{n}\left(\sum_{i=1}^{n} x_i\right) = \frac{x_1 + x_2 + \cdots + x_n}{n}$, where $\{x_1, x_2, \ldots, x_N\}$ are the observed values of the sample items, and $\bar{x}$ is the mean value of these observations.

2.2.2 Standard Deviation:

The standard deviation (SD) is a measure that is used to calculate the amount of variation of a data set. A low standard deviation indicates that the data points tend to be close to the mean of the set, while a high standard deviation indicates that the data points are spread out over a wider range of values.

The formula for SD is $s = \sqrt{\frac{1}{N-1}\sum_{i=1}^{N}(x_i - \bar{x})^2}$, where $\{x_1, x_2, \ldots, x_N\}$ are the observed values of the sample items, $\bar{x}$ is the mean value of these observations, and N is the number of observations in the sample.

2.3 Graphs

2.3.1 Line charts:

A line chart is a type of chart which displays information as a series of data points called 'markers' connected by straight line segments.

2.3.2 Histograms:

A histogram is an accurate representation of the distribution of numerical data. Its purpose is to roughly assess the probability distribution of a given variable by showing the frequencies of observations occurring in certain ranges of values.

3. Experiment

3.1 Hypothesis: Humans tend to have more accurate estimations with smaller sizes of inputs. The percentage error of time estimation decreases as the time interval decreases from 5 seconds to 3 seconds and 1 second.

3.2 Experiment Design:

Variables:

Independent variable: Time intervals

Dependent variable: Estimations of time

Controlled variable: Stopwatch, Number of estimations for each time interval

Equipment:

1. Stopwatch
2. Piece of opaque cloth

Procedures:

1. Put a piece of cloth to cover the face of the stopwatch, not allowing the participant to see the actual time
2. Ask the participant to press start
3. Ask the participant to press the “lap” button every 5 estimated seconds for 5 times
4. Record the data into a table
5. Reset the stopwatch
6. Repeat Step 2-5 but press the “lap” button every 3 estimated seconds instead of 5 seconds
7. Repeat Step 2-5 but press the “lap” button every 1 estimated second instead of 5 seconds
8. Repeat Step 1-7 with 49 other participants
9. Observe and analyze the data collected

Operational Definitions:

1. Time is measured in the SI unit of seconds

2. Error is defined as the difference between the time estimated by the pressing of the participant and the correct time.

   For instance, Person 1 is estimating in 5-second intervals for 5 times. His estimations are according to Fig 1-1.

Person	5s-1 Time (s)	5s-2 Time (s)	5s-3 Time (s)	5s-4 Time (s)	5s-5 Time (s)
1	5.19	10.57	14.83	20.27	25.31

Fig 1-1 shows an example of time estimations of Person 1

Under the heading of "5s-1 Time (s)" shows Person 1's estimated time of his first press of the 5-second interval which is 5.19 seconds. Under the heading of "5s-2 Time (s)" shows his estimated time of his second press of the 5-second interval which is 10.57.

The error of his second press of the 5-second interval is calculated as: $Error = Time\ Estimated - Correct\ Time$

$= 10.57\text{s} - 10\text{s}$

$= +0.57$ seconds

His percentage error is calculated as:

$$\%\ \text{Error} = \frac{Time\ Error}{Correct\ Time} \times 100$$

$$= \frac{+0.57\text{s}}{10\text{s}} \times 100$$

$$= +5.7\%$$

Hence, Figure 3-2 (appears in Appendix), the column of "% Error" is calculated by using the formula:

$$\%\ Error = \frac{Time\ Estimated - (Intervals \times Pressing\ Round)}{(Intervals \times Pressing\ Round)} \times 100.$$

