



PHY-765 SS18 Gravitational Lensing Week 6

Magnifying Sources

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Last week

- Expressed the time delay between multiple images of a lens:

$$\Delta t = \Delta t_{\text{Geometry}} - \Delta t_{\text{Shapiro}}$$

- For the point mass lens we saw that the difference between two images is

$$t_+ - t_- \simeq -(1 + z_L) \frac{D_L D_S}{c D_{LS}} 2c^2 \theta_E \beta$$

- This enabled us to predict the right order of appearance of SN Refsdal
 - Looked at the full lens model precisions and actual re-appearance
- Described how time delays are also useful for
 - Lens model improvements
 - Determining cosmological parameters (H_0)

The aim of today

- Explore the second consequence of the lens equation: **magnification**
- Describe magnification in terms of the Jacobian
- Define magnification, shear, convergence (again), and parity of images
- Consider how images are magnified in the point mass lens and SIS/CIS
- Applications of magnification
 - Mapping the mass distribution in lenses
 - Finding high-redshift galaxies

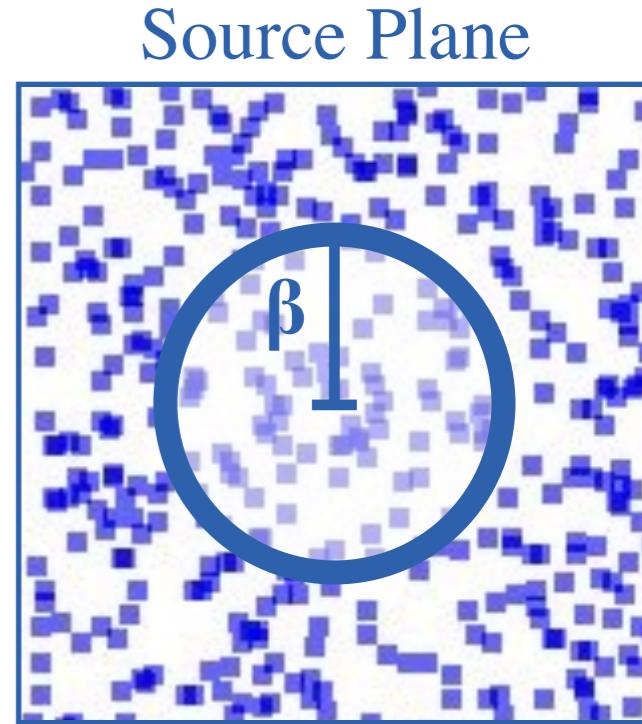
Surface Brightness Conservation

- Surface brightness: the flux density per solid angle

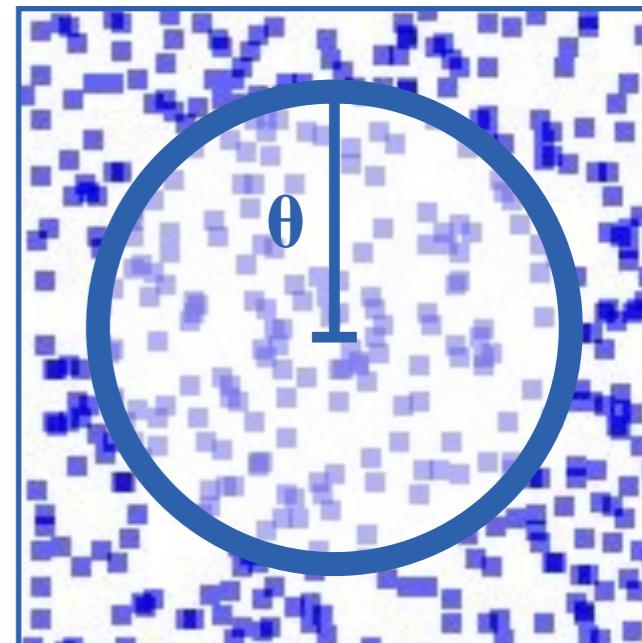
$$s = \frac{F[\text{Jy}]}{d\Omega[\text{deg}^2]}$$

- Received flux goes down for distant sources with r^2
- Physical area observed in solid angle increases with r^2
- So surface brightness of any source is independent of r
- Gravitational lensing does not change surface brightness
 - but can change the *effective* area observed of the source
- Lenses focus light from the background source
- Hence, $s(\beta) = s(\theta)$
- And the increase in observed brightness is given by

$$\frac{s(\theta)d\Omega_{\text{lens plane}}}{s(\beta)d\Omega_{\text{source plane}}} = \frac{s(\theta)d\theta^2}{s(\beta)d\beta^2} = \frac{F(\theta)}{F(\beta)} = \frac{d\theta^2}{d\beta^2} = \mu$$



Lens (Image) Plane



The Jacobian Matrix and Magnification

- The Jacobian Matrix is defined with indices $\mathbf{J}_{ij} \equiv \partial f_i / \partial x_j$
- It describes the mapping between coordinate systems via its determinant

$$|\det \mathbf{J}| du dv = dx dy$$

- $\beta = \theta - \alpha(\theta)$ describes the mapping between source and lens plane
- So the Jacobian matrix for gravitational lensing (linearized locally) is

$$\mathcal{A}(\theta) = \frac{\partial \beta}{\partial \theta} = \begin{pmatrix} \frac{\partial \beta_i}{\partial \theta_i} & \frac{\partial \beta_i}{\partial \theta_j} \\ \frac{\partial \beta_j}{\partial \theta_i} & \frac{\partial \beta_j}{\partial \theta_j} \end{pmatrix}$$

- Hence, the magnification μ (the ratio of the solid angles $d\theta$ & $d\beta$ - or fluxes) can be seen as a coordinate transformation, and expressed in terms of the Jacobian matrix:

$$\mu \equiv \det M(\theta) = \frac{1}{\det \mathcal{A}(\theta)}$$

- where the *magnification tensor* is defined as $M(\theta) = \frac{1}{\mathcal{A}(\theta)}$

The Jacobian Matrix and Magnification

- In week 3 we used that $\boldsymbol{\alpha} = \nabla\psi$
- Inserting this and the lens equation into the Jacobian matrix we get

$$\mathcal{A}(\boldsymbol{\theta}) = \begin{pmatrix} 1 - \frac{\partial\alpha_i}{\partial\theta_i} & -\frac{\partial\alpha_i}{\partial\theta_j} \\ -\frac{\partial\alpha_j}{\partial\theta_i} & 1 - \frac{\partial\alpha_j}{\partial\theta_j} \end{pmatrix} = \begin{pmatrix} 1 - \frac{\partial^2\psi}{\partial\theta_i^2} & -\frac{\partial^2\psi}{\partial\theta_i\partial\theta_j} \\ -\frac{\partial^2\psi}{\partial\theta_j\partial\theta_i} & 1 - \frac{\partial^2\psi}{\partial\theta_j^2} \end{pmatrix} \equiv (\delta_{ij} - \Psi_{ij})$$

- Where Ψ_{ij} is the *distortion tensor* defined as

$$\Psi_{ij} \equiv \begin{pmatrix} \kappa + \gamma_1 & \gamma_2 \\ \gamma_2 & \kappa - \gamma_1 \end{pmatrix}$$

- using the *convergence* (κ) and *shear* (γ)
- and that Ψ is symmetric, i.e., $M_{ij} = M_{ji}$

Convergence & Shear

- In week 3:

- we defined the convergence as $\kappa(\theta) \equiv \frac{\Sigma(D_L\theta)}{\Sigma_{\text{cr}}}$
- and noted that the convergence satisfies the Poisson equation $\nabla^2\psi = 2\kappa$

- So from the definition of Ψ_{ij} we can define:

