

```
In [1]: import matplotlib.pyplot as plt
import numpy as np
import random
import time

from math import ceil
from scipy import optimize, stats
from sympy import *
from tqdm import tqdm
```

Часть I. Основы анализа

Задача 1. Найдите предел

$$1. \lim_{x \rightarrow -\infty} \frac{\sqrt{x^2 + 1}}{x} = \lim_{x \rightarrow \infty} -\sqrt{\frac{x^2 + 1}{x^2}} = \lim_{x \rightarrow \infty} -\sqrt{1 + \frac{1}{x^2}} = -1$$

```
In [2]: x = symbols('x')
expr = sqrt(pow(x,2)+1)/x
print(f'Expressions: \n{expr}')
```

Expressions:
sqrt(x**2 + 1)/x

```
In [3]: limit_expr = limit(expr, x, -oo)
print(f'Limit: {limit_expr}')
```

Limit: -1

$$1. \lim_{n \rightarrow +\infty} \frac{(-6)^n - 5^{n+1}}{5^n - (-6)^{n+1}} = \lim_{n \rightarrow +\infty} \frac{(-6)^n / (-6)^{n+1} - 5^{n+1} / (-6)^{n+1}}{5^n / (-6)^{n+1} - 1} = \lim_{n \rightarrow +\infty} \frac{-\frac{1}{6} - (-\frac{5}{6})^{n+1}}{-\frac{1}{6}(-\frac{5}{6})^n - 1} =$$

$$= \lim_{n \rightarrow +\infty} \frac{1 + 6(-\frac{5}{6})^{n+1}}{(-\frac{5}{6})^n + 6} = \lim_{n \rightarrow +\infty} \frac{1 + 0}{0 + 6} = \frac{1}{6}$$

```
In [4]: n = symbols('n')
expr_1 = (1 + 6*pow(5/6,n+1))/(-pow(5/6,n)+6)
expr_2 = (1 - 6*pow(5/6,n+1))/(pow(5/6,n)+6)
print(f'Expressions: \n{expr_1}, \n{expr_2}')
```

Expressions:
(6*0.8333333333333333**(n + 1) + 1)/(6 - 0.8333333333333333**n),
(1 - 6*0.8333333333333333**(n + 1))/(0.8333333333333333**n + 6)

```
In [5]: limit_expr_1 = limit(expr_1, n, +oo)
limit_expr_2 = limit(expr_2, n, +oo)
print(f'Limits: \n{limit_expr_1}, \n{limit_expr_2}')
```

Limits:
1/6,
1/6

$$1. \lim_{n \rightarrow +\infty} \frac{1}{n^3} \sum_{k=1}^n k(k+1) = \lim_{n \rightarrow +\infty} \frac{1}{n^3} \left(\frac{n^3}{3} + n^2 + \frac{2}{3}n \right) = \lim_{n \rightarrow +\infty} \frac{1}{3} + \frac{1}{n} + \frac{2}{3} \frac{1}{n^2} = \frac{1}{3}$$

```
In [6]: k = symbols('k')
expr = Sum(k*(k+1), (k, 1, n)).doit()/pow(n, 3)
print(f'Expressions: \n{expr}')
```

```
Expressions:
(n**3/3 + n**2 + 2*n/3)/n**3
```

```
In [7]: limit_expr = limit(expr, n, +oo)
print(f'Limit: {limit_expr}')
```

```
Limit: 1/3
```

1.

$$\lim_{n \rightarrow +\infty} \frac{1}{n} \sqrt[n]{(n+1)(n+2) \cdots 2n} = \lim_{n \rightarrow +\infty} \frac{1}{n} \sqrt[n]{\frac{(2n)!}{n!}} = {}^{*1} \lim_{n \rightarrow +\infty} \frac{1}{n} \sqrt[n]{\frac{\sqrt{4\pi n} * (\frac{2n}{e})^{2n}}{\sqrt{2\pi n} * (\frac{n}{e})^n}} =$$

$$\lim_{n \rightarrow +\infty} \frac{1}{n} \sqrt[n]{4^n \sqrt{2} (\frac{n}{e})^n} =$$

$$\lim_{n \rightarrow +\infty} \frac{4}{e} \sqrt[n]{\frac{n^n \sqrt{2}}{n^n}} = \lim_{n \rightarrow +\infty} \frac{4}{e} 2^{\frac{1}{n+2}} = \frac{4}{e} 2^0 = \frac{4}{e}$$

$${}^{*1} \text{ Формула Стирлинга } \lim_{n \rightarrow +\infty} \frac{n!}{\sqrt{2\pi n} (\frac{n}{e})^n} = 1 \equiv n! \sim_{n \rightarrow +\infty} \sqrt{2\pi n} (\frac{n}{e})^n$$

```
In [8]: expr = pow(factorial(2*n)/factorial(n), 1/n)/n
print(expr)
```

```
(factorial(2*n)/factorial(n))**(1/n)/n
```

```
In [9]: limit_expr = limit(expr, n, +oo)
print(limit_expr)
```

```
4*exp(-1)
```

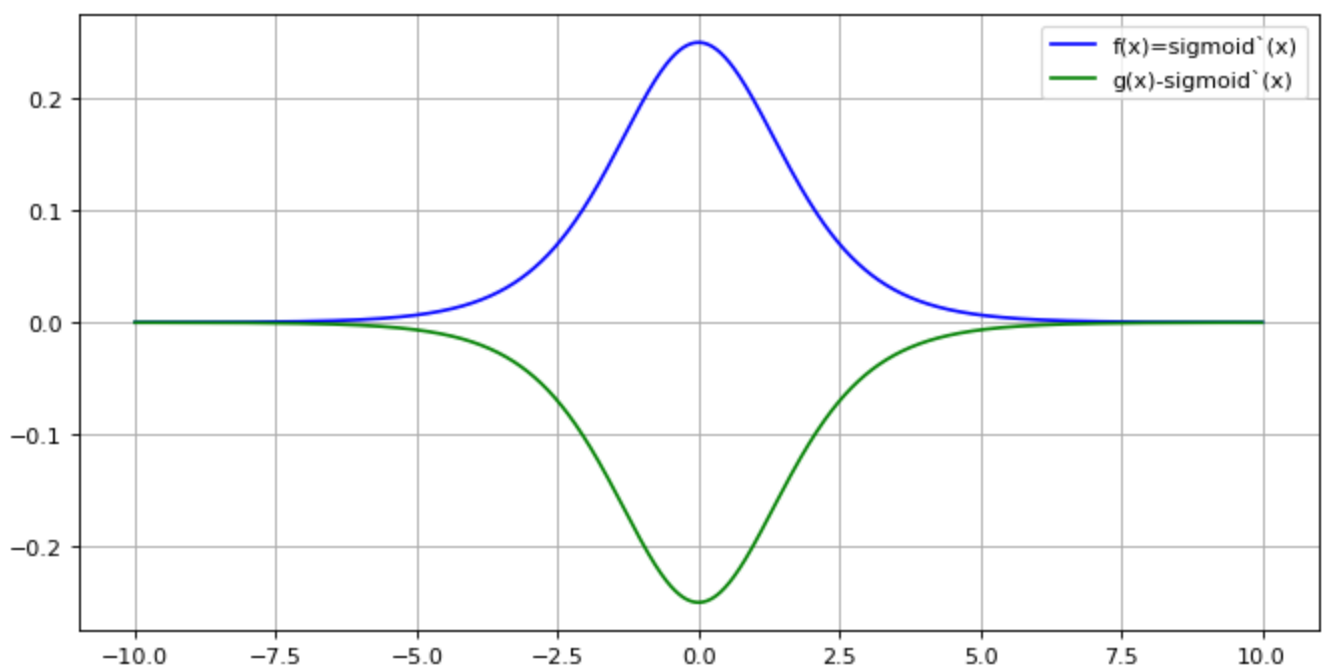
Задача 2.

Нет, не верно. $\lim_{n \rightarrow \infty} f(x)^x \geq \lim_{n \rightarrow \infty} h(x)^x$

```
In [10]: def dev_sigmoid(x):
return np.exp(-x)/(1+np.exp(-x))**2
```

```
In [11]: x = np.linspace(-10, 10, num=1000)
```

```
In [12]: fig, ax = plt.subplots(1,1,figsize=(10,5), dpi= 80)
ax.plot(x, dev_sigmoid(x), color='blue', label='f(x)=sigmoid` (x)')
ax.plot(x, -dev_sigmoid(x), color='green', label='g(x)-sigmoid` (x)')
ax.autoscale(enable=True, axis=1, tight=None)
ax.legend()
ax.grid()
```



```
In [13]: x = symbols('x')
f = exp(-x)/(1+exp(-x))**2
g = -exp(-x)/(1+exp(-x))**2
expr_f = f**x
expr_g = g**x

print(f'Functions: \n g={g}, f={f}')
print(f'Expressions: \n g={expr_g}, f={expr_f}')
print(f'lim g^x={limit(expr_g,x,oo)}={limit(expr_f,x,oo)}=lim f^x')
```

Functions:

$g = -\exp(-x) / (1 + \exp(-x))^{**2}$, $f = \exp(-x) / (1 + \exp(-x))^{**2}$

Expressions:

$g = (-\exp(-x) / (1 + \exp(-x))^{**2})^{**x}$, $f = (\exp(-x) / (1 + \exp(-x))^{**2})^{**x}$

$\lim g^x = 0 = \lim f^x$

Задача 3.

$$\square r(x) = \sin(x),$$

$$p(x) = \cos(x),$$

$$q(x) = \cos^2(x) + \sin^2(x)$$

```
In [14]: x = symbols('x')
r = symbols('r')
p = symbols('p')
q = symbols('q')
r = sin(x)
p = cos(x)
q = cos(x)**2 + sin(x)**2
```

```
In [15]: left = r*sin(x)+p*cos(x)
right = q
```

```
In [16]: left==right
```

```
Out[16]: True
```

```
In [17]: left_diff = Derivative(left, x).doit()
right_diff = Derivative(right, x).doit()
```

```

if left_diff == right_diff:
    print('The derivatives of both parts of the equality coincide.')
else:
    print('The derivatives are different.')

