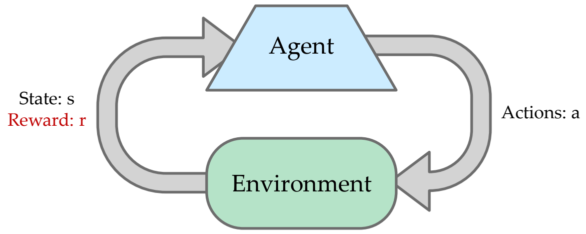
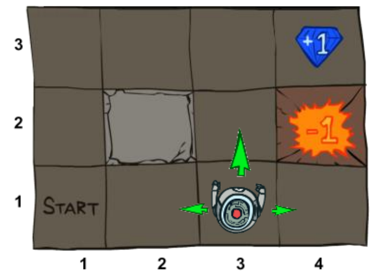
Reinforcement Learning



- Basic idea:
  - Receive feedback in the form of rewards
  - Agent's utility is defined by the reward function
  - Must (learn to) act to maximize expected rewards
  - All learning is based on observed samples of outcomes!

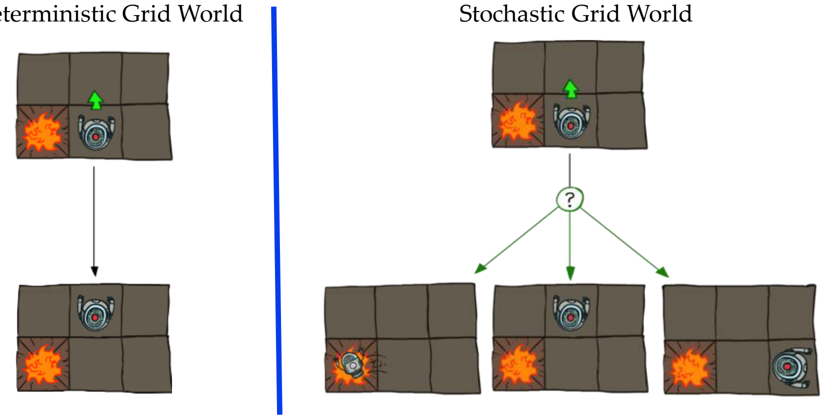
Example: Grid World

- A maze-like problem
  - The agent lives in a grid
  - Walls block the agent's path
- Noisy movement: actions do not always go as planned
  - 80% of the time, the action North takes the agent North (if there is no wall there)
  - 10% of the time, North takes the agent West; 10% East
  - If there is a wall in the direction the agent would have been taken, the agent stays put
- The agent receives rewards
  - Small "living" reward each step (can be negative)
  - Big rewards come at the end (good or bad)
- Goal: maximize sum of rewards



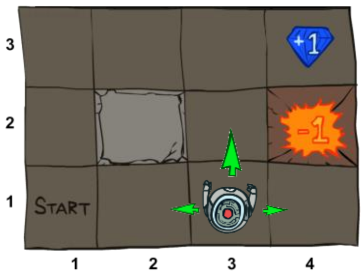
Grid World Actions

Deterministic Grid World | Stochastic Grid World



Markov Decision Processes

- An MDP is defined by:
  - A set of states s ∈ S
  - A set of actions a ∈ A
  - A transition function T(s, a, s')
    - » Probability that a from s leads to s', i.e., Pr(s'| s, a)
    - » Also called the model or the dynamics
  - A reward function R(s, a, s')
    - » Sometimes just R(s) or R(s')
  - A start state
  - Possibly one or more terminal states
  - Possibly a discount factor γ (gamma)
- MDP's are non-deterministic search problems
  - One way to solve is with expectimax search



[Demo – gridworld manual intro (L8D1)]

What is Markov about MDPs?

