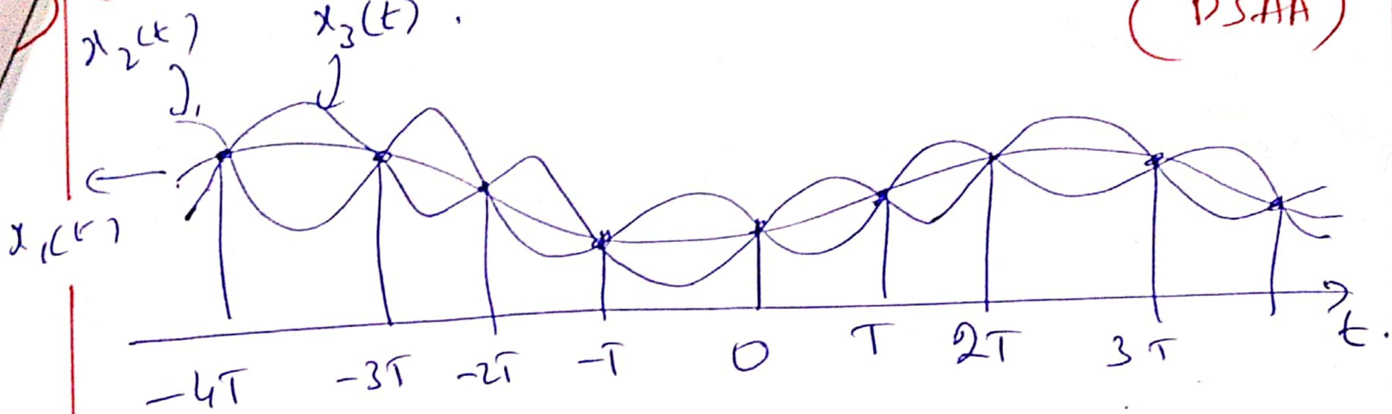
(Sampling)

(DSAA)



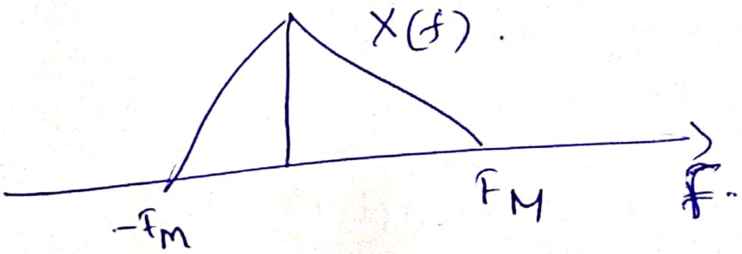
$\therefore\ x_1(kT) = x_2(kT) = x_3(kT)$.

Given $x_1(n)$, $x_2(n)$, $x_3(n)$ $\rightarrow$ we donot know which $x_i(t)$ is the source.

Sampling theorem.

