MIMO Reefer

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1 INTRODUCTION

This project deals with modeling and control of a refrigeration (reefer) trailer whose purpose is to maintain specific temperatures inside a box for the transportation of cargo. All sorts of products require a temperature-controlled environment which will depend and vary based on the specific product. Some examples are frozen food, dairy products, produce, medical supplies (including vaccines), and electronics. The need for transportation of perishable goods such as fresh meat, fish and fruits can hardly be questioned. This further highlights the need for high quality temperature-controlled transportation options. While reefer *containers* are used in shipping, reefer *trailers* are necessary when transporting to e.g. final destination facilities, warehouses, and stores.

Unlike reefer containers, reefer trailers utilize a battery as the main power supply due to the limited power delivery of the trucks they are hooked upon. Efficiency is therefore of great importance as this allows for not only greater transportation distances but also a possible reduction of the battery pack size, which is appealing because of their high cost.

The purpose of a reefer trailer is to maintain the temperature of its cargo and not actually cooling it down. It is thus expected that cargo is pre-cooled to around the desired transportation temperature before being loaded. A desired temperature is reached by cooling down air which is circulated between the cargo box and the refrigeration unit. The refrigeration unit is situated at the back of the trailer where all components besides sensors are located. Inside the refrigeration unit a refrigerant is moved between a low pressure in the evaporator and high pressure in the condenser. The evaporator extracts heat from the circulating box air and the condenser gives the heat to the ambient air outside the box.

The main disturbances in the system are: 1. The air which blows in when the door is opened, 2. The continuous convection heat transfer from the outside to the inside of the box.

This project is executed in corporation with BITZER Electronics A/S as a 2nd (8th) semester project of the Masters Program in Control and Automation at Aalborg University. BITZER supplies a high fidelity model of the reefer trailer along with other relevant documentation on the system. Furthermore Kresten Sørensen from BITZER has offered counseling throughout the project.

2 SYSTEM DESCRIPTION

The cooling trailer is an insulated trailer designed and built by Schmidt Cargobull with a cooling system developed by Termoking OR Carrier. It is attached to a truck and has a length of 13.4 meters. The trailer features room for 33 Euro-pallets of cargo and is used to transport various goods which need specific environmental conditions during transportation. The cooling system used to control the internal climate of the trailer is powered by a battery located in the trailer. This allows the trailer cooling to be powered independent of the truck.

While the trailer usually has its own cooling system made by Schmidt Cargobull, the trailer in this project utilizes a custom HVAC system designed by Bitzer. This system is built to facilitate testing of advanced control strategies.

The inputs and outputs of the system are the found by investigating the structure of the HVAC system. While the system will be described in detail in section XX, the most important components can be named as refrigerant moves around a loop through various components. These are: A scroll compressor, a condenser, a condenser throttle valve, a flash tank, an expansion valve and an evaporator. These can also be observed in Fig. 1. The trailer box is also of great importance. The evaporator fan blows cold air throughout the top of the trailer, where it travels down past the cargo to the bottom and is pulled back along the floor.

Many of the system components have variables that can be set to change their behavior. These are the controlled inputs, and include:

- The compressor speed
- The condenser fan speed
- The evaporator fan speed
- The expansion valve opening degree (OD) to the evaporator
- The condenser throttling valve opening degree to the flash tank

There are many variables such as pressure and temperature throughout the refrigeration cycle, many of which are measured and hence are outputs of the system. The controlled outputs however, are those of interest from a control point of view. These are the outputs that the control strategy seeks to keep at a set point, in this project it will be the trailer box air temperature. The aim of the project is to design a Multiple-Inputs-Multiple-Outputs (MIMO) Controller to keep the temperature inside the container constant despite exogenous

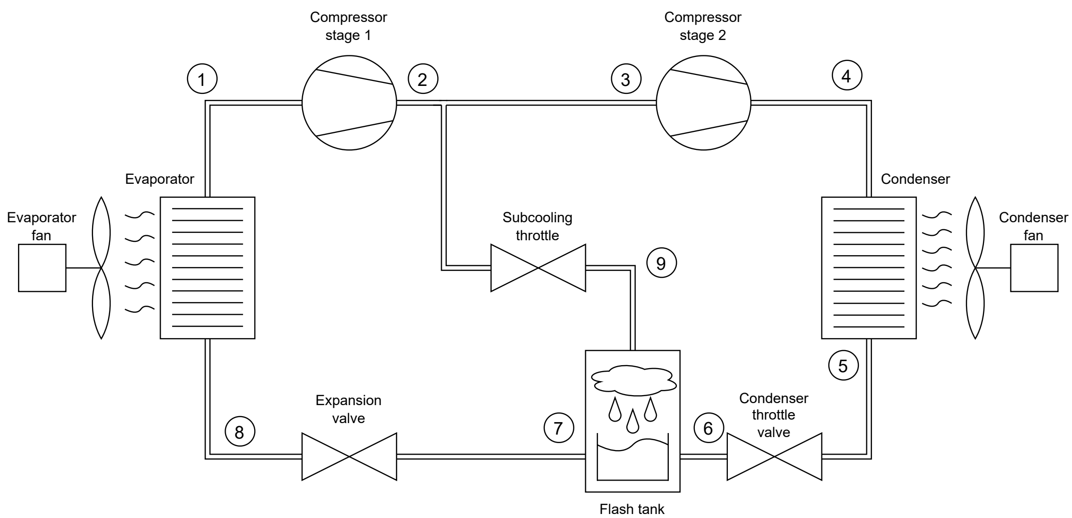

Fig. 1. Illustration of refrigeration cycle

inputs (disturbances), specifically the ambient temperature. Furthermore it should do so with the sub-goal of minimizing the energy consumption.

In Fig. 2 a p-h diagram of the refrigeration cycle can be seen. It is not accurate but merely depicts the general idea of how the enthalpy and pressure changes throughout the cycle. The numbers in the figure are also shown in Fig. 1 for reference. Two cycles can be observed on the diagram. The first is the main cycle which runs along $1 \rightarrow 8$ and the second is the cycle running along $3 \rightarrow 6 \rightarrow 9$.

