



PHY-765 SS18 Gravitational Lensing Week 13

Cosmic Shear & the CMB

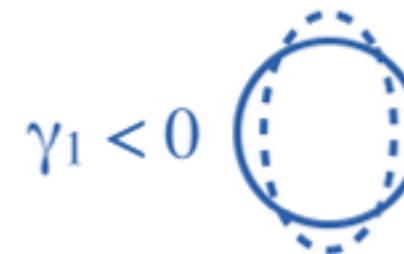
Kasper B. Schmidt

Leibniz-Institut für Astrophysik Potsdam (AIP)

Last week

- Talked about the weak lensing shearing of objects
- Used Jacobian Matrix and assumptions about sphericity to see that
 - (simple) shearing corresponds to ellipticity

$$1 = \frac{(1 - \gamma_1)^2}{\beta_0^2} \theta_1^2 + \frac{(1 + \gamma_1)^2}{\beta_0^2} \theta_2^2$$



- Described the surface brightness moments ellipticity

$$q_{ij} \equiv \int d^2\theta \, g^{\text{obs}}(\theta) \theta_i \theta_j \quad \epsilon_1 \equiv \frac{q_{11} - q_{22}}{q_{11} + q_{22}} \quad \epsilon_2 \equiv \frac{2q_{12}}{q_{11} + q_{22}}$$

- Again, the Jacobian Matrix can express this in terms of κ and γ

$$\epsilon_i = \frac{2\gamma_i}{1 - \kappa} \left[1 - \frac{\gamma^2}{(1 - \kappa)^2} \right]^{-1}$$

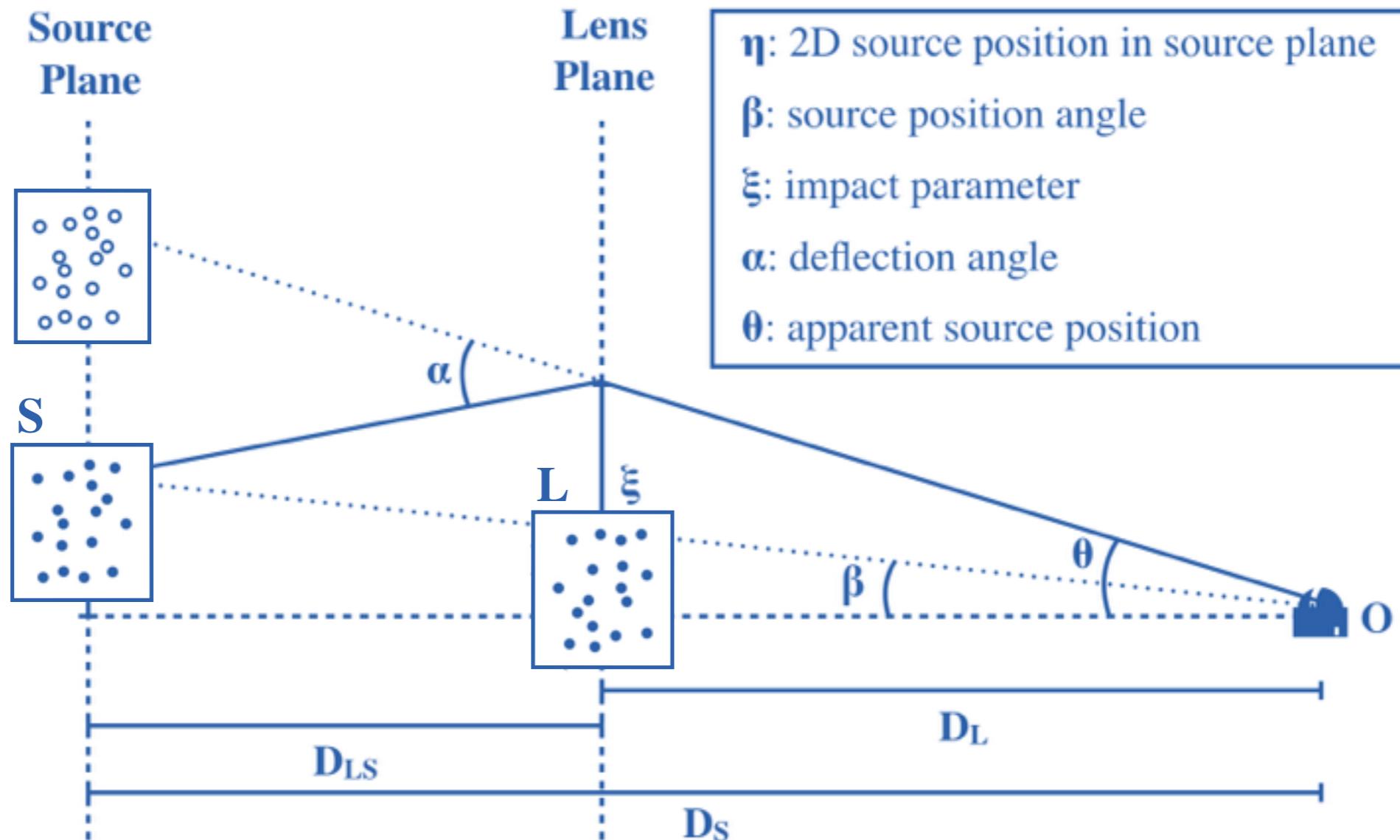
- Considered challenges with determining weak lensing
 - Intrinsic ϵ , weighing of images, accounting for PSF, etc.
- The Bullet Cluster as a proof of the existence of dark matter

The aim of today

- Deflection of ‘diffuse mass’ by ‘diffuse mass’
- The concepts of cosmic shear
- Fourier space description of cosmic shear lensing effects
- The power spectrum as a tool
- Lensing of the Cosmic Microwave Background

Light Deflection (Lensing) Regime

- Strong lensing: Concentrated source deflected by concentrated lens
- Weak lensing: Concentrated source deflected by diffuse lens
- ‘Cosmic weak lensing’: Diffuse source deflected by diffuse lens



