Boğaziçi University
Department of Mathematics
Math 132

Date: March 14, 2019	Full Name :
Time: 13:00	Student ID :
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Spring 2018/2019– Midterm 1. Part 1	

Q 1 Consider the real number e.

a) Prove that e is irrational.

b) Prove that e satisfies $2.71828182845904523536 < e < 2.71828182845904523537$.

Q 2 Let $\{a_n\}$ and $\{b_n\}$ be sequences of real numbers.

a) Show that if $\{a_n\} \to l$ and $\{b_n\} \to m$ then $\{a_n b_n\} \to lm$ as $n \to \infty$.

b) Prove or disprove: if $\{a_n\}$ converges but $\{b_n\}$ diverges then $\{a_n b_n\}$ also diverges.

c) Show that if $\sum_{n=0}^{\infty} a_n = L$ and $\sum_{n=0}^{\infty} b_n = M$, then $\sum_{i=0}^{\infty} c_n = LM$, where

$$c_n = \sum_{n=0}^{n} a_i b_{n-i}.$$

Q 3 Suppose that $\sum_{n=0}^{\infty} a_n x^n = f(x)$ and $\sum_{n=0}^{\infty} b_n x^n = g(x)$ for all $x \in (-R, R)$ where $R > 0$. Prove that for any positive integer n, the n-th derivative of the function

$$h(x) = f(x)g(x)$$

is defined at $x = 0$. Moreover find a formula which express this derivative using only the terms of $\{a_n\}$ and $\{b_n\}$.

Q 4 Let $\mathbf{r}, \mathbf{F}, \mathbf{n}$ and $\mathbf{m}$ be vectors such that $\mathbf{m}$ is a unit vector perpendicular to $\mathbf{r}$ and $\mathbf{n}$ is a unit vector in the direction of $\mathbf{r} \times \mathbf{F}$.

a) Prove that $\mathbf{r} \times \mathbf{F} = |\mathbf{r}|.|\text{Proj}_{\mathbf{m}}\mathbf{F}|.\mathbf{n}$

b) Now prove the formula $\mathbf{r} \times (\mathbf{F_1} + \mathbf{F_2}) = (\mathbf{r} \times \mathbf{F_1}) + (\mathbf{r} \times \mathbf{F_2})$ using part a).