Given

- $f \in \mathcal{E}^+$.
- $\sum_{j=0}^{M} y_j 1_{A_j}$ and $\sum_{k=0}^{N} z_k 1_{B_k}$ are two standard representations of f

Tools

$$A_i \cap A_j = \varnothing \implies \sum_{k=0}^{\infty} \mu(A_j) = \mu\Big(\bigcup_{j=1}^{\infty} A_j\Big)$$

$$\boxed{\sum_{j=0}^{M} y_j \mu(A_j) = \sum_{k=0}^{N} z_k \mu(B_k)}$$

$$\Uparrow$$

$$\sum_{j=0}^{M} y_j \sum_{k=0}^{N} \mu(A_j \cap B_k) = \sum_{k=0}^{N} z_k \sum_{j=0}^{M} \mu(A_j \cap B_k)$$

$$\Uparrow$$

$$y_j \mu(A_j \cap B_k) = z_k \mu(A_j \cap B_k) \qquad \forall (j,k)$$

$$\Uparrow$$

$$\sum_{j=0}^{M} y_j 1_{A_j}(x) = \sum_{k=0}^{N} z_k 1_{B_k}(x) \qquad \forall x \in X$$