- "Markov" generally means that given the present state, the future and the past are independent
- For Markov decision processes, "Markov" means action outcomes depend only on the current state

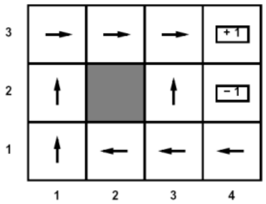
$$P(S_{t+1} = s' | S_t = s_t, A_t = a_t, S_{t-1} = s_{t-1}, A_{t-1}, \ldots S_0 = s_0)$$
$$=$$
$$P(S_{t+1} = s' | S_t = s_t, A_t = a_t)$$

Andrey Markov
(1856-1922)

- This is just like search, where the successor function could only depend on the current state (not the history)

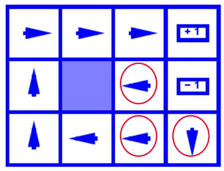
Policies

- In deterministic single-agent search problems, we wanted an optimal plan, or sequence of actions, from start to a goal
- For MDPs, we want an optimal
  policy π*: S → A
  - A policy π gives an action for each state
  - An optimal policy is one that maximizes expected utility if followed
  - An explicit policy defines a reflex agent

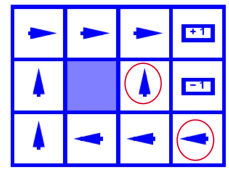


Optimal policy when R(s, a, s') = -0.03 for all non-terminals s

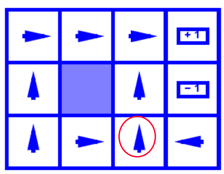
Optimal Policies for different R's



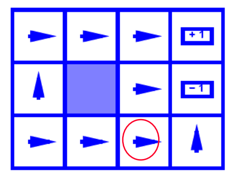
R(s) = -0.01



R(s) = -0.03



R(s) = -0.4

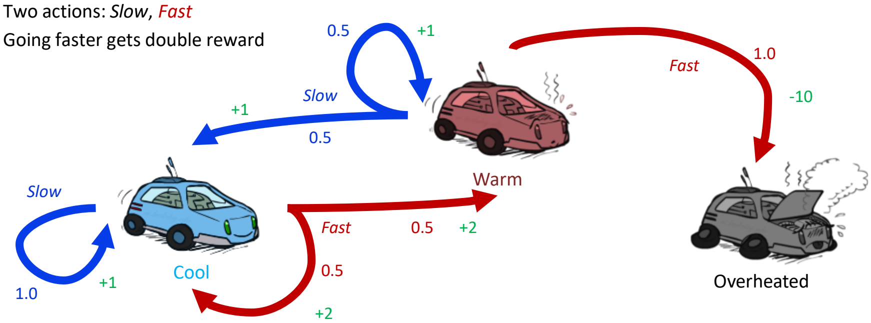


R(s) = -2.0

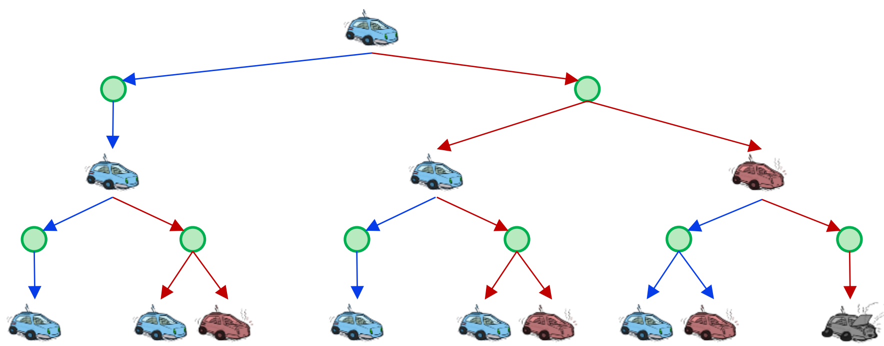
What if R(s) is > 0

Example: Racing

- A robot car wants to travel far, quickly
- Three states: Cool, Warm, Overheated
- Two actions: *Slow*, *Fast*
- Going faster gets double reward

