

Tutorial



Mental models as probabilistic programs



Kevin Smith

Kelsey Allen

August 15, 2018

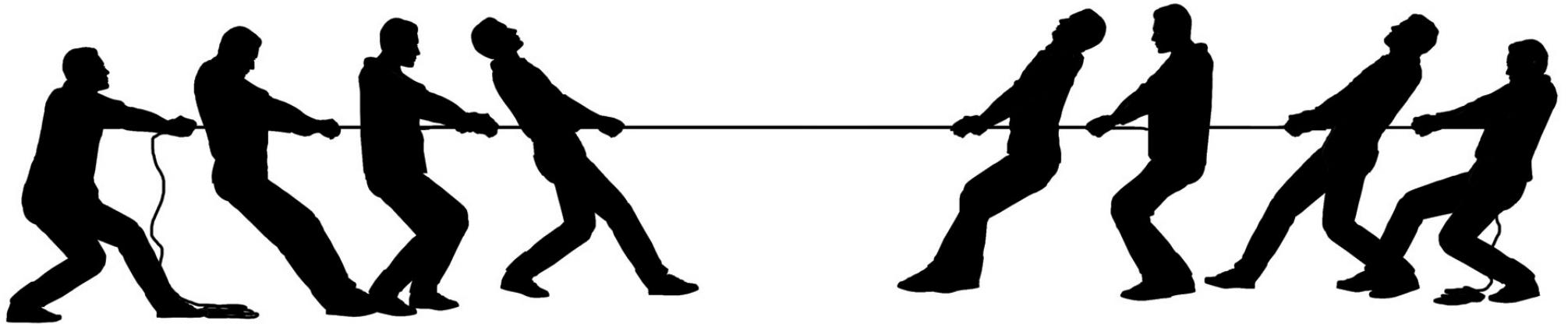
Thanks to Tobias Gerstenberg for the slides!



He's building his lab!

