CSC 365

Introduction to Database Systems

Relational Algebra

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- Operands in Relational Algebra are relations (or expressions that yield relations)
- A few additional derived operators are not listed: additional join types (Equijoin, Semijoin, Antijoin) and Division

(*derived* operators are marked with ✱)

Operator Name	Symbol
Selection	σ
Projection	π
Rename	ρ
Union	$\cup$
Difference	$-$
Intersection ✱	$\cap$
Cartesian Product	$\times$
Natural Join ✱	$\bowtie$
Theta Join ✱	$\bowtie_\theta$

Relational Algebra - Joins

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- Several types of join operations
  - Natural join
  - Theta join
  - **Equijoin**
  - **Semijoin**
  - **Antijoin**

- Selectively pair tuples from two relations.

- Joins can be expressed in terms of other basic operators we have seen (cartesian product, selection, projection)

Relational Algebra - Equijoin

A theta join where the θ condition is a conjunction of equality comparisons

FACULTY(<u>ID</u>, Name, Dept, Salary)
CHAIR(<u>Dept</u>, FacultyID)

Find salaries of department chairs:

$$\pi_{FACULTY.Dept,\ FACULTY.Salary}(FACULTY \bowtie_{FACULTY.ID\ =\ CHAIR.FacultyID} CHAIR)$$

Relational Algebra - Semijoin (⋉)

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A specialized form of natural join. The result includes attributes from only one of the relations (in the case of left semijoin: $R \ltimes S$, attributes from R) Given:

EMPLOYEE(EmpID, Name, DeptID)
DEPARTMENT(DeptID, DeptName)

EMPLOYEE ⋉ DEPARTMENT has the same schema as EMPLOYEE, lists all employees who are assigned to a department.

$$R \ltimes S = \boldsymbol{\pi}_{R.A1,\, R.A2,\, \ldots\,,\, R.An} (R \bowtie S)$$

Relational Algebra - Antijoin ($\triangleright$)

The result of $R \triangleright S$ (sometimes written as $R \overline{\ltimes} S$) includes only those tuples from R for which there is ***no tuple in S*** that is equal on the common attribute names. Antijoin is the complement of Semijoin:

$$R \triangleright S = R - R \ltimes S$$

Consider EMPLOYEE(Name, Dept) and CUSTOMER(Name, Address).
EMPLOYEE $\triangleright$ CUSTOMER would list all employees who *don't share* a name with any customers.

Relational Algebra - Semijoin & Antijoin - Why?

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- Useful shorthand, sometimes seen when discussing distributed databases, where one key goal is to minimize network transfer
- Note: Semijoin is ***not*** the same as SQL's `OUTER JOIN`
- Semijoin appears in SQL as `EXISTS` or `IN`
- Antijoin appears in SQL as `NOT EXISTS` or `NOT IN`

Relational Algebra - Division (/, ÷)

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Division allows us to compactly represent queries such as:

- Find the names of people who have visited *every* State Park in California
- Find the names of Netflix customers who have watched *every* film which features your favorite movie star.

Relational Algebra Operators

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- Five fundamental operators
  - Selection
  - Projection
  - Union
  - Difference
  - Cartesian Product
- Derived operators
  - Intersection
  - Joins: natural, theta, semi, anti
  - Division
- Utility operator
  - Rename

Relational Algebra - Algebraic Laws

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$$R \cap S = R - (R - S) = S - (S - R) = R \bowtie S$$

$$R \bowtie_{\theta} S = \sigma_{\theta}(R \times S)$$

$$\sigma_{\theta 1 \wedge \theta 2}(R) = \sigma_{\theta 1}(\sigma_{\theta 2}(R)) = \sigma_{\theta 2}(\sigma_{\theta 1}(R))$$

$$\sigma_{\theta 1 \vee \theta 2}(R) = \sigma_{\theta 1}(R) \cup \sigma_{\theta 2}(R)$$

Relational Algebra - Algebraic Laws (Set Semantics)

