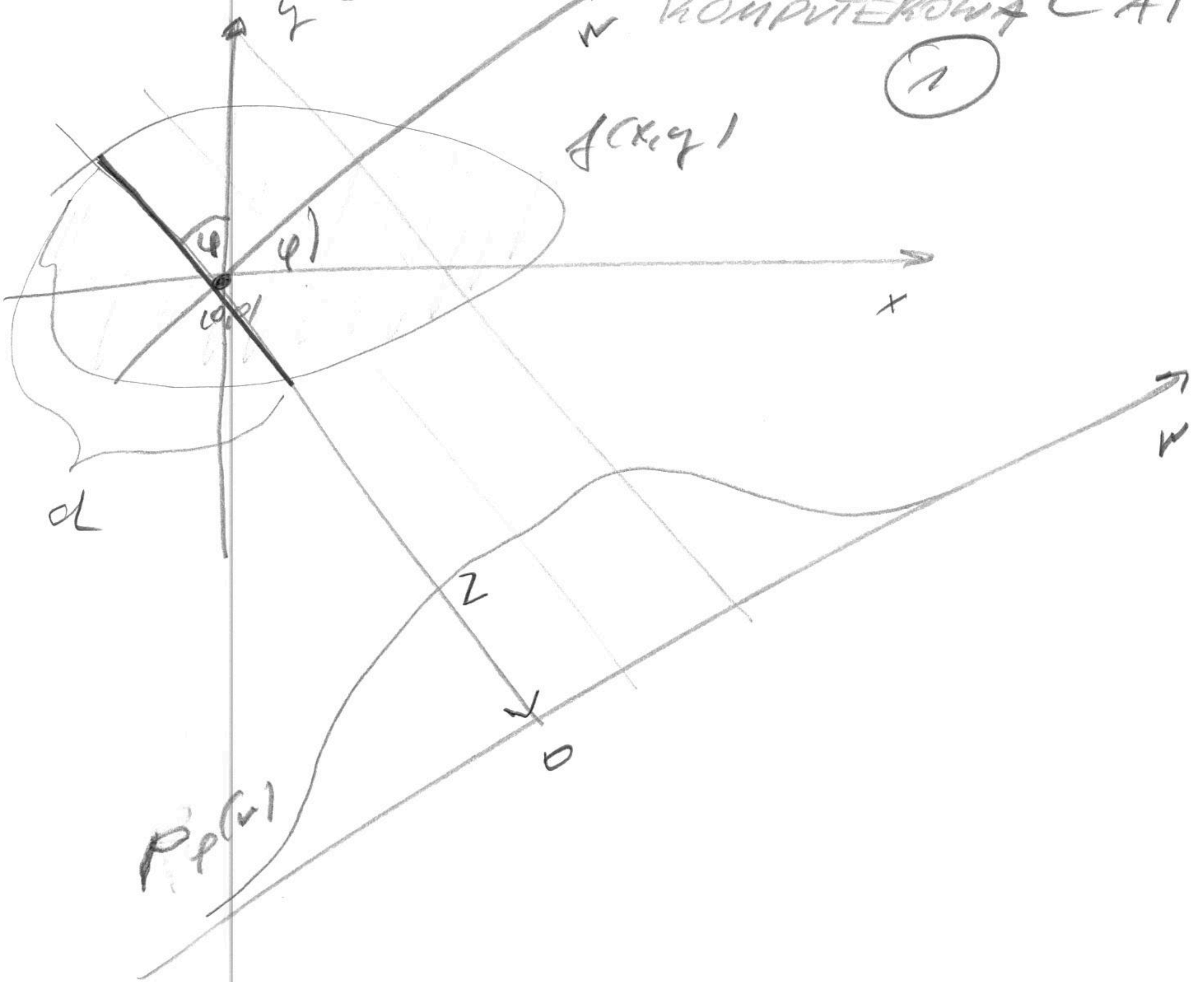
OSIOWA TOMOGRAFIA KOMPUTEROWA CAT

1



$$I = I_0 e^{-\lambda d}$$

λ - współczynnik tłumienia

$$\lambda d = \int_{-\infty}^{+\infty} f(x,y)\, dz$$

$$\frac{I}{I_0} = e^{-\int_{-\infty}^{+\infty} f(x,y)\, dz}$$

$$\ln\left|\frac{I}{I_0}\right| = -\int_{-\infty}^{+\infty} f(x,y)\, dz$$

$$\int_{-\infty}^{+\infty} f(x,y)\,dz = -\ln\left|\frac{I}{I_0}\right| = P_\varphi(r)$$

