

# Homework 9

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11/8/2020

##Problem 1 # part b

```
I=rep(1,2^3)
A=rep(c(-1,1),4)
B=rep(c(-1,-1,1,1),2)
C=c(-1,-1,-1,-1,1,1,1,1)
AB=A*B
AC=A*C
BC=B*C
ABC=A*B*C
cbind(I=rep(1,8),A,B,AB,C,AC,BC,AB,ABC)

##      I  A  B AB  C AC BC AB ABC
## [1,] 1 -1 -1 -1 -1 1 1 1 -1
## [2,] 1 1 -1 -1 -1 -1 1 1 1
## [3,] 1 -1 1 -1 -1 1 -1 -1 1
## [4,] 1 1 1 1 -1 -1 -1 1 -1
## [5,] 1 -1 -1 1 1 -1 -1 1 1
## [6,] 1 1 -1 -1 1 1 -1 -1 -1
## [7,] 1 -1 1 -1 1 -1 1 -1 -1
## [8,] 1 1 1 1 1 1 1 1 1

#design details
k=3#number of factors
n=2#number of replicates

#response
yij=c(22,31,25,32,43,29,35,34,50,55,47,46,44,45,38,40,37,36,60,50,54,39,41,47)

#coded values for treatment levels
x1c=rep(c(rep(-1,3),rep(1,3)),4)
x2c=rep(c(rep(-1,6), rep(1,6)),2)
x3c=c(rep(-1,12), rep(1,12))

#as.factor` treatment levels
x1f=as.factor(x1c)
x2f=as.factor(x2c)
x3f=as.factor(x3c)
mod.factor=aov(yij~x1f*x2f*x3f)
summary(mod.factor)

##              Df Sum Sq Mean Sq F value    Pr(>F)
## x1f           1    0.7      0.7    0.022 0.883680
## x2f           1  770.7    770.7   25.547 0.000117 ***
## x3f           1  280.2    280.2    9.287 0.007679 **
## x1f:x2f        1   16.7     16.7    0.552 0.466078
## x1f:x3f        1  468.2    468.2   15.519 0.001172 **
## x2f:x3f        1   48.2     48.2    1.597 0.224475
## x1f:x2f:x3f    1   28.2     28.2    0.934 0.348282
## Residuals     16  482.7     30.2
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

#Regression on coded levels
mod.reg=lm(yij~x1c+x2c+x3c+x1c:x2c+x1c:x3c+x2c:x3c+x1c:x2c:x3c)

#effects
l=2*k#l is the number of coefficients in the regression
effects=2*mod.reg$coeff[2:l]
effects

##      x1c      x2c      x3c      x1c:x2c      x1c:x3c      x2c:x3c
## 0.3333333 11.3333333  6.8333333 -1.6666667 -8.8333333 -2.8333333
## -2.1666667
```

Answer The effects for x2c, x3c, and x1c:x3c all appear to be large with x2c and x3c in a positive direction while x1c:x3c is in a negative direction.

#Part c

```
SS=(n*2^(k-2))*effects^2#sums of squares for effects
SST=sum((yij-mean(yij))^2)#Total sums of squares
percentSS=100*SS/SSST#percent contribution to total
cbind(effects, SS, percentSS)

##      effects      SS percentSS
## x1c 0.3333333 0.4444444 0.0212116
## x2c 11.3333333 513.7777778 24.5209757
## x3c 6.8333333 186.7777778 8.9139876
## x1c:x2c -1.6666667 11.1111111 0.53027893
## x1c:x3c -8.8333333 312.1111111 14.8953505
## x2c:x3c -2.8333333 32.1111111 1.53250610
## x1c:x2c:x3c -2.1666667 18.7777778 0.89617139
```

Answer So yes, the results do confirm what was observed in part (b) as x2c, x3c and x1c:x3c all contribute a large portion to percentSS.

