HW 7

Venkata Akshith Reddy Kasireddy (A20455209)
Sai Vishal Kodimela (A20453006)
Atharva Kadam (A20467229)

1. For a triple to be invalid for total correctness, it should be invalid for either partial correctness or termination.But in order to show $\models$ {T} S {sp(p, S)} but $\not\models$tot {T} S {sp(p, S)}, the triple must satisfy partial correctness and it shouldn't terminate. Thus, we just need a state which does not terminate.

Examples:

Since we are given that {T} S {sp(p, S)} is partially correct, we need a state in which S does not terminate. We know that if it either diverges or in case of runtime errors the program does not terminate.

1. Let W ≡ while $i \neq 0$ do i := i-1 od. Clearly, if W terminates, it's with i = 0, so for any precondition p, if i isn't free in p, we have sp(p, W)$\Leftrightarrow$ p $\wedge$ i = 0, so {p} W {p $\wedge$ i = 0} is partially correct. However, {i < 0} W {i = 0} is partially correct but never totally correct(as it does not terminate).
2. **Divergence**: {T} i := -3; while i > 0 do i := i*2 od; i := 5 {i = 5}
3. **Runtime error**: {T} x := 0; y := 1/ x {x = 0 $\wedge$ y = 1/x}

2. sp(x < y $\wedge$ x+y $\leq$ n, x := f(x+y); y := g(x*y))

≡ sp(sp(x < y $\wedge$ x+y $\leq$ n, x := f(x+y)), y := g(x*y))

≡ sp(x_0 < y $\wedge$ x_0+y $\leq$ n $\wedge$ x = f(x_0+y), y := g(x*y))

≡ (x_0 < y $\wedge$ x_0+y $\leq$ n $\wedge$ x = f(x_0+y))[y_0/y] ∧ y = (g(x*y))[y_0/y]

≡ x_0 < y_0 $\wedge$ x_0+y_0 $\leq$ n $\wedge$ x = f(x_0+y_0) $\wedge$ y = g(x*y_0)

3.

a. sp(x = 2^k, x := x/2) ≡ x_0 = 2^k $\wedge$ x = x_0/2 $\Rightarrow$ x = 2^(k-1)

As x_0 is a temporary variable from memory retention, x_0 was dropped.

b. wp(x := x/2, x = 2^k) ≡ x/2 = 2^k $\Leftrightarrow$ x = 2^(k+1)

c. $sp(x = x_0,$ if odd(x) then x := x+1 else skip fi$)$

$\equiv sp(x = x_0 \wedge odd(x), x := x+1) \vee sp(x = x_0 \wedge even(x), skip)$

$\equiv (odd(x_0) \wedge x = x_0+1) \vee (x = x_0 \wedge even(x))$

(The last predicate implies even(x) if we don't care in losing x_0)

$\Leftrightarrow$ even(x)

d. wp(S, even(x))

$\equiv$ wp(if odd(x) then x := x+1 else skip fi, even(x))

$\equiv (odd(x) \rightarrow wp(x := x+1, even(x))) \wedge (even(x) \rightarrow wp(skip, even(x)))$

$\equiv (odd(x) \rightarrow even(x+1)) \wedge (even(x) \rightarrow even(x))$

$\Leftrightarrow$ even(x)

e. $sp(p' \wedge p, S)$.

(Replacing p′ in following step)

$= sp(L = L_0 \wedge R = R_0 \wedge p, S)$.

(Replacing S in following step)

$= sp(L = L_0 \wedge R = R_0 \wedge p,$ **if** $x < b[M]$ **then** R **:=** M **else** L **:=** M **fi**)

$\equiv sp(L = L_0 \wedge R = R_0 \wedge p \wedge x < b[M], R := M) \vee sp(L = L_0 \wedge R = R_0 \wedge p \wedge x \geq b[M], L := M)$

(Replacing p in following step)

$= sp(L = L_0 \wedge R = R_0 \wedge L < R-1 \wedge M = (L+R)/2 \wedge b[L] \leq x < b[R] \wedge x < b[M], R := M) \vee$

$sp(L = L_0 \wedge R = R_0 \wedge L < R-1 \wedge M = (L+R)/2 \wedge b[L] \leq x < b[R] \wedge x \geq b[M], L := M)$

$\equiv (L = L_0 \wedge L < R_0-1 \wedge M = (L+R_0)/2 \wedge b[L] \leq x < b[R_0] \wedge x < b[M] \wedge R = M)$

$\vee (R = R_0 \wedge L_0 < R-1 \wedge M = (L_0+R)/2 \wedge b[L_0] \leq x < b[R] \wedge x \geq b[M] \wedge L = M)$

f. $wp(S, p) \equiv (x < b[M] \rightarrow wp(R := M, p)) \wedge (x \geq b[M] \rightarrow wp(L := M, p))$

$\equiv (x < b[M] \rightarrow p[M/R])) \wedge (x \geq b[M] \rightarrow p[M/L]))$

$\equiv (x < b[M] \rightarrow L < M\text{-}1 \wedge b[L] \leq x < b[M]) \wedge (x \geq b[M] \rightarrow M < R\text{-}1 \wedge b[M] \leq x < b[R])$