```

The derivatives of both parts of the equality coincide.

```

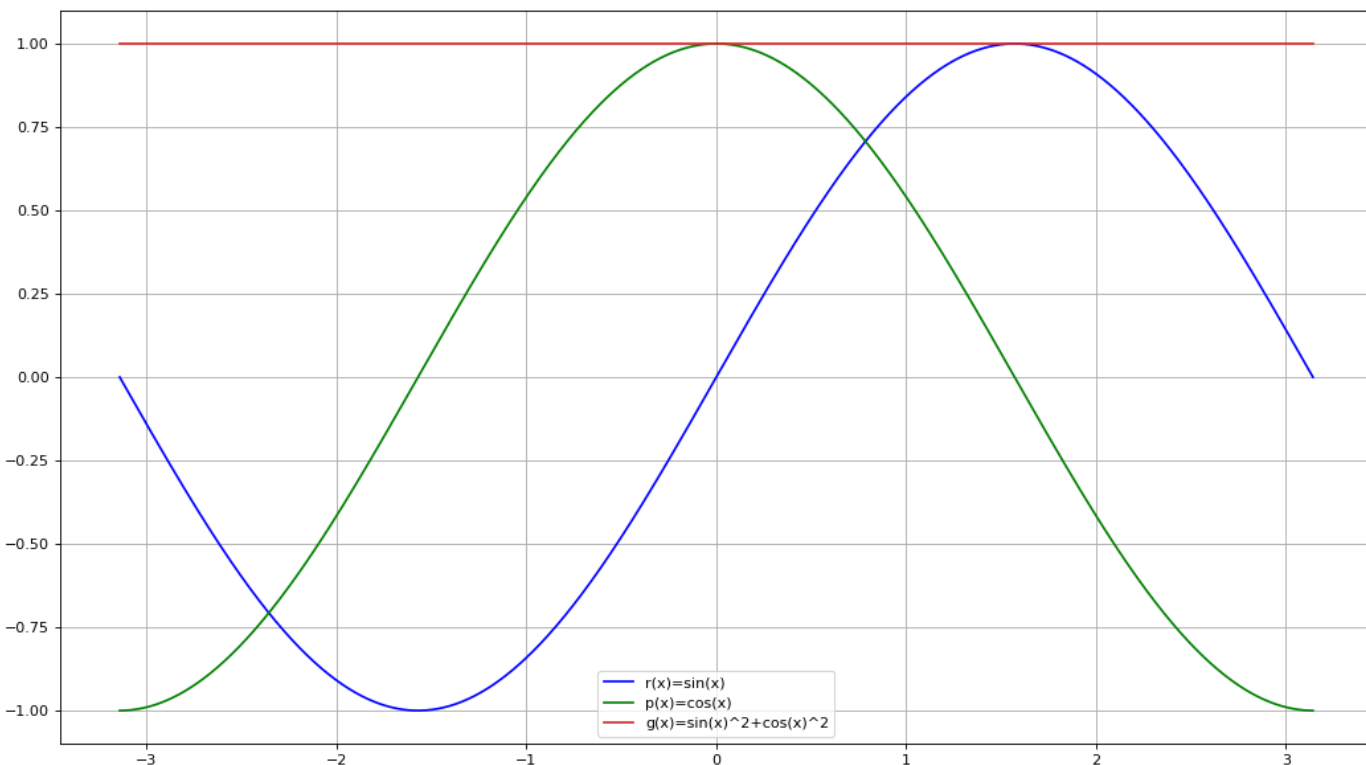
In [18]: x = np.linspace(-np.pi, np.pi, num=1000)
         y = np.sin(x)**2+np.cos(x)**2

```

```

In [19]: fig, ax = plt.subplots(1,1,figsize=(16,9), dpi= 80)
         ax.plot(x, np.sin(x), color='blue', label='r(x)=sin(x)')
         ax.plot(x, np.cos(x), color='green', label='p(x)=cos(x)')
         ax.plot(x, y, color='tab:red', label='g(x)=sin(x)^2+cos(x)^2')
         ax.autoscale(enable=True, axis=1, tight=None)
         ax.legend()
         ax.grid()

```



Задача 4.

```

In [20]: x = symbols('x')
         f = x / (1 + x**2)**2
         print(f'f(x)={f}')

```

$f(x) = x / (x^2 + 1)^2$

ряд Маклорена:

$$f(x) = f(0) + \frac{f'(0)}{1!}x + \frac{f''(0)}{2!}x^2 + \dots = \sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!}x^n$$

```

In [21]: f_diff_1 = Derivative(f, x).doit()
         print(f'f'(x)={f_diff_1}')
         f_diff_2 = Derivative(f_diff_1, x).doit()
         print(f'f''(x)={f_diff_2}')
         f_diff_3 = Derivative(f_diff_2, x).doit()
         print(f'f'''(x)={f_diff_3}')
         f_diff_4 = Derivative(f_diff_3, x).doit()
         print(f'f''''(x)={f_diff_4}')

```

```
f_diff_5 = Derivative(f_diff_4, x).doit()
print(f'f_5(x)={f_diff_5}')
f_diff_6 = Derivative(f_diff_5, x).doit()
print(f'f_6(x)={f_diff_6}')
f_diff_7 = Derivative(f_diff_6, x).doit()
print(f'f_7(x)={f_diff_7}')
```

```
f` (x)=-4*x**2/(x**2 + 1)**3 + (x**2 + 1)**(-2)
f`` (x)=24*x**3/(x**2 + 1)**4 - 12*x/(x**2 + 1)**3
f``` (x)=-192*x**4/(x**2 + 1)**5 + 144*x**2/(x**2 + 1)**4 - 12/(x**2 + 1)**3
f_4(x)=1920*x**5/(x**2 + 1)**6 - 1920*x**3/(x**2 + 1)**5 + 360*x/(x**2 + 1)**4
f_5(x)=-23040*x**6/(x**2 + 1)**7 + 28800*x**4/(x**2 + 1)**6 - 8640*x**2/(x**2 + 1)**5 +
360/(x**2 + 1)**4
f_6(x)=322560*x**7/(x**2 + 1)**8 - 483840*x**5/(x**2 + 1)**7 + 201600*x**3/(x**2 + 1)**6
- 20160*x/(x**2 + 1)**5
f_7(x)=-5160960*x**8/(x**2 + 1)**9 + 9031680*x**6/(x**2 + 1)**8 - 4838400*x**4/(x**2 +
1)**7 + 806400*x**2/(x**2 + 1)**6 - 20160/(x**2 + 1)**5
```

```
In [22]: print(f'{f_diff_1.subs(Symbol("x"), 0)/factorial(1)}, {f_diff_2.subs(Symbol("x"), 0)/fac
f' {f_diff_3.subs(Symbol("x"), 0)/factorial(3)}, {f_diff_4.subs(Symbol("x"), 0)//f
f' {f_diff_5.subs(Symbol("x"), 0)//factorial(5)}, {f_diff_6.subs(Symbol("x"), 0)//
f' {f_diff_7.subs(Symbol("x"), 0)//factorial(7)}')
```

1, 0, -2, 0, 3, 0, -4

```
In [23]: series(f, x, 0, 8, "+")
```

Out[23]: $x - 2x^3 + 3x^5 - 4x^7 + O(x^8)$

$$\frac{x}{(x^2 + 1)^2} = \sum_{n=0}^{\infty} (-1)^{n+1} n x^{2n-1}$$

Задача 5.

При рассмотрении сходимости ряда $\sum_{n=1}^{+\infty} \ln \frac{n^2 + 1}{n^2 + n + 1}$ в первом случае в знаменателе отбросили n , чего нельзя было делать так как только отбрасывание конечного числа членов ряда (или добавление конечного числа новых) не влияет на сходимость или расходимость ряда. В связи с этим была допущена ошибка, которая привела к сходимости данного ряда. Во втором же случае рассуждения верные. Этим и вызвано различие в ответах.

```
In [24]: series = Sum(ln((n**2 + 1)/(n**2+n+1)), (n, 1, +oo))
```

```
In [25]: series.doit()
```

Out[25]: $\sum_{n=1}^{\infty} \log \left(\frac{n^2 + 1}{n^2 + n + 1} \right)$

```
In [26]: series.is_convergent()
```

Out[26]: False

Задача 6.

$$\int (1 + x + x \ln x) x^x dx = \int x^x dx + \int x^{x+1} dx + \int x^{x+1} \ln x dx = x^{x+1} + \int x^{x+1} (\ln x + 1) dx = x^{x+1} + C$$

```
In [27]: x = Symbol('x')
```

```
integrate((1 + x + x*ln(x))*x**x, x)
```

Out[27]: xx^x

Задача 7.

$$\int_0^{\pi} \sin(x) \sin(nx) dx$$

```
In [28]: integrate(sin(x)*sin(n*x), x).doit()
```

Out[28]:
$$\begin{cases} -\frac{x \sin^2(x)}{2} - \frac{x \cos^2(x)}{2} + \frac{\sin(x) \cos(x)}{2} & \text{for } n = -1 \\ \frac{x \sin^2(x)}{2} + \frac{x \cos^2(x)}{2} - \frac{\sin(x) \cos(x)}{2} & \text{for } n = 1 \\ -\frac{n \sin(x) \cos(nx)}{n^2-1} + \frac{\sin(nx) \cos(x)}{n^2-1} & \text{otherwise} \end{cases}$$

```
In [29]: integrate(sin(x)*sin(n*x), x).subs(Symbol("x"), pi)-integrate(sin(x)*sin(n*x), x).subs(S
```

Out[29]:
$$\begin{cases} -\frac{\pi}{2} & \text{for } n = -1 \\ \frac{\pi}{2} & \text{for } n = 1 \\ -\frac{\sin(\pi n)}{n^2-1} & \text{otherwise} \end{cases}$$

Часть II. Комбинаторика и теоретико-числовые методы

Задача 1.

```
In [30]: def binary(n, k):
    if k == 0: # no bits set
        yield '0' * n
    elif n == k: # all bits set
        yield '1' * k
    else:
        assert n > k > 0
        for bits in binary(n - 1, k):
            yield '0' + bits
        for bits in binary(n - 1, k - 1):
            yield '1' + bits
```

```
In [31]: m = 7
k = 2
l = int(m/k)*(k-1)+m%k+1
print(l)
for bits in binary(m, l):
    print(*bits)
```

```
5
0 0 1 1 1 1 1
0 1 0 1 1 1 1
0 1 1 0 1 1 1
0 1 1 1 0 1 1
0 1 1 1 1 0 1
0 1 1 1 1 1 0
1 0 0 1 1 1 1
1 0 1 0 1 1 1
1 0 1 1 0 1 1
1 0 1 1 1 0 1
1 0 1 1 1 1 0
```

```

1 1 0 0 1 1 1
1 1 0 1 0 1 1
1 1 0 1 1 0 1
1 1 0 1 1 1 0
1 1 1 0 0 1 1
1 1 1 0 1 0 1
1 1 1 0 1 1 0
1 1 1 1 0 0 1
1 1 1 1 0 1 0
1 1 1 1 1 0 0

```

Минимальное количество единиц, которые должна содержать последовательность бит длины n , чтобы при этом обязательно нашлась подпоследовательность из m идущих подряд единиц равна:

$$\lfloor n/m \rfloor \cdot (m - 1) + \{n/m\} + 1$$

Задача 2.

