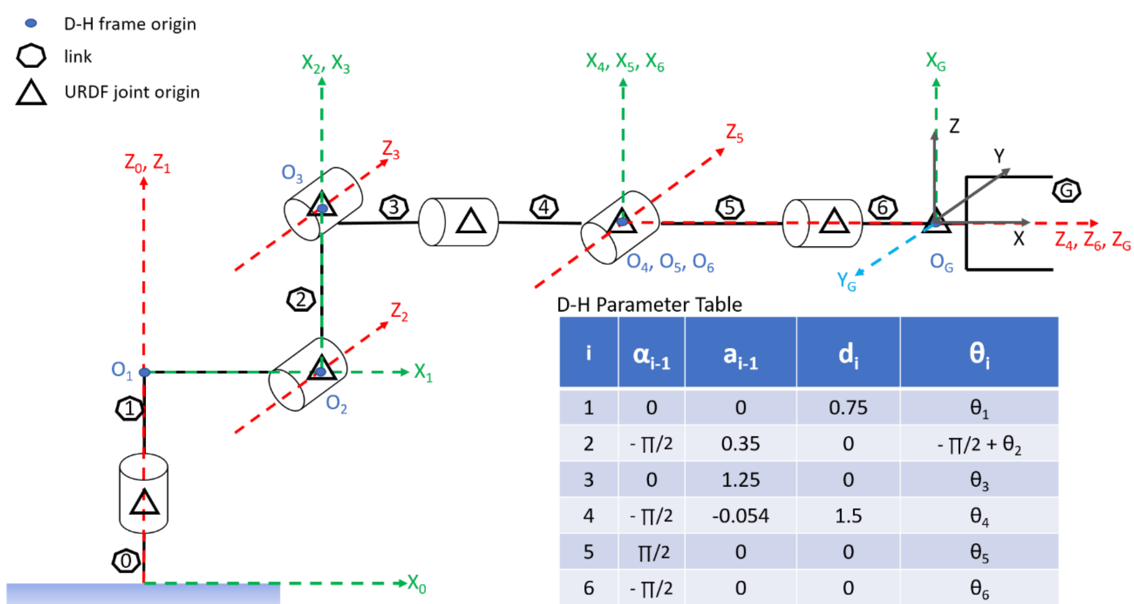
Robotic Arm: Pick and Place

The following sections describe implementation of points mentioned in the project rubric.

Kinematic Analysis

1. Forward Kinematics and DH parameter table: The goal was to determine the homogenous transforms between all adjacent links. DH convention was employed to assign the reference frames to each link from the base to the gripper.
   a. DH frame assignments:
      i. All joint axes were identified.
      ii. Z axes were assigned to coincide with the joint axes.
      iii. X_i was chosen to point from Z_i to $Z_{i+1.}$ Since Z_0 and Z_1 are coincident, X_0 was chosen arbitrarily. Since Z_3 and Z_4 were intersecting, X_3 was chosen to point in a direction perpendicular to plane containing both. Similar strategies were employed for assigning values for X_4 and X_5. X_G was chosen so that it aligned with X_6.
      iv. Origin of frame 2 was coincident with URDF origin of joint 2. Origin of frame 3 was coincident with URDF origin of join 3. Origins of 4,5,6 were coincident with URDF origin of joint5. Origin of G was coincident with URDF origin of gripper joint.

   b. DH parameter table: DH parameter values were obtained based on the DH reference frame assignments. They are listed in DH table in Figure 1 for reference frames 1 to 7 (where 7 corresponds to gripper frame). The unusual case is joint 2 rotation which has a starting position of negative 90 degrees.



D-H Parameter Table

i	α_{i-1}	a_{i-1}	d_i	θ_i
1	0	0	0.75	θ_1
2	$-\pi/2$	0.35	0	$-\pi/2 + \theta_2$
3	0	1.25	0	θ_3
4	$-\pi/2$	-0.054	1.5	θ_4
5	$\pi/2$	0	0	θ_5
6	$-\pi/2$	0	0	θ_6
7	0	0	0.303	0

Figure 1. The URDF frame origin for each joint are indicated with a triangle. Each link is marked with a number enclosed in a hexagon (base link corresponds to 0, link1 corresponds to 1 and so on), gripper link is indicated with a G. DH frames associated with each link are marked with a blue circle. X,Y, Z axes of DH frames are colored Green, Blue and Red respectively. The diagram also indicates the URDF reference frame of gripper link in gray

