Due Date:
May 29, 2020
(9 Khordad 99)

In the name of god

Advanced Robotics
Homework Assignment #6

1-Consider the model of an ideal pendulum:

$$J\ddot{q} + mgl * \sin(q) = \tau$$

Assume that we apply the PD controller with compensation

$$\tau = k_p\tilde{q} + k_v\dot{\tilde{q}} + J[\ddot{q}_d + \lambda\dot{\tilde{q}}] + mgl * \sin(q)$$

Where $\lambda = k_p / k_v$ and k_p *and* k_v are positive numbers.

a) Obtain the closed-loop equation in terms of the state vector $[\tilde{q} \;\; \dot{\tilde{q}}]^T$ Verify that the origin is its unique equilibrium point.

b) Show that the origin $[\tilde{q} \;\; \dot{\tilde{q}}]^T = 0 \in R^2$ is globally asymptotically stable.

Hint : Use the Lyapunov function candidate:

$$V(\tilde{q}, \lambda\dot{\tilde{q}}) = \frac{1}{2} J(\dot{\tilde{q}} + \lambda\tilde{q})^2 + k_p\tilde{q}^2$$

2)Develop a controller for a one-dof mass-spring-damper system of the form :

$$m\ddot{x} + b\dot{x} + kx = f$$

Where f is control force and m =4kg , b=2 Ns/m , and k=0.1 N/m .

a) What is the damping ration of the uncontrolled system? Is the uncontrolled system overdamped ,underdamped , or critically damped ? If it is underdamped , what is the damped natural frequency ?What is the time constant of convergence to the origin ?
b) Choose a P controller $f = K_p x_e$, where $x_e = x_d - x$ is the position error and $x_d = 0$ What value of Kp yields critical damping?
c) Choose a D controller $f = K_d \dot{x}_e$ where $\dot{x}_d = 0$ .What value of Kd yields critical damping?
d) Choose a PD controller that yield critical damping and a 2% settling time of 0.01 s.
e) For the PD controller above if $x_d = 1$ and $\dot{x}_d = \ddot{x}_d = 0$ what is the steady state error $x_e(t)$ as t goes to infinity ?what is the steady-state control force?
f) Now insert a PID controller for f .Assume $x_d \neq 0$ and $\dot{x}_d = \ddot{x}_d = 0$ Write the error dynamics in terms of $\ddot{x}_e, \dot{x}_e, x_e$ and $\int x_e(t)dt$ on the left-hand side and a constant forcing term on the right-hand side .(Hint : You can write kx as $-k(x_d - x) + kx_d$) .Take the time derivative of this equation and give the conditions on Kp ,Ki and Kd for stability .Show that zero steady-state error is possible with a PID controller.

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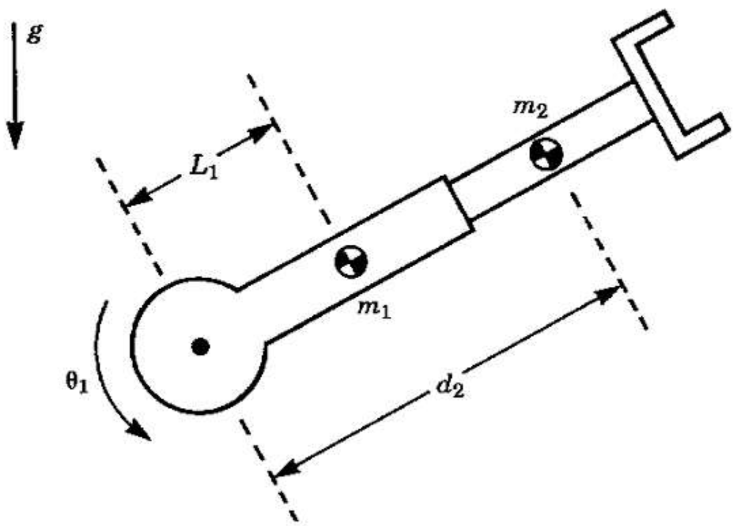
3) consider the 2-link RP shown below:



Its equation of motion shown here:

$$\tau_1 = (m_1 L_1^2 + I_{zz1} + I_{zz2} + m_2 d_2^2)\ddot{\theta}_1 + 2m_2 d_2 \dot{\theta}_1 \dot{d}_2 + (m_1 L_1 + m_2 d_2) g \cos(\theta_1)$$
$$\tau_2 = m_2 \ddot{d}_2 - m_2 d_2 \dot{\theta}_1^2 + m_2 g \sin(\theta_1)$$

The manipulated parameter have the following numerical values: L1=.2m, m1=1kg , m2=0.8kg , $I_{zz1} = .1 kgm^2$ $I_{zz2} = .07 kgm^2$ and the range of d2 is between .5m and 1.0m

a) Using inverse dynamic method and design a controller for this robot .
b) Find the value for the gains of PD compensator part such that the closed loop system for joint 1 is critically damped with natural frequency of 20 rad/sec , and the closed-loop system for joint 2 is critically damped with natural frequency of 25 rad/sec

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4) An identification has been done to design a linear controller for a 3R robot. The following were resulted for the torque , angular velocity and acceleration of the third link

$$\ddot{\theta} = [1.3\ \ 1.7\ \ 1.9\ \ 2.4\ \ 2.8]\ (rad/s^2)$$

$$\dot{\theta} = [.2\ \ .5\ \ .7\ \ .9\ \ 1.1] (rad/s)$$

$$\tau = [5.08\ \ 6.28\ \ 6.92\ \ 8.28\ \ 9.4] (N.m)$$

a) The dynamic model for this link is considered to be

$$\tau = J_e \ddot{\theta} + B_e \dot{\theta} + G_e$$

use the data from the identification test to determine the model parameters

b) Draw a block diagram of the closed-loop system with G_e as a disturbance and design a PD controller such that the system has a steady state error to the disturbance G_e less than .1 and a damping ratio $\zeta = .707$

c) How can we reduce the steady state error to zero?

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۵)ادامه تمرین حل شده در کلاس حل تمرین: اگر انعطاف (نرمی)مفاصل با یک فنر خطی به شکل

$K_1\Delta\theta + K_2(\Delta\theta)^3$ مدل شود و کنترل زیر استفاده شود $u = -k_p\tilde{\theta}_2 - k_v\dot{\theta}_2$

a) ایا سیستم حلقه بسته پایدار است؟

b)ایا سیستم پایداری مجانبی است؟

استدلال کنید . توجه کنید که θ_2 زاویه موتوری می باشد. و $\tilde{\theta}_2 = \theta_2 - \theta_d$ همچنین برای راحتی تمرین را در حالت اسکالر حل نمایید.

Good Luck!