Testing Portfolio Pptimization Strategies using Monte Carlo Simulation Methodology

Abstract

Introduction

The main goal for investors when creating an asset portfolio is to obtain the most value from their constructed portfolio. A major decision in portfolio management lies in defining how to allocate funds when constructing a portfolio. Portfolio optimization is a phenomenon widely studied in finance. It consists of determining the optimal proportions of total capital to invest in each particular asset in the portfolio. Such a problem poses a challenge to financial investors as portfolio managers seek to define the best way to distribute capital in order to yield the most favorable portfolio returns depending on the level of risk an investor is willing to take. Many different strategies exist to define optimal weights in a portfolio however, it remains hard for portfolio managers to decide which optimization strategy is best suited for a given set of risky assets.

Optimization Strategies

The optimal portfolio strategies examined in this paper are the mean variance, minimum variance, max diversification and max decorrelation portfolios. Each strategy imposes different assumptions and optimization goals in order to obtain the optimal weights for risky assets.

| Mean Variance

The mean variance portfolio optimization theory is the most popular optimization technique utilized in finance. The goal of this strategy is to determine optimal weights while considering the trade-off between risk and return. The choice of portfolio weights is one which maximizes return while avoiding unnecessary risk(Markowitz, 1952).

The objective of this optimization strategy is to maximize the ratio or mean to variance of the portfolio also known as the Sharpe ratio.

Maximize Sharpe Ratio

$$S = \frac{\mu}{\sigma_p}$$

Where μ is portfolio return and σ_p is portfolio standard deviation

| Minimum Variance

The objective of this portfolio optimization strategy is to construct a portfolio that minimizes portfolio variance. Results yield an optimized portfolio with the lowest possible volatility for a given set of risky assets.

Inputs needed to construct the minimum variance portfolio are asset mean expected return, as well as an estimation of the risk and correlation of all assets in the portfolio (Markowitz,1952).

Minimize portfolio variance:

$$\sigma^2_{minvar} = w^T \sum w$$

Where w is a set of portfolio weights and $\sum$ is tha covariance matrix

| Max Diversification

Choueifaty and Coignard proposed another portfolio optimization strategy referred to as the maximum diversification strategy. This strategy suggests that investments produced returns that are proportional to their volatility. As such, the optimization strategy aims to maximize a metric that defines a portfolio's degree of diversification. The result is a portfolio with assets that are minimally correlated and have lower risk levels.The metric used to measure diversification is a ratio of weighted average of asset volatility to the portfolio volatility (Clarke et al.,2012) where volatility is measured by variance.

The objective of this portfolio optimization strategy is to maximize the diversification ratio.

Maximize diversification ratio:

$$D(P) = \frac{w^T \sigma}{\sqrt{w^T \sum w}}$$

Where w is a set of portfolio weights, $\sum$ is the covariance matrix and σ is a vector of asset volatilities

| Max Decorrelation

The max decorrelation portfolio strategy conceived by Christoffersen et al. (2010) emphasizes that correlation is the main driver of portfolio diversification benefits (Amenc et al.,2014). As such, the optimization strategy suggests solely focusing on the correlation matrix to calculate optimal weights.

This strategy minimizes portfolio volatility with the assumption that all assets have identical volatility but heterogeneous correlations.

Minimize portfolio variance:

$$\sigma^2_{DC} = w^T A w$$

Where w is a set of portfolio weights, and A is the correlation matrix

All strategies are subject to the constraint that the sum of weights is one. Note that long-only portfolio's were considered as such all weights must be positive.

Research Question

The goal of this experiment is to determine what investment strategy performs best depending on the composition of stocks in a given portfolio. Four different scenarios are tested where portfolios are composed of stocks with different distributions. Such analysis will allow financial managers to easily choose the most favorable optimization strategy to implement when performing portfolio optimization depending on the distribution of stocks held in the portfolio.

