



PHY-765 SS19 Gravitational Lensing Week 6

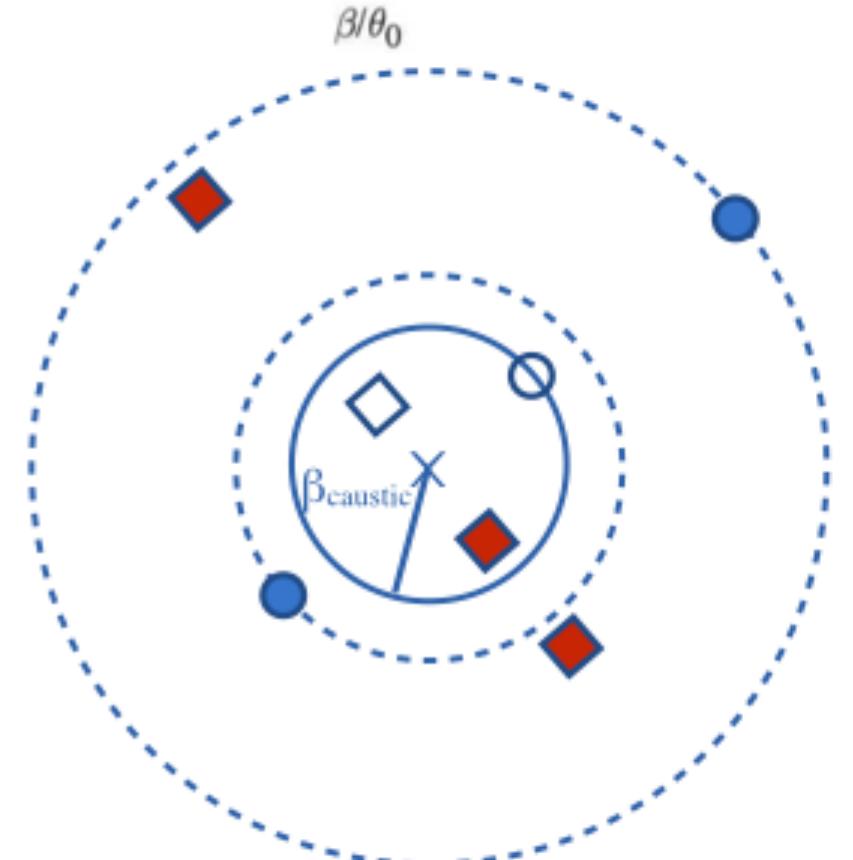
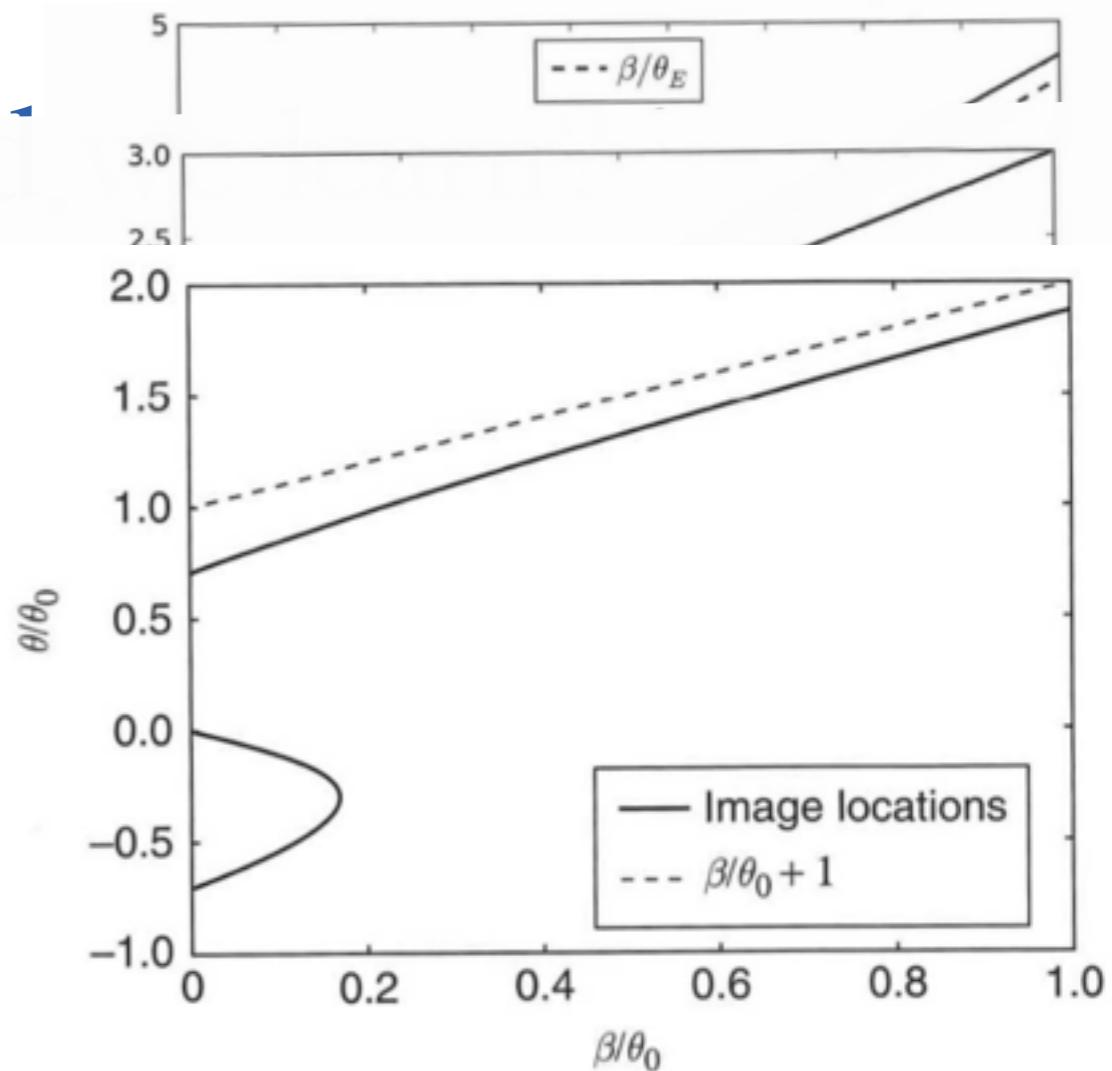
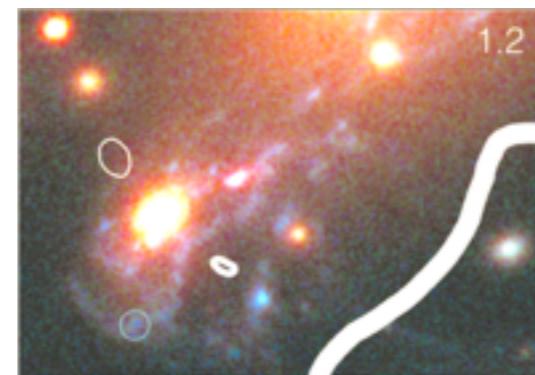
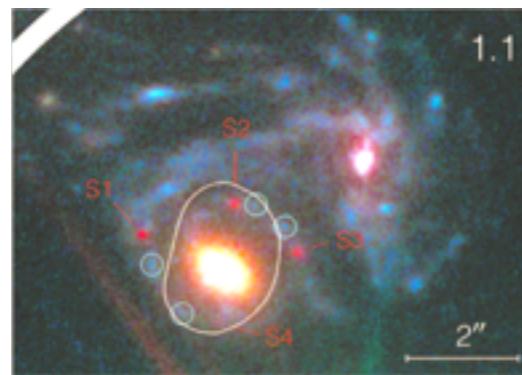
Time Delays

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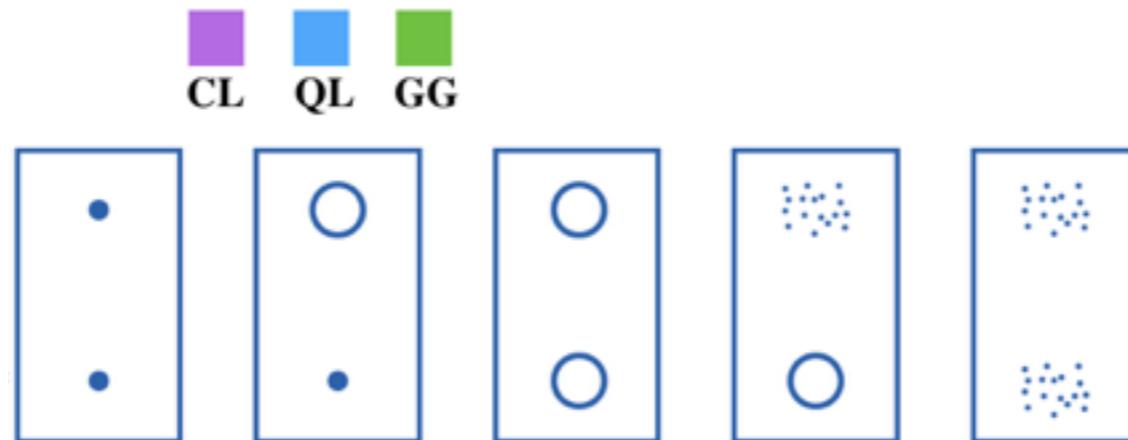
Last week - what did we do?

