M1

# Sampling (Exercises)

DSAA

~~ex. 1.1~~

$x(t) \to$ real valued signal

$\omega_s = 10{,}000\pi$ rad/s. (Sampling rate)

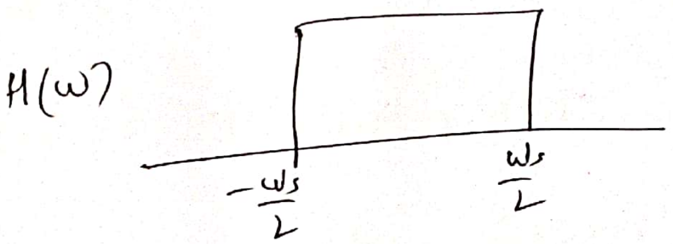
Q: What values of $\omega \to$ is $X_r(\omega) = 0$ for sure?

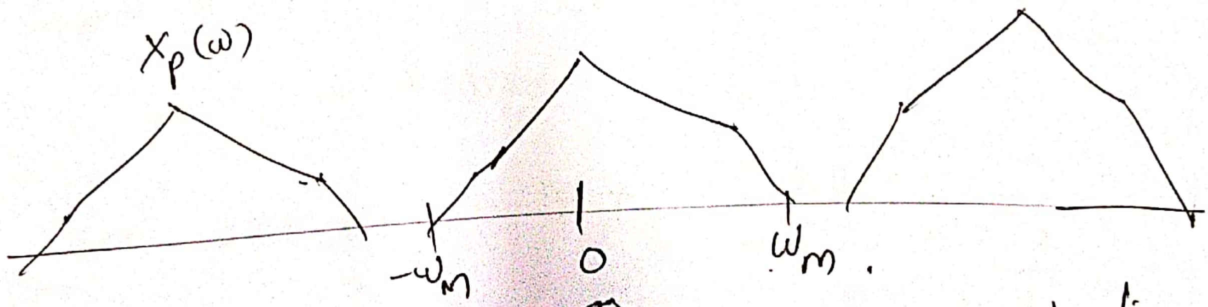
Sol: For $|\omega| > \frac{\omega_s}{2} = 5{,}000\pi$ rad/s.

Details:

$$X_p(\omega) = H(\omega)\, X_p(\omega).$$

$\omega_M = \frac{\omega_s}{2}$.





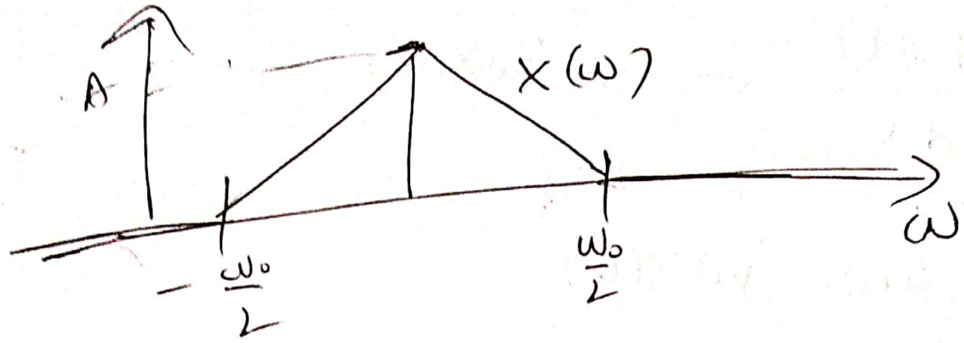
$$X_r(\omega) = \begin{cases} X(\omega) & |\omega| < 5{,}000 \text{ rad/s} \\ 0 & |\omega| > 5{,}000 \text{ rad/s.} \end{cases}$$

D SAH

# Sampling Theorem.

Given $x(t)$ with Nyquist rate $\omega_0$.

$\therefore \ \omega_0 = 2\omega_m \rightarrow \omega$ (max).




Find N.R. for

(i) $x(t) + x(t-1)$

Let $x_1(t) = x(t) + x(t-1)$

Based on time shifting property

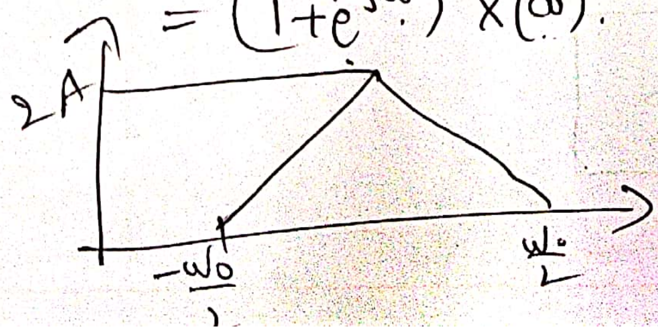
$x(t) \longleftrightarrow X(\omega)$

then $x(t-t_0) \longleftrightarrow e^{-j\omega t_0} X(\omega)$.

$X_1(\omega) = F\{x_1(t)\}$

$= X(\omega) + e^{-j\omega} . X(\omega)$

$= (1+e^{-j\omega}) X(\omega)$.