3.3 Observations and Analysis:

3.3.1 Observations:

Person	1s-1 Time (s)	1s-2 Time (s)	1s-3 Time (s)	1s-4 Time (s)	1s-5 Time (s)	3s-1 Time (s)	3s-2 Time (s)	3s-3 Time (s)	3s-4 Time (s)	3s-5 Time (s)	5s-1 Time (s)	5s-2 Time (s)	5s-3 Time (s)	5s-4 Time (s)	5s-5 Time (s)
1	1.11	2.13	3.29	4.53	6.04	2.69	5.35	8.00	9.99	13.62	5.19	10.57	14.83	20.27	25.31
2	0.86	1.91	2.23	3.32	3.89	2.48	5.03	7.50	9.98	12.67	4.71	9.38	13.01	18.33	22.59
3	1.01	2.14	3.08	3.79	5.04	2.89	5.81	8.61	11.82	14.33	3.95	7.27	11.18	16.15	19.19
4	0.77	1.77	2.39	2.99	3.85	3.11	6.14	9.35	12.38	15.35	3.55	6.81	10.25	13.95	17.57
5	0.94	1.87	2.63	3.61	4.31	2.47	4.85	7.50	9.57	11.95	2.62	5.12	7.31	10.39	12.27
6	1.04	2.18	3.11	3.94	5.61	2.43	5.29	6.96	9.14	11.45	4.48	9.09	13.34	18.24	21.65
7	1.08	2.13	3.59	4.53	5.31	2.85	5.76	8.43	11.56	14.20	5.50	10.65	16.04	21.27	26.70
8	1.05	2.00	3.17	4.38	5.36	2.22	4.55	7.01	8.27	10.80	2.52	4.86	7.39	9.41	11.28
9	0.92	1.87	3.06	3.73	4.79	2.35	4.70	6.83	9.17	11.23	3.61	6.86	10.55	14.08	17.02
10	0.98	1.96	2.94	4.01	5.38	3.00	6.32	8.54	11.87	14.38	4.08	8.26	11.35	16.09	18.83
...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
41	0.97	1.93	2.74	4.04	4.96	2.60	5.16	8.05	9.91	12.30	4.00	8.38	11.44	15.59	18.60
42	0.87	1.92	2.90	3.11	4.46	2.52	5.37	7.43	9.47	12.52	3.40	6.12	9.66	13.74	15.48
43	1.03	2.32	2.72	4.15	5.57	2.82	5.73	8.84	10.98	14.24	3.67	7.14	10.26	14.31	17.28
44	0.83	1.60	2.53	3.61	4.21	2.23	4.41	7.06	8.99	10.54	3.63	7.10	10.12	14.56	17.65
45	1.02	2.00	3.12	4.11	5.15	3.03	5.92	8.78	12.48	15.14	6.26	12.18	17.86	24.77	30.74
46	1.05	1.97	3.50	4.42	5.27	3.33	6.53	10.19	13.53	16.10	2.24	4.21	6.40	9.19	10.06
47	0.91	1.71	2.93	4.07	4.38	3.79	7.26	11.62	14.89	18.64	4.82	9.09	14.30	18.97	23.02
48	1.08	2.38	3.10	4.30	4.93	3.26	6.71	9.86	13.23	16.36	4.48	9.23	13.00	17.88	21.31
49	0.81	1.78	2.42	2.97	4.10	1.63	3.41	4.49	6.55	7.88	4.28	8.81	11.97	16.80	20.53
50	1.08	2.26	3.39	4.07	5.60	2.27	4.45	6.35	8.85	11.51	3.66	7.05	11.09	13.92	17.05

*Figure 2-1 shows the table containing each participant's time of each round of pressing of each interval (See **7. Appendix: Data Collected** for the full "raw" table)*