Methadology

Scenario 1: All Normally Distributed Stock Returns

```
## ‘stat_bin()‘ using ‘bins = 30‘. Pick better value with ‘binwidth‘.
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```

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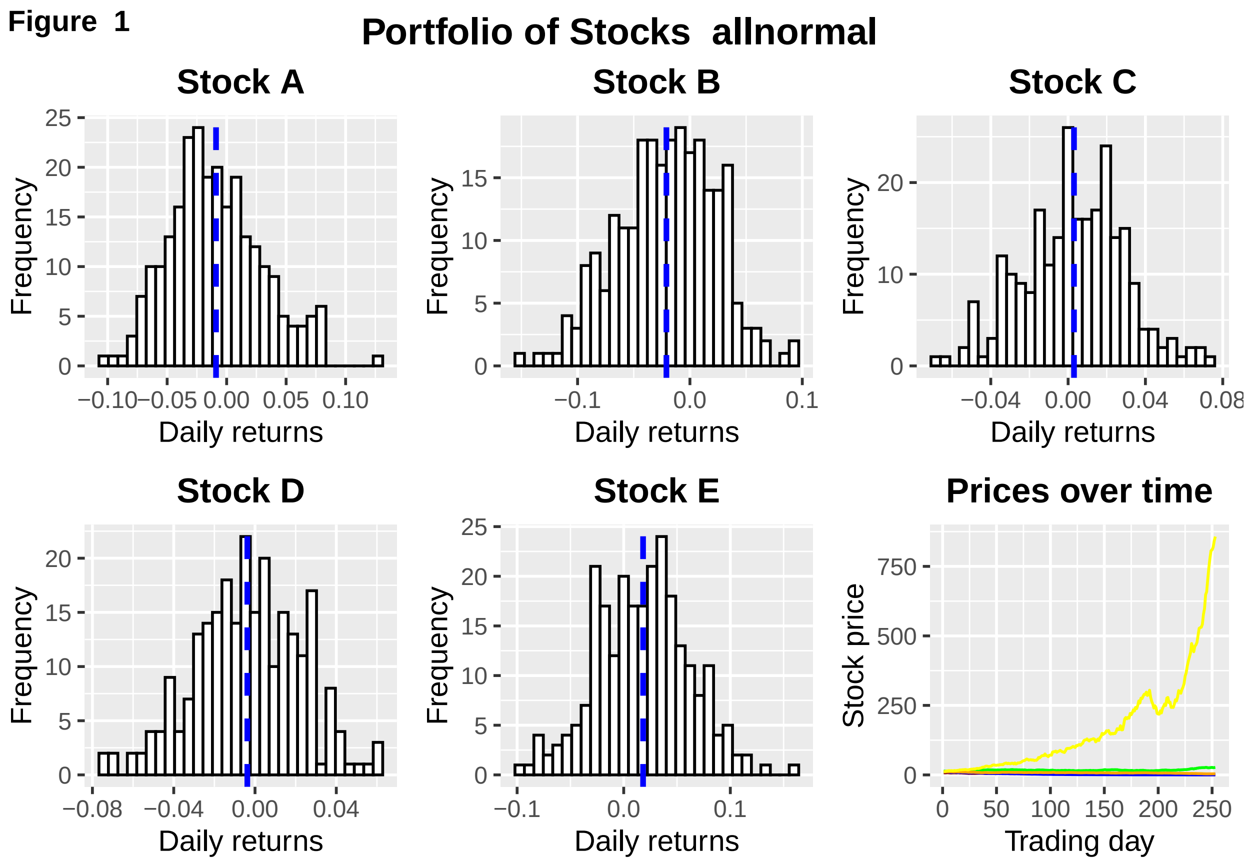
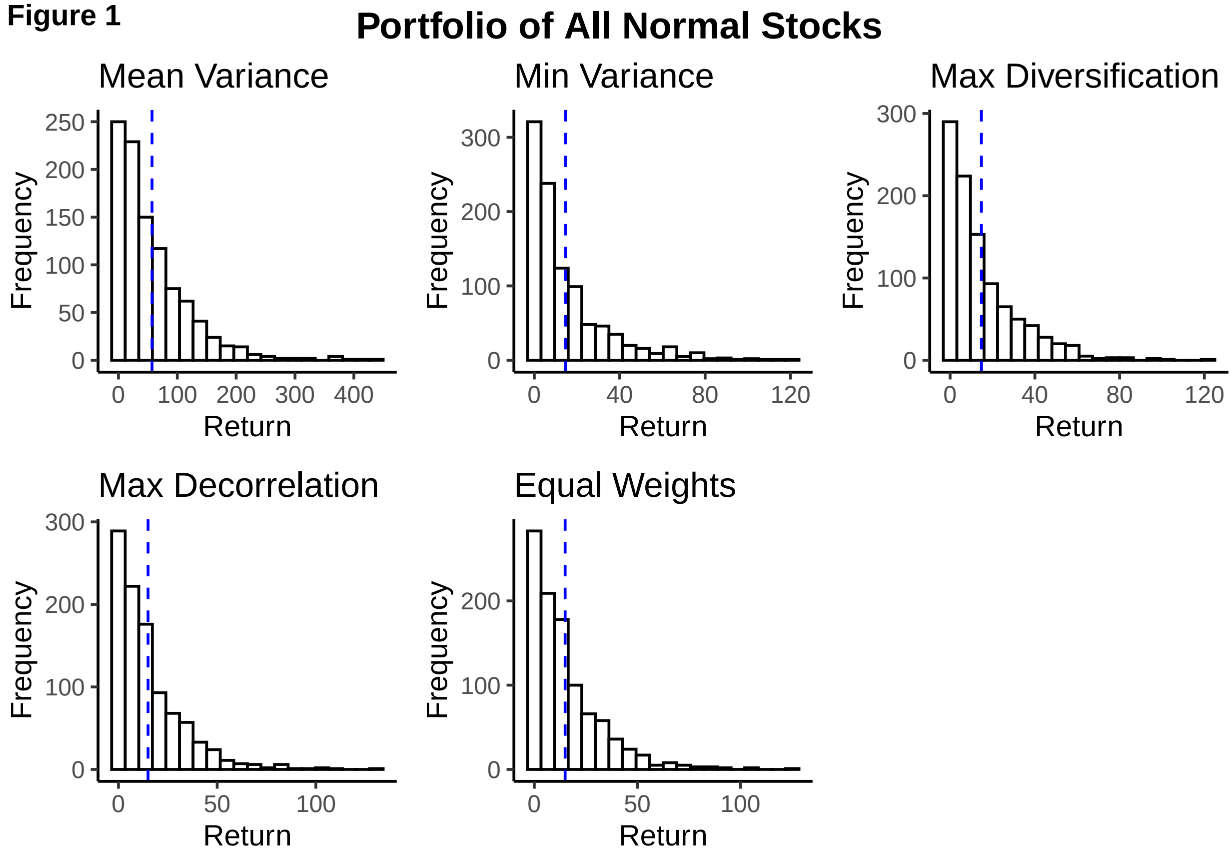


Figure 1



Scenario 2: All Normally Distributed Stock Returns with low volatility

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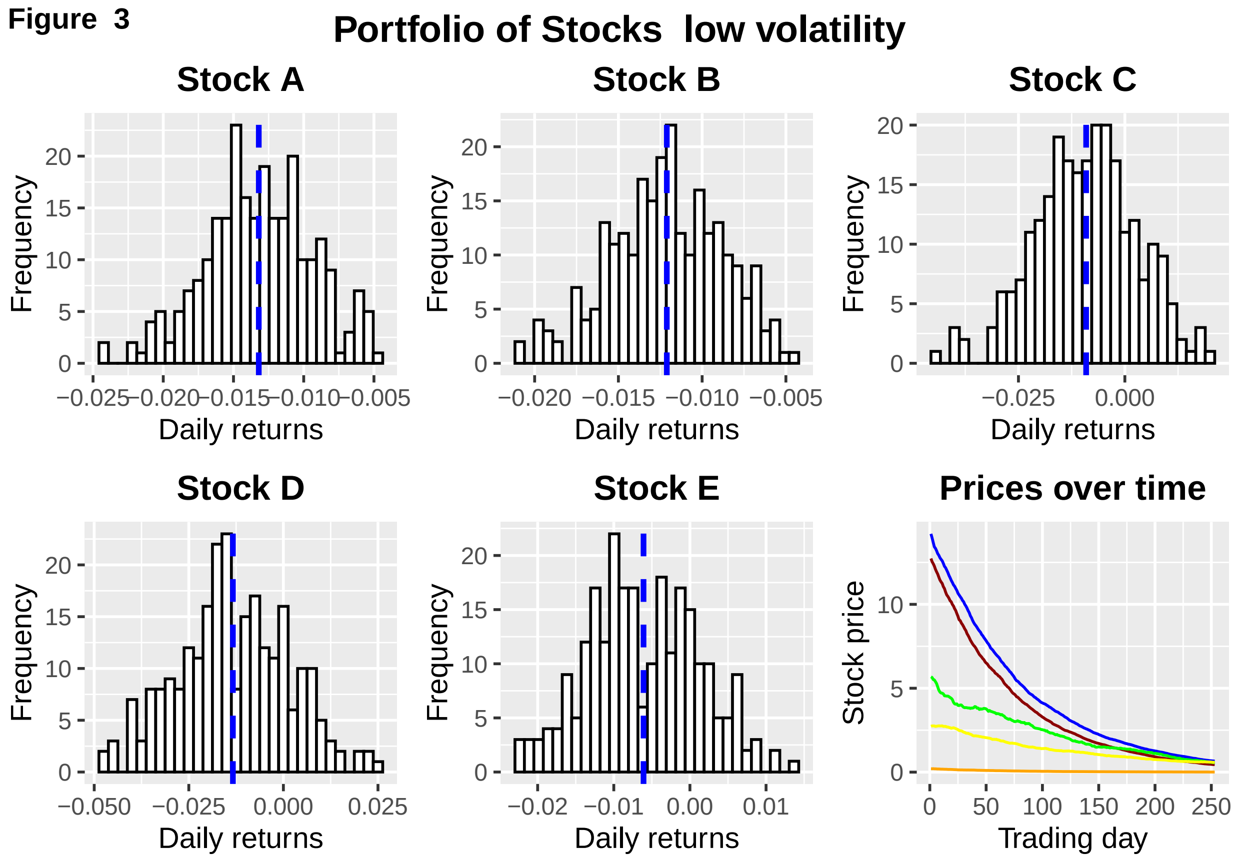
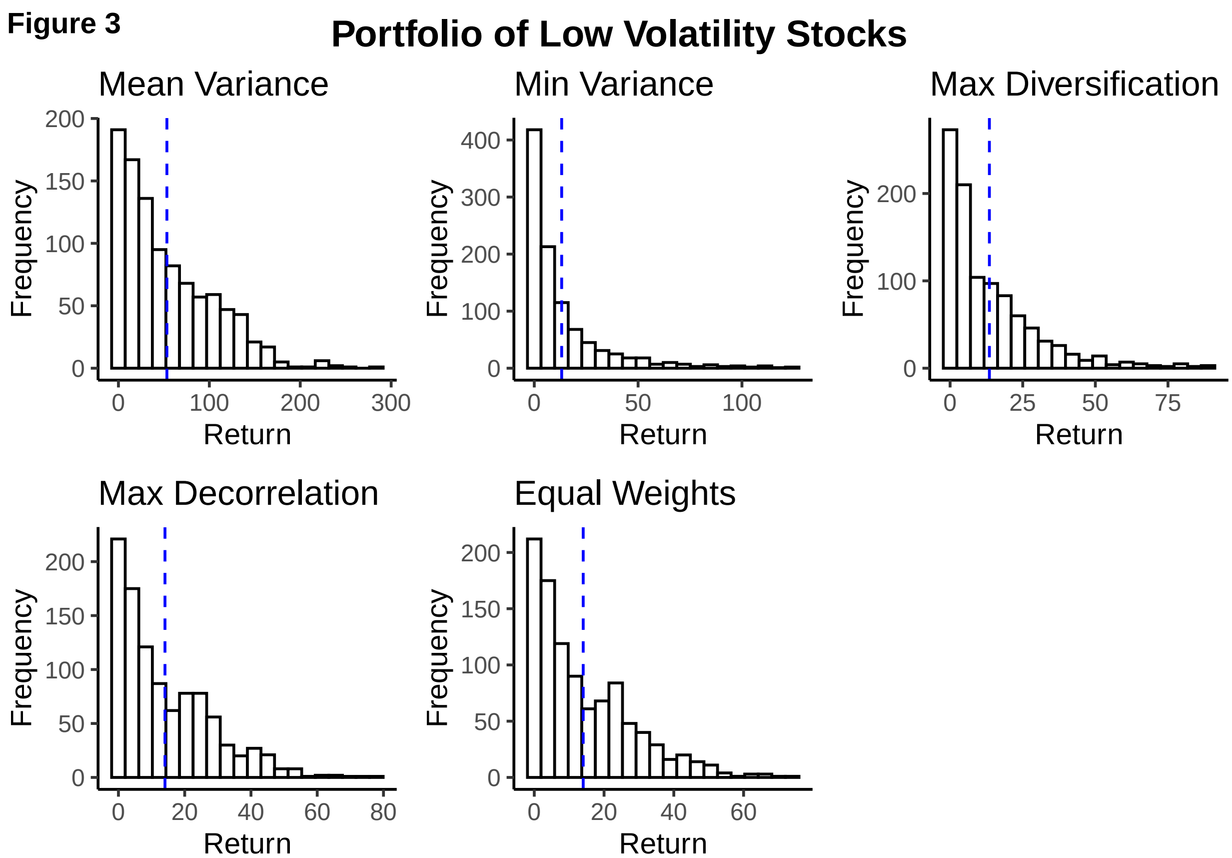