- Looked at multiple images for:
 - Point mass lens
 - Isothermal Sphere (IS)
 - Singular Isothermal Sphere (SIS)
 - Cored Isothermal Sphere (CIS)
- Introduced caustics (source plane) and critical curves (lens plane)
- Multiple images of SN refsdal and its host



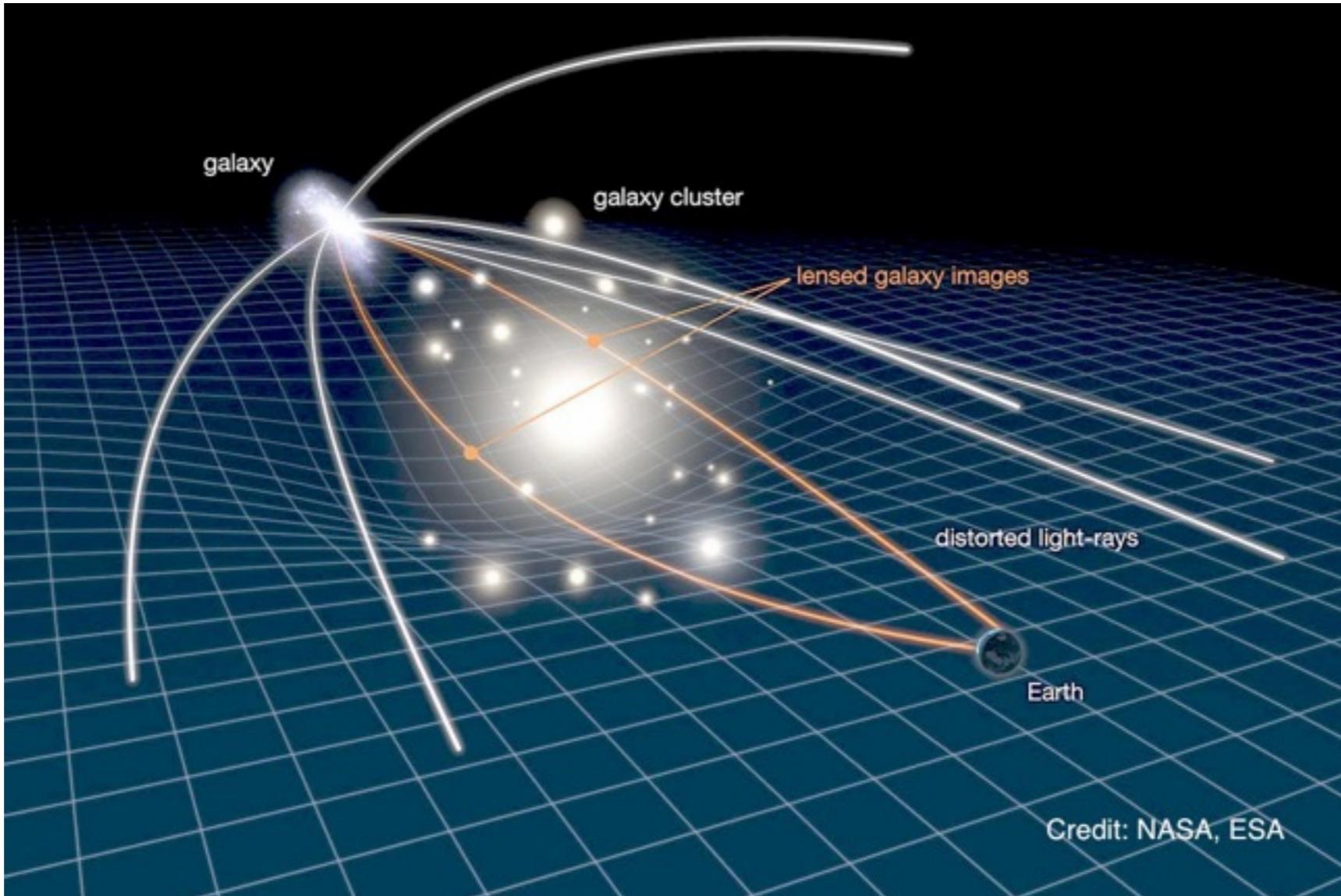
The aim of today

- Explore the second consequence of the lens equation: **time delays**
- Express time delay between the two images of the point mass lens
- Present examples of the usefulness of time delays
 - Testing GR predictions
 - The SN Refsdal re-appearance
 - Determining H_0 (COSMOGRAIL & H0LiCOW)



Time Delays

- Time delay is a natural consequence of the appearance of multiple images

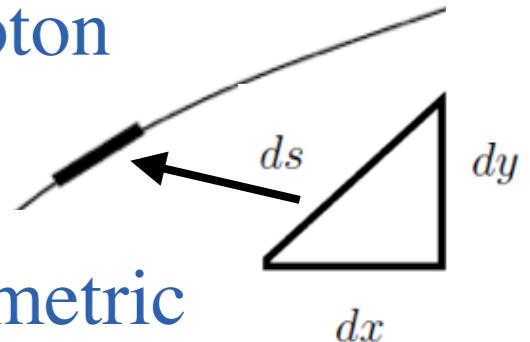


$$\Delta t = \Delta t_{\text{Geometry}} - \Delta t_{\text{Shapiro}}$$

Shapiro time delay

- “The delay of light as it passes through a gravitational potential well”
- In week 1 we were considering the GR line element for a photon

$$ds^2 = g_{00} dt^2 + g_{ij} dx^i dx^j = 0$$



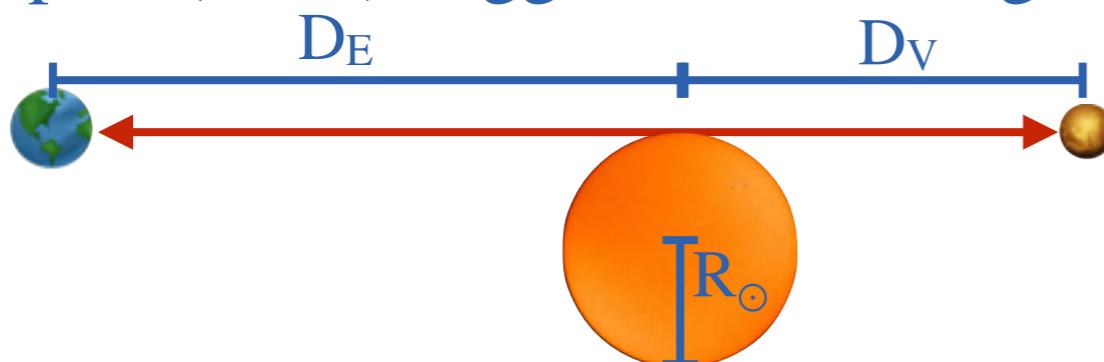
- Aligning light rays along the z -direction and again using the metric

$$g_{00} = c^2 \left(1 - \frac{2GM}{rc^2}\right) \quad g_{ij} = -\delta_{ij} \left(1 + \frac{2GM}{rc^2}\right)$$

- By Taylor expansion we find that

$$dz = c dt \left[\frac{1 - 2GM_\odot / rc^2}{1 + 2GM_\odot / rc^2} \right]^{1/2} \simeq c dt \left[1 - \frac{2GM_\odot}{rc^2} \right]$$

- So in the absence of gravity $dz/dt = c$, but with gravity $dz/dt < c$
- Shapiro (1964) suggested to use light deflection off Venus to measure this



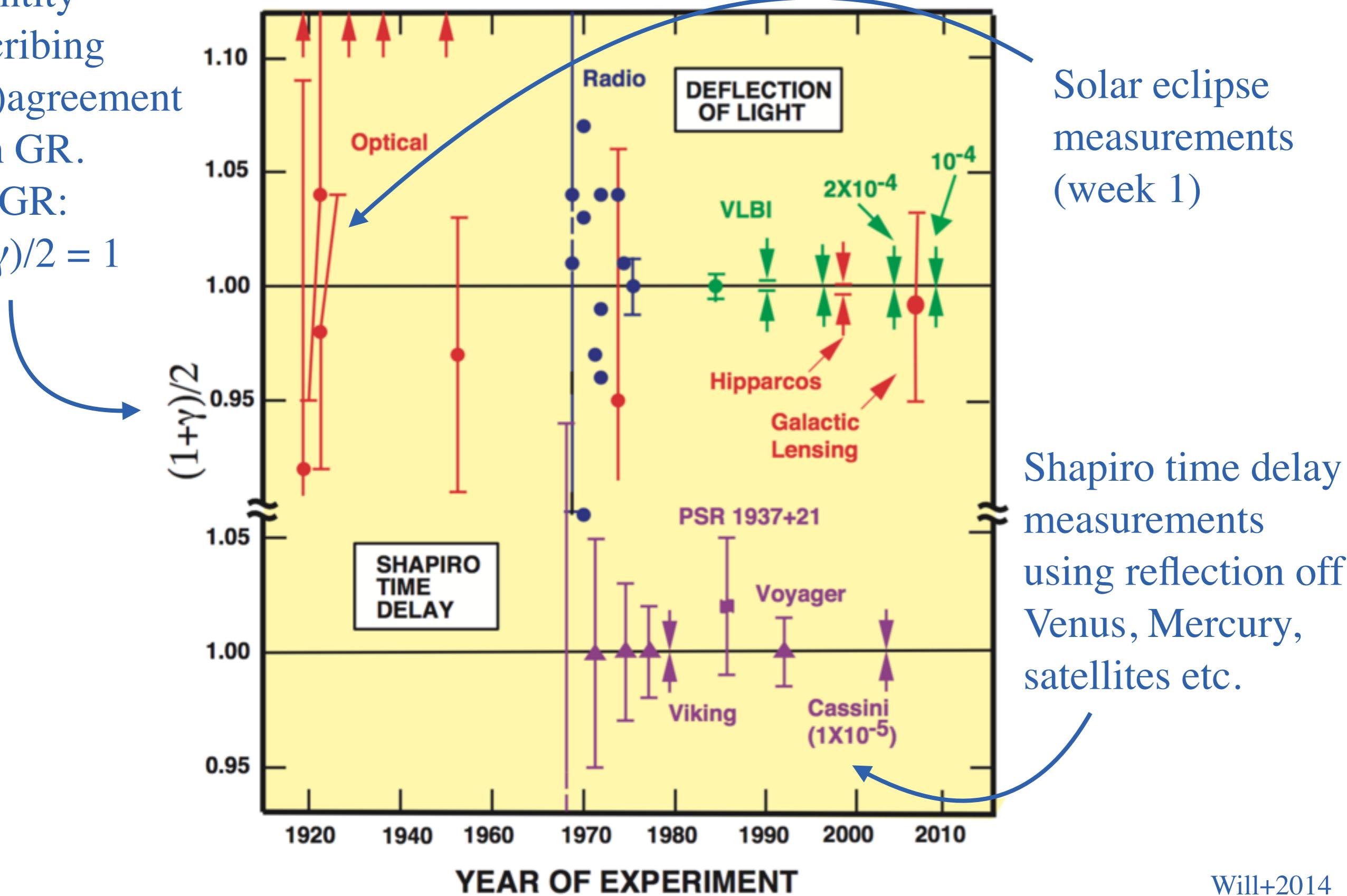
$$\Delta t_{\text{Shapiro}} = 200 \mu\text{s} \quad (\text{Exercise 2})$$

Confirming General Relativity

Quantity
describing
(dis)agreement
with GR.

