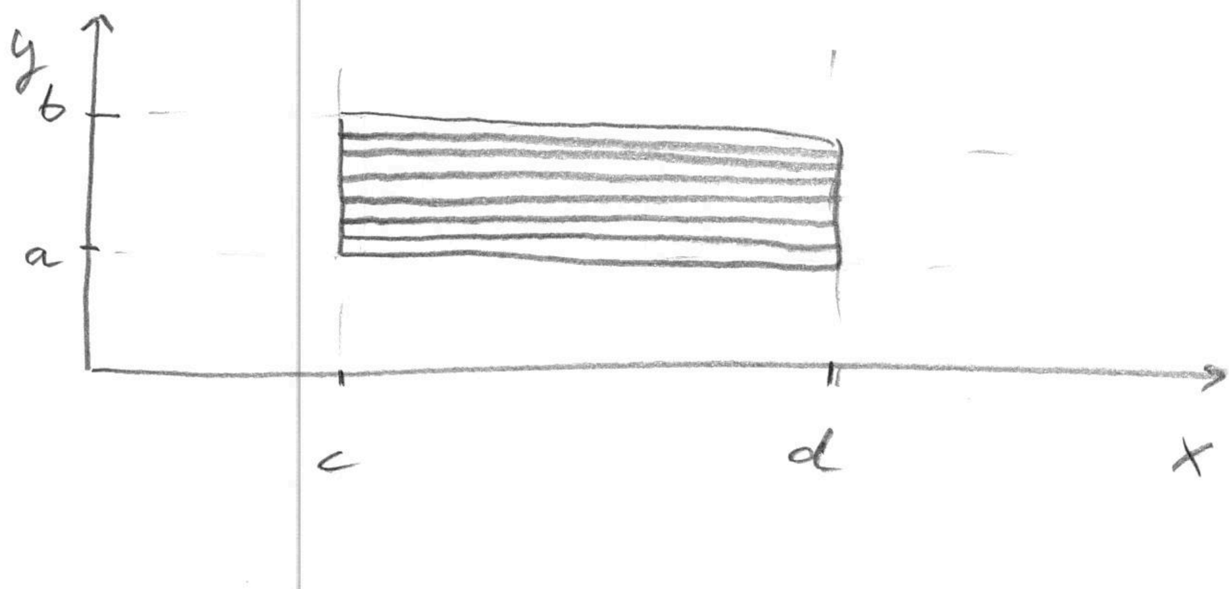
Całki wielokrotne. Całki dwukrotne CN-11