Figure 3


Portfolio of Stocks low volatility
Stock A
Stock B
Stock C
Stock D
Stock E
Prices over time
Frequency
Daily returns
Stock price
Trading day
−0.025 −0.020 −0.015 −0.010 −0.005
−0.020 −0.015 −0.010 −0.005
−0.025 0.000
−0.050 −0.025 0.000 0.025
−0.02 −0.01 0.00 0.01
0 50 100 150 200 250
0 5 10 15 20



Scenario 3: All Normally Distributed Stock Returns with high volatility

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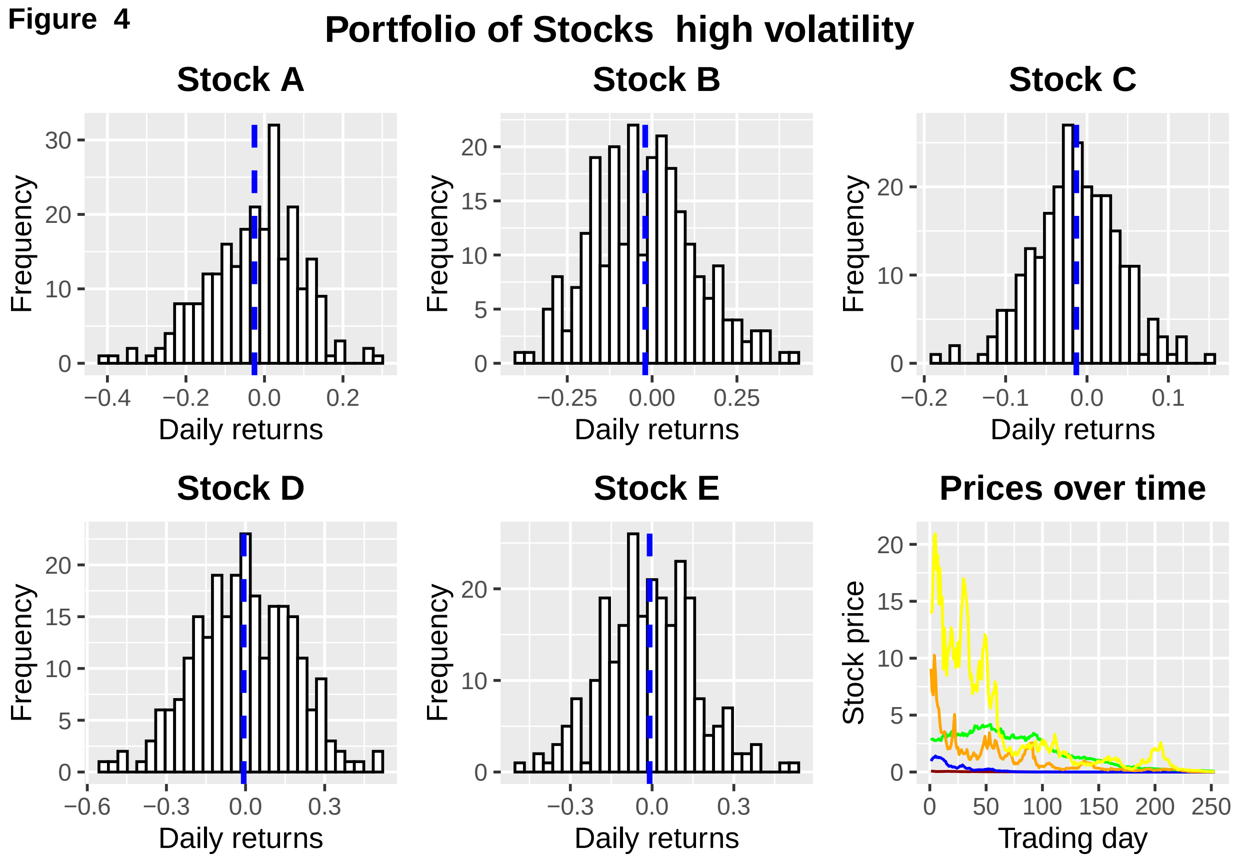
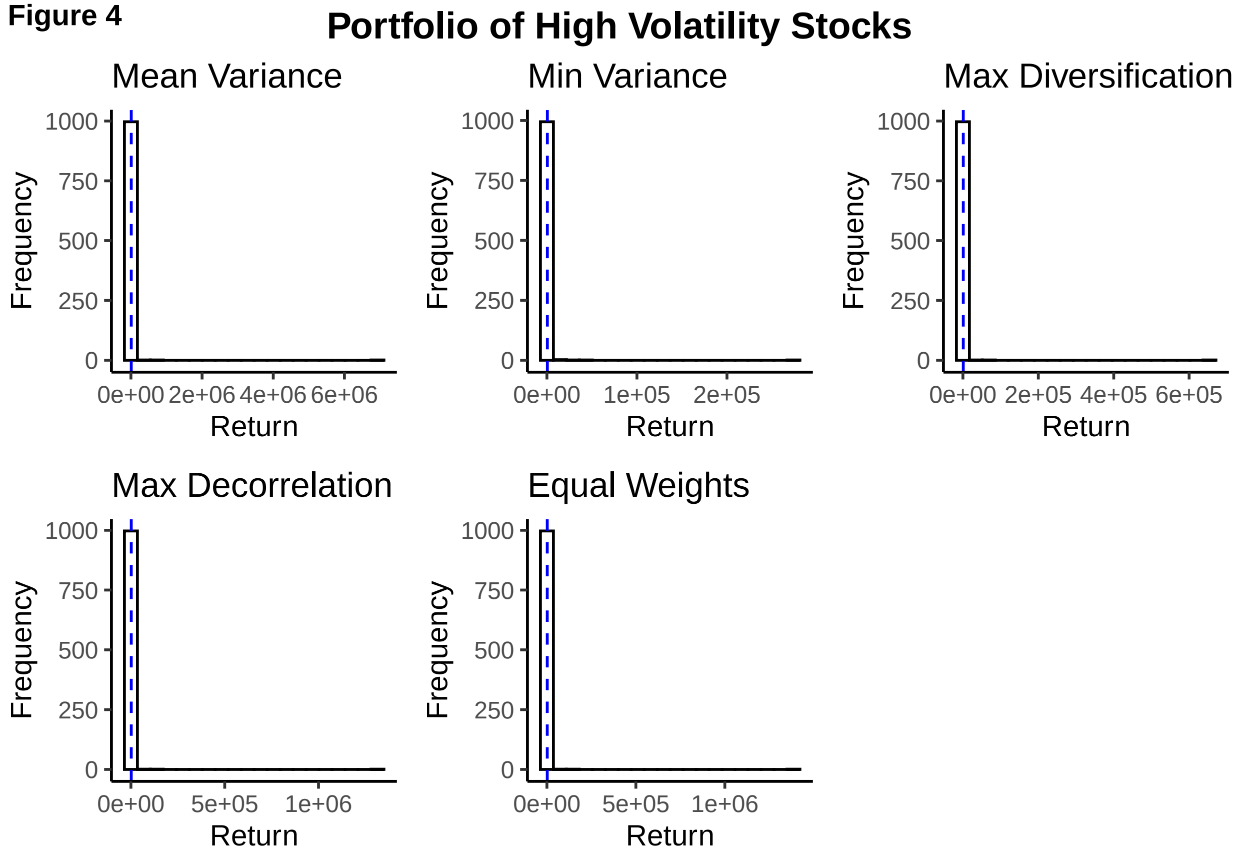


Figure 4
Portfolio of Stocks high volatility