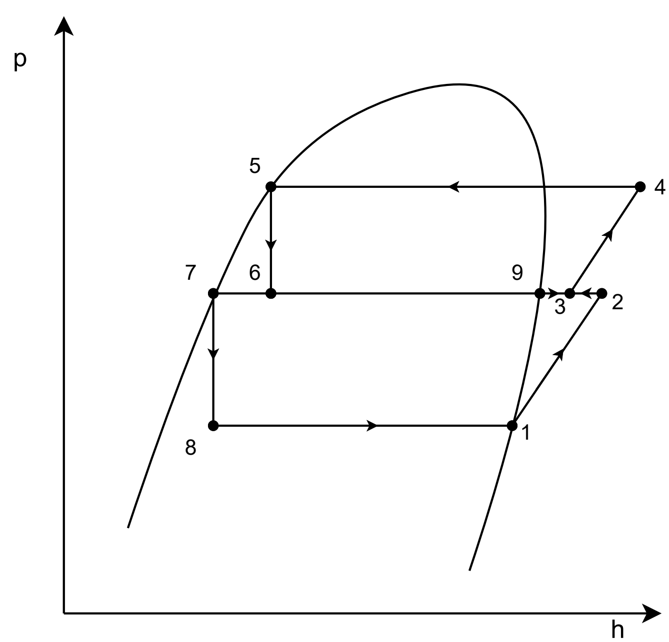

Fig. 2. p-h diagram of the refrigeration cycle

2.1 What is it used for

NOT DONE Inputs / outputs

Control objectives

- Constant temperature
  - Disturbances
- Minimum energy consumption
  - Cost of batteries and prices of electricity

3 MODELING

The most dominant dynamics of a refridgeration trailer are the large thermal capacitances, both of the metal in heat exchangers (i.e. evaporator and condensor) and of cargo and trailer walls and floor. Some components in the system will have dynamics so much faster than the dominant dynamics that they will be considered static (eg. compressor and expansion valves). Ultimately the model will be composed of a model that represents the refridgeration cycle along with a model of the thermal masses.

3.1 Component models

3.1.1 General type refrigerant control volume state equation

Many of the components in a refrigeration cycle will be based on state equations of similar structure. They will generally express the change in mass inside the control volume and/or the specific enthalpy out of the control volume. These can be constructed from the mass conservation equation and the energy balance equation of the control volume. The energy balance equations are modelled as steady state algebraic equations. This lowers accuracy but reduces complexity of the models.

Mass conservation equation

$$\frac{dM}{dt} = \dot{m}_{in} - \dot{m}_{out} \tag{1}$$

where

$\frac{dM}{dt}$	Change in mass inside control volume	[kg/s]
$\dot{m_{in}}$	Mass flow of refrigerant into control volume	[kg/s]
$\dot{m_{out}}$	Mass flow out of control volume	[kg/s]

Energy balance equation

$$h_{out} = h_{in} + \frac{Q_{in}}{\dot{m}_{in}} \tag{2}$$

where

h_{out}	Specific enthalpy out of the control volume	[J/kg]
h_{in}	Specific enthalpy into control volume	[J/kg]
Q_{in}	Energy flow from heat and work applied to control volume	[W]
$\dot{m}_{in}$	Mass flow into control volume	[kg/s]

3.1.2 Expansion valve

The flow through an expansion valve is proportional to the square root of the pressure drop across it, where the proportional constants relies on physical properties of the valve and refrigerant.

$$\dot{m} = CA\sqrt{\rho \Delta p} \tag{3}$$

where

$\dot{m}$	Flow through valve	[kg/s]
Δp	Pressure drop across valve	[Pa]
C	Discharge coefficient of valve	[·]
A	Cross sectional area of valve	[m^2]
ρ	Density of liquid	[kg/m^3]
C	Discharge coefficient of valve	[·]

To model the way that the valve is intended to be controlled, an alternative representation is introduced for the mass flow through an expansion valve in Eq. (4)

$$\begin{aligned} \dot{m} &= f_p(\Theta) \cdot K \sqrt{\frac{1}{v_{in}}(p_{in} - p_{out})} \\ K &= CA \end{aligned} \tag{4}$$

where

$\dot{m}$	Flow through valve	[kg/s]
$f_p(\Theta)$	Flow percentage as function of opening degree	[·]
Θ	Opening degree of valve	[·]
p_{in}	Absolute pressure on input side	[Pa]
p_{out}	Absolute pressure on output side	[Pa]
K	Constant, product of discharge coefficient and cross sectional area	[m^2]
v_{in}	Specific volume of liquid refrigerant into the valve	[m^3/kg]

The function $f_p()$ of the opening degree is used the model the non linear behavior of the OD - flow relationship in the valve. The valve is an equal-percentage type valve, meaning for an increase in opening degree you get a relative increase in flow. This is illustrated in Fig. 3.

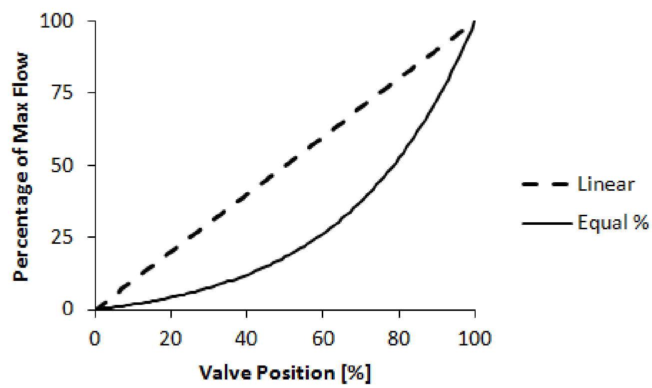

Fig. 3. Valve characteristics

3.1.3 Pipe Joining Junction

Between compressor stage 1, compressor stage 2 and the flash tank (see Fig. 1) is a Pipe Joining Junction that connects the three forementioned components.

In Eq. (5), the change of mass inside the Pipe Joining Junction can be expressed as a function of the mass flows into and out of the Pipe Joining Junction.

$$\frac{dM}{dt} = \dot{m}_{in1} + \dot{m}_{in2} - \dot{m}_{out} \tag{5}$$

where

$\frac{dM}{dt}$	Change in mass inside Pipe Joining Junction	[kg/s]
$\dot{m}_{in1}$	Flow into Pipe Joining Junction from Compressor C_1	[kg/s]
$\dot{m}_{in2}$	Flow into Pipe Joining Junction from Flash Tank	[kg/s]
$\dot{m}_{out}$	Flow into Compressor C_2 from Pipe Joining Junction	[kg/s]

In Eq. (6) the specific enthalpy of the flow out of the Pipe Joining Junction is expressed as a function of the input flows and enthalpies. This equation is based on the energy balance, assuming no heat transfer to surroundings, i.e. the Pipe Joining Junction is perfectly insulated. Additionally, it is expected that $\frac{dM}{dt}$ is zero, such that the output mass flow is equal to the sum of the input flows.

