Assignment 5 - Deadline: Nov 13, 2024, Wed 11pm

DSAI 510 Fall 2024

Complete the assignment below and upload both the .ipynb file and its pdf to https://moodle.boun.edu.tr by the deadline given above. The submission page on Moodle will close automatically after this date and time.

To make a pdf, this may work: Hit CMD+P or CTRL+P, and save it as PDF. You may also use other options from the File menu.

```
# Run this cell first

import pandas as pd
import numpy as np
import seaborn as sns
import matplotlib.pyplot as plt
import statsmodels.api as sm
from statsmodels.stats.outliers_influence import
variance_inflation_factor
import itertools
from sklearn.linear_model import Ridge

# Set the display option to show all rows scrolling with a slider
#pd.set_option('display.max_rows', None)
# To disable this, run the line below:
# pd.reset_option('display.max_rows')
```

Note:

In the problems below, if they ask "show the number of records that are nonzero", the answer is a number; so you don't need to show the records themselves. But if it asks, "show the records with NaN", it wants you to print those records (rows) containing NAN and other entries, not asking how many such records there are. So be careful about what you're asked.

Problem 1 (10 pts)

Create the dataframe by running the cell below

```
data = {'A': [1, 2, 3, 4, 5], 'B': [2.1, 4.2, 5.6, 8, 11]}
df = pd.DataFrame(data)
#df
```

Part (a)

By defining X and Y and then using the formula

$$\beta = \left(X^T X\right)^{-1} X^T Y$$

compute the β coefficients. You can use `linalg` function from `numpy` library to multiple the matrices.

```
# Break your computations into multiple cells

# Define X and Y

X = df["A"]

# Add constant terms here on X
X = sm.add_constant(X)
Y = df['B']



XtX = X.T @ X

XtX_inv = np.linalg.inv(XtX)

XtY = X.T @ Y

beta = XtX_inv @ XtY

print(beta)
```

```
[-0.3   2.16]
```

Part (b)

Use `statsmodels` or any other library to do the regression in Python. The β's you found here should be equal to or close to the ones you found in part (a).

```
# Define X and Y for regression
X = df['A']
Y = df['B']

# Add a constant to X for the intercept term
X = sm.add_constant(X)

# Fit the regression model
model = sm.OLS(Y, X).fit()

# Get the beta coefficients
beta = model.params
print("Beta coefficients:\n", beta)
```

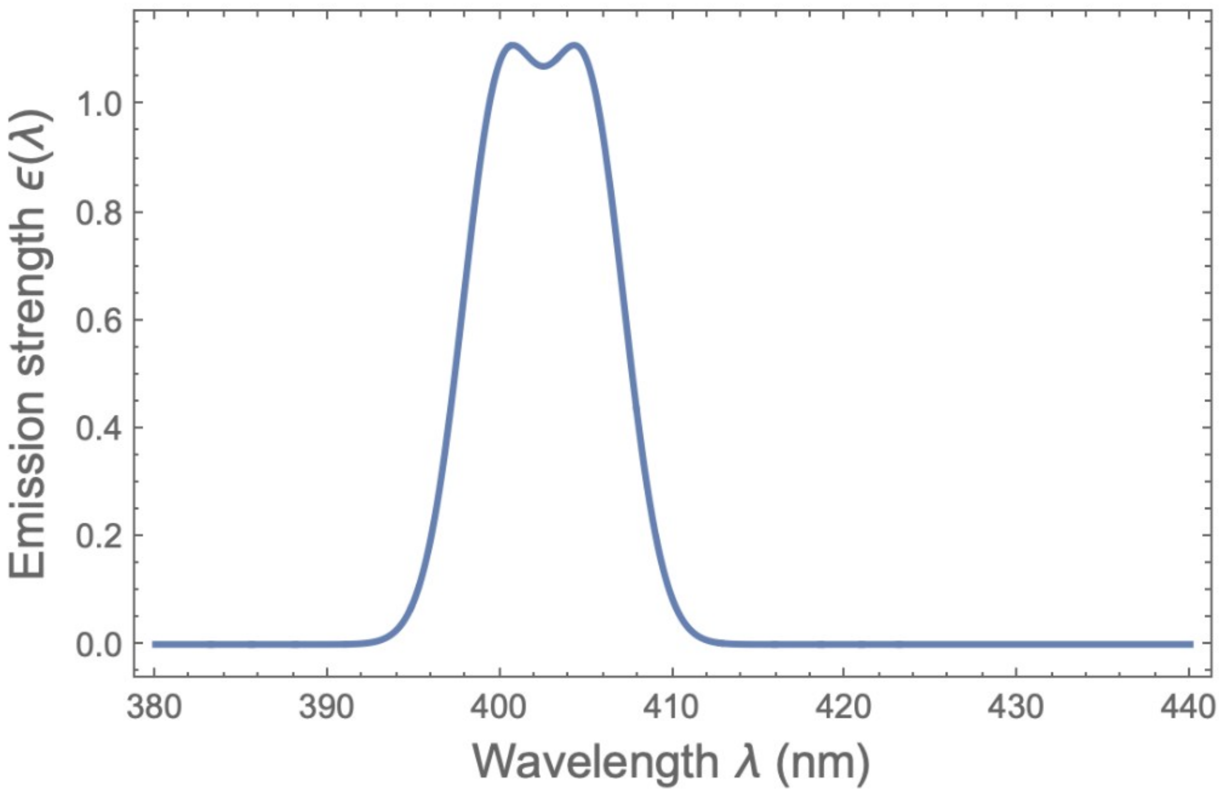
```
Beta coefficients:
 const   -0.30
A         2.16
dtype: float64
```

Problem 2 (30 pts)

A chemical substance emits light at two distinct but close wavelengths. The mean of the wavelengths are given by λ_1 and λ_2 and the spectrum of the emitted light is distributed in wavelength around these means as a Gaussian distribution. For example, if $\lambda_1 = 400$ and $\lambda_2 = 405$,

$$\epsilon(\lambda) = \exp\left[-\frac{1}{10}(\lambda - 405)^2\right) + \exp\left[-\frac{1}{10}(\lambda - 400)^2\right),$$

which is plotted below.

