1.3 Propositional Logic

Material from this section is drawn from [1, Chapter 3] and [2, Chapter 2].

1.47 Definition (Atomic Formula)
An *atomic formula* is a Boolean Variable.
(*Well-formed*) *formulas* are defined as follows.

1. All atomic formulas are formulas.
2. For every formula F, $\neg F$ is a formula, called the *negation* of F.
3. For all formulas F and G, also $(F \vee G)$ and $(F \wedge G)$ are formulas, called the *disjunction* and the *conjunction* of F and G, respectively.
4. Nothing else is a formula.

1.48 Definition (Assignment)
$\mathbb{T} = \{0, 1\}$ – the set of *truth values*: *false*, and *true*, respectively.
An *assignment* is a function $\mathcal{A} : \mathbf{D} \to \mathbb{T}$, where $\mathbf{D}$ is a set of atomic formulas.
Assignments extend to formulas, via the following *truth tables.* The following are definitions for disjunction, conjunction, and negation, respectively.

$\mathcal{A}(F)$	$\mathcal{A}(G)$	$\mathcal{A}(F \wedge G)$
0	0	0
0	1	0
1	0	0
1	1	1

$\mathcal{A}(F)$	$\mathcal{A}(G)$	$\mathcal{A}(F \vee G)$
0	0	0
0	1	1
1	0	1
1	1	1

$\mathcal{A}(F)$	$\mathcal{A}(\neg F)$
0	1
1	0

A formula F is called *satisfiable* if there exists an assignment $\mathcal{A}$ with $\mathcal{A}(F) = 1$. $\mathcal{A}$ is called a *model* of F in this case, and we write $\mathcal{A} \models F$.

1.49 Theorem (Properties for Propisitional Logic)
The following hold for all formulas F, G, and H.

$F \wedge G \equiv G \wedge F$	$F \vee G \equiv G \vee F$	Commutativity
$(F \wedge G) \wedge H \equiv F \wedge (G \wedge H)$	$(F \vee G) \vee H \equiv F \vee (G \vee H)$	Associativity
$F \wedge (G \vee H) \equiv (F \wedge G) \vee (F \wedge H)$	$F \vee (G \wedge H) \equiv (F \vee G) \wedge (F \vee H)$	Distributivity
$\neg\neg F \equiv F$		Double Negation
$\neg(F \wedge G) \equiv \neg F \vee \neg G$	$\neg(F \vee G) \equiv \neg F \wedge \neg G$	de Morgan's Laws

Proof: Straightforward using truth tables. ∎