CvP - Werkcollege 2

Exercise 1 Describe the meaning of

(a) operational semantics,

(b) denotational semantics,

(c) axiomatic semantics.

Exercise 2 Consider the following EBNF grammar G

$$
\begin{aligned}
\langle stmt \rangle &\to \langle var \rangle = \langle expr \rangle \mid \langle stmt \rangle ; \langle stmt \rangle \mid \textbf{do } \langle stmt \rangle \textbf{ until } \langle guard \rangle \textbf{ od} \\
\langle guard \rangle &\to \langle expr \rangle == \langle expr \rangle \\
\langle expr \rangle &\to \langle octal \rangle \mid \langle var \rangle \mid \langle expr \rangle (+ \mid *) \langle expr \rangle \\
\langle var \rangle &\to a \langle octal \rangle \\
\langle octal \rangle &\to 0 \mid (1 \mid 2 \mid 3 \mid 4 \mid 5 \mid 6 \mid 7)\{(0 \mid 1 \mid 2 \mid 3 \mid 4 \mid 5 \mid 6 \mid 7)\}
\end{aligned}
$$

For every non-terminal x, let $T_G(x)$ denote the set of all parse trees generated by the grammar G with start symbol x. Let $\mathbb{N} = \{0, 1, \ldots\}$ denote the set of all natural numbers. Let $\mathbb{B} = \{\textbf{false}, \textbf{true}\}$ denote the set of boolean values.

Represent the state as a function $s : L \to \mathbb{N}$, with $L \subseteq \mathbb{N}$ finite. Intuitively, a state s assigns the value $s(i)$ to the i-th memory cell, if $i \in \mathrm{dom}(s)$, and leaves the value of the i-th memory cell undefined, otherwise. Let

$$S = \{s : L \to \mathbb{N} \mid L \subseteq \mathbb{N} \text{ finite}\} \cup \{\text{error}\}$$

be the set of all possible states (including some error state).

(a) Define semantics $M_{\langle octal \rangle} : T_G(\langle octal \rangle) \to \mathbb{N}$ for octal numbers.

(b) Define semantics $M_{\langle var \rangle} : T_G(\langle var \rangle) \to \mathbb{N}$ for variable names.

(c) Define semantics $M_{\langle expr \rangle} : T_G(\langle expr \rangle) \to (S \to (\mathbb{N} \cup \{\text{error}\}))$ for expressions, and explain why $M_{\langle expr \rangle}$ is well-defined (although G is ambiguous).

(d) Define semantics $M_{\langle guard \rangle} : T_G(\langle guard \rangle) \to (S \to (\mathbb{B} \cup \{\text{error}\}))$ for guards.

(e) Define semantics $M_{\langle stmt \rangle} : T_G(\langle stmt \rangle) \to (S \to S)$ for statements, and explain why your definition is well-defined.

(Hint: use *substitution* $s[i \mapsto n] \in S$ and *composition* $f \circ g : S \to S$).

Exercise 3 Consider the statement S defined as $\{P\}\ x := 6y + 10;\ \{x > 16\}$.

(a) Compute the weakest precondition P in S.

(b) Give a different precondition of statement S.

(c) Give a different postcondition of statement S.

Exercise 4 Prove or give a counter example:

(a) $\{x < 5\}\ x := x + 4;\ \{x \leq 3\}$.

(b) $\{y \leq 1\}\ x := (2y + 2)^2;\ \{x \leq 16\}$.