(i) Freq. of signals < F_{max}

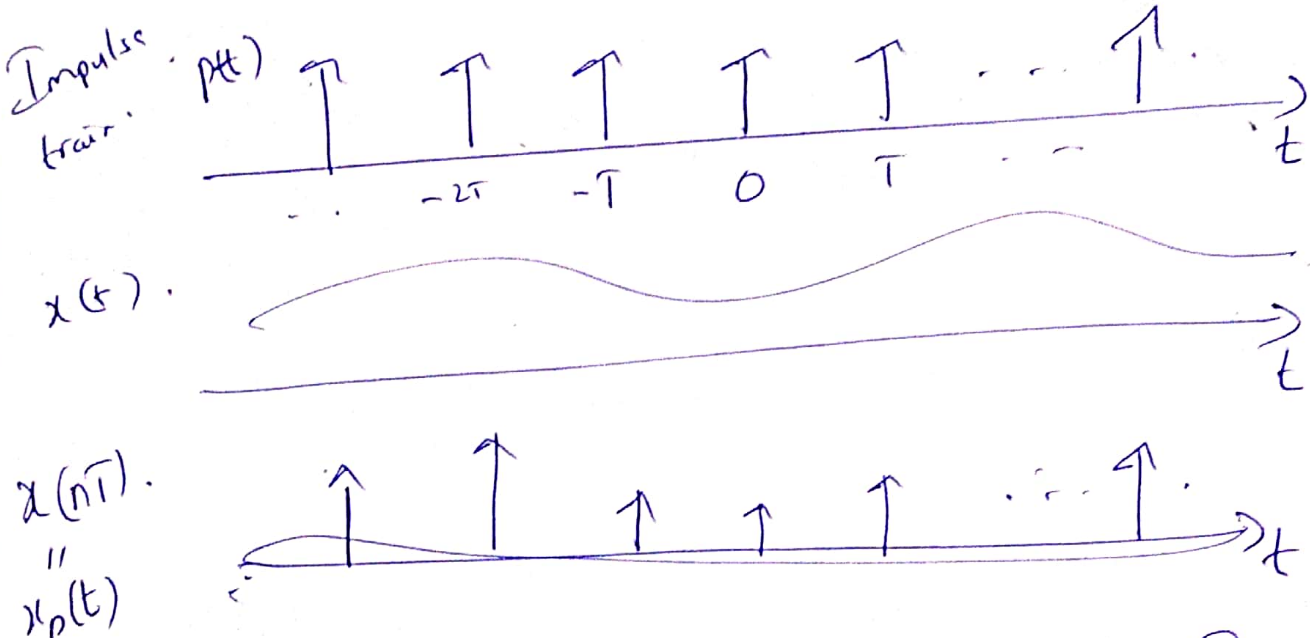
ie signal is band limited.



(ii) Sampling rate is sufficient eg. $F_s > 2F_M$.

Then $x(t)$ can be unique [recovery is possible].

7.1.1 $x(n) = x_a(nT) \rightarrow$ Sampling.



$$x_p(t) = \underset{\text{Analog}}{x(t)} \cdot \underset{\text{impulse train}}{p(t)} \quad — ①$$

$$p(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT)$$

$$\therefore x_p(t) = \sum_n \delta(t-nT)\, x(t) \longrightarrow \underline{x(nT)}.$$

$$\therefore x_p(t) = \sum_n x(nT)\, \delta(t-nT)$$

From ①. using multiplicative property CT FT

$$X_p(j\omega) = \frac{1}{2\pi} X(\omega) * P(\omega).$$

$$P(\omega) = F[p(t)] = \frac{2\pi}{T} \sum_{k=-\infty}^{\infty} \delta(\omega - k\omega_0).$$

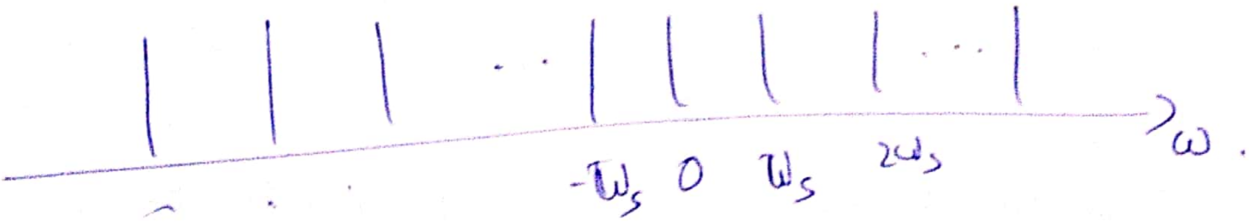
Scanned with CamScanner

$$X_p(\omega) = \frac{1}{2\pi} X(\omega) * \sum \frac{2\pi}{T}\delta(\omega - k\omega_0).$$