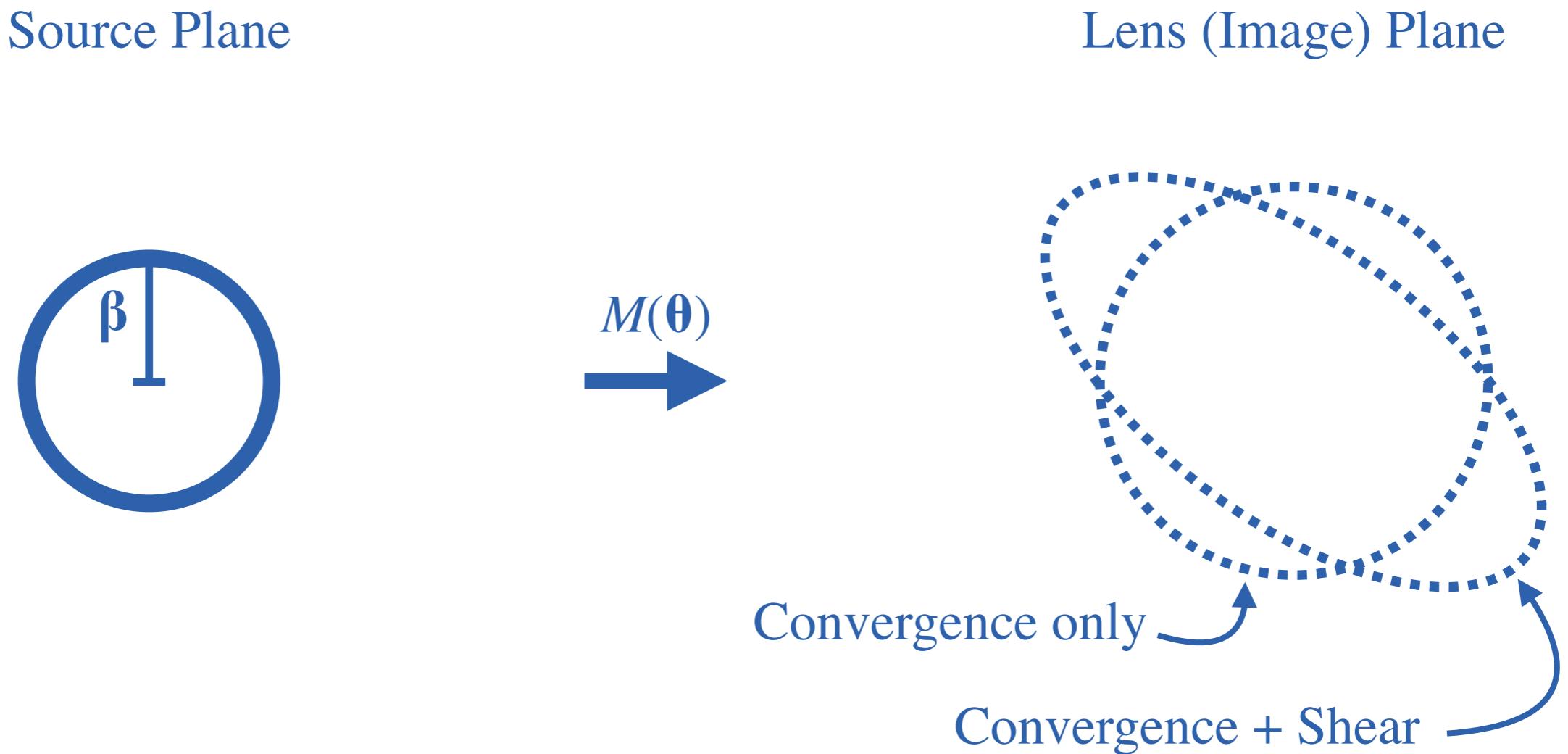
$$\kappa \equiv \frac{1}{2} \left(\frac{\partial^2\psi}{\partial\theta_i^2} + \frac{\partial^2\psi}{\partial\theta_j^2} \right)$$
$$\gamma_1 \equiv \frac{1}{2} \left(\frac{\partial^2\psi}{\partial\theta_i^2} - \frac{\partial^2\psi}{\partial\theta_j^2} \right)$$
$$\gamma_2 \equiv \frac{\partial^2\psi}{\partial\theta_i\partial\theta_j}$$

- Given that the magnification of a lensed source is given by

$$\mu = \frac{1}{(1 - \kappa)^2 - \gamma^2} \quad ; \quad \gamma^2 \equiv \gamma_1^2 + \gamma_2^2 \quad (\text{Exercise 3})$$

Convergence & Shear

- Consider a line of sight along which $\gamma_1 = \gamma_2 = 0$ and $\kappa \neq 0$ and small, then
$$\mathcal{A}(\boldsymbol{\theta}) = \delta_{ij}(1 - \kappa) \quad \Rightarrow \quad \boldsymbol{\beta} = (1 - \kappa)\boldsymbol{\theta}$$
- Dividing by $(1 - \kappa)$ and Taylor expanding (κ is small) give $\boldsymbol{\theta} = (1 + \kappa)\boldsymbol{\beta}$



Magnification for the point mass lens

- From the point mass lens we have that (week 3): $\psi(\theta) = \frac{\theta_E^2}{c^2} \ln \theta$
- Away from the origin of the point mass κ disappears so μ only depends on

$$\begin{aligned}\gamma_1 &= -\frac{\theta_E^2}{\theta^4} (\theta_x^2 - \theta_y^2) \\ \gamma_2 &= -\frac{2\theta_E^2 \theta_x \theta_y}{\theta^4}\end{aligned}\tag{Exercise 4.1}$$

- Resulting in

$$\mu = \frac{1}{1 - \frac{\theta_E^4}{\theta^4}}\tag{Exercise 4.2}$$

- For perfect alignment $\beta=0$ the Einstein ring formally has $\mu \rightarrow \infty$
 - Practically neither source nor lens is ever a point mass
 - Total magnification is not infinite but large

Magnification for the Isothermal Sphere

- For the IS with a core we have that (week 4):

$$\alpha = \frac{\theta_0}{\theta^2} \left[\sqrt{\theta^2 + \theta_{\text{core}}^2} - \theta_{\text{core}} \right] \theta$$

- But $M(\theta)$, i.e. κ , γ_1 and γ_2 are just (linear comb.) of first derivatives of α so we can derive the magnification calculating these

$$\mu = \left[\left(1 - \frac{\theta_0}{2\sqrt{\theta^2 + \theta_{\text{core}}^2}} \right)^2 - \frac{\theta_0^2 \left(2\theta_{\text{core}}\sqrt{\theta^2 + \theta_{\text{core}}^2} - 2\theta_{\text{core}}^2 - \theta^2 \right)^2}{4\theta^4 (\theta^2 + \theta_{\text{core}}^2)} \right]^{-1}$$

- Using that

$$\kappa = \frac{\theta_0}{2\sqrt{\theta^2 + \theta_{\text{core}}^2}}$$

$$\gamma^2 = \frac{\theta_0^2 \left(2\theta_{\text{core}}\sqrt{\theta^2 + \theta_{\text{core}}^2} - 2\theta_{\text{core}}^2 - \theta^2 \right)^2}{4\theta^4 (\theta^2 + \theta_{\text{core}}^2)}$$

Magnification for the SIS

- If $\theta_{\text{core}} = 0$ in the expression of μ for the (C)IS we get that

$$\begin{aligned}\kappa &\rightarrow \frac{\theta_0}{2|\theta|} \\ \gamma^2 &\rightarrow \frac{\theta_0^2}{4\theta^2}\end{aligned}$$

- Resulting in

$$\mu = \frac{1}{1 - \frac{\theta_0}{|\theta|}}$$

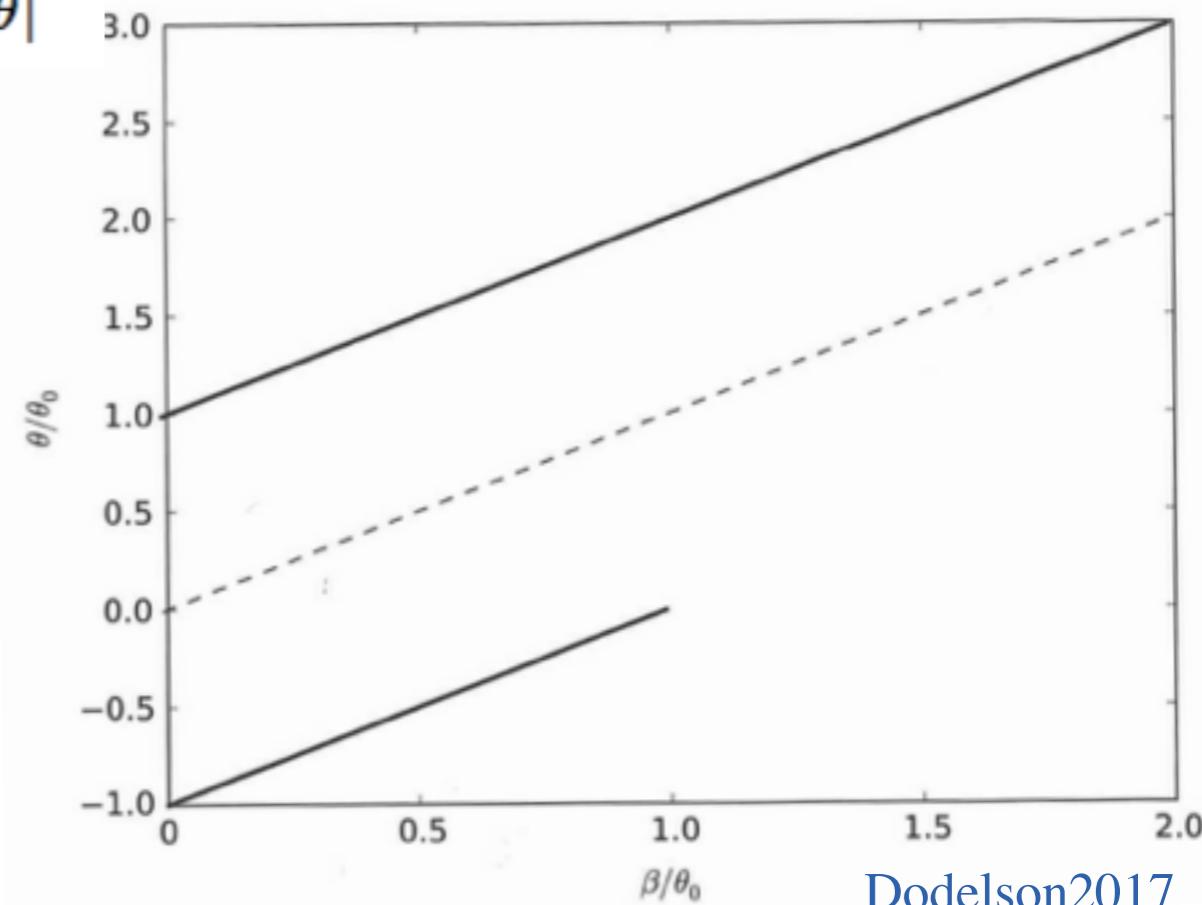
- So if $|\theta| < \theta_0$ then $\mu < 0$
- For $\beta < \theta_0$ two images appear (week 4)

