# IE 481 & IE 801
# Game Theory and Multi-Agent Reinforcement Learning: Intersection of Game Theory and Artificial Intelligence

**Jinkyoo Park**
SYSTEMS INTELLIGENCE LAB.
KAIST

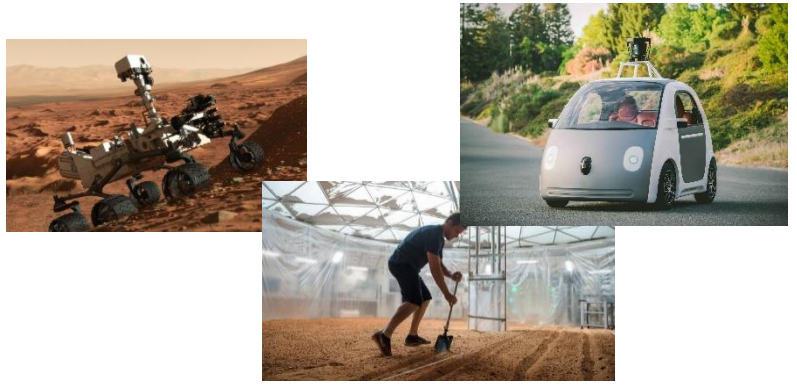
# Contents

# 1. Motivation

## Engineering is all about decision makings

- Machine Learning
- **Artificial Intelligence**
- Optimization
- Optimum Control
- Planning
- Markov Decision Process
- Influential Diagram
- Decision Tree
- Dynamic Control
- **Game Theory**
- Search
- Stochastic Programming
- Dynamic programming
- Reinforcement Learning
- Bandit problem

⋮



What are ***the differences*** in these decision-making strategies

What are ***the common aspects*** in these decision-making strategies?

# 1. Motivation

## Main Topics in Decision Makings

What type of decision making framework will be used?

- Single stage or multi stages
- Single decision maker or many decision makers
- Model based or model-free

black box

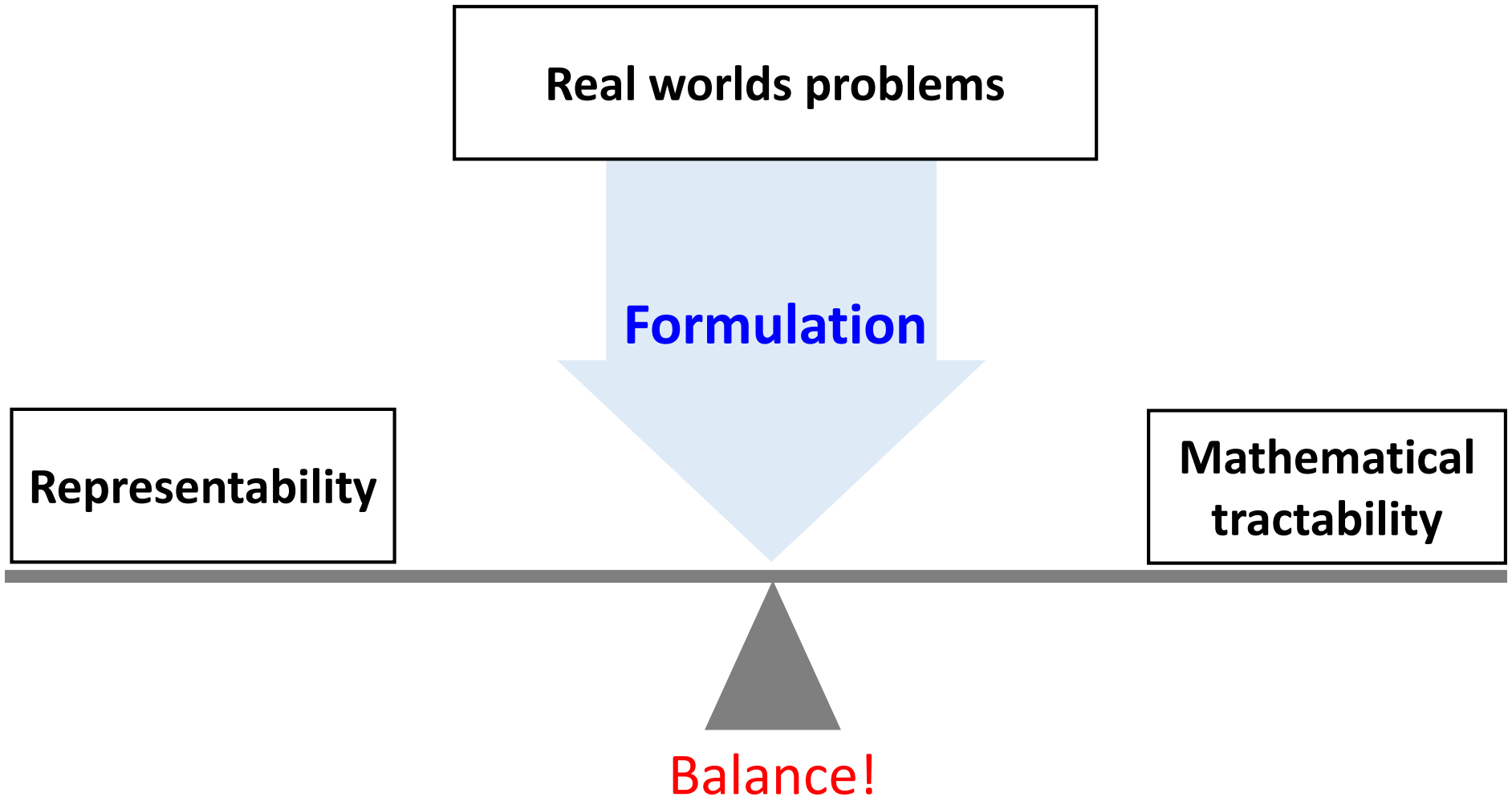
**“Decision makings under uncertainties”**

How to model uncertainties?

- **Epistemic Uncertainty** (systemic uncertainty) :
  Uncertainty arising through lack of knowledge
  - Model uncertainty
  - State uncertainty
- **Aleatoric uncertainty** (statistical uncertainty):
  Uncertainty arising through an underlying stochastic system

# 1. Motivation

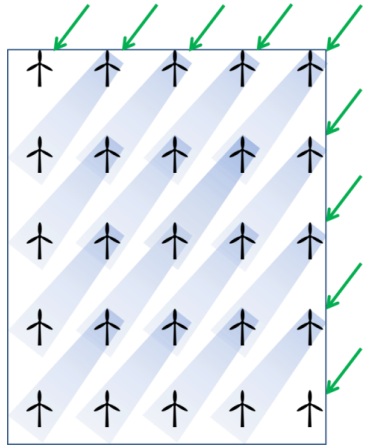
## How to Solve Complex Problems?

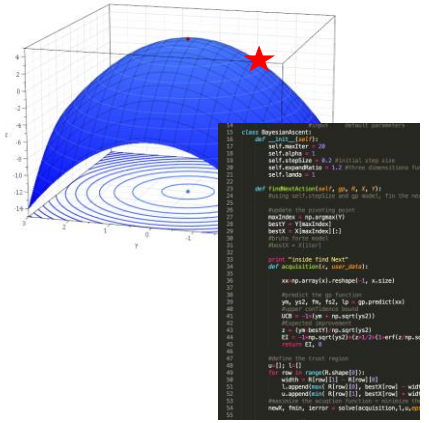


# 1. Motivation

## How to Solve Complex Problems?

**Real-world task** → **Modeling** → **Formal task (model)** → **Solving** → **Algorithm (program)**



$$\underset{\boldsymbol{x}}{\text{maximize}} \sum_{i=1}^{N} P_i(\boldsymbol{x}; \theta, U)$$

**Validation**

*Are we building the right model?*

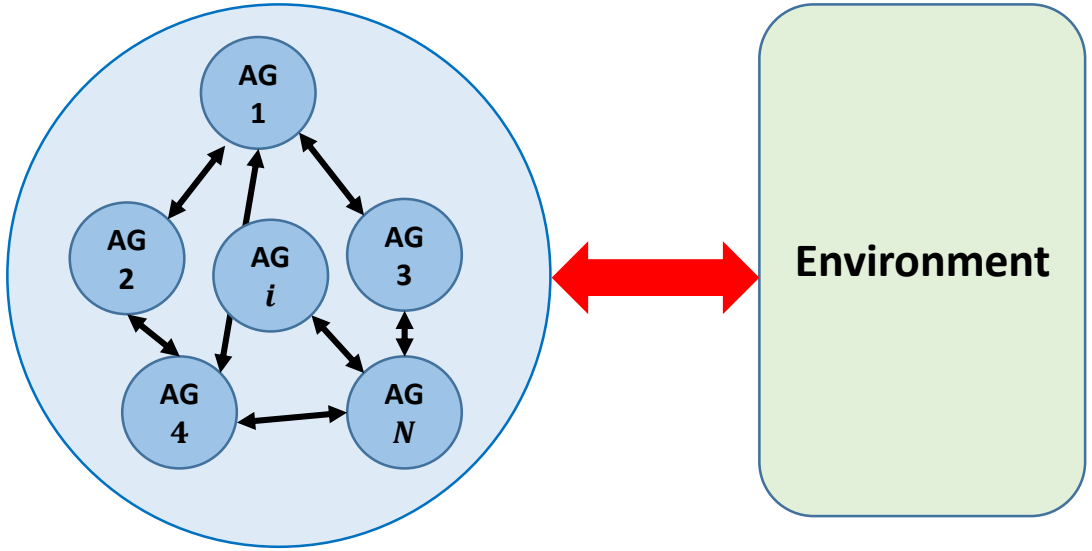
**Verification**

*Does the algorithm capture all the essential aspects of the model?*

Is this solution is good for the target system?

Data can help model more realistically and derive more accurate solution!

# 1. Motivation

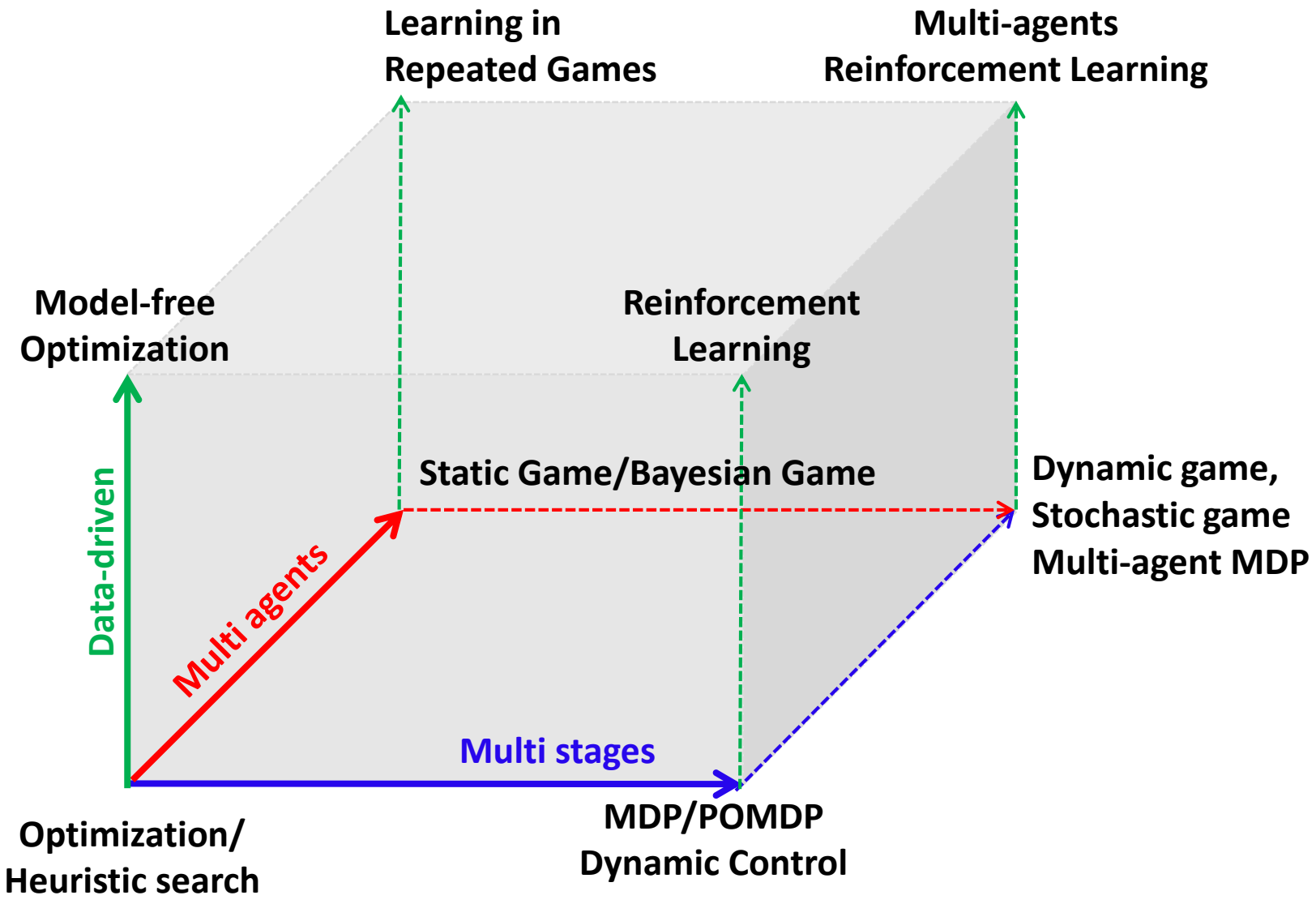



Joint control policy approximated as a decentralized control policy:

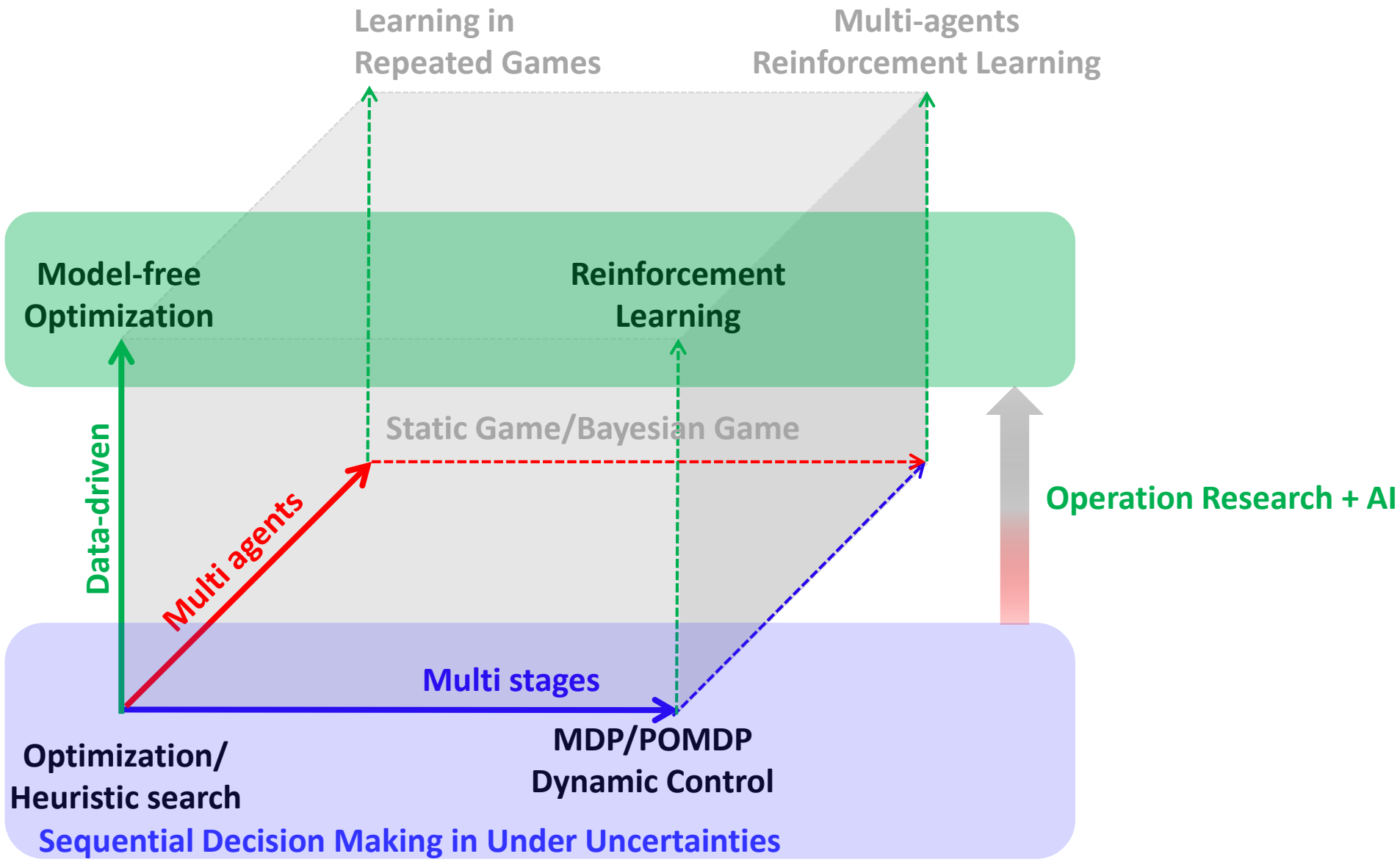
$$\pi(s, a_1, \dots, a_i, \dots, a_n) \approx \prod_{i=1}^{N} \pi_i(s, a_i) \approx \prod_{i=1}^{N} \pi_i(o_i, a_i)$$

- As a system becomes larger and more complex, it become more difficult to understand and control the target systems
- A methodology has been developed to independently model the agents that make up the entire system, to efficiently model the entire system considering their interaction, and to derive distributed control strategies
  - Decentralized MDP, Decentralized-POMDP, Decentralized Cooperative Control, Team Game, Cooperative Game..
  - Analytical solutions to these problems are limited to only special cases
- Recent advanced in Deep learning and Reinforcement learning approach can open up new solution approaches

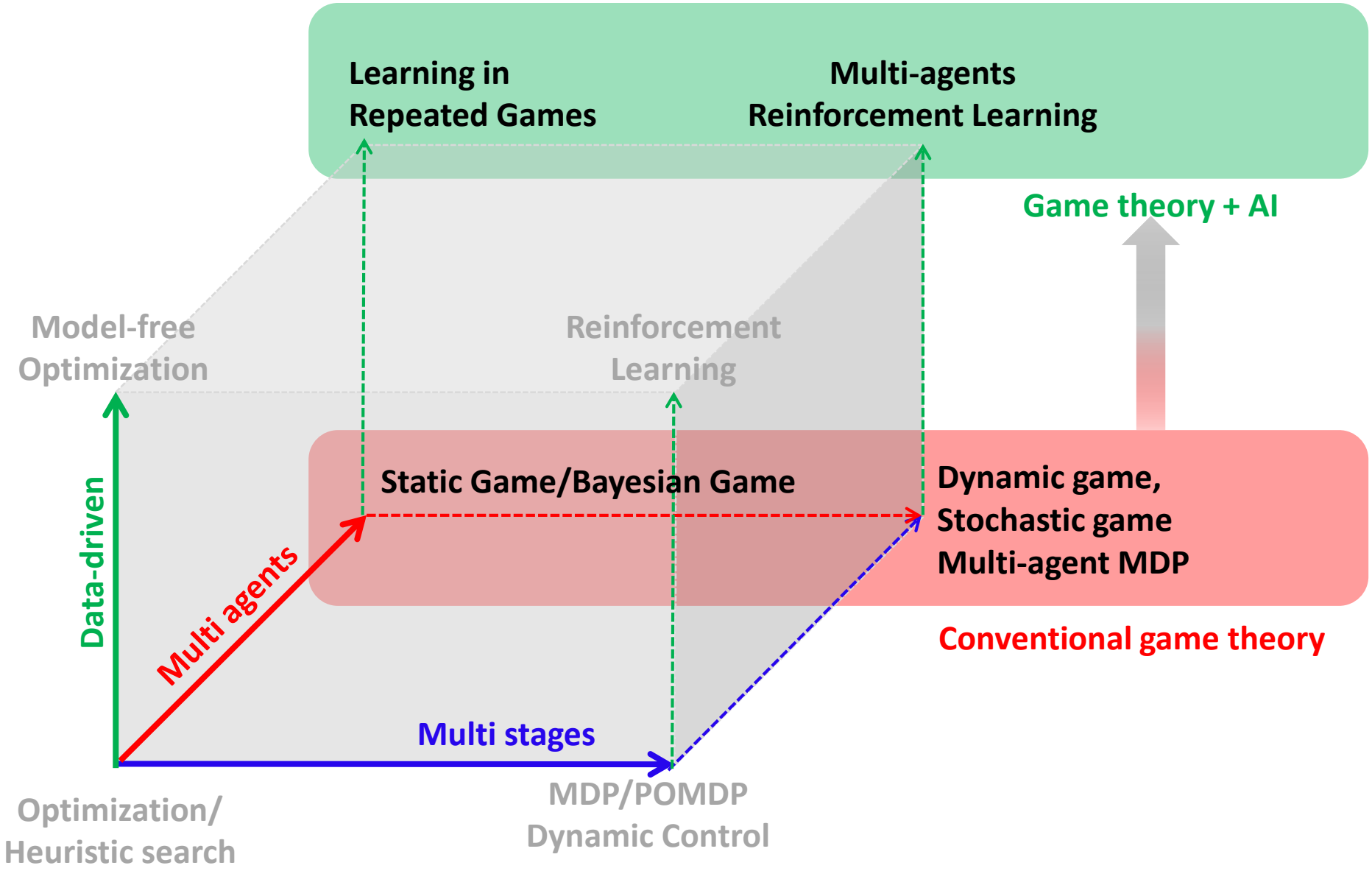
# 2. OVERVIEW


Learning in
Repeated Games
Multi-agents
Reinforcement Learning
Model-free
Optimization
Reinforcement
Learning
Static Game/Bayesian Game
Dynamic game,
Stochastic game
Multi-agent MDP
Data-driven
Multi agents
Multi stages
Optimization/
Heuristic search
MDP/POMDP
Dynamic Control

# 2. OVERVIEW


Learning in
Repeated Games
Multi-agents
Reinforcement Learning
Model-free
Optimization
Reinforcement
Learning
Static Game/Bayesian Game
Data-driven
Multi agents
Operation Research + AI
Multi stages
Optimization/
Heuristic search
MDP/POMDP
Dynamic Control
Sequential Decision Making in Under Uncertainties

# 2. OVERVIEW


Learning in
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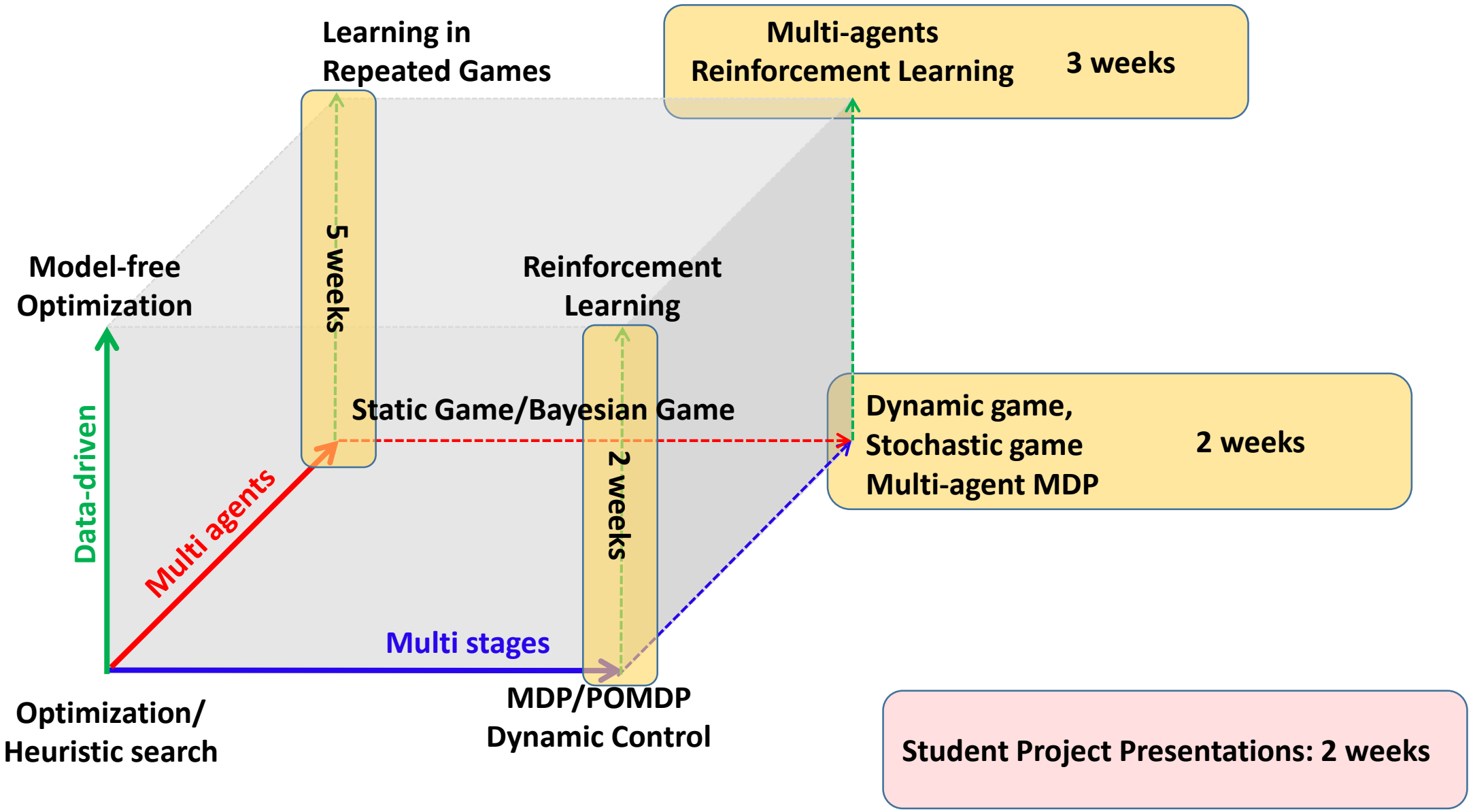
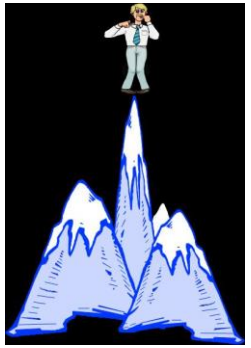
Reinforcement Learning
Game theory + AI
Model-free
Optimization
Reinforcement
Learning
Static Game/Bayesian Game
Dynamic game,
Stochastic game
Multi-agent MDP
Conventional game theory
Data-driven
Multi agents
Multi stages
Optimization/
Heuristic search
MDP/POMDP
Dynamic Control

# 2. OVERVIEW


Learning in
Repeated Games
Multi-agents
Reinforcement Learning
3 weeks
5 weeks
Model-free
Optimization
Reinforcement
Learning
Static Game/Bayesian Game
Dynamic game,
Stochastic game
Multi-agent MDP
2 weeks
2 weeks
Data-driven
Multi agents
Multi stages
Optimization/
Heuristic search
MDP/POMDP
Dynamic Control
Student Project Presentations: 2 weeks

# 2. OVERVIEW : Static Optimization to Static Game

## From Optimality to Equilibrium



- **Single agent decision making:**
  - Optimal strategy is one that maximizes the agent's expected utility for a given environment
  - Uncertainties arose from stochastic environment, partially observable states, uncertain rewards, etc., which can be dealt with probability concepts.

$$a^* = \underset{a}{\operatorname{argmax}}\, E_s[u(a, s)]$$

- **Multiagents decision making:**
  - The environment includes other agents, each of which tries to maximize its own utility
  - Thus the notion of an optimal strategy for a given agent is not meaningful because *the best strategy depends on the choices of others*
  - We need to identify certain subsets of outcomes, called solution concepts
  - Two of the most fundamental solution concepts are
    - *Pareto optimality*
    - *Nash equilibrium*

$$u_1(a_1^*, a_2^*) \geq u_1(a_1, a_2^*) \;\; \forall a_1$$
$$u_2(a_1^*, a_2^*) \geq u_2(a_1^*, a_2) \;\; \forall a_2$$

# 2. OVERVIEW : Static Optimization to Static Game

## Nash Equilibrium

**Definition (Nash Equilibrium)**

A strategy profile $s^* = (s_1^*, \dots, s_n^*)$ is a Nash Equilibrium if, for all agents $i$ and for all strategies $s_i$, $u_i(s_i^*, s_{-i}^*) \geq u_i(s_i, s_{-i}^*)$.