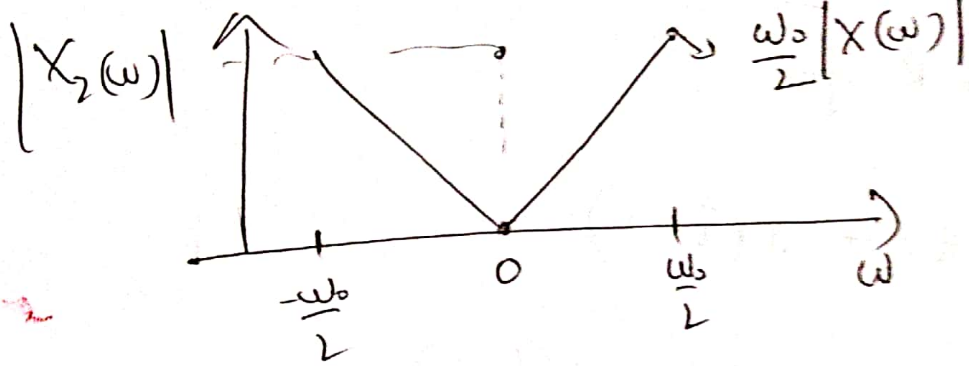



$$\boxed{NR = \omega_0}$$

(b) $x_2(t) = \frac{dx(t)}{dt}$.

$$\frac{dx(t)}{dt} \longleftrightarrow j\omega X(\omega)$$

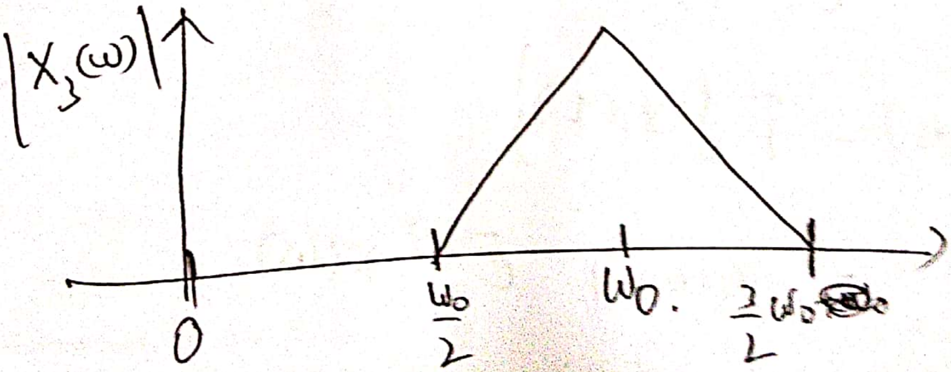
$\therefore X_2(\omega) = j\omega X(\omega)$




$\therefore \boxed{NR_2 = \omega_0}$

(c) $x_3(t) = x(t)\, e^{j\omega_0 t}$

$X_3(\omega) = X(\omega - \omega_0)$.




$\boxed{NR = \omega_0}$

# Sampling

(D'SAA)

Given $s(t) = x_1(t).x_2(t)$

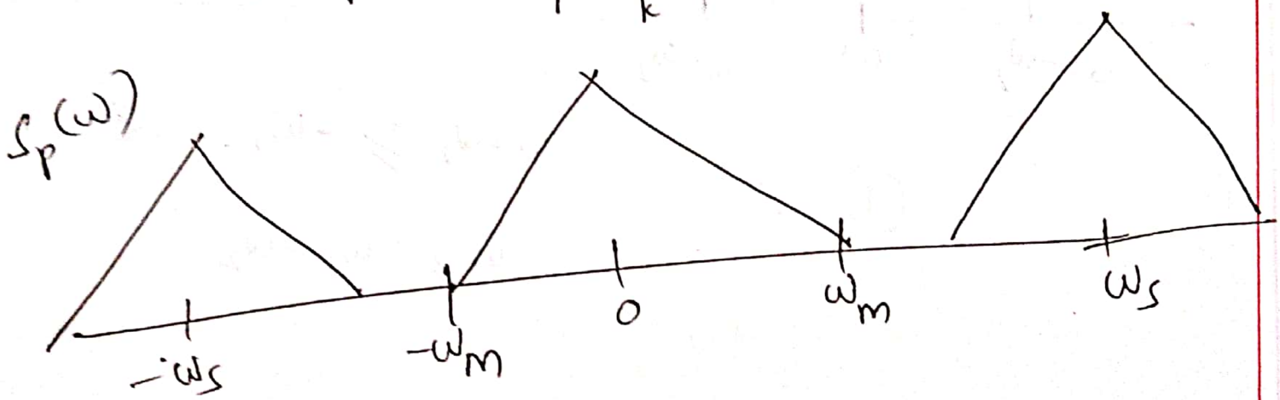
Find N.R. for $s(t)$.