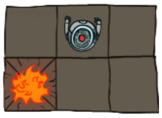


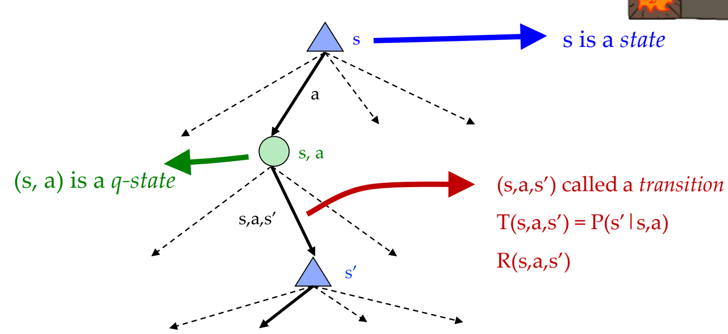
Racing Search Tree



MDP Search Trees

- Each MDP state projects an expectimax-like search tree

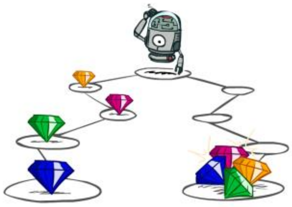




Utilities of Sequences

What preferences should an agent have over reward sequences?

- More or less? [1, 2, 2] or [2, 3, 4]
- Now or later? [0, 0, 1] or [1, 0, 0]

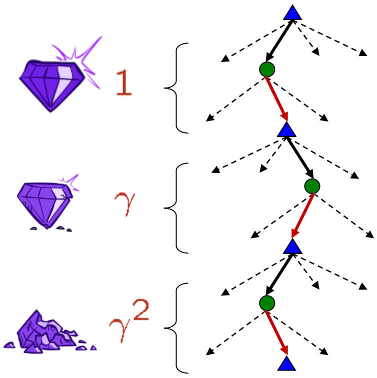


Discounting

- It's reasonable to maximize the sum of rewards
- It's also reasonable to prefer rewards now to rewards later
- One solution: values of rewards decay exponentially

Discounting

- How to discount?
  - Each time we descend a level, we multiply in the discount once
- Why discount?
  - Reward now is better than later
  - Can also think of it as a 1-gamma chance of ending the process at every step
  - Also helps our algorithms converge
- Example: discount of 0.5
  - U([1,2,3]) = 1*1 + 0.5*2 + 0.25*3
  - U([1,2,3]) < U([3,2,1])



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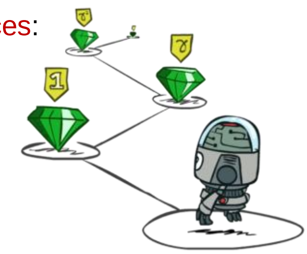
Stationary Preferences - *rational behavior* *Not always?*

- Theorem: if we assume stationary preferences:

$$[a_1, a_2, \ldots] \succ [b_1, b_2, \ldots]$$
$$\Updownarrow$$
$$[r, a_1, a_2, \ldots] \succ [r, b_1, b_2, \ldots]$$



- Then: there are only two ways to define utilities
  - Additive utility: $U([r_0, r_1, r_2, \ldots]) = r_0 + r_1 + r_2 + \cdots$
  - Discounted utilit$U([r_0, r_1, r_2, \ldots]) = r_0 + \gamma r_1 + \gamma^2 r_2 \cdots$

Quiz: Discounting

- Given:

10				1
a	b	c	d	e

  - Actions: East, West, and Exit (only available in exit states a, e)
  - Transitions: deterministic

- Quiz 1: For γ = 1, what is the optimal policy?
- Quiz 2: For γ = 0.1, what is the optimal policy?
- Quiz 3: For which γ are West and East equally good when in state d?

10	←	←	←	1

10	←	←	→	1

$1\gamma=10\ \gamma^3$

Infinite Utilities?!

- Problem: What if the game lasts forever? Do we get infinite rewards?