4. $\mathbf{p_1 \equiv (x = 2\text{^}k \wedge k \le n)[k+1/k]}$

$\mathbf{\equiv x = 2\text{^}(k+1) \wedge k+1 \le n}$

$\mathbf{p_2 \equiv p_1\, [x*2/x]}$ **(where** $\mathbf{p_1 \equiv x = 2\text{^}(k+1) \wedge k+1 \le n}$ **)**

$\mathbf{\equiv x*2 = 2\text{^}(k+1) \wedge k+1 \le n}$

$\mathbf{p_3 \equiv p \wedge k \ge n}$ **(where** $\mathbf{p \equiv x = 2\text{^}k \wedge k \le n}$ **)**

$\mathbf{\equiv x = 2\text{^}k \wedge k \le n \wedge k \ge n}$

$\mathbf{r_1}$ $\equiv$ **precondition str 4, 3**

$\mathbf{r_2}$ $\equiv$ **while loop, 5**

1. $\{p_1\}$ k := k+1 $\{x = 2\text{^}k \wedge k \le n\}$	assignment
2. $\{p_2\}$ x := x*2 $\{p_1\}$	assignment
3. $\{p_2\}$ x := x*2; k := k+1 $\{x = 2\text{^}k \wedge k \le n\}$	sequence 2, 1
4. $p \wedge k < n \rightarrow p_2$	pred logic
5. $\{ p \wedge k < n\}$ x := x*2; k := k+1 {p}	precondition str 4, 3
6. {inv p} while k < n do x := x*2; k := k+1 od $\{p_3\}$	while loop, 5

5. $\mathbf{q_1 \equiv (r = X*Y\text{-}x*2*y)[x/2/x]}$

$\mathbf{\equiv r = X*Y\text{-}(x/2)*2*y}$

$\mathbf{q_2 \equiv (r + y = X*Y\text{-}x*y)[x\text{-}1/x]}$

$\mathbf{\equiv r + y = X*Y\text{-}(x\text{-}1)*y}$

$\mathbf{r_1}$ $\equiv$ **conditional, 3, 6**

1. { r = X*Y-x*2*y} y := 2*y {r = X*Y-x*y}	assignment
2. $\{ q_1\}$ x := x/2 {r = X*Y-x*2*y}	assignment
3. $\{q_1\}$ x := x/2; y := 2*y {r = X*Y-x*y}	sequence 2, 1
4. {r+y = X*Y-x*y} r := r+y {r = X*Y-x*y}	assignment

5. { q_2} x := x-1 {r+y = X*Y-x*y} assignment
6. { q_2} x := x-1; r := r+y {r = X*Y-x*y} sequence 5, 4
7.. {(r = X*Y-x*y $\wedge$ even(x) $\rightarrow q_1$) $\wedge$ (r = X*Y-x*y $\wedge$ odd(x) $\rightarrow q_2$)} conditional 3,6
   if even(x) then x := x/2; r := 2*r
   else x := x-1; r := r+y fi {X*Y = r-x*y}

6. $\mathbf{q_1 \equiv (p_0 \wedge r = X*Y-x*y \wedge even(x))[x_0/x] \wedge x = x/2\,[x_0/x]}$

$\mathbf{\equiv r = r_0 \wedge y = y_0 \wedge r = X*Y-x_0*y \wedge even(x_0)) \wedge x = x_0/2}$

$\mathbf{q_2 \equiv q_1\,[y_0/y] \wedge y = 2*y\,[y_0/y]}$

$\mathbf{\equiv r = r_0 \wedge r = X*Y-x_0*y_0 \wedge even(x_0)) \wedge x = x_0/2 \wedge y = 2*y_0}$

q_3 isn't given in the problem.

$\mathbf{q_4 \equiv (r = r_0 \wedge x = x_0 \wedge y = y_0 \wedge r = X*Y-x*y \wedge odd(x))[x_0/x] \wedge x = x-1[x_0/x]}$

$\mathbf{\equiv r = r_0 \wedge y = y_0 \wedge r = X*Y-x_0*y \wedge odd(x_0) \wedge x = x_0-1}$

$\mathbf{q_5 \equiv q_4\,[r_0/r] \wedge r = r+y[r_0/r]}$

$\mathbf{\equiv y = y_0 \wedge r_0 = X*Y-x_0*y \wedge odd(x_0) \wedge x = x_0-1 \wedge r = r_0+y}$

r_1 ≡ conditional, 3,6

1. {$p_0 \wedge$ r = X*Y–x*y $\wedge$ even(x)} x := x/2 {q_1} assignment
2. {q_1} y := 2*y {q_2} assignment
3. {$p_0 \wedge$ r = X*Y–x*y $\wedge$ even(x)} x := x/2; y := 2*y {q_2}sequence 1,2
4. {$p_0 \wedge$ r = X*Y–x*y $\wedge$ odd(x)} x := x–1 {q_4} assignment
5. {q_4} r := r+y {q_5} assignment
6. {$p_0 \wedge$ r = X*Y–x*y $\wedge$ odd(x)} x := x–1; r := r+y {q_5} sequence 5,6
7. {$p_0 \wedge$ r = X*Y–x*y} conditional 3,6
   if even(x) then x := x/2; y := 2*y
   else x := x–1; r := r+y fi {$q_2 \vee q_5$}