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Selection distributes over the union, intersection, and set difference operations	$\sigma_\theta (R \cup S) = \sigma_\theta (R) \cup \sigma_\theta (S)$ $\sigma_\theta(R - S) = \sigma_\theta(R) - S = \sigma_\theta(R) - \sigma_\theta(S)$
Multiple applications of the same select have no effect (selection is idempotent)	$\sigma_\theta (\sigma_\theta (R)) = \sigma_\theta (R)$
Selection is commutative (order of selections does not matter)	$\sigma_{\theta 1} (\sigma_{\theta 2} (R)) = \sigma_{\theta 2} (\sigma_{\theta 1} (R))$
Projection is distributive over *set* union	$\pi_L (R \cup S) = \pi_L (R) \cup \pi_L (S)$
Projection *does not* distribute over intersection or difference	$\cancel{\pi_L (R \cap S) = \pi_L (R) \cap \pi_L (S)}$ $\cancel{\pi_L (R - S) = \pi_L (R) - \pi_L (S)}$

Relational Algebra - Algebraic Laws & Optimization

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- Laws that highlight equivalent expressions are useful during query optimization (an important task of the DBMS)

- Two equivalent expressions may have *very* different computational costs (when considering I/O & CPU)

Relational Algebra - Other Notations

Three ways to represent relational algebra expressions:

- One long expression with parentheses
- Tree representation
- Sequence of assignment statements (AKA "linear notation")

Relational Algebra - Tree Representation

We can represent a relational algebra expression as a tree:

- Internal nodes are operators
- Leaf nodes are relations

Sample Relation Instances

AIRPLANE

TailNum	Make	Model	MaxSpeed
C97W	Boeing	797	*null*
R53Q	Cessna	FG	220
T80H	Airbus	A380	634
G59K	Airbus	A320	450
P88T	Piper	Arrow	180
K30W	Boeing	707	450

FLIGHT

TailNum	PilotID	CopilotID	Runway	Date
R53Q	K407	D342	S-2	9/1/17
T80H	K407	null	W-2	9/21/17
C97W	D342	*null*	W-2	8/9/21
T80H	D342	K407	W-3	9/9/17

PILOT

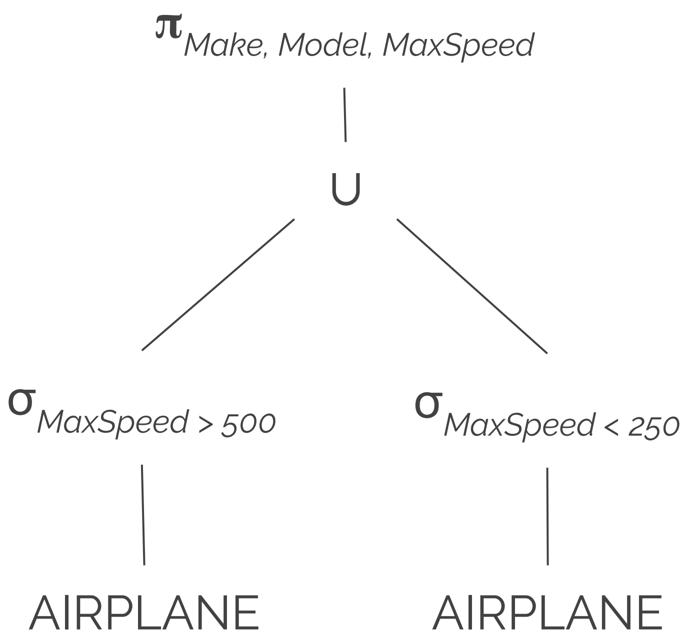
PilotID	Name
D342	Charlie
K407	Juliett
H452	Piper

Relational Algebra - Tree Representation

$$\pi_{Make,\ Model,\ MaxSpeed}(\sigma_{MaxSpeed\ >\ 500}(\text{AIRPLANE})$$
$$\cup$$
$$\sigma_{MaxSpeed\ <\ 250}(\text{AIRPLANE}))$$

Line breaks have no significance. This could be shown on one long line.

Equivalent expression?



Relational Algebra - Assignment / Linear Notation

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$\pi_{Make,\ Model,\ MaxSpeed}(\sigma_{MaxSpeed\ >\ 500}(\text{AIRPLANE})\ \cup\ \sigma_{MaxSpeed\ <\ 250}(\text{AIRPLANE}))$

Linear Notation:

R(TailNum, Make, Model, MaxSpeed) := $\sigma_{MaxSpeed\ >\ 500}$(AIRPLANE)

S(TailNum, Make, Model, MaxSpeed) := $\sigma_{MaxSpeed\ <\ 250}$(AIRPLANE)

T(TailNum, Make, Model, MaxSpeed) := R $\cup$ S

Answer(Make, Model, MaxSpeed) := $\pi_{Make,\ Model,\ MaxSpeed}$(T)

Attribute names may be omitted for brevity.