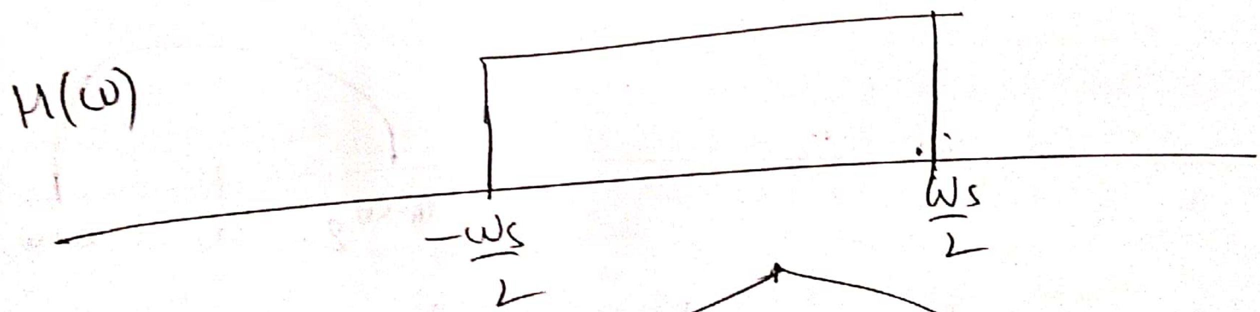
$S_p(t) = p(t)\,\cancel{\ast x(t)}\; s(t)$

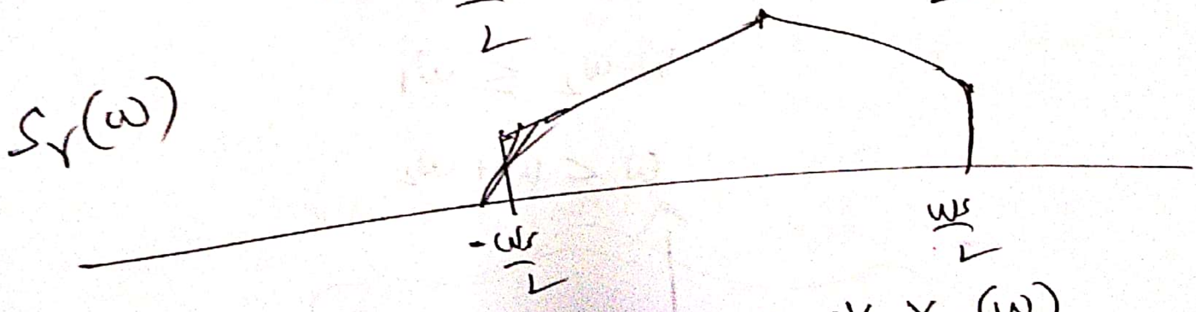
$= p(t)\,\boxed{x_1(t)\, x_2(t)} \to s(t).$

$S_r(\omega) = H(\omega).S_p(\omega).$

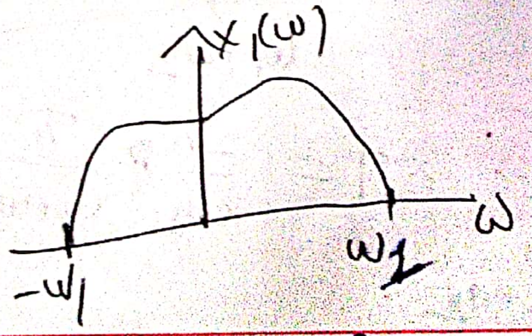
$S_p(\omega) = \frac{1}{T}\sum_k S(\omega - k\omega_s).$