$$\theta_+ = \beta + \theta_0 \quad \theta_- = \beta - \theta_0$$

- With

$$\mu(\theta_+) = +\frac{\theta_0 + \beta}{\beta}$$

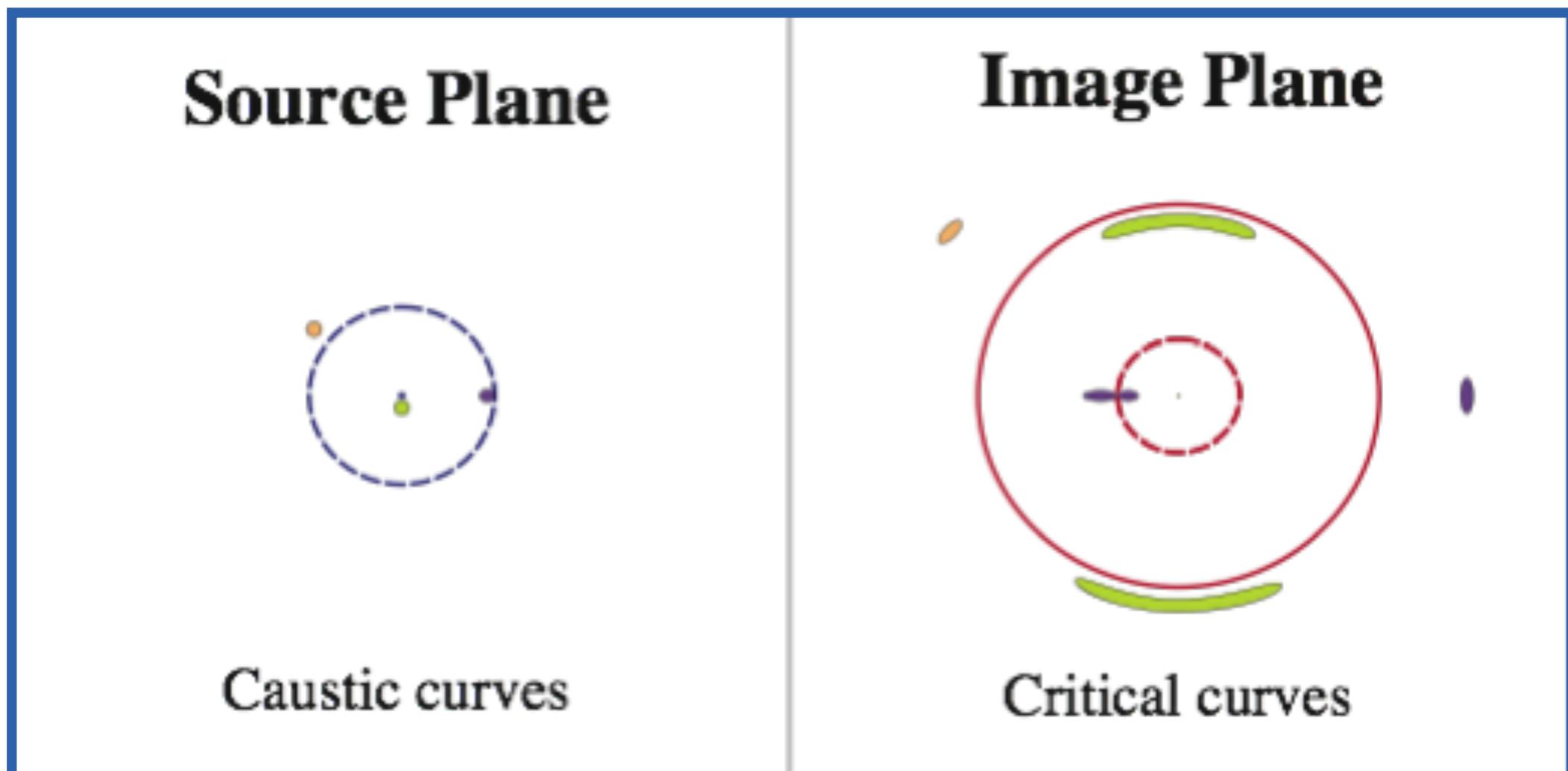
$$\mu(\theta_-) = -\frac{\theta_0 - \beta}{\beta}$$