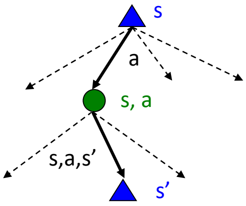
- Solutions:
  - Finite horizon: (similar to depth-limited search)
    - » Terminate episodes after a fixed T steps (e.g. life)
    - » Gives nonstationary policies (π depends on time left)
  - Discounting: use $0 < \gamma < 1$

$$U([r_0, \ldots r_\infty]) = \sum_{t=0}^{\infty} \gamma^t r_t \leq R_{\text{max}}/(1-\gamma)$$

    - » Smaller γ means smaller "horizon" – shorter term focus
  - Absorbing state: guarantee that for every policy, a terminal state will eventually be reached (like "overheated" for racing)

Recap: Defining MDPs

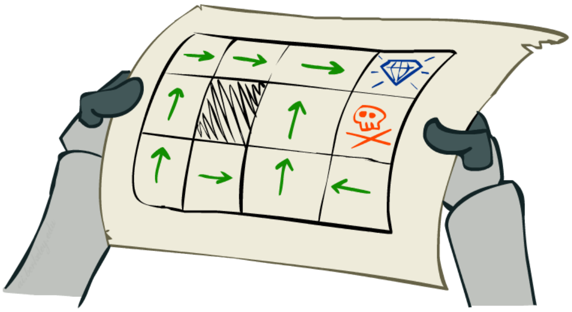
- Markov decision processes:
  - Set of states S
  - Start state s_0
  - Set of actions A
  - Transitions P(s'|s,a) (or T(s,a,s'))
  - Rewards R(s,a,s') (and discount γ)



- MDP quantities so far:
  - Policy = Choice of action for each state
  - Utility = sum of (discounted) rewards

Break? How to solve

Solving MDPs



Recall: Racing MDP

- A robot car wants to travel far, quickly
- Three states: Cool, Warm, Overheated
- Two actions: *Slow*, *Fast*
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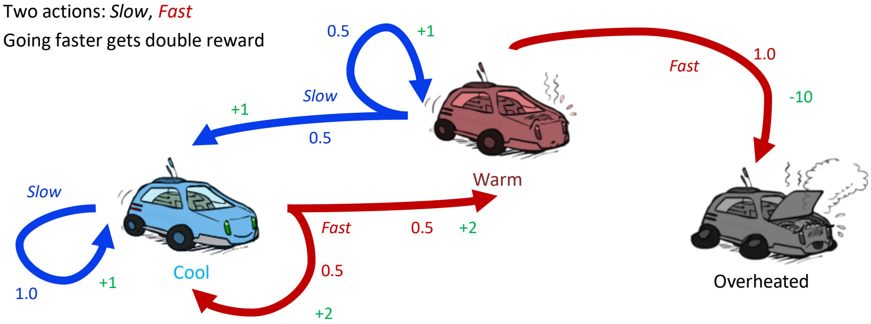
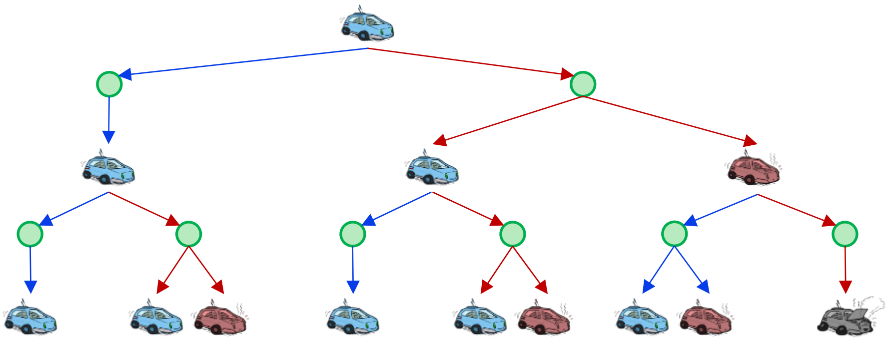
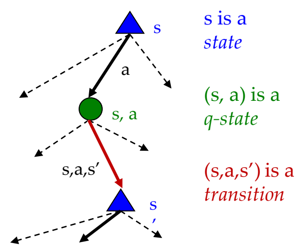


