M3 → Needed in M3

---

① $x(t) = e^{j\omega_0 t}$ $\quad \forall t$

$$X(\omega) = \int x(t)\, e^{-j\omega t}\, dt$$

$$= \int e^{j\omega_0 t}\, e^{-j\omega t}\, dt$$

$$= \int e^{-j(\omega-\omega_0)t}\, dt$$

$$= \left. \frac{e^{-j(\omega-\omega_0)t}}{-j(\omega-\omega_0)} \right|_{-\infty}^{\infty} = \infty.$$

* No F.T from basics

Reason. $\int |x(t)|\, dt < \infty$ Not Satisfied.

However. $x(t) = \delta(t)$.

$$X(\omega) = \int \delta(t)\, e^{-j\omega t}\, dt = e^{(0)} = 1.$$

based on sifting

~~[illegible]~~ Classes.

|||ly $X(\omega) = \delta(\omega)$.

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)\, e^{+j\omega t}\, d\omega$$

$$= \frac{1}{2\pi}\int \delta(\omega)\, e^{j\omega t}\, d\omega$$

$$= \frac{1}{2\pi}\, e^{(0)} = \frac{1}{2\pi}. \rightarrow \text{constant}$$

Now $x(t) = e^{j\omega_0 t}$.

$$1 \longleftrightarrow 2\pi\,\delta(\omega)$$

$$e^{j\omega_0 t} \longleftrightarrow 2\pi\,\delta(\omega-\omega_0)$$

$$\int_{-\infty}^{\infty} x(t)\, e^{j\omega_0 t}\, e^{-j\omega t}\, dt \qquad \underline{\omega' = \omega - \omega_0}$$

$$= \int_{-\infty}^{\infty} x(t)\, e^{-j\omega' t}\, dt = X(\omega').$$

$\therefore$

$$x(t) = \sin \omega_0 t \qquad \forall t$$

$$x(t) = \operatorname{Re}\{ e^{j\omega_0 t} \}. \quad -①$$

$$\int x(t) e^{-j\omega t} dt = \int \operatorname{Re}(e^{j\omega_0 t}) . e^{-j\omega t} dt.$$

$-①$ Not good.

$$x(t) = \frac{e^{j\omega_0 t} - e^{-j\omega_0 t}}{2j} \quad -②$$

$$\int_{-\infty}^{\infty} \frac{e^{j\omega_0 t} - e^{-j\omega_0 t}}{2j} e^{-j\omega t} dt$$

$$= \frac{1}{2j}\left[\int e^{-j(\omega-\omega_0)t} dt - \int e^{-j(\omega+\omega_0)t} dt \right]$$

$$= \frac{1}{2j}\left[2\pi . \delta(\omega-\omega_0) - 2\pi \delta(\omega+\omega_0) \right].$$

$$\therefore \; X(\omega) = \pi j \left[\delta(\omega+\omega_0) - \delta(\omega-\omega_0) \right]$$

Sampling theorem:

Let $x(t)$ be a band-limited signal with $X(\omega)=0$ for $|\omega| > \omega_M$. Then $x(t)$ is uniquely determined by its samples $x(nT)$, $n = 0, \pm 1, \pm 2, \ldots$

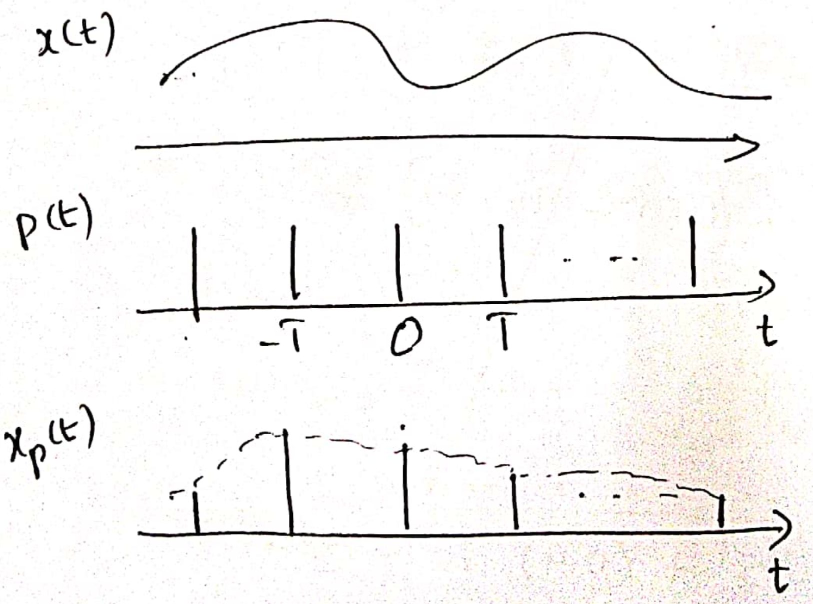
if $\omega_s > 2\omega_M$ where $\omega_s = \frac{2\pi}{T}$.

