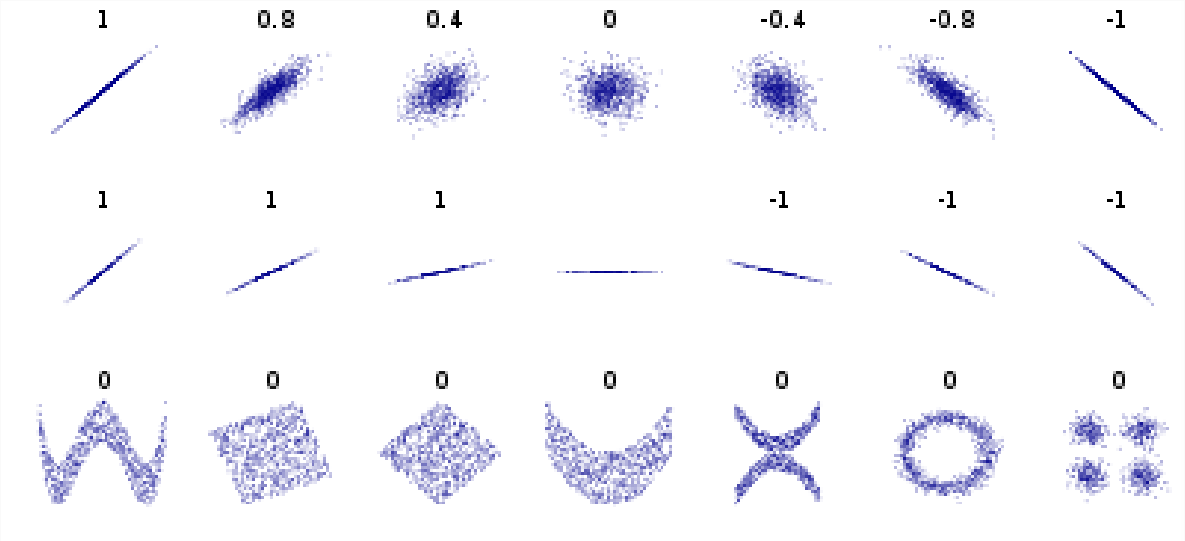
QUANTIFYING DEPENDENCE

Ella Kaye Kaspar Märtens Paul Vanetti Andi Wang

26th February 2016

Introduction

Goal: be able to quantify the level of dependence between two random variables.



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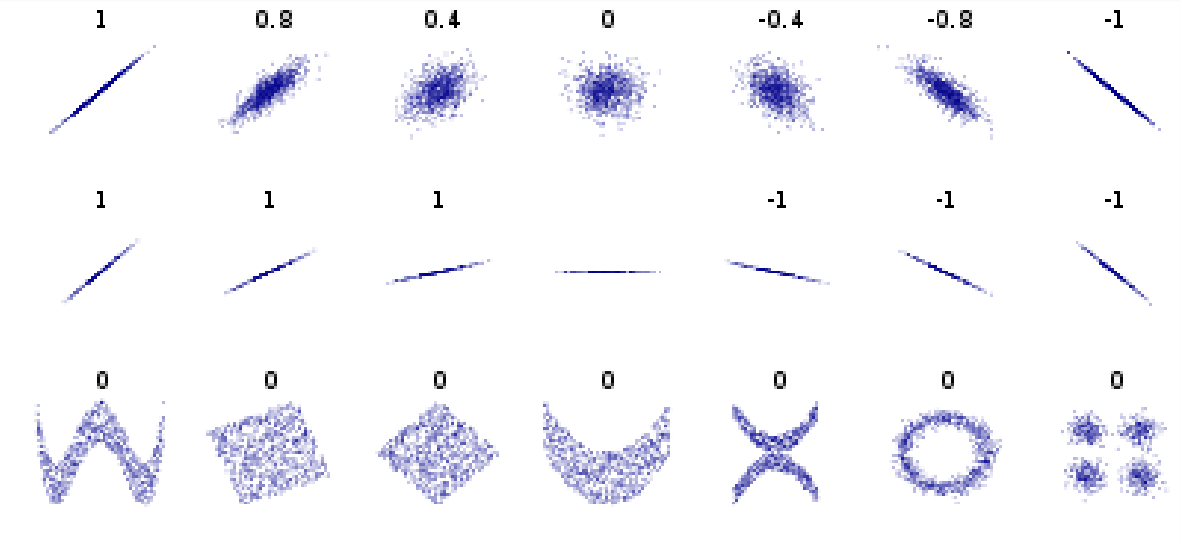

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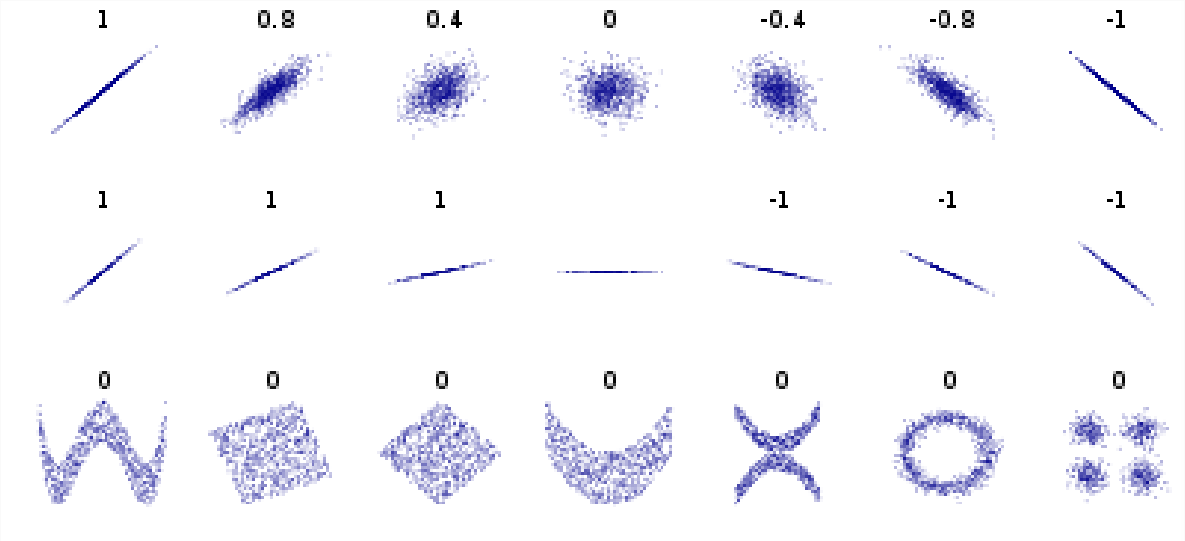

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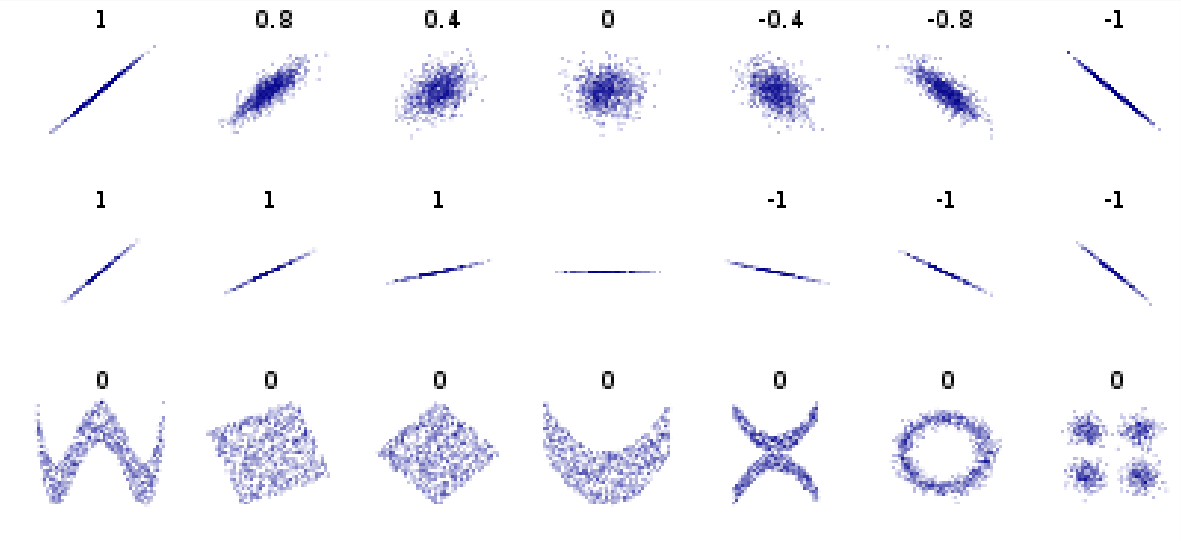
Want an *equitable* statistic; no preference for a specific relationship.



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Given a data sample, $I[X, Y]$ must be estimated.

Maximal Information Coefficient

Data set of real values $\{(x_i, y_i)\}_{i=1}^{n}$.

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Take the maximum.

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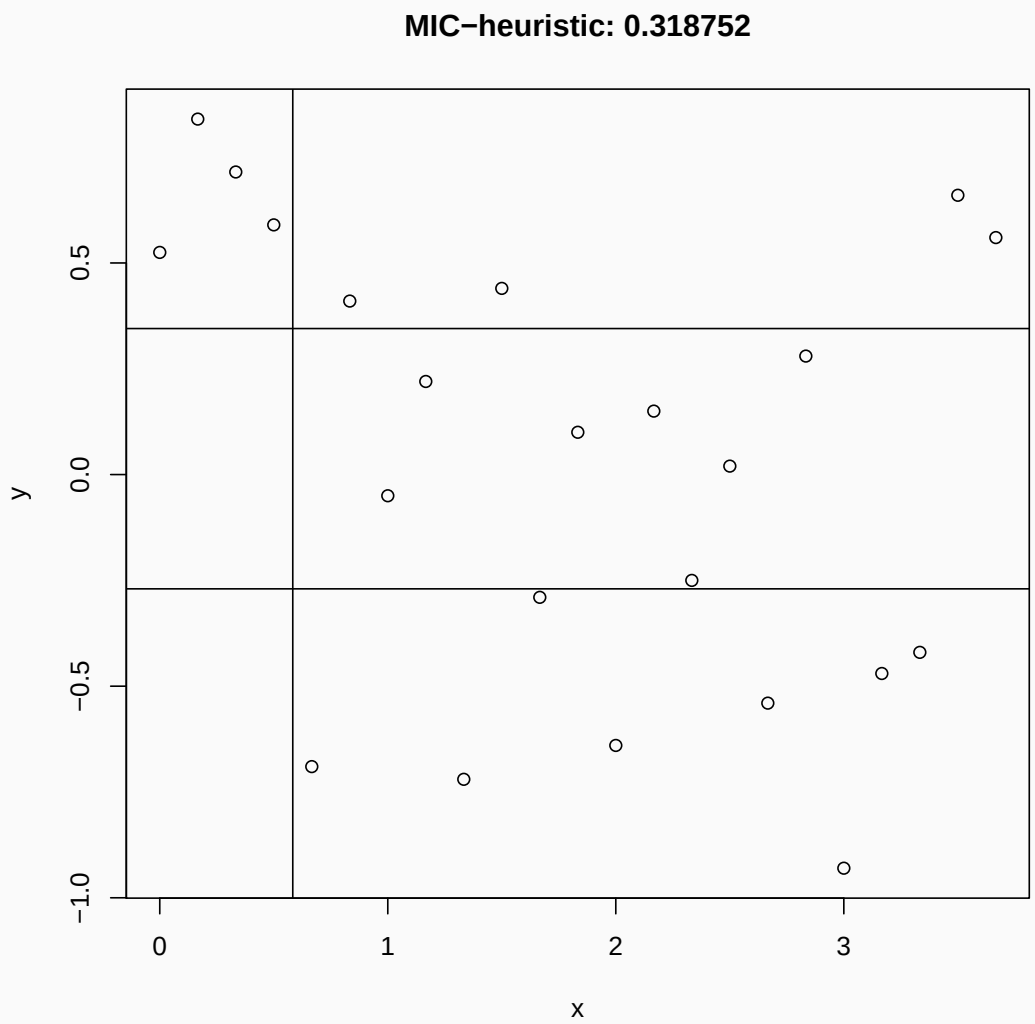

WHO data set

357 social, economic, health and political indicators from the World Health Organisation, for 202 countries.