Racing Search Tree

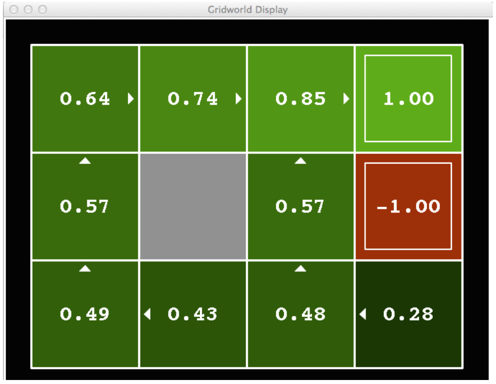


Optimal Quantities

- The value (utility) of a state s:
  V*(s) = expected utility starting in s and acting optimally
- The value (utility) of a q-state (s,a):
  Q*(s,a) = expected utility starting out having taken action a from state s and (thereafter) acting optimally
- The optimal policy:
  π*(s) = optimal **action** from state s

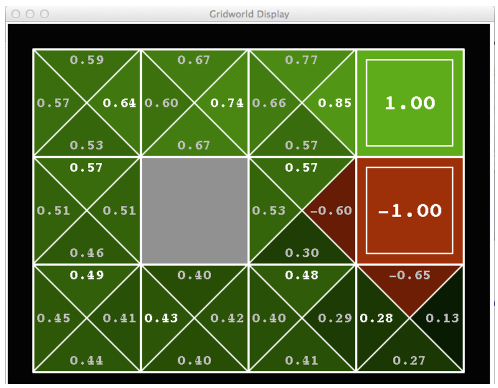


Gridworld V* Values



Noise = 0.2
Discount = 0.9
Living reward = 0

Gridworld Q* Values



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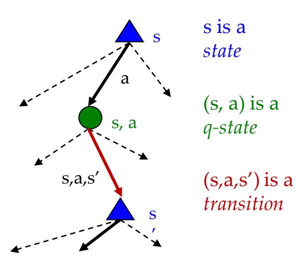
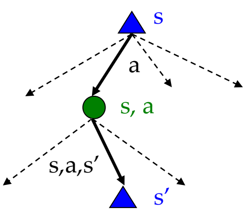


Values of States: Bellman Equations (Dynamic Programming)

Recursive definition of optimal value:

$$V^*(s) = \max_a \sum_{s'} T(s,a,s')[R(s,a,s') + \gamma V^*(s')]$$

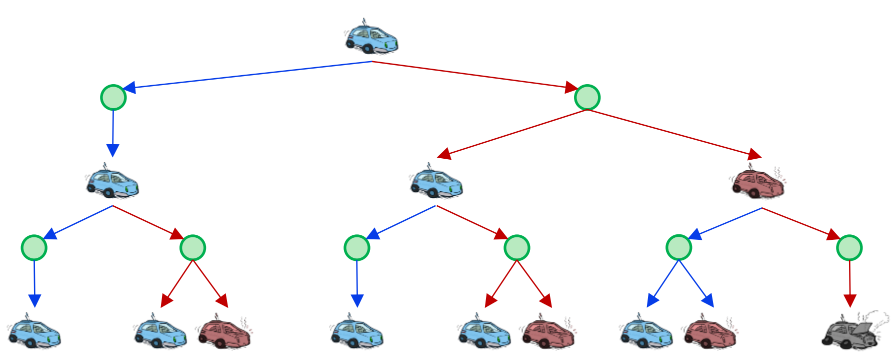


Bellman Equations: 1 step ahead equations define optimality

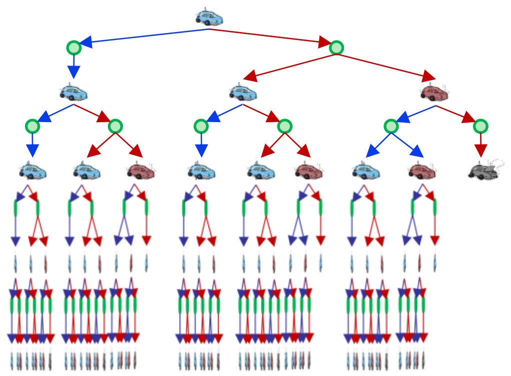
$$V^*(s) = \max_a Q^*(s,a)$$

$$Q^*(s,a) = \sum_{s'} T(s,a,s')[R(s,a,s') + \gamma V^*(s')]$$

Racing Search Tree

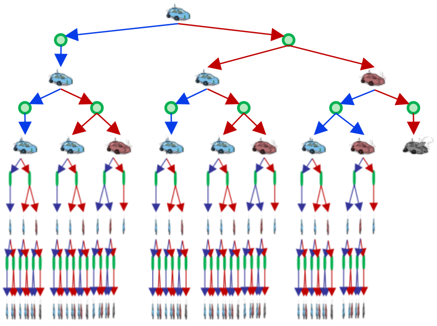


Racing Search Tree



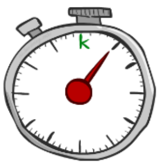
Racing Search Tree

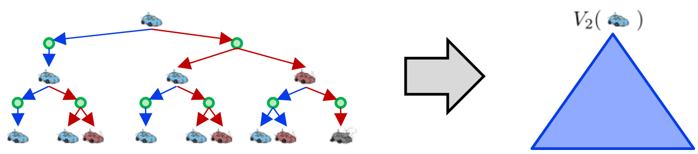
- We're doing way too much work with expectimax!
- Problem: States are repeated
  - Idea: Only compute needed quantities once
- Problem: Tree goes on forever
  - Idea: Do a depth-limited computation, but with increasing depths until change is small
  - Note: deep parts of the tree eventually don't matter if $\gamma < 1$



Time-Limited Values

- Key idea: time-limited values
- Define $V_k(s)$ to be the optimal value of s if the game ends in k more time steps
  - Equivalently, it's what a depth-k expectimax would give from s





[Demo – time-limited values (L8D4)]

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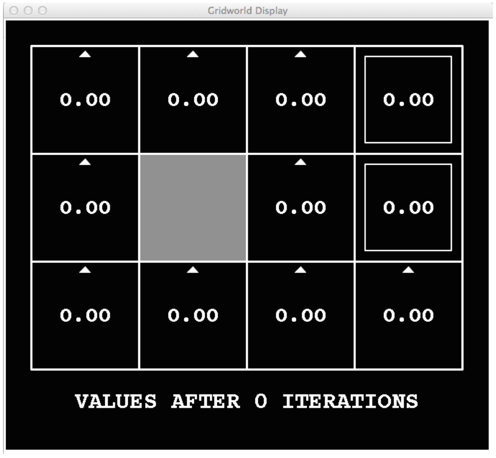
k=0



Noise = 0.2
Discount = 0.9
Living reward = 0

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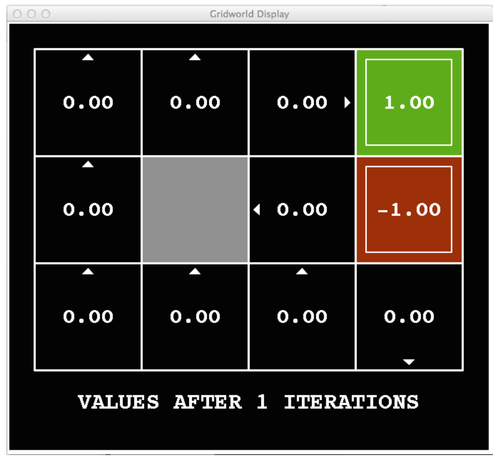
k=1