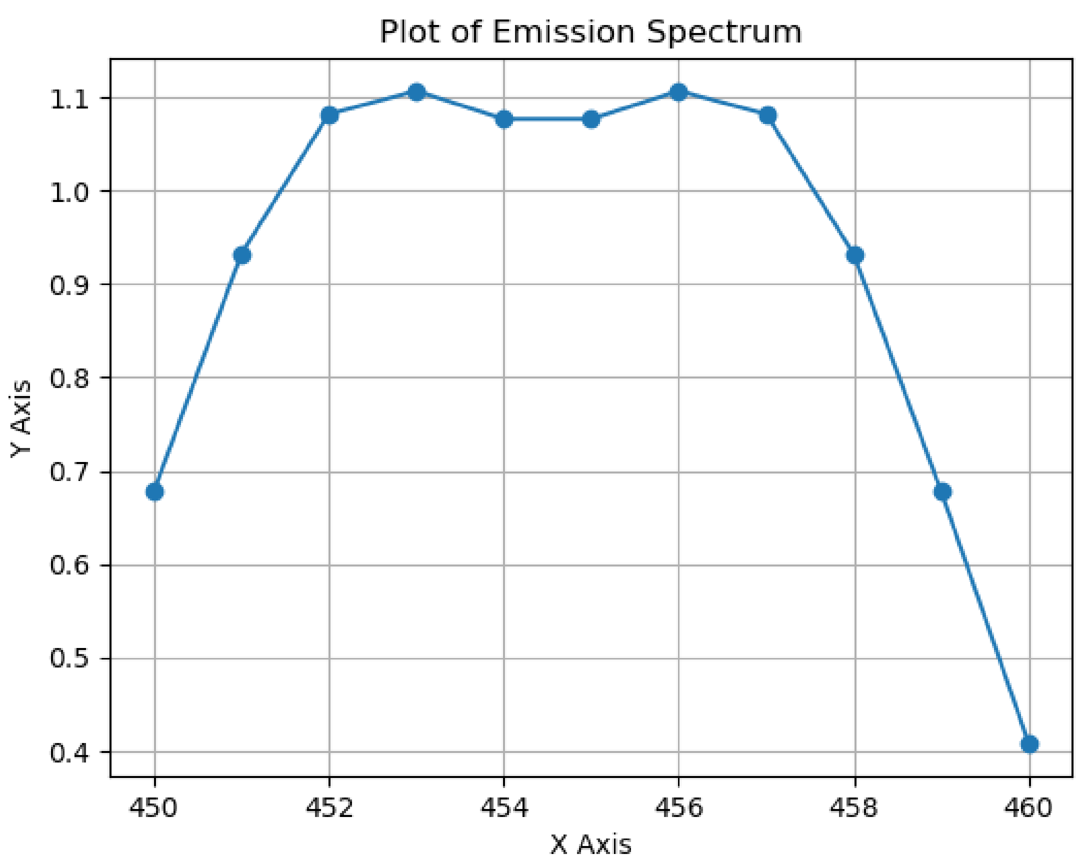


In the plot above, we see superposition of two Gaussians. Since their wavelengths are very close, they overlap and create a large bell-shaped curve with a drop in the middle. By looking at this plot, we can't see the *exact* locations of the two means λ_1 and λ_2.

The formula and plot above were just an example. In the cell below, the data given belongs to a spectrum where we don't know λ_1 and λ_2, but we know the spectrum is given by two Gaussians as such

$$\epsilon(\lambda) = \exp\left[-\frac{1}{10}(\lambda - \lambda_1)^2\right) + \exp\left[-\frac{1}{10}(\lambda - \lambda_2)^2\right).$$

```python
# This is the spectrum data
data = np.array([[450, 0.677767], [451, 0.932161], [452, 1.08208],
                 [453, 1.10673], [454, 1.07689], [455, 1.07689], [456,
1.10673],
                 [457,1.08208], [458, 0.932161], [459, 0.677767],
[460, 0.408231]])

# suppress scientific form to display numbers in fractional form
np.set_printoptions(suppress=True)

#print(data)

# Plotting
x = data[:, 0]
y = data[:, 1]


plt.plot(x, y, marker='o')  # 'o' is for circle markers
plt.xlabel('X Axis')
plt.ylabel('Y Axis')
plt.title('Plot of Emission Spectrum')
plt.grid(True)
plt.show()
```



Your task is to do linear regression by using Gaussian base functions to find the two characteristic frequecies λ_1 and λ_2 within the spectrum of which data is given in the cell below.

Part (a)

Define a function `GaussianReg(lambda1,lambda2)`. It takes two arguments, for example, `lambda1=300` and `lambda1=350`, then performs linear regression of the form

$$y=\beta_0+\beta_1\exp\left[-\frac{1}{10}(\lambda-300)^2\right)+\beta_2\exp\left[-\frac{1}{10}(\lambda-350)^2\right).$$

If for example, `lambda1=350` and `lambda1=400`, then performs linear regression of the form

$$y=\beta_0+\beta_1\exp\left[-\frac{1}{10}(\lambda-350)^2\right)+\beta_2\exp\left[-\frac{1}{10}(\lambda-400)^2\right).$$

Then this function `GaussianReg()` returns the R^2 value of this linear regression. (Here we're not interested in the β's, rather, we're interested in the particular pair (λ_1,λ_2) that would model the data well, so we use the metric R^2 to find these two values.)

```
# Define your GaussianReg() function here

def GaussianReg(lambda1,lambda2):
    # do the linear regression for the specific values of lambda1 and
lambda2
    # for the data x and y created above.
    X = np.column_stack([
        np.ones(len(x)),  # Intercept term
        np.exp(- (1/10) * (x - lambda1) ** 2),
        np.exp(- (1/10) * (x - lambda2) ** 2)
    ])

    # Fit the model
    model = sm.OLS(y, X).fit()

    # Calculate R^2
    R2 = model.rsquared

    return R2  # return R2 of this linear regression
```

Part (b)

- Create two nested for loops for λ_1 and λ_2. The for loops will iterate over all integer values of lambda1 and lambda2 in the interval [450,460] by taking a different (λ_1,λ_2) set in each iteration. Call the `GaussianReg(lambda1,lambda2)` function in each iteration
- `GaussianReg(lambda1,lambda2)` returns a R^2 value; append the three numbers (lambda1, lambda2, R2) to the list `R2values` in each iteration.

- Convert the R2values list into a numpy array. At the end, R2values will have three columns (lambda1, lambda2, R2) and 100 rows (10 values for λ_1 times 10 values for λ_1 = 100).

```
R2values = []; # create an empty list

for lambda1 in range(450, 461):
    for lambda2 in range(450, 461):
        R2values.append((lambda1, lambda2, GaussianReg(lambda1,
lambda2)))

    # Do regression by calling GaussianReg(lambda1,lambda2)
    # append (lambda1, lambda2, R2) to the list R2values

# R2values is a list; turn it into an array of 100x3.
R2values = np.array(R2values)
#print(R2values)
```

Part (c)

- Find the best fit by finding the row in `R2values` where R^2 is maximum. Print the corresponding λ_1 and λ_2 values that maximizes R^2.

