LATENT DIRICHLET ALLOCATION

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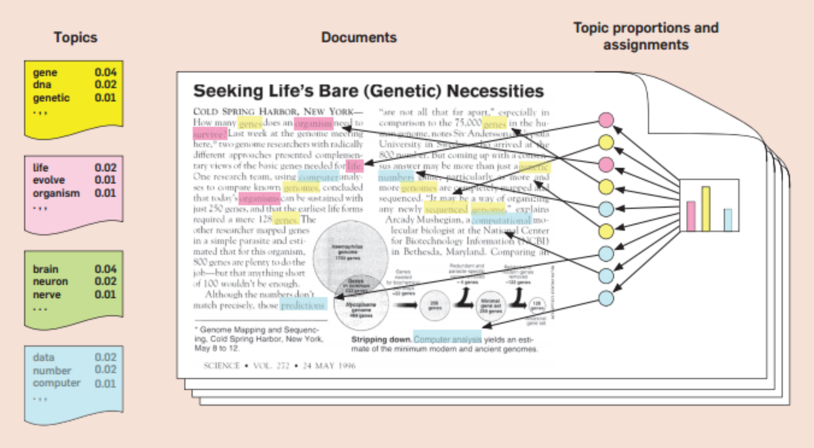
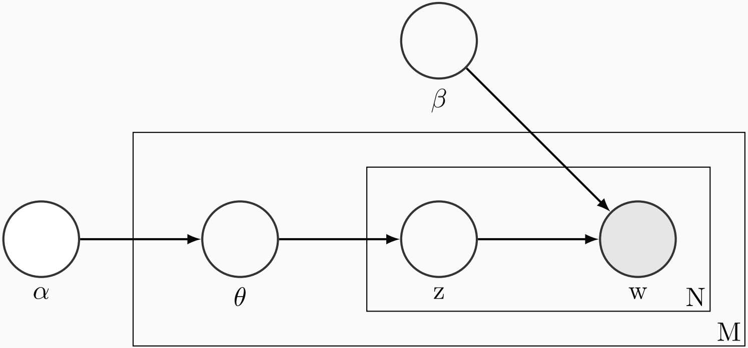
Topic Modeling



https://www.cs.princeton.edu/ blei/papers/Blei2012.pdf

Graphical model for LDA



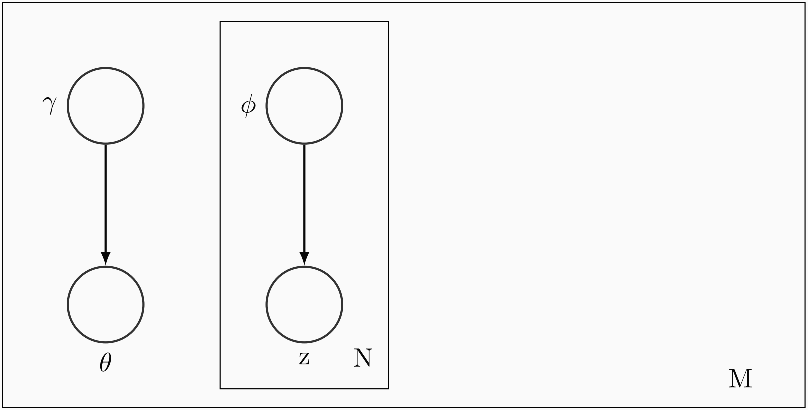
The "Bag of Words" and Exchangability

Assume documents are a "Bag of Words" leading to exchangeability in the distribution of words in a document

$$p(\mathbf{w} \mid \alpha, \beta) = \int \left(\prod_{n=1}^{N} p(w_n \mid \theta, \beta) \right) p(\theta \mid \alpha) \, d\theta$$

Documents within the corpus are also assumed to be exchangable

Variational EM



Variational EM

Likelihood, $p(\mathbf{w}\,|\,\alpha,\beta)$, required for inference on hidden parameters but intractable in this context.