For GR:

$$(1+\gamma)/2 = 1$$

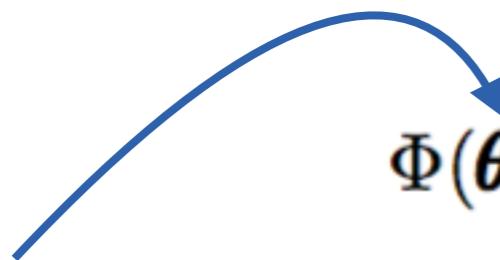


Shapiro time delay

- Considering the 3D gravitational potential $\phi(z) = -MG/r$ we have

$$\Delta t_{\text{Shapiro}} = \frac{-2}{c^3} \int dz \phi(z) = -\frac{\Phi(\boldsymbol{\theta})}{c^2} \times \frac{D_S D_L}{D_{LS} c}$$

- Where the expression $dz \simeq c dt \left[1 - \frac{2GM_\odot}{rc^2} \right]$ was divided by $1-2MG/rc^2$, Taylor expanded & integrated (see Exercise 2)
- And we introduced the *projected gravitational potential*

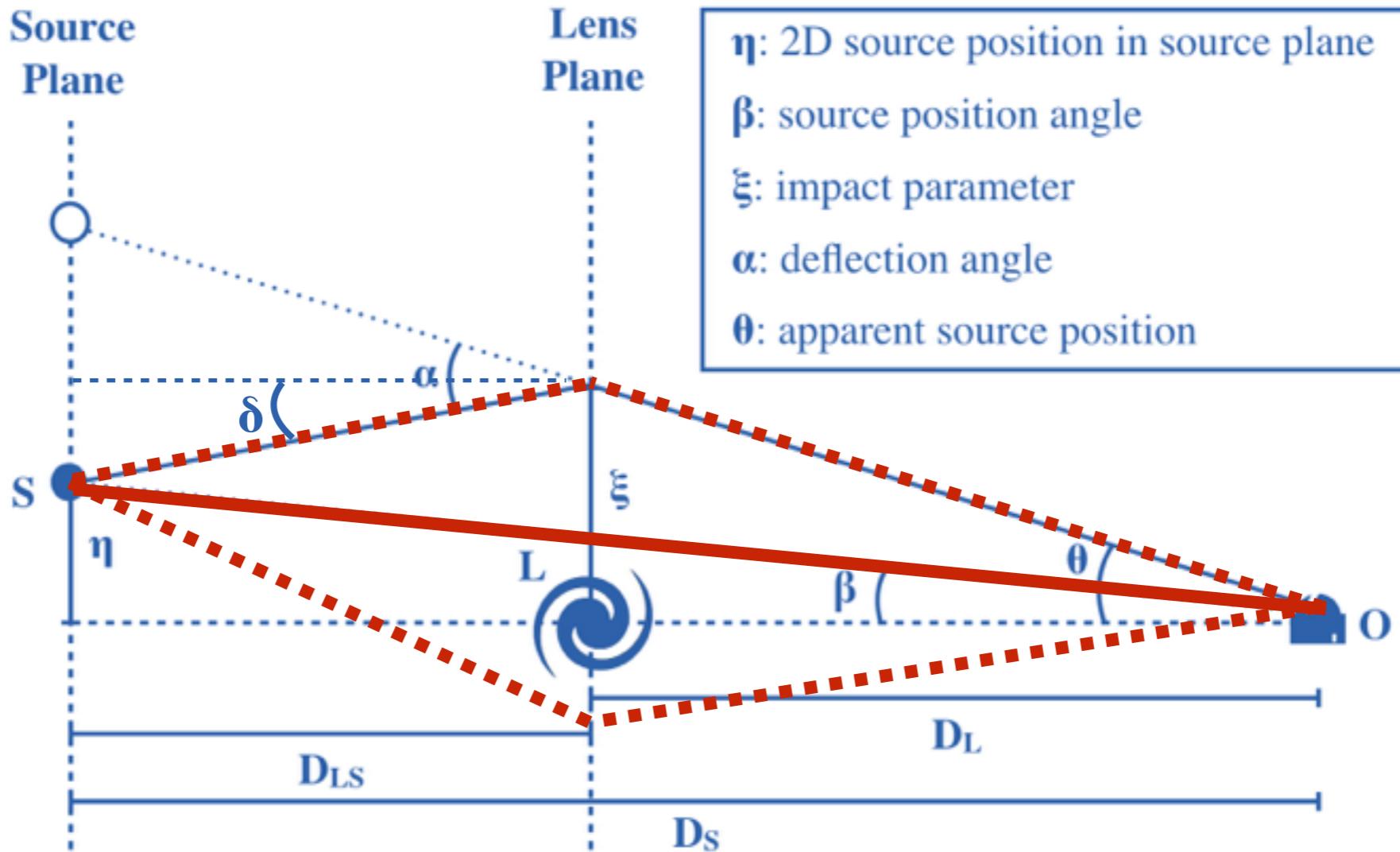

$$\Phi(\boldsymbol{\theta}) = \psi(\boldsymbol{\theta})c^2 = \frac{4MG D_{LS}}{D_S D_L} \ln |\boldsymbol{\theta}|$$

Depends on image position (light path)

Defined for the point mass in week 3

Geometric time delay

- The Geometric time delay caused by different path lengths for images



- By geometry the undeflected light path is just $D_u = D_s / \cos(\beta)$
- and the deflected light path is $D_d = D_L / \cos(\theta) + D_{LS} / \cos(\delta)$

Geometric time delay

- Combining these two expressions (and Taylor expanding cosines) we get

$$D_d - D_u = \frac{D_L D_S}{D_{LS}} \frac{(\theta - \beta)^2}{2}$$

- Dividing by c turns this light path difference into a time difference

$$\Delta t_{\text{Geometry}} = \frac{D_L D_S}{c D_{LS}} \frac{(\theta - \beta)^2}{2}$$

Time delay

$$\Delta t = \Delta t_{\text{Geometry}} - \Delta t_{\text{Shapiro}}$$

- Inserting the expressions we have the combined time delay

Only depends on distances; no lens details

$$\Delta t = \frac{D_L D_S}{c D_{LS}} \left[\frac{(\theta - \beta)^2}{2} - \frac{\Phi(\theta)}{c^2} \right]$$

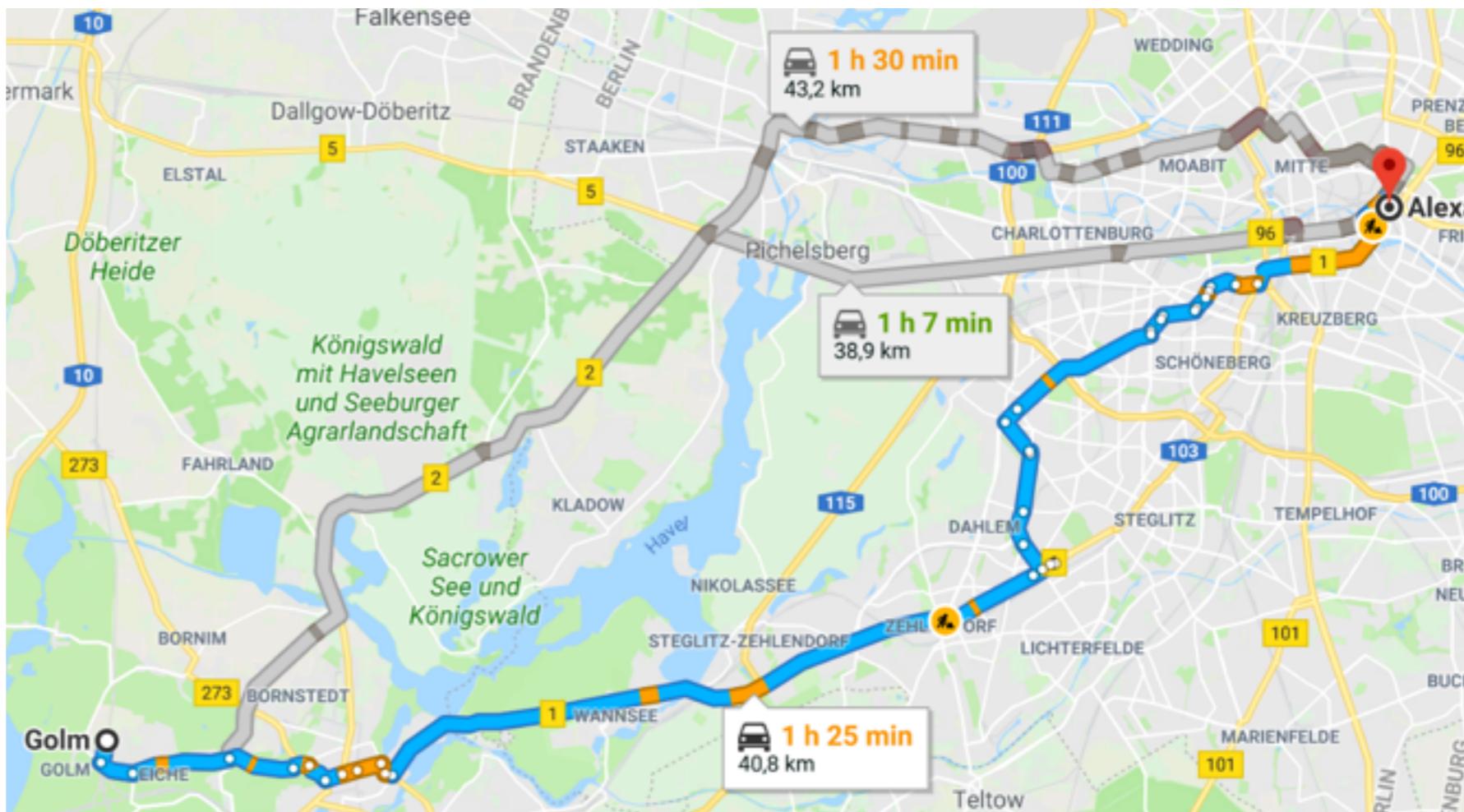
Only depends on lens mass distribution

- Where the distances are given in co-moving distances

- Add $(1+z_L)$ factor in front to have angular diameter distances

- GL time delay analogy:

- $\theta - \beta$ is “route”
- $\Phi(\theta)$ is “traffic”



Time delay reveals Lens Equation

- If we insist on minimizing the time traveled by light rays
 - Light travels on null-geodesics
- Or in other words we invoke *Fermat's Principle*:

If S is the source and O the observer in a space time defined by a metric $g_{\mu\nu}$ and mass M , then a smooth null curve γ from S to O is a light ray (null geodesic) if, and only if, its arrival time τ at O is stationary under first-order variations of γ within the set of null curves from S to O , i.e., $\delta\tau = 0$

- So differentiating Δt with respect to the angle we have:

$$\frac{d}{d\theta^i} \left[\frac{(\boldsymbol{\theta} - \boldsymbol{\beta})^2}{2} - \frac{\Phi(\boldsymbol{\theta})}{c^2} \right] = 0$$

- And since $\boldsymbol{\alpha} = \nabla\psi$ we have the lens equation $0 = \boldsymbol{\theta} - \boldsymbol{\beta} - \boldsymbol{\alpha}(\boldsymbol{\theta})$
- Hence, the lens equation is a ‘manifestation’ of Fermat’s principle.

Time delay for the point mass lens

- For images 1 and 2 of a background source being lensed we have

$$t_1 - t_2 = (1 + z_L) \frac{D_L D_S}{c D_{LS}} \left(\left[\frac{(\theta_1 - \beta)^2}{2} - \frac{\Phi(\theta_1)}{c^2} \right] - \left[\frac{(\theta_2 - \beta)^2}{2} - \frac{\Phi(\theta_2)}{c^2} \right] \right)$$

- If the lens is a point mass and β much smaller than the Einstein radius

$$\theta_{\pm} \simeq \pm \theta_E + \frac{\beta}{2} \quad (\beta \ll \theta_E) \quad (\text{cf. last week})$$

- And the geometrical time delay is insignificant (deflections \sim identical)
 - I.e., $(\theta_+ - \beta)^2 \sim (\theta_- - \beta)^2$
- The main contribution to the time delay comes from the potential $\Phi(\theta)$
- Using the point mass lens expression for the gravitational potential

$$\Phi(\theta) = \psi(\theta) c^2 = \frac{4 M G D_{LS}}{D_S D_L} \ln |\theta| = c^2 \theta_E^2 \ln |\theta|$$

Time delay for the point mass lens

- We find that in the limit of small β (Taylor expanding \ln)

$$\Phi(\theta_+) - \Phi(\theta_-) = c^2 \theta_E^2 \ln \left| \frac{\theta_+}{\theta_-} \right| \simeq 2c^2 \theta_E \beta$$

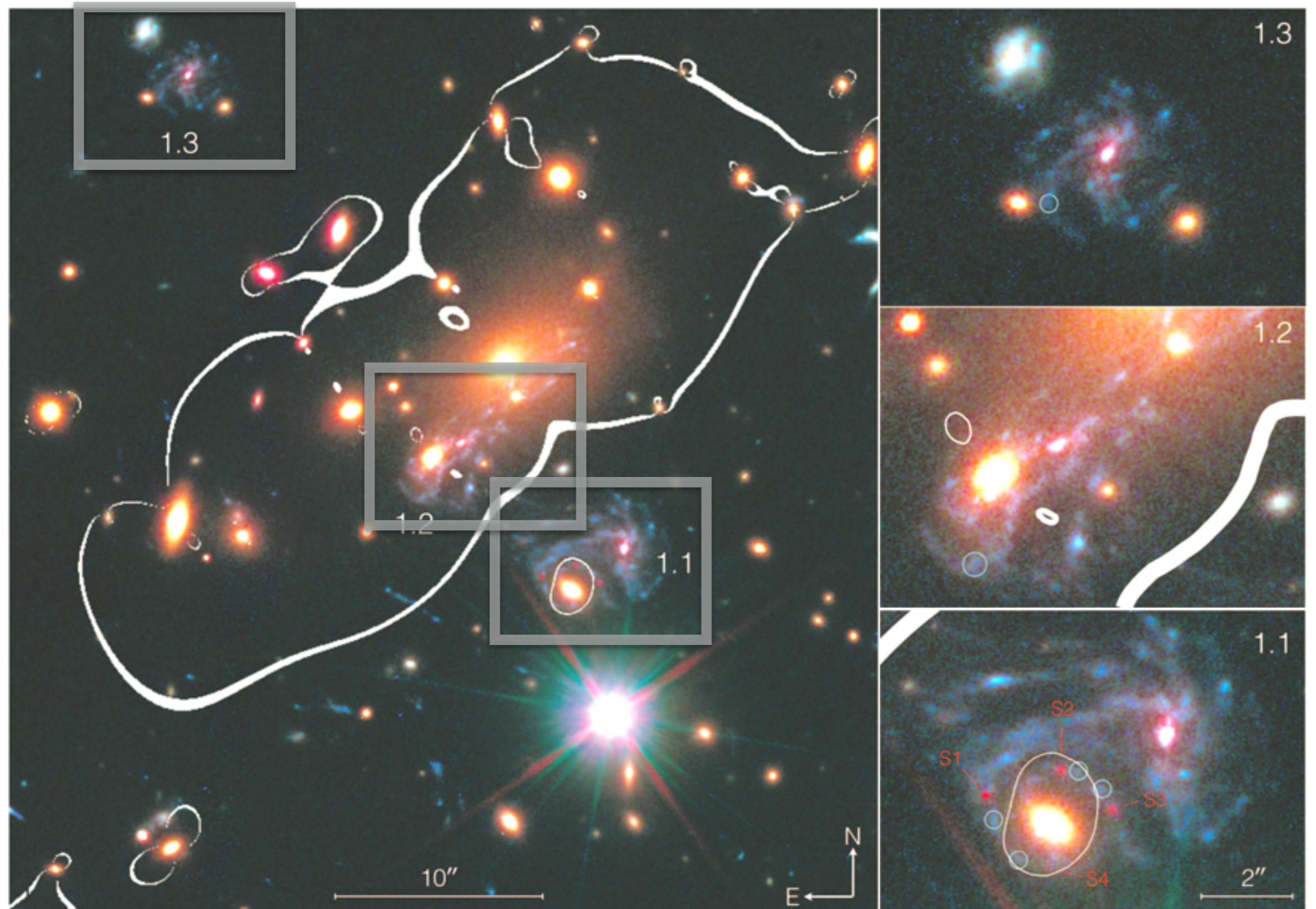
- Therefore, for a point mass lens the time difference between two images (when the geometric time delay is insignificant) is

$$t_+ - t_- \simeq -(1 + z_L) \frac{D_L D_S}{c D_{LS}} 2\theta_E \beta$$

- Light passing closest to the lens (t_-) is delayed the most
- Thus, light from image θ_+ arrive first
- Characteristic light delays between the two images are of the order months to years for cosmological lens geometries
(Exercise 3)

