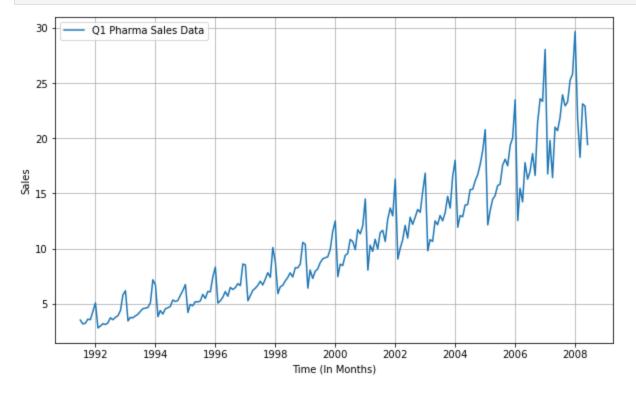
```
In [1]: import pandas as pd
         import numpy as np
         import matplotlib.pyplot as plt
         import statsmodels.api as sm
         from statsmodels.graphics.tsaplots import plot acf, plot pacf
         from datetime import datetime, timedelta
         from statsmodels.tsa.arima.model import ARIMA
         import warnings
         warnings.filterwarnings('ignore')
In [2]: # Loading the data and first look at the data
        pharma data = pd.read csv('PharmaDrugSales 1.csv')
        pharma data.head()
Out[2]:
             Month
                       Sales
            7/1/1991 3.526591
         1 8/1/1991
                    3.180891
                    3.252221
            9/1/1991
         3 10/1/1991 3.611003
         4 11/1/1991 3.565869
In [3]: pharma data.dtypes
Out[3]: Month
                 object
        Sales
                float64
        dtype: object
In [4]: def convert to datetime (date str):
             return pd.to datetime(date str, format='%m/%d/%Y')
In [5]: | pharma data['Month'] = pharma data['Month'].apply(convert to datetime)
        pharma data.dtypes
        Month datetime64[ns]
Out [5]:
        Sales
                        float64
        dtype: object
In [6]: pharma_data.head()
Out[6]:
               Month
                         Sales
         0 1991-07-01 3.526591
         1 1991-08-01 3.180891
         2 1991-09-01 3.252221
         3 1991-10-01 3.611003
         4 1991-11-01 3.565869
```

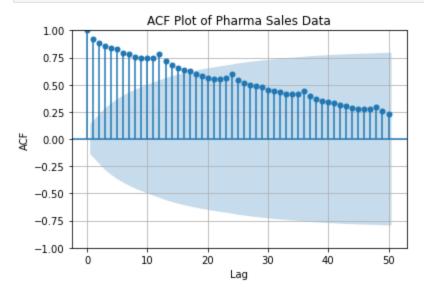
Question 1

```
In [7]: # PART 1:
    # Plotting the original time series data of sales
    plt.figure(figsize=(10,6))
    plt.plot(pharma_data['Month'], pharma_data['Sales'], label='Q1 Pharma Sales Data')
    plt.xlabel('Time (In Months)')
    plt.ylabel('Sales')
    plt.legend()
    plt.grid(True)
    plt.show()
```



2)

```
In [8]: # PART 2:
    # Plotting the ACF and finding the seasonal period from the plot
    plot_acf(pharma_data['Sales'], lags=50)
    plt.title('ACF Plot of Pharma Sales Data')
    plt.xlabel('Lag')
    plt.ylabel('ACF')
    plt.grid(True)
    plt.show()
```



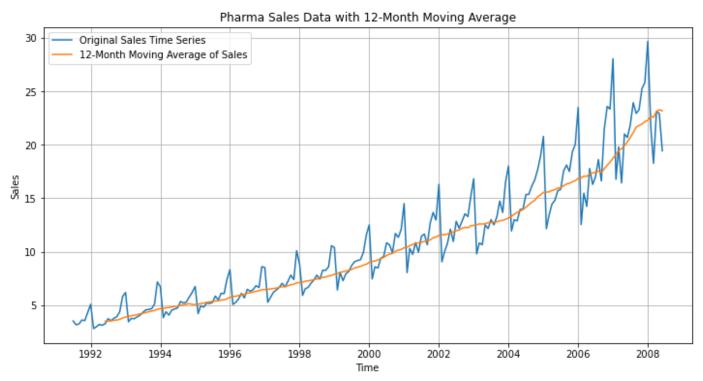
We can see from the ACF plot that the ACF is slowly decaying and much of the lags are significant which confirms that the time series of we're working with is non-stationary. This can also be seen from the plot of the data, there is a clear trend and seasonal component to it so Sales data is non-stationary. From the ACF plot, the seasonality seems to be annual, so, seasonal period = 12 months.

3)

```
In [9]: # PART 3:
    # Computing moving average of the data to determine the trend and overlaying with origin
    window_len = 12

# Calculating 12 month moving average using rolling mean
    pharma_data['Moving Average'] = pharma_data['Sales'].rolling(window=window_len).mean()

# Plotting the original sales data and the 12-month moving average data
    plt.figure(figsize=(12,6))
    plt.plot(pharma_data['Month'], pharma_data['Sales'], label='Original Sales Time Series')
    plt.plot(pharma_data['Month'], pharma_data['Moving Average'], label=f'12-Month Moving Av
    plt.xlabel('Time')
    plt.ylabel('Sales')
    plt.title('Pharma Sales Data with 12-Month Moving Average')
    plt.legend()
    plt.grid(True)
    plt.show()
```



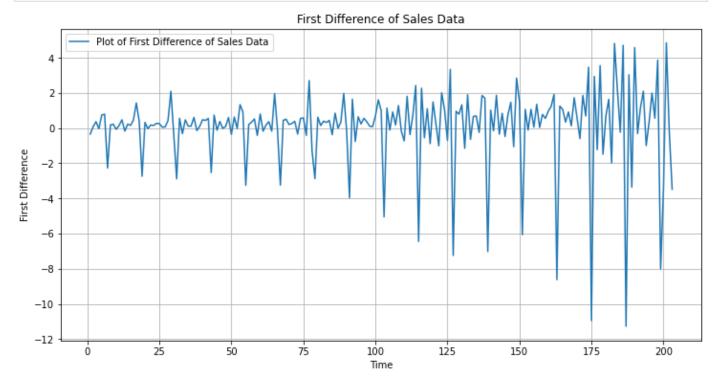
4)

From observing the moving average plot above we can say that the trend seems to be increasing.

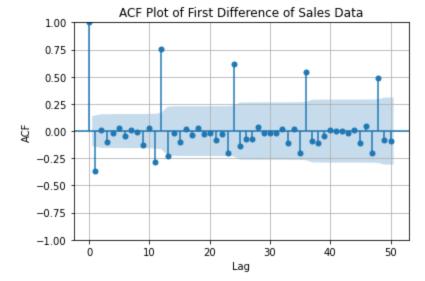
5A)

```
# Computing the first difference of the data and plotting the ACF and PACF of the differ
differenced_data = pharma_data['Sales'].diff(1).dropna()

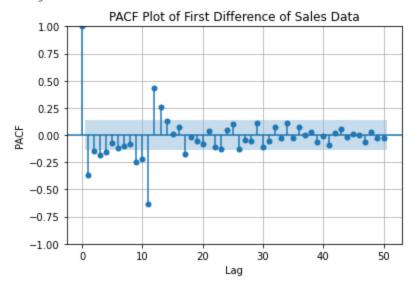
# Plotting the First Difference of Sales Data for visual inspection of data
plt.figure(figsize=(12,6))
plt.plot(differenced_data, label='Plot of First Difference of Sales Data')
plt.xlabel('Time')
plt.ylabel('First Difference')
plt.title('First Difference of Sales Data')
plt.legend()
plt.grid(True)
plt.show()
```



```
In [11]: # Plotting the ACF and PACF of the First Differenced Data
         # ACF
         plot acf(differenced data, lags=50, alpha=0.05)
         plt.title('ACF Plot of First Difference of Sales Data')
         plt.xlabel('Lag')
         plt.ylabel('ACF')
         plt.grid(True)
         plt.show()
         #PACF
         plt.figure(figsize=(10,6))
         plot pacf(differenced data, lags=50, method='ywm')
         plt.title('PACF Plot of First Difference of Sales Data')
         plt.xlabel('Lag')
         plt.ylabel('PACF')
         plt.grid(True)
         plt.show()
```



<Figure size 720x432 with 0 Axes>



5B)

In Q2 the ACF plot is slowly decaying and is significant at many lags which is a sign of the non-stationarity of the data, suggesting underlying trend or seasonality which can be seen to be present from the time series plot of the data. When we apply differencing on the data and plot it, we can see that the mean and variance seem to be constant, the differencing process helps remove the trend and seasonality. The ACF and PACF plots of the first difference of the sales data follow of decaying sinusoidal pattern which indicates the presence of seasonality and from the ACF plot we can make the seasonality to be annual i.e. seasonality period = 12 months. So after applying the first difference we removed the trend in the data but seasonality is still present.

6)

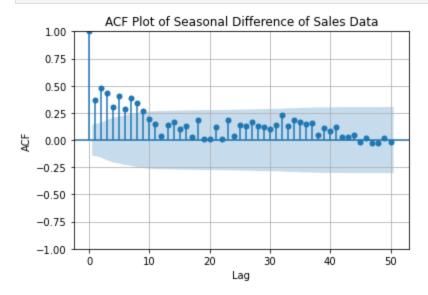
```
In [12]: # PART 6:
    # Performing seasonal differencing
    seasonal_differenced_data = pharma_data['Sales'].diff(12).dropna()

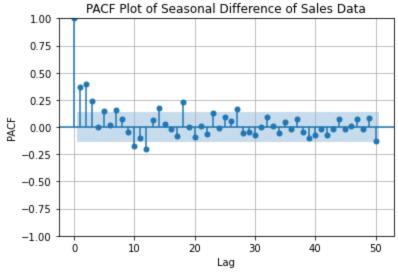
# ACF of seasonal differenced sales
    plot_acf(seasonal_differenced_data, lags=50, alpha=0.05)
    plt.title('ACF Plot of Seasonal Difference of Sales Data')
    plt.xlabel('Lag')
```

```
plt.ylabel('ACF')
plt.grid(True)
plt.show()

# PACF

plot_pacf(seasonal_differenced_data, lags=50, method='ywm')
plt.title('PACF Plot of Seasonal Difference of Sales Data')
plt.xlabel('Lag')
plt.ylabel('PACF')
plt.grid(True)
plt.show()
```





```
In [13]: # TESTING CELL TO LOOK AT THE INDEX OF 2005-06-01 ROW
    # Filtering the DataFrame for rows where 'Month' is '2005-06-01'
    desired_row = pharma_data[pharma_data['Month'] == '2005-06-01']
    index_of_desired_row = desired_row.index[0]
    print(index_of_desired_row)
```

167

7)

```
In [14]: # PART 7:
    # Finding the Best SARIMA Model
    # Specifying range of p, d and q
    p_vals = range(4)
    d = 1
    q_vals = range(4)
```

```
P VALUES = range(4)
D = 1
Q VALUES = range(4)
s = 12 # Seasonal period
best aic = np.inf
best params = None
trainset = pharma data[0 : 168] # Data until June 2005 as we found June 2005 index to be
# Grid search to find the best SARIMA model
for p in p vals:
    for q in q vals:
        for P in P VALUES:
            for Q in Q VALUES:
                try:
                    model = sm.tsa.SARIMAX(trainset['Sales'],
                                          order=(p, d, q),
                                          seasonal order=(P, D, Q, s),
                                          enforce stationarity=False,
                                          enforce invertibility=False)
                    results = model.fit()
                    aic = results.aic
                    print(aic)
                    if aic < best aic:</pre>
                       best aic = aic
                        best params = (p, d, q, P, D, Q)
                except:
                    continue
# Fitting the best SARIMA model with the identified parameters
best model = sm.tsa.SARIMAX(pharma data['Sales'][ : 168], # Data until June 2005
                            order=(best params[0], best params[1], best params[2]),
                            seasonal order=(best params[3], best_params[4], best_params[
                            enforce stationarity=False,
                            enforce invertibility=False)
best results = best model.fit()
# Displaying the best model parameters and summary
print(f"Best SARIMA Model: SARIMA{best params} (seasonal period = 12)")
print(best results.summary())
RUNNING THE L-BFGS-B CODE
           * * *
Machine precision = 2.220D-16
N =
               1
                    M =
                                   10
At X0
              O variables are exactly at the bounds
At iterate 0 f = 1.05590D + 00
                                     |proj g|= 1.13418D-01
At iterate 5 f= 1.05413D+00
                                     |proj g| = 4.79039D-07
           * * *
Tit = total number of iterations
Tnf = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip = number of BFGS updates skipped
Nact = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
   = final function value
```

F = final function value

```
Tit Inf Inint Skip Nact Projg
       5 7 1 0 0 4.790D-07 1.054D+00
   1
  F = 1.0541290038753439
CONVERGENCE: NORM OF PROJECTED GRADIENT <= PGTOL
 This problem is unconstrained.
 This problem is unconstrained.
This problem is unconstrained.
356.18734530211555
RUNNING THE L-BFGS-B CODE
         * * *
Machine precision = 2.220D-16
N =
      2 M =
                               10
At XO 0 variables are exactly at the bounds
At iterate 0 f= 9.88064D-01 |proj q|= 1.27866D-01
At iterate 5 f = 9.83434D - 01 |proj g|= 1.57418D-04
         * * *
Tit = total number of iterations
Tnf = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip = number of BFGS updates skipped
Nact = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F = final function value
          * * *
  N Tit Tnf Tnint Skip Nact Projg
                     1 0 0 2.114D-06 9.834D-01
       6
  F = 0.98343387036150409
CONVERGENCE: NORM OF PROJECTED GRADIENT <= PGTOL
334.4337804414654
RUNNING THE L-BFGS-B CODE
          * * *
Machine precision = 2.220D-16
                          10
N =
             3
                  M =
At XO
           O variables are exactly at the bounds
At iterate 0 f = 9.54113D-01 |proj g|= 2.20200D-01
At iterate 5 f= 9.06965D-01 |proj g|= 3.02160D-03
         * * *
Tit = total number of iterations
Tnf = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip = number of BFGS updates skipped
Nact = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
```

Tit = total number of iterations

Tnf = total number of function evaluations

```
N Tit Tnf Tnint Skip Nact Projg
             11 1 0 0 2.150D-06 9.070D-01
 F = 0.90695623707146211
CONVERGENCE: NORM OF PROJECTED GRADIENT <= PGTOL
310.73729565601127
RUNNING THE L-BFGS-B CODE
         * * *
Machine precision = 2.220D-16
N = 4 M =
At XO
           O variables are exactly at the bounds
At iterate 0 f= 9.89548D-01 |proj q|= 2.81533D-01
At iterate 5 f = 8.57266D - 01 |proj q| = 7.37987D - 02
This problem is unconstrained.
At iterate 10 f= 8.41735D-01
                                 |proj g| = 5.62546D-03
At iterate 15 f = 8.41721D-01 |proj g| = 2.78529D-05
         * * *
Tit = total number of iterations
Tnf = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip = number of BFGS updates skipped
Nact = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F = final function value
         * * *
      Tit Tnf Tnint Skip Nact Projg
       16
             18 1 0 0 2.193D-06 8.417D-01
 F = 0.84172134761467476
CONVERGENCE: NORM OF PROJECTED GRADIENT <= PGTOL
290.81837279853073
RUNNING THE L-BFGS-B CODE
         * * *
Machine precision = 2.220D-16
N = 2 \qquad M = 10
At XO
           O variables are exactly at the bounds
At iterate 0 f = 9.96804D - 01 |proj g|= 1.35725D - 05
This problem is unconstrained.
Warning: more than 10 function and gradient
 evaluations in the last line search. Termination
 may possibly be caused by a bad search direction.
This problem is unconstrained.
This problem is unconstrained.
```

```
Tnint = total number of segments explored during Cauchy searches

Skip = number of BFGS updates skipped

Nact = number of active bounds at final generalized Cauchy point

Projg = norm of the final projected gradient
```

= final function value

N Tit Tnf Tnint Skip Nact Projg F
2 1 13 1 0 0 1.448D-05 9.968D-01
F = 0.99680398705586859

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH 338.92613965077186
RUNNING THE L-BFGS-B CODE

* * *

Machine precision = 2.220D-16N = 3 M = 10

At XO 0 variables are exactly at the bounds

At iterate 0 f= 9.89721D-01 |proj g|= 1.24227D-01 At iterate 5 f= 9.80468D-01 |proj g|= 8.05536D-02 At iterate 10 f= 9.78750D-01 |proj g|= 2.02227D-06

* * *

Tit = total number of iterations

Tnf = total number of function evaluations

Tnint = total number of segments explored during Cauchy searches

Skip = number of BFGS updates skipped

Nact = number of active bounds at final generalized Cauchy point

Projg = norm of the final projected gradient

F = final function value

* * *

N Tit Tnf Tnint Skip Nact Projg F 3 10 13 1 0 0 2.022D-06 9.788D-01 F = 0.97875006917479634

CONVERGENCE: NORM_OF_PROJECTED_GRADIENT_<=_PGTOL 334.8600232427316
RUNNING THE L-BFGS-B CODE

* * *

Machine precision = 2.220D-16N = 4 M = 10

At XO 0 variables are exactly at the bounds

At iterate 0 f= 9.48885D-01 |proj g|= 2.11514D-01

At iterate 5 f= 9.07891D-01 |proj g|= 1.20134D-02

At iterate 10 f= 9.06644D-01 |proj g|= 2.28308D-02

At iterate 15 f= 9.06352D-01 |proj g|= 5.57299D-07

```
Tit = total number of iterations
Tnf = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip = number of BFGS updates skipped
Nact = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F = final function value
          * * *
  N Tit Tnf Tnint Skip Nact Projg
              17 1 0 0 5.573D-07 9.064D-01
 F = 0.90635210512665154
CONVERGENCE: NORM OF PROJECTED GRADIENT <= PGTOL
312.5343073225549
RUNNING THE L-BFGS-B CODE
         * * *
Machine precision = 2.220D-16
                          10
N = 5 \qquad M =
At XO
           O variables are exactly at the bounds
At iterate 0 f= 1.06359D+00 |proj g|= 4.88304D-01
This problem is unconstrained.
At iterate 5 f= 8.44257D-01
                                |proj g| = 3.85457D-02
At iterate 10 f= 8.42476D-01
                                 |proj g| = 6.34295D-03
At iterate 15 f = 8.41092D-01 |proj g|= 1.47751D-03
         * * *
Tit = total number of iterations
Tnf = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip = number of BFGS updates skipped
Nact = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F = final function value
          * * *
      Tit Tnf Tnint Skip Nact Projg
             20 1 0 0 5.012D-06 8.411D-01
       18
 F = 0.84109021131971651
CONVERGENCE: NORM OF PROJECTED GRADIENT <= PGTOL
292.60631100342476
RUNNING THE L-BFGS-B CODE
          * * *
Machine precision = 2.220D-16
N = 3 \qquad M =
           O variables are exactly at the bounds
At iterate 0 f= 9.13735D-01 |proj g|= 1.25224D-05
          * * *
Tit = total number of iterations
```

```
Tnf = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip = number of BFGS updates skipped
Nact = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F = final function value
          * * *
  N Tit Tnf Tnint Skip Nact Projg
                          0 0 1.561D-05 9.137D-01
                     1
  F = 0.91373525829533042
CONVERGENCE: REL REDUCTION OF F <= FACTR*EPSMCH
This problem is unconstrained.
This problem is unconstrained.
313.01504678723103
RUNNING THE L-BFGS-B CODE
          * * *
Machine precision = 2.220D-16
             4
                   M =
                                1 0
At XO
           O variables are exactly at the bounds
At iterate 0 f= 9.14152D-01 |proj g|= 6.29700D-02
At iterate 5 f= 9.12533D-01 |proj g|= 7.51846D-03
At iterate 10 f= 9.12091D-01
                                  |proj g| = 9.28188D-06
         * * *
Tit = total number of iterations
Tnf = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip = number of BFGS updates skipped
Nact = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F = final function value
          * * *
            Tnf Tnint Skip Nact Projg
       10
                         0 0 9.282D-06 9.121D-01
              13
                      1
  F = 0.91209094334288887
CONVERGENCE: NORM OF PROJECTED GRADIENT <= PGTOL
314.46255696321066
RUNNING THE L-BFGS-B CODE
          * * *
Machine precision = 2.220D-16
N =
              5
                  M =
                               10
At XO
           O variables are exactly at the bounds
           0 f= 9.36211D-01
                                  |proj g| = 2.03046D-01
At iterate
This problem is unconstrained.
At iterate 5 f= 9.07174D-01 |proj g|= 2.74877D-02
At iterate 10 f= 9.05712D-01 |proj g|= 4.59025D-03
```

Tit = total number of iterations

Tnf = total number of function evaluations

Tnint = total number of segments explored during Cauchy searches

Skip = number of BFGS updates skipped

Nact = number of active bounds at final generalized Cauchy point

Projg = norm of the final projected gradient

F = final function value

* * *

N Tit Tnf Tnint Skip Nact Projg F
5 14 18 1 0 0 7.568D-06 9.057D-01
F = 0.90565344930868952

CONVERGENCE: NORM OF PROJECTED GRADIENT <= PGTOL

314.2995589677197

This problem is unconstrained.

RUNNING THE L-BFGS-B CODE

* * *

Machine precision = 2.220D-16

 $N = 6 \qquad M = 10$

At XO 0 variables are exactly at the bounds

At iterate 0 f= 1.17814D+00 |proj g|= 6.50785D-01

At iterate 5 f= 8.43670D-01 |proj g|= 5.28640D-02

At iterate 10 f= 8.37323D-01 |proj g|= 1.47477D-02

At iterate 15 f= 8.34759D-01 |proj g|= 5.22156D-02

At iterate 20 f= 8.27582D-01 |proj g|= 8.25568D-02

At iterate 25 f= 8.18597D-01 |proj g|= 5.72696D-03

At iterate 30 f= 8.18551D-01 |proj g|= 1.28837D-05

* * *

Tit = total number of iterations

Tnf = total number of function evaluations

Tnint = total number of segments explored during Cauchy searches

Skip = number of BFGS updates skipped

Nact = number of active bounds at final generalized Cauchy point

Projg = norm of the final projected gradient

F = final function value

* * *

N Tit Tnf Tnint Skip Nact Projg F 6 31 42 1 0 0 4.882D-06 8.186D-01 F = 0.81855149436487751

CONVERGENCE: NORM_OF_PROJECTED_GRADIENT_<=_PGTOL

287.03330210659885

RUNNING THE L-BFGS-B CODE

N = 4 M = 10At XO 0 variables are exactly at the bounds

At iterate 0 f= 8.43483D-01 |proj g|= 1.18600D-05

Warning: more than 10 function and gradient evaluations in the last line search. Termination may possibly be caused by a bad search direction.

This problem is unconstrained.

This problem is unconstrained.

* * *

Tit = total number of iterations

Tnf = total number of function evaluations

Tnint = total number of segments explored during Cauchy searches

Skip = number of BFGS updates skipped

Nact = number of active bounds at final generalized Cauchy point

Projg = norm of the final projected gradient

F = final function value

* * *

N Tit Tnf Tnint Skip Nact Projg F 4 1 18 1 0 0 1.196D-05 8.435D-01 F = 0.84348280956592836

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH 291.41022401415194

RUNNING THE L-BFGS-B CODE

* * *

Machine precision = 2.220D-16N = 5 M = 1

At X0 0 variables are exactly at the bounds

At iterate 0 f= 8.43616D-01 |proj g|= 1.52437D-02

At iterate 5 f= 8.43415D-01 |proj g|= 7.17248D-04

At iterate 10 f= 8.43413D-01 |proj g|= 1.92698D-06

* * *

Tit = total number of iterations

Tnf = total number of function evaluations

Tnint = total number of segments explored during Cauchy searches

Skip = number of BFGS updates skipped

Nact = number of active bounds at final generalized Cauchy point

Projg = norm of the final projected gradient

F = final function value

* * *

N Tit Tnf Tnint Skip Nact Projg F
5 10 14 1 0 0 1.927D-06 8.434D-01
F = 0.84341312369450860

CONVERGENCE: NORM_OF_PROJECTED_GRADIENT_<=_PGTOL 293.3868095613549
RUNNING THE L-BFGS-B CODE

```
Machine precision = 2.220D-16
       6
At XO
           O variables are exactly at the bounds
At iterate
           0 f= 8.69154D-01 |proj g|= 1.47850D-01
This problem is unconstrained.
At iterate
           5 	 f = 8.44466D - 01
                                  |proj g| = 1.40390D-02
At iterate 10 f= 8.43581D-01
                                  |proj q| = 7.74130D-03
At iterate 15 f = 8.43384D-01 |proj g|= 1.37885D-04
          * * *
Tit = total number of iterations
Tnf = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip = number of BFGS updates skipped
Nact = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
    = final function value
          * * *
             Tnf Tnint Skip Nact Projg
                   1 0 0 3.633D-06 8.434D-01
       17
              20
  F = 0.84338413007631652
CONVERGENCE: NORM OF PROJECTED GRADIENT <= PGTOL
295.37706770564233
RUNNING THE L-BFGS-B CODE
          * * *
Machine precision = 2.220D-16
              7
                   M =
                                10
At X0
           O variables are exactly at the bounds
           0 	 f = 1.12996D + 00
At iterate
                                |proj g| = 5.88653D-01
This problem is unconstrained.
          5 	 f = 8.37087D - 01
At iterate
                                  |proj g| = 3.54428D-02
At iterate 10 f= 8.29013D-01
                                  |proj g| = 2.29499D-02
At iterate 15 f= 8.28539D-01
                                  |proj g| = 1.71291D-03
At iterate 20 f= 8.28516D-01
                                  |proj g| = 3.34127D-03
At iterate 25 f= 8.28496D-01
                                  |proj g| = 3.94142D-05
         * * *
Tit = total number of iterations
Tnf = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip = number of BFGS updates skipped
Nact = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
```

F = final function value

```
Tit
              Tnf Tnint Skip Nact Projg F
             30 1 0 0 1.152D-05 8.285D-01
       26
  F = 0.82849592339092648
CONVERGENCE: REL REDUCTION OF F <= FACTR*EPSMCH
292.3746302593513
RUNNING THE L-BFGS-B CODE
         * * *
Machine precision = 2.220D-16
N = 2
                  M =
At XO 0 variables are exactly at the bounds
At iterate 0 f = 8.30441D-01 |proj g|= 1.23660D-01
At iterate 5 f= 8.28365D-01 |proj g|= 1.03234D-04
        * * *
Tit = total number of iterations
Tnf = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip = number of BFGS updates skipped
Nact = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F = final function value
         * * *
  N Tit Tnf Tnint Skip Nact Projg F
             8 1 0 0 2.528D-06 8.284D-01
  F = 0.82836455049018998
CONVERGENCE: NORM OF PROJECTED GRADIENT <= PGTOL
282.3304889647038
RUNNING THE L-BFGS-B CODE
         * * *
Machine precision = 2.220D-16
N = 3 M = 10
At XO 0 variables are exactly at the bounds
At iterate 0 f= 7.59501D-01 |proj g|= 1.31450D-01
At iterate 5 f = 7.49855D-01 |proj g|= 1.59208D-03
        * * *
Tit = total number of iterations
Tnf = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip = number of BFGS updates skipped
Nact = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F = final function value
         * * *
```

1 0 0 2.326D-06 7.499D-01 F = 0.74985373143543121

```
CONVERGENCE: NORM OF PROJECTED GRADIENT <= PGTOL
257.95085376230486
RUNNING THE L-BFGS-B CODE
         * * *
Machine precision = 2.220D-16
                         10
N = 4 \qquad M =
At XO
           0 variables are exactly at the bounds
At iterate 0 f= 7.59472D-01 |proj g|= 2.44473D-01
This problem is unconstrained.
This problem is unconstrained.
This problem is unconstrained.
At iterate 5 f= 7.10406D-01 |proj g|= 4.36525D-03
         * * *
Tit = total number of iterations
Tnf = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip = number of BFGS updates skipped
Nact = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F = final function value
          * * *
  N Tit Tnf Tnint Skip Nact Projg
              10 1 0 0 6.555D-06 7.104D-01
 F = 0.71038670940110238
CONVERGENCE: NORM OF PROJECTED GRADIENT <= PGTOL
246.68993435877042
RUNNING THE L-BFGS-B CODE
         * * *
Machine precision = 2.220D-16
N = 5 	 M = 10
At XO 0 variables are exactly at the bounds
At iterate 0 f = 7.62157D - 01
                                 |proj g| = 2.83905D-01
This problem is unconstrained.
At iterate 5 f= 6.62167D-01 |proj g|= 2.52605D-02
At iterate 10 f = 6.59498D - 01 |proj g|= 4.23839D - 04
        * * *
Tit = total number of iterations
Tnf = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip = number of BFGS updates skipped
Nact = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F = final function value
     Tit Tnf Tnint Skip Nact Projg
       12
              15 1 0 0 1.494D-06 6.595D-01
```

F = 0.65949817977189351

```
CONVERGENCE: NORM OF PROJECTED GRADIENT <= PGTOL
231.5913884033562
RUNNING THE L-BFGS-B CODE
          * * *
Machine precision = 2.220D-16
                           10
              3
                  M =
At XO
           O variables are exactly at the bounds
At iterate 0 f = 7.78040D - 01 |proj g| = 1.15379D - 01
At iterate 5 f = 7.70057D - 01 |proj g|= 3.62184D - 04
         * * *
Tit = total number of iterations
Tnf = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip = number of BFGS updates skipped
Nact = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F = final function value
          * * *
             Tnf Tnint Skip Nact Projg
              11 1 0 0 2.899D-05 7.701D-01
 F = 0.77005685316675943
CONVERGENCE: REL REDUCTION OF F <= FACTR*EPSMCH
264.7391026640312
RUNNING THE L-BFGS-B CODE
          * * *
Machine precision = 2.220D-16
       4
                   M =
                                10
At XO
           O variables are exactly at the bounds
At iterate 0 f= 7.60520D-01 |proj g|= 1.32242D-01
At iterate 5 f = 7.52409D - 01 |proj g|= 1.83545D-02
This problem is unconstrained.
This problem is unconstrained.
At iterate 10 f= 7.50046D-01 |proj g|= 1.05148D+00
         * * *
Tit = total number of iterations
Tnf = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip = number of BFGS updates skipped
```

Nact = number of active bounds at final generalized Cauchy point Projg = norm of the final projected gradient = final function value

* * *

Tit Tnf Tnint Skip Nact Projg 14 1 0 0 1.541D-02 7.499D-01 35 F = 0.74985403097174963

```
CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH 259.9509544065079
```

RUNNING THE L-BFGS-B CODE

* * *

Machine precision = 2.220D-16

N = 5 M = 10

At XO 0 variables are exactly at the bounds

At iterate 0 f= 7.52526D-01 |proj g|= 2.39706D-01

This problem is unconstrained.

At iterate 5 f= 7.09512D-01 |proj g|= 6.82959D-03

At iterate 10 f= 7.08959D-01 |proj g|= 7.49596D-03

* * *

Tit = total number of iterations

Tnf = total number of function evaluations

Tnint = total number of segments explored during Cauchy searches

Skip = number of BFGS updates skipped

Nact = number of active bounds at final generalized Cauchy point

Projg = norm of the final projected gradient

F = final function value

* * *

N Tit Tnf Tnint Skip Nact Projg F
5 14 16 1 0 0 1.992D-06 7.089D-01
F = 0.70890555827273394

CONVERGENCE: NORM_OF_PROJECTED_GRADIENT_<=_PGTOL 248.1922675796386

This problem is unconstrained.

RUNNING THE L-BFGS-B CODE

* * *

Machine precision = 2.220D-16

 $N = 6 \qquad M = 10$

At XO 0 variables are exactly at the bounds

At iterate 0 f= 8.26350D-01 |proj g|= 4.85374D-01

At iterate 5 f = 6.61421D - 01 |proj g| = 1.82044D - 02

At iterate 10 f = 6.59936D-01 |proj g| = 3.91792D-03

At iterate 15 f = 6.59121D-01 |proj g| = 1.75667D-02

At iterate 20 f= 6.58986D-01 |proj g|= 2.24467D-05

* * *

Tit = total number of iterations

Tnf = total number of function evaluations

Tnint = total number of segments explored during Cauchy searches

Skip = number of BFGS updates skipped

Nact = number of active bounds at final generalized Cauchy point

Projg = norm of the final projected gradient

F = final function value

```
Tit Tnf Tnint Skip Nact Projg
             26 1 0 0 1.150D-06 6.590D-01
       21
  F = 0.65898620777718286
CONVERGENCE: NORM OF PROJECTED GRADIENT <= PGTOL
233.41936581313342
RUNNING THE L-BFGS-B CODE
         * * *
Machine precision = 2.220D-16
N = 4 M =
At XO
           O variables are exactly at the bounds
At iterate 0 f= 7.21917D-01 |proj g|= 8.34075D-02
At iterate 5 f= 7.13320D-01 |proj g|= 1.32730D-03
        * * *
Tit = total number of iterations
Tnf = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip = number of BFGS updates skipped
Nact = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F = final function value
         * * *
      Tit Tnf Tnint Skip Nact Projg
             11 1 0 0 4.341D-06 7.133D-01
 F = 0.71331866957844359
CONVERGENCE: NORM OF PROJECTED GRADIENT <= PGTOL
247.67507297835704
RUNNING THE L-BFGS-B CODE
         * * *
Machine precision = 2.220D-16
                          10
N =
             5 \qquad M =
           O variables are exactly at the bounds
At XO
At iterate 0 f= 7.20155D-01 |proj g|= 8.42298D-02
This problem is unconstrained.
This problem is unconstrained.
At iterate 5 f= 7.14744D-01
                                 |proj g| = 1.02042D-02
At iterate 10 f= 7.12641D-01 |proj g|= 4.62759D-04
         * * *
Tit = total number of iterations
Tnf = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip = number of BFGS updates skipped
```

Nact = number of active bounds at final generalized Cauchy point

Projg = norm of the final projected gradient

F = final function value

```
N Tit Tnf Tnint Skip Nact Projg F
5 13 16 1 0 0 4.118D-07 7.126D-01
F = 0.71264085041173042

CONVERGENCE: NORM_OF_PROJECTED_GRADIENT_<=_PGTOL
249.44732573834142
```

* * *

RUNNING THE L-BFGS-B CODE

Machine precision = 2.220D-16N = 6 M = 10

At XO 0 variables are exactly at the bounds

At iterate 0 f= 7.35494D-01 |proj g|= 1.94317D-01 This problem is unconstrained.

At iterate 5 f= 7.04269D-01 |proj g|= 5.48314D-03 At iterate 10 f= 7.03720D-01 |proj g|= 2.84442D-03 At iterate 15 f= 7.03661D-01 |proj g|= 4.42061D-04

* * *

Tit = total number of iterations

Tnf = total number of function evaluations

Tnint = total number of segments explored during Cauchy searches

Skip = number of BFGS updates skipped

Nact = number of active bounds at final generalized Cauchy point

Projg = norm of the final projected gradient

F = final function value

* * *

N Tit Tnf Tnint Skip Nact Projg F
6 17 22 1 0 0 5.554D-06 7.037D-01
F = 0.70366117986316745

CONVERGENCE: NORM_OF_PROJECTED_GRADIENT_<=_PGTOL 248.43015643402427
RUNNING THE L-BFGS-B CODE

* * *

Machine precision = 2.220D-16N = 7 M = 10

At X0 0 variables are exactly at the bounds

At iterate 0 f= 9.14556D-01 |proj g|= 6.54963D-01

This problem is unconstrained.