- A strategy profile $s^* = (s_1^*, \dots, s_n^*)$ is a Nash Equilibrium if, for all agents $i$, $s_i^*$ is a best response to $s_{-i}^*$, i.e., $s_i^* \in BR(s_{-i}^*)$
- A Nash equilibrium is a stable strategy profile:
  - no agent would want to change his strategy if he knew what strategies the other agents were following

# 2. OVERVIEW : Static Optimization to Static Game

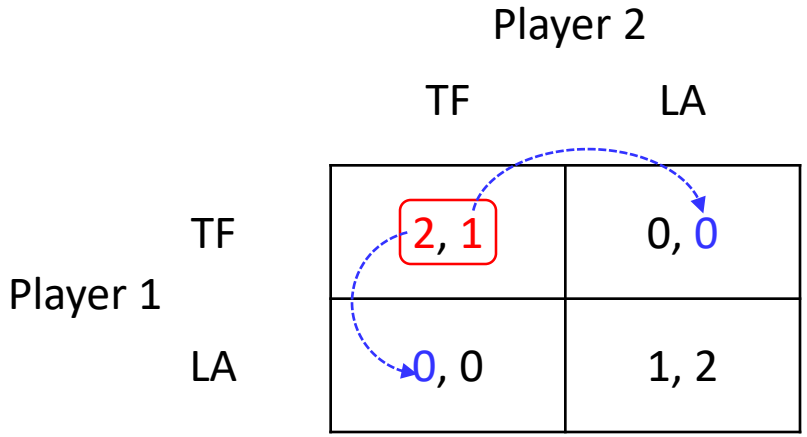
## Nash Equilibrium

| | | Player 2: TF | Player 2: LA |
|---|---|---|---|
| Player 1 | TF | 2, 1 | 0, 0 |
| Player 1 | LA | 0, 0 | 1, 2 |

- We immediately see that it has two pure-strategy Nash equilibria

# 2. OVERVIEW : Static Optimization to Static Game

## Nash Equilibrium

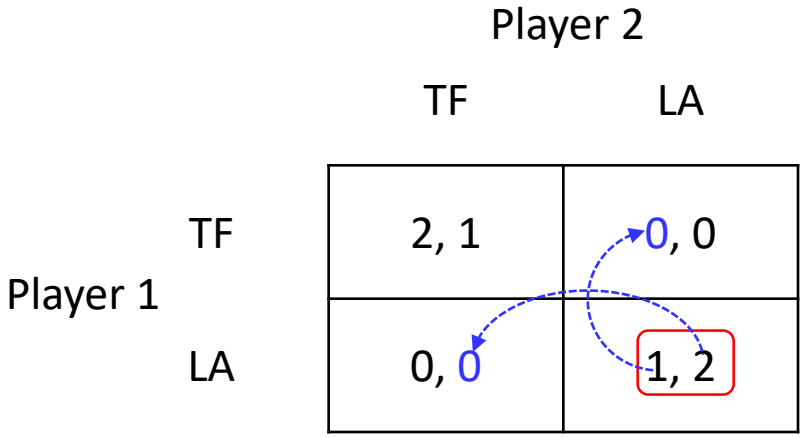



- We can check that these are Nash equilibria by confirming that whenever one of the players play the given (pure) strategy, the other player would only lose by deviating

$$a^* = (\text{TF}, \text{TF}) \qquad \begin{aligned} u_1(\text{TF}, \text{TF}) &> u_1(\text{LA}, \text{TF}) \\ u_2(\text{TF}, \text{TF}) &> u_2(\text{TF}, \text{LA}) \end{aligned}$$

# 2. OVERVIEW : Static Optimization to Static Game

## Nash Equilibrium




- We can check that these are Nash equilibria by confirming that whenever one of the players play the given (pure) strategy, the other player would only lose by deviating

$$a^* = (\text{LA}, \text{LA}) \qquad \begin{aligned} u_1(\text{LA}, \text{LA}) &> u_1(\text{TF}, \text{LA}) \\ u_2(\text{LA}, \text{LA}) &> u_2(\text{LA}, \text{TF}) \end{aligned}$$

# 2. OVERVIEW : Static Optimization to Static Game

## Max-Min Equilibrium

**Definition (Maxmin)**

The maxmin strategy for player $i$ is $s_i^* = \arg\max_{s_i} \min_{s_{-i}} u_i(s_i, s_{-i})$ and the maxmin value for player $i$ is $\max_{s_i} \min_{s_{-i}} u_i(s_i, s_{-i})$

- The ***maxmin strategy*** of player $i$ in an $n$-players game is a strategy that maximizes $i's$ worst –case payoff, in the situation where all the others players happen to play the strategies which cause the greatest harm to $i$
- The ***maxmin strategy*** is a sensible choice for a conservative agent who wants to maximize his expected utility without having to make any assumptions about the other agents
- The ***maxmin value*** (or security level) of the game for player $i$ is that minimum amount of payoff guaranteed by a maxmin strategy
- It is strategy that **defends against** other agents (defensive strategy)

# 2. OVERVIEW : Static Optimization to Static Game

## Max-Min Equilibrium

**Definition (Minmax,** two-player**)**

In an two-player game, the *minmax strategy* for player $i$ against player $-i$ is $s_i^* = \arg\min_{s_i} \max_{s_{-i}} u_{-i}(s_i, s_{-i})$ and the minmax value is $\min_{s_i} \max_{s_{-i}} u_{-i}(s_i, s_{-i})$

- The *minnmax strategy* of player $i$ in an two-players game is a strategy that keeps the maximum payoff of $-i$ at a minimum
- The *minmax value* of player $-i$ is that minimum
- It is strategy that attack against other agents (offensive strategy)

# 2. OVERVIEW : Static Optimization to Static Game

## Max-Min Equilibrium

In agent $i's$ perspective

$$\max_{s_i} \min_{s_{-i}} u_i(s_i, s_{-i})$$

- Agent always maximizes its payoff
- Defensive strategy (if max is first)

$$\min_{s_i} \max_{s_{-i}} u_{-i}(s_i, s_{-i})$$

- Agent always maximizes its payoff
- offensive strategy (if min is first)

# 2. OVERVIEW : Static Optimization to Static Game

## Correlated Equilibrium

**Definition (Correlated equilibrium)**

A correlated equilibrium of finite game is a joint probability distribution $\pi \in \Delta(A)$ such that if $R$ is random variable distributed according to $\pi$ then

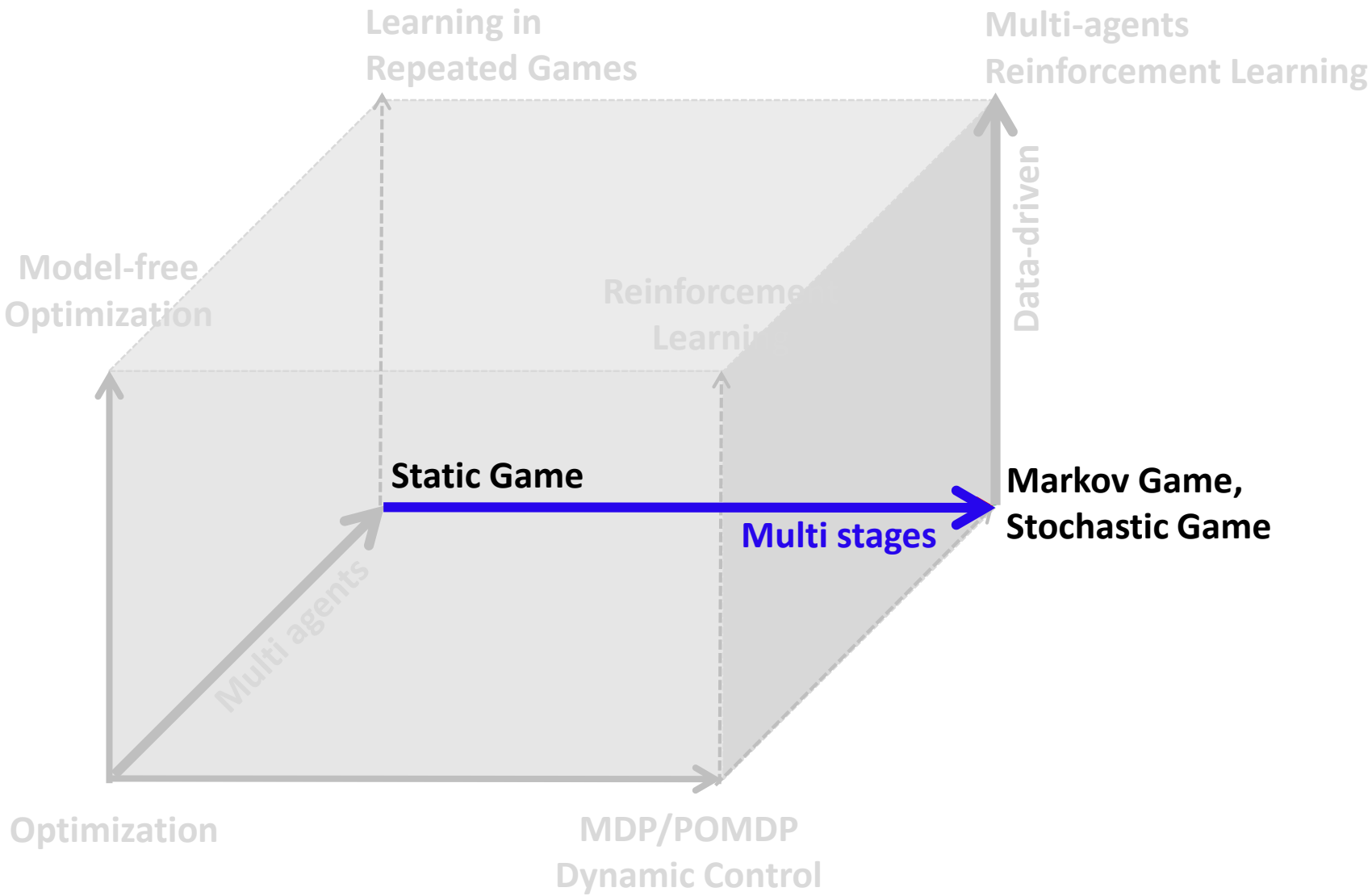
$$\sum_{a_{-i} \in A_{-i}} \text{Prob}(R = a | R_i = a_i) u_i(a_i, a_{-i}) \geq \sum_{a_{-i} \in A_{-i}} \text{Prob}(R = a | R_i = a_i) u_i(a_i', a_{-i})$$

For all players $i$, all $a_i \in A_i$ such that $\text{Prob}(R_i = a_i) > 0$ , and all $a_i' \in A_i$

- A distribution $\pi$ is defined to be a correlated equilibrium if no player can ever expect to unilaterally gain by deviating from his recommendation, assuming the other players play according to their recommendations.

  - $a_i$ is a recommendation by $R$ drawn from $\pi \in \Delta(A)$
  - $a_i'$ is a deviation from this recommendation

# 2. OVERVIEW : Static Game to Dynamic Game



# 2. OVERVIEW : Static Game to Dynamic Game

| | Single Agent | Multi Agent |
|---|---|---|
| Static | Static optimization | Static Game |
| Dynamic | **Dynamic Optimization** | Dynamic Game |

**Action space**

| **Time space** | **Model based** | Finite | Infinite |
|---|---|---|---|
| | Discrete | Discrete time MDP $P(s_{t+1}\|s_t, a_t)$ | Discrete-time dynamic system $x_{t+1} = f(x_t, u_t)$ |
| | Continuous | Continuous time MDP $P(s_{t+h}\|s_t, a_t)$ | Continuous-time dynamic system $\dot{x}_t = f(x_t, u_t)$ |

# 2. OVERVIEW : Static Game to Dynamic Game

| | Single Agent | Multi Agent |
|---|---|---|
| Static | Static optimization | Static Game |
| Dynamic | **Dynamic Optimization** | Dynamic Game |

**Action space**

| **Time space** | **Model free** | Finite | Infinite |
|---|---|---|---|
| | Discrete | Value-based Reinforcement Learning | Policy-based Reinforcement Learning |
| | Continuous | | |

# 2. OVERVIEW : Static Game to Dynamic Game

| | Single Agent | Multi Agent |
|---|---|---|
| Static | Static optimization | Static Game |
| Dynamic | Dynamic Optimization | **Dynamic Game** |

**Action space**

| **Time space** | **Model based** | Finite | Infinite |
|---|---|---|---|
| | Discrete | Markov Game (Stochastic Game) | DT Infinite dynamic game (Stochastic Game) |
| | Continuous | Continuous time Markov Game | CT-time Infinite dynamic game (differential game) |

# 2. OVERVIEW : Static Game to Dynamic Game

| | Single Agent | Multi Agent |
|---|---|---|
| Static | Static optimization | Static Game |
| Dynamic | Dynamic Optimization | **Dynamic Game** |

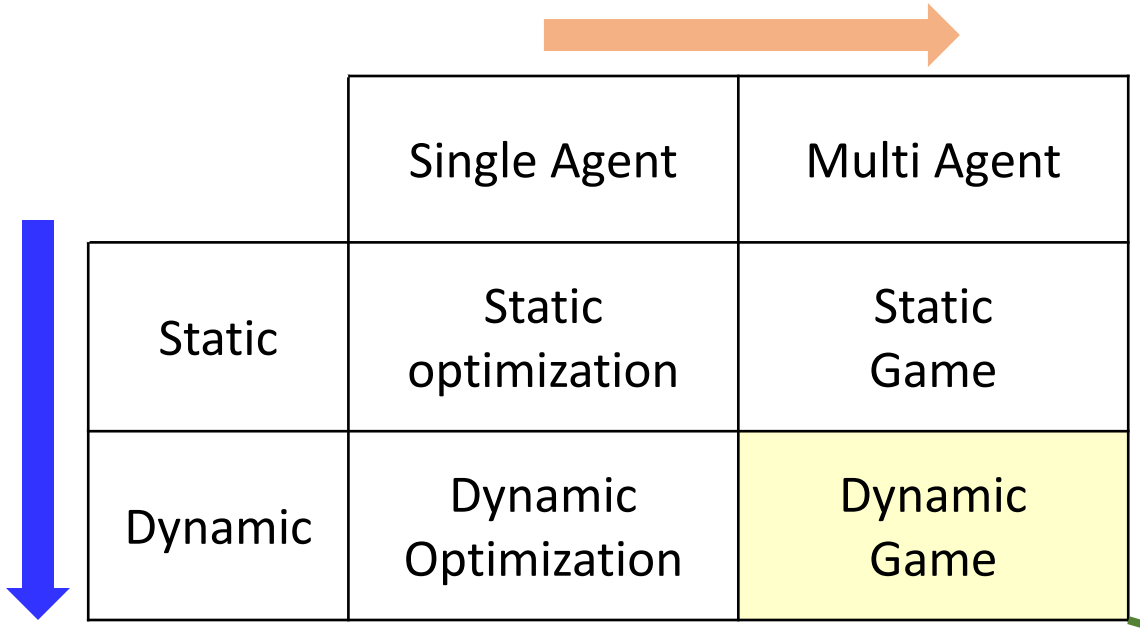
**Action space**

| **Time space** | **Model free** | Finite | Infinite |
|---|---|---|---|
| | Discrete | Multi-Agent Value-based RL | Multi-Agent Policy-based RL |
| | Continuous | | |

# 2. OVERVIEW : Static Game to Dynamic Game

**Equilibrium concept:**

-Nash; Zero-sum; Stackelberg; Correlated



| | Single Agent | Multi Agent |
|---|---|---|
| Static | Static optimization | Static Game |
| Dynamic | Dynamic Optimization | Dynamic Game |

**Dynamic optimization as a static optimization concept:**

-Minimum principle (necessary condition)

-Dynamic programming principle (sufficient condition)

-Need to specify information structure

$$
\begin{aligned}
&L^{1*} \triangleq L^1(u^{1*}; u^{2*}; \dots; u^{N*}) \le L^1(u^1; u^{2*}; \dots; u^{N*}),\\
&L^{2*} \triangleq L^2(u^{1*}; u^{2*}; \dots; u^{N*}) \le L^2(u^{1*}; u^2; \dots; u^{N*}),\\
&\dots\\
&L^{N*} \triangleq L^N(u^{1*}; u^{2*}; \dots; u^{N*}) \le L^N(u^{1*}; u^{2*}; \dots; u^{N*})
\end{aligned}
$$

**(Think in normal form game setting)**

# 2. OVERVIEW : Static Game to Dynamic Game

**Equilibrium concept:**

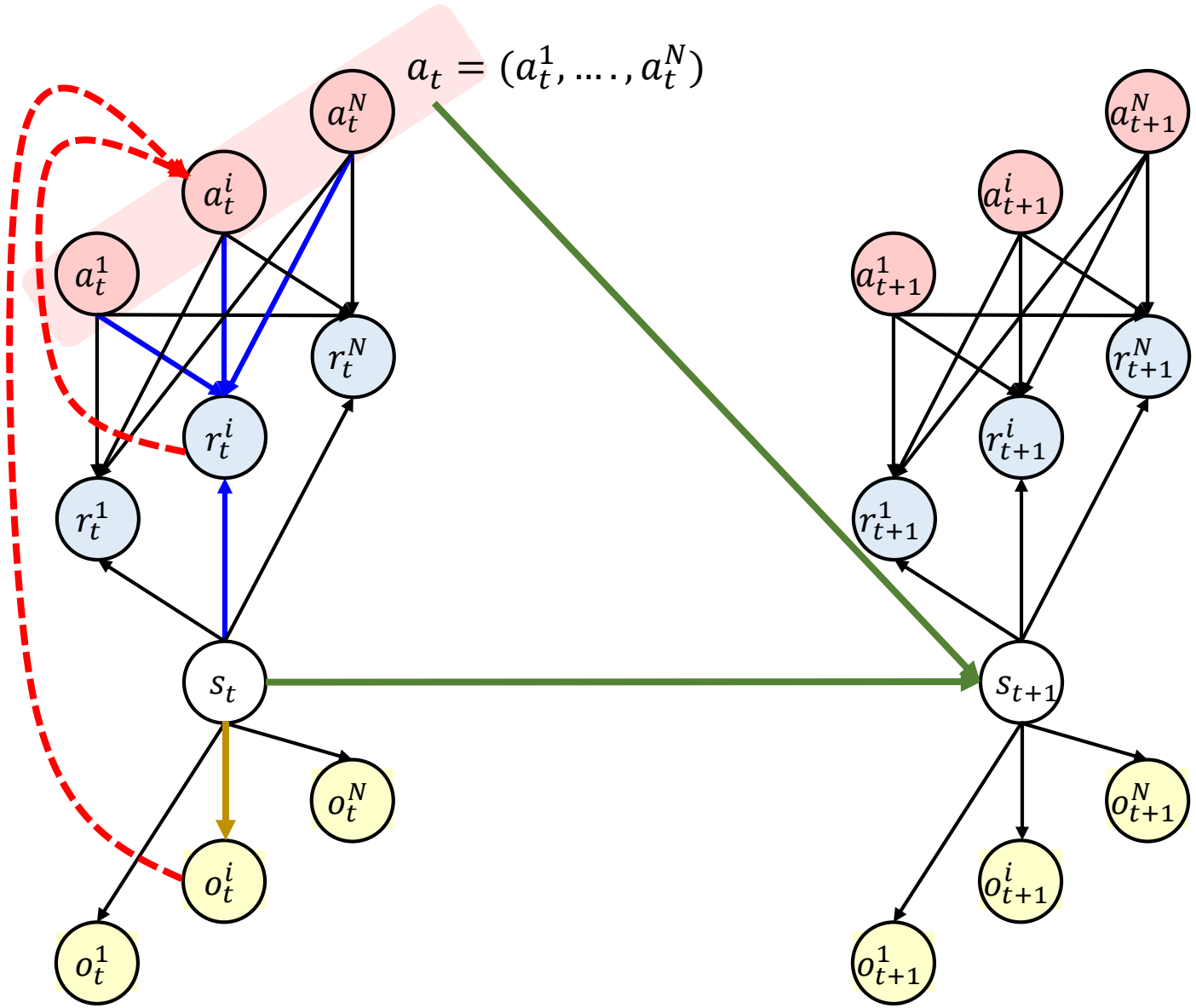
| (Dynamic) Information structure | Nash | Zero-sum | Stackelberg |
|---|---|---|---|
| **Open-loop (perfect state)** | Open-loop Nash-Strategy | Open-loop Zero-sum Strategy | |
| **Feedback (perfect state)** | Feedback Nash-Strategy | Feedback Zero-sum Strategy | |
| ⋮ | | | |

- We need to specify ***information structure***
  - ✓ Open-loop vs. close-loop (feedback)
  - ✓ Perfect vs. imperfect
- We need to ***equilibrium concept***
  - ✓ Nash, Zero-sum, Stackelberg, Correlated,…

**Equilibrium concept + information structure → solution method**

# 2. OVERVIEW : Static Game to Dynamic Game

## Stochastic Game Definition




- ***Transition:***

$$P(s_{t+1}|a_t^1, \ldots, a_t^N, s_t)$$

- ***Reward:***

$$r_i(s_t, a_t^1, \ldots, a_t^N)$$

- ***Observation:***

$$o_t^i = h^i(s_t)$$

- ***Decentralized Policy:***

$$\pi(s_t, a_t^1, \ldots, a_t^N) \approx \prod_{i=1}^{N} \pi_i(s_t, a_t^i)$$

$$\approx \prod_{i=1}^{N} \pi_i(o_t^i, a_t^i)$$

# 2. OVERVIEW : Static Game to Dynamic Game

## Stochastic Game Definition

**Definition (Stochastic game)**

A stochastic game is a tuple $(N, S, A, R, T)$, where

- $N$ is a finite set of $n$ players
- $S$ is a finite set of states (stage games),
- $A = A_1 \times \cdots \times A_n$, where $A_i$ is a finite set of actions available to player $i$,
- $T : S \times A \times S \longmapsto [0,1]$ is the transition probability function; $T(s, a, s')$ is the probability of transitioning from state $s$ to state $s'$ after joint action $a$,
- $R = r_1 \ldots, r_n$, where $r_i : S \times A \longmapsto \mathbb{R}$ is a real-valued payoff function for player $i$

# 2. OVERVIEW : Static Game to Dynamic Game

## Stochastic Game Definition : Transition

- All agents $(1, \dots, n)$ share the joint state $s$
- The transition equation is similar to the Markov Decision Process decision transition:

MDP : $$\sum_{s'} T(s, a, s') = \sum_{s'} p(s'|a, s) = 1 \,, \forall s \in S, \forall a \in A$$

SG: $$\sum_{s'} T(s, a_1, \dots, a_i, \dots, a_n, s') = \sum_{s'} p(s'|a_1, \dots, a_i, \dots, a_n, s) = 1$$

$$\forall s \in S, \forall a_i \in A_i, i = (1, \dots, n)$$

# 2. OVERVIEW : Static Game to Dynamic Game

## Stochastic Game Definition : Reward

- Reward function $r_i$ for agent $i$ depends on the current joint state $s$, the joint action $a = (a_1, \dots, a_n)$, and the next joint future state $s'$

MDP : $r(s, a, s')$

SG: $r_i(s, a_1, \dots, a_i, \dots, a_n, s')$

# 2. OVERVIEW : Static Game to Dynamic Game

## Stochastic Game Definition : State Value

- As we did in MDP, we can define value function
- Let $\pi_i$ be the policy of player $i \in N$. For a given initial state $s$, the value of state $s$ for player $i$ is defined as

$$V_i(s, \pi_1, \dots, \pi_i, \dots, \pi_n) = \sum_{t=0}^{\infty} \gamma^t E\left[r_{i,t} | \pi_1, \dots, \pi_i, \dots, \pi_n, s_0 = s\right]$$

  - The accumulated rewards depends on the policies of other agents
  - The immediate reward is expressed as expected value, because some policy $\pi_i$ can be stochastic

- In a *discounted stochastic game*, the objective of each player is to maximize the discounted sum of rewards, with discount factor $\gamma \in [0,1)$.

# 2. OVERVIEW : Static Game to Dynamic Game

## Stochastic Game Definition : Nash Equilibrium Strategy

**Definition (Nash equilibrium policy in Stochastic game)**

In a stochastic game $\Gamma = (N, S, A, R, T)$, a Nash equilibrium policy is a tuple of $n$ policies $\pi^* = (\pi_1^*, \ldots, \pi_n^*)$ such that for all $s \in S$ and $i = 1, \ldots n$,

$$V_i(s, \pi_1^*, \ldots, \pi_i^*, \ldots, \pi_n^*) \geq V_i(s, \pi_1^*, \ldots, \pi_i, \ldots, \pi_n^*) \text{ for all } \pi_i \in \Pi_i$$

- A Nash equilibrium is a joint policy where each agent's policy is a best response to the others
- For a stochastic game, each agent's policy is defined over the entire time horizon of the game
- **A Nash equilibrium state value** $V_i(s, \pi_1^*, \ldots, \pi_n^*)$is defined as the sum of discounted rewards when all agents following the Nash equilibrium policies $\pi^* = (\pi_1^*, \ldots, \pi_n^*)$
  - Notations: $V_i^*(s) = V_i^{\pi^*}(s) = V_i(s, \pi_1^*, \ldots, \pi_n^*)$

# 2. OVERVIEW : Static Game to Dynamic Game

## Single Agent Case

**Q-values**

$Q^{\pi}(s,a)$ : The expected utility of taking action $a$ from state $s$, and then following policy $\pi$

$$Q^{\pi}(s,a) = \mathbb{E}_{\pi}\left(\sum_{k=0}^{\infty}\gamma^{k} r_{t+k} \,\middle|\, S_t = s, A_t = a\right)$$

**Optimal Q-values**

$$Q^{*}(s,a) = \max_{\pi} Q^{\pi}(s,a)$$

$$= \max_{\pi} \mathbb{E}[r(s,a,s') + \gamma V^{\pi}(s') | s_t = s, a_t = a]$$

$$= \mathbb{E}\left[r(s,a,s') + \gamma \max_{\pi} V^{\pi}(s') \,|s_t = s, a_t = a\right]$$

$$= \mathbb{E}[r(s,a,s') + \gamma V^{*}(s') | s_t = s, a_t = a] \qquad \because V^{*}(s') \equiv \max_{\pi} V^{\pi}(s')$$

$$= \mathbb{E}\left[r(s,a,s') + \gamma \max_{a'} Q^{*}(s',a') \,|s_t = s, a_t = a\right] \qquad \because V^{*}(s') \equiv \max_{a'} Q^{*}(s',a')$$

Optimization over policy becomes greedy optimization over action!