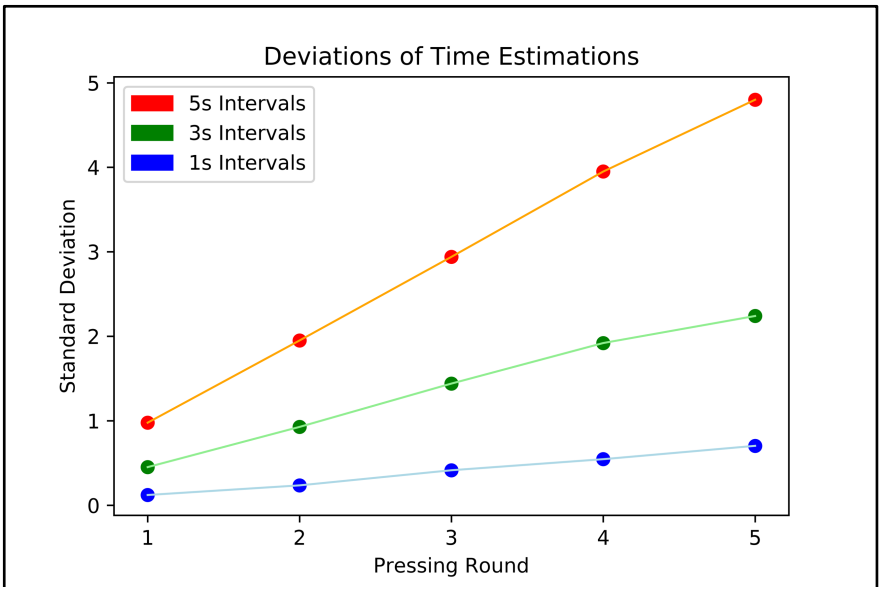

Figure 2-2 shows a line chart of the standard deviation of each round of pressing of each interval

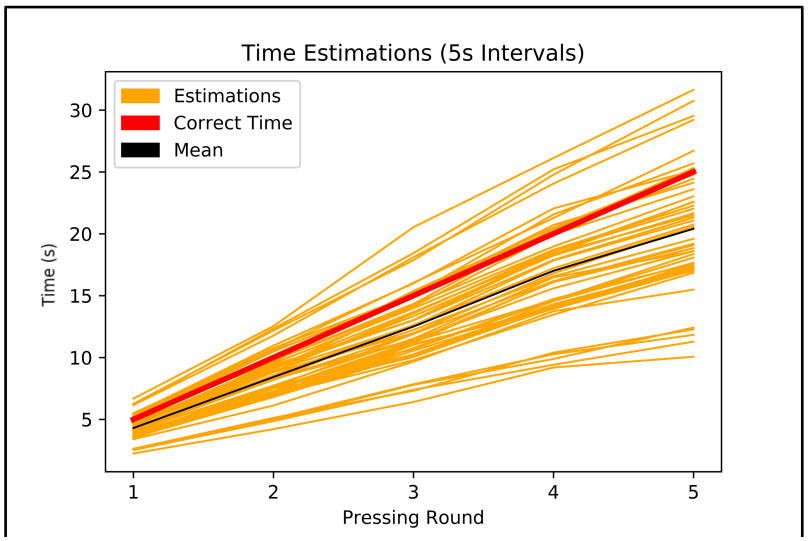

Figure 2-3 shows a line chart of each participant's estimated time of 5-second intervals

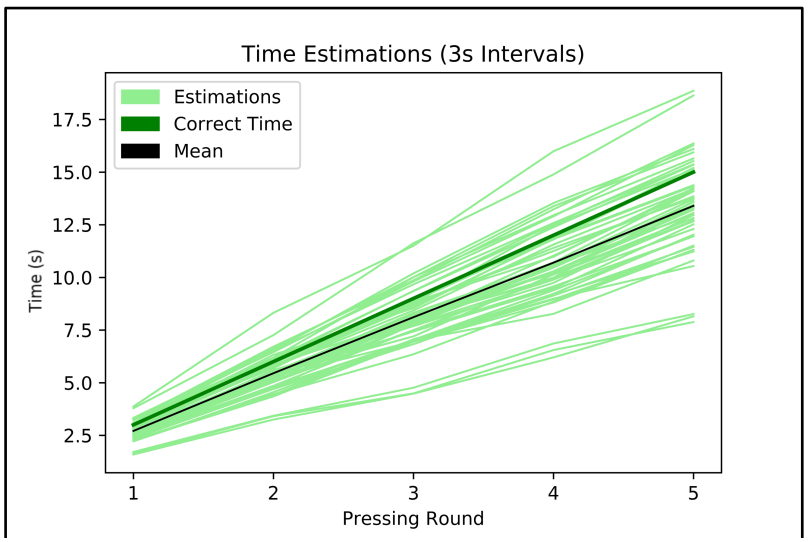

Figure 2-4 shows a line chart of each participant's estimated time of 3-second intervals

