Eigentriads and Eigenprogressions on the Tonnetz

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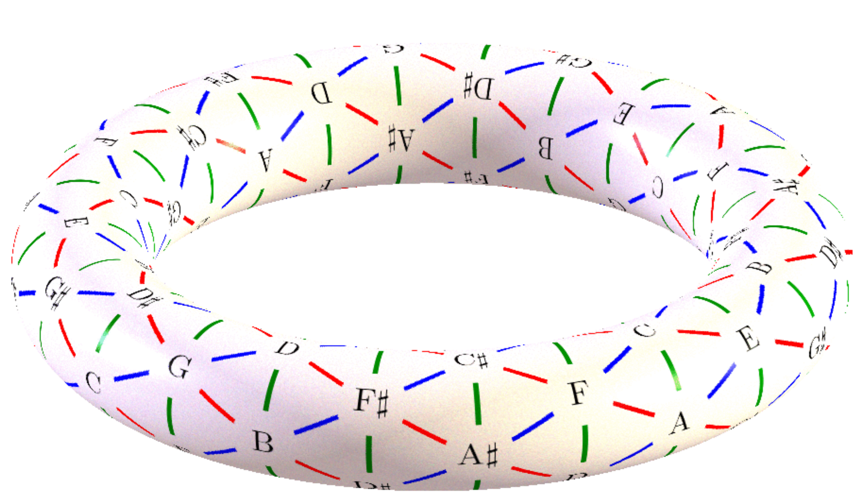
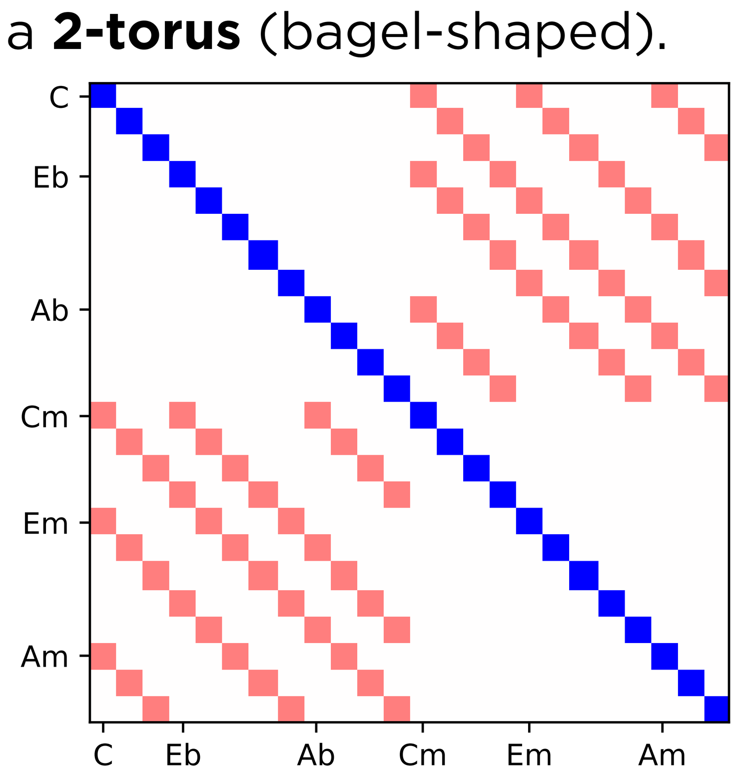
- Context: pitch-equivariant representations of **Western tonal music**.
- Prior work: 2-D Fourier transform [**Nieto**], Gabor filtering [**Velarde**].
- Problem: excessively sensitive to **triad inversion** (root position) while insufficiently sensitive to **triad quality** (major vs. minor third).
- Contribution: a **deep**, **multiscale**, **convolutional** representation in the piano-roll domain that integrates local harmonic context.
- No feature learning: suits both **supervised and unsupervised tasks**.
- How it works: by inducing **algebraic locality between triads**.
- The Tonnetz is a lattice diagram dating back to **Euler** (1739).
- Temporal integration of musical patterns by **wavelet scattering**.
- **Neo-Riemannian** music theory meets Fourier analysis on graphs.