- Optimal Q-value for a single-agent is the sum of the current reward and future discounted rewards when playing the optimal strategy from the next period onward

# 2. OVERVIEW : Static Game to Dynamic Game

## Multi Agent Case

**Q-values for agent $i$**

$Q_i^{\pi}(s, a_1, \dots, a_n)$ : The expected utility of taking joint action $(a_1, \dots, a_n)$ from state $s$, and then following policy $\pi$

$$Q_i^{\pi}(s, a_1, \dots, a_n) = \mathbb{E}_{\pi}\left(\sum_{k=0}^{\infty} \gamma^k r_{i,t+k} \,\middle|\, S_t = s, A = (a_1, \dots, a_n)\right)$$

**Optimal Q-values for agent $i$**

$$Q_i^*(s, a_1, \dots, a_n) = \max_{\pi_1, \dots, \pi_n} Q_i^{\pi}(s, a_1, \dots, a_n)$$

$$= \max_{\pi_1, \dots, \pi_n} \mathbb{E}[r_i(s, a_1, \dots, a_n, s') + \gamma V_i(s', \pi_1, \dots, \pi_n) | s_t = s, a_t = (a_1, \dots, a_n)]$$

$$= \mathbb{E}\left[r_i(s, a_1, \dots, a_n, s') + \gamma \max_{\pi_1, \dots, \pi_n} V_i(s', \pi_1, \dots, \pi_n) | s_t = s, a_t = (a_1, \dots, a_n)\right]$$

$$= \mathbb{E}[r_i(s, a_1, \dots, a_n, s') + \gamma V_i(s', \pi_1^*, \dots, \pi_n^*) | s_t = s, a_t = (a_1, \dots, a_n)]$$

$$= \mathbb{E}\left[r_i(s, a_1, \dots, a_n, s') + \gamma \max_{a_1, \dots, a_n} Q_i^*(s', a_1, \dots, a_n) \,|s_t = s, a_t = (a_1, \dots, a_n)\right]$$

- Optimal Q-value for agent $i$ occurs when all agents are jointly coordinating to maximize agent $i$'s accumulated reward
  - ➢ Rarely occurs! : Optimal Q-values for all agents are not achieved simultaneously

# 2. OVERVIEW : Static Game to Dynamic Game

## Multi Agent Case

**Q-values for agent $i$**

$Q_i^{\pi}(s, a_1, \dots, a_n)$ : The expected utility of taking joint action $(a_1, \dots, a_n)$ from state $s$, and then following policy $\pi$

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**Optimal Q-values for agent $i$**

$$Q_i^*(s, a_1, \dots, a_n) = \max_{\pi_1, \dots, \pi_n} Q_i^{\pi}(s, a_1, \dots, a_n)$$

$$= \max_{\pi_1, \dots, \pi_n} \mathbb{E}[r_i(s, a_1, \dots, a_n, s') + \gamma V_i(s', \pi_1, \dots, \pi_n) | s_t = s, a_t = (a_1, \dots, a_n)]$$

$$= \mathbb{E}\left[r_i(s, a_1, \dots, a_n, s') + \gamma \max_{\pi_1, \dots, \pi_n} V_i(s', \pi_1, \dots, \pi_n) | s_t = s, a_t = (a_1, \dots, a_n)\right]$$

$$= \mathbb{E}[r_i(s, a_1, \dots, a_n, s') + \gamma V_i(s', \pi_1^*, \dots, \pi_n^*) | s_t = s, a_t = (a_1, \dots, a_n)]$$

$$= \mathbb{E}\left[r_i(s, a_1, \dots, a_n, s') + \gamma \max_{a_1, \dots, a_n} Q_i^*(s', a_1, \dots, a_n) | s_t = s, a_t = (a_1, \dots, a_n)\right]$$

- Optimal Q-value for agent $i$ occurs when all agents are jointly coordinating to maximize agent $i$'s accumulated reward
  - ➢ Rarely occurs! : Optimal Q-values for all agents are not achieved simultaneously

# 2. OVERVIEW : Static Game to Dynamic Game

## Multi Agent Case

**Q-values for agent $i$**

$Q_i^{\pi}(s, a_1, \dots, a_n)$ : The expected utility of taking joint action $(a_1, \dots, a_n)$ from state $s$, and then following policy $\pi$

$$Q_i^{\pi}(s, a_1, \dots, a_n) = \mathbb{E}_{\pi}\left(\sum_{k=0}^{\infty} \gamma^k r_{i,t+k} \,\middle|\, S_t = s, A = (a_1, \dots, a_n)\right)$$

**Nash Q-values for agent $i$**

$$Q_i^*(s, a_1, \dots, a_n) = \underset{\pi_1,\dots,\pi_n}{\text{Nash}}\ Q_i^{\pi}(s, a_1, \dots, a_n)$$

$$= \underset{\pi_1,\dots,\pi_n}{\text{Nash}}\ \mathbb{E}[r_i(s, a_1, \dots, a_n, s') + \gamma V_i(s', \pi_1, \dots, \pi_n) | s_t = s, a_t = (a_1, \dots, a_n)]$$

$$= \mathbb{E}\left[r_i(s, a_1, \dots, a_n, s') + \gamma \underset{\pi_1,\dots,\pi_n}{\text{Nash}}\ V_i(s', \pi_1, \dots, \pi_n) | s_t = s, a_t = (a_1, \dots, a_n)\right]$$

$$= \mathbb{E}[r_i(s, a_1, \dots, a_n, s') + \gamma V_i(s', \pi_1^*, \dots, \pi_n^*) | s_t = s, a_t = (a_1, \dots, a_n)]$$

$$= \mathbb{E}\left[r_i(s, a_1, \dots, a_n, s') + \gamma \underset{a_1,\dots,a_n}{\text{Nash}}\ Q_i^*(s', a_1, \dots, a_n) \,|\, s_t = s, a_t = (a_1, \dots, a_n)\right]$$

Equilibrium over policies becomes stage game equilibrium over action!

- A **Nash Q value** $Q_i^*(s, a_1, \dots, a_n)$ is the expected sum of discounted rewards when all agents take the joint action $a = (a_1, \dots, a_n)$ at given state $s$ and follow a Nash equilibrium strategy $\pi^* = (\pi_1^*, \dots, \pi_n^*)$

# 2. OVERVIEW : Static Game to Dynamic Game

## Nash Bellman Equation

**For single agent:**

$$V^*(s') = \max_a Q^*(s', a)$$

$$Q^*(s, a) = \mathbb{E}[r(s, a, s') + \gamma V^*(s') | s_t = s, a_t = a]$$

$$= \mathbb{E}\left[r(s, a, s') + \gamma \max_{a'} Q^*(s', a') \, | s_t = s, a_t = a\right]$$

**For multiple agents:**

$$V_i(s', \pi_1^*, \dots, \pi_n^*) = \underset{a_1, \dots, a_n}{\text{Nash}} \, Q_i^*(s', a_1, \dots, a_n)$$

$$Q_i^*(s, a_1, \dots, a_n) = \mathbb{E}[r(s, a, s') + \gamma V_i(s', \pi_1^*, \dots, \pi_n^*) | s_t = s, a_t = a]$$

$$= \mathbb{E}\left[r(s, a, s') + \gamma \underset{a_1, \dots, a_n}{\text{Nash}} \, Q_i^*(s', a_1, \dots, a_n) \, | s_t = s, a_t = (a_1, \dots, a_n)\right]$$

# 2. OVERVIEW : Static Game to Dynamic Game

## Nash Bellman Equation

**For multiple agents:**

$$V_i(s', \pi_1^*, \dots, \pi_n^*) = \underset{a_1,\dots,a_n}{\text{Nash}}\ Q_i^*(s', a_1, \dots, a_n)$$

$$Q_i^*(s, a_1, \dots, a_n) = \mathbb{E}[r(s, a, s') + \gamma V_i(s', \pi_1^*, \dots, \pi_n^*) | s_t = s, a_t = a]$$

$$= \mathbb{E}\left[r(s, a, s') + \gamma \underset{a_1,\dots,a_n}{\text{Nash}}\ Q_i^*(s', a_1, \dots, a_n) \,|\, s_t = s, a_t = (a_1, \dots, a_n)\right]$$

- Nash **equilibrium** Q value $\underset{a_1,\dots,a_n}{\text{Nash}}\ Q_i^*(s', a_1, \dots, a_n)$ can be computed by computing player $i$th Nash equilibrium value for the stage game $[Q_i^*(s', a_1, \dots, a_n), \dots, Q_n^*(s', a_1, \dots, a_n)]$

  - for example when $i = 1,2$

| | $a_2^1$ | $a_2^2$ |
|---|---|---|
| $a_1^1$ | $Q_1^*(s', a_1^1, a_2^1), Q_2(s', a_1^1, a_2^1)$ | $Q_1^*(s', a_1^1, a_2^2), Q_2(s', a_1^1, a_2^2)$ |
| $a_1^2$ | $Q_1^*(s', a_1^2, a_2^1), Q_2(s', a_1^2, a_2^1)$ | $Q_1^*(s', a_1^2, a_2^2), Q_2(s', a_1^2, a_2^2)$ |

Nash equilibrium

# 2. OVERVIEW : Static Game to Dynamic Game

## Nash Bellman Equation (simple notation)

$$r_i(s, a_1, \dots, a_n, s') \rightarrow r_i(s, \vec{a}, s')$$

$$V_i(s, \pi_1^*, \dots, \pi_n^*) \rightarrow V_i^*(s)$$

$$Q_i^*(s, a_1, \dots, a_n) \rightarrow Q_i^*(s', \vec{a}\,)$$

$$\begin{aligned} Q_i^*(s, a_1, \dots, a_n) &= \mathbb{E}[r_i(s, a_1, \dots, a_n, s') + \gamma V_i(s', \pi_1^*, \dots, \pi_n^*) | s_t = s, a_t = (a_1, \dots, a_n)] \\ &= \mathbb{E}\left[r_i(s, a_1, \dots, a_n, s') + \gamma \underset{a_1,\dots,a_n}{\text{Nash}}\, Q_i^*(s', a_1, \dots, a_n) \,|s_t = s, a_t = (a_1, \dots, a_n)\right] \end{aligned}$$

$$\begin{aligned} Q_i^*(s', \vec{a}\,) &= \mathbb{E}[r_i(s, \vec{a}, s') + \gamma V_i^*(s') | s_t = s, a_t = \vec{a}] \\ &= \mathbb{E}[r_i(s, \vec{a}, s') + \gamma\, \text{Nash}\, Q_i^*(s') \,|s_t = s, a_t = \vec{a}] \end{aligned}$$

$$\underset{a_1,\dots,a_n}{\text{Nash}}\, Q_i^*(s', a_1, \dots, a_n) = Q_i^*(s', \vec{a}_{NE}) = \text{Nash}\, Q_i^*(s')$$

# 2. OVERVIEW : Static Game to Dynamic Game

## Nash Bellman Equation (simple notation)

- If we know **Nash equilibrium policy** $\pi^* = (\pi_1^*, \dots, \pi_n^*)$, we can compute the Nash equilibrium state values $V_i(s, \pi_1^*, \dots, \pi_n^*)$ (i.e., policy evaluation)

$$V_i(s, \pi_1^*, \dots, \pi_n^*) = \sum_{t=0}^{\infty} \gamma^t E\left[r_{i,t} | \pi_1^*, \dots, \pi_n^*, s_0 = s\right]$$

- If we know **Nash equilibrium state value** $V_i(s, \pi_1^*, \dots, \pi_n^*)$ and transition models $p(s'|s, a_1, \dots, a_n)$, we can compute **Nash Q-values (i.e., Nash Q-function)** using backward induction (analytical approach)

$$\begin{aligned} Q_i^*(s, a_1, \dots, a_n) &= \mathbb{E}[r_i(s, a_1, \dots, a_n, s') + \gamma V_i(s', \pi_1^*, \dots, \pi_n^*) | s_t = s, a_t = (a_1, \dots, a_n)] \\ &= r_i(s, a_1, \dots, a_n, s') + \sum_{s'} p(s'|s, a_1, \dots, a_n) V_i(s', \pi_1^*, \dots, \pi_n^*) \end{aligned}$$

# 2. OVERVIEW : Static Game to Dynamic Game

## Nash Bellman Equation for State Action Value

**Definition (Optimal Q-function)**

Optimal Q function is defined as

$$Q^*(s,a) = r(s,a,s') + \gamma \sum_{s' \in S} p(s'|s,a\,) V^*(s')$$

- $V^*(s'\,) = \max_a Q^*(s',a)$
- With **optimum** policy $\pi^*(s) = \operatorname{argmax}_a Q^*(s,a)$

**Definition (Nash Q-function)**

Nash-Q function is defined as

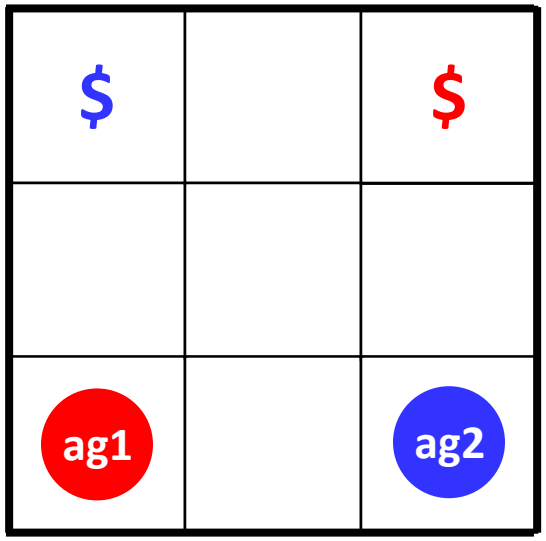
$$Q_i^*(s,\vec{a}) = r_i(s,\vec{a},s') + \gamma \sum_{s' \in S} p(s'|s,\vec{a}) V_i^*(s')$$

- $V_i^*(s') = \text{Nash}\, Q_i^*(s')$ is Nash equilibrium value that can be computed by solving the following state game

$$(Q_1^*(s',\vec{a})\,, \ldots, Q_n^*(s',\vec{a})\,)$$

# 2. OVERVIEW : Static Game to Dynamic Game

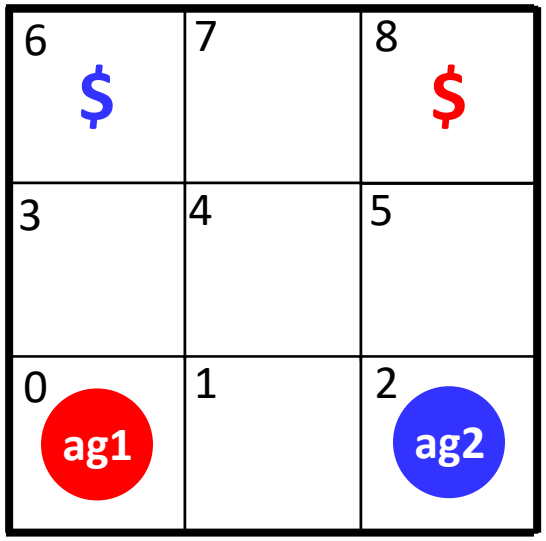
## Stochastic Game Example




- Grid game has deterministic moves
- Two agents start from respective lower corners, trying to reach their goal cells in the top row
- Agent can move only one cell a time, and in four possible directions: Left, Right, Up, Down
- If two agents attempt to move into the same cell (excluding a goal cell), they are bounced back to their previous cells
- The game ends as soon as an agent reaches its goal
  - The objective of an agent in this game is therefore to reach its goal with a minimum No. of steps
- Agents do not know
  - the locations of their goals at the beginning of the learning period
  - their own and the other agents' payoff functions
- Agent choose their action simultaneously and observe
  - the previous actions of both agents and the current joint state
  - the immediate rewards after both agents choose their actions

# 2. OVERVIEW : Static Game to Dynamic Game

## Stochastic Game Example



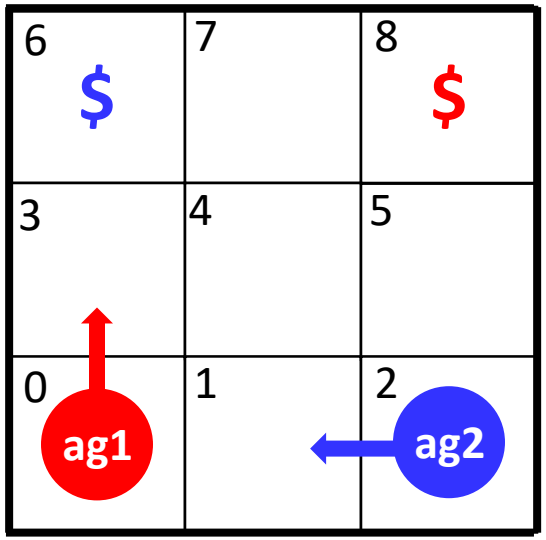

- The action space of agent $i$, $i = 1,2$, is $A_i = \{Left, Right, Down, Up\}$
- The sate space is $S = \{(0,1), (0,2), \dots, (8,7)\}$
  - $s = (l_1, l_2)$ represents the agents' joint location
  - $l_i \in \{0, 2, \dots, 8\}$ is the indexed location
- The reward function is, for $i = 1, 2$,

$$r_i = \begin{cases} 100 & \text{if } L(l_i, a_i) = Goal_i \\ -1 & \text{if } L(l_1, a_1) = L(l_2, a_2) \text{ and } L(l_i, a_i) \neq Goal_i \text{ for } i = 1,2 \\ 0 & \text{otherwise} \end{cases}$$

$l_i' = L(l_i, a_i)$ is the next location when executing $a_i$ at $l_i$

# 2. OVERVIEW : Static Game to Dynamic Game

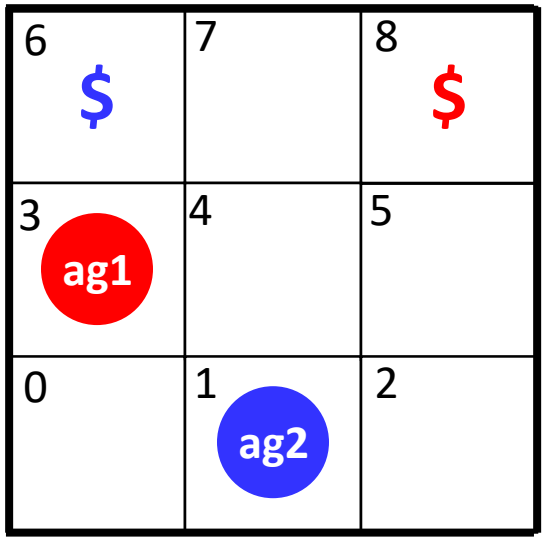
## Stochastic Game Example



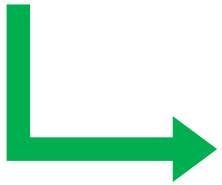

- $s = (l_1, l_2) = (0,2)$
- $a = (a_1, a_2) = (Up, Left)$

# 2. OVERVIEW : Static Game to Dynamic Game

## Stochastic Game Example

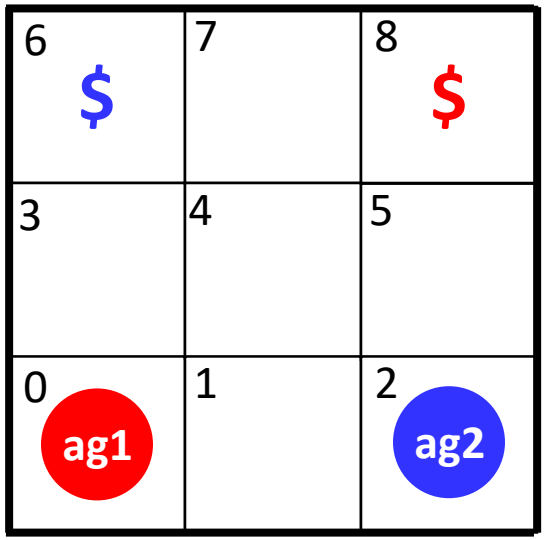



- $s = (l_1, l_2) = (0,2)$
- $a = (a_1, a_2) = (Up, Left)$



- $s' = \big(L(l_1, a_1), L(l_2, a_2)\big) = (3, 1)$
- $r_1 = 0$
- $r_2 = 0$

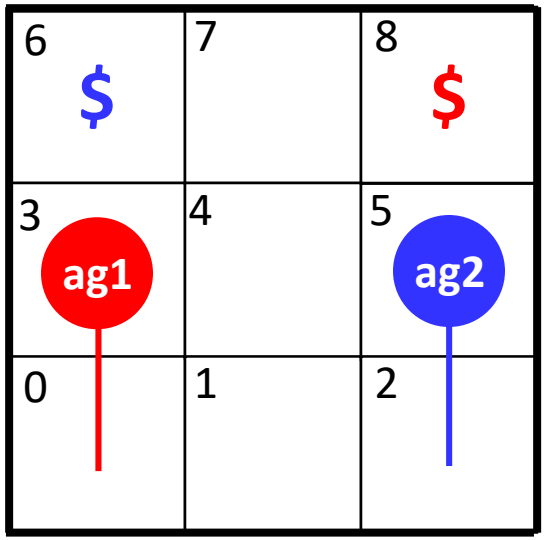
## Grid Game 1 represented as stochastic game




**Nash Equilibrium strategies**

# 2. OVERVIEW : Static Game to Dynamic Game

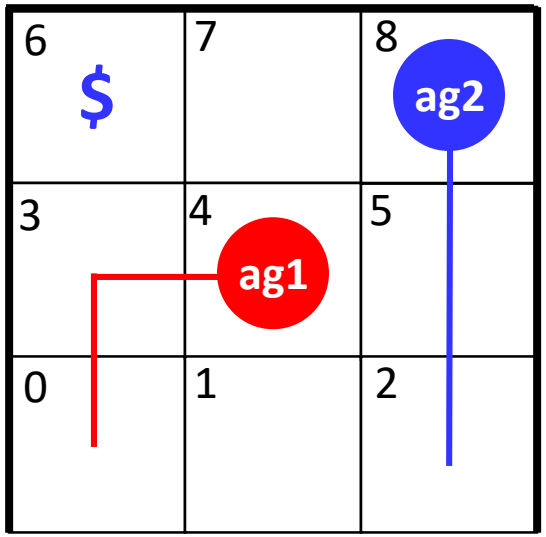
## Stochastic Game Example




Nash Equilibrium strategies

# 2. OVERVIEW : Static Game to Dynamic Game

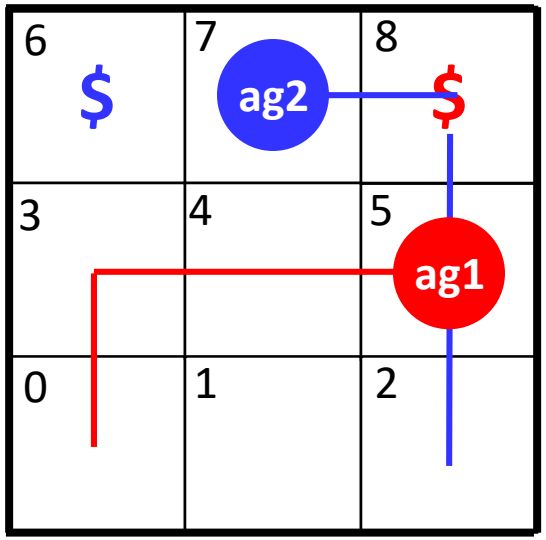
## Stochastic Game Example




Nash Equilibrium policies

# 2. OVERVIEW : Static Game to Dynamic Game

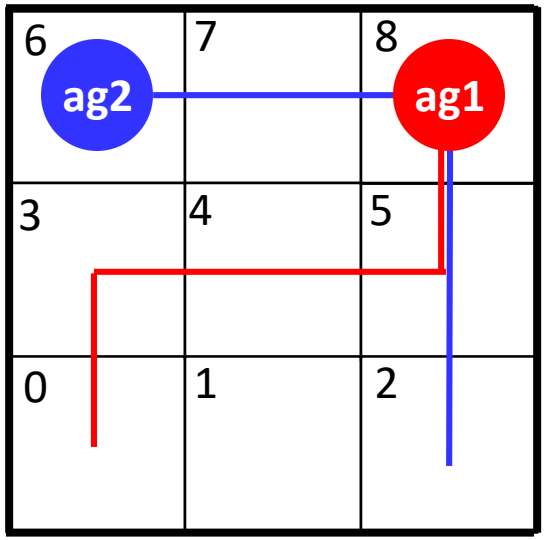
## Stochastic Game Example




Nash Equilibrium policies

# 2. OVERVIEW : Static Game to Dynamic Game

## Stochastic Game Example



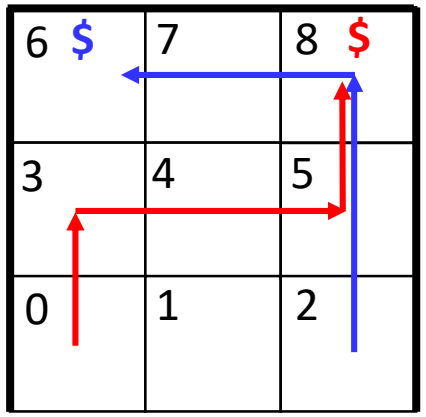

Nash Equilibrium policies

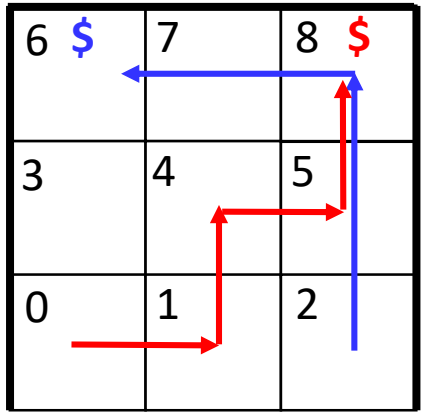
| State $s$ | $\pi_1(s)$ |
|---|---|
| $(0, any)$ | $U$ |
| $(3, any)$ | $Right$ |
| $(4, any)$ | $Right$ |
| $(5, any)$ | $Up$ |

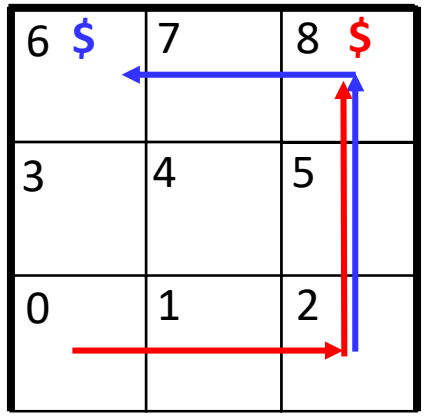
Nash strategy for agent 1

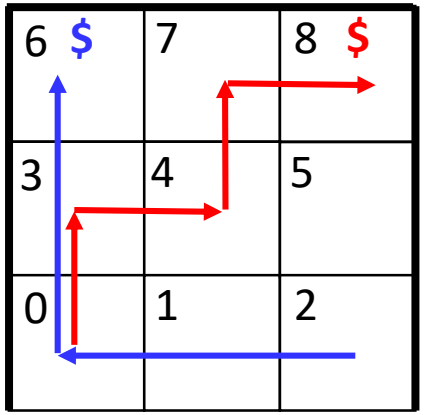
# 2. OVERVIEW : Static Game to Dynamic Game

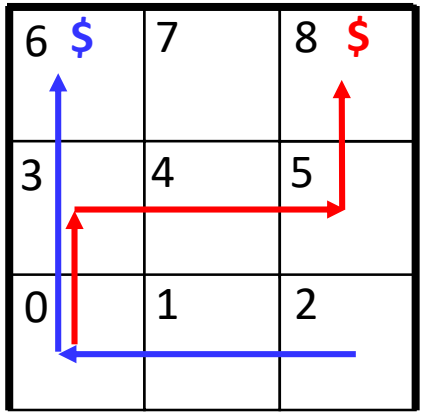
## Stochastic Game Example



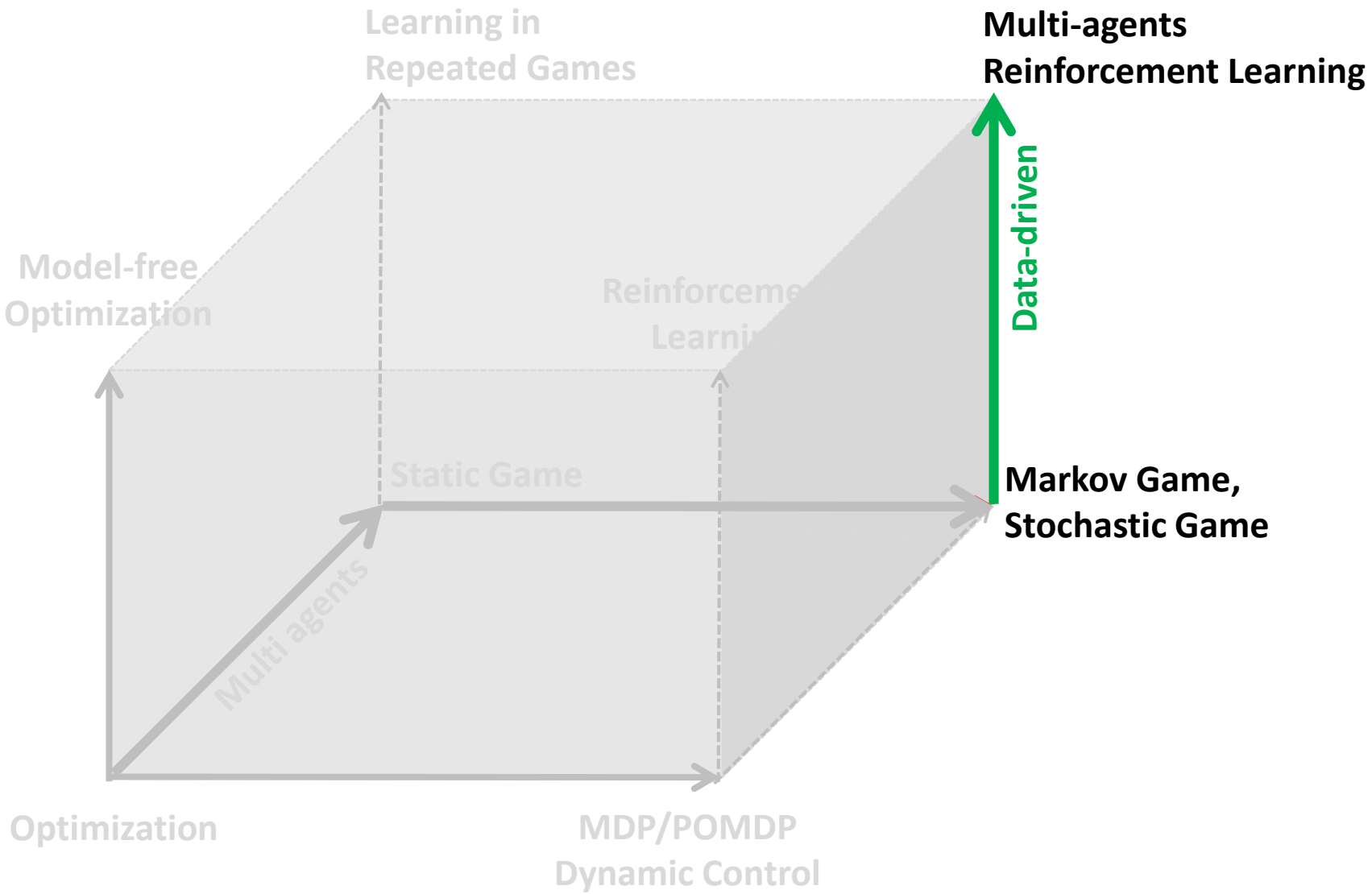



# 2. OVERVIEW : Dynamic Game to Multi-Agent Reinforcement Learning



# 2. OVERVIEW : Dynamic Game to Multi-Agent Reinforcement Learning

## Multi-Agent Reinforcement Q Learning Template

MultiQ(StochastiGame, $f, \gamma, \alpha, T$)

| | |
|---|---|
| Inputs | equilibrium selection function $f$ |
| | discounting factor $\gamma$ |
| | learning rate $\alpha$ |
| | total training time $T$ |
| Outputs | state − value functions $V_i^*$ |
| | action − value functions $Q_i^*$ |
| Initialize | $s, a_1, \dots, a_n$ and $Q_1, \dots, Q_n$ |

for $t = 1{:}T$

1. simulate actions $\vec{a} = (a_1, \dots, a_n)$in state $s$
2. observe rewards $r_1, \dots, r_n$ and next state $s'$
3. for $i = 1$ to $n$ (for each agent)
   (a) $V_i(s') = f_i(Q_1(s', \vec{a}), \dots, Q_n(s', \vec{a}))$
   (b) $Q_i(s, \vec{a}) = (1 - \alpha_i)Q_i(s, \vec{a}) + \alpha_i[r_i + \gamma V_i(s')]$
4. agent choose actions $a_1', \dots, a_n'$
5. $s = s', a_1 = a_1', \dots, a_n = a_n'$
6. adjust learning rate $\alpha = (\alpha_1, \dots, \alpha_n)$