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around 23000 missing values (32%)

Plot from Reshef et al.

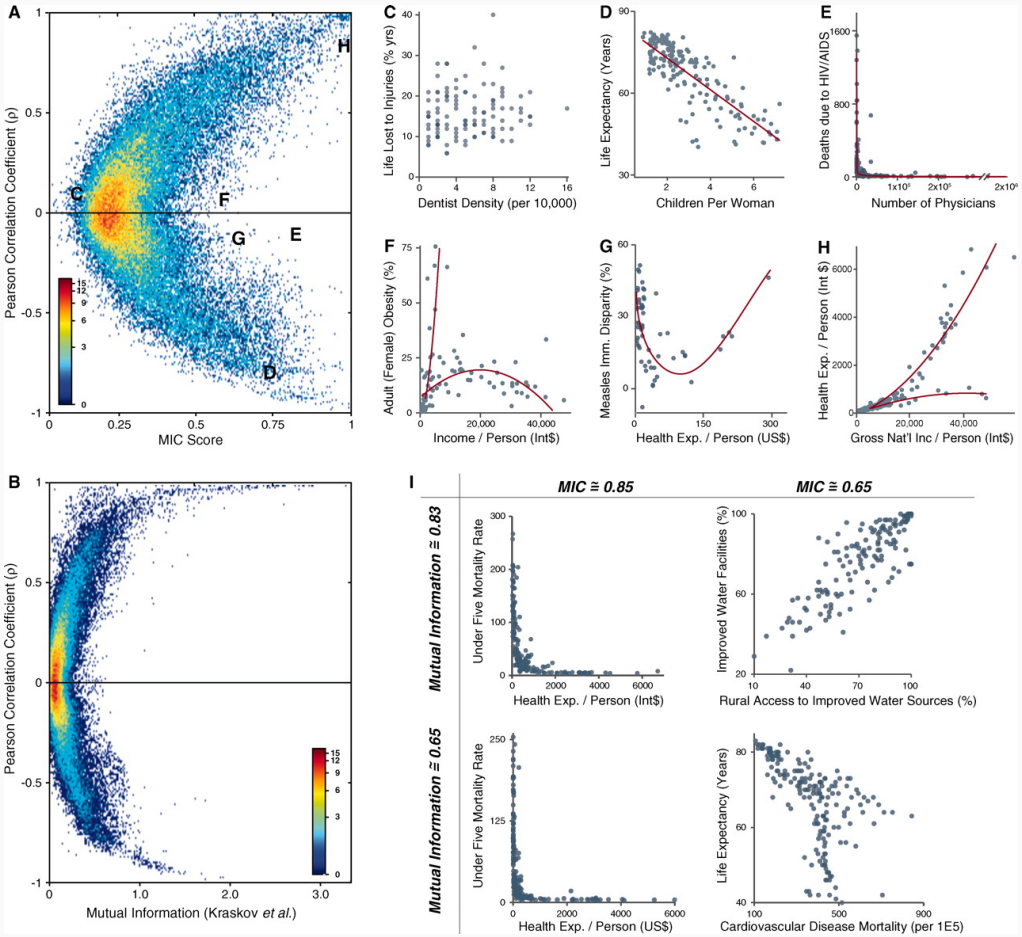
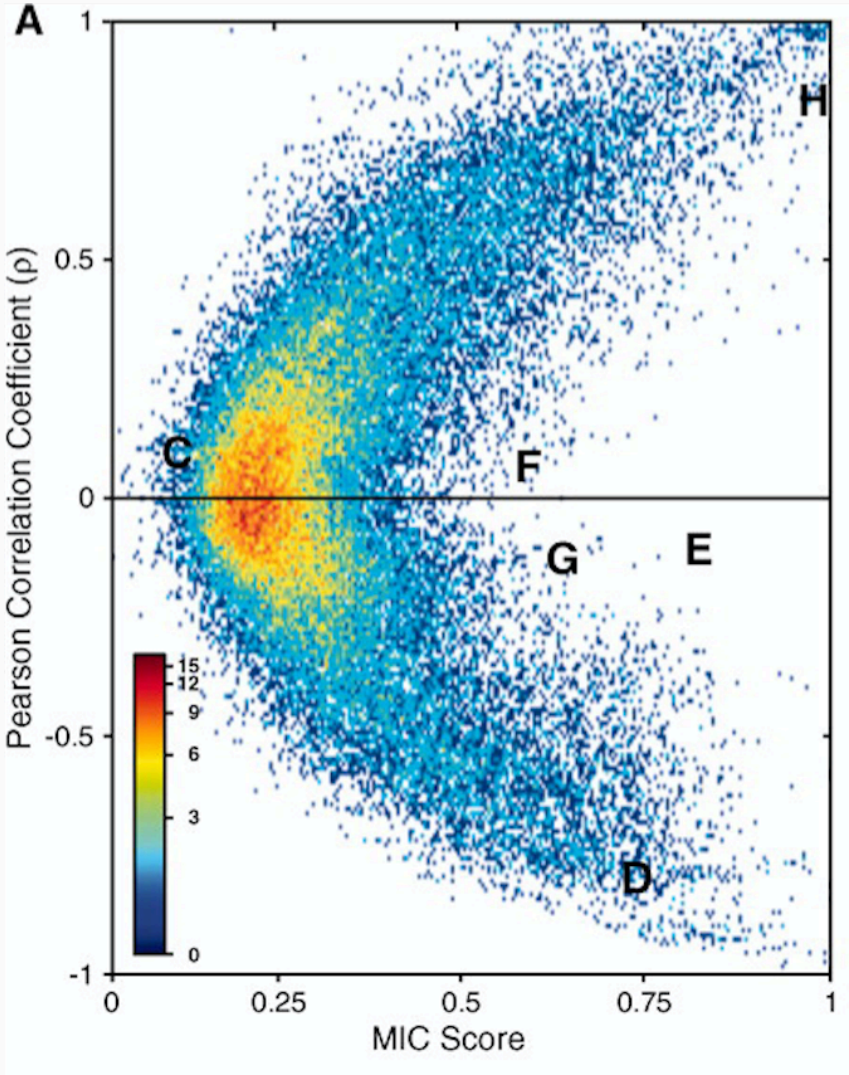


Figure: Figure 4 from [1]

MIC vs. Correlation



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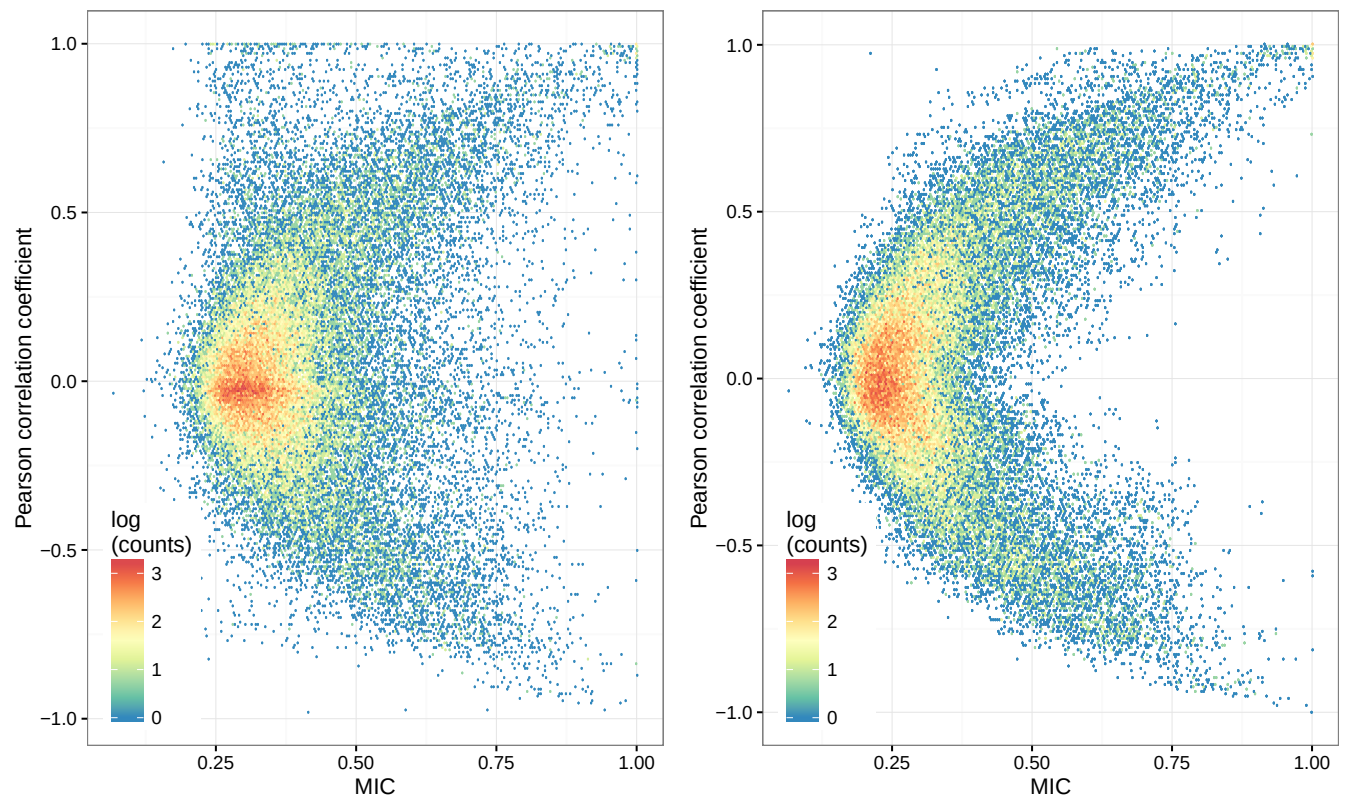
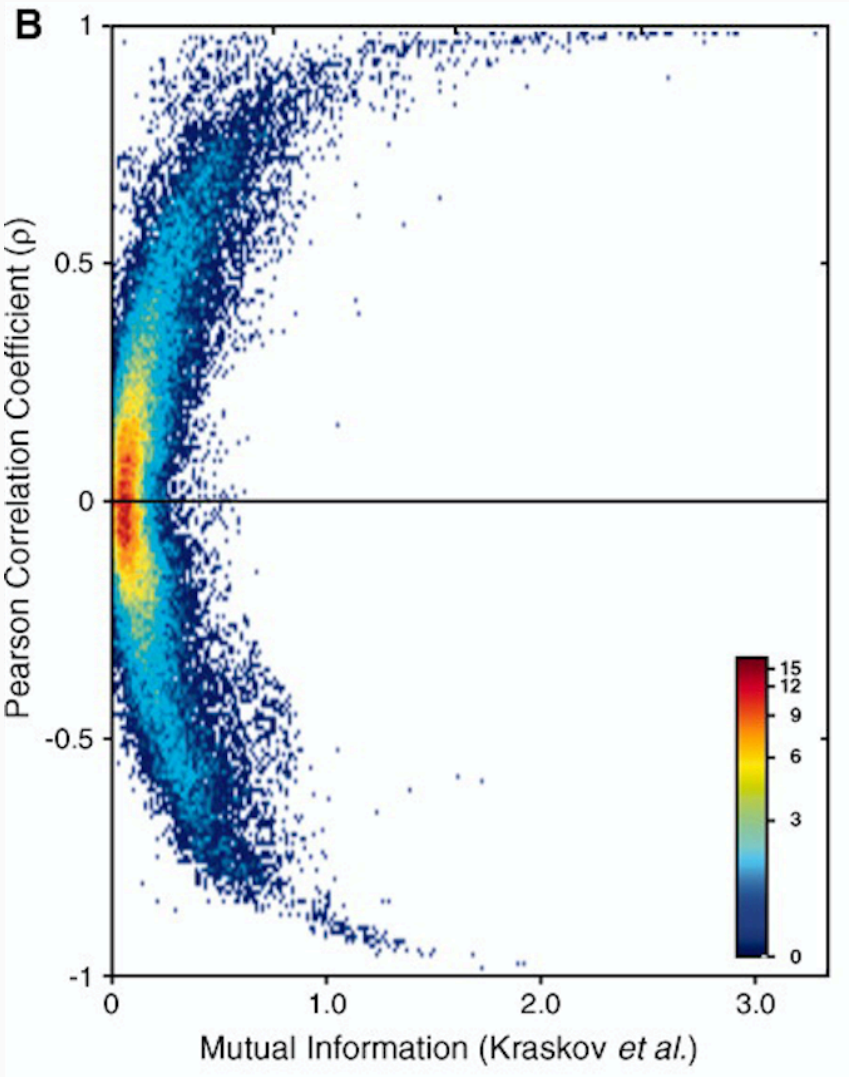