Из условия задачи следует, что надо выбирать только четное число чисел. Т.е. 2, 4 и 6. Если выбирать шесть чисел, то три нечетных числа могут быть выбраны C_{50}^3 способами и три четных числа так же C_{50}^3 .

Аналогично в остальных случаях. Следовательно ответ $\sum_{k=2,4,6} (C_{50}^{\frac{k}{2}})^2$

Задача 3.

1. Максимальное число ребер, которое можно удалить из G_n , чтобы граф остался связным равно

$$\frac{n(n-1)}{2} - (n-1) = \frac{(n-1)(n-2)}{2}, \text{ соответственно для } G_{101} \text{ их количество равно } 4950.$$

1. Число рёбер в полном двудольном графе равно $n_1 \cdot n_2$, $n_1 + n_2 = n$

```

In [32]: def max_edges(n):
          mul = 0
          for n_1 in range(1, n//2+1):
              temp = (n-n_1)*n_1
              if temp > mul:
                  mul = temp
                  mul_n_1 = n_1
          return mul_n_1, n-mul_n_1
def num_edges(n):
    n_1, n_2 = max_edges(n)
    return int(n*(n-1)/2) - n_1*n_2

```

```

In [33]: print(num_edges(101))

```

2500

Минимальное число ребер, которое можно удалить из G_{101} , чтобы граф стал двудольным равно

$$\frac{n(n-1)}{2} - n_1 \cdot n_2 = 2500.$$

Задача 4.

$$\triangleleft f(n) = (n-1)^{n-1} + n^n + (n+1)^{n+1}, \quad \mid n:3 \implies n^n:3$$

Распишем через бином Ньютона $(n-1)^{n-1}$ и $(n+1)^{n+1}$:

$$(n-1)^{n-1} = \sum_{k=0}^{n-1} C_{n-1}^k n^k \cdot (-1)^{n-1-k} = (-1)^{n-1} + \sum_{k=1}^{n-1} C_{n-1}^k n^k \cdot (-1)^{n-1-k} \equiv (-1)^{n-1} \pmod{3, \text{ т.к. } n \equiv 3}$$

$n \equiv 3$

$$(n+1)^{n+1} = \sum_{k=0}^{n+1} C_{n+1}^k n^k \cdot 1^{n+1-k} = 1 + \sum_{k=1}^{n+1} C_{n+1}^k n^k \equiv 1 \pmod{3, \text{ т.к. } n \equiv 3}$$

$$f(n) = (-1)^{n-1} + \sum_{k=1}^{n-1} C_{n-1}^k n^k \cdot (-1)^{n-1-k} + n^n + 1 + \sum_{k=1}^{n+1} C_{n+1}^k n^k \equiv 1 + (-1)^{n-1} \pmod{3}$$

$$f(777) \equiv 1 + (-1)^{777-1} \pmod{3}$$

$$f(777) \equiv 2 \pmod{3}$$

Следовательно $f(777) = 776^{776} + 777^{777} + 778^{778}$ не делится нацело на 3.

Задача 5.

$$\sum_{k=1}^n k(k+1) = \left(\frac{n^3}{3} + n^2 + \frac{2}{3}n\right)$$

$$n = 999$$

$$\frac{n^3}{3} \text{ на конце } 3,$$

$$n^2 \text{ на конце } 1,$$

$$\frac{2}{3}n \text{ на конце } 6$$

Следовательно $3+1+6=10$ на конце 0

```
In [34]: k=Symbol('k')
Sum(k*(k+1), (k, 1, 999)).doit()
```

```
Out[34]: 333333000
```

Задача 6.

$$(n^{100} - 1)^{100} (n^{101} + 1)^{101} \equiv (-1)^{100} \cdot 1^{101} = 1 \pmod{n}$$

Следовательно нужно прибавить или (-1) или $(n-1)$

Задача 7.

$$5 \cdot 2^{3n-2} + 3^{3n-1} = 5 \cdot 2^{3(n-1)+1} + 3^{3(n-1)+2} = 10 \cdot 8^{n-1} + 9 \cdot 27^{n-1} = (19-9) \cdot 8^{n-1} + 9 \cdot 27^{n-1} = 19 \cdot 8^{n-1} + 9 \cdot (27^{n-1} - 8^{n-1}) = 19 \cdot 8^{n-1} + 9 \cdot 19 \cdot (27^{n-2} + \dots + 8^{n-2}) \equiv 0 \pmod{19}$$

Задача 8.

$$x = 97^{256} \pmod{765}$$

Для решения задачи воспользуемся алгоритмом быстрого возведения в степень по модулю.


```
In [35]: import re

def dec_to_bin_inverted(degree):
    b = [int(i) for i in list(re.split('0b', bin(degree))[1])][::-1]
    return b, len(b)

def equation_solution(a_0, degree, mod):
    a = [a_0]
    b, len_b = dec_to_bin_inverted(degree)
    for i in range(len_b-1):
        a_i = (a[i]**2)%mod
        a.append(a_i)
    return sum([a*b for a, b in zip(a, b)])
```

```
In [36]: dec_to_bin_inverted(256)
```

```
Out[36]: ([0, 0, 0, 0, 0, 0, 0, 0, 1], 9)
```

```
In [37]: print(f'x={equation_solution(97, 256, 765)}')
x=511
```

Задача 9.

$$n = p_1^\alpha \cdot p_2^\beta \cdot p_3^\gamma$$

$$1080 \cdot n = 1463 \cdot \varphi(n)$$

$$1080 \cdot n = 1463 \cdot n \cdot \left(1 - \frac{1}{p_1}\right) \cdot \left(1 - \frac{1}{p_2}\right) \cdot \dots \cdot \left(1 - \frac{1}{p_k}\right)$$

$$\frac{(p_1 - 1) \cdot (p_2 - 1) \cdot \dots \cdot (p_k - 1)}{p_1 \cdot p_2 \cdot \dots \cdot p_k} = \frac{1080}{1463}$$

$$\frac{(p_1 - 1) \cdot (p_2 - 1) \cdot \dots \cdot (p_k - 1)}{p_1 \cdot p_2 \cdot \dots \cdot p_k} = \frac{6 \cdot 10 \cdot 18}{7 \cdot 11 \cdot 19} \implies p_1 = 7, \quad p_2 = 11, \quad p_3 = 19$$

```
In [38]: def calc_n(p_1, p_2, p_3, s):
    n, n_index = [], []
    for alpha in range(1,4):
        for beta in range(1,4):
            for gamma in range(1,4):
                n.append([p_1**alpha*p_2**beta*p_3**gamma, (alpha, beta, gamma)])
    n.sort(key = lambda row: row[0])
    for i, elem in enumerate(n):
        n_index.append([i+1,elem])
    return n_index[s-1]
```

```
In [39]: answer = calc_n(7, 11, 19, 9)
```

```
In [40]: print(f's_{answer[0]}=p1^{answer[1][1][0]}*p2^{answer[1][1][1]}*p3^{answer[1][1][2]}={an
s_9=p1^1*p2^2*p3^2=305767
```

Часть III. Вычисления

Задача 1.

$$A_{n \times n} = \begin{pmatrix} \alpha & \beta & \cdots & \beta \\ \beta & \alpha & \cdots & \beta \\ \vdots & \vdots & \ddots & \vdots \\ \beta & \beta & \cdots & \alpha \end{pmatrix}, \quad A^{-1} = ?$$

```
In [41]: # Initialization matrix
def init_matrix(n, a, b):
    return np.full((n,n), b) - np.eye(n)*(b-a)

# Get transpose matrix
def transpose_matrix(matrix):
    return list(map(list, zip(*matrix)))

# Get matrix minor
def get_matrix_minor(matrix, i, j):
    return [row[:j] + row[j+1:] for row in (matrix[:i] + matrix[i+1:])]

# Get matrix determinant
def get_matrix_determinant(matrix):
    if len(matrix) == 2:
        return matrix[0][0]*matrix[1][1]-matrix[0][1]*matrix[1][0]
    determinant = 0
    for c in range(len(matrix)):
        determinant += ((-1)**c)*matrix[0][c]*get_matrix_determinant(get_matrix_minor(matrix, 0, c))
    return determinant

# Get inverse matrix
def get_inverse_matrix(n, a, b):
    matrix = init_matrix(n, a, b).tolist()
    determinant = get_matrix_determinant(matrix)
    if len(matrix) == 2:
        return [[matrix[1][1]/determinant, -1*matrix[0][1]/determinant],
                [-1*matrix[1][0]/determinant, matrix[0][0]/determinant]]
    #get cofactors matrix
    cofactors = []
    for r in range(len(matrix)):
        cofactorRow = []
        for c in range(len(matrix)):
            minor = get_matrix_minor(matrix, r, c)
            cofactorRow.append((( -1)**(r+c)) * get_matrix_determinant(minor))
        cofactors.append(cofactorRow)
    cofactors = transpose_matrix(cofactors)
    for r in range(len(cofactors)):
        for c in range(len(cofactors)):
            cofactors[r][c] = cofactors[r][c]/determinant
    return cofactors
```

```
In [42]: init_matrix(5, 14, 31)
```

```
Out[42]: array([[14., 31., 31., 31., 31.],
                [31., 14., 31., 31., 31.],
                [31., 31., 14., 31., 31.],
                [31., 31., 31., 14., 31.],
                [31., 31., 31., 31., 14.]])
```

```
In [43]: %%time
np.array(get_inverse_matrix(5, 14, 31))
```

```
Wall time: 1e+03 µs
Out[43]: array([[ -0.04560955,  0.01321398,  0.01321398,  0.01321398,  0.01321398],
                [ 0.01321398, -0.04560955,  0.01321398,  0.01321398,  0.01321398],
                [ 0.01321398,  0.01321398, -0.04560955,  0.01321398,  0.01321398],
```

```
[ 0.01321398,  0.01321398,  0.01321398, -0.04560955,  0.01321398],  
[ 0.01321398,  0.01321398,  0.01321398,  0.01321398, -0.04560955]])
```

```
In [44]: %%time  
np.linalg.inv(init_matrix(5, 14, 31))
```

```
Wall time: 846 µs  
Out[44]: array([[ -0.04560955,  0.01321398,  0.01321398,  0.01321398,  0.01321398],  
[ 0.01321398, -0.04560955,  0.01321398,  0.01321398,  0.01321398],  
[ 0.01321398,  0.01321398, -0.04560955,  0.01321398,  0.01321398],  
[ 0.01321398,  0.01321398,  0.01321398, -0.04560955,  0.01321398],  
[ 0.01321398,  0.01321398,  0.01321398,  0.01321398, -0.04560955]])
```

Задача 2.