## Part d

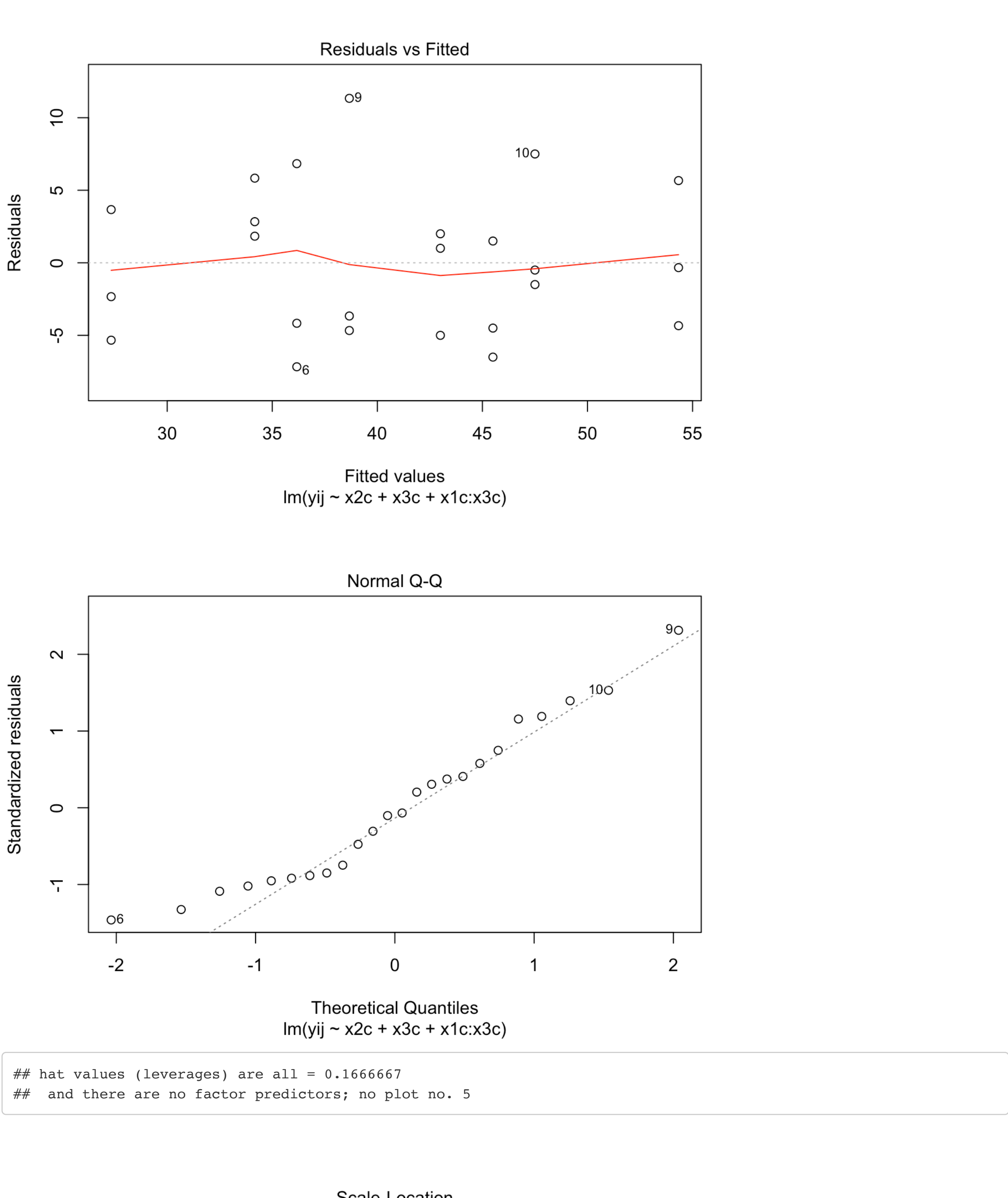
```
m1 <- lm(yij ~ x2c + x3c + x1c:x3c); m1

## Call:
## lm(formula = yij ~ x2c + x3c + x1c:x3c)
##
## Coefficients:
## (Intercept)      x2c      x3c      x3c:x1c
## 40.833      5.667      3.417     -4.417
```