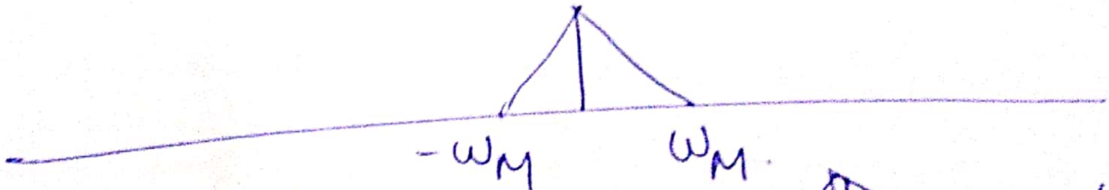
$$= \frac{1}{T}\sum_k \int X(\lambda)\,\delta(\omega - k\omega_0 - \lambda)\,d\lambda.$$

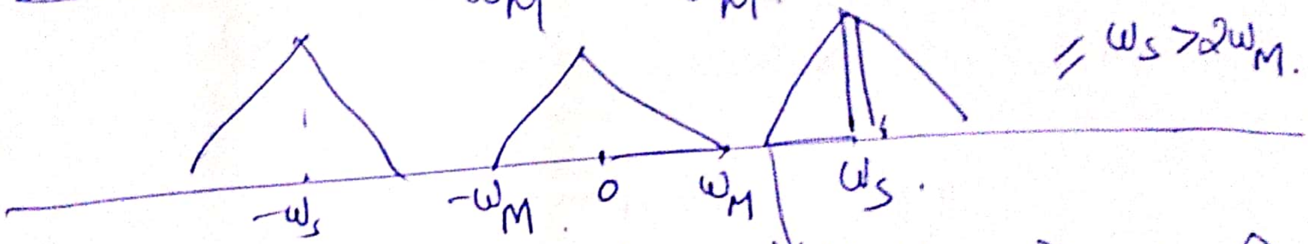
$$X_p(\omega) = \frac{1}{T}\sum_k X(\omega - k\omega_s) \longrightarrow \text{F.T. of Sampled Signal.}$$

$P(\omega)$

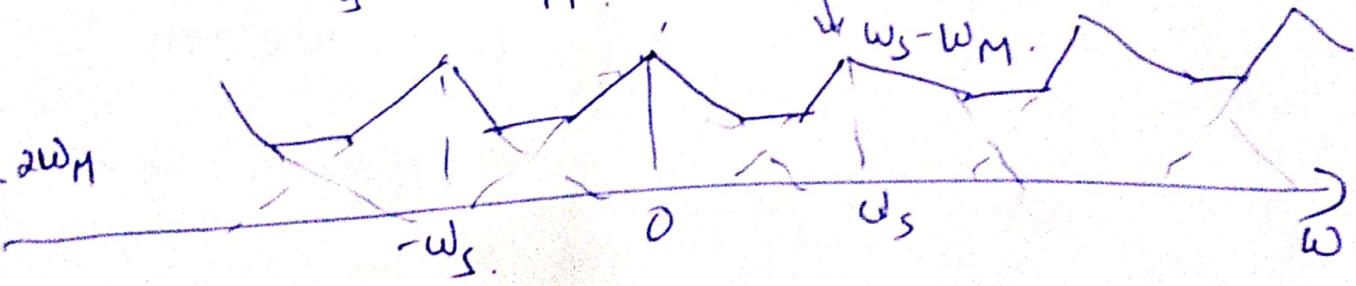


$X(\omega)$





$\omega_s < 2\omega_M$



under sampling

If $\omega_s > 2\omega_M$.

$$\hat{x}(t) = H_L(\omega)\, X_p(\omega).$$

H_L ↳ low pass filter.

~~$\omega_s > 2?$~~ ↳ cutoff $\omega_M < \omega_c < \omega_s - \omega_M$

Sampling Thm :

(1) $X(\omega)$ is $FT\{x(t)\}$.

$X(\omega)$ has $\omega < \omega_M$. band limited

(2) sampling rate or freq.

$\omega_s > 2\omega_M$. $\rightarrow$ above nyquist rate.

(3). $\omega_s = \frac{2\pi}{T}$ or $F_s = \frac{1}{T}$.

