Assignment-2

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1a. It is **illegal** because for the given conditional(ternary) operator (x < y ? x : F), the return type of two clauses are different and aren't convertible types that is x holds for integer type and F holds for the boolean type, due to which the compiler throws an error.

1b. Expression b[0] + b[1][1] is **illegal**, because from the comments we came to know that b is a 2-dimensional and in the given expression b[0] holds only one index which refers to one-dimensional.Thus, the given expression is illegal.

1c. Expression match(b1, b2, n) is **legal** and the resulting expression is **boolean**. It is legal because both b1 and b2 are of same dimensional and the resulting expression is boolean because from the comment, match asks whether first n elements of b1 and b2 match(return either true or false).

2a. The expression {x = (2), y = 4} is **well-formed**.Because the bindings of x and y are satisfied, that is both x(array of length one) and y holds the values.

2b. Expression {u = (3, 4), v = 0, w = u[1]} is **ill-formed**.Because the given expression consists of three bindings and of these 3 bindings u and v holds the values,whereas w holds the expression (u[1] is an expression) which should be a value in order to be well-formed.

2c. Expression {r = one, s = four, t = r + s} is **ill-formed**. Because the given expression consists of three bindings and among these 3 bindings r and s are satisfied but t is not satisfied as it has expression(r and s are expression variables).Thus making the whole expression as ill-formed.

3. Given $\sigma = \{x = 2, b = \beta\}$ where $\beta = ($ five, two plus two, 6 $)$. σ can be rewritten in the following two ways:

3a. $\sigma = \{x = 2, b = (5,4,6)\}$, where the value of b is provided as ordered pairs.

3b. $\sigma = \{x = 2, b[0] = 5, b[1] = 4, b[2] = 6)\}$, where the value of b is provided as separate bindings.

4. Given expression $\varphi \equiv x = y*z \wedge y = 3*z \wedge z = b[0] + b[2] \wedge 3 < b[1] < b[2] < 6$.From the question it is clear that z=5 and we know that :

$y = 3 * z$ implies $y = 3 * 5 = 15$

$x = y * z$ implies $x = 15 * 5 = 75$.

In the above expression it is clear that $3 < b[1] < b[2] < 6$, we know that b[1] and b[2] holds integers and thus b[1]=4 and b[2]=5. From the expression it is clear that:

$z = b[0] + b[2]$ implies $5 = b[0] + 5$ thus $b[0] = 0$.

Thus we got all the values of σ which are as follows:

$\sigma = \{x = 75, y = 15, z = 5, b = (0,4,5) \}$.

5. Given expression is $0 * b[b[j]]$.For the given expression a state will be termed as **well-formed** if its bindings hold similar types of values and it will be termed as **proper** if it satisfies the following two terms

- j should be an integer and
- b is an array value

5a. $\{j = 0, b = (3, 2, 5, 4), c = (3), d = 8\}$ is **well-formed and proper** (as it satisfies above mentioned conditions). Result for the expression is $0*b[b[0]]$ implies $0*4$ so the expression result is 0.

5c. For the above given expression, {j = 0, b = 0} is **well-formed**(as it satisfies above mentioned conditions) but it is **not proper.** Binding b is not holding an array value because in order to be an array value it should be represented as either b = (0) or b[0] = 0, thus the expression is **not proper**.

6. Given state is σ = {x = 2, y = 4, b = (-1, 0, 4, 2)}

6a. As σ(z) is not defined for the above state, there is no difference between σ[z ↦ 1] and σ ∪ {(z, 1)}. That is σ[z ↦ 1] = σ ∪ {(z, 1)}.

6b. As σ(x) is defined on σ[x ↦ 5] can be represented as follows (as σ already has a binding for x, its value is being replaced by 5):

σ[x ↦ 5] = {x = 2, y = 4, b = (-1, 0, 4, 2)}[x ↦ 5] so σ[x ↦ 5] = {x = 5 , y = 4, b = (-1, 0, 4, 2)}

As σ already has a binding for x, σ ∪ {(x, 5)} is ill-formed (2 values for x) and this σ ∪ {(x, 5)} can be represented as follows:

σ ∪ {(x, 5)} = {x = 2, y = 4, b = (-1, 0, 4, 2)} ∪ {(x, 5)}

so σ ∪ {(x, 5)} = {x = 2, y = 4, b = (-1, 0, 4, 2), x = 5}

Thus σ[x ↦ 5] and σ ∪ {(x, 5)} are different.

