MATH 48A - Homework II

Due: 28.04.2021

1. (a) Show that the discriminant of the cubic polynomial $x^3 + ax + b$ is given by $d = -4a^3 - 27b^2$.
   (b) Let $f(x)$ be a monic cubic polynomial with coefficients in $\mathbb{R}$ and let d be its discriminant. Explain what you can infer from $d < 0, d = 0$ and $d > 0$.
   (c) Find a formula for the discriminant of the quartic polynomial
   $$x^4 + ax^2 + bx + c.$$

2. (a) Consider the affine cubic curve $x^3 + y^3 = d$. Find all inflection points and put the equation in Weierstrass form.
   (b) Consider the affine cubic curve $y^2 + 2x^2y = 1$. Find all inflection points and put the equation in Weierstrass form.
   (c) Consider the projective cubic curve $x^2y - xy^2 - xz^2 + y^2z = 0$. Find all inflection points and put the equation in Weierstrass form.

3. Find a necessary and sufficient condition for the affine line $y = ax + b$ to be a tangent at an inflection point of $y^2 = x^3 + px + q$.

4. Show that the composition of two admissible changes of variables is again an admissible change of variable.

5. (a) Calculate the discriminant and the j-invariant of the following elliptic curves
   (b) $y^2 - xy + y = x^3$
   (c) $y^2 = x^3 - 2x - x$