$$h_{out} = \frac{h_{in1} \cdot \dot{m}_{in1} + h_{in2} \cdot \dot{m}_{in2}}{\dot{m}_{in1} + \dot{m}_{in2}} \tag{6}$$

where

h_{out}	Specific enthalpy into Compressor C_2 from Pipe Joining Junction	[J/kg]
h_{in1}	Specific enthalpy into Pipe Joining Junction from Compressor C_1	[J/kg]
h_{in2}	Specific enthalpy into Pipe Joining Junction from Flash Tank	[J/kg]
$\dot{m}_{in1}$	Flow into Pipe Joining Junction from Compressor C_1	[kg/s]
$\dot{m}_{in2}$	Flow into Pipe Joining Junction from Flash Tank	[kg/s]

3.1.4 Compressor

The compressor in the refrigeration cycle consists of two compressor stages that can be described by the same equations. The compressor type is a scroll compressor. The compressor dynamics are assumed to be fast enough compared with the refridgeration cycle that it can be considered constant. Therefore, the equations governing the compressors are algebraic equations. Adiabatic compression is assumed. The two equations describing the compression governs the mass flow and the output specific enthalpy. The output specific enthalpy is found via a lookup table (HTP).

$$\dot{m} = \left(\frac{V_1}{v_1} - \frac{V_C}{v_2}\right)\frac{\omega}{2} \tag{7}$$

$$h_{out} = \Upsilon(T_{out}, p_{out}) \tag{8}$$

where

$\dot{m}$	Flow through compressor stage	[kg/s]
h_{out}	Compressor stage output specific enthalpy	[J/kg]
V_1	Cylinder internal volume b.f. 'stroke'	[m^3]
V_C	Cylinder clearance volume after 'stroke'	[m^3]
v_1	Refrigerant specific volume b.f. 'stroke'	[m^3/kg]
v_2	Refrigerant specific volume after 'stroke'	[m^3/kg]
ω	Compressor angular velocity	[rad/s]
Υ	HTP; Lookup table of the enthalpy from temperature and pressure	[·]
T_{out}	Compressor stage output temperature	[K]
p_{out}	Compressor stage output pressure	[Pa]

In Eq. (8) v_1 is found from table lookup based on ????.

$$v_1 = \Gamma(T_{in}, p_1) \tag{9}$$

$$v_2 = \left(\frac{p_2}{p_1}\right)^{\frac{-1}{\gamma}} \tag{10}$$

$$p_1 = p_{in} - kl_1 \cdot \omega \tag{11}$$

$$p_2 = p_{out} + kl_2 \cdot \omega \tag{12}$$

$$\gamma = C_{cp}/C_{cv} \tag{13}$$

$$T_{out} = T_{in} \cdot \left(\frac{p_{out}}{p_{in}}\right)^{\frac{\gamma-1}{\gamma}} \tag{14}$$

where

Γ	VTP; Lookup table of the specific volume from temperature and pressure	[Pa]
p_{in}	Compressor stage input pressure	[Pa]
p_1	Piston input pressure	[Pa]
p_2	Piston output (discharge) pressure	[Pa]
γ	Heat capacity ratio	[·]
kl_1, kl_2	Valve loss constants	[·]
ω	Compressor angular velocity	[rad/s]
T_{in}	Compressor stage input temperature	[K]
C_{cp}	Specific heat capacity - constant pressure	[J/(kg·K)]
C_{cv}	Specific heat capacity - constant volume	[J/(kg·K)]

3.1.5 Condenser

The condenser takes in the discharge pressure vapor from the second compressor stage, at point 4 in Fig. 1. The high pressure also yields a high temperature, which enables heat transfer through the condensor to ambient air. This is done mainly through condensation of the refrigerant vapor, yielding high pressure liquid at point 5 in Fig. 1. The energy balance is modelled in Eq. (15). The mass balance is modelled in Eq. (16). Finally the temperature of the metal in the condenser is modelled in Eq. (17), as the dominant dynamics of the condenser is greatly linked to the temperature of the metal [1]. Eq. (17) is also derived from the energy balance.

$$h_{out} = h_{in} - \frac{Q_{rm}}{\dot{m}_{in}} \tag{15}$$

$$\frac{dM_r}{dt} = \dot{m}_{in} - \dot{m}_{out} \tag{16}$$

$$\frac{dT_m}{dt} = \frac{Q_{rm} - Q_{ma}}{M_m \cdot Cp_m} \tag{17}$$

where

h_{out}	Condenser output specific enthalpy	[J/kg]
h_{in}	Condenser input specific enthalpy	[J/kg]
Q_{rm}	Refrigerant to metal heat flow	[W]
Q_{ma}	Metal to air heat flow	[W]
$\dot{m_{in}}$	Condenser input mass flow	[kg/s]
$\dot{m_{out}}$	Condenser output mass flow	[kg/s]
M_r	Refrigerant mass	[kg]
M_m	Metal mass	[kg]
T_m	Metal temperature	[K]
Cp_m	Metal heat capacity	[J/K]

The pressure drop across the condenser is assumed to be linear with respect to mass flow, yielding Eq. (18). The mass flow out of the condenser is modelled in Eq. (19).

$$p_{in} = p_{out} - \lambda \cdot \dot{m}_{in} \tag{18}$$

$$\dot{m}_{out} = \dot{m}_{in} + \frac{M_r - \frac{V_i}{v}}{1s} \tag{19}$$

where

p_{in}	Condenser input pressure	[Pa]
p_{out}	Condenser output pressure	[Pa]
λ	Pressure drop constant	[$\cdot$]
$\dot{m_{in}}$	Condenser input mass flow	[kg/s]
$\dot{m_{out}}$	Condenser output mass flow	[kg/s]
M_r	Condenser refridgerant mass	[kg]
V_i	Condenser internal volume	[m^3]
v	Condenser refridgerant specific volume	[m^3/kg]

And finally the convective heat flows are modelled in Eq. (20), Eq. (21). The heat flow from metal to air is assumed to be approximately proportional to the air flow, which is why the speed of the fan is multiplied to the energy flow in Eq. (21). There is an offset of 0.05 times the energy flow to account for the fact that there will exist a heat flow even though the fan is not operating.

$$Q_{rm} = UA_{rm} \cdot (T_r - T_m) \tag{20}$$

$$Q_{ma} = UA_{ma} \cdot (T_m - T_a) \cdot (0.05 + \frac{U_{fan}}{1530}) \tag{21}$$

where

Q_{rm}	Heat flow from refridgerant to metal	[W]
Q_{ma}	Heat flow from metal to air	[W]
UA_{rm}	Heat transfer coefficient from refridgerant to metal	[J/K]
UA_{ma}	Heat transfer coefficient from metal to air	[J/K]
T_r	Temperature of refridgerant	[K]
T_m	Temperature of metal	[K]
T_a	Temperature of air	[K]
U_{fan}	Fan speed	[1/s]

3.1.6 Flash tank

The flash tank in combination with the condenser throttle valve serves to reduce the amount of high enthalpy flash gas delivered to the evaporator. The condenser throttle valve, the dynamics of which is identical to the expansion valve, lowers the pressure of the liquid from the condenser. This naturally lowers the temperature, but also generates some amount of flash gas. The flash tank then separates the liquid-vapor mixture and passes only the liquid to the expansion valve. The flash gas is returned to the second stage of the compressor, where it is reused. Thus, a lower amount of flash gas will be generated by the expansion valve, as the pressure of the liquid is already quite low.

