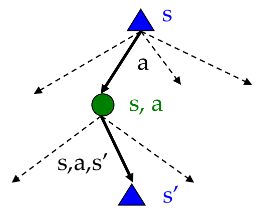
Recap: Defining MDPs

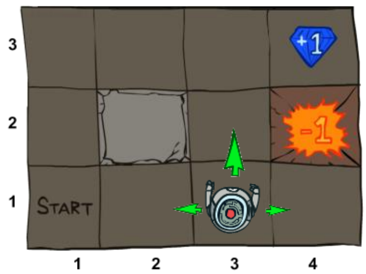
- Markov decision processes:
  - Set of states S
  - Start state s_0
  - Set of actions A
  - Transitions P(s'|s,a) (or T(s,a,s'))
  - Rewards R(s,a,s') (and discount γ)



- MDP quantities so far:
  - Policy = Choice of action for each state
  - Utility = sum of (discounted) rewards
  - Values = expected future utility from a state (max node)
  - Q-Values = expected future utility from a q-state (chance node)

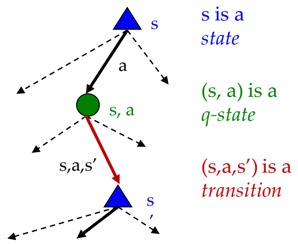
Example: Grid World

- A maze-like problem
  - The agent lives in a grid
  - Walls block the agent's path
- Noisy movement: actions do not always go as planned
  - 80% of the time, the action North takes the agent North (if there is no wall there)
  - 10% of the time, North takes the agent West; 10% East
  - If there is a wall in the direction the agent would have been taken, the agent stays put
- The agent receives rewards
  - Small "living" reward each step (can be negative)
  - Big rewards come at the end (good or bad)
- Goal: maximize sum of rewards



Optimal Quantities

- The value (utility) of a state s:
  $V^*(s)$ = expected utility starting in s and acting optimally
- The value (utility) of a q-state (s,a):
  $Q^*(s,a)$ = expected utility starting out having taken action a from state s and (thereafter) acting optimally
- The optimal policy:
  $\pi^*(s)$ = optimal action from state s

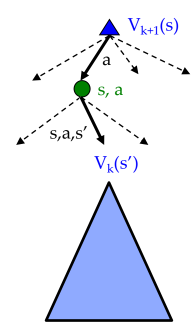


Value Iteration

- Start with $V_0(s) = 0$: no time steps left means an expected reward sum of zero
- Given vector of $V_k(s)$ values, do one ply of expectimax from each state:

$$V_{k+1}(s) \leftarrow \max_a \sum_{s'} T(s,a,s') \left[R(s,a,s') + \gamma V_k(s') \right]$$

- Repeat until convergence, which yields V*
- Complexity of each iteration: $O(S^2A)$
- Theorem: will converge to unique optimal values
  - Basic idea: approximations get refined towards optimal values
  - Policy may converge long before values do



Value Iteration

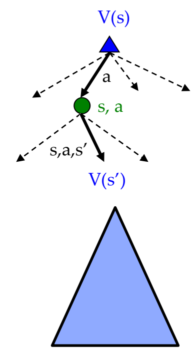
- Bellman equations characterize the optimal values:

$$V^*(s) = \max_a \sum_{s'} T(s,a,s') \left[R(s,a,s') + \gamma V^*(s')\right]$$

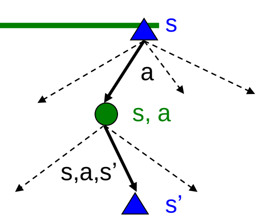
- Value iteration computes them:

$$V_{k+1}(s) \leftarrow \max_a \sum_{s'} T(s,a,s') \left[R(s,a,s') + \gamma V_k(s')\right]$$

- Value iteration is just a fixed point solution method
  - … though the V_k vectors are also interpretable as time-limited values



Value Iteration (again ☺)