At iterate 5 f= 6.61296D-01 |proj g|= 3.35109D-02

At iterate 10 f= 6.58660D-01 |proj g|= 1.50535D-02

At iterate 15 f= 6.58455D-01 |proj g|= 4.51362D-03

At iterate 20 f= 6.58163D-01 |proj g|= 2.30204D-03

At iterate 25 f= 6.58150D-01 |proj g|= 1.05825D-05

```
Tit = total number of iterations
Tnf = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip = number of BFGS updates skipped
Nact = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F = final function value
      Tit Tnf Tnint Skip Nact Projg
       26 29 1 0 0 4.716D-06 6.581D-01
 F = 0.65814980831947145
CONVERGENCE: NORM OF PROJECTED GRADIENT <= PGTOL
235.1383355953424
RUNNING THE L-BFGS-B CODE
         * * *
Machine precision = 2.220D-16
N = 5 \qquad M =
                               10
At XO
           0 variables are exactly at the bounds
At iterate 0 f = 6.77095D-01 |proj g|= 8.24161D-02
This problem is unconstrained.
At iterate 5 f= 6.66624D-01 |proj g|= 4.75941D-04
         * * *
Tit = total number of iterations
Tnf = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip = number of BFGS updates skipped
Nact = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F = final function value
          * * *
  N Tit Tnf Tnint Skip Nact Projg F
              10 1 0 0 9.166D-06 6.666D-01
  F = 0.66662357012002871
CONVERGENCE: NORM OF PROJECTED GRADIENT <= PGTOL
233.98551956032966
RUNNING THE L-BFGS-B CODE
         * * *
Machine precision = 2.220D-16
N = 6 \qquad M =
                               10
     O variables are exactly at the bounds
At iterate 0 f = 6.76422D - 01
                                 |proj g| = 6.74855D-02
This problem is unconstrained.
At iterate 5 f = 6.66981D - 01 |proj g|= 6.18803D - 03
At iterate 10 f = 6.66226D - 01 |proj g| = 4.86312D - 04
```

```
Tit = total number of iterations
Tnf = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip = number of BFGS updates skipped
Nact = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
   = final function value
          * * *
      Tit Inf Inint Skip Nact Projg
                      1 0 0 3.446D-06 6.662D-01
   6
       12
              14
  F = 0.66622587475272099
CONVERGENCE: NORM OF PROJECTED GRADIENT <= PGTOL
235.85189391691426
RUNNING THE L-BFGS-B CODE
          * * *
Machine precision = 2.220D-16
              7
                  M =
At X0
           O variables are exactly at the bounds
At iterate 0 f= 6.99958D-01 |proj g|= 1.84397D-01
At iterate 5 f = 6.66805D - 01 |proj g|= 6.81697D - 03
This problem is unconstrained.
At iterate 10 f= 6.66245D-01
                                  |proj g| = 9.23269D-04
At iterate 15 f = 6.66224D - 01 | proj g|= 1.17618D - 04
         * * *
Tit = total number of iterations
Tnf = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip = number of BFGS updates skipped
Nact = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F = final function value
         * * *
      Tit Inf Inint Skip Nact Projg
              21 1 0 0 2.977D-06 6.662D-01
       17
  F = 0.66622417115511756
CONVERGENCE: NORM OF PROJECTED GRADIENT <= PGTOL
237.8513215081195
RUNNING THE L-BFGS-B CODE
          * * *
Machine precision = 2.220D-16
N =
             8 \qquad M =
                                10
At XO
           O variables are exactly at the bounds
At iterate 0 f = 8.76710D - 01 |proj g|= 5.36461D - 01
This problem is unconstrained.
At iterate 5 f= 6.68835D-01 |proj g|= 1.72429D-02
```

```
10 f= 6.61124D-01
                                |proj g| = 2.27772D-02
At iterate
At iterate 15 f = 6.58234D - 01
                                  |proj q| = 1.75711D-03
At iterate 20 f= 6.58225D-01 |proj g|= 4.93626D-05
         * * *
Tit = total number of iterations
Tnf = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip = number of BFGS updates skipped
Nact = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F = final function value
         * * *
      Tit Tnf Tnint Skip Nact Projg
22 27 1 0 0 6.957D-07
                     1 0 0 6.957D-07 6.582D-01
  8
  F = 0.65822531039357213
CONVERGENCE: NORM OF PROJECTED GRADIENT <= PGTOL
237.16370429224023
RUNNING THE L-BFGS-B CODE
          * * *
Machine precision = 2.220D-16
                          10
             3
                  M =
At XO
           O variables are exactly at the bounds
At iterate 0 f = 8.26088D - 01 |proj g| = 7.63604D - 02
At iterate 5 f = 8.23910D-01 |proj g| = 8.23384D-04
         * * *
Tit = total number of iterations
Tnf = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip = number of BFGS updates skipped
Nact = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F = final function value
          * * *
      Tit Tnf Tnint Skip Nact Projg
       7
              10 1 0 0 5.131D-06 8.239D-01
 F = 0.82390935044678648
CONVERGENCE: NORM OF PROJECTED GRADIENT <= PGTOL
282.8335417501203
RUNNING THE L-BFGS-B CODE
          * * *
Machine precision = 2.220D-16
N = 4 \qquad M =
                               10
At XO
           0 variables are exactly at the bounds
At iterate 0 f= 7.53088D-01 |proj g|= 9.56542D-02
```

```
At iterate 10 f = 7.43008D - 01 |proj g|= 1.29871D - 06
         * * *
Tit = total number of iterations
Tnf = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip = number of BFGS updates skipped
Nact = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F = final function value
          * * *
  N Tit Tnf Tnint Skip Nact Projg
              13 1 0 0 1.299D-06 7.430D-01
       10
 F = 0.74300815112566165
CONVERGENCE: NORM OF PROJECTED GRADIENT <= PGTOL
257.6507387782223
RUNNING THE L-BFGS-B CODE
          * * *
Machine precision = 2.220D-16
N = 5
                  M =
                               10
At XO 0 variables are exactly at the bounds
               f = 7.57573D-01 |proj q|= 2.51950D-01
At iterate 0
This problem is unconstrained.
This problem is unconstrained.
This problem is unconstrained.
At iterate 5 f = 7.05660D - 01 |proj g|= 2.77819D-02
At iterate 10 f= 7.04119D-01 |proj q|= 2.54992D-05
         * * *
Tit = total number of iterations
Tnf = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip = number of BFGS updates skipped
Nact = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F = final function value
          * * *
  N Tit Tnf Tnint Skip Nact Projg
       11
                   1 0 0 2.055D-05 7.041D-01
              14
  F = 0.70411860716195240
CONVERGENCE: REL REDUCTION OF F <= FACTR*EPSMCH
246.583852006416
RUNNING THE L-BFGS-B CODE
          * * *
Machine precision = 2.220D-16
                           10
N =
             6
                  M =
```

|proj g| = 4.28046D-03

5 f = 7.43026D - 01

At XO 0 variables are exactly at the bounds

At iterate

```
At iterate 0 f = 7.50591D-01
                                 |proj g| = 3.02853D-01
This problem is unconstrained.
At iterate 5 f = 6.53814D - 01 |proj g|= 2.40697D - 02
At iterate 10 f = 6.50561D-01 |proj g| = 5.44421D-04
         * * *
Tit = total number of iterations
Tnf = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip = number of BFGS updates skipped
Nact = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
   = final function value
         * * *
      Tit Inf Inint Skip Nact Projg F
                  1 0 0 8.539D-06 6.506D-01
       13
              16
 F = 0.65055981101080151
CONVERGENCE: NORM OF PROJECTED GRADIENT <= PGTOL
230.5880964996293
RUNNING THE L-BFGS-B CODE
          * * *
Machine precision = 2.220D-16
                  M =
                               10
             4
At XO
           0 variables are exactly at the bounds
At iterate 0 f= 7.74519D-01 |proj g|= 8.68243D-02
At iterate 5 f = 7.64536D - 01 |proj g|= 1.10228D - 03
         * * *
Tit = total number of iterations
Tnf = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip = number of BFGS updates skipped
Nact = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F = final function value
          * * *
  N Tit Tnf Tnint Skip Nact Projg
              12 1 0 0 3.872D-06 7.645D-01
 F = 0.76453468565423288
CONVERGENCE: NORM OF PROJECTED GRADIENT <= PGTOL
264.88365437982225
RUNNING THE L-BFGS-B CODE
         * * *
Machine precision = 2.220D-16
N = 5 \qquad M =
                               10
At XO 0 variables are exactly at the bounds
```

```
0 f = 7.53307D - 01
                                  |proj g| = 9.73976D-02
At iterate
At iterate 5 f = 7.44066D - 01
                                  |proj q| = 1.33054D-02
At iterate 10 f = 7.43687D - 01 |proj g| = 4.28351D - 04
         * * *
Tit = total number of iterations
Tnf = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip = number of BFGS updates skipped
Nact = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F = final function value
          * * *
             Tnf Tnint Skip Nact Projg
       12
                      1 0 0 1.109D-06 7.437D-01
              15
  F = 0.74368721058678644
CONVERGENCE: NORM OF PROJECTED GRADIENT <= PGTOL
This problem is unconstrained.
This problem is unconstrained.
259.8789027571603
RUNNING THE L-BFGS-B CODE
          * * *
Machine precision = 2.220D-16
N =
             6
                  M =
                                10
At XO 0 variables are exactly at the bounds
At iterate 0 f = 7.49588D - 01 |proj g| = 2.46761D - 01
At iterate 5 f = 7.03745D-01
                                   |proj g| = 5.29776D-03
This problem is unconstrained.
At iterate 10 f = 7.02969D - 01 |proj g| = 9.30817D - 03
At iterate 15 f= 7.02896D-01 |proj g|= 1.19902D-05
         * * *
Tit = total number of iterations
Tnf = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip = number of BFGS updates skipped
Nact = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
   = final function value
             Tnf Tnint Skip Nact Projg
              20 1 0 0 8.317D-06 7.029D-01
       16
  F = 0.70289592245529808
CONVERGENCE: NORM OF PROJECTED GRADIENT <= PGTOL
248.17302994498016
This problem is unconstrained.
RUNNING THE L-BFGS-B CODE
```

```
Machine precision = 2.220D-16
N = 7
At XO
           O variables are exactly at the bounds
At iterate 0 f= 8.06662D-01 |proj g|= 4.80949D-01
At iterate 5 f = 6.52709D - 01 |proj g| = 2.53028D - 02
At iterate 10 f= 6.50904D-01
                                 |proj g| = 2.78783D-03
At iterate 15 f= 6.50277D-01
                                 |proj g| = 2.12592D-02
At iterate 20 f= 6.50110D-01 |proj g|= 9.82160D-05
         * * *
Tit = total number of iterations
Tnf = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip = number of BFGS updates skipped
Nact = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F = final function value
         * * *
      Tit Tnf Tnint Skip Nact Projg
             25 1 0 0 1.236D-04 6.501D-01
       21
 F = 0.65011048943738603
CONVERGENCE: REL REDUCTION OF F <= FACTR*EPSMCH
232.4371244509617
RUNNING THE L-BFGS-B CODE
          * * *
Machine precision = 2.220D-16
      5
                  M =
                               10
At XO
           O variables are exactly at the bounds
At iterate 0 f= 7.18787D-01 |proj g|= 9.59026D-02
At iterate 5 f = 7.08106D - 01 |proj g|= 2.06172D - 03
        * * *
Tit = total number of iterations
Tnf = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip = number of BFGS updates skipped
Nact = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F = final function value
```

N Tit Tnf Tnint Skip Nact Projg F
5 8 10 1 0 0 2.490D-05 7.081D-01
F = 0.70810558300131410

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH 247.92347588844154
RUNNING THE L-BFGS-B CODE

```
Machine precision = 2.220D-16
             6
                   M =
                                10
At XO 0 variables are exactly at the bounds
At iterate 0 f= 7.17275D-01 |proj g|= 6.92563D-02
This problem is unconstrained.
This problem is unconstrained.
At iterate 5 f = 7.09627D - 01 |proj g| = 1.51339D - 02
At iterate 10 f = 7.05947D-01 |proj g|= 6.81795D-04
         * * *
Tit = total number of iterations
Tnf = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip = number of BFGS updates skipped
Nact = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
   = final function value
         * * *
      Tit Tnf Tnint Skip Nact Projg
  6 14
             17 1 0 0 1.443D-05 7.059D-01
 F = 0.70594578908045114
CONVERGENCE: REL REDUCTION OF F <= FACTR*EPSMCH
249.19778513103157
RUNNING THE L-BFGS-B CODE
          * * *
Machine precision = 2.220D-16
N =
             7
                  M =
At XO 0 variables are exactly at the bounds
At iterate 0 f= 7.30144D-01 |proj g|= 2.09902D-01
This problem is unconstrained.
At iterate 5 f = 6.95237D - 01
                                  |proj q| = 9.62762D-03
At iterate 10 f = 6.93459D-01 |proj g|= 1.15392D-02
At iterate 15 f = 6.93333D-01 |proj g| = 1.63103D-04
         * * *
Tit = total number of iterations
Tnf = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip = number of BFGS updates skipped
Nact = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
   = final function value
             Tnf Tnint Skip Nact Projg
```

21 1 0 0 3.401D-06 6.933D-01

F = 0.69333279497058697

17

```
CONVERGENCE: NORM OF PROJECTED GRADIENT <= PGTOL
```

246.95981911011722

RUNNING THE L-BFGS-B CODE

* * *

Machine precision = 2.220D-16

 $N = 8 \qquad M = 10$

At X0 0 variables are exactly at the bounds

At iterate 0 f = 8.78307D - 01 |proj g|= 5.99739D - 01

This problem is unconstrained.

At iterate 5 f= 6.54357D-01 |proj g|= 3.53427D-02

At iterate 10 f = 6.50627D-01 |proj g| = 8.18044D-03

At iterate 15 f = 6.50192D-01 |proj g| = 2.68887D-03

At iterate 20 f= 6.49577D-01 |proj g|= 7.49018D-03

At iterate 25 f = 6.49169D - 01 |proj g|= 2.04284D-03

At iterate 30 f= 6.49165D-01 |proj g|= 2.64487D-05

* * *

Tit = total number of iterations

Tnf = total number of function evaluations

Tnint = total number of segments explored during Cauchy searches

Skip = number of BFGS updates skipped

Nact = number of active bounds at final generalized Cauchy point

Projg = norm of the final projected gradient

F = final function value

* * *

N Tit Tnf Tnint Skip Nact Projg F 8 31 36 1 0 0 7.220D-06 6.492D-01

F = 0.64916452272641945

CONVERGENCE: NORM OF PROJECTED GRADIENT <= PGTOL

234.11927963607693

RUNNING THE L-BFGS-B CODE

* * *

Machine precision = 2.220D-16

N = 6 M = 10

At XO 0 variables are exactly at the bounds

At iterate 0 f= 6.77720D-01 |proj g|= 9.54652D-02

This problem is unconstrained.

At iterate 5 f = 6.59496D - 01 | proj g|= 4.15299D - 02

At iterate 10 f = 6.58518D - 01 |proj g| = 1.97529D - 03

At iterate 15 f = 6.58516D - 01 |proj g| = 2.05153D - 03

* * *

Tit = total number of iterations

Tnf = total number of function evaluations

Tnint = total number of segments explored during Cauchy searches

Skip = number of BFGS updates skipped

Nact = number of active bounds at final generalized Cauchy point

Projg = norm of the final projected gradient

F = final function value

* * *

N Tit Tnf Tnint Skip Nact Projg F 6 15 31 1 0 0 2.052D-03 6.585D-01 F = 0.65851584470945967

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH 233.26132382237844

RUNNING THE L-BFGS-B CODE

* * *

Machine precision = 2.220D-16N = 7 M = 10

At XO 0 variables are exactly at the bounds

At iterate 0 f = 6.77125D-01 |proj g|= 8.00668D-02 This problem is unconstrained. At iterate 5 f= 6.61527D-01 |proj g| = 1.63720D-02At iterate 10 f= 6.58561D-01 |proj g|= 6.44611D-03At iterate 15 f= 6.58515D-01 |proj g| = 1.76476D-03At iterate 20 f= 6.58513D-01 |proj g| = 3.78371D-03At iterate 25 f = 6.58430D - 01 |proj g| = 8.05060D - 02At iterate 30 f= 6.58270D-01 |proj g|= 5.15799D-01|proj g| = 1.67841D-02At iterate 35 f = 6.58133D - 01At iterate 40 f= 6.58129D-01 |proj g|= 1.18111D-02 At iterate 45 f = 6.58127D-01 |proj g|= 9.97962D-04 At iterate 50 f= 6.58127D-01 |proj q|= 8.65448D-04

* * *

Tit = total number of iterations

Tnf = total number of function evaluations

Tnint = total number of segments explored during Cauchy searches

Skip = number of BFGS updates skipped

Nact = number of active bounds at final generalized Cauchy point

Projg = norm of the final projected gradient

F = final function value

* * *

N Tit Tnf Tnint Skip Nact Projg F
7 50 70 1 0 0 8.654D-04 6.581D-01
F = 0.65812728484547200

STOP: TOTAL NO. of ITERATIONS REACHED LIMIT

235.1307677080786

RUNNING THE L-BFGS-B CODE

Machine precision = 2.220D-16 N = 8 M =

At XO 0 variables are exactly at the bounds

At iterate 0 f= 7.00515D-01 |proj g|= 2.00286D-01 This problem is unconstrained.

At iterate 5 f= 6.60804D-01 |proj g|= 9.85993D-03 At iterate 10 f= 6.58362D-01 |proj g|= 7.35791D-03 At iterate 15 f= 6.58014D-01 |proj g|= 4.11838D-03 At iterate 20 f= 6.57980D-01 |proj g|= 2.12878D-03

10

* * *

Tit = total number of iterations

Tnf = total number of function evaluations

Thint = total number of segments explored during Cauchy searches

Skip = number of BFGS updates skipped

Nact = number of active bounds at final generalized Cauchy point

Projg = norm of the final projected gradient

F = final function value

* * *

N Tit Tnf Tnint Skip Nact Projg F 8 20 32 1 0 0 2.129D-03 6.580D-01 F = 0.65797968137250673

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH 237.08117294116227

RUNNING THE L-BFGS-B CODE

* * *

Machine precision = 2.220D-16

 $N = 9 \qquad M = 10$

At XO 0 variables are exactly at the bounds

At iterate 0 f= 8.51337D-01 |proj g|= 5.15679D-01

This problem is unconstrained.

At iterate 5 f= 6.62396D-01 |proj g|= 2.53038D-02

At iterate 10 f= 6.52653D-01 |proj g|= 3.49641D-02

At iterate 15 f= 6.48882D-01 |proj g|= 1.06971D-02

At iterate 20 f= 6.48531D-01 |proj g|= 2.83413D-02

At iterate 25 f= 6.48520D-01 |proj g|= 1.19757D-03

At iterate 30 f= 6.48519D-01 |proj g|= 1.14967D-04

* * *

Tit = total number of iterations

Tnf = total number of function evaluations

Thint = total number of segments explored during Cauchy searches

Skip = number of BFGS updates skipped

Nact = number of active bounds at final generalized Cauchy point

```
Projg = norm of the final projected gradient
F = final function value
      Tit Tnf Tnint Skip Nact Projg
      30 40 1 0 0 1.150D-04 6.485D-01
 F = 0.64851908959036009
CONVERGENCE: REL REDUCTION OF F <= FACTR*EPSMCH
235.90241410236098
RUNNING THE L-BFGS-B CODE
         * * *
Machine precision = 2.220D-16
N = 4 M =
                              10
At XO
           0 variables are exactly at the bounds
At iterate 0 f = 8.21138D - 01 | proj g = 1.20245D - 01
At iterate 5 f= 8.14617D-01 |proj g|= 1.40544D-02
At iterate 10 f= 8.14406D-01 |proj g|= 4.39436D-06
        * * *
Tit = total number of iterations
Tnf = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip = number of BFGS updates skipped
Nact = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F = final function value
         * * *
  N Tit Tnf Tnint Skip Nact Projg F
  4 10
             13 1 0 0 4.394D-06 8.144D-01
  F = 0.81440587199108583
CONVERGENCE: NORM OF PROJECTED GRADIENT <= PGTOL
281.64037298900485
RUNNING THE L-BFGS-B CODE
         * * *
Machine precision = 2.220D-16
5 M =
                              10
At XO 0 variables are exactly at the bounds
At iterate 0 f= 7.50224D-01 |proj q|= 1.28938D-01
This problem is unconstrained.
This problem is unconstrained.
At iterate 5 f = 7.37212D-01 |proj g| = 2.18112D-02
At iterate 10 f= 7.36504D-01 |proj g|= 6.08097D-05
        * * *
```

Tnint = total number of segments explored during Cauchy searches

Tit = total number of iterations

Tnf = total number of function evaluations

```
Skip = number of BFGS updates skipped
Nact = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
   = final function value
          * * *
      Tit Tnf Tnint Skip Nact Projg
       12
              15
                   1 0 0 8.463D-06 7.365D-01
 F = 0.73650443302061552
CONVERGENCE: NORM OF PROJECTED GRADIENT <= PGTOL
257.4654894949268
RUNNING THE L-BFGS-B CODE
          * * *
Machine precision = 2.220D-16
             6
                   M =
                                10
At XO
           O variables are exactly at the bounds
At iterate 0 f= 7.51036D-01 |proj g|= 2.39941D-01
This problem is unconstrained.
At iterate 5 f = 6.97433D-01 |proj g|= 1.84132D-02
At iterate 10 f = 6.96886D - 01 |proj q|= 1.17435D - 03
         * * *
Tit = total number of iterations
Tnf = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip = number of BFGS updates skipped
Nact = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F = final function value
          * * *
             Tnf Tnint Skip Nact Projg
       13
                  1 0 0 6.971D-06 6.969D-01
              15
 F = 0.69688601069630796
CONVERGENCE: NORM OF PROJECTED GRADIENT <= PGTOL
246.15369959395946
RUNNING THE L-BFGS-B CODE
          * * *
Machine precision = 2.220D-16
             7
                  M =
                               10
```

At X0 0 variables are exactly at the bounds

At iterate 0 f= 7.43776D-01 |proj g|= 2.68832D-01

This problem is unconstrained.

At iterate 5 f= 6.48075D-01 |proj g|= 3.58243D-02

At iterate 10 f= 6.43273D-01 |proj g|= 1.77765D-03

At iterate 15 f= 6.43259D-01 |proj g|= 6.29579D-06

```
Tit = total number of iterations
Tnf = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip = number of BFGS updates skipped
Nact = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F = final function value
          * * *
  N Tit Tnf Tnint Skip Nact Projg
              18 1 0 0 6.296D-06 6.433D-01
 F = 0.64325910432443434
CONVERGENCE: NORM OF PROJECTED GRADIENT <= PGTOL
230.13505905300994
RUNNING THE L-BFGS-B CODE
         * * *
Machine precision = 2.220D-16
                          10
N = 5 \qquad M =
At XO
           O variables are exactly at the bounds
At iterate 0 f = 7.74291D - 01 |proj g| = 1.30932D - 01
At iterate 5 f = 7.62174D - 01
                                 |proj g| = 2.00068D-02
At iterate 10 f= 7.61714D-01 |proj g|= 4.08662D-04
This problem is unconstrained.
This problem is unconstrained.
Tit = total number of iterations
Tnf = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip = number of BFGS updates skipped
Nact = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F = final function value
         * * *
  N Tit Tnf Tnint Skip Nact Projg
                     1 0 0 5.695D-06 7.617D-01
       12
              15
  F = 0.76171358963564784
CONVERGENCE: NORM OF PROJECTED GRADIENT <= PGTOL
265.93576611757766
RUNNING THE L-BFGS-B CODE
          * * *
Machine precision = 2.220D-16
                           10
N =
             6
                  M =
At XO 0 variables are exactly at the bounds
At iterate 0 f= 7.50363D-01 |proj g|= 1.30542D-01
At iterate 5 f= 7.38232D-01 |proj g|= 2.31131D-02
```

At iterate 10 f = 7.36664D - 01 |proj g|= 1.58959D-03

```
At iterate 15 f = 7.36507D-01 |proj g| = 5.42583D-04
          * * *
Tit = total number of iterations
Tnf = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip = number of BFGS updates skipped
Nact = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
    = final function value
          * * *
      Tit Tnf Tnint Skip Nact Projg
                  1 0 0 6.629D-05 7.365D-01
       17
              27
 F = 0.73650724348068641
CONVERGENCE: REL REDUCTION OF F <= FACTR*EPSMCH
259.46643380951065
RUNNING THE L-BFGS-B CODE
          * * *
Machine precision = 2.220D-16
             7
                   M =
                               10
At XO
           O variables are exactly at the bounds
At iterate
          0 f= 7.43094D-01 |proj g|= 2.32287D-01
This problem is unconstrained.
At iterate 5 f= 6.96697D-01
                                  |proj g| = 1.43016D-02
At iterate 10 f= 6.95850D-01
                                 |proj g| = 4.68647D-03
At iterate 15 f = 6.95462D - 01
                                 |proj g| = 1.83859D-03
At iterate 20 f= 6.95459D-01
                                 |proj q| = 4.72526D-06
         * * *
Tit = total number of iterations
Tnf = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip = number of BFGS updates skipped
Nact = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F = final function value
          * * *
             Tnf Tnint Skip Nact Projg
       20
                   1 0 0 4.725D-06 6.955D-01
              25
 F = 0.69545876006092111
```

CONVERGENCE: NORM OF PROJECTED GRADIENT <= PGTOL

M =

10

247.6741433804695

This problem is unconstrained.

RUNNING THE L-BFGS-B CODE

* * *

Machine precision = 2.220D-16 8

```
At iterate 0 f= 7.99865D-01 |proj q|= 4.33212D-01
At iterate 5 f = 6.47713D - 01
                                 |proj g| = 3.79531D-02
At iterate 10 f= 6.43534D-01
                                 |proj g| = 2.32821D-03
At iterate 15 f = 6.43095D-01 |proj g|= 6.44156D-03
At iterate 20 f= 6.42911D-01 |proj g|= 2.31805D-04
        * * *
Tit = total number of iterations
Tnf = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip = number of BFGS updates skipped
Nact = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F = final function value
         * * *
  N Tit Tnf Tnint Skip Nact Projg
             27 1 0 0 1.906D-05 6.429D-01
  F = 0.64291139111638829
CONVERGENCE: REL REDUCTION OF F <= FACTR*EPSMCH
232.01822741510645
RUNNING THE L-BFGS-B CODE
         * * *
Machine precision = 2.220D-16
N = 6 M =
At XO 0 variables are exactly at the bounds
At iterate 0 f = 7.17402D - 01 |proj g|= 9.48132D - 02
This problem is unconstrained.
At iterate 5 f = 7.02377D - 01 |proj g|= 1.48245D-02
At iterate 10 f= 7.02064D-01 |proj g|= 5.17010D-05
        * * *
Tit = total number of iterations
Tnf = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip = number of BFGS updates skipped
Nact = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F = final function value
          * * *
            Tnf Tnint Skip Nact Projg F
      Tit
              16 1 0 0 2.014D-05 7.021D-01
 F = 0.70206413399167156
CONVERGENCE: REL REDUCTION OF F <= FACTR*EPSMCH
```

O variables are exactly at the bounds

At X0

247.89354902120164

RUNNING THE L-BFGS-B CODE

Machine precision = 2.220D-16 7 10 M =At XO O variables are exactly at the bounds At iterate 0 f= 7.16002D-01 |proj g|= 1.00704D-01This problem is unconstrained. 5 f = 7.04714D-01At iterate |proj g| = 3.04403D-02At iterate 10 f= 7.01659D-01 |proj g| = 1.49126D-02At iterate 15 f = 7.00299D-01 |proj g| = 1.75608D-04* * * Tit = total number of iterations Tnf = total number of function evaluations Tnint = total number of segments explored during Cauchy searches Skip = number of BFGS updates skipped Nact = number of active bounds at final generalized Cauchy point Projg = norm of the final projected gradient = final function value * * * Tit Tnf Tnint Skip Nact Projg 17 21 1 0 0 3.577D-05 7.003D-01 F = 0.70029882410877642CONVERGENCE: REL REDUCTION OF F <= FACTR*EPSMCH 249.30040490054887 RUNNING THE L-BFGS-B CODE * * * Machine precision = 2.220D-16 8 M = At XO 0 variables are exactly at the bounds At iterate 0 f = 7.24489D - 01 |proj g| = 1.98986D - 01At iterate 5 f = 6.87817D - 01|proj g| = 2.32163D-02This problem is unconstrained. At iterate 10 f = 6.85942D - 01|proj g| = 5.14514D-03At iterate 15 f = 6.85128D - 01 |proj g| = 3.66570D - 04* * * Tit = total number of iterations Tnf = total number of function evaluations Tnint = total number of segments explored during Cauchy searches Skip = number of BFGS updates skipped Nact = number of active bounds at final generalized Cauchy point Projg = norm of the final projected gradient

N Tit Tnf Tnint Skip Nact Projg F 8 19 23 1 0 0 6.915D-06 6.851D-01 F = 0.68512825765437990

F = final function value

```
CONVERGENCE: NORM_OF_PROJECTED_GRADIENT_<=_PGTOL 246.20309457187165
RUNNING THE L-BFGS-B CODE
```

Machine precision = 2.220D-16N = 9 M = 10

At XO 0 variables are exactly at the bounds

At iterate 0 f= 8.69957D-01 |proj g|= 5.45411D-01

This problem is unconstrained.

At iterate 5 f= 6.46605D-01 |proj g|= 2.92369D-02

At iterate 10 f = 6.43571D-01 |proj g| = 8.03727D-03

At iterate 15 f = 6.43094D-01 |proj g| = 1.76120D-03

At iterate 20 f= 6.42773D-01 |proj g|= 7.45039D-03

At iterate 25 f = 6.41864D-01 |proj g|= 4.12166D-03

At iterate 30 f= 6.41829D-01 |proj g|= 5.70427D-04

* * *

Tit = total number of iterations

Tnf = total number of function evaluations

Tnint = total number of segments explored during Cauchy searches

Skip = number of BFGS updates skipped

Nact = number of active bounds at final generalized Cauchy point

Projg = norm of the final projected gradient

F = final function value

* * *

N Tit Tnf Tnint Skip Nact Projg F 9 34 40 1 0 0 4.696D-06 6.418D-01

F = 0.64182878746527827

CONVERGENCE: NORM OF PROJECTED GRADIENT <= PGTOL

233.6544725883335

RUNNING THE L-BFGS-B CODE

* * *

Machine precision = 2.220D-16

 $N = 7 \qquad M = 10$

At XO 0 variables are exactly at the bounds

At iterate 0 f= 6.75215D-01 |proj g|= 1.00220D-01

This problem is unconstrained.

At iterate 5 f = 6.57304D-01 |proj g| = 9.48894D-02

At iterate 10 f = 6.53962D-01 |proj g|= 1.48519D-02

Warning: more than 10 function and gradient

evaluations in the last line search. Termination

may possibly be caused by a bad search direction.

This problem is unconstrained.

* * *

Tit = total number of iterations

```
Tnf = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip = number of BFGS updates skipped
Nact = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F = final function value
         * * *
  N Tit Tnf Tnint Skip Nact Projg
                         0 0 1.485D-02 6.540D-01
       11
              34
                   1
 F = 0.65396239615634910
CONVERGENCE: REL REDUCTION OF F <= FACTR*EPSMCH
233.7313651085333
RUNNING THE L-BFGS-B CODE
         * * *
Machine precision = 2.220D-16
N = 8
                  M =
                               10
At XO 0 variables are exactly at the bounds
At iterate 0 f= 6.74648D-01 |proj q|= 1.01022D-01
At iterate 5 f = 6.57325D - 01
                                 |proj g| = 4.47852D-02
At iterate 10 f = 6.54185D-01 |proj g| = 4.21309D-02
At iterate 15 f = 6.54010D - 01 |proj g| = 8.59748D - 03
At iterate 20 f= 6.53947D-01
                                 |proj g| = 2.00456D-02
At iterate 25 f = 6.53946D-01 |proj g|= 4.38169D-04
        * * *
Tit = total number of iterations
Tnf = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip = number of BFGS updates skipped
Nact = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F = final function value
         * * *
      Tit Tnf Tnint Skip Nact Projg
       29
             53 1 0 0 1.649D-03 6.539D-01
  8
  F = 0.65394586389942277
CONVERGENCE: REL REDUCTION OF F <= FACTR*EPSMCH
235.72581027020604
RUNNING THE L-BFGS-B CODE
          * * *
Machine precision = 2.220D-16
             9 	 M = 10
At XO
           O variables are exactly at the bounds
At iterate 0 f = 6.97305D-01 |proj g|= 1.90426D-01
This problem is unconstrained.
```

```
At iterate 5 f= 6.57007D-01 |proj g|= 1.73955D-02

At iterate 10 f= 6.53847D-01 |proj g|= 1.06436D-02

At iterate 15 f= 6.53494D-01 |proj g|= 1.65000D-02

At iterate 20 f= 6.53402D-01 |proj g|= 5.00472D-03

At iterate 25 f= 6.53385D-01 |proj g|= 4.46784D-04

At iterate 30 f= 6.53385D-01 |proj g|= 4.94503D-04
```

Tit = total number of iterations

Tnf = total number of function evaluations

Tnint = total number of segments explored during Cauchy searches

Skip = number of BFGS updates skipped

Nact = number of active bounds at final generalized Cauchy point

Projg = norm of the final projected gradient

F = final function value

* * *

N Tit Tnf Tnint Skip Nact Projg F
9 31 43 1 0 0 2.739D-03 6.534D-01
F = 0.65338467475113848

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH 237.53725071638252

This problem is unconstrained.

RUNNING THE L-BFGS-B CODE

* * *

Machine precision = 2.220D-16N = 10 M =

At XO 0 variables are exactly at the bounds

At iterate 0 f= 8.44310D-01 |proj g|= 5.04630D-01 At iterate 5 f= 6.55864D-01 |proj g|= 4.33466D-02 At iterate 10 f= 6.48780D-01 |proj g|= 3.38697D-02 At iterate 15 f= 6.42363D-01 |proj g|= 1.68626D-02 At iterate 20 f= 6.41353D-01 |proj g|= 5.22322D-03 At iterate 25 f= 6.41206D-01 |proj g|= 9.18739D-04 At iterate 30 f= 6.41204D-01 |proj g|= 4.14275D-05

* * *

Tit = total number of iterations

Tnf = total number of function evaluations

Tnint = total number of segments explored during Cauchy searches

Skip = number of BFGS updates skipped

Nact = number of active bounds at final generalized Cauchy point

Projg = norm of the final projected gradient

F = final function value

* * *

```
Tit Inf Inint Skip Nact Projg
             39 1 0 0 1.952D-05 6.412D-01
 F = 0.64120363681203629
CONVERGENCE: REL REDUCTION OF F <= FACTR*EPSMCH
235.4444219688442
RUNNING THE L-BFGS-B CODE
         * * *
This problem is unconstrained.
This problem is unconstrained.
This problem is unconstrained.
Machine precision = 2.220D-16
      2
                               10
At XO
           O variables are exactly at the bounds
At iterate 0 f = 8.95902D - 01 |proj g|= 1.80231D - 05
        * * *
Tit = total number of iterations
Tnf = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip = number of BFGS updates skipped
Nact = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F = final function value
         * * *
      Tit Inf Inint Skip Nact Projg
       1
               5
                  1 0 0 1.049D-05 8.959D-01
 F = 0.89590210367205392
CONVERGENCE: REL REDUCTION OF F <= FACTR*EPSMCH
305.0231068338101
RUNNING THE L-BFGS-B CODE
          * * *
Machine precision = 2.220D-16
                               10
             3 M =
           O variables are exactly at the bounds
At iterate 0 f = 8.26672D - 01 |proj g| = 5.22545D - 02
At iterate 5 f = 8.23396D-01 |proj g|= 6.63371D-04
         * * *
Tit = total number of iterations
Tnf = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip = number of BFGS updates skipped
Nact = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
    = final function value
          * * *
       Tit Inf Inint Skip Nact Projg F
             12 1 0 0 1.093D-05 8.234D-01
```

```
F = 0.82339620999231489
CONVERGENCE: REL REDUCTION OF F <= FACTR*EPSMCH
282.6611265574178
RUNNING THE L-BFGS-B CODE
         * * *
Machine precision = 2.220D-16
N = 4
                               10
At XO
           O variables are exactly at the bounds
At iterate 0 f= 8.05545D-01 |proj g|= 2.22568D-01
At iterate 5 f = 7.69155D-01 |proj g|= 1.63081D-02
At iterate 10 f = 7.68881D-01 |proj g|= 1.51863D-05
         * * *
Tit = total number of iterations
Tnf = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip = number of BFGS updates skipped
Nact = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F = final function value
          * * *
     Tit Inf Inint Skip Nact Projg
              13 1 0 0 4.311D-07 7.689D-01
       11
  F = 0.76888147282791885
CONVERGENCE: NORM OF PROJECTED GRADIENT <= PGTOL
266.3441748701807
RUNNING THE L-BFGS-B CODE
         * * *
Machine precision = 2.220D-16
N = 5 \qquad M =
                               10
     O variables are exactly at the bounds
At iterate 0 f= 8.52855D-01 |proj q|= 4.91827D-01
```

At iterate 0 f= 8.52855D-01 |proj g|= 4.91827D-01

This problem is unconstrained.