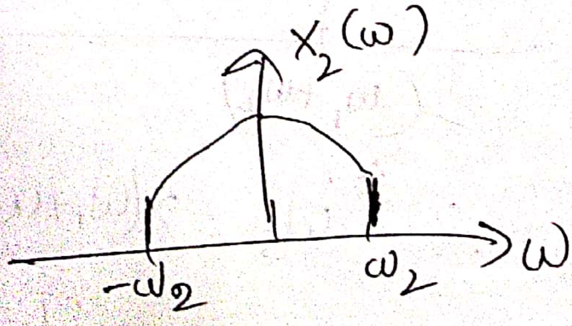


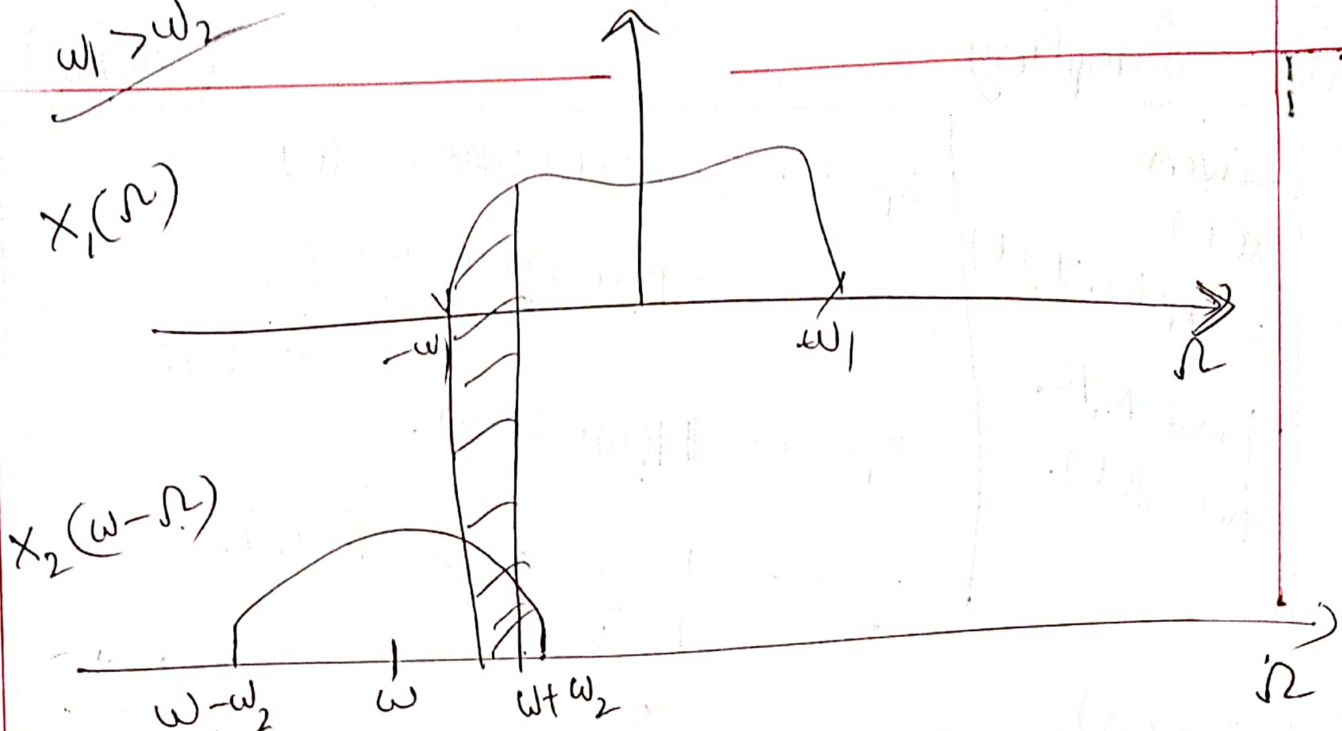

$S(\omega) = X_1(\omega) \ast X_2(\omega).$




EX | TEx.t: A.O. (S x S) Ex. 7.6 |

~~$\omega_1 > \omega_2$~~

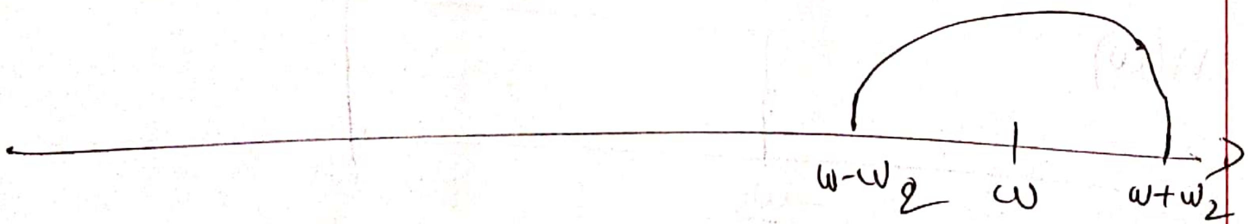



$\Omega = \omega+\omega_2$

1

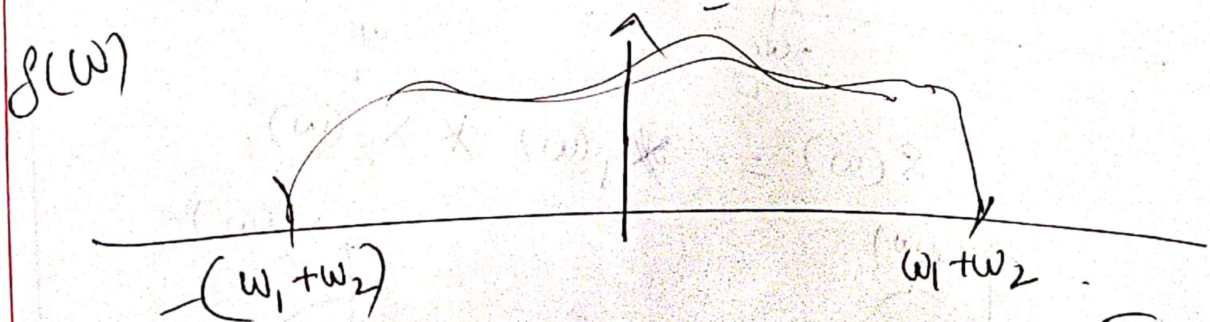
$$\omega+\omega_2 \geq -\omega_1.$$

$$\omega \geq -\omega_1 - \omega_2$$




$$\omega-\omega_2 \leq \omega_1$$

$$\omega \leq \omega_1+\omega_2$$




$$\therefore \omega_s = 2(\omega_1+\omega_2) \quad \therefore T = \frac{2\pi}{\omega_s} = \frac{\pi}{\omega_1+\omega_2}.$$