Figure: Replicating Figure 4A from [1]

I vs. Correlation



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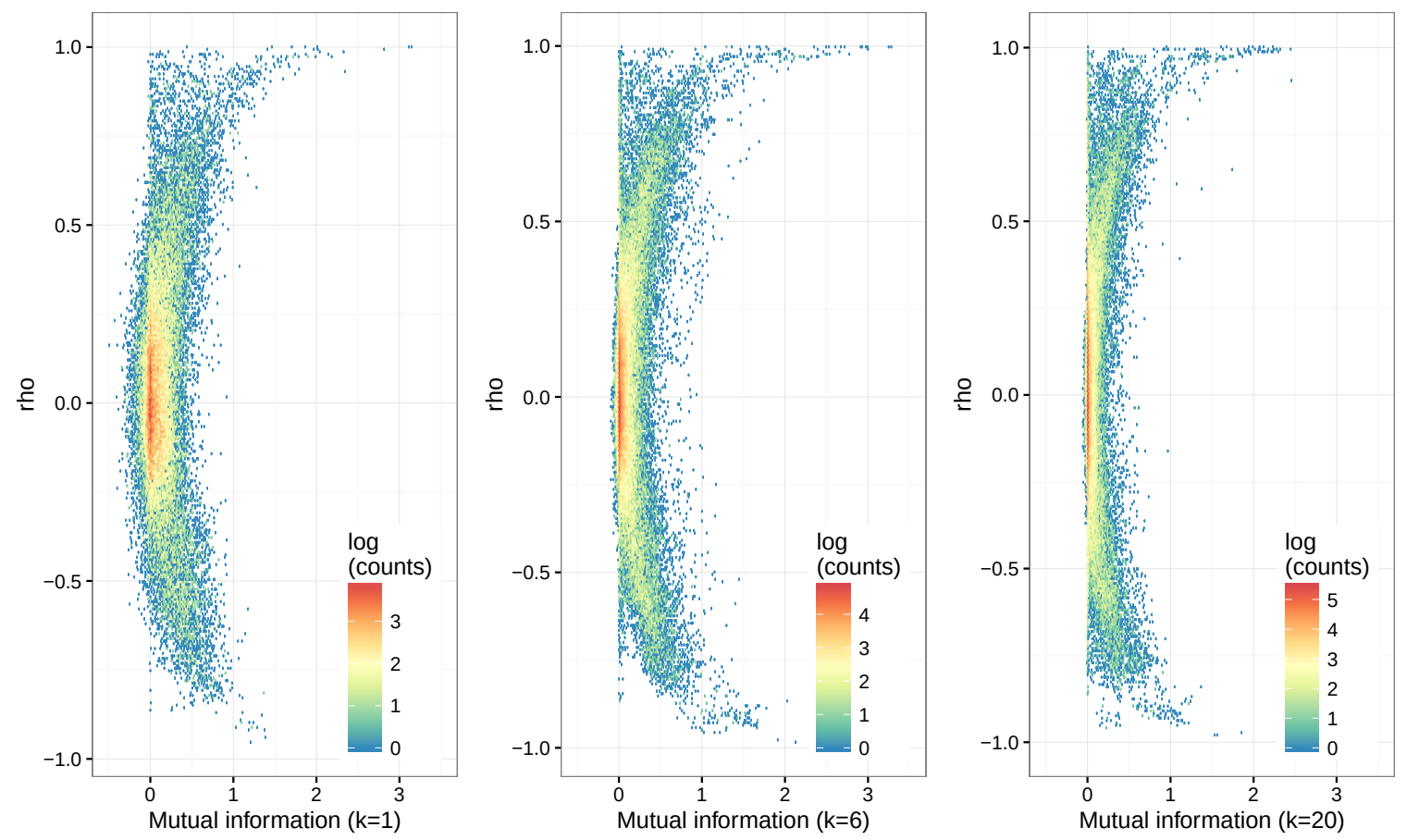
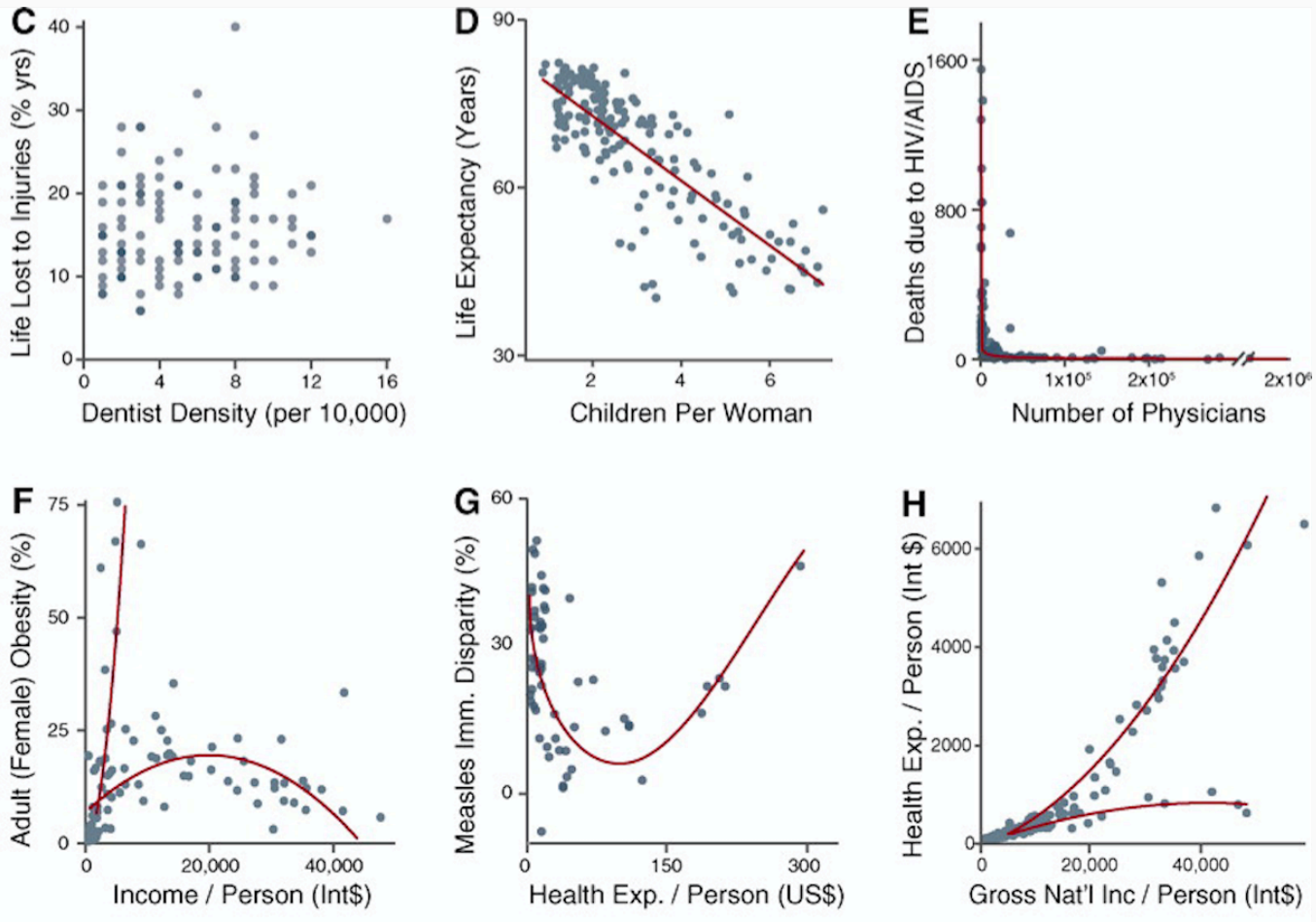


Figure: Replicating Figure 4B from [1]

Data Visualisation Choices



Yeast gene expression data

Yeast gene expression data (by Spellman et al):