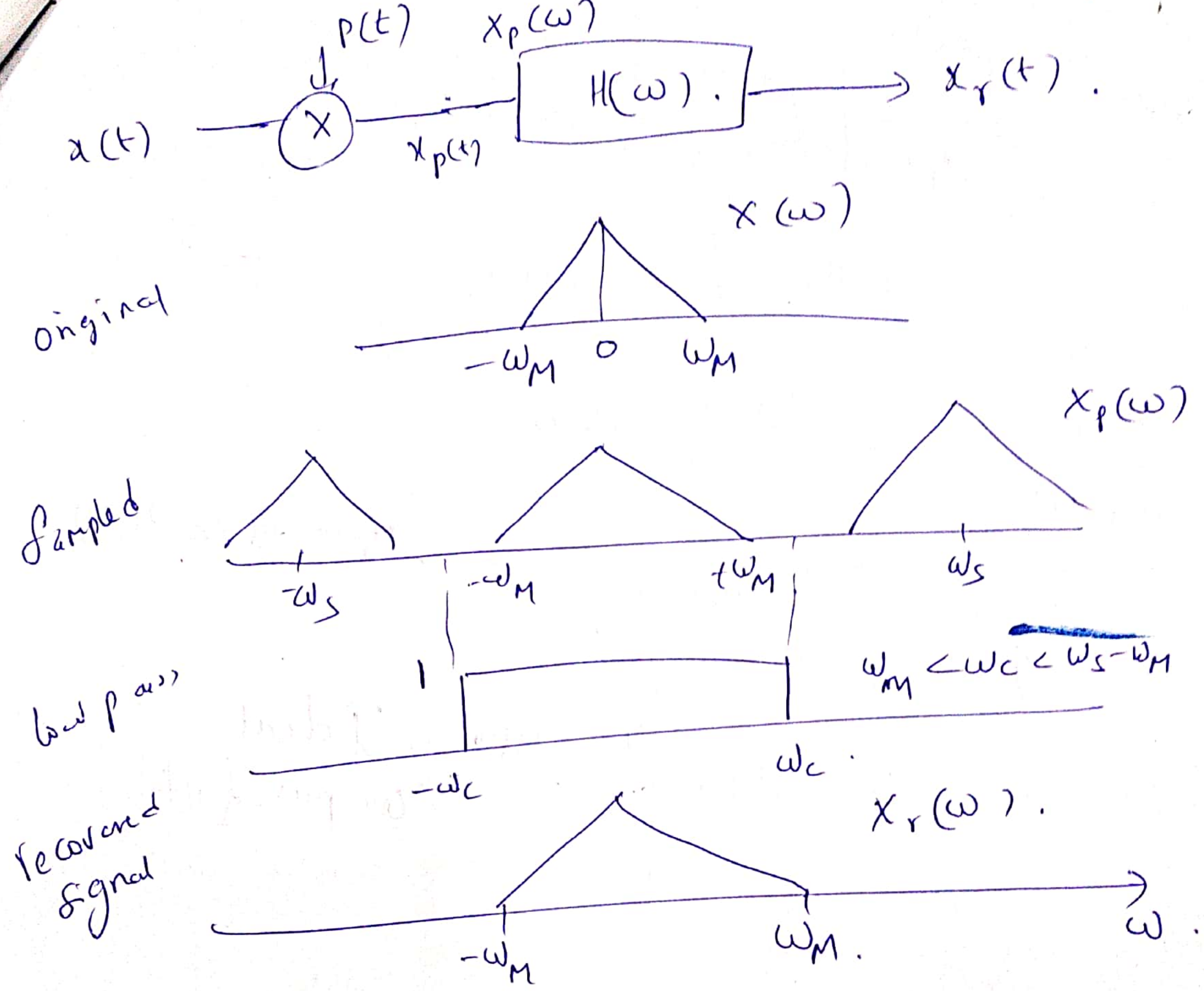
Given the samples $x(n)$ of $x(t)$. then the analog signal can be recovered by

(4) $\hat{X}(\omega) = X_p(\omega) \cdot H_L(\omega)$

$H_L(\omega) \rightarrow$ Lowpass ω_c.

$\omega_M < \omega_c < \omega_s - \omega_M$.