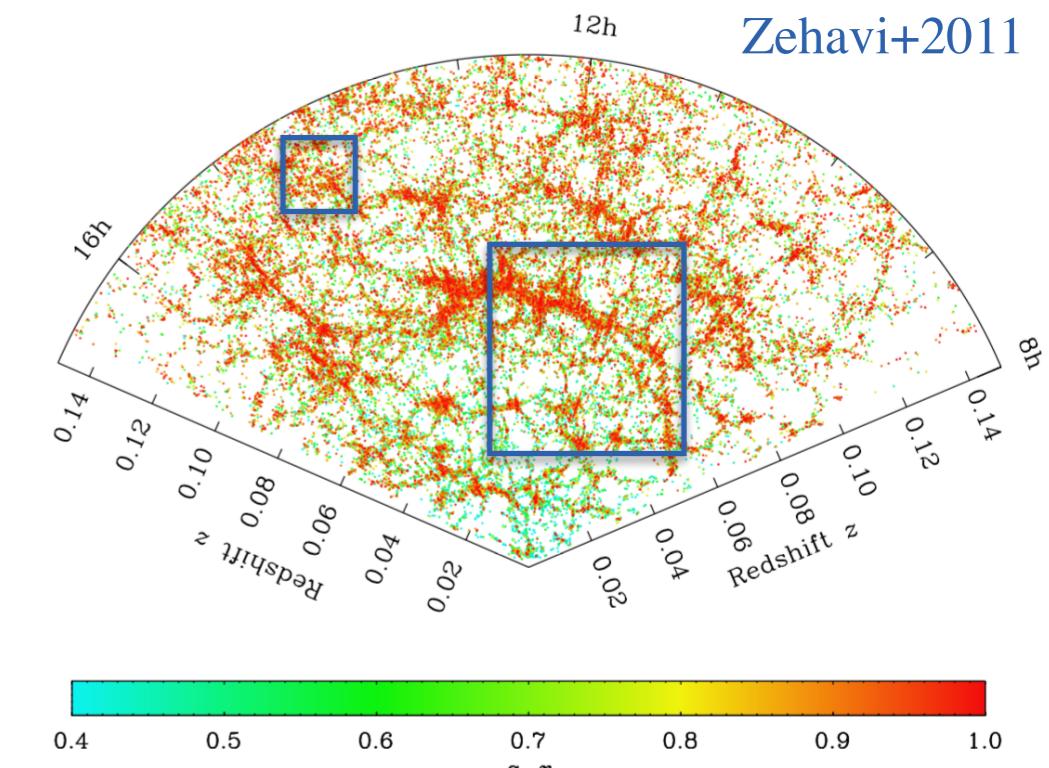
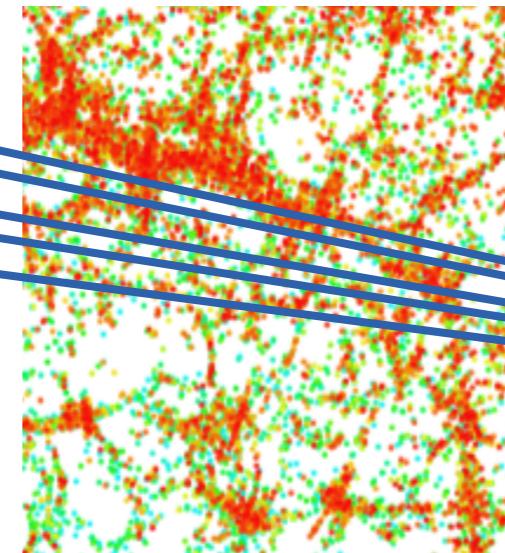
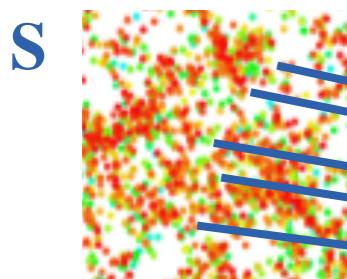
Cosmic Shear

- The large scale structure of the Universe
- Web of mass affecting background sources
- Lensing (shearing) from cosmic structure
- Assessment done in a statistical manner
- LSS predicted by cosmological models

$$p(t) = \sum w \times \rho_w(t) \quad ; \quad w = m, r, \Lambda$$

$$\rho(t) = \rho_m(t) + \rho_r(t) + \rho_\Lambda(t)$$

- Cosmic shear probes cosmology via the density contrast defined as



$$\delta(\mathbf{x}, t) \equiv \frac{\rho_m(\mathbf{x}, t) - \bar{\rho}_m(t)}{\bar{\rho}(t)}$$



Shear and the Gravitational Potential

- The Jacobina relates κ and γ to the gravitational potential (week 6)

$$\mathcal{A}(\boldsymbol{\theta}) = \begin{pmatrix} 1 - \frac{\partial \alpha_i}{\partial \theta_i} & -\frac{\partial \alpha_i}{\partial \theta_j} \\ -\frac{\partial \alpha_j}{\partial \theta_i} & 1 - \frac{\partial \alpha_j}{\partial \theta_j} \end{pmatrix} = \begin{pmatrix} 1 - \frac{\partial^2 \psi}{\partial \theta_i^2} & -\frac{\partial^2 \psi}{\partial \theta_i \partial \theta_j} \\ -\frac{\partial^2 \psi}{\partial \theta_j \partial \theta_i} & 1 - \frac{\partial^2 \psi}{\partial \theta_j^2} \end{pmatrix}$$

$$\kappa \equiv \frac{1}{2} \left(\frac{\partial^2 \psi}{\partial \theta_i^2} + \frac{\partial^2 \psi}{\partial \theta_j^2} \right) \quad \gamma_1 \equiv \frac{1}{2} \left(\frac{\partial^2 \psi}{\partial \theta_i^2} - \frac{\partial^2 \psi}{\partial \theta_j^2} \right) \\ \gamma_2 \equiv \frac{\partial^2 \psi}{\partial \theta_i \partial \theta_j}$$

- The projected gravitational potential along the line of sight is defined as

$$\psi(\boldsymbol{\theta}) \equiv \frac{2}{D_S} \int_0^{D_S} dD_L \Phi \left(x^i = D_L \theta^i, D_L; t = t_0 - \frac{D_L}{c} \right) \frac{D_S - D_L}{D_L}$$

The Project Gravitational Potential

Gives deflection angle as

$$\alpha^i(\theta) = \frac{\partial \psi(\theta)/c^2}{\partial \theta^i}$$

$$\psi(\theta) \equiv \frac{2}{D_S} \int_0^{D_S} dD_L \Phi \left(x^i = D_L \theta^i, D_L; t = t_0 - \frac{D_L}{c} \right) \frac{D_S - D_L}{D_L}$$