2. Transformation matrices calculation: Transformation matrices were generated for each adjacent DH frame based on the DH parameter values and prescribed matrix formulations (refer to Figure 2). Since the gripper frame specified in URDF does not match the DH frame associated with gripper link, to arrive at the complete transform matrix. The correction matrix is obtained by intrinsically rotating the DH gripper frame to URDF gripper frame.

$$T0\_1 = \begin{bmatrix} \cos(q1) & -\sin(q1) & 0 & 0 \\ \sin(q1)*\cos(0) & \cos(q1)*\cos(0) & -\sin(0) & -\sin(0)*0.75 \\ \sin(q1)*\sin(0) & \cos(q1)*\sin(0) & \cos(0) & \cos(0)*0.75 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \cos(q1) & -\sin(q1) & 0 & 0 \\ \sin(q1) & \cos(q1) & 0 & 0 \\ 0 & 0 & 1 & 0.75 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T1\_2 = \begin{bmatrix} \cos(-\frac{\pi}{2}+q2) & -\sin(-\frac{\pi}{2}+q2) & 0 & 0.35 \\ \sin(q2)*\cos(-\frac{\pi}{2}) & \cos(q2)*\cos(-\frac{\pi}{2}) & -\sin(-\frac{\pi}{2}) & -\sin\left(-\frac{\pi}{2}\right)*0 \\ \sin\left(-\frac{\pi}{2}+q2\right)*\sin(-\frac{\pi}{2}) & \cos\left(-\frac{\pi}{2}+q2\right)*\sin(-\frac{\pi}{2}) & \cos(-\frac{\pi}{2}) & \cos\left(-\frac{\pi}{2}\right)*0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} sin(q2) & cos(q2) & 0 & 0.35 \\ 0 & 0 & 1 & 0 \\ cos(q2) & -sin(q2) & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T2\_3 = \begin{bmatrix} \cos(q3) & -\sin(q3) & 0 & 1.25 \\ \sin(q3)*\cos(0) & \cos(q3)*\cos(0) & -\sin(0) & -\sin(0)*0 \\ \sin(q3)*\sin(0) & \cos(q3)*\sin(0) & \cos(0) & \cos(0)*0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \cos(q3) & -\sin(q3) & 0 & 1.25 \\ \sin(q3) & \cos(q3) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T3\_4 = \begin{bmatrix} \cos(q4) & -\sin(q4) & 0 & -0.054 \\ \sin(q4)*\cos(-\frac{\pi}{2}) & \cos(q4)*\cos(-\frac{\pi}{2}) & -\sin(-\frac{\pi}{2}) & -\sin\left(-\frac{\pi}{2}\right)*1.5 \\ \sin(q4)*\sin(-\frac{\pi}{2}) & \cos(q4)*\sin(-\frac{\pi}{2}) & \cos(-\frac{\pi}{2}) & \cos\left(-\frac{\pi}{2}\right)*1.5 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \cos(q4) & -\sin(q4) & 0 & -0.054 \\ 0 & 0 & 1 & 1.5 \\ -\sin(q4) & -\cos(q4) & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T4\_5 = \begin{bmatrix} \cos(q5) & -\sin(q5) & 0 & 0 \\ \sin(q5)*\cos(\frac{\pi}{2}) & \cos(q5)*\cos(\frac{\pi}{2}) & -\sin(\frac{\pi}{2}) & -\sin\left(\frac{\pi}{2}\right)*0 \\ \sin(q5)*\sin(\frac{\pi}{2}) & \cos(q5)*\sin(\frac{\pi}{2}) & \cos(\frac{\pi}{2}) & \cos\left(\frac{\pi}{2}\right)*0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \cos(q5) & -\sin(q5) & 0 & 0 \\ 0 & 0 & -1 & 0 \\ \sin(q5) & \cos(q5) & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T5\_6 = \begin{bmatrix} \cos(q6) & -\sin(q6) & 0 & 0 \\ \sin(q6)*\cos(-\frac{\pi}{2}) & \cos(q6)*\cos(-\frac{\pi}{2}) & -\sin(-\frac{\pi}{2}) & -\sin\left(-\frac{\pi}{2}\right)*0 \\ \sin(q6)*\sin(-\frac{\pi}{2}) & \cos(q6)*\sin(-\frac{\pi}{2}) & \cos(-\frac{\pi}{2}) & \cos\left(-\frac{\pi}{2}\right)*0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \cos(q6) & -\sin(q6) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\sin(q6) & -\cos(q6) & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T6\_G = \begin{bmatrix} \cos(0) & -\sin(0) & 0 & 0 \\ \sin(0)*\cos(0) & \cos(0)*\cos(0) & -\sin(0) & -\sin(0)*0.303 \\ \sin(0)*\sin(0) & \cos(0)*\sin(0) & \cos(0) & \cos(0)*0.303 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0.303 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Figure 2. Homogenous transform matrices between adjacent links from base link (link 0) to gripper link (link 7)