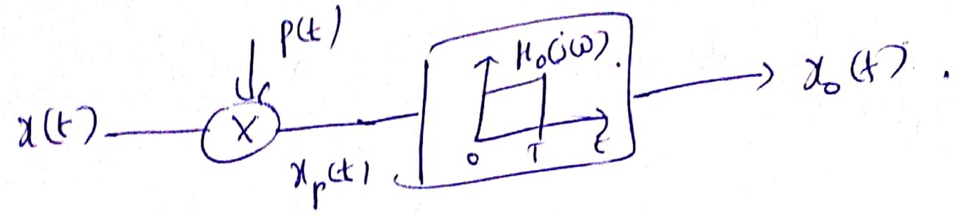
then $F^{-1}[\hat{X}(\omega)] = x(t) \rightarrow$ original analog signal.

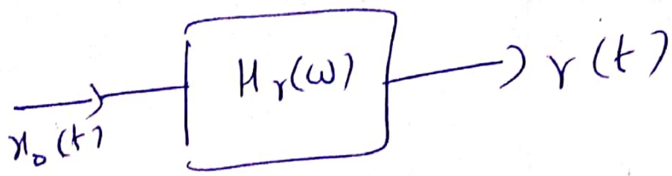


"In practice we donot have ideal-low-pass filters. we have approximated low-pass filter (with smooth transition region) leads to slight differences".

7.1.2

Sampling with a zero-order Hold.





$$H_0(\omega) = e^{-j\omega T/2} \cdot \frac{2 \cdot \sin(\omega T/2)}{\omega}$$

Reconst. filter:

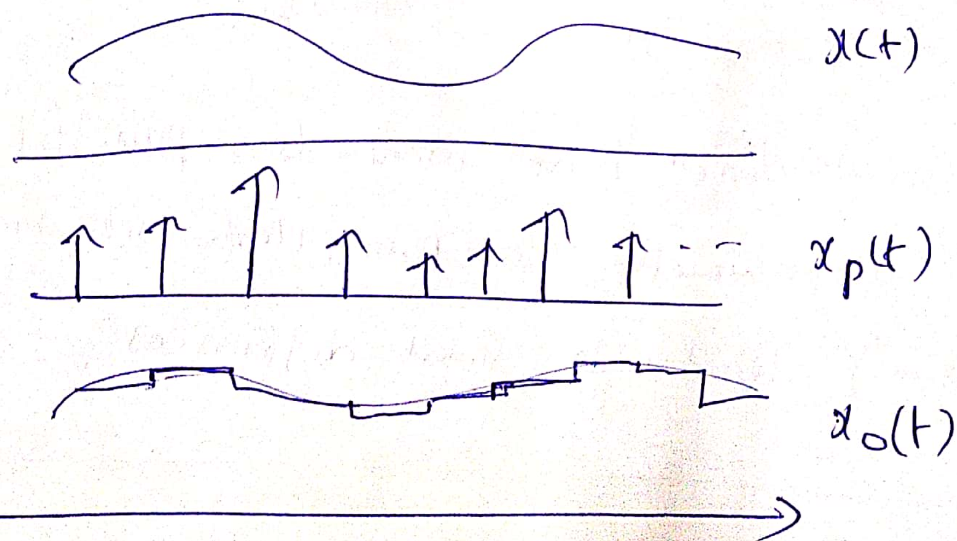
$$H_r(\omega) = \frac{e^{j\omega T/2} \cdot H(j\omega)}{\frac{2\sin(\omega T/2)}{\omega}}$$

