Propositional Logic

Set V of boolean variables

Formula (atomic)

Atomic Formula are all Formula

For every formula F, $\neg F$ is a formula

$\neg$ not (left hook)

negation

F, G are formulas

$F \vee G$ disjunction

$F \wedge G$ conjunction

} formulas

Nothing else is a formula.

complex formulas.

$F \equiv A \vee B$

$G \equiv C \wedge D$

$F \vee \neg G$

$(A \vee B) \vee \neg (C \wedge D)$

$\mathbb{T} = \{0, 1\}$ false and true

φ

Truth values.

$\mathcal{A} : \mathbb{D} \to \mathbb{T}$ assignment

Truth tables

$F \vee G$

$A(F)$	$A(G)$	$A(F \vee G)$
0	0	0
0	1	1
1	0	1
1	1	1

$F \wedge G$

$\neg F$

$A(F)$	$A(G)$	$A(F \wedge G)$
0	0	0
0	1	0
1	0	0
1	1	1

iterate through all possible assignments to the atomic formula.

$\neg F$ ← negation

$A(F)$	$A(\neg F)$
0	1
1	0

A(F)	A(G)	A(H)	A(R)
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	0
1	1	1	0

$$(\neg F \wedge \neg G \wedge H) \vee (F \wedge \neg G \wedge \neg H) \vee (F \wedge \neg G \wedge H)$$

$\equiv$ semantic equivalence

$F \equiv G$ iff $\mathcal{A}(F) = \mathcal{A}(G)$

Properties

Commutativity $F \wedge G \equiv G \wedge F$

$F \vee G \equiv G \vee F$

Associativity $(F \wedge G) \wedge H \equiv F \wedge (G \wedge H)$

Distribution $F \wedge (G \vee H) \equiv (F \wedge G) \vee (F \wedge H)$

$F \vee (G \wedge H) \equiv (F \vee G) \wedge (F \vee H)$

Double negation $\neg\neg F \equiv F$

De Morgan's Law $\neg(F \vee G) \equiv \neg F \wedge \neg G$

$\neg(F \wedge G) \equiv \neg F \vee \neg G$