Quiz 1 - Solutions

① $f(z) = u(x,y) + i(2+3x-y+x^2-y^2-4xy)$

1 Let $v(x,y) = 2+3x-y+x^2-y^2-4xy$.
For f to be analytic, the necessary and sufficient conditions are Cauchy-Riemann equations:

$$v_x = -u_y \quad \& \quad v_y = u_x .$$

1 $v_x = 3+2x-4y$ and $v_y = -1-2y-4x$.

1 So $u_y = 4y-2x-3$ and hence $u(x,y) = 2y^2-2xy-3y+\rho(x)$, where $\rho(x)$ is a function of x.

1 Then $u_x = -2y+\rho'(x)$, where $\rho'(x)$ is the derivative of ρ with respect to x.
So we need $-2y+\rho'(x) = -1-2y-4x$. Then $\rho'(x) = -1-4x$ and $\rho(x) = -x-2x^2+c$ for some $c \in \mathbb{R}$.

1 Therefore possible $u(x,y)$ are $2y^2-2xy-3y-x-2x^2+c$ for some $c \in \mathbb{R}$.

② C is given by $z(t) = \cos t + i \sin t$, $t \in [0,\pi]$. 1

Then $\displaystyle\int_C f = \int_0^{2\pi} \overline{\cos t + i \sin t}\,(\cos t + i \sin t)'\,dt$ 1

$\displaystyle = \int_0^{\pi} (\cos t - i \sin t)(-\sin t + i\cos t)\,dt$ 1

$$= \int_0^{\pi} i\,(\cos^2 t + \sin^2 t)\,dt \qquad \Big]\ 1$$

$$= i \int_0^{\pi} dt = i\left(t\,\Big|_0^{\pi}\right) = i\pi. \qquad \Big]\ 1$$