صفحه ① کسری ملکی 9523038 تمرین سری ۲ (HW4-2) درس رباتیک پیشرفته دکتر شریفی

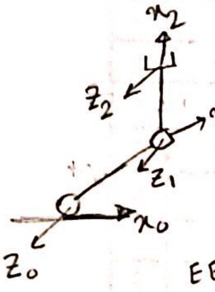
② what is the differace between DH-classic and Modified

۱- در روش modified، محور $z_i$ را به i امین مفصل اختصاص می‌دهیم ولی در کلاسیک، $z_{i-1}$ را به i امین موقعیت اختصاص می‌دهیم

۲- در modified، $x_i$ در راستای عمود مشترک $z_i$ و $z_{i+1}$ است ولی در کلاسیک $x_i$ در راستای عمود مشترک $z_{i-1}$ و $z_i$ است

۳- فرمول H یا همان T شان فرق دارد

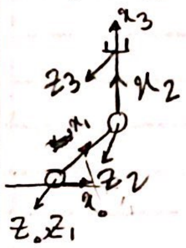
## CLASSIC




| | $\alpha_i$ | $a_i$ | $d_i$ | $\theta_i$ |
|---|---|---|---|---|
| 1 | 0 | $l_1$ | 0 | $\theta_1$ |
| EE = 2 | 0 | $l_2$ | 0 | $\theta_2$ |

$$ {}^{i-1}T_i = \begin{bmatrix} c\theta_i & -s\theta_i c\alpha_i & s\theta_i s\alpha_i & a_i c\theta_i \\ s\theta_i & c\theta_i c\alpha_i & -c\theta_i s\alpha_i & a_i s\theta_i \\ 0 & s\alpha_i & c\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

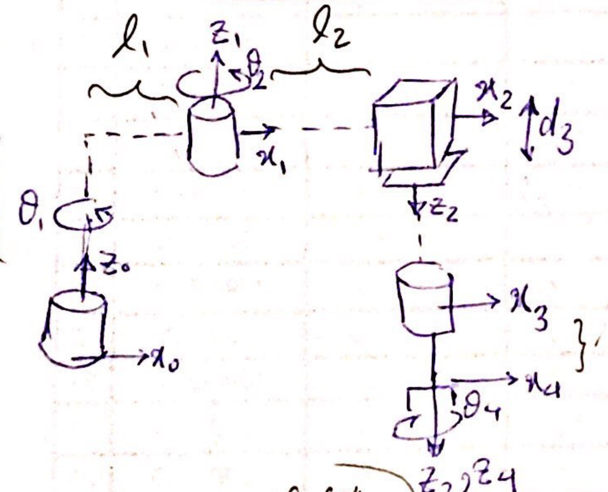
## MODIFIED




| | $\alpha_{i-1}$ | $a_{i-1}$ | $d_i$ | $\theta_i$ |
|---|---|---|---|---|
| 1 | 0 | 0 | 0 | $\theta_1$ |
| 2 | 0 | $l_1$ | 0 | $\theta_2$ |
| EE | 0 | $l_2$ | 0 | 0 |

$$ {}^{i-1}T_i = \begin{bmatrix} c\theta_i & -s\theta_i & 0 & a_{i-1} \\ s\theta_i c\alpha_{i-1} & c\theta_i c\alpha_{i-1} & -s\alpha_{i-1} & -s\alpha_{i-1} d_i \\ s\theta_i s\alpha_{i-1} & c\theta_i s\alpha_{i-1} & c\alpha_{i-1} & c\alpha_{i-1} d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

③




| | $\alpha_i$ | $a_i$ | $d_i$ | $\theta_i$ |
|---|---|---|---|---|
| 1 | 0 | $l_1$ | $d_1$ | $\theta_1$ |
| 2 | 180 | $l_2$ | 0 | $\theta_2$ |
| 3 | 0 | 0 | $d_3$ | 0 |
| 4 | 0 | 0 | $d_4$ | $\theta_4$ |