Integrate along line of sight to source

Newtonian grav. potential

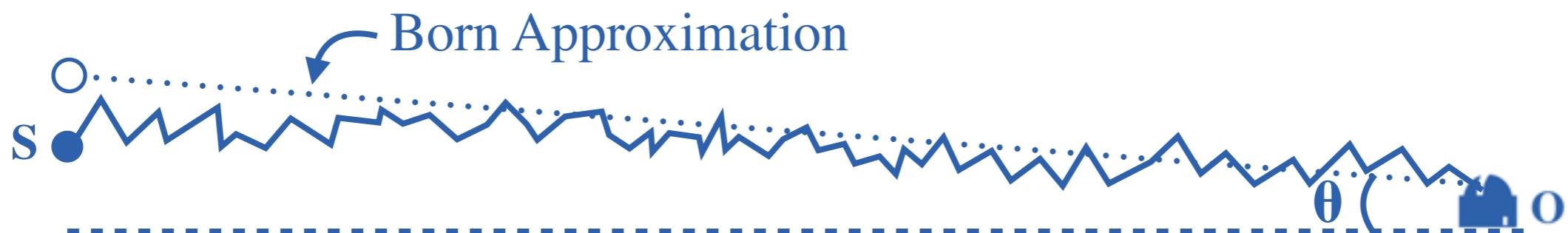
x^i is the (2D) spatial component of four vector $x^\alpha = (t, x, y, z)$ (week 2)

Grav. potential evaluated at time t when photons pass the lens plane D_L

$$D_S - D_L \over D_L$$

Mass inhomogeneities close to source ignored

Grav. potential sampled at transverse distance $D_L \theta^i$



- Gravitational potential directly relates to the density contrast via the Poisson equation:

$$\nabla^2 \Phi(\mathbf{x}, t) = \frac{4\pi G}{c^2} \bar{\rho}_m(t) a^2(t) \delta(\mathbf{x}, t)$$

$$\frac{1}{a(t)} = \frac{a(t_{\text{obs}})}{a(t_{\text{emit}})} = (1 + z)$$

Matter Distribution Described in Fourier Space

- As γ relates to the gravitational potential it relates to the density contrast
- We are interested in the statistics of these density fluctuations
- Convenient to perform such investigations in Fourier space
- Any field can be expressed in terms of its Fourier transform:

$$f(x) = \int \frac{dk}{2\pi} \tilde{f}(k) e^{ikx}$$

- In 2D taking κ from Fourier (\mathbf{l}) space to real ($\boldsymbol{\theta}$) space would then be

$$\kappa(\boldsymbol{\theta}) = \int \frac{d^2l}{(2\pi)^2} \tilde{\kappa}(\mathbf{l}) e^{i\mathbf{l}\cdot\boldsymbol{\theta}}$$

- and you can go back with

$$\tilde{\kappa}(\mathbf{l}) = \int d^2\theta \kappa(\boldsymbol{\theta}) e^{-i\mathbf{l}\cdot\boldsymbol{\theta}}$$

- Using the relations between κ , γ and Φ , you get Fourier space expressions:

$$\tilde{\kappa}(\mathbf{l}) = \frac{-l^2}{2c^2} \tilde{\psi}(\mathbf{l}) \quad \tilde{\gamma}_1(\mathbf{l}) = \frac{-l_x^2 + l_y^2}{2c^2} \tilde{\psi}(\mathbf{l}) \quad \tilde{\gamma}_2(\mathbf{l}) = \frac{-l_x l_y}{c^2} \tilde{\psi}(\mathbf{l})$$

- In Fourier space, real-space derivatives appear as powers of conjugate, \mathbf{l}

The Two-Point Correlation Function

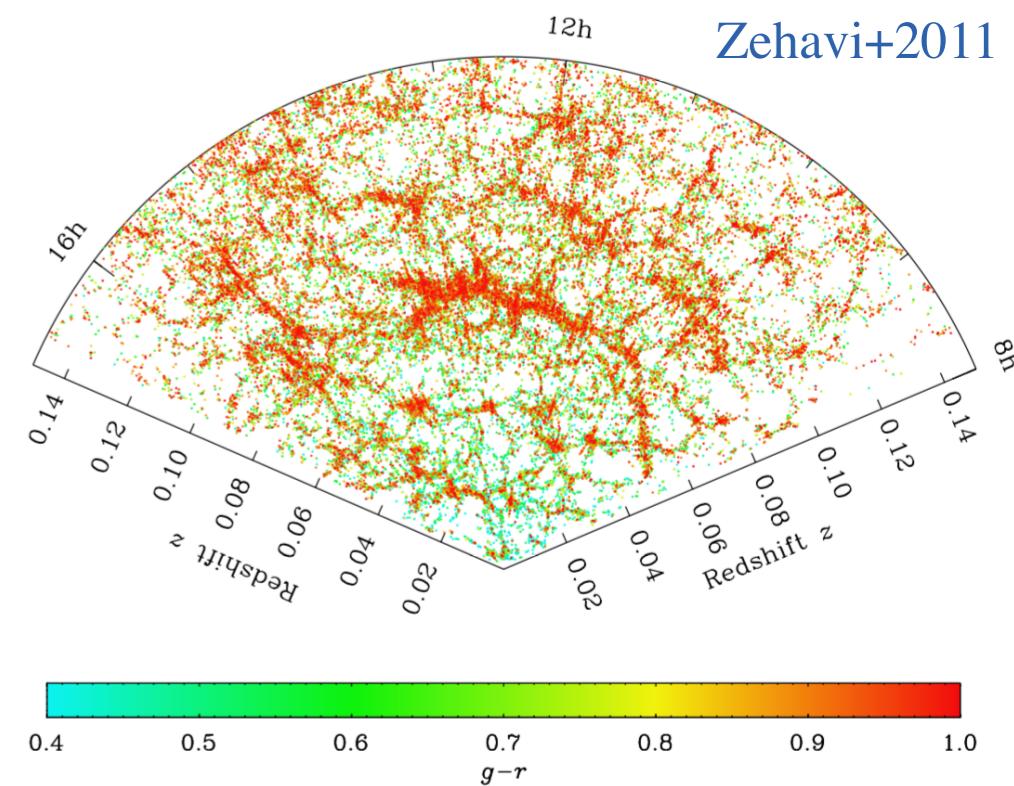
- Combining these three equations gives expression of Fourier convergence

$$\tilde{\kappa}(\mathbf{l}) = \frac{(l_x^2 - l_y^2)\tilde{\gamma}_1(\mathbf{l}) + 2l_x l_y \tilde{\gamma}_2(\mathbf{l})}{l^2} \quad (\text{Exercise 3})$$

- $l=0$ corresponds to a Fourier mode with no variation, i.e., a constant
- So up to some constant value of κ this expression holds (MSD week 10)
- We want to describe the matter density field statistically:
 - 1st order stat: $\langle \delta(\mathbf{x}) \rangle = 0$
 - 2nd order stat: $\sigma^2 \equiv \langle \delta(\mathbf{x})^2 \rangle$
- Define the two point correlation as

