QUESTION 7

Sample$_1$ $(Y) \rightarrow n_1 = 18$ # of trials

$\mu_1 = 0$

$\sigma = 3$

Sample$_2$ $(X) \rightarrow n_2 = 8$

$\mu_2 = 1$

$\sigma_2 = 2$

(i) $E[\bar{X} - \bar{Y}] = E[\bar{X}] - E[\bar{Y}] = \mu_2 - \mu_1 = 1 - 0 = 1.$

$Var[\bar{X} - \bar{Y}] = Var[\bar{X}] + Var[\bar{Y}] = \frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2} = \frac{1}{2} + \frac{1}{2} = 1$

(ii) $\bar{X} - \bar{Y} \sim N(1,1)$, since X, Y are normally distributed.

(iii) $A = \bar{X} - \bar{Y}$, $P(A \leq 1 \mid A \leq 2) = \frac{P(A \leq 1 \cap A \leq 2)}{P(A \leq 2)} = \frac{P(A \leq 1)}{P(A \leq 2)} = ?$

$\frac{A - \mu}{\sigma} \sim N(0,1)$

QUESTION 8

(i) $E[\bar{X}_n] = 1$ Since $X_i \sim Exp(\lambda)$, $\lambda = 1$

$Var[\bar{X}_n] = 1$

By CLT we have $P\left(\frac{\bar{X}_n - 1}{\frac{1}{\sqrt{n}}} \leq x\right) \rightarrow P(Z \leq x)$

where Z is standard normal variable.

QUESTION 5

(i) We know that Expected value of standard normal dist. is 0.
By the linearity of expectation, $E[X] = E[\mu + \sigma Z]$, where Z has standard normal dist.

$$= \mu + \sigma E[Z]$$
$$= \mu + \sigma \cdot 0$$
$$= \mu .$$

(ii)

$$Var[X] = Var[\mu + \sigma Z] = \sigma^2 \cdot Var[Z] = \sigma^2 \quad \boxtimes$$

QUESTION 6

Let X be the outcomes of the trials. So $X_i \in \{-3, 10\}$.

So $E[X_1] = -3 \cdot 0{,}8 + 10 \cdot 0{,}2 = -0{,}4$

$Var[X_1] = E[X^2] - (E[X])^2 = 27{,}04$

(i) $P\left(\sum_{i=1}^{36} X_i > 0\right) \approx P\left(Z > \frac{0 - (-0{,}4) \cdot 36}{6\sqrt{27{,}04}}\right) = P(\underset{\text{standard normal dist.}}{Z} > -0{,}46) = 0{,}677.$

(ii) We need to win at least 9 games so that the result will be positive.

$$P(\text{at least 9 success}) = \sum_{k=9}^{36} \binom{36}{k} (0{,}2)^k (0.8)^{36-k}$$

QUESTION 3

(i) Since all probability distributed in $(0,1)$. we have;

$$\int_0^1 f(x)\,dx = 1 \Rightarrow \int_0^1 cx^2\,dx = 1 \Rightarrow \frac{cx^3}{3}\Big|_0^1 = \frac{c}{3} = 1$$
$$\Rightarrow \underline{c = 3}$$

(ii) we have seen that $F(x) = \int_{-\infty}^{x} f(y)\,dy$, where $F(x) = cdf$, $f(y) = pdf$

for $F(0) = 0$ and $F(1) = 1$, and for $x \in (0,1)$;

$$F(a) = \int_0^a 3x^2\,dx = a^3 \Rightarrow \underline{F(x) = x^3,\ 0 < x < 1}$$

(iii) $P(1/4 \leq X \leq 3/4) = P(X \leq 3/4) - P(X \leq 1/4) = \left(\frac{3}{4}\right)^3 - \left(\frac{1}{4}\right)^3 = \underline{\frac{26}{64}}$

(iv) $P\left(X^2 - \frac{3X}{2} + 1/2 = 0\right) = P\left(X = \frac{1}{2} \vee X = 1\right) = 0$

(v) Let $Y = X^2 \Rightarrow 1 < Y = X^2 < 4$. So CDF of Y, $F_Y(y) :=$

$F(y) = P(Y < y) = P(X^2 < y) = P(-\sqrt{y} < X < \sqrt{y})$

$$\int_{-\sqrt{y}}^{\sqrt{y}} 3x^2\,dx = (\sqrt{y})^3 - [-\sqrt{y}]^3 = \underline{2y\sqrt{y} \quad \text{where } 0 \leq y < 1},$$

for $1 \leq Y < 4 \Rightarrow P(1 < X < \sqrt{y}) = \int_1^{\sqrt{y}} 3x^2\,dx = y\sqrt{y} - 3$

So $CDF(Y) = \begin{cases} 2y\sqrt{y}, & 0 \leq y < 1 \\ y\sqrt{y} - 3, & 1 < y < 4 \\ 0, & \text{else} \end{cases}$, By differentiating

$$PDF(Y) = \begin{cases} 3\sqrt{y}, & 0 < y < 1 \\ \frac{3}{2}\sqrt{y}, & 1 < y < 4 \\ 0, & \text{else} \end{cases}$$

(vi) $E[2X^2 + X + 1] = 2E[X^2] + E[X] + E[1]$.

QUESTION 2

(i) it is either $X_1 < X_2$ or $X_2 < X_1$ $\Rightarrow$ $P(X_1 < X_2) = 1/2$

(ii) 6 possible outcome we have, $\Rightarrow$ $P(X_1 < X_2 < X_3) = 1/6$

(iii) Above 6 outcome X_2 is maximum, or X_1 max, or X_2 max. So
$P(X_1 < X_2,\ X_2 > X_3) = 1/3$

! Note that results for 1, 2, 3 holds since veriables are i.i.d.

(IV)

(V)

QUESTION 1

(i) X has geometric distribution with parameter p.

(ii) $P(3 \leq X < 5) = \sum_{k=3}^{4} (1-p)^k p = (1-p)^3 p (2-p)$

(iii) $P(2|X, 5\nmid X) = ?$. for $2|X$, we have $X = 2\cdot k$ for some k.
And for $5\nmid X$, $5 \nmid 2k \Rightarrow 5 \nmid k \Rightarrow k = 5t+1, 5t+2, 5t+3, 5t+4$
So $X = 10t+1, 10t+2, 10t+3, 10t+4$. for some $t \in \mathbb{N}$

$$P(2|X, 5\nmid X) = (1-p)^{10t+1} \cancel{p} \underbrace{\left(1+(1-p)+(1-p)^2+(1-p)^3\right)}_{\frac{1-(1-p)^4}{p}} = (1-p)^{10t+1}\left(1-(1-p)^4\right)$$

(iv)
$P(Y_1 = k \mid Y_1 + Y_2 = N) = P(Y_1 = k \mid Y_2 = N-k)$ since Y_1, Y_2 are independent $\Rightarrow P(Y_1 = k \mid Y_2 = N-k) = P(Y_1 = k)$

$P(Y_1 = k) = \sum_{k=0}^{N} (1-p)^k p = p \sum_{k=0}^{N} (1-p)^k$. This is geometric series with $a_n = 1$, $r = 1-p$. So $P(Y_1=k) = \cancel{p} \cdot \frac{1-(1-p)^N}{\underbrace{1-(1-p)}_{\cancel{p}}}$

$1-(1-p)^N$.