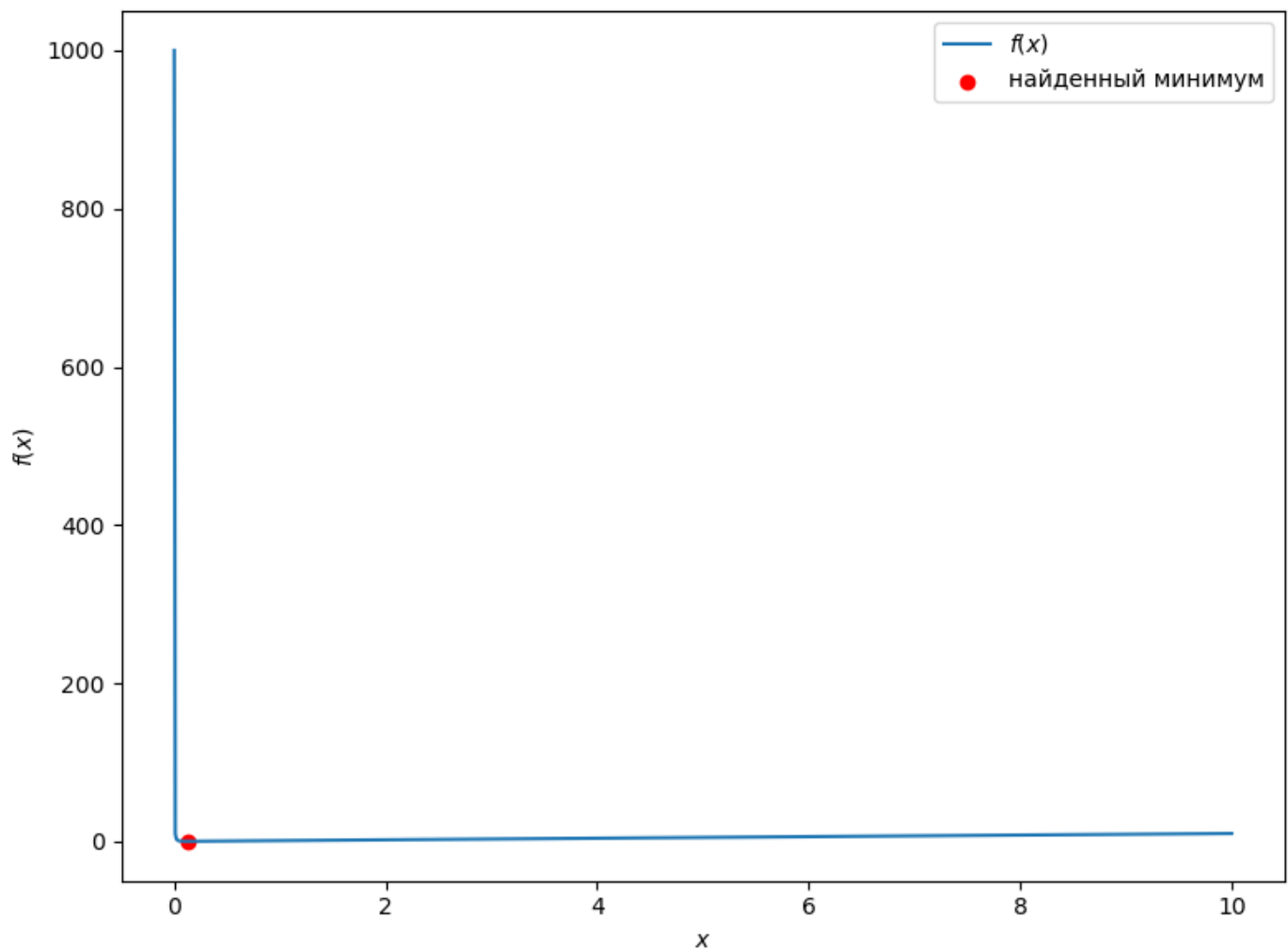
```
In [45]: def f(x):  
        return x + 10**35 * 10**(-(8*x**(0.3) + 0.9*x+32))
```

```
solution = optimize.minimize_scalar(f)  
print(solution)
```

```
fun: 0.16483667631708415  
message: '\nOptimization terminated successfully;\nThe returned value satisfies the termination criteria\n(using xtol = 1.48e-08 )'  
nfev: 23  
nit: 19  
success: True  
x: 0.12585224797201305
```

```
In [46]: x = np.linspace(0, 10, 1000)  
  
fig, ax = plt.subplots(figsize=(8, 6), layout="tight")  
ax.plot(x, f(x), label=r"$f(x)$")  
ax.scatter(solution.x, solution.fun, color="red", label="найденный минимум")  
ax.set_xlabel("$x$")  
ax.set_ylabel("$f(x)$")  
ax.legend()
```

```
Out[46]: <matplotlib.legend.Legend at 0x2a7ec178730>
```



Для решения задачи, без использования сторонних оптимизаторов, реализуем программный градиентный спуск

```
In [47]: x = symbols('x')
A = symbols('A')
a = symbols('a')
b = symbols('b')
g = symbols('g')
s = symbols('s')

f = x + A*10**(-(a*x**(1-b)+g*x+s))
f
```

Out[47]: $10^{-ax^{1-b}-gx-s}A + x$

```
In [48]: f_diff = Derivative(f, x).doit()
f_diff
```

Out[48]: $10^{-ax^{1-b}-gx-s}A \left(-\frac{ax^{1-b}(1-b)}{x} - g \right) \log(10) + 1$

```
In [49]: min_fun = []
# первоначальная точка
start_point = 0.15
# размер шага (learning rate)
learn_r = 0.0000001
# установка первоначальной точности
precision = 0.00001
for A in tqdm(range(10**35, 10**50, 10**50)):
```

```

for a in range(1, 10):
    for b in np.arange(0.1, 1, 0.2):
        for g in np.arange(0.1, 1, 0.2):
            for s in range(0, 50, 2):
                # функция градиента
                gr_func = lambda x: 1 + A*(-(a*(1-b)*x**(1-b))/x-g)*np.log(10)*10**(
                # количество итерация
                n = 1000000
                next_point = start_point
                iter_ = 0
                for i in range(n):
                    current_point = next_point
                    # движение в негативную сторону вычисляемого градиента
                    next_point = current_point - learn_r*gr_func(current_point)
                    iter_ += 1
                    if(abs(current_point - next_point) <= precision):
                        break
                if iter_ > 100:
                    print(f'fmin={next_point:2.1e}, iter={iter_},'
                          f'A={A:1.1e}, a={a}, b={b:.1f}, g={g:.1f}, s={s}')
                    min_fun.append([next_point, A, a, b, g, s])

```

0%|

| 0/1 [00:00<?, ?it/s]

```

fmin=5.6e+00, iter=49521,A=1.0e+35, a=1, b=0.1, g=0.1, s=28
fmin=3.4e+00, iter=46718,A=1.0e+35, a=1, b=0.1, g=0.1, s=30
fmin=1.3e+00, iter=38330,A=1.0e+35, a=1, b=0.1, g=0.1, s=32
fmin=4.7e+00, iter=39727,A=1.0e+35, a=1, b=0.1, g=0.3, s=28
fmin=2.8e+00, iter=37974,A=1.0e+35, a=1, b=0.1, g=0.3, s=30
fmin=1.1e+00, iter=32292,A=1.0e+35, a=1, b=0.1, g=0.3, s=32
fmin=4.0e+00, iter=33152,A=1.0e+35, a=1, b=0.1, g=0.5, s=28
fmin=2.5e+00, iter=32032,A=1.0e+35, a=1, b=0.1, g=0.5, s=30
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fmin=3.5e+00, iter=28252,A=1.0e+35, a=1, b=0.1, g=0.7, s=28
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fmin=9.2e-01, iter=24545,A=1.0e+35, a=1, b=0.1, g=0.7, s=32
fmin=3.1e+00, iter=23941,A=1.0e+35, a=1, b=0.1, g=0.9, s=28
fmin=2.0e+00, iter=24450,A=1.0e+35, a=1, b=0.1, g=0.9, s=30
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fmin=2.5e+00, iter=39913,A=1.0e+35, a=1, b=0.5, g=0.7, s=30