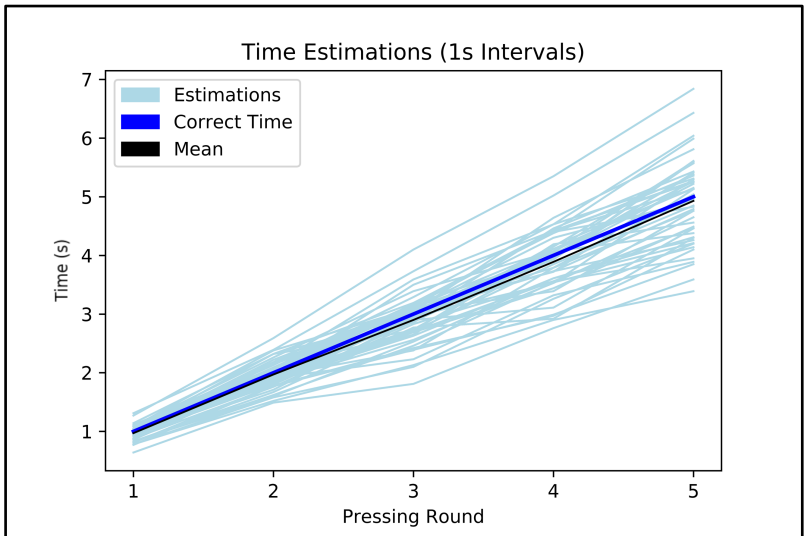

Figure 2-5 shows a line chart of each participant's estimated time of 1-second intervals

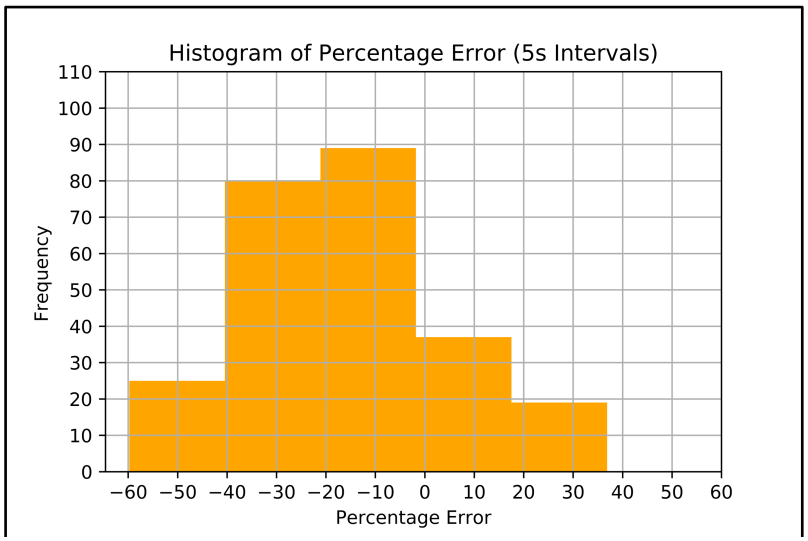

Figure 2-6 shows a histogram of percentage error of the estimated time of 5-second intervals

