Hw 6

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1. Let us consider w_1 and w_2 such that, $w_1 \equiv wp(S_1, q)$ and $w_2 \equiv wp(S_2, q)$, then $wp(IFN, q)$ $\Leftrightarrow (B_1 \wedge w_1) \vee (B_2 \wedge w_2) \vee (B_1 \wedge B_2 \wedge w_1 \wedge w_2)$,
   which $\Rightarrow$ but not $\Leftarrow (B_1 \wedge w_1) \vee (B_2 \wedge w_2)$. The third conjunct $B_1 \wedge B_2 \wedge w_1 \wedge w_2$ is needed for the *wp*
   because if B_1 and B_2 are both true then we need both w_1 and w_2 to hold so that regardless of whether the
   nondeterministic IF executes S_1 or S_2, we'll still establish q. (With deterministic **if** B …, the third conjunct never holds because it requires $B \wedge \neg B$.)

2. wp(m := m+ f(m,y), f(m,y) < g(y,m))
   $\equiv$ f(m+ f(m,y),y) < g(y,m+ f(m,y))

3. *wp (u := u*k; k := u, u > h(k))*
   $\equiv$ *wp*(u := u*k, *wp*(k := u, u > h(k)))
   $\equiv$ *wp*(u := u*k, u > h(u))
   $\equiv$ u*k > h(u*k)

4. wp(if x < 0 then x := -x fi, $x^2 \geq$ x)
   $\equiv$ (x < 0 $\rightarrow$ wp(x := -x, $x^2 \geq$ x)) $\wedge$ (x $\geq$ 0 $\rightarrow$ wp(skip, $x^2 \geq$ x))
   $\equiv$ (x < 0 $\rightarrow$ $x^2 \geq$ -x) $\wedge$ (x $\geq$ 0 $\rightarrow$ $x^2 \geq$ x).

5. $wp\left(if\ z \geq 0\ then\ x := x + \frac{a}{y}\ else\ x := y + \frac{b}{x}\ fi\ ,\ a \leq x\ <\ f(x,y)\right)$

   $$\equiv D(z \geq 0) \ \wedge\ \left(z \geq 0 \rightarrow wp\left(x := x + \tfrac{a}{y}\ ,\ a \leq x < f(x,y)\right)\right) \ \wedge \left(z < 0 \rightarrow wp\left(x := y + \tfrac{b}{x}\ ,\ a \leq x < f(x,y)\right)\right)$$

   where as

   $wp\left(x := x + \frac{a}{y}\ ,\ a \leq x < f(x,y)\right)$

$\equiv D\,(x+\frac{a}{y}) \wedge a \le x+\frac{a}{y} < f\,(x+\frac{a}{y}\,,\,y)$

$\equiv y \ne 0 \wedge a \le x+\frac{a}{y} < f\left(x+\frac{a}{y}\,,\,y\right)$

$wp\left(x := y+\frac{b}{x}\,,\; a \le x < f\,(x,y)\right)$

$\equiv D\,(y+\frac{b}{x}) \wedge a \le y+\frac{b}{x} < f\,(y+\frac{b}{x}, y)$

$\equiv x \ne 0 \wedge a \le y+\frac{b}{x} < f\left(y+\frac{b}{x}, y\right)$

So the logical significance will be as follows:

$D\,(z \ge 0) \wedge \left(z \ge 0 \rightarrow wp\left(x := x+\frac{a}{y}\,,\; a \le x < f\,(x,y)\right)\right) \wedge \left(z < 0 \rightarrow wp\left(x := y+\frac{b}{x}\,,\; a \le x < f\,(x,y)\right)\right)$

$\equiv D\,(z \ge 0) \wedge \left(z \ge 0 \rightarrow a \le x+\frac{a}{y} < f\left(x+\frac{a}{y}\,,\,y\right)\right) \wedge \left(z < 0 \rightarrow a \le y+\frac{b}{x} < f\left(y+\frac{b}{x}, y\right)\right)$

$\equiv z \ge 0 \rightarrow y \ne 0 \wedge a \le x+\frac{a}{y} < f\left(x+\frac{a}{y}\,,\,y\right) \wedge z < 0 \rightarrow x \ne 0 \wedge a \le y+\frac{b}{x} < f\left(y+\frac{b}{x}, y\right)$

6. We Know that

$wp\,(x := b\,[sqrt\,(y)]\,,\; x > 0)$

$\equiv D\,(wlp\,(x := b\,[sqrt\,(y)]\,,\; x > 0)) \wedge wlp\,(x := b\,[sqrt\,(y)]\,,\; x > 0) \wedge D\,(x := b\,[sqrt\,(y)])$

$w1 \equiv wlp\,(x := b\,[sqrt\,(y)]\,,\; x > 0)$

$\equiv b\,[sqrt\,(y)] > 0$

$D\,(wlp\,(x := b\,[sqrt\,(y)]\,,\; x > 0))$

$\equiv D\,(b\,[sqrt\,(y)] > 0)$

(**Where as** $wlp\,(x := b\,[sqrt\,(y)]\,,\; x > 0) \equiv b\,[sqrt\,(y)] > 0$)