# 2. OVERVIEW : Dynamic Game to Multi-Agent Reinforcement Learning

## Multi-Agent Reinforcement Learning Overview

Equilibrium selection function $f$ : $V_i(s') = f_i(Q_1(s', \vec{a}), \ldots, Q_n(s', \vec{a}))$

- We going to study the following equilibrium concept:
  - Value function based (Bellman function based)
    - Single agent Q-learning
    - Independent Q learning by multiple agents
    - Nash-Q learning (Hu and Wellman 1998)
    - Minmax-Q learning (Littman 1994)
    - Friend-or-Foe Q learning (Littman 2001)
    - Correlated Q learning (Greenwald and Hall 2003)
  - Policy gradient methods (direct search for policy)
    - Wind-or-Learn-Fast Policy Hill Climbing (WOLF-PHC) (Policy gradient method)
  - Deep learning based MARL (limited to a team game or cooperative game)
    - Learning to cooperate
    - Learning to communicate

# 2. OVERVIEW : Dynamic Game to Multi-Agent Reinforcement Learning

## Nash Q-Learning

- Q-learning directly find optimal Q-function (Q table) instead of optimum finding policy $\pi^*$

- Single agent Q-learning:
  - Iteratively find optimal Q values $Q^*(s,a)$ (table)

$$Q(s,a) = (1-\alpha)Q(s,a) + \alpha[r(s,a) + \gamma V(s')]$$
$$= (1-\alpha)Q(s,a) + \alpha\left[r(s,a) + \gamma \max_a Q(s',a)\right]$$

- Nah Q-learning:
  - Iteratively find Nash-Q values Nash $Q_i^*(s,a_1,\dots,a_n)$ (table for each agent)

$$Q_i(s,\vec{a}) = (1-\alpha)Q_i(s,\vec{a}) + \alpha[r_i + \gamma V_i(s')]$$
$$= (1-\alpha)Q_i(s,\vec{a}) + \alpha[r_i + \gamma \,\text{Nash}\, Q_i(s')]$$

$Q_i(s',\vec{a})$ : Nash-Q values (state-action values)
Nash $Q_i(s')$: **Nash equilibrium value** of Nash-Q values

- the learning agent updates its Nash Q-value depending on the joint strategy of all the players and not only its own expected payoff.

# 2. OVERVIEW : Dynamic Game to Multi-Agent Reinforcement Learning

## MinMax Q-Learning

For agent $i = 1{:}2$

$$\text{(a) } V_i(s') = f_i(Q_1(s', a_i, a_{-i}), Q_2(s', a_i, a_{-i}),) = \max_{\pi_i(s',\cdot)} \min_{a_{-i} \in A_{-i}} \sum_{a_i \in A_i} Q_i(s', a_i, a_{-i}) \pi_i(s', a_i)$$

$$= \text{Maxmin}\, Q_i(s')$$

$$\text{(b) } Q_i(s, a_i, a_{-i}) = (1-\alpha) Q_i(s, a_i, a_{-i}) + \alpha[r_i + \gamma V_i(s')]$$

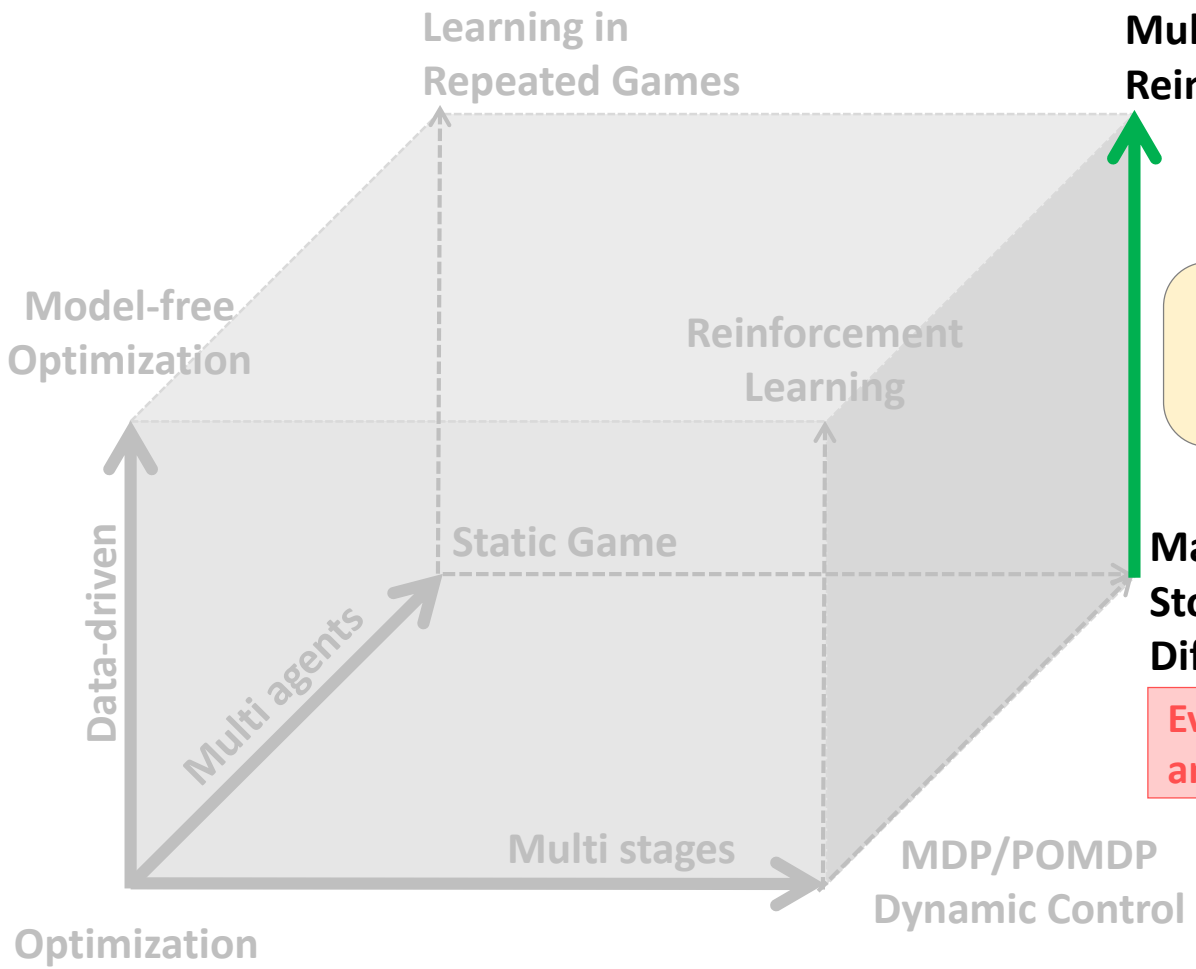
$$= (1-\alpha) Q_i(s, a_i, a_{-i}) + \alpha[r_i + \gamma\, \text{Maxmin}\, Q_i(s')]$$

Action selection strategy:

$$\pi_i(s',\cdot) = \underset{\pi_i(s',\cdot)}{\text{argmax}} \min_{a_{-i} \in A_{-i}} \sum_{a_i \in A_i} Q_i(s', a_i, a_{-i}) \pi_i(s, a_i)$$

- Note that when computing the maxmin value, each agent can consider only its own action value function $Q_i(s', a_i, a_{-i})$
- But, still each agent need to track the action taken by the other agent for updating

# 3. Deep MARL




**Multi-agents Reinforcement Learning**

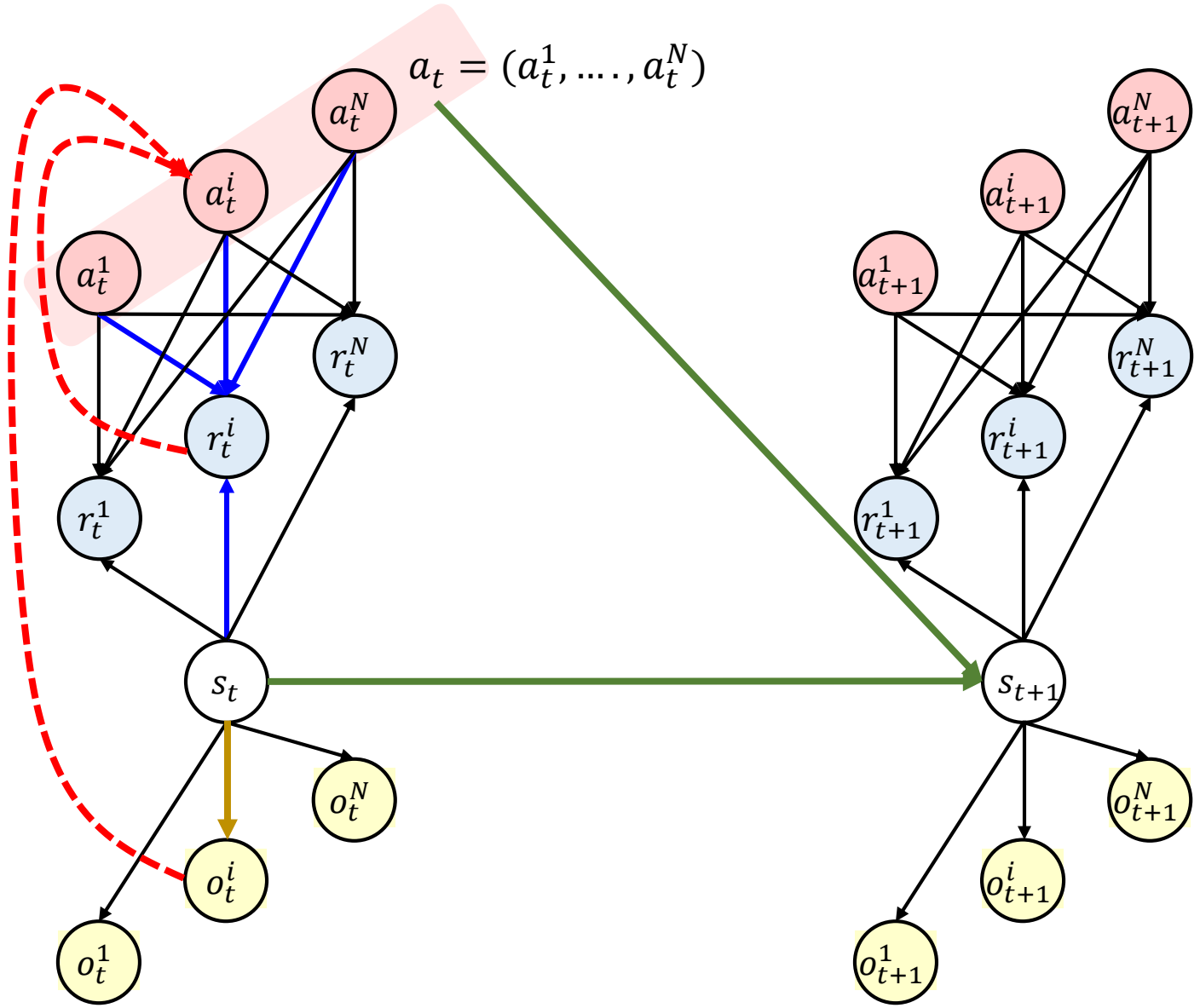
**(Control/OR + Game Theory + AI)**

How to represent a target problem so that it can be ***easily learned***, ***better generalized***, and ***intuitively interpreted***?

**Markov Game,**
**Stochastic Game**
**Differential Game**

**Even though model is known,**
**analytical solutions are only available to specific cases**

# 3. Deep MARL: Stochastic Game




- ***Transition:***

$$P(s_{t+1}|a_t^1, \ldots, a_t^N, s_t)$$

- ***Reward:***

$$r_t^i(s_t, a_t^1, \ldots, a_t^N)$$

- ***Observation:***

$$o_t^i = h^i(s_t)$$

- ***Decentralized Policy:***

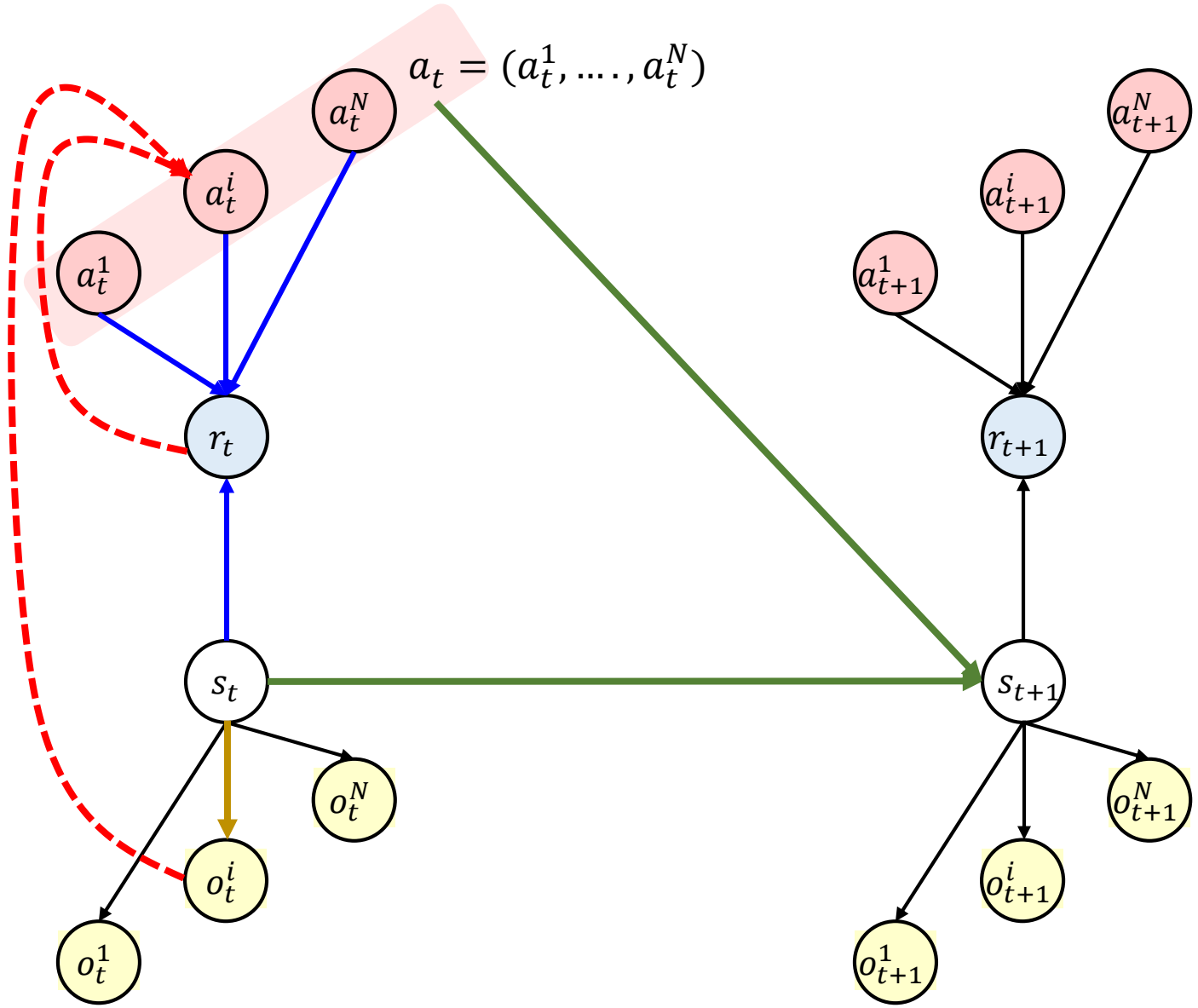
$$\pi(s_t, a_t^1, \ldots, a_t^N) \approx \prod_{i=1}^{N} \pi_i(s_t, a_t^i)$$

$$\approx \prod_{i=1}^{N} \pi_i(o_t^i, a_t^i)$$

- ***Objective of agent i:***

$$\max_{\pi_i} E\left[\sum_{t=1}^{T} r_t^i(s_t, a_t^1, \ldots, a_t^N)\right]$$

# 3. Deep MARL: Team Game




- ***Transition:***

$$P(s_{t+1}|a_t^1, \ldots, a_t^N, s_t)$$

- ***Reward:***

$$r_t(s_t, a_t^1, \ldots, a_t^N)$$

- ***Observation:***

$$o_t^i = h^i(s_t)$$

- ***Decentralized Policy:***

$$\pi(s_t, a_t^1, \ldots, a_t^N) \approx \prod_{i=1}^{N} \pi_i(s_t, a_t^i)$$

$$\approx \prod_{i=1}^{N} \pi_i(o_t^i, a_t^i)$$

- ***Objective of agent i:***

$$\max_{\pi_i} E\left[\sum_{t=1}^{T} r_t(s_t, a_t^1, \ldots, a_t^N)\right]$$

# 3. Deep MARL: MARL approach to Team Game

| | | Training | |
|---|---|---|---|
| | | Centralized Training | Decentralized Training |
| Execution | Centralized Execution | MDP | |
| | Decentralized Execution | Dec-(PO)MDP | ? |

# 3. Deep MARL: MARL approach to Team Game

| | | Training | |
|---|---|---|---|
| | | **Centralized Training** | **Decentralized Training** |
| Execution | Centralized Execution | MDP | |
| | Decentralized Execution | Dec-(PO)MDP | ? |

- **Centralized Training** vs **Decentralized Training**
  - In centralized training, we assume a central learner who can access all the global information $s = (s_1, \dots, s_N)$ and $a = (a_1, \dots, a_N)$ for modeling mutual interactions among agents
  - In decentralized training, each agent has a limited information about the global state and the joint action

# 3. Deep MARL: MARL approach to Team Game

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- Centralized Training vs Decentralized Training
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  - In decentralized training, each agent has a limited information about the global state and the joint action

- **Centralized Exestuation** vs **Decentralized Execution**
  - In centralized exestuation, a joint action $a = (a_1, \dots, a_N)$ is selected by a central agent
  - In centralized exestuation, each agent selects individual action to compose a joint action:

$$\pi(s, a) \approx \prod_{i=1}^{N} \pi_i(\text{Information of agent } i, a_i)$$

# 3. Deep MARL: MARL approach to Team Game

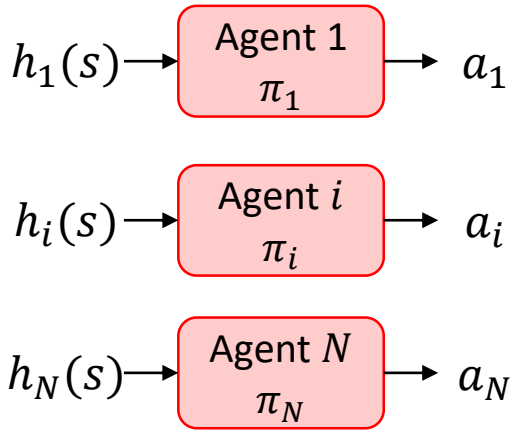
| | | Training | |
|---|---|---|---|
| | | **Centralized Training** | Decentralized Training |
| Execution | Centralized Execution | MDP | |
| | **Decentralized Execution** | Dec-(PO)MDP (CTDE) | ? |

$s = (s_1, \dots, s_N)$
$a = (a_1, \dots, a_N)$
$r = f(s, a)$

$Q(s, a)$

$\pi_1, \dots, \pi_i, \dots, \pi_N$

**Centralized Training**




**Decentralized Execution**

- **Centralized Training** and **Decentralized Execution** (CTDE) is a widely adopted to overcome the non-stationarity problem when training multi-agent systems (Oliehoek et al., 2008; Kraemer & Banerjee, 2016; Jorge et al., 2016; Foerster et al., 2018)
- CTDE enables to leverage the observations of each agent, as well as their actions, to better model the interactions among agents during training.
- Depending on the information structure $h_i(s)$ of each agent has in execution, there are various approaches in CTDE
  - Local observations: each agent can access only to its local (state) observation
  - Global observations: each agent can access to the global state (observation)

# 3. Deep MARL: CTDE framework for Team Game

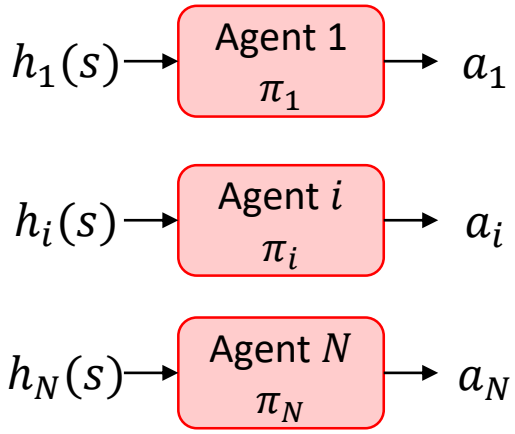
| | | Training |
|---|---|---|
| | | **Centralized Training** |
| Execution | Centralized Execution | MDP |
| | **Decentralized Execution** | Dec-(PO)MDP (CTDE) |

$s = (s_1, \dots, s_N)$
$a = (a_1, \dots, a_N)$
$r = f(s, a)$

$Q(s, a)$

$\pi_1, \dots, \pi_i, \dots, \pi_N$

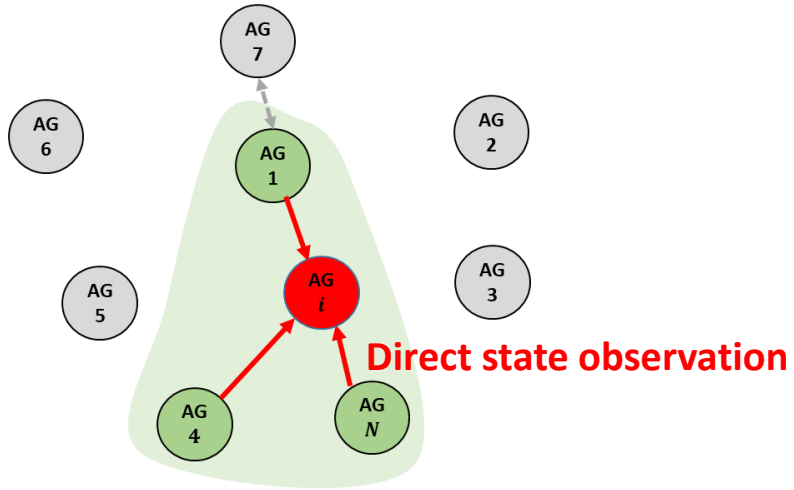
**Centralized Training**



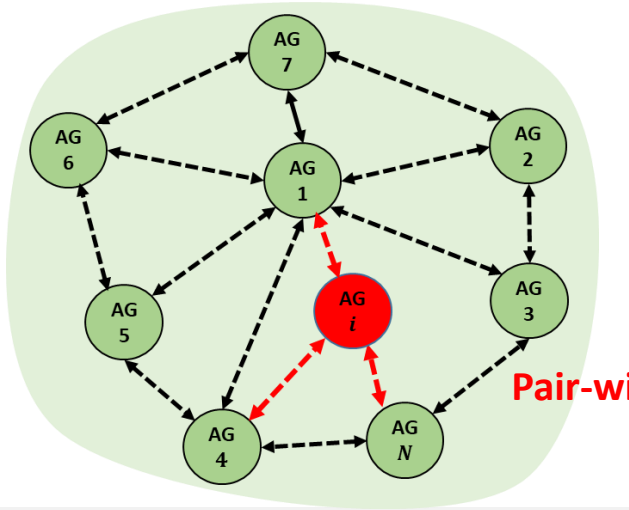

**Decentralized Execution**

- **Depending on the information that each agent can use when making a decision (information structure)**, CTDE has different approaches.
  - Local observation: $h_i(s) = s_i(\text{or } o_i)$ : (when there is partial observability and/or communication constraints)
  - Global observation: $h_i(s) = s(\text{or } o)$ : (global observation or communication is allowed)

# 3. Deep MARL: Information Structure in CTDE framework




Direct state observation




Pair-wise comm.

