(2)

$$\left\{ \text{defn: } \lim_{|h|\to 0} \frac{f(x+h_1, y+h_2) - f(x,y)}{|h|} \right\}$$

a) $f(x,y) = \sin(x,y)$ at $(0,0)$.

$$\lim_{(x,y)\to(0,0)} \frac{\sin(x+h_1, y+h_2) - \sin(0,0)}{\sqrt{x^2+y^2}} = \lim_{\substack{h_1\to 0\\ h_2\to 0}} \frac{\nabla f(x,y)\cdot h + E(h)}{|h|} \overset{!}{=} 0$$

where $\frac{E(h)}{|h|} \xrightarrow[|h|\to 0]{} 0$

$$= \frac{\cos(x)\cdot h_1 + \cos(y)\cdot h_2 + E(h)}{|h|} \xrightarrow[|h|\to 0]{} 0$$

and $\nabla f = (\cos x, \cos y)$

(b) $g(x,y,z) = x^3 + xy^2 + yz$ at $(2,3,1)$ (:))

$$\lim_{(x,y,z)\to(2,3,1)} \frac{g(x+h_1, y+h_2, z+h_3) - g(2,3,1)}{\sqrt{(x-2)^2+(y-3)^2+(z-1)^2}} = \lim_{|h|\to 0} \frac{\nabla g(x,y,z)\cdot h + E(h)}{|h|}$$

where $\frac{E(h)}{|h|} \xrightarrow[|h|\to 0]{} 0$

$$= \lim_{|h|\to 0} \frac{(3x^2+y^2)h_1 + (2xy+z)h_2 + y h_3 + E(h)}{|h|} \xrightarrow[|h|\to 0]{} 0$$

[here $|h|\to 0$ means $(h_1, h_2, h_2) \to (0,0,0)$ and $h_1 = x-2$
$h_2 = y-3$
$h_3 = z-1$]

$\partial_1 g = 3x^2+y^2$, $\partial_2 g = 2xy+z$, $\partial_3 g = y$.

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HW-7

(1) Let $0 < x_1 < x_2$ with $J_0(x_1) = J_0(x_2) = 0$ by Rolle's thm there exists $a \in (x_1, x_2)$ s.t. $J_0'(a) = 0$ since Bessel Functions are cont. and diffble. Define $f(x) = x^{-n} . J_0(x)$. For $f(x)$, continuity and differentiability are inherited by Bessel function and x^n. Also we have $f(x_1) = 0 = f(x_2)$ again by Rolle's thm, $\exists a \in (x_1, x_2)$ s.t. $f'(a) = 0 \Rightarrow f'(a) = -x^{-1} J_1(a) = 0 \Rightarrow J_1(a) = 0$. We found a root for $J_1(x)$ between the roots of $J_0(x)$.

Similarly, $f(x) = x^n . J_1(x)$ and $0 < a < b$ with $J_1(a) = 0\, J_1(b)$. We know f is cont and diffble and $f(a) = f(b) = 0$ by Rolle's thm we get $c \in (a, b)$ with $f'(c) = 0$ but $(c^n . J_1(c))' = c^n . J_0(c) = 0 \Rightarrow J_0(c) = 0$. So c is a root for $J_0(x)$.

Hence we found that a root exists for J_0 in between the roots of J_1, and vice versa. $\square$

~~(2) (a) Since $f(x,y) = \sin(x+y)$ has partial derivatives $\partial_x f = \cos x$, $\partial_y f = \cos y$ and $\partial_x f, \partial_y f$ are continous on $(0,0)$ $\Rightarrow$ f is diffble on $(0,0)$~~

~~(b) $g(x,y,z) = x^3 + xy^2 + yz$, has partial derivatives;~~

~~$$\left.\begin{aligned} \partial_x g &= 3x^2 + y^2 \\ \partial_y g &= 2xy + z \\ \partial_z g &= y \end{aligned}\right\}$$ are all continous, combination of continous functions on $(2,3,1)$. Then we have $g(x,y,z)$ is diffble on $(2,3,1)$.~~

(3-) We want to show; $|f(\vec{x}) - f(\vec{a})| < \varepsilon$ whenever $|\vec{x} - \vec{a}| < \delta$. We have all $\partial_J f$ exist and bounded; say by B'. Also S is open $\Rightarrow B(x, \vec{r}) < r'$. Using differentiability; (MVT) $|f(x_J) - f(a_J)| = |\partial_J f| . |x_J - x_a| < B' . r'$. Restricting $|x_J - a_J|$ has bound r' say $\frac{\varepsilon}{B'n}$ $\Rightarrow$ $|f(x_J) - f(a_J)| = |\partial_J f| . |x_J - x_a| < B' . \frac{\varepsilon}{B'n} < \frac{\varepsilon}{n}$ $\square$