$P(A)$

5-) Let $P(i)$ be the probability that finding the best candidate in the $i$-th trial.

We are asked to find;

$P$( $i$-th candidate is the best | $i$-th candidate has got the job).

$$P(1) = \frac{1}{N} \quad , \quad P(2) = 1 - \left(\left(1-\frac{1}{N}\right)\left(1-\frac{1}{N-1}\right)\right)$$

$$P(i) = 1 - \left(\left(1-\frac{1}{N}\right) \cdots \left(1-\frac{1}{N-i+1}\right)\right)$$

$$= 1 - \left(\frac{N-i}{N}\right) = \frac{i}{N}$$



GIPTA

8-) In a group of $n$ people, we have $\binom{n}{k}$ distinct groups of size $k$. The probability for $k$ people to have the same birthday is equal to $\left(\frac{1}{365}\right)^{k-1}$.
As a result, we have $\binom{n}{k}$ different group of size $k$.
And all have the same probability $\left(\frac{1}{365}\right)^{k-1}$
So the expected number of groups
is $\frac{\binom{n}{k}}{365^{k-1}}$ as $E(X) = X \cdot p$ by linearity.

10-) let $X = k$, for Heads $P(X=k) = p^k$ and for Tails $P(X=k) = (1-p)^k$; So $P(X=k) = p^k + (1-p)^k$.

$$* \; E(X) = \sum_{k=1}^{\infty} k\left(p^k + (1-p)^k\right) = \sum_{k=1}^{\infty} k \cdot p^k + \sum_{k=1}^{\infty} k(1-p)^k$$

6-) we are asked to find $P\left(X_1=1 \mid \sum_{i=1}^{n} X_i = k\right)$ where $k \in \{1,2,\ldots n\}$

$$\frac{P\left(X_1=1 \text{ and } \sum_{i=1}^{n} X_i = k\right)}{P\left(\sum_{i=1}^{n} X_i = k\right)} = \frac{P\left(\sum_{i=2}^{n} X_i = k-1\right)}{P\left(\sum_{i=1}^{n} X_i = k\right)}$$

$$= \frac{\frac{(n-1)!}{(n-k)!\,(k-1)!} \cdot p^{k-1} \cdot (1-p)^{n-k}}{\frac{n!}{(n-k)!\,k!}\; p^{k}\, (1-p)^{n-k}} = \frac{1}{p} \cdot \frac{k}{n} \quad \text{----}$$

8-) ... we have $\binom{n}{k}$ distinct

4-) Let,

- $C_1$ be the chosen customer is high-risk $\Rightarrow P(C_1) = 2/10$
- $C_2$ " " " " " medium risk $\Rightarrow P(C_2) = 3/10$
- $C_3$ " " " " " lower-risk $\Rightarrow P(C_3) = 5/10$

And let A be the probability that a customer has at least one accident in the current year.

a) $P(A) = \sum_{k=1}^{3} P(A|C_k)\,P(C_k)$

$$= P(A|C_1)P(C_1) + P(A|C_2)P(C_2) + P(A|C_3)P(C_3)$$

$$= 0{,}25 \times 0{,}2 + 0{,}16 \times 0{,}3 + 0{,}1 \times 0{,}5$$

$$= 0{,}148$$

b) $P(C_1|A) = \dfrac{P(A|C_1)\,P(C_1)}{P(A)} = \dfrac{0{,}05}{0{,}148} = \dfrac{25}{74}$

3-) Let $B = \{$all possible distributions of 12 calls for 7 days$\}$

$|B| = 12^7$

Now, give each day a call randomly. Then we have 5 calls to distribute. Let $A = \{$possible dist. of 5 calls for 7 days$\}$

$|A| = 5^7$

The result follows as $\frac{|A|}{|B|} = \frac{5^7}{12^7}$

4-) Let,

- $C_1$ be the chosen customer is high-risk $\Rightarrow P(C_1) = 2/10$
- $C_2$ " " " " medium risk $\Rightarrow P(C_2) = 3/10$
- $C_3$ " " " " lower-risk $\Rightarrow P(C_3) = 5/10$

2-) a) Let $P$ be the probability of the case where no match occurs at $i^{th}$ roll. So $P = \frac{n-1}{n}$ where we have $n$ sided die. So, for $n$ trial, the probability to get unmatched result is $\left(\frac{n-1}{n}\right)^n$. To get at least one match substract the result from 1, $1 - \left(\frac{n-1}{n}\right)^n$ as desired.

b) let $f(n) = 1 - \left(\frac{n-1}{n}\right)^n$, we are asked to solve the following limit,

$$\lim_{n\to\infty} f(n) = \lim_{n\to\infty} 1 - \left(\frac{n-1}{n}\right)^n$$

$$= 1 - \lim_{n\to\infty} \left(1 - \frac{1}{n}\right)^n$$

$$= 1 - \frac{1}{e}$$

Hw1

1 – $P(A) = 1/3$ , $P(B) = 1/2$ ; $P(A^c \cap B) = ?$

(i) if A and B are disjoint, then $P(A^c \cap B) = P(B) - \underbrace{P(A \cap B)}_{0} = P(B) = 1/2$

(ii) If $A \subset B$, $P(B \cap A^c) = P(B) - P(A \cap B) = P(B) - P(A) = 1/2 - \frac{1}{3} = 1/6$

(iii) if $P(A \cap B) = 1/8$ , then $P(B \cap A^c) = P(B) - P(A \cap B) = 1/2 - \frac{1}{8} = 3/8$

2-) a) Let P be the probability of the case where no match occurs at $i^{th}$ roll. So $P = \frac{n-1}{n}$ where we have n sided die. So, for n trial, the probability to get unmatched ... $(n-1)^n$ ... To get at least one match substract