$\equiv D\,(b\,[sqrt\,(y)]) \wedge D\,(0)$

$\equiv D\,(sqrt\,(y)) \wedge 0 \le sqrt\,(y) < size\,(b)$

$\equiv y \ge 0 \wedge 0 \le sqrt\,(y) < size\,(b)$

$D\,(x := b\,[sqrt\,(y)])$

$\equiv D(x) \wedge D\,(b\,[sqrt\,(y)]) \equiv D\,(b\,[sqrt\,(y)])$

$\equiv D\,(sqrt\,(y)) \wedge 0 \le sqrt\,(y) < size\,(b)$

$\equiv y \ge 0 \wedge 0 \le sqrt\,(y) < size\,(b)$

So the logical significance will be as follows:

$D\,(wlp\,(x{:=}b\,[sqrt\,(y)]\ ,\ x>0))\ \land\ wlp\,(x{:=}b\,[sqrt\,(y)]\ ,\ x>0)\ \land\ D\,(x{:=}b\,[sqrt\,(y)])$

$\equiv\ y\geq 0\ \land\ 0\leq sqrt\,(y)\ <\ size\,(b)\ \land\ b\,[sqrt\,(y)]\ >\ 0\ \land\ y\geq 0\ \land\ 0\leq sqrt\,(y)<\ size\,(b)$

$\equiv y\geq 0\ \land\ 0\leq sqrt\,(y)\ <\ size\,(b)\ \land\ b\,[sqrt\,(y)]\ >\ 0$

7. We know that,

$wp\,(k{:=}k-b\,[k]\,,\ k{\neq}0)$

$\equiv D\,(wlp\,(k{:=}k-b\,[k]\ ,\ k{\neq}0))\ \land\ wlp\,(k{:=}k-b\,[k]\ ,\ k{\neq}0)\ \land\ D\,(k{:=}k-b\,[k])$

$D\,(wlp\,(k{:=}k-b\,[k])\,,k{\neq}0)$ **(Where as** $wlp\,(k{:=}k-b\,[k]\ ,\ k{\neq}0)\equiv k-b\,[k]\neq 0$ **)**

$\equiv D\,(k-b\,[k]\ \neq 0)$

$\equiv D\,(k)\ \land\ D\,(\,b\,[\,k\,])$

$\equiv 0\leq k\ <size\,(b)$

$D\,(k{:=}k-b\,[k])$

$\equiv D\,(k)\land\ D\,(k-b\,[k])$

$\equiv D\,(k-b\,[k])$

$\equiv D\,(k)\ \land\ D\,(\,b\,[\,k\,])$

$\equiv 0\leq k\ <size\,(b)$

So the logical significance will be as follows:

$D\,(wlp\,(k{:=}k-b\,[k]\ ,\ k{\neq}0))\ \land\ wlp\,(k{:=}k-b\,[k]\ ,\ k{\neq}0)\ \land\ D\,(k{:=}k-b\,[k])$

$\equiv\ 0\leq k\ <size\,(b)\ \land\ k-b\,[k]\neq 0\ \ \land\ 0\leq k\ <size\,(b)$

$\equiv\ 0\leq k\ <size\,(b)\ \land\ k-b\,[k]\neq 0$

8. p ≡ (x+y < f(a) ∨ ∃x . x ≥ a+y → ∃y . x*y > b-y-c)[y*z / y]

p ≡ x+(y*z) < f(a) ∨ ∃x . x ≥ a+(y*z) → ∃y . x*y > b-y-c

Here the first occurrence and second occurrence of y represents the free variables, thus we substitute the value of y with y*z . Whereas, for ∃y . x*y the value of y represents the bound variable, thus making it not replaceable(value of y for third and fourth occurrence).

9. $p \equiv (x+y < f(a) \vee \exists x \,.\, x \geq a+y \rightarrow \exists y \,.\, x*y > b\text{-}y\text{-}c)[a\text{-}y/a]$

   $p \equiv x+y < f(a\text{-}y) \vee \exists x \,.\, x \geq (a\text{-}y)+y \rightarrow \exists y \,.\, x*y > b\text{-}y\text{-}c$

   Here all the occurrences of a are free variables and are not bound.
   So, a can be substituted with a-y.

10. $p \equiv (x+y < f(a) \vee \exists x \,.\, x \geq a+y \rightarrow \exists y \,.\, x*y > b\text{-}y\text{-}c)[x*y \square a][y\text{-}z \square x]$

    $p \equiv (x+y< f(a) \vee \exists u \,.\, x \geq a+y \rightarrow (\exists y \,.\, x*y > b\text{-}y\text{-}c)[u/x])[x*y \square a][y\text{-}z \square x]$

    // Here we are avoiding the capture by renaming x with u

    $p \equiv (x+y< f(a) \vee \exists u \,.\, u \geq a+y \rightarrow (\exists v \,.\, u*y > b\text{-}y\text{-}c)[v/y])[x*y \square a][y\text{-}z \square x]$

    // Here we are avoiding the capture by renaming y with v

    $p \equiv x+y< f(a) \vee \exists u \,.\, u \geq a+y \rightarrow (\exists v \,.\, u*v > b\text{-}v\text{-}c)\ [x*y \square a][y\text{-}z \square x]$

    // Replacing 'a' with'x*y'

    $p \equiv x+y< f(x*y) \vee \exists u \,.\, u \geq (x*y)+y \rightarrow (\exists v \,.\, u*v > b\text{-}v\text{-}c)\ [y\text{-}z \square x]$

    // Replacing 'x' with 'y-z'

    $p \equiv (y\text{-}z)+y< f((y\text{-}z)*y) \vee \exists u \,.\, u \geq ((y\text{-}z)*y)+y \rightarrow (\exists v \,.\, u*v > b\text{-}v\text{-}c)$