- expression for 4381 genes
- measured at 23 time points

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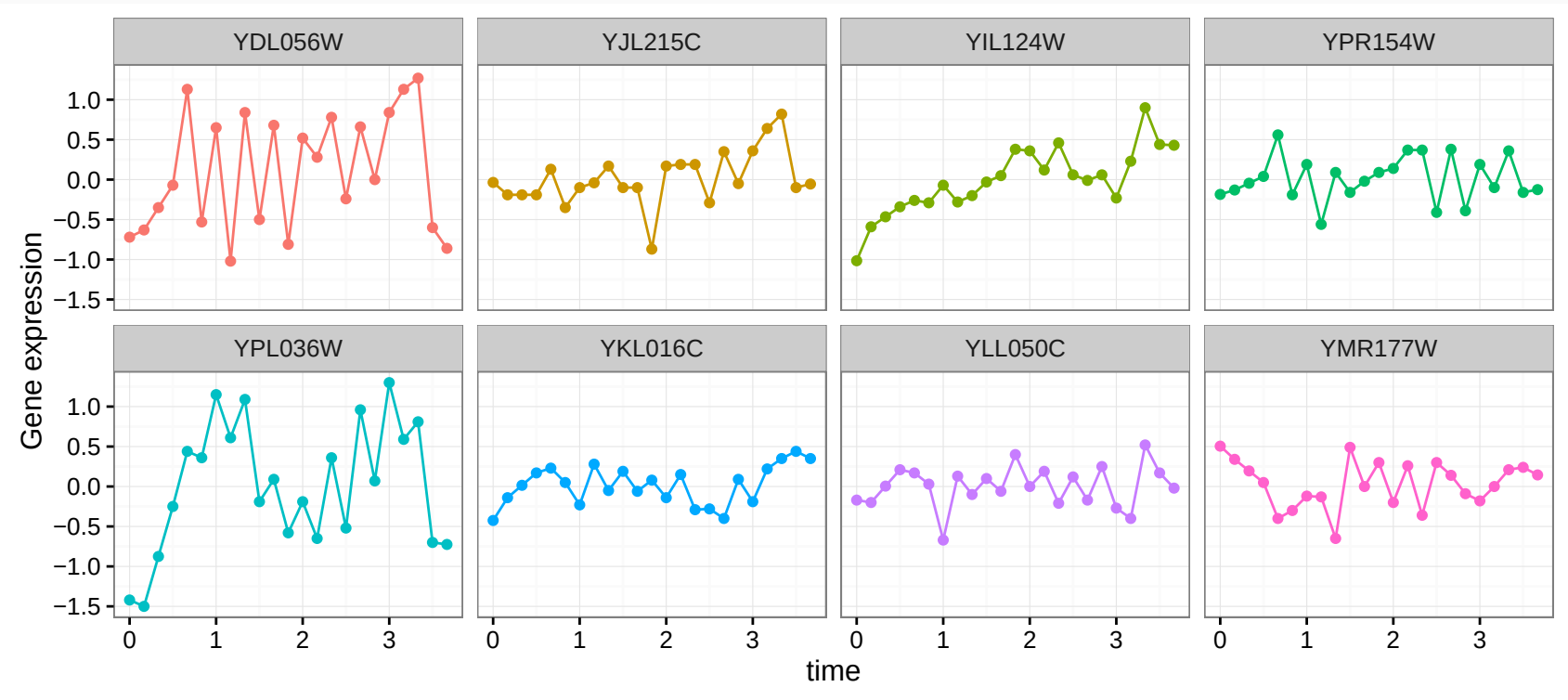
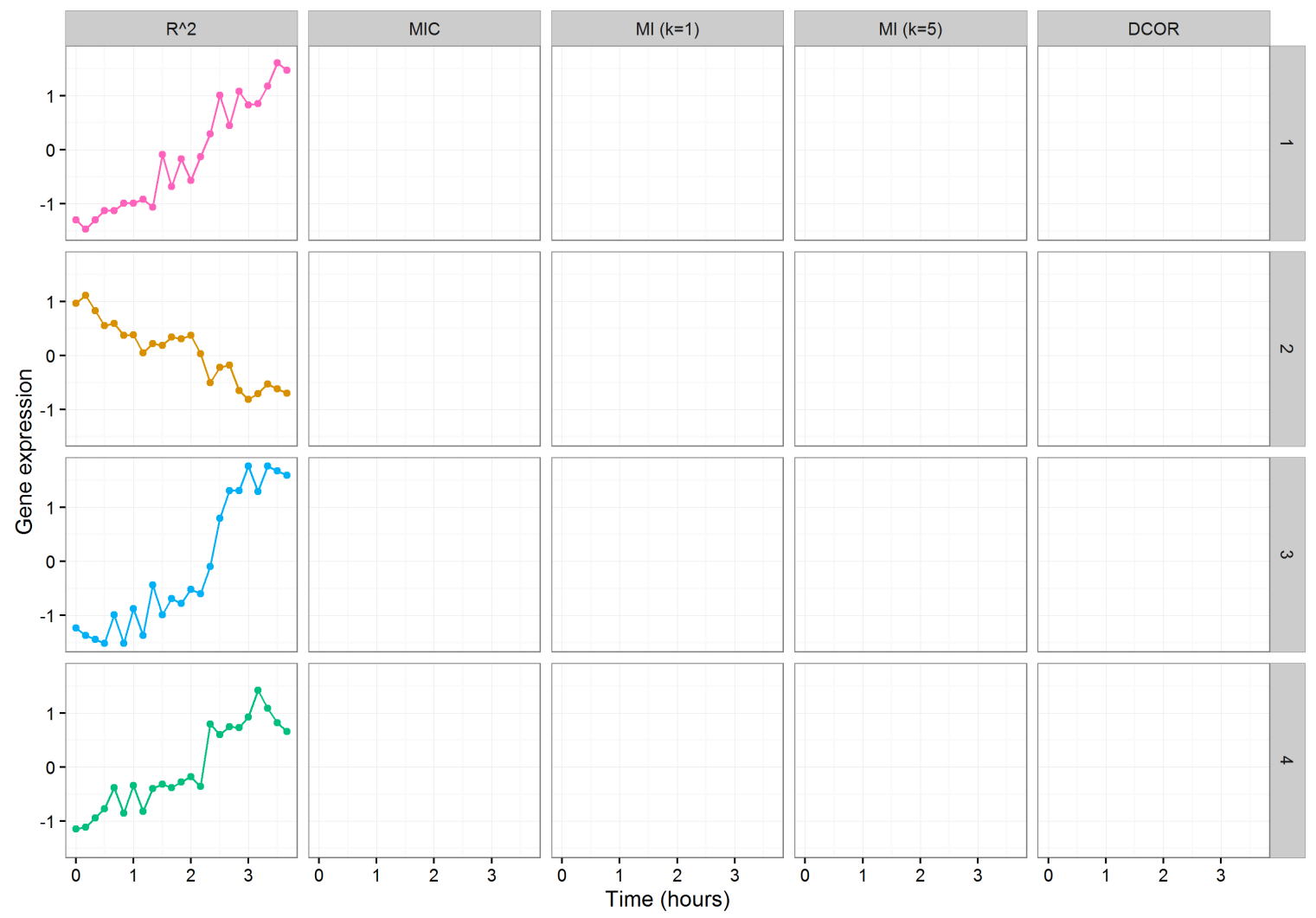


Comparing methods: TOP 4 dependencies



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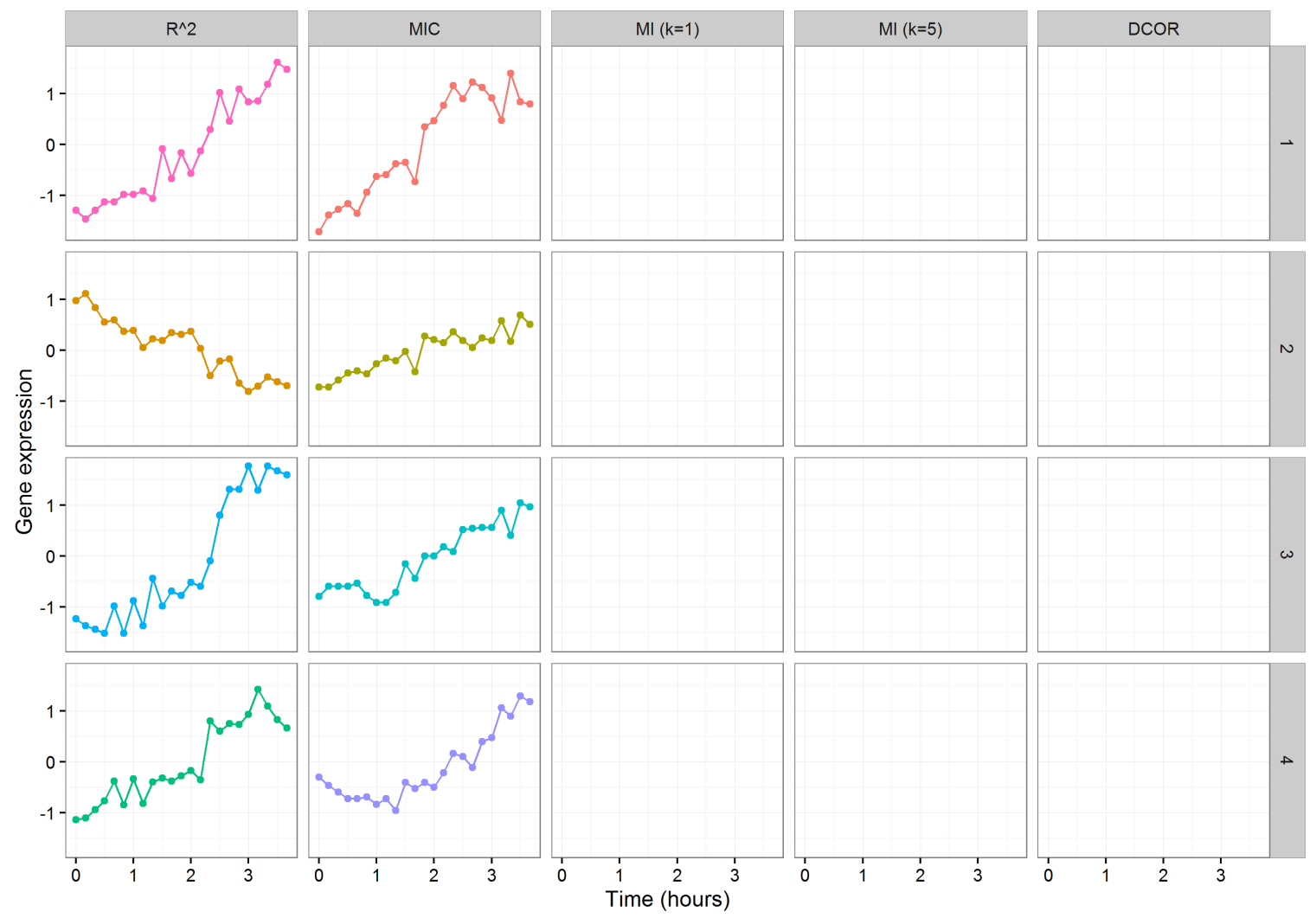

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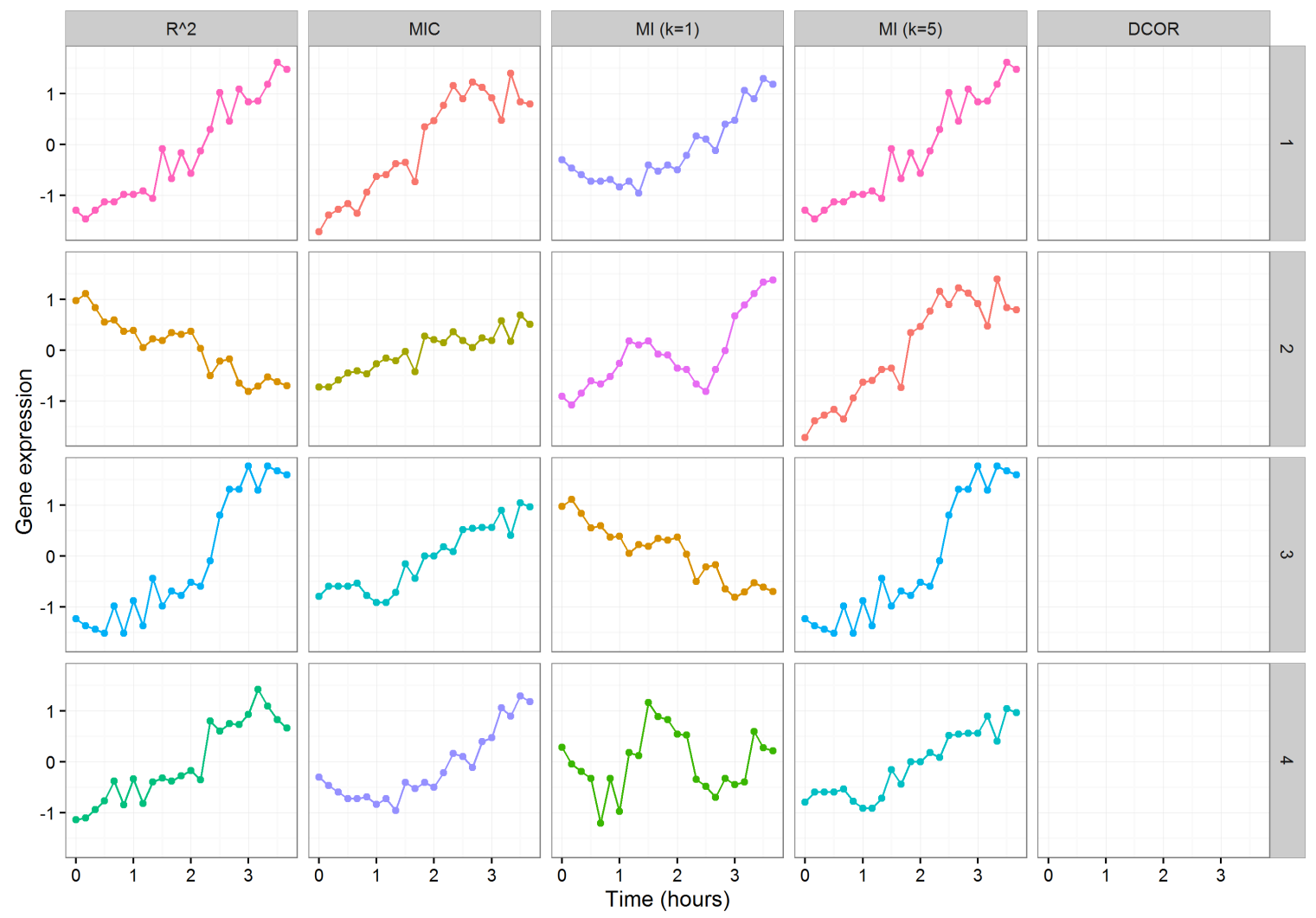

