MATH231 - HW 11

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1) $f: \overset{\text{open}}{A} \subset \mathbb{R}^n \to \mathbb{R}^n$, f is C^1 with $Jf(a) \neq 0$ $\forall a \in A$.

By InvFT, f^{-1} exists and $B \subset \mathbb{R}^n$ in which f^{-1} is cont. Assume $f(A) \subset B$ is not open, therefore for some $b \in f(A)$ we have $B(b,\varepsilon) \not\subset f(A)$ $\forall \varepsilon > 0$. Since f is invertible we can get a with f^{-1} s.t. $f(a) = b$ and since A is open $\exists \delta > 0$ s.t. $B(a,\delta) \subset A$. Since f^{-1} is continuous and $B(a,\delta)$ is open $\Rightarrow$ $f(B(a,\delta))$ is open in $f(A)$; this is a contradiction being $B(b,\varepsilon) \not\subset f(A)$ $\forall \varepsilon > 0$

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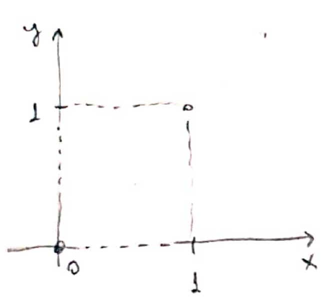
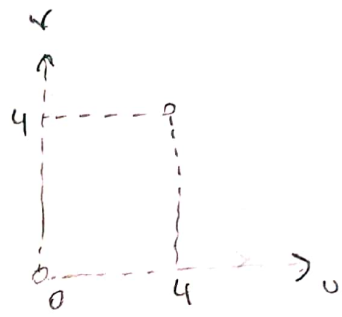
2) $g: \mathbb{R}^2 \to \mathbb{R}^2$, $g(x,y) = (\overset{u}{(x+y)^2}, \overset{v}{2(x+y)})$.



$$\left.\begin{aligned} u(x,y) &= (x+y)^2 \\ (0,1)\ (0,1) &\quad (0,4) \\ v(x,y) &= 2(x+y) \\ (0,1)\ (0,1) &\quad (0,4) \end{aligned}\right\} \Rightarrow$$



$$Jg(u_0,v_0) = \begin{vmatrix} 2x+2y & 2y+2x \\ 2 & 2 \end{vmatrix} = 0.$$

Nothing much to say. I can not say if it is a transformation of basis.

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3) a) $f(x,y) = (u,v)$, $\left.\begin{aligned} u &= x-2y \\ v &= 2x-y \end{aligned}\right\} \Rightarrow \begin{aligned} -2u &= -2x+4y \\ v &= 2x-y \end{aligned}$

$v - 2u = 3y \Rightarrow$ $\underline{y = \frac{-1}{3}(2u-v)}$

$\underline{x = \frac{-1}{3}(u-2v)}$

b) By the above result, it is the region bounded by $\boxed{v=u}$, $\boxed{v=-u}$ and

$\frac{-1}{3}(2u-v) = 1 - 2\left(\frac{-1}{3}(u-2v)\right)$

$-\frac{2}{3}u + \frac{v}{3} = 1 + \frac{2}{3}u - \frac{4}{3}v \Rightarrow \frac{5v}{3} = 1 + \frac{4u}{3} \Rightarrow \underline{5v - 4u = 3}$

c)
$$\left.\begin{aligned} (0,0) &= f(0,0) \\ (-1,2) &= f\left(\tfrac{5}{3},\tfrac{4}{3}\right) \\ \underbrace{(2,1)}_{\text{on }(u,v)\text{ plane}} &= \underbrace{f(0,-1)}_{\text{on }(x,y)\text{ plane}} \end{aligned}\right\}$$

where $(u,v) = f(x,y)$, $x = \frac{-1}{3}(u-2v)$

$y = \frac{-1}{3}(2u-v)$

Regions