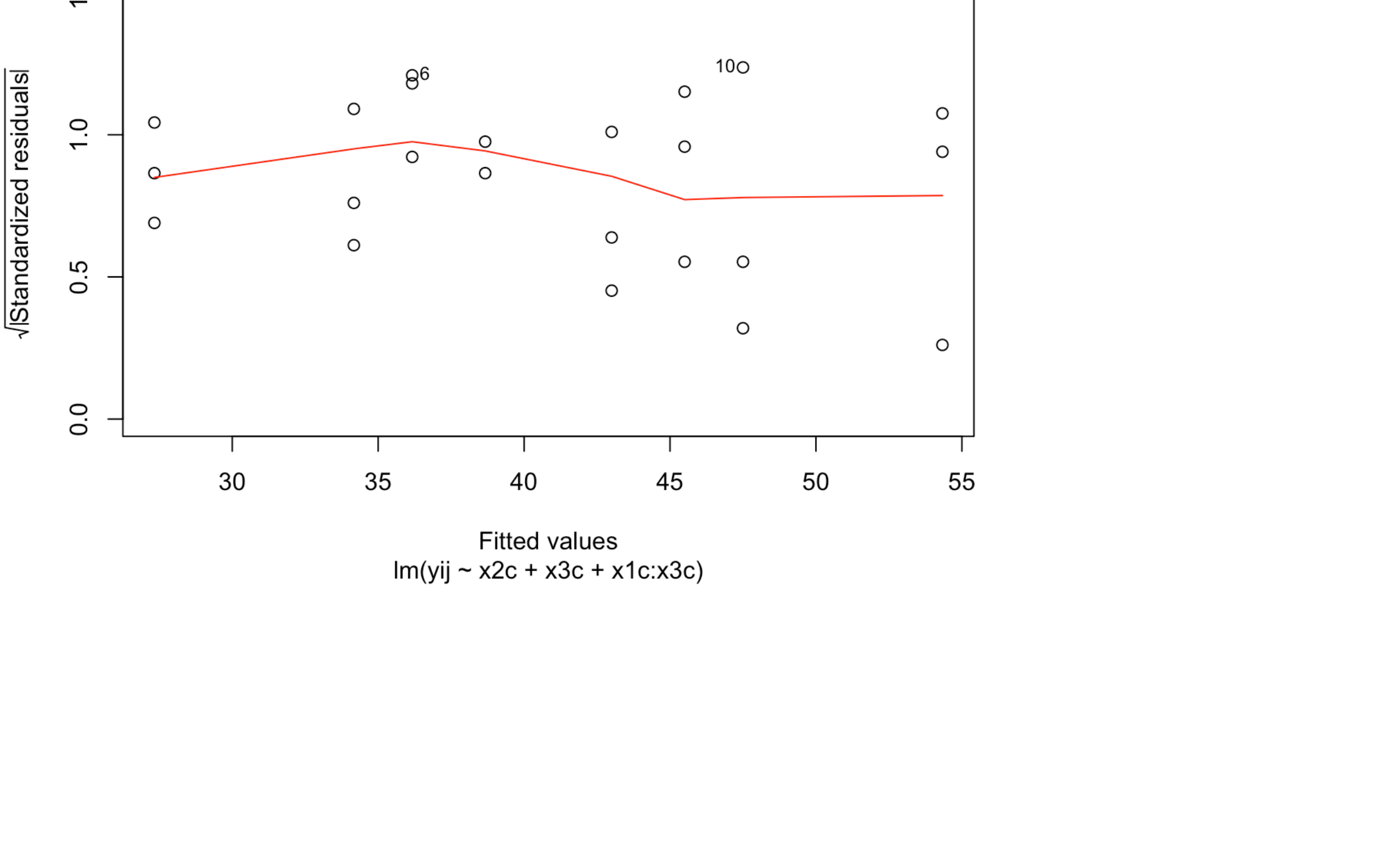
$y = 40.833 + 5.667\beta_{x2} + 3.417\beta_{x3} - 4.417\beta_{x1,x3}$

## part e

```
plot(m1)
```



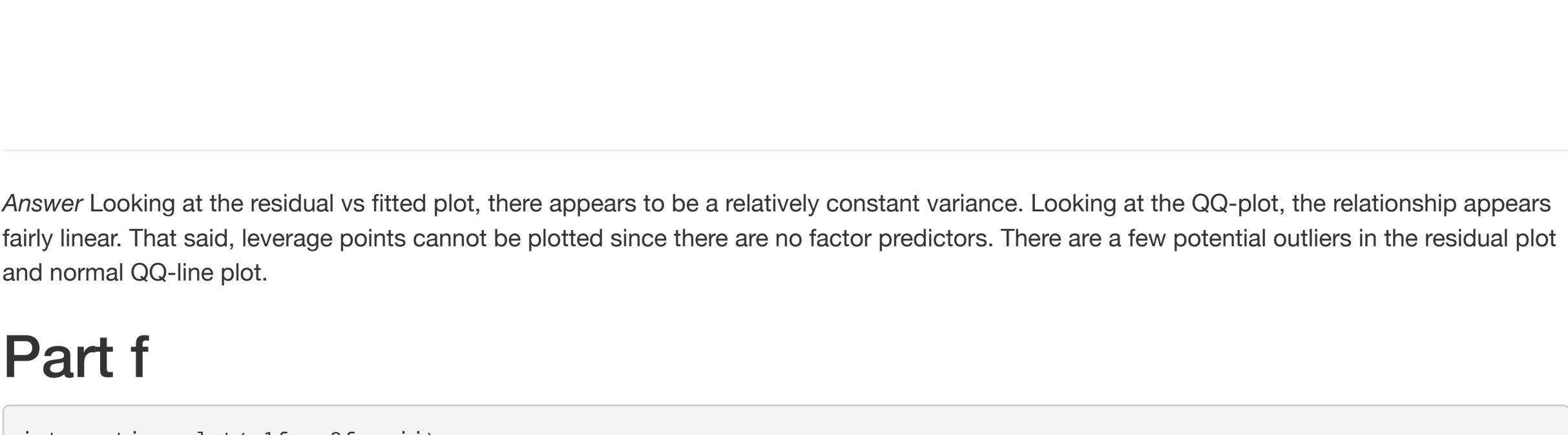
## hat values (leverages) are all = 0.1666667  
## and there are no factor predictors; no plot no. 5