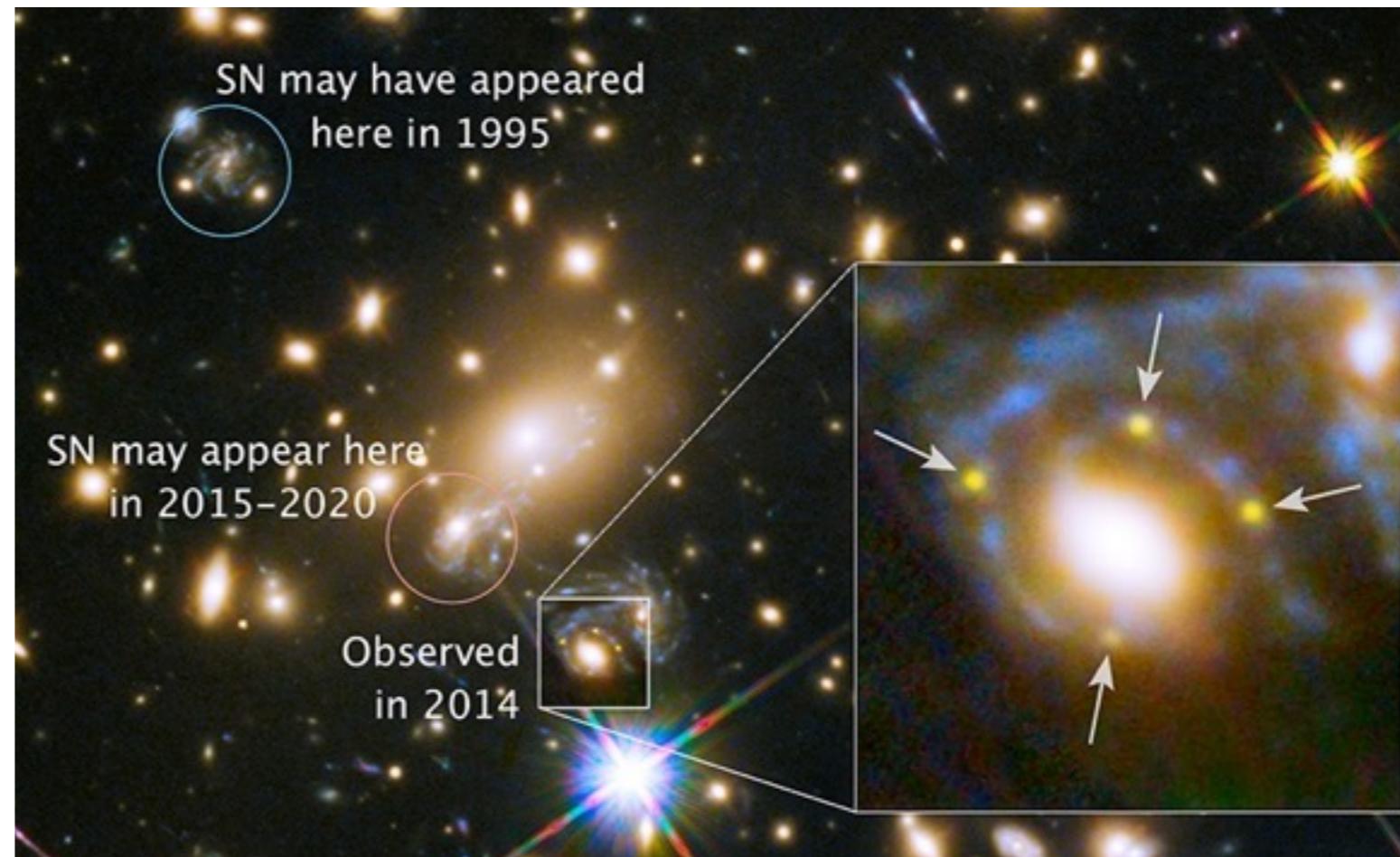
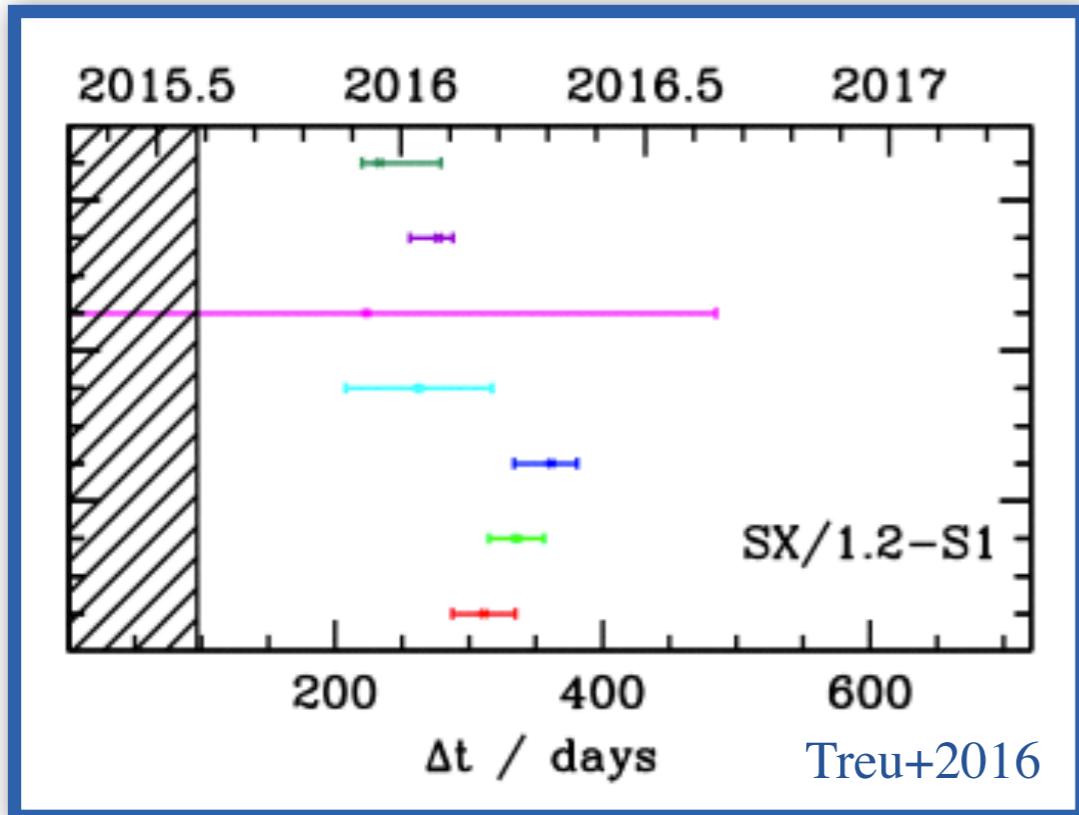
SN Refsdal - Time delay

Kelly+2015



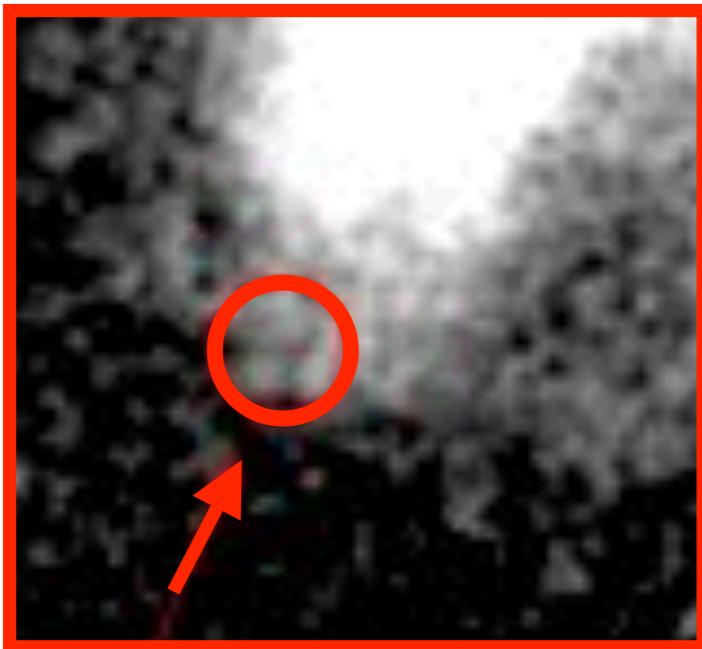
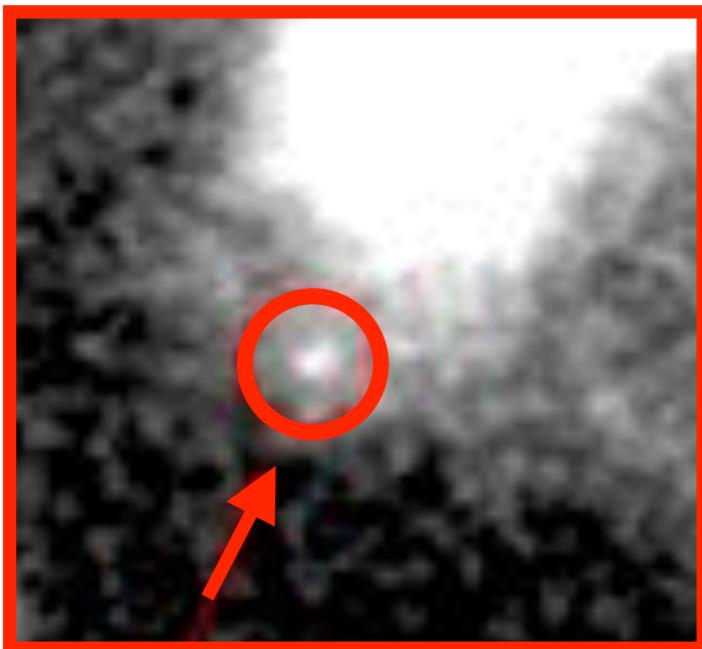
SN Refsdal - Time delay

- The SN Refsdal data can be used to strengthen/improve cluster lens models
- Treu+2016 coordinated a “blind” modeling of the MACS1149 cluster
 - Models from Zitrin+, Diego+, Oguri+, Sharon+, Grillo+
- Prediction of re-appearance of SN Refsdal and time-delay estimates