At iterate 5 f= 7.17006D-01 |proj g|= 5.92354D-02

At iterate 10 f= 7.09863D-01 |proj g|= 5.03701D-04

* * *

Tit = total number of iterations
Tnf = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip = number of BFGS updates skipped
Nact = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F = final function value

* * *

N Tit Tnf Tnint Skip Nact Projg F

```
12 14 1 0 0 3.864D-06 7.099D-01
   5
  F = 0.70986330458190028
CONVERGENCE: NORM OF PROJECTED GRADIENT <= PGTOL
248.5140703395185
RUNNING THE L-BFGS-B CODE
          * * *
Machine precision = 2.220D-16
             3
                  M =
                               1.0
At XO 0 variables are exactly at the bounds
At iterate 0 f= 8.36436D-01 |proj g|= 3.48415D-02
At iterate 5 f = 8.35310D-01 |proj g|= 1.63260D-03
        * * *
Tit = total number of iterations
Tnf = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip = number of BFGS updates skipped
Nact = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F = final function value
         * * *
  N Tit Tnf Tnint Skip Nact Projg F
             9 1 0 0 4.283D-06 8.353D-01
  3 7
 F = 0.83530920465063574
CONVERGENCE: NORM OF PROJECTED GRADIENT <= PGTOL
286.6638927626136
RUNNING THE L-BFGS-B CODE
         * * *
Machine precision = 2.220D-16
             4
                  M =
                               10
At XO 0 variables are exactly at the bounds
At iterate 0 f= 8.27300D-01 |proj g|= 4.50971D-02
At iterate 5 f= 8.21418D-01
                                 |proj g| = 9.59003D-03
At iterate 10 f= 8.21268D-01 |proj g|= 5.93968D-06
         * * *
Tit = total number of iterations
Tnf = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip = number of BFGS updates skipped
Nact = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F = final function value
```

N Tit Tnf Tnint Skip Nact Projg F 4 10 13 1 0 0 5.940D-06 8.213D-01 F = 0.82126767041329951

```
CONVERGENCE: NORM OF PROJECTED GRADIENT <= PGTOL
283.94593725886864
RUNNING THE L-BFGS-B CODE
          * * *
Machine precision = 2.220D-16
              5
                  M =
                                10
At XO
           O variables are exactly at the bounds
At iterate
           0 	 f = 7.99207D - 01
                                  |proj g| = 2.03616D-01
This problem is unconstrained.
This problem is unconstrained.
This problem is unconstrained.
          5 	 f = 7.67081D - 01
At iterate
                                  |proj g| = 3.42964D-03
At iterate 10 f= 7.66871D-01 |proj g|= 7.72442D-03
         * * *
Tit = total number of iterations
Tnf = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip = number of BFGS updates skipped
Nact = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
   = final function value
         * * *
      Tit Inf Inint Skip Nact Projg
       13
              16
                  1 0 0 8.213D-06 7.669D-01
  F = 0.76685171436551391
CONVERGENCE: NORM OF PROJECTED GRADIENT <= PGTOL
267.6621760268127
RUNNING THE L-BFGS-B CODE
          * * *
Machine precision = 2.220D-16
             6 M =
                                10
           O variables are exactly at the bounds
At iterate
          0 f= 9.46112D-01 |proj g|= 7.78727D-01
This problem is unconstrained.
At iterate 5 f = 7.15045D - 01
                                  |proj g| = 5.92912D-02
At iterate 10 f = 7.10555D-01
                                  |proj g| = 3.54398D-03
At iterate 15 f = 7.09521D-01
                                  |proj g| = 6.03635D-03
At iterate 20 f= 7.09451D-01 |proj g|= 3.07046D-05
         * * *
Tit = total number of iterations
```

Tnf = total number of function evaluations

Tnint = total number of segments explored during Cauchy searches

Skip = number of BFGS updates skipped

Nact = number of active bounds at final generalized Cauchy point

Projg = norm of the final projected gradient

```
= final function value
          * * *
      Tit Tnf Tnint Skip Nact Projg
                  1 0 0 1.371D-05 7.095D-01
       21
             25
  F = 0.70945083639773288
CONVERGENCE: REL REDUCTION OF F <= FACTR*EPSMCH
250.37548102963825
RUNNING THE L-BFGS-B CODE
          * * *
Machine precision = 2.220D-16
                  M =
             4
           O variables are exactly at the bounds
At XO
At iterate 0 f= 7.63355D-01 |proj g|= 4.93754D-02
At iterate 5 f = 7.61132D - 01 |proj g| = 1.26502D - 04
         * * *
Tit = total number of iterations
Tnf = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip = number of BFGS updates skipped
Nact = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
   = final function value
         * * *
      Tit Inf Inint Skip Nact Projg
       6
              8 1 0 0 5.554D-06 7.611D-01
  F = 0.76113207546693951
CONVERGENCE: NORM OF PROJECTED GRADIENT <= PGTOL
263.74037735689166
RUNNING THE L-BFGS-B CODE
         * * *
Machine precision = 2.220D-16
      5
                               10
At XO
           O variables are exactly at the bounds
At iterate 0 f= 7.63346D-01 |proj g|= 2.48933D-02
At iterate 5 f= 7.62080D-01 |proj g|= 2.95649D-02
This problem is unconstrained.
This problem is unconstrained.
At iterate 10 f = 7.60319D-01 |proj g|= 1.58316D-03
         * * *
Tit = total number of iterations
```

Tnf = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip = number of BFGS updates skipped
Nact = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient

= final function value

* * *

N Tit Tnf Tnint Skip Nact Projg F
5 14 23 1 0 0 1.552D-05 7.603D-01
F = 0.76031819362822073

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH

265.4669130590822

RUNNING THE L-BFGS-B CODE

* * *

Machine precision = 2.220D-16

 $N = 6 \qquad M = 10$

At XO 0 variables are exactly at the bounds

At iterate 0 f= 7.81480D-01 |proj g|= 1.64977D-01

This problem is unconstrained.

At iterate 5 f= 7.60282D-01 |proj g|= 8.60639D-03

At iterate 10 f = 7.59834D-01 |proj g|= 7.77013D-04

At iterate 15 f= 7.59824D-01 |proj g|= 4.62555D-05

* * *

Tit = total number of iterations

Tnf = total number of function evaluations

Tnint = total number of segments explored during Cauchy searches

Skip = number of BFGS updates skipped

Nact = number of active bounds at final generalized Cauchy point

Projg = norm of the final projected gradient

F = final function value

* * *

N Tit Tnf Tnint Skip Nact Projg F
6 16 20 1 0 0 1.101D-05 7.598D-01
F = 0.75982414957925215

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH

267.3009142586287

RUNNING THE L-BFGS-B CODE

* * *

Machine precision = 2.220D-16

 $N = 7 \qquad M = 10$

At XO 0 variables are exactly at the bounds

At iterate 0 f= 1.04738D+00 |proj g|= 9.81664D-01 This problem is unconstrained. At iterate 5 f= 7.12915D-01 |proj g|= 2.73809D-02

At iterate 10 f = 7.09495D-01 |proj g| = 1.83814D-03

At iterate 15 f= 7.09466D-01 |proj g|= 3.05339D-03

At iterate 20 f= 7.09081D-01 |proj g|= 4.92997D-03

At iterate 25 f= 7.09038D-01 |proj g|= 7.71599D-05

```
Tit = total number of iterations
Tnf = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip = number of BFGS updates skipped
Nact = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
    = final function value
         * * *
      Tit Tnf Tnint Skip Nact Projg
       26
                   1 0 0 6.290D-06 7.090D-01
              34
  F = 0.70903769762797708
CONVERGENCE: NORM OF PROJECTED GRADIENT <= PGTOL
252.2366664030003
RUNNING THE L-BFGS-B CODE
          * * *
Machine precision = 2.220D-16
              5
                  M =
At XO
           O variables are exactly at the bounds
At iterate 0 f= 7.15533D-01 |proj g|= 9.03745D-02
This problem is unconstrained.
At iterate 5 f = 7.09620D - 01 |proj g|= 2.45064D-03
         * * *
Tit = total number of iterations
Tnf = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip = number of BFGS updates skipped
Nact = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F = final function value
          * * *
             Tnf Tnint Skip Nact Projg
                         0 0 1.314D-05 7.096D-01
               12
                      1
  F = 0.70960396290125127
CONVERGENCE: REL REDUCTION OF F <= FACTR*EPSMCH
248.42693153482043
RUNNING THE L-BFGS-B CODE
          * * *
Machine precision = 2.220D-16
N =
              6
                   M =
                                10
At XO
           O variables are exactly at the bounds
           0 	 f = 7.15547D - 01
                                  |proj g| = 9.01102D-02
At iterate
This problem is unconstrained.
At iterate 5 f = 7.09755D-01 |proj g|= 1.25701D-02
At iterate 10 f = 7.08709D - 01 |proj g|= 2.95629D - 03
```

```
Tit = total number of iterations
Tnf = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip = number of BFGS updates skipped
Nact = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F = final function value
      Tit Tnf Tnint Skip Nact Projg
       13 16 1 0 0 4.429D-06 7.087D-01
 F = 0.70870262775345849
CONVERGENCE: NORM OF PROJECTED GRADIENT <= PGTOL
250.12408292516204
RUNNING THE L-BFGS-B CODE
         * * *
Machine precision = 2.220D-16
N = 7 \qquad M =
                               10
At XO
           O variables are exactly at the bounds
At iterate 0 f= 7.35580D-01 |proj g|= 1.60111D-01
At iterate 5 f = 7.09091D-01 |proj g| = 7.79543D-03
This problem is unconstrained.
At iterate 10 f= 7.08530D-01
                                 |proj g| = 9.52942D-04
At iterate 15 f = 7.08518D - 01 |proj g| = 9.43567D - 04
          * * *
Tit = total number of iterations
Tnf = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip = number of BFGS updates skipped
Nact = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
   = final function value
         * * *
      Tit Tnf Tnint Skip Nact Projg
              20 1 0 0 6.426D-06 7.085D-01
       18
 F = 0.70851749985015300
CONVERGENCE: NORM OF PROJECTED GRADIENT <= PGTOL
252.0618799496514
RUNNING THE L-BFGS-B CODE
          * * *
Machine precision = 2.220D-16
      8
                  M =
At XO
       0 variables are exactly at the bounds
At iterate
           0 f= 1.00784D+00 |proj g|= 8.21438D-01
This problem is unconstrained.
```

At iterate 5 f= 7.18831D-01 |proj g|= 1.82600D-02

```
At iterate 10 f = 7.13115D-01 |proj g|= 2.26027D-02
At iterate 15 f= 7.08291D-01
                                 |proj g| = 5.94671D-03
At iterate 20 f = 7.08267D - 01 |proj g|= 9.89193D-05
         * * *
Tit = total number of iterations
Tnf = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip = number of BFGS updates skipped
Nact = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F = final function value
         * * *
      Tit Tnf Tnint Skip Nact Projg
             28 1 0 0 2.420D-05 7.083D-01
       22
 F = 0.70826691161430932
CONVERGENCE: REL REDUCTION OF F <= FACTR*EPSMCH
253.97768230240794
RUNNING THE L-BFGS-B CODE
         * * *
Machine precision = 2.220D-16
N = 3 \qquad M = 10
At XO
           O variables are exactly at the bounds
At iterate 0 f= 8.33556D-01 |proj g|= 8.21707D-02
At iterate 5 f = 8.26302D - 01 |proj g|= 2.14264D-02
        * * *
Tit = total number of iterations
Tnf = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip = number of BFGS updates skipped
Nact = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F = final function value
         * * *
      Tit Tnf Tnint Skip Nact Projg
  3 9
             17 1 0 0 1.245D-05 8.261D-01
 F = 0.82613784764737463
CONVERGENCE: REL REDUCTION OF F <= FACTR*EPSMCH
283.58231680951786
RUNNING THE L-BFGS-B CODE
         * * *
Machine precision = 2.220D-16
N =
      4 	 M =
                               10
          O variables are exactly at the bounds
At XO
At iterate 0 f= 7.61693D-01 |proj g|= 1.09417D-01
```

```
This problem is unconstrained.
 This problem is unconstrained.
This problem is unconstrained.
At iterate 5 f = 7.44618D-01 |proj g|= 7.67522D-03
         * * *
Tit = total number of iterations
Tnf = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip = number of BFGS updates skipped
Nact = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F = final function value
     Tit Tnf Tnint Skip Nact Projg
             12 1 0 0 5.615D-06 7.445D-01
 F = 0.74453494734673664
CONVERGENCE: NORM OF PROJECTED GRADIENT <= PGTOL
258.1637423085035
RUNNING THE L-BFGS-B CODE
         * * *
Machine precision = 2.220D-16
N = 5 \qquad M =
                               10
At XO
           O variables are exactly at the bounds
At iterate 0 f= 7.61076D-01 |proj q|= 2.42857D-01
At iterate 5 f = 7.07892D-01 |proj g| = 3.36001D-02
At iterate 10 f= 7.06092D-01 |proj g|= 5.55644D-04
         * * *
Tit = total number of iterations
Tnf = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip = number of BFGS updates skipped
Nact = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F = final function value
          * * *
  N Tit Tnf Tnint Skip Nact Projg F
       13
              16 1 0 0 3.137D-06 7.061D-01
  F = 0.70609215005642767
CONVERGENCE: NORM OF PROJECTED GRADIENT <= PGTOL
247.2469624189597
RUNNING THE L-BFGS-B CODE
         * * *
Machine precision = 2.220D-16
N = 6 M =
                               10
At X0 0 variables are exactly at the bounds
```

```
0 	 f = 7.64468D - 01
                                 |proj g| = 3.27068D-01
At iterate
This problem is unconstrained.
At iterate
          5 f= 6.56580D-01 |proj g|= 3.94807D-02
At iterate 10 f = 6.52590D - 01 |proj g|= 9.68651D-04
At iterate 15 f= 6.52586D-01
                                 |proj g| = 3.39349D-06
         * * *
Tit = total number of iterations
Tnf = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip = number of BFGS updates skipped
Nact = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F = final function value
          * * *
  N Tit Tnf Tnint Skip Nact Projg
                          0 0 3.393D-06 6.526D-01
       15
              18
                  1
 F = 0.65258646980499468
CONVERGENCE: NORM OF PROJECTED GRADIENT <= PGTOL
231.26905385447822
RUNNING THE L-BFGS-B CODE
          * * *
Machine precision = 2.220D-16
      4
                  M =
                               10
At XO 0 variables are exactly at the bounds
At iterate 0 f = 7.75085D - 01 |proj g| = 1.03619D - 01
At iterate 5 f= 7.58853D-01 |proj g|= 6.45651D-03
         * * *
Tit = total number of iterations
Tnf = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip = number of BFGS updates skipped
Nact = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F = final function value
          * * *
     Tit Tnf Tnint Skip Nact Projg
              12 1 0 0 7.541D-06 7.588D-01
 F = 0.75879733492855062
CONVERGENCE: NORM OF PROJECTED GRADIENT <= PGTOL
262.955904535993
RUNNING THE L-BFGS-B CODE
          * * *
```

This problem is unconstrained.

Machine precision = 2.220D-16

This problem is unconstrained.

```
5 \qquad M =
At XO
           0 variables are exactly at the bounds
At iterate 0 f = 7.62049D - 01 |proj g| = 1.10516D - 01
At iterate 5 f = 7.45912D-01 |proj g| = 1.75484D-02
At iterate 10 f = 7.45718D - 01 |proj g| = 4.41902D - 04
        * * *
Tit = total number of iterations
Tnf = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip = number of BFGS updates skipped
Nact = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
   = final function value
         * * *
      Tit Tnf Tnint Skip Nact Projg
  5 13
             16 1 0 0 8.005D-08 7.457D-01
 F = 0.74571731684611797
CONVERGENCE: NORM OF PROJECTED GRADIENT <= PGTOL
260.56101846029566
RUNNING THE L-BFGS-B CODE
          * * *
Machine precision = 2.220D-16
N =
             6
                  M =
                               10
At XO 0 variables are exactly at the bounds
At iterate 0 f= 7.53376D-01 |proj g|= 2.35042D-01
This problem is unconstrained.
At iterate 5 f = 7.06126D - 01
                                 |proj g| = 2.38635D-02
At iterate 10 f = 7.04800D - 01 |proj g|= 9.13601D-03
At iterate 15 f = 7.04608D - 01 |proj g|= 1.11060D - 04
        * * *
Tit = total number of iterations
Tnf = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip = number of BFGS updates skipped
Nact = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F = final function value
          * * *
            Tnf Tnint Skip Nact Projg F
      Tit
             19 1 0 0 7.314D-06 7.046D-01
 F = 0.70460773349200356
```

CONVERGENCE: NORM OF PROJECTED GRADIENT <= PGTOL

248.7481984533132

RUNNING THE L-BFGS-B CODE

Machine precision = 2.220D-16

7

M =

At XO 0 variables are exactly at the bounds

At iterate 0 f= 8.26418D-01 |proj g|= 5.23820D-01

10

This problem is unconstrained.

At iterate 5 f= 6.54685D-01 |proj g|= 2.92359D-02

At iterate 10 f= 6.52887D-01 |proj g|= 2.74564D-03

At iterate 15 f = 6.52653D-01 |proj g| = 5.58338D-03

At iterate 20 f= 6.52375D-01 |proj g|= 3.83815D-04

At iterate 25 f = 6.52375D-01 |proj g|= 2.42551D-05

* * *

Tit = total number of iterations

Tnf = total number of function evaluations

Tnint = total number of segments explored during Cauchy searches

Skip = number of BFGS updates skipped

Nact = number of active bounds at final generalized Cauchy point

Projg = norm of the final projected gradient

F = final function value

* * *

N Tit Tnf Tnint Skip Nact Projg F
7 25 29 1 0 0 2.426D-05 6.524D-01

F = 0.65237477934055121

CONVERGENCE: REL REDUCTION OF F <= FACTR*EPSMCH

233.1979258584252

RUNNING THE L-BFGS-B CODE

* * *

Machine precision = 2.220D-16

 $N = 5 \qquad M = 10$

At XO 0 variables are exactly at the bounds

At iterate 0 f = 7.15919D-01 |proj g|= 8.89871D-02

At iterate 5 f = 7.01866D - 01 |proj g| = 1.20260D - 02

This problem is unconstrained.

This problem is unconstrained.

At iterate 10 f= 7.01475D-01 |proj g|= 2.50822D-05

* * *

Tit = total number of iterations

Tnf = total number of function evaluations

Tnint = total number of segments explored during Cauchy searches

Skip = number of BFGS updates skipped

Nact = number of active bounds at final generalized Cauchy point

Projg = norm of the final projected gradient

F = final function value

* * *

```
N Tit Tnf Tnint Skip Nact Projg F
5 11 14 1 0 0 8.801D-06 7.015D-01
F = 0.70147465413226739
```

CONVERGENCE: NORM_OF_PROJECTED_GRADIENT_<=_PGTOL

245.69548378844183

RUNNING THE L-BFGS-B CODE

* * *

Machine precision = 2.220D-16

 $N = 6 \qquad M = 10$

At XO 0 variables are exactly at the bounds

At iterate 0 f= 7.14993D-01 |proj g|= 7.25010D-02

At iterate 5 f = 7.04098D - 01 |proj g|= 1.45295D - 02

At iterate 10 f = 6.99008D-01 |proj g| = 1.01771D-02

At iterate 15 f = 6.98593D-01 |proj g| = 2.98544D-05

* * *

Tit = total number of iterations

Tnf = total number of function evaluations

Tnint = total number of segments explored during Cauchy searches

Skip = number of BFGS updates skipped

Nact = number of active bounds at final generalized Cauchy point

Projg = norm of the final projected gradient

F = final function value

* * *

N Tit Tnf Tnint Skip Nact Projg F
6 16 20 1 0 0 2.601D-06 6.986D-01
F = 0.69859308938815323

CONVERGENCE: NORM OF PROJECTED GRADIENT <= PGTOL

246.72727803441947

At iterate

RUNNING THE L-BFGS-B CODE

* * *

Machine precision = 2.220D-16

 $N = 7 \qquad M = 10$

At XO 0 variables are exactly at the bounds

5 f= 6.97022D-01

At iterate 0 f= 7.34233D-01 |proj g|= 2.00912D-01This problem is unconstrained.

|proj g| = 1.52423D-02

At iterate 10 f= 6.95423D-01 |proj q|= 7.32450D-03

At iterate 15 f = 6.95151D-01 |proj g| = 6.60199D-04

At iterate 20 f= 6.95150D-01 |proj g|= 6.21087D-06

* * *

 $\mbox{Tit} \quad = \mbox{total number of iterations}$

Tnf = total number of function evaluations

Tnint = total number of segments explored during Cauchy searches

```
Skip = number of BFGS updates skipped
Nact = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
   = final function value
          * * *
      Tit Tnf Tnint Skip Nact Projg F
       20
             22 1 0 0 6.211D-06 6.951D-01
 F = 0.69514978054584631
CONVERGENCE: NORM OF PROJECTED GRADIENT <= PGTOL
247.57032626340435
RUNNING THE L-BFGS-B CODE
          * * *
Machine precision = 2.220D-16
             8
                   M =
                                10
At XO
           0 variables are exactly at the bounds
At iterate 0 f = 9.08263D - 01 |proj g|= 6.75007D - 01
This problem is unconstrained.
At iterate 5 f = 6.56798D - 01 |proj g|= 3.13552D - 02
At iterate 10 f = 6.53026D - 01 |proj g|= 7.23304D-03
At iterate 15 f = 6.52364D - 01
                                 |proj g| = 1.92192D-03
At iterate 20 f= 6.51981D-01
                                  |proj g| = 9.43548D-03
At iterate 25 f = 6.51697D - 01
                                  |proj q| = 8.99864D-04
At iterate 30 f = 6.51695D - 01 |proj g| = 2.57783D - 05
         * * *
Tit = total number of iterations
Tnf = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip = number of BFGS updates skipped
Nact = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
   = final function value
         * * *
      Tit Inf Inint Skip Nact Projg F
                  1 0 0 1.574D-05 6.517D-01
       31
              36
  F = 0.65169543995414492
CONVERGENCE: REL REDUCTION OF F <= FACTR*EPSMCH
234.9696678245927
RUNNING THE L-BFGS-B CODE
          * * *
Machine precision = 2.220D-16
             6 M =
N =
                                10
At XO
           O variables are exactly at the bounds
At iterate 0 f = 6.75625D - 01 |proj g|= 8.84375D - 02
This problem is unconstrained.
```

```
5 	 f = 6.55403D - 01
                                  |proj g| = 3.75413D-02
At iterate
At iterate 10 f = 6.54880D - 01
                                  |proj q| = 2.56283D-02
At iterate 15 f = 6.54832D - 01 |proj g|= 1.49731D - 03
         * * *
Tit = total number of iterations
Tnf = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip = number of BFGS updates skipped
Nact = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F = final function value
          * * *
       Tit Tnf Tnint Skip Nact Projg
                      1 0 0 4.170D-04 6.548D-01
  F = 0.65483179865135865
CONVERGENCE: REL REDUCTION OF F <= FACTR*EPSMCH
232.0234843468565
RUNNING THE L-BFGS-B CODE
          * * *
Machine precision = 2.220D-16
                            10
              7
                   M =
At XO
           O variables are exactly at the bounds
At iterate
           0 	 f = 6.75223D - 01
                                  |proj g| = 8.20770D-02
 Warning: more than 10 function and gradient
 evaluations in the last line search. Termination
  may possibly be caused by a bad search direction.
This problem is unconstrained.
At iterate 5 f = 6.55017D - 01
                                  |proj g| = 1.50619D-02
At iterate 10 f = 6.54871D-01 |proj g|= 7.22712D-03
At iterate 15 f = 6.54840D-01 |proj g|= 2.12409D-04
         * * *
Tit = total number of iterations
Tnf = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip = number of BFGS updates skipped
Nact = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
    = final function value
          * * *
             Tnf Tnint Skip Nact Projg F
              36
                      1 0 0 2.552D-04 6.548D-01
       17
 F = 0.65483960492935955
```

RUNNING THE L-BFGS-B CODE

234.0261072562648

CONVERGENCE: REL REDUCTION OF F <= FACTR*EPSMCH

```
Machine precision = 2.220D-16
      8
At XO
           O variables are exactly at the bounds
At iterate
           0 f= 6.97181D-01 |proj g|= 1.92875D-01
This problem is unconstrained.
           5
               f= 6.55169D-01
                                 |proj g|= 1.21256D-02
At iterate
At iterate 10 f = 6.54665D - 01
                                 |proj q| = 1.29075D-03
 ys=-2.105E-04 -gs= 4.733E-05 BFGS update SKIPPED
At iterate 15 f= 6.54122D-01
                                 |proj g| = 1.48932D-03
At iterate 20 f= 6.54120D-01
                                 |proj g| = 1.72979D-03
At iterate 25 f= 6.54117D-01 |proj g|= 2.67144D-04
          * * *
Tit = total number of iterations
Tnf = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip = number of BFGS updates skipped
Nact = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F = final function value
          * * *
      Tit Tnf Tnint Skip Nact Projg
  N
              38 1 1 0 2.671D-04 6.541D-01
 F = 0.65411739064914509
CONVERGENCE: REL REDUCTION OF F <= FACTR*EPSMCH
235.78344325811275
RUNNING THE L-BFGS-B CODE
          * * *
Machine precision = 2.220D-16
             9 	 M =
                               10
At XO
           O variables are exactly at the bounds
           0 f= 8.73555D-01 |proj g|= 5.47507D-01
At iterate
This problem is unconstrained.
At iterate 5 f = 6.65869D - 01
                                 |proj g| = 2.64466D-02
At iterate 10 f= 6.54584D-01
                                 |proj g| = 2.92708D-02
At iterate 15 f= 6.51636D-01
                                 |proj g| = 6.99098D-03
At iterate 20 f= 6.51218D-01
                                 |proj g| = 1.06305D-03
At iterate 25 f= 6.51210D-01
                                 |proj g|= 8.34611D-06
         * * *
Tit = total number of iterations
Tnf = total number of function evaluations
```

Skip = number of BFGS updates skipped
Nact = number of active bounds at final generalized Cauchy point

Thint = total number of segments explored during Cauchy searches

```
Projg = norm of the final projected gradient
F = final function value
```

N Tit Tnf Tnint Skip Nact Projg F 9 25 29 1 0 0 8.346D-06 6.512D-01 F = 0.65121048486497723

CONVERGENCE: NORM_OF_PROJECTED_GRADIENT_<=_PGTOL 236.80672291463236

RUNNING THE L-BFGS-B CODE

* * *

Machine precision = 2.220D-16

N = 4 M = 10

At XO 0 variables are exactly at the bounds

At iterate 0 f= 8.25638D-01 |proj q|= 4.42460D-02

At iterate 5 f= 8.24086D-01 |proj g|= 1.24405D-02

At iterate 10 f= 8.23281D-01 |proj g|= 3.74921D-03

* * *

Tit = total number of iterations

Tnf = total number of function evaluations

Tnint = total number of segments explored during Cauchy searches

Skip = number of BFGS updates skipped

Nact = number of active bounds at final generalized Cauchy point

Projg = norm of the final projected gradient

F = final function value

* * *

N Tit Tnf Tnint Skip Nact Projg F 4 13 16 1 0 0 6.062D-06 8.233D-01 F = 0.82326992037347269

CONVERGENCE: NORM_OF_PROJECTED_GRADIENT_<=_PGTOL 284.61869324548684

This problem is unconstrained. This problem is unconstrained.

RUNNING THE L-BFGS-B CODE

* * *

Machine precision = 2.220D-16

 $N = 5 \qquad M = 10$

At XO 0 variables are exactly at the bounds

At iterate 0 f= 7.53120D-01 |proj g|= 7.42466D-02

At iterate 5 f = 7.43995D-01 |proj g|= 7.92191D-03

At iterate 10 f = 7.42165D-01 |proj g|= 4.71674D-03

At iterate 15 f = 7.42154D-01 |proj g|= 9.91780D-06

* * *

```
Tit = total number of iterations
Tnf = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip = number of BFGS updates skipped
Nact = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F = final function value
          * * *
     Tit Tnf Tnint Skip Nact Projg
              19 1 0 0 9.918D-06 7.422D-01
 F = 0.74215384844692656
CONVERGENCE: NORM OF PROJECTED GRADIENT <= PGTOL
259.36369307816733
RUNNING THE L-BFGS-B CODE
         * * *
Machine precision = 2.220D-16
N = 6  M =
                           10
At XO
           O variables are exactly at the bounds
At iterate 0 f= 7.57556D-01
                                 |proj g| = 2.56532D-01
This problem is unconstrained.
At iterate 5 f = 7.06233D - 01
                                |proj g| = 3.44966D-02
At iterate 10 f = 7.04647D - 01
                                 |proj g| = 1.99385D-02
At iterate 15 f = 7.03319D-01 |proj g|= 4.38723D-03
         * * *
Tit = total number of iterations
Tnf = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip = number of BFGS updates skipped
Nact = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F = final function value
          * * *
      Tit Tnf Tnint Skip Nact Projg
              20 1 0 0 4.877D-06 7.033D-01
       18
 F = 0.70331595350715359
CONVERGENCE: NORM OF PROJECTED GRADIENT <= PGTOL
248.3141603784036
RUNNING THE L-BFGS-B CODE
          * * *
Machine precision = 2.220D-16
N = 7 \qquad M =
           O variables are exactly at the bounds
           0 	 f = 7.54990D - 01
At iterate
                                 |proj g| = 3.47445D-01
This problem is unconstrained.
At iterate 5 f = 6.54252D - 01 |proj g|= 3.83409D - 02
At iterate 10 f = 6.50705D-01 |proj g|= 7.51279D-03
```

```
At iterate 15 f= 6.49488D-01 |proj g|= 4.85340D-03
```

Tit = total number of iterations

Tnf = total number of function evaluations

Tnint = total number of segments explored during Cauchy searches

Skip = number of BFGS updates skipped

Nact = number of active bounds at final generalized Cauchy point

Projg = norm of the final projected gradient

F = final function value

* * *

N Tit Tnf Tnint Skip Nact Projg F
7 19 23 1 0 0 2.276D-05 6.495D-01
F = 0.64946776394862826

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH 232.2211686867391
RUNNING THE L-BFGS-B CODE

* * *

Machine precision = 2.220D-16

 $N = 5 \qquad M = 10$

At XO 0 variables are exactly at the bounds

At iterate 0 f= 7.69717D-01 |proj g|= 7.91779D-02

At iterate 5 f= 7.61247D-01 |proj g|= 8.58270D-03

At iterate 10 f= 7.58812D-01 |proj g|= 4.51726D-03

At iterate 15 f = 7.58797D - 01 |proj g| = 7.61546D - 06

* * *

Tit = total number of iterations

Tnf = total number of function evaluations

Tnint = total number of segments explored during Cauchy searches

Skip = number of BFGS updates skipped

Nact = number of active bounds at final generalized Cauchy point

Projg = norm of the final projected gradient

F = final function value

* * *

N Tit Tnf Tnint Skip Nact Projg F
5 15 18 1 0 0 7.615D-06 7.588D-01
F = 0.75879716831911370

CONVERGENCE: NORM_OF_PROJECTED_GRADIENT_<=_PGTOL 264.9558485552222

RUNNING THE L-BFGS-B CODE

* * *

Machine precision = 2.220D-16

 $N = 6 \qquad M = 10$

At X0 0 variables are exactly at the bounds

At iterate 0 f= 7.53464D-01 |proj g|= 6.90645D-02

```
At iterate 5 f = 7.45210D - 01
                                 |proj g| = 2.54624D-02
This problem is unconstrained.
This problem is unconstrained.
At iterate 10 f= 7.43235D-01
                                 |proj g| = 2.05847D-02
At iterate 15 f = 7.42837D-01 |proj g|= 4.85807D-04
         * * *
Tit = total number of iterations
Tnf = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip = number of BFGS updates skipped
Nact = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
   = final function value
         * * *
            Tnf Tnint Skip Nact Projg
      Tit
             23 1 0 0 3.853D-06 7.428D-01
  6 18
 F = 0.74283689098510497
CONVERGENCE: NORM OF PROJECTED GRADIENT <= PGTOL
261.59319537099526
RUNNING THE L-BFGS-B CODE
          * * *
Machine precision = 2.220D-16
N =
      7
                  M =
                               10
At XO 0 variables are exactly at the bounds
At iterate 0 f= 7.49751D-01 |proj g|= 2.50725D-01
This problem is unconstrained.
At iterate 5 f = 7.04208D - 01
                                 |proj g| = 8.95698D-03
At iterate 10 f = 7.02275D-01 |proj g| = 1.22198D-02
At iterate 15 f = 7.02010D - 01
                                 |proj g| = 4.67186D-04
At iterate 20 f= 7.02003D-01 |proj g|= 2.72302D-04
          * * *
Tit = total number of iterations
Tnf = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip = number of BFGS updates skipped
Nact = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
   = final function value
          * * *
      Tit Tnf Tnint Skip Nact Projg
       22
              27 1 0 0 3.637D-06 7.020D-01
 F = 0.70200287532816308
CONVERGENCE: NORM OF PROJECTED GRADIENT <= PGTOL
249.87296611026278
This problem is unconstrained.
```

RUNNING THE L-BFGS-B CODE

= final function value

```
Machine precision = 2.220D-16
                   M =
              8
                                10
At XO
           0 variables are exactly at the bounds
At iterate 0 f = 8.16402D - 01 |proj g| = 5.45084D - 01
At iterate 5 f = 6.53417D - 01
                                  |proj g| = 2.17724D-02
At iterate 10 f= 6.51268D-01
                                  |proj g| = 7.57050D-03
At iterate 15 f = 6.49421D - 01 |proj g| = 9.66003D - 03
At iterate 20 f= 6.49309D-01
                                  |proj g| = 6.34530D-03
At iterate 25 f= 6.49132D-01
                                  |proj q| = 4.29694D-03
At iterate 30 f = 6.49128D - 01 |proj g|= 5.55785D - 05
         * * *
Tit = total number of iterations
Tnf = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip = number of BFGS updates skipped
Nact = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F = final function value
          * * *
      Tit Tnf Tnint Skip Nact Projg
                  1 0 0 3.427D-05 6.491D-01
       34
              39
  F = 0.64912808516182186
CONVERGENCE: REL REDUCTION OF F <= FACTR*EPSMCH
234.10703661437213
RUNNING THE L-BFGS-B CODE
          * * *
Machine precision = 2.220D-16
                  M =
N =
              6
                                10
           O variables are exactly at the bounds
At XO
           0 f= 7.13212D-01 |proj g|= 9.58051D-02
At iterate
This problem is unconstrained.
At iterate 5 f = 7.03462D - 01
                                  |proj g| = 1.05917D-02
At iterate 10 f = 7.01458D-01 |proj g| = 9.35678D-04
         * * *
Tit = total number of iterations
Tnf = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip = number of BFGS updates skipped
Nact = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
```

* * * Tit Tnf Tnint Skip Nact Projg 13 0 0 7.263D-06 7.015D-01 17 1 F = 0.70145614948136747CONVERGENCE: NORM OF PROJECTED GRADIENT <= PGTOL 247.68926622573946 RUNNING THE L-BFGS-B CODE * * * Machine precision = 2.220D-16 7 M =At XO 0 variables are exactly at the bounds 0 f = 7.11667D - 01At iterate |proj g| = 6.88739D-02This problem is unconstrained. At iterate 5 f= 7.05915D-01 |proj g|= 2.16045D-02 At iterate 10 f= 6.99390D-01 |proj q| = 1.21170D-02At iterate 15 f= 6.98586D-01 |proj g| = 4.88878D-03* * * Tit = total number of iterations Tnf = total number of function evaluations Tnint = total number of segments explored during Cauchy searches Skip = number of BFGS updates skipped Nact = number of active bounds at final generalized Cauchy point Projg = norm of the final projected gradient F = final function value * * * Tit Tnf Tnint Skip Nact Projg
19 23 1 0 0 2.881D-00 1 0 0 2.881D-06 6.986D-01 F = 0.69856798788516150CONVERGENCE: NORM OF PROJECTED GRADIENT <= PGTOL 248.71884392941425 RUNNING THE L-BFGS-B CODE * * * Machine precision = 2.220D-16 10 N =M =8 At XO O variables are exactly at the bounds At iterate 0 f = 7.30310D - 01 |proj g|= 2.10093D - 01 This problem is unconstrained. At iterate 5 f= 6.96598D-01 |proj q| = 8.47574D-03At iterate 10 f = 6.92621D-01|proj g| = 1.21858D-02

|proj g| = 2.36039D-04

* * *

At iterate 15 f= 6.92183D-01

At iterate 20 f= 6.92183D-01 |proj g|= 3.64190D-05

```
Tit = total number of iterations
Tnf = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip = number of BFGS updates skipped
Nact = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F = final function value
          * * *
     Tit Tnf Tnint Skip Nact Projg
             24 1 0 0 7.375D-06 6.922D-01
 F = 0.69218328899415105
```

CONVERGENCE: NORM OF PROJECTED GRADIENT <= PGTOL

248.57358510203474

RUNNING THE L-BFGS-B CODE

* * *

Machine precision = 2.220D-16 $N = 9 \qquad M =$ 10

At XO O variables are exactly at the bounds

At iterate 0 f= 8.94229D-01 |proj g| = 6.85459D-01This problem is unconstrained. At iterate 5 f = 6.54099D - 01|proj g| = 3.00307D-02At iterate 10 f= 6.50176D-01 |proj g| = 9.50072D-03At iterate 15 f= 6.49478D-01 |proj g| = 3.57097D-03At iterate 20 f= 6.48826D-01 |proj g|= 2.04205D-03At iterate 25 f= 6.48607D-01 |proj g| = 7.63244D-03At iterate 30 f= 6.48213D-01 |proj g| = 3.14660D-03At iterate 35 f = 6.48203D-01 |proj g|= 2.83185D-04

Tit = total number of iterations

Tnf = total number of function evaluations

Tnint = total number of segments explored during Cauchy searches

Skip = number of BFGS updates skipped

Nact = number of active bounds at final generalized Cauchy point

Projg = norm of the final projected gradient

F = final function value

* * *

* * *

Tit Tnf Tnint Skip Nact Projg 9 39 45 1 0 0 3.597D-06 6.482D-01 F = 0.64820257589095165

CONVERGENCE: NORM OF PROJECTED GRADIENT <= PGTOL 235.79606549935974

RUNNING THE L-BFGS-B CODE

* * *

Machine precision = 2.220D-16 N = 7 M =10 234.02071998589193 RUNNING THE L-BFGS-B CODE

* * *

Machine precision = 2.220D-16 8 M =

O variables are exactly at the bounds At XO

At iterate 0 f = 6.71513D-01 |proj g| = 7.89745D-02

At iterate 5 f = 6.58518D - 01 |proj g|= 9.27068D - 03

This problem is unconstrained.