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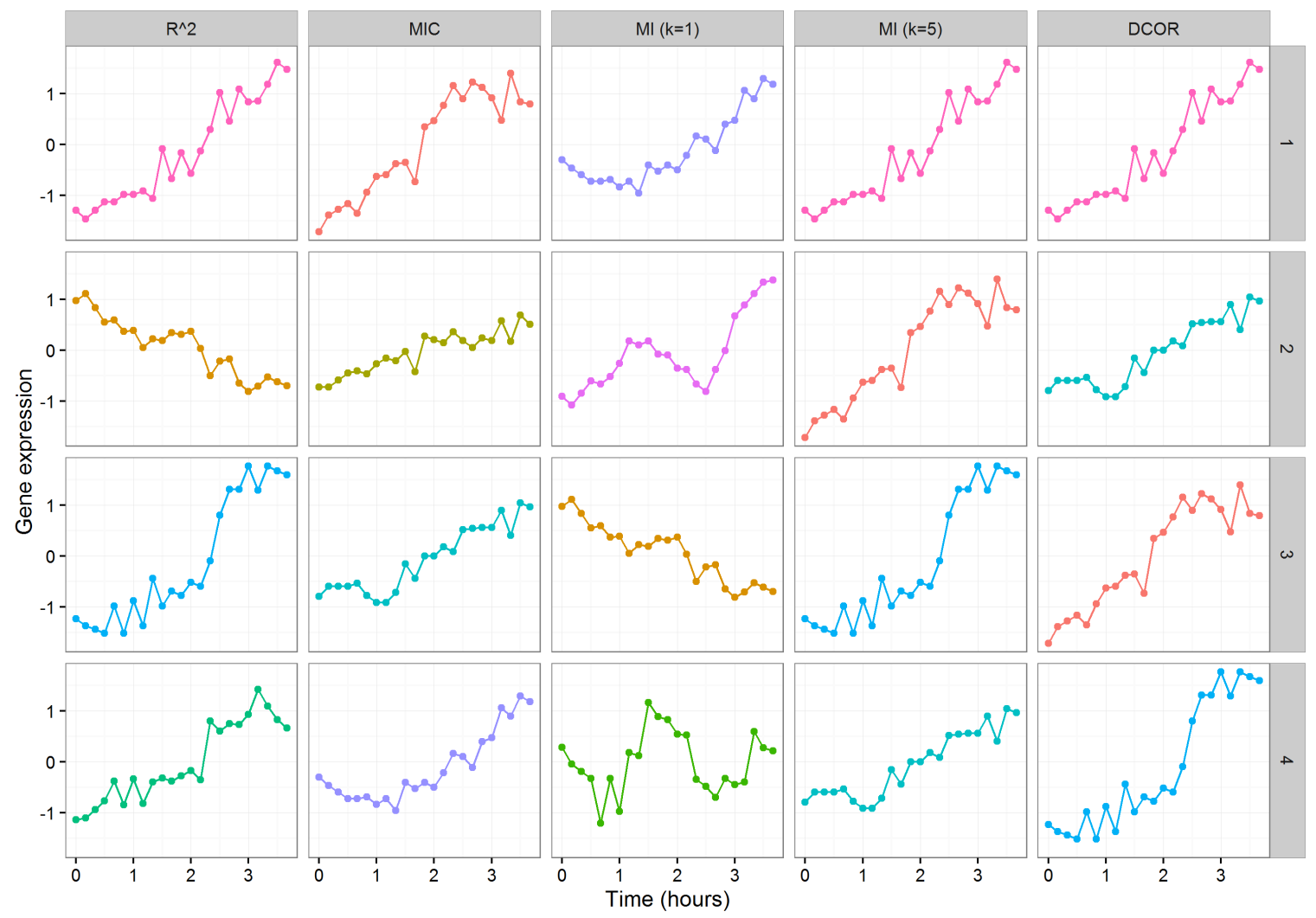

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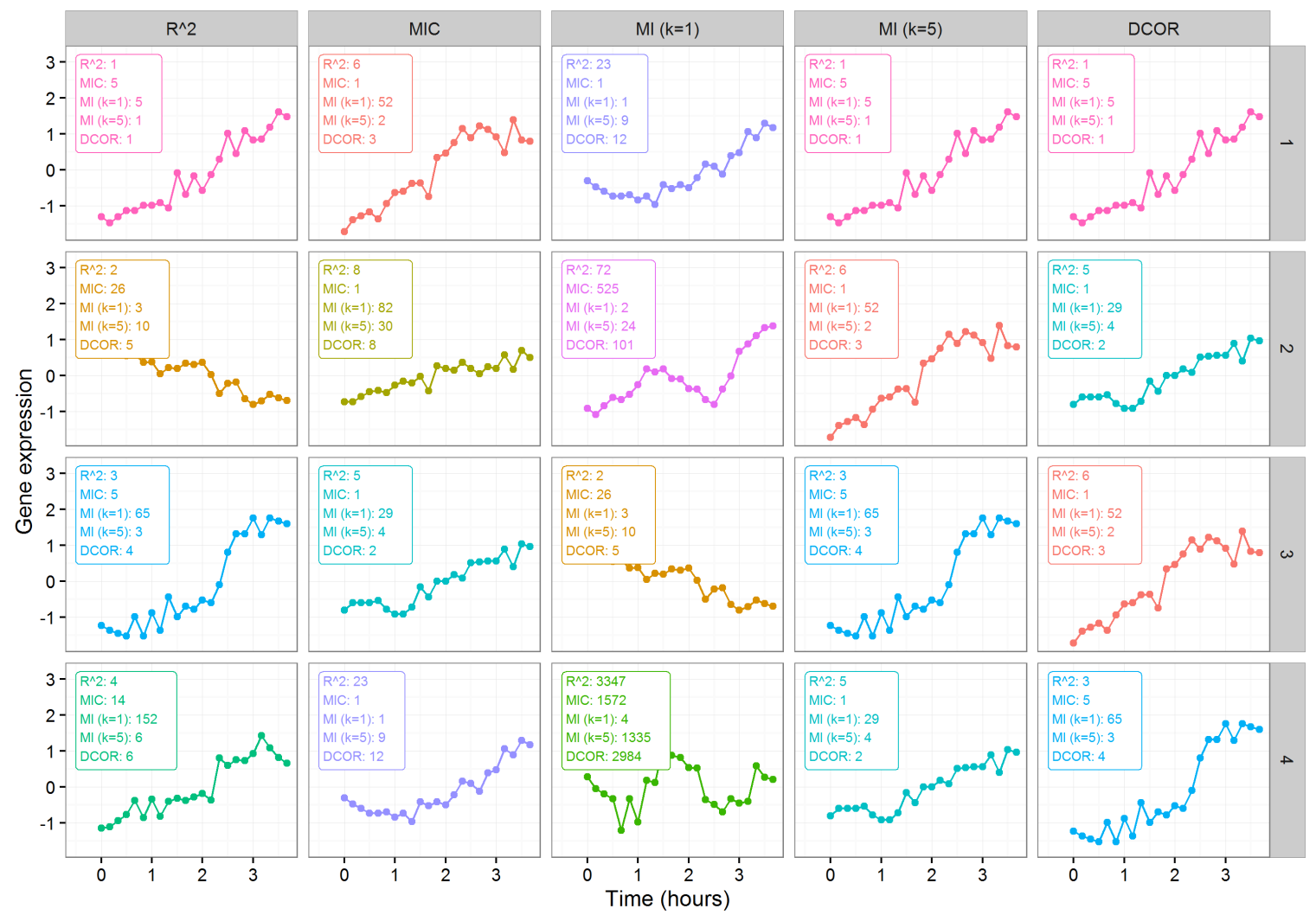

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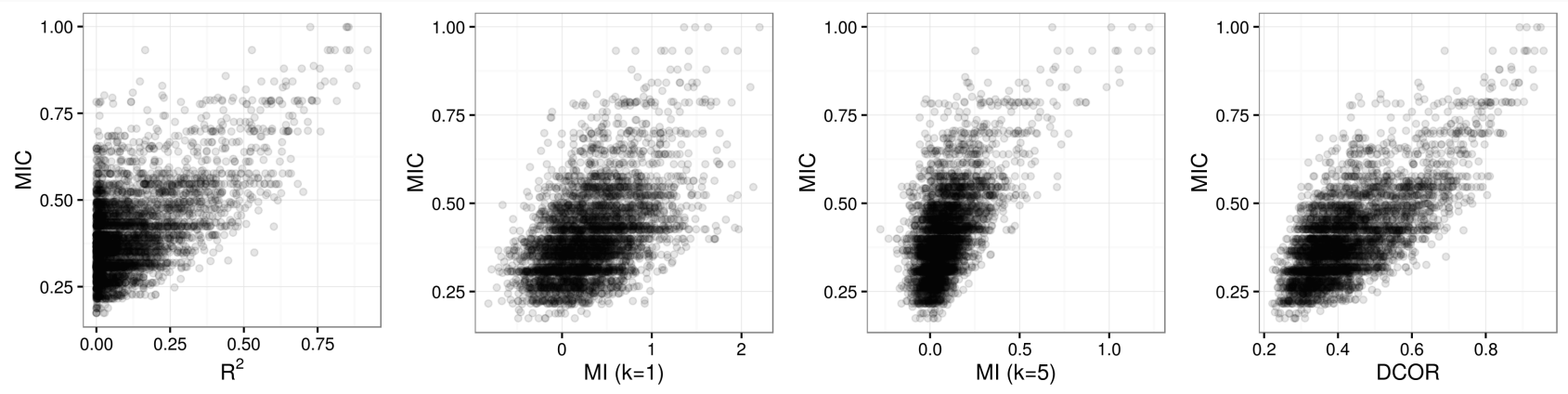

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Repeat this many times. Estimate the power by the proportion of values calculated from noisy data which are above the 95th-percentile of values from null data.