~~$H_r(\omega) \cdot H_0(\omega) = H(\omega)$~~

$$\boxed{H(\omega) = H_r(\omega) \cdot H_0(\omega)}$$

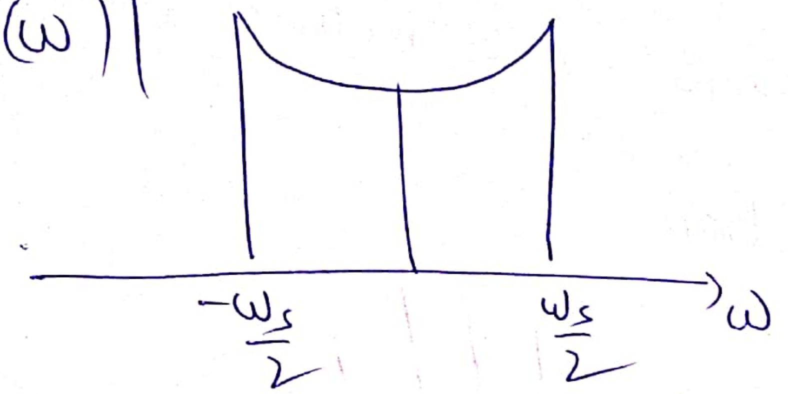
$H(\omega) \rightarrow$ Ideal low pass filter

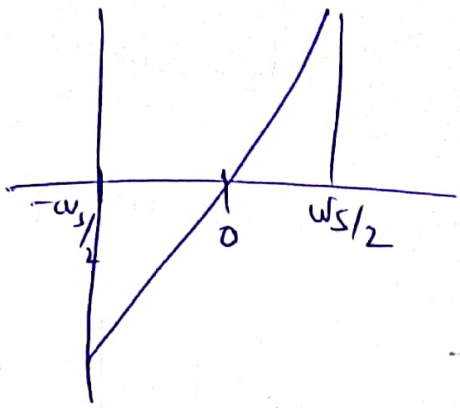
$x_0(t)$



$$x_0(t) = x_p(t) * h_0(t)$$

(4)





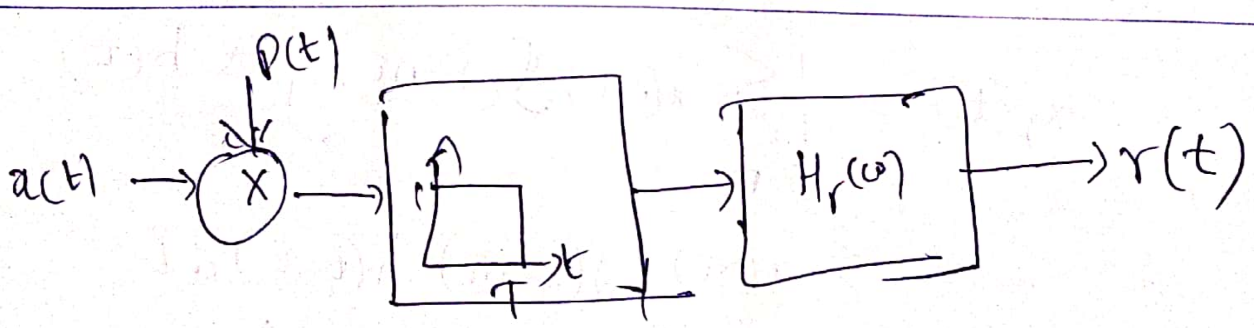
reconstruction filter.

$$H_0(\omega) = e^{-j\omega T/2} \cdot \frac{2\sin(\omega T/2)}{\omega}$$

$$F^{-1}\left\{2\frac{\sin \omega T/2}{\omega}\right\} = F^{-1}\left\{\frac{e^{j\omega T/2} - e^{-j\omega T/2}}{j\omega}\right\}.$$

$$\therefore \quad = 1 \qquad -\frac{T}{2} < t < \frac{T}{2}.$$

$$\therefore h_0(t) = \begin{cases} 1 & 0 < t < T. \\ 0 & \text{else} \end{cases}$$