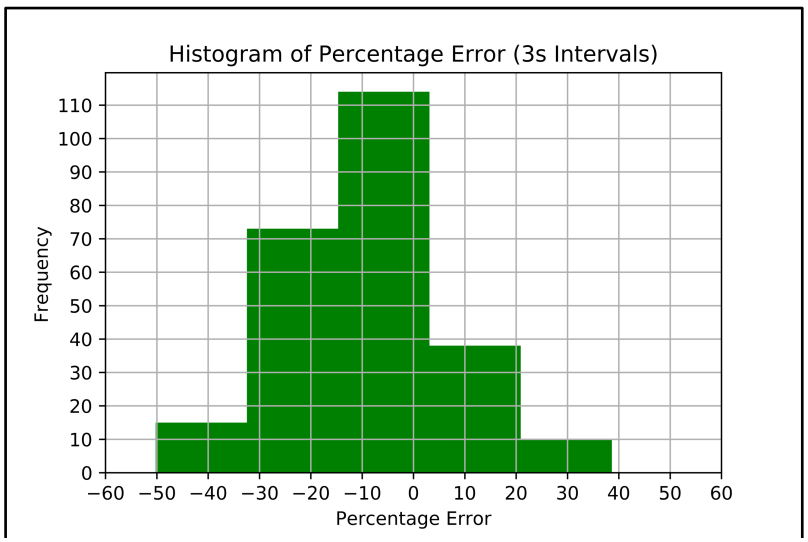

Figure 2-7 shows a histogram of percentage error of the estimated time of 3-second intervals

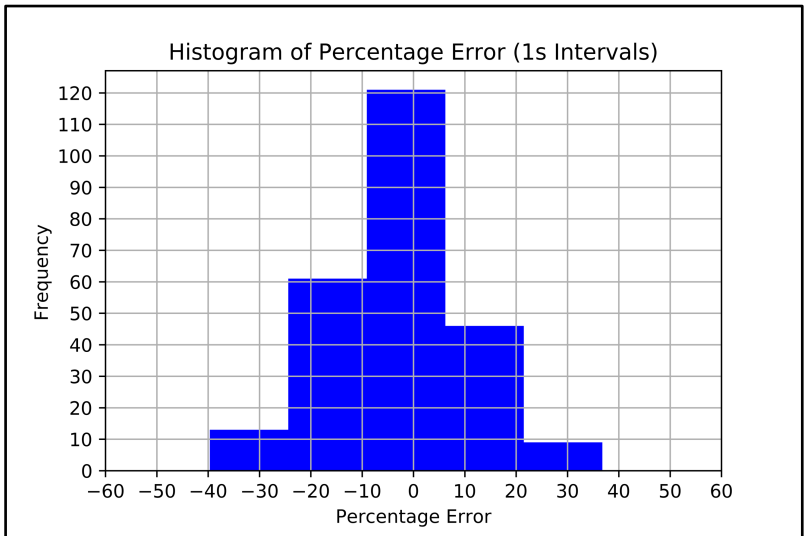

Figure 2-8 shows a histogram of percentage error of the estimated time of 1-second intervals

3.3.2 Analysis:

According to *Figure 2-3* to *Figure 2-5*, the mean moving closer to the correct time as the time interval decreases, showing that as the time interval decreases, the more accurate the estimations will be. *Figure 2-6* to *Figure 2-8* also reveals that as the time interval gets smaller, the number of participants who estimated the time with less than 10% error increases while the number of participants who estimated the time with more than 40% error decreases. This supports the fact that the accuracy of estimation improves as the input size gets smaller.

Moreover, *Figure 2-2* shows that the standard deviation is directly proportional to the time interval. It suggests that the smaller the input size is, the more precise (or the less deviate) the estimations will be.

4. Reflection

Firstly, I have learnt that Mathematics is not only about learning theories and doing proofs on pieces of papers but rather is a tool that helps us to understand how things work in the real world. Mathematics allows humanity to explore many topics deeply as we try to understand them. For example, Mathematics allows us to be able to understand Big Data by providing us with a logical and visual representation of graphs.