x(t) = ((-67/37 sin(39/25-12 t)-112/51 sin(25/16-10 t)-61/14 sin(69/44-6 t)+1164/17 sin(t+11/7)+611/131 sin(2 t+179/38)+103/41 sin(3 t+179/38)+189/20 sin(4 t+41/26)+355/41 sin(5 t+41/26)+121/26 sin(7 t+30/19)+6/29 sin(8 t+75/16)+11/20 sin(9 t+49/31)+43/33 sin(11 t+41/26)+2690/13) theta(67 pi-t) theta(t-63 pi)+(-89/79 sin(58/41-12 t)-41/16 sin(19/13-11 t)-10/13 sin(101/67-10 t)-151/33 sin(65/43-7 t)-120/31 sin(58/39-6 t)-31/46 sin(93/65-5 t)-113/33 sin(239/159-2 t)-1517/25 sin(25/16-t)+33/8 sin(3 t+43/27)+17/7 sin(4 t+103/22)+23/21 sin(8 t+38/23)+57/23 sin(9 t+68/41)-725/121) theta(63 pi-t) theta(t-59 pi)+(-73/38 sin(134/89-10 t)-181/40 sin(91/58-9 t)-77/54 sin(42/31-4 t)+1417/33 sin(t+61/39)+328/19 sin(2 t+63/40)+727/51 sin(3 t+113/24)+2971/27 sin(5 t+85/54)+11/9 sin(6 t+21/17)+70/9 sin(7 t+35/22)+62/23 sin(8 t+241/52)+73/26 sin(11 t+11/7)+37/17 sin(12 t+73/45)-31973/87) theta(59 pi-t) theta(t-55 pi)+(-4429/46 sin(69/44-t)+1722/29 sin(2 t+63/40)+276/37 sin(3 t+49/31)+379/18 sin(4 t+30/19)+40/19 sin(5 t+147/92)+281/29 sin(6 t+49/31)+23/25 sin(7 t+47/29)+161/31 sin(8 t+19/12)+11/49 sin(9 t+51/29)+98/31 sin(10 t+74/47)+3/5 sin(11 t+164/35)+17/22 sin(12 t+37/24)-3180/29) theta(55 pi-t) theta(t-51 pi)+(-23/4 sin(58/39-17 t)-118/23 sin(26/17-16 t)-48/29 sin(190/127-15 t)-86/39 sin(49/32-14 t)-421/31 sin(87/56-9 t)-1201/50 sin(48/31-6 t)-417/5 sin(53/34-4 t)-18496/349 sin(53/34-3 t)-2629/31 sin(25/16-2 t)+7245/88 sin(t+69/44)+13676/291 sin(5 t+41/26)+191/11 sin(7 t+49/31)+1550/141 sin(8 t+83/52)+90/17 sin(10 t+136/29)+190/49 sin(11 t+29/18)+14/19 sin(12 t+50/31)+331/24 sin(13 t+8/5)-9905/27) theta(51 pi-t) theta(t-47 pi)+(-926/161 sin(1/58-11 t)-137/19 sin(1/167-9 t)-1241/98 sin(1/38-5 t)-257/12 sin(1/62-3 t)+797/13 sin(t+1/339)+364/11 sin(2 t+81/26)+641/40 sin(4 t+22/7)+491/46 sin(6 t+103/33)+251/28 sin(7 t+1/88)+47/6 sin(8 t+47/15)+158/25 sin(10 t+117/37)+75/14 sin(12 t+85/27)-22535/33) theta(47 pi-t) theta(t-43 pi)+(-11/17 sin(22/25-12 t)-44/31 sin(62/41-11 t)-35/19 sin(87/56-8 t)-58/33 sin(98/65-5 t)+1948/41 sin(t+5/31)+682/27 sin(2 t+155/43)+247/15 sin(3 t+33/25)+156/31 sin(4 t+16/11)+75/26 sin(6 t+78/25)+339/97 sin(7 t+101/36)+3/28 sin(9 t+119/43)+21/32 sin(10 t+1381/345)-16712/25) theta(43 pi-t) theta(t-39 pi)+(-5/12 sin(20/41-6 t)-23/26 sin(1/35-5 t)-38/31 sin(58/39-4 t)+587/24 sin(t+49/22)+425/63 sin(2 t+281/78)+50/29 sin(3 t+91/31)+4/7 sin(7 t+29/37)+8/37 sin(8 t+87/31)+10/51 sin(9 t+47/28)+3/7 sin(10 t+63/17)+1/17 sin(11 t+29/12)+5/29 sin(12 t+67/15)-11925/17) theta(39 pi-t) theta(t-35 pi)+(-9/25 sin(41/36-5 t)-39/38 sin(4/3-4 t)-27/13 sin(1/48-3 t)-79/33 sin(77/60-2 t)+818/29 sin(t+19/15)+3/8 sin(6 t+194/47)+7/23 sin(7 t+71/23)-9803/20) theta(35 pi-t) theta(t-31 pi)+(-397/22 sin(2/15-t)+62/21 sin(2 t+2/11)+58/27 sin(3 t+37/9)+11/17 sin(4 t+65/38)+9/14 sin(5 t+89/31)+114/229 sin(6 t+27/31)+3/35 sin(7 t+185/42)-34395/43) theta(31 pi-t) theta(t-27 pi)+(-93/70 sin(1/20-7 t)-97/28 sin(86/115-3 t)+27721/616 sin(t+5/43)+221/44 sin(2 t+5/51)+96/35 sin(4 t+10/3)+14/15 sin(5 t+77/50)+41/42 sin(6 t+11/5)-33646/41) theta(27 pi-t) theta(t-23 pi)+(-34/25 sin(25/29-6 t)-59/27 sin(31/26-4 t)-23/4 sin(38/33-2 t)+2071/35 sin(t+24/13)+101/42 sin(3 t+22/27)+30/17 sin(5 t+57/40)+13/18 sin(7 t+27/10)-35021/76) theta(23 pi-t) theta(t-19 pi)+(-19/33 sin(25/18-4 t)-47/30 sin(54/37-3 t)+1195/14 sin(t+21/26)+12/13 sin(2 t+93/34)+18/13 sin(5 t+43/27)-7251/20) theta(19 pi-t) theta(t-15 pi)+(-247/45 sin(53/40-10 t)-107/24 sin(19/51-8 t)+3579/23 sin(t+237/71)+7181/68 sin(2 t+77/51)+1574/61 sin(3 t+80/29)+...

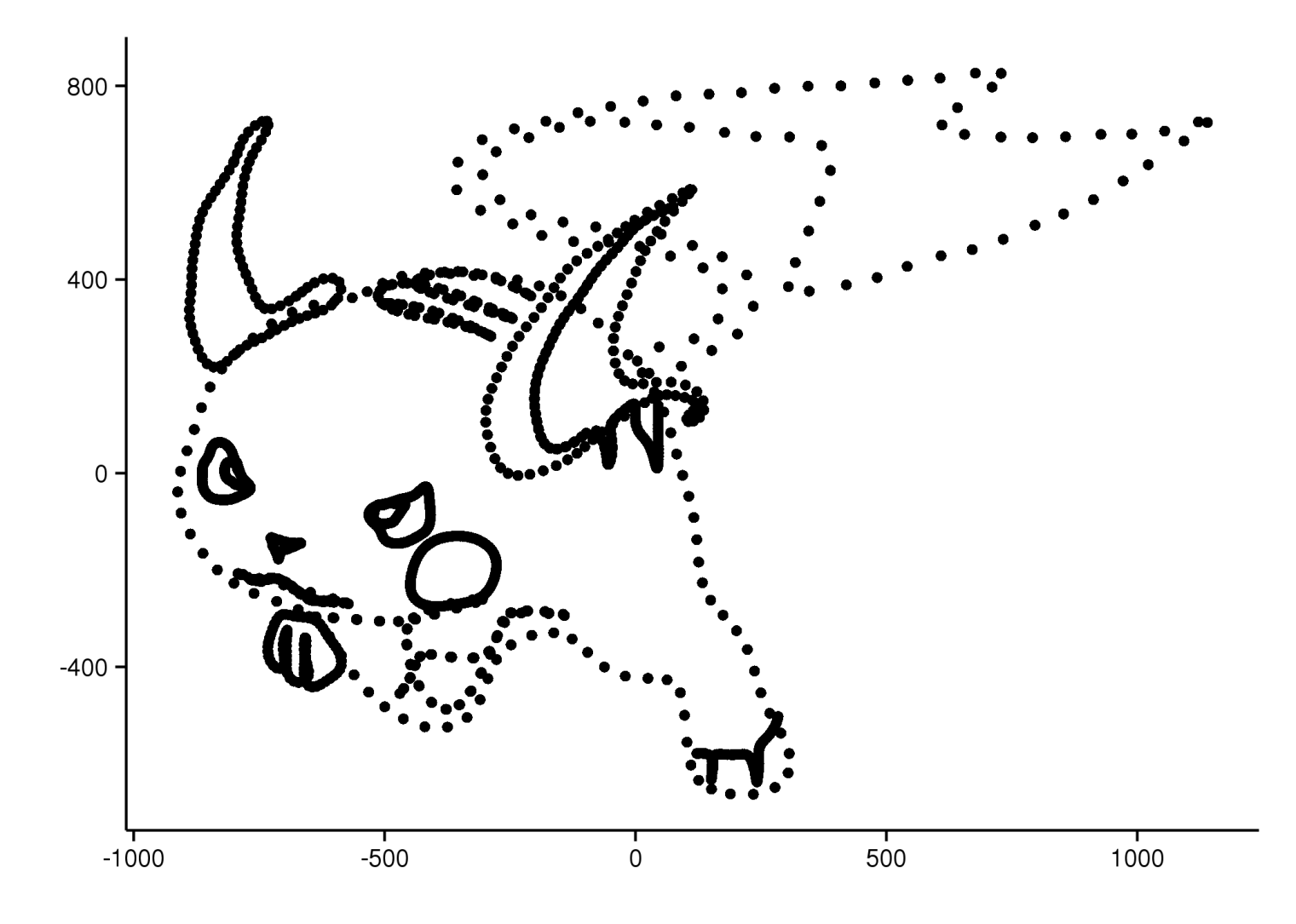
...+466/23 sin(4 t+72/19)+97/39 sin(5 t+128/43)+201/29 sin(6 t+11/5)+329/31 sin(7 t+127/31)+48/37 sin(9 t+99/31)+74/27 sin(11 t+23/28)+53/20 sin(12 t+143/31)-771/16) theta(15 pi-t) theta(t-11 pi)+(-1/13 sin(2/3-16 t)-2/39 sin(43/46-14 t)-26/17 sin(9/22-9 t)-219/58 sin(41/33-7 t)+4135/46 sin(t+115/28)+2992/43 sin(2 t+80/43)+943/42 sin(3 t+159/38)+174/35 sin(4 t+301/150)+671/84 sin(5 t+137/34)+67/19 sin(6 t+13/25)+51/16 sin(8 t+3/11)+49/16 sin(10 t+46/47)+21/22 sin(11 t+108/31)+75/43 sin(12 t+31/38)+16/15 sin(13 t+36/13)+34/31 sin(15 t+37/14)-37942/49) theta(11 pi-t) theta(t-7 pi)+(-25/49 sin(33/32-14 t)-67/5 sin(13/33-4 t)-1711/46 sin(31/77-3 t)+2714/5 sin(t+5/8)+1370/23 sin(2 t+56/19)+179/9 sin(5 t+57/17)+81/11 sin(6 t+118/41)+244/23 sin(7 t+83/41)+1069/91 sin(8 t+66/35)+233/37 sin(9 t+115/43)+106/31 sin(10 t+12/17)+63/19 sin(11 t+5/8)+145/22 sin(12 t+4/13)+87/37 sin(13 t+6/13)+73/33 sin(15 t+62/21)+77/26 sin(16 t+25/12)+49/29 sin(17 t+49/15)-17486/67) theta(7 pi-t) theta(t-3 pi)+(-35/29 sin(19/26-27 t)-102/41 sin(12/25-26 t)-255/47 sin(41/36-18 t)-1242/73 sin(13/23-12 t)-1113/32 sin(15/17-6 t)-2824/31 sin(49/32-3 t)+9066/19 sin(t+33/20)+3402/13 sin(2 t+52/27)+981/11 sin(4 t+32/19)+1561/17 sin(5 t+37/25)+46 sin(7 t+137/60)+1062/35 sin(8 t+159/40)+135/7 sin(9 t+178/59)+127/25 sin(10 t+169/36)+109/8 sin(11 t+63/47)+359/29 sin(13 t+88/41)+103/15 sin(14 t+92/29)+367/54 sin(15 t+63/22)+181/23 sin(16 t+98/29)+74/35 sin(17 t+427/95)+65/49 sin(19 t+19/24)+4/11 sin(20 t+25/29)+25/18 sin(21 t+33/41)+180/43 sin(22 t+37/11)+63/23 sin(23 t+56/17)+38/23 sin(24 t+202/47)+44/37 sin(25 t+52/15)+47/25 sin(28 t+73/16)+7/6 sin(29 t+90/31)+853/3) theta(3 pi-t) theta(t+pi)) theta(sqrt(sgn(sin(t/2))))