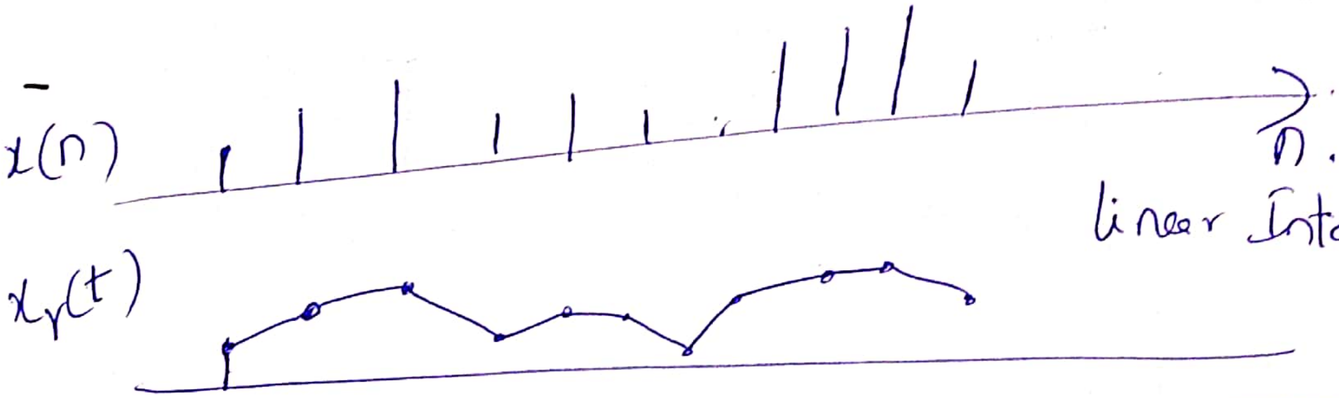


$r(t) \Rightarrow x(t)$ ($\rightarrow$ goal)

7.2 Reconstruction.

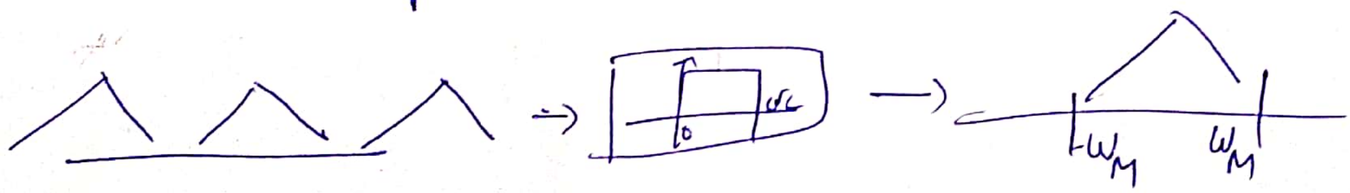
- Show slides on 'sample' hold method. (zero-order hold).

- linear interpolation.



Other-methods eg: the exact method.

$$X_p(\omega)\, H_L(\omega) = X_r(\omega).$$



$$x_r(t) = x_p(t) * h(t)$$

$$x_r(t) = \left[\sum_n x(nT)\,\delta(t-nT)\right] * h(t)$$

$$= \sum x(nT) \int \delta(\tau - nT)\, h(t-\tau)\, d\tau.$$

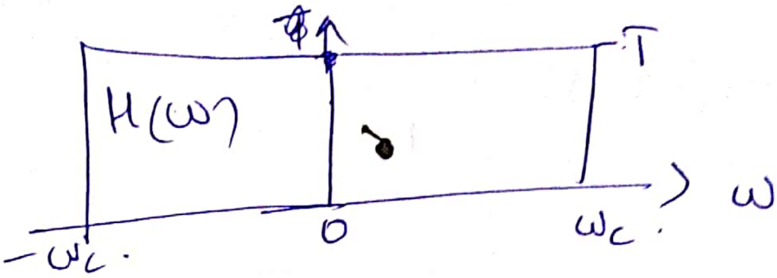
$$\boxed{x_r(t) = \sum_n x(nT)\, h(t-nT)}$$

$\sum x(nT) \int \delta(t-\tau-nT)\, h(\tau)\, d\tau$

Ls.