Stock A
Stock B
Stock C
Stock D
Stock E
Prices over time
Frequency
Daily returns
Stock price
Trading day
30
20
15
10
5
0
−0.4
−0.2
0.0
0.2
−0.25
0.00
0.25
−0.1
0.1
−0.6
−0.3
0.3
50
100
150
200
250

Figure 4



Scenario 4: Mixed distributed Stock Returns

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Figure 2

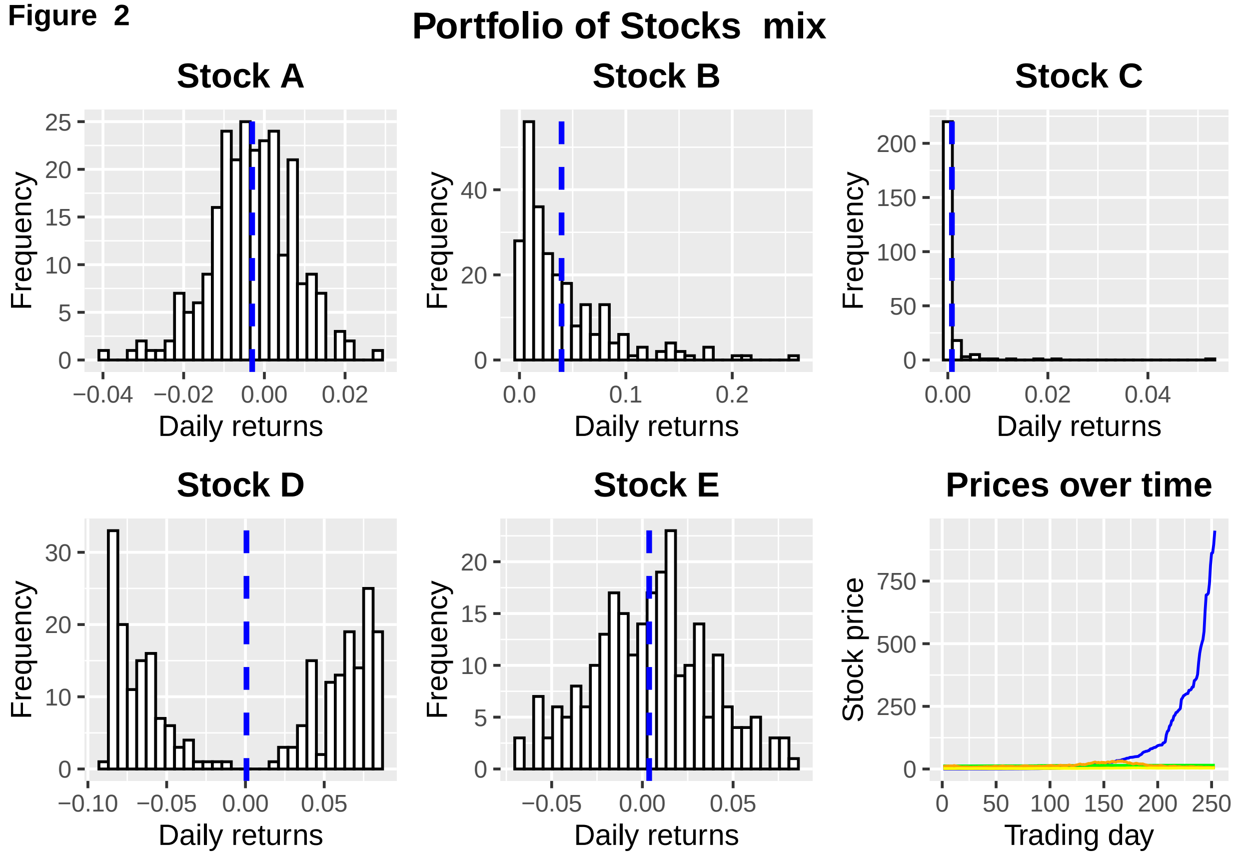
Portfolio of Stocks mix
Stock A
Stock B
Stock C
Stock D
Stock E
Prices over time
Frequency
Daily returns
Stock price
Trading day

Table 1: Table 1: Portfolio of All Normal Stocks