```

fmin=8.1e-01, iter=25546,A=1.0e+35, a=1, b=0.5, g=0.7, s=32
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fmin=3.0e+00, iter=34832,A=1.0e+35, a=2, b=0.7, g=0.9, s=28
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fmin=3.3e+00, iter=132386,A=1.0e+35, a=2, b=0.9, g=0.1, s=30
fmin=1.4e+01, iter=133673,A=1.0e+35, a=2, b=0.9, g=0.3, s=26
fmin=8.0e+00, iter=125580,A=1.0e+35, a=2, b=0.9, g=0.3, s=28
fmin=2.5e+00, iter=82526,A=1.0e+35, a=2, b=0.9, g=0.3, s=30
fmin=9.2e+00, iter=75397,A=1.0e+35, a=2, b=0.9, g=0.5, s=26
fmin=5.4e+00, iter=77160,A=1.0e+35, a=2, b=0.9, g=0.5, s=28
fmin=2.0e+00, iter=58055,A=1.0e+35, a=2, b=0.9, g=0.5, s=30
fmin=4.2e+00, iter=55803,A=1.0e+35, a=2, b=0.9, g=0.7, s=28
fmin=1.7e+00, iter=44688,A=1.0e+35, a=2, b=0.9, g=0.7, s=30
fmin=3.4e+00, iter=43750,A=1.0e+35, a=2, b=0.9, g=0.9, s=28
fmin=1.4e+00, iter=36331,A=1.0e+35, a=2, b=0.9, g=0.9, s=30
fmin=1.3e+00, iter=15332,A=1.0e+35, a=3, b=0.1, g=0.1, s=30
fmin=5.6e-01, iter=13115,A=1.0e+35, a=3, b=0.1, g=0.1, s=32
fmin=1.2e+00, iter=14243,A=1.0e+35, a=3, b=0.1, g=0.3, s=30
fmin=5.3e-01, iter=12287,A=1.0e+35, a=3, b=0.1, g=0.3, s=32
fmin=1.1e+00, iter=13302,A=1.0e+35, a=3, b=0.1, g=0.5, s=30
fmin=5.1e-01, iter=11556,A=1.0e+35, a=3, b=0.1, g=0.5, s=32
fmin=1.1e+00, iter=12480,A=1.0e+35, a=3, b=0.1, g=0.7, s=30
fmin=4.9e-01, iter=10905,A=1.0e+35, a=3, b=0.1, g=0.7, s=32
fmin=1.0e+00, iter=11757,A=1.0e+35, a=3, b=0.1, g=0.9, s=30
fmin=4.7e-01, iter=10322,A=1.0e+35, a=3, b=0.1, g=0.9, s=32
fmin=2.3e+00, iter=22685,A=1.0e+35, a=3, b=0.3, g=0.1, s=28
fmin=1.3e+00, iter=18961,A=1.0e+35, a=3, b=0.3, g=0.1, s=30
fmin=4.6e-01, iter=11993,A=1.0e+35, a=3, b=0.3, g=0.1, s=32
fmin=2.1e+00, iter=19654,A=1.0e+35, a=3, b=0.3, g=0.3, s=28
fmin=1.2e+00, iter=17144,A=1.0e+35, a=3, b=0.3, g=0.3, s=30
fmin=4.4e-01, iter=11236,A=1.0e+35, a=3, b=0.3, g=0.3, s=32
fmin=1.9e+00, iter=17114,A=1.0e+35, a=3, b=0.3, g=0.5, s=28
fmin=1.1e+00, iter=15659,A=1.0e+35, a=3, b=0.3, g=0.5, s=30
fmin=4.3e-01, iter=10563,A=1.0e+35, a=3, b=0.3, g=0.5, s=32
fmin=1.8e+00, iter=14865,A=1.0e+35, a=3, b=0.3, g=0.7, s=28
fmin=1.0e+00, iter=14422,A=1.0e+35, a=3, b=0.3, g=0.7, s=30
fmin=4.1e-01, iter=9962,A=1.0e+35, a=3, b=0.3, g=0.7, s=32
fmin=1.7e+00, iter=12782,A=1.0e+35, a=3, b=0.3, g=0.9, s=28
fmin=9.8e-01, iter=13373,A=1.0e+35, a=3, b=0.3, g=0.9, s=30
fmin=4.0e-01, iter=9422,A=1.0e+35, a=3, b=0.3, g=0.9, s=32
fmin=2.9e+00, iter=37970,A=1.0e+35, a=3, b=0.5, g=0.1, s=28

fmin=1.3e+00, iter=24153,A=1.0e+35, a=3, b=0.5, g=0.1, s=30
fmin=3.4e-01, iter=9422,A=1.0e+35, a=3, b=0.5, g=0.1, s=32
fmin=2.5e+00, iter=30787,A=1.0e+35, a=3, b=0.5, g=0.3, s=28
fmin=1.2e+00, iter=21128,A=1.0e+35, a=3, b=0.5, g=0.3, s=30
fmin=3.3e-01, iter=8910,A=1.0e+35, a=3, b=0.5, g=0.3, s=32
fmin=2.2e+00, iter=25986,A=1.0e+35, a=3, b=0.5, g=0.5, s=28
fmin=1.1e+00, iter=18796,A=1.0e+35, a=3, b=0.5, g=0.5, s=30
fmin=3.2e-01, iter=8433,A=1.0e+35, a=3, b=0.5, g=0.5, s=32
fmin=2.0e+00, iter=22536,A=1.0e+35, a=3, b=0.5, g=0.7, s=28
fmin=1.0e+00, iter=16944,A=1.0e+35, a=3, b=0.5, g=0.7, s=30
fmin=3.1e-01, iter=7991,A=1.0e+35, a=3, b=0.5, g=0.7, s=32
fmin=1.8e+00, iter=19929,A=1.0e+35, a=3, b=0.5, g=0.9, s=28
fmin=9.4e-01, iter=15437,A=1.0e+35, a=3, b=0.5, g=0.9, s=30
fmin=3.0e-01, iter=7582,A=1.0e+35, a=3, b=0.5, g=0.9, s=32
fmin=4.1e+00, iter=76709,A=1.0e+35, a=3, b=0.7, g=0.1, s=28
fmin=1.2e+00, iter=31157,A=1.0e+35, a=3, b=0.7, g=0.1, s=30
fmin=1.9e-01, iter=3083,A=1.0e+35, a=3, b=0.7, g=0.1, s=32
fmin=3.2e+00, iter=52162,A=1.0e+35, a=3, b=0.7, g=0.3, s=28
fmin=1.1e+00, iter=26510,A=1.0e+35, a=3, b=0.7, g=0.3, s=30
fmin=1.9e-01, iter=2933,A=1.0e+35, a=3, b=0.7, g=0.3, s=32
fmin=2.6e+00, iter=39766,A=1.0e+35, a=3, b=0.7, g=0.5, s=28
fmin=1.0e+00, iter=22998,A=1.0e+35, a=3, b=0.7, g=0.5, s=30
fmin=1.8e-01, iter=2766,A=1.0e+35, a=3, b=0.7, g=0.5, s=32
fmin=2.3e+00, iter=32259,A=1.0e+35, a=3, b=0.7, g=0.7, s=28
fmin=9.2e-01, iter=20294,A=1.0e+35, a=3, b=0.7, g=0.7, s=30
fmin=1.8e-01, iter=2586,A=1.0e+35, a=3, b=0.7, g=0.7, s=32
fmin=2.0e+00, iter=27206,A=1.0e+35, a=3, b=0.7, g=0.9, s=28
fmin=8.6e-01, iter=18160,A=1.0e+35, a=3, b=0.7, g=0.9, s=30
fmin=1.8e-01, iter=2397,A=1.0e+35, a=3, b=0.7, g=0.9, s=32
fmin=2.3e+01, iter=346442,A=1.0e+35, a=3, b=0.9, g=0.1, s=26
fmin=8.2e+00, iter=225323,A=1.0e+35, a=3, b=0.9, g=0.1, s=28
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fmin=1.0e+01, iter=124333,A=1.0e+35, a=3, b=0.9, g=0.3, s=26
fmin=4.7e+00, iter=101680,A=1.0e+35, a=3, b=0.9, g=0.3, s=28
fmin=8.1e-01, iter=30307,A=1.0e+35, a=3, b=0.9, g=0.3, s=30
fmin=6.9e+00, iter=76279,A=1.0e+35, a=3, b=0.9, g=0.5, s=26
fmin=3.5e+00, iter=66027,A=1.0e+35, a=3, b=0.9, g=0.5, s=28
fmin=7.7e-01, iter=27238,A=1.0e+35, a=3, b=0.9, g=0.5, s=30
fmin=5.3e+00, iter=55132,A=1.0e+35, a=3, b=0.9, g=0.7, s=26
fmin=2.8e+00, iter=49041,A=1.0e+35, a=3, b=0.9, g=0.7, s=28
fmin=7.2e-01, iter=24188,A=1.0e+35, a=3, b=0.9, g=0.7, s=30
fmin=4.3e+00, iter=43198,A=1.0e+35, a=3, b=0.9, g=0.9, s=26
fmin=2.3e+00, iter=39073,A=1.0e+35, a=3, b=0.9, g=0.9, s=28
fmin=6.7e-01, iter=21559,A=1.0e+35, a=3, b=0.9, g=0.9, s=30
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fmin=4.4e-01, iter=9677,A=1.0e+35, a=4, b=0.1, g=0.1, s=32
fmin=9.1e-01, iter=10705,A=1.0e+35, a=4, b=0.1, g=0.3, s=30
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fmin=8.7e-01, iter=10163,A=1.0e+35, a=4, b=0.1, g=0.5, s=30
fmin=4.1e-01, iter=8774,A=1.0e+35, a=4, b=0.1, g=0.5, s=32
fmin=8.4e-01, iter=9674,A=1.0e+35, a=4, b=0.1, g=0.7, s=30
fmin=4.0e-01, iter=8380,A=1.0e+35, a=4, b=0.1, g=0.7, s=32
fmin=8.1e-01, iter=9232,A=1.0e+35, a=4, b=0.1, g=0.9, s=30
fmin=3.9e-01, iter=8019,A=1.0e+35, a=4, b=0.1, g=0.9, s=32
fmin=1.6e+00, iter=15411,A=1.0e+35, a=4, b=0.3, g=0.1, s=28
fmin=9.1e-01, iter=13107,A=1.0e+35, a=4, b=0.3, g=0.1, s=30
fmin=3.5e-01, iter=8285,A=1.0e+35, a=4, b=0.3, g=0.1, s=32
fmin=1.5e+00, iter=13938,A=1.0e+35, a=4, b=0.3, g=0.3, s=28
fmin=8.6e-01, iter=12206,A=1.0e+35, a=4, b=0.3, g=0.3, s=30
fmin=3.4e-01, iter=7882,A=1.0e+35, a=4, b=0.3, g=0.3, s=32
fmin=1.4e+00, iter=12683,A=1.0e+35, a=4, b=0.3, g=0.5, s=28
fmin=8.2e-01, iter=11427,A=1.0e+35, a=4, b=0.3, g=0.5, s=30
fmin=3.3e-01, iter=7512,A=1.0e+35, a=4, b=0.3, g=0.5, s=32
fmin=1.3e+00, iter=11603,A=1.0e+35, a=4, b=0.3, g=0.7, s=28
fmin=7.9e-01, iter=10746,A=1.0e+35, a=4, b=0.3, g=0.7, s=30
fmin=3.2e-01, iter=7171,A=1.0e+35, a=4, b=0.3, g=0.7, s=32