$$ {}^{i-1}T_i = \begin{bmatrix} c\theta_i & -s\theta_i c\alpha_i & s\theta_i s\alpha_i & a_i c\theta_i \\ s\theta_i & c\theta_i c\alpha_i & -c\theta_i s\alpha_i & a_i s\theta_i \\ 0 & s\alpha_i & c\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad {}^{0}T_1 = \begin{bmatrix} c\theta_1 & -s\theta_1 & 0 & l_1 \\ s\theta_1 & c\theta_1 & 0 & 0 \\ 0 & 0 & 1 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ \begin{cases} {}^{i}\omega_i = {}^{i}R_{i-1}\,{}^{i-1}\omega_{i-1} + \dot{\theta}_i\,{}^{i}\hat{Z}_i \\ {}^{i}V_i = {}^{i}R_{i-1}\left({}^{i-1}V_{i-1} + {}^{i-1}\omega_{i-1} \times {}^{i-1}P_i\right) + \dot{d}_i\,{}^{i}\hat{Z}_i \end{cases} \qquad {}^{1}T_2 = \begin{bmatrix} c\theta_2 & s\theta_2 & 0 & -l_2 \\ s\theta_2 & -c\theta_2 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^{2}T_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \theta_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad {}^{3}T_4 = \begin{bmatrix} c\theta_4 & -s\theta_4 & 0 & 0 \\ s\theta_4 & c\theta_4 & 0 & 0 \\ 0 & 0 & 1 & d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^{0}T_2 = {}^{0}T_1 \, {}^{1}T_2 = \begin{bmatrix} c(\theta_1+\theta_2) & s(\theta_1+\theta_2) & 0 & l_1 - l_2 c\theta_1 \\ s(\theta_1+\theta_2) & -c(\theta_1+\theta_2) & 0 & -l_2 s\theta_1 \\ 0 & 0 & -1 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^{0}T_3 = \begin{bmatrix} c(\theta_1+\theta_2) & s(\theta_1+\theta_2) & 0 & l_1 - l_2 c(\theta_1) \\ s(\theta_1+\theta_2) & -c(\theta_1+\theta_2) & 0 & -l_2 s(\theta_1) \\ 0 & 0 & -1 & d_1 - \theta_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad {}^{0}T_4 = \begin{bmatrix} c(\theta_1+\theta_2-\theta_4) & s(1+2-4) & 0 & l_1 - l_2 c(\theta_1) \\ s(\theta_1+\theta_2-\theta_4) & -c(1+2-4) & 0 & -l_2 s(\theta_1) \\ 0 & 0 & -1 & (d_1-\theta_3-d_4) \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ J_1 = \begin{bmatrix} z_0 \times ({}^{0}O_n - {}^{0}O_0) \\ {}^{0}z_0 \end{bmatrix} = \begin{bmatrix} l_2 s\theta_1 \\ l_1 - l_2 c\theta_1 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \qquad J_2 = \begin{bmatrix} {}^{0}z_1 \times ({}^{0}O_{EE} - {}^{0}O_1) \\ {}^{0}z_1 \end{bmatrix} = \begin{bmatrix} l_2 s\theta_1 \\ -l_2 c\theta_1 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} $$

$$ J_3 = \begin{bmatrix} {}^{0}z_2 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \qquad J_4 = \begin{bmatrix} {}^{0}z_3 \times ({}^{0}O_{EE} - {}^{0}O_3) \\ {}^{0}z_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -1 \end{bmatrix} $$

$$ J = [J_1 \; J_2 \; J_3 \; J_4] = \begin{bmatrix} l_2 s\theta_1 & l_2 s\theta_1 & 0 & 0 \\ l_1 - l_2 c\theta_1 & -l_2 c\theta_1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & -1 \end{bmatrix} $$

$l_1 = a_1 = 250\,mm$

$l_2 = a_2 = 150\,mm$

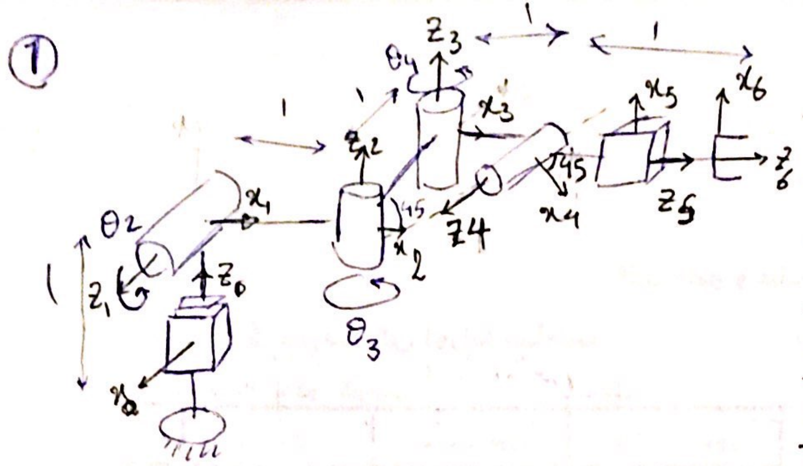
$d_4 = 50\,mm$

$0 < d_3 < 300\,mm$

$$ \Rightarrow {}^{0}V = {}^{0}J_v \dot{q} = \begin{bmatrix} l_2 s\theta_1 & l_2 s\theta_1 & 0 & 0 \\ l_1 - l_2 c\theta_1 & -l_2 c\theta_1 & 0 & 0 \\ 0 & 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \\ \dot{\theta}_4 \end{bmatrix} $$