**Learning to *cooperate* (only local observation is allowed)**

$$\pi(s,a) \approx \prod_{i=1}^{N} \pi_i(o_i,a_i)$$

$$\approx \prod_{i=1}^{N} \pi_i(o_i,a_i;\theta_i)$$ :Function approximation

$$= \prod_{i=1}^{N} \pi(o_i,a_i;\theta_{g(i)}) \quad g(i) \in \{1,\dots,M\}$$

Parameter sharing among **a group of agents**

$$= \prod_{i=1}^{N} \pi(h_i = GNN(o_i;\phi),a_i;\theta_{g(i)})$$

(State representation with inductive biases)

**Learning to *communicate* (more than local observation)**

$$\pi(s,a) \approx \prod_{i=1}^{N} \pi_i(s,a_i)$$

$$\approx \prod_{i=1}^{N} \pi_i(s,a_i;\theta_i)$$ :Function approx.:

→**difficult** to process $s$ (high dim)

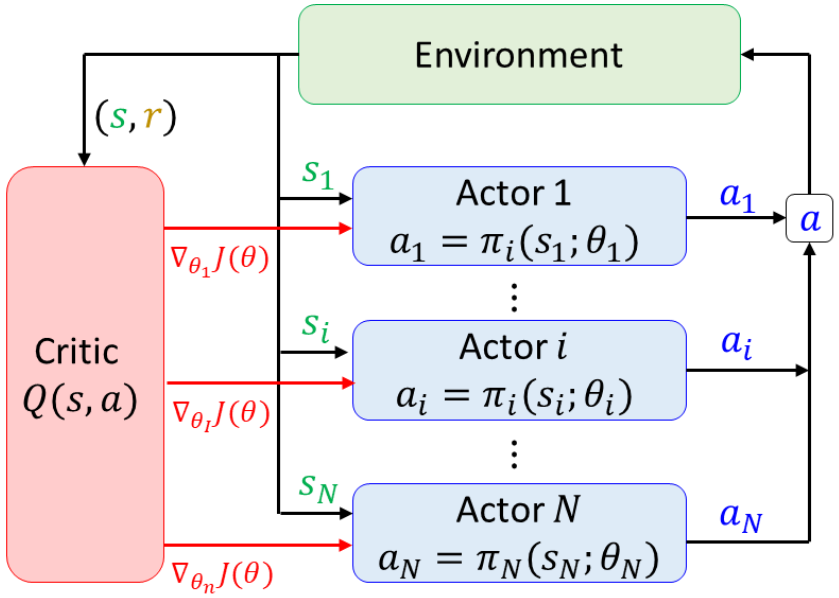
$$\approx \prod_{i=1}^{N} \pi_i(h_i = GNN(s;\phi),a_i;\theta_i)$$

(State representation with inductive biases)

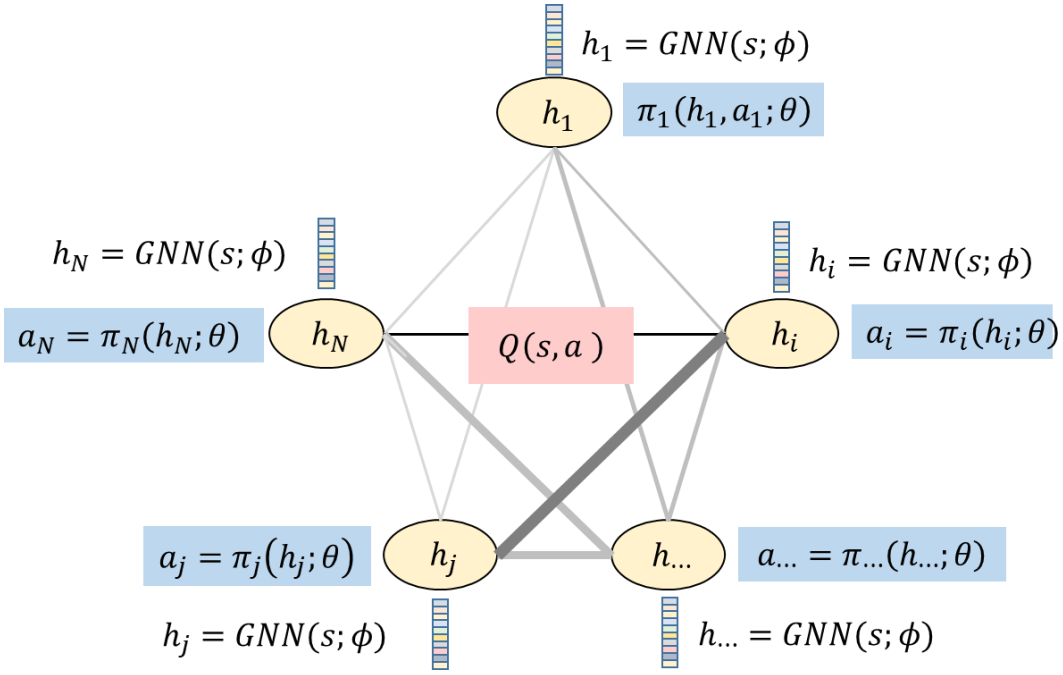
$$\approx \prod_{i=1}^{N} \pi(h_i = GNN(s;\phi),a_i;\theta_{g(i)}) \quad g(i) \in \{1,\dots,M\}$$

Parameter sharing among **a group of agents**

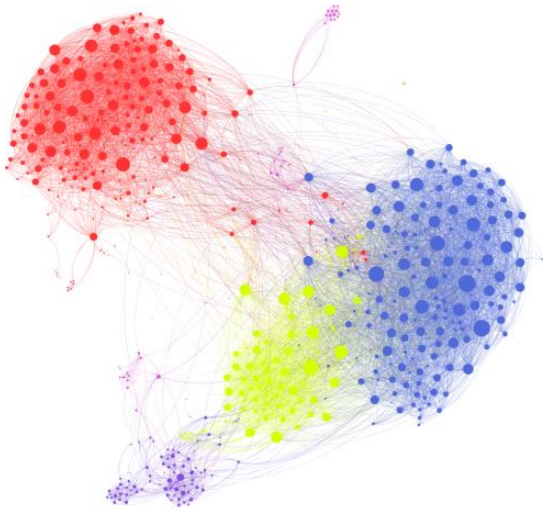
# 3. Deep MARL: MARL + Graph Neural Network (GNN)



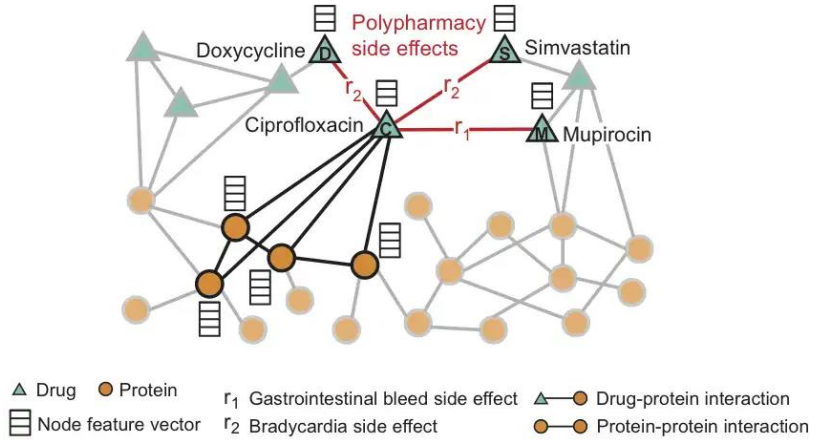

**Apply relative inductive biases using graph**



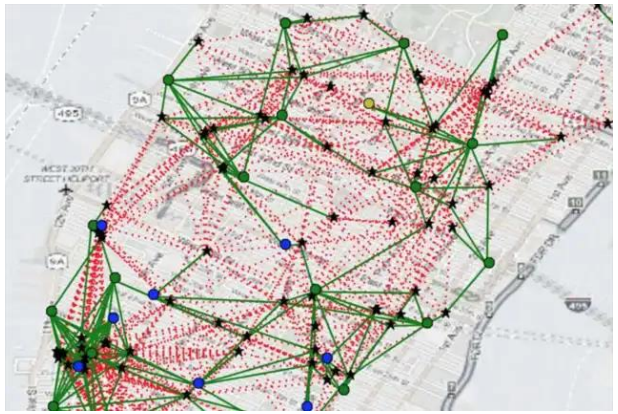
# 3. Deep MARL: Graph Neural Network



<GNN for Social network analysis>




<GNN for modelling polypharmacy>

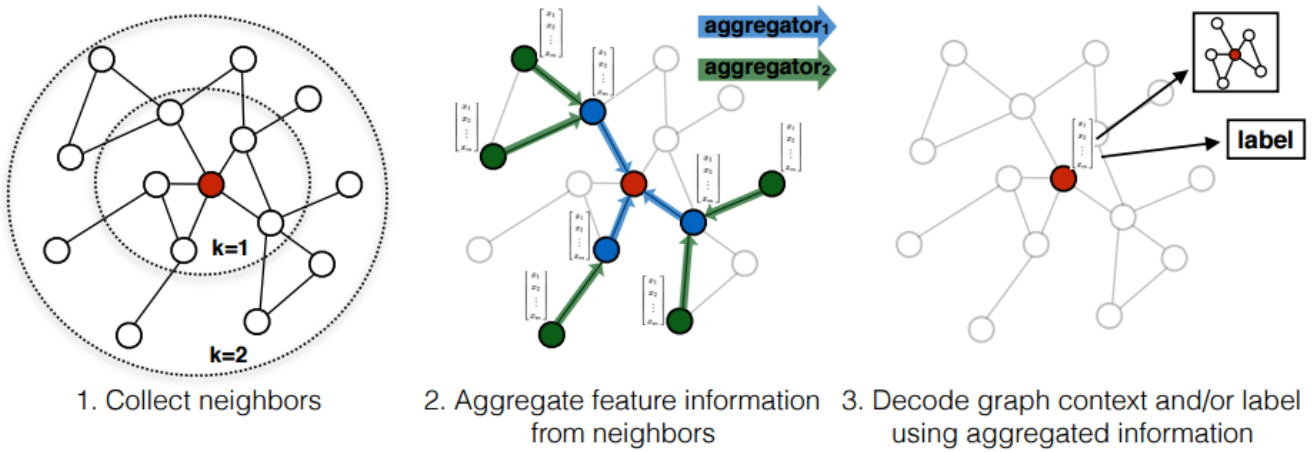


<GNN for modelling traffic system>

- Graph Neural Network (GNN) is a powerful tool for processing graph represented data.
- GNN employ sub-neural networks for processing information (Generalization)
- GNN computations employ the optimized batch computation for reducing computation time (Scalability)

Ref <https://griffsgraphs.wordpress.com/2012/07/02/a-facebook-network/> , <http://snap.stanford.edu/decagon/>, <http://news.mit.edu/2016/how-ride-sharing-can-improve-traffic-save-money-and-help-environment-0104>

# 3. Deep MARL: Graph Neural Network




(Figure: example showing how GNN process and propagate data among nodes and edges)

- Graph The Neural Network (GNN) accepts a graph $G = (u,\ V,\ E)$ as an input
  - ✓ $u$ is global attribute
  - ✓ $V$ is a set of nodes (each represented by attributes)
  - ✓ $E$ is a set of edges (also represented by attributes)
- Graph Neural Network (GNN) outputs a graph or vector as predictions
- GNN learns how to propagate and process the data among neighboring nodes and edges using NN
- Because it learns the relationships among nodes, it can applied to any size of graph with different nodes =

Ref:"Representation Learning on Graphs: Methods and Applications", Hamilton et al., Bulletin of the IEEE Computer Society Technical Committee on Data Engineering

# 4. APPLICATIONS

## Overhead Hoist Transport (OHT) Control in Semiconductor Fab.


https://wccftech.com/globalfoundries-puts-7nm-on-hold-indefinitely-spins-off-asic-division-as-subsidiary/

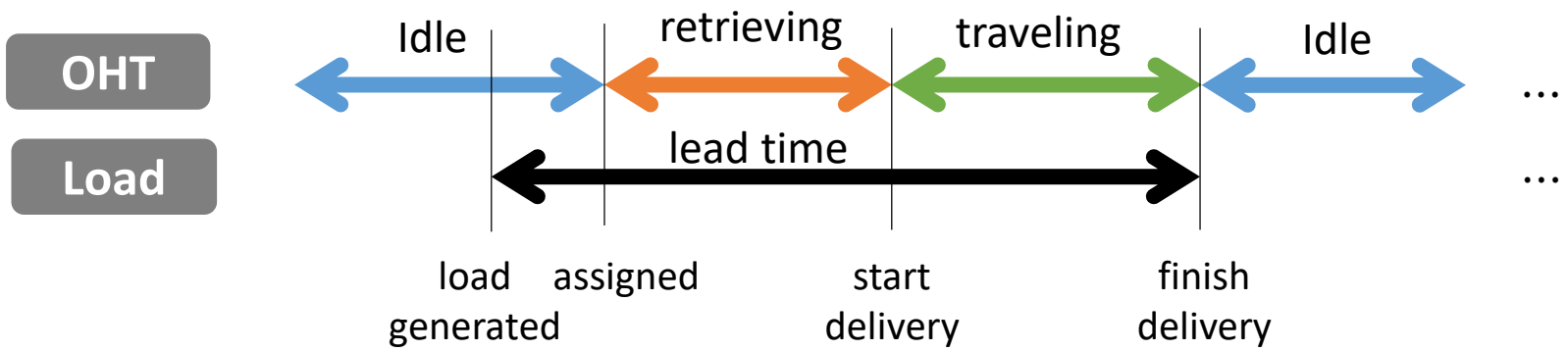
# Overhead hoist transport (OHT)

- **OHTs** in semiconductor FAB deliver **semiconductor load** from tool to tool

http://samsung.com/sec

# Motivation

- OHTs in semiconductor fab make the following decisions to increase productivity:
  - Which job to pick up? (**Dispatching**)
  - Which route to take? (**Routing**)
  - Where should wait to take a new semiconductor load? (**Rebalancing**)




**Reduce congestion**

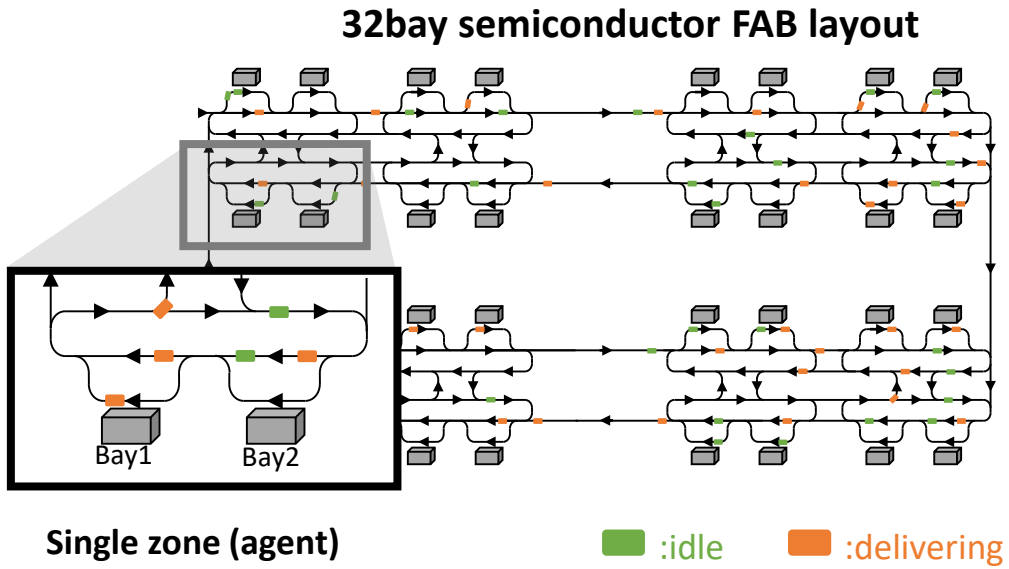
**+**

**Reduce load waiting time (lead time)**

# Motivation

## Zone-based rebalancing algorithm

- Entire FAB is discretized into number of smaller **zones**
- Each zone determines rebalancing policy, depend on local information




- The cooperative rebalancing strategy is represented as decentralized policies of all zones

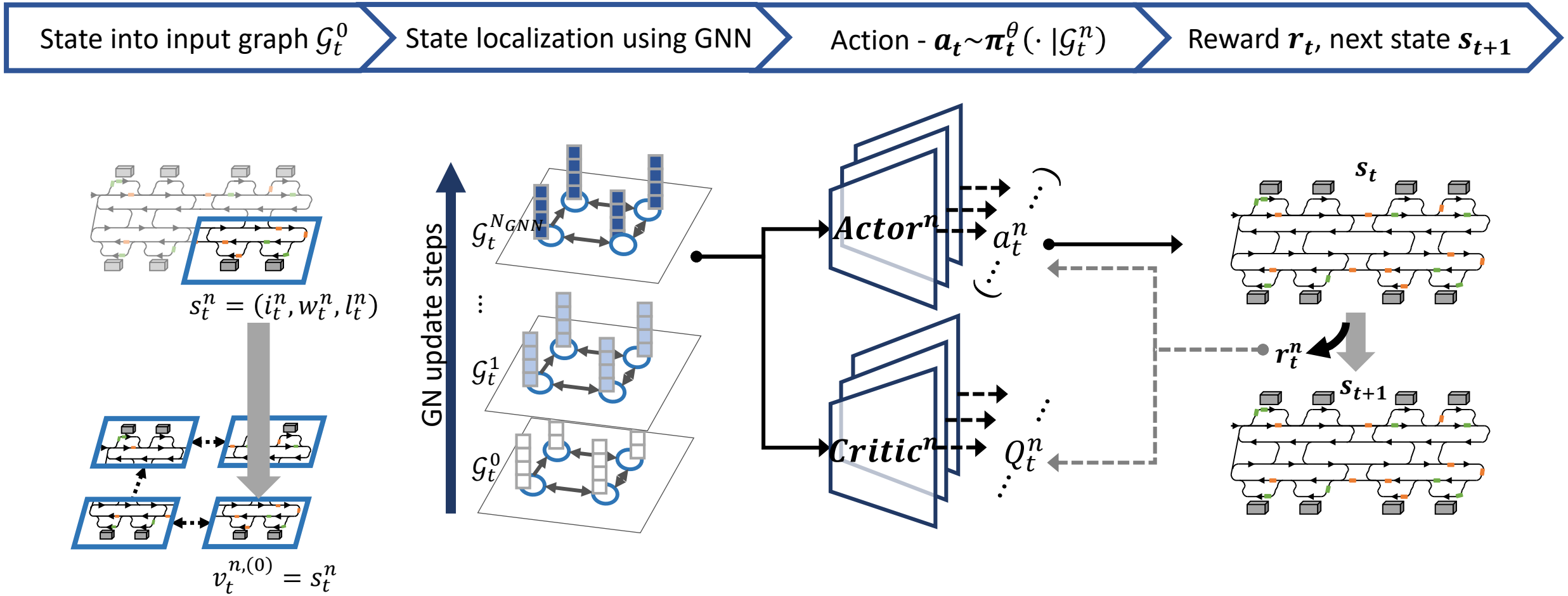
$$\boldsymbol{\pi}(a^1, \ldots, a^N \,|\boldsymbol{s}) \approx \prod_{n=1}^{N} \pi^n(a^n|h^n)$$

**Coordinated strategy of entire system** **Individual strategy**

- MARL effectively derives such centralized policies with an aid of graph neural network

$$o^n(\boldsymbol{s}) \approx h^n = GNN(\boldsymbol{s})$$

# Overall architecture



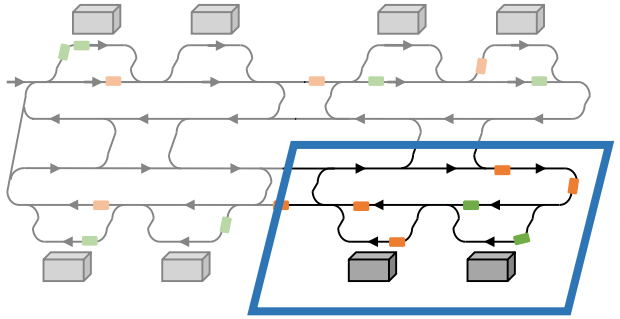
# State into input graph $\mathcal{G}_t^0$

## State, action representation ($N$ zones)

- **state** $s_t^n = (i_t^n, w_t^n, l_t^n)$ **/ global state** $\boldsymbol{s_t} = (s_t^1, \dots, s_t^N)$
  - $i_t^n$ : # idle OHTs on zone $i$
  - $w_t^n$ : # working OHTs on zone $i$
  - $l_t^n$ : # waiting loads on zone $i$
- **observation** $o_t^n(\boldsymbol{s_t})$
- **action** $a_t^n \in \{n, Nb(n)\}$ **/ joint action** $\boldsymbol{a_t} = (a_t^1, \dots, a_t^N)$
  - $a_t^n \in \{Nb(n)\}$ : agent sends vehicle to adjacent zone
  - $a_t^n = n$ : leave the vehicle on the same zone
- **reward** $r_t^n$

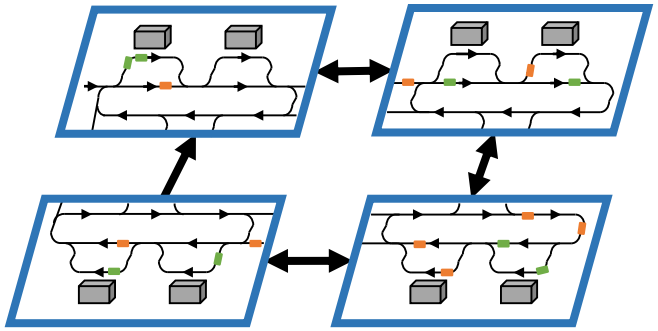
$$r_t^n(\boldsymbol{s_t}, \boldsymbol{a_t}) = \begin{cases} 1 - \alpha \cdot dist(n, a_t^n), & if\ job\ assigned \\ -\alpha \cdot dist(n, a_t^n), & otherwise \end{cases}$$

Global state $\boldsymbol{s_t}$



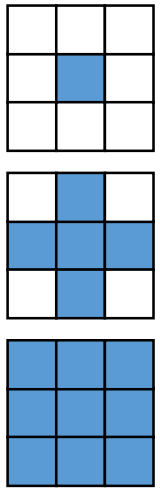
$s_t^n = (i_t^n, w_t^n, l_t^n)$

Input graph $\mathcal{G}_t^0$



# State localization using GNN

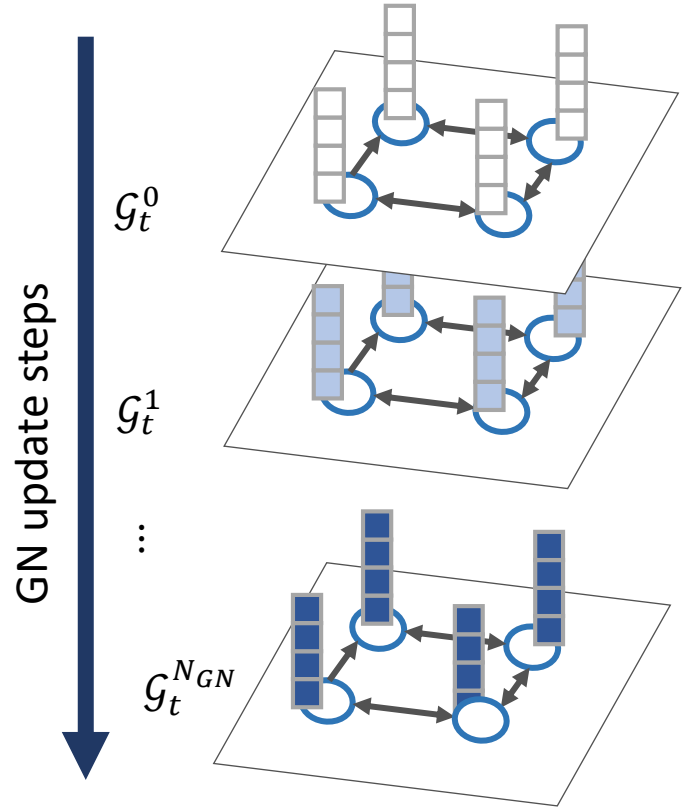
- Observation $o_t^n(\boldsymbol{s_t})$ is each agent's localized view of global state $\boldsymbol{s_t}$



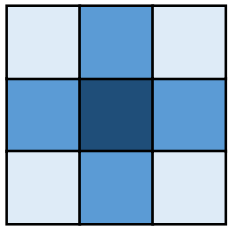
$$o_t^n(\boldsymbol{s_t}) = s_t^n \quad \text{if locally observable,}$$

$$o_t^n(\boldsymbol{s_t}) = \left\{ s_t^j | \forall j \in \{n, Nb(n)\} \right\} \quad \text{if partially observable,}$$

$$o_t^n(\boldsymbol{s_t}) = \boldsymbol{s_t} \quad \text{if globally observable}$$




- GNN **can approximate global state** into agent' localized view



$$v_t^{n,(N_{GN})} \approx o^n(\boldsymbol{s_t}), \qquad where\ v_t^{n,(N_{GN})} \in \mathcal{G}_t^{N_{GN}}$$

# Actor-Critic

(Critic) Q-function is reward sum of all agents

$$Q_t(\boldsymbol{s_t}, \boldsymbol{a_t}) = \sum_{n=1}^{N} Q_t^n(\boldsymbol{s_t}, \boldsymbol{a_t})$$

(Actor) Joint policy $\boldsymbol{\pi}$ is product of individual policy $\pi^n$

$$\boldsymbol{\pi}(\boldsymbol{a_t}|\boldsymbol{s_t}) = \prod_{n=1}^{N} \pi^n\left(a_t^n | o_t^n(\boldsymbol{s_t})\right)$$

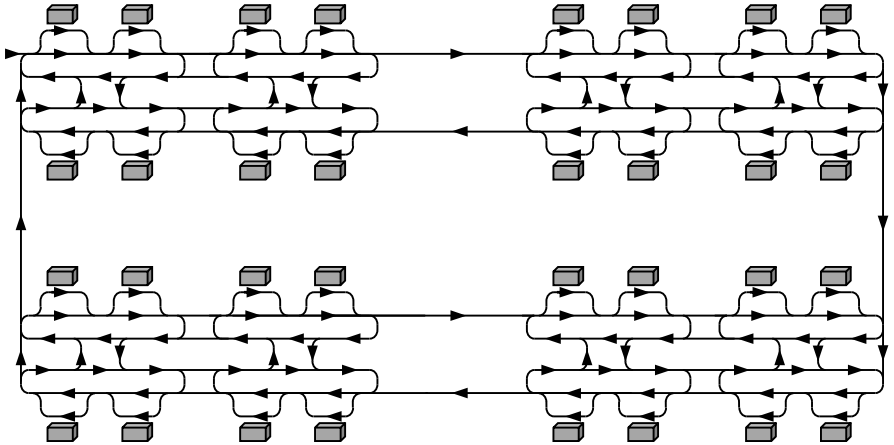
GNN node embedding vector $v_t^n$ approximates local observation $o_t^n(\boldsymbol{s_t})$

$$Q_t^n(\boldsymbol{s_t}, \boldsymbol{a_t}) \approx Q_t^n(v_t^n, a_t^n)$$
$$\pi^n\left(a^n | o_t^n(\boldsymbol{s_t})\right) \approx \pi^n(a^n | v_t^n)$$

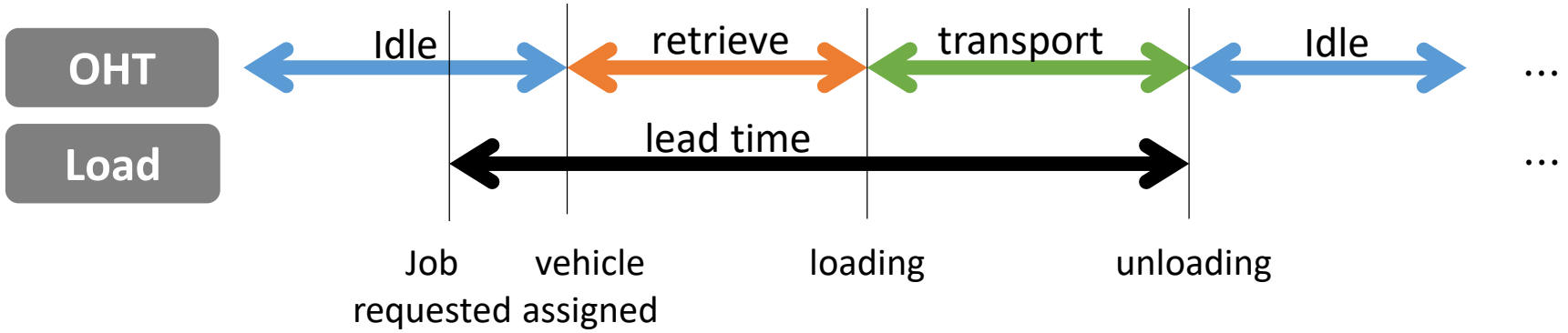
# Experiment details

- 32 bays semiconductor system, 2bays/zone
- Baseline algorithms
  - Push (used in real fab) : idle vehicle moves to the next bay when other vehicle comes
  - Heuristic : $\pi(j|i) \propto \frac{LF(j)}{l_t^j}$, policy proportion to the load generation ratio divided by current number of jobs

32bay semiconductor FAB layout



- Key performance metrics: retrieve time, delivery time (lead time)



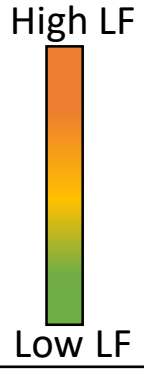
# Experiment 1: Varying Number of vehicles and loads