The modeling will only evaluate the steady state behaviour of the flash tank due to the limited scope of the project.

In steady state it is first assumed that the pressure of the liquid-vapor mixture entering is the same as the separated liquid and vapor leaving the tank.

$$p_{lv} = p_l = p_v \tag{22}$$

where

p_{lv}	Liquid-vapor mixture pressure	[Pa]
p_l	Liquid pressure	[Pa]
p_v	Vapor pressure	[Pa]

Secondly, it is assumed that the energy of the mixture does not change, meaning the energy flow in equals the energy flow out.

$$\dot{m}_{lv} \cdot h_{lv} - \dot{m}_l \cdot h_l - \dot{m}_v \cdot h_v = 0 \tag{23}$$

where

$\dot{m}_{lv}$	Liquid-vapor mixture mass flow	[kg/s]
$\dot{m}_l$	Liquid mass flow	[kg/s]
$\dot{m}_v$	Vapor mass flow	[kg/s]
h_{lv}	Liquid-vapor mixture specific enthalpy	[J/kg]
h_l	Liquid specific enthalpy	[J/kg]
h_v	Vapor specific enthalpy	[J/kg]

Lastly, it is assumed that the separated liquid and vapor leaves at boiling point and flash point respectively. This last assumption allows us to find the enthalpy of the two substances purely from investigation of the p-h diagram since the pressure is known. In practice, this is performed by a software tool as MATLAB. We express this as:

$$h_l = M(p) \tag{24}$$

$$h_l = N(p) \tag{25}$$

$$\tag{26}$$

where

$M(p)$	Lookup table of the enthalpy of saturated liquid at pressure p	[J/kg]
$N(p)$	Lookup table of the enthalpy of saturated vapour at pressure p	[J/kg]

3.1.7 Subcooling throttle

The subcooling throttle valve is used differently from the two other valves in the system. During normal operation it is fully open, acting as a pipe. It can be closed fully allowing for various speciel functionalities. Firstly, shutting it off allows for detecting the amount of refridgerant in the system. This is a convenient diagnostic feature which helps ensuring that the system has enough refridgerant to properly function, and enabling leakage detection. Secondly, closing the valve while fully opening the condenser throttle valve, allows the system to operate as a standard refridgeration system, with only one valve between the evaporator and condenser.

3.1.8 Evaporator

The superheat of the evaporator is an important and difficult state to control. It is important as the compressor can be damaged if the refrigerant contains liquid. Additionally the superheat is important from an efficiency point of view. The superheat is the difference between the vapor saturation temperature and the actual temperature at the compressor suction inlet. It is a measure of excess energy transferred to the refrigerant.

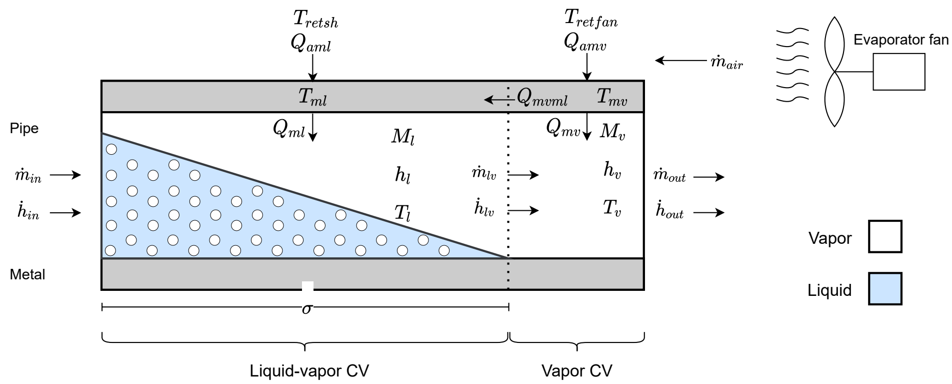

Fig. 4. Diagram of evaporator control volumes

The evaporator is split into two control volumes, divided by a moving CV boundary σ which divides liquid-vapor mixture and the superheated vapor, see Fig. 4.

Because the heat transfer coefficient between liquid and metal and vapor and metal is different, the metal is likewise split by the σ boundary. The modeling of σ is based on an assumption that the refrigerant has a constant average quality throughout the liquid-vapor mixture.

The calculation of the boundary location can be seen in Eq. (27)

$$\sigma = \frac{M_l \cdot v_1}{V_i} \tag{27}$$

where

σ	Control Volume boundary	[$\cdot$]
M_l	Mass of liquid-vapor CV	[kg]
v_1	Refrigerant specific volume of vapor-liquid CV	[m^3/kg]
V_i	Evaporator volume	[m^3]

The temperatures of the air which is blown over the evaporator is modeled by the two equations below. The fan has some loss in the form of heat which is transferred to the air.

$$T_{retfan} = T_{ret} + \frac{Q_{fan}}{\dot{m}_{air} \cdot Cp_{air}} \tag{28}$$

$$T_{retsh} = T_{retfan} - \frac{Q_{amv}}{\dot{m}_{air} \cdot Cp_{air}} \tag{29}$$

where

T_{retfan}	Temperature of return air after passing through fan	[K]
T_{retsh}	Temperature of air over superheated vapor CV	[K]
T_{ret}	Return temperature of air coming from trailer	[K]
Q_{fan}	Heat added from fan to air (heatloss)	[W]
Q_{amv}	Heat flow from air to metal surrounding vapor CV	[W]
$\dot{m}_{air}$	Mass flow of air through fan	[kg/s]
Cp_{air}	Specific heat capacity of air	[J/(K$\cdot$kg)]

The heat flow from air to metal of evaporator is modeled based on the assumption that the mass flow of air is cooled down to the metal temperature as seen in Eq. (32) and Eq. (33).