<http://cicl.stanford.edu/>

Tug of War

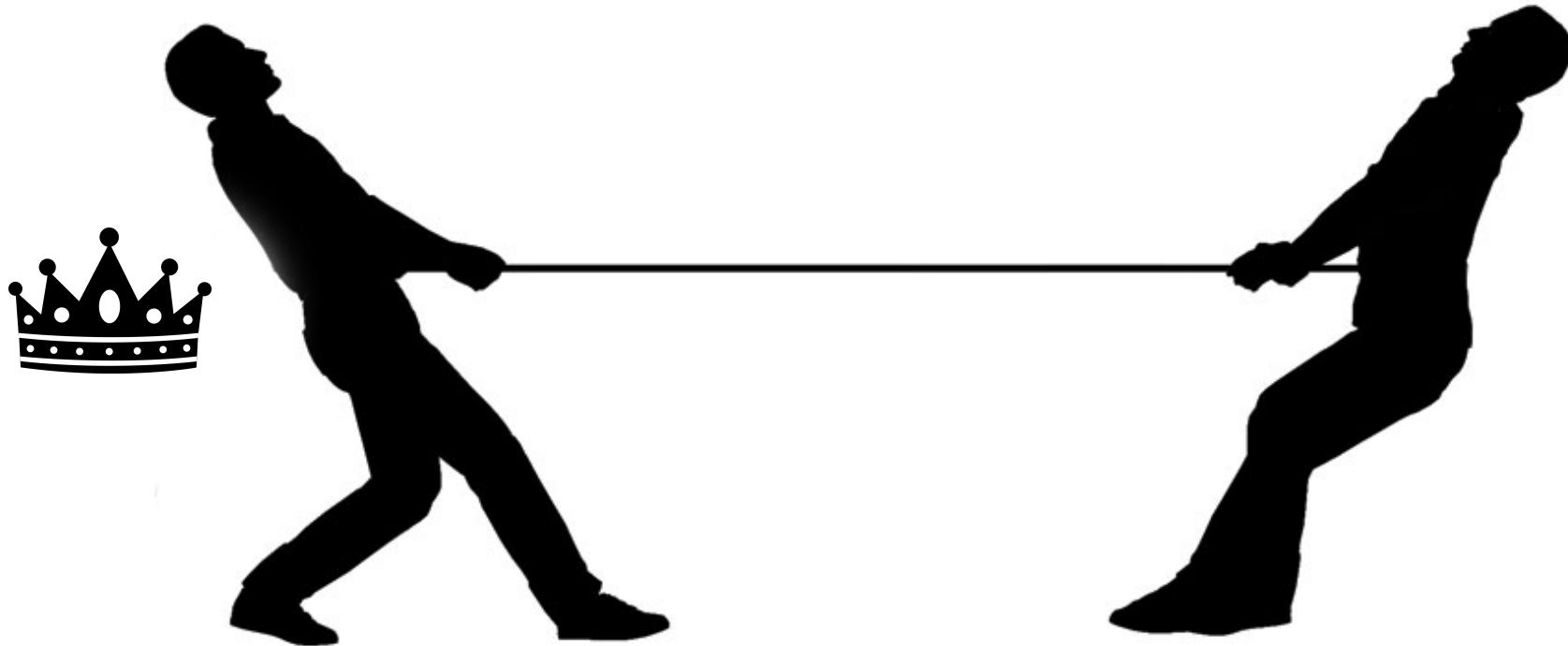




Tom

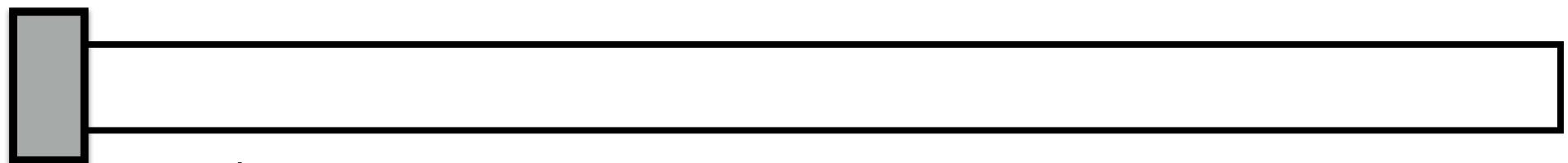


Steve



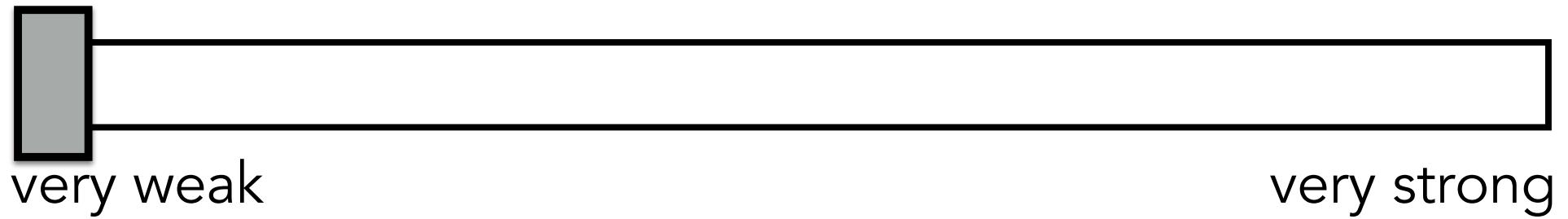
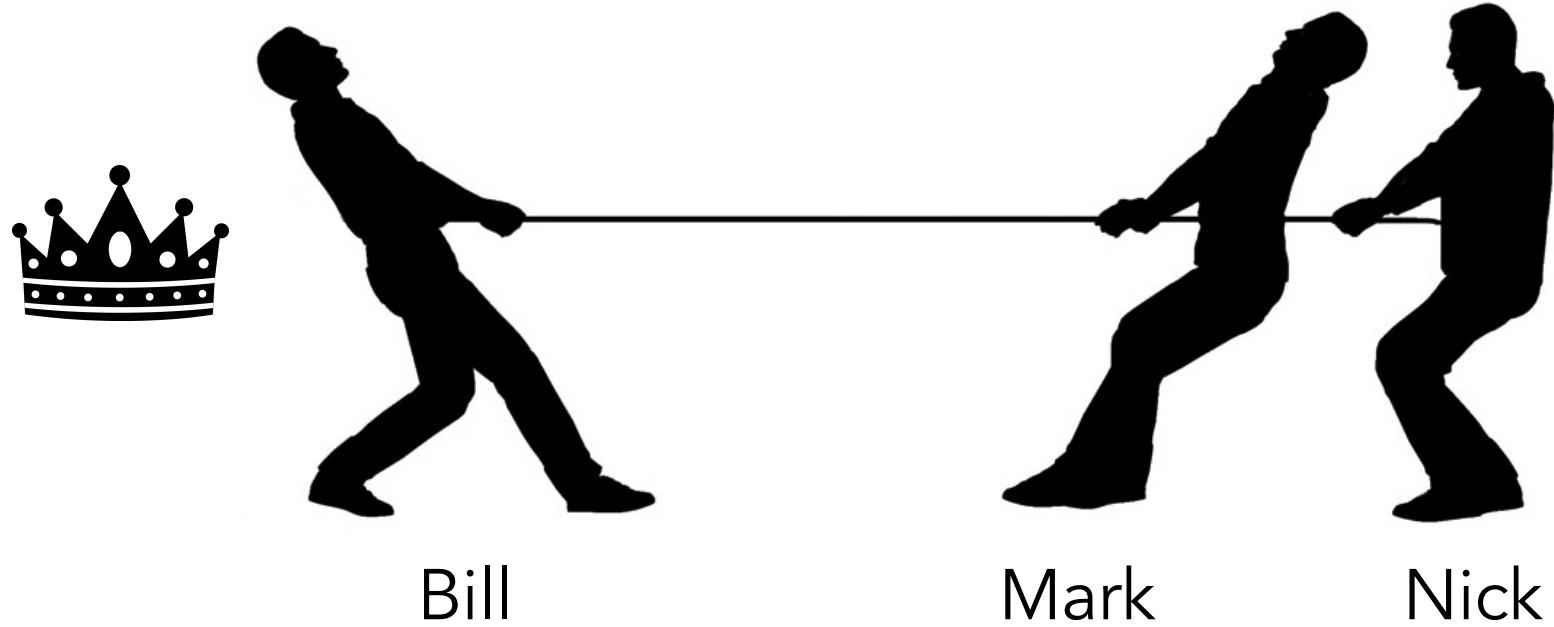
Tom

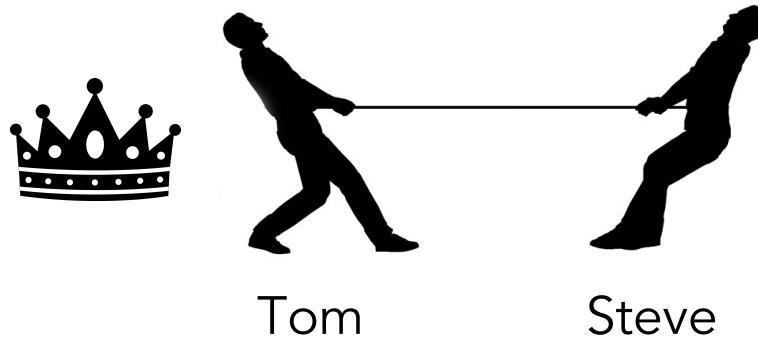
Steve



very weak

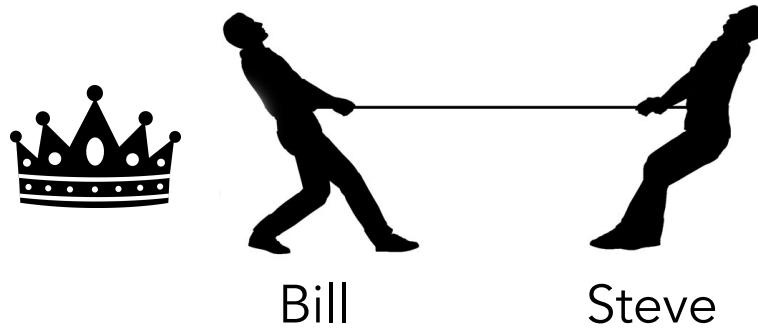
very strong





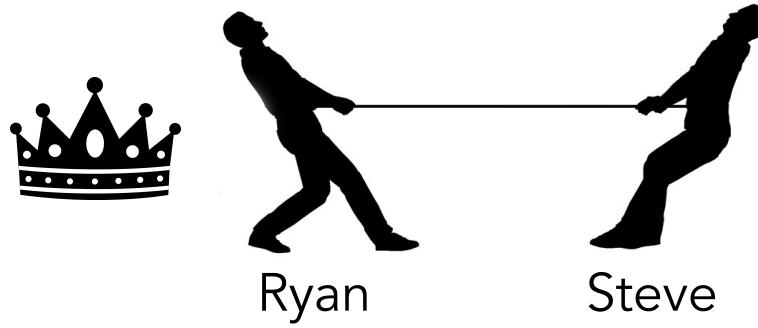
Tom

Steve



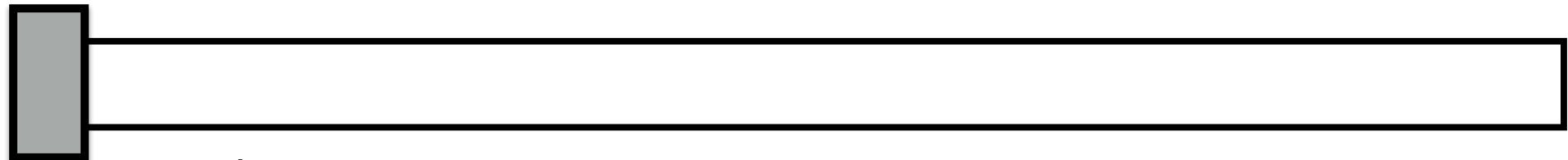
Bill

Steve



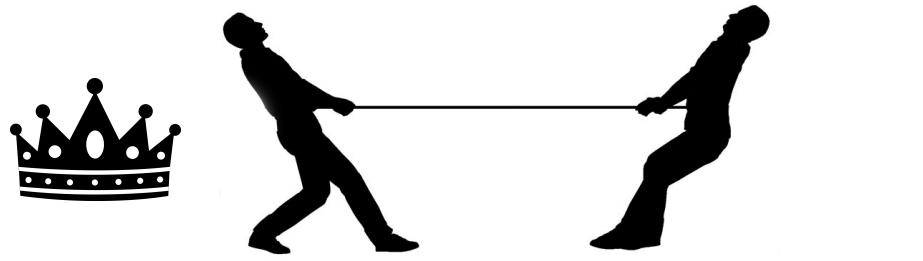
Ryan

Steve



very weak

very strong



Tom

Steve



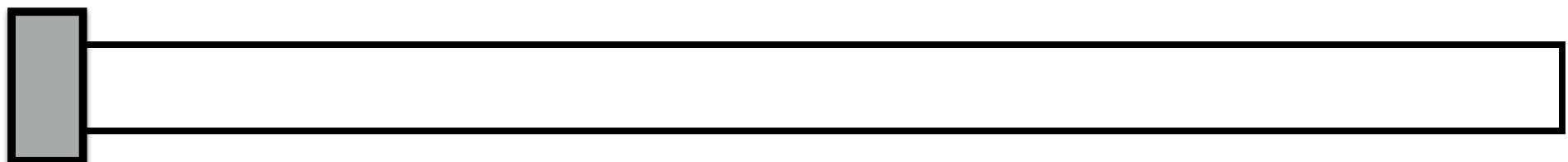
Bill

Steve



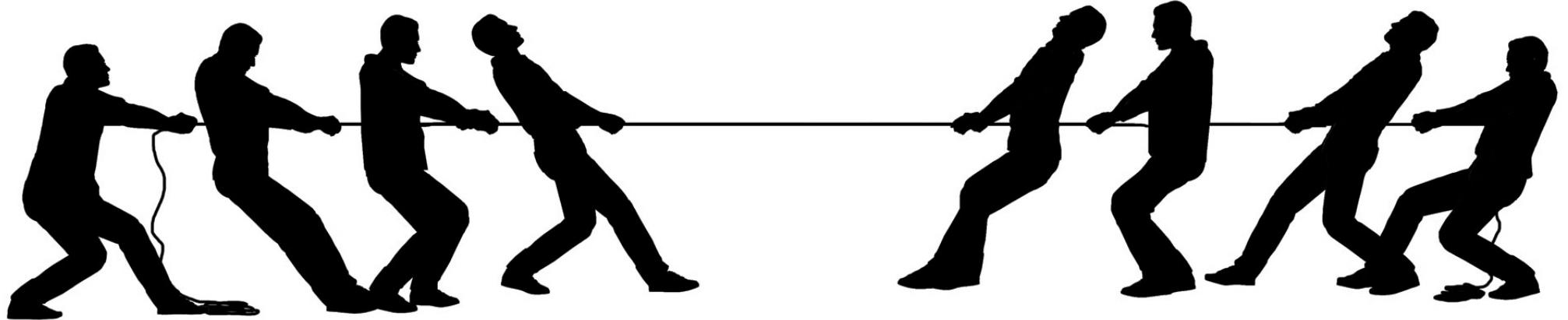
Ryan

Steve



very weak

very strong



Concepts:

