MATH 231 HW - 9

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a) $e^x = \sum_{J=0}^{k} \frac{x^J}{J!}$, $\cos x = \sum_{J=0}^{2k} \frac{(-1)^J x^{2J}}{2J!}$ $\underset{\substack{\downarrow \\ \text{By uniqueness} \\ \text{of Taylor polyn.}}}{\Rightarrow}$ $\cos(2x) = 4\sum_{J=0}^{2k} \frac{(-1)^J x^{2J}}{2J!}$, $\sin x = \sum_{J=0}^{k} \frac{(-1)^J . x^{2J+1}}{(2J+1)!}$

$f(x) = x^2 [x - (x - \frac{x^3}{6} + o(x^3))] = \frac{x^5}{6} + o(x^3)$

$g(x) = [(1+x+o(x)) - 1][(1-2x^2+o(x^2)) - 1]^2 = [x + o(x)][4x^4 + o(x^4)] = 4x^5 + o(x^5)$

b) $\lim_{x\to 0} \frac{f(x)}{g(x)} = \frac{x^5/6 + \cancel{o(x^5)}}{4x^5 + \cancel{o(x^5)}} \overset{\to 0}{} \underset{\substack{\downarrow \\ \text{THM 3}}}{=} \frac{x^5/6}{4x^5} \xrightarrow[x\to 0]{} \underline{\underline{1/24}}$

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2 $f(x) = \log x$, $f'(x) = x^{-1}$, $f''(x) = -1.x^{-2}$, $f'''(x) = 2.x^{-3}$, $f^{(4)}(x) = -6.x^{-4}$

$P_{1,3}(x-1) = (x-1)^{-1} - \frac{1}{2}(x-1)^{-2} + \frac{1}{3}(x-1)^{-3} \underbrace{-6(x-1)^{-4}}_{R_{1,3}(h)}$, $|-6(x-1)^{-4}| \leq 96$ as $|x-1| \leq \frac{1}{2}$

So $\underline{\underline{C = 4}}$.

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3 $e^x = \sum_{J=0}^{k} \frac{x^J}{J!}$ $\underset{\substack{\downarrow \\ \text{By oniqueness} \\ \text{of Taylor.}}}{\Rightarrow}$ $e^{-x^2} = \sum_{J=0}^{k} \frac{(-1)^J . x^{2J}}{J!}$. $f(x) = e^{-x^2}$. f is C^{k+1} and we have $|f^{(k+1)}(x)| \leq 1$ on $[0,1]$

By $\boxed{\text{thm 4}}$ $|R_{0,k}(x^2)| \leq \frac{|x^2|^{k+1}}{(k+1)!} \cdot 1$ for $x \in [0,1]$

$\Rightarrow |\int_0^1 R_{0,k}(x^2)\,dx| \leq \int_0^1 \frac{x^{2k+2}}{(k+1)!} \cdot dx = \frac{1}{(2k+3)(k+1)!}$. This is less then 0,00011 when $k \geq 5$. So $\int_0^1 e^{-x^2} dx = \sum_{k=0}^{5} (-1)^k \int_0^1 \frac{x^{2k}}{k!} dx = \sum_{k=0}^{5} \frac{(-1)^k}{(2k+1)k!} = 1 - \frac{-1}{3.1!} + \frac{1}{5.2!} - \frac{1}{7.3!} + \frac{1}{9.4!} - \frac{1}{11.5!}$

$= \frac{2}{3} + \frac{1}{11} - \frac{1}{42} + \frac{1}{216} - \frac{1}{120.11} = \underline{\underline{0,738}}$