fmin=1.3e+00, iter=10665,A=1.0e+35, a=4, b=0.3, g=0.9, s=28
fmin=7.5e-01, iter=10145,A=1.0e+35, a=4, b=0.3, g=0.9, s=30
fmin=3.1e-01, iter=6855,A=1.0e+35, a=4, b=0.3, g=0.9, s=32
fmin=1.8e+00, iter=23583,A=1.0e+35, a=4, b=0.5, g=0.1, s=28
fmin=8.3e-01, iter=15080,A=1.0e+35, a=4, b=0.5, g=0.1, s=30
fmin=2.4e-01, iter=5343,A=1.0e+35, a=4, b=0.5, g=0.1, s=32
fmin=1.6e+00, iter=20559,A=1.0e+35, a=4, b=0.5, g=0.3, s=28
fmin=7.8e-01, iter=13826,A=1.0e+35, a=4, b=0.5, g=0.3, s=30
fmin=2.3e-01, iter=5081,A=1.0e+35, a=4, b=0.5, g=0.3, s=32
fmin=1.5e+00, iter=18258,A=1.0e+35, a=4, b=0.5, g=0.5, s=28
fmin=7.4e-01, iter=12771,A=1.0e+35, a=4, b=0.5, g=0.5, s=30
fmin=2.3e-01, iter=4830,A=1.0e+35, a=4, b=0.5, g=0.5, s=32
fmin=1.4e+00, iter=16445,A=1.0e+35, a=4, b=0.5, g=0.7, s=28
fmin=7.0e-01, iter=11871,A=1.0e+35, a=4, b=0.5, g=0.7, s=30
fmin=2.2e-01, iter=4589,A=1.0e+35, a=4, b=0.5, g=0.7, s=32
fmin=1.3e+00, iter=14976,A=1.0e+35, a=4, b=0.5, g=0.9, s=28
fmin=6.7e-01, iter=11094,A=1.0e+35, a=4, b=0.5, g=0.9, s=30
fmin=2.2e-01, iter=4358,A=1.0e+35, a=4, b=0.5, g=0.9, s=32
fmin=2.2e+00, iter=40455,A=1.0e+35, a=4, b=0.7, g=0.1, s=28
fmin=6.6e-01, iter=16072,A=1.0e+35, a=4, b=0.7, g=0.1, s=30
fmin=1.9e+00, iter=32251,A=1.0e+35, a=4, b=0.7, g=0.3, s=28
fmin=6.2e-01, iter=14712,A=1.0e+35, a=4, b=0.7, g=0.3, s=30
fmin=1.7e+00, iter=26873,A=1.0e+35, a=4, b=0.7, g=0.5, s=28
fmin=5.9e-01, iter=13544,A=1.0e+35, a=4, b=0.7, g=0.5, s=30
fmin=1.5e+00, iter=23080,A=1.0e+35, a=4, b=0.7, g=0.7, s=28
fmin=5.6e-01, iter=12537,A=1.0e+35, a=4, b=0.7, g=0.7, s=30
fmin=1.4e+00, iter=20257,A=1.0e+35, a=4, b=0.7, g=0.9, s=28
fmin=5.4e-01, iter=11665,A=1.0e+35, a=4, b=0.7, g=0.9, s=30
fmin=2.8e+01, iter=206309,A=1.0e+35, a=4, b=0.9, g=0.1, s=24
fmin=1.3e+01, iter=265685,A=1.0e+35, a=4, b=0.9, g=0.1, s=26
fmin=3.0e+00, iter=95107,A=1.0e+35, a=4, b=0.9, g=0.1, s=28
fmin=2.6e-01, iter=7370,A=1.0e+35, a=4, b=0.9, g=0.1, s=30
fmin=6.9e+00, iter=108117,A=1.0e+35, a=4, b=0.9, g=0.3, s=26
fmin=2.3e+00, iter=63814,A=1.0e+35, a=4, b=0.9, g=0.3, s=28
fmin=2.6e-01, iter=7430,A=1.0e+35, a=4, b=0.9, g=0.3, s=30
fmin=4.9e+00, iter=68547,A=1.0e+35, a=4, b=0.9, g=0.5, s=26
fmin=1.9e+00, iter=47218,A=1.0e+35, a=4, b=0.9, g=0.5, s=28
fmin=2.6e-01, iter=7348,A=1.0e+35, a=4, b=0.9, g=0.5, s=30
fmin=3.8e+00, iter=50367,A=1.0e+35, a=4, b=0.9, g=0.7, s=26
fmin=1.6e+00, iter=37449,A=1.0e+35, a=4, b=0.9, g=0.7, s=28
fmin=2.6e-01, iter=7167,A=1.0e+35, a=4, b=0.9, g=0.7, s=30
fmin=3.2e+00, iter=39884,A=1.0e+35, a=4, b=0.9, g=0.9, s=26
fmin=1.4e+00, iter=31052,A=1.0e+35, a=4, b=0.9, g=0.9, s=28
fmin=2.5e-01, iter=6924,A=1.0e+35, a=4, b=0.9, g=0.9, s=30
fmin=7.7e-01, iter=8914,A=1.0e+35, a=5, b=0.1, g=0.1, s=30
fmin=3.7e-01, iter=7575,A=1.0e+35, a=5, b=0.1, g=0.1, s=32
fmin=7.4e-01, iter=8533,A=1.0e+35, a=5, b=0.1, g=0.3, s=30
fmin=3.6e-01, iter=7270,A=1.0e+35, a=5, b=0.1, g=0.3, s=32
fmin=7.2e-01, iter=8184,A=1.0e+35, a=5, b=0.1, g=0.5, s=30
fmin=3.5e-01, iter=6985,A=1.0e+35, a=5, b=0.1, g=0.5, s=32
fmin=6.9e-01, iter=7863,A=1.0e+35, a=5, b=0.1, g=0.7, s=30
fmin=3.4e-01, iter=6720,A=1.0e+35, a=5, b=0.1, g=0.7, s=32
fmin=6.7e-01, iter=7568,A=1.0e+35, a=5, b=0.1, g=0.9, s=30
fmin=3.3e-01, iter=6473,A=1.0e+35, a=5, b=0.1, g=0.9, s=32
fmin=1.2e+00, iter=11656,A=1.0e+35, a=5, b=0.3, g=0.1, s=28
fmin=7.0e-01, iter=9796,A=1.0e+35, a=5, b=0.3, g=0.1, s=30
fmin=2.8e-01, iter=6008,A=1.0e+35, a=5, b=0.3, g=0.1, s=32
fmin=1.2e+00, iter=10878,A=1.0e+35, a=5, b=0.3, g=0.3, s=28
fmin=6.7e-01, iter=9281,A=1.0e+35, a=5, b=0.3, g=0.3, s=30
fmin=2.7e-01, iter=5757,A=1.0e+35, a=5, b=0.3, g=0.3, s=32
fmin=1.1e+00, iter=10203,A=1.0e+35, a=5, b=0.3, g=0.5, s=28
fmin=6.5e-01, iter=8821,A=1.0e+35, a=5, b=0.3, g=0.5, s=30
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fmin=6.0e-01, iter=8030,A=1.0e+35, a=5, b=0.3, g=0.9, s=30
fmin=2.5e-01, iter=5086,A=1.0e+35, a=5, b=0.3, g=0.9, s=32
fmin=1.3e+00, iter=16061,A=1.0e+35, a=5, b=0.5, g=0.1, s=28
fmin=5.9e-01, iter=10343,A=1.0e+35, a=5, b=0.5, g=0.1, s=30
fmin=1.8e-01, iter=2312,A=1.0e+35, a=5, b=0.5, g=0.1, s=32
fmin=1.2e+00, iter=14588,A=1.0e+35, a=5, b=0.5, g=0.3, s=28
fmin=5.6e-01, iter=9730,A=1.0e+35, a=5, b=0.5, g=0.3, s=30
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fmin=1.1e+00, iter=13376,A=1.0e+35, a=5, b=0.5, g=0.5, s=28
fmin=5.4e-01, iter=9188,A=1.0e+35, a=5, b=0.5, g=0.5, s=30
fmin=1.7e-01, iter=1926,A=1.0e+35, a=5, b=0.5, g=0.5, s=32
fmin=1.0e+00, iter=12360,A=1.0e+35, a=5, b=0.5, g=0.7, s=28
fmin=5.2e-01, iter=8705,A=1.0e+35, a=5, b=0.5, g=0.7, s=30
fmin=1.7e-01, iter=1734,A=1.0e+35, a=5, b=0.5, g=0.7, s=32
fmin=1.0e+00, iter=11497,A=1.0e+35, a=5, b=0.5, g=0.9, s=28
fmin=5.0e-01, iter=8272,A=1.0e+35, a=5, b=0.5, g=0.9, s=30
fmin=1.7e-01, iter=1543,A=1.0e+35, a=5, b=0.5, g=0.9, s=32
fmin=3.0e+00, iter=40292,A=1.0e+35, a=5, b=0.7, g=0.1, s=26
fmin=1.3e+00, iter=23337,A=1.0e+35, a=5, b=0.7, g=0.1, s=28
fmin=4.0e-01, iter=9213,A=1.0e+35, a=5, b=0.7, g=0.1, s=30
fmin=2.6e+00, iter=28542,A=1.0e+35, a=5, b=0.7, g=0.3, s=26
fmin=1.2e+00, iter=20314,A=1.0e+35, a=5, b=0.7, g=0.3, s=28
fmin=3.8e-01, iter=8720,A=1.0e+35, a=5, b=0.7, g=0.3, s=30
fmin=2.2e+00, iter=19654,A=1.0e+35, a=5, b=0.7, g=0.5, s=26
fmin=1.1e+00, iter=17998,A=1.0e+35, a=5, b=0.7, g=0.5, s=28
fmin=3.7e-01, iter=8270,A=1.0e+35, a=5, b=0.7, g=0.5, s=30
fmin=2.0e+00, iter=11640,A=1.0e+35, a=5, b=0.7, g=0.7, s=26
fmin=1.0e+00, iter=16170,A=1.0e+35, a=5, b=0.7, g=0.7, s=28
fmin=3.6e-01, iter=7858,A=1.0e+35, a=5, b=0.7, g=0.7, s=30
fmin=1.8e+00, iter=3557,A=1.0e+35, a=5, b=0.7, g=0.9, s=26
fmin=9.4e-01, iter=14690,A=1.0e+35, a=5, b=0.7, g=0.9, s=28
fmin=3.5e-01, iter=7481,A=1.0e+35, a=5, b=0.7, g=0.9, s=30
fmin=1.8e+01, iter=283716,A=1.0e+35, a=5, b=0.9, g=0.1, s=24
fmin=6.3e+00, iter=157935,A=1.0e+35, a=5, b=0.9, g=0.1, s=26
fmin=1.0e+00, iter=33549,A=1.0e+35, a=5, b=0.9, g=0.1, s=28
fmin=9.0e+00, iter=109048,A=1.0e+35, a=5, b=0.9, g=0.3, s=24
fmin=4.1e+00, iter=82757,A=1.0e+35, a=5, b=0.9, g=0.3, s=26
fmin=9.6e-01, iter=29486,A=1.0e+35, a=5, b=0.9, g=0.3, s=28
fmin=6.3e+00, iter=61890,A=1.0e+35, a=5, b=0.9, g=0.5, s=24
fmin=3.2e+00, iter=56368,A=1.0e+35, a=5, b=0.9, g=0.5, s=26
fmin=8.9e-01, iter=25678,A=1.0e+35, a=5, b=0.9, g=0.5, s=28
fmin=4.9e+00, iter=13445,A=1.0e+35, a=5, b=0.9, g=0.7, s=24
fmin=2.6e+00, iter=42904,A=1.0e+35, a=5, b=0.9, g=0.7, s=26
fmin=8.2e-01, iter=22554,A=1.0e+35, a=5, b=0.9, g=0.7, s=28
fmin=2.2e+00, iter=34709,A=1.0e+35, a=5, b=0.9, g=0.9, s=26
fmin=7.6e-01, iter=20042,A=1.0e+35, a=5, b=0.9, g=0.9, s=28
fmin=6.5e-01, iter=7329,A=1.0e+35, a=6, b=0.1, g=0.1, s=30
fmin=3.1e-01, iter=6142,A=1.0e+35, a=6, b=0.1, g=0.1, s=32
fmin=6.3e-01, iter=7070,A=1.0e+35, a=6, b=0.1, g=0.3, s=30
fmin=3.1e-01, iter=5925,A=1.0e+35, a=6, b=0.1, g=0.3, s=32
fmin=6.1e-01, iter=6828,A=1.0e+35, a=6, b=0.1, g=0.5, s=30
fmin=3.0e-01, iter=5721,A=1.0e+35, a=6, b=0.1, g=0.5, s=32
fmin=5.9e-01, iter=6603,A=1.0e+35, a=6, b=0.1, g=0.7, s=30
fmin=2.9e-01, iter=5528,A=1.0e+35, a=6, b=0.1, g=0.7, s=32
fmin=5.8e-01, iter=6392,A=1.0e+35, a=6, b=0.1, g=0.9, s=30
fmin=2.9e-01, iter=5345,A=1.0e+35, a=6, b=0.1, g=0.9, s=32
fmin=9.8e-01, iter=9272,A=1.0e+35, a=6, b=0.3, g=0.1, s=28
fmin=5.6e-01, iter=7701,A=1.0e+35, a=6, b=0.3, g=0.1, s=30
fmin=2.3e-01, iter=4373,A=1.0e+35, a=6, b=0.3, g=0.1, s=32
fmin=9.4e-01, iter=8796,A=1.0e+35, a=6, b=0.3, g=0.3, s=28
fmin=5.4e-01, iter=7378,A=1.0e+35, a=6, b=0.3, g=0.3, s=30
fmin=2.3e-01, iter=4192,A=1.0e+35, a=6, b=0.3, g=0.3, s=32
fmin=9.0e-01, iter=8371,A=1.0e+35, a=6, b=0.3, g=0.5, s=28
fmin=5.3e-01, iter=7082,A=1.0e+35, a=6, b=0.3, g=0.5, s=30
fmin=2.2e-01, iter=4018,A=1.0e+35, a=6, b=0.3, g=0.5, s=32