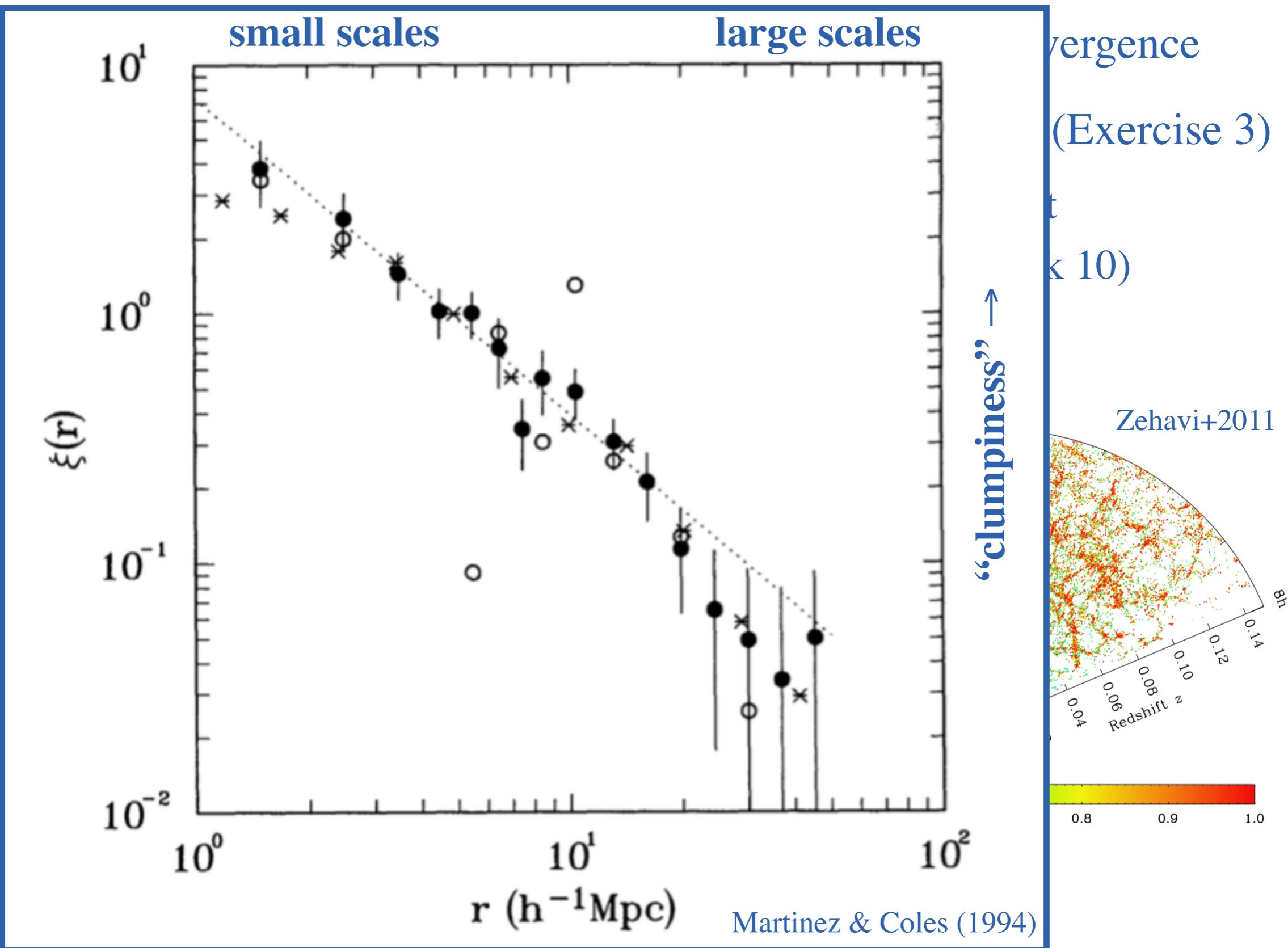
$$\xi(\mathbf{x}, \mathbf{y}) \equiv \langle \delta(\mathbf{x}) \delta(\mathbf{y}) \rangle$$

- Then from homogeneity: $\xi(\mathbf{x}, \mathbf{y}) = \xi(\mathbf{x} - \mathbf{y})$
- And from isotropy: $\xi(\mathbf{x} - \mathbf{y}) = \xi(|\mathbf{x} - \mathbf{y}|)$



The Two-Point Correlation Function

- Combining
- $l=0$ corresponds to the two-point correlation function
- So up to now we have been looking at the power spectrum
- We want to look at the two-point correlation function
- 1st order
- 2nd order
- Define the two-point correlation function
- Then from the power spectrum
- And from the two-point correlation function



The Power Spectrum

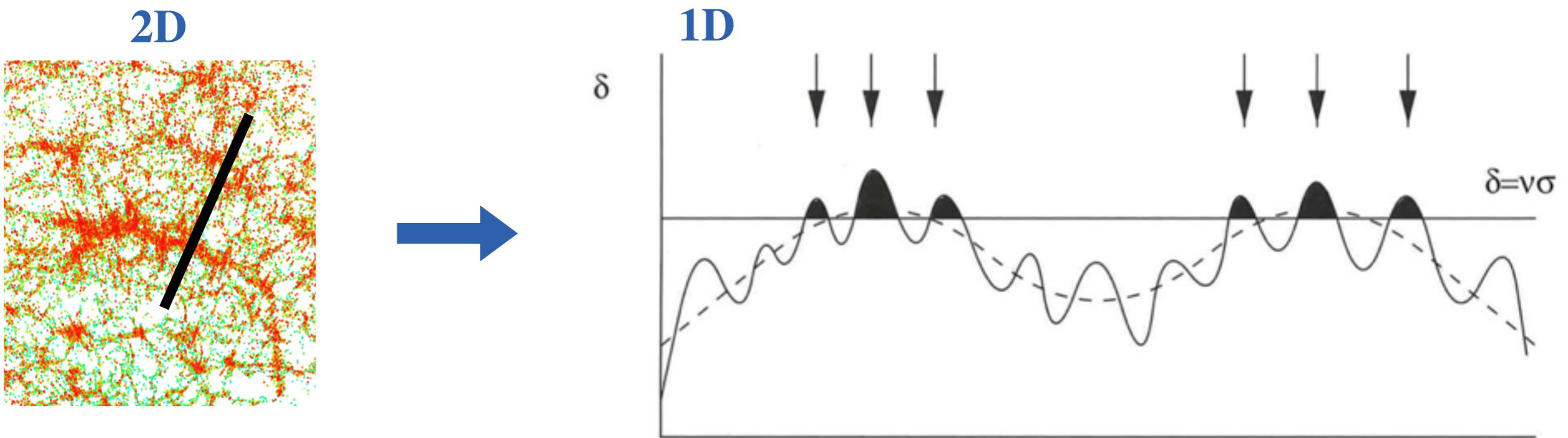
- The Fourier transform of the correlation function, is the Power Spectrum