Noise = 0.2
Discount = 0.9
Living reward = 0

CAL POLY
SAN LUIS OBISPO

Computer Science Department

33

33

k=2

Noise = 0.2
Discount = 0.9
Living reward = 0

CAL POLY
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k=3

Noise = 0.2
Discount = 0.9
Living reward = 0

35

k=4

Noise = 0.2
Discount = 0.9
Living reward = 0

36

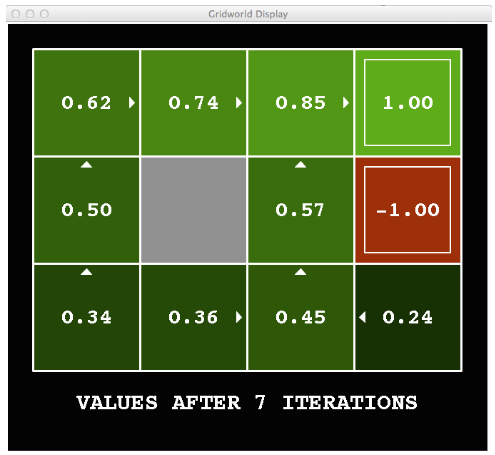
k=5

Noise = 0.2
Discount = 0.9
Living reward = 0

k=6

Noise = 0.2
Discount = 0.9
Living reward = 0

k=7



Noise = 0.2
Discount = 0.9
Living reward = 0

39

k=8

Noise = 0.2
Discount = 0.9
Living reward = 0

40

k=9

Noise = 0.2
Discount = 0.9
Living reward = 0

k=10

Noise = 0.2
Discount = 0.9
Living reward = 0

k=11

Noise = 0.2
Discount = 0.9
Living reward = 0

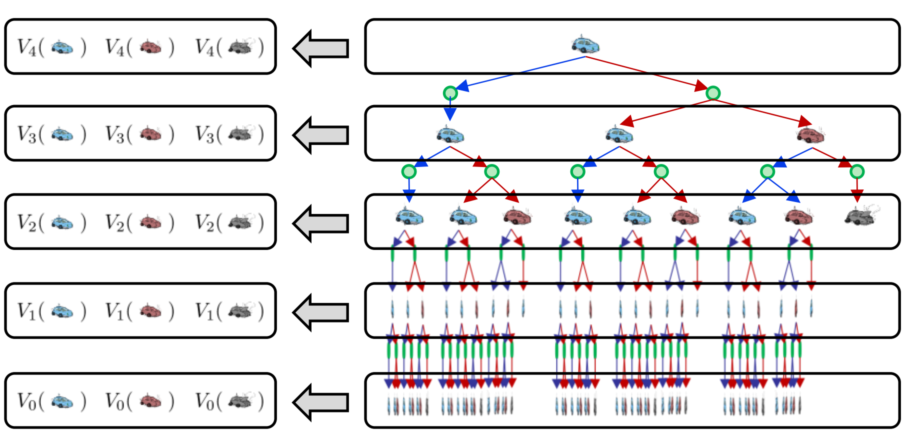
k=12

Noise = 0.2
Discount = 0.9
Living reward = 0

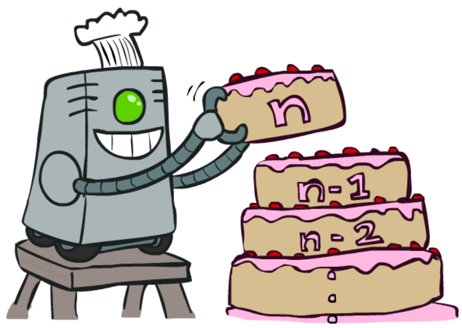
k=100

Noise = 0.2
Discount = 0.9
Living reward = 0

Computing Time-Limited Values



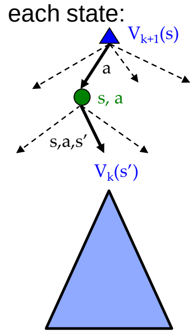
Value Iteration



Value Iteration

- Start with $V_0(s) = 0$: no time steps left means an expected reward sum of zero
- Given vector of $V_k(s)$ values, do one ply of expectimax from each state:

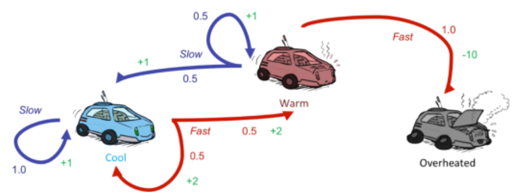
$$V_{k+1}(s) \leftarrow \max_a \sum_{s'} T(s,a,s') \left[R(s,a,s') + \gamma V_k(s') \right]$$



