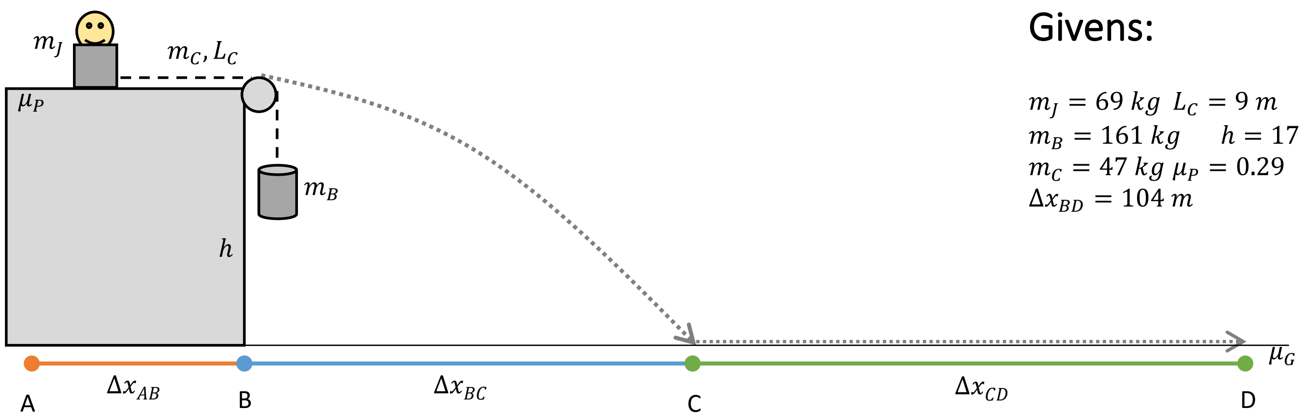
Uber Pulley | Calculus

"Jerky" Jerry decided to make a jabberwocky jumper using a pulley system. His method was to attach one end of a chain to a barrel of rocks, and the other end to the jumper. He placed the barrel and chain over a massless frictionless pulley, and then walked along a platform away from the pulley to point A (the full length of the chain). When he sat in the jumper, he accelerated along the platform to point B and then launched off it while releasing the chain from the jumper and avoiding the pulley. He flew through the air as a projectile to point C, transitioning 75% of his speed into the horizontal direction, and eventually slid to a stop at point D. Calculate the coefficient of kinetic friction between the jumper and the ground. Ignore any heights of the jumper, pulley, and barrel. Ignore any frictional and normal forces of the chain.



Givens:

$m_J = 69\ kg$ $L_C = 9\ m$
$m_B = 161\ kg$ $h = 17\ m$
$m_C = 47\ kg$ $\mu_P = 0.29$
$\Delta x_{BD} = 104\ m$

Strategy:

The goal of this problem is to find the coefficient of friction μ_G between Jerry m_J and the ground during Stage CD. It can be split up into four stages as shown in the diagram:

Stage AB:

In this stage, the goal is to find the velocity of Jerry at point B. Jerry is attached to a chain on a frictionless and massless pulley that is not touching the platform. A weight, m_B, is attached to the end of the chain. The chain has a length L_C and a mass m_C. When the length of the chain in the x direction is 0, Jerry releases himself and has the velocity of the chain at that moment. First, the mass of the chain and items attached in the x- and y-directions needs to be found.* After that, Newton's Second Law can be used to show the tension in the chain in terms of the acceleration and position. Using a system of equations for the x- and y-directions, an equation for acceleration in terms of position can be found. This can be multiplied by $\frac{dy}{dy}$ and integrated to find the velocity when the y-position equals L_C.

*To find the mass as a function of position: The linear mass density of the chain, which is the total mass of the chain m_C divided by its length L_C, can be multiplied by the y-position (the number of meters of the chain in the y-direction) to obtain the mass of the chain. Then, constant masses should be added. For the x-direction, the linear mass density should me multiplied by the length of the chain minus the y-length.

$$m_{ABy}[y] = \frac{m_C}{L_C} y + m_B$$
$$m_{ABx}[y] = \frac{m_C}{L_C}(L_C - y) + m_J$$

Stage BC:

After the velocity of Jerry at point B is found, he travels in projectile motion through the air until he reaches point C. The velocity at point C should be found so that the acceleration for Stage CD can be calculated. When the horizontal displacement Δx_{BC} is found, it can be subtracted from given Δx_{BD} to find Δx_{CD}.

Stage CD:

Once the velocity at point C and the displacement in this stage is found, the acceleration can be calculated using a kinematic formula. Newton's Second Law should be used to set the sum of the forces acting on Jerry while he is sliding to a stop on the ground equal to his mass times the acceleration just found. The force of friction will be in the terms of Normal force times μ_G, which can easily be solved for at this point since it will be the only unknown in the equation.

