

## Relations & Functions

$R$  defined over  $A, B$

$$R = \{ (a, b) \mid a \in A, b \in B \}$$

$$R \subseteq A \times B$$

$R$  defined over  $A \times B$

$$\boxed{a R b}$$

infix notation

$$\text{if } \underbrace{(a, b) \in R}$$

set notation

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let  $R$  be a relation,

$$R \subseteq \mathbb{Z} \times \mathbb{Z}$$

if  $R$  is defined a set  $A$

$$R \subseteq A \times A$$

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$$A = \{3, 4, 5\}$$

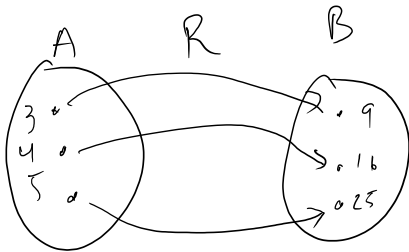
$$S = \{(a, b) \mid a \in A, b \in B, \\ b = a^2\}$$

$$B = \{9, 16, 25\}, \quad B_2 = \{36\} \cup B.$$

# Arrow Diagrams

$$A = \{3, 4, 5\}$$

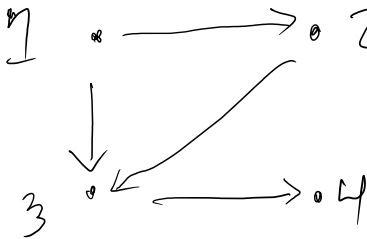
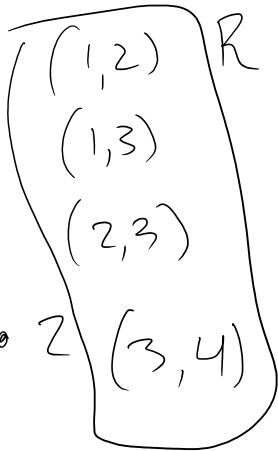
$$B = \{9, 16, 25\}$$



Directed Graph

$R$  is defined A

$$A = \{1, 2, 3, 4\}$$



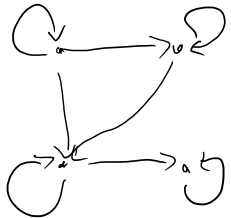
Reflexivity of a relation

if  $R$  is defined over  $A$

then  $R$  is reflexive

iff

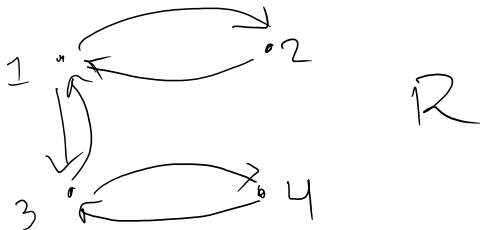
$$\forall a \in A \quad a R a$$



# Symmetry

A Relation  $R$  is symmetric  
iff

$\forall a R b$ , then  $b R a$



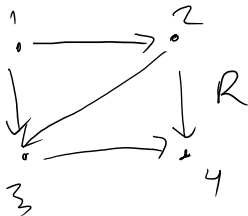
# Transitivity

$R$  is transitive

iff

$\forall a, b, c \quad aRb, bRc$

then  
 $aRc$





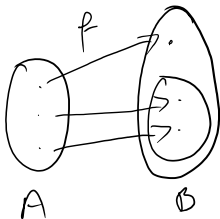
# Functions

a type of Relation

$$f: A \rightarrow B$$

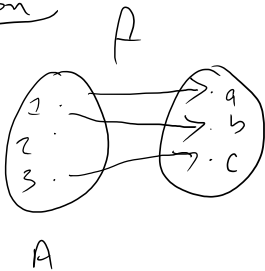
$A, B$  are sets

$$\forall a \in A, \exists f(a) \in B.$$



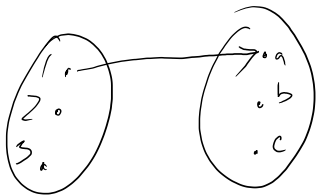
not a function

function



$$f(1) = a \text{ or } b$$

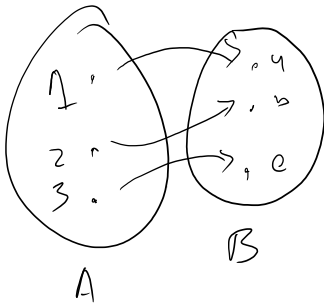
injective function



$$f(x) = \cancel{f} a$$

$$f(y) = \cancel{f} a, \quad x = y = 1.$$

# Surjective function



Surjective  
onto

## Bijective

A function that is  
surjective  
and  
injective