$$\begin{aligned}\langle \tilde{\delta}(\mathbf{k})\tilde{\delta}(\mathbf{k}') \rangle &= \int d^3x e^{-i\mathbf{k}\cdot\mathbf{x}} \int d^3y e^{-i\mathbf{k}'\cdot\mathbf{y}} \xi(\mathbf{x} - \mathbf{y}) \quad (\text{Exercise 4}) \\ \dots &= (2\pi)^3 \delta_{\text{Dirac}}^3(k - k') P(k) \quad \text{for } \mathbf{x}_- = \mathbf{x} - \mathbf{y}\end{aligned}$$

with

$$P(|\mathbf{k}|) \equiv \int d^3x_- e^{i\mathbf{k}\cdot\mathbf{x}_-} \xi(\mathbf{x}_-)$$

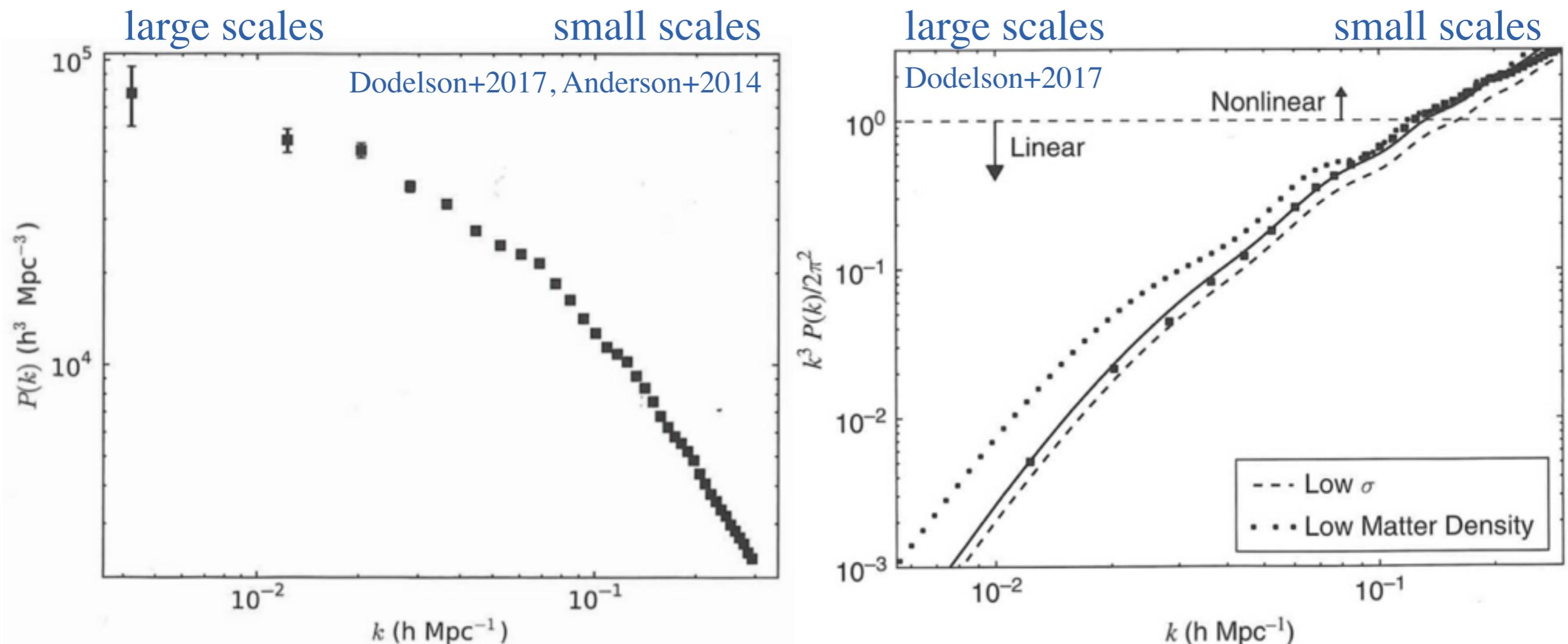
- Describes the scales ($1/l$) at which there is “power” in the density field



Schneider+2006

The Power Spectrum

- Baryon Oscillation Spectroscopic Survey (BOSS) power spectrum
 - $P(k)$ hides the expected correlation (power) at small scales
- Power spectrum needs to me unless to reveal correlation
 - Scaled by dimensionality; in this case k^3



Cosmic Shear Decomposition

- Considering the shear components in Fourier space

$$\tilde{\gamma}_1(\mathbf{l}) = \frac{-l_x^2 + l_y^2}{2c^2} \tilde{\psi}(\mathbf{l}) \quad \tilde{\gamma}_2(\mathbf{l}) = \frac{-l_x l_y}{c^2} \tilde{\psi}(\mathbf{l})$$

- The defining angle ϕ that \mathbf{l} makes with the (arbitrary) x-axis we have

$$\tilde{\gamma}_1(\mathbf{l}) = -\frac{l^2 \tilde{\psi}(\mathbf{l})}{2c^2} \cos(2\phi) \quad \text{as} \quad \cos(2\phi) = \cos^2(\phi) - \sin^2(\phi)$$

$$\tilde{\gamma}_2(\mathbf{l}) = -\frac{l^2 \tilde{\psi}(\mathbf{l})}{2c^2} \sin(2\phi) \quad \text{as} \quad \sin(2\phi) = 2 \cos(\phi) \sin(\phi)$$