teams

person

winner

strength

pulling

effort

A computational model of tug of war

```
var towModel = function() {
    var strength = mem(function (person) {return gaussian(50, 10)})
    var lazy = function(person) {return flip(0.1) }

    var pulling = function(person) {
        return lazy(person) ? strength(person) / 2 : strength(person) }

    var totalPulling = function (team) {return sum(map(pulling, team))}

    var winner = function (team1, team2) {
        totalPulling(team1) > totalPulling(team2) ? team1 : team2 }

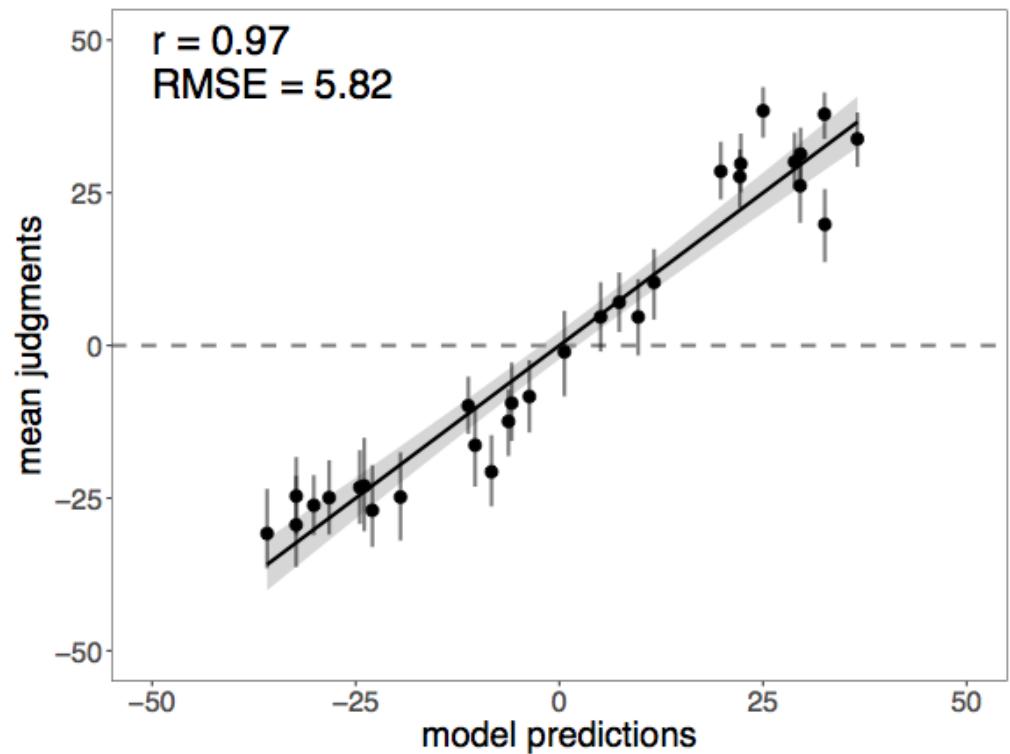
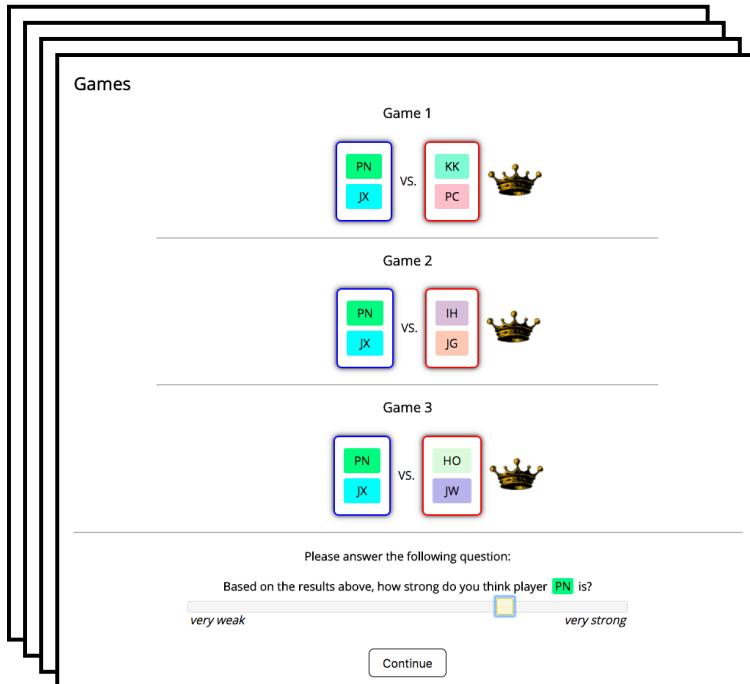
    var beat = function(team1, team2){winner(team1, team2) == team1}

    condition(beat(['bob'], ['tom']))

    return strength('bob')
}
```

Experiment

How strong is Player X?



Gerstenberg & Goodman (2012) Ping Pong in Church: Productive use of concepts in human probabilistic inference. *Cognitive Science Proceedings*

Gerstenberg & Tenenbaum (2017) Intuitive Theories. *Oxford Handbook of Causal Reasoning*

Goodman, Tenenbaum, & Gerstenberg (2015) Concepts in a probabilistic language of thought. *The Conceptual Mind: New Directions in the Study of Concepts*

Experiment

Did player X try hard in Game 1?

Games

Game 1

Kevin VS. Jason 

Game 2

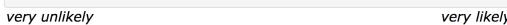
 Kevin VS. Jason

Game 3

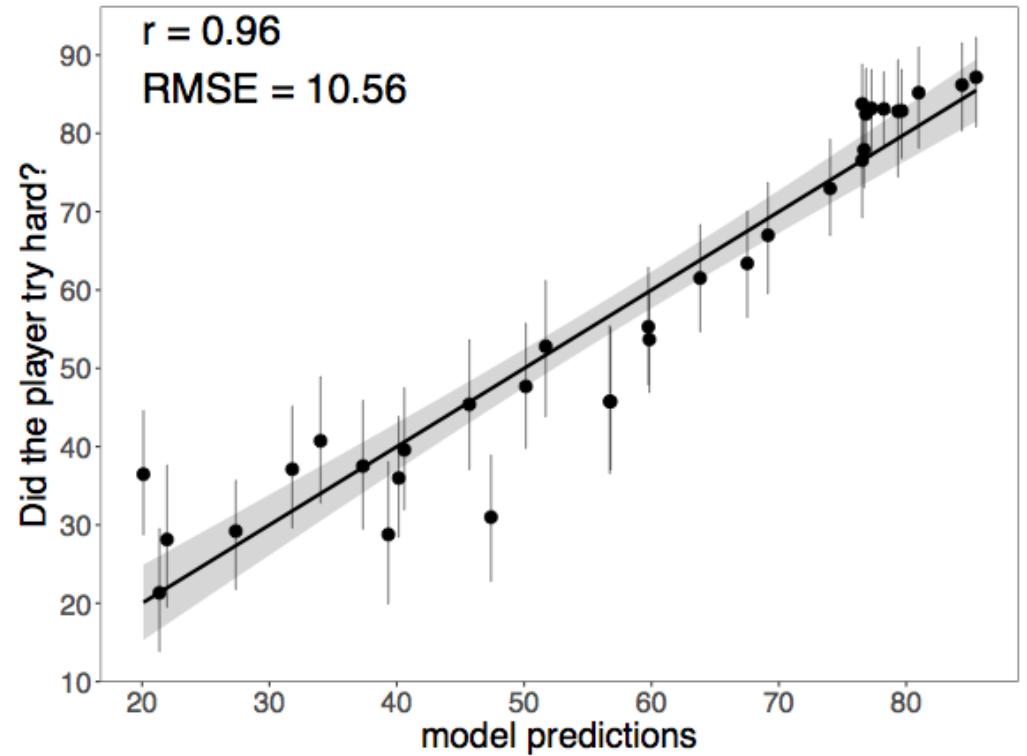
 Kevin VS. Jason

Please answer the following question:

How likely is it that **Jason** tried hard in Game 1?

very unlikely  very likely

Continue



Outline

- What is thinking?
 - What kind of a program?
 - Probabilistic language of thought hypothesis
 - Compositionality
 - Productivity
 - Inference
- **Do it yourself!**
 1. WebPPL basics
 2. Building generative models
 3. Doing inference
- Questions, Resources, some cool examples

What is thinking?



How can we describe the intelligent
inferences made in everyday human
reasoning and learning?



How can we engineer
intelligent machines?

What is thinking?

Computational theory of mind



mind = computer

```
a.length;c++) {    0 ==  
& b.push(a[c]);    }  ret  
function h() {  for (var  
#User_logged").a(), a = q()  
place(/ +(?= )/g, "", a =  
, b = [], c = 0;c < a.length;  
0 == r(a[c], b) && b.push(c);  
}  c = {};  
= b.length - 1;  
= a();  b = q();  
}
```

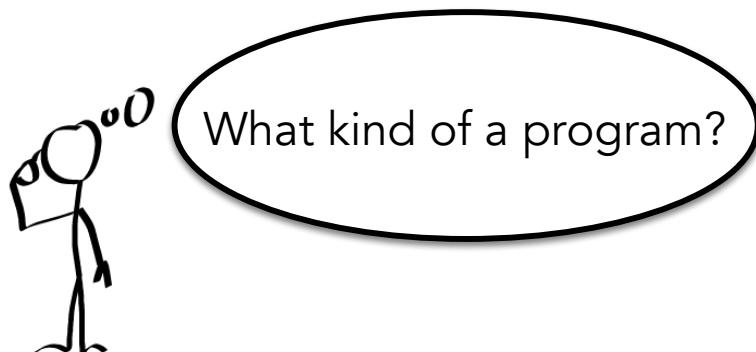
run(program)

mental representations =

computer programs

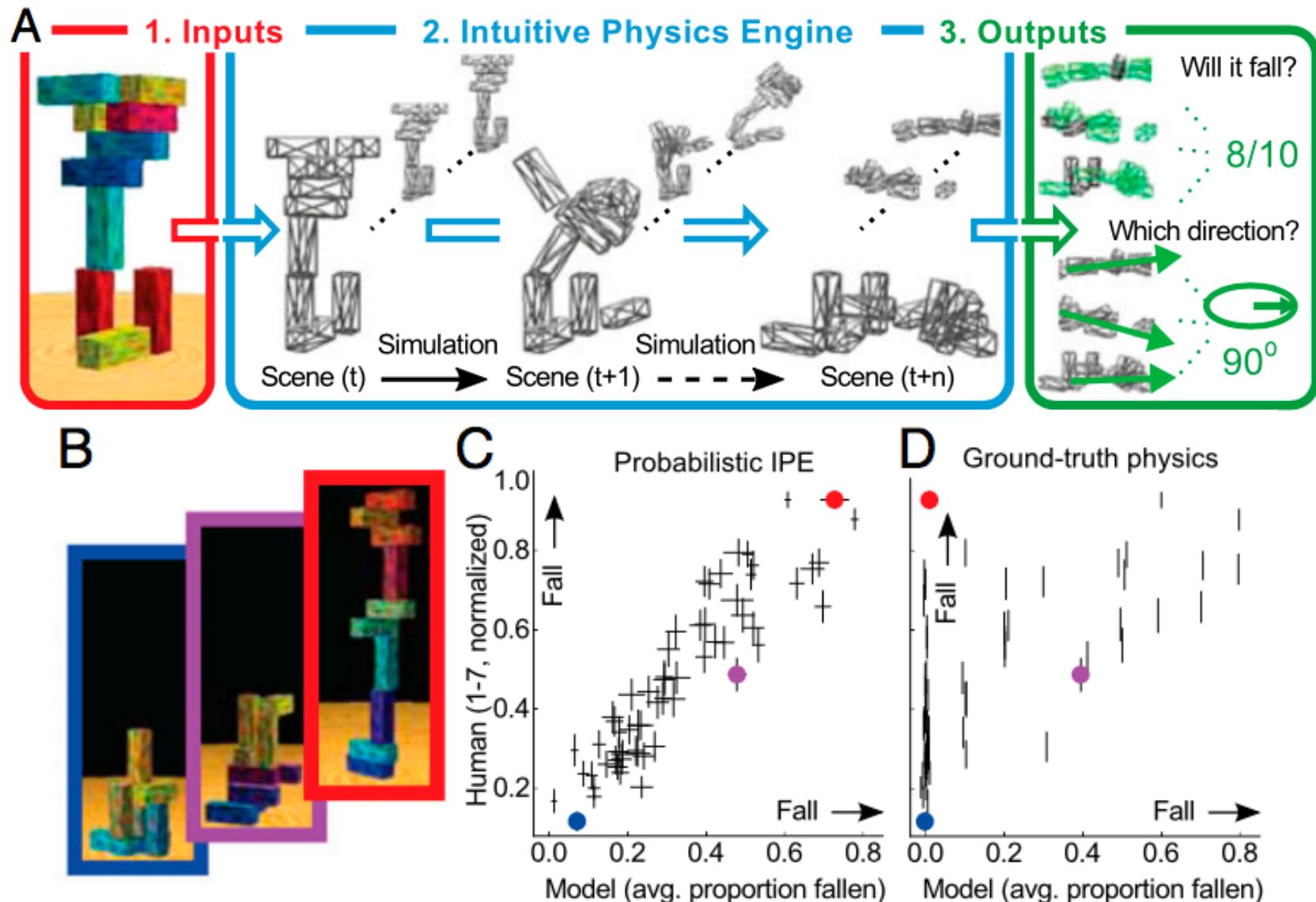
thinking =

running a program



What kind of a program?

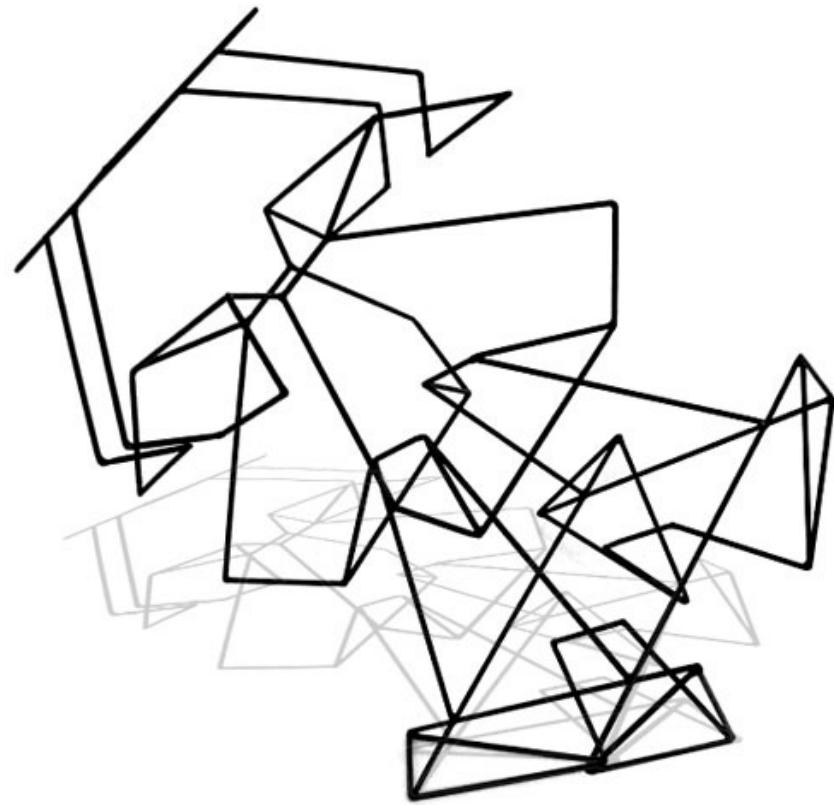
Working models of a domain



Battaglia, Hamrick, & Tenenbaum (2013) Simulation as an engine of physical scene understanding.
Proceedings of the National Academy of Sciences

What kind of a program?