$$R\_Z = \begin{bmatrix} \cos(\pi) & -\sin(\pi) & 0 \\ \sin(\pi) & \cos(\pi) & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$R\_Y = \begin{bmatrix} \cos\left(-\frac{\pi}{2}\right) & 0 & \sin\left(-\frac{\pi}{2}\right) \\ 0 & 1 & 0 \\ -\sin\left(-\frac{\pi}{2}\right) & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$R\_Yaw = \begin{bmatrix} \cos(\Psi) & -\sin(\Psi) & 0 \\ \sin(\Psi) & \cos(\Psi) & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$R\_Pitch = \begin{bmatrix} \cos(\theta) & 0 & \sin(\theta) \\ 0 & 1 & 0 \\ -\sin(\theta) & 0 & \cos(\theta) \\ 0 & 0 & 0 \end{bmatrix}$$

$$R\_Roll = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\phi) & -\sin(\phi) \\ 0 & \sin(\phi) & \cos(\phi) \\ 0 & 0 & 0 \end{bmatrix}$$

$$R\_total = R\_Yaw * R\_Pitch * R\_Roll * R\_Z * R\_Y$$

Figure 3. Rotation matrices to account for relative orientation of URDF gripper frame-DH gripper frame and orientation of gripper with respect to base link frame.

3. Inverse kinematics calculation: Wrist center was assigned to DH frame origin 4. Since last three joint axes intersect at a single point, we can decouple the inverse kinematic problem to translation and orientation of wrist center.

The wrist center position can be determined as follows

$^0R_{WC/0} = {}^0R_{G/0} + {}^0R_{WC/G}$

$^0R_{G/0}$ (gripper position) is provided by the simulator. The rotation matrix R_{total} provides the rotation of the DH gripper frame with respect to the origin. The wrist center position with respect to gripper is obtained moving along the z-axis in the negative direction by the distance to the wrist center (d6 + d7).

$$^0R_{WC/G} = {}^0R_{G/0} - (d_6 + d_7)*R_{total}.\begin{bmatrix}0\\0\\1\end{bmatrix} \qquad - (1)$$

The wrist center position is used to determine the value of first 3 joint angles. Figure 4 details the solution of joint angles in terms of wrist center position.