y(t) = ((-122/39 sin(36/23-10 t)-227/16 sin(47/30-5 t)+171/8 sin(t+11/7)+887/37 sin(2 t+63/40)+794/35 sin(3 t+52/33)+425/19 sin(4 t+52/33)+25/24 sin(6 t+52/33)+34/31 sin(7 t+35/22)+39/17 sin(8 t+51/32)+115/17 sin(9 t+49/31)+33/46 sin(11 t+35/22)+45/58 sin(12 t+27/17)-27039/46) theta(67 pi-t) theta(t-63 pi)+(-97/19 sin(59/39-11 t)-2/21 sin(33/82-10 t)-674/23 sin(36/23-3 t)-677/39 sin(66/43-2 t)-347/73 sin(304/203-t)+219/5 sin(4 t+51/32)+283/39 sin(5 t+39/23)+326/37 sin(6 t+47/29)+104/27 sin(7 t+43/26)+41/23 sin(8 t+31/19)+103/49 sin(9 t+43/26)+152/91 sin(12 t+35/23)+4799/64) theta(63 pi-t) theta(t-59 pi)+(-99/28 sin(95/61-12 t)-53/17 sin(25/16-7 t)-638/21 sin(58/37-5 t)+1635/46 sin(t+85/54)+72/23 sin(2 t+25/16)+181/22 sin(3 t+64/41)+367/19 sin(4 t+63/40)+226/25 sin(6 t+202/43)+8/7 sin(8 t+38/25)+80/23 sin(9 t+113/24)+240/49 sin(10 t+174/37)+23/33 sin(11 t+5/3)+5633/16) theta(59 pi-t) theta(t-55 pi)+(-457/45 sin(80/51-3 t)-8497/36 sin(69/44-t)+11/18 sin(2 t+117/25)+137/15 sin(4 t+30/19)+79/18 sin(5 t+49/31)+249/37 sin(6 t+41/26)+155/36 sin(7 t+30/19)+155/34 sin(8 t+65/41)+44/15 sin(9 t+165/103)+73/31 sin(10 t+46/29)+47/38 sin(11 t+13/8)+37/34 sin(12 t+51/32)+6758/23) theta(55 pi-t) theta(t-51 pi)+(-873/238 sin(106/71-14 t)-108/35 sin(118/79-13 t)-179/24 sin(38/25-12 t)-242/21 sin(49/32-11 t)-13/25 sin(43/36-10 t)-225/52 sin(59/38-8 t)-1016/47 sin(67/43-7 t)-1823/30 sin(39/25-4 t)-1150/29 sin(64/41-3 t)-454/13 sin(36/23-t)+1079/59 sin(2 t+36/23)+1417/58 sin(5 t+63/40)+313/14 sin(6 t+41/26)+336/73 sin(9 t+45/29)+10/9 sin(15 t+11/7)+111/43 sin(16 t+59/36)+34/29 sin(17 t+56/33)-9919/29) theta(51 pi-t) theta(t-47 pi)+(-37/19 sin(5/31-10 t)+790/31 sin(t+64/21)+81/16 sin(2 t+14/39)+169/31 sin(3 t+249/71)+649/130 sin(4 t+7/48)+122/31 sin(5 t+218/67)+43/15 sin(6 t+3/41)+105/37 sin(7 t+173/55)+172/69 sin(8 t+5/36)+55/29 sin(9 t+137/43)+62/37 sin(11 t+81/25)+51/37 sin(12 t+2/23)-16525/69) theta(47 pi-t) theta(t-43 pi)+(-28/17 sin(19/27-7 t)-785/22 sin(17/47-3 t)+2329/65 sin(t+58/13)+565/22 sin(2 t+59/31)+691/31 sin(4 t+23/70)+53/35 sin(5 t+39/25)+99/29 sin(6 t+93/20)+23/18 sin(8 t+15/23)+23/28 sin(9 t+95/26)+11/29 sin(10 t+73/20)+92/35 sin(11 t+191/53)+14/47 sin(12 t+138/37)-44969/122) theta(43 pi-t) theta(t-39 pi)+(-3/17 sin(1/38-12 t)-6/5 sin(19/22-4 t)-83/46 sin(47/59-3 t)+439/29 sin(t+119/33)+300/41 sin(2 t+47/29)+38/27 sin(5 t+296/71)+21/29 sin(6 t+53/28)+5/32 sin(7 t+16/23)+7/19 sin(8 t+45/91)+3/13 sin(9 t+107/25)+9/41 sin(10 t+17/9)+2/35 sin(11 t+69/32)-4835/32) theta(39 pi-t) theta(t-35 pi)+(-9/14 sin(1/7-7 t)-37/36 sin(41/47-5 t)-157/59 sin(19/47-3 t)+1717/74 sin(t+1/15)+161/50 sin(2 t+78/35)+25/19 sin(4 t+47/16)+23/24 sin(6 t+71/24)-5095/63) theta(35 pi-t) theta(t-31 pi)+(-9/49 sin(38/29-5 t)+375/16 sin(t+163/39)+13/14 sin(2 t+17/21)+18/19 sin(3 t+67/26)+22/39 sin(4 t+49/30)+13/30 sin(6 t+16/9)+10/41 sin(7 t+114/31)-29/36) theta(31 pi-t) theta(t-27 pi)+(-83/28 sin(29/20-3 t)+1369/25 sin(t+107/23)+131/22 sin(2 t+24/23)+47/21 sin(4 t+61/48)+19/18 sin(5 t+146/35)+43/42 sin(6 t+23/20)+29/37 sin(7 t+123/31)-347/81) theta(27 pi-t) theta(t-23 pi)+(-201/100 sin(68/91-4 t)+1526/29 sin(t+135/43)+149/18 sin(2 t+97/38)+101/19 sin(3 t+137/30)+14/13 sin(5 t+32/31)+85/64 sin(6 t+31/27)+24/23 sin(7 t+50/17)-1227/14) theta(23 pi-t) theta(t-19 pi)+(-68/69 sin(5/12-5 t)-6417/85 sin(20/33-t)+311/74 sin(2 t+94/33)+107/35 sin(3 t+124/29)+27/43 sin(4 t+46/11)-5948/29) theta(19 pi-t) theta(t-15 pi)+(-72/35 sin(8/21-11 t)-113/31 sin(14/19-8 t)-92/15 sin(4/7-5 t)+7212/31 sin(t+59/31)+...

...+1707/20 sin(2 t+42/25)+875/33 sin(3 t+138/31)+945/43 sin(4 t+177/62)+68/33 sin(6 t+131/54)+278/29 sin(7 t+123/32)+47/23 sin(9 t+29/35)+15/4 sin(10 t+147/38)+35/23 sin(12 t+21/29)+12683/49) theta(15 pi-t) theta(t-11 pi)+(-52/31 sin(32/29-10 t)-89/33 sin(27/28-8 t)-128/13 sin(24/35-6 t)+5680/29 sin(t+32/15)+1160/17 sin(2 t+39/20)+877/34 sin(3 t+128/29)+139/20 sin(4 t+62/23)+1319/120 sin(5 t+380/117)+166/31 sin(7 t+61/18)+73/38 sin(9 t+24/95)+45/31 sin(11 t+152/71)+56/25 sin(12 t+6/23)+31/24 sin(13 t+106/39)+3/14 sin(14 t+430/123)+21/22 sin(15 t+56/33)+53/80 sin(16 t+142/33)+11153/25) theta(11 pi-t) theta(t-7 pi)+(-13548/29 sin(101/72-t)+2803/21 sin(2 t+67/28)+1713/29 sin(3 t+50/19)+1951/49 sin(4 t+123/44)+569/18 sin(5 t+139/49)+255/29 sin(6 t+164/37)+269/21 sin(7 t+4/43)+505/39 sin(8 t+19/52)+302/41 sin(9 t+35/34)+34/11 sin(10 t+26/21)+373/71 sin(11 t+32/27)+106/33 sin(12 t+74/49)+41/18 sin(13 t+163/109)+9/46 sin(14 t+270/61)+106/31 sin(15 t+13/9)+65/37 sin(16 t+47/36)+62/25 sin(17 t+43/31)-2707/19) theta(7 pi-t) theta(t-3 pi)+(-106/39 sin(17/60-26 t)-27/26 sin(5/24-20 t)-269/36 sin(7/50-11 t)-308/17 sin(9/20-6 t)+6803/48 sin(t+67/30)+4066/25 sin(2 t+183/43)+2096/27 sin(3 t+145/67)+498/17 sin(4 t+43/31)+1181/36 sin(5 t+32/27)+327/14 sin(7 t+47/20)+293/18 sin(8 t+40/9)+1190/69 sin(9 t+81/29)+181/26 sin(10 t+68/33)+263/81 sin(12 t+41/44)+8/7 sin(13 t+116/31)+43/17 sin(14 t+105/43)+145/39 sin(15 t+154/51)+2623/874 sin(16 t+14/5)+64/23 sin(17 t+135/29)+27/22 sin(18 t+47/19)+15/19 sin(19 t+37/13)+23/19 sin(21 t+59/18)+17/29 sin(22 t+39/10)+68/31 sin(23 t+122/33)+61/71 sin(24 t+61/81)+9/7 sin(25 t+126/31)+81/37 sin(27 t+117/31)+27/67 sin(28 t+106/27)+17/16 sin(29 t+93/22)+27524/47) theta(3 pi-t) theta(t+pi)) theta(sqrt(sgn(sin(t/2))))

for $t \in [0, 68\pi]$, and where theta denotes the indicator function of $[0, \infty)$.

Testing Statistical Power



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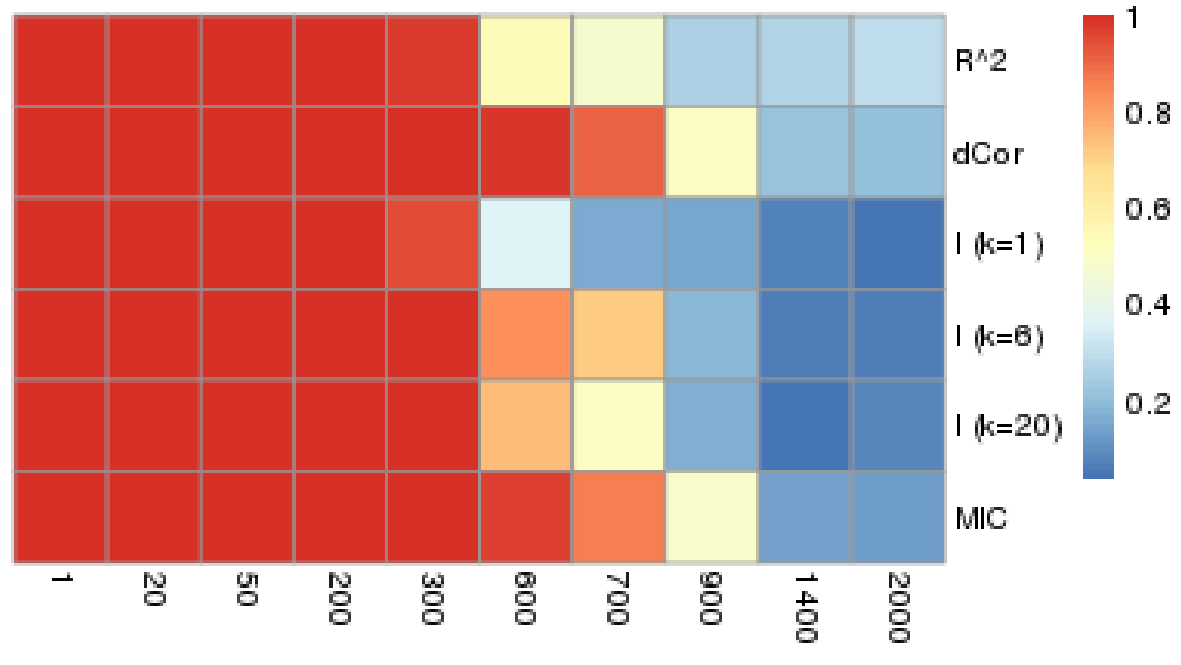

Figure: Heatmap of the statistical power (indicated by color) of various methods (y-axis) for the Raichu-like curve, for different noise levels (the standard deviation of the Gaussian noise added to the y_{R}-components on the x-axis)

Computing MIC

MIC is defined as a maximum over all grids of less than a certain size:

$$\max_{|R||C|<B(n)} MI(R, C)$$

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Too many possibilities to check!