- Do these λ_1 and λ_2 you found from the linear regression make sense when you look at the plot with the plot label "Plot of Emission Spectrum" above?

```
R2values = pd.DataFrame(R2values)

R2values.columns=["lambda1", "lambda2", "R_square"]

R2values.sort_values(by="R_square").iloc[len(R2values) - 1]

# The below values are around peaks on the emission spctrum.

lambda1     452.0
lambda2     457.0
R_square      1.0
Name: 29, dtype: float64
```

Problem 3 (20 pts)

Use the approach from Problem 2 (I don't mean exactly the same functions you defined, but the approach in general) to solve this problem:

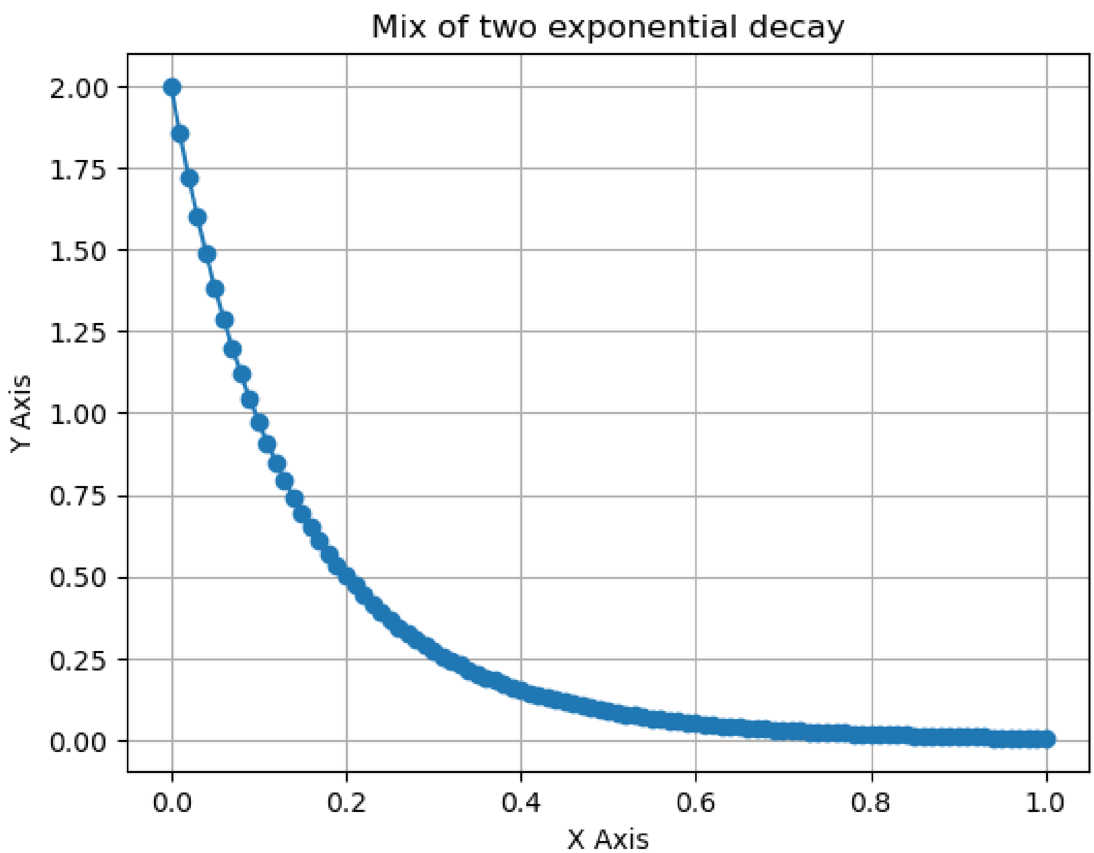
`data2` includes (x,z) pairs of a mixture of two exponential decay functions in the form

$$f(x) = e^{-ax} + e^{-bx}$$

For a and b are integers in the interval [1,10], find the (a,b) pair that makes the model f(x) fit the `data2` the best.

(Make sure you don't mistakenly use the varible `data` variable; it's `data2` here.)

```
# Run this cell
data2 = np.array([[0., 2.], [0.01, 1.85607], [0.02, 1.72357], [0.03,
1.60153], [0.04,
  1.48905], [0.05, 1.38533], [0.06, 1.28963], [0.07, 1.20127], [0.08,
  1.11965], [0.09, 1.0442], [0.1, 0.97441], [0.11, 0.909821], [0.12,
  0.850006], [0.13, 0.794578], [0.14, 0.743182], [0.15,
  0.695497], [0.16, 0.651225], [0.17, 0.610098], [0.18,
  0.571869], [0.19, 0.53631], [0.2, 0.503215], [0.21,
  0.472394], [0.22, 0.443674], [0.23, 0.416896], [0.24,
  0.391912], [0.25, 0.36859], [0.26, 0.346805], [0.27,
  0.326446], [0.28, 0.307407], [0.29, 0.289594], [0.3,
  0.272917], [0.31, 0.257297], [0.32, 0.242659], [0.33,
  0.228933], [0.34, 0.216057], [0.35, 0.203971], [0.36,
  0.192623], [0.37, 0.181961], [0.38, 0.171939], [0.39,
  0.162516], [0.4, 0.153651], [0.41, 0.145308], [0.42,
  0.137452], [0.43, 0.130053], [0.44, 0.12308], [0.45,
  0.116508], [0.46, 0.110311], [0.47, 0.104464], [0.48,
  0.0989477], [0.49, 0.0937402], [0.5, 0.0888229], [0.51,
  0.0841784], [0.52, 0.0797901], [0.53, 0.0756428], [0.54,
  0.0717221], [0.55, 0.0680146], [0.56, 0.0645079], [0.57,
  0.0611903], [0.58, 0.0580508], [0.59, 0.0550792], [0.6,
  0.0522658], [0.61, 0.0496018], [0.62, 0.0470786], [0.63,
  0.0446884], [0.64, 0.0424238], [0.65, 0.0402776], [0.66,
  0.0382435], [0.67, 0.0363153], [0.68, 0.034487], [0.69,
  0.0327534], [0.7, 0.0311093], [0.71, 0.0295497], [0.72,
  0.0280703], [0.73, 0.0266667], [0.74, 0.0253348], [0.75,
  0.0240708], [0.76, 0.0228712], [0.77, 0.0217326], [0.78,
  0.0206516], [0.79, 0.0196254], [0.8, 0.0186511], [0.81,
  0.0177259], [0.82, 0.0168473], [0.83, 0.0160129], [0.84,
  0.0152204], [0.85, 0.0144677], [0.86, 0.0137527], [0.87,
  0.0130734], [0.88, 0.0124281], [0.89, 0.011815], [0.9,
  0.0112324], [0.91, 0.0106789], [0.92, 0.0101529], [0.93,
  0.00965303], [0.94, 0.009178], [0.95, 0.00872655], [0.96,
  0.00829748], [0.97, 0.00788966], [0.98, 0.00750203], [0.99,
  0.00713358], [1., 0.00678335]])

# Plotting
x = data2[:, 0]
y = data2[:, 1]

plt.plot(x, y, marker='o')  # 'o' is for circle markers
plt.xlabel('X Axis')
plt.ylabel('Y Axis')
plt.title('Mix of two exponential decay')
plt.grid(True)
plt.show()
```




```
# your soluion here

def Regression(a,b):
    # do the linear regression for the specific values of a and b
    # for the data x and y created above.
    X = np.column_stack([
        np.ones(len(x)),  # Intercept term
        np.exp(-a * x),
        np.exp(-b * x)
    ])

    # Fit the model
    model = sm.OLS(y, X).fit()

    # Calculate R^2
    R2 = model.rsquared

    return R2  # return R2 of this linear regression

R2values2 = []; # create an empty list

for a in range(1, 11):
    for b in range(1, 11):
        R2values2.append((a, b, Regression(a, b)))
```

```
    # Do regression by calling GaussianReg(a,b)
    # append (a, b, R2) to the list R2values2

# R2values is a list; turn it into an array of 100x3.
R2values2 = np.array(R2values2)
#print(R2values2)

R2values2 = pd.DataFrame(R2values2)

R2values2.columns=["a", "b", "R_square"]

R2values2.sort_values(by="R_square").iloc[len(R2values2) - 1]

# Here R2 = 1 means the model overfitted. We need check for VIF
([multi]colinearity), or do some regularization

a             5.0
b            10.0
R_square      1.0
Name: 49, dtype: float64
```

Problem 4 - Housing dataset (20 pts)

- **Part (a)** Load the housing dataset. By using the `statsmodel` library, perform a linear regression of 'SalePrice' on the variable 'GarageArea'. The R^2 you will find will be less than 0.4.