$$T_s \leq \frac{\pi}{\omega_1+\omega_2}$$

$x(t) = \text{sinc}^2(200\pi t)$

$X(\omega) = \,?$

$h(t) \rightleftharpoons H(\omega)$


$\rightleftharpoons$

$$\frac{1}{T} \cdot \frac{\sin^2\left(\frac{\omega}{2} T\right)}{(\omega/2)^2}$$

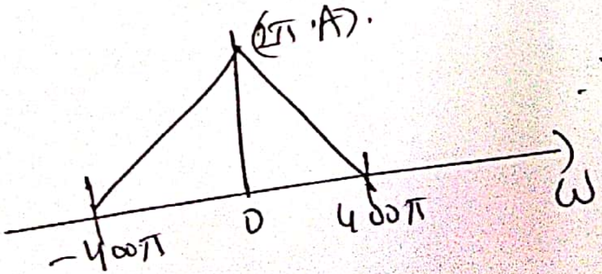
duality: $H(t) \rightleftharpoons 2\pi\, h(-\omega)$

$\therefore F[x(t)] = \,?$

$$x(t) = \frac{\sin^2(200\pi t)}{(200\pi t)^2} = \frac{1}{\cancel{A}} \cdot \frac{\sin^2\left(\frac{400\pi}{\cancel{A}} \cdot \frac{t}{2}\right)}{\left(400\pi \cdot \frac{t}{2}\right)^2} \cdot (A)$$

$$\therefore X(\omega) = (2\pi) \cdot 1 - \frac{|-\omega|}{400\pi}$$

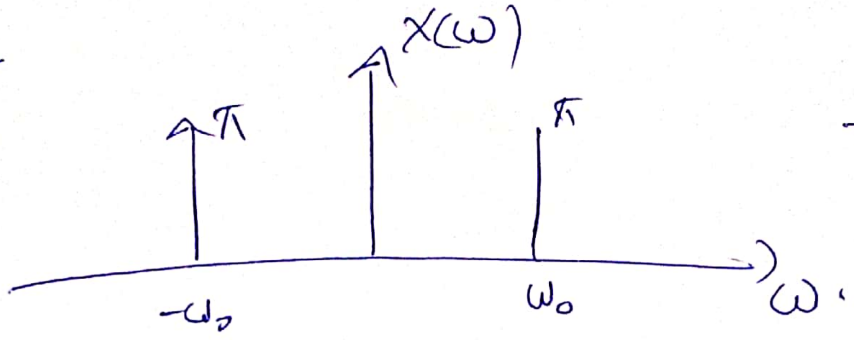
$$\therefore X(\omega) = 1 - \frac{|\omega|}{400\pi}$$



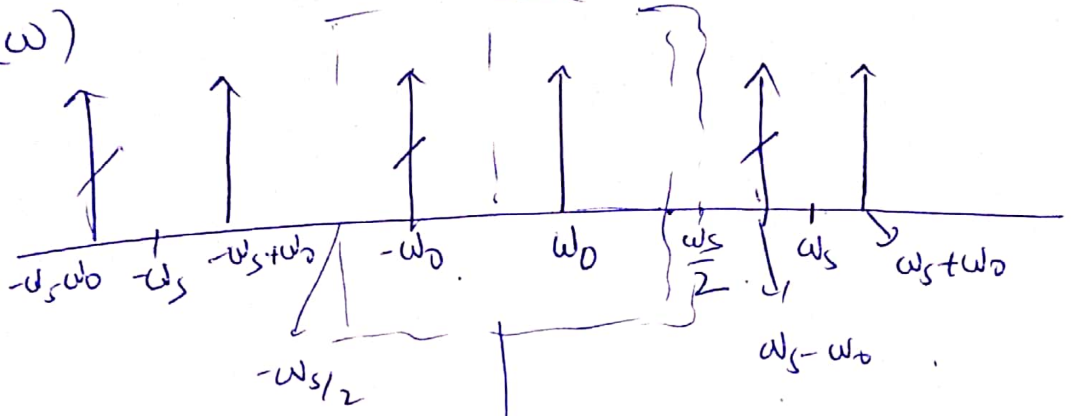

$\therefore \underline{BW = 400\pi}$

$\therefore \boxed{\omega_s = 800\pi}$

Simple eg.