- Init:

$$\forall s: \quad V(s) = 0$$

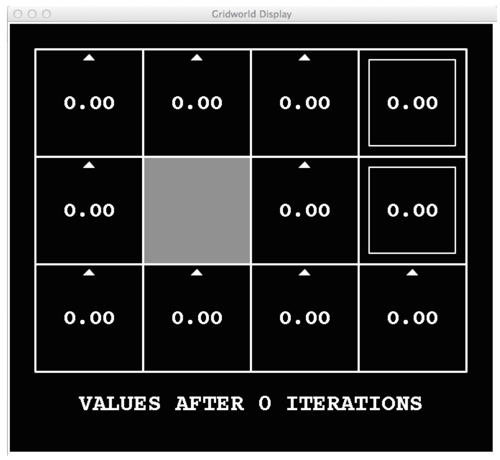
- Iterate:

$$\forall s: \quad V_{new}(s) = \max_a \sum_{s'} T(s,a,s')[R(s,a,s') + \gamma V(s')]$$

$$V = V_{new}$$

Note: can even directly assign to *V(s)*, which will not compute the sequence of V_k but will still converge to V^*

k=0



Noise = 0.2
Discount = 0.9
Living reward = 0

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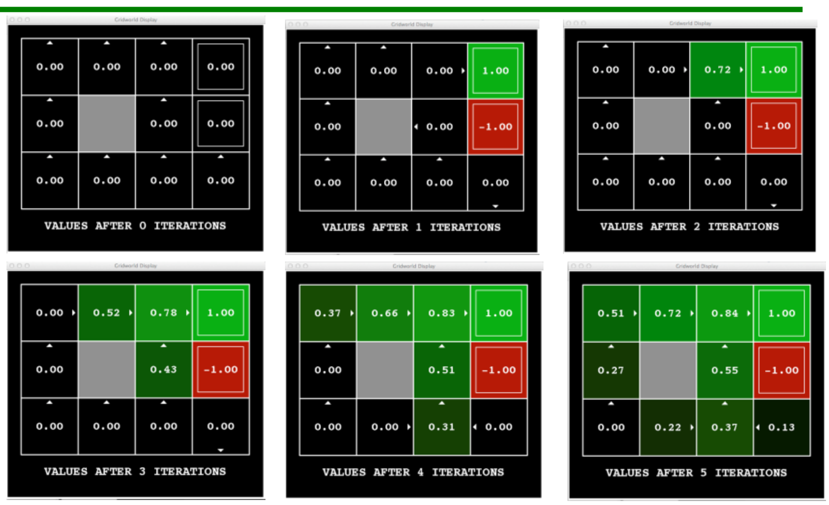
Noise = 0.2 Discount = 0.9 Living reward = 0



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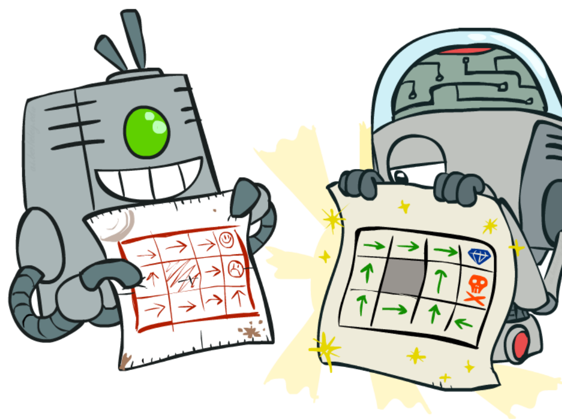
Noise = 0.2 Discount = 0.9 Living reward = 0

Let's think.

- Take a minute, think about value iteration.
- Write down the biggest question you have about it.

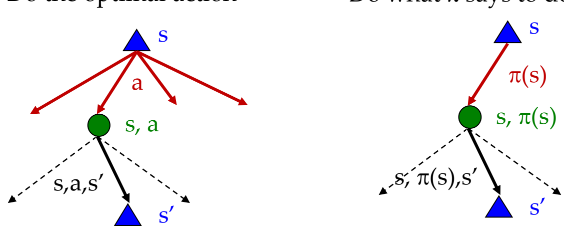
Policy Methods



Fixed Policies

Do the optimal action

Do what π says to do



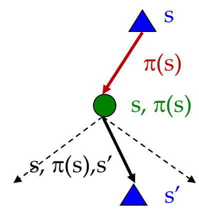
- Expectimax trees max over **all actions** to compute the optimal values
- If we fixed some policy π(s), then the tree would be simpler – **only one action per state**
  - ... **though the tree's value would depend on which policy we fixed**

