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in the name of God

Statistical signal Processing

2,1

$$\begin{cases} H_0 : x_{(0)} = w_{(0)} \\ H_1 : x_0 = 1 + w_{(0)} \end{cases} \qquad w_{(0)} \sim N(\sigma^2)$$

remind that: $P\{x_1 \leqslant x \leqslant x_2\} = \mathcal{G}\left(\frac{x_2-\mu}{\sigma}\right) - \mathcal{G}\left(\frac{x_1-\mu}{\sigma}\right)$
for Gussian dist.

if H_0 is True $\rightarrow$ A_i detected

$$P_{(H_1|H_0)} = P\{x_{(0)} \geqslant 1/2 \,; H_0\} = \int_{\frac{1}{2}}^{\infty} \frac{e^{-\frac{x^2}{2\sigma^2}}}{\sqrt{2\pi\sigma^2}} dx = \mathcal{G}^{1}_{(\infty)} - \mathcal{G}\left(\frac{\frac{1}{2}-0}{\sigma}\right)$$

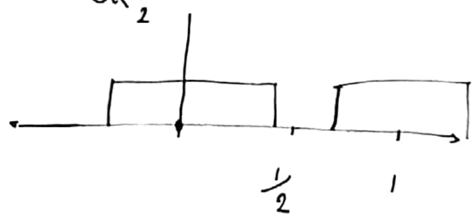
$$= \boxed{1 - \mathcal{G}\left(\frac{1}{2\sigma}\right)}$$

$$P_0 = 0.001 = 1 - \mathcal{G}\left(\frac{1}{2\sigma}\right) \rightarrow \frac{1}{2\sigma} = 3.09 \rightarrow \boxed{\sigma^2 = .026}$$

1,2

$$\begin{cases} H_0 \quad x_0 = w_{(0)} \\ H_1 \quad x_1 = w_{(0)} + 1 \end{cases} \qquad w_0 \sim U[-a, a]$$

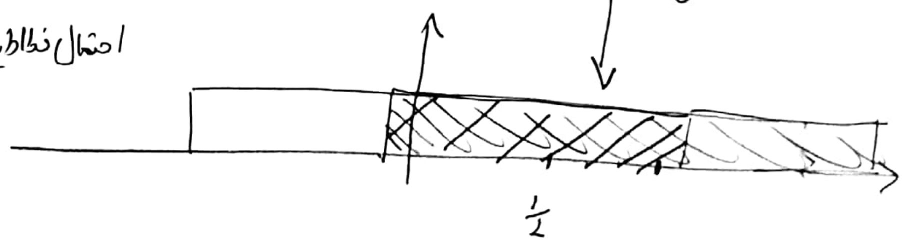
$a < \frac{1}{2}$ ✓



if $a < \frac{1}{2}$ $\rightarrow$ excellent
non-overlapping area

if $a > \frac{1}{2}$ $\rightarrow$ overlapping area

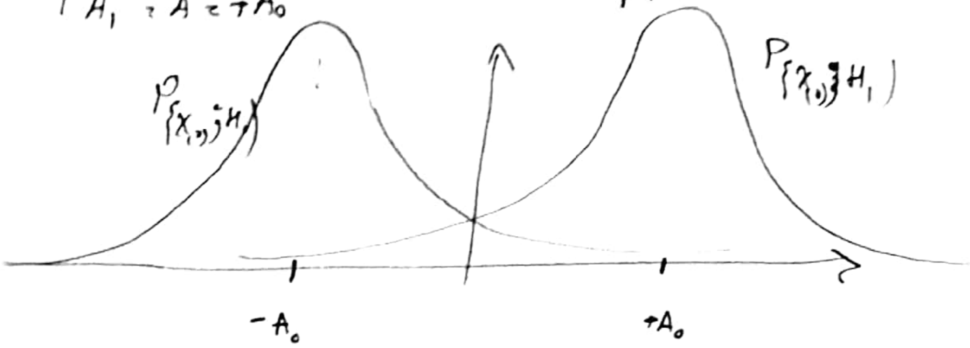
احتمال خطا داریم.