$$I = \int_a^b \int_c^d f(x,y)\,dx\,dy =$$

$$= \int_a^b \underbrace{\left(\int_c^d f(x,y)\,dx \right)}_{=F(y)} dy =$$

$$= \int_a^b F(y)\,dy$$

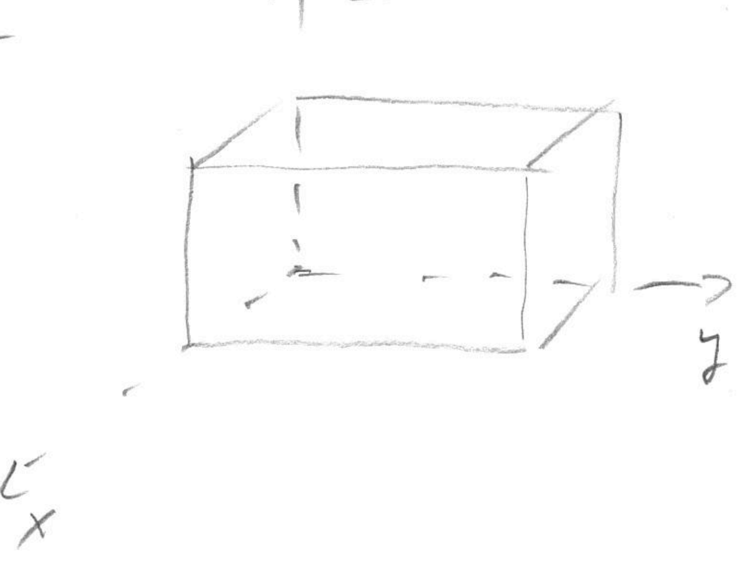


Całki trójkrotne

$$I = \int_a^b \int_c^d \underbrace{\int_e^f f(x,y,z)\,dx}_{=F(y,z)}\,dy\,dz =$$

$$= \int_a^b \underbrace{\int_c^d F(y,z)\,dy}_{=G(z)}\,dz =$$

$$= \int_a^b G(z)\,dz$$



Całkowanie Monte-Carlo

$$I=\int_a^b f(x)\,dx=(b-a)\langle f\rangle_n \pm (b-a)\,\sigma_n$$

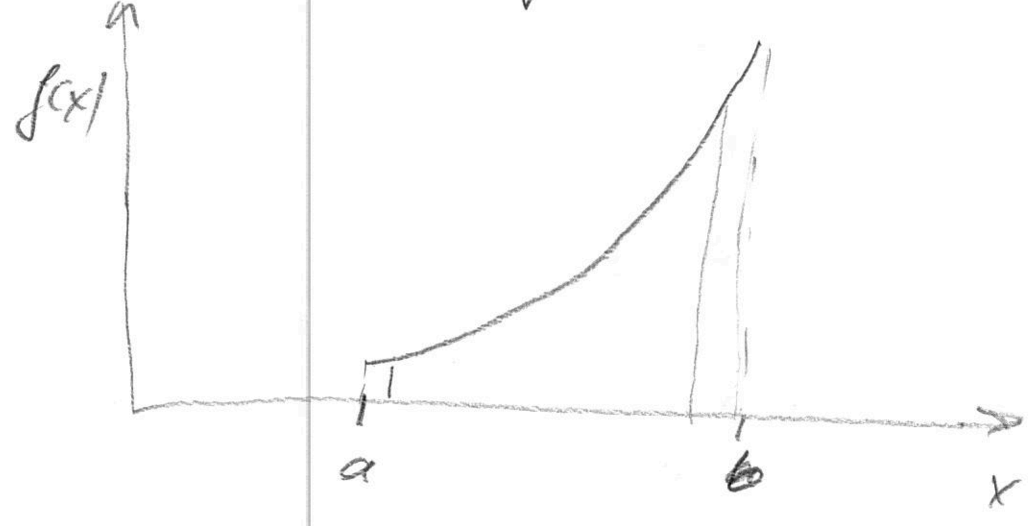
$$\sigma_n=\sqrt{\frac{\frac{1}{n}\sum_{i=1}^{n} f_i^2-\left(\frac{1}{n}\sum_{i=1}^{n} f_i\right)^2}{n-1}}$$

$$\langle f\rangle_n=\frac{1}{n}\sum_{i=1}^{n} f_i$$

$\langle f\rangle_n$ - wartość średnia funkcji

σ_n - średnie odchylenie standardowe

Równomierność próbkowania



$$I = \int_a^b f(x)\,dx \approx (b-a)\langle f\rangle_n$$

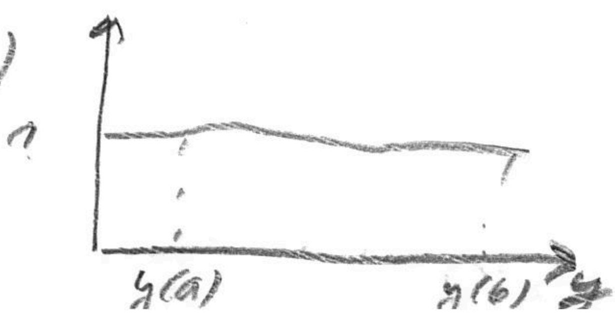
Lepiej dla $g(x) \approx f(x)$

$$I = \int_a^b \frac{f(x)}{g(x)}\, g(x)\,dx = \therefore$$

$$dy = g(x)\,dx, \quad y(x) = \int^x g(t)\,dt$$

$$\therefore = \int_{y(a)}^{y(b)} \frac{f(x(y))}{g(x(y))}\,dy = \int_{y(a)}^{y(b)} F(y)\,dy$$

$F(y) \approx 1$

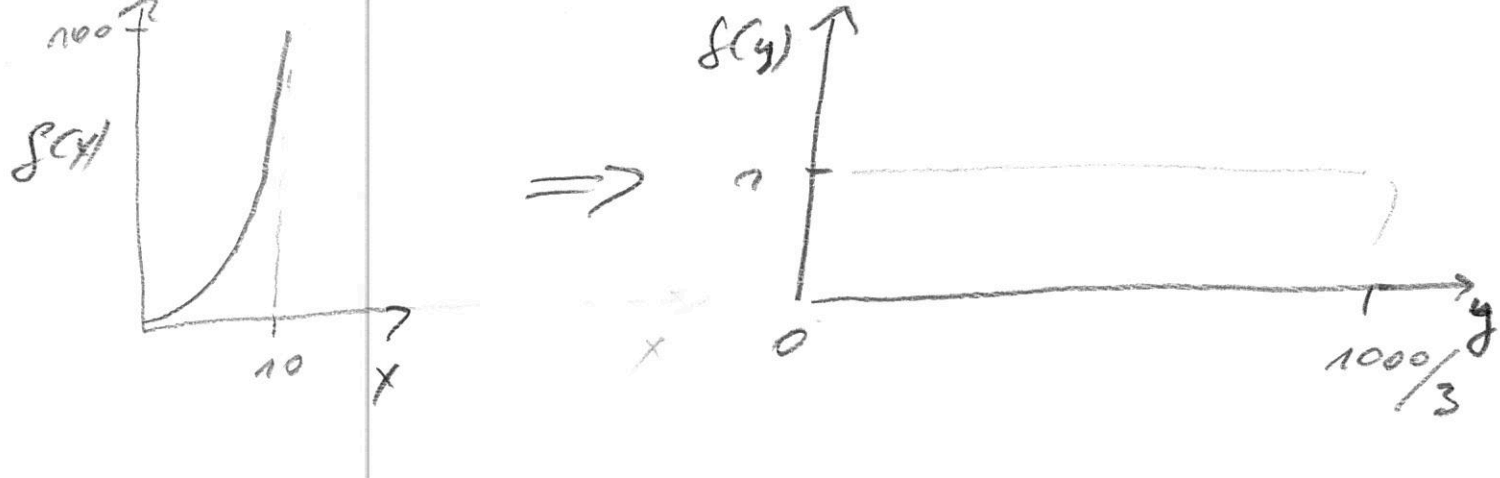


Przykład

$$\int_0^{10} x^2\,dx = \int_0^{10} \frac{x^2}{x^2}\, x^2\,dx = \therefore$$

$$dy = x^2\,dx, \quad y = \int^{x} t^2\,dt = \frac{x^3}{3}$$

$$\therefore = \int_{y(0)}^{y(10)} dy = \int_0^{\frac{1000}{3}} dy$$



Inne tematy:

(1) całki wielokrotne

(2) symulacje Monte-Carlo