Utilities for a Fixed Policy

- Another basic operation: compute the utility of a state s under a fixed (generally non-optimal) policy
- Define the utility of a state s, under a fixed policy π:
  $V^\pi(s)$ = expected total discounted rewards starting in s and following π
- Recursive relation (one-step look-ahead / Bellman equation):

$$V^\pi(s) = \sum_{s'} T(s, \pi(s), s')[R(s, \pi(s), s') + \gamma V^\pi(s')]$$

No Max



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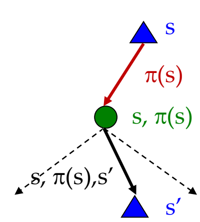
Policy Evaluation

- How do we calculate the V's for a fixed policy π?
- Idea 1: Turn recursive Bellman equations into updates (like value iteration)

$$V_0^\pi(s) = 0$$
$$V_{k+1}^\pi(s) \leftarrow \sum_{s'} T(s, \pi(s), s')[R(s, \pi(s), s') + \gamma V_k^\pi(s')]$$



- Efficiency: $O(S^2)$ per iteration
- Idea 2: Without the maxes, the Bellman equations are just a linear system
  - Solve with Matlab (or your favorite linear system solver)

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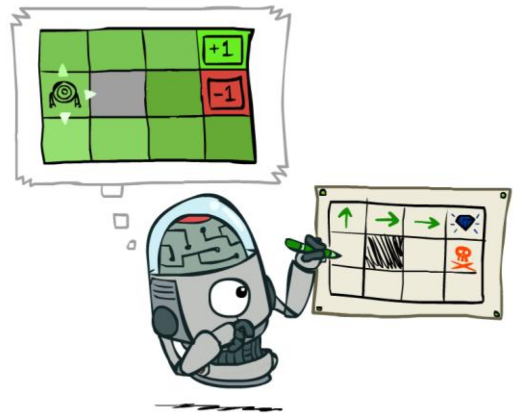
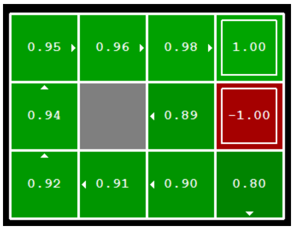
Policy Extraction



Policy Extraction:
Computing Actions from Values

- Given the optimal values V*(s). How should we act?
  - It's not obvious!
- We need to do a (one step) mini-expectimax
- This is called policy extraction, since it gets the policy implied by the values



$$\pi^*(s) = \arg\max_a \sum_{s'} T(s,a,s')[R(s,a,s') + \gamma V^*(s')]$$

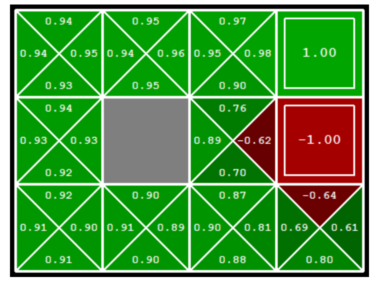
Computing Actions from Q-Values

- Let's imagine we have the optimal q-values:

- How should we act?
  - Completely trivial to decide!

$$\pi^*(s) = \arg\max_a Q^*(s,a)$$

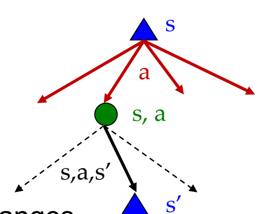
- Important lesson: actions are easier to select from q-values than values!



Problems with Value Iteration

- Value iteration repeats the Bellman updates:

$$V_{k+1}(s) \leftarrow \max_a \sum_{s'} T(s,a,s') \left[R(s,a,s') + \gamma V_k(s') \right]$$