Structure



Knowledge

Probability



Uncertainty

Probabilistic language of thought hypothesis

Informal version:

“Concepts have a language-like compositionality and encode probabilistic knowledge. These features allow them to be extended productively to new situations and support flexible reasoning and learning by probabilistic inference.”

Fodor (1975). The language of thought

Goodman, Tenenbaum, & Gerstenberg (2015) Concepts in a probabilistic language of thought. *The Conceptual Mind: New Directions in the Study of Concepts*

Probabilistic language of thought hypothesis

Informal version:

"Concepts have a language-like **compositionality** and encode probabilistic knowledge. These features allow them to be extended **productively** to new situations and support flexible reasoning and learning by **probabilistic inference**."

Compositionality

"A giraffe with a blue ribbon around its neck eats ice cream."



Productivity

"infinite use of finite means"
(Wilhelm von Humboldt)

Language

limited number of letters
fixed set of grammatical rules

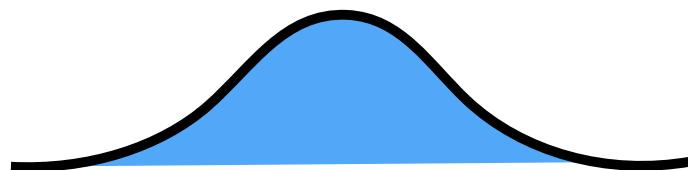
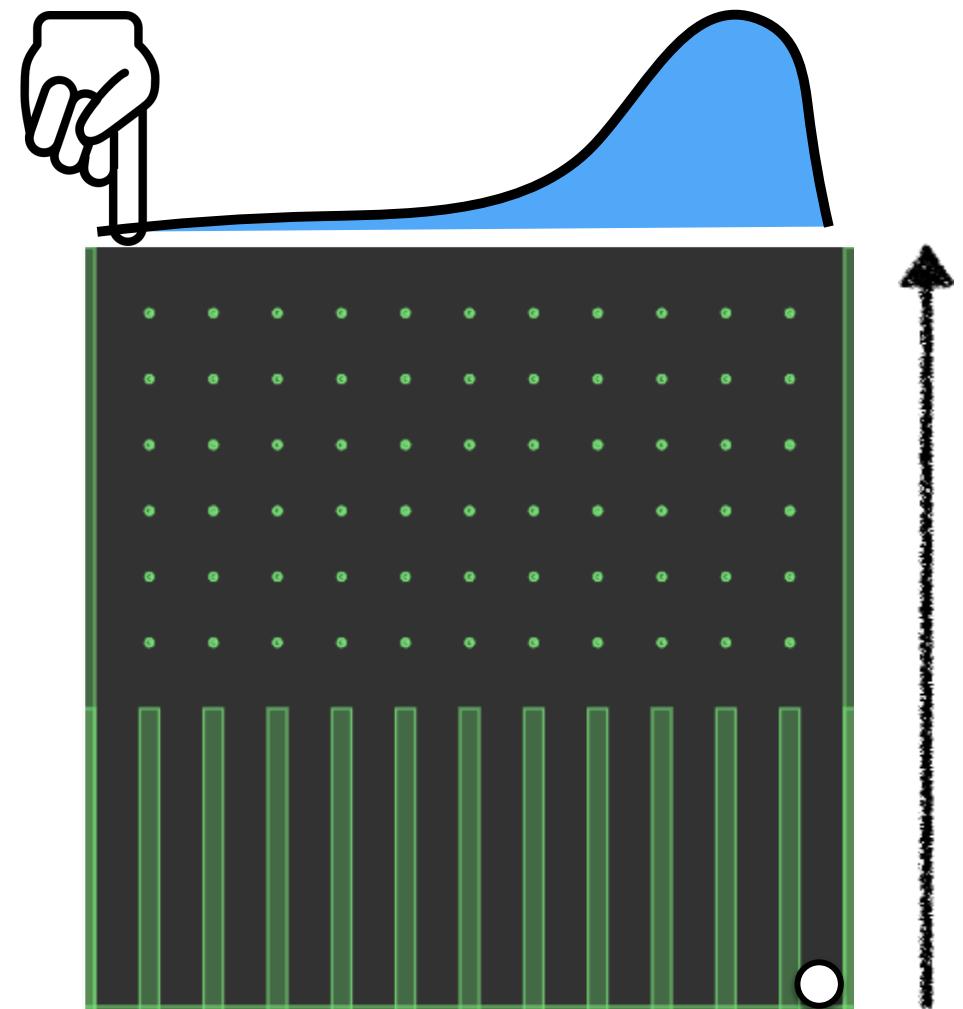
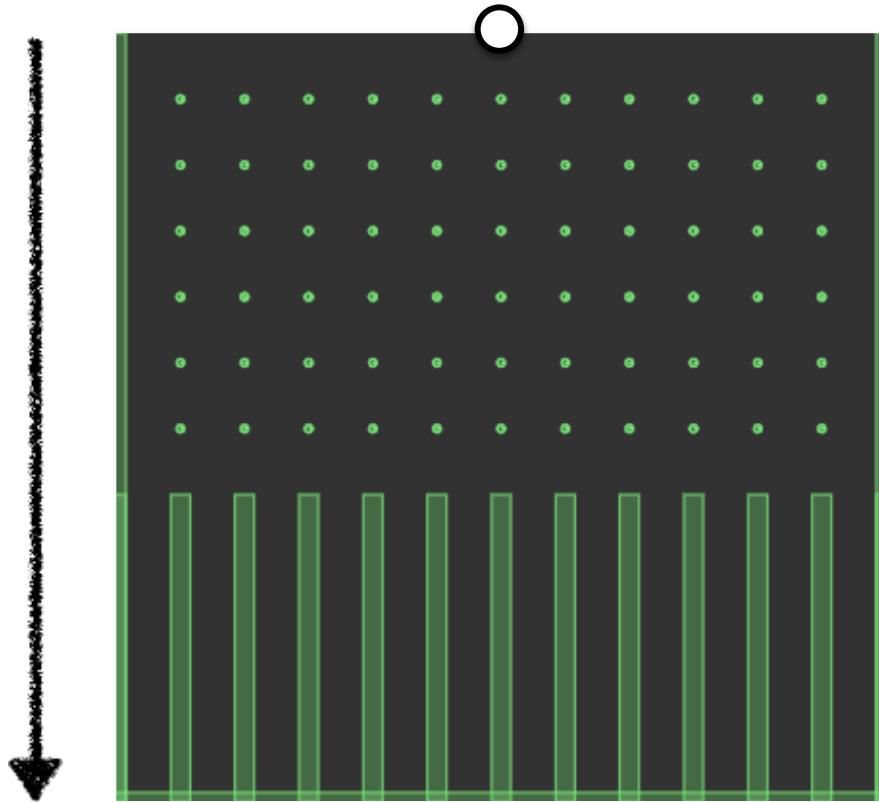
can express an infinite number of thoughts

Goodman, Tenenbaum, & Gerstenberg (2015) Concepts in a probabilistic language of thought. *The Conceptual Mind: New Directions in the Study of Concepts*

Probabilistic inference

Run forward

Where will the ball land?