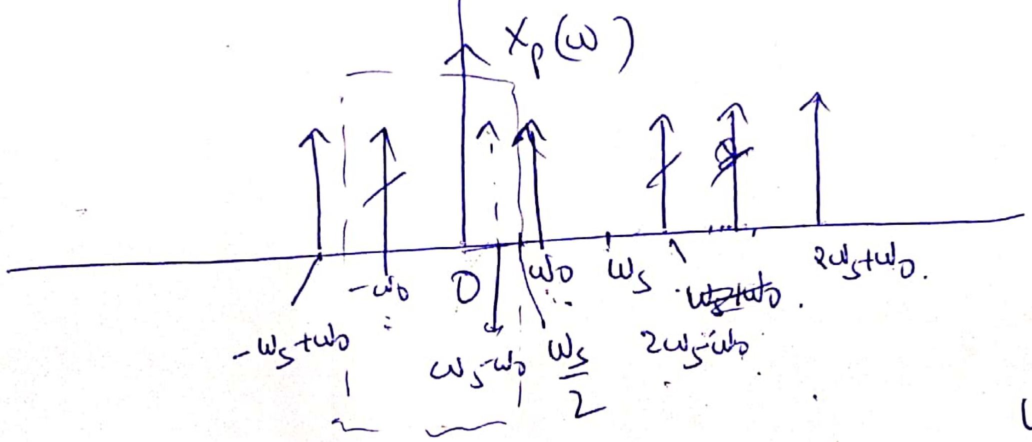



$\rightarrow \omega_s$ fn.




$\omega_s > 2\omega_0$

$\omega_s = 6\omega_0$




$\omega_s = \frac{3}{2}\omega_0$

$\frac{\omega_s}{2} = \frac{3}{4}\omega_0$

$\omega_s-\omega_0 = \frac{1}{2}\omega_0$.

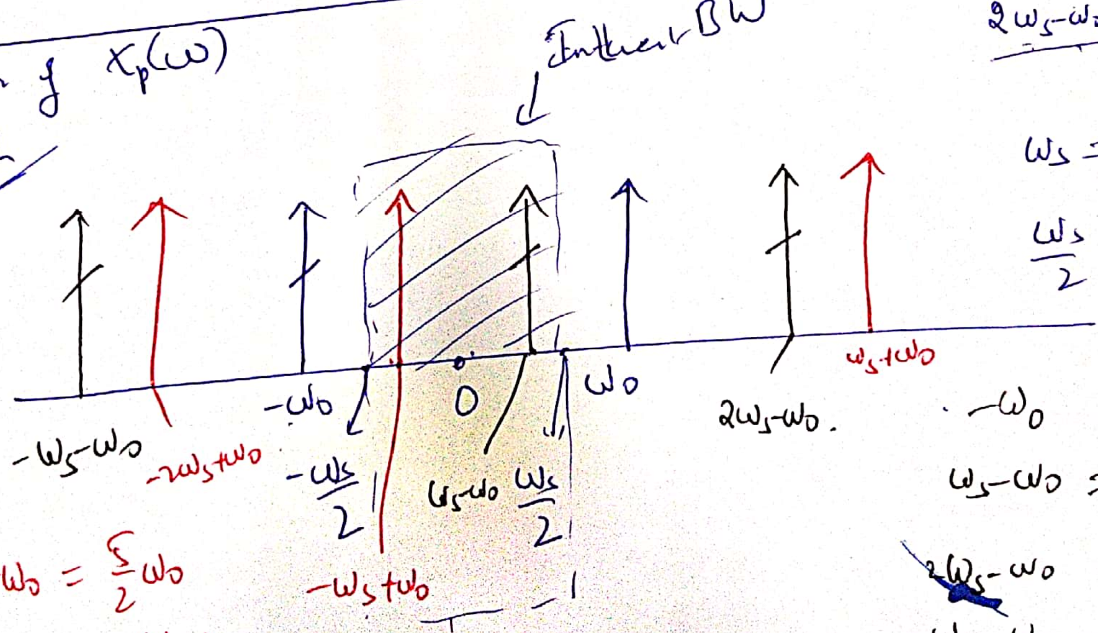
$\omega_s+\omega_0 = \frac{5}{2}\omega_0$

$2\omega_s-\omega_0 = 3\omega_0-\omega_0 = 2\omega_0$.

Construction of $X_p(\omega)$

ALIASING

Interest BW




$\omega_s = \frac{3}{2}\omega_0$

$\frac{\omega_s}{2} = \frac{3}{4}\omega_0$

$\omega_s-\omega_0 = \frac{\omega_0}{2}$

$2\omega_s-\omega_0 = 2\omega_0$

$-\omega_s-\omega_0 = -\frac{5}{2}\omega_0$.