	Return (%)	Variance (%)	Sharpe	Lower CI	Upper CI
Mean Variance	5709.072	25.3679	141.42784	5321.090	6097.054
Min Variance	1463.656	3.1273	89.97104	1347.456	1579.855
Max Diversification	1478.492	3.5649	84.13362	1373.827	1583.156
Max Decorrelation	1501.562	5.2390	68.33846	1395.068	1608.056
Equal Weights	1496.801	5.2716	67.88538	1391.815	1601.787

Table 2: Table 2: Portfolio of Mixed Distribution Stocks

	Return (%)	Variance (%)	Sharpe	Lower CI	Upper CI
Mean Variance	303537.81	24.4473	5223.228	259051.47	348024.16
Min Variance	30972.20	2.8523	1692.363	27341.00	34603.41
Max Diversification	44287.14	3.3907	2409.752	39181.36	49392.93
Max Decorrelation	61827.96	6.6956	2377.325	52773.15	70882.77
Equal Weights	61968.90	6.7090	2378.796	53072.82	70864.98

Figure 2

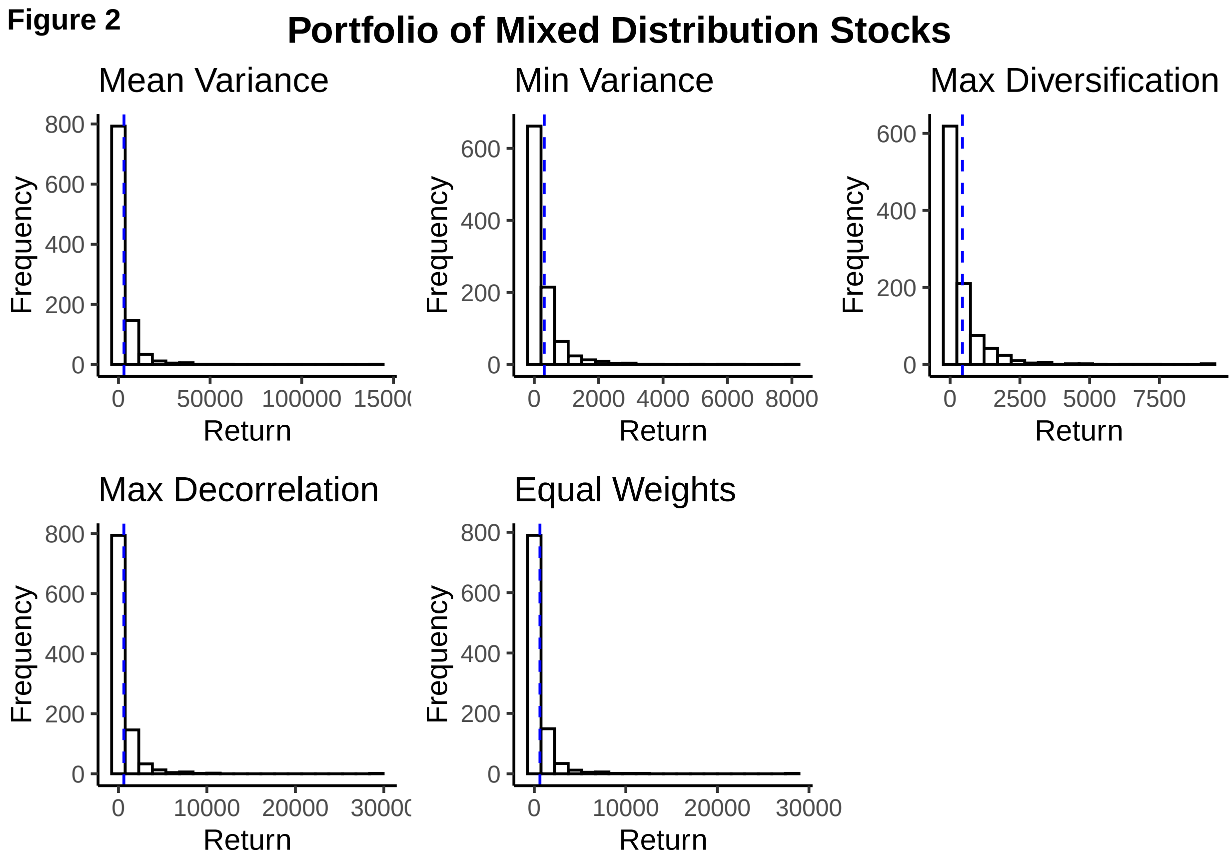

Table 3: Table 3: Portfolio of Low Volatility Stocks

	Return (%)	Variance (%)	Sharpe	Lower CI	Upper CI
Mean Variance	5329.661	3.6028	511.3011	5021.916	5637.407
Min Variance	1319.318	0.2566	377.7116	1194.238	1444.398
Max Diversification	1352.597	0.3207	305.6494	1253.760	1451.433
Max Decorrelation	1404.413	0.6935	179.5355	1318.130	1490.696
