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Tuesday, January 14, 2020 9:07 AM

O notation (upper bound)

$$f(n) \in O(g(n)) \text{ if } \exists c, n_0 \text{ s.t. } f(n) \le c \cdot g(n) \ \forall n > n_0 \text{ and } c > 0$$

Ω notation (lower bound)

$$f(n) \in \Omega(g(n)) \text{ if } \exists c, n_0 \text{ s.t. } c \cdot g(n) \le f(n) \ \forall n, n_0 \text{ and } c > 0$$

Θ notation (tight bound)

$$f(n) \in \Theta(g(n)) \text{ if } \exists c_1, c_2, n_0 \text{ s.t. } c_1 \cdot g(n) \le f(n) \le c_2 \cdot g(n) \ \forall n > n_0 \text{ and } c_1 > 0, c_2 > 0$$

ex) $1000n^2 + 10n - 50$ is $O(n^3)$ ← loose upper bound

$1000n^2 + 10n - 50$ is $\Omega(n)$ ← "at least n", loose lower bound

$1000n^2 + 10n - 50$ is $\Theta(n^2)$ ← only possibility for Θ

ex) $3n^2 - 50n + 10 \in \Theta(n^2)$

$$c_1 \cdot n^2 \le 3n^2 - 50n + 10 \le c_2 \cdot n^2 \quad \forall n > n_0$$

$$c_1 \le 3 - \frac{50}{n} + \frac{10}{n^2} \le c_2 \quad \forall n > n_0$$

~~Let n_0 be 4~~ → yields negative # in the middle, can't do that

Let n_0 be 50 → $2 \le 3 - \frac{50}{n} + \frac{10}{n^2}$ so $c_1 = 2$

$3 - \frac{50}{n} + \frac{10}{n^2} \le 4$ so $c_2 = 4$

ex) $9n^3 - 10n^2 + 15n - 5 = \Theta(n^3)$

$$c_1 \cdot n^3 \le 9n^3 - 10n^2 + 15n - 5 \le c_2 \cdot n^3 \quad \forall n > n_0$$

$$c_1 \le 9 - \frac{10}{n} + \frac{15}{n^2} - \frac{5}{n^3} \le c_2 \quad \forall n > n_0$$

* Need to verify this is correct → Let n_0 be 1 → $8 \le 9 - \frac{10}{n} + \frac{15}{n^2} - \frac{5}{n^3}$ so $c_1 = 8$

$9 - \frac{10}{n} + \frac{15}{n^2} - \frac{5}{n^3} \le 10$ so $c_2 = 10$?

Let n_0 be 5 → $7 \le 9 - \frac{10}{n} + \frac{15}{n^2} - \frac{5}{n^3}$ so $c_1 = 7$

$9 - \frac{10}{n} + \frac{15}{n^2} - \frac{5}{n^3} \le 10$ so $c_2 = 10$

$an^2 + bn + c = \Theta(n^2)$ where $a > 0$

$c_1 = a/4$, $c_2 = \frac{7a}{4}$, $n_0 = 2 \cdot \max\left(\frac{|b|}{a}, \sqrt{|c|/a}\right)$

$$c_1 \cdot n^2 \le an^2 + bn + c \le c_2 \cdot n^2$$

$$\frac{a}{4} \cdot n^2 \le an^2 + bn + c \le \frac{7a}{4} \cdot n^2 \quad \forall n > 2 \cdot \max\left(\frac{|b|}{a}, \sqrt{|c|/a}\right)$$

$$\frac{a}{4} \le a + \frac{b}{n} + \frac{c}{n^2} \le \frac{7a}{4}$$

$\frac{b}{n} \le \frac{b}{2 \cdot \frac{|b|}{a}} \rightarrow \frac{b}{n} \le \frac{b \cdot \frac{|b|}{a}}{2} \rightarrow \frac{b}{n} \le \frac{a}{2}$ ← 2nd term in center of eq. equated to $n = \frac{|b|}{a}$

$\frac{c}{n^2} \le \frac{c}{\left(2 \cdot \sqrt{|c|/a}\right)^2} \rightarrow \frac{c}{n^2} \le \frac{c}{4 \cdot \frac{|c|}{a}} \rightarrow \frac{c}{n^2} \le \frac{c \cdot \frac{|c|}{a}}{4} \rightarrow \frac{c}{n^2} \le \frac{a}{4}$ ← 3rd term in center of eq. equated to $n = \sqrt{|c|/a}$

o notation ("little-o")

$$f(n) \in o(g(n)) \text{ if } \forall c_1 > 0, \exists n_0 \text{ s.t. } f(n) \le c_1 \cdot g(n) \ \forall n > n_0$$

ω notation ("little-omega")

ω notation ("little-omega")

$f(n) \in \omega(g(n))$ if $\forall c_1 > 0, \exists n_0$ s.t. $c_1 \cdot g(n) \le f(n) \; \forall n > n_0$

Recurrence Relation

$T(n) = 2 \cdot T\left(\frac{n}{2}\right) + \Theta(n)$ $\leftarrow$ Mergesort recurrence relation

$T(n) = \Theta(n \lg n)$