- **Part (b)** Add some more variables from 'LotArea', 'YearBuilt', 'YearRemodAdd', 'BsmtUnfSF', 'TotalBsmtSF', '1stFlrSF', 'GarageArea', 'WoodDeckSF', 'OpenPorchSF', 'EnclosedPorch', '3SsnPorch' and redo the linear regression until the adjusted R^2 comes out larger than 0.5. (Here you're not adding all, just choose some until adjusted R^2>0.5.)

- **Part (c)** Perform a linear regression of 'SalePrice' on all of these variables at the same time (i.e., use all of them, not some of them) 'LotArea', 'YearBuilt', 'YearRemodAdd', 'BsmtUnfSF', 'TotalBsmtSF', '1stFlrSF', 'GarageArea', 'WoodDeckSF', 'OpenPorchSF', 'EnclosedPorch', '3SsnPorch'. Calculate adjusted R^2.

- **Part (d)** Look at the output table. Remove the variables that has no effect on 'SalePrice', and then redo the linear regression. Note down the variables you removed.

```
houses = pd.read_csv('../Assignment-4/house-prices/train.csv')

print("Number of duplicate records in
'houses':",houses.duplicated().sum()  )

quantitative = [f for f in houses.columns if houses.dtypes[f] !=
```

```
'object']
qualitative = [f for f in houses.columns if houses.dtypes[f] ==
'object']
#print("Quantitative columns :","\n",quantitative,"\n\n\
n","Qualitative columns :","\n", qualitative)
```

```
Number of duplicate records in 'houses': 0
```

```
# Part (a)

X = houses[['GarageArea']]
X = sm.add_constant(X)  # Add intercept terms
y = houses['SalePrice']

# Fit the model
model = sm.OLS(y, X).fit()

print(model.rsquared)
```

```
0.3886667590318186
```

```
# Part (b)

# I choose the next field with a method inspired from forward subset
selection
# I listed all possible subsets containing "GarageArea" and train the
model for each subset
# search for first result passes 0.5

fields = [
    'LotArea', 'YearBuilt', 'YearRemodAdd', 'BsmtUnfSF',
'TotalBsmtSF',
    '1stFlrSF', 'GarageArea', 'WoodDeckSF', 'OpenPorchSF',
'EnclosedPorch', '3SsnPorch'
]

element = "GarageArea"

subsets_with_element = [list(subset) for r in range(1, len(fields) +
1)
                        for subset in itertools.combinations(fields,
r)
                        if element in subset]

threshold = 0.5
for subset in subsets_with_element:

    X = houses[subset]
    X = sm.add_constant(X)  # Add intercept terms
    y = houses['SalePrice']
```

```
    # Fit the model
    model = sm.OLS(y, X).fit()
    if model.rsquared_adj > threshold:
        threshold = model.rsquared_adj
        print(subset, threshold)

        if threshold > 0.5:
            break

# Here the field that causing the R2 value to pass the threshold ->
TotalBsmtSF
```

```
['TotalBsmtSF', 'GarageArea'] 0.5140696278363066
```

```
# Part(c)


X = houses[fields]
X = sm.add_constant(X)  # Add intercept terms
y = houses['SalePrice']

# Fit the model
model = sm.OLS(y, X).fit()

print(model.summary())
```

```
                            OLS Regression Results

======================================================================
========
Dep. Variable:              SalePrice   R-squared:
0.629
Model:                            OLS   Adj. R-squared:
0.626
Method:                 Least Squares   F-statistic:
223.1
Date:                Wed, 13 Nov 2024   Prob (F-statistic):
3.42e-302
Time:                        21:43:36   Log-Likelihood:
-17820.
No. Observations:                1460   AIC:
3.566e+04
Df Residuals:                    1448   BIC:
3.573e+04
Df Model:                          11

Covariance Type:            nonrobust

======================================================================
===========
                    coef    std err          t      P>|t|      [0.025
```

```
0.975]
------------------------------------------------------------------------------
-----------
const          -2.282e+06   1.45e+05    -15.784      0.000   -2.57e+06
-2e+06
LotArea            0.6916      0.137      5.066      0.000       0.424
0.959
YearBuilt        381.3979     61.324      6.219      0.000     261.104
501.691
YearRemodAdd     791.4815     78.553     10.076      0.000     637.392
945.571
BsmtUnfSF         -5.7319      3.225     -1.777      0.076     -12.059
0.595
TotalBsmtSF       26.4899      5.564      4.761      0.000      15.576
37.404
1stFlrSF          44.5745      5.977      7.457      0.000      32.849
56.300
GarageArea        93.6271      7.605     12.311      0.000      78.709
108.546
WoodDeckSF        59.3462     10.825      5.482      0.000      38.112
80.581
OpenPorchSF      113.7501     20.254      5.616      0.000      74.019
153.481
EnclosedPorch     63.3492     22.779      2.781      0.005      18.666
108.032
3SsnPorch         23.2259     43.584      0.533      0.594     -62.269
108.721
======================================================================
========
Omnibus:                      548.054   Durbin-Watson:
1.951
Prob(Omnibus):                  0.000   Jarque-Bera (JB):
22358.817
Skew:                           1.030   Prob(JB):
0.00
Kurtosis:                      22.060   Cond. No.
1.67e+06
======================================================================
========

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is
correctly specified.
[2] The condition number is large, 1.67e+06. This might indicate that
there are
strong multicollinearity or other numerical problems.

# Part(d)

removed_fields = [
```

```
    "3SsnPorch", # t-value = 0.533 ( bkz. p value also )
    "BsmtUnfSF", # t-value was -1.7 ( bkz. p value also )
]

for i in removed_fields:
    if i in fields:
        fields.remove(i)

X = houses[fields]
X = sm.add_constant(X)  # Add intercept terms
y = houses['SalePrice']

# Fit the model
model = sm.OLS(y, X).fit()

print(model.summary())
```

```
                            OLS Regression Results

======================================================================
========
Dep. Variable:              SalePrice   R-squared:
0.628
Model:                            OLS   Adj. R-squared:
0.626
Method:                 Least Squares   F-statistic:
272.0
Date:                Wed, 13 Nov 2024   Prob (F-statistic):
6.95e-304
Time:                        21:43:40   Log-Likelihood:
-17822.
No. Observations:                1460   AIC:
3.566e+04
Df Residuals:                    1450   BIC:
3.572e+04
Df Model:                           9

Covariance Type:            nonrobust

======================================================================
===========
                    coef    std err          t      P>|t|      [0.025
0.975]
----------------------------------------------------------------------
-----------
const          -2.27e+06   1.44e+05    -15.727      0.000   -2.55e+06
-1.99e+06
LotArea           0.7187      0.136      5.294      0.000       0.452
0.985
YearBuilt       386.6061     61.278      6.309      0.000     266.402
```

```
506.810
YearRemodAdd    779.5071     78.168      9.972      0.000     626.173
932.841
TotalBsmtSF      23.6251      5.335      4.428      0.000      13.160
34.090
1stFlrSF         44.9475      5.973      7.525      0.000      33.231
56.664
GarageArea       93.7656      7.609     12.324      0.000      78.841
108.691
WoodDeckSF       60.9762     10.757      5.669      0.000      39.876
82.077
OpenPorchSF     112.6244     20.252      5.561      0.000      72.899
152.350
EnclosedPorch    61.8896     22.769      2.718      0.007      17.226
106.553
======================================================================
========
Omnibus:                      560.213   Durbin-Watson:
1.954
Prob(Omnibus):                  0.000   Jarque-Bera (JB):
20870.469
Skew:                           1.097   Prob(JB):
0.00
Kurtosis:                      21.392   Cond. No.
1.67e+06
======================================================================
========

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is
correctly specified.
[2] The condition number is large, 1.67e+06. This might indicate that
there are
strong multicollinearity or other numerical problems.
```

