CS201 Data Structures and Algorithms

Summations

Printable Version

Prove $\sum$ k = 1 n k = n (n + 1) 2 .

By induction on *n*.

Basis: Let n = 1.

$\sum$ k = 1 n k = n (n + 1) 2

$\sum$ k = 1 1 k = 1 (1 + 1) 2

1 = 1 × 2 2 = 2 2

1 = 1

∎

Inductive Hypothesis: Assume true for 1 < k < n.

Induction:

$\sum$ k = 1 n k = $\sum$ k = 1 n - 1 k + n

$\sum$ k = 1 n k = (n - 1) ((n - 1) + 1) 2 + n (By The Inductive Hypothesis)

$\sum$ k = 1 n k = (n - 1) n 2 + n

$\sum$ k = 1 n k = n 2 - n 2 + 2 n 2

$\sum$ k = 1 n k = n 2 - n + 2 n 2

$\sum$ k = 1 n k = n 2 + n 2

$\sum$ k = 1 n k = n (n + 1) 2

∎

Prove $\sum$ k = 0 n 2 k = 2 n + 1 - 1.

By induction on *n*.

Basis: Let $n = 0$.

$$\sum_{k=0}^{n} 2^k = 2^{n+1} - 1$$

$$\sum_{k=0}^{0} 2^k = 2^{0+1} - 1$$

$$2^0 = 2^1 - 1 = 2 - 1$$

$$1 = 1$$

∎

Inductive Hypothesis: Assume true for $1 < d < n$.

Induction:

$$\sum_{k=0}^{n} 2^k = \sum_{k=0}^{n-1} 2^k + 2^n$$

$$\sum_{k=0}^{n} k = 2^{(n-1)+1} - 1 + 2^n \text{ (By The Inductive Hypothesis)}$$

$$\sum_{k=0}^{n} k = 2^n - 1 + 2^n$$

$$\sum_{k=0}^{n} k = 2 \times 2^n - 1$$

$$\sum_{k=0}^{n} k = 2^{n+1} - 1$$

∎

Problems:

- 2.2-3
- 2.2-7
- 3.1-1
- 3.1-5
- 3.1-7