Dodelson2017

Caustics and Critical Curves Pt. 2

- The SIS with shear and convergence

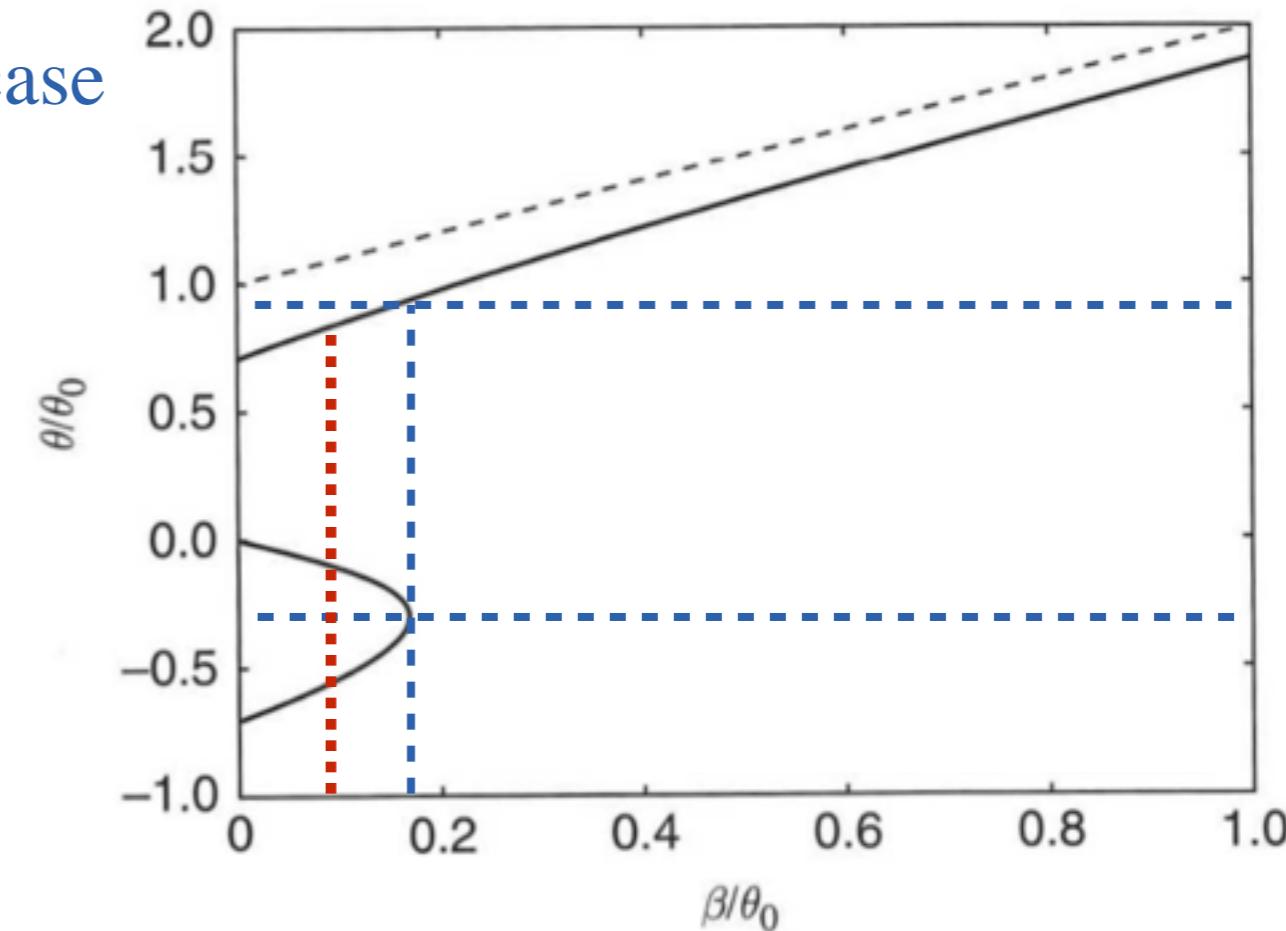
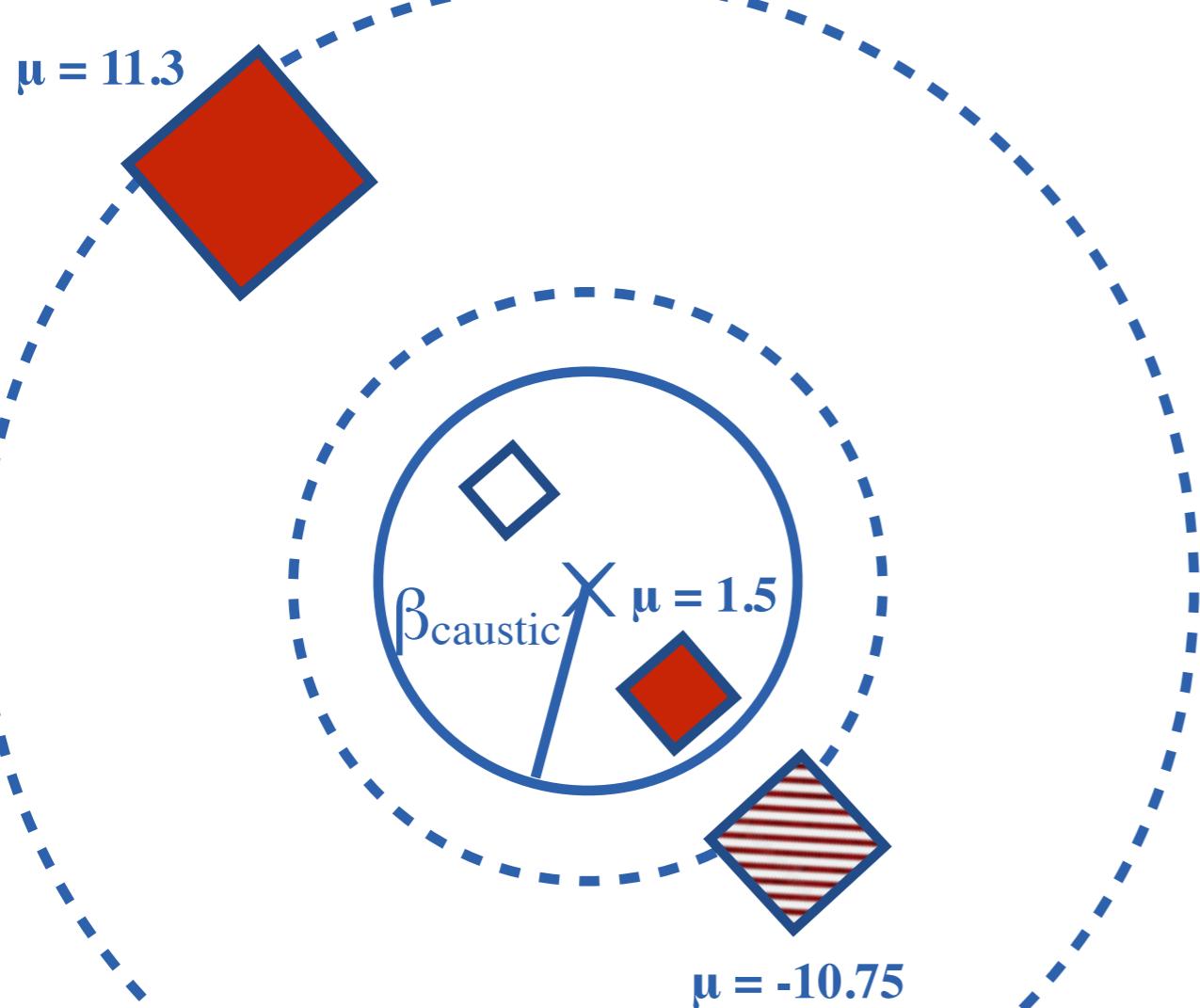


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Caustics and Critical Curves Pt. 2

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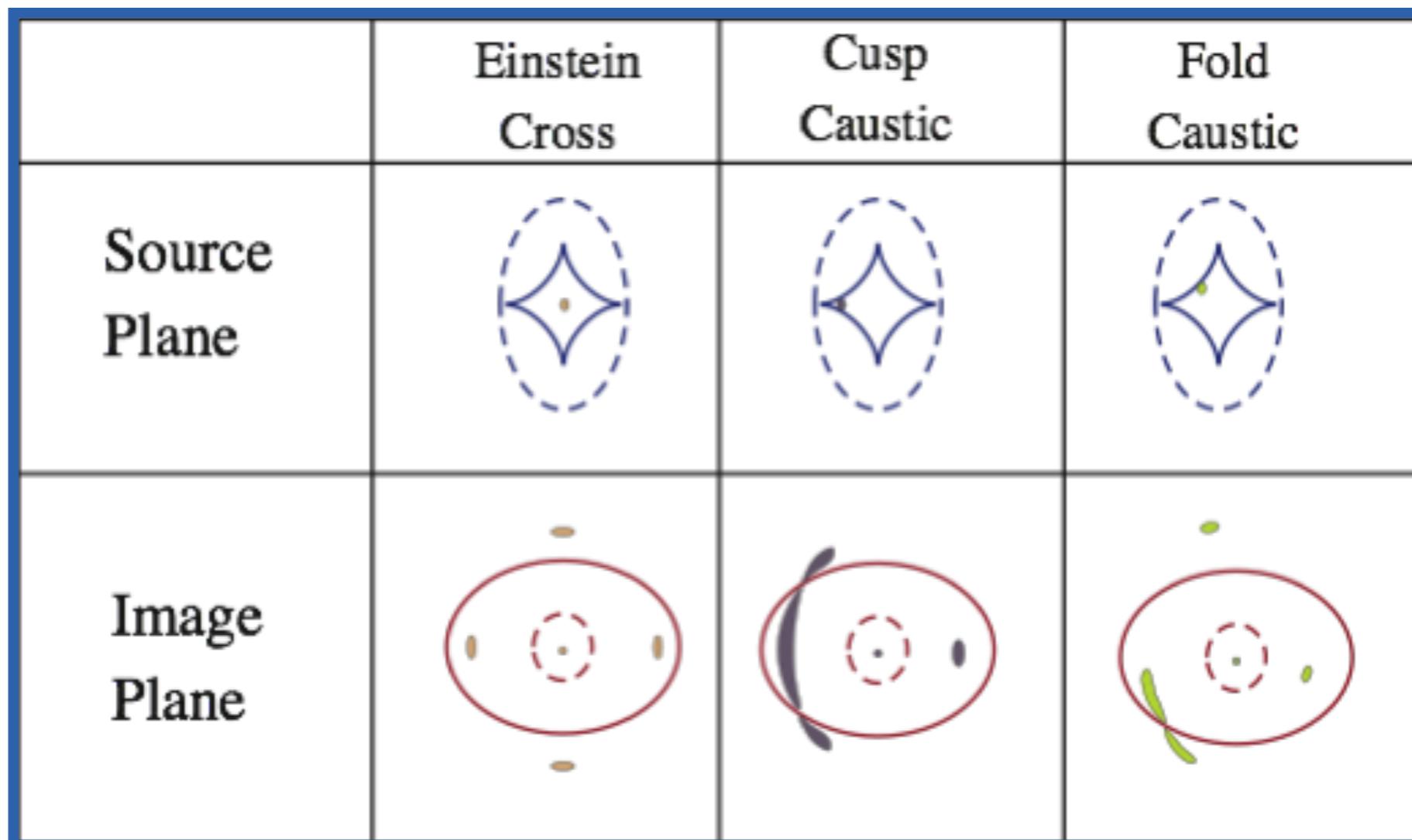
- For the CIS we were considering the case
- The number of images changes by 2 when crossing the caustic



- Calculating magnifications of images and scaling accordingly
 - Absolute magnitude not known
 - So the *observable* for is the flux ratios (not μ) for multiple images

