HW1 CS536

Akhilesh Datar A20452932

Shirish Mecheri Vogga A20456808

Venkata Akshith Reddy Kasireddy A20455209

1. The basic rules for parenthesis are:

    a. Precedence of propositional operators

    b. Associativity (-> and <-> are right associative and V(or), ∧(and) are left associative (to avoid multiple answers))

    c. Number of parenthesis are equal to number of operators including outermost parenthesis.

    d. Precedence of ∀ and ∃ are taken last, since they're quantifiers

Thus

A. $p \wedge \neg r \wedge s \rightarrow \neg q \vee r \rightarrow \neg p \leftrightarrow \neg s \rightarrow t$, becomes.

$((((p \wedge (\neg r)) \wedge s) \rightarrow (((\neg q) \vee r) \rightarrow (\neg p))) \leftrightarrow ((\neg s) \rightarrow t))$

B. $\exists\ m\ .\ 0 \le m < n \wedge \forall\ j\ .\ 0 \le j < m \rightarrow b[0] \le b[j] \le b[m]$ * becomes

$(\exists\ m\ .\ ((0 \le m < n) \wedge (\forall\ j\ .\ ((0 \le j < m) \rightarrow (b[0] \le b[j] \le b[m])))))$

2. Given the expression as follows

2a. $((\neg(p \vee q) \vee r) \rightarrow (((\neg q) \vee r) \rightarrow ((p \vee (\neg r)) \vee (q \wedge s))))$

In order to remove the redundant parentheses we label each of the parentheses since we can have a better understanding of which parentheses belongs to which expression then we get

$(_1\ (_2\ \neg(_3\ p \vee q)_3 \vee r\ )_2 \rightarrow (_4(_5(_6\ \neg q)_6 \vee r\ )_5 \rightarrow (_7(_8\ p \vee (_9\ \neg r\ )_9)_8 \vee (_{10}\ q \wedge s\ )_{10})_7)_4)_1$

We proceed on analyzing the innermost parentheses and then extend to the outer ones

$(_6$, $(_9$, $(_{10}$ are redundant given that $(_6$ is on ¬ which is higher precedence than ∨ similarly for parenthesis $(_9$. For $(_{10}$ since we know that ^ has higher precedence than v hence it is redundant on removing we get the following expression

$$(_1\ (_2\ \neg(_3\ p \vee q)_3 \vee r\ )_2 \rightarrow (_4(_5\ \neg q \vee r\ )_5 \rightarrow (_7(_8\ p \vee \neg r\ )_8 \vee q \wedge s\ )_7)_4)_1$$

Now we can remove $(_2$ (∨ over →) and $(_5$ (∨ over →) $(_8$ (∨ is associative) hence we get

$$(_1\ \neg(_3\ p \vee q)_3 \wedge r \rightarrow (_4\ \neg q \vee r \rightarrow (_7\ p \vee \neg r \vee q \wedge s\ )_7)_4)_1$$

Lastly now we can remove $(_1$ being the outermost pair , $(_4$ since → is right associative and $(_7$ since ∨ over → then we get the final expression as

$$\mathbf{\neg(p \vee q) \wedge r \rightarrow \neg q \vee r \rightarrow p \vee \neg r \vee q \wedge s}$$

2b. The expression is as shown below

$$(\exists\ i\ .\ (((0 \leq i) \wedge (i < m)) \wedge (\forall\ j\ .\ (((m \leq j) \wedge (j < n)) \rightarrow (b[i] = b[j])))))$$

Since we know that in predicate logic >,<,= have a higher precedence than the logical operators then using the above definition we can simplify the expressions as shown below

$$(\exists\ i\ .\ ((0 \leq i \wedge i < m) \wedge (\forall\ j\ .\ ((m \leq j \wedge j < n)) \rightarrow b[i] = b[j]))))$$

Since the body of ∀ j ends at its matching expression and since ∧ is stronger than → we get

$$(\exists\ i.((0 \leq i \wedge i < m) \wedge (\forall\ j.m \leq j \wedge j < n \rightarrow b[i] = b[j]\ )))$$

Lastly for ∃ i the parentheses ends at the expression so removing the outer parenthesis , since ∧ is associative then we can combine the expression as follows to get the result

$$\mathbf{\exists\ i.0 \leq i \wedge i < m \wedge \forall\ j.m \leq j \wedge j < n \rightarrow b[i] = b[j]}$$

2c. Given the Expression

$$(\ \forall x.((\exists y.(p \rightarrow q)) \rightarrow (\forall z.(q \vee (r \wedge s)))))$$

We first start by removing the innermost parentheses since we know that ∧ over v and since for parentheses around ∀z is redundant as its end parentheses is at the end of the expression we can remove that.

Also since we know that → is right associative the parentheses around p→q is also redundant however we dont remove the parentheses around ∃y since the expression would end up becoming p → q → expression hence we get the result as

$\forall x.(\exists y.p \rightarrow q) \rightarrow (\forall z.q \lor r \land s)$

3. Here are the solutions to the 2 problems.

   A. The $p \land q \lor \neg r \rightarrow \neg p \rightarrow q \equiv ((p \land q) \lor ((\neg r \rightarrow ((\neg p) \rightarrow q))))$ **IS NOT** $\not\equiv$ due to no parenthesis on the LHS(left hand side).

   B. $\forall x . p \rightarrow \exists y . q \rightarrow r \equiv ((\forall x . p) \rightarrow (\exists y . q)) \rightarrow r$ **IS NOT** $\not\equiv$ due to incorrect parenthesis on the LHS (left hand side).