At iterate 10 f = 6.55371D-01|proj g| = 2.05756D-02At iterate 15 f= 6.54217D-01 |proj g| = 3.91408D-02At iterate 20 f = 6.54095D - 01|proj g| = 3.02485D-03At iterate 25 f = 6.54080D - 01 |proj g|= 7.48825D-04

* * *

Tit = total number of iterations

Tnf = total number of function evaluations

Tnint = total number of segments explored during Cauchy searches

Skip = number of BFGS updates skipped

Nact = number of active bounds at final generalized Cauchy point

Projg = norm of the final projected gradient

F = final function value

* * *

Tit Tnf Tnint Skip Nact Projg

```
27 63 1
                          0 0 2.179D-04 6.541D-01
   8
  F = 0.65407981404455440
CONVERGENCE: REL REDUCTION OF F <= FACTR*EPSMCH
235.7708175189703
RUNNING THE L-BFGS-B CODE
          * * *
Machine precision = 2.220D-16
             9
                  M =
                               10
At XO 0 variables are exactly at the bounds
At iterate 0 f = 6.95265D-01 |proj g|= 2.01665D-01
Warning: more than 10 function and gradient
  evaluations in the last line search. Termination
  may possibly be caused by a bad search direction.
This problem is unconstrained.
At iterate 5 f = 6.57776D - 01 |proj g| = 7.43623D - 03
At iterate 10 f= 6.55467D-01 |proj q|= 1.87462D-02
 ys=-5.963E-04 -gs= 2.300E-04 BFGS update SKIPPED
At iterate 15 f = 6.54235D-01 |proj g| = 2.07145D-02
At iterate 20 f= 6.54162D-01 |proj g|= 7.59959D-03
At iterate 25 f = 6.54106D - 01 | proj g| = 8.19217D - 03
At iterate 30 f = 6.54065D - 01
                                 |proj g| = 3.84370D-03
At iterate 35 f= 6.54057D-01
                                 |proj q| = 5.20266D-04
```

|proj g| = 5.24425D-04

Bad direction in the line search; refresh the lbfgs memory and restart the iteration.

Warning: more than 10 function and gradient evaluations in the last line search. Termination may possibly be caused by a bad search direction. This problem is unconstrained.

* * *

* * *

At iterate 40 f = 6.54056D - 01

Tit = total number of iterations
Tnf = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip = number of BFGS updates skipped
Nact = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F = final function value

N Tit Tnf Tnint Skip Nact Projg F
9 41 88 2 1 0 5.244D-04 6.541D-01
F = 0.65405626842087816

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH 237.76290618941505
RUNNING THE L-BFGS-B CODE

* * *

```
10 	 M =
At XO
           0 variables are exactly at the bounds
At iterate
          0 f= 8.65976D-01 |proj g|= 5.61359D-01
At iterate 5 f= 6.61771D-01 |proj g|= 1.60679D-02
At iterate 10 f = 6.53270D-01 |proj g|= 1.57522D-02
At iterate 15 f = 6.47824D - 01
                                  |proj g| = 6.58472D-03
At iterate 20 f= 6.47540D-01
                                 |proj g| = 7.20954D-03
At iterate 25 f = 6.47510D - 01 |proj g|= 9.23983D - 04
At iterate 30 f = 6.47509D-01 |proj g|= 7.12384D-05
         * * *
Tit = total number of iterations
Tnf = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip = number of BFGS updates skipped
Nact = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
   = final function value
         * * *
            Tnf Tnint Skip Nact Projg
                  1 0 0 7.454D-06 6.475D-01
       32
              39
 F = 0.64750904022324152
CONVERGENCE: NORM OF PROJECTED GRADIENT <= PGTOL
237.56303751500914
RUNNING THE L-BFGS-B CODE
          * * *
Machine precision = 2.220D-16
              5
                   M =
                                10
At XO
     O variables are exactly at the bounds
At iterate 0 f= 8.23060D-01 |proj g|= 6.80524D-02
At iterate 5 f= 8.18939D-01
                                  |proj g| = 2.31602D-03
At iterate 10 f= 8.18470D-01
                                 |proj g| = 9.22994D-03
At iterate 15 f = 8.18435D-01 |proj q| = 1.16863D-05
          * * *
Tit = total number of iterations
Tnf = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip = number of BFGS updates skipped
Nact = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
   = final function value
```

N Tit Tnf Tnint Skip Nact Projg F

```
16 31 1
                          0 0 1.109D-05 8.184D-01
   5
  F = 0.81843507975067253
CONVERGENCE: REL REDUCTION OF F <= FACTR*EPSMCH
284.99418679622596
RUNNING THE L-BFGS-B CODE
          * * *
Machine precision = 2.220D-16
             6
                  M =
                               10
At XO
           O variables are exactly at the bounds
At iterate 0 f= 7.51681D-01 |proj g|= 7.86058D-02
This problem is unconstrained.
Warning: more than 10 function and gradient
 evaluations in the last line search. Termination
  may possibly be caused by a bad search direction.
This problem is unconstrained.
At iterate 5 f = 7.40111D - 01 |proj g|= 1.15138D-03
At iterate 10 f= 7.40026D-01
                                 |proj g| = 6.72606D - 03
At iterate 15 f= 7.40002D-01 |proj g|= 8.24296D-05
         * * *
Tit = total number of iterations
Tnf = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip = number of BFGS updates skipped
Nact = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F = final function value
         * * *
      Tit Inf Inint Skip Nact Projg
       17
              21 1 0 0 9.083D-06 7.400D-01
  F = 0.74000215749271148
CONVERGENCE: NORM OF PROJECTED GRADIENT <= PGTOL
260.64072491755104
RUNNING THE L-BFGS-B CODE
          * * *
Machine precision = 2.220D-16
N =
             7 	 M =
                               10
At XO
           O variables are exactly at the bounds
At iterate 0 f= 7.55212D-01 |proj q|= 2.49738D-01
This problem is unconstrained.
At iterate 5 f = 7.03268D - 01
                                 |proj g| = 3.64541D-02
At iterate 10 f= 7.01276D-01
                                 |proj g| = 7.89264D-04
At iterate 15 f= 7.01231D-01 |proj g|= 3.63161D-03
          * * *
```

Tit = total number of iterations

```
Tnf = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip = number of BFGS updates skipped
Nact = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F = final function value
          * * *
  N Tit Tnf Tnint Skip Nact Projg
                          0 0 2.094D-05 7.012D-01
       19
              22
                     1
  F = 0.70122577636247674
CONVERGENCE: REL REDUCTION OF F <= FACTR*EPSMCH
249.61186085779218
RUNNING THE L-BFGS-B CODE
          * * *
This problem is unconstrained.
Machine precision = 2.220D-16
              8
                  M =
                                10
At XO
           O variables are exactly at the bounds
At iterate 0 f = 7.49433D-01 |proj g|= 3.20682D-01
At iterate 5 f = 6.51288D - 01 |proj g|= 2.42622D - 02
At iterate 10 f = 6.47579D - 01
                                 |proj g| = 8.07171D-04
At iterate 15 f = 6.47559D - 01
                                  |proj g| = 1.88501D-03
At iterate 20 f= 6.47545D-01 |proj q|= 6.17491D-06
         * * *
Tit = total number of iterations
Tnf = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip = number of BFGS updates skipped
Nact = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F = final function value
         * * *
      Tit Inf Inint Skip Nact Projg
                  1 0 0 6.175D-06 6.475D-01
       20
              25
  F = 0.64754534666170238
CONVERGENCE: NORM OF PROJECTED GRADIENT <= PGTOL
233.57523647833202
RUNNING THE L-BFGS-B CODE
          * * *
Machine precision = 2.220D-16
N =
             6 	 M =
                               10
           0 variables are exactly at the bounds
At XO
At iterate 0 f = 7.70791D - 01 |proj g| = 7.92932D - 02
At iterate 5 f = 7.58419D-01 |proj g|= 7.60446D-04
```

```
At iterate 10 f = 7.58418D-01 |proj g|= 7.68206D-05
         * * *
Tit = total number of iterations
Tnf = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip = number of BFGS updates skipped
Nact = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
   = final function value
         * * *
     Tit Tnf Tnint Skip Nact Projg
             19 1 0 0 2.110D-05 7.584D-01
       14
 F = 0.75841814884106828
CONVERGENCE: REL REDUCTION OF F <= FACTR*EPSMCH
266.82849801059893
RUNNING THE L-BFGS-B CODE
          * * *
Machine precision = 2.220D-16
                               10
             7
                  M =
At XO 0 variables are exactly at the bounds
At iterate 0 f= 7.51638D-01 |proj g|= 8.11355D-02
This problem is unconstrained.
This problem is unconstrained.
At iterate 5 f = 7.40523D - 01 |proj g|= 5.84578D - 03
At iterate 10 f = 7.40209D-01 |proj g| = 1.79691D-03
At iterate 15 f = 7.40176D-01 |proj g|= 6.85636D-05
        * * *
Tit = total number of iterations
Tnf = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip = number of BFGS updates skipped
Nact = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F = final function value
          * * *
  N Tit Tnf Tnint Skip Nact Projg
             20 1 0 0 8.233D-06 7.402D-01
       16
 F = 0.74017552396423003
```

CONVERGENCE: NORM OF PROJECTED GRADIENT <= PGTOL

At X0 0 variables are exactly at the bounds

10

262.6989760519813

RUNNING THE L-BFGS-B CODE

* * *

Machine precision = 2.220D-16N = 8 M =

```
|proj g| = 2.42136D-01
                f = 7.46862D - 01
At iterate
           0
This problem is unconstrained.
           5 	 f = 7.00730D - 01
                                   |proj g| = 4.31871D-03
At iterate
At iterate 10 f = 7.00131D-01
                                   |proj g| = 1.49275D-02
At iterate 15 f = 6.99977D - 01
                                   |proj g| = 2.62481D-04
At iterate 20 f= 6.99973D-01
                                   |proj g| = 3.69308D-04
         * * *
Tit = total number of iterations
Tnf = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip = number of BFGS updates skipped
Nact = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
   = final function value
          * * *
             Tnf Tnint Skip Nact Projg
   8
        23
                       1 0 0 1.777D-05 7.000D-01
               26
  F = 0.69997321537238288
CONVERGENCE: REL REDUCTION OF F <= FACTR*EPSMCH
251.19100036512066
RUNNING THE L-BFGS-B CODE
          * * *
Machine precision = 2.220D-16
                   M =
At XO
            O variables are exactly at the bounds
            0 	 f = 8.07109D - 01
                                   |proj g| = 5.04206D-01
At iterate
This problem is unconstrained.
At iterate
           5 	 f = 6.49655D - 01
                                   |proj g| = 2.49402D-02
At iterate 10 f = 6.47977D - 01
                                   |proj g| = 3.17687D-03
At iterate 15 f = 6.47032D - 01
                                   |proj q| = 9.15601D-03
At iterate 20 f= 6.46740D-01
                                   |proj g| = 1.20828D-03
At iterate 25 f = 6.46736D - 01
                                   |proj g| = 8.45898D-04
At iterate 30 f = 6.46732D - 01 |proj g|= 2.46977D - 05
          * * *
Tit = total number of iterations
Tnf = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip = number of BFGS updates skipped
Nact = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
   = final function value
```

N Tit Tnf Tnint Skip Nact Projg F

```
30 36 1
                          0 0 2.470D-05 6.467D-01
  F = 0.64673213905213467
CONVERGENCE: REL REDUCTION OF F <= FACTR*EPSMCH
235.30199872151726
RUNNING THE L-BFGS-B CODE
          * * *
Machine precision = 2.220D-16
             7
                  M =
                               10
At XO 0 variables are exactly at the bounds
At iterate 0 f= 7.14714D-01 |proj g|= 9.67135D-02
This problem is unconstrained.
At iterate 5 f = 7.02695D - 01
                                 |proj g| = 2.98171D-03
At iterate 10 f = 7.02568D - 01 |proj g|= 2.12563D - 04
         * * *
Tit = total number of iterations
Tnf = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip = number of BFGS updates skipped
Nact = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F = final function value
         * * *
      Tit Inf Inint Skip Nact Projg
       14
              19 1 0 0 3.720D-05 7.026D-01
  F = 0.70256773997789490
CONVERGENCE: REL REDUCTION OF F <= FACTR*EPSMCH
250.0627606325727
RUNNING THE L-BFGS-B CODE
          * * *
Machine precision = 2.220D-16
             8 M =
                               10
           O variables are exactly at the bounds
At iterate 0 f= 7.13242D-01 |proj g|= 7.00539D-02
This problem is unconstrained.
At iterate 5 f = 7.04623D - 01
                                 |proj g| = 1.31248D-02
At iterate 10 f= 6.99517D-01
                                 |proj g| = 2.86045D-03
At iterate 15 f = 6.99511D-01 |proj g|= 1.91004D-04
At iterate 20 f= 6.99511D-01 |proj g|= 3.35856D-06
         * * *
Tit = total number of iterations
Tnf = total number of function evaluations
```

Tnint = total number of segments explored during Cauchy searches
Skip = number of BFGS updates skipped
Nact = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient

```
F = final function value
```

```
N Tit Tnf Tnint Skip Nact Projg F
8 20 23 1 0 0 3.359D-06 6.995D-01
F = 0.69951132055410858
```

CONVERGENCE: NORM_OF_PROJECTED_GRADIENT_<=_PGTOL

251.0358037061805

RUNNING THE L-BFGS-B CODE

* * *

Machine precision = 2.220D-16

N = 9 M = 10

At XO 0 variables are exactly at the bounds

At iterate 0 f = 7.28647D - 01 |proj g|= 2.08554D-01

This problem is unconstrained.

At iterate 5 f= 6.92862D-01 |proj g|= 8.29729D-03

At iterate 10 f= 6.91144D-01 |proj g|= 3.00316D-03

At iterate 15 f = 6.91114D-01 |proj g| = 2.37979D-03

At iterate 20 f = 6.91075D-01 |proj g|= 5.92340D-03

At iterate 25 f= 6.90993D-01 |proj g|= 2.29208D-04

At iterate 30 f= 6.90993D-01 |proj g|= 1.60751D-05

* * *

Tit = total number of iterations

Tnf = total number of function evaluations

Tnint = total number of segments explored during Cauchy searches

Skip = number of BFGS updates skipped

Nact = number of active bounds at final generalized Cauchy point

Projg = norm of the final projected gradient

F = final function value

* * *

N Tit Tnf Tnint Skip Nact Projg F 9 30 32 1 0 0 1.608D-05 6.910D-01

F = 0.69099311035071342

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH

250.1736850778397

RUNNING THE L-BFGS-B CODE

* * *

Machine precision = 2.220D-16

N = 10 M = 1

At XO 0 variables are exactly at the bounds

At iterate 0 f= 8.79681D-01 |proj g|= 6.26436D-01

This problem is unconstrained.

At iterate 5 f= 6.50455D-01 |proj g|= 3.52407D-02

At iterate 10 f= 6.47122D-01 |proj g|= 9.39636D-03

```
At iterate 15 f= 6.46717D-01
                                |proj g| = 3.61458D-03
At iterate 20 f= 6.45895D-01
                                 |proj g| = 8.08618D-03
At iterate 25 f= 6.45790D-01
                                 |proj g| = 1.66289D-03
At iterate 30 f= 6.45621D-01 |proj g|= 2.67284D-03
At iterate 35 f= 6.45611D-01 |proj q|= 4.34154D-05
        * * *
Tit = total number of iterations
Tnf = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip = number of BFGS updates skipped
Nact = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F = final function value
          * * *
            Tnf Tnint Skip Nact Projg
             41
                  1 0 0 2.761D-05 6.456D-01
 F = 0.64561059834485823
CONVERGENCE: REL REDUCTION OF F <= FACTR*EPSMCH
236.92516104387238
RUNNING THE L-BFGS-B CODE
          * * *
Machine precision = 2.220D-16
             8
                  M =
                               10
At XO 0 variables are exactly at the bounds
At iterate 0
               f = 6.73709D-01 |proj g|= 9.56160D-02
This problem is unconstrained.
At iterate 5 f = 6.55887D - 01 |proj g|= 2.28750D - 02
 ys=-6.130E-04 -gs= 7.641E-05 BFGS update SKIPPED
At iterate 10 f= 6.55463D-01
                                 |proj q| = 3.30509D-03
At iterate 15 f = 6.55401D-01
                                 |proj g| = 5.06185D-03
At iterate 20 f= 6.55320D-01 |proj g|= 1.04080D-03
At iterate 25 f= 6.55318D-01 |proj g|= 1.68003D-05
         * * *
Tit = total number of iterations
Tnf = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip = number of BFGS updates skipped
Nact = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
    = final function value
          * * *
      Tit Inf Inint Skip Nact Projg F
             42 1 1 0 1.680D-05 6.553D-01
     25
```

F = 0.65531788700028326

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH 236.18681003209517

This problem is unconstrained.

RUNNING THE L-BFGS-B CODE

* * *

Machine precision = 2.220D-16

N = 9 M = 10

At XO 0 variables are exactly at the bounds

At iterate 0 f= 6.73164D-01 |proj q|= 8.01753D-02

At iterate 5 f= 6.56623D-01 |proj g|= 3.17832D-02

At iterate 10 f = 6.55597D-01 |proj g| = 1.21111D-02

At iterate 15 f = 6.54602D-01 |proj g|= 3.16283D-03

At iterate 20 f= 6.54577D-01 |proj g|= 1.44399D-02

At iterate 25 f = 6.54524D-01 |proj g|= 8.22958D-03

At iterate 30 f = 6.54516D - 01 |proj g| = 9.23669D - 05

* * *

Tit = total number of iterations

Tnf = total number of function evaluations

Tnint = total number of segments explored during Cauchy searches

Skip = number of BFGS updates skipped

Nact = number of active bounds at final generalized Cauchy point

Projg = norm of the final projected gradient

F = final function value

* * *

N Tit Tnf Tnint Skip Nact Projg F 9 34 45 1 0 0 1.806D-05 6.545D-01

F = 0.65451600366008700

CONVERGENCE: REL REDUCTION OF F <= FACTR*EPSMCH

237.91737722978922

RUNNING THE L-BFGS-B CODE

* * *

Machine precision = 2.220D-16

 $N = 10 \qquad M = 10$

At XO 0 variables are exactly at the bounds

At iterate 0 f= 6.96085D-01 |proj g|= 2.00676D-01

This problem is unconstrained.

At iterate 5 f = 6.56003D-01 |proj g| = 1.12813D-02

At iterate 10 f = 6.54718D - 01 |proj g| = 4.51794D - 03

At iterate 15 f = 6.54600D-01 |proj g| = 2.64217D-03

At iterate 20 f= 6.54523D-01 |proj g|= 2.00055D-03

```
At iterate 25 f = 6.54513D-01 |proj g| = 5.14284D-04
          * * *
Tit = total number of iterations
Tnf = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip = number of BFGS updates skipped
Nact = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
    = final function value
          * * *
      Tit Tnf Tnint Skip Nact Projg F
                   1 0 0 3.074D-05 6.545D-01
       28
              38
  F = 0.65451256665135782
CONVERGENCE: REL REDUCTION OF F <= FACTR*EPSMCH
239.91622239485622
RUNNING THE L-BFGS-B CODE
          * * *
Machine precision = 2.220D-16
             11
                   M =
                                10
At XO
           O variables are exactly at the bounds
At iterate 0 f= 8.51433D-01 |proj g|= 5.24901D-01
This problem is unconstrained.
At iterate 5 f= 6.58361D-01
                                  |proj g| = 2.01056D-02
At iterate 10 f= 6.49380D-01
                                  |proj g| = 6.04783D-02
 ys=-1.100E-03 -gs= 2.422E-04 BFGS update SKIPPED
At iterate 15 f = 6.46669D - 01
                                  |proj g| = 1.12736D-02
At iterate 20 f= 6.46488D-01
                                   |proj g| = 1.77674D-02
Warning: more than 10 function and gradient
  evaluations in the last line search. Termination
  may possibly be caused by a bad search direction.
This problem is unconstrained.
This problem is unconstrained.
Tit = total number of iterations
Tnf = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip = number of BFGS updates skipped
Nact = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F = final function value
```

Projg Tnf Tnint Skip Nact Tit 23 1 1 0 3.362D-03 6.465D-01 11 50 F = 0.64645097996622192

CONVERGENCE: REL REDUCTION OF F <= FACTR*EPSMCH 239.20752926865055 RUNNING THE L-BFGS-B CODE

* * *

```
Machine precision = 2.220D-16
N = 3 M =
At XO
           O variables are exactly at the bounds
At iterate 0 f= 8.39722D-01 |proj g|= 2.25422D-05
        * * *
Tit = total number of iterations
Tnf = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip = number of BFGS updates skipped
Nact = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F = final function value
         * * *
      Tit Inf Inint Skip Nact Projg
              6 1 0 0 1.653D-05 8.397D-01
       1
 F = 0.83972178513771145
CONVERGENCE: REL REDUCTION OF F <= FACTR*EPSMCH
288.14651980627104
RUNNING THE L-BFGS-B CODE
         * * *
Machine precision = 2.220D-16
                               10
        4 M =
           O variables are exactly at the bounds
At iterate 0 f = 7.66583D - 01 |proj g| = 8.12553D - 02
At iterate 5 f = 7.59050D - 01 |proj g| = 1.56624D - 03
        * * *
Tit = total number of iterations
Tnf = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip = number of BFGS updates skipped
Nact = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
   = final function value
         * * *
      Tit Inf Inint Skip Nact Projg F
                  1 0 0 2.888D-05 7.590D-01
       8
              11
 F = 0.75904608925704109
CONVERGENCE: REL REDUCTION OF F <= FACTR*EPSMCH
263.03948599036585
RUNNING THE L-BFGS-B CODE
         * * *
Machine precision = 2.220D-16
N =
             5
                   M =
                               10
```

At XO 0 variables are exactly at the bounds

```
At iterate
           0 	 f = 7.71858D - 01
                                  |proj g| = 2.53463D-01
At iterate 5 f = 7.22043D - 01
                                  |proj q| = 4.38235D-02
This problem is unconstrained.
At iterate 10 f = 7.18398D - 01
                                 |proj g| = 1.42527D-04
         * * *
Tit = total number of iterations
Tnf = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip = number of BFGS updates skipped
Nact = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F = final function value
         * * *
      Tit Tnf Tnint Skip Nact Projg
       12
              14
                  1 0 0 3.897D-06 7.184D-01
  5
  F = 0.71839844791509999
CONVERGENCE: NORM OF PROJECTED GRADIENT <= PGTOL
251.3818784994736
RUNNING THE L-BFGS-B CODE
          * * *
Machine precision = 2.220D-16
             6
                  M =
                                10
At XO
           O variables are exactly at the bounds
At iterate 0 f = 7.62983D - 01 |proj g|= 3.52049D - 01
This problem is unconstrained.
          5 f= 6.64324D-01
At iterate
                                  |proj q| = 3.64215D-02
At iterate 10 f = 6.58270D - 01 |proj g| = 7.60822D - 04
         * * *
Tit = total number of iterations
Tnf = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip = number of BFGS updates skipped
Nact = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F = final function value
         * * *
      Tit Tnf Tnint Skip Nact Projg
       14
              19
                  1 0 0 9.492D-06 6.583D-01
  F = 0.65826848740128996
CONVERGENCE: NORM OF PROJECTED GRADIENT <= PGTOL
233.1782117668334
RUNNING THE L-BFGS-B CODE
          * * *
Machine precision = 2.220D-16
N =
             4
                  M =
                                10
```

O variables are exactly at the bounds

At XO

```
At iterate 0 f = 7.75613D-01 |proj g|= 9.07789D-02
At iterate 5 f = 7.67241D - 01 |proj q| = 9.19884D - 04
Tit = total number of iterations
Tnf = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip = number of BFGS updates skipped
Nact = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F = final function value
          * * *
  N Tit Tnf Tnint Skip Nact Projg
              11 1 0 0 5.199D-06 7.672D-01
  F = 0.76724036531686846
CONVERGENCE: NORM OF PROJECTED GRADIENT <= PGTOL
265.7927627464678
RUNNING THE L-BFGS-B CODE
         * * *
Machine precision = 2.220D-16
N = 5 \qquad M = 10
At XO 0 variables are exactly at the bounds
At iterate 0 f = 7.63821D - 01 |proj g| = 7.92854D - 02
This problem is unconstrained.
This problem is unconstrained.
At iterate 5 f= 7.56275D-01 |proj g|= 8.89229D-03
At iterate 10 f = 7.56085D - 01 |proj q| = 4.02316D - 04
         * * *
Tit = total number of iterations
Tnf = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip = number of BFGS updates skipped
Nact = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F = final function value
         * * *
  N Tit Tnf Tnint Skip Nact Projg
       12
                   1 0 0 8.051D-06 7.561D-01
              15
  F = 0.75608543357786206
CONVERGENCE: NORM OF PROJECTED GRADIENT <= PGTOL
264.04470568216163
RUNNING THE L-BFGS-B CODE
          * * *
Machine precision = 2.220D-16
                          10
N =
             6
                  M =
```

At XO 0 variables are exactly at the bounds

```
|proj g| = 2.45423D-01
At iterate
          0 	 f = 7.63026D - 01
This problem is unconstrained.
At iterate
          5 	 f= 7.18610D-01
                                 |proj g| = 6.10722D-03
At iterate 10 f = 7.17644D-01 |proj g|= 1.37145D-02
At iterate 15 f= 7.17422D-01
                                 |proj g| = 6.39937D-06
         * * *
Tit = total number of iterations
Tnf = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip = number of BFGS updates skipped
Nact = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F = final function value
          * * *
             Tnf Tnint Skip Nact Projg
                          0 0 6.399D-06 7.174D-01
       15
              19
                     1
 F = 0.71742203841172458
CONVERGENCE: NORM OF PROJECTED GRADIENT <= PGTOL
253.05380490633945
RUNNING THE L-BFGS-B CODE
          * * *
Machine precision = 2.220D-16
             7
                  M =
                                10
At XO
           O variables are exactly at the bounds
At iterate
          0 	 f = 8.11349D - 01
                                 |proj g| = 5.07697D-01
This problem is unconstrained.
At iterate 5 f= 6.61334D-01 |proj g|= 2.74736D-02
At iterate 10 f= 6.58741D-01
                                 |proj g| = 4.28188D-03
At iterate 15 f= 6.57538D-01
                                  |proj g| = 1.30854D-02
At iterate 20 f= 6.57350D-01
                                 |proj q| = 2.98453D-04
         * * *
Tit = total number of iterations
Tnf = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip = number of BFGS updates skipped
Nact = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F = final function value
          * * *
      Tit Inf Inint Skip Nact Projg
              28 1 0 0 4.931D-06 6.574D-01
   7
       22
  F = 0.65735036923298584
CONVERGENCE: NORM OF PROJECTED GRADIENT <= PGTOL
```

234.86972406228324

RUNNING THE L-BFGS-B CODE

Machine precision = 2.220D-16 5 10 At XO O variables are exactly at the bounds At iterate 0 f= 7.15593D-01 |proj g|= 1.11031D-01At iterate 5 f= 7.03483D-01 |proj g|= 8.19928D-03* * * Tit = total number of iterations Tnf = total number of function evaluations Tnint = total number of segments explored during Cauchy searches Skip = number of BFGS updates skipped Nact = number of active bounds at final generalized Cauchy point Projg = norm of the final projected gradient F = final function value * * * N Tit Tnf Tnint Skip Nact Projg 11 1 0 0 3.395D-06 7.034D-01 F = 0.70344617037541912CONVERGENCE: NORM OF PROJECTED GRADIENT <= PGTOL This problem is unconstrained. This problem is unconstrained. 246.3579132461408 RUNNING THE L-BFGS-B CODE * * * Machine precision = 2.220D-16 6 M =10 At XO O variables are exactly at the bounds At iterate 0 f= 7.15240D-01 |proj g|= 8.34969D-02At iterate 5 f= 7.05852D-01 |proj q|= 2.41294D-02At iterate 10 f= 6.95759D-01 |proj g| = 2.87573D-03At iterate 15 f = 6.95750D - 01 |proj g| = 5.15143D - 07* * * Tit = total number of iterations Tnf = total number of function evaluations Tnint = total number of segments explored during Cauchy searches Skip = number of BFGS updates skipped Nact = number of active bounds at final generalized Cauchy point Projg = norm of the final projected gradient F = final function value

* * *

N Tit Tnf Tnint Skip Nact Projg F 6 15 18 1 0 0 5.151D-07 6.958D-01 F = 0.69575005581227878

```
CONVERGENCE: NORM_OF_PROJECTED_GRADIENT_<=_PGTOL
```

245.77201875292567

RUNNING THE L-BFGS-B CODE

* * *

Machine precision = 2.220D-16

 $N = 7 \qquad M = 10$

At XO 0 variables are exactly at the bounds

At iterate 0 f= 7.38017D-01 |proj g|= 2.25560D-01

This problem is unconstrained.

At iterate 5 f = 6.98767D-01 |proj g| = 1.93647D-02

At iterate 10 f = 6.94612D - 01 |proj g|= 3.92555D - 03

At iterate 15 f = 6.94493D-01 |proj g|= 1.04570D-04

* * *

Tit = total number of iterations

Tnf = total number of function evaluations

Tnint = total number of segments explored during Cauchy searches

Skip = number of BFGS updates skipped

Nact = number of active bounds at final generalized Cauchy point

Projg = norm of the final projected gradient

F = final function value

* * *

N Tit Tnf Tnint Skip Nact Projg F 7 16 19 1 0 0 7.241D-06 6.945D-01

F = 0.69449285732556576

CONVERGENCE: NORM OF PROJECTED GRADIENT <= PGTOL

247.3496000613901

RUNNING THE L-BFGS-B CODE

* * *

Machine precision = 2.220D-16

N = 8 M = 10

At XO 0 variables are exactly at the bounds

At iterate 0 f= 8.65626D-01 |proj g|= 5.62549D-01

This problem is unconstrained.

At iterate 5 f = 6.66180D - 01 |proj g| = 3.55506D - 02

At iterate 10 f= 6.59184D-01 |proj g|= 1.87300D-02 ys=-1.861E-03 -gs= 1.510E-04 BFGS update SKIPPED

At iterate 15 f = 6.57643D - 01 |proj g|= 3.21873D-03

At iterate 20 f = 6.57544D-01 |proj g|= 6.79516D-03

Warning: more than 10 function and gradient

evaluations in the last line search. Termination

may possibly be caused by a bad search direction.

This problem is unconstrained.

At iterate 25 f = 6.57351D-01 |proj g|= 2.19460D-03

* * *

```
Tit = total number of iterations
Tnf = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip = number of BFGS updates skipped
Nact = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F = final function value
          * * *
      Tit Tnf Tnint Skip Nact Projg
              51 1 1 0 2.195D-03 6.574D-01
  F = 0.65735114100698711
CONVERGENCE: REL REDUCTION OF F <= FACTR*EPSMCH
236.86998337834768
RUNNING THE L-BFGS-B CODE
         * * *
Machine precision = 2.220D-16
N = 6 M =
                           10
At XO
           O variables are exactly at the bounds
At iterate 0 f= 6.82538D-01 |proj g|= 1.32774D-01
At iterate 5 f = 6.53735D-01
                                 |proj g| = 2.58331D-02
At iterate 10 f = 6.51634D - 01 |proj q| = 3.44879D - 04
         * * *
Tit = total number of iterations
Tnf = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip = number of BFGS updates skipped
Nact = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F = final function value
         * * *
      Tit Inf Inint Skip Nact Projg F
              14 1 0 0 4.058D-06 6.516D-01
       12
  F = 0.65163323894686032
CONVERGENCE: NORM OF PROJECTED GRADIENT <= PGTOL
230.94876828614508
This problem is unconstrained.
RUNNING THE L-BFGS-B CODE
          * * *
Machine precision = 2.220D-16
              7
N =
                   M =
                               10
At XO
           O variables are exactly at the bounds
At iterate 0 f= 6.82204D-01 |proj g|= 1.31381D-01
At iterate 5 f = 6.52646D - 01 |proj g| = 2.48520D - 02
```

At iterate 10 f= 6.50478D-01 |proj g|= 1.07426D-02

```
At iterate 15 f = 6.50207D-01 |proj g| = 3.56526D-05
         * * *
Tit = total number of iterations
Tnf = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip = number of BFGS updates skipped
Nact = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
   = final function value
          * * *
     Tit Inf Inint Skip Nact Projg F
              18 1 0 0 3.565D-05 6.502D-01
       15
 F = 0.65020701050586771
CONVERGENCE: REL REDUCTION OF F <= FACTR*EPSMCH
232.46955552997156
RUNNING THE L-BFGS-B CODE
          * * *
Machine precision = 2.220D-16
                               10
             8
                  M =
At XO
       O variables are exactly at the bounds
At iterate 0 f= 7.04427D-01 |proj q|= 2.16766D-01
This problem is unconstrained.
                                 |proj g| = 8.64022D-02
At iterate 5 f = 6.51975D - 01
At iterate 10 f = 6.50282D - 01 |proj g| = 1.05987D - 02
At iterate 15 f = 6.50214D-01 |proj g|= 2.25077D-03
At iterate 20 f= 6.50205D-01 |proj g|= 5.84978D-06
         * * *
Tit = total number of iterations
Tnf = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip = number of BFGS updates skipped
Nact = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F = final function value
          * * *
  N Tit Tnf Tnint Skip Nact Projg
```

N Tit Thi Thint Skip Nact Projg F 8 20 24 1 0 0 5.850D-06 6.502D-01 F = 0.65020494594032552

CONVERGENCE: NORM_OF_PROJECTED_GRADIENT_<=_PGTOL 234.46886183594938
RUNNING THE L-BFGS-B CODE

* * *

Machine precision = 2.220D-16N = 9 M = 10

At XO 0 variables are exactly at the bounds

```
|proj g| = 4.84148D-01
At iterate
          0 	 f = 8.41528D - 01
This problem is unconstrained.
At iterate
          5 f= 6.75736D-01 |proj g|= 5.29095D-02
At iterate 10 f = 6.53303D - 01 |proj g| = 1.55482D - 02
At iterate 15 f = 6.50299D - 01
                                 |proj g| = 1.68951D-02
At iterate 20 f= 6.49953D-01
                                 |proj g| = 1.90573D-03
At iterate 25 f= 6.49945D-01 |proj g|= 3.08553D-05
        * * *
Tit = total number of iterations
Tnf = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip = number of BFGS updates skipped
Nact = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
   = final function value
         * * *
      Tit Tnf Tnint Skip Nact Projg
       27
                  1 0 0 8.371D-05 6.499D-01
             34
  9
  F = 0.64994538572596927
CONVERGENCE: REL REDUCTION OF F <= FACTR*EPSMCH
236.38164960392567
This problem is unconstrained.
This problem is unconstrained.
RUNNING THE L-BFGS-B CODE
         * * *
Machine precision = 2.220D-16
N = 4 M =
                         10
At XO
           0 variables are exactly at the bounds
           0 f= 8.38943D-01 |proj g|= 1.65595D-02
At iterate
At iterate 5 f= 8.37067D-01
                                 |proj q| = 4.83410D-02
At iterate 10 f = 8.22449D-01 |proj g|= 1.82869D-02
At iterate 15 f = 8.22152D-01 |proj g|= 5.97618D-03
        * * *
Tit = total number of iterations
Tnf = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip = number of BFGS updates skipped
Nact = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
   = final function value
            Tnf Tnint Skip Nact Projg
       18
             27 1 0 0 9.826D-06 8.221D-01
  F = 0.82214781564494643
```

```
CONVERGENCE: NORM OF PROJECTED GRADIENT <= PGTOL
```

284.241666056702

RUNNING THE L-BFGS-B CODE

* * *

Machine precision = 2.220D-16

 $N = 5 \qquad M = 10$

At XO 0 variables are exactly at the bounds

At iterate 0 f= 7.63175D-01 |proj g|= 8.18540D-02

At iterate 5 f= 7.54210D-01 |proj g|= 1.33034D-02

At iterate 10 f = 7.46167D-01 |proj g| = 4.73747D-02

At iterate 15 f = 7.44687D - 01 |proj g|= 4.59527D - 04

* * *

Tit = total number of iterations

Tnf = total number of function evaluations

Tnint = total number of segments explored during Cauchy searches

Skip = number of BFGS updates skipped

Nact = number of active bounds at final generalized Cauchy point

Projg = norm of the final projected gradient

F = final function value

* * *

N Tit Tnf Tnint Skip Nact Projg F 5 18 31 1 0 0 5.890D-06 7.447D-01

F = 0.74468644531553374

CONVERGENCE: NORM OF PROJECTED GRADIENT <= PGTOL

260.2146456260193

RUNNING THE L-BFGS-B CODE

* * *

Machine precision = 2.220D-16

 $N = 6 \qquad M = 10$

At XO 0 variables are exactly at the bounds

At iterate 0 f= 7.68731D-01 |proj g|= 2.53054D-01

This problem is unconstrained.

At iterate 5 f= 7.18273D-01 |proj g|= 4.59441D-02

At iterate 10 f= 7.13882D-01 |proj g|= 1.67849D-02

At iterate 15 f = 7.07112D-01 |proj g|= 6.00352D-02

At iterate 20 f= 7.05468D-01 |proj g|= 5.55484D-04

* * *

Tit = total number of iterations

Tnf = total number of function evaluations

Tnint = total number of segments explored during Cauchy searches

Skip = number of BFGS updates skipped

Nact = number of active bounds at final generalized Cauchy point

Projg = norm of the final projected gradient

' = final function value

* * *

N Tit Tnf Tnint Skip Nact Projg F 6 23 27 1 0 0 5.800D-06 7.055D-01 F = 0.70546812991739449

CONVERGENCE: NORM_OF_PROJECTED_GRADIENT_<=_PGTOL

249.03729165224456

RUNNING THE L-BFGS-B CODE

* * *

Machine precision = 2.220D-16

 $N = 7 \qquad M = 10$

At XO 0 variables are exactly at the bounds

At iterate 0 f= 7.57659D-01 |proj g|= 3.40248D-01

This problem is unconstrained.