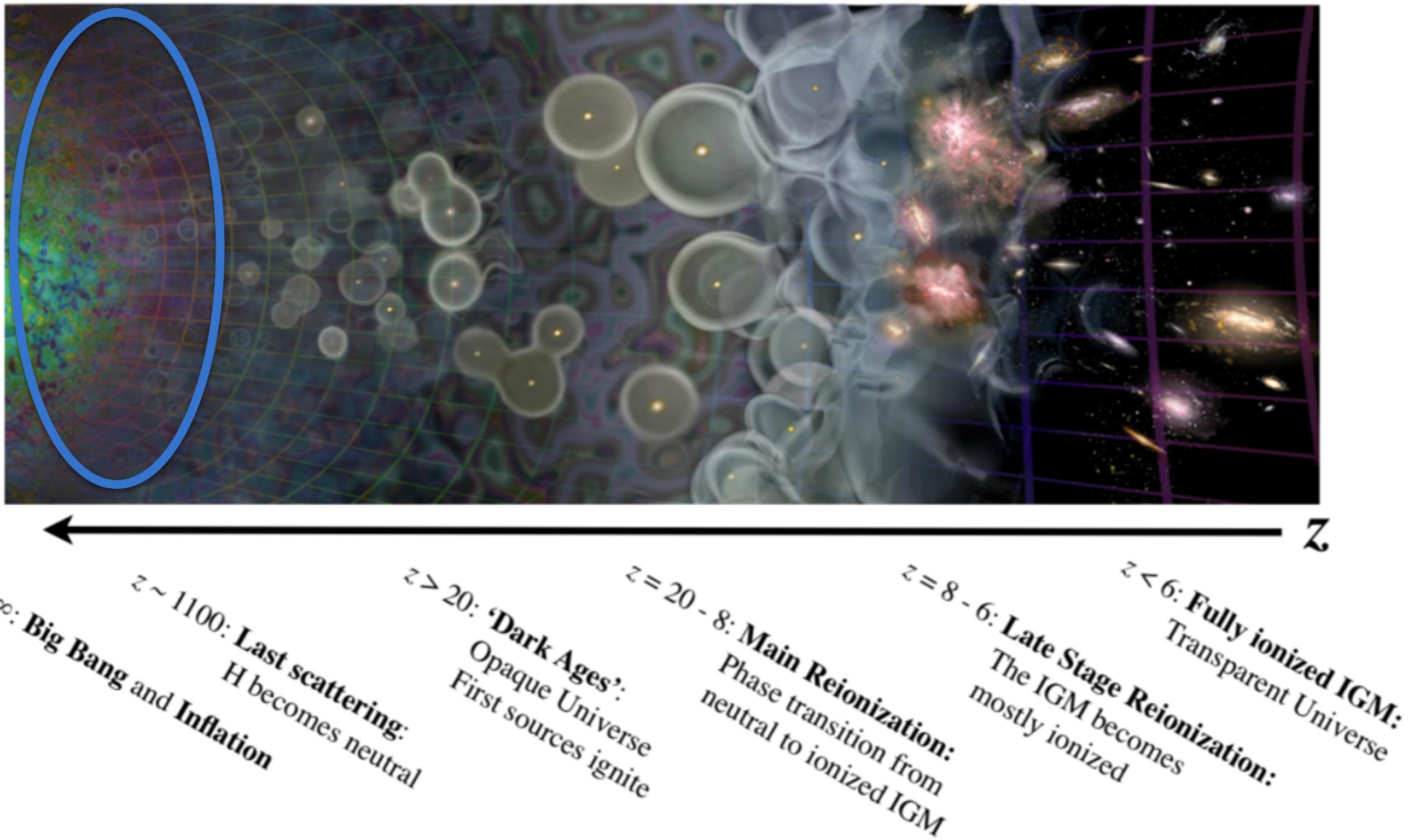
- Considering linear combinations of the shear components we arrive at

$$\tilde{E} \equiv -\tilde{\gamma}_1(\mathbf{l}) \cos(2\phi) - \tilde{\gamma}_2(\mathbf{l}) \sin(2\phi) = -\frac{l^2 \tilde{\psi}(\mathbf{l})}{2c^2}$$

$$\tilde{B} \equiv \tilde{\gamma}_1(\mathbf{l}) \sin(2\phi) - \tilde{\gamma}_2(\mathbf{l}) \cos(2\phi) = 0$$