fmin=8.7e-01, iter=7988,A=1.0e+35, a=6, b=0.3, g=0.7, s=28
fmin=5.1e-01, iter=6811,A=1.0e+35, a=6, b=0.3, g=0.7, s=30
fmin=2.2e-01, iter=3850,A=1.0e+35, a=6, b=0.3, g=0.7, s=32
fmin=8.4e-01, iter=7641,A=1.0e+35, a=6, b=0.3, g=0.9, s=28
fmin=5.0e-01, iter=6560,A=1.0e+35, a=6, b=0.3, g=0.9, s=30
fmin=2.1e-01, iter=3688,A=1.0e+35, a=6, b=0.3, g=0.9, s=32
fmin=9.3e-01, iter=11654,A=1.0e+35, a=6, b=0.5, g=0.1, s=28
fmin=4.4e-01, iter=7542,A=1.0e+35, a=6, b=0.5, g=0.1, s=30
fmin=8.8e-01, iter=10853,A=1.0e+35, a=6, b=0.5, g=0.3, s=28
fmin=4.3e-01, iter=7205,A=1.0e+35, a=6, b=0.5, g=0.3, s=30
fmin=8.4e-01, iter=10162,A=1.0e+35, a=6, b=0.5, g=0.5, s=28
fmin=4.1e-01, iter=6898,A=1.0e+35, a=6, b=0.5, g=0.5, s=30
fmin=8.1e-01, iter=9559,A=1.0e+35, a=6, b=0.5, g=0.7, s=28
fmin=4.0e-01, iter=6616,A=1.0e+35, a=6, b=0.5, g=0.7, s=30
fmin=7.8e-01, iter=9027,A=1.0e+35, a=6, b=0.5, g=0.9, s=28
fmin=3.9e-01, iter=6357,A=1.0e+35, a=6, b=0.5, g=0.9, s=30
fmin=1.9e+00, iter=27486,A=1.0e+35, a=6, b=0.7, g=0.1, s=26
fmin=8.0e-01, iter=14538,A=1.0e+35, a=6, b=0.7, g=0.1, s=28
fmin=2.6e-01, iter=5374,A=1.0e+35, a=6, b=0.7, g=0.1, s=30
fmin=1.7e+00, iter=23195,A=1.0e+35, a=6, b=0.7, g=0.3, s=26
fmin=7.6e-01, iter=13293,A=1.0e+35, a=6, b=0.7, g=0.3, s=28
fmin=2.5e-01, iter=5152,A=1.0e+35, a=6, b=0.7, g=0.3, s=30
fmin=1.6e+00, iter=20105,A=1.0e+35, a=6, b=0.7, g=0.5, s=26
fmin=7.2e-01, iter=12248,A=1.0e+35, a=6, b=0.7, g=0.5, s=28
fmin=2.5e-01, iter=4940,A=1.0e+35, a=6, b=0.7, g=0.5, s=30
fmin=1.4e+00, iter=17771,A=1.0e+35, a=6, b=0.7, g=0.7, s=26
fmin=6.9e-01, iter=11359,A=1.0e+35, a=6, b=0.7, g=0.7, s=28
fmin=2.4e-01, iter=4738,A=1.0e+35, a=6, b=0.7, g=0.7, s=30
fmin=1.3e+00, iter=15943,A=1.0e+35, a=6, b=0.7, g=0.9, s=26
fmin=6.6e-01, iter=10594,A=1.0e+35, a=6, b=0.7, g=0.9, s=28
fmin=2.4e-01, iter=4546,A=1.0e+35, a=6, b=0.7, g=0.9, s=30
fmin=1.0e+01, iter=200723,A=1.0e+35, a=6, b=0.9, g=0.1, s=24
fmin=2.7e+00, iter=73628,A=1.0e+35, a=6, b=0.9, g=0.1, s=26
fmin=4.0e-01, iter=12202,A=1.0e+35, a=6, b=0.9, g=0.1, s=28
fmin=6.0e+00, iter=92446,A=1.0e+35, a=6, b=0.9, g=0.3, s=24
fmin=2.2e+00, iter=52054,A=1.0e+35, a=6, b=0.9, g=0.3, s=26
fmin=3.9e-01, iter=11669,A=1.0e+35, a=6, b=0.9, g=0.3, s=28
fmin=4.4e+00, iter=60729,A=1.0e+35, a=6, b=0.9, g=0.5, s=24
fmin=1.8e+00, iter=39926,A=1.0e+35, a=6, b=0.9, g=0.5, s=26
fmin=3.8e-01, iter=11077,A=1.0e+35, a=6, b=0.9, g=0.5, s=28
fmin=3.6e+00, iter=45423,A=1.0e+35, a=6, b=0.9, g=0.7, s=24
fmin=1.6e+00, iter=32392,A=1.0e+35, a=6, b=0.9, g=0.7, s=26
fmin=3.7e-01, iter=10479,A=1.0e+35, a=6, b=0.9, g=0.7, s=28
fmin=3.0e+00, iter=36363,A=1.0e+35, a=6, b=0.9, g=0.9, s=24
fmin=1.4e+00, iter=27280,A=1.0e+35, a=6, b=0.9, g=0.9, s=26
fmin=3.6e-01, iter=9901,A=1.0e+35, a=6, b=0.9, g=0.9, s=28
fmin=5.6e-01, iter=6207,A=1.0e+35, a=7, b=0.1, g=0.1, s=30
fmin=2.8e-01, iter=5080,A=1.0e+35, a=7, b=0.1, g=0.1, s=32
fmin=5.4e-01, iter=6020,A=1.0e+35, a=7, b=0.1, g=0.3, s=30
fmin=2.7e-01, iter=4916,A=1.0e+35, a=7, b=0.1, g=0.3, s=32
fmin=5.3e-01, iter=5843,A=1.0e+35, a=7, b=0.1, g=0.5, s=30
fmin=2.6e-01, iter=4759,A=1.0e+35, a=7, b=0.1, g=0.5, s=32
fmin=5.2e-01, iter=5677,A=1.0e+35, a=7, b=0.1, g=0.7, s=30
fmin=2.6e-01, iter=4608,A=1.0e+35, a=7, b=0.1, g=0.7, s=32
fmin=5.1e-01, iter=5520,A=1.0e+35, a=7, b=0.1, g=0.9, s=30
fmin=2.5e-01, iter=4464,A=1.0e+35, a=7, b=0.1, g=0.9, s=32
fmin=8.0e-01, iter=7583,A=1.0e+35, a=7, b=0.3, g=0.1, s=28
fmin=4.7e-01, iter=6272,A=1.0e+35, a=7, b=0.3, g=0.1, s=30
fmin=2.0e-01, iter=3001,A=1.0e+35, a=7, b=0.3, g=0.1, s=32
fmin=7.8e-01, iter=7264,A=1.0e+35, a=7, b=0.3, g=0.3, s=28
fmin=4.6e-01, iter=6055,A=1.0e+35, a=7, b=0.3, g=0.3, s=30
fmin=1.9e-01, iter=2848,A=1.0e+35, a=7, b=0.3, g=0.3, s=32
fmin=7.5e-01, iter=6973,A=1.0e+35, a=7, b=0.3, g=0.5, s=28
fmin=4.4e-01, iter=5853,A=1.0e+35, a=7, b=0.3, g=0.5, s=30
fmin=1.9e-01, iter=2698,A=1.0e+35, a=7, b=0.3, g=0.5, s=32
fmin=7.3e-01, iter=6706,A=1.0e+35, a=7, b=0.3, g=0.7, s=28

