MATH 234 - HW1.

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(1) Given P and Q are distinct partitions of $[a,b]$ (bounded)

$a = a_0 \leq a_1 \dots \leq a_n = b$

$a = a_0 \leq a_1 \dots \leq a_m = b$

Set $M_i = \sup f([a_i, a_{i+1}])$, $M_J = \sup f([a_J, a_{J+1}])$ and $\bar{S}_P f = \sum M_i [a_{i+1} - a_i]$, $\bar{S}_Q = \sum M_J . [a_{J+1} - a_J]$.

Also we have $P \cup Q$ is a refinement of $[a,b]$, considering the partitioning of P and Q. we get

$$\left.\begin{array}{l} \bullet\ \bar{S}_P f \geq \bar{S}_{P \cup Q} f \\ \bullet\ \bar{S}_Q f \geq \bar{S}_{P \cup Q} f \end{array}\right\} \Rightarrow 2 \bar{S}_{P \cup Q} f \leq \bar{S}_P f + \bar{S}_Q f$$

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Ex $f(x) = x^2$ on $[0,6]$, $P = [0,3,6]$

$Q = [0,2,4,6]$

$P \cup Q = [0,1,2,3,4,5,6]$

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(2) let f be integrable on $[a,b]$. (Construction of P, m_i, M_i, $\bar{S}_P f$ and $\underline{S}_P f$ are the same as above Q.) take $h = c.f \Rightarrow$ we have $c m_i$ and $c M_i$

So, $\bar{S}_P h = \sum c M_i (x_{i+1} - x_i)$, $\underline{S}_P h = \sum c . m_i (x_{i+1} - x_i)$. Since for a given ε,

$\bar{S}_P f - \underline{S}_P f < \varepsilon \Rightarrow \bar{S}_P h - \underline{S}_P h < c . \varepsilon \Rightarrow c.f$ intble.

This also shows $\int_a^b c.f = c . \int_a^b f.$

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(3) Given P and ε. we have $\bar{S}_P f - \underline{S}_P f < \varepsilon$.
$\int$ on $[a,b]$.

Let P' be the refinement of P considering the partitioning on $[c,d]$. So,

$\underline{S}_{P'} f \geq \underline{S}_P f$ and $\bar{S}_{P'} f \leq \bar{S}_P f$. By this we have $\bar{S}_{P'} - \underline{S}_P f < \varepsilon$, Just omitting the subintervals $[a,c]$ and $[d,b]$, say Q, we obtain;

$\bar{S}_Q f - \underline{S}_Q f < \varepsilon \Rightarrow$

f is intble. on $[c,d]$.

□