Reason backward
Where did the ball come from?

"Where is this in the brain?"



```
var towModel = function() {
  var strength = mem(function (person) {return gaussian(50, 10)})

  var lazy = function(person) {return flip(0.1) }

  var pulling = function(person) {
    return lazy(person) ? strength(person) / 2 : strength(person) }

  var totalPulling = function (team) {return sum(map(pulling, team))}

  var winner = function (team1, team2) {
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  var beat = function(team1,team2){winner(team1,team2) == team1}

  condition(beat(['bob'], ['tom']))

  return strength('bob')
}
```



Probabilistic programs

D
O
I
T
Y
OURSELF

Why WebPPL?

WebPPL

- written by cognitive scientists for cognitive scientists
- goal: building computational models of cognition
- extremely flexible
- can be slow

other PPLs

- Stan, Anglican, Alchemy, BUGS, Edward, PyMC
- goal: building rich models for data analysis
- flexible but limited
- faster (for the models that can be expressed)

1. WebPPL basics
2. Building generative models
3. Doing inference

<http://bit.do/webppl>

→ [notes/1_webppl_basics.md](#)

→ <http://webppl.org>

1. WebPPL basics

WebPPL basics

- declare variables
- data formats: numbers, strings, logicals, objects, arrays
- if-then-statements
- defining functions
- higher order functions
 - `map`

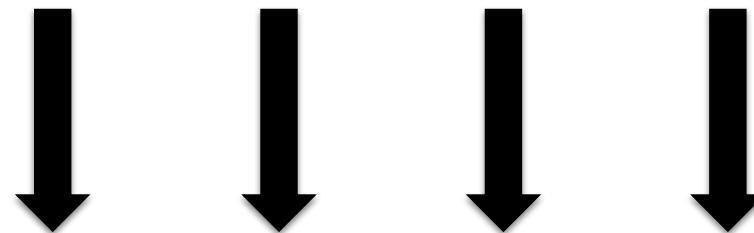
WebPPL basics

- The map function

```
var mySquare = function(x){return x*x}  
var someNumbers = [1,2,3,4]  
display(map(mySquare, someNumbers))
```

In:

```
[ 1, 2, 3, 4 ]
```



Out:

```
[ 1, 4, 9, 16 ]
```

WebPPL basics



- WebPPL = purely functional programming language
 - **can't** write `for` loops or `while` statements
 - **can** create higher-order functions and recursive functions
 - for example: `map` = apply a function to each element of a list (like a `for` loop)

2. Building generative models

Building generative models

- forward sampling and random primitives
- building simple models
- sample from probability distributions
- memoization: `mem`
- recursive functions

Building generative models



- WebPPL is a language to formally describe how the world works
- random choices capture our uncertainty or ignorance
- the language is **universal**: it can express any computable process
- causal dependence is important: the program describes what influences what

Stuhlmüller, Tenenbaum, & Goodman (2010) Learning Structured Generative Concepts. Cognitive Science Proceedings

Stuhlmüller & Goodman (2014) Reasoning about reasoning by nested conditioning: Modeling theory of mind with probabilistic programs. Cognitive Systems Research

Building generative models

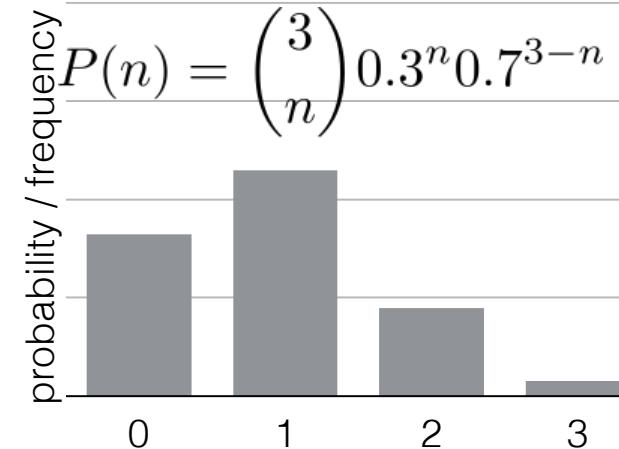


relationship between sampling and probability distributions

Random primitives:

```
var a = flip(0.3)
var b = flip(0.3)
var c = flip(0.3)
return a + b + c
```

=> 1 0 0
=> 0 0 0 ...
=> 1 0 1
=> 2 0 1



Sampling

≈



Distributions

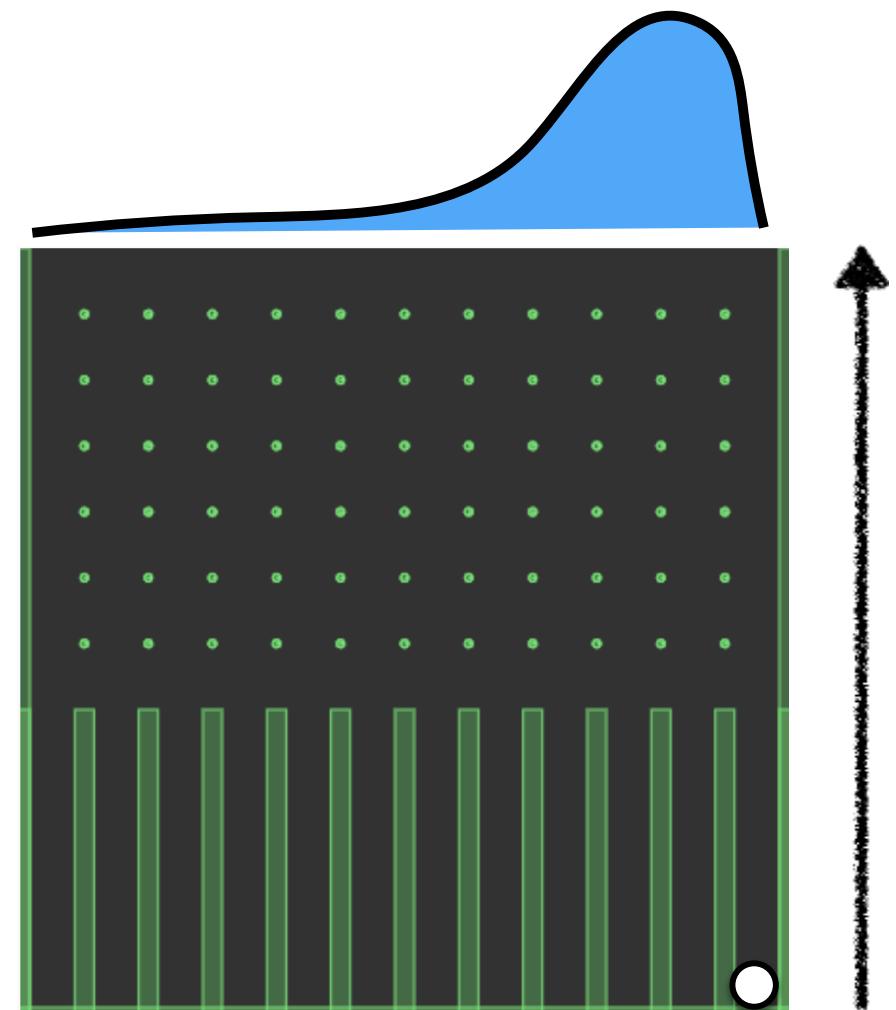
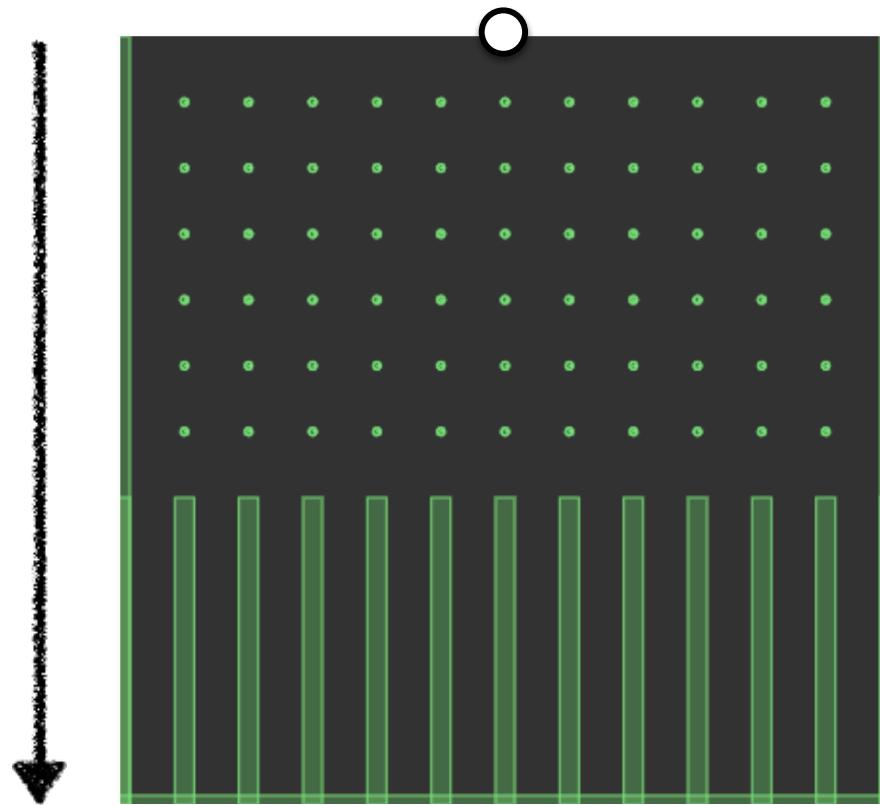
any computable distribution can be represented as the distribution induced by sampling from a probabilistic program