Relational Algebra - Examples

Given the database schema:

Relation	**Sample Tuple**
PRODUCT(maker, <u>model</u>, type)	(Dell, 1001, pc)
PC(<u>model</u>, speed, ram, hd, price)	(1001, 3.1, 8192, 512, 2114)
LAPTOP(<u>model</u>, speed, ram, hd, screen, price)	(2001, 1.7, 4096, 256, 13,3, 1699)
PRINTER(<u>model</u>, color, type, price)	(3001, true, laser, 239)

Relational Algebra - Examples

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PRODUCT

maker	<u>model</u>	type
Dell	1001	pc
Dell	2001	laptop
Orange	1002	pc
HP	1003	pc
HP	2002	laptop

PC

<u>model</u>	speed	ram	hd	price
1001	3.1	8	512	1200
1002	2.8	16	512	1300
1003	2.9	8	1024	1500

LAPTOP

<u>model</u>	speed	ram	hd	screen	price
2001	1.7	8	512	15	1599
2002	2.0	16	1024	13.3	999

Relational Algebra - Examples

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(1) What PC models have a speed of at least 3.00?

(2) Find the model number and price of all products (of any type) made by manufacturer *Orange*

Relational Algebra - Examples

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(3) Find the manufacturers (makers) that sell laptops, but not PCs.

Hint: Recall relational algebra's set operators ($\cup$, $\cap$, $-$)

Relational Algebra - Examples

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(3) Find the manufacturers (makers) that sell laptops, but not PCs.

Hint: Recall relational algebra's set operators (∪ , ∩, –)

$$\pi_{maker}(\text{PRODUCT} \bowtie \text{LAPTOP}) - \pi_{maker}(\text{PRODUCT} \bowtie \text{PC})$$

(4) Find hard-disk sizes that occur in two or more PCs.

Hint: It's entirely acceptable to join a relation to itself (a "self-join")

Relational Algebra - Examples

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(4) Find hard-disk sizes that occur in two or more PCs.

Hint: It's entirely acceptable to join a relation to itself (a "self-join")

$$\boldsymbol{\pi}_{hd}\,(\sigma_{\text{PC1.model != PC2.model} \wedge \text{PC1.hd = PC2.hd}}(\boldsymbol{\rho}_{\text{PC1}}(\text{PC}) \times \boldsymbol{\rho}_{\text{PC2}}(\text{PC})))$$

Relational Algebra - Examples

(5) Find the names of all manufacturers that sell products of <u>all</u> types (pc and laptop in our example; the query should not be specifically limited to these values)

Relational Algebra - Division (/, ÷)

Informally, division is related to the Cartesian produc as follows:

$C = A \times B$

$A = C \div B$

$B = C \div A$

(This is an oversimplification that does not hold in the case of empty tables.)

Relational Algebra - Division Example

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R

A
4
8

S

B
3
1
7

R x S

A	B
4	3
4	1
4	7
8	3
8	1
8	7

(R x S) ÷ S

A
4
8

(R x S) ÷ R

B
3
1
7

Relational Algebra - Division Example

CAL POLY

R

A
4
8

S

B
3
1
7

T

A	B
4	3
4	1
~~4~~	~~7~~
8	3
8	1
8	7

In this example, the row (4,7) has been removed from T

T ÷ S

A
~~4~~
8

T ÷ R

B
3
1
~~7~~

Relational Algebra - Division

Division can be represented as a combination of projection, difference and cross product.

$\boldsymbol{R \div S}$ =

$\boldsymbol{\pi}_{Schema(R)-Schema(S)}(R) - \boldsymbol{\pi}_{Schema(R)-Schema(S)}((\boldsymbol{\pi}_{Schema(R)-Schema(S)}(R) \times S) - R)$

(here, Schema(R) and Schame(S) represent the attribute sets of relation R and S, respectively)

Relational Model / SQL

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	Relational Model	SQL
Structure	Attributes, Tuples, Relations	Columns, Rows, Tables
Manipulation	Relational Algebra	DML, Queries

Constraints	Primary Key, Referential (Foreign Key), Domain, Relational Algebra expressions to define constraints	`CREATE TABLE`
		`ALTER TABLE`
		Triggers