MATH 231 HW-5 Ahmet Kasım ERBAY - 2017205108

1)

a) ($\Longrightarrow$) Assume $f: A \to R$ is continous where A is compact. There exists $\{x_n\} \in A$ with $f(x_n) = y_n$. Since A is compact, by Bo-We $\exists \{x_{n_k}\}$ of $\{x_n\}$ s.t. $\lim_{k\to\infty} x_{n_k} = x \in A$. Since f is continous $f(x) = \lim_{k\to\infty} f(x_{n_k}) = y_{n_k}$, where $\{y_{n_k}\}$ is a subsequence of $\{y_n\}$. Hence $(\{x_{n_k}\}, \{y_{n_k}\}) \subset R^2$ is a compact

(Above proof inherited by lecture notes)

($\Longleftarrow$) Take some $x \in A$ and a sequence $\{x_n\} \subset A$ converging to x. We want to show $f(x)$ is continous; that is, $\lim_{k\to\infty} f(x_n) = x$. Assume this is not the case; that is, $(x_n, f(x_n))$ does not converge to $(x, f(x))$. We know Γ_f is compact; by Bo-We, we have $(x_{n_k}, f(x_{n_k}))$ which is a convergent subsequence in Γ_f, say $(a, f(a)) \neq (x, f(x))$. We know that $\{x_n\} \to x \Rightarrow \{x_{n_k}\} \to x$. This gives us that $a = x$. Is $f(a) = f(x)$?

I couldn't finish it. I didn't see a path to limit definition of cont.

b) $f(x) = \sin(\frac{1}{x})$ not cont on $[0,1]$ and Γ_f not closed since $\{(0,y) : y \in [-1,1]\} \subset \partial \Gamma_f$ but not in Γ_f.

c) $f(x) = \begin{cases} x \geq 1/2, & 2x \\ x < 1/2 & 0 \end{cases}$ is not continous on $1/2$ and Γ_f is closed.

2)

(i) TRUE.

We have $A \in R^n$ is connected. Look at $\overline{A}$ and assume $B_1, B_2 \subset \overline{A}$ nonempty subsets of $\overline{A}$ s.t $\overline{A} = B_1 \cup B_2$ and

$$\overline{B_1} \cap B_2 = \emptyset = B_1 \cap \overline{B_2} \Rightarrow A = A \cap \overline{A} = (B_1 \cap A) \cup (B_1 \cap A).$$

Also we have $\overline{(B_1 \cap A)} \cap (B_2 \cap A) \subset B_1 \cap B_2 = \emptyset$,

$$(B_1 \cap A) \cap \overline{(B_2 \cap A)} \subset B_1 \cap B_2 = \emptyset$$

This is a contradiction for A being connected. Hence $\overline{A}$ is connected.

(ii) FALSE

$A = (1,2) \cup (2,3) \to$ not connected $(1,2), (2,3)$ is a disconnection

But $\overline{A} = [1,2] \cup [2,3] = [1,3]$ is an interval $\Longleftrightarrow$ connected.

3) ($\Longrightarrow$) Assume A is disconnected and let A_1, A_2 is a connection for A.

Define $f: A \to \{2,3\}$; $f(x) = \begin{cases} 2 & \text{if } x \in A_1 \\ 3 & \text{if } x \in A_2 \end{cases}$. f is well defined since $\overline{A_1} \cap A_2 = \emptyset = A_1 \cap \overline{A_2}$. Also continous since $f_1: A_1 \to 2$, $f_2: A_2 \to 3$ are constant; continous, functions.

($\Longleftarrow$) ?