Given these samples we can reconstruct $x(t)$ by generating a periodic impulse train in which successive impulses have amplitudes that are successive sample values. This impulse train is then processed through an ideal low-pass filter with gain T and cut-off frequency greater than ω_M and less than $\omega_s - \omega_M$. The result is exactly $x(t)$.

3M

Proof:

Sampling function. $p(t)$ (Impulse train)



In time domain

$$x_p(t) = x(t)\,p(t)$$

where

$$p(t) = \sum_{n=-\infty}^{\infty} \delta(t-nT)$$

∴ Sampler in time domain.

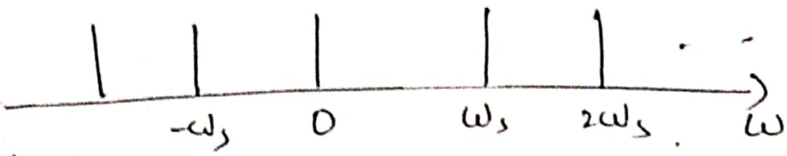
$$x_p(t) = \sum_n x(nT)\,\delta(t-nT).$$

In frequency domain. (by applying Fourier transform)

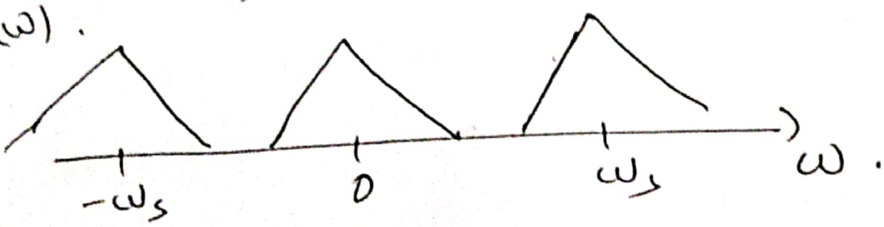
$$X_p(\omega) = \frac{1}{T}\sum_{k=-\infty}^{\infty} X(\omega - k\omega_s).$$

$X(\omega)$

$P(\omega)$ $\qquad$ $P(\omega) = \mathcal{F}\{p(t)\}.$



$X_p(\omega)$.



3M

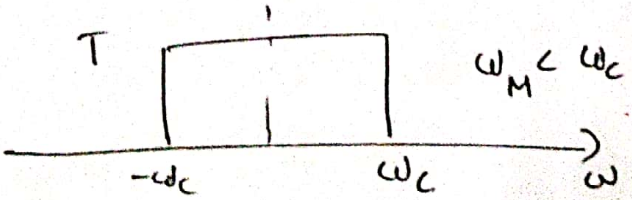
For reconstruction:

$$x_r(t) = x_p(t) * h(t).$$

i.e; $X_r(\omega) = X_p(\omega)H(\omega)$.

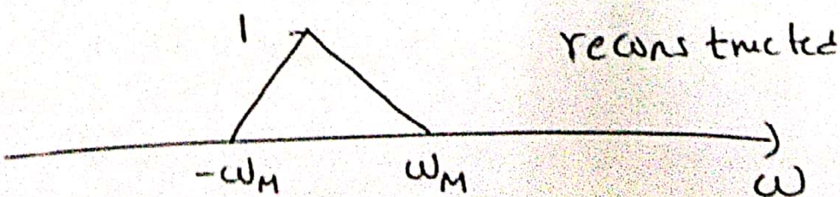
$H(\omega) \rightarrow$ low pass filter from Sampling theorem.

