CvP - Werkcollege 2

Exercise 1 Describe the meaning and purpose of

(a) operational semantics,

(b) denotational semantics,

(c) axiomatic semantics.

Exercise 2 Consider the following EBNF grammar G

$$
\begin{aligned}
\langle stmt\rangle &\to \langle var\rangle = \langle expr\rangle \mid \langle stmt\rangle ; \langle stmt\rangle \mid \textbf{do } \langle stmt\rangle \textbf{ until } \langle guard\rangle \textbf{ od}\\
\langle guard\rangle &\to \langle expr\rangle == \langle expr\rangle\\
\langle expr\rangle &\to \langle octal\rangle \mid \langle var\rangle \mid \langle expr\rangle(+\mid *)\langle expr\rangle\\
\langle var\rangle &\to a\langle octal\rangle\\
\langle octal\rangle &\to 0 \mid (1\mid 2\mid 3\mid 4\mid 5\mid 6\mid 7)\{(0\mid 1\mid 2\mid 3\mid 4\mid 5\mid 6\mid 7)\}
\end{aligned}
$$

For every non-terminal x, let $T_G(x)$ denote the set of all parse trees generated by the grammar G with start symbol x. Let $\mathbb{N} = \{0, 1, \ldots\}$ denote the set of all natural numbers. Let $\mathbb{B} = \{\textbf{false}, \textbf{true}\}$ denote the set of boolean values.

Represent the state as a function $s : L \to \mathbb{N}$, with $L \subseteq \mathbb{N}$ finite. Intuitively, a state s assigns the value $s(i)$ to the i-th memory cell, if $i \in \mathrm{dom}(s)$, and leaves the value of the i-th memory cell undefined, otherwise. Let

$$S = \{s : L \to \mathbb{N} \mid L \subseteq \mathbb{N} \text{ finite}\} \cup \{\text{error}\}$$

be the set of all possible states (including some error state).

(a) Define semantics $M_{\langle octal\rangle} : T_G(\langle octal\rangle) \to \mathbb{N}$ for octal numbers.

(b) Define semantics $M_{\langle var\rangle} : T_G(\langle var\rangle) \to \mathbb{N}$ for variable names.

(c) Define semantics $M_{\langle expr\rangle} : T_G(\langle expr\rangle) \to (S \to \mathbb{N})$ for expressions, and explain why ambiguity of G is not a problem.

(d) Define semantics $M_{\langle guard\rangle} : T_G(\langle guard\rangle) \to (S \to \mathbb{B})$ for guards.

(e) Define semantics $M_{\langle stmt\rangle} : T_G(\langle stmt\rangle) \to (S \to S)$ for statements, and explain why your definition is well-defined.

(Hint: use *substitution* $s[i \mapsto n]$ and *composition* $s \circ s'$).

Exercise 3 Consider the statement S defined as $\{P\}\ x := 6y + 10;\ \{x > 16\}$.

(a) Compute the weakest precondition P in S.

(b) Give a different precondition of statement S.

(c) Give a different postcondition of statement S.

Exercise 4 Prove or give a counter example:

(a) $\{x < 5\}\ x := x + 4;\ \{x \leq 3\}$.

(b) $\{y \leq 1\}\ x := (2y + 2)^2;\ \{x \leq 16\}$.