Eq. (31) is the heat loss from the fan that is being added to the air flow.

$$U_{*P} = \left(\frac{U_{fan}}{3060} \cdot 100 - 55.56\right) \cdot 0.0335 \tag{30}$$

$$Q_{fan} = 177.76 + 223.95 \cdot U_{*P} + 105.85 \cdot U_{*P}^2 + 16.74 \cdot U_{*P}^3 \tag{31}$$

$$Q_{amv} = Cp_{air} \cdot \dot{m}_{air} \cdot (T_{retfan} - T_{mv}) \tag{32}$$

$$Q_{aml} = Cp_{air} \cdot \dot{m}_{air} \cdot (T_{retsh} - T_{ml}) \tag{33}$$

where

U_{*P}	Scaled fan speed	[1/s]
U_{fan}	Fan speed	[1/s]
Q_{fan}	Heat flow from fan to air (heatloss)	[W]
Q_{amv}	Heat flow from air to metal surrounding vapor CV	[W]
Cp_{air}	Specific heat capacity of air	[J/(K·kg)]
$\dot{m}_{air}$	Mass flow of air through fan	[kg/s]
T_{retfan}	Temperature of return air after passing through fan	[K]
T_{mv}	Temperature of metal on the vapor CV	[K]

The fans used to move air over the condenser and evaporator are driven by VFD allowing for an assumed continous range of speed settings from 0% to 100%. The mass flow as a function of fan speed can be modelled with a 2nd order polynomial, as shown below.

The airflow over the evaporator and condenser are dynamic because they are driven by fans that have rotational inertia. Additionally, as the air is a fluid itself, it contains some inertia too. This behavior is modeled by:

$$U_{*\dot{m}} = (U_{fan} - 2270.4) \cdot 0.0017 \tag{34}$$

$$\bar{\dot{V}}_{air} = 0.7273 + 0.1202 \cdot U_{*\dot{m}} - 0.0044 \cdot U_{*\dot{m}}^2 \tag{35}$$

$$\bar{\dot{m}}_{air} = \bar{\dot{V}}_{air} \cdot \rho_{air} \tag{36}$$

$$\frac{\Delta \dot{m}_{air}}{\Delta t} = \frac{\bar{\dot{m}}_{air} - \dot{m}_{air}}{10s} \tag{37}$$

where

$U_{*\dot{m}}$	Intermediate variable	[1/s]
$\bar{\dot{V}}_{air}$	Estimated steady state volume flow of air for a given fan speed	[m^3/s]
$\bar{\dot{m}}_{air}$	Estimated steady state mass flow of air for a given fan speed	[kg/s]
$\dot{m}_{air}$	Actual mass flow of air	[kg/s]
U_{fan}	Fan speed	[1/s]
ρ_{air}	Density of air	[kg/m^3]
$\frac{\Delta \dot{m}_{air}}{\Delta t}$	The rate of change of air flow	[kg/s^2]

Eq. (36) calculates the steady state air mass flow at new speed. Eq. (37) approximates the rate of change of the air mass flow as a first-order difference with time constant of 10 seconds.

The temperatures of the evaporator vapor-liquid and vapor metal CVs are modeled by the two equations below respectively.

$$\frac{dT_{ml}}{dt} = \frac{Q_{aml} - Q_{ml} + Q_{mvml}}{M_m \cdot Cp_m \cdot \sigma} \tag{38}$$

$$\frac{dT_{mv}}{dt} = \frac{Q_{amv} - Q_{mv} - Q_{mvml}}{M_m \cdot Cp_m \cdot (1 - \sigma)} \tag{39}$$

where

T_{ml}	Metal temperature in liquid-vapor CV	[K/s]
T_{mv}	Metal temperature in vapor CV	[K/s]
Q_{aml}	Heat flow from air to metal surrounding liquid-vapor CV	[W]
Q_{ml}	Heat flow from evaporator metal to liquid-vapor CV	[W]
Q_{mvml}	Heat flow from through from metal surrounding vapor CV to metal surrounding liquid-vapor CV	[W]
Q_{amv}	Heat flow from air to metal surrounding vapor CV	[W]
Q_{mv}	Heat flow from evaporator metal to vapor CV	[W]
M_m	Mass of metal	[kg]
Cp_m	Specific heat capacity of metal	[J/(K·kg)]
σ	Control Volume boundary	[·]

Eq. (40), Eq. (41) and Eq. (42) all model convection heat flows.

$$Q_{mvml} = UA_3 \cdot (T_{mv} - T_{ml}) \tag{40}$$
$$Q_{ml} = UA_1 \cdot (T_{ml} - T_l) \cdot \sigma \tag{41}$$
$$Q_{mv} = UA_2 \cdot (T_{mv} - T_v) \cdot (1 - \sigma) \tag{42}$$

Q_{aml}	Heat flow from air to metal surrounding liquid-vapor CV	[W]
Q_{mvml}	Heat flow from through from metal surrounding vapor CV to metal surrounding liquid-vapor CV	[W]
Q_{ml}	Heat flow from evaporator metal to liquid-vapor CV	[W]
Q_{mv}	Heat flow from evaporator metal to vapor CV	[W]
U_{fan}	Fan speed	[1/s]
Cp_{air}	Specific heat capacity of air	[J/(K·kg)]
$\dot{m}_{air}$	Mass flow of air through fan	[kg/s]
T_{retsh}	Temperature of air over superheated vapor CV	[K]
T_{ml}	Temperature of metal on the liquid-vapor CV	[K]
T_{mv}	Temperature of metal on the vapor CV	[K]
T_l	Saturation temperature for evaporation of the refrigerant	[K]
T_v	Temperature of refrigerant (vapor) leaving the evaporator	[K]
UA_1	Heat transfer coefficient from metal to liquid	[J/K]
UA_2	Heat transfer coefficient from metal to vapor	[J/K]
UA_3	Heat transfer coefficient from metal surrounding vapor CV to metal surrounding liquid-vapor CV	[J/K]

Output pressure, specific enthalpies and mass balances are given by equations Eq. (43) $\rightarrow$ Eq. (47). Eq. (48) describes the temperature of the air leaving the evaporator and Eq. (49) describes the flow from the liquid-vapor CV to the vapor CV. The dew point specific enthalpy is the point at the evaporator pressure where liquid turns to vapor.