Use approximation from statistical physics to bound the log-likelihood

$$\begin{aligned}\log p(\mathbf{w}\,|\,\alpha,\beta) &\geq E_q[\log p(\theta, z, \mathbf{w}\,|\,\alpha,\beta)] - E_q[\log q(\theta, z\,|\,\gamma,\phi)] \\ &=: \mathcal{L}(\gamma,\phi;\alpha,\beta)\end{aligned}$$

$$\log p(\mathbf{w}\,|\,\alpha,\beta) = \mathcal{L}(\gamma,\phi;\alpha,\beta) + \mathrm{D}(\,q(\theta, z\,|\,\gamma,\phi)\,||\,p(\theta, z\,|\,\mathbf{w},\alpha,\beta))$$

Variational EM

repeat

    # E-step

    for d in documents **do**

        repeat

            update ϕ_d (loop over all words and topics)

            update γ_d

            compute $\mathcal{L}_d$

        until convergence (i.e. relative change in $\mathcal{L}_d$ is less than ε)

    # M-step

    update α (Newton's method)

    update β

until convergence

How to update α

Blei et al (2003) describe a scheme for updating α with Newton's method in linear time

$$\alpha^{t+1} = \alpha^t - \mathbf{H}(f(\alpha^t))^{-1} \nabla f(\alpha^t)$$

How to update α

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However,

1. They provide implementation with $\alpha_i = \alpha_j$
2. It is better to optimize α on log-scale
3. On the log-scale, we cannot invert the Hessian in linear time

Data

Dataset of 1500 NIPS papers from 1988 to 1999, split into 90% train and 10% test portions.

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We constructed a *document* $\times$ *term* matrix, used as input to our R function.

	network	model	learning	function	input	neural	set	algorithm	system	data
[1,]	4	56	4	1	6	1	5	6	24	26
[2,]	25	71	6	5	1	15	15	0	3	40
[3,]	0	11	20	9	4	1	4	6	15	10
[4,]	18	0	36	8	5	1	1	18	3	1
[5,]	6	39	67	12	7	7	1	1	10	0

Example implementation

Topic 1	Topic 2	Topic 4	Topic 6	Topic 8
network	function	model	network	object
system	learning	data	training	visual
model	network	distribution	unit	model
learning	weight	gaussian	input	image
neural	error	parameter	hidden	motion
control	algorithm	algorithm	set	field
input	result	function	error	direction
dynamic	set	method	output	unit
output	neural	mean	neural	map
recurrent	number	component	data	position
rules	parameter	probability	weight	system
rule	case	likelihood	learning	eye
attractor	input	density	performance	images
point	bound	mixture	layer	representation
trajectory	training	matrix	model	view

$\gamma(w_{18}) =$
$(\mathbf{601.97}, \mathbf{556.37}, 0.06, 0.06, 0.06, \mathbf{366.60}, 0.06, 0.06, 99.42, 32.93)$

This paper provides a systematic analysis of the recurrent backpropagation (RBP) algorithm, introducing a number of new results. The main limitation of the RBP algorithm is that it assumes the convergence of the network to a stable fixed point in order to backpropagate the error signals. We show by experiment and eigenvalue analysis that this condition can be violated and that chaotic behavior can be avoided. Next we examine the advantages of RBP over the standard backpropagation algorithm. RBP is shown to build stable fixed points corresponding to the input patterns. This makes it an appropriate tool for content addressable memories, one-to-many function learning, and inverse problems.