$H(\omega)$



$\omega_M < \omega_c < \omega_s - \omega_M$

$X_r(\omega)$ $\qquad$ reconstructed signal.



In time domain

$$h(t) = \frac{\omega_c T \sin \omega_c t}{\pi \omega_c t} \quad \rightarrow \text{Inverse F.T. of } H(\omega).$$

$$\boxed{\therefore x_r(t) = \sum_{n=-\infty}^{\infty} x(nT) \frac{\omega_c T}{\pi} \frac{\sin \omega_c (t-nT)}{\omega_c (t-nT)}.}$$

3M

M1

Quantization

DSAA

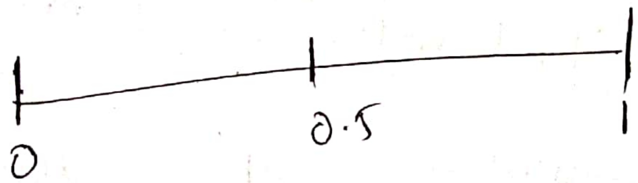
main idea.

Analog Value $\longrightarrow$ Quantizer $\longrightarrow$ Digital codewords.

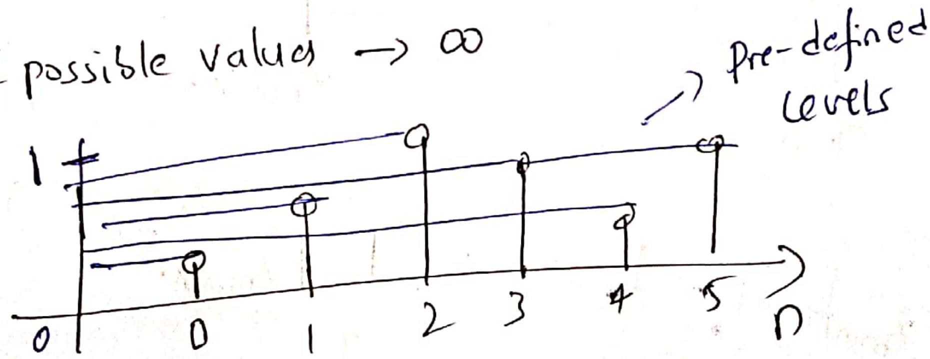
Basic

e.g. Analog values

$x(n) \rightarrow$



No. of possible values $\rightarrow \infty$



digital value.

$$x_q(n) = \begin{cases} 0 & \text{if } x(n) < 0.5 \\ 1 & x(n) \geq 0.5 \end{cases}$$

$x_q(n) = [x(n)] \rightarrow$ round off.

only 2-possible values $\{0, 1\}$.

i/p $(0,1) \longrightarrow \{0,1\}$ o/p.

TH

Generalization to multiple intervals

$$x_q(n) = Q[x(n)] = \hat{x}_k$$

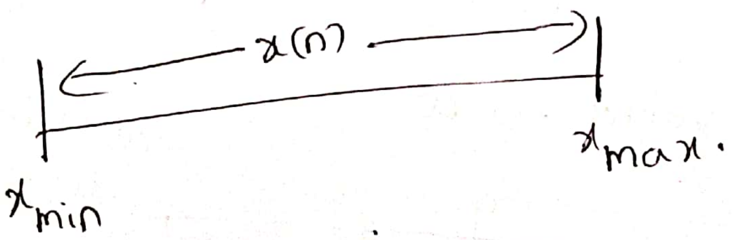
Q → Quantizer

$\hat{x}_k$ → Quantization levels.

main idea.

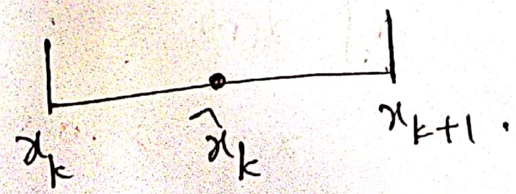
Intervals. → 'L' nr.

$x(n) \in (x_{min}, x_{max})$



→ L-intervals.

I_k $\quad k = 1, \dots, L.$



$$\therefore \; I_k = \{x_k < x(n) < x_{k+1}\}$$

operation.

If $x(n) \in (x_k, x_{k+1})$

then $x_q(n) = \hat{x}_k$.

$x_k \to$ decision level.