Problem 5 - More Housing data (20 pts)

- **Part (a)** Create a new dataframe `houses2` with the columns 'LotArea', 'YearBuilt', 'YearRemodAdd', 'TotalBsmtSF', '1stFlrSF', 'GarageArea', 'WoodDeckSF', 'OpenPorchSF', 'EnclosedPorch' and 'SalePrice' from the `houses` dataframe.
- **Part (b)** Check the correlation matrix with `houses2.corr()` and identify the pair that has the maximum correlation; let's call these featureA and featureB.
- **Part (c)** A linear regression model may suffer from collinearity. Perform linear regression for SalePrice by using all features except featureA. Perform linear regression again by using all features except featureB. In both cases, keep all other variables included in the model.
- **Part (d)** Now you've three models with: (i) All variables, (ii) All varibles except featureA and (iii) All varibles except featureB. Compare the adjusted R^2's and report which one is the best performing model.

```python
# Part (a)

fields = [
    'LotArea', 'YearBuilt', 'YearRemodAdd', 'TotalBsmtSF',
    '1stFlrSF', 'GarageArea', 'WoodDeckSF', 'OpenPorchSF',
    'EnclosedPorch', 'SalePrice']

houses2 = houses[fields]

# Part (b)

max = houses2.corr()["LotArea"]["YearBuilt"]

# Finds max value in the corrolation matrix and corresponding features
for i in range(len(fields)):
    for j in range(i):
        if houses2.corr()[fields[i]][fields[j]] > max:
            max, featureA, featureB = houses2.corr()[fields[i]]
[fields[j]], fields[i], fields[j]

print(max, featureA, featureB)
```

```
0.8195299750050339 1stFlrSF TotalBsmtSF
```

```python
# Linear Regression with all fields

X = houses2[[
    'LotArea', 'YearBuilt', 'YearRemodAdd', 'TotalBsmtSF',
    '1stFlrSF', 'GarageArea', 'WoodDeckSF', 'OpenPorchSF',
    'EnclosedPorch']]
X = sm.add_constant(X)  # Add intercept terms
y = houses2['SalePrice']

# Fit the model
model = sm.OLS(y, X).fit()

print(model.summary())
```

```
                            OLS Regression Results

======================================================================
========
Dep. Variable:              SalePrice   R-squared:
0.628
Model:                            OLS   Adj. R-squared:
0.626
Method:                 Least Squares   F-statistic:
272.0
Date:                Wed, 13 Nov 2024   Prob (F-statistic):
6.95e-304
Time:                        21:46:34   Log-Likelihood:
```

```
-17822.
No. Observations:                1460   AIC:
3.566e+04
Df Residuals:                    1450   BIC:
3.572e+04
Df Model:                           9

Covariance Type:            nonrobust

======================================================================
===========
                    coef    std err          t      P>|t|      [0.025
0.975]
----------------------------------------------------------------------
-----------
const          -2.27e+06   1.44e+05    -15.727      0.000   -2.55e+06
-1.99e+06
LotArea           0.7187      0.136      5.294      0.000       0.452
0.985
YearBuilt       386.6061     61.278      6.309      0.000     266.402
506.810
YearRemodAdd    779.5071     78.168      9.972      0.000     626.173
932.841
TotalBsmtSF      23.6251      5.335      4.428      0.000      13.160
34.090
1stFlrSF         44.9475      5.973      7.525      0.000      33.231
56.664
GarageArea       93.7656      7.609     12.324      0.000      78.841
108.691
WoodDeckSF       60.9762     10.757      5.669      0.000      39.876
82.077
OpenPorchSF     112.6244     20.252      5.561      0.000      72.899
152.350
EnclosedPorch    61.8896     22.769      2.718      0.007      17.226
106.553
======================================================================
========
Omnibus:                      560.213   Durbin-Watson:
1.954
Prob(Omnibus):                  0.000   Jarque-Bera (JB):
20870.469
Skew:                           1.097   Prob(JB):
0.00
Kurtosis:                      21.392   Cond. No.
1.67e+06
======================================================================
========

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is
```

```
correctly specified.
[2] The condition number is large, 1.67e+06. This might indicate that
there are
strong multicollinearity or other numerical problems.

# Part(c)

fields1 = [
    'LotArea', 'YearBuilt', 'YearRemodAdd', 'TotalBsmtSF',
    'GarageArea', 'WoodDeckSF', 'OpenPorchSF',
    'EnclosedPorch'
]

fields2 = [
    'LotArea', 'YearBuilt', 'YearRemodAdd',
    '1stFlrSF', 'GarageArea', 'WoodDeckSF', 'OpenPorchSF',
    'EnclosedPorch']

# Linear Regression without "1stFlrSF" column

X1 = houses2[fields1]
X1 = sm.add_constant(X1)  # Add intercept terms
y1 = houses2['SalePrice']

# Fit the model
model1 = sm.OLS(y1, X1).fit()

print(model1.summary())

                            OLS Regression Results

======================================================================
========
Dep. Variable:              SalePrice   R-squared:
0.614
Model:                            OLS   Adj. R-squared:
0.611
Method:                 Least Squares   F-statistic:
287.9
Date:                Wed, 13 Nov 2024   Prob (F-statistic):
4.23e-293
Time:                        21:46:39   Log-Likelihood:
-17850.
No. Observations:                1460   AIC:
3.572e+04
Df Residuals:                    1451   BIC:
3.577e+04
Df Model:                           8

Covariance Type:            nonrobust
```

```
==============================================================================
===========
                    coef    std err          t      P>|t|      [0.025
0.975]
------------------------------------------------------------------------------
-----------
const          -2.17e+06   1.46e+05    -14.819      0.000   -2.46e+06
-1.88e+06
LotArea           0.8425      0.137      6.135      0.000       0.573
1.112
YearBuilt       324.8668     61.880      5.250      0.000     203.483
446.250
YearRemodAdd    797.3282     79.616     10.015      0.000     641.154
953.502
TotalBsmtSF      53.7066      3.600     14.917      0.000      46.644
60.769
GarageArea      105.4107      7.591     13.886      0.000      90.520
120.301
WoodDeckSF       65.6750     10.943      6.002      0.000      44.210
87.140
OpenPorchSF     111.5796     20.636      5.407      0.000      71.100
152.059
EnclosedPorch    59.7352     23.200      2.575      0.010      14.227
105.243
==============================================================================
========
Omnibus:                      580.350   Durbin-Watson:
1.954
Prob(Omnibus):                  0.000   Jarque-Bera (JB):
20929.066
Skew:                           1.172   Prob(JB):
0.00
Kurtosis:                      21.400   Cond. No.
1.66e+06
==============================================================================
========

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is
correctly specified.
[2] The condition number is large, 1.66e+06. This might indicate that
there are
strong multicollinearity or other numerical problems.

# Linear Regression without "TotalBsmtSF" column

X2 = houses2[fields2]
X2 = sm.add_constant(X2)  # Add intercept terms
y2 = houses2['SalePrice']
```

```python
# Fit the model
model2 = sm.OLS(y2, X2).fit()

print(model2.summary())
```

```
                            OLS Regression Results

==============================================================================
========
Dep. Variable:              SalePrice   R-squared:
0.623
Model:                            OLS   Adj. R-squared:
0.621
Method:                 Least Squares   F-statistic:
299.8
Date:                Wed, 13 Nov 2024   Prob (F-statistic):
6.42e-301
Time:                        21:46:41   Log-Likelihood:
-17832.
No. Observations:                1460   AIC:
3.568e+04
Df Residuals:                    1451   BIC:
3.573e+04
Df Model:                           8

Covariance Type:            nonrobust

==============================================================================
===========
                    coef    std err          t      P>|t|      [0.025
0.975]
------------------------------------------------------------------------------
-----------
const          -2.378e+06   1.43e+05    -16.606      0.000   -2.66e+06
-2.1e+06
LotArea            0.7458      0.136      5.464      0.000       0.478
1.014
YearBuilt        445.8174     60.184      7.408      0.000     327.760
563.874
YearRemodAdd     775.1645     78.661      9.854      0.000     620.862
929.467
1stFlrSF          64.7652      3.981     16.269      0.000      56.956
72.574
GarageArea        95.1595      7.651     12.438      0.000      80.152
110.167
WoodDeckSF        61.8388     10.824      5.713      0.000      40.607
83.071
OpenPorchSF      120.9227     20.294      5.959      0.000      81.114
160.731
EnclosedPorch     66.6654     22.889      2.913      0.004      21.767
```

```
111.564
==============================================================================
========
Omnibus:                      597.405   Durbin-Watson:
1.961
Prob(Omnibus):                  0.000   Jarque-Bera (JB):
16512.815
Skew:                           1.306   Prob(JB):
0.00
Kurtosis:                      19.267   Cond. No.
1.64e+06
==============================================================================
========

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is
correctly specified.
[2] The condition number is large, 1.64e+06. This might indicate that
there are
strong multicollinearity or other numerical problems.
```

```
# Part (d)

# In terms of R2, both field added version has better performance
```