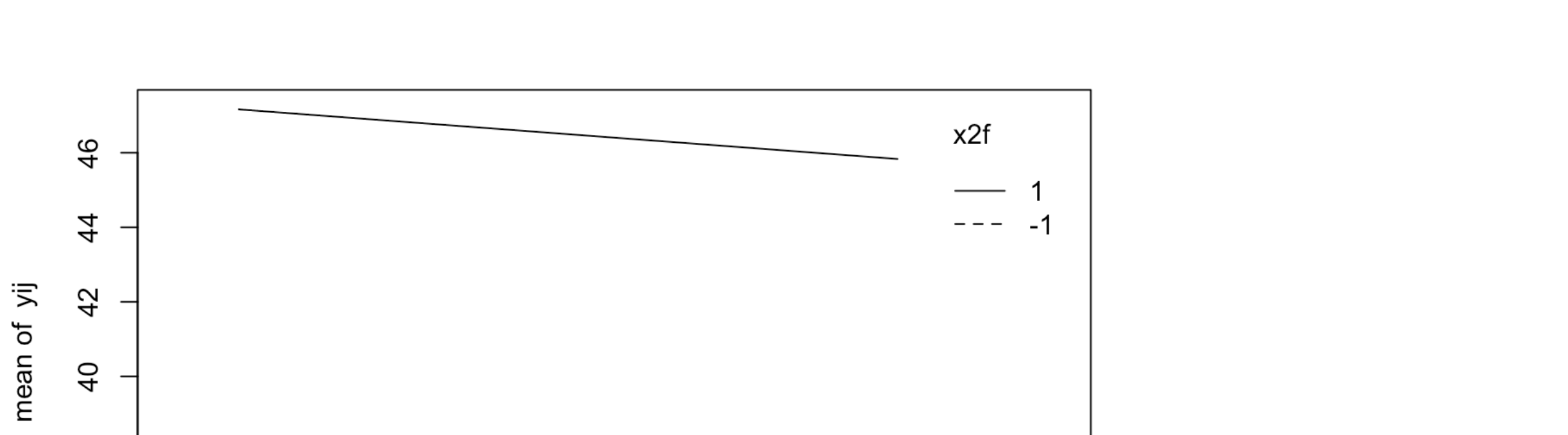
Answer Looking at the residual vs fitted plot, there appears to be a relatively constant variance. Looking at the QQ-plot, the relationship appears fairly linear. That said, leverage points cannot be plotted since there are no factor predictors. There are a few potential outliers in the residual plot and normal QQ-line plot.

## Part f

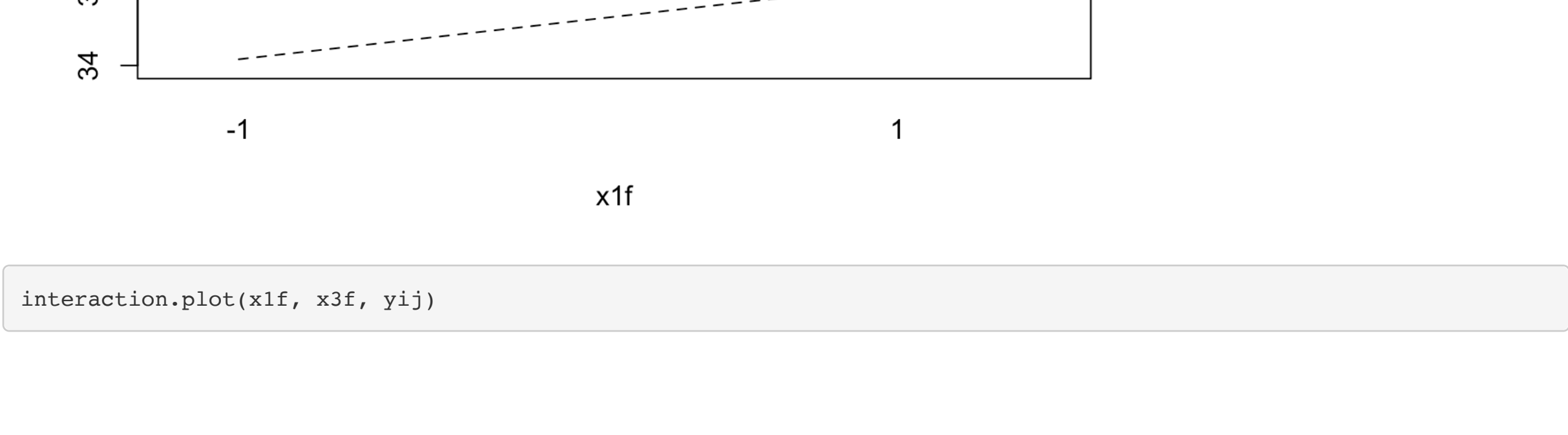
```
interaction.plot(x1f, x2f, yij)
```



```
interaction.plot(x1f, x3f, yij)
```



```
interaction.plot(x2f, x3f, yij)
```