$$P_\varphi(r) = \int_{-\infty}^{+\infty} f(x,y)\,dz$$

transformata Radona

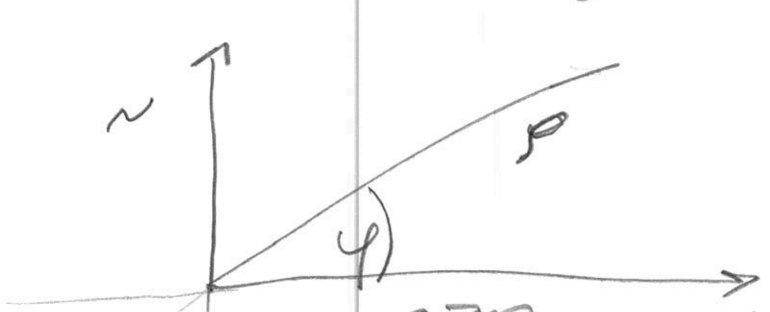
$x = r\cos\varphi - z\sin\varphi$ lub $r = x\cos\varphi + y\sin\varphi$

$y = r\sin\varphi + z\cos\varphi$ $\quad z = -x\sin\varphi + y\cos\varphi$

Transformaty Fouriera (DFT lub FFT)

$$P_\varphi(\rho) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty} p_\varphi(r)\, e^{-ir\rho}\,dr$$

$$F(u,v) = \frac{1}{2\pi}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} f(x,y)\, e^{-i(ux+vy)}\,dx\,dy$$



$u = \rho\cos\varphi$

$v = \rho\sin\varphi \Rightarrow$

$\rho \geq 0$, $0 \leq \varphi \leq \pi$

$du\,dv = |\rho|\,d\varphi\,d\rho \Rightarrow \rho = u\cos\varphi + v\sin\varphi$

$$P_\varphi(\rho) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} \left(\int_{-\infty}^{+\infty} f(x,y)\, dz \right) e^{-ir\rho}\, dr =$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} \left(\int_{-\infty}^{+\infty} f(x,y)\, dz \right) e^{-i\rho(x\cos\varphi + y\sin\varphi)}\, dr =$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} \left(\int_{-\infty}^{+\infty} f(x,y)\, dz \right) e^{-i(xu + yv)}\, dr =$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} f(x,y)\, e^{-i(ux + vy)}\, dr\, dz$$, ponieważ $dr\,dz = dx\,dy$ itd

$$P_\varphi(\rho) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} dx\, dy\, f(x,y)\, e^{-i(ux + vy)} = \sqrt{2\pi}\, F(u,v)$$

Transformata odwrotna

$$f(x,y) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} du\, dv\, \underbrace{F(u,v)}_{= \frac{1}{\sqrt{2\pi}} P_\varphi(\rho)} e^{i(ux+vy)} =$$

$$= \frac{1}{(2\pi)^{3/2}} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} du\, dv\, P_\varphi(\rho)\, e^{i(ux+vy)} =$$

$$= \frac{1}{(2\pi)^{3/2}} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} du\, dv\, P_\varphi(u\cos\varphi + v\sin\varphi)\, e^{i(ux+vy)}$$

lub ponieważ $du\,dv = |\rho|\,d\varphi\,d\rho$

$$f(x,y) = \frac{1}{(2\pi)^{3/2}} \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} |\rho|\,d\varphi\,d\rho\; P_\varphi(\rho)\, e^{i(ux+vy)} =$$

$$= \frac{1}{(2\pi)^{3/2}} \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} |\rho|\,d\varphi\,d\rho\; P_\varphi(\rho)\, e^{i\rho(x\cos\varphi + y\sin\varphi)}$$