Problem 6 - Even More Housing Data (20 pts)

- **Part (a)** Perform two linear regressions, first as **SalePrice** ~ **YearBuilt** and then **SalePrice** ~ **YearBuilt** + **YearBuilt**^{2} and create residuals plots for both case.
- **Part (b)** Which model do you prefer? Discuss from the perspective of both adjusted R^2 and residual plots.
- **Part (c)** Make a scatterplot for the data with the axes SalePrice (y-axis) vs YearBuilt (x-axis). On the same plot, plot the model polynomial model (a curve) **SalePrice** ~ **YearBuilt** + **YearBuilt**^{2} so that you can see how well this curve approximates the data.

```
# Part (a)


### 1
X1 = houses2[['YearBuilt']]
X1 = sm.add_constant(X1)  # Add intercept terms
y1 = houses2['SalePrice']

# Fit the model
model1 = sm.OLS(y1, X1).fit()

print(model1.summary())

# Predicted values
X_pred1 = model1.predict(X1)
```

```
residuals1 = y1 - X_pred1


### 2
houses2["year_squared"] = houses2[['YearBuilt']] ** 2
X2 = houses2[['YearBuilt']] + houses2["year_squared"]
X2 = sm.add_constant(houses2[['YearBuilt', 'year_squared']])  # Add
intercept terms
y2 = houses2['SalePrice']

# Fit the model
model2 = sm.OLS(y2, X2).fit()

print(model2.summary())

# Predicted values
X_pred2 = model2.predict(X2)
residuals2 = y2 - X_pred2
```