$\omega_s+\omega_0 = \frac{5}{2}\omega_0$

$-\omega_s+\omega_0 = \frac{-1}{2}\omega_0$

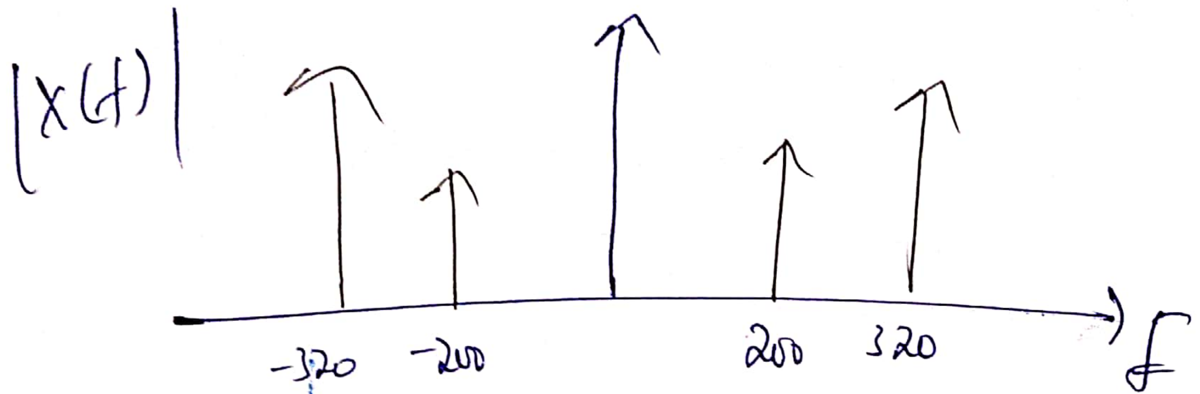
$-2\omega_s+\omega_0 = -2\omega_0$

CW

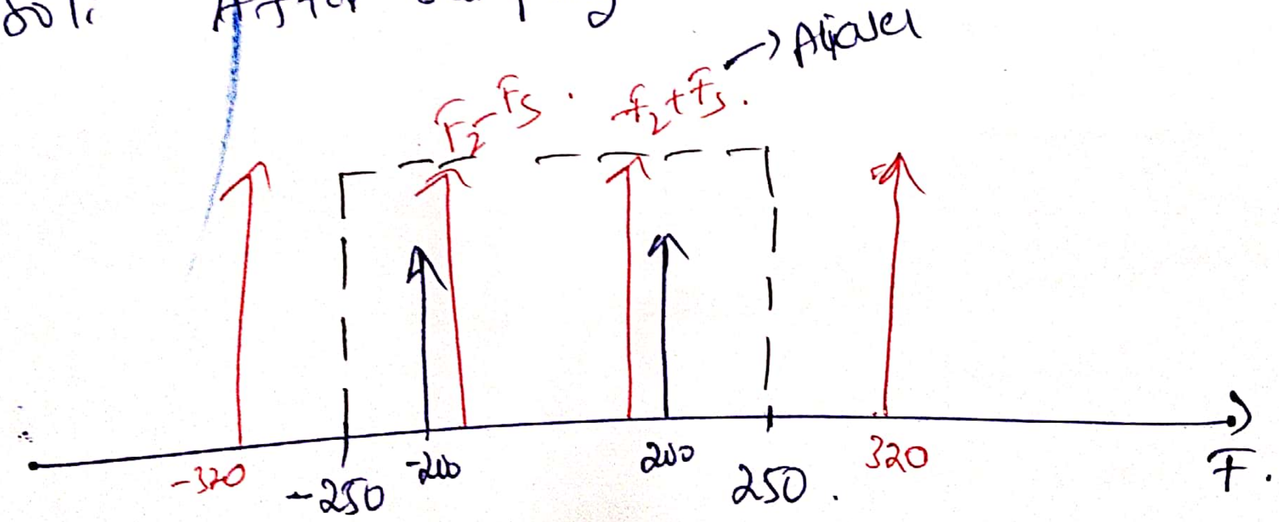
$x(t) = 2\cos 400\pi t + 6\cos 640\pi t$

$f_1 = 200 \; , \; F_2 = 320$

Given $F_s = 500$




Sol: After sampling




Freqs: $F_1 \; , \; -f_1$ ; Correct

$F_2 - F_s \; , \; -f_2 + F_s$ Aliases.

4)

discuss in the class

$x(t) = 1 + \cos(2000\pi t) + 10\sin(10000\pi t) + 20\cos(5000\pi t)$

Highest freq component in '1' is zero

Highest freq component in $2000\pi t$ is

$\omega m_1 = 2000\pi$

$fm_1 = \frac{2000\pi}{2\pi} = 1000$

Highest freq component in $(10000\pi t)$ is

$\omega m_2 = 10,000\pi$

$fm_2 = 5,000\,Hz$

Highest freq component in $(5000\pi t)$ is

$fm_3 = \frac{5000\pi}{2\pi}$

$2500\,Hz$

(3)

Nyquist rate $f_N = 2f_m$

$= 2 \times 5{,}000$

$= 10{,}000\,Hz \longrightarrow$ 4M

~~Ny~~ Nyquist Interval $= \dfrac{1}{10{,}000\,Hz}$

$= 1 \times 10^{-4}$ sec $= 0.1$ msec

Note : upto Nyquist Rate calculations : 4M

Total $\longrightarrow$ 8M

Problem:

Find the Nyquist sampling rate of a given signal.

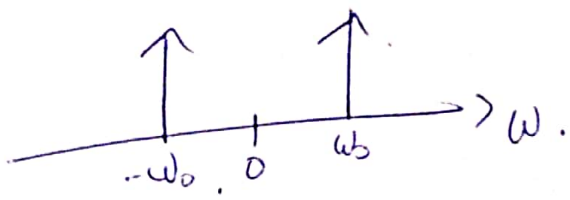
$$x(t) = \text{sinc}^2(200\pi t)$$

Sol: We need to find the frequency content of the signal.