Moreover, I have learnt to persevere, be ambitious, and always stay positive. Before I started collecting data, I had set a goal for myself to collect data from 50 people. As 30 minutes passed by waiting for people in front of an elevator, I started to feel exhausted and was about to give up since I only collected data from 10 people. However, I knew I had to finish what I started. I was patient and persevere as I waited for people. At the end of the day, I finally finished collecting the data from 50 people and I felt accomplished. I knew if I had quit collecting data at that particular moment, my insights would not be as accurate as if I had collected data from 50 people.

Lastly, I have learnt to have appreciation for everyone. I learnt to appreciate the participants as they sacrifice their time to help me. I believe that showing appreciation by smiling and saying thank you helps people to feel happier and feel that they have gracefully helped someone. By doing this, I also received some smiles back from some participants which also motivated me to not give up on waiting for people to pass by.

5. Conclusion

Before I did this project, I procrastinated quite a lot. Sometimes I plan to study but I end up watching YouTube for more than an hour. Thus, I wanted to find a technique that would help me procrastinate less.

At the end of the day, I have gained a lot of things from doing this project. I have learnt from the experiment that as input size decreases, the accuracy of estimation increases as well as the precision of estimation. This proves why experts recommend people to break big tasks into smaller subtasks. By evaluating smaller subtasks, humans tend to be more accurate with the time they estimated. Hence, productivity will increase as a result of accurate time estimation. I have personally tried the following technique to help me fight against my procrastination and it actually helps me to complete more tasks and be more productive.

This project does not only give me motivation and some excitement for studying math since this project brings math to the real world but also help me to reflect some of the values that are important in life such as perseverance, appreciation, and tenacity.