$$ = \begin{bmatrix} \dot{\theta}_1 l_2 s\theta_1 + \dot{\theta}_2 l_2 s\theta_1 \\ \dot{\theta}_1 (l_1 - l_2 c\theta_1) - \dot{\theta}_2 l_2 c\theta_1 \\ 0 \end{bmatrix} = {}^{0}V_{EE} $$

1




| | $\alpha_i$ | $a_i$ | $d_i$ | $\theta_i$ |
|---|---|---|---|---|
| 1 | 0 | 0 | $\theta_1$ | 90 |
| 2 | −90 | 1 | 0 | $\theta_2$ |
| 3 | 0 | 1 | 1 | $\theta_3$ |
| 4 | +90 | 1 | 0 | $\theta_4$ |
| 5 | 135 | 0 | 0 | $\theta_5+90$ |
| 6 | 0 | 0 | $\theta_6$ | 0 |

$$
{}^{i-1}T_i = \begin{bmatrix} C\theta_i & -S\theta_i C\alpha_i & S\theta_i S\alpha_i & a_i C\theta_i \\ S\theta_i & C\theta_i C\alpha_i & -C\theta_i S\alpha_i & a_i S\theta_i \\ 0 & S\alpha_i & C\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^{0}T_1 = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & \theta_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad
{}^{1}T_2 = \begin{bmatrix} C\theta_2 & 0 & -S\theta_2 & C\theta_2 \\ S\theta_2 & 0 & C\theta_2 & S\theta_2 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad
{}^{2}T_3 = \begin{bmatrix} C\theta_3 & -S\theta_3 & 0 & C\theta_3 \\ S\theta_3 & C\theta_3 & 0 & S\theta_3 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^{3}T_4 = \begin{bmatrix} C\theta_4 & 0 & S\theta_4 & C\theta_4 \\ S\theta_4 & 0 & C\theta_4 & S\theta_4 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad
{}^{4}T_5 = \begin{bmatrix} -S\theta_5 & +C\theta_5 \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} C\theta_5 & 0 \\ C\theta_5 & +S\theta_5 \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} S\theta_5 & 0 \\ 0 & \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad
{}^{5}T_6 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \theta_6 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^{0}T_2 = \begin{bmatrix} -S\theta_2 & 0 & -C\theta_2 & -S\theta_2 \\ C\theta_2 & 0 & -S\theta_2 & C\theta_2 \\ 0 & -1 & 0 & \theta_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad
{}^{0}T_3 = \begin{bmatrix} -C\theta_3 S\theta_2 & S\theta_2 S\theta_3 & -C\theta_2 & -C\theta_2 - S\theta_2 - C\theta_3 S\theta_2 \\ C\theta_2 C\theta_3 & -C\theta_2 S\theta_3 & -S\theta_2 & C\theta_2 - S\theta_2 + C\theta_2 C\theta_3 \\ -S\theta_3 & -C\theta_3 & 0 & \theta_1 - S\theta_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^{0}T_4 = \begin{bmatrix} -C(3+4) S\theta_2 & -C\theta_2 & S(3-4) S2 & S2S3S4 - S2 - S2C_3 - C2 - C3C4S2 \\ C(3+4) C\theta_2 & -S\theta_2 & -S(3-4) C2 & C2 - S2 + C2C3 - C2S3S4 + C2C3C4 \\ -S(\theta_3+\theta_4) & 0 & -C(3-4) & \theta_1 - S(3+4) - S3 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^{0}T_5 = \begin{bmatrix} \text{——} , & -\frac{\sqrt{2}}{2} C2S5 - \frac{\sqrt{2}}{2} S(3-4) S2 - \frac{\sqrt{2}}{2} C(3+4) C5 S2, & S2S3S4 - S2 - C3S2 - C2 - C3C4S2 \\ \text{——} & \frac{\sqrt{2}}{2} S(3-4) C2 - \frac{\sqrt{2}}{2} S2S5 + \frac{\sqrt{2}}{2} C(3+4) C2C5, & C2 - S2 + C2C3 - C2S3S4 + C2C3C4 \\ \text{——} & \frac{\sqrt{2}}{2} C(3-4) - \frac{\sqrt{2}}{2} S(3+4) \times C5 \quad , & t_1 - S(3+4) - S3 \\ \text{——} & 0 \quad , & 1 \end{bmatrix}
$$