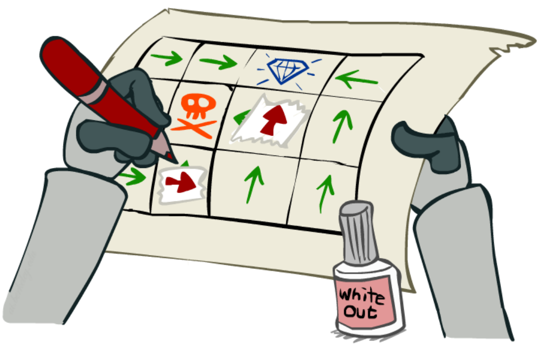


- Problem 1: It's slow – $O(S^2A)$ per iteration

- Problem 2: The "max" at each state rarely changes

- Problem 3: The policy often converges long before the values

Policy Iteration



Policy Iteration

- Alternative approach for optimal values:
  - Step 1: Policy Evaluation: calculate utilities for some fixed policy (not optimal utilities!) until convergence
  - Step 2: Policy Improvement: update policy using one-step look-ahead with resulting converged (but not optimal!) utilities as future values
  - Repeat steps until policy converges
- This is Policy Iteration
  - It's still optimal!
  - Can converge (much) faster under some conditions

Policy Iteration

- Evaluation: For fixed current policy π, find values with policy evaluation:
  - Iterate until values converge:

$$V_{k+1}^{\pi_i}(s) \leftarrow \sum_{s'} T(s, \pi_i(s), s') \left[R(s, \pi_i(s), s') + \gamma V_k^{\pi_i}(s')\right]$$

- Improvement: For fixed values, get a better policy using policy extraction
  - One-step look-ahead:

$$\pi_{i+1}(s) = \arg\max_a \sum_{s'} T(s, a, s') \left[R(s, a, s') + \gamma V^{\pi_i}(s')\right]$$

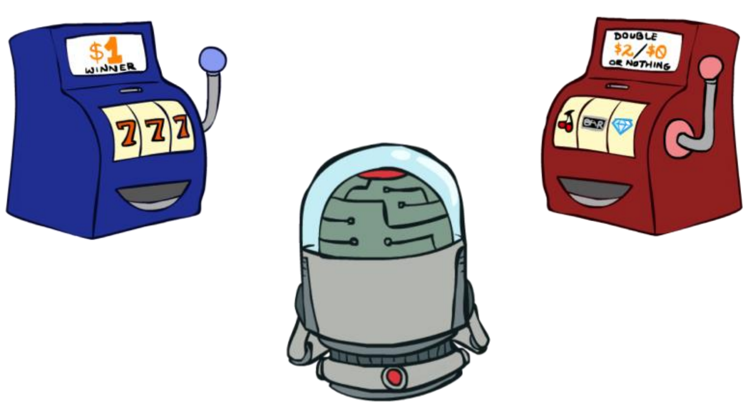
Comparison

- Both value iteration and policy iteration compute the same thing (all optimal values)
- In value iteration:
  - Every iteration updates both the values and (implicitly) the policy
  - We don't track the policy, but taking the max over actions implicitly recomputes it
- In policy iteration:
  - We do several passes that update utilities with fixed policy (each pass is fast because we consider only one action, not all of them)
  - After the policy is evaluated, a new policy is chosen (slow like a value iteration pass)
  - The new policy will be better (or we're done)
- Both are dynamic program solutions for solving MDPs

Summary: MDP Algorithms

- So you want to….
  - Compute optimal values: use value iteration or policy iteration
  - Compute values for a particular policy: use policy evaluation
  - Turn your values into a policy: use policy extraction (one-step lookahead)
- These all look the same!
  - They basically are – they are all variations of Bellman updates
  - They all use one-step look-a-head expectimax fragments
  - They differ only in whether we plug in a fixed policy or max over actions

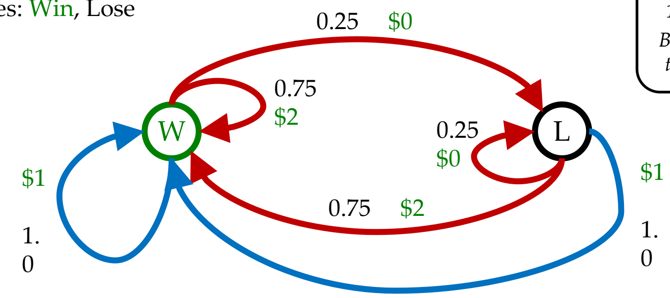
Double Bandits



Double-Bandit MDP

- Actions: *Blue*, *Red*
- States: Win, Lose

No discount
100 time steps
Both states have the same value

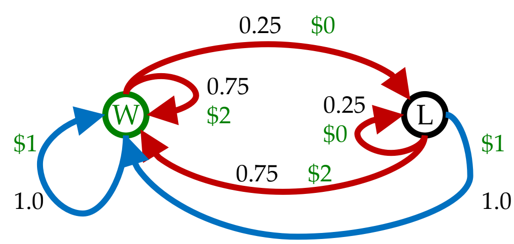


Offline Planning

- Solving MDPs is offline planning
  - You determine all quantities through computation
  - You need to know the details of the MDP
  - You do not actually play the game!