1-3]

$x \sim N\{A, \sigma^2 = 1\}$

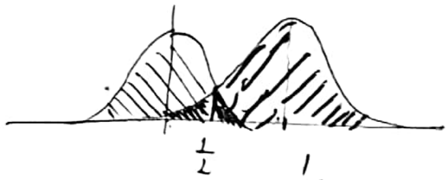
$$\begin{cases} H_0 : A = -A_0 \\ A_1 : A = +A_0 \end{cases} \qquad P_{x_{(0)}} = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}(x_{(0)}-A)^2}$$



$$\boxed{\delta_{(x)} \begin{cases} x_{(0)} > 0 \longrightarrow H_1 \\ x_{(0)} < 0 \longrightarrow H_0 \end{cases}}$$

1-4]

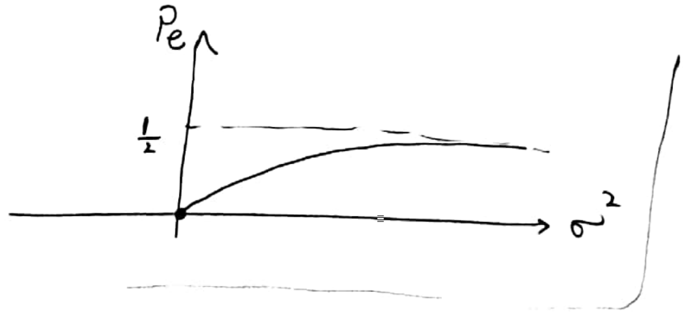
$P_{(H_0)} = P_{(H_1)} = 1/2$



$$P_e = P\{x_0 > \tfrac{1}{2} \mid H_0\} \overbrace{P_{(H_0)}}^{\frac{1}{2}} + P\{x_{(0)} < \tfrac{1}{2} \mid H_1\} \overbrace{P_{(H_1)}}^{\frac{1}{2}}$$

$$= \Big(\underbrace{P\{x_0 > \tfrac{1}{2} \mid H_0\}}_{1 - G(\frac{1}{2\sigma})} + \underbrace{P\{x_0 < \tfrac{1}{2} \mid H_1\}}_{1 - G(\frac{1}{2\sigma})}\Big)\frac{1}{2} = 1 - G\left(\frac{1}{2\sigma}\right)$$

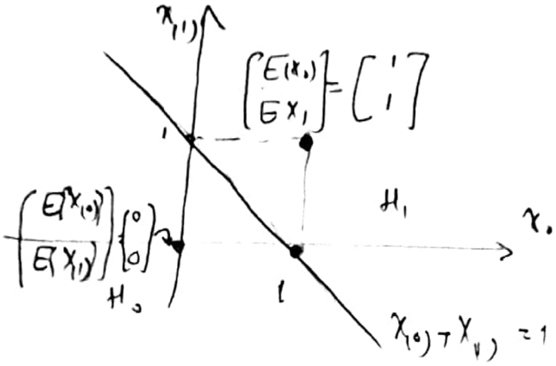
$$\longrightarrow P_e = 1 - G\left(\frac{1}{2\sqrt{\sigma^2}}\right) \qquad \sigma \to \infty \Rightarrow P_e = \frac{1}{2}$$



if $\sigma^2 \to \infty$ it means that we are adding tail of Gussian dist. with the head and easily we can verify our answer $\frac{1}{2}((1-a)+a) = \boxed{\frac{1}{2}}$

Scanned by CamScanner

1,5 Signal is Present if $\frac{1}{2}(x_{(1)} + x_{0}) > \frac{1}{2}$ H_1



$H_0 \to E[x_{(0)}] = E(x_{(1)}) = 0 \to \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$r.\ H_1 \to E\{x_{(0)}\} = E(x_1) = 1 \to \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

1,6

$$T = \frac{1}{N} \sum_{0}^{N-1} x_{(n)} > \gamma$$

$$E\{T \mid H_0\} = E\{\frac{1}{N} \sum_{0}^{N-1} w_{(n)}\} = 0$$

$$E\{T \mid H_1\} = E\{\frac{1}{N} \sum_{0}^{N} E(A + w)\} = \frac{NA}{N} = A$$

$$\mathrm{Var}\{T_i \mid H_0\} = E\{(T - \bar{T})^2 \mid H_0\} = E(T^2 / H_0) - 0 - 0$$

$$E\{(\frac{1}{N} \sum_{n} w_{(n)})^2\} = \frac{1}{N^2} E\{\sum_{m} \sum_{n} w_{(n)} w_{(m)}\} = \frac{1}{N^2} \sum \sum \overbrace{E(w_{(n)} w_{(m)})}^{\sigma^2 \delta_{(m-n)}}$$

$$= \frac{1}{N^2} \times N \sigma^2 = \boxed{\frac{\sigma^2}{N}}$$

$$d^2 = \frac{(A - 0)^2}{\sigma^2 / N} = \frac{NA^2}{\sigma^2} = \frac{2A^2}{\sigma^2} = \boxed{\frac{2}{\sigma^2}}$$

$\begin{cases} N = 2 \\ A = 1 \end{cases}$

1,7

$$SNR \to \frac{P_S}{P_N} = \frac{(A)^2}{\sigma^2} = \left(\frac{A}{\sigma}\right)^2 = 10^{-\frac{20}{10}} = 0.01$$

$$d^2 = \frac{NA^2}{\sigma^2} = 100 \to \boxed{N = 10^4}$$