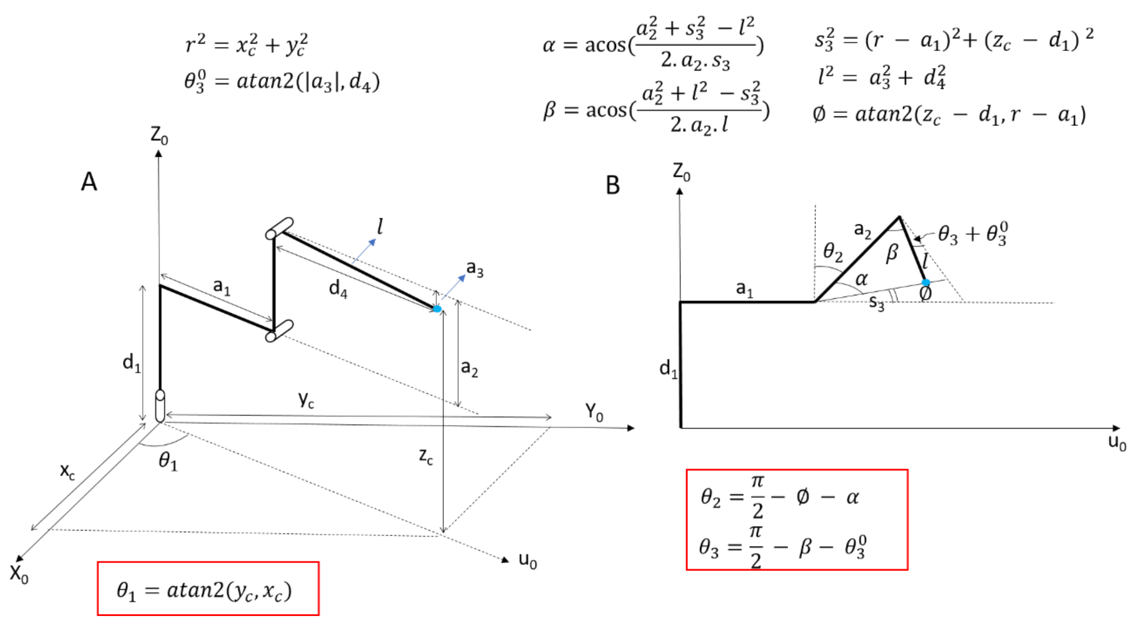


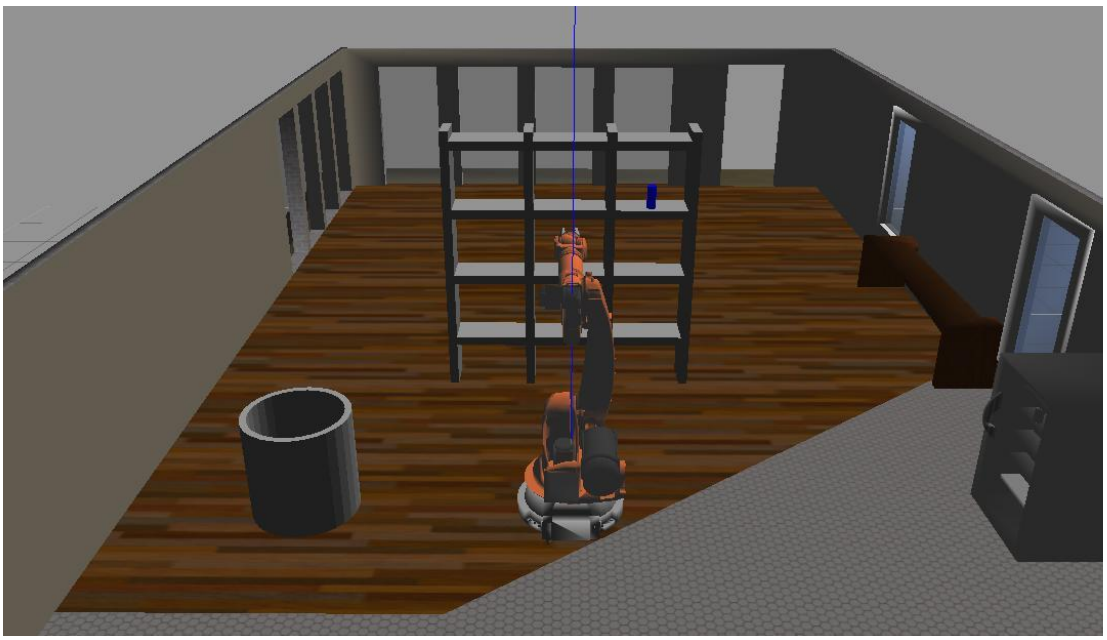
Figure 4. (A) shows links 1, 2 and 3 in base link reference frame (X_0-Y_0); θ_1 is determined from the x and y location (x_c, y_c) of the wrist center. (B) shows the projection of the links onto z0-u0 plane. Sides of the triangle (s_3, a_2, l) and angles Φ and α are determined. θ_2 is determined by subtracting Φ and α from π/2. θ_3 is determined by subtracting β and θ_3^0 from π/2.

Code implementation

1. Transformation matrices were calculated only once outside the “for loop”. This was done to ensure faster IK calculations.
2. Inside the for loop:
   a. Rotation matrix was calculated, and wrist center positions were estimated using above equation for wrist center.
   b. θ_1, θ_2, θ_3 were calculated using geometric method
   c. θ_4, θ_5, θ_6 were calculated by calculating ${}^3_6R$ by multiplying inverse of ${}^0_3R$ to ${}^0_6R$. Then Euler angles were determined from the rotation matrix ${}^3_6R$