- So any survey has to prove B -mode constant with 0 and E -mode > 0

The Cosmic Microwave Background



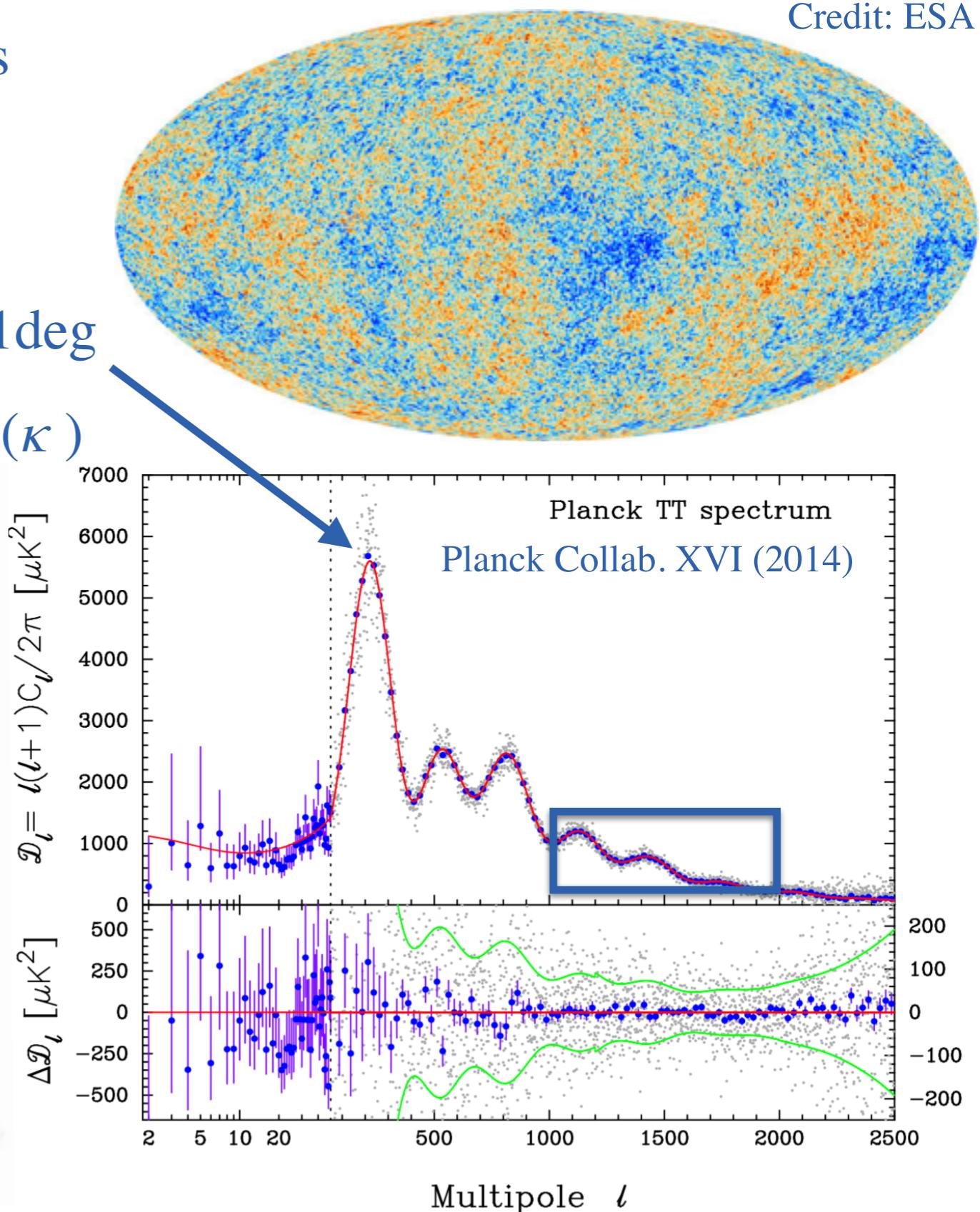
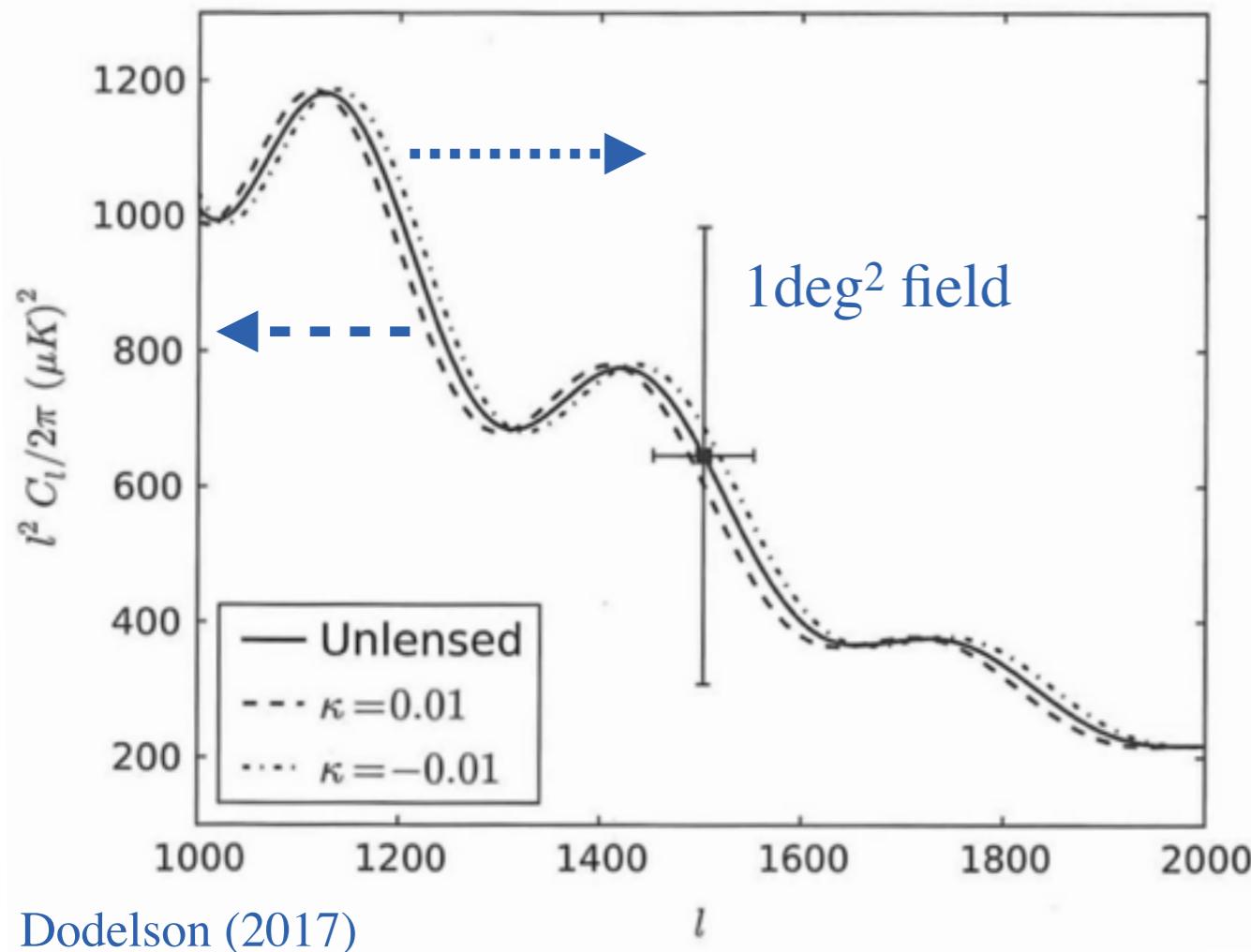
Schmidt 2016, Loeb 2006

The Cosmic Microwave Background

- Black body with $T = 2.725\text{K}$
- Peak (maximum power) at 160Ghz in the microwave today
 - But has been redshifted from universe expansion
- At recombination $T \sim 3000\text{K}$ ($z \sim 1100$)
 - CMB is snapshot of Universe when photons started traveling freely
- Surface of last scattering - the edge of the observable Universe
- Temperature contrast, i.e., the CMB temperature fluctuations are $\delta_T \sim 10^{-4}$

