(1) R : irreflexive, symmetric & transitive relation. Show $R = \emptyset$

Pf: Suppose for a contradiction that $R \neq \emptyset$. Then since R is irreflexive we have $(a,b) \in R$ s.t $a \neq b$. Then we have $(b,a) \in R$ by symmetry of R. Then by transitivity, (a,b) & $(b,a) \in R \to (a,a) \in R$ which is a contradiction since R was irreflexive

thus, $R = \emptyset$

! Check the irreflexivity, sym. & transitivity of the empty set. (No problem here.)

(2) aRb if $a = b^x$, $a,b \in \mathbb{N}$ & $x \in \mathbb{Q}$.

(a) Show R is an eq. rel.

✓ reflexive : $aRa \to a = a^1$ for all $a \in \mathbb{N}$, $1 \in \mathbb{Q}$

✓ sym. : $aRb \to a = b^{x_1}$, then for [an x_2 s.t] $x_1 \cdot x_2 = 1$,

$b = a^{x_2}$. [So that $a = (\underbrace{a^{x_2}}_{b})^{x_1} = a^1$.]

✓ transitive: aRb & $bRc \to aRc$

$\underbrace{a = b^{x_1} \text{ \& } b = c^{x_2}}$

$a = (c^{x_2})^{x_1}$ where $x_2 \cdot x_1 \in \mathbb{Q}$ since $x_1, x_2 \in \mathbb{Q}$.

(b) List all eq. classes of R if $a,b \in \{1,2,\ldots,20\}$.

$\bar{0} = \{0\}$

$\bar{1} = \{1\}$

$\bar{2} = \{2,4,8,16\} = \bar{4} = \bar{8} = \overline{16}$

$\bar{3} = \{3,9\} = \bar{9}$

$\bar{5} = \{5\}$

$\bar{6} = \{6\}$

$\bar{7} = \{7\}$

$\overline{10} = \{10\}$

$\overline{11} = \{11\}$

$\overline{12} = \{12\}$

$\vdots$

$\overline{15} = \{15\}$

$\overline{17} = \{17\}$

$\vdots$

$\overline{20} = \{20\}$

(3) Problem 1.4 in TB

(4) Problem 1.2 in TB.