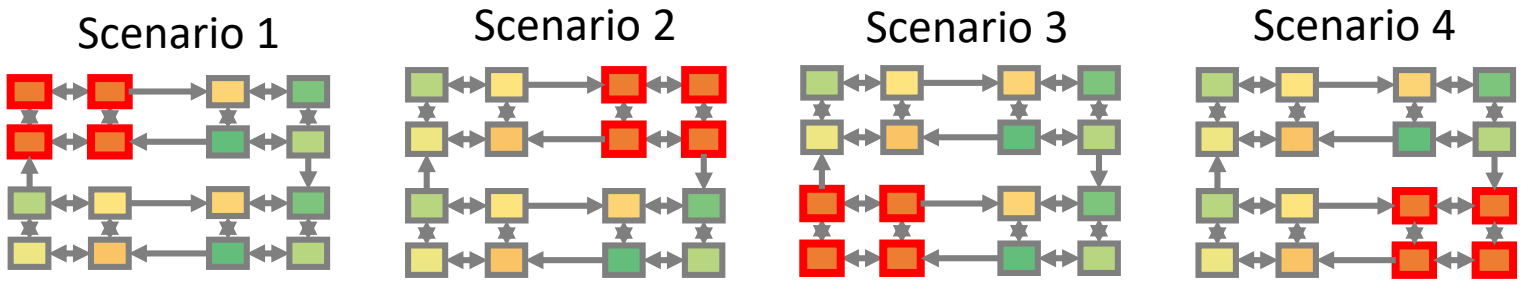
Retrieve time varying on scenarios

| #veh → # jobs/day↓ | 70 | | | 75 | | | 80 | | | 85 | | | 90 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | push | heur-istic | **ours** | push | heur-istic | **ours** | push | heur-istic | **ours** | push | heur-istic | **ours** | push | heur-istic | **ours** |
| 17,500 | 119.7 | 115.8 | **112.1** | 121.1 | 119.0 | **115.0** | 122.5 | 122.4 | **117.9** | 125.2 | 122.4 | **121.0** | 126.8 | 130.4 | **125.5** |
| 21,000 | 122.3 | 112.7 | **108.2** | 126.0 | 116.1 | **112.2** | 125.3 | 119.7 | **114.3** | 128.4 | 119.7 | **118.0** | 130.5 | 127.0 | **121.4** |
| 24,500 | 121.7 | 108.9 | **104.1** | 125.1 | 112.4 | **107.4** | 127.9 | 116.3 | **110.7** | 127.9 | 120.0 | **114.6** | 133.3 | 123.7 | **118.1** |
| 27,000 | 115.3 | 103.4 | **99.0** | 120.4 | 107.7 | **102.9** | 125.5 | 112.7 | **106.8** | 129.3 | 112.1 | **110.3** | 133.7 | 120.2 | **113.9** |
| 31,500 | 105.5 | 97.3 | **92.8** | 122.3 | 102.6 | **97.1** | 118.6 | 107.3 | **101.2** | 124.5 | 107.3 | **105.6** | 129.2 | 116.0 | **109.7** |
| 35,000 | 94.1 | 89.9 | **86.2** | 102.1 | 95.6 | **91.3** | 109.0 | 101.0 | **96.0** | - | 100.9 | **100.4** | - | 110.6 | **104.5** |
| 38,500 | 84.1 | 81.6 | **78.9** | 91.3 | 87.9 | **84.7** | 98.8 | 93.6 | **90.0** | - | 93.6 | **94.6** | - | 104.6 | **98.9** |

- GNN-MARL outperforms a heuristic algorithm in all scenarios
- GNN-MARL reduces mean retrieval time of the vehicles, **at most 25 secs (20% gap)**

# Experiment 2: Transfer policy to different congestion scenarios

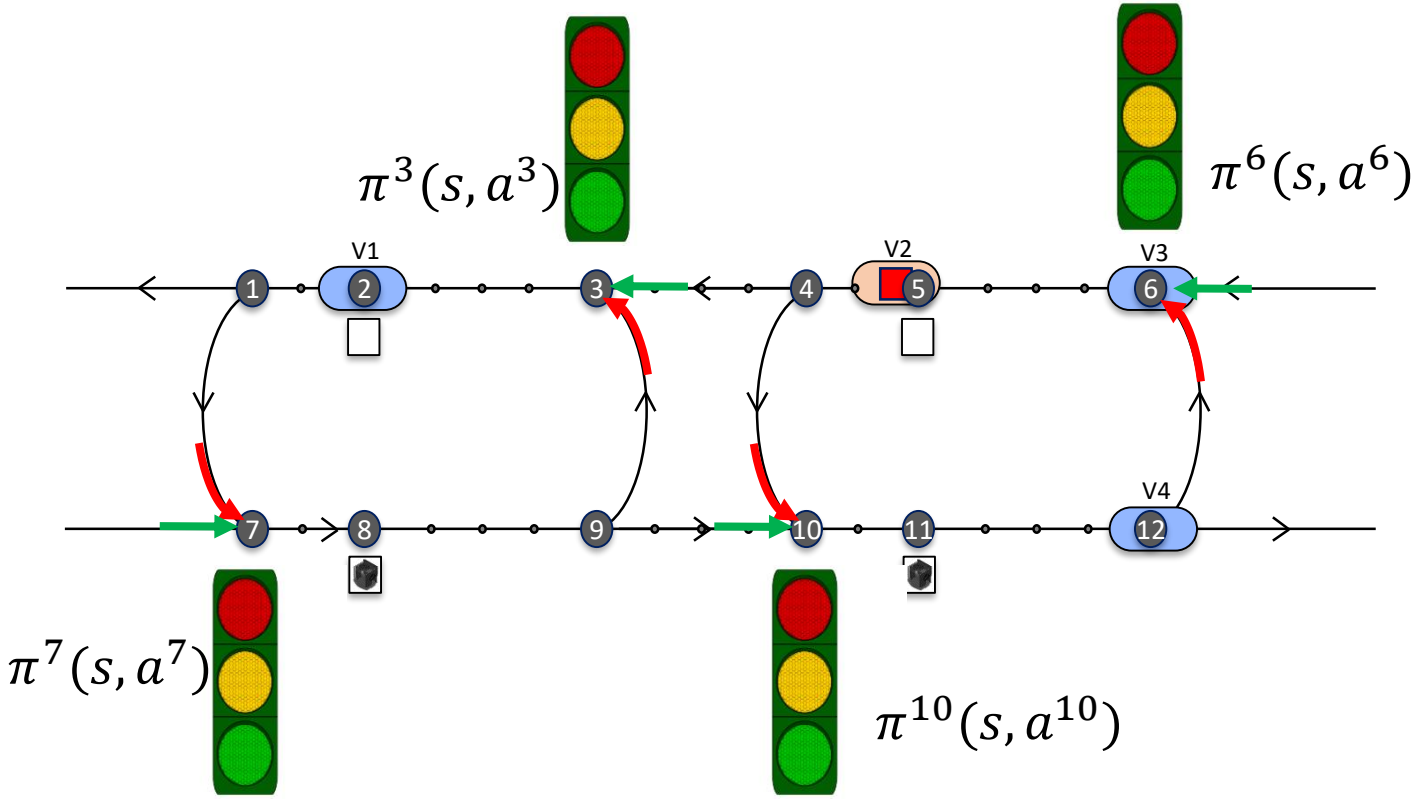



| Rebalancing algorithm | | | Test case | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | s1 | | s2 | | s3 | | s4 | |
| | | | Retrieve time | Lead time | Retrieve time | Lead time | Retrieve time | Lead time | Retrieve time | Lead time |
| Push | | | 136.3 | 196.5 | 137.1 | 197.0 | 136.0 | 195.7 | 136.2 | 196.2 |
| Heuristic | | | 117.5 | 181.2 | 117.2 | 180.9 | 118.2 | 181.8 | 118.1 | 181.1 |
| MARL approach | Train case | S1 | **107.2** | **169.5** | **107.1** | **170.0** | **107.5** | **169.9** | **107.6** | **170.1** |
| | | S2 | **108.3** | **170.9** | **107.9** | **170.8** | **108.0** | **170.8** | **107.6** | **170.5** |
| | | S3 | **107.2** | **169.8** | **107.1** | **170.6** | **107.2** | **169.8** | **106.8** | **169.6** |
| | | S4 | **107.3** | **170.0** | **107.0** | **170.1** | **107.1** | **170.2** | **107.3** | **170.3** |

- GNN-MARL reduces retrieval time and lead time **about 30 secs (about 20% gap)**
- GNN-MARL achieves similar performance to various congestion scenario, **even in unseen scenarios (transfer)**

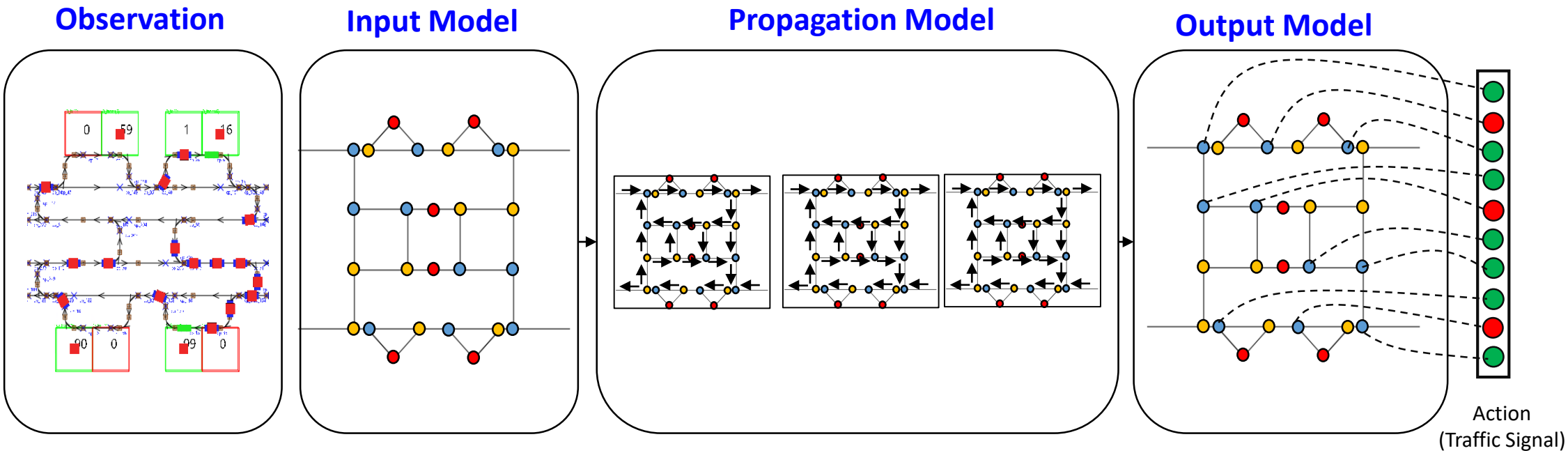
# Cooperative Traffic Light Control



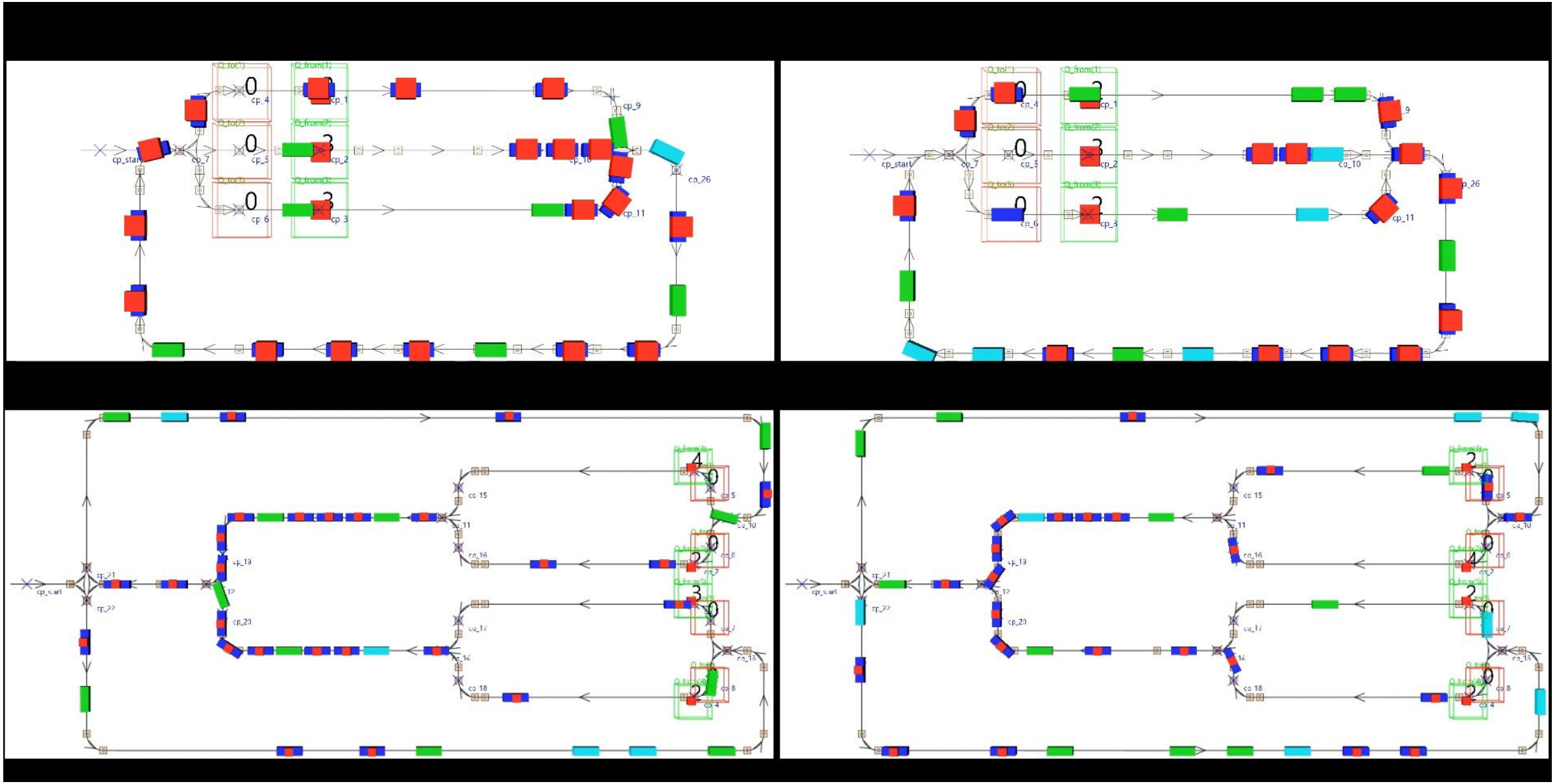

- Derive the decentralized control policy for multiple traffic lights to minimize the congestion:

$$\pi(s, a) = \prod_{i=1}^{N} \pi(s, a^i)$$

# Cooperative Traffic Light Control



# Cooperative Traffic Light Control

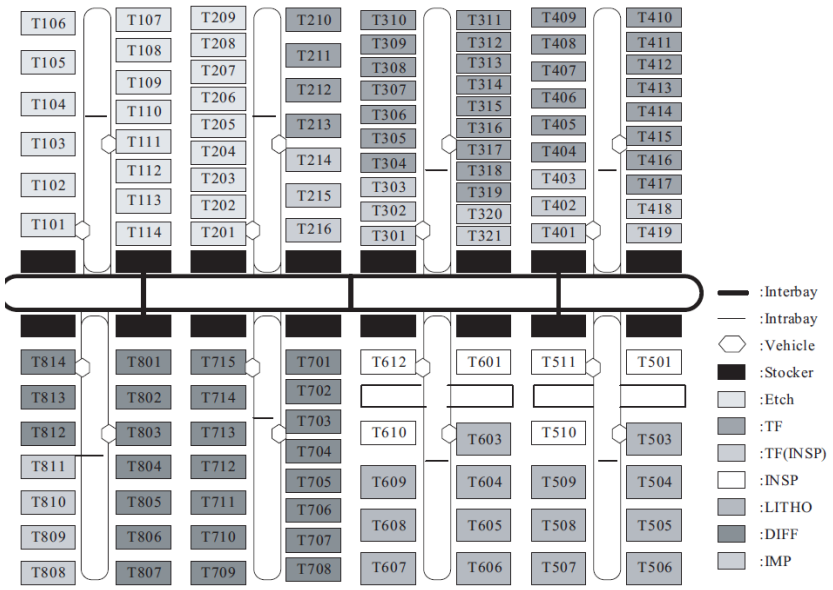


# 4. APPLICATIONS

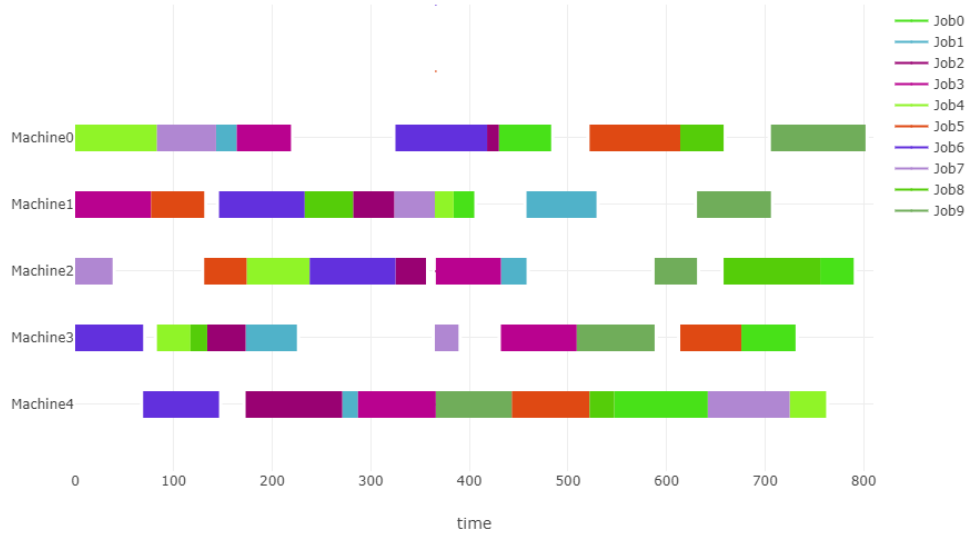
## Decentralized Job Shop Scheduling


https://en.letsgodigital.org/semiconductor-industry/samsung-semiconductor-business/

# Scheduling complex fab




<A simplified layout of an AMHS in a 300mm wafer fab>




## Scheduling complex fab is becoming harder and harder as

- Size of fab increases
- Number of tools increases
- Types of product increases
- Production lines become more denser

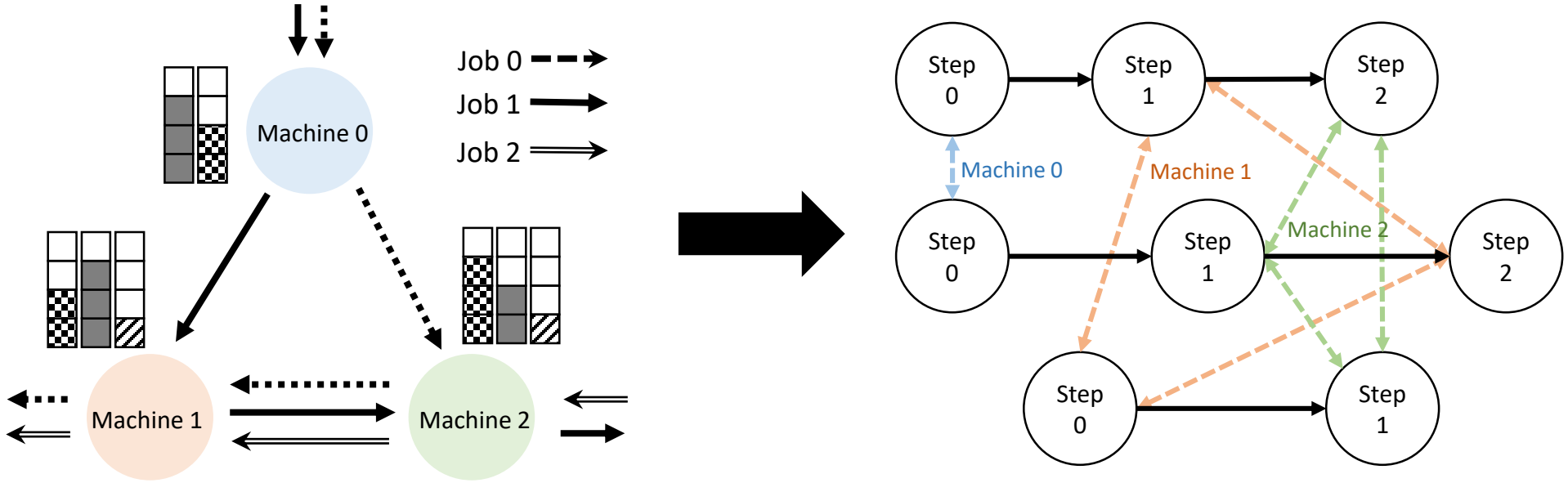
Figure : <The performance of the number of vehicles in a dynamic connecting transport AMHS by J. T. Lin , F. K. Wang & C. J. Yang>

# Motivations

| | Optimality guarantee | Empirical performance | Scalability | Adaptation (Feedback) | Generalization to unseen problem (transfer) |
|---|---|---|---|---|---|
| **Optimization methods** | Yes for small problem | N/A | Poor | No | No |
| **Heuristic Algorithm** | No | OK | Good | No | No |
| **RL** | No | Acceptable | Good | Yes | Yes |

- Optimization methods are not applicable to real-time decision making scenarios in practice.
  - Problem size are often large -> Exploding computational time
  - Problem structure varies on time (random machine failure, stochastic processing times, etc.)
- Dispatching rule approach **does not consider** problem's **spatial** and **temporal** structure
- Efficient RL based scheduler need to be
  - Scheduler should solve a large-scale problem **(Scalability)**
  - Scheduler should make decision based on current status of problem **(Adaptive)**
  - **Scheduler should solve any unseen problem (transferable)**

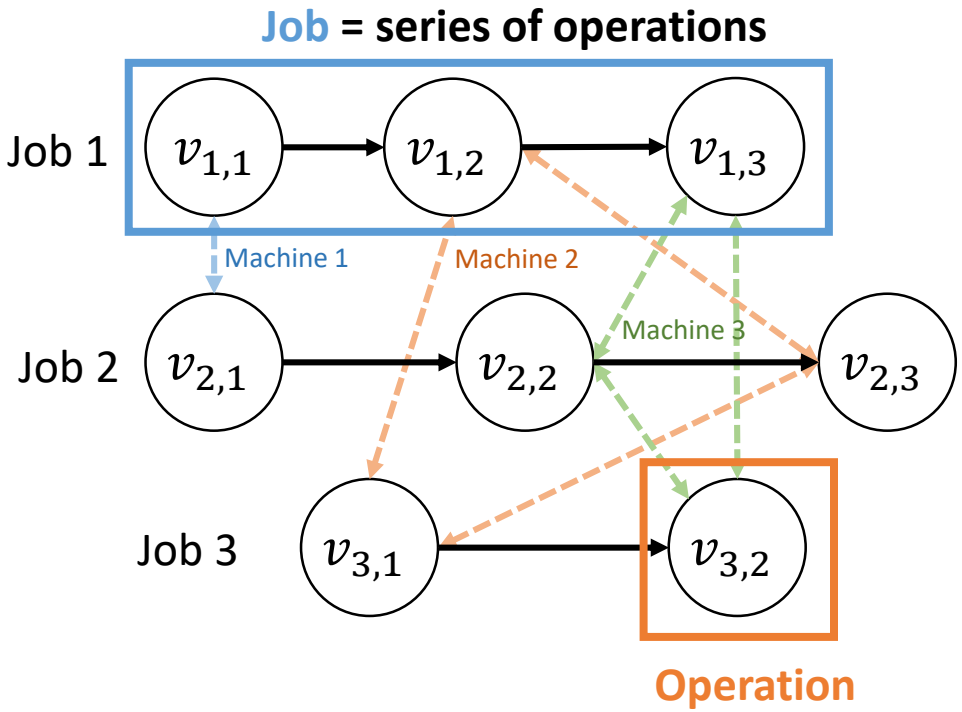
# Target Problem




3 Machines X 3 Jobs JSSP instance, (3x3 instance)

- Job shop scheduling or the job-shop problem (JSP) is an optimization problem in computer science and operations research in which jobs are assigned to resources at particular times.
- the job-shop problem is a combinatorial optimization problem and is NP-hard problem (difficult to solve)
- We derive RL policy that can solve any JSP problem

# Job-Shop Scheduling Problem (JSSP)




3 Machines X 3 Jobs JSSP instance, (3x3 instance)

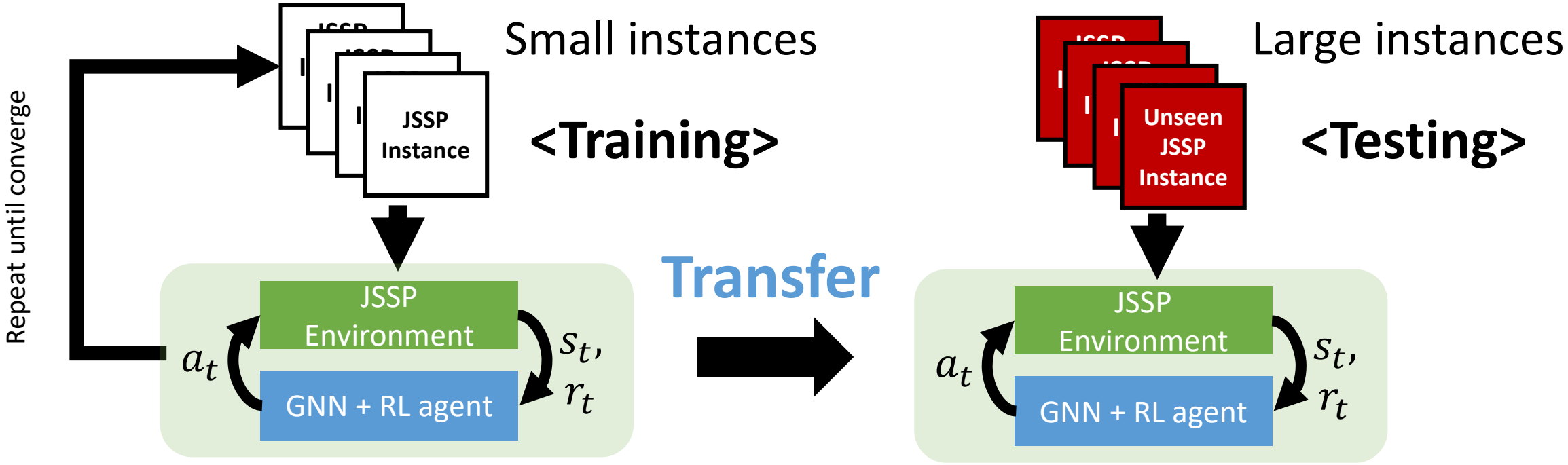
**Objective:**

- **Find the sequence of dispatching jobs which minimize total production time (makespan)**

**Two constraints:**

- **Precedence constraints : An operation can be processed only if the preceding operations are all done.**
- **Machine-sharing constraints : The operations that share a machine cannot be processed at the same time.**

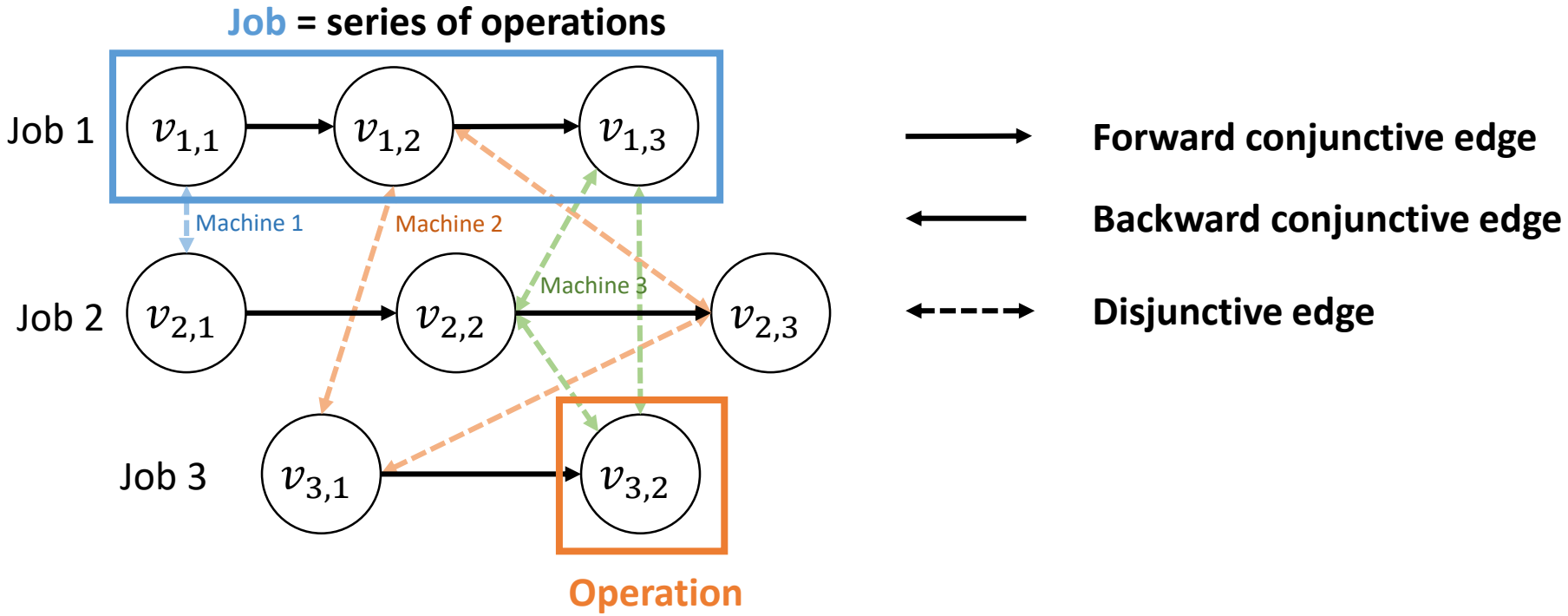
# Learning based adaptive JSSP solver




We propose **Graph neural network (GNN) – Reinforcement learning (RL)** based learnable solver.