4. Contingency is a proposition which isn't either tautology or contradiction. Tautology is a proposition that holds only true values, whereas contradiction is a proposition which is always false (opposite of tautology is contradiction).

4a.The proposition $(p \rightarrow (q \rightarrow r)) \longleftrightarrow ((p \rightarrow q) \rightarrow r)$ is termed as a contingency. For example, when p,q and r holds Truth values as T,T and F respectively, the above proposition becomes as follows:

F$\longleftrightarrow$ F which is in turn T.

Similarly if p,q and r holds F,T and F values respectively, the proposition becomes as follows:

T$\longleftrightarrow$F which is in turn F.

Thus the above proposition is contingency which is neither tautology nor contradiction.

4b. $(\forall x \in \mathbb{Z}.\forall y \in \mathbb{Z}.f(x, y) > 0) \rightarrow (\exists x \in \mathbb{Z}.\exists y \in \mathbb{Z}.f(x, y) > 0)$ is a tautology.

For the given proposition in order to be a tautology both the expressions should hold the same values as mentioned below:

T→T is T

F→F is T

Given proportion is tautology because, if $f(x,y) > 0$ is satisfied for all values of $x \in \mathbb{Z}$ and $y \in \mathbb{Z}$ then there exists a value of x and y which also satisfies $f(x,y) > 0$.Similarly, if $f(x,y) > 0$ is not satisfied for all values of $x \in \mathbb{Z}$ and $y \in \mathbb{Z}$ then there exists a value of x and y which is not satisfied for $f(x,y) > 0$.Thus for the given proportion the expressions hold the similar values, which will be either T→T or F→F which will in turn become T(which is termed as tautology).

5. Here's the solution to the 4 statements:

   A. p is sufficient for q: $p \rightarrow q$

   B. p only if q: $p \rightarrow q$

   C. p if q: $q \rightarrow p$

   D. p is necessary for q: $q \rightarrow p$

6. $e_1 \equiv e_2$ is termed as a syntactic equality for expressions e_1 and e_2. Whereas $e_1 = e_2$ is termed as a semantic equality for expressions e_1 and e_2

6a. Yes it satisfies because, For the proposition $e_1 \neq e_2$ imply $e_1 \not\equiv e_2$, in order to verify let us consider contrapositive for it, which will be as follows:

$e_1 \equiv e_2$ imply $e_1 = e_2$

If expressions e_1, e_2 satisfies syntactic equality(i.e., $e_1 \equiv e_2$), then those expressions must have the same values(syntactic equality satisfies equality in text) and differ only in the redundant parentheses.

6b.No, the proposition $e_1 = e_2$ does not imply $e_1 \equiv e_2$ because, if e_1 and e_2 are the expressions satisfying semantic equality and syntactic equality, then their semantic equality does not imply syntactic equality.For example, let us consider 1+2 and 3 as two different expression then $1+2=3$ but $1+2 \not\equiv 3$.

7. Here's the proof that $p \land \neg(q \land r) \rightarrow q \land r \rightarrow \neg p$ is a tautology:

$p \land \neg(q \land r) \rightarrow q \land r \rightarrow \neg p$

$\Leftrightarrow p \land \neg(q \land r) \rightarrow \mathbf{(\neg(q \land r) \lor \neg p)}$ **Definition of →**

$\Leftrightarrow \mathbf{\neg\ (p \land \neg\ (q \land r))} \lor (\neg\ (q \land r) \lor \neg p)$ **Definition of →**

$\Leftrightarrow \mathbf{(\neg p \lor (q \land r))} \lor (\neg(q \land r) \lor \neg p)$ **DeMorgan's law (on ¬(… ∧ …)) and ¬¬)**

$\Leftrightarrow \mathbf{(q \land r) \lor \neg(q \land r) \lor \neg p \lor \neg p}$ **Rule of ∨ associative and commutative**

$\Leftrightarrow \mathbf{T} \lor \neg p \lor \neg p$ Excluded Middle (**(q ∧ r) ∨ ¬(q ∧ r) becomes T**)

$\Leftrightarrow \mathbf{T}$ **Domination by T**

9. Given the definition of a predicate function GT(b, x, m, k) that yields true iff $x > b[m]$, …$b[m+k-1]$, also the function is true if $k < 0$ then the predicate function can be written as follows

$$GT(b, x, m, k) \equiv \forall\ i.m \le i < m+k \wedge k > 0 \rightarrow x > b[i]$$

Where i is the loop variable which satisfies the above conditions and the other condition is necessary since by function definition it should return true if $k < 0$ since this above predicate function can be formulated as $p \rightarrow q$ operation then this is always true if p is false hence the above condition $k > 0$ guarantees that.

8. Simplify $\neg(\forall\ x\ .\ (\exists\ y\ .\ x \le y) \vee \exists\ z\ .\ x \ge z)$

Since we know that by De morgans Law $\neg(e1 \le e2) = e1 > e2$

To apply the negation we first reduce it to a simpler form as shown below

$= \quad (\forall\ x\ .\neg(\exists\ y\ .\ x \le y) \vee \exists\ z\ .\ x \ge z)$

Since this is of the form $\neg(p \vee q) = \neg p \wedge \neg q$ by Demorgan's laws we get

$= \quad (\forall\ x\ .\ \exists\ y\ \neg(x\le y)\ \wedge\ \exists\ z\ \neg(x \ge z))$

$= \quad (\forall\ x\ .\exists\ y\ .\ (x > y) \wedge \exists\ z\ (x<z))$ By Demorgan's Law $\neg(e1 \le e2) = e1 > e2$