Caustics and Critical Curves Pt. 2

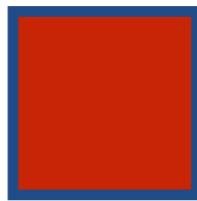
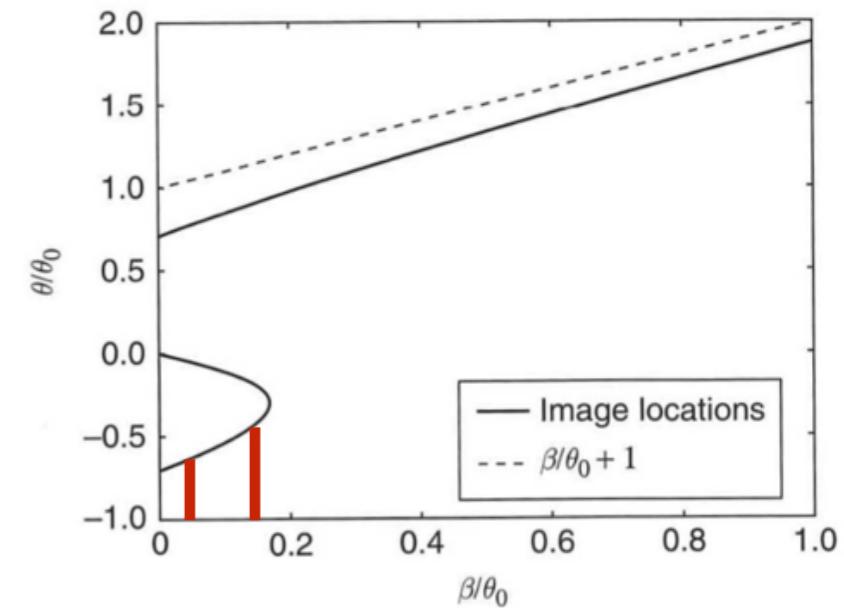
- So what about all the quads (and doubles) we are seeing?
- Consequence of asymmetric mass distribution
 - The Singular Isothermal Ellipsoid (SIE) attempt to account for this
 - Roughly the SIE added an asymmetric potential (or external shear)



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Parity

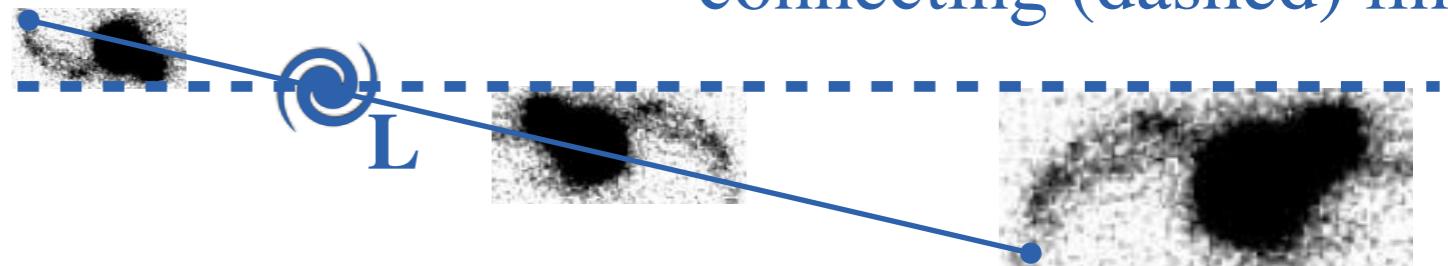
- Critical curves are the dividing lines between
 - Images with $\mu = \det M < 0$ have *negative parity*
 - Images with $\mu = \det M > 0$ have *positive parity*



Symmetrical lens so bottom of tail touches connecting (dashed) line



All points are deflected with $\alpha > 0$



Symmetrical lens so also tail on connecting (solid) line
→ image flipped around (dashed) axis

The vertical axis flip:
An image closer to the lens (in the source plane) appear farther away in the lens plane
This flip is captured by $\mu < 0$

Determining the Mass of Lenses

- The observable for magnification is the flux ratio
- This translates into a difference in magnitudes

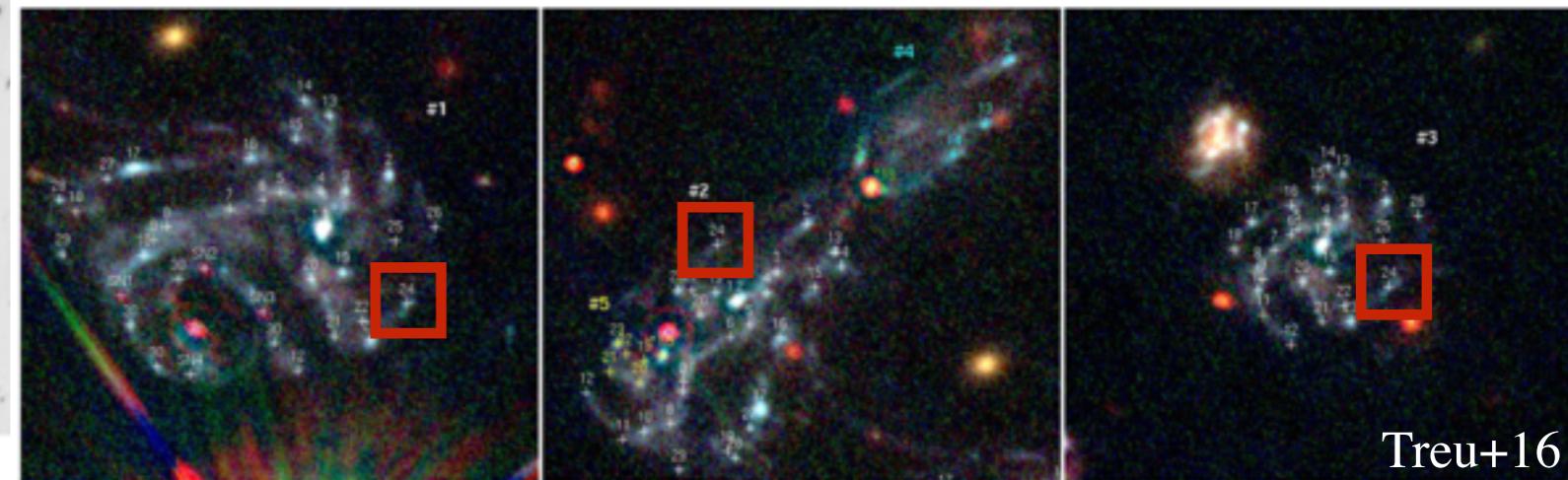
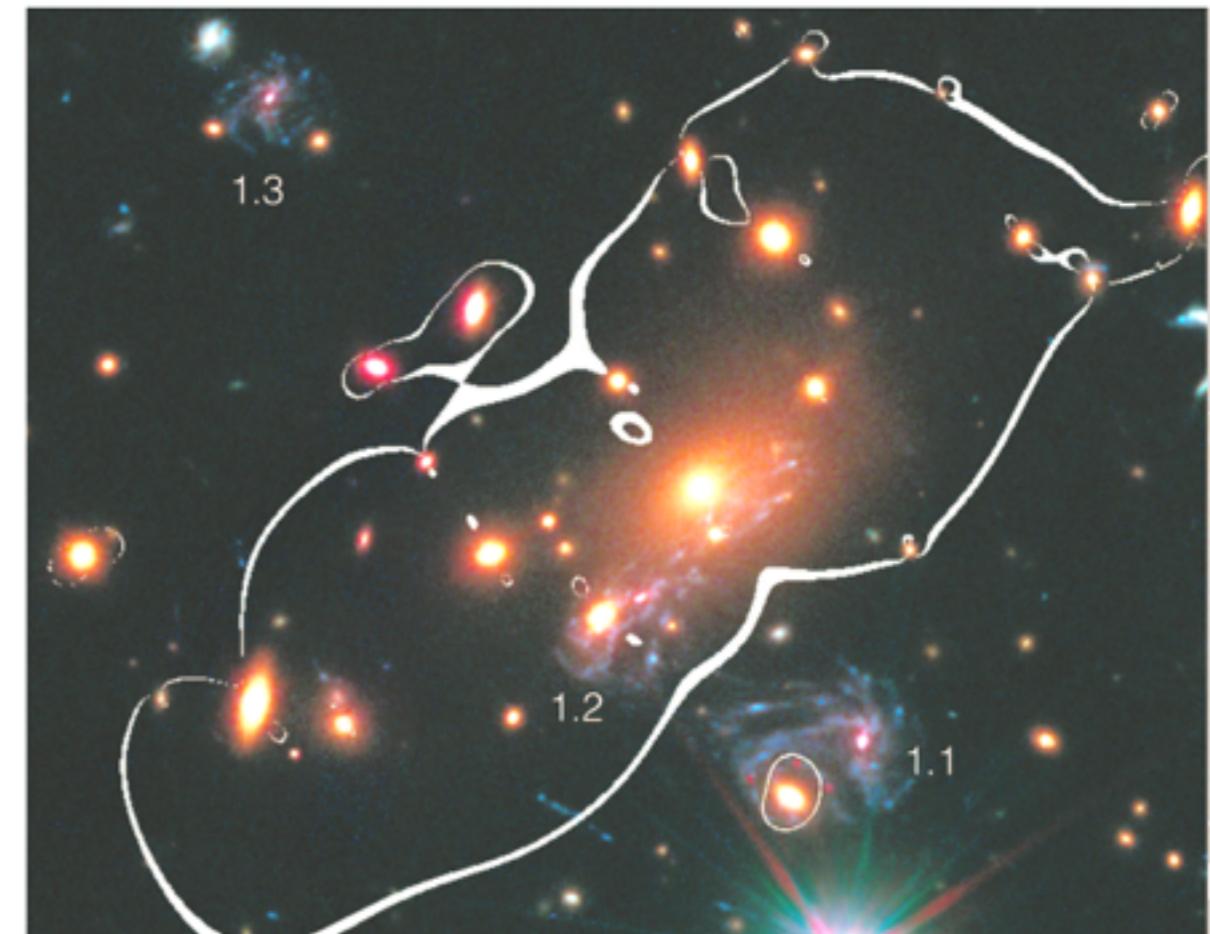
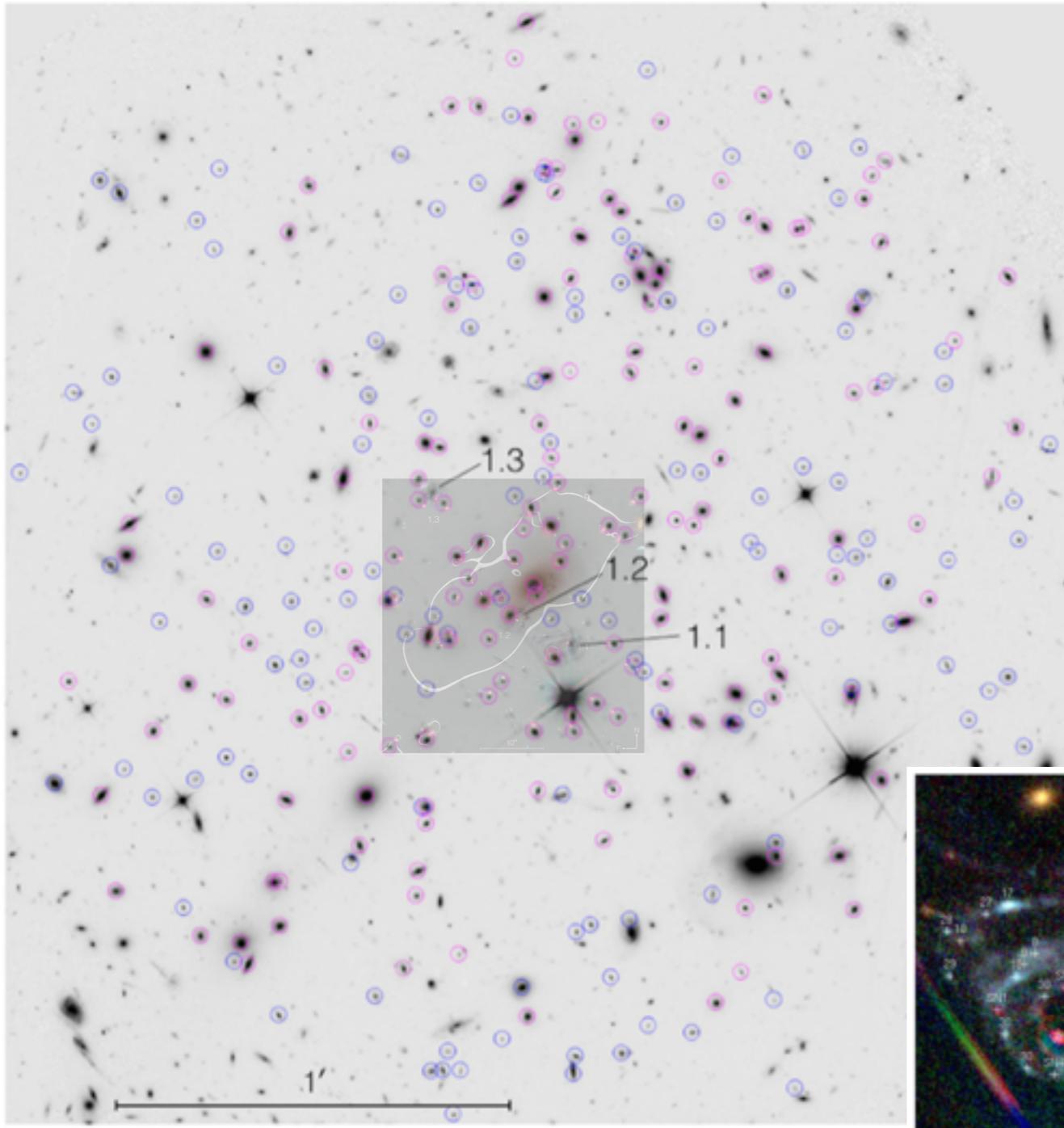
$$m_1 - m_2 = -2.5 \log_{10} \left(\frac{F_1}{F_2} \right) = -2.5 \log_{10} (\mu)$$