- The solver is adaptive
- The solver appreciate problems' spatial, temporal structure
- The solver is transferable to the unseen JSSP instances.

# Graph representation of JSSP snapshot

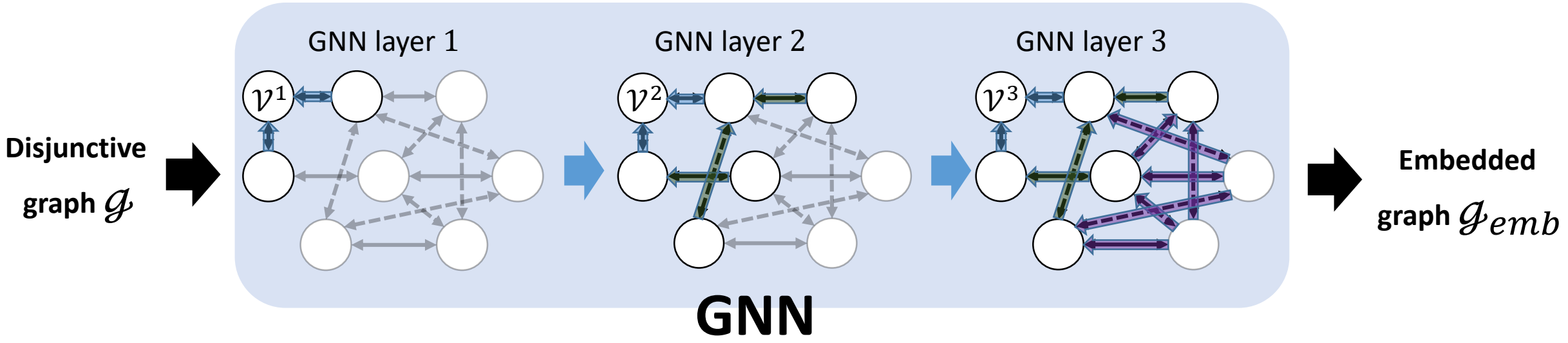



**3 Machines X 3 Jobs JSSP instance, (3x3 instance)**

We formulate Disjunctive graph, $\mathcal{g}$, that represent current snapshot of JSSP as a directed graph.

- Node feature $v = \{$node types, processing time, degree of completion, #. remaining operations, waiting time, remain time$\}$
- Edge connectivity has 3 types: Conjunctive edge forward, backward (precedence constraints) and disjunctive edges (machine sharing constraints)

## Graph neural network (GNN) learns state representation that considers problem structure




**Embedding step**

$$v_{i,j}^{\tau+1} = F_{\theta_{\tau+1}}\left(\left[v_{i,j}^{\tau}; v_{i,j-1}^{\tau}; v_{i,j+1}^{\tau}; \sum_{(p,q)\in\mathcal{N}_{disj}} v_{p,q}^{\tau}, \sum_{v^{\tau}} v^{\tau}\right]\right)$$

Precedence feature

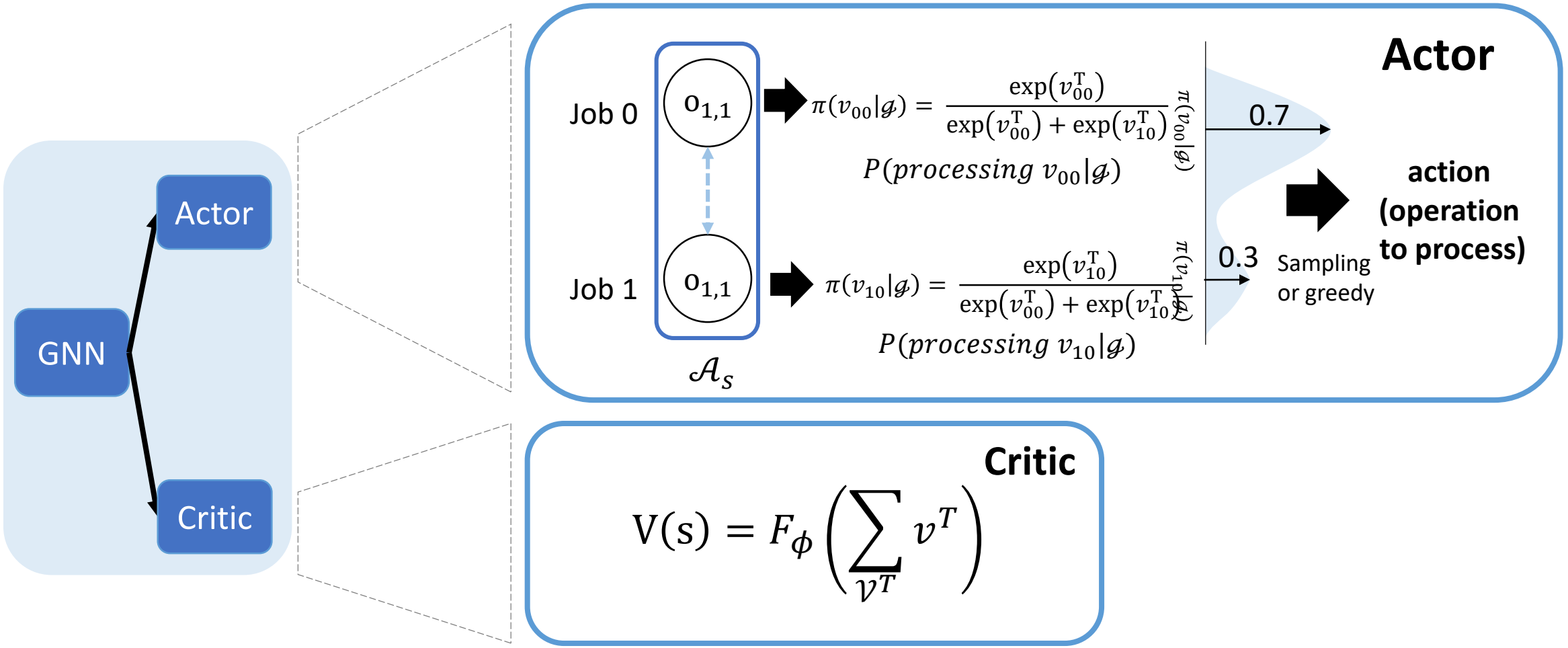
Disjunctive-relationship node features

Successor feature

Graph level feature

- **N** layer propagation can update node features with the features of **N hop** neighborhood nodes.
  Therefore, our embedded node values in $\mathcal{G}_{emb}$ effectively consider the node feature that might affect to the quality of the dispatching.

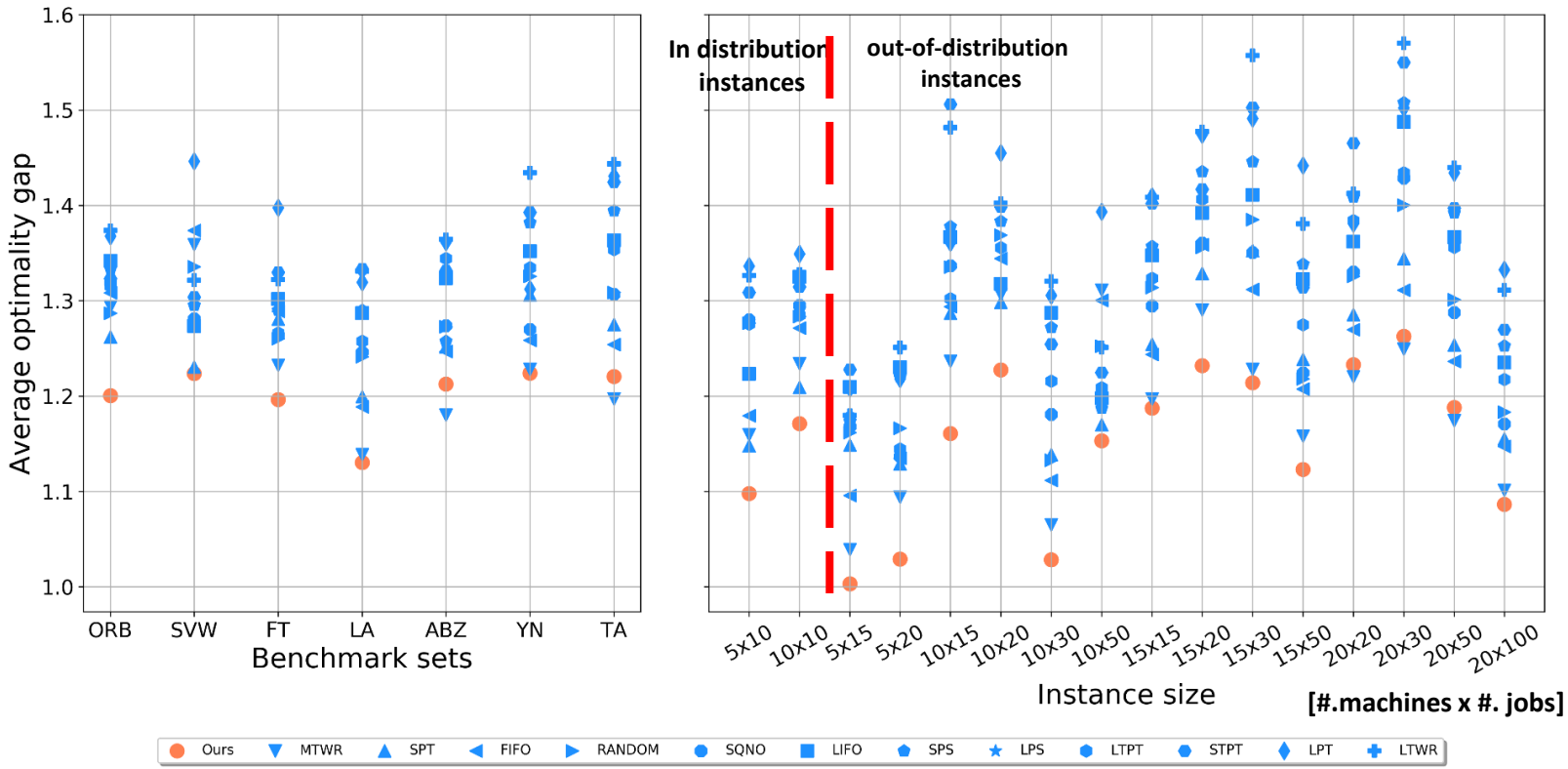
# Reinforcement Learning (RL) for training dispatching policy $\pi$




We employed proximal policy optimization (PPO)[1] to train entirely neural network. Due to the objective function of PPO,
RL agent make decisions based on the state while considering future scheduling consequences and the corresponding rewards. (**Adaptive**)

# Testing result on the benchmark JSSP instances

- Performance metric: $\frac{Dispatched\ schedule's\ makespan}{optimal\ makespan}$; **(lower is better)**
- Testing JSSP benchmark sets: ORB, SVW, FT, LA, ABZ, YN, TA
- Baseline dispatching rules: **MTWR**, **SPT, FIFO**, RANDOM, SQNO, LIFO, SPS, LPS, LTPT, STPT, LPT, LTWR

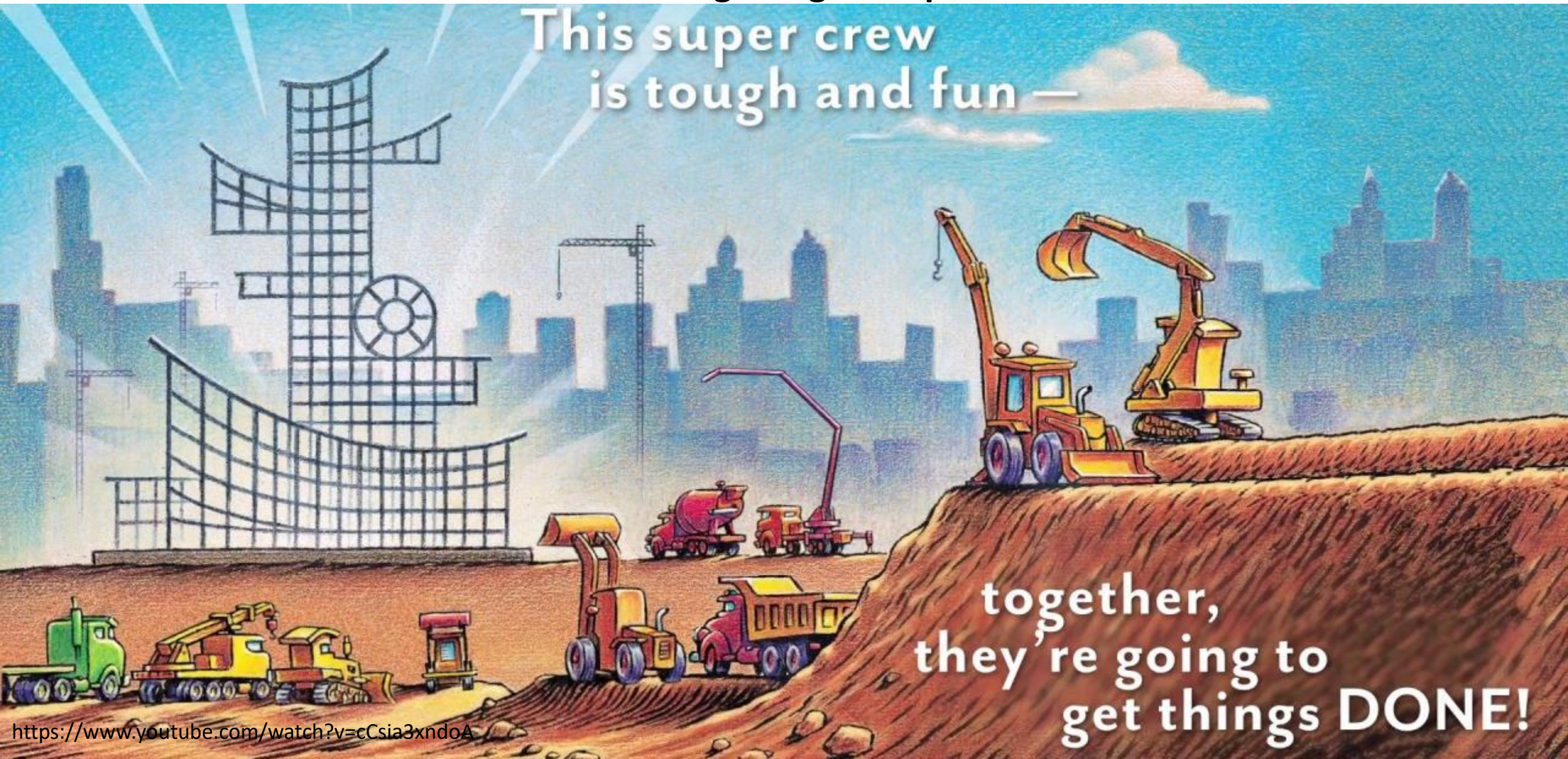



- **MTWR** – Most Total Work Remaining
- **SPT** – Shortest processing time
- **FIFO** – First-in-First-Out

- GNN-RL solver, in general, outperforms other dispatching rules across different benchmark sets.
- GNN-RL solver shows leading performances even in out-of-training distribution. (**Transferable**)

# 4. APPLICATIONS

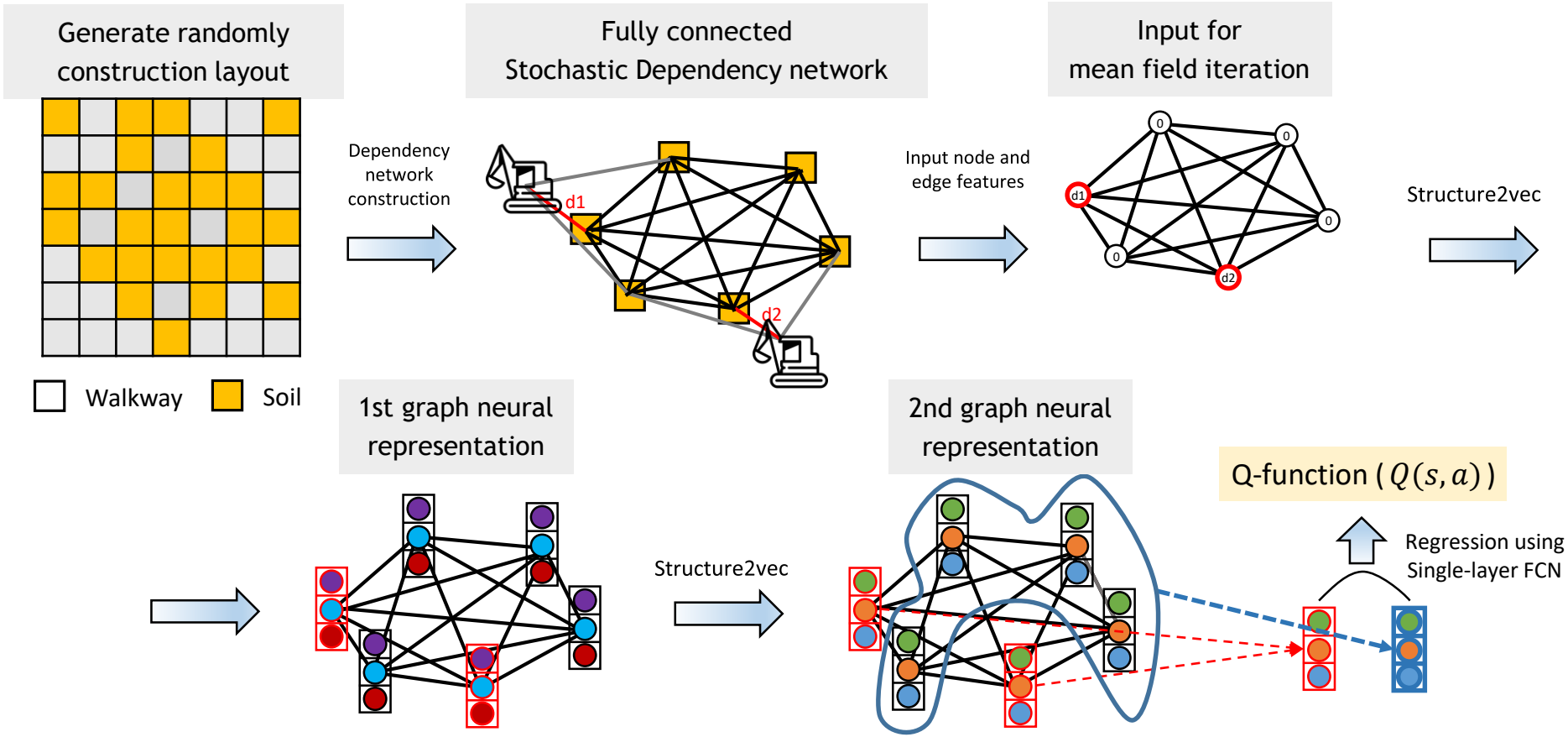
## Earthwork Planning using Multiple Excavators




https://www.youtube.com/watch?v=cCsia3xndoA

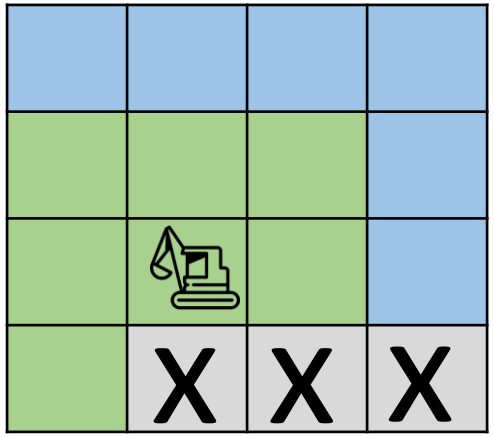
# 4. Applications : Multi-Agent Vehicle Routing Problem

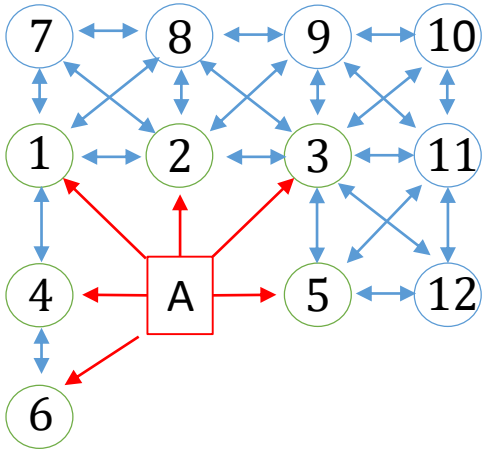
## Overview



# 4. Applications : Multi-Agent Vehicle Routing Problem

## Graph & State Representation



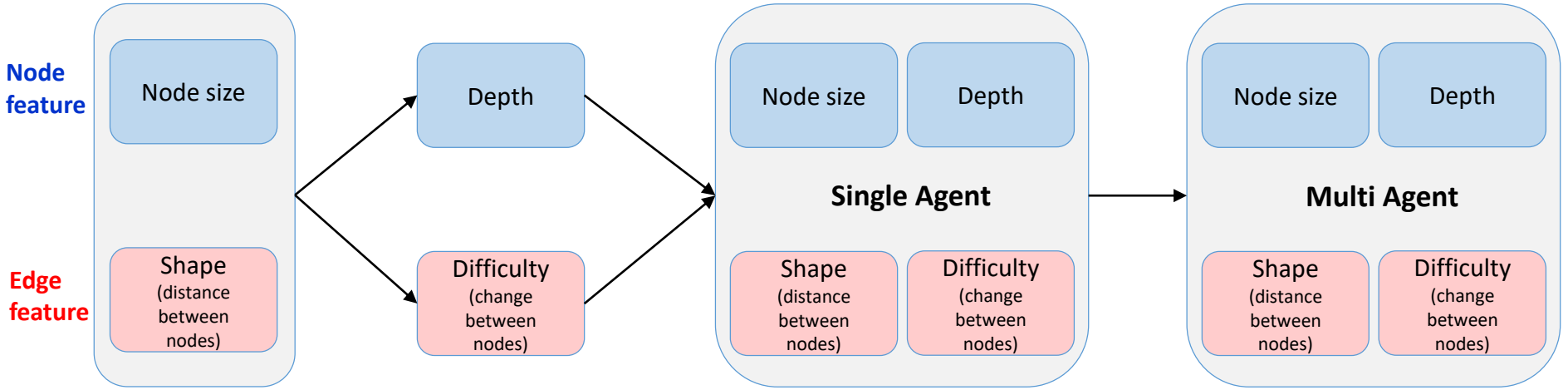

decision edge

travel edge: soil node

soil node adjacent to agent

soil node not adjacent to agent

A agent



# 4. Applications : Multi-Agent Vehicle Routing Problem

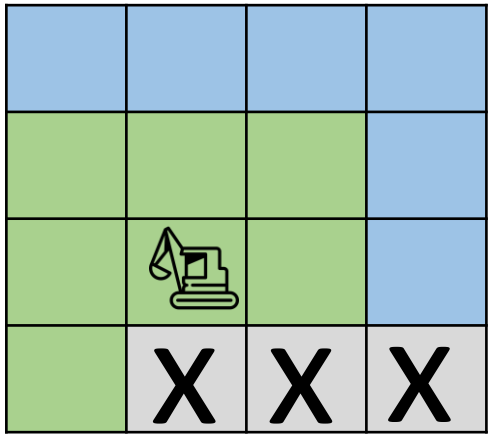
## Node Embedding

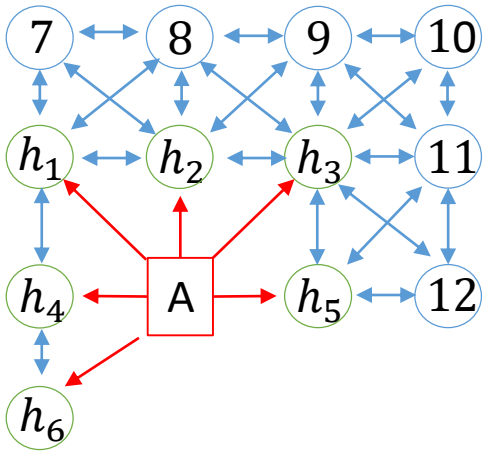
$$for\ t = 1{:}T,$$
$$for\ all\ node\ v \in V,$$
$$\mu_v^{(t+1)} \leftarrow relu\left(\theta_1 x_v + \theta_2 \sum_{u \in \mathcal{N}(v)} \mu_u{}^{(t)} + \theta_3 \sum_{u \in \mathcal{N}(v)} relu\big(\theta_4 w(u,v)\big)\right)$$

- $\mu_v^{(t+1)}$: Embedding vector (Hidden vector)
- $x_v$: Node attribute
- $\mu_u{}^{(t)}$: Previous hidden vectors
- $\mathcal{N}(v)$: Neighbors of $v$
- $w(u,v)$: Edge attribute

# 4. Applications : Multi-Agent Vehicle Routing Problem

## Action Selection



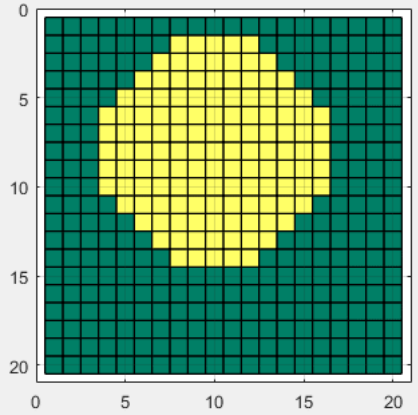

$$Q(s, a_1) \approx f(h_1; \theta) = f(GNN(s); \theta)$$

$$a^* = \max_{a \in Neigh} Q(s, a)$$

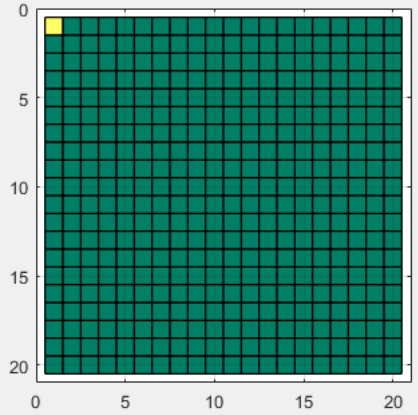
# 4. Applications : Multi-Agent Vehicle Routing Problem