Recall. for simple sinusoids.

e.g $\sin(2\pi f_0 t)$ or $\sin \omega_0 t$.

F.T look like




∴ The bandwidth is $\omega_0$ and sampling rate is $2\omega_0$ rad/sec or $2f_0$ cycles/sec.

$$\therefore F_s = 2f_0.$$

problem

Now for $\text{sinc}^2(200\pi t)$ - we cannot directly identify the frequencies as $200\pi$ or $100$.

Sol: Find the F.T using the FFT algorithm and empirically identify the 3dB BW. and then find the Nyquist rate.

$$g(t) \rightleftharpoons G(f)$$

Duality:

$$G(t) \rightleftharpoons 2\pi g(-f).$$

$$G(\omega) = \int_{-\infty}^{\infty} g(t)\, e^{-j\omega t}\, dt.$$

$$F[G(t)] = \int_{-\infty}^{\infty} G(t)\, e^{-j\omega t}\, dt. \quad -①$$

recall $g(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} G(\omega)\, e^{j\omega t}\, d\omega. \quad -②$

$\omega \to t'.$

$t \to \Omega.$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} G(t').\, e^{-j(-\Omega)t'}\, dt'$$

matcha ①

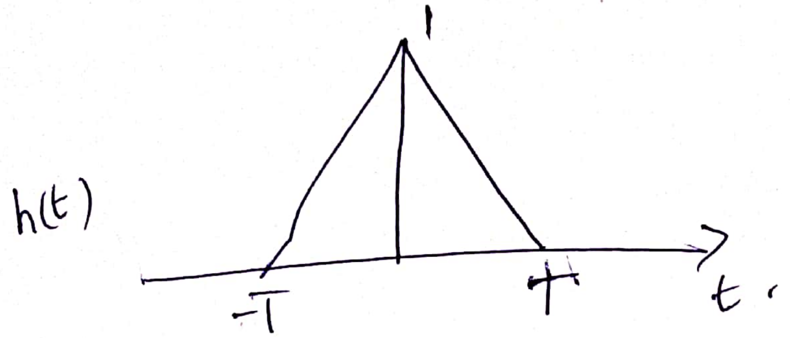
freq $\to -\Omega$.

$$\Rightarrow F[G(t)] = 2\pi\, g(-\Omega).$$

$$\therefore \boxed{G(t) \rightleftharpoons 2\pi\, g(-\omega).}$$

Fourier transform.

(i)



$\frac{1-0}{0+T} \cdot t$

$$h(t) = \begin{cases} 1 + \frac{t}{T} & -T < t < 0 \\ 1 - \frac{t}{T} & 0 \le t < T \end{cases}$$

$$H(\omega) = F\{h(t)\}$$

$$= \int_{-T}^{0} \left(1 + \frac{t}{T}\right) e^{-j\omega t}\, dt + \int_{0}^{T} \left(1 - \frac{t}{T}\right) e^{-j\omega t}\, dt$$

$$= \frac{e^{-j\omega t}}{-j\omega}\Bigg|_{-T}^{0} + \frac{1}{T} \cdot \frac{t e^{-j\omega t}}{-j\omega}\Bigg|_{-T}^{0} - \frac{1}{T}\int_{-T}^{0} \frac{e^{-j\omega t}}{-j\omega}\, dt$$

$$+ \frac{e^{-j\omega t}}{-j\omega}\Bigg|_{0}^{T} - \frac{1}{T} \cdot t \cdot \frac{e^{-j\omega t}}{-j\omega}\Bigg|_{0}^{T} - \frac{1}{T}\int_{0}^{T} \frac{e^{-j\omega t}}{j\omega}\, dt.$$

$$= \frac{e^{j\omega T}}{j\omega} - \frac{e^{-j\omega T}}{j\omega} - \frac{e^{+j\omega T}}{j\omega} + \frac{e^{-j\omega T}}{j\omega}$$

$$+ \frac{1}{T} \cdot \frac{e^{-j\omega t}}{\omega^2}\Big|_{-T}^{0} - \frac{1}{T} \frac{e^{-j\omega t}}{\omega^2}\Big|_{0}^{T}$$

$$= \frac{1}{T} \cdot \frac{1 - e^{j\omega T}}{\omega^2} - \frac{1}{T} \frac{e^{-j\omega T} - 1}{\omega^2}$$

$$= \frac{1}{T\omega^2}\left[ 2 - 1\left(e^{j\omega T} + e^{-j\omega T}\right)\right]$$

$$= \frac{2(1 - \cos\omega T)}{T\omega^2}$$

$$= \frac{2 \cdot 2\sin^2(\omega T/2)}{T\omega^2}$$

$$= \frac{1}{T} \cdot \frac{\left[\sin\left(\frac{\omega}{2} T\right)\right]^2}{(\omega/2)^2}$$

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