- To directly detect magnification you want to compare $m_{\text{intrinsic}}$ & m_{observed}
- Lens is stationary (except for the sun and ulensing) so you want to compare
 - Magnitudes of a sample of galaxies being lensed
 - Magnitude of a similar sample of galaxies not being lensed
- Δm between these two m -distributions then estimate $\sim \mu$ and hence M_{lens}
- However, numbers of strongly lensed ($\theta \lesssim \theta_E$) galaxies per lens is small
- Hence, you have to observe galaxies outside θ_E , i.e., weak lensing regime
- Increasing area to $O(\text{arcmin}^2)$ pushes S/N above a few
 - Annuli give estimates of surface density $\Sigma_{\text{lens}} = \Sigma_{\text{cr}} \kappa$ as a function of r

Determining the Mass of Lenses

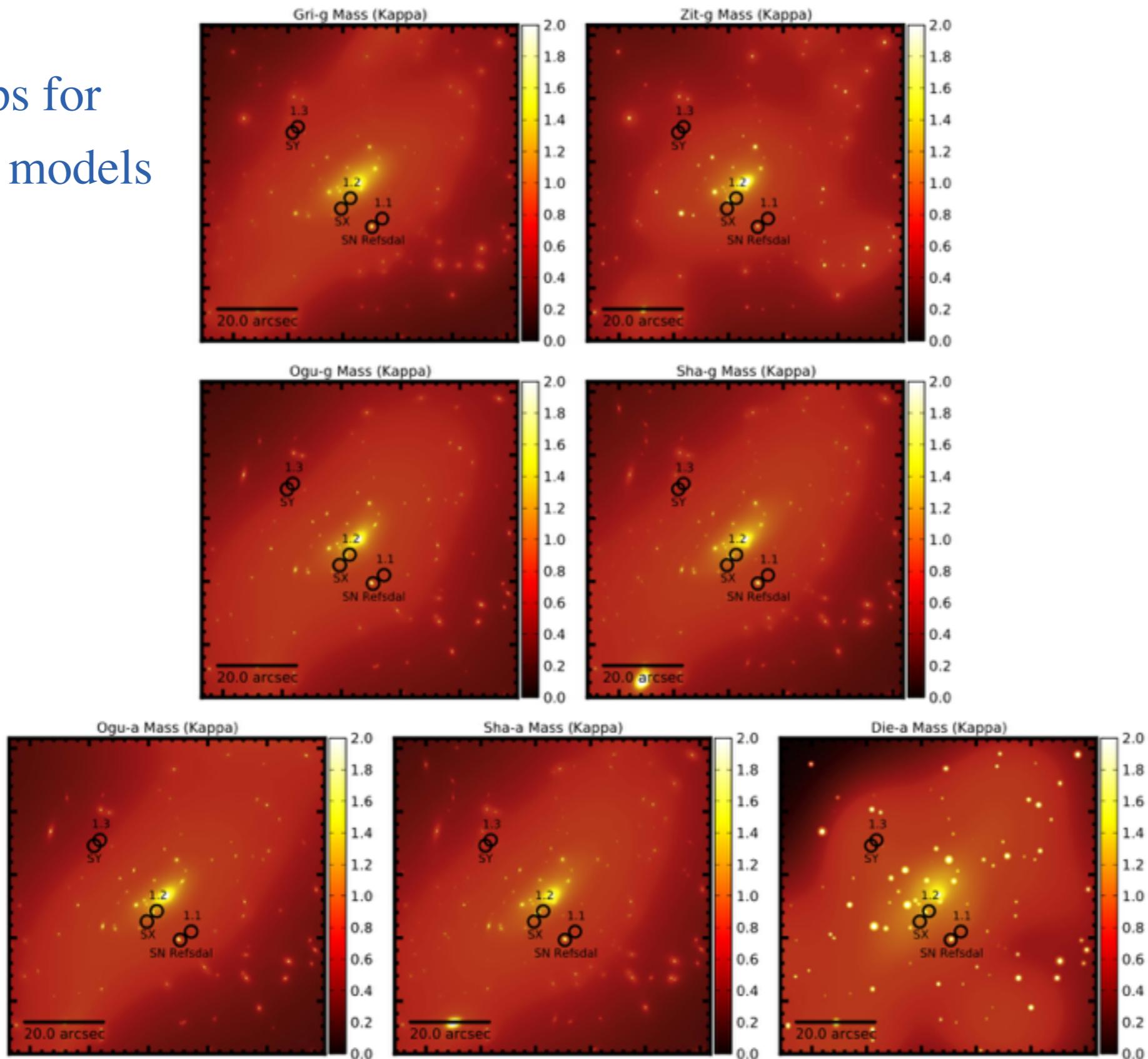
- The position of multiple images, their magnification and morphology, can be used to constrain mass maps of lenses (no reference sample needed).

