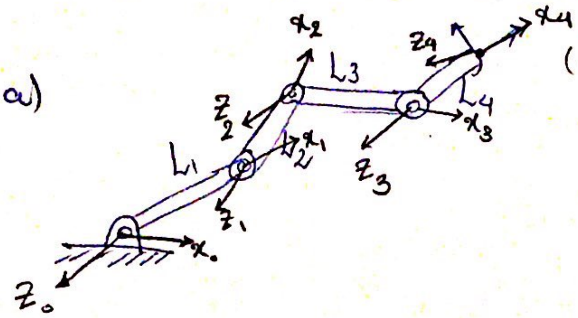
کسری حلفی تمرین سری ۴ (الف) 9523038 رباتیک پیشرفته دکتر شریفی صفحه (1)

(4) a)



classic DH

(لینک)	α_i	a_i	d_i	θ_i
1	0	L_1	0	θ_1
2	0	L_2	0	θ_2
3	0	L_3	0	θ_3
4	0	L_3	0	θ_4

$$^{i-1}T_i = \begin{bmatrix} c\theta_i & -s\theta_i c\alpha_i & s\theta_i s\alpha_i & a_i c\theta_i \\ s\theta_i & c\theta_i c\alpha_i & -c\theta_i s\alpha_i & a_i s\theta_i \\ 0 & s\alpha_i & c\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^0T_1 = \begin{bmatrix} c\theta_1 & -s\theta_1 & 0 & L_1 \\ s\theta_1 & c\theta_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad {}^1T_2 = \begin{bmatrix} c\theta_2 & -s\theta_2 & 0 & L_2 \\ s\theta_2 & c\theta_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad {}^2T_3 = \begin{bmatrix} c\theta_3 & -s\theta_3 & 0 & L_3 \\ s\theta_3 & c\theta_3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^3T_4 = \begin{bmatrix} c\theta_4 & -s\theta_4 & 0 & L_4 \\ s\theta_4 & c\theta_4 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \Rightarrow {}^0T_2 = {}^0T_1\,{}^1T_2 = \begin{bmatrix} c(\theta_1+\theta_2) & -s(\theta_1+\theta_2) & 0 & L_1 + L_2 c\theta_1 \\ s(\theta_1+\theta_2) & c(\theta_1+\theta_2) & 0 & L_2 s\theta_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^0T_3 = {}^0T_2\,{}^2T_3 = \begin{bmatrix} c_{1+2+3} & -s_{1+2+3} & 0 & L_1 + L_2 c_1 + L_3 c_{1+2} \\ s_{1+2+3} & c_{1+2+3} & 0 & L_2 s_1 + L_3 s(\theta_1+\theta_2) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^0T_4 = {}^0T_{EE} = {}^0T_3\,{}^3T_4 = \begin{bmatrix} c(1+2+3+4) & -s(1+2+3+4) & 0 & L_1 + L_2 c_1 + L_3 c_{1+2} + L_4 c(1+2+3) \\ s(1+2+3+4) & c(1+2+3+4) & 0 & L_2 s_1 + L_3 s_{1+2} + L_4 s(1+2+3) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

a) Spong's method

all joints are Revolute $\xrightarrow{\text{so}}$ $J_i = \begin{bmatrix} {}^0z_{i-1} \times ({}^0O_n - {}^0O_{z_{i-1}}) \\ {}^0z_{i-1} \end{bmatrix} \Rightarrow J_1 = \begin{bmatrix} -L_2 s_1 - L_3 s_{1+2} - L_4 \cdots \\ L_1 + L_2 c_1 + L_3 c_{1+2} + \cdots \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$

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$$\Rightarrow J_2 = \begin{bmatrix} {}^0z_1 \times ({}^0O_{EE} - {}^0O_1) \\ z_1 \end{bmatrix} = \begin{bmatrix} -L_2 S_1 - L_3 S_{(1+2)} - L_4 S(1+2+3) \\ L_2 C(\theta_1) + L_3 C(\theta_1+\theta_2) + L_4 C_{1+2+3} \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$J_3 = \begin{bmatrix} {}^0Z_2 \times ({}^0O_{EE} - {}^0O_2) \\ {}^0Z_2 \end{bmatrix} = \begin{bmatrix} -L_3 \sin(\theta_1+\theta_2) - L_4 S(1+2+3) \\ L_3 C_{1+2} + L_4 C_{1+2+3} \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

از روش عمومی

$$J_4 = \begin{bmatrix} -L_4 \sin(\theta_1+\theta_2+\theta_3) \\ L_4 \cos(\theta_1+\theta_2+\theta_3) \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \Rightarrow \boxed{{}^0J_{EE} = \begin{bmatrix} J_1 & J_2 & J_3 & J_4 \end{bmatrix}_{6\times 4}}$$

روش دوم ← روش انتشار سرعت Velocity Propagation

$$\Rightarrow \begin{cases} {}^i\omega_i = {}^iR_{i-1}\, {}^{i-1}\omega_{i-1} + \dot q_i\, {}^iZ_i \\ {}^iV_i = {}^iR_{i-1}\left({}^{i-1}V_{i-1} + {}^{i-1}\omega_{i-1} \times {}^{i-1}P_i\right) \end{cases}$$

$$\Rightarrow \begin{cases} {}^1\omega_1 = {}^1R_0\, \cancel{{}^0\omega_0} + \begin{bmatrix} 0 \\ 0 \\ \dot\theta_1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \dot\theta_1 \end{bmatrix} \\ {}^1V_1 = {}^1R_0(0+0) = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \end{cases}$$

$$\begin{cases} {}^2\omega_2 = {}^2R_1 \begin{bmatrix} 0 \\ 0 \\ \dot\theta_1 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \dot\theta_2 \end{bmatrix} = ({}^1R_2)^T \begin{bmatrix} 0 \\ 0 \\ \dot\theta_1 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \dot\theta_2 \end{bmatrix} = \begin{bmatrix} C\theta_2 & S\theta_2 & 0 \\ -S\theta_2 & C\theta_2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ \dot\theta_1 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \dot\theta_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \dot\theta_1 + \dot\theta_2 \end{bmatrix} \\ {}^2V_2 = {}^2R_1 \left(0 + \begin{bmatrix} 0 & -\dot q_1 & 0 \\ \dot q_1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} L_2 \\ 0 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} C\theta_2 & S\theta_2 & 0 \\ -S\theta_2 & C\theta_2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ L_2 \dot q_1 \\ 0 \end{bmatrix} = \begin{bmatrix} L_2 \theta_1 S(\theta_2) \\ L_2 \theta_1 C(\theta_2) \\ 0 \end{bmatrix} \end{cases}$$

$$\begin{cases} {}^3\omega_3 = {}^3R_2\, {}^2\omega_2 + \begin{bmatrix} 0 \\ 0 \\ \dot\theta_3 \end{bmatrix} = \begin{bmatrix} C\theta_3 & S\theta_3 & 0 \\ -S\theta_3 & C\theta_3 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ \dot\theta_1+\dot\theta_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \dot\theta_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \dot\theta_1+\dot\theta_2+\dot\theta_3 \end{bmatrix} \\ {}^3V_3 = {}^3R_2\left({}^2V_2 + {}^2\omega_2 \times {}^3P_2\right) = \begin{bmatrix} C\theta_3 & S\theta_3 & 0 \\ -S\theta_3 & C\theta_3 & 0 \\ 0 & 0 & 1 \end{bmatrix} \left(\begin{bmatrix} 0 & -(\dot\theta_1+\dot\theta_2) & 0 \\ \dot\theta_1+\dot\theta_2 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} L_3 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} L_2 \theta_1 S(\theta_2) \\ L_2 \theta_1 C(\theta_2) \\ 0 \end{bmatrix} \right) \end{cases}$$

$$\Rightarrow {}^{3}V_3 = \begin{bmatrix} C\theta_3 & S\theta_3 & 0 \\ -S\theta_3 & C\theta_3 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} L_2\dot{\theta}_1 S\theta_2 \\ L_2\dot{\theta}_1 C_2 + l_3(\dot{\theta}_1+\dot{\theta}_2) \\ 0 \end{bmatrix} = \begin{bmatrix} C\theta_3(L_2\dot{\theta}_1 S\theta_2) + S\theta_3(L_1\dot{\theta}_1 C_2 + l_2(\dot{\theta}_1+\dot{\theta}_2)) \\ -S\theta_3(L_2\dot{\theta}_1 S\theta_2) + C\theta_3(L_1\dot{\theta}_1 C_2 + l_2(\dot{\theta}_1+\dot{\theta}_2)) \\ 0 \end{bmatrix}$$

$$\begin{cases} {}^{4}\omega_4 = {}^{4}R_3\, {}^{3}\omega_3 + \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_1+\dot{\theta}_2+\dot{\theta}_3+\dot{\theta}_4 \end{bmatrix} \\ {}^{4}V_4 = {}^{4}R_3\left(\overset{V_3}{\begin{bmatrix} \\ \\ \\ \end{bmatrix}} + \begin{bmatrix} 0 & -(\dot{\theta}_1+\dot{\theta}_2+\dot{\theta}_3) & 0 \\ (\dot{\theta}_1+\dot{\theta}_2+\dot{\theta}_3) & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} l_3 \\ 0 \\ 0 \end{bmatrix} \right) \end{cases}$$

$$= \begin{bmatrix} C\theta_4 & S\theta_4 & 0 \\ -S\theta_4 & C\theta_4 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C\theta_3(L_2\dot{\theta}_1 S\theta_2) + S(\theta_3)\times(L_1\dot{\theta}_1 C\theta_2 + l_2(\dot{\theta}_1+\dot{\theta}_2)) \\ l_3(\dot{\theta}_1+\dot{\theta}_2+\dot{\theta}_3) - S\theta_3(L_2\dot{\theta}_1 S_2) + C_3(l_1\dot{\theta}_1 C_2 + L_2(\dot{\theta}_1+\dot{\theta}_2)) \\ 0 \end{bmatrix}$$

$$\Rightarrow {}^{EE}\omega_{EE} = {}^{4}\omega_4 = \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_1+\dot{\theta}_2+\dot{\theta}_3+\dot{\theta}_4 \end{bmatrix}$$

$${}^{EE}V_{EE} = {}^{4}V_4 + \begin{bmatrix} 0 & (\dot{\theta}_1+\dot{\theta}_2+\dot{\theta}_3+\dot{\theta}_4) & 0 \\ (\dot{\theta}_1+\dot{\theta}_2+\dot{\theta}_3+\dot{\theta}_4) & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} l_4 \\ 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow {}^{EE}J_\omega = \underbrace{\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}}_{{}^{EE}J_\omega} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \\ \dot{q}_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \dot{q}_1+\dot{q}_2+\dot{q}_3+\dot{q}_4 \end{bmatrix}$$

${}^{EE}J_V \longrightarrow$ مثل بالا محاسبه میکنیم

$${}^{EE}J_V = \begin{bmatrix} C\theta_4(C\theta_3)(l_2 S\theta_2) + S\theta_4 l_3 + C\theta_4 S\theta_3(l_1 C\theta_2 + l_2) & C\theta_4 S\theta_3 l_2 + S\theta_4 l_3 + S\theta_4 C_3 l_2 & S\theta_4 l_3 & 0 \\ -S\theta_4(C\theta_3)(l_2 S\theta_2) + C\theta_4 l_3 + S\theta_3 S\theta_3(l_1 C\theta_2 + l_2) + l_4 & l_4 - S\theta_4 S\theta_3 l_2 + C\theta_4 l_3 + C\theta_4 C_3 l_2 & l_4 + C\theta_4 l_3 & l_4 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\Rightarrow {}^{EE}J = \begin{bmatrix} {}^{EE}J_V \\ {}^{EE}J_\omega \end{bmatrix}$$

(b) $\tau = J^T F$

$$F = \begin{bmatrix} f \\ m \end{bmatrix} = \begin{bmatrix} 10 \\ 10 \\ 0 \\ 0 \\ 0 \\ 10 \end{bmatrix} \Rightarrow \tau = J^T \begin{bmatrix} 10 \\ 10 \\ 0 \\ 0 \\ 0 \\ 10 \end{bmatrix}$$

$$\Rightarrow J = \begin{bmatrix} -L_2S_1 - L_3S_{1+2} - L_4S_{1+2+3} & -L_2S_1 - L_3S_{1+2} - L_4S_{1+2+3} & -L_3S_{1+2} - L_4S_{1+2+3} & -L_4S_{1+2+3+4} \\ L_1 + L_2C_1 + L_3C_{1+2} + L_4C_{1+2+3} & L_2C_1 + L_3C_{1+2} + L_4C_{1+2+3} & L_3C_{1+2} + L_4C_{1+2+3} & L_4C_{1+2+3+4} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

$$\Rightarrow \tau = \begin{bmatrix} 10l_1 + 10L_2(C_1 - S_1) + 10L_3(C_{1+2} - S_{1+2}) + 10L_4(C_{1+2+3} - S_{1+2+3}) + 10 \\ 10 + 10L_2(C_1 - S_1) + 10L_3(C_{1+2} - S_{1+2}) + 10L_4(C_{1+2+3} - S_{1+2+3}) \\ 10 + 10L_3(C_{1+2} - S_{1+2}) + 10L_4(C_{1+2+3} - S_{1+2+3}) \\ 10 + 10L_4(C_{1+2+3+4} - S_{1+2+3+4}) \end{bmatrix}$$

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(1) a) $S(\alpha a + \beta b) = \alpha S(a) + \beta S(b)$

ضرب بردار V لازم است

$$S(\alpha a + \beta b) \cdot V \overset{(b)}{=} (\alpha a + \beta b) \times V = (\alpha a) \times V + (\beta b) \times V = \alpha (a \times V) + \beta (b \times V)$$

$$= \alpha R(a) \cdot V + \beta R(b) \cdot V = \underbrace{(\alpha R(a) + \beta R(b))}_{= R(\alpha a + \beta b)} \cdot V$$

(b) $R S(a) R^T = S(Ra)$

$$R S(a) R^T \cdot V \overset{(b)}{=} R \cdot (a \times R^T V) \xrightarrow{R(a \times b) = R(a) \times R(b)} Ra \times RR^T b = Ra \times b = S(Ra) \cdot b$$

$R S(a) R^T$

(b) $S(\vec{a}) \cdot \vec{p} = \vec{a} \times \vec{p}$

$$\rightarrow S\left(\begin{bmatrix} \Omega_x \\ \Omega_y \\ \Omega_z \end{bmatrix}\right) = \begin{bmatrix} 0 & -\Omega_z & \Omega_y \\ \Omega_z & 0 & -\Omega_x \\ -\Omega_y & \Omega_x & 0 \end{bmatrix} \Rightarrow S\left(\begin{bmatrix} \Omega_x \\ \Omega_y \\ \Omega_z \end{bmatrix}\right) \cdot \begin{bmatrix} P_x \\ P_y \\ P_z \end{bmatrix} = \begin{bmatrix} 0 & -\Omega_z & \Omega_y \\ \Omega_z & 0 & -\Omega_x \\ -\Omega_y & \Omega_x & 0 \end{bmatrix} \begin{bmatrix} P_x \\ P_y \\ P_z \end{bmatrix} = \begin{bmatrix} -\Omega_z P_y + P_z \Omega_y \\ P_x \Omega_z - P_z \Omega_x \\ -P_x \Omega_y + P_y \Omega_x \end{bmatrix}$$

$$= (\Omega_x i + \Omega_y j + \Omega_z k) \times (P_x i + P_y j + P_z k) = \begin{bmatrix} \Omega_x \\ \Omega_y \\ \Omega_z \end{bmatrix} \times \begin{bmatrix} P_x \\ P_y \\ P_z \end{bmatrix}$$

ⓒ $\overset{3}{S}(K) = -S(K)$

$$S(K) = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \longrightarrow \overset{3}{S}(K) = \underbrace{\begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}}_{\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix}} \overset{S(K)}{\Rightarrow} \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = -S(K)$$

$$\Downarrow$$

$$\overset{3}{S}(K) = -S(K)$$

ⓓ $Y = SX \xrightarrow[\text{Product}]{\text{iner}} X^T Y = Y^T X \xrightarrow{Y=SX} X^T S X = X^T S^T X$

$$X^T(S+S^T)X = 0 = X^T S X + X^T S^T X = 2X^T S X = 0 \Rightarrow X^T S X = 0$$

II) $R = R_{z,\psi} R_{y,\theta} R_{z,\varphi} \xrightarrow{\text{show}} \frac{d}{dt}R = S(\omega)R \rightsquigarrow \omega = \begin{bmatrix} C\psi S\theta \dot{\varphi} - S\psi \dot{\theta} \\ S\psi S\theta \dot{\varphi} + C\psi \dot{\theta} \\ \dot{\psi} + C\theta \dot{\varphi} \end{bmatrix}$

$$\Rightarrow \frac{dR}{d\theta} = S(K)R \Rightarrow \frac{dR}{dt} = \frac{dR}{d\theta} \cdot \frac{d\theta}{dt} = S(K)R\dot{\theta}$$

$$\dot{R} = \dot{R}_z R_y R_z + R_z \dot{R}_y R_z + R_z R_y \dot{R}_z$$

$$= [S(\dot{\psi}K) R_{z,\psi}] R_{y,\theta} R_z + R_z [S(\dot{\theta}j) \underbrace{R_{y,\theta}}_{R_{z,\psi}^T R_{z,\psi}}] R_z + \underbrace{R_z R_y [S(\dot{\varphi}K) \overbrace{R_{z,\varphi}}^{R_{y,\theta}^T R_{y,\theta}}]}_{R_z[S(R_{y,\theta}\dot{\varphi}K)] R_y R_z} \; R_{z,\varphi}^T R_{z,\varphi}$$

$$= [S(\dot{\psi}K)] R_{z,\psi} R_y R_z + [S(R_{z,\psi}\dot{\theta}j)] R_z R_y R_z + [S(R_{z,\psi} R_{y,\theta} \dot{\varphi}K)] R_z R_y R_z$$

$$= \Big(\underbrace{S(\dot{\psi}K) + S(R_{z,\psi}\dot{\theta}j) + S(R_{z,\psi} R_{y,\theta} \dot{\varphi}K)}_{S(\omega)} \Big] R_z R_y R_z$$

$$\Rightarrow \omega = \dot{\psi}K + R_{z,\psi}\dot{\theta}j + R_{z,\psi} R_{y,\theta} \dot{\varphi}K$$

$$= (C\psi S\theta \dot{\varphi} - S\psi \dot{\theta})i + (S\psi S\theta \dot{\varphi} + C\psi \dot{\theta})j + (\dot{\psi} + C\theta \dot{\varphi})K$$

III) Eeuler angle transformation $\rightarrow R = R_{z,\varphi} R_{y,\theta} R_{x,\psi}$

مشتق می‌گیریم $\longrightarrow$ $\dot{R} = \left[S(\dot{\varphi} k) + S(R_z \dot{\theta} j) + S(R_z R_y \dot{\psi} x) \right] R = S(\omega) R$

$$\Rightarrow \omega = \dot{\varphi} k + R_z \dot{\theta} j + R_z R_y \dot{\psi} x$$

$$= (c_\varphi c_\theta \dot{\psi} - s_\varphi \dot{\theta}) i + (c_\varphi \dot{\theta} + c_\theta s_\varphi \dot{\psi}) j + (\dot{\varphi} - s_\theta \dot{\psi}) k$$

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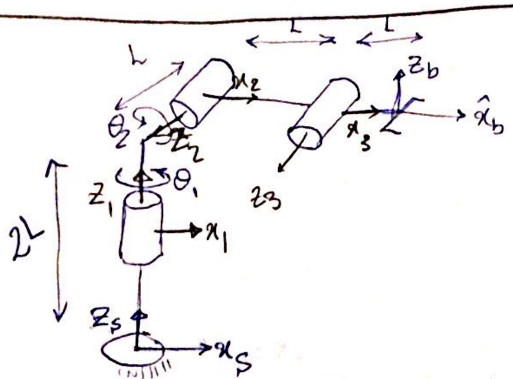
(3)



modified D-H parameters

	α_{i-1}	a_{i-1}	d_i	θ_i
1	0	0	L	θ_1
2	90	0	L	θ_2
3	0	L	0	θ_3
EE	-90	L	0	0

$${}^{i-1}T_i = \begin{bmatrix} c\theta_i & -s\theta_i & 0 & a_{i-1} \\ s\theta_i c\alpha_{i-1} & c\theta_i c\alpha_{i-1} & -s\alpha_{i-1} & -s\alpha_{i-1} d_i \\ s\theta_i s\alpha_{i-1} & c\theta_i s\alpha_{i-1} & c\alpha_{i-1} & c\alpha_{i-1} d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \Rightarrow {}^{0}T_1 = \begin{bmatrix} c\theta_1 & -s\theta_1 & 0 & 0 \\ s\theta_1 & c\theta_1 & 0 & 0 \\ 0 & 0 & 1 & L \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^{1}T_2 = \begin{bmatrix} c\theta_2 & -s\theta_2 & 0 & 0 \\ 0 & 0 & -1 & -L \\ s\theta_2 & c\theta_2 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad {}^{2}T_3 = \begin{bmatrix} c\theta_3 & -s\theta_3 & 0 & L \\ s\theta_3 & c\theta_3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad {}^{3}T_{EE} = \begin{bmatrix} 1 & 0 & 0 & L \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow {}^{0}T_2 = \begin{bmatrix} c_1 c_2 & -c_1 s_2 & s_1 & L s_1 \\ s_1 c_2 & -s_1 s_2 & -c_1 & -L c_1 \\ s_2 & c_2 & 0 & L \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad {}^{0}T_3 = \begin{bmatrix} c_1 c_{2+3} & -c_1 s_{2+3} & s_1 & L(c_1 c_2 + s_1) \\ s_1 c_{2+3} & -s_1 s_{2+3} & -c_1 & L(s_1 c_2 - c_1) \\ s_{2+3} & c_{2+3} & 0 & L(1 + s_2) \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^{0}T_4 = \begin{bmatrix} c_1 c_{2+3} & -s_1 & -c_1 s_{2+3} & L(c_1 c_{2+3} + c_1 c_2 + s_1) \\ s_1 c_{2+3} & c_1 & -s_1 s_{2+3} & L(s_1 c_{2+3} + s_1 c_2 - c_1) \\ s_{2+3} & 0 & c_{2+3} & L(s_{2+3} + 1 + s_2) \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

a) $J_i = \begin{bmatrix} \vec{z}_i \times ({}^0O_{EE} - {}^0O_i) \\ {}^0\vec{z}_i \end{bmatrix}$ ← because all joints are Revolute

$$\Rightarrow J_1 = \begin{bmatrix} z_1 \times \left(O_{EE} - \begin{bmatrix} 0 \\ 0 \\ L \end{bmatrix}\right) \\ z_1 \end{bmatrix} = \begin{bmatrix} -L(s_1 c_{2+3} + s_1 c_2 - c_1) \\ L(c_1 c_{2+3} + c_1 c_2 + s_1) \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$J_2 = \begin{bmatrix} {}^0\vec{z}_2 \times ({}^0O_{EE} - {}^0O_2) \\ {}^0\vec{z}_2 \end{bmatrix} = \begin{bmatrix} -Lc_1(s_2 + s_{2+3}) \\ -Ls_1(s_2 + s_{2+3}) \\ L(s_1^2 c_{2+3} + s_1^2 c_2 + c_1^2 c_{2+3} + c_1^2 c_2) \\ s_1 \\ -c_1 \\ 0 \end{bmatrix} \rightarrow L(c_{2+3} + c_2)$$

$$J_3 = \begin{bmatrix} {}^0z_3 \times ({}^0O_{EE} - {}^0O_3) \\ {}^0z_3 \end{bmatrix} = \begin{bmatrix} -Lc_1 s_{2+3} \\ -Ls_1 s_{2+3} \\ L(s_1^2 c_{2+3} + c_1^2 c_{2+3}) = Lc_{2+3} \\ s_1 \\ -c_1 \\ 0 \end{bmatrix} \Rightarrow {}^0J_{EE} = [J_1 \; J_2 \; J_3]$$

(b) as we Know $\Rightarrow V = J_v \dot{q} \rightsquigarrow \dot{q} = J_v^{-1} V$

$${}^bV_p = \begin{bmatrix} 10 \\ 0 \\ 0 \end{bmatrix} \;\&\; {}^0J_{EE} = \text{as above} \Longrightarrow {}^bJ_{EE} = {}^{EE}J_{EE} = ?$$

$${}^{EE}J_1 = {}^{EE}R_0 \, {}^0J_{EE} = \begin{bmatrix} c_1 c_{2+3} & s_1 c_{2+3} & s_{2+3} \\ -s_1 & c_1 & 0 \\ -c_1 s_{2+3} & -s_1 s_{2+3} & c_{2+3} \end{bmatrix} \begin{bmatrix} -L(s_1 c_{2+3} + s_1 c_2 - c_1) \\ L(c_1 c_{2+3} + c_1 c_2 + s_1) \\ 0 \end{bmatrix} = {}^{EE}J_{1v}$$

$({}^0R_{EE})^T$ که 3x3 است

$({}^{EE}J_v = {}^{EE}R_0 \, {}^0J_v$ (یا متلب) را با ضرب یا دستی حساب می‌کنیم ${}^{EE}J_{3v}$ و ${}^{EE}J_{2v}$ مثل روش بالا

$$\rightarrow J_{New} = {}^{EE}J_v \checkmark \Rightarrow \dot{q} = \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \end{bmatrix} = {}^{EE}J_v^{-1} \begin{bmatrix} 10 \\ 0 \\ 0 \end{bmatrix} \Rightarrow \dot{q}$$ بدست می‌آید ✓

① $P_b = \begin{bmatrix} 10 \\ 0 \\ 0 \end{bmatrix}$ & $\theta_1 = 0$ & $\theta_2 = 45$ & $\theta_3 = -45$

$$\rightarrow {}^{EE}R_0 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I \Rightarrow {}^{EE}J = {}^{EE}R_0\,{}^{0}J = {}^{0}J$$

$$\Rightarrow {}^{EE}J = \begin{bmatrix} L & 0 & 0 \\ \frac{3}{2}L & 0 & 0 \\ 0 & L(1+\frac{\sqrt{2}}{2}) & L \\ 0 & 0 & 0 \\ 0 & -1 & -1 \\ 1 & 0 & 0 \end{bmatrix}$$

که $\tau = J^T F \Rightarrow J^T = \begin{bmatrix} L & \frac{3}{2}L & 0 & 0 & 0 & 1 \\ 0 & 0 & L(1+\frac{\sqrt{2}}{2}) & 0 & -1 & 0 \\ 0 & 0 & L & 0 & -1 & 0 \end{bmatrix}$

$$\Rightarrow \tau = \begin{bmatrix} L & \frac{3}{2}L & 0 \\ 0 & 0 & L(1+\frac{\sqrt{2}}{2}) \\ 0 & 0 & L \end{bmatrix} \begin{bmatrix} 10 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 10L \\ 15L \\ 0 \end{bmatrix} = \tau$$

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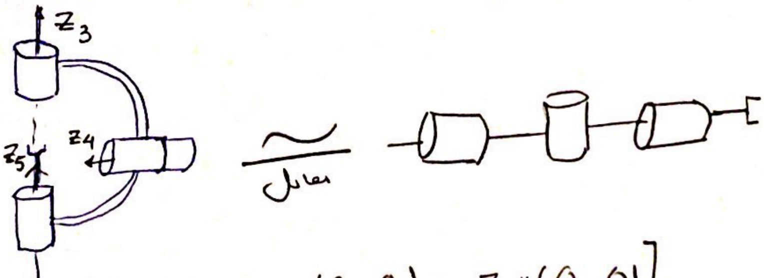
② a)



$$\text{spherical wrist} \rightarrow J = \begin{bmatrix} z_3 \times (O_6 - O_3) & z_4 \times (O_6 - O_4) & z_5 \times (O_6 - O_5) \\ z_3 & z_4 & z_5 \end{bmatrix}$$

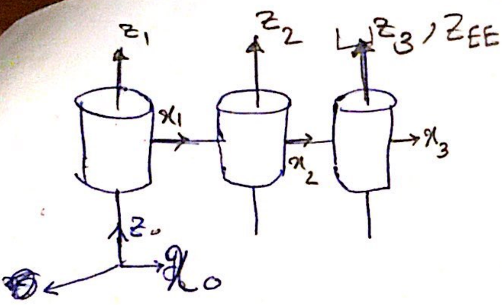
می‌توان دستگاه های مختصات را طوری انتخاب کنیم که $O_3 = O_4 = O_5 = O_6 = 0$

$$\Rightarrow J = \begin{bmatrix} 0 & 0 & 0 \\ z_3 & z_4 & z_5 \end{bmatrix}$$ $\Rightarrow$ با توجه به اینکه z_3 و z_5 هم راستا هستند در نتیجه

مقادیر بردارهای بالا صفر در نتیجه دو سطر وابسته‌ی خطی هستند به هم

در نتیجه سینگولار است.

(b)



	α_{i-1}	a_{i-1}	d_i	θ_i
1	0	0	0	θ_1
2	0	L_1	0	θ_2
3	0	L_2	0	θ_3
EE	0	0	L_1	0

$$\Rightarrow {}^0T_1 = \begin{bmatrix} c\theta_1 & -s\theta_1 & 0 & 0 \\ s\theta_1 & c\theta_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad {}^1T_2 = \begin{bmatrix} c\theta_2 & -s\theta_2 & 0 & L_1 \\ s\theta_2 & c\theta_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad {}^2T_3 = \begin{bmatrix} c\theta_3 & -s\theta_3 & 0 & L_2 \\ s\theta_3 & c\theta_3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow {}^0T_2 = \begin{bmatrix} C_{1+2} & -S_{1+2} & 0 & L_1C_1 \\ S_{1+2} & C_{1+2} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad {}^0T_3 = \begin{bmatrix} C_{1+2+3} & -S_{1+2+3} & 0 & L_2C_{1+2}+L_1C_1 \\ S_{1+2+3} & C_{1+2+3} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad {}^3T_{EE} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^0T_{EE} = \begin{bmatrix} C_{1+2+3} & -S_{1+2+3} & 0 & L_2C_{1+2}+L_1C_1 \\ S_{1+2+3} & C_{1+2+3} & 0 & 0 \\ 0 & 0 & 1 & d \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

ادامه صفحه
بعدی

(b) ادامه

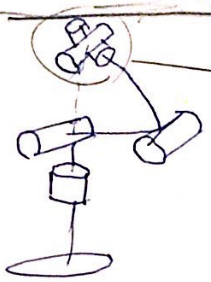
$$J_1 = \begin{bmatrix} z_1 \times (O_{EE} - z_1) \\ z_1 \end{bmatrix} = \begin{bmatrix} 0 \\ L_2 C_{1+2} + L_1 C_1 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$J_2 = \begin{bmatrix} z_2 \times (O_{EE} - O_2) \\ z_2 \end{bmatrix} = \begin{bmatrix} 0 \\ L_2 C_{1+2} \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} , \quad J_3 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\longrightarrow J = \begin{bmatrix} 0 & 0 & 0 \\ L_2 C_{1+2} + L_1 C_1 & L_2 C_{1+2} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

ستون ها وابسته → $Rank < 3$ → سینگولار

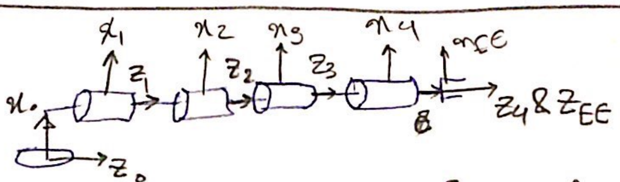
(c)



→ spherial wrist → در بالا اثبات شد که اگر z های sw که هم محور شوند singular می‌شود

همچنین اگر مثل بالا جدول تشکیل دهیم مشاهده می‌شود دو ستون (ممکن است حتی بیشتر) به هم وابسته خطی هستند

(d)



	α_{i-1}	a_{i-1}	d_i	θ_i
1	0	0	L_1	θ_1
2	0	0	L_2	θ_2
3	0	0	L_3	θ_3
4	0	0	L_4	θ_4
EE	0	0	L_5	0

$$ {}^{0}T_1 = \begin{bmatrix} c\theta_1 & -s\theta_1 & 0 & 0 \\ s\theta_1 & c\theta_1 & 0 & 0 \\ 0 & 0 & 1 & L_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \& \quad {}^{1}T_2 = \begin{bmatrix} c\theta_2 & -s\theta_2 & 0 & 0 \\ s\theta_2 & c\theta_2 & 0 & 0 \\ 0 & 0 & 1 & L_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^{2}T_3 = \begin{bmatrix} c\theta_3 & -s3 & 0 & 0 \\ s\theta_3 & c3 & 0 & 0 \\ 0 & 0 & 1 & L_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad {}^{3}T_4 = \begin{bmatrix} c4 & -s4 & 0 & 0 \\ s4 & c4 & 0 & 0 \\ 0 & 0 & 1 & L_4 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^{4}T_{EE} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & L_5 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$\Rightarrow {}^{0}T_2 = \begin{bmatrix} C_{1+2} & -S_{1+2} & 0 & 0 \\ S_{1+2} & C_{1+2} & 0 & 0 \\ 0 & 0 & 1 & L_1+L_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad {}^{0}T_3 = \begin{bmatrix} C_{1+2+3} & -S_{1+2+3} & 0 & 0 \\ S_{1+2+3} & C_{1+2+3} & 0 & 0 \\ 0 & 0 & 1 & L_1+L_2+L_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

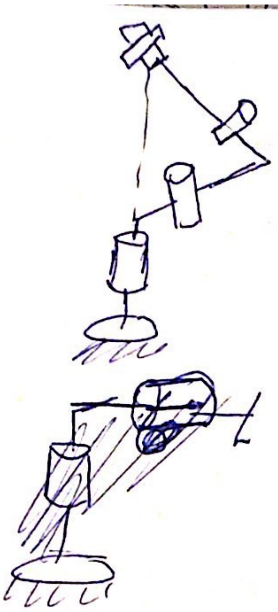
$${}^{0}T_4 = \begin{bmatrix} C_{1+2+3+4} & -S_{1+2+3+4} & 0 & 0 \\ S_{1+2+3+4} & C_{1+2+3+4} & 0 & 0 \\ 0 & 0 & 1 & L_1+L_2+L_3+L_4 \\ 0 & 0 & 0 & 1 \end{bmatrix} \; \& \; {}^{0}T_{EE} = \begin{bmatrix} C_{1+2+3+4} & -S_{1+2+3+4} & 0 & 0 \\ S_{1+2+3+4} & C_{1+2+3+4} & 0 & 0 \\ 0 & 0 & 1 & L_1+L_2+L_3+L_4+L_5 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow {}^{0}J_1 = \begin{bmatrix} Z_1 \times (O_{EE} - O_1) \\ Z_1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \qquad {}^{0}J_2 = \begin{bmatrix} {}^{0}Z_2 \times ({}^{0}O_{EE} - {}^{0}O_2) \\ {}^{0}Z_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

وابسته ی خطی اند

سینگولار $\leftarrow \det(J) = 0$

e



6 عدد مفصل Revolute

که سه تای آخری spherical wrist است

دقیقاً ماشین پوما ج