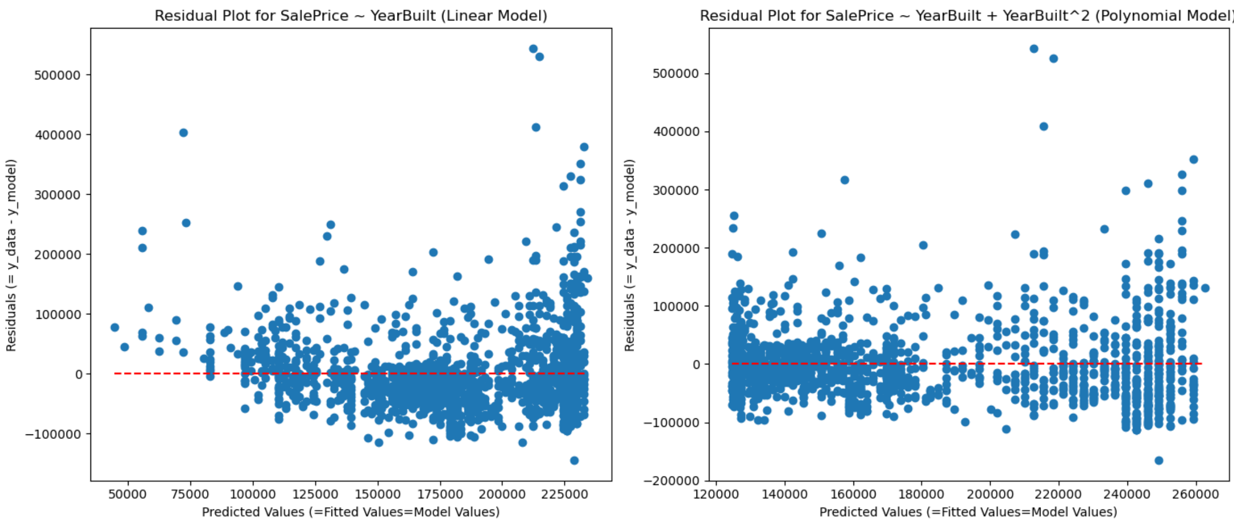
```
                            OLS Regression Results

======================================================================
========
Dep. Variable:              SalePrice   R-squared:
0.273
Model:                            OLS   Adj. R-squared:
0.273
Method:                 Least Squares   F-statistic:
548.7
Date:                Wed, 13 Nov 2024   Prob (F-statistic):
2.99e-103
Time:                        21:46:53   Log-Likelihood:
-18311.
No. Observations:                1460   AIC:
3.663e+04
Df Residuals:                    1458   BIC:
3.664e+04
Df Model:                           1

Covariance Type:            nonrobust

======================================================================
========
                 coef    std err          t      P>|t|      [0.025
0.975]
----------------------------------------------------------------------
--------
const       -2.53e+06   1.16e+05    -21.858      0.000   -2.76e+06
-2.3e+06
YearBuilt   1375.3735     58.717     23.424      0.000    1260.194
```

```
1490.553
==============================================================================
========
Omnibus:                      761.903   Durbin-Watson:
1.984
Prob(Omnibus):                  0.000   Jarque-Bera (JB):
6856.947
Skew:                           2.264   Prob(JB):
0.00
Kurtosis:                      12.603   Cond. No.
1.29e+05
==============================================================================
========

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is
correctly specified.
[2] The condition number is large, 1.29e+05. This might indicate that
there are
strong multicollinearity or other numerical problems.
                            OLS Regression Results

==============================================================================
========
Dep. Variable:              SalePrice   R-squared:
0.348
Model:                            OLS   Adj. R-squared:
0.348
Method:                 Least Squares   F-statistic:
389.6
Date:                Wed, 13 Nov 2024   Prob (F-statistic):
2.98e-136
Time:                        21:46:53   Log-Likelihood:
-18231.
No. Observations:                1460   AIC:
3.647e+04
Df Residuals:                    1457   BIC:
3.648e+04
Df Model:                           2

Covariance Type:            nonrobust

==============================================================================
==========
                   coef    std err          t      P>|t|      [0.025
0.975]
------------------------------------------------------------------------------
----------
const         8.191e+07   6.52e+06     12.561      0.000    6.91e+07
9.47e+07
```

```
YearBuilt    -8.473e+04   6648.816    -12.743      0.000   -9.78e+04
-7.17e+04
year_squared    21.9421      1.694     12.950      0.000      18.619
25.266
======================================================================
========
Omnibus:                       758.973   Durbin-Watson:
1.983
Prob(Omnibus):                   0.000   Jarque-Bera (JB):
7721.057
Skew:                            2.204   Prob(JB):
0.00
Kurtosis:                       13.368   Cond. No.
1.51e+10
======================================================================
========

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is
correctly specified.
[2] The condition number is large, 1.51e+10. This might indicate that
there are
strong multicollinearity or other numerical problems.

/var/folders/ld/m60xnjf52cndhf07bcb4br040000gn/T/
ipykernel_34034/3873165136.py:20: SettingWithCopyWarning:
A value is trying to be set on a copy of a slice from a DataFrame.
Try using .loc[row_indexer,col_indexer] = value instead

See the caveats in the documentation:
https://pandas.pydata.org/pandas-docs/stable/user_guide/indexing.html#
returning-a-view-versus-a-copy
  houses2["year_squared"] = houses2[['YearBuilt']] ** 2
```

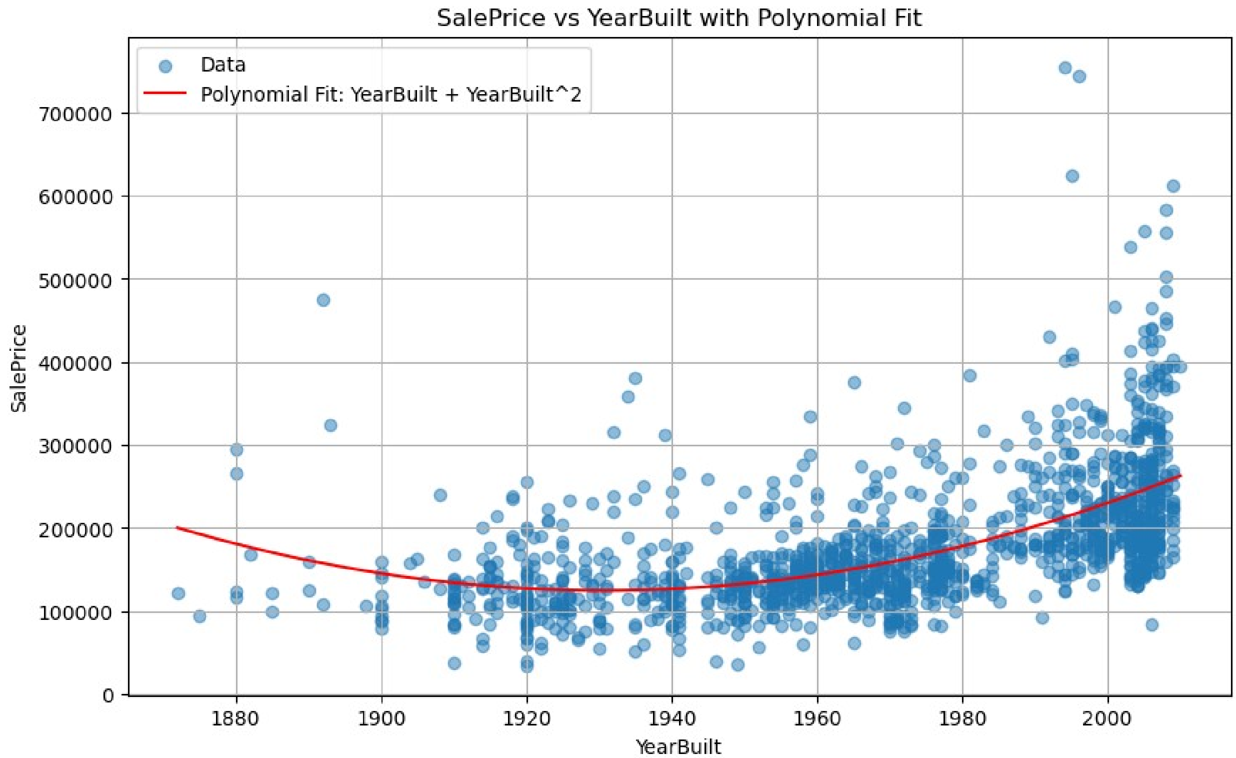
```python
# Plot residuals for both models
plt.figure(figsize=(14, 6))

# Create a residual plot
plt.subplot(1, 2, 1)
plt.scatter(X_pred1, residuals1)
plt.hlines(y=0, xmin=X_pred1.min(), xmax=X_pred1.max(), colors='red',
linestyles='--')
plt.xlabel('Predicted Values (=Fitted Values=Model Values)')
plt.ylabel('Residuals (= y_data - y_model)')
plt.title('Residual Plot for SalePrice ~ YearBuilt (Linear Model)')

# Create a residual plot
plt.subplot(1, 2, 2)
plt.scatter(X_pred2, residuals2)
plt.hlines(y=0, xmin=X_pred2.min(), xmax=X_pred2.max(), colors='red',
```

```
linestyles='--')
plt.xlabel('Predicted Values (=Fitted Values=Model Values)')
plt.ylabel('Residuals (= y_data - y_model)')
plt.title('Residual Plot for SalePrice ~ YearBuilt + YearBuilt^2
(Polynomial Model)')

plt.tight_layout()
plt.show()
```




```
## Part (b)

# In terms of R2_adj model 2 has better. Also it is easier to see the
data on the 2. plot where data point are distinguished better

# Part (c)

plt.figure(figsize=(10, 6))

# Scatterplot of data
plt.scatter(houses2['YearBuilt'], houses2['SalePrice'], alpha=0.5,
label='Data')

# Plotting the polynomial model
year_range = np.linspace(houses2['YearBuilt'].min(),
houses2['YearBuilt'].max(), 200)
year_range_squared = year_range ** 2
X_poly =  np.column_stack([
                            np.ones(len(year_range)),
                            year_range,
                            year_range_squared
                            ])

y_pred = model2.predict(X_poly)
```

```
plt.plot(year_range, y_pred, color='red', label='Polynomial Fit:
YearBuilt + YearBuilt^2')
plt.xlabel('YearBuilt')
plt.ylabel('SalePrice')
plt.title('SalePrice vs YearBuilt with Polynomial Fit')
plt.legend()
plt.grid(True)
plt.show()
```



