(1) Determine whether the "function" $g : \mathbb{Z}_{13} \longrightarrow \mathbb{N}$ is well-defined if g is "defined" as follows (here "gcd" denotes the greatest common divisor).

(1) $g([a]) = \gcd(a, 13)$.
(2) $g([a]) = \gcd(a, 26)$.
(3) $g([a]) = \gcd(a, 13^2)$.
(4) $g([a]) = \gcd(a^2, 13)$.
(5) $g([a]) = \gcd(a^3, 13^2)$.
(6) $g([a]) = \gcd(a, 6)$.
(7) $g([a]) = \gcd(a^2, 65)$.

(2) (1) Let $f : \mathbb{Z}_{12} \times \mathbb{Z} \longrightarrow \mathbb{Z}_{12}$ be "defined" by $f([a], b) = [a^2 + ab + b^2]$. Is f well-defined?

(3) For an integer a, we denote by $\bar{a}$ the residue class of a modulo 12, by $\tilde{a}$ the residue class of a modulo 6 and by $\hat{a}$ the residue class of a modulo 5. Thus $\bar{a} \in \mathbb{Z}_{12}$ and $\tilde{a} \in \mathbb{Z}_6$ and $\hat{a} \in \mathbb{Z}_5$. Determine whether the following "functions" are well-defined.

(1) $f : \mathbb{Z}_{12} \longrightarrow \mathbb{Z}_6, \quad f(\bar{a}) = \tilde{a}$.
(2) $f : \mathbb{Z}_6 \longrightarrow \mathbb{Z}_{12}, \quad f(\tilde{a}) = \bar{a}$.
(3) $f : \mathbb{Z}_{12} \longrightarrow \mathbb{Z}_5, \quad f(\bar{a}) = \hat{a}$.
(4) $f : \mathbb{Z}_5 \longrightarrow \mathbb{Z}_6, \quad f(\hat{a}) = \tilde{a}$.
(5) $f : \mathbb{Z}_5 \longrightarrow \mathbb{Z}_6, \quad f(\hat{a}) = \widetilde{a+1}$.