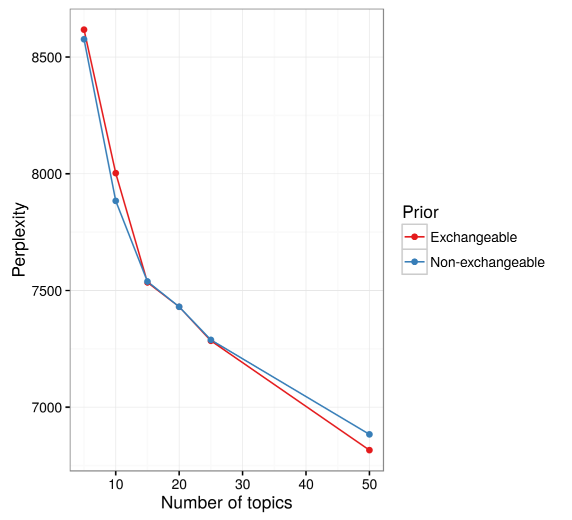
Document Modeling - Perplexity

For an unseen set of M documents,

$$\text{perplexity}(\mathcal{D}_{test}) = \exp\left\{-\frac{\sum_{d=1}^{M}\log p(w_d|\alpha,\beta)}{\sum_{d=1}^{M} N_d}\right\} = \exp\left\{-\frac{\mathcal{L}(\mathbf{w}_d)}{\sum_{d=1}^{M} N_d}\right\}$$

where N_d is the number of words in document d.

Document Modeling - Perplexity



Collaborative Filtering

At random remove one word from an unseen document

Find 'optimal' ϕ and γ using the $(N_d - 1)$ words left in the unseen document

Use these to calculate the likelihood of the removed word based on the rest of the document

Using likelihood

$$p(w|w_{obs}) = \sum_z \beta_{z,w} \frac{\gamma(w_{obs})_z}{\sum_i \gamma(w_{obs})_i} \tag{1}$$

Collaborative Filtering - An Example

e.g.

Random removed word "hand"

36th most probable word in topic 7 and the 51st most probable word in topic 1

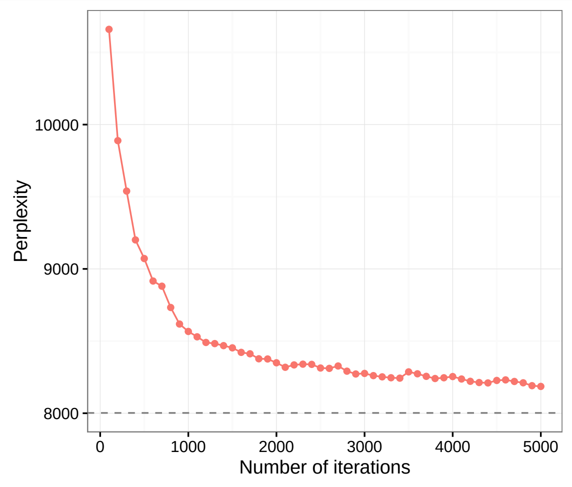
$\gamma(w_{obs})$ =(**458.57**, 0.06, **152.18**, 0.06, 71.04, 4.13, 0.06, **530.90**, 0.06, 55.51)

92^{nd} most likely word to be the final word

Number of intersections between 20 most likely words to be removed word and 50 most likely words for each topic (12, 8, 6, 5, 10, 9, 8, 15, 7, 5)

Define $\rho_t = (\tau_0 + t)^{-k}$, with $k \in (0.5, 1]$
Initialize λ randomly
for $t = 0$ to ∞ **do**
  # *E-step*:
  Initialize γ
  repeat
    update ϕ (loop over all words and topics)
    update γ
  until $\frac{1}{k}\sum_k |\text{change in } \gamma_{tk}| < \epsilon$
  # *M-step*:
  update λ

Stochastic Variational EM



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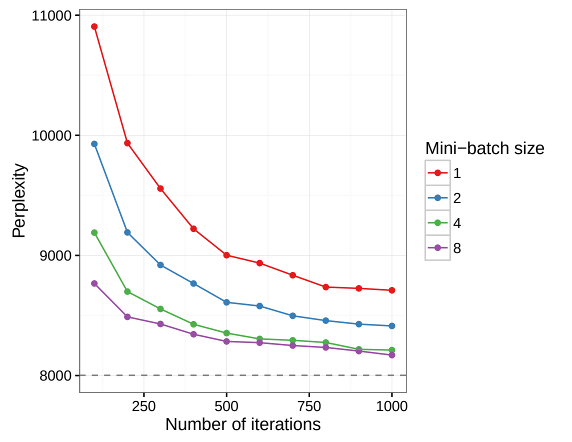

Discussion and Further Work

1. Choice for number of topics: Hierarchical Dirichlet Processes (Teh et al. 2006)
2. Measures of human interpretability for topic models (Chang et al. 2009)