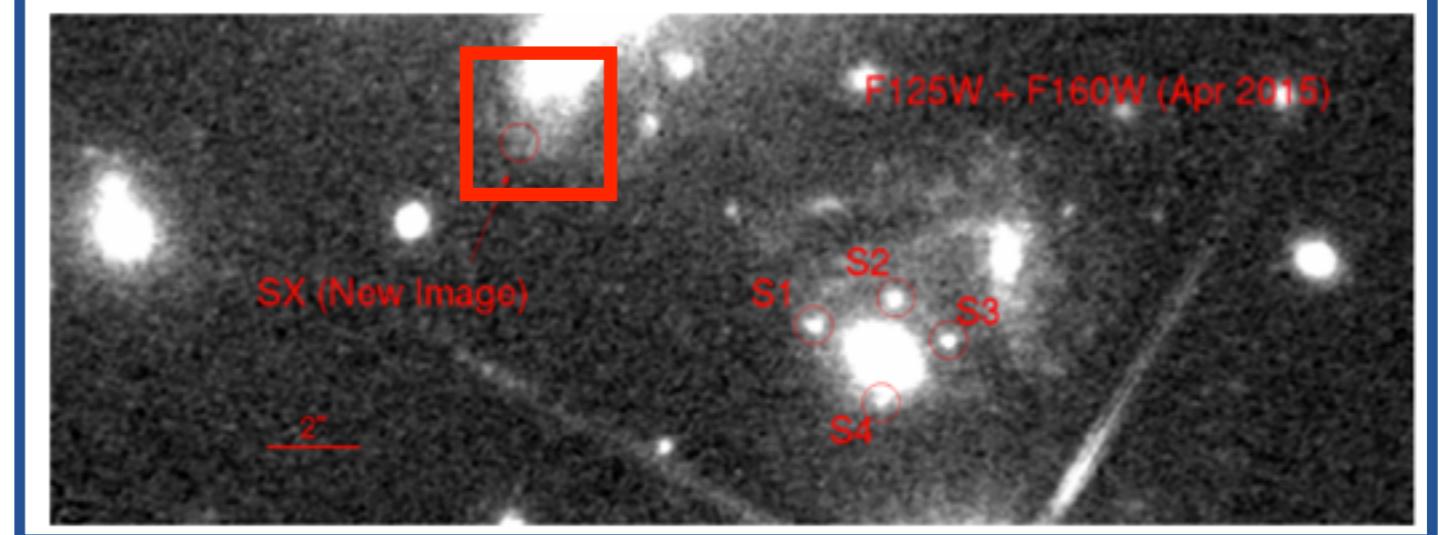
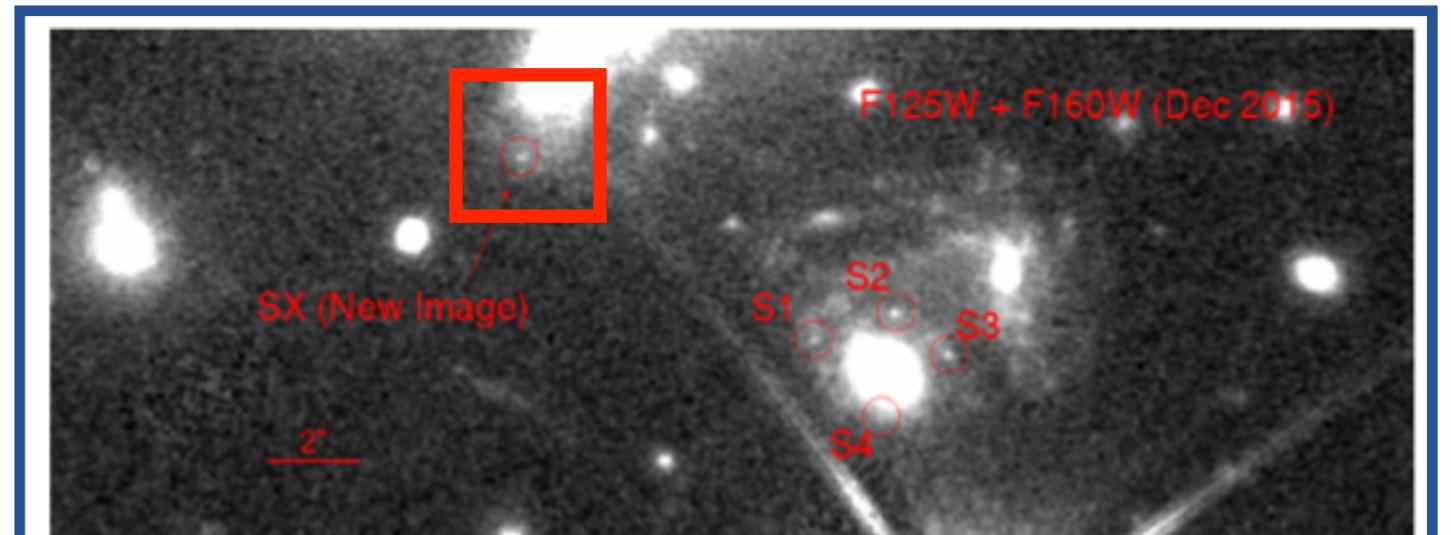


SN Refsdal - Time delay

- When MACS1149 Became re-observable ~Oct. 30 2015 (after Treu+2016 came out!) the hunt for the predicted re-appearance began.

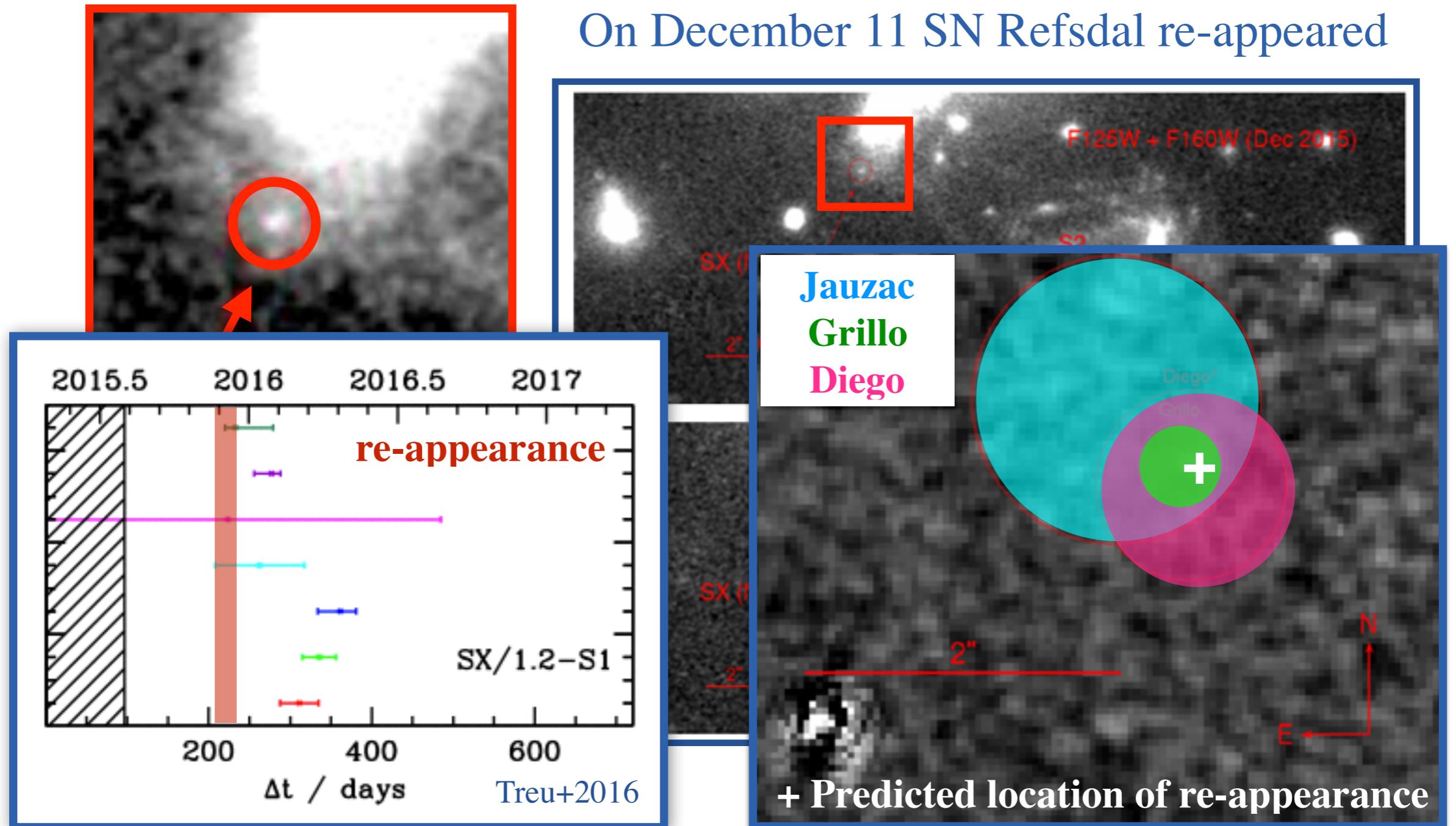


On December 11 SN Refsdal re-appeared



SN Refsdal - Time delay

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SN Refsdal - Time delays



<http://hubblesite.org/newscenter/archive/releases/2015/08/video/>

Estimating H_0 from time delays

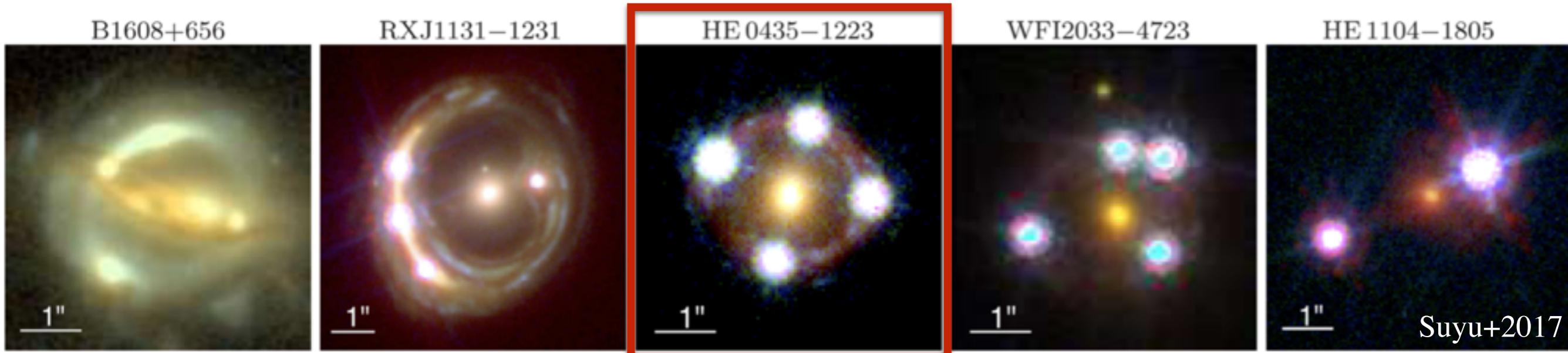
- Time delay lenses can also be used for estimating H_0
- The $\Delta t_{\text{Geometry}}$ is proportional to path lengths, i.e., scales with $1/H_0$
- The $\Delta t_{\text{Shapiro}}$ is also proportional to the path lengths, i.e., scales with $1/H_0$
- Hence, for any gravitational lens $H_0(t_1-t_2)$ depends only on geometry

$$t_1 - t_2 = (1 + z_L) \frac{D_L D_S}{c D_{LS}} \left(\left[\frac{(\theta_1 - \beta)^2}{2} - \frac{\Phi(\theta_1)}{c^2} \right] - \left[\frac{(\theta_2 - \beta)^2}{2} - \frac{\Phi(\theta_2)}{c^2} \right] \right)$$