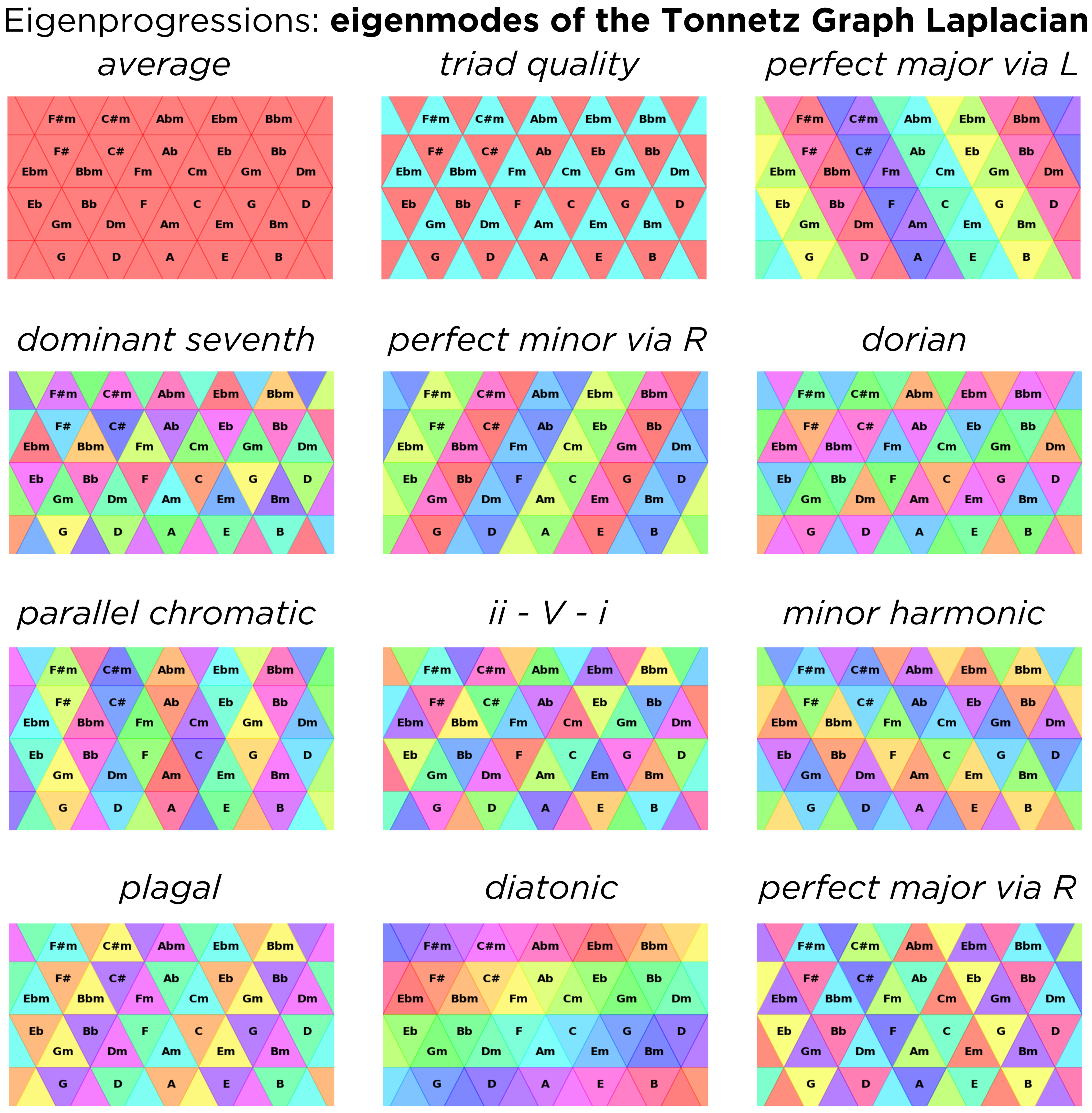
Diagonalizing the Tonnetz Graph Laplacian

- Laplacian matrix of a graph = **Adjacency matrix - Degree diagonal**.
- It plays the role of a partial derivative in the **d'Alembert equation**.
- Tonnetz graph: **24 vertices**, i.e. 12 major and 12 minor triads.
- Any two triads are connected iff they share 2/3 of their pitch classes.
- Ex: C major is connected to C minor, E minor, and A minor.
- The Tonnetz has the topology of a **2-torus** (bagel-shaped).