(5)

For ideal low-pass filter



inverse transform

$$h(t) = \frac{\omega_c T \sin \omega_c t}{\pi \omega_c t}$$

$$h(t) = \frac{1}{2\pi} \int_{-\omega_c}^{\omega_c} T \cdot e^{j\omega t} \, d\omega = \frac{T}{2\pi} \left. \frac{e^{j\omega t}}{jt} \right|_{-\omega_c}^{\omega_c}$$

$$= \frac{T}{2\pi} \cdot \frac{e^{jt\omega_c} - e^{-jt\omega_c}}{jt}$$

$$= \frac{T}{\pi t} \cdot \sin \omega_c t = \frac{\omega_c T \cdot \sin \omega_c t}{\pi \omega_c t}$$

$$\therefore x_r(t) = \sum_{n=-\infty}^{\infty} x(nT) \cdot \frac{\omega_c T}{\pi} \cdot \frac{\sin \omega_c (t - nT)}{\omega_c (t - nT)}$$

↳ Reconstruction formula

"Infinite no. of samples".

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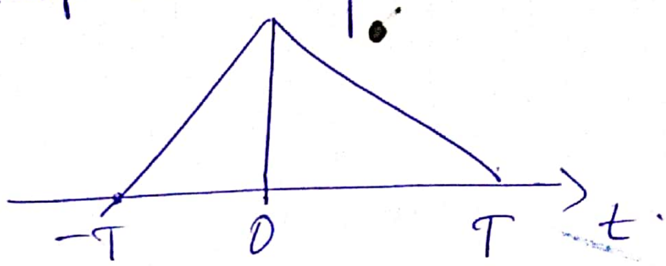
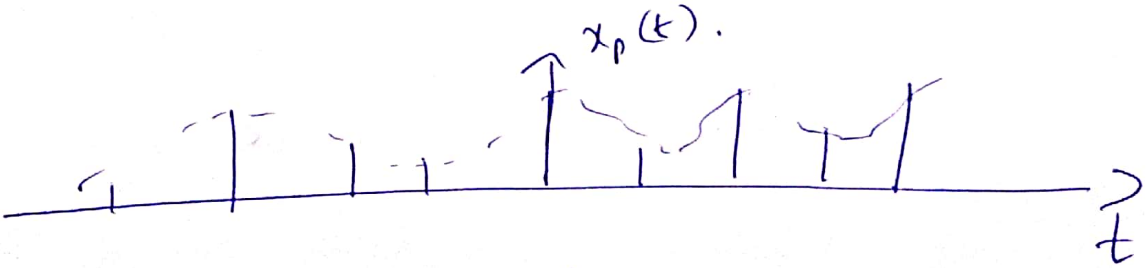
other filters for interpolation

— linear interpolation

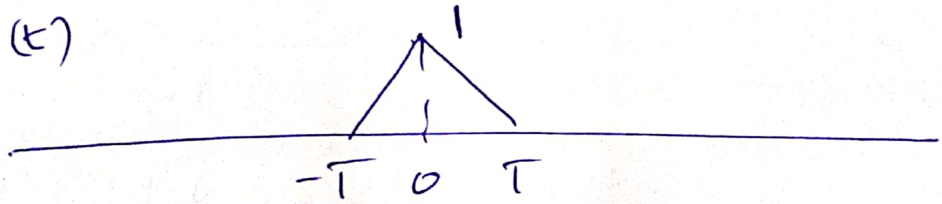
~~h(ω)~~ → h(t)



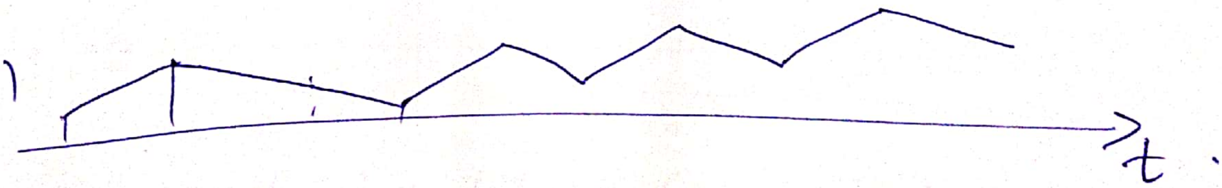
$$H(\omega) \rightarrow \frac{1}{T} \cdot \frac{\left(\sin \omega T/2\right)^2}{(\omega/2)^2} \longrightarrow \text{linear interpolation.}$$



h(t)



$x_r(t)$



H(ω)