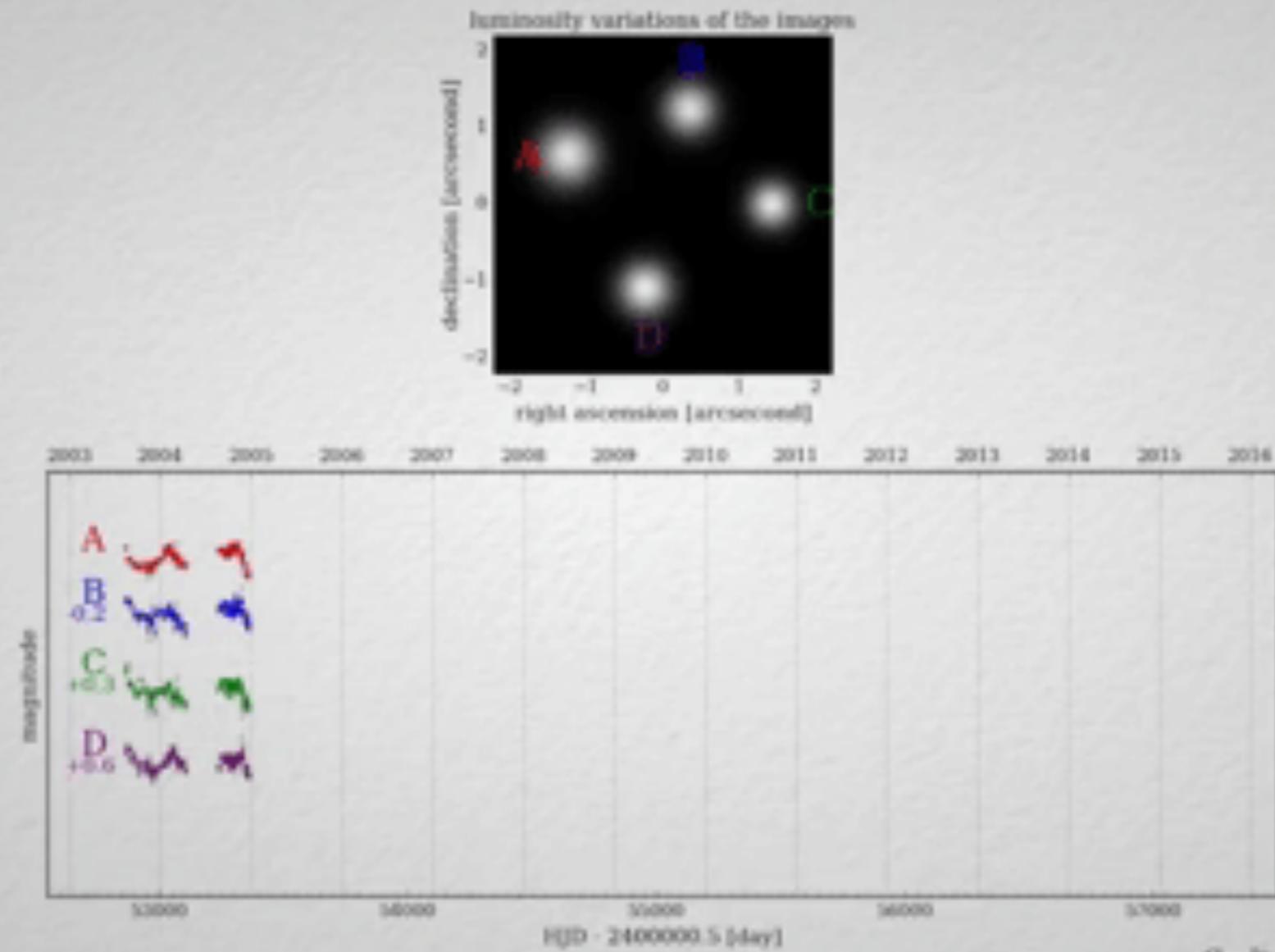
- Lens models provide β , θ and Φ (mass of lens) and predict $H_0(t_1-t_2)$
- This is compared to measurements of (t_1-t_2) from light curve monitoring

COSMOGRAIL & H0LiCOW

- COSmological MOnitoring of GRAVItational Lenses (www.cosmograin.org)
 - Imaging campaign to sample lensed QSO light curves
 - Time delay measurements [e.g., WFI J2033-4723 (Vuissoz+08), RXJ1132 (Suyu+13)]
- H₀ Lenses in COSMOGRAIL's Wellspring (www.h0licow.org)
 - Extending work from COSMOGRAIL with focus on estimating H₀
- H0LiCOW is focusing on 5 lensed QSOs
- First set of papers from 2017 focused on HE0435-1223
 - Bonvin+17, Wang+17, Rusu+17, Sluse+17