Answer I would recommend a high

tool geometry, low cutting angle, and low cutting speed.

##Problem 2

```
#se.effect
se.effect=summary(m1)$sigma/sqrt(n*2^(k-2))
moe=qt(0.975,m1$df.residual)*se.effect
cbind(effects, effects-moe, effects+moe)#confidence intervals

##      effects      SS percentSS
## x1c 0.3333333 -5.265513 5.932180
## x2c 11.3333333 5.734487 16.932180
## x3c 6.8333333 1.234487 12.432180
## x1c:x2c -1.6666667 -7.265513 3.932180
## x1c:x3c -8.8333333 -14.432180 -3.234487
## x2c:x3c -2.8333333 -8.432180 2.765513
## x1c:x2c:x3c -2.1666667 -7.765513 3.432180
```

Answer So the intervals in question are x2c, x3c, and x1c:x3c interaction (these intervals do not contain zero), which are all of the ones we used in the m1 model. These intervals are significant.

## Problem 3

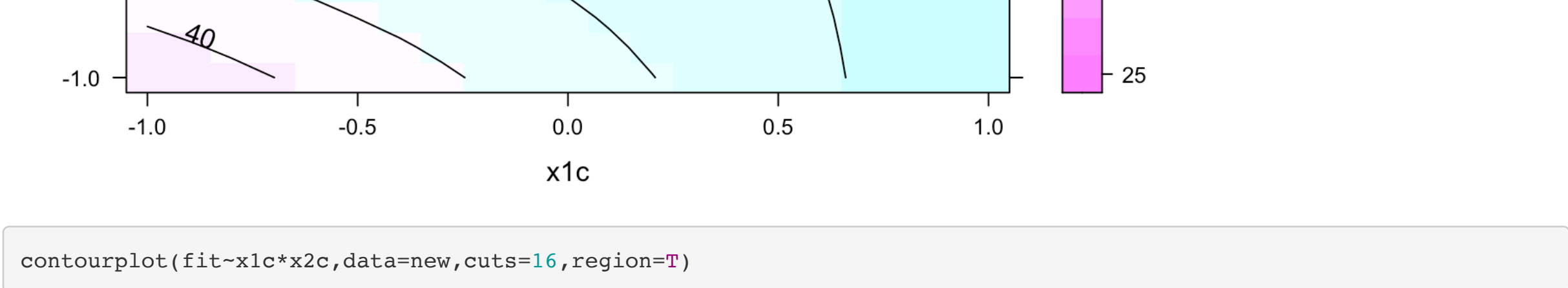
```
library(lattice)

## Warning: package 'lattice' was built under R version 3.6.2

x1.new=(rep(seq(-1,1,by=0.1),5))#create a sequence within range of A
x2.new=sort((rep(seq(-1,1,by=0.1),5)))#create a sequence within range of B
x3.new=sort((rep(seq(-1,1,by=0.1),5)))#create a sequence within range of B
tmp=list(x1c=x1.new,x2c=x2.new,x3c=x3.new)
new=expand.grid(tmp)#create grid
fit=c(predict(m1,new))#predict response for grid
contourplot(fit~x1c*x3c,data=new,cuts=16,region=T)
```



```
contourplot(fit~x2c*x3c,data=new,cuts=16,region=T)
```



```
contourplot(fit~x2c*x3c,data=new,cuts=16,region=T)
```

