

**Given**

- $f \in \mathcal{E}^+$ .
- $\sum_{j=0}^M y_j 1_{A_j}$  and  $\sum_{k=0}^N z_k 1_{B_k}$  are two standard representations of  $f$

**Tools**

$$A_i \cap A_j = \emptyset \implies \sum_{k=0}^{\infty} \mu(A_j) = \mu\left(\bigcup_{j=1}^{\infty} A_j\right)$$

$$\sum_{j=0}^M y_j \mu(A_j) = \sum_{k=0}^N z_k \mu(B_k)$$

$$\Uparrow$$

$$\sum_{j=0}^M y_j \sum_{k=0}^N \mu(A_j \cap B_k) = \sum_{k=0}^N z_k \sum_{j=0}^M \mu(A_j \cap B_k)$$

$$\Uparrow$$

$$y_j \mu(A_j \cap B_k) = z_k \mu(A_j \cap B_k) \quad \forall (j, k)$$

$$\Uparrow$$

$$\sum_{j=0}^M y_j 1_{A_j}(x) = \sum_{k=0}^N z_k 1_{B_k}(x) \quad \forall x \in X$$