The Cosmic Microwave Background

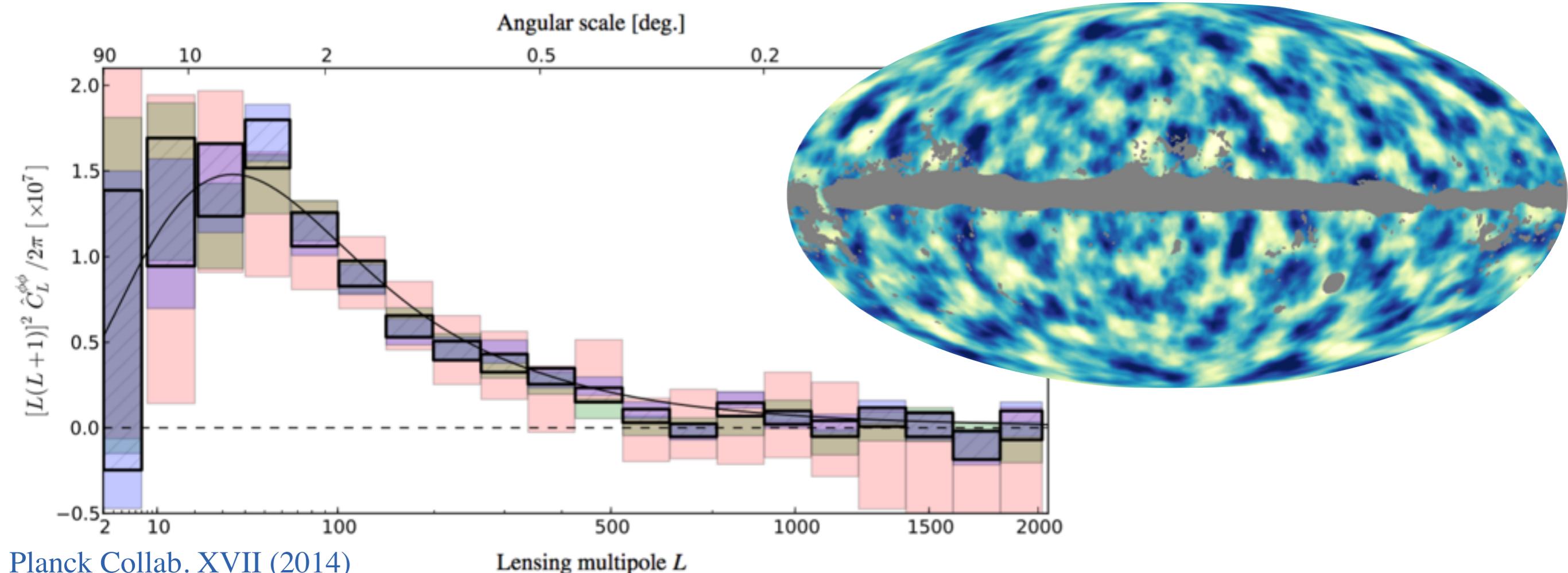
- Map shows temperature fluctuations
- Power spectrum describes their oscillations around average density
- Hot and cold spots are of the order 1deg
- Shape sensitive to lensing potential (κ)



Dodelson (2017)

Lensing Potential from CMB

- Relate T-T CMB map to the gravitational potential, Φ
- As we have seen Φ is related to δ , κ , γ_1 , and γ_2
 - Hence a lensing power spectrum can be estimated
- Measured lensing deflection angles: Due to density fluctuations of $\sim 1'$
- Structures that perform these deflections are of the order degrees on sky



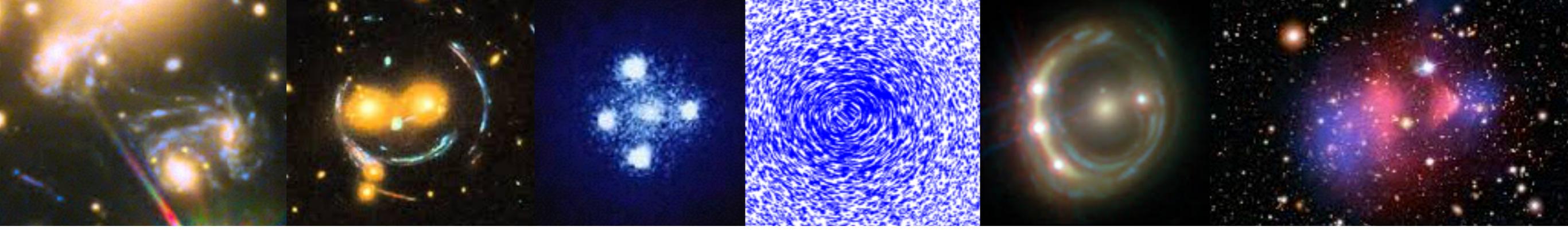
So in summary...

- The cosmic energy density maps are lensed by matter along line of sight
 - Lensing of diffuse source by diffuse lens

- The density contrast, δ is related to κ and γ via the gravitational potential

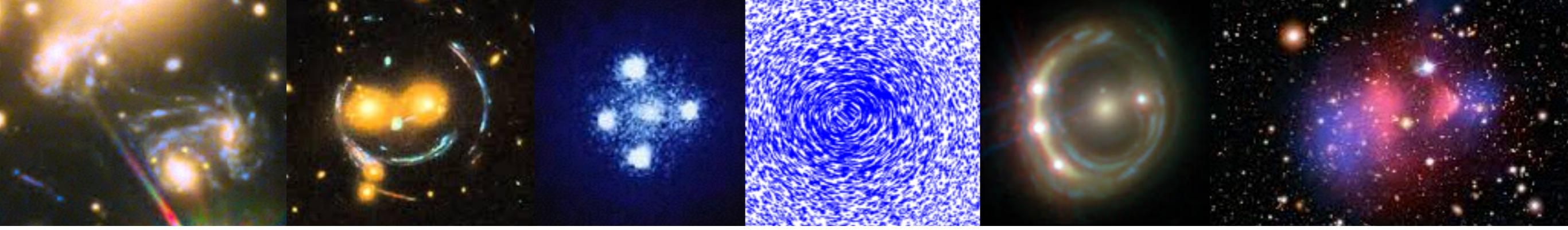
$$\boldsymbol{\delta}(\mathbf{x}, t) \leftrightarrow \Phi(\mathbf{x}, t) \leftrightarrow \psi(\theta) \leftrightarrow \kappa \quad \gamma_1 \quad \gamma_2$$

- The variance (2-point correlation function) of δ provides statistic on pattern
- In Fourier space, this results in the Power Spectrum
 - Describing (size) scales containing most power, i.e., correlation
- E and B mode decomposition of γ are useful sanity checks of results
- Cosmic Microwave Background T maps provide information about
 - Cosmology via TT Power Spectrum
 - Lensing potential (matter distribution) via matter Power Spectrum



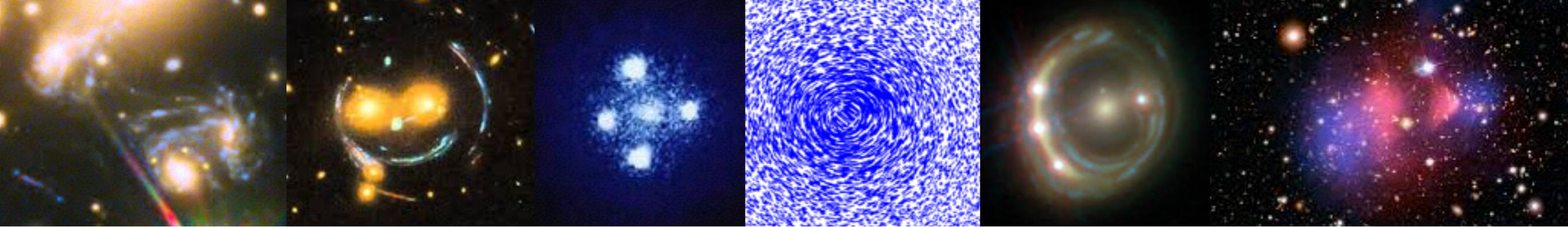
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Questions?



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Last Week's Worksheet



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This Week's Worksheet