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What they do: MIC-Heuristic

Optimization can be done efficiently in *one axis only*

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MIC-Heuristic $\leq$ MIC-Exact

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MIC-Exact feasible in this setting

Can compare MIC-Exact and MIC-Heuristic

Computing MIC

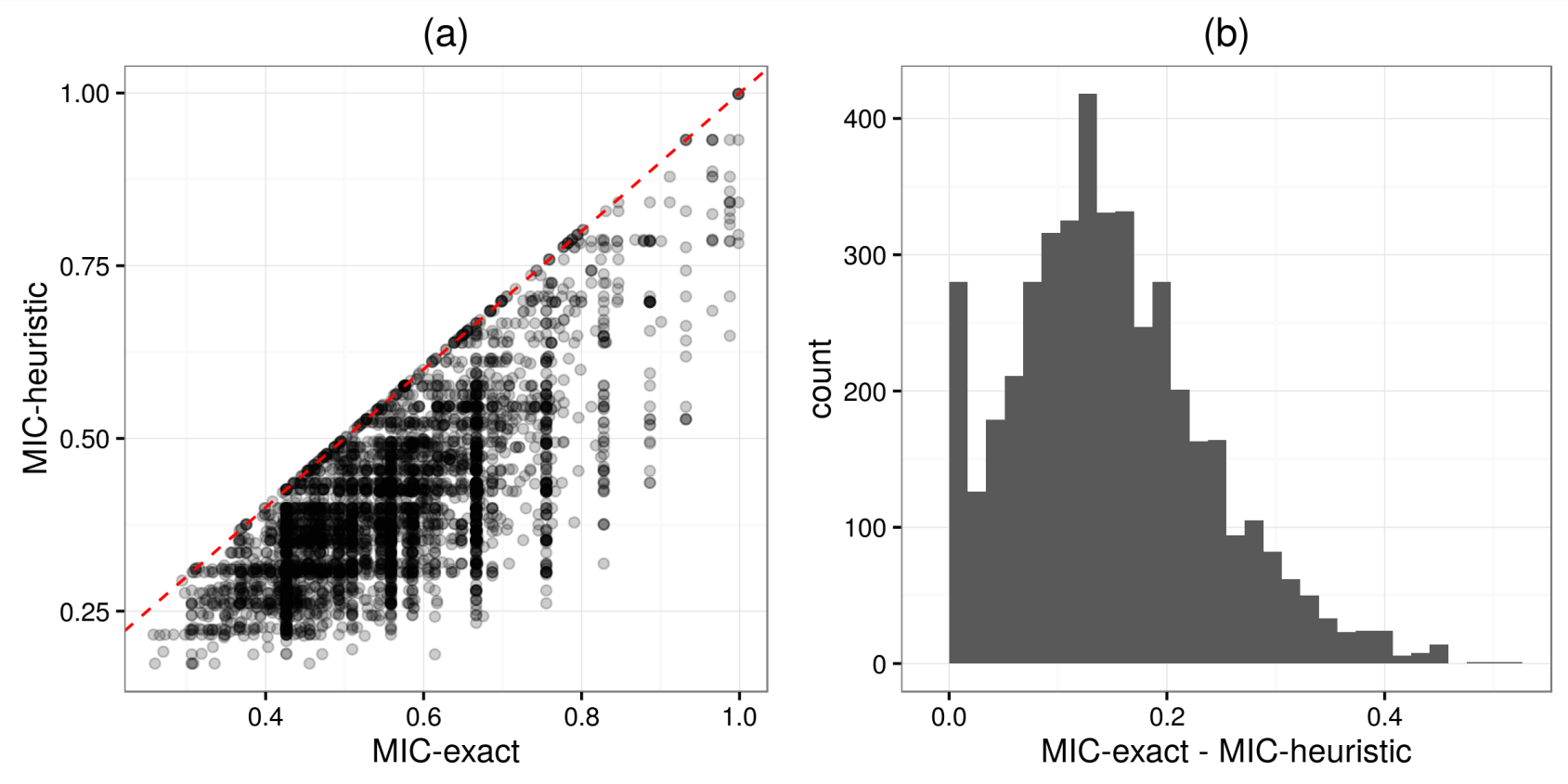
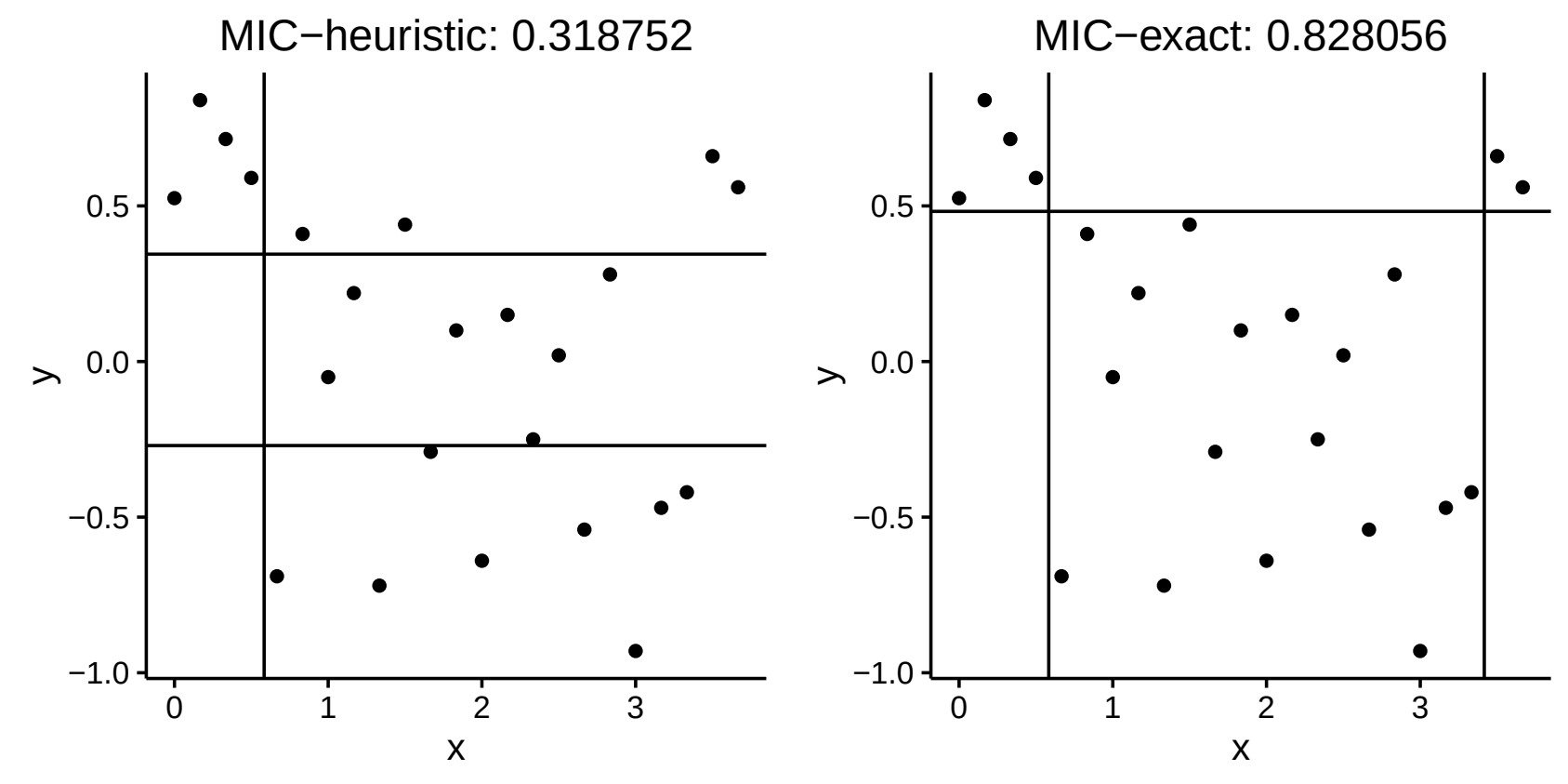


Figure: Discrepancies for Spellman yeast data

Computing MIC



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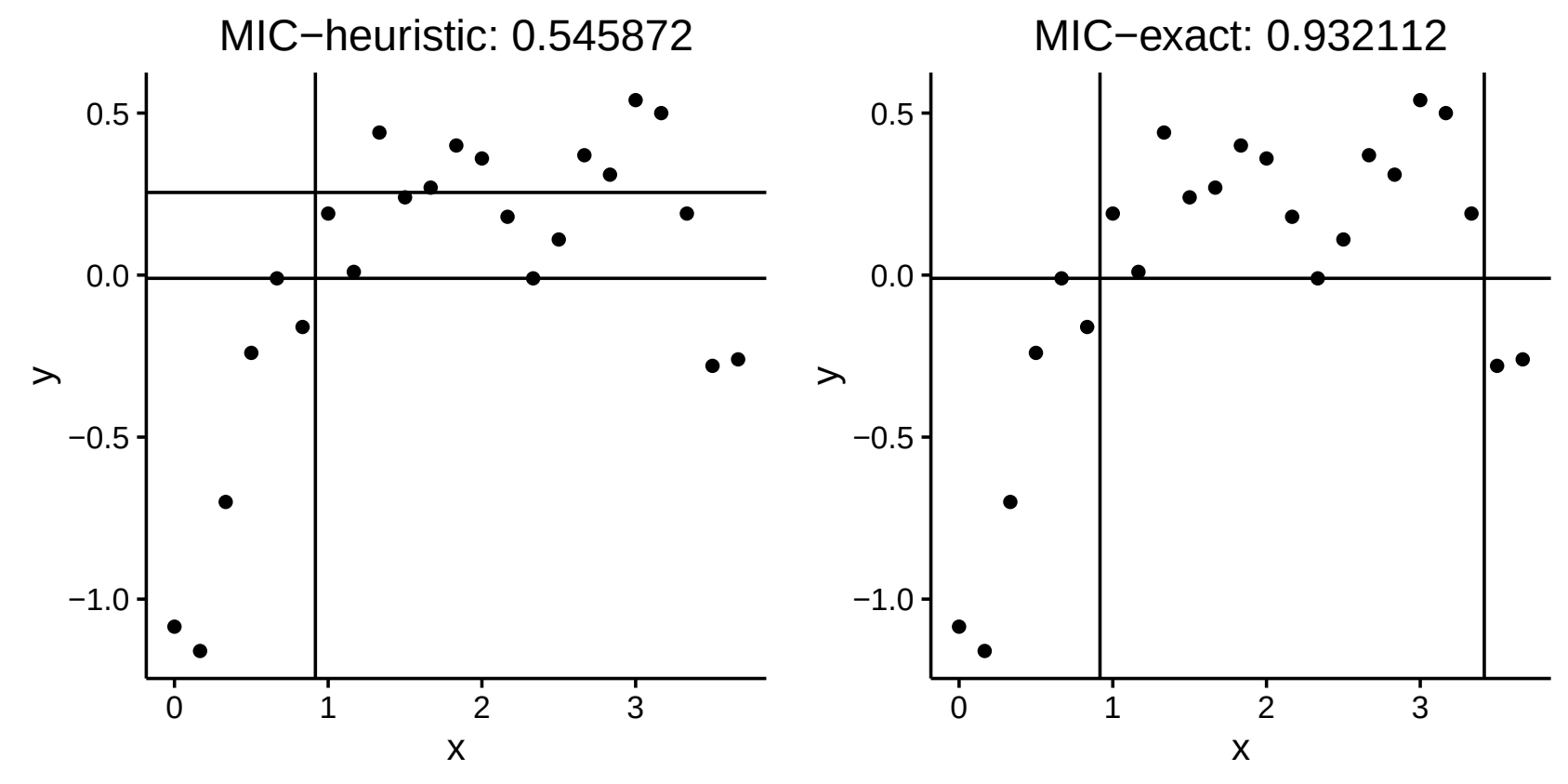

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Better algorithms possible?

Bibliography

[1] David N Reshef, Yakir A Reshef, Hilary K Finucane, Sharon R Grossman, Gilean McVean, Peter J Turnbaugh, Eric S Lander, Michael Mitzenmacher, and Pardis C Sabeti. Detecting novel associations in large data sets. *Science*, 334(6062):1518–1524, 2011.