3. Doing inference

Doing inference

Run forward

Where will the ball land?



Reason backward
Where did the ball come from?

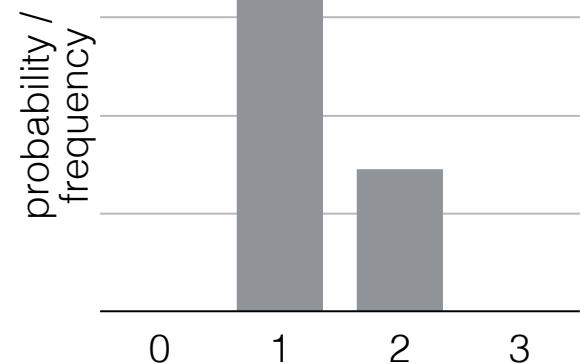
Doing inference

Conditional inference:

```
Infer(  
  function(){  
    var a = flip(0.3)  
    var b = flip(0.3)  
    var c = flip(0.3)  
    condition(a + b == 1)  
    return a + b + c})
```

=> 1 0 0 1
=> 0 0 0 0
=> 1 0 1 0
=> T F F T
=> 2 0 1 1

Posterior distribution



“It is an old maxim of mine that when you have excluded the impossible, whatever remains, however improbable, must be the truth.”



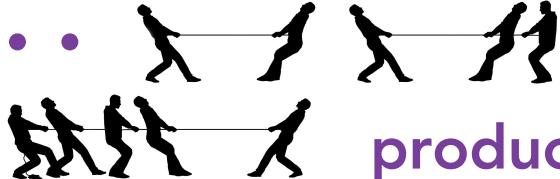
Doing inference



- conditioning on variables
 - rejection sampling
 - WebPPL's inference procedures
- conditioning on arbitrary expressions
- other inference procedures
- `forward`, `rejection`, `enumerate`, `MH`, ...
- you don't have to worry about inference. **very nice!**
- WebPPL allows us to parsimoniously describe rich generative model structures and explore the inference patterns that emerge from the interaction of model and data

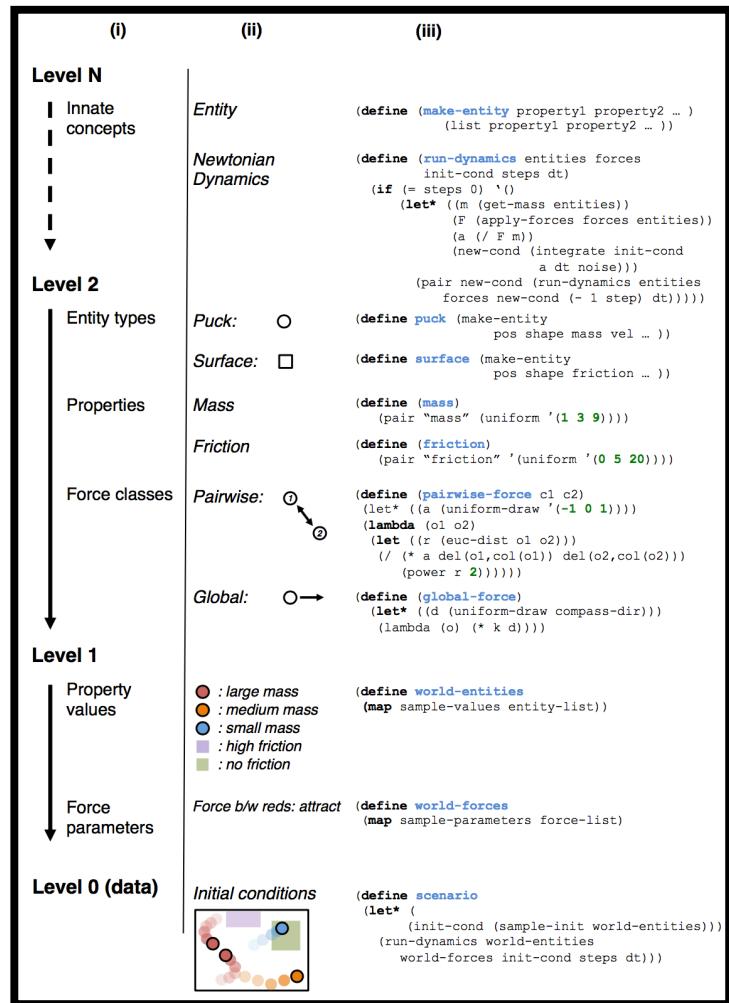
"Concepts have a language-like **compositionality** and encode probabilistic knowledge. These features allow them to be extended **productively** to new situations and support flexible reasoning and learning by **probabilistic inference**."

```
var towModel = function() {  
    var strength = mem(function (person) {return gaussian(50, 10)})  
  
    var lazy = function(person) {return flip(0.1)}  
    compositionality  
  
    var pulling = function(person) {  
        return lazy(person) ? strength(person) / 2 : strength(person)  
    }  
  
    var totalPulling = function (team) {return sum(map(pulling, team))}  
  
    var winner = function (team1, team2) {  
        totalPulling(team1) > totalPulling(team2) ? team1 : team2 }  
  
    var beat = function(team1, team2){winner(team1, team2) == team1}  
  
condition(beat(['bob'], ['tom'])) • • •  
  
return strength('bob') • • •  
}
```