$$
{}^{\bullet}T_6 = \begin{bmatrix}
\sim, -\frac{\sqrt2}{2}c2\,s5 - \frac{\sqrt2}{2}s(3-4)s2 - \frac{\sqrt2}{2}c(3+4)c5\,s2 & , s2s3s4 - s2 - c3s2 - c2, -c3c4s2 + \frac{\sqrt2}{2}(-\theta_6 s(3-4)s2 - \theta_6 \ldots \\
\sim, \frac{\sqrt2}{2}s(3-4)c2 - \frac{\sqrt2}{2}s2\,s5 + \frac{\sqrt2}{2}c(3+4)c2\,c5 & , c2 - s2 + c2c3 - c2s3s4 + c2c3c4 + \frac{\sqrt2}{2}(-\theta_6 s2s5 + \theta_6 s(3-4)) + \ldots \\
\sim, \frac{\sqrt2}{2}c(3-4) - \frac{\sqrt2}{2}s(3+4)c5 & , t1 - s(3+4) - s3 + \theta_6 \times \left(\frac{\sqrt2}{2}c(3-4) - \frac{\sqrt2}{2}s(3+4)c5\right) \\
\sim, \quad 0 & , 1
\end{bmatrix}
$$

رابطه استرسرت

$$
\begin{cases}
{}^{i+1}\omega_{i+1} = {}^{i+1}_{i}R\,{}^{i}\omega_i + \dot\theta_{i+1}\,{}^{i+1}Z_{i+1} \\
{}^{i+1}V_{i+1} = {}^{i+1}_{i}R\,({}^{i}V_i + {}^{i}\omega_i \times {}^{i}P_{i+1}) + \dot d_{i+1}\,{}^{i+1}Z_{i+1}
\end{cases}
$$

← برای Modified

برای کلاسیک هم تقریبا همینه

فقط z ها یه دونه شیفت می‌خورند

$$
\Rightarrow \begin{cases} {}^{0}\omega_0 = \begin{bmatrix}0\\0\\0\end{bmatrix} \\ {}^{0}V_0 = \begin{bmatrix}0\\0\\\dot\theta_1 = \dot d_1\end{bmatrix}\end{cases}
\Rightarrow \begin{cases}
{}^{1}\omega_1 = {}^{1}_{0}R\,\cancel{{}^{0}\omega_0} + \dot\theta_2\,{}^{1}Z_1 = \begin{bmatrix}0\\0\\\dot\theta_2\end{bmatrix} \\
{}^{1}V_1 = {}^{1}_{0}R\left(\begin{bmatrix}0\\0\\\dot\theta_1\end{bmatrix} + 0 \times {}^{0}P_1\right) = \begin{bmatrix}0&1&0\\-1&0&0\\0&0&1\end{bmatrix}\begin{bmatrix}0\\0\\\dot\theta_1\end{bmatrix} = \begin{bmatrix}0\\0\\\dot\theta_1\end{bmatrix}
\end{cases}
$$

$$
\begin{cases}
{}^{2}\omega_2 = {}^{2}_{1}R\,{}^{1}\omega_1 + \dot\theta_3\,{}^{2}Z_2 = \begin{bmatrix}c\theta_2 & s\theta_2 & 0\\ 0 & 0 & -1\\ -s\theta_2 & c\theta_2 & 0\end{bmatrix}\begin{bmatrix}0\\0\\\dot\theta_2\end{bmatrix} + \begin{bmatrix}0\\0\\\dot\theta_3\end{bmatrix} = \begin{bmatrix}\dot\theta_3 s\theta_2\\ \dot\theta_2 + \dot\theta_2 c\theta_2\\ \dot\theta_3\end{bmatrix} \\
{}^{2}V_2 = {}^{2}_{1}R\left({}^{1}V_1 + \begin{bmatrix}0\\0\\\dot\theta_2\end{bmatrix} \times {}^{1}P_2\right) = \begin{bmatrix}c\theta_2 & s\theta_2 & 0\\ 0 & 0 & -1\\ -s\theta_2 & c\theta_2 & 0\end{bmatrix}\left(\begin{bmatrix}0\\0\\\dot\theta_1\end{bmatrix} + \begin{bmatrix}-\dot\theta_2 s\theta_2\\ \dot\theta_2 c\theta_2\\ 0\end{bmatrix}\right) = \begin{bmatrix}0\\-\dot\theta_1\\ \dot\theta_3 + \dot\theta_2\end{bmatrix}
\end{cases}
$$

$$
\begin{cases}
{}^{3}\omega_3 = {}^{3}_{2}R\,{}^{2}\omega_2 + \dot\theta_4\,{}^{3}Z_3 = \begin{bmatrix}c\theta_2 & s\theta_2 & 0\\ -s\theta_2 & c\theta_2 & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}0\\\dot\theta_2\\0\end{bmatrix} + \begin{bmatrix}0\\0\\\dot\theta_4\end{bmatrix} = \begin{bmatrix}\dot\theta_2 s\theta_2\\ \dot\theta_2 c\theta_2\\ \dot\theta_4\end{bmatrix} \\
{}^{3}V_3 = {}^{3}_{2}R\left({}^{2}V_2 + \begin{bmatrix}0\\\dot\theta_2\\0\end{bmatrix} \times {}^{2}P_3\right) = \begin{bmatrix}c\theta_2 & s\theta_2 & 0\\ -s\theta_2 & c\theta_2 & 0\\ 0 & 0 & 1\end{bmatrix}\left(\begin{bmatrix}0\\-\dot\theta_1\\ \dot\theta_2 + \dot\theta_3\end{bmatrix} + \begin{bmatrix}+1\\0\\ c\theta_3 \cdot \dot\theta_2\end{bmatrix}\right)
\end{cases}
$$

$$
\begin{cases}
{}^{4}\omega_4 = {}^{4}_{3}R\,{}^{3}\omega_3 + \dot\theta_5\,{}^{4}Z_4 \\
{}^{4}V_4 = {}^{4}_{3}R\,({}^{3}V_3 + {}^{3}\omega_3 \times {}^{3}P_4)
\end{cases}
,\quad
\begin{cases}
{}^{5}\omega_5 = {}^{5}_{4}R\,{}^{4}\omega_4\ \cancel{+\dot\theta_6} \\
{}^{5}V_5 = {}^{5}_{4}R\,({}^{4}V_4 + {}^{4}\omega_4 \times {}^{4}P_5)
\end{cases}
\quad
\begin{cases}
{}^{EE}\omega_{EE} = {}^{6}_{5}R\,{}^{5}\omega_5 \\
{}^{EE}V_{EE} = {}^{6}_{5}R\,({}^{5}V_5 + {}^{5}\omega_5 \times {}^{5}P_6)
\end{cases}
$$

$$
\Rightarrow {}^{EE}\omega_{EE} = \begin{bmatrix}\Box & \Box & \Box\\ & & \\ & & \end{bmatrix}\begin{bmatrix}\dot q_1\\\dot q_2\\\dot q_3\\\vdots\\\dot q_6\end{bmatrix} = {}^{EE}J_{\omega}\,\dot q \ \checkmark
$$

درست شد

$$
{}^{EE}V_{EE} = \begin{bmatrix}\Box & \Box & \Box\\ & & \end{bmatrix}\dot q = {}^{EE}J_V\,\dot q \ \checkmark
$$

برای روش دوم از روش اسپانگ استفاده می‌کنیم.

برای Revolut $\Rightarrow J_i = \begin{bmatrix} z_i \times (o_n - o_i) \\ z_i \end{bmatrix}$ و Prismatic $\Rightarrow \begin{bmatrix} 0 \\ 0 \\ 0 \\ z_i \end{bmatrix}$

شش روش های حل شده، جاگذاری می‌کنیم

ب) برای سینگولاریتی ← $J$ را چک می‌کنیم ⇐ اگر رنک از دست بدهد سینگولار است

در غیر این صورت غیر سینگولار است

ج) طبق فرمول داریم $\tau = J^T F$ که

$F = \begin{bmatrix} 0 \\ 1 \\ -1 \\ 1 \\ 0 \\ 0 \end{bmatrix} \Rightarrow$ $\tau$ به دست می‌آید