6. References

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7. Appendix: Data Collected

Person	1s-1 Time (s)	1s-2 Time (s)	1s-3 Time (s)	1s-4 Time (s)	1s-5 Time (s)	3s-1 Time (s)	3s-2 Time (s)	3s-3 Time (s)	3s-4 Time (s)	3s-5 Time (s)	5s-1 Time (s)	5s-2 Time (s)	5s-3 Time (s)	5s-4 Time (s)	5s-5 Time (s)
1	1.11	2.13	3.29	4.53	6.04	2.69	5.35	8.00	9.99	13.62	5.19	10.57	14.83	20.27	25.31
2	0.86	1.91	2.23	3.32	3.89	2.48	5.03	7.50	9.98	12.67	4.71	9.38	13.01	18.33	22.59
3	1.01	2.14	3.08	3.79	5.04	2.89	5.81	8.61	11.82	14.33	3.95	7.27	11.18	16.15	19.19
4	0.77	1.77	2.39	2.99	3.85	3.11	6.14	9.35	12.38	15.35	3.55	6.81	10.25	13.95	17.57
5	0.94	1.87	2.63	3.61	4.31	2.47	4.85	7.50	9.57	11.95	2.62	5.12	7.31	10.39	12.27
6	1.04	2.18	3.11	3.94	5.61	2.43	5.29	6.96	9.14	11.45	4.48	9.09	13.34	18.24	21.65
7	1.08	2.13	3.59	4.53	5.31	2.85	5.76	8.43	11.56	14.20	5.50	10.65	16.04	21.27	26.70
8	1.05	2.00	3.17	4.38	5.36	2.22	4.55	7.01	8.27	10.80	2.52	4.86	7.39	9.41	11.28
9	0.92	1.87	3.06	3.73	4.79	2.35	4.70	6.83	9.17	11.23	3.61	6.86	10.55	14.08	17.02
10	0.98	1.96	2.94	4.01	5.38	3.00	6.32	8.54	11.87	14.38	4.08	8.26	11.35	16.09	18.83
11	0.92	1.95	3.01	3.73	4.26	2.63	5.59	7.69	9.74	12.96	4.49	8.93	12.56	17.84	21.07
12	0.88	1.93	2.71	3.85	4.85	2.98	6.05	8.87	11.89	15.20	4.38	8.26	12.63	17.01	20.55
13	0.83	1.62	2.46	3.54	4.14	2.57	5.07	7.74	10.57	12.96	6.15	12.33	18.43	25.20	29.52
14	0.96	1.82	2.96	3.53	5.13	2.99	5.68	8.98	11.80	15.07	6.16	11.77	18.10	24.05	29.20
15	0.94	1.84	2.70	3.56	4.47	3.87	8.32	11.50	15.99	18.86	4.07	7.73	11.52	16.47	18.58
16	0.98	1.94	2.65	4.19	4.56	2.74	5.53	8.16	10.49	13.58	2.59	4.95	7.79	9.88	12.42
17	0.93	1.83	3.04	3.39	4.76	1.59	3.25	4.48	6.20	8.15	4.67	9.51	13.98	18.70	22.27
18	1.02	2.22	2.86	4.17	5.14	2.22	4.43	6.99	8.96	11.26	4.61	9.21	13.67	18.39	21.48
19	0.98	2.08	2.69	3.95	5.41	2.61	5.05	7.92	10.27	13.22	4.09	8.19	11.90	16.16	19.07
20	1.03	2.32	2.96	3.93	5.59	2.40	4.81	7.02	9.22	12.03	4.31	8.56	12.63	17.21	20.74
21	0.87	1.94	2.38	3.79	4.76	2.90	5.80	8.71	11.40	13.77	5.12	10.48	15.07	20.71	24.12
22	1.03	1.98	2.94	4.46	5.43	2.71	5.52	8.00	10.57	13.35	3.83	7.49	11.29	14.64	18.36
23	1.04	2.05	3.17	3.98	5.22	3.10	6.51	9.67	12.58	15.37	4.20	8.51	12.23	16.56	19.59
24	0.88	1.76	2.83	3.56	3.95	2.60	5.15	8.21	10.17	12.82	6.68	12.53	20.53	26.12	31.63
25	1.14	2.32	3.19	4.42	5.99	2.85	6.13	8.79	11.25	13.85	3.51	6.97	9.78	13.46	16.85
26	1.27	2.59	4.10	5.35	6.84	2.56	5.39	7.20	10.14	12.69	2.63	5.07	7.84	10.26	11.83
27	0.95	1.86	2.56	3.53	4.65	2.26	4.35	6.95	8.81	11.31	4.09	7.61	11.90	16.17	19.10
28	1.02	2.22	3.14	4.42	5.25	3.28	6.45	9.81	12.90	16.28	4.89	9.78	14.12	19.94	23.60
29	1.02	1.97	3.16	4.40	4.83	1.71	3.43	4.76	6.86	8.26	4.65	9.15	13.03	18.54	22.56
30	1.01	2.01	3.09	3.72	5.30	2.77	5.85	8.06	11.03	14.13	3.68	6.83	10.77	14.73	17.60
31	1.04	2.09	3.17	4.41	5.13	3.11	6.06	9.36	12.54	15.53	5.26	9.91	15.27	20.27	24.44
32	1.02	1.96	3.05	4.07	5.26	2.61	5.28	7.82	10.26	13.06	5.06	9.78	14.33	20.37	24.79
33	0.94	1.88	2.69	3.44	4.49	2.69	5.46	8.32	10.62	13.72	3.65	6.80	11.03	14.04	17.18
34	0.78	1.52	2.13	2.90	3.39	3.23	6.61	10.01	13.38	15.94	4.63	8.87	13.56	18.62	22.02
35	0.64	1.49	1.81	2.76	3.59	3.23	6.19	10.01	12.95	15.65	5.34	10.76	15.09	21.56	25.68
36	1.09	2.16	3.04	4.64	5.81	2.67	5.47	7.95	10.46	13.45	5.43	10.96	15.99	22.04	25.12
37	0.78	1.58	2.10	3.26	4.20	2.78	5.50	8.26	10.71	14.09	5.21	9.62	15.38	20.49	25.02
38	0.94	1.80	2.87	3.99	4.97	2.70	5.22	7.99	10.29	13.30	3.78	7.62	10.70	14.63	18.07
39	1.31	2.39	3.73	5.02	6.43	2.42	4.73	7.04	9.39	11.96	3.57	7.29	10.04	14.03	17.38
40	0.83	1.67	2.77	2.92	4.30	2.54	4.77	7.77	9.89	12.72	3.60	7.49	10.24	14.23	17.41
41	0.97	1.93	2.74	4.04	4.96	2.60	5.16	8.05	9.91	12.30	4.00	8.38	11.44	15.59	18.60
42	0.87	1.92	2.90	3.11	4.46	2.52	5.37	7.43	9.47	12.52	3.40	6.12	9.66	13.74	15.48
43	1.03	2.32	2.72	4.15	5.57	2.82	5.73	8.84	10.98	14.24	3.67	7.14	10.26	14.31	17.28
44	0.83	1.60	2.53	3.61	4.21	2.23	4.41	7.06	8.99	10.54	3.63	7.10	10.12	14.56	17.65
45	1.02	2.00	3.12	4.11	5.15	3.03	5.92	8.78	12.48	15.14	6.26	12.18	17.86	24.77	30.74
46	1.05	1.97	3.50	4.42	5.27	3.33	6.53	10.19	13.53	16.10	2.24	4.21	6.40	9.19	10.06
47	0.91	1.71	2.93	4.07	4.38	3.79	7.26	11.62	14.89	18.64	4.82	9.09	14.30	18.97	23.02
48	1.08	2.38	3.10	4.30	4.93	3.26	6.71	9.86	13.23	16.36	4.48	9.23	13.00	17.88	21.31
49	0.81	1.78	2.42	2.97	4.10	1.63	3.41	4.49	6.55	7.88	4.28	8.81	11.97	16.80	20.53
50	1.08	2.26	3.39	4.07	5.60	2.27	4.45	6.35	8.85	11.51	3.66	7.05	11.09	13.92	17.05