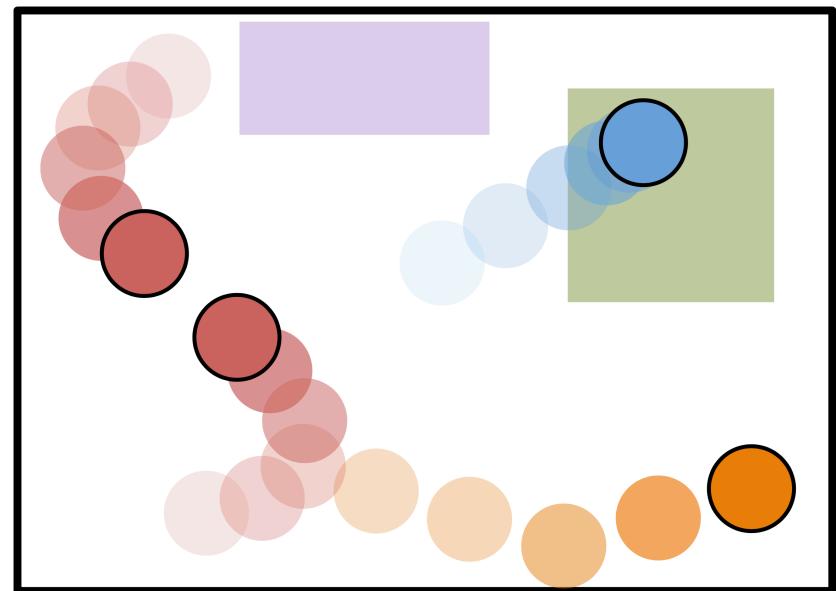


productivity

Some more cool examples: Program induction

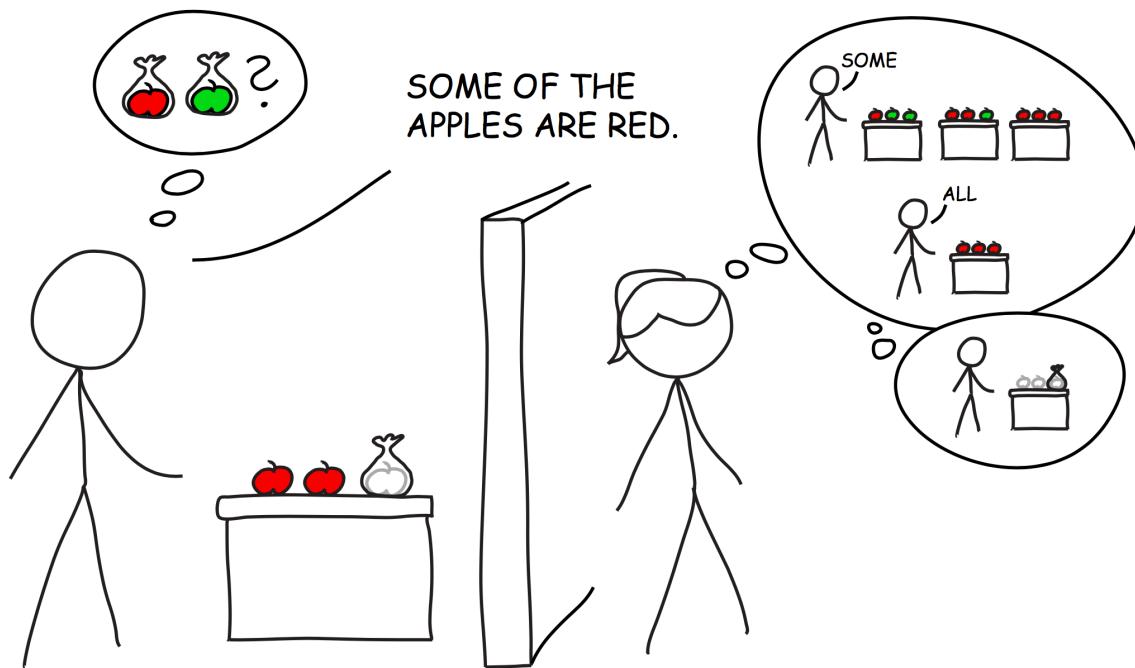


Learning at theory of physics



Ullman, Stuhlmüller, Goodman, & Tenenbaum (2018) Learning physical parameters from dynamic scenes. Cognitive Psychology

Some more cool examples: Pragmatic inference

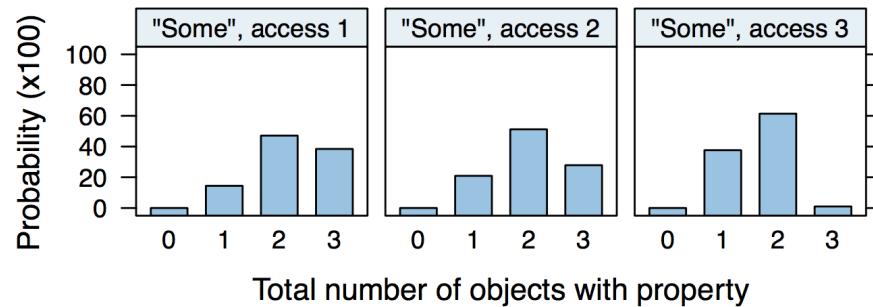


Scalar implicature

("some" but not "all")

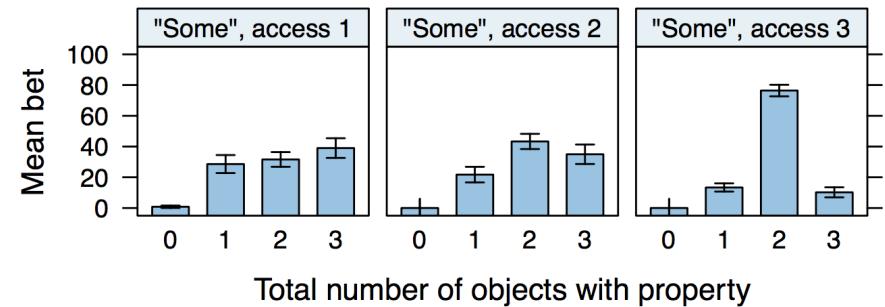
Model

(a)



Experiment

(c)



Goodman & Stuhlmüller (2013) Knowledge and implicature: Modeling language understanding as social cognition. Topics in Cognitive Science

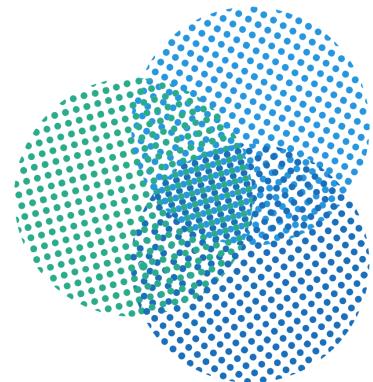
Resources: theory

- Goodman, Mansinghka, Roy, Bonawitz, & Tenenbaum (2008) Church: A language for generative models. *Uncertainty in Artificial Intelligence*
 - https://stanford.edu/~ngoodman/papers/churchUAI08_rev2.pdf
- Goodman, Tenenbaum, & Gerstenberg (2015) Concepts in a probabilistic language of thought. *The Conceptual Mind: New Directions in the Study of Concepts*
 - [web.mit.edu/tger/www/papers/Concepts in a probabilistic language of thought \(Goodman, Tenenbaum, Gerstenberg, 2014\).pdf](http://web.mit.edu/tger/www/papers/Concepts_in_a_probabilistic_language_of_thought_(Goodman,_Tenenbaum,_Gerstenberg,_2014).pdf)
- Freer, Roy, & Tenenbaum (2012) Towards common-sense reasoning via conditional simulation: legacies of Turing in Artificial Intelligence. *arXiv preprint arXiv:1212.4799*
 - <https://arxiv.org/pdf/1212.4799.pdf>
- Lake, Ullman, Tenenbaum, & Gershman (2016) Building machines that learn and think like people. *arXiv preprint arXiv:1604.00289*
 - <https://arxiv.org/pdf/1604.00289.pdf>
- Gerstenberg & Tenenbaum (2017) Intuitive Theories. *Oxford Handbook of Causal Reasoning*
 - [web.mit.edu/tger/www/papers/Intuitive Theories, Gerstenberg, Tenenbaum, 2017.pdf](http://web.mit.edu/tger/www/papers/Intuitive_Theories,_Gerstenberg,_Tenenbaum,_2017.pdf)
- Chater & Oaksford (2013) Programs as causal models: Speculations on mental programs and mental representation. *Cognitive Science*
 - <http://onlinelibrary.wiley.com/doi/10.1111/cogs.12062/abstract>
- Ghahramani (2015) Probabilistic machine learning and artificial intelligence. *Nature*
 - <http://www.nature.com/nature/journal/v521/n7553/full/nature14541.html>

Resources: practice

- <https://probmods.org/>
 - many more cool chapters to play around with
- <http://webppl.org/>
 - Editor to play around with code
- <http://dippl.org/>
 - Details about WebPPL
- <https://github.com/probmods/webppl>
 - github repository with latest developments
- <http://webppl.readthedocs.io/en/master/>
 - function reference for the webppl language
- <http://agentmodels.org/>
 - Great web book that focuses on how to model agents and inferences about agents
- <http://probabilistic-programming.org/wiki/Home>
 - homepage comparing different probabilistic programming languages

Thanks !



CENTER FOR
Brains
Minds +
Machines