



1) Prove each of the following:

I) For a skew-symmetric matrix  $S(a)$  of a vector  $(a)$  and a rotation matrix  $R$  and a vector  $X$

a-  $S(\alpha a + \beta b) = \alpha S(a) + \beta S(b)$

b-  $S(a)p = a \times p$

c-  $RS(a)R^T = S(Ra)$

d-  $X^T SX = 0$

e-  $S(k)^3 = -S(k)$

II) Given the euler angle transformation

$$R = R_{z,\psi} R_{y,\theta} R_{z,\phi}$$

Show that  $\frac{d}{dt}R = S(\omega)R$  where

$$\omega = \{c_\psi s_\theta \dot{\phi} - s_\psi \dot{\theta}\}i + \{s_\psi s_\theta \dot{\phi} + c_\psi \dot{\theta}\}j + \{\dot{\psi} + c_\theta \dot{\phi}\}k$$

III) Repeat Problem part (II) this time for the Roll-Pitch-Yaw transformation. In

other words find an explicit expression for  $\omega$  such that  $\frac{d}{dt}R = S(\omega)R$

Due Date:  
March 31, 2020  
(12 Farvardin 98)

In the name of god

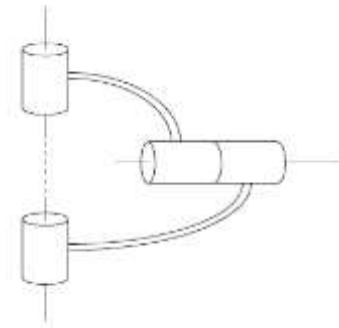
**Advanced Robotics**  
Homework Assignment #4-1



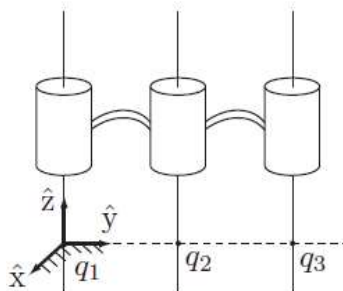
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2) some common kinematic singularities that occur in 6-dof manipulator with revolute and prismatic joints are mentioned below. Use Geometric Jacobian and prove singularity existence in each of the following :

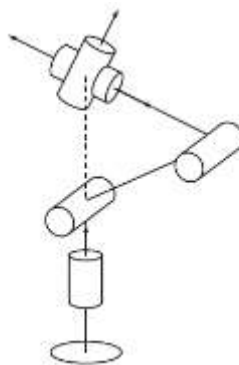
a) Two collinear Revolute joint Axes



b) Three Coplanar and Parallel Revolute Joint Axes



c) Four Revolute Joint Axes Intersecting at a Common Point

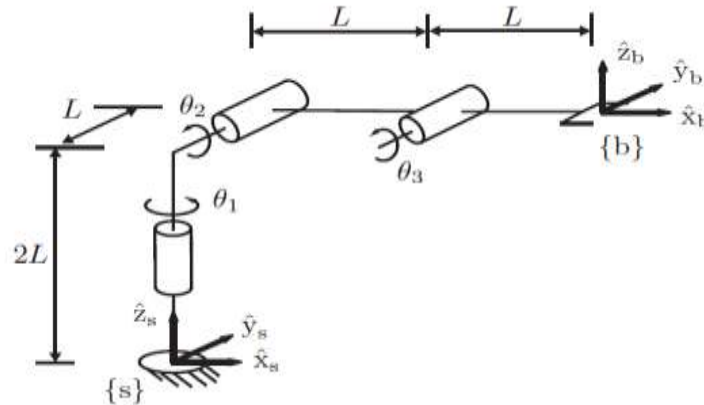


d) Four Coplanar Revolute Joints

e) Six Revolute Joints Intersecting a Common Line



3) The spatial 3R manipulator is shown in its zero position. Let  $p$  be the coordinates of the origin of  $\{b\}$  expressed in  $\{s\}$ .



- Write down the Jacobian (Geometric Jacobian). (use method of Spong's book)
- In its zero position (configuration shown in figure), suppose we wish to make the end-effector move with linear velocity  $V = (10; 0; 0)$  ( $V$  is expressed in  $\{s\}$  frame). What are the required input joint velocities?
- Suppose that the robot is in the configuration  $\theta_1 = 0$ ,  $\theta_2 = 45^\circ$ ,  $\theta_3 = -45^\circ$ . Assuming static equilibrium, suppose that we wish to generate an end-effector force  $f_b = (10; 0; 0)$ , where  $f_b$  is expressed with respect to the end-effector frame ( $\{b\}$  frame). What are the required input joint torques?

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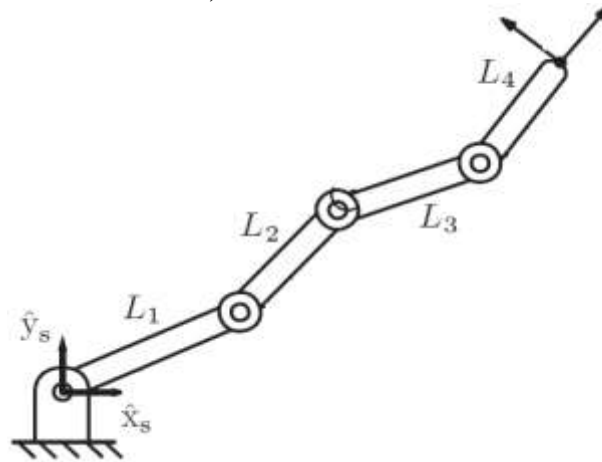
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4) Answer the following questions for the 4R planar manipulator (this planar 4-DOF robot is a redundant robot)



- a) Write down the Jacobian (Geometric Jacobian). (with **two different methods** arbitrary for sure)
- b) Suppose that the manipulator is in static equilibrium at the configuration and that a force  $f = (10; 10; 0)$  and a moment  $m = (0; 0; 10)$  are applied to the tip (both  $f$  and  $m$  are expressed with respect to the fixed frame  $\{s\}$ ). What are the torques experienced at each joint?