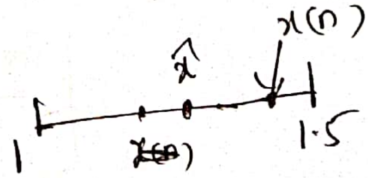
(or) $\boxed{\begin{array}{l} \text{If } x(n) \in I_k. \\ \text{then } x_q(n) = \hat{x}_k. \end{array}}$

Important properties.

1. quantization is non-invertible.

$$x(n) \longrightarrow \boxed{Q} \longrightarrow x_q(n)$$

$$x_q(n) \longrightarrow \boxed{\cancel{Q^{-1}}} \longrightarrow x(n)$$ Not Possible.



e.g. $x(n) = 1.45$

$x_q(n) = 1.25$.

$I_2 = \{1 < x(n) < 1.5\}$

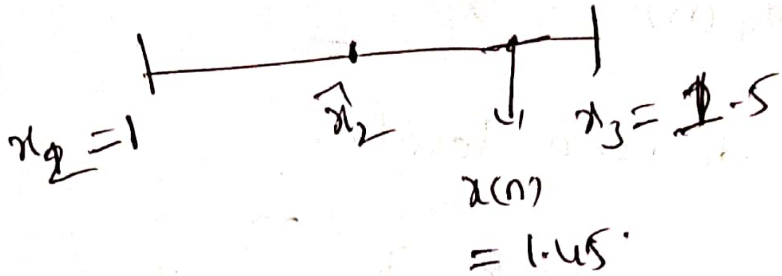
$\boxed{\hat{x}_2 = 1.25}$

Scanned with CamScanner

Problem: We cannot recover 1.45 $(x(n))$ from 1.25 $(x_q(n))$

why?

I_2

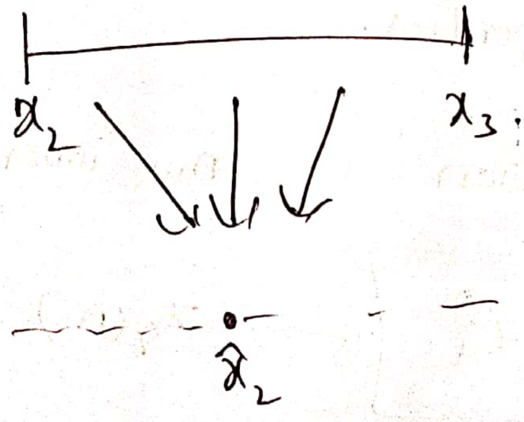


$\hat{x}_2 = 1.25$

Now for $x(n) = 1.40 \rightarrow x_q(n) = 1.25$

Illy for all $x(n) \in I_2$.

$\therefore$



many to one mapping

$\therefore$ Not invertible.

9. (a). Non-linear nature of quantizer.

Prop (2)

$$x(n) \longrightarrow \boxed{\text{Truncation}} \longrightarrow y(n)$$

(1)

$\cancel{y[n]}$

$\rightarrow \alpha$

$$y(n) = 2^{-b} \left\lfloor \frac{x(n)}{2^{-b}} \right\rfloor$$

$\lfloor \cdot \rfloor$ ↳ floor function.

Let $x_1(n) \rightarrow y_1(n)$

$x_2(n) \rightarrow y_2(n)$

$$y(n) = a\, y_1(n) + b\, y_2(n) \quad \text{--- (1)}$$

(2M)

Let $x(n) = a\, x_1(n) + b\, x_2(n)$.

$$y'(n) = 2^{-b} \left\lfloor \frac{x(n)}{2^{-b}} \right\rfloor$$

Ex

$$= 2^{-b} \left\lfloor \frac{a x_1(n) + b\, x_2(n)}{2^{-b}} \right\rfloor \quad - ②$$

$$y(n) = a \cdot 2^{-b} \left\lfloor \frac{x_1(n)}{2^{-b}} \right\rfloor + b \cdot 2^{-b} \left\lfloor \frac{x_2(n)}{2^{-b}} \right\rfloor$$

$$\therefore y[n] = 2^{-b} \left\{ a \left\lfloor \frac{x_1(n)}{2^{-b}} \right\rfloor + b \left\lfloor \frac{x_2(n)}{2^{-b}} \right\rfloor \right\} \quad - ③$$

Since eq. ② & ③ are usually unequal

∴ Truncation is non-linear. (2 M)

(PTO)

② Rounding.

$$y(n) = 2^{-b} \left[\frac{x(n)}{2^{-b}} \right]$$

$[\cdot]$ ↳ rounding fn.