- Repeat until convergence, which yields V*
- Complexity of each iteration: $O(S^2A)$
- Theorem: will converge to unique optimal values
  - Basic idea: approximations get refined towards optimal values
  - Policy may converge long before values do

Example: Value Iteration

V_2			
V_1	S: 1 F: .5*2+.5*2=2		
V_0	0	0	0

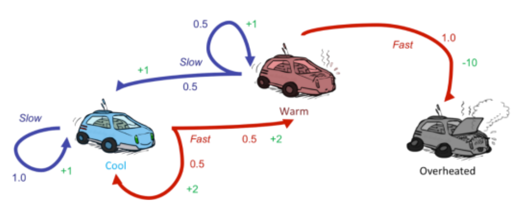


Assume no discount!

$$V_{k+1}(s) \leftarrow \max_a \sum_{s'} T(s,a,s') \left[R(s,a,s') + \gamma V_k(s')\right]$$

Example: Value Iteration

V_2			
V_1	2	S: .5*1+.5*1=1 F: -10	
V_0	0	0	0



Assume no discount!

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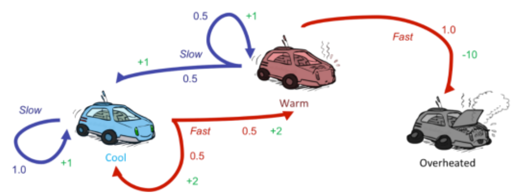
Example: Value Iteration

V_2			
V_1	2	1	0
V_0	0	0	0



Assume no discount!

$$V_{k+1}(s) \leftarrow \max_a \sum_{s'} T(s,a,s') \left[R(s,a,s') + \gamma V_k(s')\right]$$

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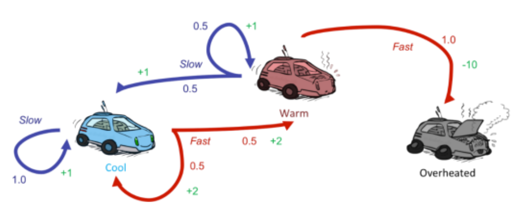
Example: Value Iteration

V_2	S: 1+2=3 F: .5*(2+2)+.5*(2+1)=3.5		
V_1	2	1	0
V_0	0	0	0



Assume no discount!

$$V_{k+1}(s) \leftarrow \max_a \sum_{s'} T(s,a,s') \left[R(s,a,s') + \gamma V_k(s')\right]$$

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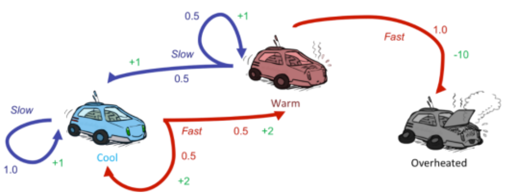
Example: Value Iteration

V_2	3.5	2.5	0
V_1	2	1	0
V_0	0	0	0

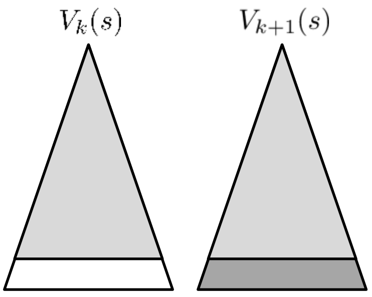


Assume no discount!

$$V_{k+1}(s) \leftarrow \max_a \sum_{s'} T(s,a,s') \left[R(s,a,s') + \gamma V_k(s')\right]$$

Convergence*

- How do we know the V_k vectors are going to converge? (assuming $0 < \gamma < 1$)
- Proof Sketch:
  - For any state V_k and V_{k+1} can be viewed as depth k+1 expectimax results in nearly identical search trees
  - The difference is that on the bottom layer, V_{k+1} has actual rewards while V_k has zeros
  - That last layer is at best all R_{MAX}
  - It is at worst R_{MIN}
  - But everything is discounted by γ^k that far out
  - So V_k and V_{k+1} are at most γ^k max|R| different
  - So as k increases, the values converge



Next Lecture: Policy-Based Methods