At iterate 5 f = 6.57902D-01 |proj g| = 3.92932D-02

At iterate 10 f = 6.54013D-01 |proj g| = 2.61711D-03

At iterate 15 f = 6.52963D-01 |proj g| = 1.49874D-02

At iterate 20 f= 6.52496D-01 |proj g|= 5.91874D-04

* * *

Tit = total number of iterations

Tnf = total number of function evaluations

Tnint = total number of segments explored during Cauchy searches

Skip = number of BFGS updates skipped

Nact = number of active bounds at final generalized Cauchy point

Projg = norm of the final projected gradient

F = final function value

* * *

N Tit Tnf Tnint Skip Nact Projg F 7 23 26 1 0 0 1.301D-05 6.525D-01

F = 0.65249602484735647

CONVERGENCE: REL REDUCTION OF F <= FACTR*EPSMCH

233.23866434871178

RUNNING THE L-BFGS-B CODE

* * *

Machine precision = 2.220D-16

 $N = 5 \qquad M = 10$

At X0 0 variables are exactly at the bounds

At iterate 0 f = 7.74768D - 01 |proj g|= 9.02952D - 02

At iterate 5 f= 7.63502D-01 |proj g|= 1.85802D-02

This problem is unconstrained.

This problem is unconstrained.

At iterate 10 f= 7.55961D-01 |proj g|= 1.64044D-02

At iterate 15 f = 7.55720D-01 |proj g| = 4.11658D-05

```
Tit = total number of iterations
Tnf = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip = number of BFGS updates skipped
Nact = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F = final function value
      Tit Tnf Tnint Skip Nact Projg
      16 25 1 0 0 1.551D-06 7.557D-01
 F = 0.75572043776861175
CONVERGENCE: NORM OF PROJECTED GRADIENT <= PGTOL
263.92206709025356
RUNNING THE L-BFGS-B CODE
         * * *
Machine precision = 2.220D-16
N = 6 \qquad M =
                               10
At XO
           0 variables are exactly at the bounds
At iterate 0 f= 7.63138D-01 |proj g|= 7.83606D-02
At iterate 5 f = 7.54235D-01 |proj g| = 1.23743D-02
At iterate 10 f= 7.49421D-01
                                |proj g| = 6.87273D-02
At iterate 15 f = 7.45652D - 01
                                 |proj g| = 3.38878D-03
At iterate 20 f= 7.45624D-01 |proj g|= 4.72080D-06
         * * *
Tit = total number of iterations
Tnf = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip = number of BFGS updates skipped
Nact = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F = final function value
      Tit Tnf Tnint Skip Nact Projg
             26 1 0 0 4.721D-06 7.456D-01
       20
 F = 0.74562426918967417
CONVERGENCE: NORM OF PROJECTED GRADIENT <= PGTOL
262.5297544477305
RUNNING THE L-BFGS-B CODE
         * * *
Machine precision = 2.220D-16
N = 7 \qquad M =
                               10
At XO
           0 variables are exactly at the bounds
```

At iterate 0 f = 7.59731D-01 |proj g|= 2.45036D-01

```
This problem is unconstrained.
                                 |proj g| = 5.89648D-03
At iterate 5 f = 7.14658D - 01
At iterate 10 f= 7.10686D-01
                                 |proj g| = 5.58666D - 02
At iterate 15 f = 7.06004D - 01
                                 |proj g| = 4.15584D-02
At iterate 20 f= 7.03970D-01 |proj g|= 4.38439D-03
At iterate 25 f= 7.03913D-01 |proj q|= 1.05189D-05
        * * *
Tit = total number of iterations
Tnf = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip = number of BFGS updates skipped
Nact = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F = final function value
          * * *
  N Tit Tnf Tnint Skip Nact Projg
       25
             28 1 0 0 1.052D-05 7.039D-01
 F = 0.70391315617984040
CONVERGENCE: REL REDUCTION OF F <= FACTR*EPSMCH
250.5148204764264
RUNNING THE L-BFGS-B CODE
          * * *
Machine precision = 2.220D-16
                  M =
             8
                               10
At XO 0 variables are exactly at the bounds
At iterate 0 f= 8.06508D-01 |proj g|= 4.96754D-01
This problem is unconstrained.
At iterate 5 f = 6.56587D-01 |proj g|= 2.57727D-02
At iterate 10 f = 6.54351D-01 |proj g|= 5.40956D-03
At iterate 15 f= 6.52548D-01
                                 |proj g| = 1.06053D-02
At iterate 20 f = 6.52308D - 01 |proj g|= 5.17201D-05
        * * *
Tit = total number of iterations
Tnf = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip = number of BFGS updates skipped
Nact = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F = final function value
         * * *
      Tit Tnf Tnint Skip Nact Projg
             26 1 0 0 2.337D-05 6.523D-01
  8
       21
  F = 0.65230814271602477
```

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH

```
235.17553595258434
RUNNING THE L-BFGS-B CODE
```

Machine precision = 2.220D-16N = 6 M =

At XO 0 variables are exactly at the bounds

At iterate 0 f= 7.15290D-01 |proj g|= 1.10565D-01

10

At iterate 5 f = 7.02491D-01 |proj g|= 3.37413D-03

At iterate 10 f = 7.01136D-01 |proj g|= 3.56698D-02

This problem is unconstrained.

At iterate 15 f = 6.98404D-01 |proj g|= 7.90862D-03

At iterate 20 f= 6.98306D-01 |proj g|= 3.96454D-06

* * *

Tit = total number of iterations

Tnf = total number of function evaluations

Tnint = total number of segments explored during Cauchy searches

Skip = number of BFGS updates skipped

Nact = number of active bounds at final generalized Cauchy point

Projg = norm of the final projected gradient

F = final function value

* * *

N Tit Tnf Tnint Skip Nact Projg F 6 20 27 1 0 0 3.965D-06 6.983D-01 F = 0.69830583509314048

CONVERGENCE: NORM_OF_PROJECTED_GRADIENT_<=_PGTOL

246.6307605912952

RUNNING THE L-BFGS-B CODE

* * *

Machine precision = 2.220D-16

 $N = 7 \qquad M = 10$

At XO 0 variables are exactly at the bounds

At iterate 0 f= 7.14781D-01 |proj g|= 8.31139D-02

This problem is unconstrained.

At iterate 5 f= 7.05306D-01 |proj g|= 2.30665D-02

At iterate 10 f= 6.94733D-01 |proj g|= 9.78098D-03

At iterate 15 f= 6.94652D-01 |proj g|= 2.74866D-04

At iterate 20 f= 6.94636D-01 |proj g|= 1.65451D-03

At iterate 25 f= 6.94633D-01 |proj g|= 3.31173D-05

* * *

Tit = total number of iterations

Tnf = total number of function evaluations

Tnint = total number of segments explored during Cauchy searches

Skip = number of BFGS updates skipped

Nact = number of active bounds at final generalized Cauchy point Projg = norm of the final projected gradient = final function value * * * Tit Inf Inint Skip Nact Projg F 1 0 0 3.312D-05 6.946D-01 25 28 F = 0.69463317305464678CONVERGENCE: REL REDUCTION OF F <= FACTR*EPSMCH 247.39674614636132 RUNNING THE L-BFGS-B CODE * * * Machine precision = 2.220D-16 8 M =10 At XO 0 variables are exactly at the bounds At iterate 0 f = 7.37789D-01 |proj g|= 2.24506D-01 This problem is unconstrained. At iterate 5 f = 6.98278D - 01|proj g| = 1.62491D-02At iterate 10 f= 6.93389D-01 |proj g| = 2.83444D-03At iterate 15 f = 6.93347D-01 |proj g| = 3.22731D-04At iterate 20 f = 6.93346D-01 |proj g|= 1.35695D-04 * * * Tit = total number of iterations Tnf = total number of function evaluations Tnint = total number of segments explored during Cauchy searches Skip = number of BFGS updates skipped Nact = number of active bounds at final generalized Cauchy point Projg = norm of the final projected gradient F = final function value * * * N Tit Tnf Tnint Skip Nact Projq 28 1 0 0 2.067D-05 6.933D-01 24 F = 0.69334634539431816CONVERGENCE: REL REDUCTION OF F <= FACTR*EPSMCH 248.96437205249092 RUNNING THE L-BFGS-B CODE * * * Machine precision = 2.220D-16

Machine precision = 2.220D-16N = 9 M = 10

At XO 0 variables are exactly at the bounds

At iterate 0 f= 8.61009D-01 |proj g|= 5.57499D-01

This problem is unconstrained.

At iterate 5 f= 6.59954D-01 |proj g|= 2.63434D-02

At iterate 10 f= 6.53683D-01 |proj g|= 1.83561D-02

At iterate 15 f= 6.53192D-01 |proj g|= 3.63585D-03

```
At iterate 20 f= 6.52496D-01
                                 |proj g| = 1.79007D-02
At iterate 25 f= 6.52189D-01
                                  |proj g| = 4.50870D-03
At iterate 30 f = 6.51937D - 01
                                  |proj g| = 1.34746D-03
At iterate 35 f= 6.51894D-01
                                 |proj g|= 2.91388D-03
At iterate 40 f= 6.51669D-01
                                  |proj q| = 3.69745D-03
At iterate 45 f= 6.51179D-01
                                  |proj g| = 1.31881D-02
At iterate 50 f= 6.50577D-01 |proj q|= 4.94050D-03
         * * *
Tit = total number of iterations
Tnf = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip = number of BFGS updates skipped
Nact = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F = final function value
          * * *
      Tit Inf Inint Skip Nact Projg
       50
              64
                     1 0 0 4.941D-03 6.506D-01
  F = 0.65057745041272597
STOP: TOTAL NO. of ITERATIONS REACHED LIMIT
236.59402333867592
RUNNING THE L-BFGS-B CODE
          * * *
Machine precision = 2.220D-16
N =
             7 	 M =
                               10
At XO
           O variables are exactly at the bounds
At iterate
          0 f= 6.81200D-01 |proj q|= 1.25233D-01
This problem is unconstrained.
At iterate 5 f = 6.51689D - 01
                                  |proj g| = 1.41215D-02
At iterate 10 f= 6.51071D-01
                                  |proj g| = 1.76070D-04
At iterate 15 f = 6.51071D-01 |proj g| = 2.52155D-04
         * * *
Tit = total number of iterations
Tnf = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip = number of BFGS updates skipped
Nact = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
    = final function value
          * * *
             Tnf Tnint Skip Nact Projg
```

1 0 0 3.796D-06 6.511D-01

F = 0.65107062824359707

19

22

```
CONVERGENCE: NORM OF PROJECTED GRADIENT <= PGTOL
```

232.7597310898486

RUNNING THE L-BFGS-B CODE

* * *

Machine precision = 2.220D-16

 $N = 8 \qquad M = 10$

At X0 0 variables are exactly at the bounds

At iterate 0 f= 6.80823D-01 |proj g|= 1.24561D-01

This problem is unconstrained.

At iterate 5 f= 6.51226D-01 |proj g|= 1.11029D-02

At iterate 10 f = 6.49932D-01 |proj g| = 1.20147D-02

At iterate 15 f = 6.49856D - 01 |proj g|= 6.13564D-04

At iterate 20 f = 6.49851D-01 |proj g|= 8.93703D-05

* * *

Tit = total number of iterations

Tnf = total number of function evaluations

Tnint = total number of segments explored during Cauchy searches

Skip = number of BFGS updates skipped

Nact = number of active bounds at final generalized Cauchy point

Projg = norm of the final projected gradient

F = final function value

* * *

N Tit Tnf Tnint Skip Nact Projg F 8 22 26 1 0 0 1.481D-05 6.499D-01

F = 0.64985080575643883

CONVERGENCE: REL REDUCTION OF F <= FACTR*EPSMCH

234.34987073416343

RUNNING THE L-BFGS-B CODE

* * *

Machine precision = 2.220D-16

N = 9 M = 10

At XO 0 variables are exactly at the bounds

At iterate 0 f= 7.03130D-01 |proj g|= 2.15347D-01

This problem is unconstrained.

At iterate 5 f = 6.51244D - 01 |proj g|= 1.47875D - 02

At iterate 10 f = 6.49910D-01 |proj g| = 2.45790D-03

At iterate 15 f = 6.49856D-01 |proj g| = 1.86553D-03

At iterate 20 f= 6.49852D-01 |proj g|= 2.86055D-04

At iterate 25 f= 6.49850D-01 |proj g|= 1.69987D-04

* * *

Tit = total number of iterations

Tnf = total number of function evaluations

```
Tnint = total number of segments explored during Cauchy searches
Skip = number of BFGS updates skipped
Nact = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
   = final function value
          * * *
      Tit Tnf Tnint Skip Nact Projg
  9 27
             31 1 0 0 1.406D-05 6.499D-01
  F = 0.64985022259640401
CONVERGENCE: REL REDUCTION OF F <= FACTR*EPSMCH
236.34967479239174
RUNNING THE L-BFGS-B CODE
         * * *
Machine precision = 2.220D-16
                               10
N =
            10 M =
At XO 0 variables are exactly at the bounds
At iterate 0 f= 8.40749D-01 |proj g|= 4.82518D-01
This problem is unconstrained.
At iterate 5 f = 6.73058D - 01
                                 |proj g| = 5.69232D-02
At iterate 10 f = 6.53101D-01 |proj g| = 1.23879D-02
At iterate 15 f= 6.49916D-01
                                 |proj g| = 1.13013D-02
At iterate 20 f= 6.49632D-01
                                 |proj g|= 1.91665D-03
At iterate 25 f = 6.49606D - 01 |proj g| = 2.69844D - 03
At iterate 30 f= 6.49588D-01 |proj g|= 7.86912D-05
At iterate 35 f = 6.49588D-01 |proj g|= 5.18690D-05
         * * *
Tit = total number of iterations
Tnf = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip = number of BFGS updates skipped
Nact = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F = final function value
         * * *
      Tit Tnf Tnint Skip Nact Projg
  10
       38
                   1 0 0 8.734D-06 6.496D-01
              44
  F = 0.64958834620366657
CONVERGENCE: NORM OF PROJECTED GRADIENT <= PGTOL
238.26168432443197
```

238.26168432443197
RUNNING THE L-BFGS-B CODE

* * *

Machine precision = 2.220D-16N = 5 M = 10

At XO 0 variables are exactly at the bounds

```
At iterate 0 f= 8.24388D-01
                                 |proj g| = 5.00447D-02
At iterate 5 f= 8.21483D-01
                                 |proj g| = 3.09802D-02
At iterate 10 f= 8.20626D-01
                                 |proj g| = 2.57239D-02
At iterate 15 f = 8.20345D-01 |proj q| = 1.29488D-04
         * * *
Tit = total number of iterations
Tnf = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip = number of BFGS updates skipped
Nact = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
   = final function value
          * * *
             Tnf Tnint Skip Nact Projg
                  1 0 0 1.686D-05 8.203D-01
       16
              19
  F = 0.82034529412787427
CONVERGENCE: REL_REDUCTION OF F <= FACTR*EPSMCH
This problem is unconstrained.
This problem is unconstrained.
285.63601882696577
RUNNING THE L-BFGS-B CODE
          * * *
Machine precision = 2.220D-16
                           10
N =
             6
                  M =
At XO
           O variables are exactly at the bounds
At iterate 0 f = 7.49758D - 01 |proj g| = 7.91825D - 02
At iterate 5 f = 7.39779D-01 |proj g|= 9.23093D-03
At iterate 10 f = 7.39193D-01 |proj g|= 1.12480D-02
At iterate 15 f= 7.39132D-01 |proj g|= 3.21685D-05
         * * *
Tit = total number of iterations
Tnf = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip = number of BFGS updates skipped
Nact = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F = final function value
          * * *
  N Tit Tnf Tnint Skip Nact Projg
       15
              19 1 0 0 3.217D-05 7.391D-01
  F = 0.73913221703009446
CONVERGENCE: REL REDUCTION OF F <= FACTR*EPSMCH
260.34842492211175
```

This problem is unconstrained.

Machine precision = 2.220D-16N = 7 M =

At XO 0 variables are exactly at the bounds

At iterate 0 f= 7.58275D-01 |proj g|= 2.59501D-01

10

At iterate 5 f = 7.04519D - 01 |proj g| = 4.47921D - 02

At iterate 10 f= 7.01084D-01 |proj g|= 3.90020D-03

At iterate 15 f = 7.00800D - 01 |proj g|= 6.74135D-03

At iterate 20 f= 7.00767D-01 |proj g|= 4.31987D-04

* * *

Tit = total number of iterations

Tnf = total number of function evaluations

Tnint = total number of segments explored during Cauchy searches

Skip = number of BFGS updates skipped

Nact = number of active bounds at final generalized Cauchy point

Projg = norm of the final projected gradient

F = final function value

* * *

N Tit Tnf Tnint Skip Nact Projg F
7 23 27 1 0 0 2.328D-05 7.008D-01
F = 0.70076663227526459

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH 249.45758844448892

This problem is unconstrained.

RUNNING THE L-BFGS-B CODE

* * *

Machine precision = 2.220D-16

 $N = 8 \qquad M = 10$

At XO 0 variables are exactly at the bounds

At iterate 0 f= 7.47078D-01 |proj g|= 3.45769D-01

At iterate 5 f = 6.48813D - 01 |proj g| = 3.79286D - 02

At iterate 10 f = 6.45322D - 01 |proj g|= 3.53179D-03

At iterate 15 f = 6.44973D-01 |proj g|= 4.38942D-03

At iterate 20 f = 6.44958D - 01 |proj g|= 1.46297D - 04

* * *

Tit = total number of iterations

Tnf = total number of function evaluations

Tnint = total number of segments explored during Cauchy searches

Skip = number of BFGS updates skipped

Nact = number of active bounds at final generalized Cauchy point

Projg = norm of the final projected gradient

= final function value

* * *

N Tit Tnf Tnint Skip Nact Projg F 8 23 27 1 0 0 6.977D-06 6.450D-01 F = 0.64495784549293156

CONVERGENCE: NORM_OF_PROJECTED_GRADIENT_<=_PGTOL

232.705836085625

RUNNING THE L-BFGS-B CODE

* * *

Machine precision = 2.220D-16

 $N = 6 \qquad M = 10$

At XO 0 variables are exactly at the bounds

At iterate 0 f= 7.63511D-01 |proj g|= 9.02804D-02

At iterate 5 f= 7.51681D-01 |proj g|= 7.11154D-03

This problem is unconstrained. This problem is unconstrained.

At iterate 10 f = 7.50994D-01 |proj g|= 4.97314D-03

At iterate 15 f = 7.50964D - 01 |proj q|= 2.78343D-05

* * *

Tit = total number of iterations

Tnf = total number of function evaluations

Tnint = total number of segments explored during Cauchy searches

Skip = number of BFGS updates skipped

Nact = number of active bounds at final generalized Cauchy point

Projg = norm of the final projected gradient

F = final function value

* * *

N Tit Tnf Tnint Skip Nact Projg F 6 15 20 1 0 0 2.783D-05 7.510D-01 F = 0.75096397143545568

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH 264.3238944023131

RUNNING THE L-BFGS-B CODE

* * *

Machine precision = 2.220D-16

N = 7 M = 10

At XO 0 variables are exactly at the bounds

At iterate 0 f = 7.49716D - 01 |proj g| = 7.57833D - 02

At iterate 5 f = 7.39877D - 01 |proj g|= 1.05299D - 02

At iterate 10 f = 7.39417D-01 |proj g| = 5.64913D-03

At iterate 15 f = 7.39293D-01 |proj g|= 1.16530D-03

* * *

```
Tit = total number of iterations
Tnf = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip = number of BFGS updates skipped
Nact = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F = final function value
          * * *
     Tit Inf Inint Skip Nact Projg
              23 1 0 0 1.039D-05 7.393D-01
  F = 0.73929139021391099
CONVERGENCE: REL REDUCTION OF F <= FACTR*EPSMCH
262.4019071118741
RUNNING THE L-BFGS-B CODE
         * * *
Machine precision = 2.220D-16
N = 8 M =
                          10
At XO
           O variables are exactly at the bounds
At iterate 0 f = 7.49553D - 01
                                 |proj g| = 2.54444D-01
This problem is unconstrained.
At iterate 5 f = 7.01525D-01
                                |proj g| = 1.12274D-02
At iterate 10 f= 7.00398D-01
                                 |proj g| = 5.95245D-03
At iterate 15 f= 6.99464D-01
                                 |proj g| = 3.83765D-03
At iterate 20 f = 6.99448D - 01 |proj g|= 5.73119D - 04
At iterate 25 f = 6.99447D-01 |proj g|= 1.77729D-04
         * * *
Tit = total number of iterations
Tnf = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip = number of BFGS updates skipped
Nact = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
   = final function value
          * * *
            Tnf Tnint Skip Nact Projg F
      Tit
       27
              31 1 0 0 1.057D-05 6.994D-01
 F = 0.69944655688583646
CONVERGENCE: REL REDUCTION OF F <= FACTR*EPSMCH
251.01404311364104
RUNNING THE L-BFGS-B CODE
          * * *
Machine precision = 2.220D-16
                   M =
                               10
At XO
       O variables are exactly at the bounds
```

At iterate 0 f= 7.97347D-01 |proj g|= 5.08364D-01

```
This problem is unconstrained.
At iterate 5 f = 6.47394D - 01
                                   |proj g| = 2.37179D-02
At iterate 10 f= 6.45622D-01
                                  |proj g| = 3.54850D - 03
At iterate 15 f= 6.44978D-01
                                  |proj g| = 3.88027D-03
At iterate 20 f= 6.44897D-01
                                  |proj q| = 5.88066D-03
At iterate 25 f = 6.44706D - 01
                                  |proj q| = 1.04490D-02
At iterate 30 f= 6.44666D-01
                                  |proj g| = 5.88094D-04
At iterate 35 f = 6.44666D - 01 |proj g|= 1.62026D - 05
         * * *
Tit = total number of iterations
Tnf = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip = number of BFGS updates skipped
Nact = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F = final function value
          * * *
      Tit Inf Inint Skip Nact Projg
       35
              41
                   1 0 0 1.620D-05 6.447D-01
  F = 0.64466554500882312
CONVERGENCE: REL REDUCTION OF F <= FACTR*EPSMCH
234.60762312296455
RUNNING THE L-BFGS-B CODE
          * * *
Machine precision = 2.220D-16
N =
                  M =
                                10
              7
           O variables are exactly at the bounds
At XO
At iterate 0 f = 7.08043D - 01 |proj q|= 1.09012D - 01
At iterate 5 f = 6.95345D-01 |proj g|= 5.03333D-03
At iterate 10 f = 6.94795D - 01
                                  |proj g| = 1.97093D-03
This problem is unconstrained.
At iterate 15 f = 6.94773D-01 |proj g|= 2.70074D-04
         * * *
Tit = total number of iterations
Tnf = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip = number of BFGS updates skipped
Nact = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
```

= final function value

N Tit Tnf Tnint Skip Nact Projg F 7 18 22 1 0 0 8.297D-06 6.948D-01 F = 0.69477278292316724

```
CONVERGENCE: NORM OF PROJECTED GRADIENT <= PGTOL
```

247.44365506218418

RUNNING THE L-BFGS-B CODE

* * *

Machine precision = 2.220D-16

8 M =10

At XO O variables are exactly at the bounds

At iterate 0 f= 7.06944D-01 |proj g|= 8.09483D-02

At iterate 5 f = 6.98096D - 01 |proj q|= 1.91829D - 02

This problem is unconstrained.

At iterate 10 f= 6.90548D-01 |proj g| = 1.31731D-02

At iterate 15 f = 6.89314D-01|proj g| = 3.01673D-04

At iterate 20 f= 6.89314D-01 |proj g|= 2.95797D-04

* * *

Tit = total number of iterations

Tnf = total number of function evaluations

Tnint = total number of segments explored during Cauchy searches

Skip = number of BFGS updates skipped

Nact = number of active bounds at final generalized Cauchy point

Projg = norm of the final projected gradient

= final function value

* * *

Tit Tnf Tnint Skip Nact Projg 27 1 0 0 6.720D-06 6.893D-01 24 8

F = 0.68931413564018218

CONVERGENCE: NORM OF PROJECTED GRADIENT <= PGTOL

247.6095495751012

RUNNING THE L-BFGS-B CODE

* * *

Machine precision = 2.220D-16

N =9 M = 10

O variables are exactly at the bounds

At iterate 0 f= 7.28357D-01 |proj g|= 2.24517D-01This problem is unconstrained.

At iterate 5 f = 6.89698D - 01

|proj g| = 1.10513D-02

At iterate 10 f= 6.85736D-01 |proj g| = 4.46536D-03

At iterate 15 f = 6.85654D - 01|proj q| = 1.52684D-03

At iterate 20 f = 6.85568D - 01|proj g| = 9.93077D-03

At iterate 25 f= 6.85496D-01 |proj g| = 3.74918D-03

At iterate 30 f= 6.85491D-01 |proj g|= 8.42333D-05

* * *

```
Tit = total number of iterations
Tnf = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip = number of BFGS updates skipped
Nact = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F = final function value

* * *
```

N Tit Tnf Tnint Skip Nact Projg F
9 33 40 1 0 0 1.795D-05 6.855D-01
F = 0.68549066358329314

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH 248.3248629639865

RUNNING THE L-BFGS-B CODE

* * *

Machine precision = 2.220D-16N = 10 M = 10

At X0 0 variables are exactly at the bounds

At iterate 0 f= 8.61765D-01 |proj g| = 6.05707D-01This problem is unconstrained. At iterate 5 f = 6.50113D-01|proj g| = 2.64243D-02At iterate 10 f = 6.46489D - 01|proj g| = 7.12553D-03At iterate 15 f= 6.44866D-01 |proj g| = 1.18430D-02At iterate 20 f= 6.44720D-01 |proj g| = 2.39770D-03At iterate 25 f= 6.44688D-01 |proj g| = 7.86102D-04At iterate 30 f= 6.44311D-01 |proj g| = 2.11484D-02At iterate 35 f= 6.42643D-01 |proj g| = 8.04453D-03At iterate 40 f= 6.42450D-01 |proj g| = 3.13687D-03At iterate 45 f= 6.42439D-01|proj q| = 4.55196D-04At iterate 50 f= 6.42439D-01 |proj g|= 4.36985D-05

* * *

Tit = total number of iterations

Tnf = total number of function evaluations

Tnint = total number of segments explored during Cauchy searches

Skip = number of BFGS updates skipped

Nact = number of active bounds at final generalized Cauchy point

Projg = norm of the final projected gradient

F = final function value

* * *

N Tit Tnf Tnint Skip Nact Projg F 10 50 61 1 0 0 4.370D-05 6.424D-01 F = 0.64243925850723671

STOP: TOTAL NO. of ITERATIONS REACHED LIMIT 235.85959085843152

Machine precision = 2.220D-16N = 8 M = 10

At XO 0 variables are exactly at the bounds

At iterate 0 f = 6.71963D-01 |proj g|= 1.07188D-01

At iterate 5 f = 6.48363D - 01 |proj g|= 2.60401D - 02

This problem is unconstrained.

At iterate 10 f = 6.46892D-01 |proj g|= 9.12600D-03

At iterate 15 f = 6.46637D-01 |proj g| = 8.12665D-03

At iterate 20 f= 6.46555D-01 |proj g|= 2.15977D-04

* * *

Tit = total number of iterations

Tnf = total number of function evaluations

Tnint = total number of segments explored during Cauchy searches

Skip = number of BFGS updates skipped

Nact = number of active bounds at final generalized Cauchy point

Projg = norm of the final projected gradient

F = final function value

* * *

N Tit Tnf Tnint Skip Nact Projg F 8 24 27 1 0 0 2.811D-06 6.466D-01 F = 0.64655458988796544

CONVERGENCE: NORM_OF_PROJECTED_GRADIENT_<=_PGTOL 233.24234220235638

This problem is unconstrained.

RUNNING THE L-BFGS-B CODE

* * *

Machine precision = 2.220D-16

N = 9 M = 10

At XO 0 variables are exactly at the bounds

At iterate 0 f= 6.71461D-01 |proj g|= 1.05003D-01

At iterate 5 f = 6.50053D - 01 |proj g|= 7.29340D-01

At iterate 10 f = 6.49013D - 01 |proj g| = 1.55569D - 01

At iterate 15 f = 6.48801D-01 |proj g|= 3.49302D-01

At iterate 20 f= 6.48544D-01 |proj g|= 4.32593D-02

At iterate 25 f = 6.48532D-01 |proj g| = 5.71740D-02

At iterate 30 f= 6.48462D-01 |proj g|= 2.64571D-02

At iterate 35 f = 6.48336D-01 |proj g|= 1.05043D-02

At iterate 40 f= 6.48137D-01 |proj g|= 8.75492D-02

```
At iterate 45 f= 6.48046D-01 |proj g|= 4.90163D-03

At iterate 50 f= 6.47407D-01 |proj g|= 2.21626D-01

* * * *
```

Tit = total number of iterations

Tnf = total number of function evaluations

Tnint = total number of segments explored during Cauchy searches

Skip = number of BFGS updates skipped

Nact = number of active bounds at final generalized Cauchy point

Projg = norm of the final projected gradient

F = final function value

* * *

N Tit Tnf Tnint Skip Nact Projg F 9 50 62 1 0 0 2.216D-01 6.474D-01 F = 0.64740676094543037

STOP: TOTAL NO. of ITERATIONS REACHED LIMIT

235.52867167766462

RUNNING THE L-BFGS-B CODE

* * *

Machine precision = 2.220D-16

= 10 M = 10

At XO 0 variables are exactly at the bounds

At iterate 0 f= 6.94210D-01 |proj g|= 2.15509D-01

This problem is unconstrained.

At iterate 5 f= 6.47830D-01 |proj g|= 1.66857D-02

At iterate 10 f= 6.45767D-01 |proj g|= 9.39967D-03

At iterate 15 f= 6.45469D-01 |proj g|= 9.96037D-03

At iterate 20 f= 6.45409D-01 |proj g|= 2.47431D-04

At iterate 25 f= 6.45409D-01 |proj g|= 2.76892D-05

* * *

Tit = total number of iterations

Tnf = total number of function evaluations

Tnint = total number of segments explored during Cauchy searches

Skip = number of BFGS updates skipped

Nact = number of active bounds at final generalized Cauchy point

Projg = norm of the final projected gradient

F = final function value

* * *

N Tit Tnf Tnint Skip Nact Projg F 10 26 31 1 0 0 1.247D-05 6.454D-01 F = 0.64540911002917867

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH 236.85746096980404 RUNNING THE L-BFGS-B CODE

* * *

```
Machine precision = 2.220D-16

N = 11 M = 10

At X0 0 variables are exactly at the bounds

At iterate 0 f= 8.35847D-01 |proj g|= 4.

This problem is unconstrained.
```

At iterate 0 f= 8.35847D-01 |proj g|= 4.98082D-01

This problem is unconstrained.

At iterate 5 f= 6.62143D-01 |proj g|= 4.52857D-02

At iterate 10 f= 6.49629D-01 |proj g|= 2.62182D-02

At iterate 15 f= 6.42541D-01 |proj g|= 6.46539D-03

At iterate 20 f= 6.41700D-01 |proj g|= 5.63180D-03

At iterate 25 f= 6.41650D-01 |proj g|= 6.19746D-04

At iterate 30 f= 6.41649D-01 |proj g|= 1.35095D-04

* * *

Tit = total number of iterations

Tnf = total number of function evaluations

Tnint = total number of segments explored during Cauchy searches

Skip = number of BFGS updates skipped

Nact = number of active bounds at final generalized Cauchy point

Projg = norm of the final projected gradient

F = final function value

* * *

N Tit Tnf Tnint Skip Nact Projg F 11 33 39 1 0 0 1.032D-04 6.416D-01 F = 0.64164914636174297

10

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH 237.59411317754564
RUNNING THE L-BFGS-B CODE

* * *

Machine precision = 2.220D-16 N = 6 M =

At XO 0 variables are exactly at the bounds

At iterate 0 f= 8.17524D-01 |proj g|= 4.37851D-01 At iterate 5 f= 8.08520D-01 |proj g|= 4.07463D-02 At iterate 10 f= 8.07016D-01 |proj g|= 1.09598D-02 At iterate 15 f= 8.05292D-01 |proj g|= 1.57632D-02 At iterate 20 f= 8.04984D-01 |proj g|= 6.34353D-03 At iterate 25 f= 8.04930D-01 |proj g|= 2.05200D-04

* * *

Tit = total number of iterations

Tnf = total number of function evaluations

Tnint = total number of segments explored during Cauchy searches

Skip = number of BFGS updates skipped

Nact = number of active bounds at final generalized Cauchy point

```
Projg = norm of the final projected gradient
F = final function value
      Tit Tnf Tnint Skip Nact Projg
       27 35 1 0 0 3.149D-05 8.049D-01
 F = 0.80492964478337348
CONVERGENCE: REL REDUCTION OF F <= FACTR*EPSMCH
282.4563606472135
RUNNING THE L-BFGS-B CODE
         * * *
Machine precision = 2.220D-16
N = 7 \qquad M =
                               10
At XO
           0 variables are exactly at the bounds
At iterate
           0 	 f= 7.51725D-01
                                 |proj q| = 6.54266D-01
This problem is unconstrained.
This problem is unconstrained.
At iterate
          5 	 f = 7.30600D - 01
                                  |proj g| = 5.08173D-02
At iterate 10 f = 7.27582D - 01 |proj g| = 3.52557D - 01
At iterate 15 f = 7.24187D-01 |proj g|= 8.25292D-02
At iterate 20 f= 7.22583D-01
                                  |proj g| = 5.78572D-03
Bad direction in the line search;
 refresh the lbfgs memory and restart the iteration.
Warning: more than 10 function and gradient
 evaluations in the last line search. Termination
  may possibly be caused by a bad search direction.
This problem is unconstrained.
At iterate 25 f= 7.22562D-01 |proj g|= 2.62104D-05
         * * *
Tit = total number of iterations
Tnf = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip = number of BFGS updates skipped
Nact = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F = final function value
         * * *
      Tit Tnf Tnint Skip Nact Projg
       25
             66 2 0 0 2.621D-05 7.226D-01
  7
 F = 0.72256181326340552
CONVERGENCE: REL REDUCTION OF F <= FACTR*EPSMCH
256.78076925650424
RUNNING THE L-BFGS-B CODE
```

At XO 0 variables are exactly at the bounds

10

M =

* * *

N =

Machine precision = 2.220D-16

8

Tnf = total number of function evaluations

Tnint = total number of segments explored during Cauchy searches

Skip = number of BFGS updates skipped

Nact = number of active bounds at final generalized Cauchy point

Projg = norm of the final projected gradient

F = final function value

* * *

N Tit Tnf Tnint Skip Nact Projg F 8 33 39 1 0 0 8.175D-06 6.729D-01 F = 0.67293180935326558

CONVERGENCE: NORM OF PROJECTED GRADIENT <= PGTOL

242.10508794269722

RUNNING THE L-BFGS-B CODE

This problem is unconstrained.