Solving:

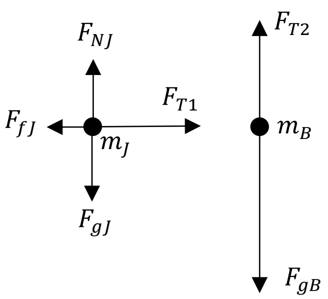
Stage AB:

Recall these equations created in Strategy to represent the mass in the y-direction and x-direction of the platform:

$$m_{ABy}[y] = \frac{m_C}{L_C} y + m_B$$
$$m_{ABx}[y] = \frac{m_C}{L_C}(L_C - y) + m_J$$

Next, force body diagrams for the barrel and rocks and Jerry in the y- and x-directions respectively can be drawn. F_{T1} is assumed to be equal to F_{T2} and a_1 is assumed to be equal to $-a_2$ for calculation purposes and using standard axes.

x-direction y-direction



$$\Sigma F_{Jy}: F_{NJ} - F_{gJ} = m_J a_{Jy}$$
$$F_{NJ} = m_J g$$
$$F_{NJ} = 69 * 9.8$$
$$F_{NJ} = 676.2\ N\ \uparrow$$

$$\Sigma F_{Jx}:\ \ F_{T1} - F_{fJ} = m_{ABx} a_1$$
$$F_{T1} = \mu_P F_{NJ} + \left[\frac{m_C}{L_C}(L_C - y) + m_J\right] a_1$$
$$F_{T1} = 0.29 * 676.2 + \left[\frac{47}{9}(9 - y) + 69\right] a_1$$
$$F_{T1} = -5a_1 y + 116a_1 + 196$$

$$\Sigma F_{By}:\ \ F_{T2} - F_{gB} = m_{ABy} a_2$$
$$F_{T1} = \left[\frac{m_C}{L_C} y + m_B\right] g - \left[\frac{m_C}{L_C} y + m_B\right] a_1$$
$$F_{T1} = \left[\frac{47}{9} y + 161\right] * 9.8 - \left[\frac{47}{9} y + 161\right] a_1$$
$$F_{T1} = -5a_1 y + 51.2y - 161a_1 + 1578$$

Set each F_{T1} equation equal and solve for a_1 in terms of y.

$$-5a_2 y + 116a_2 + 196 = -5a_2 y + 51.2y - 161a_2 + 1578$$
$$a_2[y] = 0.1848y + 4.989$$

Switch $a_2[y]$ to $\frac{dv}{dt}$ and multiply by $\frac{dy}{dy}$ so that $\frac{dy}{dt}$ becomes v. Then integrate with the limits for y set from 0 to 9 to find the velocity when the y-length is 9 and the x-length is 0.

$$\frac{dv}{dt} * \frac{dy}{dy} = 0.1848y + 4.989$$
$$v * dv = (0.1848y + 4.989) * dy$$
$$\int_{v=0}^{v} v\, dv = \int_{y=0}^{y=9} 0.1848y + 4.989\, dy$$
$$\frac{v_{Bx}{}^2}{2} = 52.38$$
$$\underline{v_{Bx} = 10.2348 \tfrac{m}{s}\ \longrightarrow}$$

Stage BC:

Here, Jerry enters a projectile. The horizontal velocity v_{Bx} is constant, the acceleration in the y-direction is gravity, and the y-displacement is h. First, v_C is solved for, then the time to find a missing parameter, and finally Δx_{BC}.

$$v_{Cy}{}^2 = v_{By}{}^2 + 2a_y \mathrm{h}$$
$$v_{Cy}{}^2 = 0^2 + 2 * -9.8 * -17$$
$$v_{Cy} = 18.2538 \tfrac{m}{s} \downarrow$$

$$v_C = 0.75\sqrt{v_{Bx}{}^2 + v_{Cy}{}^2}$$
$$v_C{}^2 = 0.75\sqrt{(10.2348)^2 + (-18.2538)^2}$$
$$\underline{v_C = 15.6955 \tfrac{m}{s} \rightarrow}$$

$$v_{Cy} = v_{By} + a_y t_{BC}$$
$$-18.2538 = 0 - 9.8t_{BC}$$
$$t_{BC} = 1.8623\ s$$

$$\Delta x_{BC} = \tfrac{1}{2} a_x t_{BC}{}^2 + v_{Bx} t_{BC}$$
$$\Delta x_{BC} = 0 + 10.2348 * 1.8623$$
$$\underline{\Delta x_{BC} = 19.06\ m\ \rightarrow}$$

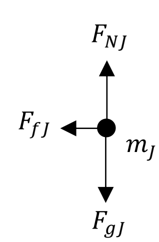
Stage CD:

In this stage, the horizontal displacement needs to be found to find the acceleration. Using Newton's Second Law, the acceleration multiplied by Jerry's mass can be set equal to the sum of the forces including the unknown μ_G in F_{fG}.

$$\Delta x_{CD} = \Delta x_{BD} - \Delta x_{BC}$$
$$\Delta x_{CD} = 104 - 19.06$$
$$\Delta x_{CD} = 84.94\ m \rightarrow$$

$$v_D{}^2 = v_C{}^2 + 2a_{CD} \Delta x_{CD}$$
$$0^2 = (15.6955)^2 + 2a_{CD} * 84.94$$
$$\underline{a_{CD} = -1.4502 \tfrac{m}{s^2} \leftarrow}$$

Jerry at CD



$F_{NJ} = 676.2\ N\ \uparrow$ (from Stage AB)

$$\Sigma F_{Jx}:\ \ -F_{fJ} = m_J a_{CD}$$
$$-\mu_G F_{NJ} = m_J a_{CD}$$
$$-676.2\mu_G = 69 * -1.4502$$

$$\underline{\mu_G = 0.1480}$$

The coefficient of friction between Jerry and the ground is 0.1480.