Problem 7 - Small datasets and R^2 (20 pts)

Let's assume `IQ` (target) does not depend on `Shoe Size` (predictor) at all. To simulate this situation, we can create a sample dataset where the two columns have no correlation, i.e., two columns are random numbers.

Part (a) Define a function `fnc(num)` that does the following.

1. It takes the argument 'num' and create 'num' number of random x and y values between [1,100] with np.random.uniform(). For example, if num=100, it will create 100 random numbers for x and also another 100 random numbers for y.
2. It then puts these two uncorrelated x and y values on a dataframe with `df = pd.DataFrame(np.column_stack((x_values, y_values)), columns=['Shoe Size','IQ'])`
3. Then it runs a linear regression by using the `statsmodels` package, and finally returns the R^2 value of the model with the last line `return model.rsquared`

Define this function and try `fnc(100)`; you should be getting a very small R^2 value because we're creating a linear regression model on a completely random data without any trend between `Shoe Size` and `IQ`. Also try fnc(2); this time you should be getting $R^2=1$ or very close to 1.

```
def fnc(num):

    # Generate random x and y values between 0 and 100
    x_s = np.random.uniform(0, 100, num)
    y_s = np.random.uniform(0, 100, num)

    # x and y values into pairs
    df = pd.DataFrame(np.column_stack((
        x_s,
        y_s
    )), columns=['Shoe Size', 'IQ'])

    # linear regression
    X = np.column_stack([
        np.ones(len(df['Shoe Size'])),
        df['Shoe Size']
    ])
    model = sm.OLS(df['IQ'], X).fit()
    return model.rsquared

# try the model
print(fnc(100))
print(fnc(2))
```

```
0.0003008539909610253
1.0
```

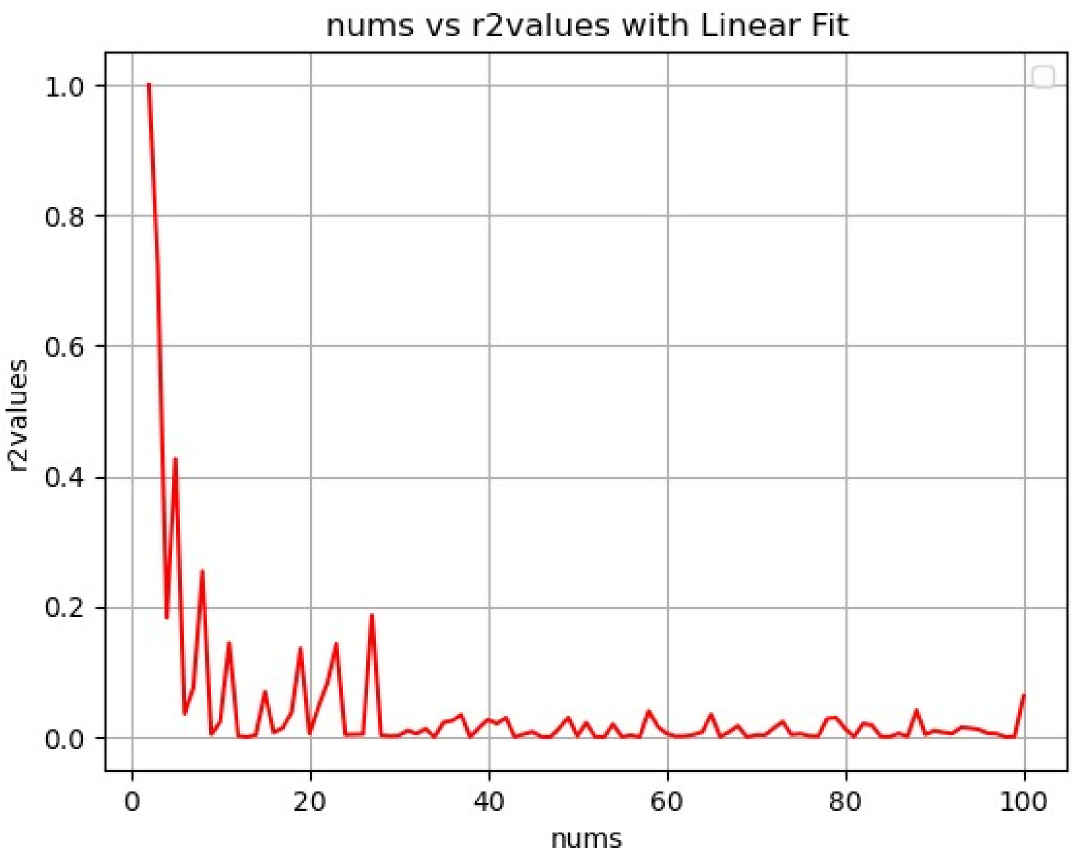
Part (b)

- Create an empty list `R2values = []`
- Create a for loop where the fnc(num) will be called for num=2,3,..,100: `for num in range(2,101):`
- In each iteration, append the calculated pairs (num,fnc(num)) into `R2values`.
- Convert the list `R2values` into numpy array.
- Plot num (x-axis) vs R2values (y-axis). You should be getting R^2=1 for num=2, and it should decrease down to zero (or very small number) as num increases.
- Write a sentence or two as a cautionary piece of advice for your grandchildren regarding the pitfalls of modeling with very small datasets.

```
r2values = []
for num in range(2,101):
    r2values.append((num, fnc(num)))

r2values = np.array(r2values)
```

```
plt.plot(r2values[:, 0], r2values[:, 1], color='red')
plt.xlabel('nums')
plt.ylabel('r2values')
plt.title('nums vs r2values with Linear Fit')
plt.legend()
plt.grid(True)
plt.show()
```

```
No artists with labels found to put in legend.  Note that artists
whose label start with an underscore are ignored when legend() is
called with no argument.
```




```
# My dear grandy boy, please do not have concise predictions about
situations with a small amount of data. Do not be judgy!
# especially with less data at hand. Conditions can be fit well on
small data sets. Be patient wait for the right moment for your
decisions
```