Kelly+15



Determining the Mass of Lenses

Mass (κ) maps for
different lens models

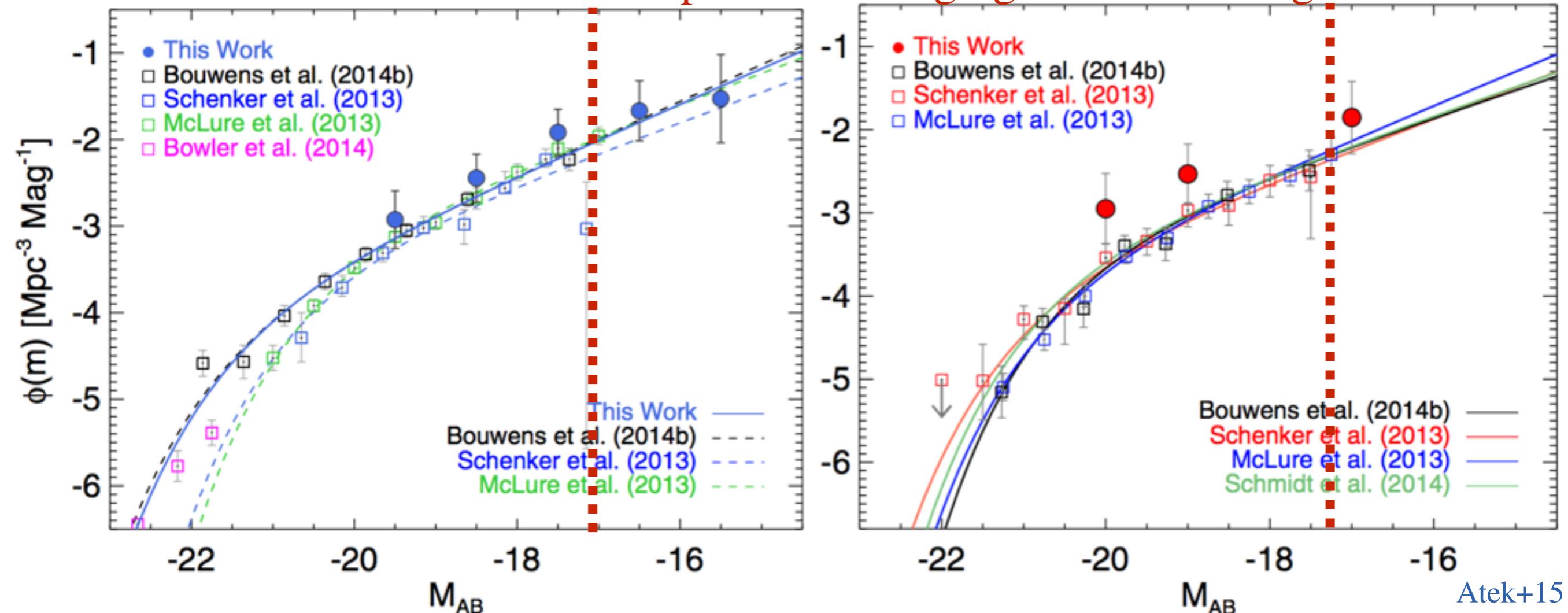


Treu+16

Finding Lensed High- z Galaxies

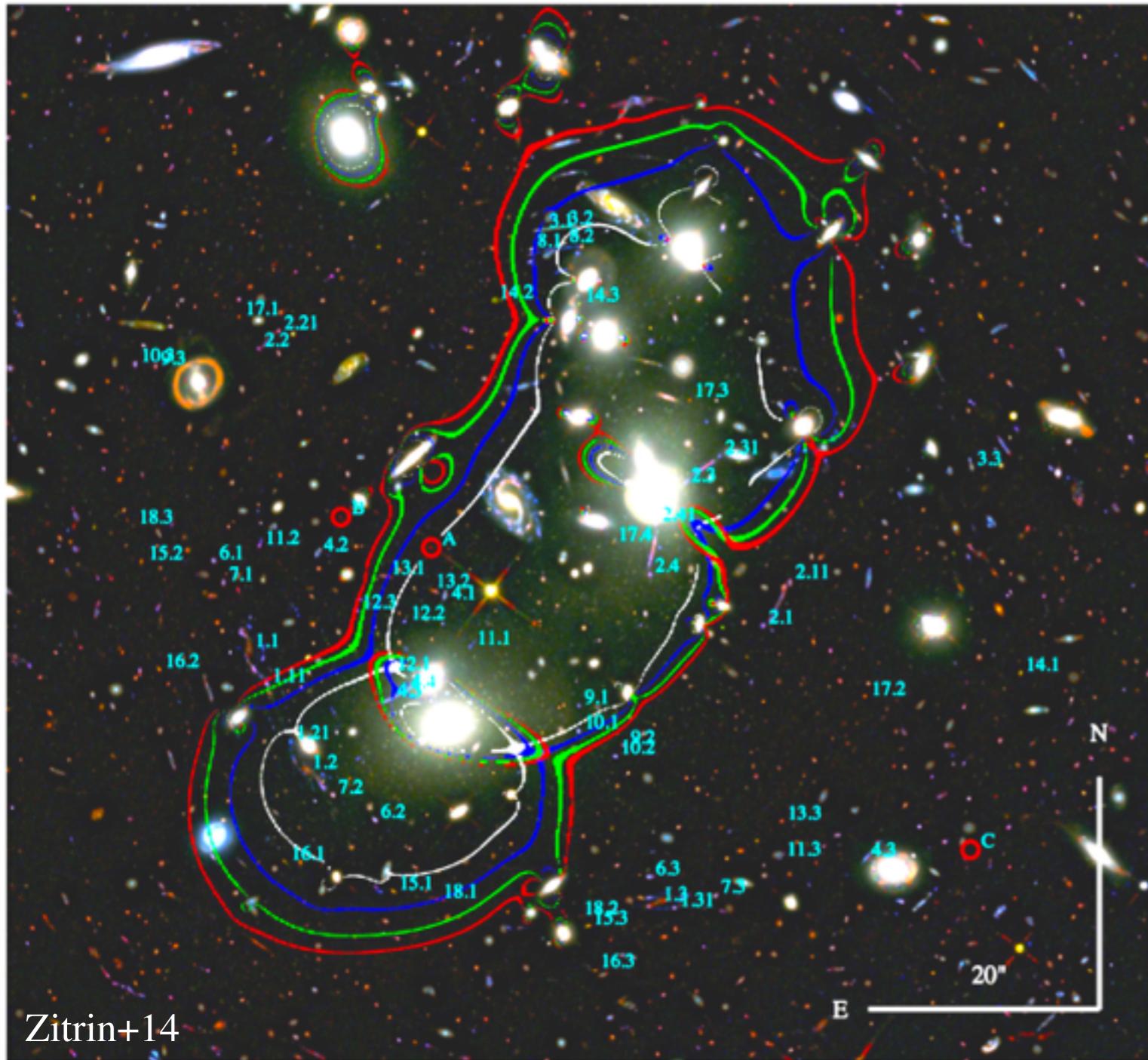
- The magnification of sources enable detection of faint & distant sources
- High- z galaxies are faint both because of intrinsic brightness and distances
- Even the deepest Hubble images ($m \sim 29$) do not reach these depths
- GL to the rescue: $\mu \sim 10$ implies $\Delta m = 2.5$ mag

Deepest HST imaging without lensing



Confirming High- z Galaxies

- The standard way to confirm high- z galaxies is Ly α in spectrum
- But the lensing geometry can also provide confirmation