Along similar lines as in previous case (truncation) rounding fn. is also non-linear. (1 M)

Scanned with CamScanner

Binary coding schemes:

- 2's complement representation is most common.

e.g. of 2's complement rep.

$\beta_0 \, \beta_1 \, \beta_2 \cdots \beta_b . \rightarrow$ b+1 bits.

$$-\beta_0 \cdot 2^0 + \beta_1 2^{-1} + \beta_2 2^{-2} + \cdots + \beta_b 2^{-b}$$

$\beta_0 \rightarrow$ most significant bit. (MSB)

$\beta_b \rightarrow$ LSB

Note: Coding is only important for A/D conversion. hardware design. quantization ~~is not~~ performance is independent of coding.

e.g. of 2's comp. rep (Integers)

+7 → 0111.0

+ve nrs no conversions other than bin to dec.

+1 → 0001.0

-1 → 1111

→ -1 × [(000) Inversion
+(1)]

= -1×001 = -1.

<u>fractions</u>

$x = -0.125$

$x_{2} = 1.111$

verify ① $(-1)1 + 2^{-1} + 2^{-2} + 2^{-3}$.

$= -1 + 0.875 = -0.125$

② $1.111 \rightarrow (-1) \times [000 + 1] =$

$-(0.001)_2 = -0.125_d$.

$$y_1(n) = 2^{-b}\left[\frac{x(n)}{2^{-b}}\right] \quad — (1)$$

$$y_1(n) = 2^{-b}\left[\frac{a_1 x_1(n) + b_1 x_2(n)}{2^{-b}}\right]$$

$$y(n) = 2^{-b}\left\{ a_1\left[\frac{x_1(n)}{2^{-b}}\right] + b_1\left[\frac{x_2(n)}{2^{-b}}\right]\right\}$$

1 1 1 1

− 0 0 0
  1

− 1

1 0 0 0

→ 0 1 1 1
  + 1

= −1 0 0 0

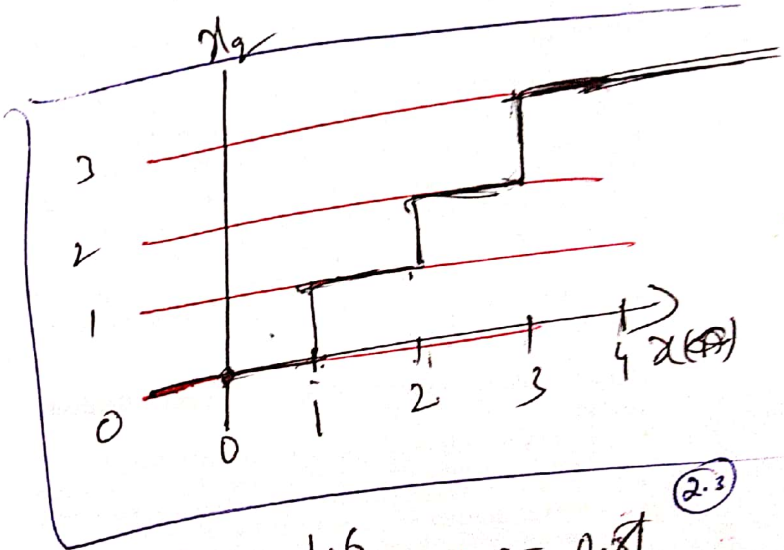
−8

Let

$b = 2$

$2^{-b} = 1/4$

00 − 0
01 − 1
10 − 2
11 − 3



2.3

$x_1(n) = 1.6$ $\quad a = 0.4$

$x_2(n) = 2.1$ $\quad b = 0.6$

$$y_1(n) = \frac{1}{4}\left[[0.4(1.6) + 0.6(2.1)]4\right] = 1.75$$

01.11

$\frac{1}{8}$

$$y(n) = \frac{1}{4}\left[0.4(6) + 0.6(8)\right] = 1.8$$

1 + 0.5
+ 0.25
+ 0.005

1.111