7a. Yes, that is {x = 4, y = 6, b = (4,2,8)} ⊨ (∃x.∃j. b[j] < x < y) using j = 1 and x = 3 (for j = 1 as a witness value in order to satisfy, x can also have multiple values like 4 and 5). Let σ = {x = 4, y = 6, b = (4,2,8)} then the state σ[j ↦ 1][x ↦ 3] = {x = 3, y = 6, b = (4,2,8), j = 1} satisfies the condition b[j] < x < y so that the condition can be reduced to 2<3<6(as j = 1, b[1] = 2 and bindings x and y are 3 and 6 respectively).

For j = 0 and x = 5 also $\{x = 4, y = 6, b = (4,2,8)\} \vDash (\exists x.\exists j.\ b[j] < x < y)$. That is $\sigma[j \mapsto 0][x \mapsto 5] = \{x = 5, y = 6, b = (4,2,8), j = 0\}$ satisfies the condition b[j] < x < y which can be reduced to 4<5<6.

7b. Let us consider $\sigma = \{x = 0, y = 7, b = (4,2,8)\}$, then $\sigma \nvDash (\forall x.\forall k.0 < k < 3 \rightarrow x < b[k])$. We can prove the above statement by providing a counter example that is by showing that there is some value of x for which there is some value of k such that the condition $0 < k < 3 \rightarrow x < b[k]$ is not satisfied .From the given condition 0 < k < 3, the value of k can be either 0 or 1 and for these k values there are a lot of x values(because the condition specifies for all x) that do not satisfy x < b[k]. For example let us consider the bindings x = 10 and k = 2 : $\sigma[x \mapsto 10][k \mapsto 2] \nvDash (0 < k < 3 \rightarrow x < b[k])$.

8a. Let us consider some state σ such that $\sigma \nvDash (\forall x \in U . (\exists y \in V . P(x, y)))$.

The above statement holds when there is some value a ∈ U for x where for each and every value of b ∈ V for y, the condition P(x, y) is not satisfied, that is it can be represented as: $\sigma[x \mapsto a][y \mapsto b] \nvDash (\forall x \in U . (\exists y \in V . P(x, y)))$. Importantly, we can have $\sigma = \varnothing$ if variables that have bindings in σ are not used in P(x, y).

8b. Let us consider some state σ such that $\sigma \nvDash \forall y .((\exists x \in U. P(x, y)) \rightarrow (\exists y \in U. Q(x, y)))$.

In order to follow the above condition, We need a value β for y such that $\sigma[y \mapsto \beta] \vDash (\exists x \in U. P(x, y))$ and also $\sigma[y \mapsto \beta] \nvDash (\exists y \in U. Q(x, y))$.

Firstly, let us consider $\sigma[y \mapsto \beta] \vDash (\exists x \in U. P(x, y))$. For this condition apart from β for y, we need a value $\alpha \in U$ for x such that P(x, y) is satisfied, that is it can be represented as follows:

$\sigma[y \mapsto \beta][x \mapsto \alpha] \vDash (\exists x \in U. P(x, y))$

Secondly, let us consider $\sigma[y \mapsto \beta] \nvDash (\exists y \in U. Q(x, y))$. For this condition we need a value $\delta \in U$ for y such that $\sigma[y \mapsto \alpha] [y \mapsto \delta] \nvDash Q(x, y)$. We know that $\sigma[y \mapsto \alpha] [y \mapsto \delta] = \sigma[y \mapsto \delta]$ because 2nd update overwrites the 1st update.Since $\sigma[y \mapsto \delta] \nvDash Q(x, y)$, we can get false boolean value for Q(x, y) over $\sigma(x)$ and δ.

9. In order to define a predicate function $P(b, c, d, s, t) \equiv \ldots$, let us first consider and define helper predicate R(x,y) that is true if x and y are legal indexes for b: $R(x, y) \equiv x \wedge y$ where $0 < x < n$ and $0 < y < n$. Now coming to our predicate function we can define it as follows $P(b, c, d, s, t) \equiv R(c, d) \wedge R(s, t) \wedge \forall\, c \le i < d. \exists s \le j < t. b[i] < b[j]$, that is the indexes c,d,s and t are all legal indexes for b, and for all indexes i between c and d (including c and excluding d), there is some index j between s and t (including s and excluding t) such that b[i] < b[j].