* * *

Machine precision = 2.220D-16 N = 9 M =

 $N = 9 \qquad M = 10$

At XO 0 variables are exactly at the bounds

At iterate 0 f= 7.94638D-01 |proj g|= 1.70528D+00

At iterate 5 f = 6.56616D-01 |proj g| = 2.63793D-01

At iterate 10 f = 6.41457D-01 |proj g| = 7.16469D-02

At iterate 15 f = 6.36303D - 01 |proj g| = 4.89783D - 02

At iterate 20 f= 6.33765D-01 |proj g|= 6.14359D-02

At iterate 25 f = 6.33677D-01 |proj g| = 7.14105D-04

At iterate 30 f = 6.33676D-01 |proj g| = 9.78957D-04

At iterate 35 f = 6.33676D-01 |proj g|= 2.04526D-04

* * *

Tit = total number of iterations

Tnf = total number of function evaluations

Tnint = total number of segments explored during Cauchy searches

Skip = number of BFGS updates skipped

```
Nact = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
   = final function value
         * * *
      Tit Tnf Tnint Skip Nact Projg F
             45 1 0 0 9.103D-05 6.337D-01
       38
  F = 0.63367620848451134
CONVERGENCE: REL REDUCTION OF F <= FACTR*EPSMCH
230.9152060507958
RUNNING THE L-BFGS-B CODE
          * * *
Machine precision = 2.220D-16
             7
                  M =
At XO
           O variables are exactly at the bounds
At iterate 0 f = 7.70033D-01 |proj g| = 5.49979D-01
At iterate 5 f = 7.47482D-01 |proj g|= 3.52716D-02
At iterate 10 f= 7.45551D-01
                                 |proj g| = 4.95700D-02
At iterate 15 f= 7.39571D-01
                                 |proj g| = 1.52188D-02
This problem is unconstrained.
At iterate 20 f = 7.34092D - 01
                                 |proj g| = 5.43128D-02
At iterate 25 f= 7.29363D-01
                                 |proj g| = 1.65265D-02
At iterate 30 f = 7.29322D - 01 |proj g| = 1.10048D - 04
         * * *
Tit = total number of iterations
Tnf = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip = number of BFGS updates skipped
Nact = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
   = final function value
         * * *
            Tnf Tnint Skip Nact Projg
             43 1 0 0 1.331D-05 7.293D-01
       32
 F = 0.72932151762723796
CONVERGENCE: REL REDUCTION OF F <= FACTR*EPSMCH
259.05202992275196
RUNNING THE L-BFGS-B CODE
          * * *
Machine precision = 2.220D-16
      8
                  M =
At XO
       O variables are exactly at the bounds
At iterate
           0 f= 7.56582D-01 |proj g|= 6.53619D-01
This problem is unconstrained.
At iterate 5 f = 7.34620D - 01 |proj g|= 2.58577D-02
```

```
At iterate 10 f = 7.31043D-01 |proj g| = 6.81083D-02
At iterate 15 f= 7.25540D-01
                                |proj g| = 1.49251D-02
At iterate 20 f= 7.21921D-01
                                |proj g| = 6.43004D-02
At iterate 25 f = 7.14178D-01 |proj g|= 2.01877D-02
At iterate 30 f = 7.11905D - 01
                                |proj g| = 4.70813D-03
At iterate 35 f = 7.11896D - 01 |proj g|= 3.29662D-05
        * * *
```

Tit = total number of iterations

Tnf = total number of function evaluations

Tnint = total number of segments explored during Cauchy searches

Skip = number of BFGS updates skipped

Nact = number of active bounds at final generalized Cauchy point

Projg = norm of the final projected gradient

F = final function value

* * *

Tit Tnf Tnint Skip Nact Projg
35 47 1 ^ ^ 1 0 0 3.297D-05 7.119D-01 F = 0.71189632451044249

CONVERGENCE: REL REDUCTION OF F <= FACTR*EPSMCH

255.19716503550868

RUNNING THE L-BFGS-B CODE

* * *

Machine precision = 2.220D-1610 N =9 M =

At XO 0 variables are exactly at the bounds At iterate 0 f= 7.44000D=01 | hroi gl= 3.04722D=01

| Αt | ıterate | U | i= | 7.44000D-01 | proj g = | 3.04/22D-01 | | | | |
|--------------------------------|---------|----|----|-------------|----------|-------------|--|--|--|--|
| This problem is unconstrained. | | | | | | | | | | |
| At | iterate | 5 | f= | 6.93822D-01 | proj g = | 4.71814D-02 | | | | |
| At | iterate | 10 | f= | 6.88518D-01 | proj g = | 4.36336D-02 | | | | |
| At | iterate | 15 | f= | 6.84472D-01 | proj g = | 3.04689D-02 | | | | |
| At | iterate | 20 | f= | 6.72399D-01 | proj g = | 4.74643D-02 | | | | |
| At | iterate | 25 | f= | 6.71653D-01 | proj g = | 1.83537D-03 | | | | |
| At | iterate | 30 | f= | 6.71652D-01 | proj g = | 6.80350D-04 | | | | |

* * *

Tit = total number of iterations

Tnf = total number of function evaluations

Tnint = total number of segments explored during Cauchy searches

Skip = number of BFGS updates skipped

Nact = number of active bounds at final generalized Cauchy point

Projg = norm of the final projected gradient

= final function value

* * *

```
N Tit Tnf Tnint Skip Nact Projg
  9 34
             43 1 0 0 2.385D-05 6.717D-01
  F = 0.67165134188353459
CONVERGENCE: REL REDUCTION OF F <= FACTR*EPSMCH
243.6748508728676
RUNNING THE L-BFGS-B CODE
         * * *
Machine precision = 2.220D-16
N =
            10
                  M =
                               10
At XO 0 variables are exactly at the bounds
At iterate 0 f = 8.45082D - 01 |proj g|= 2.07458D+00
This problem is unconstrained.
At iterate 5 f = 6.48959D - 01
                                 |proj g| = 2.33925D-01
At iterate 10 f = 6.38806D - 01 |proj g|= 1.69271D-01
At iterate 15 f = 6.34626D - 01
                                 |proj g| = 3.76141D-02
At iterate 20 f= 6.33992D-01
                                 |proj g| = 1.71153D-02
At iterate 25 f = 6.33246D-01 |proj g|= 7.46104D-02
At iterate 30 f = 6.32481D - 01
                                 |proj g| = 6.99241D-03
At iterate 35 f= 6.32466D-01
                                 |proj g| = 1.94727D-03
At iterate 40 f= 6.32465D-01 |proj g|= 2.08272D-04
        * * *
Tit = total number of iterations
Tnf = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip = number of BFGS updates skipped
Nact = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F = final function value
         * * *
      Tit Inf Inint Skip Nact Projg
                  1 0 0 1.374D-04 6.325D-01
       42
             49
 F = 0.63246496141146369
CONVERGENCE: REL REDUCTION OF F <= FACTR*EPSMCH
232.5082270342518
RUNNING THE L-BFGS-B CODE
         * * *
Machine precision = 2.220D-16
N =
             8 	 M =
                               10
           O variables are exactly at the bounds
At XO
At iterate 0 f = 7.21760D - 01 |proj g| = 5.76916D - 01
At iterate 5 f = 6.92341D - 01 |proj g|= 5.88817D - 02
```

This problem is unconstrained.

```
10 f= 6.87633D-01
                                  |proj g| = 4.65714D-02
At iterate 15 f = 6.82089D - 01
                                  |proj q| = 1.29591D-02
At iterate 20 f = 6.73072D - 01
                                  |proj g| = 7.29658D-02
At iterate 25 f= 6.67732D-01
                                  |proj g| = 2.26180D-02
At iterate 30 f = 6.67082D - 01 |proj g| = 3.15431D - 04
         * * *
Tit = total number of iterations
Tnf = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip = number of BFGS updates skipped
Nact = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
    = final function value
          * * *
      Tit Tnf Tnint Skip Nact Projg
       34
              42 1 0 0 5.773D-05 6.671D-01
  8
  F = 0.66708232063934769
CONVERGENCE: REL REDUCTION OF F <= FACTR*EPSMCH
240.13965973482084
RUNNING THE L-BFGS-B CODE
          * * *
Machine precision = 2.220D-16
N =
             9
                   M =
                                10
At XO 0 variables are exactly at the bounds
At iterate 0 f= 7.21966D-01 |proj g|= 6.14540D-01
At iterate 5 f= 6.96701D-01
                                   |proj g| = 2.72145D-02
This problem is unconstrained.
At iterate 10 f= 6.86464D-01
                                  |proj g| = 2.89616D-02
At iterate 15 f = 6.77456D - 01
                                  |proj q| = 1.65005D-02
At iterate 20 f = 6.70685D - 01
                                  |proj g| = 3.16023D-02
At iterate 25 f= 6.68009D-01
                                  |proj g| = 1.80525D-02
At iterate 30 f = 6.64694D - 01
                                  |proj g| = 2.03787D-02
At iterate 35 f= 6.64399D-01
                                  |proj g| = 2.13054D-03
At iterate 40 f = 6.64395D - 01
                                  |proj q| = 8.02996D-05
         * * *
Tit = total number of iterations
Tnf = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip = number of BFGS updates skipped
Nact = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
     = final function value
```

At iterate

Tit Tnf Tnint Skip Nact Projg 0 0 5.787D-06 6.644D-01 55 1 F = 0.66439487141070586CONVERGENCE: NORM OF PROJECTED GRADIENT <= PGTOL 241.23667679399716 RUNNING THE L-BFGS-B CODE * * * Machine precision = 2.220D-16 10 M =At XO 0 variables are exactly at the bounds At iterate 0 f = 7.29185D-01|proj g| = 3.69423D-01This problem is unconstrained. At iterate 5 f= 6.83587D-01 |proj g|= 3.92640D-02 At iterate 10 f= 6.74108D-01 |proj q| = 5.24125D-02At iterate 15 f= 6.64656D-01 |proj g| = 4.58713D-02At iterate 20 f= 6.57451D-01 |proj g| = 4.62172D-02At iterate 25 f= 6.53701D-01 |proj g| = 4.65571D-02At iterate 30 f= 6.50071D-01 |proj g| = 1.60377D-02At iterate 35 f= 6.49963D-01|proj g| = 2.01298D-03At iterate 40 f = 6.49957D-01 |proj g| = 3.46575D-04* * * Tit = total number of iterations Tnf = total number of function evaluations Tnint = total number of segments explored during Cauchy searches Skip = number of BFGS updates skipped Nact = number of active bounds at final generalized Cauchy point Projg = norm of the final projected gradient = final function value * * * Tnf Tnint Skip Nact Projg 1 0 0 1.405D-05 6.500D-01 43 53 10 F = 0.64995742105777177CONVERGENCE: REL REDUCTION OF F <= FACTR*EPSMCH 238.38569347541133 This problem is unconstrained. RUNNING THE L-BFGS-B CODE * * * Machine precision = 2.220D-16 $N = 11 \qquad M =$ At XO 0 variables are exactly at the bounds

At iterate 0 f = 8.97581D-01 |proj g| = 2.36935D+00

```
5
                f= 6.56285D-01
                                   |proj g| = 1.58625D-01
At iterate
At iterate 10 f = 6.37608D - 01
                                  |proj g| = 2.50380D-02
At iterate
          15 f= 6.30630D-01
                                  |proj g| = 1.76606D-02
At iterate
          20 f= 6.24395D-01
                                  |proj g| = 5.75407D-02
At iterate 25 f = 6.20581D-01
                                  |proj g| = 2.22518D-02
At iterate 30 f = 6.12826D - 01
                                  |proj g| = 6.55121D-02
At iterate 35 f = 6.07742D - 01
                                  |proj g| = 2.52122D-02
At iterate 40 f= 6.05760D-01
                                  |proj g| = 2.06324D-02
At iterate 45 f= 6.04960D-01
                                  |proj g| = 1.36755D-03
At iterate 50 f = 6.04954D - 01
                                  |proj q| = 1.95574D-04
         * * *
Tit = total number of iterations
Tnf = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip = number of BFGS updates skipped
Nact = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F = final function value
          * * *
             Tnf Tnint Skip Nact
                                      Projg
       50
                          0 0 1.956D-04 6.050D-01
  11
               62
                      1
  F = 0.60495385060557627
STOP: TOTAL NO. of ITERATIONS REACHED LIMIT
225.26449380347364
RUNNING THE L-BFGS-B CODE
          * * *
Machine precision = 2.220D-16
N =
              9
                   M =
                                10
At XO
           O variables are exactly at the bounds
           0 f = 6.88847D - 01
                                  |proj g| = 4.66988D-01
At iterate
This problem is unconstrained.
          5 	 f = 6.51794D - 01
At iterate
                                |proj g| = 2.20176D-01
At iterate 10 f= 6.39186D-01
                                  |proj g| = 9.20231D-02
At iterate
          15 f= 6.35070D-01
                                  |proj g| = 2.34518D-02
At iterate 20 f = 6.23216D-01
                                  |proj g| = 5.92061D-02
At iterate 25 f= 6.17183D-01
                                  |proj g| = 4.93565D-03
At iterate 30 f= 6.17166D-01
                                 |proj g| = 1.81272D-03
```

= total number of iterations

* * *

Tnf = total number of function evaluations

```
Tnint = total number of segments explored during Cauchy searches
Skip = number of BFGS updates skipped
Nact = number of active bounds at final generalized Cauchy point
```

Projg = norm of the final projected gradient

F = final function value

* * *

N Tit Tnf Tnint Skip Nact Projg F 9 34 59 1 0 0 1.797D-04 6.172D-01 F = 0.61716576684230184

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH 225.3676976590134

RUNNING THE L-BFGS-B CODE

* * *

Machine precision = 2.220D-16N = 10 M = 10

At XO 0 variables are exactly at the bounds

| At iterate | 0 | f= | 6.89794D-01 | proj g = | 4.99645D-01 | | | | | |
|--------------------------------|----|----|-------------|----------|-------------|--|--|--|--|--|
| This problem is unconstrained. | | | | | | | | | | |
| At iterate | 5 | f= | 6.54270D-01 | proj g = | 2.47238D-01 | | | | | |
| At iterate | 10 | f= | 6.39522D-01 | proj g = | 1.01073D-01 | | | | | |
| At iterate | 15 | f= | 6.35081D-01 | proj g = | 1.38614D-02 | | | | | |
| At iterate | 20 | f= | 6.26231D-01 | proj g = | 2.36545D-01 | | | | | |
| At iterate | 25 | f= | 6.19613D-01 | proj g = | 4.86854D-01 | | | | | |
| At iterate | 30 | f= | 6.18385D-01 | proj g = | 2.63244D-01 | | | | | |
| At iterate | 35 | f= | 6.16716D-01 | proj g = | 1.21970D-02 | | | | | |
| At iterate | 40 | f= | 6.16622D-01 | proj g = | 4.00369D-02 | | | | | |
| At iterate | 45 | f= | 6.16444D-01 | proj g = | 3.36727D-02 | | | | | |
| At iterate | 50 | f= | 6.16393D-01 | proj g = | 3.11292D-03 | | | | | |

* * *

Tit = total number of iterations

Tnf = total number of function evaluations

Tnint = total number of segments explored during Cauchy searches

Skip = number of BFGS updates skipped

Nact = number of active bounds at final generalized Cauchy point

Projg = norm of the final projected gradient

F = final function value

* * *

N Tit Tnf Tnint Skip Nact Projg F 10 50 70 1 0 0 3.113D-03 6.164D-01 F = 0.61639265383326702

STOP: TOTAL NO. of ITERATIONS REACHED LIMIT

227.10793168797773

RUNNING THE L-BFGS-B CODE

Machine precision = 2.220D-16 10 11 M =O variables are exactly at the bounds At XO At iterate 0 f = 7.02016D - 01|proj g| = 2.53779D-01This problem is unconstrained. 5 At iterate f= 6.48968D-01 |proj g| = 5.91398D-02At iterate 10 f = 6.34852D - 01|proj g| = 1.45475D-02At iterate 15 f = 6.29883D - 01|proj g| = 7.43794D-02At iterate 20 f= 6.22194D-01 |proj g| = 3.57969D-02At iterate f = 6.17904D - 01|proj g| = 6.02018D-0130 f = 6.17119D - 01|proj g| = 9.66752D-02At iterate |proj g| = 2.08873D-02At iterate 35 f= 6.15821D-01 At iterate 40 f= 6.15372D-01 |proj g|= 1.62933D-01 At iterate 45 f= 6.14826D-01 |proj g| = 3.16350D-02At iterate 50 f= 6.13925D-01 |proj g|= 4.37196D-01* * * Tit = total number of iterations Tnf = total number of function evaluations Tnint = total number of segments explored during Cauchy searches Skip = number of BFGS updates skipped Nact = number of active bounds at final generalized Cauchy point Projg = norm of the final projected gradient = final function value Tnf Tnint Skip Nact Projg 1 0 0 4.372D-01 6.139D-01 50 70 F = 0.61392492747753580STOP: TOTAL NO. of ITERATIONS REACHED LIMIT 228.27877563245204 This problem is unconstrained. RUNNING THE L-BFGS-B CODE * * * Machine precision = 2.220D-16 N =12 M =10

At X0 0 variables are exactly at the bounds
At iterate 0 f= 8.81462D-01 |proj g|= 2.04929D+00
At iterate 5 f= 6.65944D-01 |proj g|= 7.01318D-02
At iterate 10 f= 6.35624D-01 |proj g|= 7.08545D-02
At iterate 15 f= 6.26292D-01 |proj g|= 2.60788D-02

```
20 f= 6.17495D-01 |proj g|= 5.52551D-02
At iterate 25 f = 6.12429D - 01
                                 |proj q| = 2.08574D-02
At iterate 30 f = 6.06532D - 01
                                 |proj g| = 5.63287D-01
At iterate 35 f= 6.06244D-01
                                 |proj g| = 7.45172D-02
At iterate 40 f= 6.05922D-01
                                 |proj g| = 1.15508D-01
At iterate 45 f= 6.05535D-01
                                 |proj g| = 9.66310D-02
At iterate 50 f= 6.05437D-01 |proj g|= 8.05429D-02
         * * *
Tit = total number of iterations
Tnf = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip = number of BFGS updates skipped
Nact = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F = final function value
          * * *
      Tit Tnf Tnint Skip Nact Projg
  N
             71 1 0 0 8.054D-02 6.054D-01
  12
       50
 F = 0.60543685552474591
STOP: TOTAL NO. of ITERATIONS REACHED LIMIT
227.42678345631464
RUNNING THE L-BFGS-B CODE
          * * *
Machine precision = 2.220D-16
N = 4 \qquad M = 10
At XO
           O variables are exactly at the bounds
At iterate 0 f= 8.36832D-01 |proj g|= 2.31877D-05
         * * *
Tit = total number of iterations
Tnf = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip = number of BFGS updates skipped
Nact = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F = final function value
         * * *
      Tit Inf Inint Skip Nact Projg
             6 1 0 0 2.069D-05 8.368D-01
       1
   4
  F = 0.83683220573141526
CONVERGENCE: REL REDUCTION_OF_F_<=_FACTR*EPSMCH
 This problem is unconstrained.
 This problem is unconstrained.
This problem is unconstrained.
```

At iterate

289.1756211257555

RUNNING THE L-BFGS-B CODE

Machine precision = 2.220D-16N = 5 M = 10

At XO 0 variables are exactly at the bounds

At iterate 0 f= 7.66204D-01 |proj g|= 8.11612D-02

At iterate 5 f = 7.58354D - 01 |proj g|= 5.32639D - 03

* * *

Tit = total number of iterations

Tnf = total number of function evaluations

Tnint = total number of segments explored during Cauchy searches

Skip = number of BFGS updates skipped

Nact = number of active bounds at final generalized Cauchy point

Projg = norm of the final projected gradient

F = final function value

* * *

N Tit Tnf Tnint Skip Nact Projg F
5 9 12 1 0 0 1.054D-05 7.583D-01
F = 0.75833409940331875

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH 264.8002573995151

RUNNING THE L-BFGS-B CODE

* * *

Machine precision = 2.220D-16

 $N = 6 \qquad M = 10$

At XO 0 variables are exactly at the bounds

At iterate 0 f = 7.71904D-01 |proj g|= 2.54881D-01

At iterate 5 f = 7.21340D-01 |proj g|= 4.32286D-02

At iterate 10 f = 7.17788D-01 |proj g| = 6.33726D-03

* * *

Tit = total number of iterations

Tnf = total number of function evaluations

Tnint = total number of segments explored during Cauchy searches

Skip = number of BFGS updates skipped

Nact = number of active bounds at final generalized Cauchy point

Projg = norm of the final projected gradient

F = final function value

* * *

N Tit Tnf Tnint Skip Nact Projg F 6 14 22 1 0 0 1.422D-05 7.178D-01 F = 0.71776447064874760

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH 253.16886213797918

RUNNING THE L-BFGS-B CODE

* * *

```
Machine precision = 2.220D-16
N = 7
At XO
           O variables are exactly at the bounds
At iterate 0 f = 7.60970D - 01 |proj g| = 3.37825D - 01
This problem is unconstrained.
At iterate
          5 	 f = 6.62455D - 01
                                 |proj g| = 3.44878D-02
At iterate 10 f = 6.57866D - 01 |proj g|= 3.00699D - 03
         * * *
Tit = total number of iterations
Tnf = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip = number of BFGS updates skipped
Nact = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F = final function value
          * * *
      Tit Tnf Tnint Skip Nact Projg
             22 1 0 0 1.146D-05 6.579D-01
 F = 0.65785214819365501
CONVERGENCE: REL REDUCTION OF F <= FACTR*EPSMCH
235.03832179306806
RUNNING THE L-BFGS-B CODE
          * * *
Machine precision = 2.220D-16
N = 5 \qquad M =
                               10
At XO
           O variables are exactly at the bounds
At iterate 0 f = 7.72570D - 01 |proj g| = 9.10085D - 02
At iterate 5 f = 7.63905D-01 |proj g|= 7.73677D-03
        * * *
Tit = total number of iterations
Tnf = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip = number of BFGS updates skipped
Nact = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F = final function value
          * * *
      Tit Inf Inint Skip Nact Projg
              13 1 0 0 4.495D-06 7.639D-01
       9
  F = 0.76387189882128326
CONVERGENCE: NORM OF PROJECTED GRADIENT <= PGTOL
266.6609580039512
```

This problem is unconstrained. This problem is unconstrained.

RUNNING THE L-BFGS-B CODE

```
Machine precision = 2.220D-16
                                10
             6
                   M =
At XO
       O variables are exactly at the bounds
At iterate 0 f= 7.60776D-01 |proj g|= 8.00199D-02
At iterate 5 f= 7.52950D-01 |proj g|= 2.04231D-02
At iterate 10 f = 7.52673D-01 |proj g|= 4.01627D-03
         * * *
Tit = total number of iterations
Tnf = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip = number of BFGS updates skipped
Nact = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F = final function value
          * * *
      Tit Tnf Tnint Skip Nact Projg
14 17 1 0 0 4.024D-06
                      1 0 0 4.024D-06 7.527D-01
  F = 0.75267007588293589
CONVERGENCE: NORM OF PROJECTED GRADIENT <= PGTOL
264.89714549666644
RUNNING THE L-BFGS-B CODE
          * * *
Machine precision = 2.220D-16
                           10
             7
                  M =
At XO
           0 variables are exactly at the bounds
           0 f = 7.63167D - 01
                                  |proj q| = 2.47685D-01
At iterate
This problem is unconstrained.
At iterate 5 f= 7.18242D-01 |proj g|= 9.63779D-03
At iterate 10 f = 7.17467D - 01
                                  |proj g| = 7.54015D-03
At iterate 15 f = 7.16811D-01 |proj g|= 8.11962D-04
         * * *
Tit = total number of iterations
Tnf = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip = number of BFGS updates skipped
Nact = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F = final function value
         * * *
      Tit Inf Inint Skip Nact Projg
              23 1 0 0 5.921D-06 7.168D-01
       18
   7
  F = 0.71681055611114153
```

CONVERGENCE: NORM OF PROJECTED GRADIENT <= PGTOL

254.84834685334354 RUNNING THE L-BFGS-B CODE

* * *

Machine precision = 2.220D-16

N = 8 M = 10

At XO 0 variables are exactly at the bounds

* * *

Tit = total number of iterations

Tnf = total number of function evaluations

Tnint = total number of segments explored during Cauchy searches

Skip = number of BFGS updates skipped

Nact = number of active bounds at final generalized Cauchy point

Projg = norm of the final projected gradient

F = final function value

* * *

N Tit Tnf Tnint Skip Nact Projg F 8 25 31 1 0 0 4.229D-06 6.571D-01 F = 0.65707613309015900

CONVERGENCE: NORM OF PROJECTED GRADIENT <= PGTOL

236.77758071829342

RUNNING THE L-BFGS-B CODE

* * *

Machine precision = 2.220D-16

N = 6 M = 10

At XO 0 variables are exactly at the bounds

At iterate 0 f= 7.13099D-01 |proj g|= 1.11413D-01

At iterate 5 f = 7.00730D-01 |proj g| = 5.48413D-03

At iterate 10 f = 7.00663D - 01 |proj g|= 1.15424D-06

* * *

Tit = total number of iterations

Tnf = total number of function evaluations

Tnint = total number of segments explored during Cauchy searches

Skip = number of BFGS updates skipped

Nact = number of active bounds at final generalized Cauchy point

Projg = norm of the final projected gradient

F = final function value

```
Tit Tnf Tnint Skip Nact Projg
       10
                         0 0 1.154D-06 7.007D-01
              12
                     1
  F = 0.70066340869459243
CONVERGENCE: NORM OF PROJECTED GRADIENT <= PGTOL
247.42290532138304
RUNNING THE L-BFGS-B CODE
          * * *
Machine precision = 2.220D-16
      7
                  M =
At XO 0 variables are exactly at the bounds
At iterate 0 f = 7.12546D - 01
                                 |proj g| = 8.38524D-02
This problem is unconstrained.
This problem is unconstrained.
At iterate 5 f = 7.03417D - 01
                                |proj g| = 2.72964D-02
At iterate 10 f = 6.93005D - 01
                                 |proj g| = 9.88843D-03
At iterate 15 f = 6.92930D - 01 |proj g|= 9.13512D-05
         * * *
Tit = total number of iterations
Tnf = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip = number of BFGS updates skipped
Nact = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F = final function value
          * * *
      Tit Tnf Tnint Skip Nact Projg
       17
             20 1 0 0 7.534D-06 6.929D-01
 F = 0.69292971567359507
CONVERGENCE: NORM OF PROJECTED GRADIENT <= PGTOL
246.82438446632796
RUNNING THE L-BFGS-B CODE
         * * *
Machine precision = 2.220D-16
                         10
N = 8 \qquad M =
At XO
           O variables are exactly at the bounds
At iterate 0 f= 7.35558D-01 |proj g|= 2.25822D-01
This problem is unconstrained.
At iterate 5 f = 6.96364D - 01
                                 |proj g| = 2.01195D-02
At iterate 10 f= 6.92077D-01
                                 |proj g| = 1.14797D-02
At iterate 15 f = 6.91716D-01 |proj g| = 2.17269D-03
          * * *
```

Tit = total number of iterations

```
Tnf = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip = number of BFGS updates skipped
Nact = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F = final function value
          * * *
  N Tit Tnf Tnint Skip Nact Projg
       19
                         0 0 3.008D-06 6.917D-01
              22
                  1
  F = 0.69170952131778674
CONVERGENCE: NORM OF PROJECTED GRADIENT <= PGTOL
248.41439916277633
RUNNING THE L-BFGS-B CODE
          * * *
Machine precision = 2.220D-16
                  M =
N = 9
                               10
At XO 0 variables are exactly at the bounds
At iterate 0
               f= 8.65982D-01
                                 |proj g|= 5.55489D-01
This problem is unconstrained.
At iterate 5 f = 6.64922D-01 |proj g|= 3.06650D-02
At iterate 10 f = 6.58513D-01 |proj g|= 1.82902D-02
 ys=-2.911E-03 -gs= 1.169E-04 BFGS update SKIPPED
At iterate 15 f = 6.57355D-01 |proj g| = 2.73451D-03
At iterate 20 f= 6.57296D-01 |proj g|= 4.63503D-03
At iterate 25 f = 6.57082D - 01 |proj g|= 9.51559D-03
At iterate 30 f = 6.57076D-01 |proj g|= 3.27509D-04
         * * *
Tit = total number of iterations
Tnf = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip = number of BFGS updates skipped
Nact = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F = final function value
          * * *
  N Tit Tnf Tnint Skip Nact Projg
       32
                  1 1 0 2.020D-04 6.571D-01
              48
  F = 0.65707613524266639
CONVERGENCE: REL REDUCTION OF F <= FACTR*EPSMCH
238.77758144153592
RUNNING THE L-BFGS-B CODE
```

Machine precision = 2.220D-16N = 7 M =

* * *

At XO 0 variables are exactly at the bounds

10

```
0 	 f = 6.78205D - 01
At iterate
                                  |proj g| = 1.29679D-01
This problem is unconstrained.
          5 	 f = 6.49560D - 01
                                  |proj g| = 2.19614D-02
At iterate
At iterate 10 f= 6.47891D-01
                                  |proj g| = 1.50907D-03
At iterate 15 f= 6.47877D-01
                                  |proj g| = 1.62045D-06
         * * *
Tit = total number of iterations
Tnf = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip = number of BFGS updates skipped
Nact = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F = final function value
          * * *
             Tnf Tnint Skip Nact Projg
                          0 0 1.620D-06 6.479D-01
        15
               17
                      1
  F = 0.64787734049869050
CONVERGENCE: NORM OF PROJECTED GRADIENT <= PGTOL
231.68678640756
RUNNING THE L-BFGS-B CODE
          * * *
Machine precision = 2.220D-16
             8
                   M =
                                10
At XO
      O variables are exactly at the bounds
At iterate 0 f= 6.77816D-01 |proj q|= 1.28141D-01
           5 	 f = 6.48724D - 01
                                  |proj g| = 1.83273D-02
At iterate
This problem is unconstrained.
At iterate 10 f = 6.46730D - 01 |proj g|= 6.05589D - 03
At iterate 15 f= 6.46443D-01
                                  |proj g| = 2.55280D-04
         * * *
Tit = total number of iterations
Tnf = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip = number of BFGS updates skipped
Nact = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F = final function value
          * * *
                                      Projg
      Tit
             Tnf Tnint Skip Nact
       17
                    1 0 0 1.225D-05 6.464D-01
              21
  F = 0.64644266216655166
CONVERGENCE: REL REDUCTION OF F <= FACTR*EPSMCH
```

RUNNING THE L-BFGS-B CODE

233.20473448796136

```
Machine precision = 2.220D-16
      9 M =
At XO
           O variables are exactly at the bounds
          0 f= 7.00038D-01 |proj g|= 2.16709D-01
At iterate
This problem is unconstrained.
At iterate
           5 	 f = 6.49361D - 01
                                  |proj g| = 2.21168D-02
At iterate 10 f = 6.46553D - 01
                                  |proj q| = 3.97619D-03
At iterate 15 f = 6.46450D - 01 |proj g| = 1.85887D - 03
At iterate 20 f= 6.46442D-01
                                 |proj g| = 7.05125D-04
         * * *
Tit = total number of iterations
Tnf = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip = number of BFGS updates skipped
Nact = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
   = final function value
          * * *
     Tit Tnf Tnint Skip Nact Projg
                     1 0 0 1.161D-05 6.464D-01
       24
              29
 F = 0.64644148386457678
CONVERGENCE: REL REDUCTION OF F <= FACTR*EPSMCH
235.20433857849778
RUNNING THE L-BFGS-B CODE
          * * *
Machine precision = 2.220D-16
            10
                  M =
                                10
At XO 0 variables are exactly at the bounds
               f= 8.36508D-01
At iterate
          0
                                 |proj q| = 4.83725D-01
This problem is unconstrained.
At iterate 5 f = 6.69848D - 01 |proj g| = 3.55075D - 02
At iterate 10 f = 6.50489D-01 |proj g|= 3.81441D-02
At iterate 15 f = 6.47208D - 01
                                  |proj g| = 2.50640D-02
At iterate 20 f= 6.46184D-01
                                  |proj g| = 1.71807D-03
At iterate 25 f = 6.46174D-01 |proj g|= 3.36457D-04
         * * *
Tit = total number of iterations
Tnf = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
```

Skip = number of BFGS updates skipped

= final function value

Projg = norm of the final projected gradient

Nact = number of active bounds at final generalized Cauchy point

```
* * *
     Tit Tnf Tnint Skip Nact Projg
                         0 0 9.316D-06 6.462D-01
       28
              37
                     1
  F = 0.64617352611971968
CONVERGENCE: NORM OF PROJECTED GRADIENT <= PGTOL
237.1143047762258
RUNNING THE L-BFGS-B CODE
          * * *
Machine precision = 2.220D-16
             5
At XO 0 variables are exactly at the bounds
At iterate 0 f= 8.34486D-01 |proj g|= 6.19014D-02
At iterate 5 f= 8.27380D-01
                                 |proj g| = 3.46718D-02
This problem is unconstrained.
This problem is unconstrained.
At iterate 10 f= 8.12938D-01
                                 |proj g| = 7.39607D-02
At iterate 15 f = 8.06122D - 01 |proj g|= 4.97604D-03
          * * *
Tit = total number of iterations
Tnf = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip = number of BFGS updates skipped
Nact = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F = final function value
          * * *
      Tit Tnf Tnint Skip Nact Projg
             28 1 0 0 3.483D-06 8.061D-01
       19
 F = 0.80609084566844469
CONVERGENCE: NORM OF PROJECTED GRADIENT <= PGTOL
280.8465241445974
RUNNING THE L-BFGS-B CODE
         * * *
Machine precision = 2.220D-16
N = 6 M =
At XO
           O variables are exactly at the bounds
```

Machine precision = 2.220D-16 N = 6 M = 10At X0 0 variables are exactly at the bounds At iterate 0 f = 7.61872D-01 |proj g| = 8.42296D-02At iterate 5 f = 7.47037D-01 |proj g| = 2.51119D-02At iterate 10 f = 7.38780D-01 |proj g| = 5.41409D-02At iterate 15 f = 7.33188D-01 |proj g| = 2.20967D-02At iterate 20 f = 7.32566D-01 |proj g| = 5.39668D-05

* * *

```
Tit = total number of iterations
Tnf = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip = number of BFGS updates skipped
Nact = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F = final function value
          * * *
     Tit Inf Inint Skip Nact Projg
              32 1 0 0 3.343D-06 7.326D-01
 F = 0.73256611014581219
CONVERGENCE: NORM OF PROJECTED GRADIENT <= PGTOL
258.14221300899294
RUNNING THE L-BFGS-B CODE
         * * *
Machine precision = 2.220D-16
N = 7 \qquad M =
                           10
At XO
           O variables are exactly at the bounds
                                 |proj g| = 2.51453D-01
At iterate 0 f= 7.66817D-01
This problem is unconstrained.
At iterate 5 f = 7.08499D - 01
                                 |proj g| = 4.51547D-02
At iterate 10 f= 6.99000D-01
                                 |proj g| = 8.45247D-02
At iterate 15 f= 6.93582D-01
                                 |proj g| = 9.29351D-03
At iterate 20 f = 6.93568D - 01 |proj g| = 1.75797D - 05
          * * *
Tit = total number of iterations
Tnf = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip = number of BFGS updates skipped
Nact = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
   = final function value
          * * *
             Tnf Tnint Skip Nact Projg
              28 1 0 0 1.758D-05 6.936D-01
       20
 F = 0.69356794346256934
CONVERGENCE: REL REDUCTION OF F <= FACTR*EPSMCH
247.0388290034233
RUNNING THE L-BFGS-B CODE
          * * *
Machine precision = 2.220D-16
       8
                  M =
At XO
           0 variables are exactly at the bounds
At iterate
           0 f= 7.56956D-01 |proj g|= 3.37880D-01
This problem is unconstrained.
At iterate 5 f = 6.52256D - 01 |proj g|= 3.39754D-02
```

```
At iterate 10 f= 6.47007D-01
                                 |proj g| = 1.72677D-02
At iterate 15 f= 6.38699D-01
                                 |proj g| = 2.27746D-02
At iterate 20 f= 6.38482D-01 |proj g|= 2.20088D-04
         * * *
Tit = total number of iterations
Tnf = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip = number of BFGS updates skipped
Nact = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F = final function value
          * * *
      Tit Tnf Tnint Skip Nact Projg
             29 1 0 0 5.983D-05 6.385D-01
 F = 0.63848204529084374
CONVERGENCE: REL REDUCTION OF F <= FACTR*EPSMCH
230.52996721772348
RUNNING THE L-BFGS-B CODE
         * * *
Machine precision = 2.220D-16
N = 6 M =
                         10
At XO
           O variables are exactly at the bounds
At iterate 0 f= 7.70565D-01 |proj g|= 8.92913D-02
At iterate 5 f = 7.53941D - 01 |proj g| = 2.05962D - 02
This problem is unconstrained.
At iterate 10 f= 7.43267D-01
                                 |proj g| = 3.94023D-02
At iterate 15 f = 7.42874D-01 |proj g|= 1.19270D-03
          * * *
Tit = total number of iterations
Tnf = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip = number of BFGS updates skipped
Nact = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F = final function value
      Tit Tnf Tnint Skip Nact Projg
       19
                          0 0 1.718D-05 7.429D-01
             23 1
 F = 0.74287381817846243
CONVERGENCE: REL REDUCTION OF F <= FACTR*EPSMCH
261.6056029079634
RUNNING THE L-BFGS-B CODE
```

```
At XO
           O variables are exactly at the bounds
At iterate
           0 f= 7.58978D-01 |proj g|= 8.66470D-02
At iterate 5 f = 7.44396D - 01
                                  |proj g| = 2.48974D-02
This problem is unconstrained.
At iterate
               f = 7.32046D - 01
                                  |proj g| = 1.73258D-01
           10
At iterate 15 f = 7.29726D-01
                                  |proj q| = 1.39074D-01
At iterate 20 f= 7.29214D-01
                                  |proj g| = 2.41961D-01
At iterate 25 f = 7.29147D-01
                                  |proj g| = 1.65171D-02
At iterate 30 f = 7.29139D-01
                                  |proj g| = 2.10166D-02
At iterate 35 f= 7.29137D-01 |proj g|= 4.57621D-03
         * * *
Tit = total number of iterations
Tnf = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip = number of BFGS updates skipped
Nact = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
   = final function value
          * * *
      Tit Inf Inint Skip Nact Projg
       38
                          0 0 1.278D-03 7.291D-01
              51
                   1
  F = 0.72913733297733363
CONVERGENCE: REL REDUCTION OF F <= FACTR*EPSMCH
258.9901438803841
RUNNING THE L-BFGS-B CODE
          * * *
Machine precision = 2.220D-16
             8
                  M =
                                10
           O variables are exactly at the bounds
At iterate
           0 	 f = 7.57996D - 01
                                  |proj g| = 2.43142D-01
This problem is unconstrained.
At iterate
           5 	 f = 7.08522D - 01
                                  |proj g| = 1.83267D-02
At iterate 10 f= 6.97676D-01
                                  |proj g| = 7.33356D-02
At iterate 15 f = 6.92053D - 01
                                  |proj g| = 5.37947D-03
At iterate 20 f = 6.92008D - 01
                                  |proj q| = 2.23454D-03
At iterate 25 f= 6.91990D-01
                                  |proj g| = 1.58279D-04
          * * *
Tit = total number of iterations
Tnf = total number of function evaluations
```

Tnint = total number of segments explored during Cauchy searches

Skip = number of BFGS updates skipped

M =

```
Nact = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
   = final function value
         * * *
      Tit Tnf Tnint Skip Nact Projg
             36 1 0 0 8.122D-06 6.920D-01
      29
  F = 0.69198979665468374
CONVERGENCE: NORM OF PROJECTED GRADIENT <= PGTOL
248.50857167597374
RUNNING THE L-BFGS-B CODE
         * * *
Machine precision = 2.220D-16
             9
                  M =
At XO 0 variables are exactly at the bounds
At iterate 0 f = 8.06004D - 01 |proj g|= 4.95051D-01
This problem is unconstrained.
At iterate 5 f= 6.51817D-01
                                 |proj g| = 2.39582D-02
At iterate 10 f= 6.42976D-01
                                 |proj g| = 3.71135D-02
At iterate 15 f = 6.38862D-01 |proj g| = 3.82521D-02
At iterate 20 f= 6.38546D-01
                                 |proj g| = 6.17011D-03
At iterate 25 f = 6.37473D - 01
                                 |proj g| = 4.95594D-03
At iterate 30 f = 6.37427D-01 |proj g| = 3.80267D-05
         * * *
Tit = total number of iterations
Tnf = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip = number of BFGS updates skipped
Nact = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
   = final function value
         * * *
      Tit Tnf Tnint Skip Nact Projg
             53 1 0 0 1.805D-05 6.374D-01
       32
 F = 0.63742651828348706
CONVERGENCE: REL REDUCTION OF F <= FACTR*EPSMCH
232.17531014325166
RUNNING THE L-BFGS-B CODE
          * * *
Machine precision = 2.220D-16
      7
                 M =
At XO
       0 variables are exactly at the bounds
At iterate
           0 f= 7.11319D-01 |proj g|= 1.09094D-01
This problem is unconstrained.
At iterate 5 f = 6.95052D - 01 |proj g|= 1.48011D-02
```

```
At iterate 10 f= 6.87161D-01
                                 |proj g| = 3.18044D-02
At iterate 15 f= 6.83999D-01
                                 |proj g| = 1.52627D-02
At iterate 20 f = 6.83896D - 01 |proj g|= 4.15776D-05
         * * *
Tit = total number of iterations
Tnf = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip = number of BFGS updates skipped
Nact = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F = final function value
          * * *
      Tit Tnf Tnint Skip Nact Projg
             30 1 0 0 3.267D-05 6.839D-01
 F = 0.68389618760981230
CONVERGENCE: REL REDUCTION OF F <= FACTR*EPSMCH
243.78911903689695
RUNNING THE L-BFGS-B CODE
         * * *
Machine precision = 2.220D-16
N = 8 M =
                         10
At XO
           O variables are exactly at the bounds
At iterate 0 f = 7.10785D-01 |proj g|= 8.17712D-02
This problem is unconstrained.
At iterate 5 f = 6.97779D - 01
                                 |proj g| = 2.54547D-02
At iterate 10 f= 6.81808D-01
                                 |proj g| = 2.62770D-02
At iterate 15 f = 6.79512D-01 |proj g| = 3.77688D-02
At iterate 20 f = 6.77801D-01 |proj g|= 2.72031D-03
At iterate 25 f = 6.77779D-01 |proj g| = 9.28247D-05
         * * *
Tit = total number of iterations
Tnf = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip = number of BFGS updates skipped
Nact = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F = final function value
          * * *
  N Tit Tnf Tnint Skip Nact Projg
       28
              39 1 0 0 1.197D-05 6.778D-01
  F = 0.67777853432755231
CONVERGENCE: REL REDUCTION OF F <= FACTR*EPSMCH
243.73358753405756
```

RUNNING THE L-BFGS-B CODE

```
Machine precision = 2.220D-16
                  M =
             9
                                10
At X0
           O variables are exactly at the bounds
At iterate 0 f = 7.33437D - 01
                                 |proj g| = 2.22583D-01
This problem is unconstrained.
At iterate
          f = 6.91288D-01 |proj g|= 1.58709D-02
At iterate 10 f= 6.82034D-01 |proj g|= 2.11854D-02
At iterate 15 f = 6.81006D - 01
                                 |proj g| = 9.13206D-03
At iterate 20 f= 6.77498D-01
                                  |proj g| = 8.87065D-03
At iterate 25 f= 6.77101D-01
                                 |proj g| = 4.70778D-04
At iterate 30 f = 6.77101D - 01 |proj g|= 6.37906D - 06
         * * *
Tit = total number of iterations
Tnf = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip = number of BFGS updates skipped
Nact = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
   = final function value
          * * *
      Tit Inf Inint Skip Nact Projg
              41 1 0 0 6.379D-06 6.771D-01
       30
  9
  F = 0.67710053771907963
CONVERGENCE: NORM OF PROJECTED GRADIENT <= PGTOL
245.50578067361076
This problem is unconstrained.
RUNNING THE L-BFGS-B CODE
          * * *
Machine precision = 2.220D-16
             10
                  M =
                                10
           O variables are exactly at the bounds
At iterate
           0 f= 8.62207D-01 |proj g|= 5.56312D-01
At iterate
          5 f= 6.58249D-01 |proj g|= 3.39038D-02
At iterate 10 f= 6.45040D-01
                                 |proj g| = 2.92732D-02
At iterate 15 f = 6.40082D - 01
                                  |proj g| = 3.54308D-02
At iterate 20 f= 6.38647D-01
                                  |proj g| = 1.87154D-02
At iterate 25 f = 6.37315D-01
                                 |proj g| = 3.23148D-02
```

|proj g| = 8.40405D-03

|proj g| = 6.31660D - 04

At iterate 30 f= 6.34681D-01

At iterate 35 f= 6.34538D-01

```
At iterate 40 f= 6.34536D-01 |proj g|= 1.31331D-05
```

Tit = total number of iterations

Tnf = total number of function evaluations

Tnint = total number of segments explored during Cauchy searches

Skip = number of BFGS updates skipped

Nact = number of active bounds at final generalized Cauchy point

Projg = norm of the final projected gradient

F = final function value

* * *

N Tit Tnf Tnint Skip Nact Projg F 10 40 55 1 0 0 1.313D-05 6.345D-01 F = 0.63453553164989385

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH 233.20393863436433 RUNNING THE L-BFGS-B CODE

* * *

Machine precision = 2.220D-16

 $M = 8 \qquad M = 1$

At XO 0 variables are exactly at the bounds

At iterate 0 f= 6.76237D-01 |proj g|= 1.23923D-01 This problem is unconstrained.