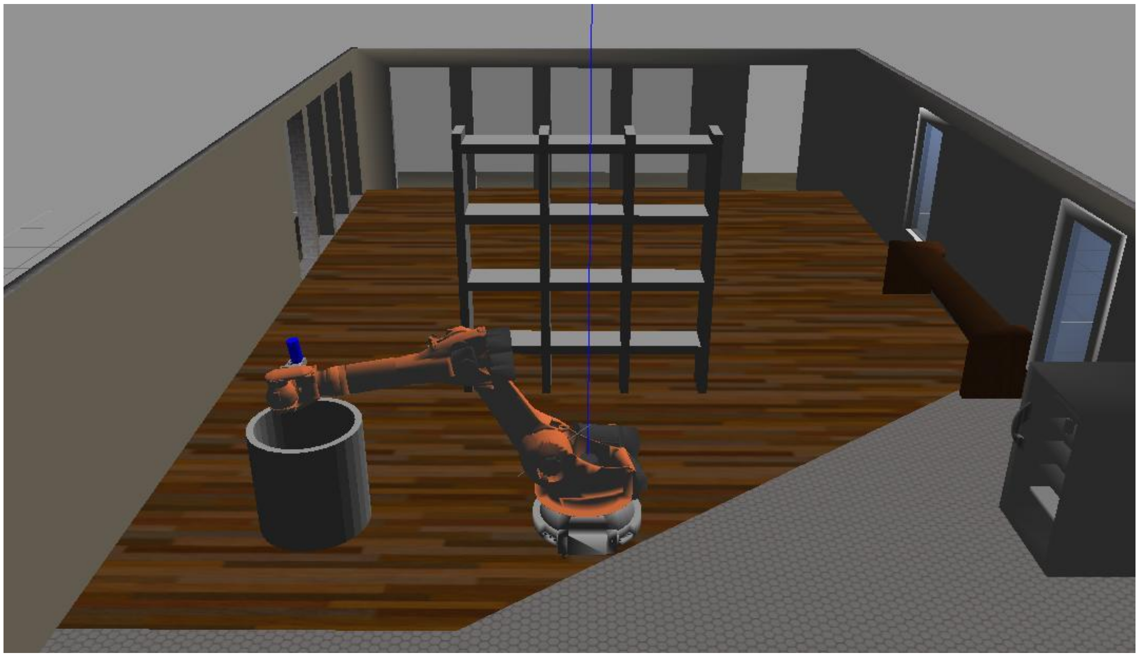
Results

1. Pick and place operation was performed for few runs one of which is displayed in Figure 5. The arm seemed to track the desired trajectory reasonably well. The performance of pick and place operation was not quantified for more runs because the computer was slow for the graphics operations.

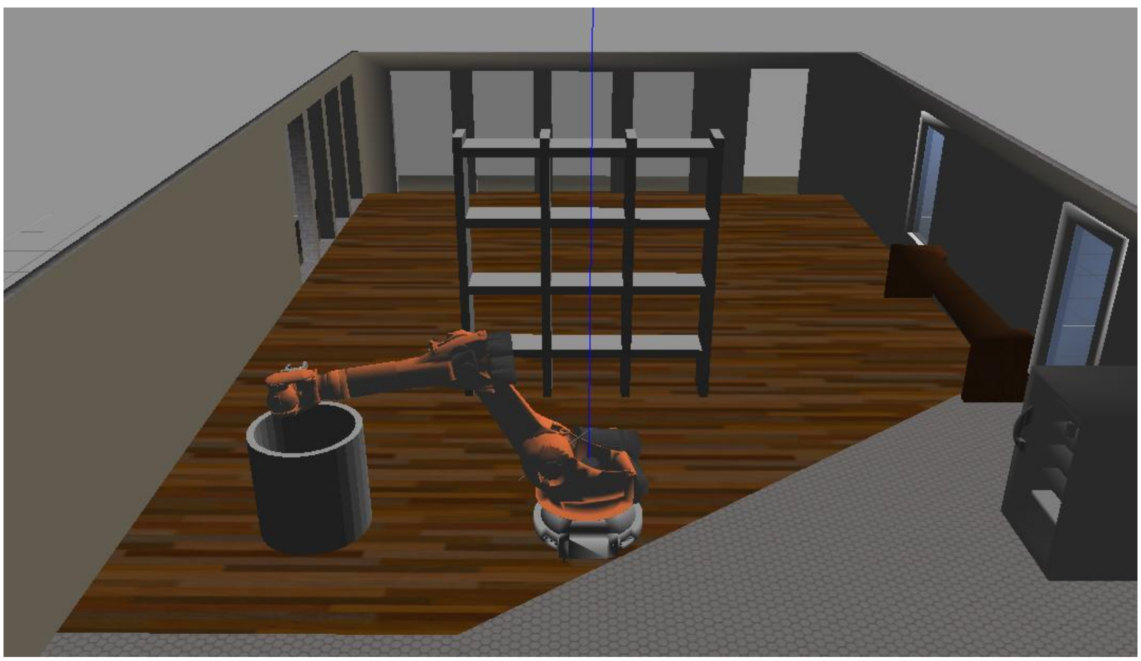


A

B



C



D

Figure 5. (A) Arm at starting position. (B) Arm follows trajectory to reach sample location, reaches out, grabs and retrieves the sample. (C) Arm follows trajectory to reach bin location (D) Arm drops sample into the bin thus completing the pick and place operation

2. It was observed that even for reasonably simple trajectory, all joints (including wrist joints) were exercised throughout the trajectory. This seemed wasteful as the wrist needs to be in the correct orientation only at the last pose in the trajectory. The need for wrist movement during the trajectory is justified only if the arm must avoid obstacles during its motion.