Equal Weights	1402.633	0.6983	178.9473	1316.452	1488.814

Table 4: Table 4: Portfolio of High Volatility Stocks

	Return (%)	Variance (%)	Sharpe	Lower CI	Upper CI
Mean Variance	932880.08	613.8309	2570.8555	-436548.472	2302308.6
Min Variance	49291.47	77.8187	591.1767	-5441.578	104024.5
Max Diversification	99595.38	88.5490	1149.8546	-30032.850	229223.6
Max Decorrelation	182517.61	129.4078	1632.5315	-77600.349	442635.6
Equal Weights	188498.80	130.3586	1620.9829	-85397.959	462395.6

Comparison of Portfolio Strategies

References

Choueifaty, Y., & Coignard, Y. (2008). Toward Maximum Diversification. *The Journal of Portfolio Management*, 35(1), 40-51. doi:10.3905/jpm.2008.35.1.40

Clarke, R., Silva, H. D., & Thorley, S. (2010). Minimum-Variance Portfolio Composition. *The Journal of Portfolio Management*, 101116223821055. doi:10.3905/jpm.2010.2010.1.009

Clarke, R., Silva, H. D., & Thorley, S. (2012). Risk Parity, Maximum Diversification, and Minimum Variance: An Analytic Perspective. *SSRN Electronic Journal*. doi:10.2139/ssrn.1977577

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