At iterate 5 f= 6.45223D-01 |proj g|= 2.46916D-02 At iterate 10 f= 6.40510D-01 |proj g|= 4.23892D-02 At iterate 15 f= 6.36135D-01 |proj g|= 5.08437D-03 At iterate 20 f= 6.36062D-01 |proj g|= 2.12804D-04

* * *

Tit = total number of iterations

Tnf = total number of function evaluations

Tnint = total number of segments explored during Cauchy searches

Skip = number of BFGS updates skipped

Nact = number of active bounds at final generalized Cauchy point

Projg = norm of the final projected gradient

F = final function value

* * *

N Tit Tnf Tnint Skip Nact Projg F 8 24 37 1 0 0 6.799D-06 6.361D-01 F = 0.63606184264134957

CONVERGENCE: NORM_OF_PROJECTED_GRADIENT_<=_PGTOL 229.71677912749345

RUNNING THE L-BFGS-B CODE

* * *

Machine precision = 2.220D-16

N = 9 M = 10

```
At iterate
           0
                 f = 6.75865D - 01
                                   |proj q| = 1.23260D-01
This problem is unconstrained.
At iterate
           5
                f= 6.45230D-01
                                   |proj g| = 2.72752D-02
At iterate 10 f = 6.39639D - 01
                                   |proj g| = 3.93681D-02
At iterate
           15 f= 6.33983D-01
                                   |proj g| = 8.15779D-02
           20 f= 6.31084D-01
                                   |proj q| = 7.02148D-02
At iterate
At iterate
           25 f= 6.30879D-01
                                  |proj g| = 1.01849D-02
At iterate
           30
               f= 6.30815D-01
                                   |proj g| = 2.55708D-02
At iterate
           35
               f = 6.30790D - 01
                                   |proj g| = 3.94191D-03
At iterate
           40 f= 6.30752D-01
                                   |proj g| = 1.65920D-02
At iterate 45 f= 6.30733D-01
                                  |proj g| = 5.79764D-03
At iterate 50 f = 6.30727D - 01
                                  |proj g| = 3.89609D-04
          * * *
Tit = total number of iterations
Tnf = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip = number of BFGS updates skipped
Nact = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
    = final function value
          * * *
             Tnf Tnint Skip Nact Projg
  N
                          0 0 3.896D-04 6.307D-01
               72
                   1
  F = 0.63072722602333420
STOP: TOTAL NO. of ITERATIONS REACHED LIMIT
229.9243479438403
RUNNING THE L-BFGS-B CODE
          * * *
Machine precision = 2.220D-16
            10
                   M =
                                10
            O variables are exactly at the bounds
At iterate
           0
                f= 6.97794D-01
                                   |proj g| = 2.13755D-01
This problem is unconstrained.
At iterate
           5
               f = 6.44753D - 01
                                 |proj g| = 2.14745D-02
               f= 6.39658D-01
          10
                                  |proj g| = 2.93839D-02
 ys=-6.235E-03 -gs= 4.027E-04 BFGS update SKIPPED
At iterate
           15
               f = 6.34465D - 01
                                   |proj g| = 3.16476D-02
At iterate
          20 f= 6.32452D-01
                                  |proj g| = 6.25892D-02
At iterate 25 f = 6.31223D - 01
                                  |proj g| = 3.06024D-02
```

|proj g| = 7.36585D-02

At iterate 30 f = 6.30991D-01

O variables are exactly at the bounds

At X0

```
At iterate 35 f= 6.30719D-01 |proj g|= 6.82753D-03

At iterate 40 f= 6.30690D-01 |proj g|= 5.91251D-03

At iterate 45 f= 6.30682D-01 |proj g|= 7.63117D-03

At iterate 50 f= 6.30681D-01 |proj g|= 1.65329D-03

* * * *
```

Tit = total number of iterations

Tnf = total number of function evaluations

Tnint = total number of segments explored during Cauchy searches

Skip = number of BFGS updates skipped

Nact = number of active bounds at final generalized Cauchy point

Projg = norm of the final projected gradient

F = final function value

* * *

N Tit Tnf Tnint Skip Nact Projg F 10 50 75 1 1 0 1.653D-03 6.307D-01 F = 0.63068100664786353

O variables are exactly at the bounds

10

STOP: TOTAL NO. of ITERATIONS REACHED LIMIT 231.90881823368215

RUNNING THE L-BFGS-B CODE

At XO

* * *

Machine precision = 2.220D-16 N = 11 M =

At iterate 0 f= 8.35760D-01 |proj g|= 4.83405D-01This problem is unconstrained. At iterate 5 f= 6.62680D-01 |proj g| = 5.77626D-02At iterate 10 f= 6.43816D-01 |proj g| = 5.65565D-02At iterate 15 f = 6.34432D - 01|proj g| = 1.74535D-01At iterate 20 f= 6.31255D-01 |proj g| = 2.52666D-01At iterate 25 f = 6.30953D - 01|proj g| = 4.53654D-03At iterate 30 f= 6.30789D-01 |proj g| = 1.48480D-02At iterate 35 f= 6.30676D-01|proj g| = 6.80681D-02At iterate 40 f= 6.30642D-01 |proj g| = 7.64932D-03At iterate 45 f= 6.30580D-01|proj q| = 3.27705D-03At iterate 50 f = 6.30574D - 01|proj g| = 1.93028D-02

* * *

Tit = total number of iterations

Tnf = total number of function evaluations

Tnint = total number of segments explored during Cauchy searches

Skip = number of BFGS updates skipped

Nact = number of active bounds at final generalized Cauchy point

```
Projg = norm of the final projected gradient
F = final function value
```

N Tit Tnf Tnint Skip Nact Projg F 11 50 72 1 0 0 1.930D-02 6.306D-01 F = 0.63057418559273037

STOP: TOTAL NO. of ITERATIONS REACHED LIMIT

233.87292635915742

RUNNING THE L-BFGS-B CODE

* * *

Machine precision = 2.220D-16

 $N = 6 \qquad M = 10$

At XO 0 variables are exactly at the bounds

At iterate 0 f= 8.30042D-01 |proj g|= 2.50323D-01

At iterate 5 f = 8.06862D - 01 |proj g|= 2.06987D-01

This problem is unconstrained. This problem is unconstrained.

At iterate 10 f = 8.01311D-01 |proj g|= 6.33265D-03

At iterate 15 f= 8.01234D-01 |proj g|= 6.01545D-03

At iterate 20 f= 8.01109D-01 |proj g|= 1.83237D-03

* * *

Tit = total number of iterations

Tnf = total number of function evaluations

Tnint = total number of segments explored during Cauchy searches

Skip = number of BFGS updates skipped

Nact = number of active bounds at final generalized Cauchy point

Projg = norm of the final projected gradient

F = final function value

* * *

N Tit Tnf Tnint Skip Nact Projg F
6 23 28 1 0 0 1.527D-05 8.011D-01
F = 0.80110838083812097

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH

281.17241596160864

RUNNING THE L-BFGS-B CODE

* * *

Machine precision = 2.220D-16

N = 7 M = 10

At X0 0 variables are exactly at the bounds

At iterate 0 f= 7.58443D-01 |proj g|= 2.59136D-01

At iterate 5 f = 7.31099D-01 |proj g|= 9.92952D-02

At iterate 10 f = 7.26800D-01 |proj g| = 9.16212D-04

At iterate 15 f = 7.26800D - 01 |proj g|= 1.19900D - 04

```
At iterate 20 f = 7.26800D - 01 |proj g|= 2.46166D-05
```

Tit = total number of iterations

Tnf = total number of function evaluations

Tnint = total number of segments explored during Cauchy searches

Skip = number of BFGS updates skipped

Nact = number of active bounds at final generalized Cauchy point

Projg = norm of the final projected gradient

F = final function value

* * *

N Tit Tnf Tnint Skip Nact Projg F
7 20 25 1 0 0 2.462D-05 7.268D-01
F = 0.72679954582450279

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH

258.20464739703294

RUNNING THE L-BFGS-B CODE

* * *

Machine precision = 2.220D-16

M = 0.01

At XO 0 variables are exactly at the bounds

At iterate 0 f= 7.46482D-01 |proj g|= 2.03222D-01

This problem is unconstrained.

At iterate 5 f = 6.88756D-01 |proj g|= 3.14457D-02

At iterate 10 f= 6.88199D-01 |proj g|= 3.06602D-03

At iterate 15 f = 6.88181D - 01 |proj q| = 3.35145D - 03

At iterate 20 f = 6.88176D - 01 |proj g|= 1.46209D - 05

* * *

Tit = total number of iterations

Tnf = total number of function evaluations

Tnint = total number of segments explored during Cauchy searches

Skip = number of BFGS updates skipped

Nact = number of active bounds at final generalized Cauchy point

Projg = norm of the final projected gradient

F = final function value

* * *

N Tit Tnf Tnint Skip Nact Projg F 8 20 24 1 0 0 1.462D-05 6.882D-01

F = 0.68817576899250721

CONVERGENCE: REL REDUCTION OF F <= FACTR*EPSMCH

247.22705838148244

RUNNING THE L-BFGS-B CODE

* * *

Machine precision = 2.220D-16

 $N = 9 \qquad M = 10$

```
At iterate
          0
                f = 7.65435D - 01
                                    |proj q| = 2.99018D-01
This problem is unconstrained.
           5 	 f = 6.47427D - 01
At iterate
                                    |proj g| = 1.52145D-01
At iterate 10 f = 6.33691D - 01
                                   |proj g| = 9.13904D-03
At iterate
           15 f= 6.33451D-01
                                    |proj g| = 1.31613D-03
At iterate 20 f = 6.33433D - 01
                                   |proj q| = 8.83507D-04
At iterate 25 f= 6.33432D-01 |proj g|= 5.48169D-05
          * * *
Tit = total number of iterations
Tnf = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip = number of BFGS updates skipped
Nact = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
    = final function value
          * * *
             Tnf Tnint Skip Nact Projg
                   1 0 0 5.482D-05 6.334D-01
       25
               30
  F = 0.63343234258508097
CONVERGENCE: REL REDUCTION OF F <= FACTR*EPSMCH
230.8332671085872
RUNNING THE L-BFGS-B CODE
          * * *
Machine precision = 2.220D-16
              7
                    M =
                                 10
At X0
            O variables are exactly at the bounds
At iterate
            0 	 f = 7.70607D - 01
                                   |proj g| = 2.40294D-01
This problem is unconstrained.
           5 	 f = 7.44394D - 01
At iterate
                                   |proj g| = 4.70840D-02
At iterate 10 f = 7.39483D - 01
                                   |proj g| = 4.60124D-03
At iterate 15 f= 7.38527D-01
                                   |proj g| = 7.59452D-03
At iterate 20 f= 7.38313D-01
                                   |proj g| = 9.42093D-05
          * * *
Tit = total number of iterations
Tnf = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip = number of BFGS updates skipped
Nact = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F = final function value
          * * *
```

O variables are exactly at the bounds

At X0

N Tit Tnf Tnint Skip Nact Projg F 7 22 31 1 0 0 2.084D-05 7.383D-01

F = 0.73831269340120032CONVERGENCE: REL REDUCTION OF F <= FACTR*EPSMCH 262.0730649828033 RUNNING THE L-BFGS-B CODE * * * Machine precision = 2.220D-16 8 M = 10 At XO O variables are exactly at the bounds At iterate 0 f= 7.62041D-01 |proj q|= 2.48112D-01At iterate 5 f = 7.34480D - 01 |proj g|= 9.65442D - 02 At iterate 10 f= 7.27501D-01 |proj g| = 8.61475D-03This problem is unconstrained. At iterate 15 f= 7.26566D-01 |proj g|= 3.90128D-02 At iterate 20 f = 7.26460D - 01|proj q| = 2.86677D-03At iterate 25 f= 7.26457D-01 |proj g| = 2.03427D-03At iterate 30 f= 7.26440D-01 |proj g| = 3.46102D-03At iterate 35 f = 7.26431D - 01 |proj g|= 1.03356D - 03 * * * Tit = total number of iterations Tnf = total number of function evaluations Tnint = total number of segments explored during Cauchy searches Skip = number of BFGS updates skipped Nact = number of active bounds at final generalized Cauchy point Projg = norm of the final projected gradient = final function value * * * Tit Inf Inint Skip Nact Projg 1 0 0 8.535D-05 7.264D-01 8 39 55 F = 0.72643057200300798CONVERGENCE: REL REDUCTION OF F <= FACTR*EPSMCH 260.0806721930107 RUNNING THE L-BFGS-B CODE * * * Machine precision = 2.220D-16 N =9 M =10 At XO 0 variables are exactly at the bounds At iterate 0 f = 7.39457D-01 |proj g|= 1.93540D-01 This problem is unconstrained. At iterate 5 f = 6.87274D - 01|proj g| = 2.04616D-02

|proj g| = 5.40451D-03

|proj g| = 9.76074D-04

|proj g| = 1.03731D-04

At iterate 10 f= 6.86951D-01

At iterate 15 f = 6.86603D - 01

At iterate 20 f= 6.86602D-01

At iterate 25 f = 6.86601D-01 |proj g|= 8.05327D-06

* * *

Tit = total number of iterations

Tnf = total number of function evaluations

Tnint = total number of segments explored during Cauchy searches

Skip = number of BFGS updates skipped

Nact = number of active bounds at final generalized Cauchy point

Projg = norm of the final projected gradient

F = final function value

* * *

N Tit Tnf Tnint Skip Nact Projg F
9 25 28 1 0 0 8.053D-06 6.866D-01

F = 0.68660104771001917

CONVERGENCE: NORM OF PROJECTED GRADIENT <= PGTOL

248.69795203056646

RUNNING THE L-BFGS-B CODE

* * *

Machine precision = 2.220D-16

I = 10 M = 1

At XO 0 variables are exactly at the bounds

At iterate 0 f= 8.20655D-01 |proj g|= 4.77485D-01

This problem is unconstrained.

At iterate 5 f= 6.43924D-01 |proj g|= 3.75964D-02

At iterate 10 f= 6.34092D-01 |proj g|= 1.00258D-02

At iterate 15 f= 6.33779D-01 |proj g|= 1.01853D-02

At iterate 20 f= 6.32704D-01 |proj g|= 1.53366D-02

At iterate 25 f= 6.32539D-01 |proj g|= 1.98348D-03

At iterate 30 f= 6.32475D-01 |proj g|= 1.29713D-03

At iterate 35 f= 6.32472D-01 |proj g|= 5.83726D-05

* * *

Tit = total number of iterations

Tnf = total number of function evaluations

Tnint = total number of segments explored during Cauchy searches

Skip = number of BFGS updates skipped

Nact = number of active bounds at final generalized Cauchy point

Projg = norm of the final projected gradient

F = final function value

* * *

N Tit Tnf Tnint Skip Nact Projg F
10 36 42 1 0 0 1.906D-05 6.325D-01
F = 0.63247176943453398

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH 232.51051453000343

RUNNING THE L-BFGS-B CODE

Tit = total number of iterations

Skip = number of BFGS updates skipped

Tnf = total number of function evaluations

Tnint = total number of segments explored during Cauchy searches

Machine precision = 2.220D-16M = 8 10 At XO O variables are exactly at the bounds At iterate 0 f = 7.08596D - 01 |proj g| = 2.12617D - 01This problem is unconstrained. f = 6.85114D-01 |proj g|= 8.43632D-02 At iterate At iterate 10 f = 6.80599D - 01 |proj g| = 6.61335D - 03At iterate 15 f = 6.80141D-01|proj q| = 1.35644D-02At iterate 20 f = 6.79797D - 01|proj g| = 5.05597D-03At iterate 25 f = 6.79765D-01 |proj g| = 4.98789D-05* * * Tit = total number of iterations Tnf = total number of function evaluations Tnint = total number of segments explored during Cauchy searches Skip = number of BFGS updates skipped Nact = number of active bounds at final generalized Cauchy point Projg = norm of the final projected gradient = final function value * * * Tit Inf Inint Skip Nact Projg 26 37 1 0 0 1.282D-05 6.798D-01 8 F = 0.67976504045126929CONVERGENCE: REL REDUCTION OF F <= FACTR*EPSMCH 244.40105359162646 RUNNING THE L-BFGS-B CODE * * * Machine precision = 2.220D-16 N =9 M =10 O variables are exactly at the bounds At iterate 0 f= 7.09300D-01 |proj g|= 2.10836D-01At iterate 5 f = 6.89760D - 01|proj g| = 8.35777D-02This problem is unconstrained. At iterate 10 f = 6.79064D - 01|proj g| = 4.40881D-02At iterate 15 f= 6.75445D-01 |proj g| = 1.32793D-02At iterate 20 f= 6.75291D-01 |proj q| = 4.12830D-04At iterate 25 f = 6.75290D-01 |proj g|= 6.03674D-06 * * *

```
Nact = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
   = final function value
         * * *
      Tit Tnf Tnint Skip Nact Projg
                  1 0 0 6.037D-06 6.753D-01
       25
  9
             30
  F = 0.67529031573941134
CONVERGENCE: NORM OF PROJECTED GRADIENT <= PGTOL
244.89754608844223
RUNNING THE L-BFGS-B CODE
          * * *
Machine precision = 2.220D-16
            10 	 M =
                               10
At XO
           O variables are exactly at the bounds
At iterate 0 f = 7.24832D - 01 |proj g|= 2.00217D - 01
This problem is unconstrained.
At iterate 5 f= 6.78415D-01
                                 |proj g| = 2.37920D-02
At iterate 10 f= 6.75522D-01
                                 |proj g| = 1.31025D-02
At iterate 15 f = 6.75019D-01 |proj g| = 7.73715D-03
At iterate 20 f= 6.74881D-01
                                 |proj g| = 4.95632D-03
At iterate 25 f= 6.74821D-01
                                 |proj g| = 3.79109D-04
At iterate 30 f = 6.74819D-01 |proj g| = 3.47853D-05
          * * *
Tit = total number of iterations
Tnf = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip = number of BFGS updates skipped
Nact = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
   = final function value
         * * *
      Tit Tnf Tnint Skip Nact Projg
                  1 0 0 3.594D-05 6.748D-01
       31
             35
  10
 F = 0.67481947337860271
CONVERGENCE: REL REDUCTION OF F <= FACTR*EPSMCH
246.7393430552105
RUNNING THE L-BFGS-B CODE
          * * *
Machine precision = 2.220D-16
      11
                  M =
At XO
           0 variables are exactly at the bounds
At iterate
           0 f= 8.77254D-01 |proj g|= 5.49950D-01
This problem is unconstrained.
At iterate 5 f = 6.46160D - 01 |proj g|= 9.74186D-02
```

```
At iterate 10 f= 6.37158D-01 |proj g|= 1.35689D-02

At iterate 15 f= 6.33653D-01 |proj g|= 1.93962D-02

At iterate 20 f= 6.33065D-01 |proj g|= 4.87704D-03

At iterate 25 f= 6.30034D-01 |proj g|= 2.05143D-02

At iterate 30 f= 6.29332D-01 |proj g|= 1.75780D-03

At iterate 35 f= 6.29252D-01 |proj g|= 4.05318D-03

At iterate 40 f= 6.29232D-01 |proj g|= 2.88827D-04

At iterate 45 f= 6.29232D-01 |proj g|= 6.41240D-06

* * * *
```

Tit = total number of iterations

Tnf = total number of function evaluations

Tnint = total number of segments explored during Cauchy searches

Skip = number of BFGS updates skipped

Nact = number of active bounds at final generalized Cauchy point

Projg = norm of the final projected gradient

F = final function value

* * *

N Tit Tnf Tnint Skip Nact Projg F 11 45 51 1 0 0 6.412D-06 6.292D-01 F = 0.62923190889535419

CONVERGENCE: NORM_OF_PROJECTED_GRADIENT_<=_PGTOL 233.421921388839

RUNNING THE L-BFGS-B CODE

* * *

Machine precision = 2.220D-16N = 9 M = 10

At X0 0 variables are exactly at the bounds

At iterate 0 f= 6.69917D-01 |proj g|= 1.62871D-01

This problem is unconstrained.

At iterate 5 f= 6.33211D-01 |proj g|= 3.31141D-02

At iterate 10 f= 6.32189D-01 |proj g|= 2.71576D-02

At iterate 15 f= 6.32036D-01 |proj g|= 8.50721D-03

At iterate 20 f= 6.31977D-01 |proj g|= 1.61998D-02

At iterate 25 f= 6.31877D-01 |proj g|= 3.08304D-03

At iterate 30 f= 6.31858D-01 |proj g|= 1.80708D-04

At iterate 35 f= 6.31858D-01 |proj g|= 9.16310D-05

* * *

Tit = total number of iterations

Tnf = total number of function evaluations

Tnint = total number of segments explored during Cauchy searches

Skip = number of BFGS updates skipped

Nact = number of active bounds at final generalized Cauchy point

Projg = norm of the final projected gradient

F = final function value

* * *

N Tit Tnf Tnint Skip Nact Projg F 9 35 53 1 0 0 9.163D-05 6.319D-01

F = 0.63185791313520223

CONVERGENCE: REL REDUCTION OF F <= FACTR*EPSMCH

230.30425881342794

RUNNING THE L-BFGS-B CODE

* * *

Machine precision = 2.220D-16

N = 10 M = 10

At XO 0 variables are exactly at the bounds

At iterate 0 f = 6.70177D - 01|proj g| = 1.62209D-01This problem is unconstrained. At iterate 5 f = 6.33832D - 01 |proj g|= 2.77706D-02 At iterate 10 f = 6.30497D - 01 |proj g| = 5.63551D - 02At iterate 15 f= 6.29120D-01 |proj g| = 1.02792D-02At iterate 20 f= 6.28992D-01 |proj g| = 9.83547D-03At iterate 25 f= 6.28867D-01 |proj g| = 3.43260D-03At iterate 30 f= 6.28827D-01 |proj g| = 1.39090D-02At iterate 35 f= 6.28756D-01|proj g| = 4.53673D-02At iterate 40 f= 6.28716D-01 |proj g| = 4.14478D-03At iterate 45 f = 6.28706D - 01 |proj g| = 3.48579D - 03At iterate 50 f= 6.28701D-01 |proj g|= 5.81875D-04

* * *

Tit = total number of iterations

Tnf = total number of function evaluations

Tnint = total number of segments explored during Cauchy searches

Skip = number of BFGS updates skipped

Nact = number of active bounds at final generalized Cauchy point

Projg = norm of the final projected gradient

F = final function value

* * *

N Tit Tnf Tnint Skip Nact Projg F 10 50 64 1 0 0 5.819D-04 6.287D-01 F = 0.62870142736563273

STOP: TOTAL NO. of ITERATIONS REACHED LIMIT

231.24367959485258

RUNNING THE L-BFGS-B CODE

* * *

```
Machine precision = 2.220D-16
N = 11 M = 10
```

At XO 0 variables are exactly at the bounds

At iterate 0 f= 6.86381D-01 |proj g|= 1.66357D-01This problem is unconstrained. At iterate 5 f = 6.31414D - 01|proj g| = 3.02208D-02At iterate 10 f = 6.29672D - 01|proj q| = 1.64554D-02At iterate 15 f= 6.29382D-01 |proj g| = 1.21353D-02At iterate 20 f= 6.29146D-01 |proj g| = 1.35464D-02At iterate 25 f= 6.28867D-01 |proj g| = 9.96185D-03At iterate 30 f= 6.28848D-01 |proj g| = 1.85846D-02At iterate 35 f= 6.28725D-01|proj g| = 1.10331D-02At iterate 40 f= 6.28694D-01|proj g| = 1.75355D-03At iterate 45 f= 6.28692D-01 |proj g| = 1.21199D-03At iterate 50 f= 6.28692D-01 |proj q|= 2.44249D-04

* * *

Tit = total number of iterations

Tnf = total number of function evaluations

Tnint = total number of segments explored during Cauchy searches

Skip = number of BFGS updates skipped

Nact = number of active bounds at final generalized Cauchy point

Projg = norm of the final projected gradient

F = final function value

* * *

N Tit Tnf Tnint Skip Nact Projg F 11 50 58 1 0 0 2.442D-04 6.287D-01 F = 0.62869209498985812

STOP: TOTAL NO. of ITERATIONS REACHED LIMIT 233.24054391659232

This problem is unconstrained.

RUNNING THE L-BFGS-B CODE

* * *

Machine precision = 2.220D-16 N = 12 M = 10At X0 0 variables are exactly at the bounds At iterate 0 f = 8.59004D-01 |proj g| = 5.03077D-01At iterate 5 f = 6.63468D-01 |proj g| = 7.89790D-02At iterate 10 f = 6.40202D-01 |proj g| = 5.52426D-02At iterate 15 f = 6.35749D-01 |proj g| = 4.54096D-02At iterate 20 f = 6.34901D-01 |proj g| = 6.02553D-02

```
At iterate 25 f= 6.33520D-01
                                  |proj g| = 1.53121D-01
At iterate 30 f= 6.32938D-01
                                  |proj g| = 7.21193D-02
At iterate 35 f= 6.32798D-01
                                  |proj g| = 6.83890D-01
At iterate 40 f= 6.32747D-01
                                  |proj g|= 1.28995D-02
At iterate 45 f= 6.32668D-01
                                  |proj q| = 3.76814D-01
At iterate 50 f= 6.32616D-01
                                  |proj g| = 7.31052D-03
         * * *
Tit = total number of iterations
Tnf = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip = number of BFGS updates skipped
Nact = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F = final function value
          * * *
       Tit Tnf Tnint Skip Nact Projg 50 80 1 ^ ^
                      1 0 0 7.311D-03 6.326D-01
  12
  F = 0.63261608236925049
STOP: TOTAL NO. of ITERATIONS REACHED LIMIT
236.55900367606816
RUNNING THE L-BFGS-B CODE
          * * *
Machine precision = 2.220D-16
                            10
N =
              7
                   M =
At XO
           O variables are exactly at the bounds
           0 	 f = 8.04769D - 01
At iterate
                                  |proj q| = 9.74835D-02
At iterate
          5 	 f= 8.00660D-01
                                  |proj q| = 2.20762D-02
This problem is unconstrained.
This problem is unconstrained.
                                  |proj g| = 5.05323D-02
At iterate
           10 	 f = 7.93980D - 01
At iterate 15 f= 7.92679D-01 |proj g|= 4.52286D-03
At iterate 20 f= 7.92412D-01 |proj g|= 3.99709D-04
          * * *
Tit = total number of iterations
Tnf = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip = number of BFGS updates skipped
Nact = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
    = final function value
          * * *
       Tit Tnf Tnint Skip Nact Projg
```

30 1 0 0 1.403D-05 7.924D-01

24

F = 0.79241146689386066

CONVERGENCE: REL REDUCTION OF F <= FACTR*EPSMCH

280.2502528763372

RUNNING THE L-BFGS-B CODE

* * *

Machine precision = 2.220D-16

 $N = 8 \qquad M = 10$

At X0 0 variables are exactly at the bounds

At iterate 0 f = 7.37902D-01 |proj g|= 1.30386D-01

At iterate 5 f = 7.22830D-01 |proj g|= 3.91060D-02

At iterate 10 f= 7.19084D-01 |proj g|= 5.15643D-02

At iterate 15 f= 7.10368D-01 |proj g|= 2.62782D-02

At iterate 20 f= 7.09419D-01 |proj g|= 1.28682D-02

At iterate 25 f = 7.09276D-01 |proj g|= 4.08824D-05

* * *

Tit = total number of iterations

Tnf = total number of function evaluations

Tnint = total number of segments explored during Cauchy searches

Skip = number of BFGS updates skipped

Nact = number of active bounds at final generalized Cauchy point

Projg = norm of the final projected gradient

F = final function value

* * *

N Tit Tnf Tnint Skip Nact Projg F 8 26 31 1 0 0 1.792D-05 7.093D-01

F = 0.70927609701455441

CONVERGENCE: REL REDUCTION OF F <= FACTR*EPSMCH

254.31676859689028

RUNNING THE L-BFGS-B CODE

* * *

Machine precision = 2.220D-16

 $N = 9 \qquad M = 10$

At XO 0 variables are exactly at the bounds

At iterate 0 f= 7.38084D-01 |proj g|= 2.27892D-01

This problem is unconstrained.

At iterate 5 f = 6.91405D - 01 |proj q| = 5.35793D - 02

At iterate 10 f = 6.84550D - 01 |proj g| = 1.82388D - 02

At iterate 15 f = 6.76527D - 01 |proj g| = 2.53501D - 02

At iterate 20 f = 6.74104D-01 |proj g| = 1.77186D-02

At iterate 25 f = 6.72789D-01 |proj g| = 1.65631D-03

At iterate 30 f = 6.72788D-01 |proj g|= 7.22233D-05

* * *

```
Tit = total number of iterations
```

Tnf = total number of function evaluations

Tnint = total number of segments explored during Cauchy searches

Skip = number of BFGS updates skipped

Nact = number of active bounds at final generalized Cauchy point

Projg = norm of the final projected gradient

F = final function value

* * *

N Tit Tnf Tnint Skip Nact Projg F 9 30 32 1 0 0 7.222D-05 6.728D-01

F = 0.67278766754389019

CONVERGENCE: REL REDUCTION OF F <= FACTR*EPSMCH

244.0566562947471

RUNNING THE L-BFGS-B CODE

* * *

Machine precision = 2.220D-16

N = 10 M = 1

At XO 0 variables are exactly at the bounds

At iterate 0 f = 7.65450D - 01 |proj g|= 4.77316D-01

This problem is unconstrained.