Figure 3-1 shows the "raw" table containing each participant's time of each round of pressing of each interval

	Person	Intervals (s)	Pressing Round	Time (s)	% Error
0	1	1	1	1.11	11.00
1	1	1	2	2.13	6.50
2	1	1	3	3.29	9.67
3	1	1	4	4.53	13.25
4	1	1	5	6.04	20.80
5	1	3	1	2.69	-10.33
6	1	3	2	5.35	-10.83
7	1	3	3	8.00	-11.11
8	1	3	4	9.99	-16.75
9	1	3	5	13.62	-9.20
...	...	...	...	...	...
740	50	3	1	2.27	-24.33
741	50	3	2	4.45	-25.83
742	50	3	3	6.35	-29.44
743	50	3	4	8.85	-26.25
744	50	3	5	11.51	-23.27
745	50	5	1	3.66	-26.80
746	50	5	2	7.05	-29.50
747	50	5	3	11.09	-26.07
748	50	5	4	13.92	-30.40
749	50	5	5	17.05	-31.80

Figure 3-2 shows the "clean" table containing each participant's time of each round of pressing of each interval with the percentage error of each time

	Interval (s)	Round of Pressing	Min (s)	Max (s)	Mean (s)	Standard Deviation
0	1	1	0.64	1.31	0.97	0.121
1	1	2	1.49	2.59	1.97	0.235
2	1	3	1.81	4.10	2.90	0.414
3	1	4	2.76	5.35	3.89	0.545
4	1	5	3.39	6.84	4.93	0.703
5	3	1	1.59	3.87	2.71	0.451
6	3	2	3.25	8.32	5.45	0.927
7	3	3	4.48	11.62	8.11	1.440
8	3	4	6.20	15.99	10.70	1.920
9	3	5	7.88	18.86	13.40	2.240
10	5	1	2.24	6.68	4.30	0.977
11	5	2	4.21	12.53	8.42	1.950
12	5	3	6.40	20.53	12.50	2.940
13	5	4	9.19	26.12	17.00	3.950
14	5	5	10.06	31.63	20.40	4.800

Figure 3-3 shows the summary of statistics of each round of pressing of each interval