## Learning & Simulation Results : Transfer to different Agent Number

DQN-20

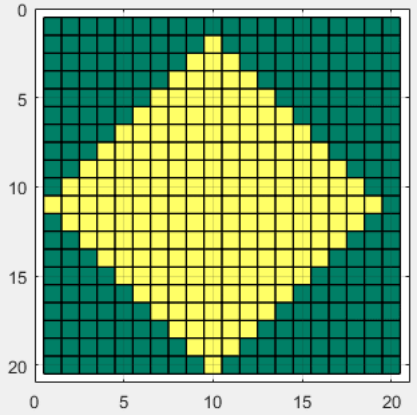



Soil Status - tick 1 , ACTION : 0 0

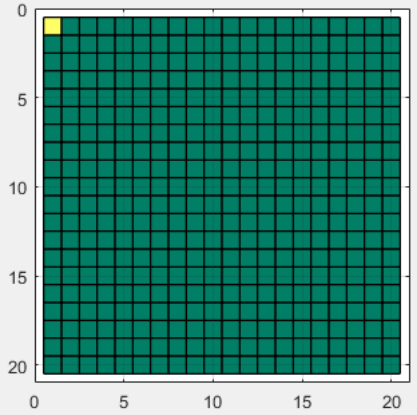



Digger Postion - tick 1 , ACTION : 0 0

DQN-20



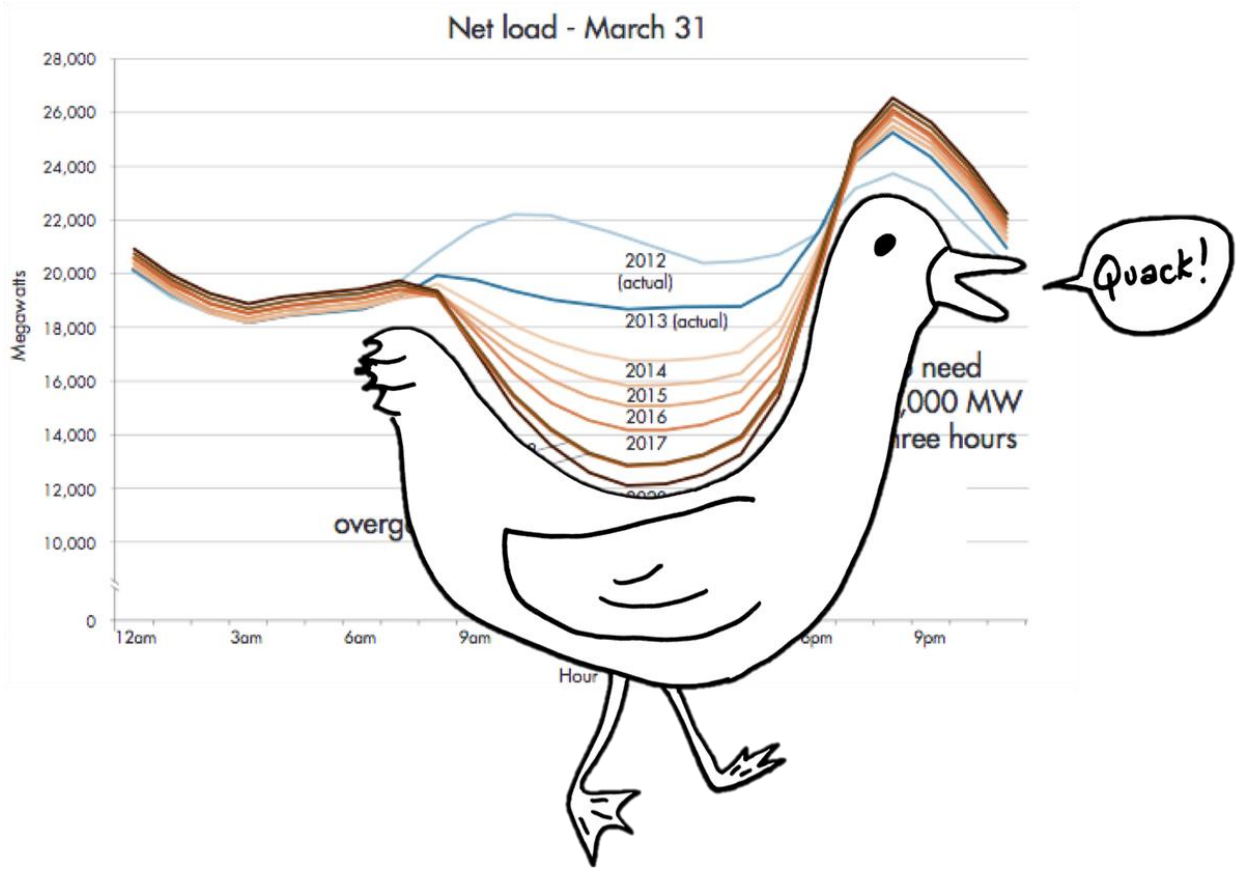

Soil Status - tick 1 , ACTION : 0 0




Digger Postion - tick 1 , ACTION : 0 0

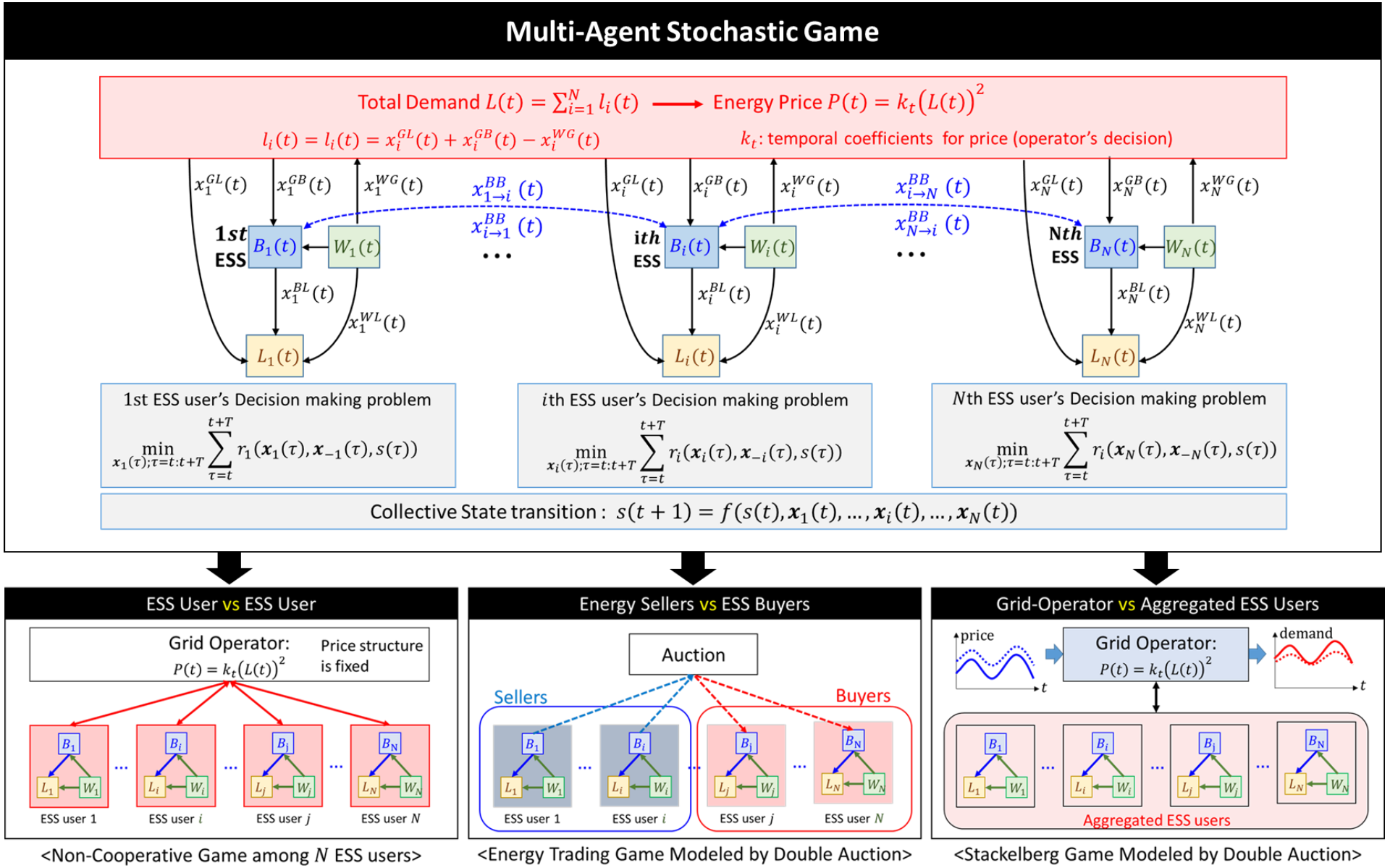
# 4. Applications : Multiple Energy Storage System Control

## Motivation



# 4. Applications : Multiple Energy Storage System Control

## Formulation




<Non-Cooperative Game among $N$ ESS users>

<Energy Trading Game Modeled by Double Auction>

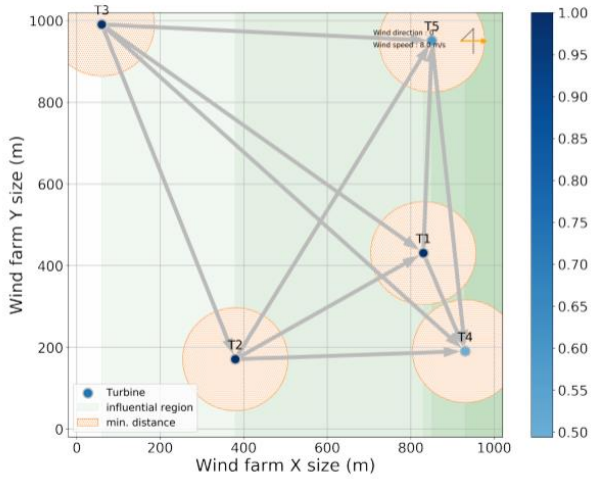
<Stackelberg Game Modeled by Double Auction>

# 4. APPLICATIONS

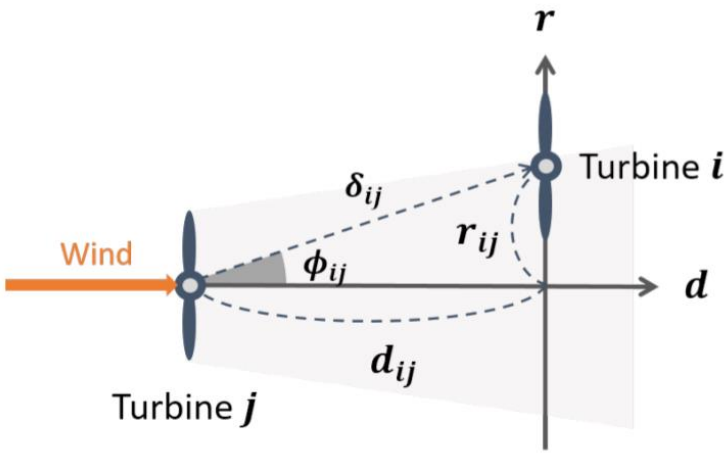
## Cooperative Wind Farm Control

https://wattsupwiththat.com/2015/08/31/study-wind-farms-offer-diminishing-returns-as-they-grow-more-widespread/

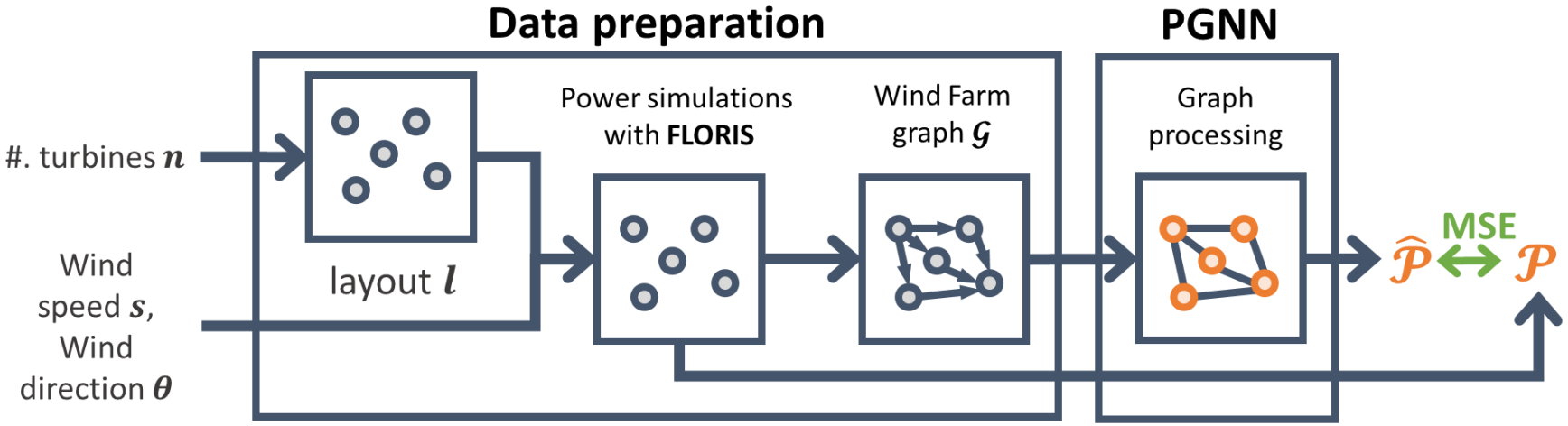
# Physics-Induced Graph Neural Network: An Application to wind-farm power estimation




(a) A randomly generated feasible wind farm

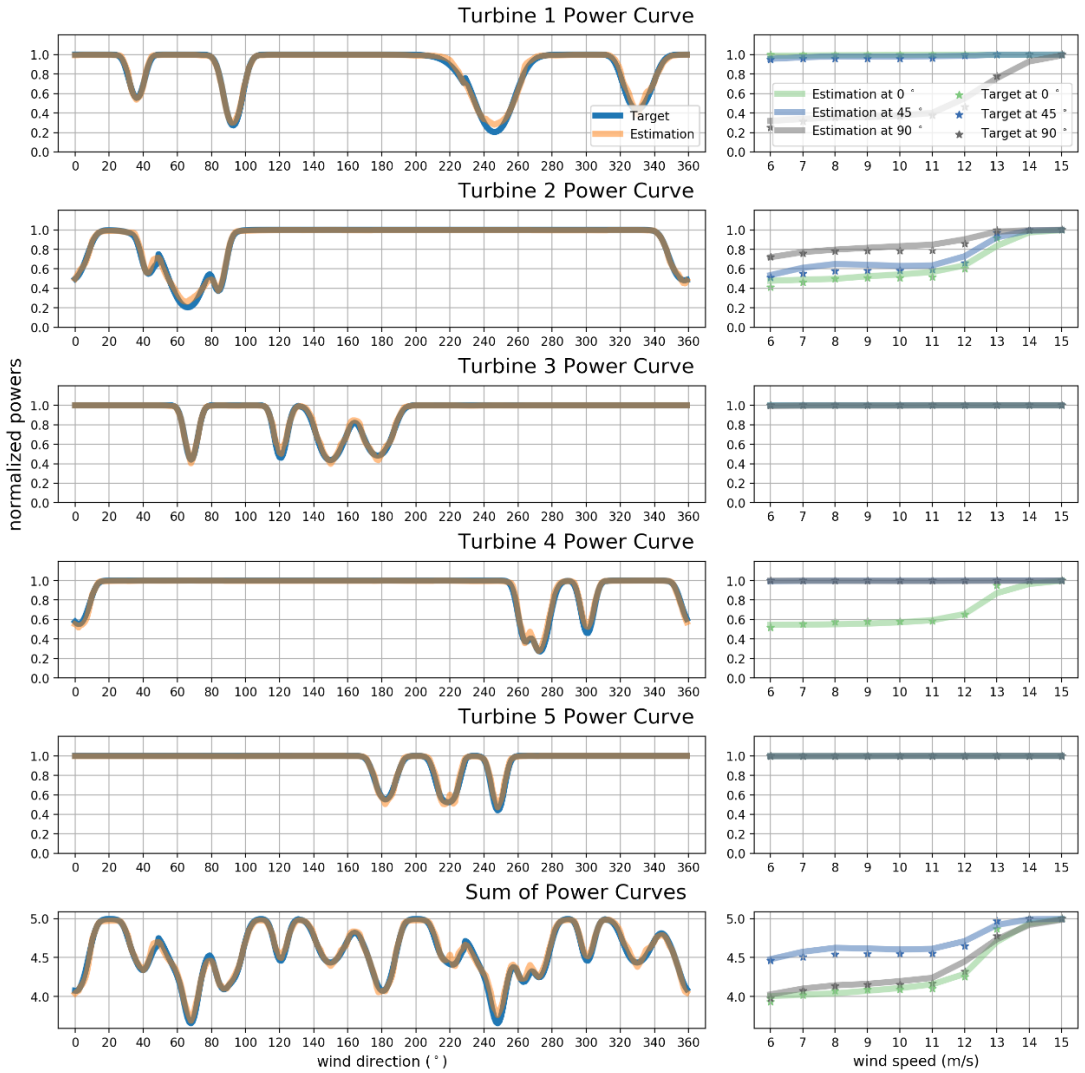


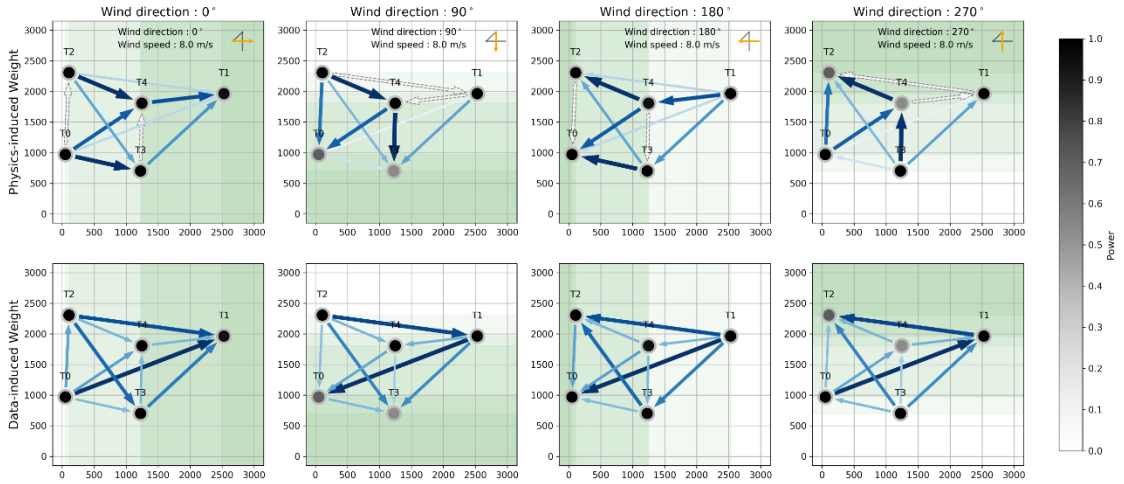

(b) Graphical description of the downstream wake distances

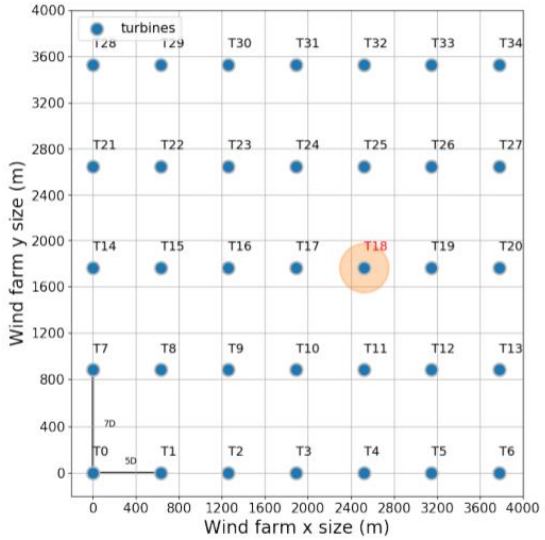


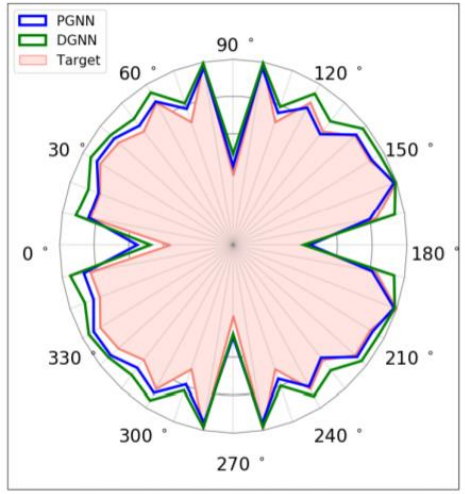

Jounyoung Park and Jinkyoo Park, "Physics-Induced Graph Neural Network: An Application to wind-farm power estimation," Energy (accepted)

# Physics-Induced Graph Neural Network: An Application to wind-farm power estimation




Jounyoung Park and Jinkyoo Park, “Physics-Induced Graph Neural Network: An Application to wind-farm power estimation,” Energy (accepted)

# StarCraft2 Minigame

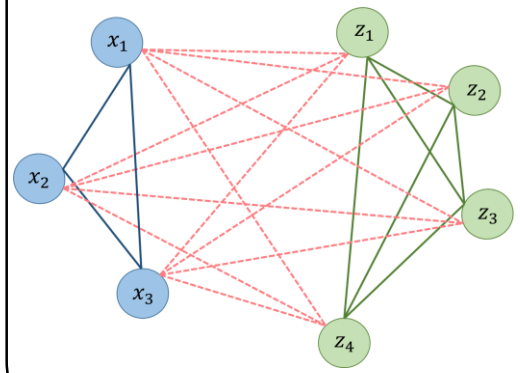
Mikayel Samvelyan et el. (Axriv), “The StarCraft Multi-Agent Challenge (SMAC)”,

- Oxford Computer Science research team has developed StarCraft2 learning environment: https://github.com/oxwhirl/smac
- The team also provide pyMARL, a PyTorch based MARL code library : https://github.com/oxwhirl/pymarl

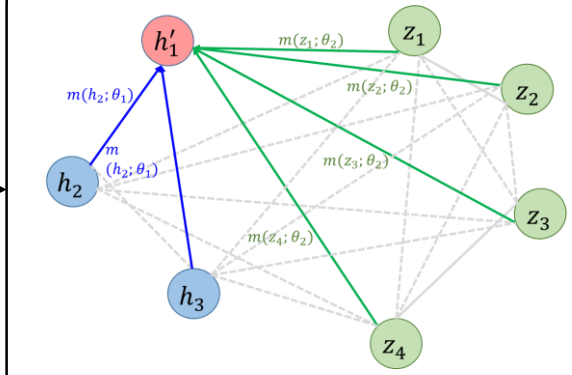
# StarCraft2 Minigame

**0. Observation**

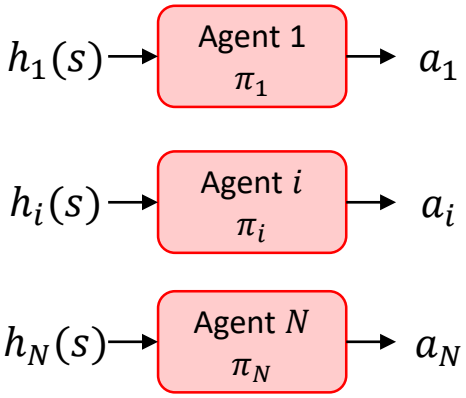
**1. State representation using graph**




**2. Graph Embedding using Graph Neural Network**

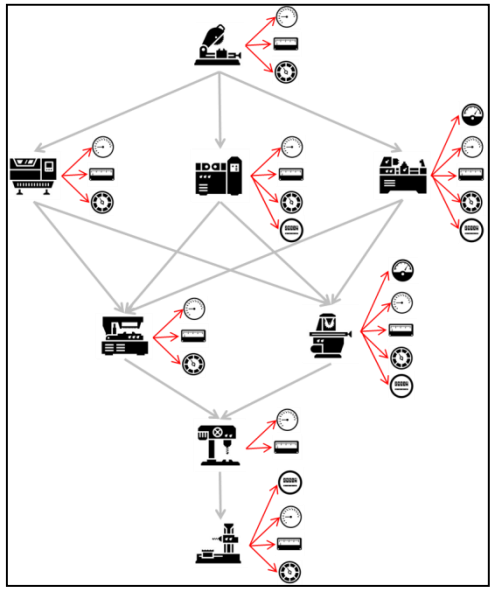


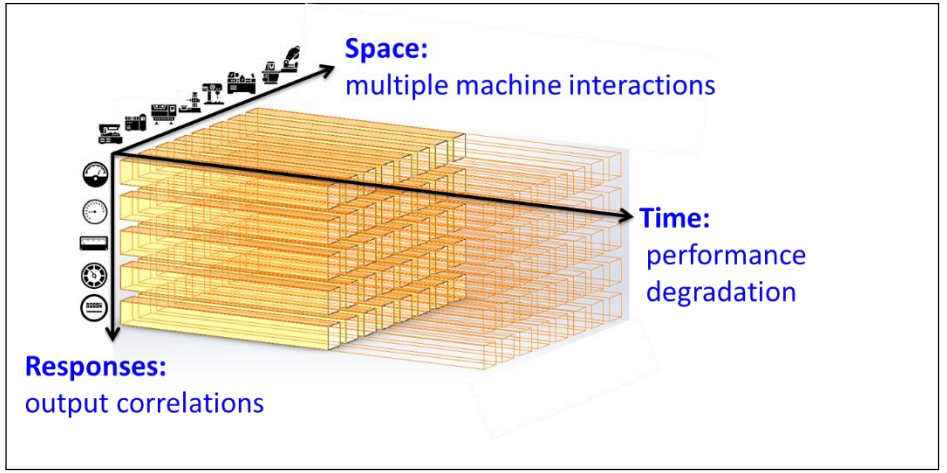

**3. Decentralized Actor policy**

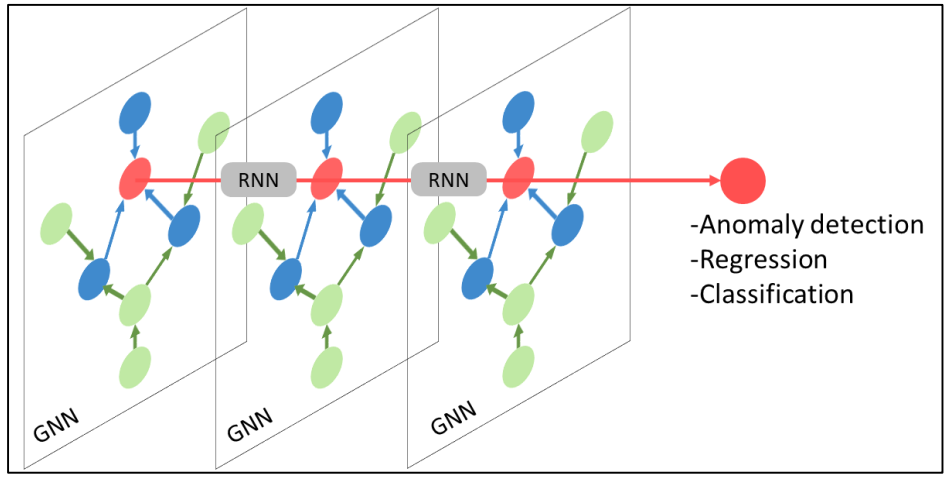


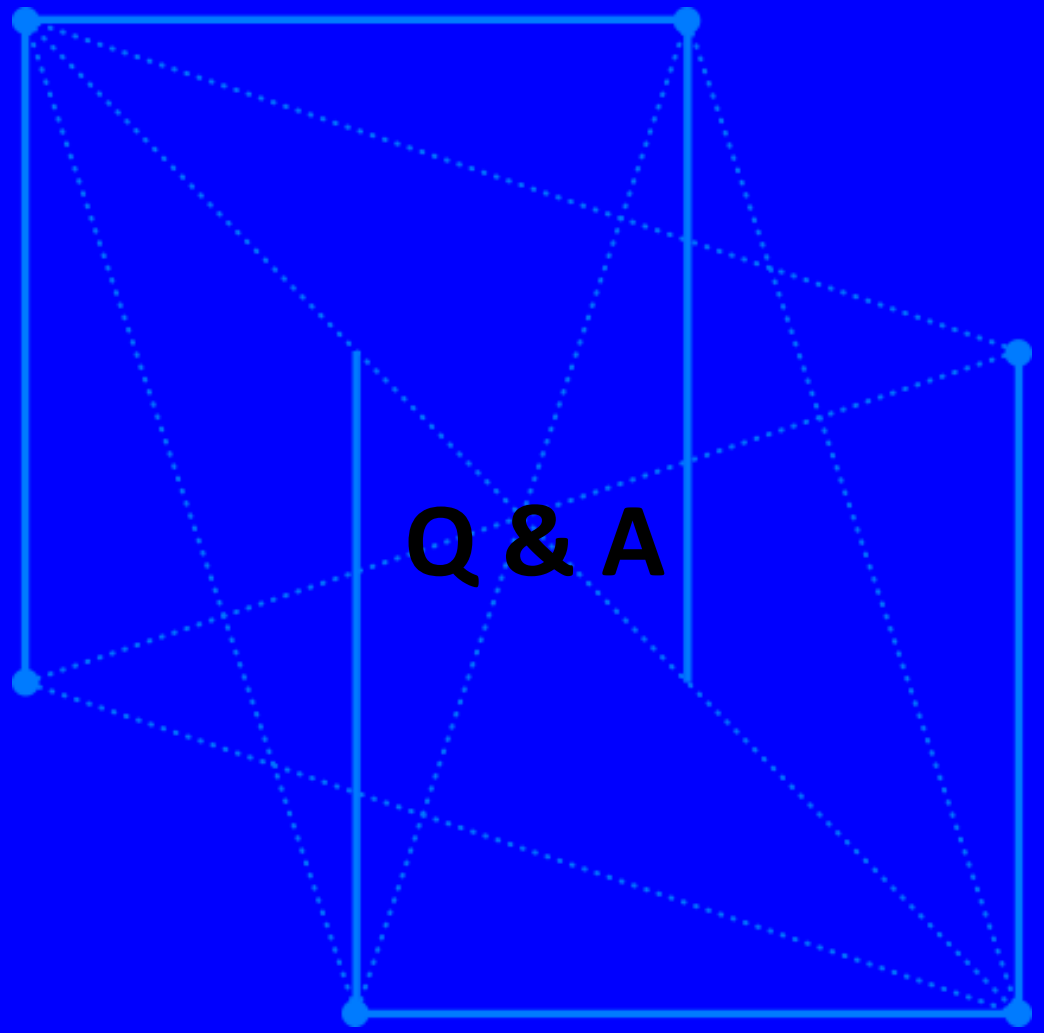

1. 게임 상황을 잘 표현할 수 있는 그래프 형태의 state표현법 도출
2. 그래프로 연결된 agent들과 환경 요소들간의 관계를 Graph Neural Network기반 node embedding을 통해 함축적으로 표현
3. node embedding을 통해 함축적으로 표현된 전체 state을 활용하여 agent별로 분산적으로 의사결정 도출

# Monitoring Spatial and Temporal Patterns in Sensor Networks






Q & A