At iterate 5 f= 6.38528D-01 |proj g|= 4.52922D-02

At iterate 10 f = 6.27333D-01 |proj g|= 1.86361D-02

At iterate 15 f = 6.21855D-01 |proj g| = 1.30383D-02

At iterate 20 f = 6.21651D-01 |proj g|= 8.06338D-04

At iterate 25 f= 6.21643D-01 |proj g|= 3.67193D-04

* * *

Tit = total number of iterations

Tnf = total number of function evaluations

Tnint = total number of segments explored during Cauchy searches

Skip = number of BFGS updates skipped

Nact = number of active bounds at final generalized Cauchy point

Projg = norm of the final projected gradient

F = final function value

* * *

N Tit Tnf Tnint Skip Nact Projg F 10 28 33 1 0 0 1.299D-05 6.216D-01

F = 0.62164250093878359

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH

228.87188031543127

RUNNING THE L-BFGS-B CODE

* * *

Machine precision = 2.220D-16

 $N = 8 \qquad M = 10$

```
At iterate 0 f = 7.51840D - 01
                                  |proj q| = 1.07904D-01
At iterate 5 f = 7.35801D - 01
                                  |proj g| = 3.45323D-02
At iterate 10 f = 7.33579D-01
                                  |proj g| = 3.39569D-02
At iterate 15 f = 7.26031D - 01
                                  |proj g| = 9.43749D-03
This problem is unconstrained.
At iterate 20 f= 7.24871D-01 |proj q|= 3.54977D-03
At iterate 25 f= 7.24863D-01 |proj g|= 3.31063D-06
          * * *
Tit = total number of iterations
Tnf = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip = number of BFGS updates skipped
Nact = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
    = final function value
          * * *
             Tnf Tnint Skip Nact Projg
                   1 0 0 3.311D-06 7.249D-01
       25
              29
  F = 0.72486325385543171
CONVERGENCE: NORM OF PROJECTED GRADIENT <= PGTOL
259.5540532954251
RUNNING THE L-BFGS-B CODE
          * * *
Machine precision = 2.220D-16
              9
                   M =
                                10
At XO
           O variables are exactly at the bounds
At iterate 0 f= 7.39883D-01 |proj g|= 1.25303D-01
At iterate 5 f = 7.24286D - 01
                                  |proj q| = 3.29304D-02
At iterate 10 f = 7.20656D - 01
                                  |proj g| = 2.90111D-02
This problem is unconstrained.
At iterate 15 f = 7.12994D-01 |proj g|= 1.36577D-02
At iterate 20 f = 7.11478D - 01
                                  |proj g| = 1.57974D-02
At iterate 25 f = 7.11300D - 01
                                  |proj g| = 2.74658D-03
At iterate 30 f= 7.11291D-01
                                 |proj g| = 3.44215D-06
         * * *
Tit = total number of iterations
Tnf = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip = number of BFGS updates skipped
Nact = number of active bounds at final generalized Cauchy point
```

Projg = norm of the final projected gradient

= final function value

O variables are exactly at the bounds

At X0

* * *

Tit Tnf Tnint Skip Nact Projg 0 0 3.442D-06 7.113D-01 35 1 F = 0.71129097453117807CONVERGENCE: NORM OF PROJECTED GRADIENT <= PGTOL 256.9937674424758 RUNNING THE L-BFGS-B CODE * * * Machine precision = 2.220D-16 10 M =At XO 0 variables are exactly at the bounds f = 7.30111D - 01At iterate 0 |proj g| = 2.18478D-01This problem is unconstrained. At iterate 5 f= 6.87330D-01 |proj g|= 2.87247D-02 At iterate 10 f= 6.82438D-01 |proj g|= 2.08135D-02 At iterate 15 f = 6.73875D - 01|proj g| = 1.44280D-02At iterate 20 f= 6.71856D-01 |proj g| = 2.84840D-02At iterate 25 f = 6.71602D-01|proj g| = 2.87756D-03At iterate 30 f= 6.71418D-01 |proj g| = 1.22922D-03At iterate 35 f = 6.71413D-01 |proj g|= 1.00852D-04 * * * Tit = total number of iterations Tnf = total number of function evaluations Tnint = total number of segments explored during Cauchy searches Skip = number of BFGS updates skipped Nact = number of active bounds at final generalized Cauchy point Projg = norm of the final projected gradient = final function value * * * Tit Tnf Tnint Skip Nact Projg 42 1 0 0 3.374D-05 6.714D-01 10 F = 0.67141329618032430CONVERGENCE: REL REDUCTION OF F <= FACTR*EPSMCH 245.59486751658898 This problem is unconstrained. RUNNING THE L-BFGS-B CODE * * * Machine precision = 2.220D-16 N =11 M =10 At XO 0 variables are exactly at the bounds At iterate 0 f = 8.24410D-01 |proj g|= 6.82577D-01

At iterate 5 f = 6.35984D - 01 |proj g|= 2.53360D - 02

```
At iterate 15 f= 6.22311D-01
                                 |proj q| = 4.03835D-02
At iterate
          20 f= 6.19795D-01
                                  |proj g| = 5.20944D-02
At iterate 25 f = 6.19074D - 01
                                  |proj g| = 4.97659D-02
At iterate 30 f = 6.16954D - 01
                                 |proj g| = 3.55556D-02
At iterate 35 f= 6.16821D-01
                                  |proj g| = 2.11905D-03
At iterate 40 f= 6.16815D-01
                                 |proj g| = 1.59050D-04
At iterate 45 f= 6.16815D-01 |proj g|= 3.18893D-04
At iterate 50 f= 6.16815D-01
                                 |proj g| = 1.31605D-05
         * * *
Tit = total number of iterations
Tnf = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip = number of BFGS updates skipped
Nact = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
   = final function value
          * * *
      Tit
             Tnf Tnint Skip Nact Projg
       50
              62
                  1 0 0 1.316D-05 6.168D-01
 F = 0.61681455746827984
STOP: TOTAL NO. of ITERATIONS REACHED LIMIT
229.249691309342
RUNNING THE L-BFGS-B CODE
          * * *
Machine precision = 2.220D-16
              9
                   M =
                                10
At X0
           O variables are exactly at the bounds
          0 	 f = 6.96262D - 01
                                 |proj g| = 9.95407D-02
At iterate
This problem is unconstrained.
At iterate
          5 f= 6.78835D-01 |proj g|= 3.80642D-02
At iterate 10 f = 6.74554D-01 |proj g| = 4.04843D-02
At iterate 15 f= 6.66990D-01
                                  |proj g| = 2.15945D-02
At iterate 20 f= 6.64500D-01
                                  |proj g| = 4.37596D-02
At iterate 25 f = 6.62408D - 01
                                 |proj g| = 6.49605D-03
At iterate 30 f = 6.62333D - 01 |proj g|= 9.85369D-05
          * * *
```

|proj g| = 1.97282D-02

Tit = total number of iterations

At iterate

10

f= 6.28913D-01

Tnf = total number of function evaluations

Tnint = total number of segments explored during Cauchy searches

Skip = number of BFGS updates skipped

CONVERGENCE: REL REDUCTION OF F <= FACTR*EPSMCH

240.543722020516

RUNNING THE L-BFGS-B CODE

* * *

F = 0.66233250601344040

Machine precision = 2.220D-16N = 10 M = 10

At XO 0 variables are exactly at the bounds

At iterate 0 f= 6.95976D-01 |proj g|= 1.12468D-01

This problem is unconstrained.

At iterate 5 f= 6.82731D-01 |proj g|= 3.89593D-02

At iterate 10 f= 6.74790D-01 |proj g|= 6.07799D-02

At iterate 15 f= 6.63891D-01 |proj g|= 2.12523D-02

At iterate 20 f= 6.61872D-01 |proj g|= 4.71447D-02

At iterate 25 f= 6.59891D-01 |proj g|= 5.64550D-03

At iterate 30 f= 6.59601D-01 |proj g|= 9.13829D-03

At iterate 35 f= 6.59527D-01 |proj g|= 3.36313D-04

At iterate 40 f= 6.59526D-01 |proj g|= 4.97678D-05

Tit = total number of iterations

Tnf = total number of function evaluations

Tnint = total number of segments explored during Cauchy searches

Skip = number of BFGS updates skipped

Nact = number of active bounds at final generalized Cauchy point

Projg = norm of the final projected gradient

F = final function value

* * *

* * *

N Tit Tnf Tnint Skip Nact Projg F
10 42 49 1 0 0 1.810D-05 6.595D-01
F = 0.65952590079297890

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH 241.6007026664409

RUNNING THE L-BFGS-B CODE

* * *

Machine precision = 2.220D-16N = 11 M = 10

At XO 0 variables are exactly at the bounds

```
At iterate 0 f= 7.11889D-01
                                  |proj g| = 1.87952D-01
           f = 6.74808D - 01
                                  |proj g| = 2.54732D-02
At iterate
This problem is unconstrained.
           10 f= 6.65844D-01
                                 |proj g| = 5.66634D-02
At iterate
At iterate 15 f = 6.55174D - 01
                                  |proj g| = 2.93046D-02
At iterate 20 f = 6.51687D - 01
                                  |proj g| = 3.61222D-02
At iterate 25 f= 6.50113D-01
                                  |proj g| = 1.46415D-02
At iterate 30 f = 6.49726D - 01
                                  |proj g|= 1.22233D-03
At iterate 35 f= 6.49707D-01
                                 |proj g| = 3.73798D-04
          * * *
Tit = total number of iterations
Tnf = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip = number of BFGS updates skipped
Nact = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F = final function value
          * * *
      Tit
             Tnf Tnint Skip Nact
                                      Projg
  N
                  1 0 0 1.822D-05 6.497D-01
              43
  11
 F = 0.64970712086531157
CONVERGENCE: REL REDUCTION OF F <= FACTR*EPSMCH
240.30159261074468
RUNNING THE L-BFGS-B CODE
          * * *
Machine precision = 2.220D-16
N =
            12 M =
                               10
At XO
           O variables are exactly at the bounds
At iterate 0 f = 8.65906D - 01
                                  |proj g| = 8.44488D-01
This problem is unconstrained.
At iterate
          5 	 f = 6.39088D - 01
                                 |proj q| = 6.51201D-02
At iterate 10 f = 6.27816D - 01
                                  |proj g| = 3.66421D-02
At iterate
          15 f= 6.21238D-01
                                  |proj g| = 6.03552D-03
At iterate
          20 f= 6.14059D-01
                                  |proj g| = 8.73396D-02
At iterate 25 f = 6.07993D - 01
                                  |proj g| = 2.29269D-02
At iterate 30 f= 6.05208D-01
                                  |proj g| = 7.14988D-03
At iterate 35 f= 6.04804D-01
                                  |proj g| = 6.55792D-03
At iterate 40 f= 6.04776D-01
                                 |proj g| = 4.58614D-04
          * * *
```

Tit = total number of iterations

```
Tnf = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip = number of BFGS updates skipped
Nact = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F = final function value
          * * *
  N Tit Tnf Tnint Skip Nact Projg
       43
                          0 0 1.712D-05 6.048D-01
              60
                     1
  F = 0.60477543672550116
CONVERGENCE: REL REDUCTION OF F <= FACTR*EPSMCH
227.2045467397684
RUNNING THE L-BFGS-B CODE
          * * *
Machine precision = 2.220D-16
N = 10 M =
                               10
At XO 0 variables are exactly at the bounds
At iterate 0
               f = 6.63095D - 01
                                 |proj q| = 1.38363D-01
This problem is unconstrained.
At iterate 5 f = 6.35228D - 01 |proj g|= 4.88677D - 02
At iterate 10 f = 6.30618D - 01 |proj g| = 2.16243D - 02
At iterate 15 f= 6.18691D-01
                                 |proj g| = 9.43610D-02
At iterate 20 f= 6.15866D-01
                                 |proj q| = 6.63346D-03
At iterate 25 f = 6.15646D - 01 |proj g|= 2.36231D-03
At iterate 30 f= 6.15632D-01 |proj q|= 1.55673D-04
         * * *
Tit = total number of iterations
Tnf = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip = number of BFGS updates skipped
Nact = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F = final function value
          * * *
     Tit Inf Inint Skip Nact Projg F
             42 1 0 0 1.732D-05 6.156D-01
       33
 F = 0.61563170644194332
CONVERGENCE: REL REDUCTION OF F <= FACTR*EPSMCH
```

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH 226.85225336449295 RUNNING THE L-BFGS-B CODE

* * *

Machine precision = 2.220D-16N = 11 M = 10

At XO 0 variables are exactly at the bounds

```
|proj g| = 1.38131D-01
           0
                f= 6.63221D-01
At iterate
This problem is unconstrained.
At iterate
           f = 6.38027D - 01
                                  |proj g| = 9.90343D-02
At iterate 10 f = 6.30116D-01
                                  |proj g| = 2.60184D-02
At iterate 15 f = 6.16397D - 01
                                   |proj g| = 2.08301D-01
At iterate
          20 f= 6.14183D-01
                                   |proj g| = 3.39047D-02
At iterate
          25 f= 6.14050D-01
                                   |proj g| = 3.02629D-02
At iterate 30 f= 6.14003D-01
                                   |proj g| = 8.48627D-02
At iterate 35 f= 6.12986D-01
                                   |proj g| = 1.67169D-01
At iterate
          40 f= 6.12908D-01
                                  |proj g| = 6.53702D-02
At iterate 45 f= 6.12813D-01
                                   |proj g| = 6.93446D-03
At iterate 50 f = 6.12771D-01
                                  |proj g| = 1.49755D-02
         * * *
Tit = total number of iterations
Tnf = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip = number of BFGS updates skipped
Nact = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
   = final function value
          * * *
             Tnf Tnint Skip Nact Projg
      Tit
       50
              75
                     1 0 0 1.498D-02 6.128D-01
  F = 0.61277100250757366
```

STOP: TOTAL NO. of ITERATIONS REACHED LIMIT 227.89105684254474

This problem is unconstrained.

RUNNING THE L-BFGS-B CODE

* * *

```
Machine precision = 2.220D-16
N =
             12
                  M =
                                10
            O variables are exactly at the bounds
At XO
At iterate
           0 	 f = 6.81251D - 01
                                  |proj g| = 1.90922D-01
At iterate
          5 f= 6.35368D-01
                                  |proj g| = 2.56550D-02
At iterate 10 f = 6.30245D - 01
                                  |proj q| = 3.72932D-02
At iterate
           15
               f= 6.27068D-01
                                  |proj g| = 2.99829D-02
At iterate
          20 f= 6.17778D-01
                                  |proj g| = 5.17909D-02
At iterate
          25 f= 6.15165D-01
                                  |proj g| = 2.47000D-02
At iterate 30 f = 6.11408D - 01
                                  |proj g| = 8.66904D-01
```

```
f= 6.10046D-01
                                  |proj g| = 9.40404D-02
At iterate
           35
At iterate 40 f= 6.09059D-01
                                  |proj q| = 2.49295D-01
At iterate 45 f= 6.08750D-01
                                  |proj g| = 5.93195D-01
At iterate 50 f= 6.08337D-01 |proj g|= 3.33542D-02
         * * *
Tit = total number of iterations
Tnf = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip = number of BFGS updates skipped
Nact = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
    = final function value
          * * *
      Tit Inf Inint Skip Nact Projg
       50
                          0 0 3.335D-02 6.083D-01
              73
                     1
  12
  F = 0.60833700521985945
STOP: TOTAL NO. of ITERATIONS REACHED LIMIT
228.4012337538728
RUNNING THE L-BFGS-B CODE
          * * *
Machine precision = 2.220D-16
            13 M =
                                10
At XO
           O variables are exactly at the bounds
At iterate
           0 f= 8.50669D-01 |proj q|= 7.54837D-01
This problem is unconstrained.
At iterate
           5 f= 6.53688D-01
                                  |proj g| = 6.01917D-02
At iterate 10 f = 6.29831D - 01
                                  |proj g| = 3.99901D-02
At iterate 15 f= 6.18902D-01
                                  |proj g| = 8.53974D-02
At iterate 20 f= 6.10738D-01
                                  |proj g| = 1.36038D-02
At iterate
          25 f= 6.10285D-01
                                  |proj g| = 5.93247D-03
At iterate
          f = 6.09926D - 01
                                  |proj g| = 5.19697D-03
At iterate 35 f= 6.09741D-01
                                  |proj g| = 5.20491D-03
                                  |proj g| = 3.60026D-04
At iterate 40 f= 6.09729D-01
At iterate 45 f= 6.09728D-01
                                  |proj g| = 1.07301D-04
          * * *
Tit = total number of iterations
Tnf = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip = number of BFGS updates skipped
Nact = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
     = final function value
```

* * *

```
N Tit Tnf Tnint Skip Nact Projg
    48
         54 1 0 0 1.513D-05 6.097D-01
F = 0.60972824685717153
```

CONVERGENCE: REL REDUCTION OF F <= FACTR*EPSMCH 230.86869094400964

RUNNING THE L-BFGS-B CODE

* * *

Machine precision = 2.220D-16 N = 11

M = 10

At XO 0 variables are exactly at the bounds

| At iterate | 0 | f= | 8.97581D-01 | proj g = | 2.36935D+00 | | |
|--------------------------------|----|----|-------------|----------|-------------|--|--|
| This problem is unconstrained. | | | | | | | |
| At iterate | 5 | f= | 6.56285D-01 | proj g = | 1.58625D-01 | | |
| At iterate | 10 | f= | 6.37608D-01 | proj g = | 2.50380D-02 | | |
| At iterate | 15 | f= | 6.30630D-01 | proj g = | 1.76606D-02 | | |
| At iterate | 20 | f= | 6.24395D-01 | proj g = | 5.75407D-02 | | |
| At iterate | 25 | f= | 6.20581D-01 | proj g = | 2.22518D-02 | | |
| At iterate | 30 | f= | 6.12826D-01 | proj g = | 6.55121D-02 | | |
| At iterate | 35 | f= | 6.07742D-01 | proj g = | 2.52122D-02 | | |
| At iterate | 40 | f= | 6.05760D-01 | proj g = | 2.06324D-02 | | |
| At iterate | 45 | f= | 6.04960D-01 | proj g = | 1.36755D-03 | | |
| At iterate | 50 | f= | 6.04954D-01 | proj g = | 1.95574D-04 | | |
| | | | | | | | |

* * *

Tit = total number of iterations

Tnf = total number of function evaluations

Tnint = total number of segments explored during Cauchy searches

Skip = number of BFGS updates skipped

Nact = number of active bounds at final generalized Cauchy point

Projg = norm of the final projected gradient

F = final function value

* * *

N Tit Tnf Tnint Skip Nact Projg 50 11 1 0 0 1.956D-04 6.050D-01 62 F = 0.60495385060557627

STOP: TOTAL NO. of ITERATIONS REACHED LIMIT Best SARIMA Model: SARIMA(2, 1, 3, 2, 1, 3) (seasonal period = 12)

SARIMAX Results

Dep. Variable: Sales No. Observations:

68 32

Model: SARIMAX(2, 1, 3) \times (2, 1, 3, 12) Log Likelihood

-101.6

1

```
Date: Mon, 30 Oct 2023 AIC 225.2
64

Time: 18:48:49 BIC 255.4
59

Sample: 0 HQIC 237.5
20
- 168
```

opg

| ========= | ======== | ======== | | | | ======= |
|-------------|--------------|----------|--------|-------------|---------|---------|
| | coef | std err | Z | P> z | [0.025 | 0.975] |
| ar.L1 | -0.5518 | 0.080 | -6.871 | 0.000 | -0.709 | -0.394 |
| ar.L2 | -0.7734 | 0.084 | -9.179 | 0.000 | -0.939 | -0.608 |
| ma.L1 | -0.3141 | 9.833 | -0.032 | 0.975 | -19.587 | 18.959 |
| ma.L2 | 0.6866 | 39.072 | 0.018 | 0.986 | -75.892 | 77.265 |
| ma.L3 | -0.7385 | 34.220 | -0.022 | 0.983 | -67.808 | 66.331 |
| ar.S.L12 | -0.5556 | 0.195 | -2.851 | 0.004 | -0.938 | -0.174 |
| ar.S.L24 | -0.4890 | 0.172 | -2.843 | 0.004 | -0.826 | -0.152 |
| ma.S.L12 | 0.1805 | 0.223 | 0.810 | 0.418 | -0.256 | 0.617 |
| ma.S.L24 | 0.3033 | 0.202 | 1.498 | 0.134 | -0.094 | 0.700 |
| ma.S.L36 | -0.0547 | 0.125 | -0.437 | 0.662 | -0.300 | 0.191 |
| sigma2 | 0.3267 | 15.141 | 0.022 | 0.983 | -29.349 | 30.002 |
| Ljung-Box (| L1) (Q): | ======= | 0.00 | Jarque-Bera | (JB): | 8.7 |
| Prob(Q): | | | 0.95 | Prob(JB): | | 0.0 |
| Heteroskeda | sticity (H): | | 5.13 | Skew: | | -0.4 |
| Prob(H) (tw | o-sided): | | 0.00 | Kurtosis: | | 3.9 |

Warnings:

Covariance Type:

[1] Covariance matrix calculated using the outer product of gradients (complex-step).

8A)

```
In [15]: # PART 8A:
    testset = pharma_data[168 : ]
    p = 2
    d = 1
    q = 3
    P = 2
    D = 1
    Q = 3

    best_model = sm.tsa.SARIMAX(trainset['Sales'], order=(p, d, q), seasonal_order=(P, D, Q, best_model_fit = best_model.fit()
    predictions=best_model_fit.forecast(36)
```

This problem is unconstrained.

RUNNING THE L-BFGS-B CODE

* * *

```
Machine precision = 2.220D-16 N = 11 M = 10  
At X0 0 variables are exactly at the bounds  
At iterate 0 f= 1.14948D+00 |proj g|= 2.95380D+00  
At iterate 5 f= 8.58155D-01 |proj g|= 1.37487D-01
```

```
At iterate
              f = 7.83707D - 01
                                |proj g| = 3.11468D-02
           10
At iterate 15 f = 7.82873D - 01
                                |proj q| = 1.56837D-02
At iterate 20 f= 7.82753D-01
                                |proj g| = 4.53157D-03
At iterate 25 f= 7.82722D-01
                                |proj g| = 3.08611D-03
At iterate 30 f= 7.82210D-01 |proj g|= 2.16453D-02
At iterate 35 f= 7.78648D-01
                                |proj g| = 6.21804D - 02
                                |proj g| = 2.38767D-02
At iterate 40 f= 7.72752D-01
At iterate 45 f = 7.70817D-01 |proj g|= 1.69276D-02
At iterate 50 f = 7.64224D-01 |proj g|= 6.27346D-02
```

* * *

Tit = total number of iterations

Tnf = total number of function evaluations

Tnint = total number of segments explored during Cauchy searches

Skip = number of BFGS updates skipped

Nact = number of active bounds at final generalized Cauchy point

Projg = norm of the final projected gradient

F = final function value

* * *

N Tit Tnf Tnint Skip Nact Projg F 11 50 63 1 0 0 6.273D-02 7.642D-01 F = 0.76422448717798208

STOP: TOTAL NO. of ITERATIONS REACHED LIMIT

In [16]: testset["prediction"] = predictions testset

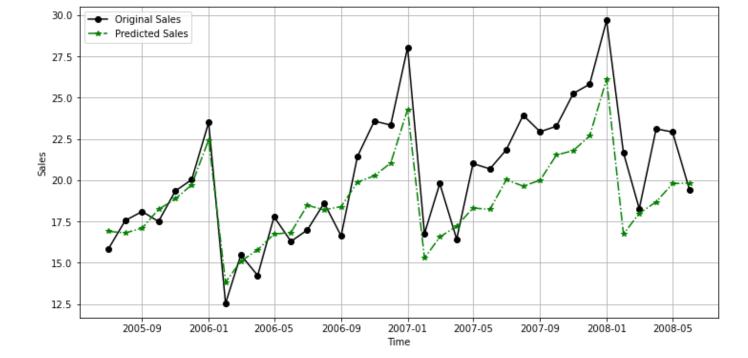
Out[16]:

| | Month | Sales | Moving Average | prediction |
|-----|------------|-----------|----------------|------------|
| 168 | 2005-07-01 | 15.829550 | 15.985782 | 16.902176 |
| 169 | 2005-08-01 | 17.554701 | 16.167777 | 16.802333 |
| 170 | 2005-09-01 | 18.100864 | 16.331015 | 17.091382 |
| 171 | 2005-10-01 | 17.496668 | 16.398591 | 18.229050 |
| 172 | 2005-11-01 | 19.347265 | 16.541136 | 18.868575 |
| 173 | 2005-12-01 | 20.031291 | 16.637966 | 19.720662 |
| 174 | 2006-01-01 | 23.486694 | 16.863631 | 22.437812 |
| 175 | 2006-02-01 | 12.536987 | 16.895500 | 13.817875 |
| 176 | 2006-03-01 | 15.467018 | 17.067552 | 15.088746 |
| 177 | 2006-04-01 | 14.233539 | 17.048744 | 15.787315 |
| 178 | 2006-05-01 | 17.783058 | 17.297740 | 16.733295 |
| 179 | 2006-06-01 | 16.291602 | 17.346603 | 16.830020 |
| 180 | 2006-07-01 | 16.980282 | 17.442497 | 18.475998 |
| 181 | 2006-08-01 | 18.612189 | 17.530621 | 18.206811 |
| 182 | 2006-09-01 | 16.623343 | 17.407495 | 18.395711 |

| 183 | 2006-10-01 | 21.430241 | 17.735292 | 19.877506 |
|-----|------------|-----------|-----------|-----------|
| 184 | 2006-11-01 | 23.575517 | 18.087647 | 20.265329 |
| 185 | 2006-12-01 | 23.334206 | 18.362890 | 21.033709 |
| 186 | 2007-01-01 | 28.038383 | 18.742197 | 24.284290 |
| 187 | 2007-02-01 | 16.763869 | 19.094437 | 15.281701 |
| 188 | 2007-03-01 | 19.792754 | 19.454915 | 16.554246 |
| 189 | 2007-04-01 | 16.427305 | 19.637729 | 17.211212 |
| 190 | 2007-05-01 | 21.000742 | 19.905869 | 18.314535 |
| 191 | 2007-06-01 | 20.681002 | 20.271653 | 18.228071 |
| 192 | 2007-07-01 | 21.834890 | 20.676203 | 20.019287 |
| 193 | 2007-08-01 | 23.930204 | 21.119371 | 19.639391 |
| 194 | 2007-09-01 | 22.930357 | 21.644956 | 19.989743 |
| 195 | 2007-10-01 | 23.263340 | 21.797714 | 21.521625 |
| 196 | 2007-11-01 | 25.250030 | 21.937257 | 21.789612 |
| 197 | 2007-12-01 | 25.806090 | 22.143247 | 22.679834 |
| 198 | 2008-01-01 | 29.665356 | 22.278828 | 26.108347 |
| 199 | 2008-02-01 | 21.654285 | 22.686363 | 16.737836 |
| 200 | 2008-03-01 | 18.264945 | 22.559045 | 17.990453 |
| 201 | 2008-04-01 | 23.107677 | 23.115743 | 18.684940 |
| 202 | 2008-05-01 | 22.912510 | 23.275057 | 19.786641 |
| 203 | 2008-06-01 | 19.431740 | 23.170952 | 19.823521 |

8B)

```
In [17]: # PART 8B:
    # Plot of predicted values overlaid on the original data
    plt.figure(figsize=(12,6))
    plt.plot(testset['Month'], testset['Sales'], label='Original Sales', c='black', linestyl
    plt.plot(testset['Month'], testset['prediction'], label='Predicted Sales', c='green', li
    plt.xlabel('Time')
    plt.ylabel('Sales')
    plt.legend()
    plt.grid(True)
    plt.show()
```



8C)

```
In [18]: # PART 8C:
    # Calculating the MSE between the original data and the predicted values
    from sklearn.metrics import mean_squared_error

# Compute the MSE
mse = mean_squared_error(pharma_data['Sales'].iloc[168:], testset['prediction'])
print(f"Mean Squared Error (MSE): {mse}")
```

Mean Squared Error (MSE): 5.462130003412936

From the table and the plot we can see that the predicted values are very close to the actual/original data points for the Sales column. The MSE value of 5.462130003412936 indicates that our model captures the underlying patterns and trends in the data fairly well. It is noteworthy that the discrepancies between the predicted values and actual values is quite common and our model does a decent job, further analysis might even reduce this MSE and we might be able to get better predictions.

Problem 2

A)

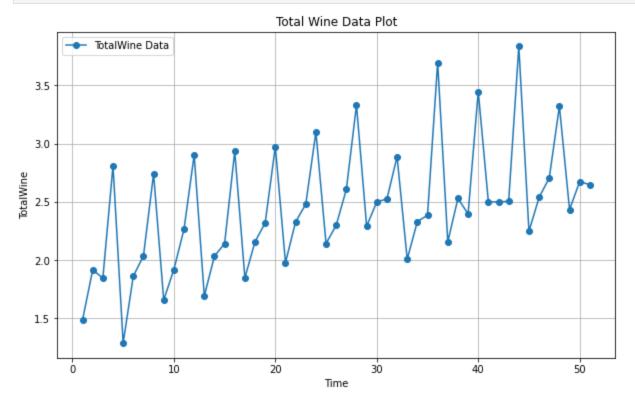
```
In [19]: wine_data = pd.read_csv('TotalWine.csv')
    wine_data
```

| Out[19]: | | Time (Quarter) | TotalWine |
|----------|---|----------------|-----------|
| | 0 | 1 | 1.486 |
| | 1 | 2 | 1.915 |
| | 2 | 3 | 1.844 |
| | 3 | 4 | 2.808 |

| 4 | 5 | 1.287 |
|----|----|-------|
| 5 | 6 | 1.861 |
| 6 | 7 | 2.034 |
| 7 | 8 | 2.739 |
| 8 | 9 | 1.656 |
| 9 | 10 | 1.918 |
| 10 | 11 | 2.265 |
| 11 | 12 | 2.902 |
| 12 | 13 | 1.691 |
| 13 | 14 | 2.033 |
| 14 | 15 | 2.141 |
| 15 | 16 | 2.932 |
| 16 | 17 | 1.847 |
| 17 | 18 | 2.157 |
| 18 | 19 | 2.318 |
| 19 | 20 | 2.974 |
| 20 | 21 | 1.977 |
| 21 | 22 | 2.328 |
| 22 | 23 | 2.479 |
| 23 | 24 | 3.099 |
| 24 | 25 | 2.141 |
| 25 | 26 | 2.299 |
| 26 | 27 | 2.606 |
| 27 | 28 | 3.330 |
| 28 | 29 | 2.290 |
| 29 | 30 | 2.499 |
| 30 | 31 | 2.524 |
| 31 | 32 | 2.887 |
| 32 | 33 | 2.007 |
| 33 | 34 | 2.330 |
| 34 | 35 | 2.384 |
| 35 | 36 | 3.696 |
| 36 | 37 | 2.157 |
| 37 | 38 | 2.529 |
| 38 | 39 | 2.395 |
| 39 | 40 | 3.447 |
| 40 | 41 | 2.499 |
| 41 | 42 | 2.499 |
| 42 | 43 | 2.504 |

| 43 | 44 | 3.834 |
|----|----|-------|
| 44 | 45 | 2.246 |
| 45 | 46 | 2.538 |
| 46 | 47 | 2.704 |
| 47 | 48 | 3.321 |
| 48 | 49 | 2.433 |
| 49 | 50 | 2.673 |
| 50 | 51 | 2.647 |

```
In [20]: plt.figure(figsize=(10,6))
    plt.plot(wine_data['Time (Quarter)'], wine_data['TotalWine'], label="TotalWine Data", ma
    plt.xlabel('Time')
    plt.ylabel('TotalWine')
    plt.title('Total Wine Data Plot')
    plt.legend()
    plt.grid(True)
    plt.show()
```

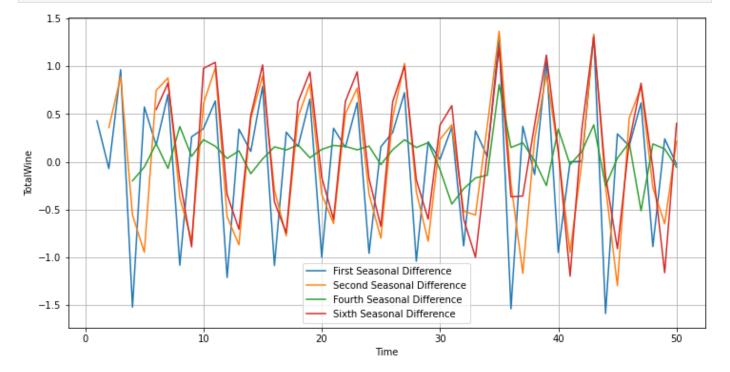


The seasonal period of this time series seems to be every 4 quarters so one possible interpretation of this would be annual seasonality as there are 4 quarters in a fiscal year.

B)

```
In [21]: # Seasonal Differencing
    seasonal_diff1 = wine_data['TotalWine'].diff(1)
    seasonal_diff2 = wine_data['TotalWine'].diff(2)
    seasonal_diff4 = wine_data['TotalWine'].diff(4)
    seasonal_diff6 = wine_data['TotalWine'].diff(6)
    # pos 0 for diff1 nan, pos 0 and 1 for diff2 nan, pos 0,1,2,3 for diff4 nan, pos 0,1,2,3
# Plots
```

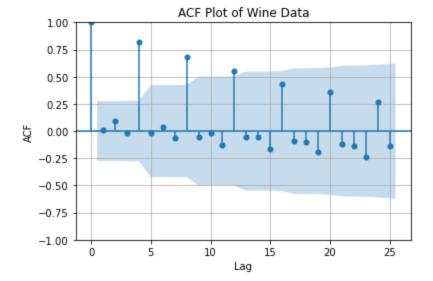
```
plt.figure(figsize=(12,6))
plt.plot(seasonal_diff1, label='First Seasonal Difference')
plt.plot(seasonal_diff2, label='Second Seasonal Difference')
plt.plot(seasonal_diff4, label='Fourth Seasonal Difference')
plt.plot(seasonal_diff6, label='Sixth Seasonal Difference')
plt.xlabel('Time')
plt.ylabel('TotalWine')
plt.legend()
plt.grid(True)
plt.show()
```



From the above plot the fourth seasonal difference i.e. lag 4 seems to be the most suitable to remove the seasonality.

C)

```
In [22]: # Plotting ACF of the Original Time Series
    plot_acf(wine_data['TotalWine'], lags=25)
    plt.title('ACF Plot of Wine Data')
    plt.xlabel('Lag')
    plt.ylabel('ACF')
    plt.grid(True)
    plt.show()
```



From the above ACF plot of the wine data, the significant peaks seem to be at lag 4, lag 8, lag 12 and so on. Therefore, the seasonal period is every 4 quarters or in other words annual but in terms of quarters not months.

```
In [23]:
         from statsmodels.tsa.ar model import AutoReg
         best order = None
         min aic = float('inf')
         # Iterate through different lag orders and calculate AIC
         # NOTE: I am running the following search for best AR order on the fourth seasonal data
         ###### If it is run on the original time series the best AR order found by minimum AIC
         ###### value method comes out to be 8, however, here we find it to be 4.
         for p in range(1, 11):
             model = sm.tsa.AutoReg(seasonal diff4.dropna(), lags=p)
             results = model.fit()
             aic = results.aic
             print(f"AIC for order {p}: {aic}")
             if aic < min aic:</pre>
                 min aic = aic
                 best order = p
         # Printing the optimal AR order and its corresponding AIC
         print("Minimum AIC:", min aic)
         print("Optimal AR Order (p) based on Minimum AIC:", best order)
         AIC for order 1: -3.2585296001103874
         AIC for order 2: -0.43612691994517405
         AIC for order 3: 2.2955947939000376
         AIC for order 4: -18.571190665860307
         AIC for order 5: -16.137237852006194
         AIC for order 6: -13.649083439021595
         AIC for order 7: -14.197955461006565
```

E

AIC for order 8: -11.229617621206952 AIC for order 9: -8.072504350825529 AIC for order 10: -5.124304330227638 Minimum AIC: -18.571190665860307

Optimal AR Order (p) based on Minimum AIC: 4



```
In [24]:
           ar model = ARIMA(seasonal diff4.dropna(),order=(4,0,0))
           model fit = ar model.fit()
           model fit.summary()
                                  SARIMAX Results
Out [24]:
                                     TotalWine No. Observations:
                                                                      47
              Dep. Variable:
                     Model:
                                 ARIMA(4, 0, 0)
                                                   Log Likelihood
                                                                   16.370
                      Date: Mon, 30 Oct 2023
                                                                 -20.739
                                                            AIC
                      Time:
                                     18:48:53
                                                             BIC
                                                                   -9.638
                    Sample:
                                            0
                                                           HQIC -16.562
                                          - 47
           Covariance Type:
                                          opg
                                          z P>|z| [0.025 0.975]
                       coef std err
                              0.018
             const
                     0.0725
                                      4.034
                                             0.000
                                                     0.037
                                                             0.108
              ar.L1 -0.0072
                              0.112
                                    -0.064
                                             0.949
                                                    -0.227
                                                             0.213
                    0.0470
                                                    -0.280
                                                             0.374
             ar.L2
                              0.167
                                      0.282
                                             0.778
             ar.L3
                    0.0600
                              0.097
                                      0.618
                                             0.537
                                                    -0.130
                                                             0.250
             ar.L4 -0.6652
                              0.120
                                    -5.546
                                             0.000
                                                    -0.900
                                                            -0.430
                     0.0277
                              0.007
                                      4.119 0.000
                                                     0.015
                                                             0.041
           sigma2
               Ljung-Box (L1) (Q): 0.44 Jarque-Bera (JB):
                                                            0.43
                         Prob(Q): 0.51
                                                            0.80
                                                 Prob(JB):
```

Warnings:

Heteroskedasticity (H): 3.07

Prob(H) (two-sided): 0.03

[1] Covariance matrix calculated using the outer product of gradients (complex-step).

Skew:

Kurtosis:

-0.18

3.29

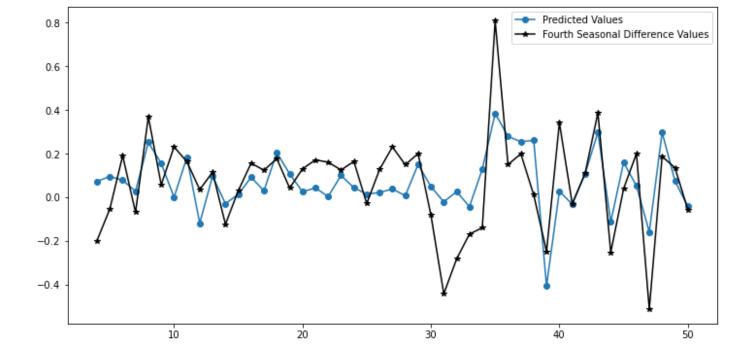
ii)

```
In [25]: | preds = model fit.predict()
          # Taking a look at the prediction values starting from the optimal lag
          preds
                0.072507
Out[25]:
                0.095271
          6
                0.077843
          7
                0.027467
          8
                0.252057
          9
                0.154932
          10
               -0.000086
          11
               0.182566
          12
               -0.118841
          13
                0.096858
          14
               -0.029558
          15
                0.013466
```

```
16
      0.091069
17
      0.029851
18
      0.204228
19
      0.107467
20
      0.025191
      0.042679
21
      0.003167
22
23
      0.100249
24
      0.043960
      0.014115
25
26
      0.021822
27
      0.037916
      0.006983
28
29
     0.150205
30
     0.048456
31
    -0.021228
32
     0.025701
33
   -0.043271
34
     0.129355
35
     0.384244
36
     0.279226
37
     0.254483
38
     0.260804
39
    -0.406360
40
     0.027960
    -0.032376
41
42
     0.107534
43
     0.297461
44
   -0.113444
45
     0.160005
46
     0.052038
47
    -0.158714
48
     0.297208
49
      0.074091
     -0.042502
50
Name: predicted mean, dtype: float64
```

iii)

```
In [26]: plt.figure(figsize=(12,6))
    plt.plot(preds,label = 'Predicted Values', marker='o')
    plt.plot(seasonal_diff4,label = 'Fourth Seasonal Difference Values', c='black', marker='
    plt.legend()
    plt.show()
```



iv)

In [27]: # Calculating the Mean Absolute Error (MAE) by comparing predicted results with the seas
mae = np.mean(np.abs(preds - seasonal_diff4))
print('The Mean Absolute Error (MAE) is', mae)

The Mean Absolute Error (MAE) is 0.13461884335362367

In []: