All of Statistic
1 - Probability

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Summary

Ω - sample space: The set of possible outcomes of an experiment
A - Event: Subsets of Ω

Example. If we toss a coin twice, then $\Omega = \{HH, HT, TH, TT\}$. The event that the first toss is heads is $A = \{HH, HT\}$.

A^c **- complement of A** - Not A. $A^c = \{\omega \in \Omega : \omega \notin A\}$.
$A \cup B$**- Union**: $\{\omega \in \Omega : \omega \in A$ or $\omega \in B$ or $\omega \in Both\}$.
$A \cap B$ **- Intersection** :$\{\omega \in \Omega : \omega \in A$ and $\omega \in B\}$.
$\emptyset$ **- Empty event**.
Disjoint events : We say that $A1, A2, ...$ are disjoint or are mutually exclusive if $A_i \cap A_j = \emptyset$ whenever $i \neq j$.
Partitions of Ω : a sequence of disjoint sets $A_1, A_2..$ such that $\cup_{i=1}^{\infty} A_i = \Omega$.

Probability - We will assign a real number $P(A)$ to every event A, called the probability of A.

3 Axiom of probability distribution:

1: $P(A) \leq 0$ for every A.
2: $P(\Omega) = 1$.
3: if A_1, A_2...are disjoint then $P(\cup_{i=1}^{\infty} A_i) = \sum_{i=1}^{\infty} P(A_i)$

Lemma: $P(A \cup B)$= $P(A)$ + $P(B)$ - $P(AB)$ for any events A and B.

Example:

Two coin tosses. Let H_1 be the event that heads occurs on toss 1 and let H_2 be the event that heads occurs on toss 2. If all outcomes are equally likely, then $P(H_1 \cup H_2) = P(H_1) + P(H_2) - P(H_1 H_2) = \frac{1}{2} + \frac{1}{2} - \frac{1}{4} = \frac{3}{4}$

Independent events: Two events A and B are Independent if $\{(AB) =$

$P(A)P(B)\}$ Independence can arise in two distinct ways. Sometimes, we explicitly assume that two events are independent. For example, in tossing a coin twice, In other instances, we derive independence by verifying that $P(AB) = P(A)P(B)$ holds. For example, in tossing a fair die, let $A = \{2, 4, 6\}$ and let $B = \{1, 2, 3, 4\}$. Then, $A \cap B = 2, 4$, $P(AB) = \frac{2}{6} = P(A)P(B) = \frac{1}{2} * \frac{2}{3}$

Disjoint events with positive probability are not independent.

Conditional Probability: Assuming that $P(B) > 0$, we define the conditional probability of A given that B has occurred as follows:

$$P(A|B) = \frac{P(AB)}{P(B)}$$

For any fixed B such that $P(B) > 0, P(: |B)$ is a probability (i.e., it satisfies the three axioms of probability)

Lemma: If A and B are independent events, then:
$P(A|B) = P(A)$
$P(AB) = P(A|B)P(B) = P(B|A)P(A)$

The Law of Total Probability:
Let $A_1, ..., A_k$ be a partition of Ω. Then, for any event B,

$$P(B) = \sum_{i=1}^{\infty} P(B|A_i)P(A_i)$$

Bayes' Theorem
Let $A_1, ..., A_k$ be a partition of Ω such that $P(A_i) > 0$ for each i. If $P(B) > 0$ then, for each $i = 1, ..., k$,

$$P(A_i|B) = \frac{P(B|A_i)P(A_i)}{P(B)} = \frac{P(B|A_i)P(A_i)}{\sum_j P(B|A_j)P(A_j)}$$