CS201 Data Structures and Algorithms

Order Notation

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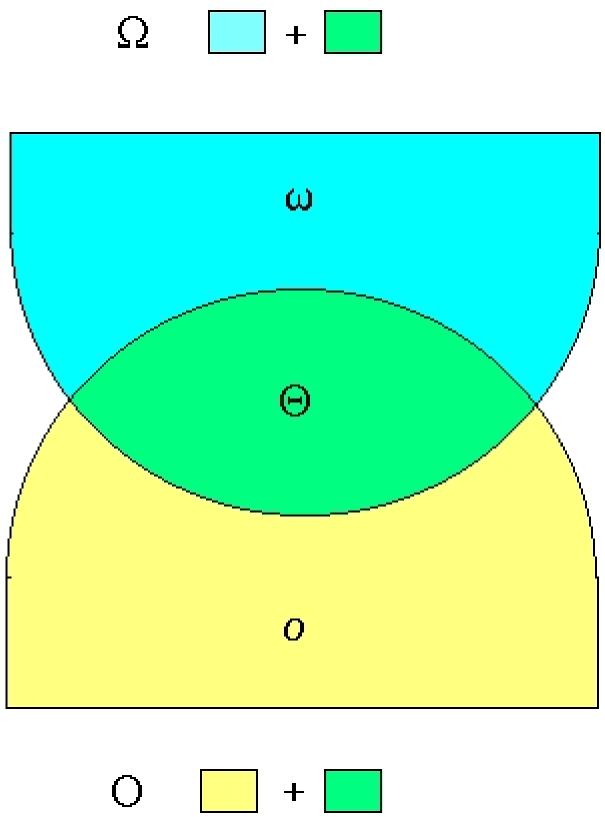
Rough ideas

Consider this Venn diagram:



We can assign the following meanings:

ω	always worse than	lower bound
o	always better than	upper bound
Ω	never better than	lower bound
O	never worse than	upper bound
Θ	never better/worse than	upper and lower (tight) bound

under the following conditions:

- the problem size (usually denoted *n*) is sufficiently large (NSL)
- we are comparing worst case behavior (WCB)

For the Θ, Ω, and *O* regions, we apply one more condition:

- we ignore constant factors and other lower order terms (ICF)

If we are comparing the running times of two algorithms, *f* and *g*, we place one of them, say *g*, in the Θ region. If algorithm *f* is in the ω region, then:

$$f = \omega (g)$$

The English interpretation is that *f* is always slower than *g* (NSL,WCB). If *f* is in the Θ or *o* regions, then:

$$f = O (g)$$

The English interpretation of this statement is that *f* is never slower than *g* (NSL,WCB,ICF).

When using order notation, one generally assumes that NSL, WCB, and ICF always apply, unless otherwise stated.

Formal Statements

A formal definition of *O* is:

$$\text{if } \lim n \rightarrow \infty \; f \; g = 0$$

The definitions of the other symbols are similar.