Why is $\frac{\partial J(W)}{\partial W_{i,j}^{(out)}} = (A^{(h)})^T \delta^{(out)}$?

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1 Introduction

In this LaTeX document, I want to show my step-by-step process to derive the partial derivative of all the weights in $W^{(out)}$.

I'll start simple with a 2-2-2 MLP, which is a Multi-Layer Perceptron with 2 inputs, 2 hidden units, and 2 outputs, and then generalize what we learn to a general $m - d - t$ MLP.

In both cases, I will use the following loss/error function:

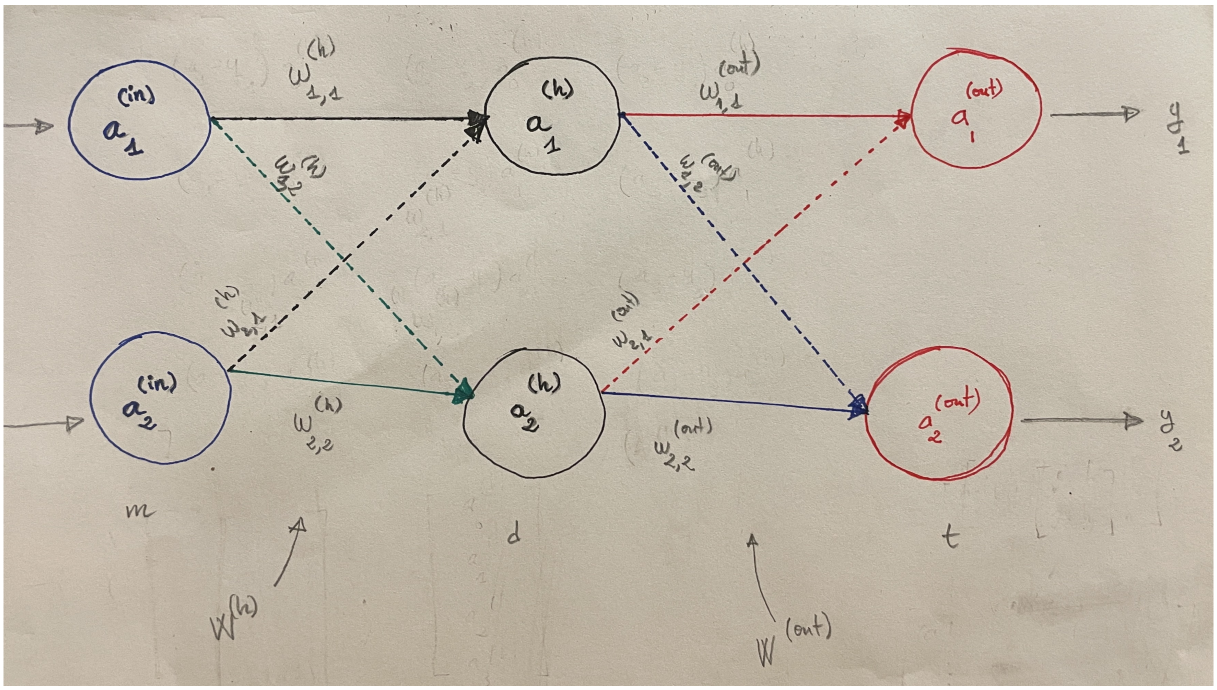
$$J(w) = -\sum_{i=1}^{n} \sum_{j=1}^{t} y_j^{[i]} ln(a_j^{[i]}) + (1 - y_j^{[i]}) ln(1 - a_j^{[i]})$$

Where the superscript $[i]$ is an index for training examples, and j is the number of output units.

Ready? Let's go!

2 With a 2-2-2 MLP

For this example, we will use the following Neural network:



We'll ignore biase units in the input and hidden layers, and consider only ONE training example for simplicity purposes.

Since one training example is considered and the network has two output units, then $n = 1$ and $t = 2$. So, the loss function becomes like this:

$$J(w) = -\sum_{j=1}^{t} y_j^{[1]} ln(a_j^{[1]}) + (1 - y_j^{[1]}) ln(1 - a_j^{[1]})$$

The journey of a SINGLE training example from the input layer to the output layer goes like this:

$$
\begin{gathered}
[a_1^{(in)}, a_2^{(in)}] \\
\downarrow \\
W^{(h)} \\
\downarrow \\
[z_1^{(h)}, z_2^{(h)}] \\
\downarrow \\
\phi(\bullet) \\
\downarrow \\
[a_1^{(h)}, a_2^{(h)}] \\
\downarrow \\
W^{(out)} \\
\downarrow \\
[z_1^{(out)}, z_2^{(out)}] \\
\downarrow \\
\phi(\bullet) \\
\downarrow \\
[a_1^{(out)}, a_2^{(out)}]
\end{gathered}
$$

3 With a general $m - d - t$ MLP