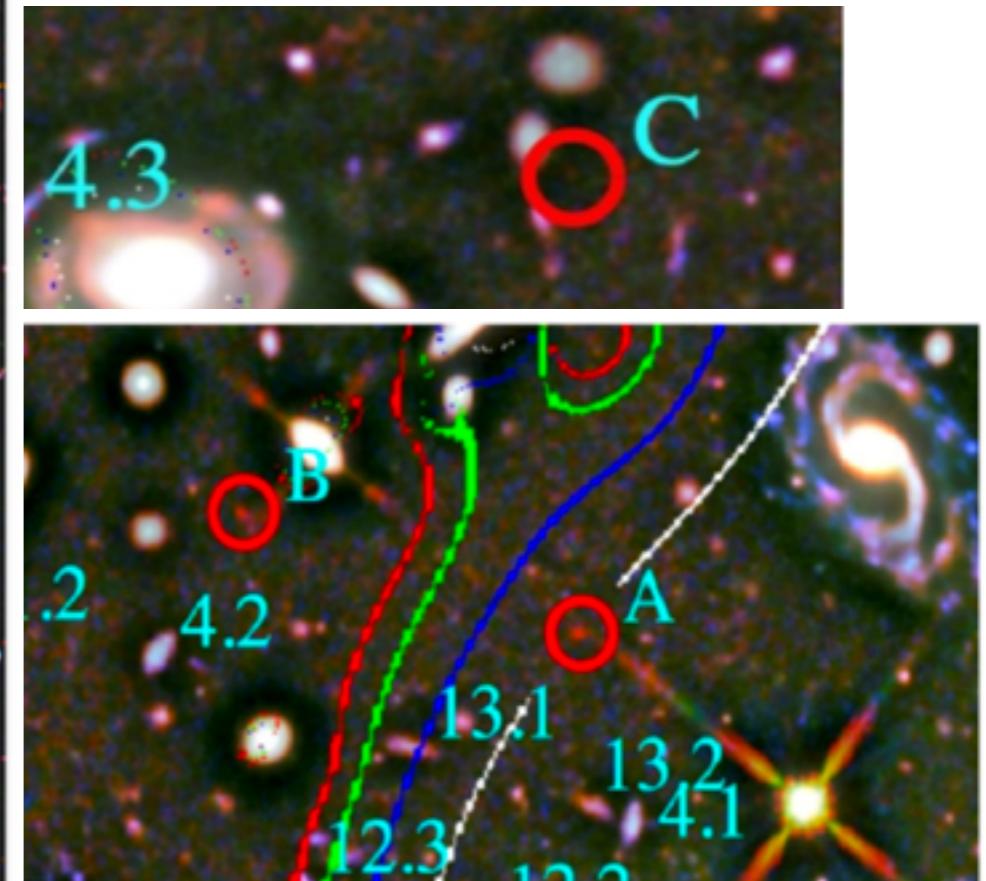
Critical curves:

$z = 1.3$ (white)

$z = 2.0$

$z = 3.6$

$z = 10.0$



So in summary...

- Lensed sources are magnified, i.e. apparent fluxes increased (light focused)
- The magnification is as the inverse determinant of the Jacobian matrix

$$\frac{F(\boldsymbol{\theta})}{F(\boldsymbol{\beta})} = \mu \equiv \det M(\boldsymbol{\theta}) = \frac{1}{\det \mathcal{A}(\boldsymbol{\theta})}$$

- It can be expressed in terms of the convergence, κ , and the shear, γ

$$\mu = \frac{1}{(1 - \kappa)^2 - \gamma^2} \quad ; \quad \gamma^2 \equiv \gamma_1^2 + \gamma_2^2$$

- The magnification for standard lens models are:

- Point mass lens:

$$\mu = \frac{1}{1 - \frac{\theta_E^4}{\theta^4}}$$

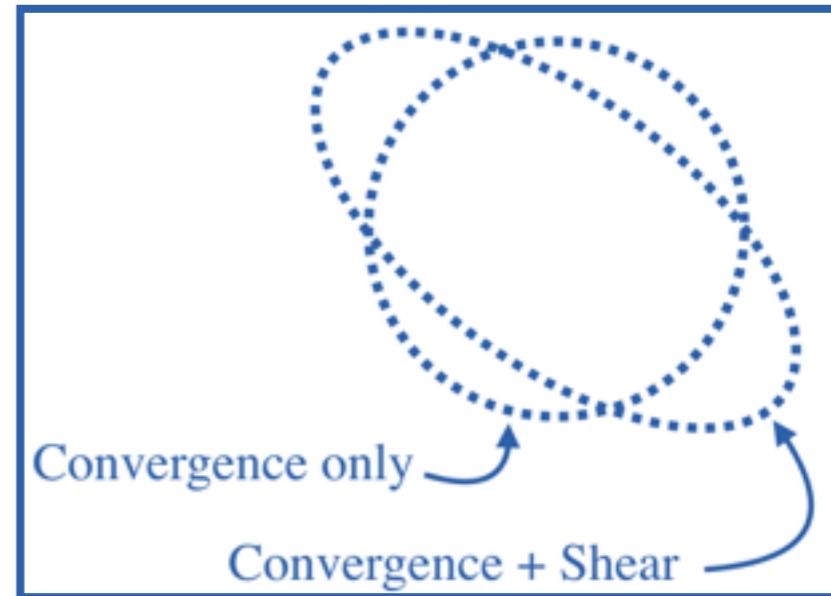
- SIS:

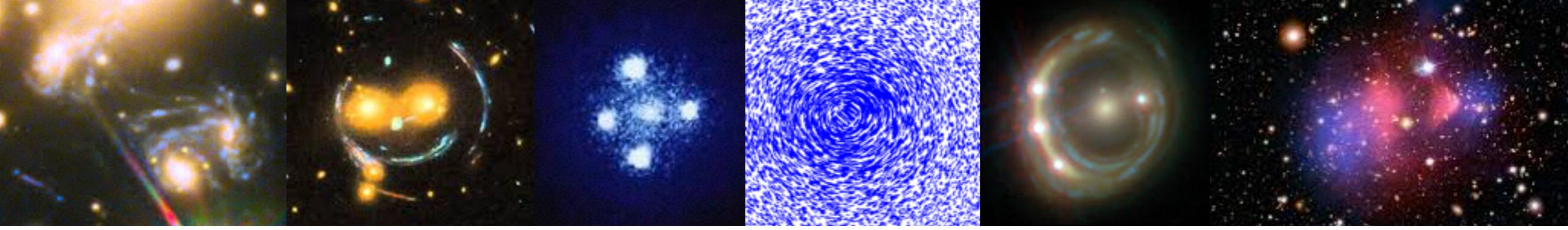
$$\mu = \frac{1}{1 - \frac{\theta_0}{|\theta|}}$$

- CIS:

$$\mu = \left[\left(1 - \frac{\theta_0}{2\sqrt{\theta^2 + \theta_{\text{core}}^2}} \right)^2 - \frac{\theta_0^2 \left(2\theta_{\text{core}}\sqrt{\theta^2 + \theta_{\text{core}}^2} - 2\theta_{\text{core}}^2 - \theta^2 \right)^2}{4\theta^4 (\theta^2 + \theta_{\text{core}}^2)} \right]^{-1}$$

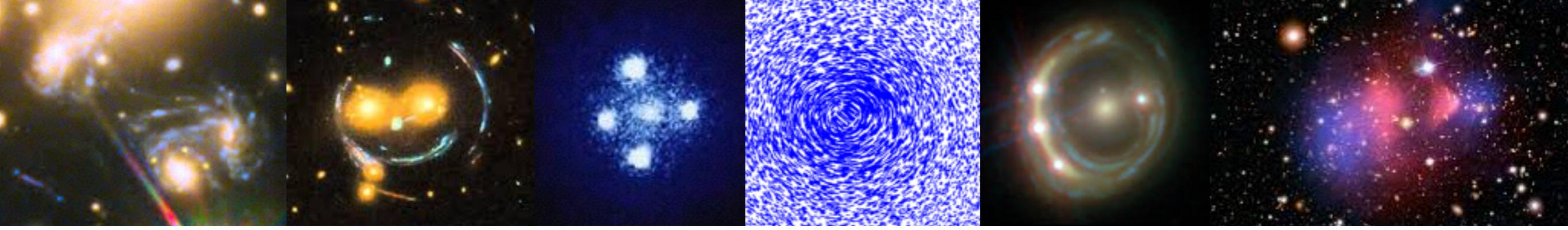
- Magnification and geometry useful for lens mass measurements and modeling, and for faint object (high- z) searches.





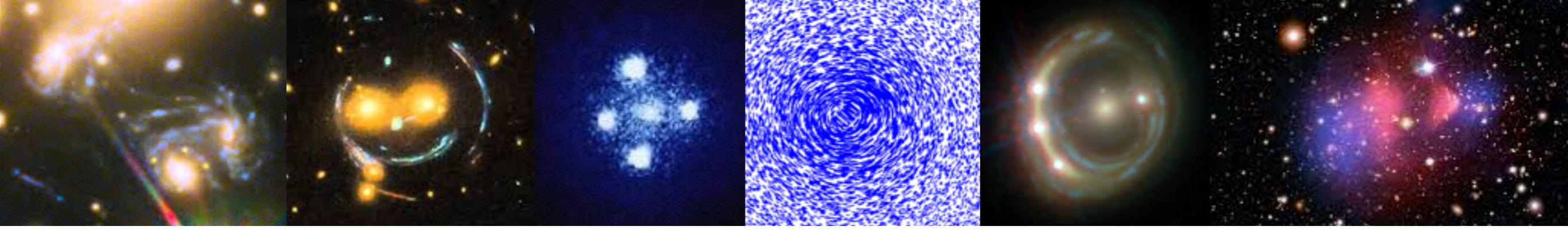
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Questions?



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Last Week's Worksheet



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This Week's Worksheet