$$p_{out} = \Pi\left(h_v, \frac{V_i - V_l}{M_v}\right) \tag{43}$$

$$h_l = h_{in} + \frac{Q_{ml}}{\dot{m}_{in}} \tag{44}$$

$$h_v = h_{lv} + \frac{Q_{mv}}{\dot{m}_{lv}} \tag{45}$$

$$\frac{dM_l}{dt} = \dot{m}_{in} - \dot{m}_{lv} \tag{46}$$

$$\frac{dM_v}{dt} = \dot{m}_{lv} - \dot{m}_{out} \tag{47}$$

$$T_{sup} = T_{retfan} + \frac{Q_{aml} + Q_{amv}}{Cp_{air} \cdot \dot{m}_{air}} \tag{48}$$

$$\dot{m}_{lv} = \frac{Q_{ml}}{h_{dew} - h_{in}} \tag{49}$$

where

p_{out}	Pressure in evaporator	[Pa]
$\Pi(h, \rho)$	Table lookup of pressure, where inputs are specific enthalpy and density	[Pa]
h_v	Specific enthalpy of vapor CV	[J/kg]
h_l	Specific enthalpy of liquid-vapor CV	[J/kg]
h_{in}	Specific enthalpy of input liquid refrigerant	[J/kg]
h_{lv}	Specific enthalpy of refrigerant moving from liquid-vapor CV to vapor CV	[J/kg]
h_{dew}	Specific enthalpy of dew point	[J/kg]
V_i	Total volume of evaporator	[m^3]
V_l	Volume of liquid refrigerant	[m^3]
M_v	Mass in in vapor CV	[kg/s]
M_l	Mass in in liquid-vapor CV	[kg/s]
Q_{ml}	Heat flow from evaporator metal to liquid-vapor CV	[W]
Q_{mv}	Heat flow from evaporator metal to vapor CV	[W]
Q_{aml}	Heat flow from air to metal surrounding liquid-vapor CV	[W]
Q_{amv}	Heat flow from air to metal surrounding vapor CV	[W]
M_m	Mass of metal	[kg]
M_v	Mass of vapor	[kg]
Cp_{air}	Specific heat capacity of air	[J/(K·kg)]
$\dot{m}_{in}$	Mass flow of input refrigerant	[kg/s]
$\dot{m}_{lv}$	Mass flow of refrigerant from liquid-vapor CV to vapor CV	[kg/s]
$\dot{m}_{out}$	Mass flow of output refrigerant	[kg/s]
$\dot{m}_{air}$	Actual mass flow of air	[kg/s]
T_{sup}	Temperature of air flowing into trailer box	[K]
T_{retfan}	Temperature of return air after passing through fan	[K]

3.1.9 Box

The trailer box contains by far the greatest thermodynamic capacities due to the large mass of the cargo. The cargo temperature is strongly coupled to the surrounding air temperature due to its large surface area. The temperatures of the two main thermal capacities are modeled and their state space equations are given as below:

$$\frac{dT_{air}}{dt} = \frac{Q_{ca} + Q_{ba} + Q_{fan} - Q_{cool}}{M_{air} \cdot Cp_{air}} \tag{50}$$

$$\frac{dT_{box}}{dt} = \frac{Q_{amb} - Q_{ba}}{M_{box} \cdot Cp_{box}} \tag{51}$$

$$\frac{dT_{cargo}}{dt} = \frac{-Q_{ca}}{M_{cargo} \cdot Cp_{cargo}} \tag{52}$$

where

Q_{ca}	Cargo to air heat flow	[W]
Q_{ba}	Box to air heat flow	[W]
Q_{fan}	Fan to air heat flow	[W]
Q_{cool}	Air to evaporator heat flow	[W]
Q_{amb}	Ambient to box heat flow	[W]
T_{air}	Air temperature	[K]
T_{box}	Box temperature	[K]
T_{cargo}	Cargo temperature	[K]
M_{air}	Air mass	[kg]
M_{box}	Trailer box aluminum mass	[kg]
M_{cargo}	Cargo mass	[kg]
Cp_{air}	Air specific heat capacity	[J/kg K]
Cp_{cargo}	Cargo specific heat capacity	[J/kg K]
Cp_{box}	Cargo specific heat capacity	[J/kg K]

The heat flows are modeled as seen in Eq. (53) $\rightarrow$ Eq. (57). U_* in Eq. (58) is simply an intermediate scaled fan speed used in modeling the fan heat loss Q_{fan}.

$$Q_{cool} = Cp_{air} \cdot \dot{m_{air}} \cdot (T_{ret} - T_{sup}) \tag{53}$$

$$Q_{amb} = (T_{ambi} - T_{box}) \cdot UA_{amb} \tag{54}$$

$$Q_{ba} = (T_{box} - T_{air}) \cdot UA_{ba} \tag{55}$$

$$Q_{ca} = (T_{cargo} - T_{air}) \cdot UA_{cargo} \tag{56}$$

$$Q_{fan} = 177.76 + 223.95 \cdot U_* + 105.85 \cdot U_*^2 + 16.74 \cdot U_*^3 \tag{57}$$

$$U_* = \left(\frac{U_{fan}}{3060} \cdot 100 - 55.56\right) \cdot 0.0335 \tag{58}$$

where

$\dot{m}_{air}$	Air mass flow	[kg/s]
T_{ret}	Return air temperature	[K]
T_{sup}	Supply air temperature	[K]
T_{ambi}	Ambient air temperature	[K]
T_{box}	Box aluminum temperature	[K]
UA_{amb}	Ambient air to box heat transfer coefficient	[W/K]
UA_{ba}	Box to air heat transfer coefficient	[W/K]
UA_{cargo}	Cargo to air heat transfer coefficient	[W/K]
U_{fan}	Fan speed	[·]
U_*	Scaled fan speed	[·]

Q_{cool} is the cooling provided by the evaporator. It is calculated based on the difference between the temperature of the air returning from the box (T_{ret}) and the temperature of the air supplied to the box T_{sup} as seen in Eq. (53)