fmin=4.3e-01, iter=5665,A=1.0e+35, a=7, b=0.3, g=0.7, s=30
fmin=1.9e-01, iter=2550,A=1.0e+35, a=7, b=0.3, g=0.7, s=32
fmin=7.1e-01, iter=6460,A=1.0e+35, a=7, b=0.3, g=0.9, s=28
fmin=4.2e-01, iter=5489,A=1.0e+35, a=7, b=0.3, g=0.9, s=30
fmin=1.8e-01, iter=2403,A=1.0e+35, a=7, b=0.3, g=0.9, s=32
fmin=7.1e-01, iter=8851,A=1.0e+35, a=7, b=0.5, g=0.1, s=28
fmin=3.4e-01, iter=5723,A=1.0e+35, a=7, b=0.5, g=0.1, s=30
fmin=6.9e-01, iter=8380,A=1.0e+35, a=7, b=0.5, g=0.3, s=28
fmin=3.4e-01, iter=5520,A=1.0e+35, a=7, b=0.5, g=0.3, s=30
fmin=6.6e-01, iter=7959,A=1.0e+35, a=7, b=0.5, g=0.5, s=28
fmin=3.3e-01, iter=5331,A=1.0e+35, a=7, b=0.5, g=0.5, s=30
fmin=6.4e-01, iter=7581,A=1.0e+35, a=7, b=0.5, g=0.7, s=28
fmin=3.2e-01, iter=5154,A=1.0e+35, a=7, b=0.5, g=0.7, s=30
fmin=6.2e-01, iter=7240,A=1.0e+35, a=7, b=0.5, g=0.9, s=28
fmin=3.1e-01, iter=4988,A=1.0e+35, a=7, b=0.5, g=0.9, s=30
fmin=1.3e+00, iter=18253,A=1.0e+35, a=7, b=0.7, g=0.1, s=26
fmin=5.4e-01, iter=9620,A=1.0e+35, a=7, b=0.7, g=0.1, s=28
fmin=1.8e-01, iter=2252,A=1.0e+35, a=7, b=0.7, g=0.1, s=30
fmin=1.2e+00, iter=16254,A=1.0e+35, a=7, b=0.7, g=0.3, s=26
fmin=5.2e-01, iter=9053,A=1.0e+35, a=7, b=0.7, g=0.3, s=28
fmin=1.8e-01, iter=2093,A=1.0e+35, a=7, b=0.7, g=0.3, s=30
fmin=1.1e+00, iter=14667,A=1.0e+35, a=7, b=0.7, g=0.5, s=26
fmin=5.0e-01, iter=8549,A=1.0e+35, a=7, b=0.7, g=0.5, s=28
fmin=1.8e-01, iter=1935,A=1.0e+35, a=7, b=0.7, g=0.5, s=30
fmin=1.0e+00, iter=13376,A=1.0e+35, a=7, b=0.7, g=0.7, s=26
fmin=4.9e-01, iter=8100,A=1.0e+35, a=7, b=0.7, g=0.7, s=28
fmin=1.7e-01, iter=1777,A=1.0e+35, a=7, b=0.7, g=0.7, s=30
fmin=9.8e-01, iter=12305,A=1.0e+35, a=7, b=0.7, g=0.9, s=26
fmin=4.7e-01, iter=7697,A=1.0e+35, a=7, b=0.7, g=0.9, s=28
fmin=1.7e-01, iter=1619,A=1.0e+35, a=7, b=0.7, g=0.9, s=30
fmin=5.2e+00, iter=116872,A=1.0e+35, a=7, b=0.9, g=0.1, s=24
fmin=1.2e+00, iter=31836,A=1.0e+35, a=7, b=0.9, g=0.1, s=26
fmin=1.7e-01, iter=2108,A=1.0e+35, a=7, b=0.9, g=0.1, s=28
fmin=3.7e+00, iter=68495,A=1.0e+35, a=7, b=0.9, g=0.3, s=24
fmin=1.0e+00, iter=27311,A=1.0e+35, a=7, b=0.9, g=0.3, s=26
fmin=1.7e-01, iter=1969,A=1.0e+35, a=7, b=0.9, g=0.3, s=28
fmin=2.9e+00, iter=48681,A=1.0e+35, a=7, b=0.9, g=0.5, s=24
fmin=9.6e-01, iter=23654,A=1.0e+35, a=7, b=0.9, g=0.5, s=26
fmin=1.7e-01, iter=1817,A=1.0e+35, a=7, b=0.9, g=0.5, s=28
fmin=2.4e+00, iter=37908,A=1.0e+35, a=7, b=0.9, g=0.7, s=24
fmin=8.8e-01, iter=20786,A=1.0e+35, a=7, b=0.9, g=0.7, s=26
fmin=1.7e-01, iter=1655,A=1.0e+35, a=7, b=0.9, g=0.7, s=28
fmin=2.1e+00, iter=31116,A=1.0e+35, a=7, b=0.9, g=0.9, s=24
fmin=8.2e-01, iter=18516,A=1.0e+35, a=7, b=0.9, g=0.9, s=26
fmin=1.7e-01, iter=1485,A=1.0e+35, a=7, b=0.9, g=0.9, s=28
fmin=4.9e-01, iter=5372,A=1.0e+35, a=8, b=0.1, g=0.1, s=30
fmin=2.5e-01, iter=4237,A=1.0e+35, a=8, b=0.1, g=0.1, s=32
fmin=4.8e-01, iter=5231,A=1.0e+35, a=8, b=0.1, g=0.3, s=30
fmin=2.4e-01, iter=4104,A=1.0e+35, a=8, b=0.1, g=0.3, s=32
fmin=4.7e-01, iter=5097,A=1.0e+35, a=8, b=0.1, g=0.5, s=30
fmin=2.4e-01, iter=3975,A=1.0e+35, a=8, b=0.1, g=0.5, s=32
fmin=4.6e-01, iter=4970,A=1.0e+35, a=8, b=0.1, g=0.7, s=30
fmin=2.3e-01, iter=3850,A=1.0e+35, a=8, b=0.1, g=0.7, s=32
fmin=4.5e-01, iter=4849,A=1.0e+35, a=8, b=0.1, g=0.9, s=30
fmin=2.3e-01, iter=3728,A=1.0e+35, a=8, b=0.1, g=0.9, s=32
fmin=6.8e-01, iter=6344,A=1.0e+35, a=8, b=0.3, g=0.1, s=28
fmin=4.0e-01, iter=5242,A=1.0e+35, a=8, b=0.3, g=0.1, s=30
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fmin=3.7e-01, iter=4807,A=1.0e+35, a=8, b=0.3, g=0.7, s=30

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fmin=3.3e-01, iter=4141,A=1.0e+35, a=9, b=0.3, g=0.7, s=30
fmin=5.3e-01, iter=4811,A=1.0e+35, a=9, b=0.3, g=0.9, s=28
fmin=3.2e-01, iter=4043,A=1.0e+35, a=9, b=0.3, g=0.9, s=30
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fmin=4.5e-01, iter=5424,A=1.0e+35, a=9, b=0.5, g=0.3, s=28

```
fmin=2.3e-01, iter=3302,A=1.0e+35, a=9, b=0.5, g=0.3, s=30
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fmin=2.4e-01, iter=5298,A=1.0e+35, a=9, b=0.9, g=0.5, s=26
fmin=2.3e+00, iter=33791,A=1.0e+35, a=9, b=0.9, g=0.7, s=22
fmin=9.3e-01, iter=19187,A=1.0e+35, a=9, b=0.9, g=0.7, s=24
fmin=2.4e-01, iter=5065,A=1.0e+35, a=9, b=0.9, g=0.7, s=26
```

100%|

| 1/1 [02:05<00:00, 125.82s/it]

```
fmin=2.0e+00, iter=28109,A=1.0e+35, a=9, b=0.9, g=0.9, s=22
fmin=8.6e-01, iter=17158,A=1.0e+35, a=9, b=0.9, g=0.9, s=24
fmin=2.3e-01, iter=4838,A=1.0e+35, a=9, b=0.9, g=0.9, s=26
```

```
In [50]: min_fun.sort(key = lambda row: row[0])
```

```
In [51]: print(f'fmin={min_fun[0][0]:.3f}, A={min_fun[0][1]:.1e}, a={min_fun[0][2]}, b={min_fun[0][3]}, g={min_fun[0][4]:.1f}, s={min_fun[0][5]}')
```

```
fmin=0.162, A=1.0e+35, a=8, b=0.3, g=0.9, s=32
```

Задача 3.

Не понял условие: Сумма S считается по обычным правилам сложения десятичных чисел — таким образом, она находится в диапазоне от 0 до $N(q - 1)$? Пусть $q=7$ последовательность длины $N = 1000$.

$$0 < \sum_i q_i < 233317$$

```
In [52]: def convert_base(num, to_base=10, from_base=10):
# first convert to decimal number
if isinstance(num, str):
    n = int(num, from_base)
else:
    n = int(num)
# now convert decimal to 'to_base' base
alphabet = "0123456789ABCDEFGHIJKLMNOPQRSTUVWXYZ"
if n < to_base:
    return alphabet[n]
else:
    return convert_base(n // to_base, to_base) + alphabet[n % to_base]
```

```
In [53]: def sum_rand_n(n):
```

```

q = 0
for _ in range(1000):
    q += random.randint(0, 7)
return q

```

```

In [55]: q = 6066
convert_base(q, to_base=7, from_base=10)

```

```

Out[55]: '23454'

```

$$M(x_i) = \frac{\sum_{i=1}^{q-1} i}{q} = \frac{q-1}{2}, \quad M(x_i^2) = \frac{\sum_{i=1}^{q-1} i^2}{q} = \frac{\frac{1}{6}q(q-1)(2q-1)}{q} = \frac{(q-1)(2q-1)}{6},$$

$$D(x_i) = M(x_i^2) - M(x_i)^2 = \frac{q^2-1}{12} \implies$$

$$M(S_n) = \frac{n(q-1)}{2}, \quad D(S_n) = \frac{n(q^2-1)}{12}, \quad P\left(\left\{\frac{S_n - M(S_n)}{\sqrt[2]{D(S_n)}} < d\right\}\right) \approx 2\Phi_0(d)$$

```

In [56]: def ppf(p):
        return round(stats.norm.ppf(0.5+p/200), 2)

def segment_calc(q, N, p):
    S_n = (q-1)/2*N
    ppf_p = ppf(p)
    return ceil(((sqrt(ppf_p**2*(q**2-1)/12+2*(q-1)*S_n)-ppf_p*sqrt((q**2-1)/12))/(q-1))

def segment_out(q, N, p):
    seg = segment_calc(q, N, p)
    a = '{'
    b = '}'
    print(f'{a}d1, d2{b}={a}k, k+{seg}{b} \forall k \in {a}0,1,...,{N-seg}{b}')

```

```

In [57]: segment_out(7, 2022, 95)

{d1, d2}={k, k+1965} \forall k \in {0,1,...,57}

```

```

In [60]: # проверка
(sum_rand_n(segment_calc(7, 2022, 95))/sum_rand_n(2022))*100

```

```

Out[60]: 98.61268403171007

```

```

In [ ]:

```