No discount
100 time steps
Both states have the same value

	Value
Play Red	150
Play Blue	100



Let's Play!

$2 $2 $0 $2 $2
$2 $2 $0 $0 $0

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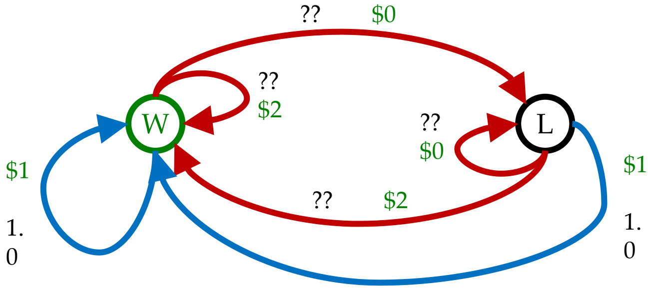
Online Planning

- Rules changed! Red's win chance is different.



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Let's Play!

$0 $0 $0 $2 $0
$2 $0 $0 $0 $0

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What Just Happened?

- That wasn't planning, it was learning!
  - Specifically, reinforcement learning
  - There was an MDP, but you couldn't solve it with just computation
  - You needed to actually act to figure it out
- Important ideas in reinforcement learning that came up
  - Exploration: you have to try unknown actions to get information
  - Exploitation: eventually, you have to use what you know
  - Regret: even if you learn intelligently, you make mistakes
  - Sampling: because of chance, you have to try things repeatedly
  - Difficulty: learning can be much harder than solving a known MDP

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Reinforcement Learning -- Overview

- **Passive Reinforcement Learning (= how to learn from experiences)**
  - **Model-based Passive RL**
    - Learn the MDP model from experiences, then solve the MDP
  - Model-free Passive RL
    - Forego learning the MDP model, directly learn V or Q:
      - Value learning – learns value of a fixed policy; 2 approaches: Direct Evaluation & TD Learning
      - Q learning – learns Q values of the optimal policy (uses a Q version of TD Learning)
- Active Reinforcement Learning (= agent also needs to decide how to collect experiences)
  - Key challenges:
    - How to efficiently explore?
    - How to trade off exploration <> exploitation
  - Applies to both model-based and model-free. In CS188 we'll cover only in context of Q-learning

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Model-Based Reinforcement Learning

- Model-Based Idea:
  - Learn an approximate model based on experiences
  - Solve for values as if the learned model were correct
- Step 1: Learn empirical MDP model
  - Count outcomes s' for each s, a
  - Normalize to give an estimate of $\hat{T}(s,a,s')$
  - Discover each $\hat{R}(s,a,s')$ when we experience (s, a, s')
- Step 2: Solve the learned MDP
  - For example, use value iteration, as before

(and repeat as needed)

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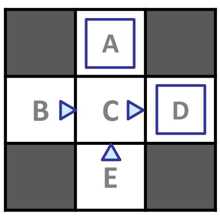
Example: Model-Based RL

Episode: start from a state and continue to end state

Input Policy π



Assume: $\gamma = 1$

Observed Episodes (Training)

Episode 1

B, east, C, -1
C, east, D, -1
D, exit, x, +10

Episode 2

B, east, C, -1
C, east, D, -1
D, exit, x, +10

Episode 3

E, north, C, -1
C, east, D, -1
D, exit, x, +10

Episode 4

E, north, C, -1
C, east, A, -1
A, exit, x, -10

Learned Model

$\widehat{T}(s, a, s')$

T(B, east, C) = 1.00
T(C, east, D) = 0.75
T(C, east, A) = 0.25
...

$\widehat{R}(s, a, s')$

R(B, east, C) = -1
R(C, east, D) = -1
R(D, exit, x) = +10
...