3.2 Collection of components

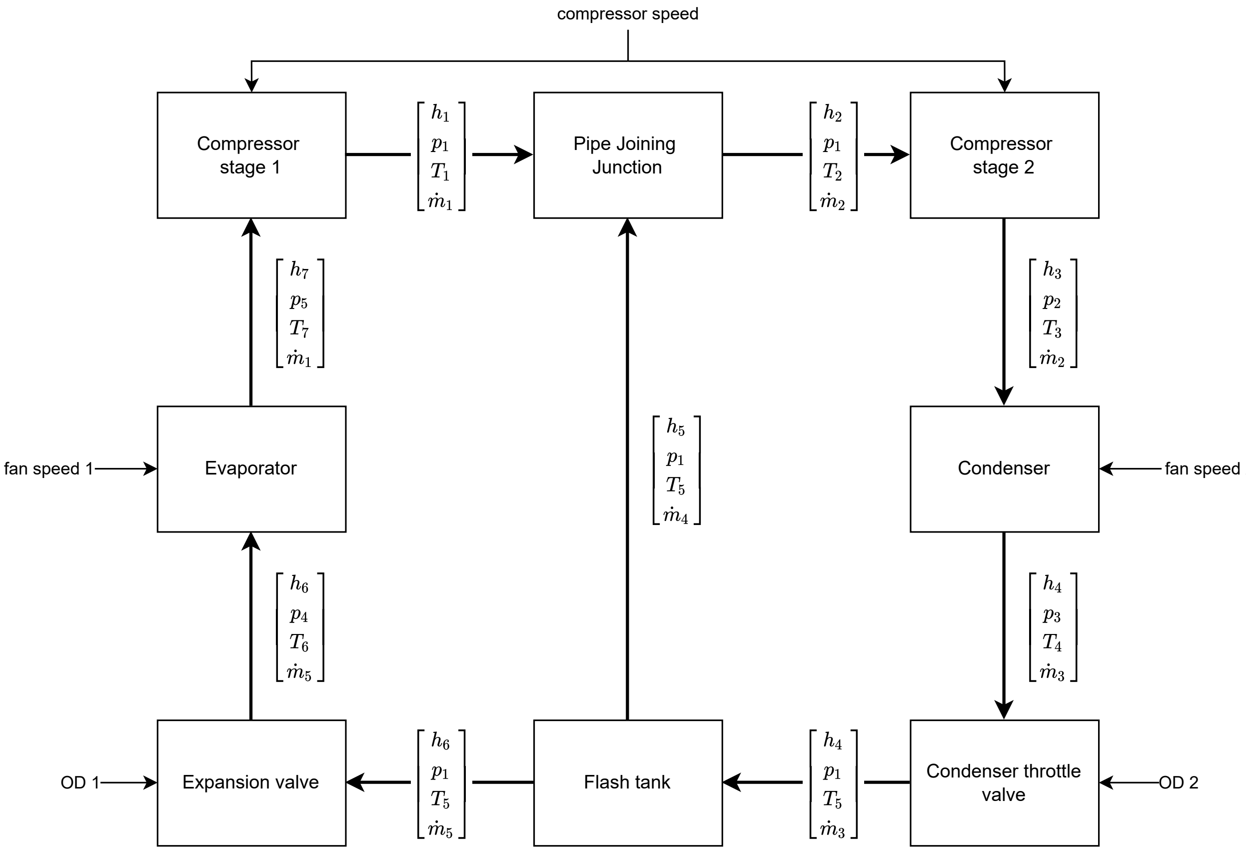

Fig. 5. Initial Block Diagram with states

$$f(x,u)=\frac{d}{dt}\begin{bmatrix} M_{PJJ} \\ M_{Con} \\ T_m \\ \dot{m}_{air} \\ T_{ml} \\ T_{mv} \\ M_l \\ M_v \end{bmatrix} = \begin{bmatrix} \left(\frac{V_1}{v_{1COM1}}-\frac{V_C}{v_{2COM1}}\right)\frac{\omega}{2}+f_p(\Theta_1)\cdot K\sqrt{\frac{1}{v_{CTV_{in}}}(p_3-p_1)}-\left(\frac{V_1}{v_{1COM2}}-\frac{V_C}{v_{2COM2}}\right)\frac{\omega}{2} \\ \left(\frac{V_1}{v_{1COM2}}-\frac{V_C}{v_{2COM2}}\right)\frac{\omega}{2}-f_p(\Theta_1)\cdot K\sqrt{\frac{1}{v_{CTV_{in}}}(p_3-p_1)} \\ \frac{UA_{rm}\cdot(T_r-T_m)-UA_{ma}\cdot(T_m-T_a)\cdot\left(0.05+\frac{U_{fan_1}}{1530}\right)}{M_m\cdot Cp_m} \\ \frac{(0.7273+0.1202\cdot U_{*\dot{m}}-0.0044\cdot U_{*\dot{m}}^2)\cdot\rho_{air}-\dot{m}_{air}}{10s} \\ \frac{Cp_{air}\cdot\dot{m}_{air}\cdot(T_{retsh}-T_{ml})-UA_1\cdot(T_{ml}-T_l)\cdot\sigma+UA_3\cdot(T_{mv}-T_{ml})}{M_m\cdot Cp_m\cdot\sigma} \\ \frac{Cp_{air}\cdot\dot{m}_{air}\cdot(T_{retfan}-T_{mv})-UA_2\cdot(T_{mv}-T_v)\cdot(1-\sigma)-UA_3\cdot(T_{mv}-T_{ml})}{M_m\cdot Cp_m\cdot(1-\sigma)} \\ f_p(\Theta_2)\cdot K\sqrt{\frac{1}{v_{EV_{in}}}(p_1-p_4)}-\frac{UA_1\cdot(T_{ml}-T_l)\cdot\sigma}{h_{dew}-h_6} \\ \frac{UA_1\cdot(T_{ml}-T_l)\cdot\sigma}{h_{dew}-h_6}-\left(\frac{V_1}{v_{1COM1}}-\frac{V_C}{v_{2COM1}}\right)\frac{\omega}{2} \end{bmatrix} \tag{59}$$

$$g_{PJJ}(x,u)=\begin{bmatrix} h_2 \\ \dot{m}_2 \end{bmatrix}-\begin{bmatrix} \frac{h_1\cdot\dot{m}_1+h_5\cdot\dot{m}_3}{\dot{m}_2} \\ \dot{m}_1+\dot{m}_3 \end{bmatrix}=\mathbf{0} \tag{60}$$

$$g_{COM2}(x,u)=\begin{bmatrix} \dot{m}_2 \\ h_3 \\ v_{1COM2} \\ v_{2COM2} \\ p_{i1COM2} \\ p_{i2COM2} \\ \gamma \\ T_3 \end{bmatrix}-\begin{bmatrix} \left(\frac{V_1}{v_{1COM2}}-\frac{V_C}{v_{2COM2}}\right)\frac{\omega}{2} \\ \Upsilon(T_3,p_2) \\ \Gamma(T_2,p_{i1COM2}) \\ \left(\frac{p_{i2COM2}}{p_{i1COM2}}\right)^{\frac{-1}{\gamma}} \\ p_1-kl_1\cdot\omega \\ p_2+kl_2\cdot\omega \\ \frac{C_{cp}}{C_{cv}} \\ T_2\cdot\left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} \end{bmatrix}=\mathbf{0} \tag{61}$$

$$g_{Con}(x,u) = \begin{bmatrix} h_4 \\ p_2 \\ \dot{m}_3 \\ Q_{rm} \\ Q_{ma} \end{bmatrix} - \begin{bmatrix} h_3 - \dfrac{Q_{rm}}{\dot{m}_2} \\ p_3 - \lambda \cdot \dot{m}_2 \\ \dot{m}_2 + \dfrac{M_{con} - \frac{V_i}{v_{Con}}}{1s} \\ UA_{rm} \cdot (T_r - T_m) \\ UA_{ma} \cdot (T_m - T_a) \cdot \left(0.05 + \dfrac{U_{fan_1}}{1530}\right) \end{bmatrix} = \mathbf{0} \tag{62}$$

