In[43]:=

In[44]:=
```
k1 = Mod[441 + 380 + 305 + 379 , 2]
k2 = Mod[441 + 380 + 305 + 379 , 2 ^ 2]
k3 = Mod[441 + 380 + 305 + 379 , 2 ^ 3]
```
Out[44]= 1

Out[45]= 1

Out[46]= 1

In[47]:=
```
θ0 = π / k2
ϕ0 = 2 π / k3
```
Out[47]= π

Out[48]= 2π

In[49]:=
```
(Ket[1] = {{0}, {1}} ) // MatrixForm
(Ket[0] = {{1}, {0}}) // MatrixForm
```
Out[49]//MatrixForm=

$$\begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

Out[50]//MatrixForm=

$$\begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

In[51]:=
```
(u1[ϕ_] = {{1, 0}, {0, e ^(ı ϕ )}}) // MatrixForm
(u3[θ_, 0, 0] = {{Cos[θ / 2], - Sin[θ / 2]}, {Sin[θ / 2], Cos[θ / 2]}}) // MatrixForm
Bra[ı_] := Simplify [Ket[ı]†]
```
Out[51]//MatrixForm=

$$\begin{pmatrix} 1 & 0 \\ 0 & e^{\imath \phi} \end{pmatrix}$$

Out[52]//MatrixForm=

$$\begin{pmatrix} \mathrm{Cos}\left[\frac{\theta}{2}\right] & -\mathrm{Sin}\left[\frac{\theta}{2}\right] \\ \mathrm{Sin}\left[\frac{\theta}{2}\right] & \mathrm{Cos}\left[\frac{\theta}{2}\right] \end{pmatrix}$$

In[54]:=
```
(u1[ϕ0].u3[θ0, 0, 0].Ket[1]) // MatrixForm
```
Out[54]//MatrixForm=

$$\begin{pmatrix} -1 \\ 0 \end{pmatrix}$$

```
In[55]:= pxd[0 x] = 0.49125
         pxd[1 x] = 0.50875
         pyd[0 y] = 0.49925
         pyd[1 y] = 0.50075
         pzd[0 z] = 1.0
         pzd[1 z] = 0.0

Out[55]= 0.49125

Out[56]= 0.50875

Out[57]= 0.49925

Out[58]= 0.50075

Out[59]= 1.

Out[60]= 0.

In[61]:= Pxd = pxd[0 x] - pxd[1 x]
         Pyd = pyd[0 y] - pyd[1 y]
         Pzd = pzd[0 z] - pzd[1 z]

Out[61]= -0.0175

Out[62]= -0.0015

Out[63]= 1.

In[64]:= Pdn = {Pxd , Pyd , Pzd}

Out[64]= {-0.0175 , -0.0015 , 1.}

In[65]:= nPd2 = Pdn . Pdn

Out[65]= 1.00031

In[66]:= Pd = (1 / Sqrt[nPd2]) * Pdn

Out[66]= {-0.0174973 , -0.00149977 , 0.999846 }
```

In[67]:=
```
σx = PauliMatrix [1]
σy = PauliMatrix [2]
σz = PauliMatrix [3]
σ = {{σx}, {σy}, {σz}}
(σPd = Flatten [Pd.σ, 1]) // MatrixForm
```

Out[67]= {{0, 1}, {1, 0}}

Out[68]= {{0, -ⅈ}, {ⅈ, 0}}

Out[69]= {{1, 0}, {0, -1}}

Out[70]= {{{{0, 1}, {1, 0}}}, {{{0, -ⅈ}, {ⅈ, 0}}}, {{{1, 0}, {0, -1}}}}

Out[71]//MatrixForm=

$$\begin{pmatrix} 0.999846 + 0.\,i & -0.0174973 + 0.00149977\,i \\ -0.0174973 - 0.00149977\,i & -0.999846 + 0.\,i \end{pmatrix}$$

In[72]:=
```
(wwd = Eigensystem [σPd]) // MatrixForm
```

Out[72]//MatrixForm=

$$\begin{pmatrix} -1. & 1. \\ \{-0.00874899 + 0.000749913\,i, -0.999961 + 0.\,i\} & \{0.996308 - 0.0853978\,i, -0.00878107 + 0.\,i\} \end{pmatrix}$$

In[73]:=
```
(Ket[ψnd] = {{wwd[[2, 2, 1]]}, {wwd[[2, 2, 2]]}}) // MatrixForm
```

Out[73]//MatrixForm=

$$\begin{pmatrix} 0.996308 - 0.0853978\,i \\ -0.00878107 + 0.\,i \end{pmatrix}$$

In[74]:=
```
normad2 = (Flatten [Chop [Bra[ψnd].Ket[ψnd]]][[1]])
```

Out[74]= 1.

In[75]:=
```
(Ket[ψud] = (1 / Sqrt [normad2 ]) * Ket[ψnd]) // MatrixForm
```

Out[75]//MatrixForm=

$$\begin{pmatrix} 0.996308 - 0.0853978\,i \\ -0.00878107 + 0.\,i \end{pmatrix}$$

In[76]:=
```
C0 = (Ket[ψud][[1, 1]]);
C1 = (Ket[ψud][[2, 1]]);
ϕ0 = Arg[C0];
ϕ1 = Arg[C1];
mC0 = Abs[C0];
mC1 = Abs[C1];
ϕw = ϕ1 - ϕ0
```

Out[82]= 3.2271

In[83]:=
```
(Ket[ψd] = Chop[mC0 * Ket[0] + (ℯ^(Defer[ⅈ] ϕw)) * mC1 * Ket[1]]) // MatrixForm
(Ket[ψd]) // MatrixForm
```
$$\begin{pmatrix} 0.9999614456768875\grave{} \\ 0.008781068260154172\grave{}\ \ e^{3.227097947267998\grave{}\, i} \end{pmatrix} \text{ // N // MatrixForm}$$

Out[83]//MatrixForm=

$$\begin{pmatrix} 0.999961 \\ 0.00878107\ \ e^{3.2271 i} \end{pmatrix}$$

Out[84]//MatrixForm=

$$\begin{pmatrix} 0.999961 \\ 0.00878107\ \ e^{3.2271 i} \end{pmatrix}$$

Out[85]//MatrixForm=

$$\begin{pmatrix} 0.999961 \\ -0.00874899 - 0.000749913\ i \end{pmatrix}$$