HW 12 (Compensation for HW3) Ahmet Kasım ERBAY.

- The lectures I attended lastly, → 23.01.2021
  19.01.2021
  → second week of November
- I had to move to a new appartment due to the crowdedness at home. This cost me lower grades. Thank you so much for your kindness.

1.) a) if $\sin x = a$ and $\cos x = b$ for some constant lines. Then we have the cannonical form of hyperbola which is $\left(\frac{u}{a}\right)^2 - \left(\frac{v}{b}\right)^2 = 1$.

For fixed y we get $\cosh y = a$, $\sinh y = b \Rightarrow \left(\frac{u}{a}\right)^2 + \left(\frac{v}{b}\right)^2 = 1$.

b) $\frac{\partial(u,v)}{\partial(x,y)} = \det\begin{pmatrix} \cos x.\cosh y & \sin x \sinh y \\ -\sin x \sinh y & \cos x \cosh y \end{pmatrix} = \cos^2 x.\cosh^2 y + \sin x^2 \sinh^2 y$

$= \cos^2 x(1 + \sinh^2 y) + \sin x^2.\sinh^2 y$

$= \cos^2 x + \sinh^2 y$

This vanishes when $\cos x = 0$ and $\sinh y = 0 \Rightarrow x = \{\frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2} \ldots\}$
$= \{\frac{k\pi}{2} : k \in \mathbb{Z}, k \text{ odd } \#\}$

$y = 0$

This corresponds to $u = \pm 1$, $v = 0$ ($u = \sin x \cosh y$, $v = \cos x \sinh y$)

c) Foci for the hyperbola are $\pm(c,0)$ where $c^2 = a^2 + b^2$
" " " ellypses are $\pm(c,0)$. " $c^2 = a^2 - b^2$ (as a in (a))
so c's are either -1 or $+1$

2.) Let $(u,v) = f(x,y) = \left(\frac{y}{x^4}, x^2 y^2\right)$. $Df = \begin{pmatrix} -\frac{4y}{x^5} & 1/x^4 \\ 2y & 2x \end{pmatrix}$, $\det Df = \frac{10y}{x^4}$

$y = ux^4$,
$v = x^2 y^2 \Rightarrow x = \frac{\sqrt{v}}{ux^4} \Rightarrow$ $x = v^{1/10}.u^{-1/5}$
$y = u.v^{2/5}.u^{-4/5} = u^{1/5}.v^{2/5}$