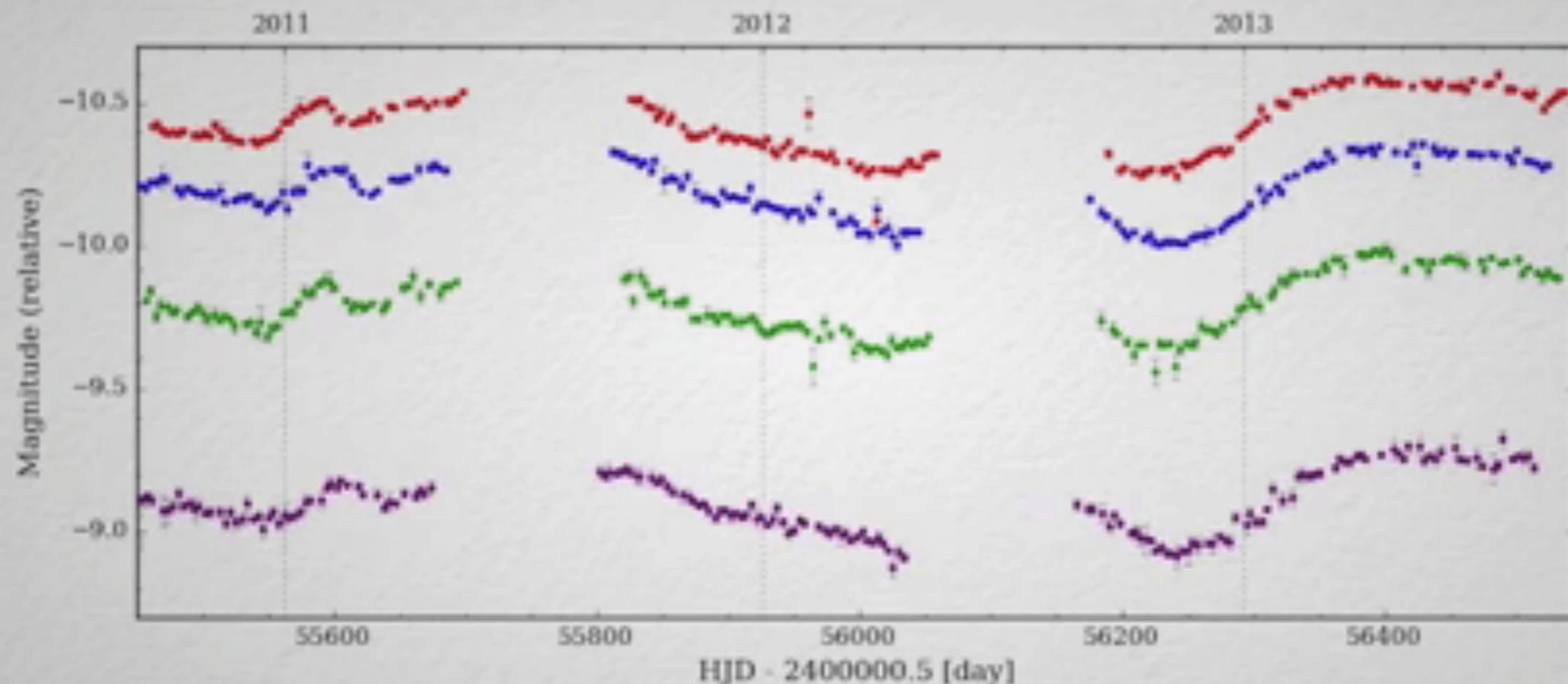


Light Curve for HE0435-1223 images



<https://www.youtube.com/watch?v=qoVQ8f5nVOw>

Matching Fluxes and Obtain Δt

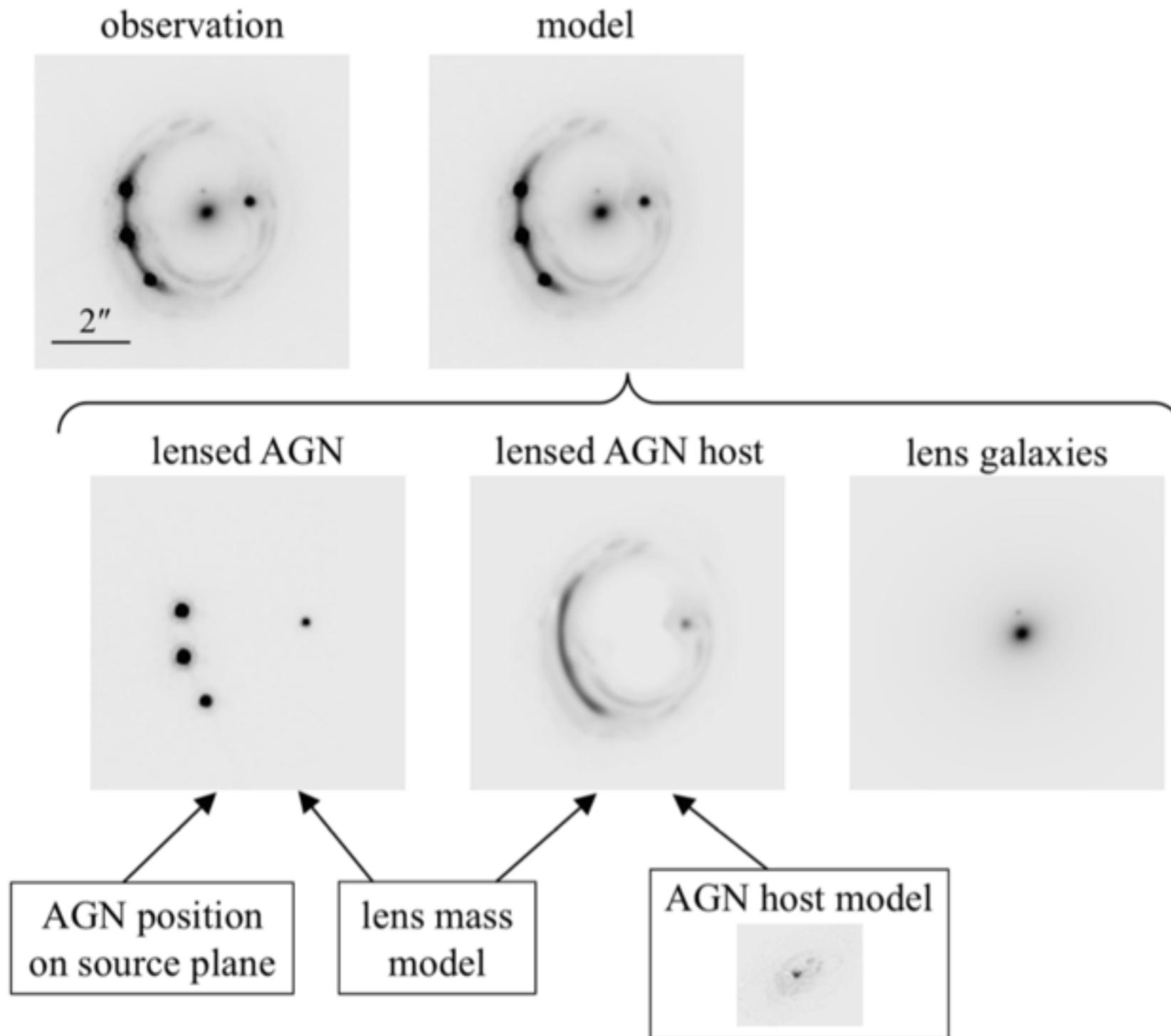


Credits: V. Bozzo - HÖLCOW

<https://www.youtube.com/watch?v=qoVQ8f5nVOw>

A good lens model is key

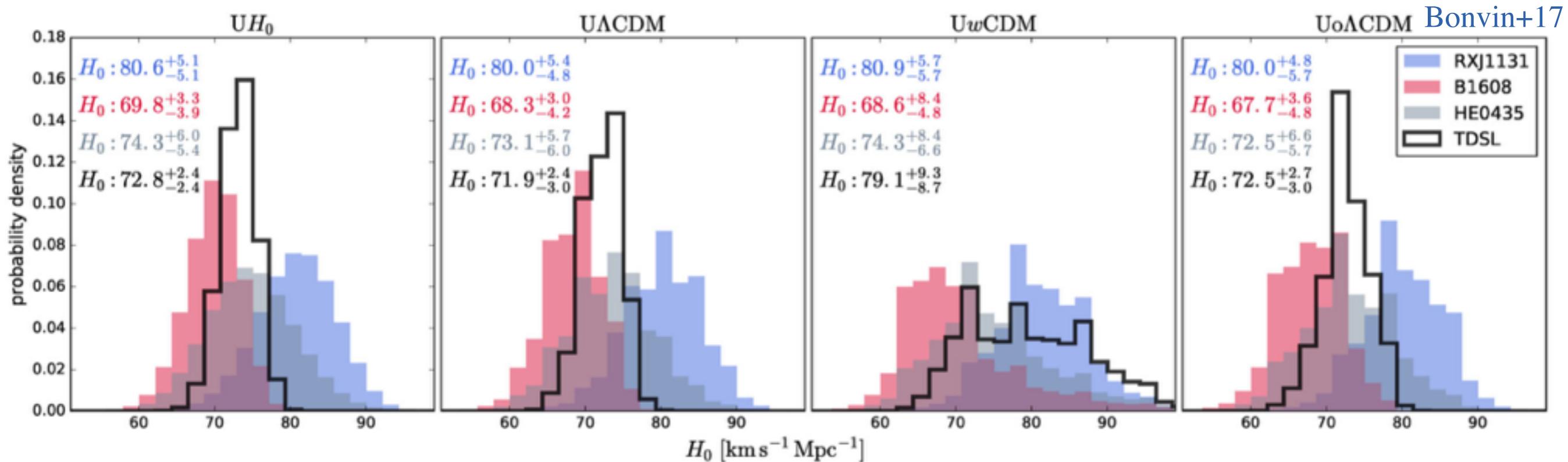
- The obtainable constraint on H_0 from comparing model-predicted $H_0(t_1-t_2)$ to observed (t_1-t_2) is set by model precision/accuracy



Suyu+2018

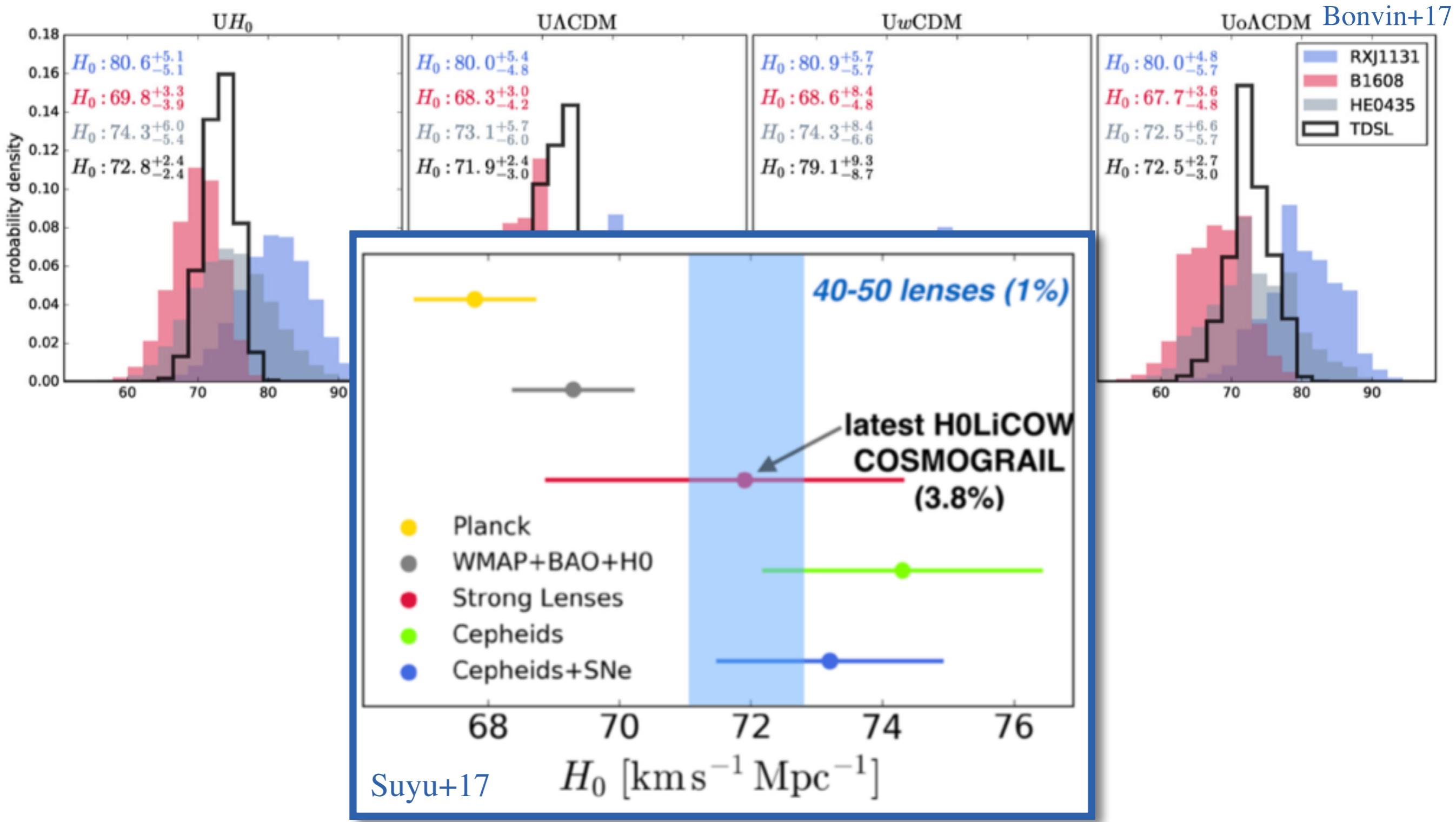
H_0 From HE0435-1223

- Then, comparing $\Delta t(\text{observed})$ with $H_0 \Delta t(\text{model})$ H_0 can be estimated



H_0 From HE0435-1223

- Then, comparing $\Delta t(\text{observed})$ with $H_0 \Delta t(\text{model})$ H_0 can be estimated



So in summary...

- Time delays are a natural consequence of appearance of multiple images

$$\Delta t = \Delta t_{\text{Geometry}} - \Delta t_{\text{Shapiro}}$$

- The Shapiro time delay is caused by gravitational potential (“traffic”)

$$\Delta t_{\text{Shapiro}} = -\frac{\Phi(\theta)}{c^2} \times \frac{D_S D_L}{D_{LS} c}$$

- The Geometric time delay is caused by differences in path lengths (“route”)

$$\Delta t_{\text{Geometry}} = \frac{D_L D_S}{c D_{LS}} \frac{(\theta - \beta)^2}{2}$$

- For the point mass lens, the time delay between the two images is

$$t_+ - t_- \simeq -(1 + z_L) \frac{D_L D_S}{c D_{LS}} 2\theta_E \beta$$

- Time delays are useful for:

- Confirming GR (Shapiro time delay & SN Refsdal)
- Improving lens models (SN Refsdal)
- Determining cosmological parameters, in particular H_0 (H0LiCOW)