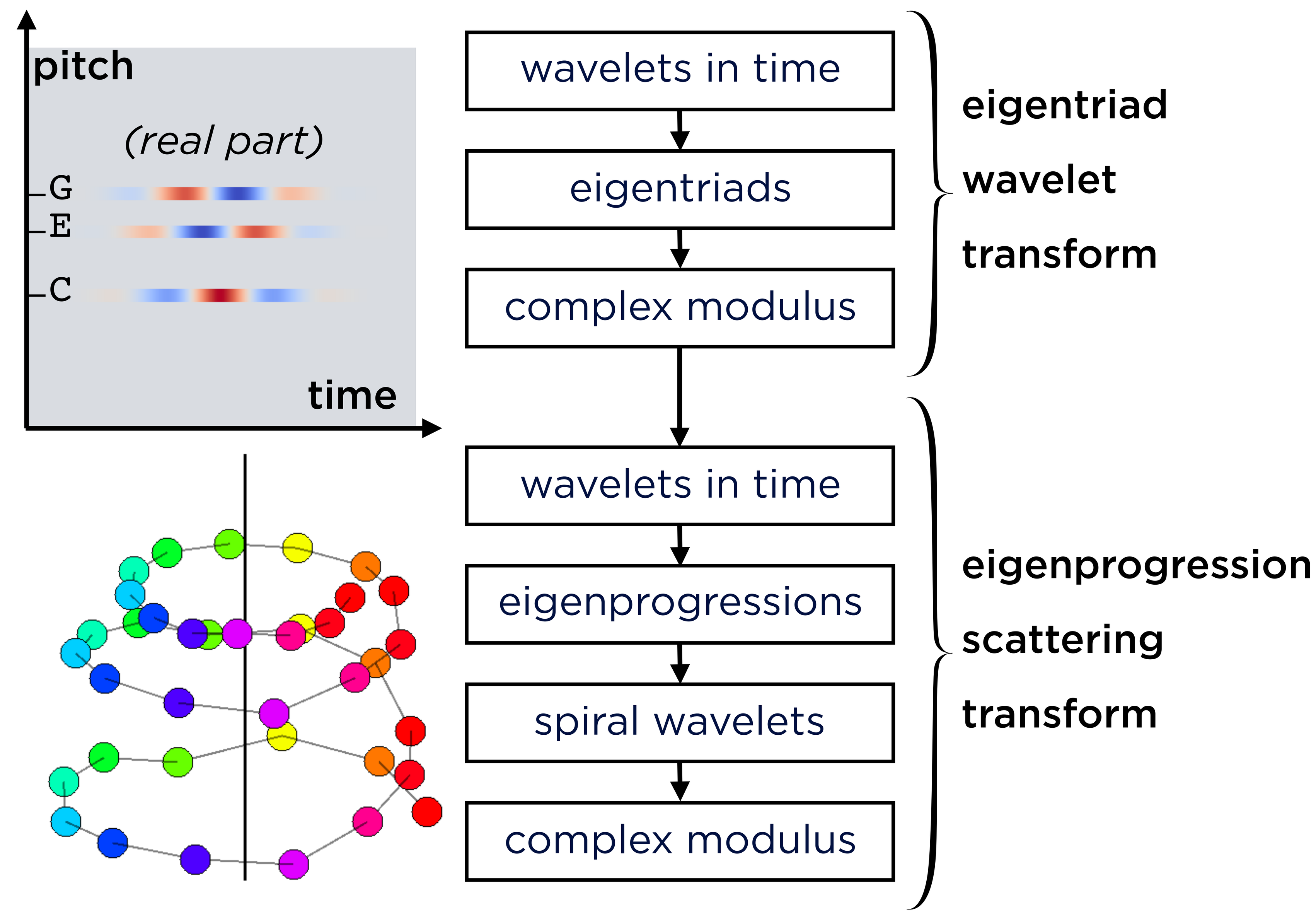


- Such progressions are known as parallel, leading tone, and relative.
- Eigenprogressions: **eigenmodes of the Tonnetz Graph Laplacian**.



- The above eigenmodes are comparable to a **Fourier basis** of sinusoids, taking complex values over the vertices of the Tonnetz.
- **Visualizing phase** as hue, especially in motion (see demo), reveals some well-known **archetypes** of Western tonal harmony.

Deep spectral networks meet spiral scattering



- Multivariable scattering equation [**Andén, L., and Mallat**]

$$\mathbf{U}_2(x)[t,p,q,\alpha_1,\beta_1,\alpha_2,\beta_2,\gamma_2] = \left| \left| \boldsymbol{x} \overset{t}{*} \boldsymbol{\psi}_{\alpha_1} \overset{p}{*} \boldsymbol{\psi}_{\beta_1} \right| \overset{t}{*} \boldsymbol{\psi}_{\alpha_2} \overset{p}{*} \boldsymbol{\psi}_{\beta_2} \overset{p,q}{*} \boldsymbol{\psi}_{\gamma_2} \right| [t,p]$$

(time, triads) *(time, progressions, spiral)*

How to turn eigenprogressions into a convolutional operator?
by regarding them as the kernel weights of a **spectral network** [**Bruna**]

How to induce a Tonnetz geometry onto the piano roll input?
with **eigentriads**, i.e. a 3-sample discrete Fourier transform

How to integrate multiple scales of harmonic progression?
with **scattering**, i.e. two layers of temporal wavelet modulus

How to induce robustness to chord inversion?
with **spiral wavelets** [L.], convolving across octaves

Haydn vs. Mozart piano-roll classification

- In Western tonal music, **musical style** is transposition-invariant.
- Therefore, we **average in pitch and time** scattering coefficients.
- **Ablation study**: all five variables are beneficial to **sparsity**.
- We train a **support vector machine** on standardized features.

					dim.	sparsity (ℓ_1/ℓ_2 ratio)	acc (%)
[**Van Kranenburg**]							79.4
[**Velarde**]							80.4
α_1					8	2.6	67.3
α_1	β_1				24	4.6	71.0
α_1	β_1	α_2			129	6.1	72.0
α_1	β_1	α_2	β_2		1677	17.0	76.7
α_1	β_1	α_2	β_2	γ_2	8385	42.4	77.6
α_1	β_1	α_2	β_2	γ_2	1119	22.3	**82.2**

- **Wavelet shrinkage** denoising, i.e. keeping the 1119 most energetic coefficients, accounting for 50% of the total energy, is a near-optimal feature selection procedure [**Donoho**].
- State-of-the-art results on Haydn vs. Mozart string quartets.
- The eigenprogression transform is **not restricted to symbolic** inputs. It might also work with **deep salience** input [**Bittner**].
- Possible **future applications**: cover song retrieval, key estimation, structure analysis.
- **Contributors welcome**!

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