$$g_{Val}(x,u) = \begin{bmatrix} \dot{m}_3 \\ \dot{m}_5 \end{bmatrix} - \begin{bmatrix} f_p(\Theta_1) \cdot K\sqrt{\dfrac{1}{v_{CTV_{in}}}(p_3 - p_1)} \\ f_p(\Theta_2) \cdot K\sqrt{\dfrac{1}{v_{EV_{in}}}(p_1 - p_4)} \end{bmatrix} = \mathbf{0} \tag{63}$$

$$g_{FT}(x,u) = \begin{bmatrix} \dot{m}_5 \\ \dot{m}_5 \\ h_5 \\ h_6 \end{bmatrix} - \begin{bmatrix} \dfrac{\dot{m}_3 \cdot h_4 - \dot{m}_4 \cdot h_5}{h_6} \\ \dot{m}_3 - \dot{m}_4 \\ N(p_1) \\ M(p_1) \end{bmatrix} = \mathbf{0} \tag{64}$$

$$g_{_{Eva}}(x,u) = \begin{bmatrix} \sigma \\ T_{retfan} \\ T_{retsh} \\ U_{*_P} \\ Q_{fan} \\ Q_{amv} \\ Q_{aml} \\ U_{*_{\dot{m}}} \\ \bar{\dot{V}}_{air} \\ \bar{\dot{m}}_{air} \\ Q_{mvml} \\ Q_{ml} \\ Q_{mv} \\ p_{out} \\ h_l \\ h_v \\ T_{sup} \\ \dot{m}_{lv} \end{bmatrix} - \begin{bmatrix} \dfrac{M_l \cdot v_{_{Eva}}}{V_i} \\ T_{ret} + \dfrac{Q_{fan_2}}{\dot{m}_{air} \cdot Cp_{air}} \\ T_{retfan} - \dfrac{Q_{amv}}{\dot{m}_{air} \cdot Cp_{air}} \\ \left(\dfrac{U_{fan_2}}{3060} \cdot 100 - 55.56\right) \cdot 0.0335 \\ 177.76 + 223.95 \cdot U_{*_P} + 105.85 \cdot U_{*_P}^2 + 16.74 \cdot U_{*_P}^3 \\ Cp_{air} \cdot \dot{m}_{air} \cdot (T_{retfan} - T_{mv}) \\ Cp_{air} \cdot \dot{m}_{air} \cdot (T_{retsh} - T_{ml}) \\ (U_{fan_2} - 2270.4) \cdot 0.0017 \\ 0.7273 + 0.1202 \cdot U_{*_{\dot{m}}} - 0.0044 \cdot U_{*_{\dot{m}}}^2 \\ \bar{\dot{V}}_{air} \cdot \rho_{air} \\ UA_3 \cdot (T_{mv} - T_{ml}) \\ UA_1 \cdot (T_{ml} - T_l) \cdot \sigma \\ UA_2 \cdot (T_{mv} - T_v) \cdot (1 - \sigma) \\ \Pi\left(h_v, \dfrac{V_i - V_l}{M_v}\right) \\ h_6 + \dfrac{Q_{ml}}{\dot{m}_{in}} \\ h_{lv} + \dfrac{Q_{mv}}{\dot{m}_{lv}} \\ T_{retfan} + \dfrac{Q_{aml} + Q_{amv}}{Cp_{air} \cdot \dot{m}_{air}} \\ \dfrac{Q_{ml}}{h_{dew} - h_6} \end{bmatrix} = \mathbf{0} \qquad (65)$$

$$g_{COM1}(x,u) = \begin{bmatrix} \dot{m}_1 \\ h_1 \\ v_{1_{COM1}} \\ v_{2_{COM1}} \\ p_{i1_{COM1}} \\ p_{i2_{COM1}} \\ \gamma \\ T_1 \end{bmatrix} - \begin{bmatrix} \left(\frac{V_1}{v_{1_{COM1}}} - \frac{V_C}{v_{2_{COM1}}}\right)\frac{\omega}{2} \\ \Upsilon(T_1, p_1) \\ \Gamma(T_7, p_{i1_{COM1}}) \\ \left(\frac{p_{i2_{COM1}}}{p_{i1_{COM1}}}\right)^{\frac{-1}{\gamma}} \\ p_5 - kl_1 \cdot \omega \\ p_1 + kl_2 \cdot \omega \\ \frac{C_{cp}}{C_{cv}} \\ T_7 \cdot \left(\frac{p_1}{p_5}\right)^{\frac{\gamma-1}{\gamma}} \end{bmatrix} = \mathbf{0} \tag{66}$$

$$g(x,u) = \begin{bmatrix} g_{PJJ}(x,u) \\ g_{COM2}(x,u) \\ g_{Con}(x,u) \\ g_{Val}(x,u) \\ g_{FT}(x,u) \\ g_{Eva}(x,u) \\ g_{COM1}(x,u) \end{bmatrix} = \mathbf{0} \tag{67}$$

3.2.1 Linearisation

4 Controller Design

4.1 State space controller - Pole placement

4.2 LMI / MPC controller

5 Robustness of Controller

6 Test - Simulation

Benchmark against Bitzers current controller

7 Test - Real System

8 Conclusion

References

[1] K. K. Sørensen, "Model Based Control of Reefer Container Systems," 2013.
[2] P. Koelet, *Industrial Refrigeration: Principles, Design and Applications*. The MacMillan Press LTD, 1992.

9 APPENDIX

9.1 Explanation of relevant terms for thermodynamics

9.1.1 Enthalpy

Enthalpy is an energy term that is defined as the sum of the flow work [2].

$$h = u + pv \tag{68}$$

where

h	specific enthalpy	$[\mathrm{Joule/kg}]$
u	internal energy	$[\mathrm{Joule/kg}]$
p	absolute pressure	$[\mathrm{Pa}]$
v	specific volume	$[\mathrm{m^3/kg}]$

Assuming the reader is familiar with the definition of absolute pressure and specific volumes, it remains to define internal energy.

9.1.2 Internal energy

The internal energy of a mass can be viewed as primarily the 'sensible heat' kinetic energy that dictates the movement of molecules. From a higher 'movement' of molecules, e.g. more energy, the temperature of a mass (that consists of the molecules) increases. The amount of energy required to increase a unit of mass by one degree is known as the specific heat capacity of the given material. Secondly the internal energy also consists of the latent energy. The latent energy is the energy that is embedded in phase changes of a systems state, e.g. from or to solid, liquid and vapor from either one. Analogously to the specific heat capacity for the sensible heat, for latent energy has a similar definition. The amound of energy required to change phase from one state to another is called the latent heat capacity of the material. It differs to material to material and also from whether its from solid to liquid or from liquid to vapor/gas.